THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

In conversation with simulation

The application of numerical simulation to the design of structural nodal connections

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Nodal connection approximated with 100000 randomly scattered particles. Image created by the author.

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ABSTRACT

The thesis explores methods for integration of structural analysis, design and production in a digital design environment. The somewhat ambiguous title implies the ambition to make such integration in relation to the explorative phase of the design process which is described by Donald Schön as having a conversational character. A conversation between the designer and the representation by the means of the tool. The tool is in this context a simulation and instead of exploring the potential of automatic optimisation, the simulation is used for designer driven exploration.

The aim of the thesis is to give an overview of how this type of integration is currently being approached and to contribute with new tools and methods in that pursuit. The motivation behind the work is to lower the threshold for the application of structural analysis in early-stage design, with an ambition of architectural qualities and resource efficiency in mind.

An overview of the historical context is portrayed with broad brush strokes, followed by a more precise account of the mathematical and physical context, which is complemented by an attempt to describe how our tools and roles tend to interplay in the composition of the design process. Methods such as the finite element method, isogeometric analysis, smoothed particle hydrodynamics and peridynamics, including their related geometrical representations are introduced in relation to this context. A variety of production techniques are also discussed in relation to material mechanical properties for conventional building materials such as steel, concrete and wood.

The method development is approached through the use of numerical and physical experiments which are applied for design of material-efficient structural components, with a particular design process perspective. The nodal connection is chosen as an application because it combines geometrical and structural complexity in an element that is of crucial importance for a holistic spatial setting, while often being produced in a material inefficient way, with poor attention to detail.

The three articles that are included follow a trajectory from large to small, from the holistic to the particular. The first article is a description of the computational design work with the roof for the new international airport of Mexico City. The second article aims to address one of the challenges that were faced in that project with material inefficiency for nodal connections, with a critical perspective on optimisation. The final article presents an extension/modification for the peridynamics theory enabling variable particle sizes and an irregular particle distribution through the introduction of a concept called *force flux density*. The development is motivated by limitations found in the present theory through numerical experiments. The method enables simulation of phenomena such as brittle fracture, for which correlation with Griffith's theory of fracture is shown.

Further work includes an extension of the force flux method from 2D to 3D, including calibration of material a model for 3D printed steel. Other possibilities involve the exploration of how such a method can adapt to the various stages of the design process, where requirements of accuracy, speed and interactivity will vary.

Keywords: Structural design, Digital design, Simulation, 3D Printing, Peridynamics, Steel structures, Nodal connections, Structural efficiency, Design process, Conceptual design, Design theory, Force flux density.

PREFACE

During my time as an intern at British engineering consultancy Buro Happold, in the year 2010-2011, I got introduced to the practice for the first time. My background in architecture and engineering with basic programming skills gave me a place as a specialist in the research group where I worked on projects but also with the development of tools for integration of structural optimisation in a digital design environment. Perhaps due to my junior positions, or because of my somewhat specialist niche I often experienced a significant gap between holistic understanding of the project I was involved in and the specificity of the given task. I would wonder about the usefulness of panel layout optimisation on specific geometries to save construction cost when the initial shape driver seemed unclear. Was it just a bit arbitrary, carefully crafted, or perhaps driven by other design constraints?

The slight frustration of working with computational design tools that often had a great impact on project outcome but constantly a little disconnected from the vision carrying capacity of the architectural team, lead me after graduation to apply for a similar role but in an architectural context. As an employee in the team of Applied Research and Development at Foster + Partners, my tasks were similar but always closely connected to the project vision. After all, I was part of the architectural team and would participate in all the relevant meetings to understand the projects I was working on. But other challenges arose with the shift of context. The focus now was more often to generate multiple comparative options in order to evaluate and assess an idea. That meaning that less time could be spent on each design and the process of working and the way in which tools and methods were applied needed to adjust to the high phase.

The common denominator of much of the work I have been involved with has revolved around the introduction of material properties and physics into a digital design environment, whether that is to do form-finding of a fabric structure, size optimisation of a gridshell or analysis of thin carbon fibre structure. The challenge in many of those cases is that the input geometry needs to be designed prior to the simulation from which you realise that the input geometry wasn't quite right. Here you need to start over again and rebuild the input to rerun the simulation. That feedback loop can seemingly be closed using a model that is parametric. However, the parametric models only contain the type of variability that can be anticipated at the inception of the model. Making it problematic enough for an individual designer (because you might not know where your intuition might take you) and almost useless in a collaborative design process, unless the design domain is very clearly defined from the beginning. But the definition of such a design domain is often a significant part of the design work itself.

Thus, it seemed one should investigate how to bridge between analysis and design, in a slightly different way than what is commonly the case. A way the takes intuition and exploration seriously as important drivers in the creative process. That was what I set out to do when I started my PhD at Chalmers. It is a huge undertaking and a topic that reaches far beyond the discipline of architecture, but perhaps there are gaps to identify where a contribution can be made.

My interest in the relationship between form and structure and thus the background to this thesis project is shaped by this threefold experience. The experience of working as an engineer in the context of computational design, on the side of the engineering consultant. The experience around similar projects and subject matters but on the side of the architect, most often the initiator and carrier of a project vision. The third influential experience comes from participation in the research community and from my role as a teacher in an academic context. The motivation behind the project is furthermore driven by a curiosity to learn more about the subject but also to further practice and learn the process of academic scholarly work.

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Thesis

This thesis consists of an extended summary and the following appended papers:

Paper A	M. Tsigkari et al. “The computational challenges of a mega space frame”. *ACADIA Conferance proceedings.* Association for Computer Aided Design in Architecture (ACADIA). 2017
Paper B	F. Aish et al. “Form Finding Nodal Connections in Grid Structures”. *Conference proceedings: IASS Boston 2018.* 2018
Paper C	J. Olsson, M. Ander, and C. J. K. Williams. The use of peridynamic virtual fibres to simulate yielding and brittle fracture [Manuscript submitted for publication]. *Journal of Peridynamics and Non-local Methods* (2020)

Other publication that are related to the thesis are:

1. J. Olsson et al. “Sculptural form finding with bending action”. *Conference processdings: IASS Hamburg 2017.* 2017
2. J. Hilmersson et al. “Isogeometric analysis and form nding for thin elastic shells”. *Conference processdings: IASS Barcelona 2019.* 2019
3. E. Adiels et al. “The use of virtual work for the formfinding of fabric, shell and gridshell structures”. *Proceedings of the Advances in Architectural Geometry conference.* 2018

Contents

1 Introduction

The introduction here consists of four parts. The first part is an introduction to the design work behind the roof for the new international Mexico city airport. The narrative is told from my own personal experience and the project is introduced here because it has had an important influence on the research project. The airport story is finalised with the posing of a challenge from which the second part picks up, which describes the scope of the challenge. The third part is a motivation for the specific research application and leads up the formulation of the research questions. These questions are then used as a springboard into the broad contextualisation chapter in 2. The context is then summarised in chapter 3.1 and a more narrow focus is presented in 3.2. But before the contextualisation starts the last part of the introduction contains a brief description of the milieu in which the research has been conducted.

1.1 Mexico city airport

In September 2014 British architects Foster + Partners in collaboration with a Mexican firm FREE architects where announced winners of the competition for the new international airport in Mexico City. The initial master plan for the project suggested a plan layout with two terminal buildings connected with an underground train link, as was common for airports of such a scale, and suggested by the competition brief. During the initial stages of design work, the split terminal idea was scrutinised and the team came to question the master plan strategy. Due to the complicated soil conditions at the site, it was concluded that a single terminal building would be more efficient, in terms of people flow, logistics and even cost. The final proposal which was handed to the competition jury was a single terminal building, with a continuous roof embracing the whole building as a gesture of the unifying strategy. The roof also functions to unify the often spatially fragmented structure of an airport with the structural member of the space frame system as the repeated and recognisable element.

The choice to work with lightweight roof structure is partly a response to the complicated soil condition at the site and partly a strategy to design for flexibility, where the enclosed space is uninterrupted, except for the 21 protruding columns. The roof covers a space with a 0.5 million square meter footprint stretching 1.6 x 0.5 km in the length and width directions and creates a continuous space with spans between structural supports that exceeded 100 m. The shape of the roof is based on structural and spatial concerns like a tightly wrapped skin that also acts as walls, roof, daylight intake and environmental shield for the building. With high aspirations in terms of sustainability, material efficiency, flexibility and concerns about loading the continuously settling soil, the aim was to create a light-weight, structurally sound envelope. Hence, form-finding was adopted as the main strategy for shaping the roof, which was also the reason for my involvement with the project in the stage after the competition was won. Due to heavy seismic activity in the area, the idea of a single layer grid shell was early on found unsuitable in favour of a double layer space frame, with a shear and bending stiff tetrahedral configuration.

From a longer-term perspective, the project is part of an iterative design development

Figure 1.1: *Render showing the exterior of the roof for the proposal of the new Mexico city airport, shown from a bird's-eye view. Image © Foster + Partners.*

of the Airport as a building typology being the 8th project in that category for Foster + Partners. This process is described in the keynote lecture by Norman Foster and Francis Aish at the conference Advances of Architectural Geometry in 2016 [57]. Stansted airport (from 1981) was the first airport that was designed by the office and it was a reinvention of the airport typology by reversing the typical organisational diagram. Technical and mechanical systems were placed underground to facilitate the technical needs from below, enabling a lightweight roof that functions more as a filter of daylight. The main space of the building was thus re-envisioned through this strategy into a space of light and drama. The Chek Lap Kok Airport continued the development of the same principle while increasing the spans to suit a significantly larger airport. Beijing Airport became the third iteration of a similar scheme but this time in the form of an even larger structure. Adorned with the symbolic features such as the dragon skin like cladding and the particular colouring scheme, the airport rationale is woven together with elements of local folklore [57].

The ambitions of structural lightness with the Mexico City airport puts the project in yet another context. The development of lightweight gravity structures owes gratitude to a whole range of people, among them Gaudi, Gustavino, Frei Otto, Buckmister Fuller, Heins Isler, Nervi and many others. But there is one project in particular which influenced the design objectives and the project workflow more than the rest, which is the British museum great court roof, shown in 2.33, for which the computational design was carried out by Chris J.K. Williams, as described in [118]. This paper became an important inspiration for how to approach the design of such a complex, undulating and lightweight roof structure.

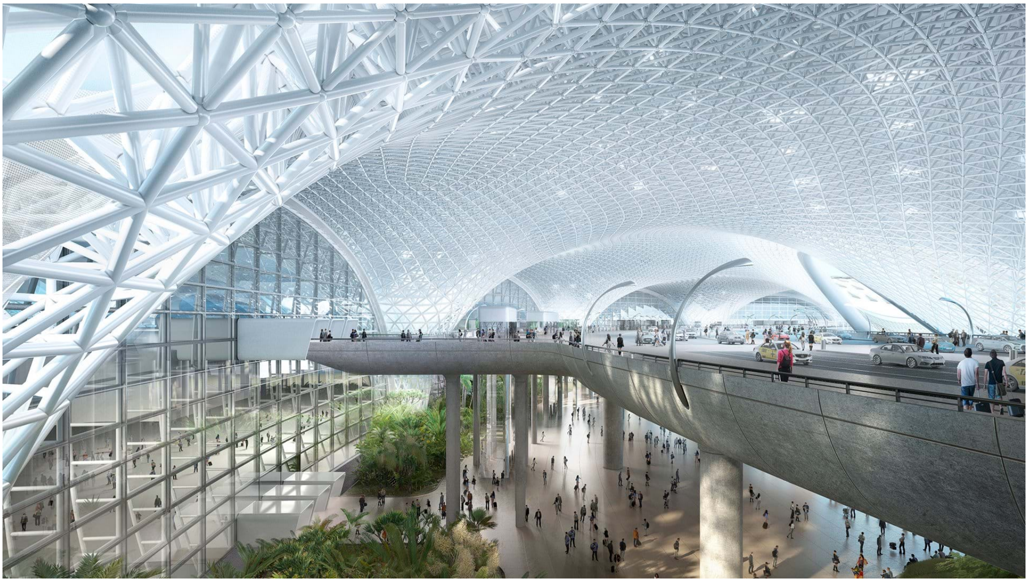


Figure 1.2: *Render for the Mexico city airport proposal showing the roof over the drop of area. Image © Foster + Partners.*

During the design development from the competition until production drawings, a couple of dozen versions of the roof geometry were drawn using various computational design techniques. Due to the multi-functional nature of the roof, the way in which it connects to other architectural elements, and the ambitions sustainability goals, the initial phase of design development was much about understanding the constraints and how they were interlinked. As a strategy for natural daylighting, an even pattern of scattered triangular skylights were distributed over the roof which put constraints on the roof geometry due to glass panel size manufacturing limits.

The same triangle size also drives the depth of the space frame due to concerns of structural efficiency. An equilateral tetrahedral configuration with 60-degree angles in between the bars was found to provide the most lightweight solution because it ensures the smallest possible nodes. The constraint to get as close to equilateral as possible links the triangle size on the surface, with the triangles on the innermost layer but also the triangles in the depth direction. To achieve a continuous roof all the way down to the ground, the choice of triangle size also needed to consider interfaces such as doors and other apertures that penetrate the roof surface. All in all, such requirements of strict geometry control meant that the roof needed to be designed starting from the outside where the inside of the structure was created through a shifted offset where adjustments were made such that the inside layer of the space frame would align with slab edges and walls. For a free form undulating surface like the airport roof, it is difficult to define the tetrahedral geometry using only geometrical primitives or rational shapes. The variation in surface curvature that comes from patching together parts of geometrical primitives with a partial form found structure, driven by load cases, spatial consideration and geometrical constraints results in a geometry that does not follow simple mathematical logic. Therefore the design strategies needed to be flexible enough to allow for compromise with the many design drivers. One important goal that was identified early in the project was the aim to keep the angles between the bars in the space frame as close to 60 degrees as possible. In the ideal 60-degree angle case, the tetrahedral space frame bars are distributed symmetrically around the node and the angle between adjacent bars is maximised. The bar angles together with the bar diameters are the two primary drivers for the node size. By maximising the bar angle and minimising the bar diameter the conditions are set right to achieve the smallest possible node. The nodes in a Mero-like space frame system are milled out of solid steel spheres and are therefore the heaviest component of the structure. Thus reducing the node size was identified to be the most efficient way to save weight for the structure. Smaller nodes meant a reduction in weight of the structure and therefore a reduction in the bar diameters, enabling further node size reduction. Even though the Mexico airport project was designed for a Mero type ball node space frame which utilises material rather efficiently, there were still problem areas with large nodes as a result of structurally disadvantageous compromises. After the completion of the project in terms of design work, the question was then raised what would be the next step in development for such a structure. The obvious answer was to deal with the reduction of materials used for some of the large and heavy nodes. Could they have been 3D printed? How would one go about shaping such a large number of connections to reduce the weight?

1.2 The scope of the challenge

The motivation behind much research into lightweight structure often leans on the idea that *less is more* famously uttered by modernist architect Ludwig Mies van der Rohe. The same idea is expressed in the other words by the structural pioneer R. Buckminster Fuller where he formulates an ambition *to do more and more with less and less until eventually you can do everything with nothing.* A similar attraction to the elegance of simplicity and also an important problem-solving strategy in the sciences is sometimes referred to as the Occam's razor. It suggests to always chose the simplest of among competing hypothesis that give the same prediction. No doubt is there benefits to such an approach.

But simplicity in its own right is simply not enough. Or as Robert Venturi puts it — in *Complexity and Contradiction*, which has become one of the important writings for the postmodern movement [82] — *less is bore.* The discussion in architecture of the simple and the complex, obvious unity or messy vitality, clarity of meaning or richness of meaning, is one that will surely go on, but has simultaneously been sidetracked by a different perspective altogether, the Anthropocene. The modernist ambitions to do more with less is suddenly put in a different light. But the light shines on a matter of great complexity, the question of a sustainable way of building leads to the question of a sustainable way of living and the scope grows out of proportion for a thesis. However, a strategy for a sustainable built environment arguably relies on holistic thinking, including long term perspectives, and cannot be reduced to only questions of embodied energy, which is sometimes a prevalent undertone in the *less is more* paradigm. Is it not a question of how to, at one hand build things that last, but more importantly to better integrate our built man-made systems with the cycles of *nature*? In that case, one could argue that a good source of inspiration would be *nature* itself. Not only does *nature* produce materials, structures and organisms which are integrated with the cycles of growth and decay it does so presumably in a sustainable way. D'Arcy Thompson, the Scottish naturalist and mathematician and author of the influential work titled On Growth and Form, was among the first to suggest looking at the natural environment for solutions to engineering problems.

Other designers and engineers also developed a fascination with biology and nature in the context of form and structure, among them Frei Otto. Together with his group at the Institute of lightweight structures in Stuttgart he studied material properties and structural behaviour as a source of form-generation using physical models. They developed an understanding and knowledge that lead to the realisation of some groundbreaking projects like the Olympic stadium in Munich and the Mannheim grid-shell. They started from the material phenomenon and posed the question, of what the form should be? A process of creation turned on its head to a process of finding.

But, in order to design buildings that last, material durability is arguably not enough. Is it not so that what we appreciate is also what we will preserve? Appreciation, in this case, is not only to be thought of as matter taste or style but perhaps better understood as a matter of care. Do we not value that where the effort of hardship and expression of intention can be seen, more than we value that which seems to lack just such qualities? Such intentions are arguably not expressed through the use of a specific tool, material, method or style but a result of the orchestration of the design process as a whole. To design with intention and care requires knowledge of various kinds especially in the category of lightweight structures. It is a typology that presumes an understanding of material properties, of structural phenomena, tools for design and production, but also knowledge of the type that concerns every architectural endeavour. That includes spatial composition, people movement, contextual understanding of a place that is geographical, but also empathy for the context that is cultural, political and value-based. Some knowledge of which can be formulated almost as strict rules and integrated with the tools that we use (material properties, physics, geometry), and some of which are enabled through the hand of the designer, while requiring non-rule-based unrestricted freedom to deal with the complexity of a holistic perspective. Much of initiatives in the field of structural analysis/optimisation with design aspiration seem to strive for design process automation. The ambition of this project is rather to explore designer driven integration. The integration of material simulation, design and production.

It is said that Louis Kahn, another of the big modernist architects, used to tell his students "if you get stuck, ask the material what it wants". To paraphrase, he then goes on to ask the brick, "what do you want to be"? The brick answers that "I want to be an arch", perhaps as a response to his inner material composition? Because a structure is at its most fundamental some type of material distribution. At the smaller scale, the border between structure and material starts to blur, where in fact the material is a structural organisation of small building blocks, with different characteristics depending on the lens and perspective from where it is observed. To predict the behaviour of various materials, mathematical modelling is usually applied by effectively mapping the phenomenon of the material properties that are observed in experiments, by smearing the properties and treating the material as continuous.

Knowledge of practical skills related to production is another important aspect in any design endeavour and more so than ever in the case of lightweight structures. Precision and control of production are paramount with ambitions to save weight and resource while doing so in a way that is safe and controlled. Most techniques for production naturally start from the material properties for which manipulation is sought. Just think of the saw that is made to cut wood perpendicular to the grain, or the axe for which splitting is the manipulative action. These processes are today increasingly controlled by digital information processing which has become an enabler in the dream of the custom made and the bespoke.

