A Grey Mathematical Programming model to Time-Cost Trade-offs in Project Management under Uncertainty

Hannan Amoozad Mahdiraji
Department of Management
Kashan Branch, Islamic Azad University
Kashan, Iran
hannanvictor@gmail.com

Seyyed Hossein Razavi Hajiagha
Department of Systematic Research
Institute For Trade Studies And Research
Tehran, Iran
s.hossein.r@gmail.com

Rouzbeh Pourjam
Department of Management
Kashan Branch, Islamic Azad University
Kashan, Iran

***Abstract*— Time and cost are two salient elements indicative of success in project management. This importance obliges the project managers to seek for the best feasible amalgamation of time and cost regarding project's activities. This condition engenders a trade-off problem in terms of creating a required balance between time and cost considerations to execute all activities in a project efficiently. Such problem relates to time and cost trade-offs issue. Time and cost trade-offs model is based on estimated values of time and cost required for a given activity to be complete in a normal or crashed form. Current models of time and cost trade-offs have made use of crisp values for these estimations. In this paper, we extend a model for time and cost trade-offs based on grey numbers to deal with the uncertain nature of time and cost estimation. The proposed method has also been applied in an example and interpretations pertaining to offered solutions have been examined.**

Keywords-project management, time and cost tradeoffs, uncertainty, grey numbers, grey linear programming

I. INTRODUCTION

Project management is one of the most important fields in business and industry. Every task in an organization can be taken into account as a project, i.e. a temporary endeavor undertaken to produce a unique product, service or result [1]. In this context, the purpose of project management is to foresee or predict as many of the dangers and problems as possible and to plan, organize and control activities so that projects are completed successfully in spite of all the risks involved[2]. According to PMBOK Guide, the nine areas of knowledge in project management are project integration, time, cost, quality, human resource, communication, and risk and procurement management respectively [3]. There are three main points which are of highest significance to a successful project: (1) a project must meet the customer requirements, (2) it has to be within the budget, and (3) it has to be carried out based on right timings [4].

Time and cost are the two most important resources that any project manager deals with. Both of these resources have constraints and the job of a project manager is to create a sensible balance between the two. The judicious balance between time and cost is called Time-Cost Trade-off [5]. The main concept of Time-Cost Trade-off is the choice of adding resources to selected activities to reduce project completion time. Added resources (such as more workers, overtime, and so on) generally tend to increase project costs, so any decision on reducing time of activities must take into consideration the additional cost involved. In effect, the project manager must make a decision that involves trading reduced activity time for additional project cost [6]. This is the focal point of Time-Cost Trade-off. Complexity and importance of such decision makes it an interesting field to researchers and a wide range of heuristic and mathematical optimization models are presented to such problem.

Many researchers have developed mathematical programming model for the cost and time tradeoff problems. Reference [7] presents the relationship between project's total cost and project's total duration for a given type of project and represents this relationship in mathematical form. Reference [8] also offers an application oriented procedure for solving the project management duration/ resource trade-off problem. A procedure is proposed for reducing a project from a normal to a crash duration state at a minimum amount of additional resource expenditure assuming a linear utilization function. The procedure was a network based on using a graphical cut search approach to locate the minimal resource level at each reduction in total project duration. Reference [9] describes a new exact procedure for the discrete time/cost trade-off problem in deterministic activity-on-the-arc networks of the CPM type, where the duration of each activity is a discrete, non-increasing function of the amount of a single resource (money) committed to it. The objective is to construct a complete and efficient time/cost profile for the set of feasible project durations. Reference [10] implements a linear programming formulation to solve the problem through a truly interactive computational environment. Reference [11] presents an electromagnetic meta-heuristic algorithm for the discrete time/cost trade-off problem. Reference [12] considered the problem of allocating resources to projects performed under given due dates and stochastic time–cost trade-off settings. In particular, they show how to implement a state-of-the-art methodology known as “robust optimization” to solve the aforementioned problem. Reference [13] investigates a new approach in solving time-cost trade-off problem because of uncertainties affecting activity’s cost. Fuzzy logic theory is employed to consider impact of uncertainties in total direct and indirect cost of a construction project. Non-dominated Sorting Genetic Algorithm (NSGA) is applied to make a trade-off between time and total cost. Reference [14] attempts to determine optimal solutions from which the project managers would select their desirable choice to run the project effectively. Their problem is multi-objective and the purpose is finding the Pareto optimal front of time, cost and quality of a project, therefore a meta-heuristic algorithm is developed based on a version of genetic algorithm specially adapted to solve multi-objective problems named fast PGA. Reference [15] determines the optimal levels of activity durations and activity costs which satisfy the project goal(s), leading to a balance between the project completion time and the project’s total cost. In this paper, TCTP (Time-cost trade off problem) will be studied while considering the influence of discount on the re-source price and using genetic algorithm (GA). Reference [16] presents a multi objective optimization model that provides new and unique capabilities including generating and evaluating optimal/near-optimal construction resource utilization and scheduling plans that simultaneously minimize the time and maximize the profit of construction projects. Reference [17] analyzes a project’s scheduling problem including time-cost trade-offs and proposes a new technique based on computer simulation and interactive approach. Reference [18] develops a model by considering time value of money (TVOM) which was ignored in previous researches. Reference [19] develops a model for discrete time-cost-quality trade-off problem that uses the planner-specified weights for handling a multi-objective optimization problem. They propose a new metaheuristic-based genetic algorithm, called NHGA, for optimizing a multi-objectives time-cost-quality trade-off problem and scrutinize it through the analysis of variance (ANOVA) method. Reference [20] developed a Meta heuristic multi-colony ant algorithm for optimization of three objectives time-cost-risk as trade-off problem.