There is little doubt about the protagonist in that context of designer tools today. The introduction of the computer as a design tool in the '90s has been a liberating force in terms of formal expression but simultaneously made the shaping of objects a rather abstract activity. The form no longer relates by necessity to the material which it is made from, or for that sake the process through which it is produced. Whereas the separation has been felt in fields of automotive design, furniture and product design, architecture is arguably a little different. On the one hand, that same gap between representation and the artefact already existed for the architect due to the matter of scale. The drawing or the physical model is by necessity a scaled-down representation of the building and thus the notion of the gap is part of the architectural DNA. On the other hand, the seductive potential of computer modelling with form exploration as an easily accessible driver has seemingly lead to a similar separation between the design, material and production, also in the architectural field.

However, since the computer environment is well suited for integration of various mathematical models it has the potential of hosting form representation (design), material simulation (analysis) and robotic information processing (production). Various form representation techniques have been developed for the computational environment with real-time interaction, even while displaying rendered textures and shadows. Simulation techniques are increasingly being integrated in our digital design environment to guide the choices of the designer with information such as stress distribution and utilisation levels. Various optimisation techniques can be used with the purpose of guidance to suggest minimal material solutions. Rapid prototyping and digital manufacturing can be applied to make physical testing and experiments. In a profession where the gap between representation and artefact is intrinsic to the task (due to scale), it seems the computational digital environment offers an opportunity to work for the unification of material, form and production as opposed to further that separation. The challenge however, is to make such tools for integration inviting for intuitive exploration.

1.3 Application and research questions

This research project aims at bridging between digital modelling and material simulation with respect to material mechanical properties, and link the modelling context to digital production. In the exploration of methods and tools for this particular pursuit a suitable application seemed important, and the experience from the airport project came to mind. Structural nodal connections are often complex both in terms of geometry and in terms of structural performance and load conditions. They are often manufactured in a rather material inefficient way with poor attention to detailing, so it seemed like an area worth the attention. Various techniques for form optimisation has been applied to the global form of structures, justified as a good strategy for material savings. But only a few buildings are built with such budgets and ambition, so these techniques remain rather niche and applied in small numbers. Structural joints however are needed in all structures and perhaps there is potential for application of new strategies to make material savings here.

The following research question has been formulated as a response to ambitions discussed in the introduction. These questions should be understood as a motivation behind the contextualisation in chapter 2. The context is then summarised in section 3.1 and a focus is presented in section 3.2 leading the reformulation of more narrow and precise questions in section 3.3. The broad research questions which sets the direction for the next chapter are the following:

i. Which numerical methods for analysis and related geometrical representation are suitable for the integration of modelling and simulations in the design process?

ii. What is a suitable production technique for material-efficient structural nodal connections?

iii. What are the desirable properties of a quantitative numerical method (such as simulation) to enable integration in a design process which has both quantitative and qualitative character?

iv. What characteristics of the design process are important to consider in the design of structural nodal connections, in the stages ranging from concept to final production?

1.4 Research milieu

The research which aims to position itself between the traditions of architecture and engineering is situated in an environment where this is already an established mindset amongst both architects and engineering scholars. The architecture and engineering research group, from which context this thesis springs, started in 2016 as a continuation of the architecture and engineering double degree program (MArch, MSc Eng) at Chalmers, which was introduced 10 years earlier. The philosophy of the program is about introducing natural science phenomena and knowledge (of structures, climate, acoustics etc.) as a backbone to the creative process in the design of our built environment through project work, mathematical modelling and artistic exploration.

2 Context

The contextualisation in this chapter starts with an overview of historical development that is related to the research in some way or another using very broad brush strokes. This is done by dividing history in three eras which are sorted in terms of production capabilities and technological advancement namely the pre-industrial, the industrial and the digital era. From such a categorisation a variety of tendencies are portrayed in short, more or less in line with the main theme. The broad historical overview is then followed by a complementary section on geometry which is a useful descriptive language for anything of material form. The next section is about the development of production techniques sorted in terms of material type, where the focus is concentrated on the structural node. Such a perspective seemed reasonable, partly because of the research application on structural nodes, but also because of the importance of joining techniques in the development structures more generally. The following section contains a short overview of tools for design and representation in the architectural venture. The focus here is on the state of the art digital modelling and analysis with a focus on form and structure. The final section which wraps up this contextualisation chapter deals with the design process, including a note on roles, design theory, design drivers and values. This is a general description of the context in which any project takes shape and thus seemed to be a reasonable endnote.

2.1 Historical background

For the broad historical perspective, an additional set of handles seemed necessary to get a hold on the narrative and to bridge to the rest of the contextualisation. Each era is therefore summarised in terms of four themes which are formulated to mirror the following main sections of this chapter, namely

i. Understanding of mathematics and geometry (the language of natural sciences).

ii. Technological possibilities in terms of production.

iii. Tools and representations for architectural/engineering design.

iv. Design process, including driving ambitions and values of the culture but also the roles of the actors involved in the design process.

The historical background that follows should be understood as a humble attempt to define the characteristics, of what is in actuality, a vast and complex historical account, so what is presented is naturally short, simplified and with many gaps.

2.1.1 Pre-industrial

One could wonder why we see such a rich variation in form and expression when it comes to the handcrafts, the sculptural arts and product/furniture design. Partly the smaller scale (often even 1:1) likely plays an important role here, especially in terms of production. But the designer also works close to the material, the shaping and the production, all tied together in an embodied process which is that of design.

Much of architectural history up until the industrialisation rely on skilled craftsmanship in one way or another. But it is not the type of craftsmanship that is necessarily small scale and hand made. There are many astonishing examples of engineering achievements that would challenge such a simplified view. Built examples for which the realisation is difficult to comprehend today without imagining the use of modern machinery [1]. There is the mystery of the pyramids, the heavy stonework of the old Greek and Roman architecture, the temples and cathedrals of the medieval period etc. The pre-industrial is understood here in relation to technological achievements and has less to do with architectural theory and related philosophy but there is arguably an interesting angle which relates the technological and the philosophical and that is mathematics.

Architectural history is full of attempts to formulate geometrical systems of proportion to systematise beauty or essence of form. Pythagoras tried to formulate a theory of everything on the basis that everything is numbers. Plato famously talked about mathematics as a divine realm where the non-physical essence prevails, of which our lived experience is yet but "shadows on a cave wall". More famously in architecture and design is the formulation of the golden section, initially defined by Platonic Academy student Euclides in 300 B.C. It was presented together with a comprehensive collection of known geometrical theories from a variety of authors in a book called Elementa. Yet another example of an influential proportional system is the Vitruvian man famously illustrated by Leonardo Da Vinci but based on the writings of Roman architect Marcus Vitruvius in his book *De architectura*. Vitruvius believed that the use of geometrical properties of the body as a point of departure for architecture would lead to universal appeal. Mathematics in the pre-industrial era is not only used to motivate value-based judgement—even though such tendencies reappear once more during the Renaissance with the rediscovery of the antique thinkers, among those the works of Vitruvius—it is also a field of practical use for measurements and construction work. After all, architecture and mathematics are tightly linked during many of the phases in the pre-industrial era and many of the aforementioned pioneers are experts in what today is regarded as separate fields. The pre-industrial era is further marked by the lack of machinery. Not in the sense that there were no machines and clever inventions to assist the designer and builder. But rather a time were machines where fueled predominantly by human and animal muscle power (with the exception of windmills and water wheels). The limited power resource put practical limits on the building process and the speed of production. The manual production also meant a lack of high precision production but allowed for bespoke tailoring, adjustments and a variation that the later industrial production struggles with. This is also a time in the history of architecture with many examples of tight integration of material, space and form. That can be seen in the many impressive cathedrals from the era showcasing an intuitive understanding of vault and shell action for a compression-only material such as stone. An integration that is arguably made possible because of the lesser building complexity and the tightly integrated role of the architect as master builder and engineer. It is furthermore a time of and agrarian/handicraft economy, soon to be replaced by the market economy capitalism based on mass production. The key characteristics to be further associated with the pre-industrial era can be summarised as;

i. Geometry and mathematics is used by architects as an essential tool in the design and construction process but also as the basis for various motivations of value and meaning.

ii. Manual production was slow but flexible to enable bespoke solutions based on individual circumstances.

iii. Tight integration of building material, structure, space and form.

iv. The role of the architect is integrated with the role of the engineer as the master builder.

In the transition to industrialisation various counter-reactions can be seen, notably the arts and crafts movement in England. It was a reaction to the dehumanising conditions of workers in the industries, emphasising a return to the skilled craftsmanship and attention to design in the decorative arts. It is a rejection of the excessive ornamentation of the past (in particular the Victorian era) but celebrates a closer relationship between designer, maker and object.

2.1.2 Industrialisation

An important moment in the popularisation of the industrial revolution in architecture is the 1851 Great exhibition in London. It was a celebration of modern industrial technology and design and it was held in the famously groundbreaking cast-iron and glass Victorian exhibition space called the Crystal Palace. The building itself reviled a new type of architecture that is light-weight, slender and transparent, a stark contrast to the heavy and dark stone buildings of the time.

The main characteristic of the industrial revolution can be summarised quite well in terms such as standardisation, specialisation and mass production. The industrialisation of the production meant a subdivision of the steps in the making processes such that it can be performed using machines that increase the rate of production at a lower cost. The term mass production was popularised in 1926 with an article in the Encyclopedia Britannica supplement that was written based on correspondence with Ford Motor Company. Some aspects of this making process and the thinking behind predate the Industrial Revolution by centuries but it was not until the introduction of machine tools in the mid-19^{th} century, that modern version of mass production was possible.

The foundation to much of the development during the industrialisation was based on new discoveries in mathematics and science. Including Isaac Newton's formulation of classical mechanics which could unify the explanation of movement from human scale to celestial bodies, Newton and Leibniz formulation of calculus, Robert Hooke's formulation of elasticity, the generalisation of elasticity theory by Claud-Louise Navier just to mention a few. As a response to a variety of geometrical issues that arise with the introduction of calculus a special field of geometry concerning curves and surfaces called differential geometry starts developing in the end of the 17th century. It becomes the language behind Einstein's theory of relativity, but also important for later development of computer-based geometric modelling.

For the built environment the industrial era meant a huge transformation. People started moving from the countryside to the cities while leaving the rural lifestyle and starting to work in factories instead. New materials like reinforced concrete and glass where invented giving rise to new opportunities in terms of building techniques and architectural qualities. The heavy stone buildings could now be replaced with the curtain wall of glass hung off a steel or concrete skeleton frame. These new building possibilities stood in great contrast to the heavy and dark stone building of the time, for which the Crystal Palace is one such example. The introduction of mechanical and electrical systems, but also the separation of the different functional aspects into different layers, meant that buildings became more complex. Thus the role for the architect changed, and it became more separated from the builder and the engineer, and the methods from drawing, analysing and building started developing in parallel tracks. The specialisation and standardisation also put requirements on documentation, and a more precise process for drawing was developed and applied [74].

The machine-based manufacturing that was adopted during this time allowed for increasing design complexity due to increased possibilities in terms of accuracy, precision and scale, especially in the machine industry and for product development. It was now possible manufacturing rather complex parts with fine details due to the economy of mass production. This splitting of the production process also means that more actors could be involved. In the early day of industrialisation, this is done in a rather comprehensible scale, but over the years the complexity of the process has been magnified to reach across continents. Today it is normal that complex product such as a car or another piece of technology comes with many birth certificates. Due to the complexity and site-specificity of building projects, mass production was not immediately as successfully adopted by architects and engineers [20]. But the ambitions were clearly there as famously articulated by Swiss-French architect Le Corbusier in his book *Towards a New Architecture* published in 1923 [18]. Three key characteristics of the industrialisation era can be summarised as

i. Many great advancements are achieved in a multiple of scientific fields, including the methods for structural analysis and description of curves and surfaces with differential geometry.

ii. The manufacturing process is divided into discrete steps, for which the economy of mass production allows a high level of efficiency, detailing and quality.

iii. Buildings are getting more complex and organised in layers that each performs separate tasks and requires a special type of representation.

iv. Distinct separation in between the role of the architect, builder and engineer as a result of specialisation.

2.1.3 Digitalisation

Just like the machines replaced the muscle power in the early days of industrialisation the computer is sometimes described as the replacement of the mind in the era of digitalisation. The digital age means a shift from an agricultural and power centric economy towards one centred on information. The repetitive logic of mass production is loosened up to become more flexible as the manufacturing industry makes a move towards what is today referred to as industry 4.0. A logic of production that builds on flexibility and information technology.

The digital era began in Britain and the US in the post-war days as a result of inventions such as the transistor, the integrated circuit and the silicon chip [74]. The ability to control the whole of the production sequence using computer code makes it possible to mass-produce unique products at a cost comparable to traditional mass production [20]. Engineers in the automobile industry during the late 1950s wanted to expand the repertoire of classical geometric shapes (such as the circle, the ellipse, parabola etc.) for descriptive purpose, with a curve that could take any shape and form. Bezier and Casteljau both made breakthroughs in what came to be known as spline geometry, which was later refined to Non-Uniform Rational B-Splines (NURBS). But their inventions required large amounts of calculation and were impractical until computers were available, and it is not until the development of interactive graphical user interfaces that the computers start to be thought of as a design tool, and the term Computer Aided Design (CAD) is coined.

The introduction of CAD becomes a design enabler in many ways, various software packages are developed for the design of cars, ships and aeroplanes, but also for animations. Frank Gehry is one of the first architects to make use of such tools in a design process for buildings, notably with the golden fish sculpture in Barcelona [20]. But computers also takes the process of making too another level of abstraction. The artefact to be produced is now often designed in a virtual computer environment where software is used to represent its three-dimensional form and various behaviours can be simulated in realistic scenarios. The process puts requirements on the designer to simultaneously imagine the real object with its tactility and materiality while controlling the design through control points, weighting factors and other more or less abstract thinking. The inevitable emphasis that is put on the visual interface with computer-based design tools has been famously critiqued by finish architect Juhani Pallasmaa in *The Eyes of The Skin*. Pallasmaa is critical of the way in which the computer tools tend to flatten the multi-sensory experience of architecture to merely a visual image, increasing the distance between the designer and the object [87].

The introduction of computers also opens up for a new field of science namely scientific computing. It is an area with many disciplines but the common base can be understood as the modelling and simulation of natural behaviour. Numerical analysis becomes an increasingly important field of mathematics concerning the approximation of solutions to mathematical problems, and applications range from biology, physics and social science to economy. In the field of engineering, numerical approximation of partial differential equations leads to the development of software tools that enables analysis of various complex behaviour for solids and fluids. The finite element method is the most widely used such method for application in engineering and it has rendered the most unimaginable building structure feasible.

The debate on digitalisation in architecture in the last two decades has circled around on parametric design, artificial intelligence and automation of things like floor plan layout, structural sizing, and creation of design options. There has also been significant interest to explore the new design language that is enabled through computational tools and attempts have been made to name the current style. Yet other tendencies that has attracted attention during this time can be summarised in terms of digital based bio-mimicry and bionics. Inspired by the works of Darcy Thompson, it can be understood as a strategy to gain knowledge in terms of design based on material compositions and processes found in nature and translate that into the building scale through the rational of mathematics and algorithms. Examples of that can be found in works of Neri Oxman [86] and Achim Menges. Further explorative tendencies with an emphasis on design computation can be found in the writings of Mario Carpo notably in *The second digital turn* [20]. Carpo argues for a new design logic that builds on the enabling potential of the recent development in computational hardware and the availability of big data for a type of heuristics driven design. Recent works of Michael Hansmeyer with the Digital Grotesque exemplifies a new type of expression that is enabled through such a design logic [79].

Considerable effort is also spent in academia and by various large actors in the market to develop tools for workflow integration such that digital models can be seamlessly translated between software for analysis and design. Two examples of such initiatives are Flux and Speckle. The creative process in the digital design space seems to put emphasis on and enable collaboration in a new way. The characters of the digital era is summarised here in terms of:

i. The application of geometry and mathematics in a numerical computational setting is a revolutionary step that is fundamental to digitalisation as a whole.

ii. Mass production of custom made components is made possible with the integration of computational design software and digitally controlled manufacturing.

iii. There are speculative tendencies looking at artificial intelligence-powered computational design, parametric design and bio-mimicry.

iv. Tools for architects, engineers and manufacturers are based on a common foundation, the computer, enabling a more integrated of workflow between different disciplines.

2.2 Differential geometry

In this section a geometrical base is introduced for further discussion of structures, production and digital form representation. The section ends with a classification of surface types based on the mathematics of differential geometry. Differential geometry is the field in mathematics that deal with description of curves and surfaces using a combination of differential calculus, integral calculus and linear algebra. It is usually divided into smooth and discrete differential geometry, where the smooth setting assumes infinitely differentiable equations, and discrete setting works on geometries that are approximated using a mesh of quadrilaterals or triangles. The following section will give a brief introduction to some of the concepts in differential geometry followed by a collection of some key concepts that are useful in further shape discussion. The notations from the book *Elementary Differential Geometry* will be used for the equations [93].

2.2.1 Curves

A curve in 3D space in its most general sense can be described by the following parametric equation,

$$\gamma(t) = (x(t), y(t), z(t)). \tag{2.1}$$

For example the 3d helix which is described by

$$x(t) = r\cos(t), \tag{2.2}$$

$$y(t) = r\sin(t), \tag{2.3}$$

$$z(t) = at, \tag{2.4}$$

giving the full curve equation as

$$\gamma(t) = (rcos(t), rsin(t), at). \tag{2.5}$$

The interesting properties for a curve are typically directions such as the tangent, normal and bi-normal vectors which span a frame that moves along the curve. It is also interesting to know the curvature which tells us something about its shape, and how it varies along its length. If we assume that the curve has unit speed, the Frenet-Serret equations give the relationship between curvature, torsion and directional vectors according to,

$$\dot{\boldsymbol{T}} = \kappa \boldsymbol{N}, \tag{2.6}$$

$$\dot{\boldsymbol{N}} = -\kappa \boldsymbol{T} + \tau \boldsymbol{B}, \tag{2.7}$$

$$\dot{\boldsymbol{B}} = -\tau \boldsymbol{N}. \tag{2.8}$$

Where the curvature κ and the torsion t are computed from

$$\kappa = \frac{\|\ddot{\gamma} \times \dot{\gamma}\|}{\|\dot{\gamma}^3\|} \tag{2.9}$$

and

$$\tau = \frac{(\dot{\gamma} \times \ddot{\gamma}) \cdot \dddot{\gamma}}{\|\dot{\gamma} \times \ddot{\gamma}\|^2} \tag{2.10}$$

respectively.

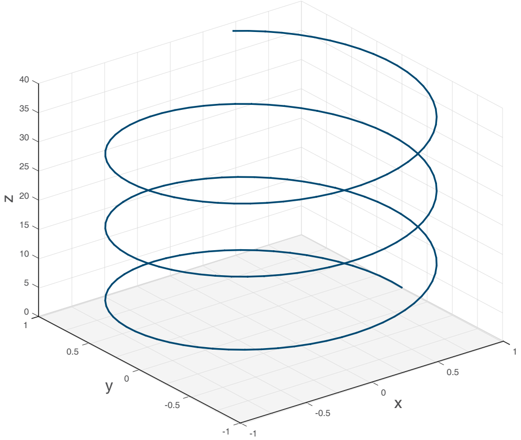


Figure 2.1: *The helix curve computed and drawn in a 3D coordinate system for r = 1, a = 2.*

2.2.2 Surfaces

The analogy of the curve that can be further expanded into 2D as a description of the surface. Similarly a parametric equation for the surface in the notation used in [93] reads,

$$\sigma(u, v) = (x(u, v), y(u, v), z(u, v)). \tag{2.11}$$

An example surface of in this format in analogy with the helix is the helicoid. It is formulate by the following set of equations,

$$x(u, v) = v\cos(u), \tag{2.12}$$

$$y(u, v) = v\sin(u), \tag{2.13}$$

$$z(u, v) = au. \tag{2.14}$$

Following the notations in [93] in order to compute the orientation of the surface, the normal direction can be computed from,

$$\boldsymbol{N}_\sigma = \frac{\boldsymbol{\sigma}_u \times \boldsymbol{\sigma}_v}{\|\boldsymbol{\sigma}_u \times \boldsymbol{\sigma}_v\|}, \tag{2.15}$$

where $\boldsymbol{\sigma}_u$ and $\boldsymbol{\sigma}_v$ are the derivatives of $\boldsymbol{\sigma}$ with respect to the u and v direction. They represent two linearly independent vectors that span the tangent plane at each point on the surface. In order to understand and measure lengths on a surface the first fundamental form is introduced. In short, it is computed from the components

$$E = \|\boldsymbol{\sigma}_u\|^2, \tag{2.16}$$

$$F = \boldsymbol{\sigma}_u \cdot \boldsymbol{\sigma}_v, \tag{2.17}$$

$$G = \|\boldsymbol{\sigma}_v\|^2, \tag{2.18}$$

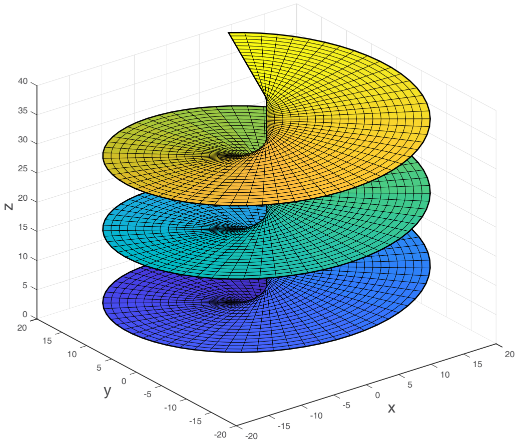


Figure 2.2: *The helicoid is the surface swept out by an aeroplane propeller, when both the aeroplane (z-axis movement) and its propeller (the rotation) move at constant speed.*

traditionally expressed as,

$$Edu^2 + 2Fdudv + Gdv^2. \tag{2.19}$$

In order to further describe the geometric properties of a generic surface such as curvature, it is suitable also to introduce the concept of the second fundamental form. In short the components of the second fundamental form read as follows,

$$L = \boldsymbol{\sigma}_{uu} \cdot \boldsymbol{N}, \tag{2.20}$$

$$M = \boldsymbol{\sigma}_{uv} \cdot \boldsymbol{N}, \tag{2.21}$$

$$N = \boldsymbol{\sigma}_{vv} \cdot \boldsymbol{N}, \tag{2.22}$$

where $\boldsymbol{\sigma}_{uu}$ represents a two time differentiation of $\boldsymbol{\sigma}$ with respect to u and the same logic applies for other subscripts. The final expression for the second fundamental form the becomes,

$$Ldu^2 + 2Mdudv + Ndv^2. \tag{2.23}$$

From the first and second fundamental form it is then possible to compute some important metrics for surface classification. These metric include principle curvature and corresponding directions, gaussian curvature and mean curvature. The first and second fundamental form can be summarised in the symmetric matrices $\mathcal{F}_I$ and $\mathcal{F}_{II}$ as,

$$\mathcal{F}_I = \begin{pmatrix} E & F \\ F & G \end{pmatrix}, \quad \mathcal{F}_{II} = \begin{pmatrix} L & M \\ M & N \end{pmatrix}, \tag{2.24}$$

where the relationship between $\mathcal{F}_I$, $\mathcal{F}_{II}$ is defined as,

$$\mathcal{F}_{II} = \mathcal{F}_I \mathcal{W}, \tag{2.25}$$

and where $\mathcal{W}$ is the Weingarten map defined as

$$\mathcal{W} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}. \tag{2.26}$$

Giving the following expression as written out with the individual components,

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}. \tag{2.27}$$

From the Weingarten map $\mathcal{W}$ the Gaussian curvature K, and the mean curvature H can be computed as,

$$K = det(W), \tag{2.28}$$

$$H = \frac{1}{2} trace(W). \tag{2.29}$$

2.2.3 The minimal surface

As another example of a surface that can be defined with the differential geometry which also has an important place in the field of structural engineering is the minimal surface. It can be understood as a special case in the family of constant mean curvature (CMC) surfaces, with the condition that the mean curvature is zero. The mean curvature is calculate as the average of the principle curvature such that,

$$H = \frac{1}{2}(\kappa_1 + \kappa_2) = 0, \tag{2.30}$$

where the principle curvatures κ_1 and κ_2 can be calculated from the surfaces Weingarten map $\mathcal{W}$ which links the first and second fundamental form of the surface geometry. The symmetric matrices $\mathcal{F}_I$ and $\mathcal{F}_{II}$ as well as the Weingarten map are defined as,

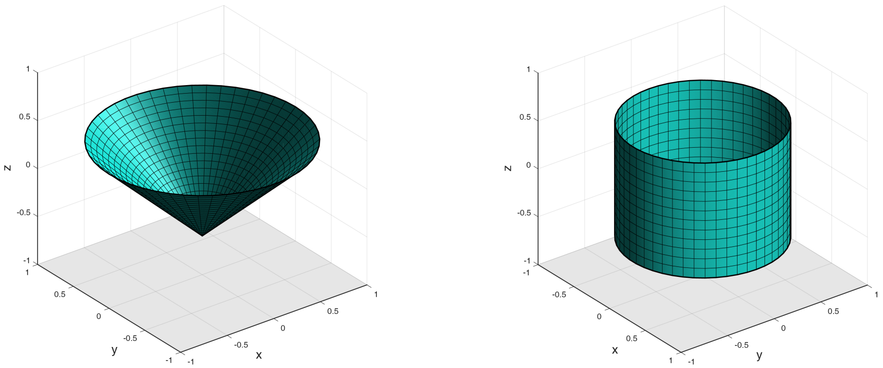
$$\mathcal{F}_I = \begin{pmatrix} E & F \\ F & G \end{pmatrix}, \ \mathcal{F}_{II} = \begin{pmatrix} L & M \\ M & N \end{pmatrix}, \ \mathcal{W} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}, \tag{2.31}$$

where E, F, G are calculate from (2.16) - (2.18), and L, M, N are calculated according to (2.20) - (2.22). The principle curvature is computed from the roots of κ for the equation,

$$det(\mathcal{F}_I^{-1}\mathcal{F}_{II} - \kappa I) = 0. \tag{2.32}$$

From equation (2.30) the anticlastic nature of a minimal surface can be understood rather intuitively since the two components need to cancel out they need to have the same magnitude but different signs. The minimal surface is applicable for tension only structures [92], [25], [117], [109].

Surface Types

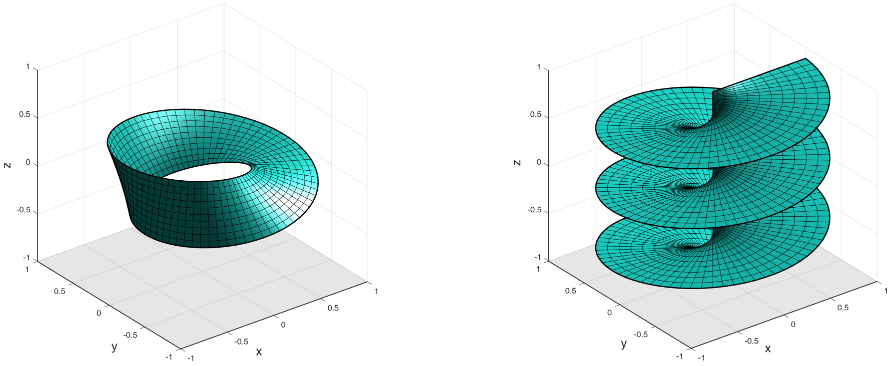


(a) *Cone* (b) *Cylinder*

Figure 2.3: *The two simplest developable surfaces. Note that the mathematical cylinder does not need to be round but any planar curve in a straight extrusion is cylindrical.*

The surface equations on parametrised form for the cone and the cylinder read

$$\begin{aligned} \sigma_{cylinder}(\theta, v) &= (r\cos(\theta), r\sin(\theta), v), && (2.33) \\ \sigma_{cone}(u, v) &= (v\cos(u), v\sin(u), v). && (2.34) \end{aligned}$$

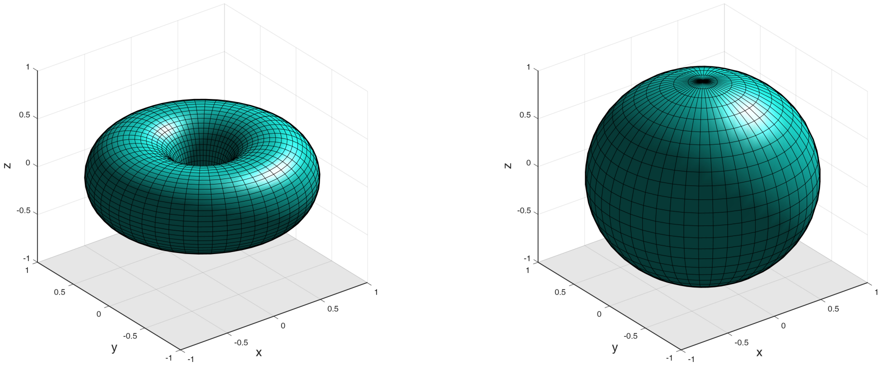


(a) *Möbius* (b) *Helicoid*

Figure 2.4: *Two complex looking ruled surfaces, the Möbius is also developable.*

The surface equations on parametrised form for the möbius and the helicoid read

$$\begin{aligned} \sigma_{\text{möbius}}(s, t) &= \left(\left(r + s\cos\left(\frac{1}{2}t\right)\right)\cos(t), \left(r + s\cos\left(\frac{1}{2}t\right)\right)\sin(t), s\sin\left(\frac{1}{2}t\right)\right), && (2.35) \\ \sigma_{helicoid}(u, v) &= (v\cos(u), v\sin(u), au). && (2.36) \end{aligned}$$



(a) *Torus* (b) *Sphere*

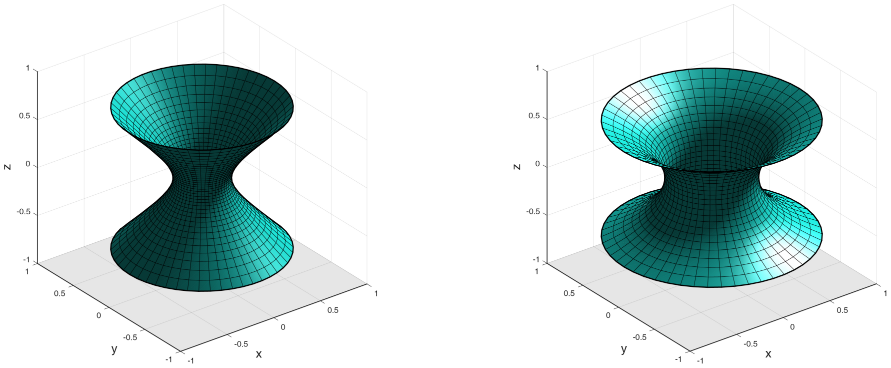
Figure 2.5: *Surface often used to approximate complex forms due to the row vise repetition of same panels.*

The surface equations on parametrised form for the torus and the sphere read

$$\sigma_{torus}(\theta, \phi) = (a + b\cos(\theta)\cos(\phi), a + b\cos(\theta)\sin(\phi), b\sin(\theta)), \tag{2.37}$$

$$\sigma_{sphere}(u, v) = (r\cos(v)\cos(u), r\cos(v)\sin(u), r\sin(v)). \tag{2.38}$$

The surface equations on parametrised form for the hyperboloid and the catenoid read



(a) *Hyperboloid* (b) *Catenoid*

Figure 2.6: *Similarly lookig but rather different surfaces. The catenoid is a minimal surface with constant zero Gaussian curvature. The hyperboloid is a ruled surface and can therefore be built using straight lines.*

$$\sigma_{catenoid}(u, v) = (c\cosh\left(\frac{v}{c}\right)\cos(u), c\cosh\left(\frac{v}{c}\right)\sin(u), v), \tag{2.39}$$

$$\sigma_{hyperboloid}(\theta, v) = (a\cosh(v)\cos(\theta), b\cosh(v)\sin(\theta), c\sinh(v)). \tag{2.40}$$

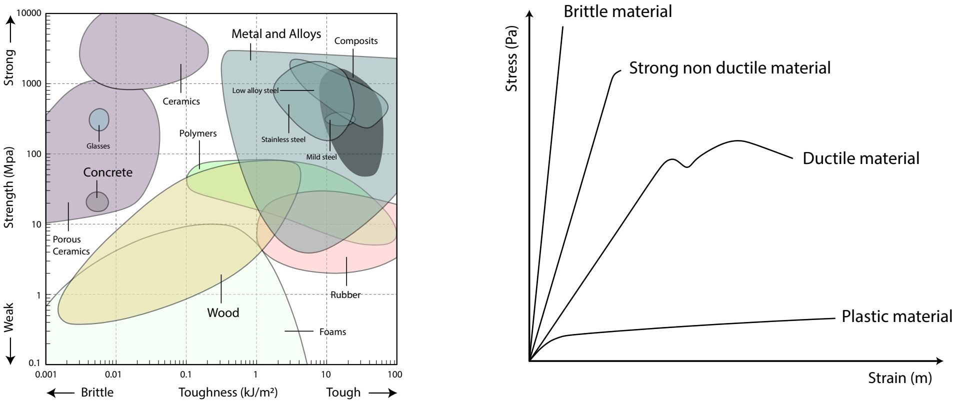
Surface properties						
Surface	k_1	k_2	K	Rul	Dev	CMC
Cone	0	+	0	yes	yes	no
Cylinder	0	+	0	yes	yes	yes
Möbius	0	+	0	yes	yes	no
Helicoid	0	+	0	yes	no	no
Sphere	+	+	+	no	no	yes
Torus	+/-	+/-	+/-	no	no	no
Hyperboloid	0	+	0	no	yes	no
Catenoid	-	+	0	no	no	yes (0)
Unduloid	-	+	0	no	no	yes

Table 2.1: Table showing a lists of the surfaces that were presented above and their geometrical properties where k_1, k_2 refer to the min, max principle curvature, K is the gaussian curvature, *Rul* stands for ruled, *Dev* stands for developable, *CMC* for constant mean curvature.

2.3 Material phenomena paramount to design

This section touches upon some of the physical material considerations that are paramount to structural design. That includes material properties and manufacturing which is linking back to the historical narrative in the beginning of the chapter but also to the section on geometry. Each manufacturing technique is described in relation to the geometric primitives that are gathered in table 2.1 and the focus is aimed at the relationship between material properties, manufacturing technique and formal possibilities.

The section is sorted in terms of material type based on the three classical building materials, steel, concrete and wood. This choice is made partly because of the wide use of these materials in construction, but also due to the characteristic differences between the three. Steel which is an isotropic, ductile and strong material, concrete which is strong but brittle and therefore most often used as a composite with steel, and timber which is orthotropic and organic. The characteristics of these three types of material are summarised in figure 2.7a, in terms of toughness (the ability to absorb energy without fracture) and strength (the ability to withstand load without plastic deformation).



(a) *Strength toughness graph*

(b) *Principal stress strain curves*

Figure 2.7: *A variety of materials described in terms of strength and toughness in the figure to the left. The figure to the right is the stress and strain graph for a set of different types of materials.*

2.3.1 Steel

The development of steel as a structural material has played an important role in the growth of the industrialised modern world. It was used to build the machines that enabled factory-based mass production, but it also enabled for a whole new typology of infrastructure with railway tracks and bridges. It was used to build the first skyscrapers but was also as a key material in the invention of reinforced concrete.

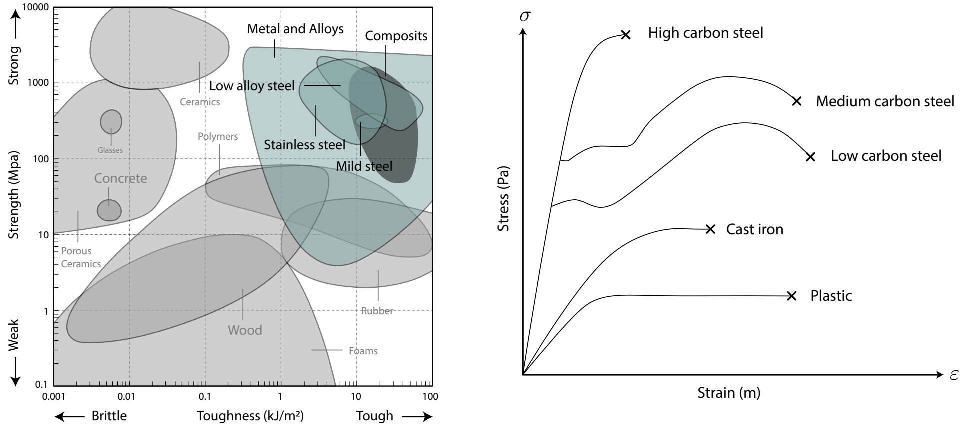
Cast iron was initially used to replace wood in everyday items such as the plough, sewing machines and pots for cooking. Steel was at the time a known material but was regarded as a premium product. It was expensive to produce and was used for small items such as watches, springs, knives and swords. When train tracks were developed as an infrastructure to assist the expanding industries in Great Britain in the middle of the 18^{th} century they were initially built of timber, later on, reinforced with a layer of cast iron plates to enable transport of heavier loads, but the brittle character of cast iron meant that the tracks often cracked under heavy loads. The Wrought iron was invented in 1820 and the production process was improved over the following decade making production more efficient and therefore more economical [10].

At the inception of iron as a structural material, the structures were design based on techniques used for wooden carpentry so naturally, this new building material was first applied using similar joining techniques. That can be seen in the detailing of the first bridge that was built with iron, conveniently called the Iron Bridge, from which a close-up view can be seen in figure 2.8. Mortise, tendon and blind dovetail connections are used for joining the precast structural elements [99].

Figure 2.8: *The first bridge build with iron is located in Shropshire in England and is called the Iron Bridge. The bridge is built using joining techniques from traditional wooden crafts. ©Photo by Uncle Silver.*

Mechanical properties

Steel is today an important material for the construction industry. It has the advantage of being strong, predictable and ductile with rather isotropic properties. The specific properties can also be altered in the refinement process when raw iron is converted to steel. By increasing the carbon content in the iron a stronger but more brittle material is gained whereas a lesser carbon content results in a more ductile and soft material. Typical construction steel has a carbon content of less than 0.25 percentage, for which a higher value would make welding problematic. The crystalline microstructure of the material can be altered through various techniques such as heat treatments and cold working.



(a) *Strength toughness graph*

(b) *Principal stress strain curves*

Figure 2.9: *Shows a collection of metallic materials in terms of strength and toughness in the figure to the left [19]. The figure to the right is the stress and strain graph for various metal qualities to give a rough overview of similarities and differences.*

By rapidly cooling the steel, it will become stronger but more brittle, whereas a slowly cooling specimen will become more ductile. Using various cold treatment techniques, such as compression and twisting beyond the yield limit, the grain size will be reduced and in that way increase the strength of the steel.