The main contribution of our work is to consider the uncertainty concept in Time and Cost tradeoff. In real world projects it is so difficult to determine the values of resources needed to decrease the time of an activity and decrease in the range of this activity's time. So, in this paper we developed a grey multi objective program for Time-Cost Trade-off in projects in which the approximations used as parameters of model can't be expressed as crisp numbers. The rest of the paper is organized as follows: In Section 2, we briefly introduce the problem and models of optimization applied to Time and Cost Tradeoffs. Section 3 involves a review of the concepts and arithmetic operations of grey numbers. Section 4 presents our proposed grey mathematical model and its solution. An application of proposed model will be illustrated in a numerical example in section 5. Finally Section 6 incorporates conclusions and future work.

II. MULTI OBJECTIVE TIME – COST TRADEOFF PROBLEM

A. Problem Definition

Balancing the cost and duration of individual project tasks (or activities) with the consequences of schedule slippages and project completion delays has been a classic optimization problem since the introduction of project network models in the late 1950s. The tradeoffs (or compromises) involving the duration and cost of project activities take place in an environment constrained by operational considerations such as due dates, sequencing requirements, and budgetary limitations [21]. Reference [22] argues that as technology advances, the commitment of time and money becomes inflexible, and this is the driving force behind most of project management approaches. Therefore, in view of the extremely large and costly systems which are being developed today, the concept of project time–cost tradeoff models can have considerable economic importance. As long as the objective is to perform the activity in the most economical way, the combination of resources and the material lead-times assigned to the activity is assumed to minimize the cost of the activity. However, in many cases the activity duration should be shortened. The critical path method (CPM) of time–cost tradeoffs [23, 24] assumes that by changing the resources and lead-times combination, i.e. increasing the performance speed by moving from the most economical combination to a combination of higher cost, a shortened duration can be obtained. The CPM model enables us to compute the activity duration and the activity cost within the interval whose lower boundary is the most economical performance and the upper boundary is the shortest possible activity duration. The computed relation between the activity time and the activity cost enables us to minimize the project duration under budgetary constraints or to minimize the budget that is required to accomplish the project on the due date, assuming deterministic durations and deterministic costs [25]. The objective of cost/time tradeoff is to balance direct and indirect costs and thereby reduce overall project costs [26] Project scheduling with time-cost tradeoff decisions plays a significant role in project management. In particular, discrete time-cost tradeoff models with deadline or budget constraints are important tools for project managers to perform time planning and budgeting for their projects. As a result, efficient and effective solution procedures for such models are highly attractive to those practitioners [27]. Shorter time, lower cost, and higher quality of the project is the main aim in project management. These three factors influence each other [25]. Most models dealing with the three factors assume that the parameters about activities such as EF (Early Finish) and LF (Late Finish) are all known and constant, and try to find the optimal deterministic or approximate program [28].

B. Problem Formulation

Time and cost trade-off is one of the most important issues in project planning and project control. This problem can be solved by heuristic models such as [29] and also by linear and nonlinear programming models. In this paper we have described two types of mathematical linear models that are used in time and cost trade-off problems in project and network situations. Depending on the objective, two types of models can be used as pointed below:

1. Predefined time for finalizing the project (T_S) by considering the minimum direct/Indirect costs of each activity.
2. Identifying the minimum time needed for finalizing the project by considering the minimum direct/Indirect costs of each activity.

The variables and parameters used in the two models are listed in table 1.