Riveting

Riveting is an early joining technique for steel structures that became popular in the 19th century during the industrialisation. The concept of clamping pieces of material together using rivets have been used since at least 3000 BC but often for wooden structures using wooden rivets. Rivets were for example used in the construction of the Eiffel Tower where all the 2.5 million rivets are placed by hand. It is typically applied in the assembly of layered sheets of steel even though it is not a necessary restriction intrinsic to the joining technique itself. Structures made of rivets are nevertheless often based on the geometric logic of flat sheets which can be altered into the surface shape of a *cone*, a *cylinder*, or the *mobius* i.e. developable surfaces. The elastic properties of steel also make it possible to shape the material using plastic deformation such as cold pressing and a wide range of geometric shapes are then possible. The ductile property of steel is also what makes riveting possible since it involves the plastic deformation of an initially cylindrical pin.

Bolting

The possibility of using high-preload bolts in steel structures for buildings was first introduced by Batho and Bateman in 1934 [31]. Bolts are mechanically similar to rivets but with the beneficial property of enabling disassembly. A quality that may result in an upsurge of bolted construction due to ambitions of circularity. As opposed to rivets, bolts are often used in the assembly of more elaborate structural members such as hot rolled beams and columns.

(a) *Eiffel tower rivets*

(b) *Bolting the empire state building.*

Figure 2.10: *To left a close up photo of the rivets on the Eiffel tower, photo by Prabhu B Doss, licensed by (CC BY-NC-ND 2.0). To the right a worker attaching one of many bolts at the Empire state building during construction in 1930. The Chrysler building can be seen in the background. The photo is taken by Lewis Hine.*

Welding

Welding is the process of joining pieces of metal through fusion and the technique can be traced back to the bronze age. During the industrialisation, arc welding was first used by the automobile industry and the first all-welded building structure is from 1924. The new technology effectively replaced the previous techniques such as riveting and bolting, especially in the machine industry, where welding resulted in cost savings, increased speed of production and more refined products. Like other steel base techniques, it often applied as a technique to stitch together plates of steel that are formed prior to assembly. Due to the logic of the method, it is also better at joining plates than solid faces.

The British museum great court roof which was mentioned in the introduction and which is shown in figure 2.33 employs welding between the thick flame cut star-shaped steel plate nodes and the rectangular steel beams that are tapered rectangular sections, also made from welded steel plates.

Casting

One of the first architects to make substantial use of steel castings were Renzo Piano and Richard Rogers together with Peter Rice in the design of the Centre Pompidou in the Paris that was finished in 1977. The casting technique was used for the production of the gerberettes that connect the trusses carrying the floor slabs, with the lateral support system in the facade. The technique allowed the design team to make a multi-functional nodal connection which was carefully crafted into an almost bone-like structure through a meticulous process using different tools and techniques including models made of insulation polyester [42]. The casting technique comes with few formal restrictions which are mainly due to the choice of mould material and strategy for mould removal, both of which are dependent on whether or not the mould should be used for multiple casts.

Figure 2.11: *The Centre Pompidou in paris from 1978. The casted steel gerberettes Can be seen connecting the trusses and the facade structure. Photo by Upic/Tonyjoe.Gardener.*

Milling

The nodes in the Mero space frame (which was described in relation to the airport project in 1) are created using a subtractive manufacturing technique called milling. It is a technique where a drill is used to remove material from an initial assembly, either with the top or with the side of the drill. The nodes in the Mero system are initially solid casts from which material is removed with the milling process, the connecting bars consists of hollow extruded steel tube onto which conical castings are welded. The bar is finally attached to the node with a bolt from the inside of the cone. In that way, the Mero system combines a range of standard steel manufacturing techniques.

The formal logic of milling follow the principle of a ruling line which means that shapes such as the *cone*, the *cylinder*, the *hyperboloid*, but also the *möbius* and the *helicoid* can be constructed using a milling or flank milling process, assuming that the tool path is arranged such that the milling head can be introduced and removed and move freely around the object.

Additive manufacturing

Additive manufacturing (AM) is considered by some to be one of the key drivers in the current industrial revolution. AM provides a greater degree of design freedom compared to most other manufacturing strategies and comes with benefits such as increased production

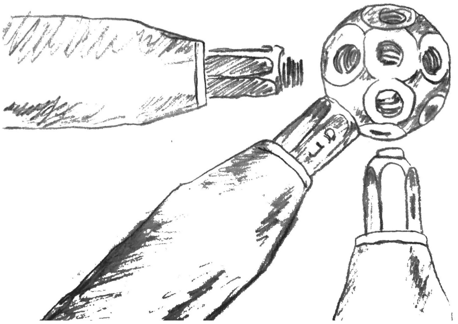


Figure 2.12: *Sketch of a Mero space frame ball node and three attaching bars. The Mexico city airport project was design with this type of system in mind. Sketch created by the author.*

flexibility, product customisation, reduced lead-time, efficient material utilisation, reduced weight [34] . For the architecture and construction industry which is widely relying on construction steel, AM has the potential to remove many limitations posed by other production techniques related to geometrical complexity with the potential to enable a new approach to the design of modern structures. However, until now there are only a few examples of AM application by architects and civil engineers. One such example is the MX3D bridge that was created with an AM technique referred to as Wire and Arc Additive Manufacturing (WAAM) [77]. As opposed to more common techniques for metal AM such as powder bed laser fusion and WAAM is not restricted by a 3D printer bounding box.

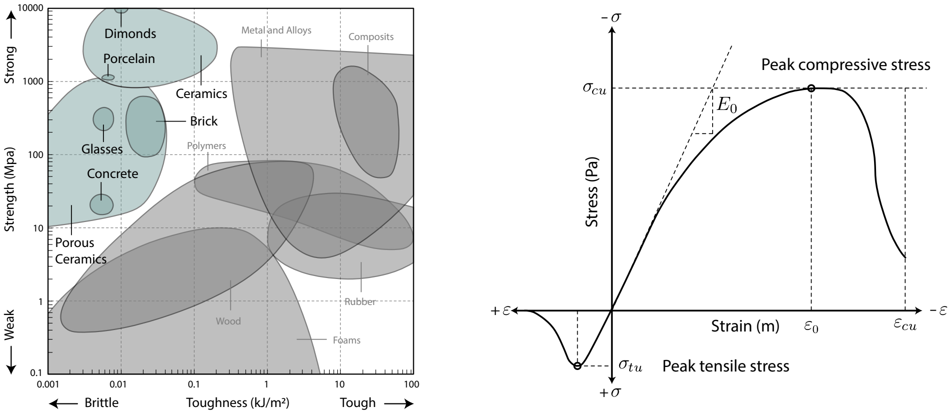
Figure 2.13: *Physical experiments of 3D printed nodal connection in steel created with powder bed laser fusion printing. The shape of the node is found through a topology optimisation process as described in [4]. Photo taken by the author.*

Assuming that there will be an economy in scaling up the process of metal 3D printing, it seems reasonable to think this could be an important contributor to the production of our future built environment. Especially considering the mass customisation need that is intrinsic to the architectural task and the matching capabilities of the AM process.

2.3.2 Concrete

Concrete is another important building material and today it is often found combined with steel as reinforcement. In its earlier forms, the ancient Romans used a material that is rather closely related to modern concrete for buildings like the Colosseum and the Pantheon. The fact that some of these buildings stand to this day testify to the robustness of the material and the skill and the knowledge of builders of the time. Joseph Aspdine of England was credited with the invention of the portland concrete in 1824 and the real breakthrough for concrete happened with the invention of steel reinforcement. W. B. Wilkinson got the first patents for reinforced concrete slabs from 1854 aimed at improving fire resistance of buildings, but french horticulturalists Lambot and Monier developed similar systems and got patents from the same decade [15]. Development of this new construction technique continued to spread around the world and design codes were developed to bridge from practice to theory, notably by Matthias Koenen and Edmond Coignet. Concrete is interesting in the way that it combines both a construction technique and a material. Unlike wood, steel or stone it is the result of innovation rather than discovery. It can be thought of as a composite material where the different constituents are placed purposefully to perform different structural tasks. Initially, the introduction of steel re-bars was primarily thought of as a safety precaution, but the focus then shifted towards seeing it also as a load-carrying design strategy. The next big step in this field was the invention of prestressed concrete often credited to French engineer Eugène Freyssinet [39].

Material mechanics



(a) *Strength Toughness Ceramics*

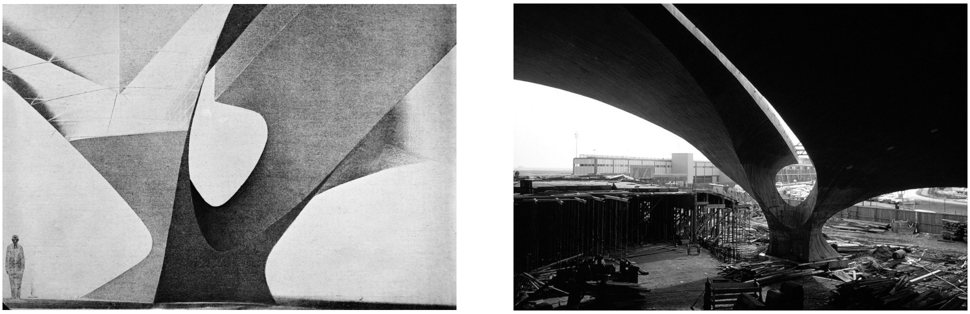
(b) *Typical stress strain for concrete*

Figure 2.14: *The diagram to the left is showing the properties of the family of ceramic material, including concrete [19]. The figure to the right is showing a typical stress strain curve for concrete.*

Concrete was originally a mix of three ingredients: waters, cement and aggregate. The cement reacts with water through a process called hydration and acts as a binder between the hard rock aggregates, creating a strong and durable material. In modern concrete the mix is complemented with admixtures and other additives to tailor the properties and performance of the wet mix but also the final product. Concrete is a mineral based brittle-elastic material with high compressive strength but a with only a fraction of that capacity under tensile load, which is illustrated in figure 2.14b. As a precaution to avoid abrupt failures in tension most structural analysis of concrete assume a non-existing capacity in tension. The tension forces introduced in a concrete component are therefore taken care of using steel reinforcement [89].

Casting

Casting is the primary construction technique for concrete structures. It can be performed onsite, so-called in-situ casting or in a factory environment often referred to as pre-casting. Whereas in-situ castings are not limited in size due to transport the onsite conditions may be challenging in terms of climate control and precision. Pre-casting on the other hand can be done with high accuracy but sizes are limited due to transport and the process of erection. The particular characteristics of concrete connections produced from in-situ casting constriction is the seamless integration with adjacent elements, shown clearly in the photographs from the construction of the JFK terminal building. There is no definite distinction between column, node and roof structure which are all integrated in one continuous structural system. ´



(a) *Sketch model* (b) *Photography from Construction*

Figure 2.15: *Construction of the roof for the John F. Kennedy terminal building in New York in the early 1960s. The architect for the project is Eero Saarinen and photographs are taken by Balthazar Korab.*

The geometrical possibilities with cast concrete structures is quite literally a mirrored reflection of the material used for the mould. For moulds based on timber planks and plywood sheets the geometry is restricted to differential geometry primitives such as the *cone*, *cylinder*, *möbius* and the *helicoid*, all of which are ruled surfaces (thus can be constructed with straight plank lines) and some of which are developable (can be constructed by bending sheet material such as plywood). But there are other casting techniques, such as fabric casting, for which shape of the structure can obtain the characteristics of the *catenoid*, a minimal surface, again based on the mould material properties [113]. An example of fabric casting is the *KnitCandela* shell which is described in [91] by the Block research group.

Additive manufacturing

Another technique that is getting attention in research regarding construction techniques with concrete is additive extrusion-based 3D printing. Either as an in-situ based extrusion where the printer if borough to the site and a wheel-base is used to increase the immediate reach of the printer. Or in terms of pre-printing in a factory environment where the printed object needs to fit within the printer range. One early example of such manufacturing is the *Radiolaria Pavilion* as described in [16]. It is made using a concrete-like material based on sand and with a cement-like binder.

There are two main challenges to this technique that need to be overcome for application in large scale. The first one is about creating a mix that hardens fast enough to not require a supporting mould while still retaining as much of the high strength as possible that can be achieved with traditional casting. Since the concrete mix needs to be distributed through a nozzle, the opening size will dictate the maximum size of aggregate that can be used. The second challenge is to introduce reinforcement to improve the tensile strength and to do that with good bonding between the concrete and the reinforcing material.[88] [80] Various attempts have been made in that regard and are described in [41]. Some of which are; placement of reinforcement material horizontally in between printed layers, metal cables placement along with the concrete extrusion, the use of concrete mixed with short fibres (of carbon or glass). Another strategy is to print the form-work, within which reinforcement is placed and concrete is poured, like in conventional casting.

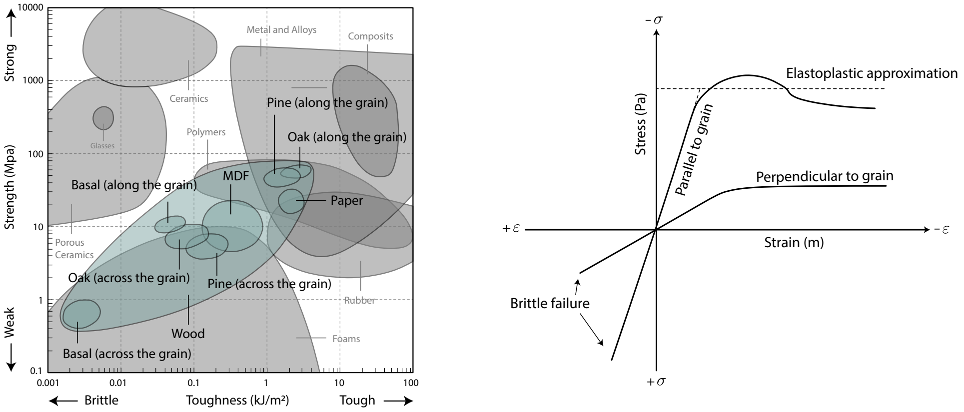
The application of 3D printing with concrete for components such as structural nodes will likely rely on the success of reinforcement strategy. For a complex structural node it is unlikely to suffice with only capabilities to handle compressive force. That would render the design space for such components too limited.

2.3.3 Wood

Wood has been an important material for the construction of buildings in many cultures and regions, so there are incredibly rich resources of various ways in which the construction of such buildings has been carried out. Sometimes the skills have travelled between continents along trade routes or due to wars and take over of land. But as Elias Cornell points out in [24], the travel of technology is sometimes overemphasised, and it is likely that similar building techniques also developed independently of each other. After all, the techniques are developed from the material properties point of view, and even though timber qualities vary with climate (even within the same species), the main characteristics of the orthotropic nature of timber will be the same independent of place.

Material mechanics

Wood is typically described as an orthotropic material, meaning the mechanical properties vary with the direction of a specimen. Due to the cylindrical nature of the log, these directions are usually referred as the longitudinal direction (parallel to the grain), the transversal direction (along with the rings in the cross-section) and the radial direction (perpendicular to the surface of the cylindrical shape).



(a) *Strength and toughness for wood*

(b) *Typical stress strain for timber*

Figure 2.16: *The diagram to the left is showing the strength and toughness properties for the family of wood based products [19]. The figure to the right showing a typical stress strain curve for wood. [116]*

Traditional carpentry

The log house is traditionally built from stacking of horizontal logs that are connected at the corners through notching, and it has been an important building technique for the Nordic countries. Peter Sjömar gives a detailed description of how construction and design of such buildings progressed in Sweden with case studies of log houses from the

13^{th} and 14^{th} century. The picture in figure 2.17 shows a collection of various timber connections from roof trusses. It shows carefully detailed connections that were possible to make only through skilled carpentry, and even so, the detailing requires both time and effort [104].

As alluded to in the chapter on the industrial revolution, the type of handcraft detailing that Sjömar portrays was deemed obsolete due to increased pressure of standardisation, higher production rates and reduced cost. Plugs were replaced by iron nails, skilfully crafted timber detailing, which required shaping of complicated three-dimensional forms, were replaced by standardised steel plates. However, as production has improved with the introduction of computer-controlled machines there is a tendency to look back at the traditional handcraft. Aspiring to rejuvenate the detailing of a skilled craftsman but with the aid of computational technology and the efficiency of industrial mass production.

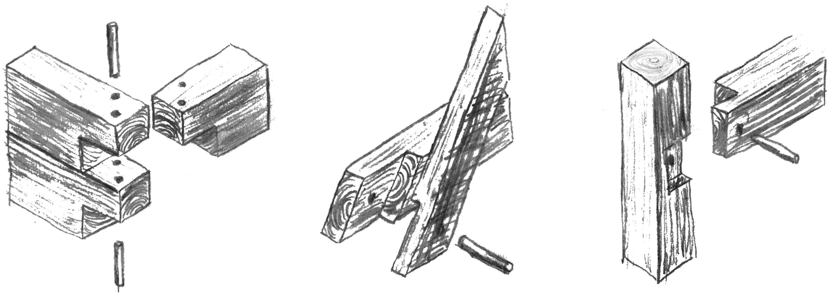


Figure 2.17: *Nodal connections for a truss structure in wood inspired from timber trusses used in medieval church buildings as depicted in [104]. Sketch by the author.*

Digital technology

The ability to control machines with the aid of computer code eliminates the constraints set by classical mass production. If a parametric model which generates machine instructions is set up so that its parameter space maps to the range of manufacturing possibilities for a specific computer-controlled machine, a virtually infinite number of variations can be produced from that setup. The limiting factor is the storage of numbers in the computer (which is vast) and the precision of the machine. Thus, a variable production can be achieved with the same efficiency as in mass production, enabling a new type of mass customisation.

The first machines that were controlled through the flow of digital information, although using punch cards, was the Jaquard loom, developed by Joseph Marie Jaquard in 1804. It was invented to simplify the process of textile manufacturing and could handle brocade, damask and even matelassé patterns. The first digitally controlled machine for timber structures was invented in the 1980s and was initially used to improve accuracy in production, but with the advent of the 5 axis CNC machines with automatic tool switching that came in the '90s, manufacturing of complex joinery with inspiration from traditional carpentry building techniques started gaining traction. Examples from practice can be seen in projects like the Tamedia office Building by Shigure Ban and Centre Pompidou Metz by the same architect, and in academia by researcher Christopher Robeller which goes into depth on the possibilities and challenges with these manufacturing techniques in his PhD Thesis [96].

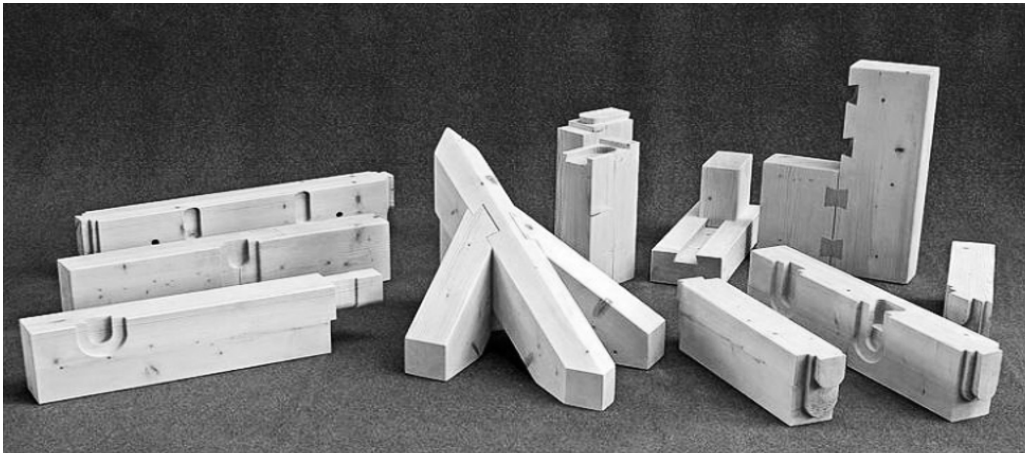


Figure 2.18: *Sophisticated timber connection details produced with a Hundegger K1 joinery machine. Photo © Hans Hundegger GmbH.*

2.4 Tools and representation

The output of the architectural design process is often some form of representation (drawing, images or today digital model) and one could argue that this is the primary communicative device in the repertoire of the architect. Inspired by the reasoning by Bryan Lawson in [61], the representation can be divided into *instructive drawings* which is aimed to the builder, *communicative drawing* which is aimed for at the client, and the *design drawing* which is that process drawing used by in the act of making, integral to the thinking process itself.

The tools that are used to create a representation are naturally closely linked to the outcome. The choice of a tool is in itself a choice that can be understood as an indirect design decision. Furthermore, one could argue that any tool can be integrated as part of a design process because what is important is the mindset and the attitude of how the tool is valued and placed in relation to the composition of the process as a whole. However, the way in which the tools are developed can make the design process more of less efficient and allow for more or fewer perspectives on the subject matter.

David Hockney takes on a particular discussion about the overseen influence of tools within the arts by the European painters during the middle ages in his book *Secret Knowledge* [48]. He builds a case for the usage of tools such as the optical lenses, mirrors and technology such as the camera obscura for many of the celebrated master painters that were active hundreds of year before the invention of the camera. The tools allowed the artists to capture the texture of cloth, patterned garments in complex folds, glare and reflections in metal armour and complex shapes, with an almost camera-like precision. To what extent Hockney is right in his claims is a disputed matter [71], but it raises interesting questions regarding the role of tools and technology in the creative process, also in the arts.

Due to the scale of buildings, architectural representations are, just like paintings often made in a scale different from reality and various drawing techniques and scales measures have been used to bridge between the representation and the real. Just like in the arts, there is more than measurements of distance and size that can be conveyed in an architectural representation, notably of importance for the *communicative drawing*. Materiality, mass, void, light, atmosphere, and even the sound of a space or other sensory impressions may be communicated in a drawn format, and as the architect has got more distanced from the making, the communicative capacity of the drawing has become more significant [61].

The descriptive drawing was introduced as a fundamental core subject at The Ecole Polytechnique in Paris and enabled for systematic reduction of a 3D object onto a 2D drawing with precise measurements. It was a key invention and an enabler for the industrial revolution for which production required control and precision [94]. The pre-industrial speculations of the transcendental in geometry which was common in discussions of philosophy and theology became a marginal perspective, and geometry was treated as a pragmatic representation of empirical reality. Descriptive geometry became the foundation to many modern endeavours ranging from the artistic drawings with the particular style of the Ecole des Beaux-Arts to the functionalism of the Bauhaus [94]. The computer-based representation of modern CAD systems is building on the same geometric principles but with a higher degree of sophistication. The objects in a computer can be rotated much like in the real world and the new perspectives are calculated and projected on to the screen fast enough to enable a virtual real-time interaction.

With the use of computers as primary design tools in much of contemporary practice, exchange of information between collaborative parties in a design situation is often shared directly as digital representations, typically in a format that will reduce the information to parameters that can be converted to a binary form (distance, colour, names, areas etc.) which means that other forms of representations are indirectly contained or omitted by the "infrastructure" of the process. The drawing as it is represented in terms of the binary data and the abstract act of drawing with computer programs is discussed by Austin and Perin in *Drawing the Glitch* in [115]. They go on describing the two dominating techniques for computer graphics, namely vector and raster graphics, and their numerical representations. Whereas the handmade drawing only exists in its visual form, the digital version is also represented as a matrix of numbers, allowing for an altogether different set of possibilities in terms of sharing and manipulation.

Whereas the precise nature of vector-based computer representation comes with clear benefits in terms of precision, the same precision puts requirements on the drafter to give precise instructions, literally demanding the coordinates for the pixels to be drawn on the screen. For raster graphics on the other hand the manipulation of precise positions of points and vectors is replaced by manipulations of pixel fields without clear borders and edges. It is a technique that is gaining traction with the development of photo editing and sketching pads, enabling a form of digital sketching. Whereas CAD modelling builds on the logic of vector graphics, image manipulation is more likely to be associate with raster graphics. There are methods for transitioning between the two formats (with more or less loss of information), after all, what is shown on the computer screen is always a rasterized image. [115].

The rest of this section is dedicated to a state of the art overview of digital form representation techniques that are used in computer modelling to enable a discussion of its relationship to the simulation of material mechanics and production. The aim is to find the potential in the joint effort of representation and simulation in a way that facilitates intuitive and explorative ways of creating. The conceptual modes by Lawson, as described in length here [62] and in condensed form here 2.5.3, will be used to discuss the various techniques and their relation to the design process. The discussion is furthermore limited to focus primarily on the early stage of the design process, conceptual design, and the relation between the designer and the object.

2.4.1 From representation in a digital environment

In order to deal with geometry in a computational environment, a suitable geometric representation needs to be chosen. Such representation also need to fulfil various requirements in terms of user interactivity, capabilities to model certain forms, allowing for required precision, and much more. Like any other tool in the design process, the computer the modelling environment — its capabilities and shortcomings — will influence decision making, and thus the outcome. Whereas differential geometry in the smooth setting, which was introduced in 2.2, is a powerful foundation to the researcher, it is a rather cumbersome representation technique when dealing with a design process of multiple constraints. That realisation has historically lead to the development of various modelling techniques, of which the two dominating techniques, surface modelling and polygon modelling, will be introduced here, followed by a complimentary introduction to point clouds.

Surface modelling

Surface modelling was developed with the ambition to represent 3D geometry in a precise way and its main contributors come from the vehicle and machine industry, where the challenge has been to represent geometry for car bodies, air plane frames and boat hulls with high precision. The general idea is to model surface geometry through the interpolation of a bi-directional net of points. The most commonly used technique is called Non-Uniform Rational B-Splines and will be described in short here as background to the section on Iso-Geometric Analysis.

The basic equation behind the shape functions used for NURBS modelling reads,

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \tag{2.41}$$

and is solved using a recursive algorithm, such as Cox-De Boor [90]. For the application on 2D-surfaces these shape functions are applied in the two parameter direction and associated with a control point to provide user interaction. The description for the surface S is obtained through multiplication of the two shape functions such that,

$$S(\xi_1, \xi_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p}(\xi_1) M_{j,q}(\xi_2) P_{i,j} \tag{2.42}$$

where n, m are the number of control points in direction 1 and 2, N, M are the shape functions in the two directions and $P_{i,j}$ is the bidirectional control point net [90].

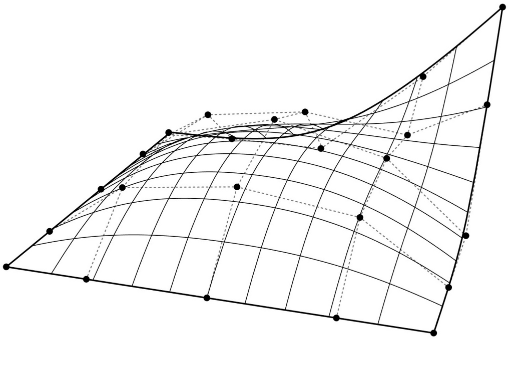


Figure 2.19: *A bi-directional grid of points and the resulting NURBS surface*

Polygon modelling

Polygon modelling is another technique that has developed in order to represent free forms in a digital environment, but this technique comes out of the film industry striving to perfect the animated illusions. It is a technique that prioritises artistic freedom over precision which is regarded as important in the creative process. A mesh is essentially a collection of points with topological information that describes the connectivity which creates edges and eventually triangles, quads or ngons. Each vertex is given three coordinates which are all stored in a matrix. The edge and face matrices are then just a collection of vertex indices, thus the mesh becomes rather lightweight and straight forward. However, in order to approximate curved shapes in a good way, a high mesh density may be required and recursive subdivision may be applied.

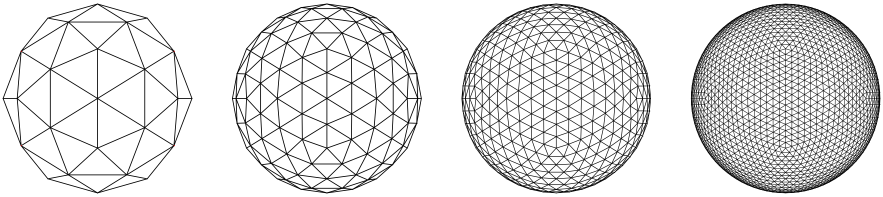


Figure 2.20: *Illustrating increasing subdivision for a polygon approximation of a sphere.*

Point clouds

The third way in which geometry tends to be represented in a digital environment is through point clouds. Just like it sounds it is essentially a collection of points that represent the geometry at hand. Thus it is a rather rough representation and is often converted to a mesh for further data processing. Point clouds are particularly relevant for 3D scanners as it is the output data of such scanning process. Typical techniques that are applied to convert points clouds to mesh data are for example Delaunay triangulation, marching cubes or alpha shapes.

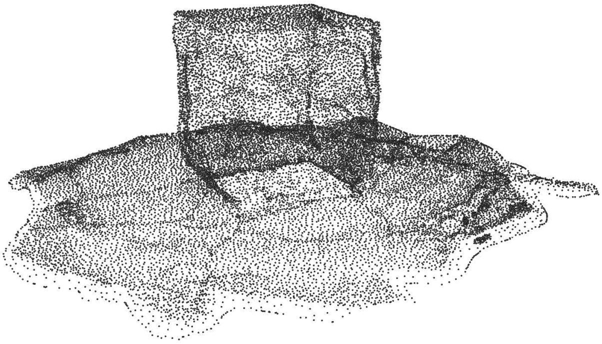


Figure 2.21: *Digital Representation of a stone cube with a point cloud. Image produced by the author.*

Computer graphics

The demands in terms of entertainment from in the film industry has pushed the development of the computational methods for digital representation in terms of realistic images and animations. The progress in this field has made big leaps as computational power has improved. At the 2014 GTC conference, which is dedicated to Graphics Processing Unit (GPU) computation [81], Nvidia CEO Jen-Hsun Huang, show the progress of computer graphics in three steps. The first image is a render from 1984. One of the first computer renders used in graphics software for Pixar animations. It shows a table lamp in a dark room with the light on. Material and texture on the surrounding objects are captured, including soft shadows and highlight reflections. It took 1.5 hours to render one single frame for that animation sequence. The next example is from the film that won the Academy Award for best visual effects in 2013. The animation shows a whale jumping out of a dark blue sea at night in the moonlight. The water and the splashes are simulated using particles, which are also used to simulate the mist and fog in the scene. Collision detection between the particles and the whale ensure a realistic interaction between the movement of the body and its surroundings. Even though computational power has increased by a factor of 1 million compared to the lamp animation, the rendering time per frame has increased to 250 hours. Exactly when the animation was created and on which hardware is not disclosed but in the next sequence, the same animation is shown on what was modern hardware in 2014. But this time in real-time using the GPU for integrated simulation and rendering. The development of GPU hardware has been an enabler for development in artificial intelligence and machine learning, and the popularisation of these techniques mutually drives the hardware development.

Parametric design

Parametric design is a collective name for approaches to computational design. At the most essential it is computer code which is wrapped in a user interface that aims to expose access to the logic of the algorithm for the purpose of design. Thus, the parametric tool makes the abstract nature of coding a little less abstract by visualising and exposing the decision tree (or the graph) as something interactive. What goes on behind the interface is no different from the logic of regular imperative programming where statements are executed in sequential order. When parameters are updated the code is re-executed. Attempts have been made, notably with the development of Design Script, to change the nature of parametric design by grounding it on an associative language base. The difference between imperative and associative programming and its potential in parametric design is explained more thoroughly in [6]. To the knowledge of the author, no major breakthrough has been achieved on this front.

The parameters in a parametric model are often of geometric or relation-based character. A geometric parameter can, for example, be the radius of a circle, and a relation based parameter the distance from the centre of the same circle to some other object. Parameters can be further classified as dependent or independent. If the circle is used as a base for a cylindrical column, and the radius is a function of the height of the column, where only the height is exposed to user input. The radius is then a dependant parameter whereas the height is independent. Parameters can also be input to analysis which could

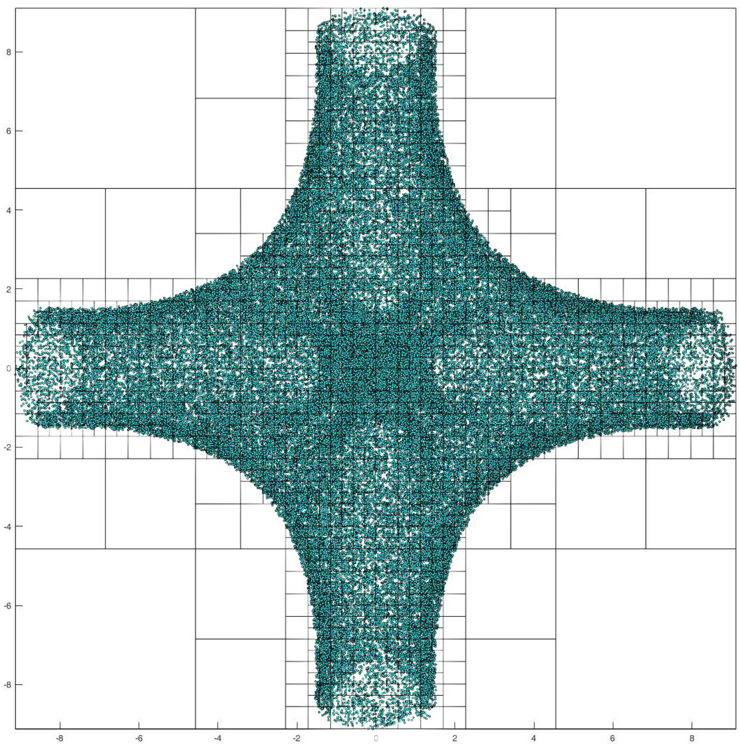


Figure 2.22: *Top view of a structural node with an hypar like shape where 100000 particles are distributed within the thickness of a containing polygon mesh which is used to define its shape. The particles are stored in rectangular zones which are sized for an upper limit of particle count. A computer graphics algorithm called an Octree is used to generate these zones through a subdivided scheme. Image created by the author.*

be integrated with a feedback loop where results from the analysis affect geometric and relational parameters.

Parametric design is often associated with narratives related to the conceptual mode of Lawson's Oracle, and described as a foundational technique in the making of a super "automator". Especially when it is integrated with various forms of analysis and optimisation. Such approaches are certainly useful in later stages of design refinement, but arguably less suitable for conceptual design. The flexibility of the parametric model only mirrors the understanding of the design situation at the creation of the model. Thus the type of variations of the model which is achieved through parameter adjustments only creates fined tuned versions of that same idea. Other types of changes that require a rewiring of the graph are often complicated and it becomes difficult to imagine the parametric logic as *conversational* at the inception of design concepts.

2.4.2 Structures and analysis

In this section, a number of structural analysis techniques that could be related to the project are introduced. Numerical methods for analysis can be applied in different ways depending on the objectives at hand. On one hand, there is the study of the phenomena of material behaviour under various loading conditions where mathematical models are developed to mimic natural behaviour. On the other hand, there is the development of design codes that assist the engineer in the daily work where safety is the highest priority. That involves calibration of safety coefficients as a response to theoretical limitations and natural variation of material properties that create uncertainty in performance. In order to introduce material properties to the design process for the structural nodal connection, a choice of continuum theory need to be made as well as a choice of the analysis technique.

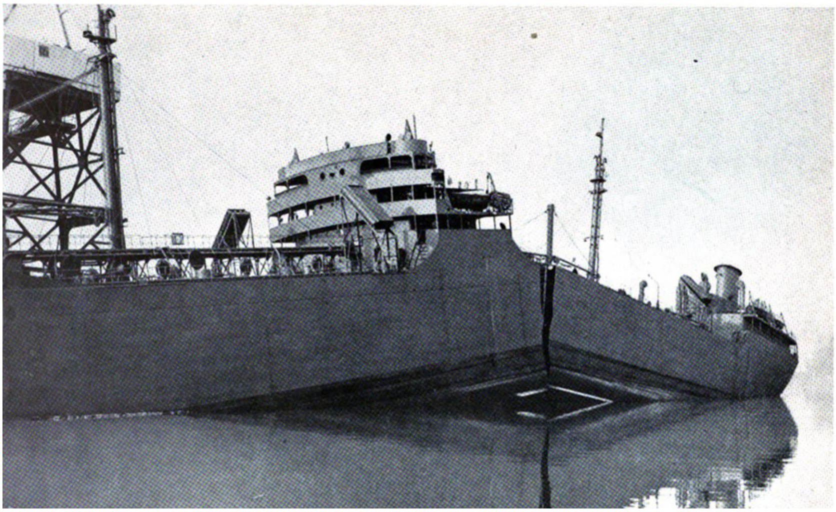


Figure 2.23: *View of S.S. Schenectady after splitting in two at her outfitting dock. Brittle fracture of the steel hull due to temperature changes in the water is understood to have caused the failure. Photo from the Final report from the board of investigation [27].*

Material modelling

The mathematical material model links some state of stress and measure of strain and is needed for all analysis of structures. It is the constitutive equations which are used to define the constitutive model and thus the material properties of the body. The constitutive models aim to approximation the particular microscopic phenomenon using mathematical relationships retrieved from empirical testing. Some of these relationships are common across many material types and others are uniquely found in a specific material class, such as fibre delamination in wood. However, independent of the particular material class, it is the microscopic structures all the way down to the atomic level that will determine the macroscopic behaviour and every model that is used for structural analysis will be smearing the real atomic behaviour over huge areas compared to the atomic scale. Material behaviour varies not only between material classes (should it be crystalline, amorphous or polycrystalline etc.) but also temperature, sometimes moisture content and loading rate will impact the mechanical properties [97]. Loading cycles, radiation and decay are other factors that may impact the material performance, some of which are difficult to predict. One such (at the time) unpredictable material phenomenon is exemplified with the structural failure of the S.S. Schenectady that broke in half just as she had returned from her sea trail. The common explanation for the indecent became known as defected welding. But later investigations concluded that the butt welded joints were fine and that the failure was more likely due to brittle fracture. The steel that was used for the hull structure contained high levels of Sulfur and low levels of Manganese effectively weakening the material and making it more brittle at lower temperatures such as in cold weather [76].

The finite element method

The Finite Element Method (FEM) is a numerical analysis technique used to approximate the solution of partial differential equations that model physical phenomena. The problem region is modelled using a set of discrete elements, effectively a polygon mesh, which is then assembled into an approximation of the whole region. The elements can be one, two or three dimensional and will be interconnected at nodal points. The variation of the field variable over the elements is approximated using interpolating polynomials of various kinds. Once the problem domain is set up and the equations for the whole continuum are established the global unknowns will be the nodal values of the field variables. By solving what is most often formulated in a matrix form the nodal values will be found and the variation of the field variable over the elements can be found using the chosen interpolation scheme [78].