As the objectives, variables and parameters mentioned above, two linear models have been described. Notice that we assume that all of the activities have linear cost-time relationship, if not, nonlinear cost and time tradeoff models should be used.

First Model) Predefined time for finalizing the project (T_S) by considering the minimum direct/Indirect costs of each activity.

Second Model) Identifying the minimum time for finalizing the project by considering the minimum direct/Indirect costs of each activity.

TABLE I. PARAMETERS AND VARIABLES

Row	Variable/Parameter	Definition	Type
1	T_S	Final Time Defined For Project	Parameter
2	D_{ij}	Normal Duration Of Activity ij	Parameter
3	d_{ij}	Crash(Minimum) Duration Of Activity	Parameter
4	CD_{ij}	Normal Direct Cost Of Activity ij	Parameter
5	Cd_{ij}	Crash(Minimum) Direct Cost Of	Parameter
6	$CS_{ij} = \frac{Cd_{ij} - CD_{ij}}{D_{ij} - d_{ij}}$	Cost Slope Of Activity ij	Parameter
7	B	Maximum Budget Defined For Time	Parameter
8	O	Indirect Cost For Each Time Unit	Parameter
9	X_{ij}	Actual Time Of Activity ij	Variable
10	T_n	Time Of Last Node In Project	Variable
11	T_1	Time Of First Node In Project	Variable

$$Max\ Z = Min\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} CS_{ij}.(D_{ij} - X_{ij})$$

$$\begin{aligned} & S.T: \\ & T_n - T_1 \le T_S \\ & T_i + X_{ij} \le T_j \\ & X_{ij} \le D_{ij} \\ & X_{ij} \ge d_{ij} \\ & T_i, T_j \ge 0 \\ & i, j \in \{1,2,...,n\} \end{aligned} \tag{1}$$

$$MinZ = O(T_n - T_1) + \sum_{i=1}^{n}\sum_{j=1}^{n} CS_{ij}.(D_{ij} - X_{ij}) + \sum_{i=1}^{n}\sum_{j=1}^{n} CD_{ij}$$

$$\begin{aligned} & S.T: \\ & T_i + X_{ij} \le T_j \\ & X_{ij} \le D_{ij} \\ & X_{ij} \ge d_{ij} \\ & T_i, T_j \ge 0 \\ & i, j \in \{1,2,...,n\} \end{aligned} \tag{2}$$

Now, we have two objectives and a set of constraints are imposed on models (1) and (2). In order to obtain an aggregate model, we combine these two models into a single one using a weighting method which assumes the weight of w_i for ith goal in contrast to other goals. By combining two models mentioned above, the final linear model used for cost-time tradeoff is acquired.

$$MinZ = w_1.\left[O(T_n - T_1) + \sum_{i=1}^{n}\sum_{j=1}^{n} CD_{ij} \right] + w_1.\left[\sum_{i=1}^{n}\sum_{j=1}^{n} CS_{ij}(D_{ij} - X_{ij}) \right]$$

$$\begin{aligned} & S.T: \\ & T_n - T_1 \le T_S \\ & T_i + X_{ij} \le T_j \\ & X_{ij} \le D_{ij} \\ & X_{ij} \ge d_{ij} \\ & \sum_{i=1}^{n}\sum_{j=1}^{n} [CD_{ij} + CS_{ij}.(D_{ij} - X_{ij})] + O.(T_n - T_1) \le B \\ & T_i, T_j \ge 0 \\ & i, j \in \{1,2,...,n\} \end{aligned} \tag{3}$$

III. GREY NUMBERS

Under many conditions, exact data are inadequate to model the real-life situations. These situations are called uncertainty and many researchers developed some structures such as bounded data, ordinal data, fuzzy data, and grey numbers in response to such situations. In fact, most of the decisions aren't made on the basis of precise calculations and there is a lot of ambiguity and uncertainty in decision making problems [30]. Time – Cost tradeoff problem is a kind of problem which consists of uncertain data. Often, an analyst neither has enough information about exact costs required to reduce the time of an activity nor does he has about the amount of time which will be reduced from activity's required time. Application of grey data in such situations will improve the ability of current model in response to ambiguity of data.

Reference [31] developes the Grey system theory and presents grey decision making systems [32]. Many researchers applied this concept in decision making problems. In a simple word, a grey number is a number whose exact value is unknown, but a range within which the value lies is known [33]. In this section, we briefly define grey numbers and some of their characteristics.