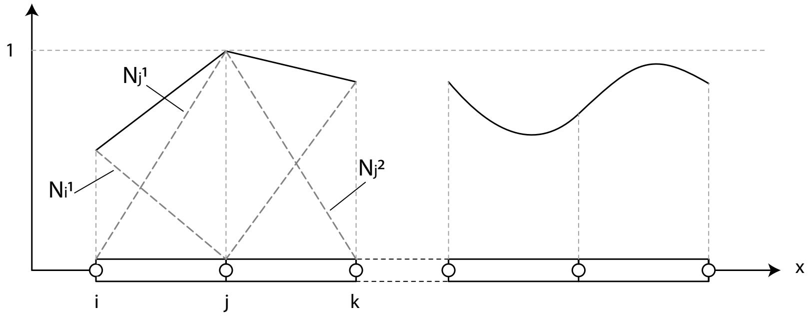


Figure 2.24: *Linear and non-linear Shape functions for a 1D FEM model. The graphics are inspired by the descriptive diagrams in [98]*

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Isogeometric analysis

Isogeometric analysis can be understood of as a special case of the finite element method. The ambition behind the development of the method is to unify the geometry used for design and analysis where the NURBS surface definition is used as the common ground. The shape function as shown in equation 2.41 and figure 2.25 which describe the distribution of the unknown field variable for the analysis, is chosen as the same shape functions that determine the geometry of the object. Elements in an IGA model does not look the same as in FEM because the shape functions are non-local, i.e. they span across more than one element.

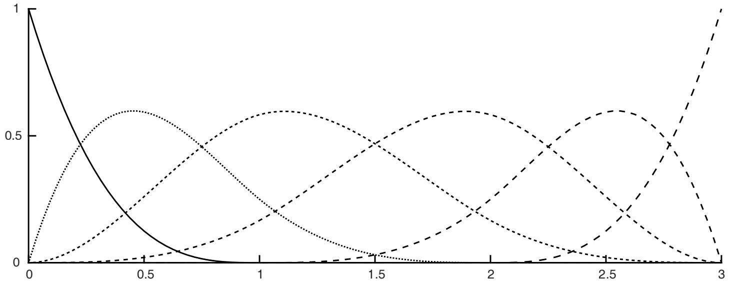


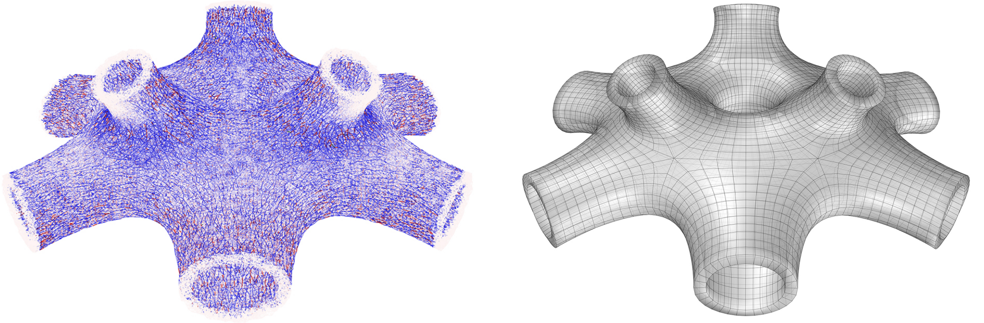
Figure 2.25: *Illustrating the shape functions that are used to interpolate between control points to compute curve and surface geometry for NURBS.*

Isogeometric Analysis was introduced by Huges et. al in 2004 and has since gained a lot of attention [55]. Some implementations into the field of structural design and architecture can be found in [58], [59], [8].

Meshless methods

Another class of method for the analysis of structural phenomena is meshless methods. Whereas FEM relies on a mesh to construct the local approximation of functions and their derivatives for solving a partial differential equation, meshless methods approximate the unknowns based on a field of scattered points without mesh connectivity [23]. The invention behind the method was to figure out how to take the derivative of a function without using a mesh. Previously a mesh was needed to compute spatial derivatives using the finite difference method. Using a class of approximation functions that are not directly related to the elements (such as in FEM) the strong tie between the quality of the discretization and the quality of the approximation can be relaxed. Furthermore, the simplicity in terms of implementation without requirements for numerical integration while simultaneously reaching capabilities of solving highly complex problems makes this class of methods attractive.

The influence function w is used to estimate the contribution of the field variable from the neighbouring particles. Various functions can be used to this end but are typically chosen as smooth, compactly supported functions [23]. One example of that is the b-spline



(a) *Bonds between particles coloured by force* (b) *Polygon mesh container for the particles*

Figure 2.26: *Showing one of the numerical experiments for the nodal connection in figure 2.32 which is subjected to a prescribed displacement along the direction of each tube-shaped opening. The simulation is carried out using an implementation of bond-based Peridynamics. The bonds that are coloured in blue are in compression, the bonds that are coloured red are in tension, and the white bonds are unloaded. Images created by the author.*

function,

$$w_a(x-s) = \begin{cases} \frac{2}{3} - 4z^2 + 4z^3 & for \;\; 0 \le z \le \frac{1}{2}, \\ \frac{4}{3} - 4z + 4z^2 - \frac{4}{3}z^3 & for \;\; \frac{1}{2} \le z \le 1, \\ 0 & for \;\; z > 1, \end{cases} \tag{2.43}$$

where

$$z = \frac{|x-s|}{a}. \tag{2.44}$$

Another influence function that is attractive due to its simplicity is the Gauss bell function,

$$w(x) = e^{-\frac{x^2}{a^2}}, \qquad for \;\; x \le a. \tag{2.45}$$

However, in its simplest form it does not provide compact support as can be seen in fig. 2.27. To achieve that the function need to be altered by as scale factor such that

$$w(x) = 0.4^2(e^{-\frac{x^2}{a^2}}), \qquad for \;\; x \le a. \tag{2.46}$$

There are today more than 30 different meshless methods that have been developed since the concept was introduced in 1977 [37] [23]. Only two of these methods will be mentioned here, due to their importance for the overall field and for their relevance for the project. That is Smoothed Particle Hydrodynamics (SPH) and Peridynamics.

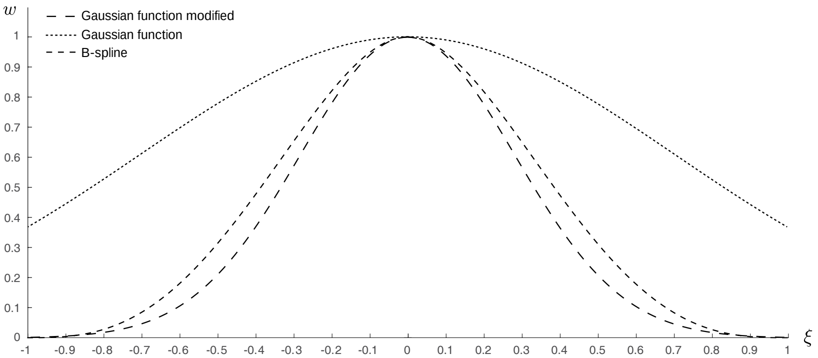


Figure 2.27: *Three different influence functions for meshless analysis, two of which are based the Gaussian function, and one which is the B-spline function. Image created by the author.*

Smoothed particle hydrodynamics

The theory of SPH was published for the first time in 1977 by developed R.A. Gingold and J.J. Monaghan [37]. It was developed for the simulation of astrophysics but has been applied frequently in the simulation of other phenomena such as fluid dynamics. It offers several advantages over grid-based methods for fluid simulations, such as automatic mass preservation, but also the possibility to simulate more complex phenomena, including two-phase materials and solid-fluid interaction. SPH was also the starting point for the development of other meshless methods. The drawback with many particle methods including SPH is computational cost. A rather large amount of particles are needed for accurate simulations. However, just like other particle methods, SPH lends itself well to parallelisation and computation on the graphics card.

Peridynamics

Peridynamics (PD) is another meshless method and also sometimes referred to as a continuum mechanics theory. It was introduced by Stewart Silling in 1999 as an attempt to improve modelling of spontaneously forming cracks in fracture mechanics. It is a particle-based method where the particles are connected to its neighbours through a set of bonds spanning a distance which is called the horizon. Since the bonds reach further than their immediate neighbours the method is referred to as non-local. A dense network of bonds makes up the domain of interest for the analysis. FEM can be understood as a special case of peridynamics when the horizon is set such that is only adjacent particles are connected.

As opposed to classical continuum mechanics (CCM), for which assumptions are made that the domain of interest is spatially differentiable, no such assumption is made in PD. The differentiation of the strain tensor is replaced by the integration of an already discrete setting. Therefore the equations are valid also in the presence of discontinuities, making it a suitable for the modelling of phenomenon such as the spontaneous formation

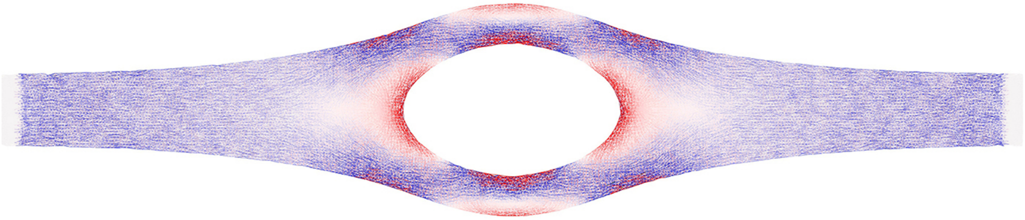


Figure 2.28: *Numerical experiment showing a 2D Peridynamics simulation of a bar with an internal void exerted to a prescribed displacement of the two ends that induce a compressive force in the member. The bonds in between the particles are coloured blue in compression and red in tension. Image created by the author.*

of cracks. Since the introduction of PD the theory has developed into 3 main tracks [54].

- Bond-Based Peridynamics (BB-PD). The bond-based theory is the simplest implementation where the bond between two particles always push or pull with a force of the same magnitude but opposite direction on each connecting particle. The result of such an assumption is that theory is limited to the modelling materials with a constant Poisson's ratio of 0.33 for 2D and 0.25 in 3D [54].

- Ordinary State-Based Peridynamics (OSB-PD). In order to address the limitation with bond-based PD a generalisation called state-based PD was introduce in 2007. The core concept here is a mathematical object called a *state* which operates like a mapping. The state makes it possible for an unbalance in the bond force in relation to the adjacent particles while fulfilling conservation of balance and momentum for each particle as a whole.

- Non-Ordinary State-Based Peridynamics (NOSB-PD). State-based PD has been further developed into what is called a non-ordinary state based formulation, where neither the force magnitude nor the force direction needs to be the same but opposite between two bonded particles. Again, this is done while simultaneously preserving conservation of momentum and balance as per requirements of the fundamental laws of thermodynamics

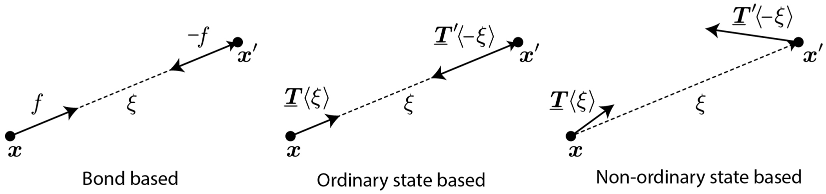


Figure 2.29: *Illustration of the 3 types of peridynamics theory, where* $\boldsymbol{x}$ *and* $\boldsymbol{x}'$ *are two particles,* $\boldsymbol{f}$ *is the bond force,* $\underline{\mathbf{T}}\langle\xi\rangle$*, and* $\underline{\mathbf{T}}'\langle-\xi\rangle$ *represent the state at each particle. Image inspired from [54].*

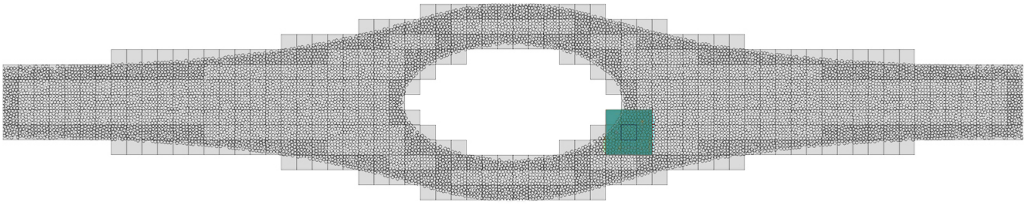


Figure 2.30: *Showing the set up of data storage and zoning for the example which was shown in figure 2.28. Each zone acts as a host for the particles within its borders and has references to the neighbouring zones for calculations of bond connectivity. Image created by the author.*

2.4.3 Geometry and structural analysis

One crucial step in bridging the gap between digital design and analysis is to simultaneously facilitate the needs of form representation for the purpose of shaping and analysing. In the shaping process high resolution and smoothness, visual appeal, is preferred, whereas the analysis relies on a discretisation of time and space for the approximation of the equations of physics to simulate mechanical behaviour. There are a few different ways in which that can be achieved.

Iso Geometric analysis: Unifying shape functions for the form and approximation of analysis variable as described in 2.4.2. Thus, the form representation for the design is also used for the analysis, there is no mesh generation. However, if the geometry undergoes large changes and the NURBS patches are not kept square-like, the distribution of points that are evaluated on the geometry may be unsatisfactory and produce ill results. It can also be difficult to work with NURBS geometry in an interactive and intuitive way, thus modelling expertise is paramount.

The finite element method: Combined with automatic meshing generation can be used to bridge between design and analysis. However, FEM is rather sensitive to the quality of the elements for evaluation of spatial derivatives which makes the automatic generation tricky. IGA was developed just to overcome this issue.

Calculus: Unifying the theory for shape definition and elasticity with calculus of differential geometry. Notably done in the research by Chris Williams in [117], [2], based on the elasticity formulation from [7]. This development is axiomatic i character for the understanding of relations between form and structure. But the differential geometry makes the application in practice rather difficult.

2.4.4 Geometry and production

The *design for production* and the *design for assembly* paradigms suggest that objects should be designed in such a way that production and assembly can be performed easily and therefor also cheaply [3]. As computational optimisation techniques are getting more used in industry for the conceptual design of parts, some researchers are focusing their attention on rationalisation of the geometry to enable the production. One such example is the integration of production rationality constraints within the algorithms of topology optimisation. An overview of that process including production techniques such as forging, casting, rolling extrusion and turning is described in [112].

Casting: Since casting is such a well-established production process which also works well for mass production there are good reasons that it gets continuous attention also in relation to the development of shape optimisation algorithms. One such exploration is found in [66], where the authors present a technique by which the topology optimisation algorithm is tweaked using a virtual temperature method to tackle the problem of mould removal. There are shape restrictions with casting because of the challenge to remove the mould while keeping it intact. That requires the absence of interior voids and undercuts in the geometry as enabled using the virtual temperature method. The other possibility is to increase the number of mould segments, which will simultaneously increase the complexity of assembly and disassemble [3]. A review of these cast removal problem can be found in [3],[44],[121].

Milling: Milling is a subtractive production process by which an object is extracted by the removal of material from a workpiece. There are two main types of milling, which is top milling and flank milling. Top milling works by removing material with the top of the drilling head, whereas flank milling removes material with the side of the drill. Since flank milling has a larger contact surface it is faster at removing material. Flank milling is effectively a drill that moves around to remove material its geometrical condition is a sweeping movement along a ruled surface, where the drilling head represents the straight line. In order to prepare an object for this type of production, the model needs to go through some type of rationalisation. A free form object can be made developable through various techniques such as presented in [108].

Additive: The process of generating a physical model from a digital model using 3d printing involves a couple of geometrical steps. If the model of the object is created using a 3d modelling software and the object is typically represented with an exact surface model, the first step is then to simplify that geometry to a mesh using some type of meshing algorithm. The next step is to run a slicing algorithm which cuts the object into slices using intersection operations between the object and a set of cutting planes which are separated by the material layer thickness. Thirdly the slices are used to generate a tool path for the printer head. The next challenge for the geometry of a 3D printed object is to calculate the need for supports. Exactly what the geometrical limitation are in this context seems difficult to pin down, due to the complexity of the process.

2.5 Design process

The design process is the activity through which a project takes shape. In architecture, it involves a variety of actors with different roles, responsibilities and knowledge. It is often a rather non-linear process where the project takes shape during a number of iterations, with constantly shifting perspectives, from the partial to the holistic. Whether the most important aspects are related to a social agenda, environmental strategy, specific expression, sensory experience, resource efficiency, material and related manufacturing techniques, or other architectural qualities.

The design process is to some extent subjective which is described by Jadwiga Krupinska in [60] as a prerequisite for the artistic expression/ambition. The choice of tools and ways of working are also closely linked to personal preference and skill. Krupinska points out how the subjective dimension of the design task is furthermore ladened with uncertainty for the architect and poses a challenge in terms of communication to others. Because of the complexity and time frames for an architectural project, the process is also shaped by how collaboration is organised, within the design team itself, but also in relation to external partners and clients. Thus, oral and drawn communication becomes a backbone to the process as a whole.

Those aspects of architecture that are understood to have a subjective nature are according to Krupinska [60] often undervalued in comparison to what is referred to as objective values. But the so-called objective values are usually built on a subjective value judgement which might have been the basis for the initial formulation of the task. The weak status of the subjective judgement means that it is sometimes hidden or camouflaged using various linguistic formulations [60]. The design process is furthermore driven by values which historically often have been formulated in terms of doctrines. Perhaps as a way to cover over the difficulties aspects of the inevitable subjective dimension in the design process. Some values are less obvious, which is typically values of normative character given a particular time and the culture. Other values could probably be classified as conscious pursuits for the architect, the office, the team and the particular project.

The discussion on the design in this thesis is first and foremost limited to the individual designer (architect/engineer) and the personal activity of making, with a particular focus on the digital environment. This section starts with a short note of the changing roles of the architects and engineers followed by an overview of the development of design theory and a note on the designer computer relationship. The chapter continues to make a note on the sustainability challenge, which is approached here from the partial perspective of embodied carbon. The final part of the chapter is therefore devoted to a description of various strategies that are commonly applied in such a pursuit.

2.5.1 Roles

As was mentioned briefly in the historical context in 2.1, the roles of the actors in the process of architectural design have gone through a variety of phases as a response to factors of which some are societal, technological and political. The relationship between theory and practice is another important aspect that has shaped the role of the architect and could almost be described as a dance that sends oscillating waves through the recent history of the architectural profession. Tendencies are furthermore regional, cultural and historical but the large trajectory follows the rest of knowledge development within society with increasing specialisation, virtually exploding with the enlightenment and the industrialisation [51].

The pre-industrial architects were knowledgeable of most steps in the building process, acting also as engineers and project leaders in the role of the master builder. Project time frames and the hands-on approach to architecture was important for building such a generalist type of knowledge. These were also times when mathematics and geometry were most highly regarded and natural know-how of the architect. During the Renaissance, it was the role of the architect to materialise the holistic unity of all things with a mathematical foundation, often with an undertone of platonic thinking. The architect was furthermore expected to be knowledgeable in music, art, philosophy, astronomy and linguistics.

The industrial era as described in 2.1.2 meant that the role of the master builders was split into architect, engineer and builder as a result of specialisation. Civil engineering also developed as its own discipline in the 18^{th} century and unlike the role of the scientist whose aim it is to know, the role of the engineer is focused on the doing. Whereas the scientist tends toward the study of phenomenon out of pure knowledge pursuit the engineer tend towards work that is addressing problems as they arise in a practical situation. Usually in a way were multiple conflicting criteria are prioritised and compromised through the application of predominantly two types of natural resources, material and energy. [106]. The architect got furthermore distanced from the act of making, resulting in a more abstract type of work. The drawings were expected to be delivered in a type ready to build fashion, to meet the needs of the increasingly industrialised construction process [60].

The roles of architects and engineers in the digital era are continuously adapting to new conditions, partly affected by the tools for drawing and the possibilities enabled by new means of production. The digitalisation of the production process may enable a new type of bespoke mass production, and the shared digital workflow seems to blur the borders between professions that has for a while been drifting apart. The emphasis on computer modelling makes the expertise in mathematics once more relevant know-how in architects toolbox.

2.5.2 Design theory

Initial attempts to understand and systematise the design process through a scientific inquiry sprung from the structuralist and functionalist thinking from the '60s and '70s [60]. The goal was to rationalise the activity of design and remove its somewhat mystical connotation. The design process was split into discrete stages such as analysis, synthesis and evaluation and structured in a more or less linear fashion. But the model was soon to be rejected as insufficient. Other attempts were made to understand this process starting from the perspective of the designer or artist, who were asked to document their own process. It was however noted that the focus on documentation interrupted the process, effectively invalidating the subject of study itself. Later studies attempted to place architects and designers in what was understood as objectively controlled environments, a laboratory, where their behaviour could be observed in a systematic way. Other studies focused on observing the designer in the more natural studio environment, but neither of these strategies could successfully reveal what was going on in the head of the designer [60]. The identifiable stages of the design process were organised in various scheme's, with inbuilt iterative loops, but the same linear assumption characterised these attempts. During the second generation of design theory research in the '70s the linear model was replaced with a structure of a dialectic character. The assumptions of an analytical point of departure were replaced by a criteria-based solution focus. But the end results were deemed problematic. The attempts to capture the objective core of the design process up until then appeared more focused on creating theoretical systems of logic, then to look at what the studies that were undertaken actually indicated [60].

The third generation of design theory rejected the assumptions of design as a process of rational problem-solving. Notably exemplified and explained in *How Designers Think - The Design Process Demystified* by Bryan Lawson [61]. Now the focus is shifted towards a thinking about the design process that is both rational and irrational, driven by intuition in a process of analysis and synthesis. Design is understood as a particular way of thinking including knowledge that is indirect, implied and contextual. Analysis in this context is the exploration of patterns and relations, the sorting and classification of information. Synthesis is the move towards a solution of a problem, the creation of a solution. The simultaneous process of analysis through synthesis is a key concept in Lawson's approach to the challenge of complex design situations.

2.5.3 Designer and computer

Conceptual design has been understood to play the most significant role in the design process of new products [49], and the capabilities of the computer to represent form and simulate material in an "immaterial" way, make it an attractive tool for such a pursuit. However, the conceptual design work varies to some extent between different fields such as architecture, product design and mechanical design. Architects often work with bespoke solutions for unique situations whereas product developers more often design for mass production. Imre Horváth outlines some common denominators in [49] and points out how the process is characterised by "abstraction, incompleteness and uncertainty". Horváth describes the process as an iterative search where the designer gathers, transform, generate, manipulate, and communicate information and knowledge in relation to the external constraints of the task. He describes the process as characterised by an inherent abstraction, which is a challenging prerequisite for a binary machine like the computer.

Bryan Lawson describes how computers can support the design process in [62] by defining three conceptual modes, the oracle, the draughtsman and the agent. The oracle can be understood as the super rationaliser of automatic design, which Lawson describes as merely wishful thinking, as he refers to early design automation ambitions from the '60s and '70s. The computer as draughtsman refers to the use of computer-based geometric drawing/modelling, including the mesmerising and almost seductive potential these tools bring to the table, according to Lawson, often without actual gains in terms of architectural quality. It is furthermore described as a mode in which design is approached as a problem-solving process based on procedural knowledge. The computer as an agent is the final and somewhat speculative mode where Lawson describes the computer as an assistant to the thinking designer. A useful type of agent, according to Lawson, is one that supports the designer having "a conversation with the drawing", to cite Donald Schön from his book *The reflective practitioner* [100]. This type of interaction does not rely on a procedural structure but rather on a reflective practice that builds on episodic knowledge.

2.5.4 Sustainability

As alluded to in the introduction, the notion of sustainability has seemingly saturated the contemporary discourse in architectural both in academia and in practice. By now the discourse is often framed from the perspective of the Anthropocene, which is a term that was outed for the first time at a geo/biosphere conference in the year 2000 by scientist Paul Crutzen. It refers to the time in history when humans have started making changes not only to our environment but to the earth system as a whole [64]. In this context, the environment is referred to as local systems of plants, vegetation, biogeochemical cycles etc. The earth system, on the other hand, refers to the interrelated cycles of matter (such as water, rock, atmosphere etc.) that make up the system of cycles and recycling that make the earth a habitable planet. The rapidly changing rhythm in some of these fundamental cycles is pointing towards seemingly irreversible changes in our climate, resulting in changing conditions for life in large.

The immense challenge of the Anthropocene raises fundamental questions for how to relate to the world and puts the fossil-fueled industrialisation from the 18^{th} and 19^{th} century and the justifying reasoning behind that development in question. Using the act of building as a metaphor, the historian Dipesh Chakrabarty puts it; *The mansion of modern freedom stands on an ever-expanding base of fossil fuel use* [21]. According to the *UN Environment Global Status Report 2017* [30] the building sector stands for between 36-39 % of the world's CO_2 emissions and that fraction is more or less the same if the metric is changed to energy usage instead. A large part of that is operational usage, but a not-insignificant portion is also due to embodied carbon. A detailed review of the potential gains with structural efficiency as a prioritised driver in architecture to reduce embodied carbon in our buildings is presented in the PhD thesis by Catherine De Wolf called *Low Carbon Pathways for Structural Design* [119]. The thesis contributes to a method for evaluating embodied carbon coefficients (ECC). The results for a particular project will be tied to the resources of a particular region making comparative studies difficult. But the typical buildings range between 200 - 550 kg_{CO2e}/m^2, and the results from one of the extreme low carbon projects that is mentioned shows a reduction to 30 kg_{CO2e}/m^2 for the embodied carbon [119].

2.5.5 Structural efficient forms

Geometry and structures are intrinsically intertwined, making it difficult to talk about the one without the other. Just like material is both structure and matter and distinct categories arise first when we make definitions to simplify. Every structure is geometrical but not every structure is designed to work actively with the geometrical principles to serve a structural purpose. In the following sections, examples are given of structural principles that make use of geometrical concepts with the ambition to reduce material usage and thus embodied energy. Examples of such projects are found in the work of Frei Otto, Chris Williams, Philippe Block etc [114] [118] [14]. Efficiency in terms of material reduction can be understood as partly a value-based position in line with the *Less is more* paradigm, often today linked to the aforementioned argument of sustainability.

Minimal surface

From a mathematical point of view, the minimal surface can be understood as a surface that locally minimises the area. It is also a surface where the mean curvature is zero everywhere [93]. The principle of minimal surfaces has been applied in the field of architecture for various structural applications and it was an area of great innovation in middle 20th century with Frei Otto and his research group at the institute of lightweight structures in Stuttgart. [114].

The minimal surface structure can be found in nature in various phenomena such as bubbles and spider webs (perhaps more analogous with cable net). It is also subject to studies in other fields such as molecular engineering, material science, general relativity, and the shapes have inspired artists like Robert Engman, Robert Longhurst and Charles O. Perry. Tension structures are shaped as minimal surfaces and the shape is achieved in a procedure where a pretension force is applied along the boundary, and the membrane is stretched into shape. The reason for this shape is to avoid wrinkling of the fabric which requires that the fabric is under stress at each point, and the minimal surface an equilibrium shape with equal stress in each point of the surface [5].

Figure 2.31: *Minimal surface shaped using a soup bubble and thread from a workshop at Chalmers. © Photo by Emil Adiels.*

Compression structure

The compression structure is another case of form active structures (which is not necessarily light-weight) where the geometry is key for efficient performance. It has a long history of exploration starting with the early stone vaults, progressing with the cathedral domes in the middle ages and continuously driving innovation in the field of large-span structures today. The list is long of important contributors to the development of the structural theories within this field, some of which are Felix Candela, Heinz Isler, Vladimir Shukhov, Lugi Nervi, Frei Otto, Rafael Gustavino, Eduardo Troja, and more recently Chris Williams and Philipe Block [110].

As opposed to the minimal surface structure for which the geometry is derived from the initial configuration and the applied pretension in the membrane, the shape of a compression structure is driven predominately by an external load case and the support conditions. Making the precise geometry more difficult to define since there is no elegant mathematical formulation as for some minimal surfaces. The shape is better understood as an equilibrium state where geometry is found through a simulation process driven by boundary conditions and a specific load case like the self-weight.

However, for lighter materials such as steel and timber, the self-weight is often not the governing load case, as it would be for a stone structure, hence there is potential for other form drivers. One example of that is the shaping of the great court British museum roof, shown in figure 2.33, which is partially driven by a need to push the horizontal thrust towards the corners of the roof where the thrust could be carried using tension cables [118]. It is therefore the combination of a load case and the constraints in terms of boundary conditions that are used to drive the shape. The design work with the roof for the Mexico City airport as described in [111] is another example of a challenging form-finding situation due to the discrepancy between the governing load case and the form driving load case. The governing load case is a type of horizontal force field induced in the structure through the sudden release of energy in the lithosphere during a seismic event. The form-finding load case on the other hand is applied as a scaled version of the self-weight for which the scale factors are used for reasons of spatial character. As elaborated in the paper, the undulating form that was achieved with the form-finding simulation created a rather bending stiff surface, and the shape of the columns as they blend into the roof surface was carefully controlled in response to the seismic load [111].

Structural form optimisation

With the emphasis on the relationship between form and structure, there are two main categories of optimisation techniques that are relevant to point out. These can be understood in relation to the notion of mathematical topology, which refers to the properties of a geometric object that are preserved under continuous deformation such as stretching and twisting. Not to be mistaken for the topological connectivity which deals with the relations between discrete components of a polygon mesh, a distinction that is made more clear with the design exploration in [4].

There are structural optimisation techniques that leave the mathematical topology unchanged, such as form-finding and size optimisation. These methods may involve stretching, translating and twisting and the object may change in terms of form and/or volume. There are other techniques that change both the mathematical topology and the connectivity such as topology optimisation. Such a process starts from an excess of material which is called the design domain. Boundary conditions are applied and the optimisation algorithm removes material from the design domain where it is underutilised [12]. Each of these methods for optimisation certainly has their area of application but just like Lawson's conceptual mode of the draughtsman as described in 2.4, there is a mesmerising potential in such techniques which seems to result in an overrated value in relation to a holistic perspective on design.

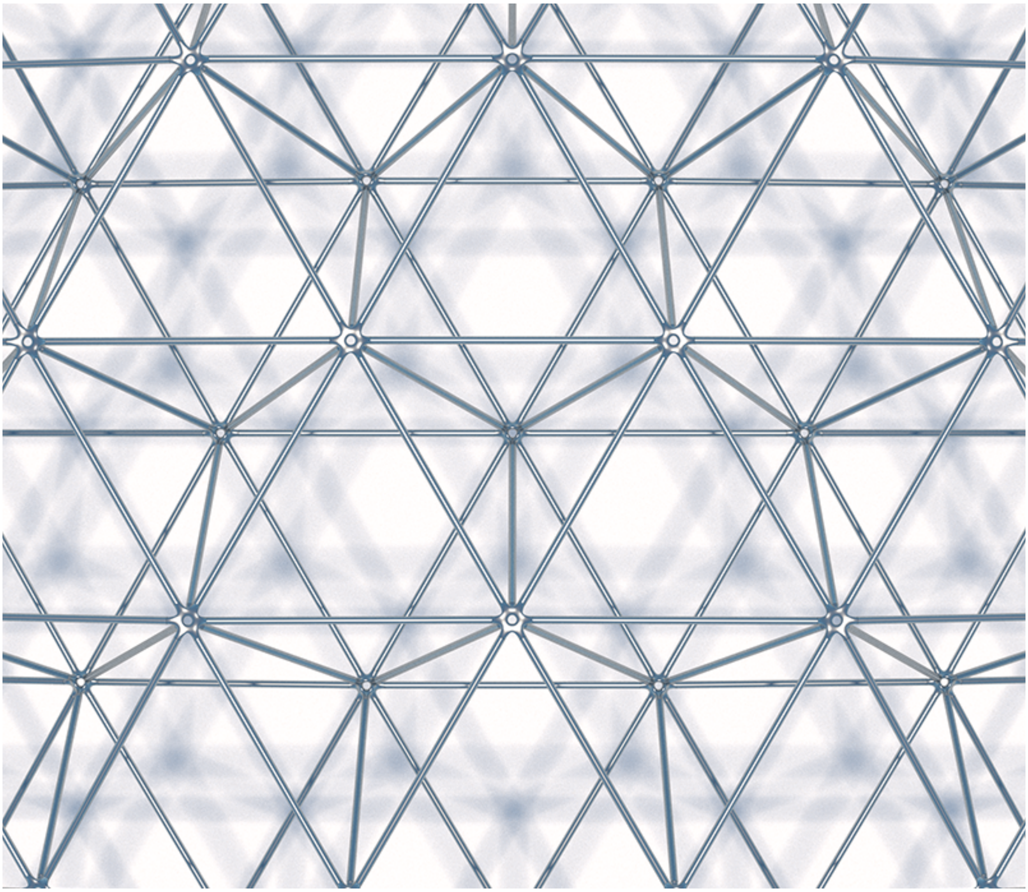


Figure 2.32: *Showing a digital prototype of a space frame where the nodes (same as in figure 2.26) have been shaped using the form finding technique that is presented in [4]. This approach allows the designer to control the topology of the object and the same nodal expression can be achieved throughout the whole structure. Image create by the author.*

Figure 2.33: *Roof structure defined by Chris Williams, where the shape is defined such that the horisontal thrust is concentrated to the corners. Photo by Andrew Dunn, http://www.andrewdunnphoto.com/. Licensed under CC-BY-SA 2.0.*

3 Limitations

This chapter aims to narrow down the research and reformulate a set of more specific research questions. The initial questions listed in 1.3 were used as a springboard into the contextualisation in chapter 2, which is repeated here in the format of a condensed summary. The summary is followed by a section of reasoning where choices are made to focus the scope of the theses, leading up to a formulation of more specific research questions that are numbered from 1-5. These questions are then addressed at the end of the methods chapter in 4.3.

3.1 Recap

The contextualisation chapter started with a broad historical narrative in order to set the canvas for the rest of the thesis. The historical part is divided into the pre-industrial, the industrial and the digital era, for which tendencies related to mathematics, production technology, tools and design process are described in short. The chapter that follows is a basic introduction to differential geometry which ends with a collection of geometrical *primitives* that are summarised in table 2.1 and used throughout the rest of the thesis as the base language of shapes and forms, relating analysis, production and design.

The next chapter contains a description of production technology which is organised in terms of material, for the three classical building materials with clearly different characteristics, namely steel, concrete and wood. These materials are discussed in relation to historical examples of structural nodal connection with an emphasis on material mechanical properties and production technology. The relationship between production and material on one hand, and shaping possibilities on the other, are approached through the language of the differential geometry primitives. The next chapter aims to address the use of tools and representations in architecture with a particular focus on the digital context of modelling and simulation. The notion of a tool and its importance for the creative endeavour takes its departure from the example with David Hockney's discoveries in the arts and continues with a description of vector and raster-based graphics. Three different families of numerical methods are then introduced in section 2.4.2, that includes the finite element method, isogeometric analysis, meshless methods such as smoothed particle hydrodynamics and peridynamics. Each of these methods is related to a particular digital form representation and modelling technique which was introduced in section 2.4.1, and included polygon meshes, the NURBS surface, and the point cloud. The chapter is concluded by listing the current attempts to unify modelling and simulation, through automatic mesh generation (FEM), shared representation (IGA), and unity in the analytical formulation.

The final chapter of the contextualisation is a continuation of the design process discussion in the aforementioned chapter but with a shift towards roles and values. The focus here is predominantly limited to the individual designer and the relationship between designer, tool and object. A short overview of the changing role of the architect/engineer is introduced, followed by a brief description of the development in design theory from the 60's until today. The three conceptual modes by Bryan Lawson are introduced to enable a closer look at the relationship between the designer and the computer, and the expectations of what the computer as a tool can contribute within a design situation. The design process is described as iterative and explorative in nature, driven partly by intuition and partly by the simultaneous analysis and synthesis, evaluated through reflection. The section is concluded with a note on the sustainability challenge, which is approached in this context from the partial perspective of embodied carbon. Thus the final part of the section describes various strategies that are commonly applied in the optimisation of material-efficient structures.

3.2 Focus

The overall aim of the project is to contribute to the development of methods and/or tools for the integration of simulation and manufacturing considerations in a digital design process. Such tool development constitutes many parts, and far from all will be addressed within the scope of this thesis. One of the key aspects of a digital design tool, which is not touched upon here specifically, is the user interface. The focus is instead on the underlying numerical methods, although with future user interactivity in mind.

Whereas the language of differential geometry has been useful as a means for classification of surfaces related to production, it seems less useful in a design situation of early-stage character. Polygon modelling on the other hand is lightweight and easy to use for calculations. Even if the situation requires a higher degree of precision in favour of surface modelling, a polygon mesh can often be extracted from a surface with relative ease. Thus, the polygon mesh seems like a wise choice in terms of form representation.

From section 2.3 which was about the physical conditions and material properties for structural nodes, conclusions are made to continue with a more narrow focus on steel. The orthotropic nature of wood makes it challenging for nodal connections in large scale due to the complex loading conditions that often occur in many of these situations. It is also a material where mathematical models are difficult to set up due to the complex buildup of fibres and vessels, sensitivity in terms of moister content and natural variations in the mechanical properties due to growth conditions and age. Although great progress has been made with 3D printing for concrete, the lack of tensile capacity means that the reinforcement strategy becomes such a dominant driver in the design of a complex node, so one might as well omit the concrete and work directly with the steel design instead.

In terms of production techniques for steel connections, there are three interesting options one could explore further: casting, milling and 3D printing. Just like with the Mero system that was illustrated in figure 2.12 a combination of techniques seems reasonable. One could, for example, imagine a combination of casting and milling where the cast object is shaped to represents the common design domain for a collection of individual nodes (which is not necessarily spherical like the Mero system), and where an individual treatment is performed using milling. However, to reach a truly material-efficient component, casting and milling still restrains the geometry due to mould removal and forbids internal voids. 3D printing of steel is therefore the methods of choice for the continuation of the project.

As described in the section on the relationship between geometry and structures 2.4.3, a number of attempts have been made to unify numerical methods for analysis with geometric form representation. That is done through automatic meshing procedures combined with FEM, and through combined basis functions for the unknown filed variable and the NURBS form definition with IGA. The problem with automatic meshing is not in the creation the mesh, which is already solved for computer visualisation (thus integrated with all surface-based modelling software), rather the problem has to do with mesh quality. A useful analysis mesh needs to provide good aspect ratio for triangles/quadrilaterals to enable the computation of reasonable spatial derivatives. IGA on the other hand has seemed like a promising alternative, effectively removing the need for a mesh. However, in the case of multi-patch models (which is most cases) and trimmed surfaces (NURBS are otherwise limited to rectangular patches), the techniques become complicated in terms of modelling and difficult to implement. The nature of NURBS modelling is already complicated enough without having to think of the implications of continuity across patches for reasonable numerical approximations.