Definition1. Such a number instead of its range whose exact value is unknown is referred to as a grey number. In applications, a grey number in fact stands for an indeterminate number that takes its possible value within an interval or a general set of numbers. This grey number is generally represented by the symbol "$\otimes$". There are several types of grey numbers with which we only define interval grey numbers. This kind of grey number $\otimes$ written $\otimes(a) \in \left[\underline{a}, \overline{a}\right]$, where $\underline{a}$ stands for the definite, known lower bound and $\overline{a}$ stands for the definite, known upper bound of $\otimes(a)$ and $\otimes(a)$ takes its number in this interval [34].

Arithmetic operations on grey numbers are defined as follow [35].

Definition2. Assume that $\otimes(a_1) \in [a,b], a \prec b$ and $\otimes(a_2) \in [c,d], c \prec d$. The sum of $\otimes(a_1)$ and $\otimes(a_2)$, written $\otimes(a_1)+\otimes(a_2)$, is defined as follows.

$$\otimes(a_1)+\otimes(a_2) \in [a+c, b+d] \quad (4)$$

Definition3. Assume that $\otimes(x) \in [a,b], a \prec b$. The negative inverse of $\otimes(x)$, written $-\otimes(x)$, is defined as follows.

$$-\otimes(x) = [-a, -b] \quad (5)$$

Definition4. Assume that $\otimes(a_1) \in [a,b], a \prec b$ and $\otimes(a_2) \in [c,d], c \prec d$. The difference of $\otimes(a_1)$ and $\otimes(a_2)$ is defined as follows

$$\otimes(a_1)-\otimes(a_2) \in [a-d, b-c] \quad (6)$$

Definition5. Assume that $\otimes(x) \in [a,b], a \prec b$ and $ab \succ 0$. The reciprocal of $\otimes(x)$, written $\otimes(x)^{-1}$, is defined as follows.

$$\otimes(x)^{-1} \in \left[\frac{1}{b}, \frac{1}{a}\right] \quad (7)$$

Definition6. Assume $\otimes(a_1) \in [a,b], a \prec b$ and $\otimes(a_2) \in [c,d], c \prec d$. The product of $\otimes(a_1)$ and $\otimes(a_2)$ is defined as follows.

$$[min(ac,ad,bc,bd), max(ac,ad,bc,bd)] \quad (8)$$

Definition7. Assume $\otimes(a_1) \in [a,b], a \prec b$ and $\otimes(a_2) \in [c,d], c \prec d$, satisfying $c \prec d$ and $cd \succ 0$. The quotient of $\otimes(a_1)$ divided by $\otimes(a_2)$ is as defined as follows.

$$\otimes(a_1)/\otimes(a_2) \in \otimes(a_1).\otimes(a_2)^{-1} \quad (9)$$

Definition 8. For grey number $\otimes(x) \in [a,b]$ we have:

$$\begin{aligned} &\otimes(x) \geq 0 \;\; iff \; a \geq 0 \; and \; b \geq 0 \\ &\otimes(x) \leq 0 \;\; iff \; a \leq 0 \; and \; b \leq 0 \end{aligned} \quad (10)$$

Definition9. For grey number $\otimes(x)$ we define $Sign(\otimes(x))$ as follows:

$$Sign(\otimes(x)) = \begin{cases} 1, & if \otimes(x) \geq 0 \\ -1, & if \otimes(x) \prec 0 \end{cases} \quad (11)$$

Definition10. For grey number $\otimes(x)$, we define its grey absolute value $\otimes(|x|)$ as follows:

$$\otimes(|x|) \in \left[\underline{\otimes}(|x|), \overline{\otimes}(|x|)\right] = \begin{cases} \otimes, & if \otimes \geq 0 \\ -\otimes, & if \otimes \prec 0 \end{cases} \quad (12)$$

IV. GREY MULTI OBJECTIVE TIME AND COST TRADEOFF MODEL

As defined in section 2, we developed an aggregate Time – Cost tradeoff model. Now suppose that in a real situation, we want to apply such model. As shown in model (4), we require some information such as normal and minimum time of an activity and costs required to reduce an activity's time. In conventional models, such information is crisp and there is an exact approximation of those data. But in real conditions, such information is often unknown and ill-defined, so analyst might be required to apply an inexact form of data. In this section, we propose a grey time and cost tradeoff model, called G-TCTM and show how to solve such model and find its optimal solution. Now suppose that our parameters in table 1 are grey numbers with a lower and an upper bound. Now the G-TCTM is as model (12), in which weights w_1, w_2 can be crisp or grey data.