Meshless methods such as PD on the other hand are independent on mesh geometry, rather simple in the theoretical formulation and can be integrated with both polygon mesh and surface form representations. Meshless methods are furthermore able to model both simple analyses but also highly complex material phenomena such as plasticity and fracture. The set up of the model can be tuned according to the accuracy needed for the analysis, and the accuracy can be related to the requirements of the current phase in the design process. Such adjustments of accuracy can be done through the formulation of the bond force and the size of the horizon as discussed in 2.4.2. PD is sometimes described as a generalisation of FEM and in the case where the horizon is chosen such that particles only connect to their closest neighbours, the PD and FEM models converge. Since the theory was developed for fracture simulation, it allows for the removal of particles and bonds during the spontaneous formation of cracks. The same ability could potentially be used to create interactive simulations, where the design domain changes during run time through interaction by the designer.

One of the drawbacks with meshless methods is that they can be computationally expensive, especially for larger horizons. But they are at the same time well suited for parallel computing. Thus the development of GPU computing as described in 2.4.1 may alleviate some of those shortcomings. The calibration of the material model is another challenging aspect, especially for irregular particle distribution and variable particle density.

The typical PD formulations as in [101], [102], [103] are most often found working with an implementation of regular lattice structure type of particle distribution, with a constant horizon [22]. Seemingly because material parameters are calibrated for particular settings in terms of particle horizon and grid density, and these are shared among researchers to simplify the comparison of results. Many applications are also concentrated on simple domains and common fracture problems. For a complex shape such as a nodal connection however, the ability to vary particle distribution and also particle size is important in order to adapt to the form and also to model stress concentrations well.

Based on the contextualisation in 2.4 and 2.5 regarding tools and design process a list of concluding criteria are presented below for what a tool should be capable of doing to fit the work of conceptual design. The criteria are mainly derived from [61], [60], [62], [49],

[40] and [100]. The ambition is to formulate a framework for how to evaluate the extent to which a particular tool can live up to a type of *conversational* character, or as Lawson puts it, become an extension of the thinking process [62]. The list of criteria is as follows.

- Real-time feedback is a pre-requisite for what Donald Schön calls having a conversation with a drawing [100], and is according to [62] equally relevant when drawing in a digital environment. It is therefore also considered an important aspect of a drawing tool that aims for *conversational* character even when it is built on a simulation.
- An ability to represent abstract ideas that vary in precision. Also to enable a transition from abstract to specific as the design matures. This is typically challenging in the computer environment which is normally aimed for high resolution and representation of finished ideas [40].
- An ability to handle ambiguity and incompleteness which is intrinsic to the development of design [40], which is another challenge for the binary computer.

Some parts of the focus that has been presented here is condensed into a set of new set of research questions in the following section.

3.3 Reformulation of research questions

Based on the recap in 3.1 and the focus as presented in 3.2 a new set of research questions with a sharper focus are formulated here and sorted under the more general questions that were posed in 1.3 at the end of the introduction.

i. Which numerical methods and related geometrical representation are suitable for the integration of modelling and simulations in the design process?

    1. Meshless methods seem to provide interesting abilities for simulation of a variety of structural phenomena. How can meshless methods be integrated with drawing and modelling?

ii. What is a suitable production technique for material-efficient structural nodal connections?

    2. Additive manufacturing seems to be a promising technique for the production of steel nodal connections. How can meshless methods support the design and production of such objects?

iii. What are the desirable properties of a quantitative numerical method (such as simulation) to enable integration in a design process which has both quantitative and qualitative character?

    3. Much of peridynamics implementation seems to rely on regular particle distribution and constant particle sizes. What needs to be changed in order to enable irregular particle distributions and variable particle sizes?
    4. The equations behind Peridynamics remain valid even in the occurrence of discontinuities enabling the simulation of progressive fracture. How can such capabilities be used in the context of design for material-efficient structural components that are produced through metal 3D printing?
    5. The relaxed relationship between particle distribution and calculation of spatial derivatives in meshless methods (as compared to grid-based methods) seems to suggest that a method such as peridynamics would lend it selves well for automatic particle distribution, with the possibility for user-controlled particle density, allowing for adjustments of accuracy in the simulation as an object is taking shape during a design process. To what extent can that be a realistic and useful feature in peridynamics driven design exploration?

iv. What characteristics of the design process are important to consider in the design of structural nodal connections, in the stages ranging from concept to final production?

4 Methods

With the aim to contribute to the development of tools and methods for integrated design, simulation and production, the question of research methods becomes a question of how to evaluate the development of a method. What are the criteria by which that evaluation is carried out? Surely the evaluation of a method must be done in relation to what the method is aimed to be used for. A method that is aimed at establishing correlations between loading and buckling for pin jointed columns, is arguably best evaluated thought mathematical correlation with other established methods that do the same thing and take the same point of departure or through comparison with physical experiments.

Due to the complexity of the design process that was touched upon in 2.5, and the multi-perspectival nature of architecture the evaluation of a method that aims to contribute in that process is arguably less straightforward. The design processes can perhaps be better understood to be based on a methodology, which will incorporate a variety of methods depending on the situation at hand. Each such method then needs to be integrated in a way such that it allows for the other perspectives that are explored using other methods.

The methods that are used to develop the tools and methods in this project are mainly of two types, experiments and simulations, each of which will be elaborated in more detail in the following sections. The research is also supported by a literature review in fields such as numerical methods for structural engineering, computer modelling techniques, mathematics, architectural history and design theory. With a particular focus on the theory of peridynamics and design theory related to conceptual design.

After the research methods are introduced in 4.1 and 4.2 the following section aims to connect the research questions that were posed in 3.3 to the methods that were introduced including a note on the results that are obtained.

4.1 Simulation

Numerical methods for structural analysis are typically used to simulate or emulate a particular behaviour, such that predictions can be made, patterns can be understood and the underlying mechanisms revealed. Two common objectives in the application of simulations in structural engineering were discussed in the beginning of 2.4.2. On the one hand, the simulation as applied by the scientist in the study of natural phenomena, and on the other hand the simulation as applied by the structural engineer for practical application with the design for safety as a first priority. Both of these approaches however rely on the fundamental assumption that the object to be simulated already can be conceived of as existing with definite borders and characteristics that are supposed to be emulated. In such a situation questions about the accuracy of the copy, and what it doesn't capture about the real thing, become important. These types of simulations are attractive methods for many practical reasons. It is for example much cheaper to perform multiple car crash simulations in a computer than to perform the physical experiment with real cars. The same goes for many other applications, should that be evacuation fire drills, induced loads from earthquakes on tall buildings or simply in the design of car components. The outcome of a simulation can be of the type that predicts a certain event or quantifies a certain measurement or even the projection of a behavioural pattern [38].

However, there is a third approach in which the simulation of structural phenomena can be tailored more specifically to be used as design drivers. But what does it mean to simulate something that doesn't already exist other than, at best in the head of the designer, whom at worst doesn't have a clue yet of what to make? The three approaches to the simulation that have been mentioned here can be summarised as follows.

a). Simulation to model structural/material phenomena, conformity with an observation from nature as the main objective.

b). Techniques and methods for sizing of structures with practical application and safety as the main objective.

c). Simulation and analysis to drive design decision. Holistic integration of design solutions with material properties and structural behaviour as the main objective.

The simulation as design driver comes to its right first when it is integrated with a methodology which has an iterative character in which objects come into existence and cease to exist with the "pen-strokes" at the drawing board. Such a methodology is furthermore best complemented with the process of prototyping. As the design takes shape the emphasis of design driven simulations (type c) should gradually shift towards simulations that put emphasis on practical application (type b).

4.2 Numerical and physical experiments

Experimental research is sometimes referred to as the standard to which all other research strategies should be judged [38]. The experiment becomes a bridge between theory or hypothesis and reality. However, experiments can be conducted in so many different ways and some would argue that the shortcomings in intersubjective matters would disqualify such an exalting claim [38].

The experiments that are applied in this project are of two different categories, numerical experiments and physical experiments the latter of which will be referred to as prototypes. A further distinction is made between two different types of numerical experiments, these are defined as *narrow*, and *speculative* numerical experiments.

The *narrow* and precise numerical experiment is typically applied in theory development, aiming to reduce the complexity of a problem such that the correlation between parameters can be established. This is the type of experiments that are used in science and engineering. It is furthermore based on a form of a controlled environment for which the variables are regulated in the setup of the simulation to reduce complexity. This approach to experiments aims to prove or disprove theories and hypothesis that make claims of the type - *how things are.*

The *speculative* numerical experiments are typically applied in the case of design exploration. These types of experiments are aimed to spark the imagination and to contribute with examples that broaden the horizon of possibilities of the topic at hand.

This approach can be understood as related to the concept of research by design which aims to address questions of the type - *how things could be.*

Prototyping is a type of physical experiment aimed to bridge the theoretical and digital with the physical by integrating the making process. It is essential to the making of real structures and used to address questions related to - *how things can be made.*

4.3 Research questions and achieved results

As a final note for this chapter, the specific research questions that were introduced in 3.3 are paired with the different research methods that are presented here and the results obtained thus far are discussed. Some questions that are yet to be tackled specifically in the thesis, are still posed with future work in mind.

Question 1: *Meshless methods seem to provide interesting abilities for simulation of a variety of structural phenomena. How can meshless methods be integrated with drawing and modelling?*

This question has been approached through numerical experiments, of which results for a 2D implementation of bond-based peridynamics is shown in figures 2.28, 2.30, and results of 3D implementations can be seen in figures 2.22, 2.26. These simulations are implemented in the form of a plug-in for the 3D modelling software Rhinoceros, and the focus has been on exploring the integration of particle clouds with NURBS curves for 2D experiments and polygon mesh for 3D experiments. That includes data storage for zoning of the particle domain as shown in figure 2.30, visualisation of particles and bonds as shown in figure 2.28, and the implementation of a central difference scheme for the numerical simulation.

Two different strategies for the creation of zones are tested for the more challenging 3D case. In the first case which is shown in 2.26, zones are created to fill the entire bounding box of the initial polygon mesh shown in figure 2.26b which is used to define the object. Zones that intersect with the polygon mesh are filled with particles but only such that the particles also fall inside of the polygon mesh which is used to define the object. In this case, all the zones are created in the same size which is defined to be larger than the horizon for the particles (which are all of the same size).

For the other strategy which is illustrated in figure 2.22, the particles are generated in a randomised fashion inside of the polygon mesh that defines the object before the zones are created. An octree algorithm is then used to create zones of variable sizes such that a target number of particles are allowed in each zone. The approach works on the basis of zone subdivision and the results of that process in a 3D case can be seen in figure 2.22.

Both of these strategies are found to work well but have different pros and cons, so the method of choice will depend on other constraints. The direct implementation into the 3D modelling environment makes the simulation easily accessible for design exploration, and for the 2D case, real-time interaction is possible even for a basic implementation which does not incorporate multi-threading. A choice of strategy for zoning is necessary whether the simulations are purposed for accurate analysis type b), or when the objective is to provide guidance in terms of early-stage design as described in type c).

Question 2: *Additive manufacturing seems to be a promising technique for the production of steel nodal connections. How can meshless methods support the design and production of such objects?*

This question has been approached through the choice of setup for the numerical experiments which are created with round and organic shapes. Additive manufacturing enables precisely such freedom for production so the designer is liberated to work actively with the form of an object for a structural purpose, such as described in 2.5.5. Structurally efficient forms usually follow a type of natural organic shapes for which a good polygon mesh is difficult to define. Particle distribution with meshless methods on the other hand can be integrated with relative ease for complex shapes, which has been explored in 2D and 3D with the numerical experiments show in 2.22, 2.28, 2.30 and 2.26. Since these methods take the spatial derivative on the influence function as opposed to on the geometry, they are less sensitive to the particle distribution. Which also enables irregularity in the particle distribution as a strategy to reduce the otherwise grid-like pattern in the results, which seems unnatural, especially in the case of fracture. The noise is introduced either through a random placement which is the case for the 2.22, or by "shaking" the particles using a Monte Carlo inspired algorithm. These examples can be understood best under the category of speculative numerical experiments.

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Question 3:*Much of peridynamics implementation seems to rely on regular particle distribution and constant particle sizes. What needs to be changed in order to enable irregular particle distributions and variable particle sizes?*

This question has been approached fist and foremost through mathematical derivations as shown in [83]. The irregular distribution and variable particle size are achieved through the introduction of a quantity called force flux density. The results are verified partly using mathematical correlation with already established theories and partly with numerical experiments. The numerical experiments are performed for an elliptical disk with a prescribed crack spanning between the two focus points. The disk is setup using an elliptical coordinate system and the zones are generated by steeping with even intervals in the two parameter directions. The resulting zones are smaller towards the focus points of the ellipse and greater towards the perimeter. By inserting a constant number of particles per zone, the sought density variation is achieved. The numerical experiment for the elastic case is compared to the analytical solution of the same problem with acceptable small deviations. The numerical implementation which includes plasticity and fracture furthermore demonstrates that the expected proportional relationship between fracture stress and the square root of the crack length as predicted by Griffith's theory. The experiments, in this case, could be classified as the *narrow* type and primarily used to show correlation with established theories.

Question 4: *The equations behind Peridynamics remain valid even in the occurrence of discontinuities enabling the simulation of progressive fracture. How can such capabilities be used in the context of design for material-efficient structural components that are produced through metal 3D printing?*

The question is yet to be tackled properly but is still included with future work in mind. There seems to be two way in which the simulation of fracture can be approached in this context. On one hand, the ability to simulate stress concentration and fracture is something important for type b) simulations in later stages of design, especially for structural nodal connections with ambitions of material efficiency. Such an effort is arguably best approached with a type of *narrow* experiment which is complemented with physical prototypes. The theory development in [83] could be used in that pursuit.

On the other hand, the lack of continuity assumptions for these methods which becomes the enabler to the simulation of fracture might also be used in a design exploration manner, for type c) simulation. One type of large change in the design domain could be imposed, not by external loads, but though the designer interacting with the simulation at run time. Something which is inherently difficult with methods that build on classical continuum mechanics continuity assumptions. Such exploration would have to rely on near to real-time feedback and therefore first approached for the 2D case.

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Question 5: *The relaxed relationship between particle distribution and calculation of spatial derivatives in meshless methods (as compared to grid-based methods) seems to suggest that a method such as peridynamics would lend it selves well for automatic particle distribution, with the possibility for user-controlled particle density, allowing for adjustments of accuracy in the simulation as an object is taking shape during a design process. To what extent can that be a realistic and useful feature in peridynamics driven design exploration?*

The influence of particle distribution on the results from the simulations is best tested through a type of narrow numerical experiment where comparisons are made with idealised cases and/or other simulation techniques where the accuracy is known. Convergence tests also need to be performed for various settings of the material model. This is another question that remains in the periphery of the work that has been carried out so far.

5 Summary of Papers

5.1 Paper A

The first paper discusses the modelling of the envelope and space frame for the new Mexico City Airport, including the development of methods, tools and processes to deal with the complexity and scale of the project. The shape of the envelope was created through form finding, leading to an all-encompassing lightweight shell with internal spans reaching 170 m. Lending from a variety of computational techniques developed in mathematics and computer science, the paper further discusses how these techniques are applied in a design situation with requirements on visually continuity and smoothness for the space-frame elements and the exterior envelope, while simultaneously complying with very strict spatial and programmatic constraints and structural optimisation criteria. The paper thus exemplifies the integration of such computational techniques in a design context with many conflicting drivers and constraints. The paper furthermore includes a first time (to the knowledge of the authors) application of Optimal Delaney triangulation smoothing in a design context.

5.2 Paper B

The second paper applies the form finding strategy usually used in the optimisation of roof structures, such as presented in [111], for the optimisation of the structural nodal connections. A spline-based bending element to is used to control the shape to oppose the "will" of the form finding simulation when needed. The technique works by modelling the node as a hollow shell with a mesh, applying a set of tensile forces derived from the structural action from elements adjacent to the node (where compression is converted to tension) and running a form finding simulation. After the simulation, the shell is then thickened and analysed for the real load case (which consider both tension and compression) using FE-analysis. The paper takes a critical stance towards the application of methods such as topology optimisation in the design of structural nodal connections which tends to dictate too much of the design. For these speculative numerical experiments, simulations in terms of type c) are used in the shaping of the object, and of type b) in the final assessment.

5.3 Paper C

The third paper presents a modification of the Peridynamics 2.4.2 that allows for irregular particle distribution and variable particle sizes in the analysis of yielding and brittle fracture. This is made possible through the introduction of a concept called the *force flux density*. The modified PD theory has a strong resemblance with SPH which was introduced in 2.4.2 and can be used to simulate both solids and fluids. These modifications simplify the peridynamics theory and improve implementation for analysis of complex 3D shapes, although the paper only exemplifies the 2D case. The method that is applied in this paper is a simulation of type a), and numerical experiments of the narrow sort, with the aim to establish correlation. The results from the numerical experiments demonstrate that the fracture stress is inversely proportional to the square root of the crack length as predicted by Griffith's theory of fracture.

6 Conclusions and discussion

This thesis aims to discuss, problematise and exemplify the application of computational tools in the design process. On the one hand specifically related to structural design aspects of architecture but the hope is also to contribute with a general comment on the notion of computer tools integrated in the design process. The development of numerical methods, specifically with the reformulation/extension of peridynamcis using *force flux density* as shown in [83], serves to contribute with a new type of approach that could be useful in the design of material-efficient structural components.

However, in order for material-efficient structures to become a viable option in terms of sustainable design, it is arguably not sufficient with automated optimisation for material reduction. There needs to be space made for the subjective aspects of architecture and design, so that qualities that are difficult to formulate in an algorithmic fashion can be integrated. The choice to work with analysis informed design should arguably not have to be a choice of style.

Meshless methods seem to enable a new type of integration of material simulations that could suit the design process well. Potentially alleviating the designer from the task of converting NURBS to mesh and diminish duties model cleaning, so that more time can be spent on design development. The drawback with the non-local nature of meshless nature is that computational cost increases compared to traditional FEM. These are however methods that are well suited to parallelisation, for example using the graphics card. To further reduce computational cost it could be interesting to introduce rotations and bending capability with the force flux method. That would enable a significant reduction of particles for simulations of thin-walled structural components.

7 Future work

The natural next step in the development of the meshless force flux method would be to incorporate an extension to 3D, for which the theory is already provided in [83]. Further numerical experiments on structural nodal connections could then be performed to evaluate the usefulness of the method from a design process point of view. Calibration of material models also needs to be carried out for comparison with other analysis techniques. Another interesting possibility is to continue developing a 2D version of the theory by shifting focus to user interaction. One could imagine creating tools in app-format for a type of sketchpad interface where the user could interact with the model with a digital pen. Effectively blurring the borders between sketch and simulation. Stiffness could be varied through the density of colour, and the user could interact with the simulation in run time by adding, removing ”material” or guiding the forces through imposed changes in stiffness. Perhaps a type of structural sketching in a conversational manner.

A third potential focus for further work could be to explore the possibilities that are gained with the force flux method when materials undergo phase shifting between a solid and a fluid state. One could for example imagine simulating the process of melting and hardening of metal powder as a laser bed fusion 3D printer builds an object. That would shift the focus towards modeling of heat equations and potentially the resulting inner stress resulting from temperature changes in the material.

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