$$\begin{aligned} &Min Z = \otimes w_1 \left[\otimes O(\otimes T_n - \otimes T_1) + \sum_{i=1}^{n}\sum_{j=1}^{n} \otimes CD_{ij}\right] \\ &+ \otimes w_2 \left[\sum_{i=1}^{n}\sum_{j=1}^{n} \otimes CS_{ij}(\otimes D_{ij} - \otimes X_{ij})\right] \\ &S.T: \\ &\otimes T_n - \otimes T_1 \leq \otimes T_S \\ &\otimes T_i + \otimes X_{ij} \leq \otimes T_j \\ &\otimes X_{ij} \leq \otimes D_{ij} \\ &\otimes X_{ij} \geq \otimes d_{ij} \\ &\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\otimes CD_{ij} + \otimes CS_{ij}(\otimes D_{ij} - \otimes X_{ij})\right] \\ &+ \otimes O(\otimes T_n - \otimes T_1) \leq \otimes B \\ &\otimes T_i, \otimes T_j \geq 0 \\ &i, j \in \{1,2,\ldots,n\} \end{aligned} \quad (13)$$

As observed in (13), we define all the cost and time parameters of our model as grey numbers which reflect a better picture of uncertainty in project control. The model (13) is a grey linear programming (GLP) model, a linear programming model with grey parameters, which is needed for a suitable approach to solve the problem. Generally, a GLP can be defined as follows:

$$
\begin{aligned}
&Max(min) \otimes f = \otimes C.\otimes X \\
&S.T. \quad \otimes A.\otimes X \le (\ge) \otimes B \\
&\qquad\quad \otimes X \ge 0
\end{aligned} \tag{14}
$$

Where $\otimes B \in \otimes(R)^{m\times 1}$, $\otimes C \in \otimes(R)^{1\times n}$, $\otimes X \in \otimes(R)^{n\times 1}$. The proposed approach to solve GLP model is taken from [36] which in his thesis proposed an approach to this kind of problems. As objective function is a grey variable, we went to find its optimal value as $\otimes f \in [\underline{f}, \overline{f}]$. For the upper and lower bounds of $\otimes f$, we have:

$$\overline{f} = \sum_{j=1}^{k_1} \overline{c}_j.\overline{x}_j + \sum_{j=k_1+1}^{n} \overline{c}_j.\underline{x}_j \tag{15}$$

$$\underline{f} = \sum_{j=1}^{k_1} \underline{c}_j.\underline{x}_j + \sum_{j=k_1+1}^{n} \underline{c}_j.\overline{x}_j \tag{16}$$

In which $\otimes(c_j) \in [\underline{c}_j, \overline{c}_j], j = 1,2,\ldots,k_1$ are positive and $\otimes(c_j), j = k_1+1, k_1+2,\ldots,n$ are negative coefficient as illustrated in Def.8. In order to obtain grey solutions for model (14), constraints corresponding to $\overline{f}$ are developed as follows:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|).Sign(\underline{a_{ij}})\overline{x}_j + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|).Sign(\overline{a_{ij}})\underline{x}_j \le \otimes(b_i), \forall i \tag{17}$$

Similarly for $\underline{f}$ the relevant constraints are:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|).Sign(\overline{a_{ij}})\underline{x}_j + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|).Sign(\underline{a_{ij}})\overline{x}_j \le \otimes(b_i), \forall i \tag{18}$$

For right hand side (RHS) constraint, the possible relationships can be analyzed as follows. When RHS is a deterministic number, thus in (17) and (18), b_i is replaced with $\otimes b_i$. When RHS is a grey number, and does not contain a zero. The grey properties of $\otimes b_i$ can be easily incorporated into left hand side coefficients as follows:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|).Sign(\underline{a_{ij}})\overline{x}_j \Big/ \overline{b}_i + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|).Sign(\overline{a_{ij}})\underline{x}_j \Big/ \underline{b}_i \le 1, \forall i \tag{19}$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|).Sign(\overline{a_{ij}})\underline{x}_j \Big/ \underline{b}_i + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|).Sign(\underline{a_{ij}})\overline{x}_j \Big/ \overline{b}_i \le 1, \forall i \tag{20}$$

Based on above definition, solution algorithm for proposed G-TCTM consists of the following steps. First a whitened sub model corresponding to $\underline{f}$ (because objective is to be minimized), based on (16) and (20), is formulated and solved. Then the relevant sub model corresponding to $\overline{f}$ can be formulated based on the generated lower bound solution. First model determined the $\underline{x}_j, j = 1,2,\ldots,k_1$ and $\overline{x}_j, j = k_1+1, k_1+2,\ldots,n$. The value of $\overline{x}_j, j = 1,2,\ldots,k_1$ and $\underline{x}_j, j = k_1+1, k_1+2,\ldots,n$ is determined by solving second sub model, by adding two constraints:

$$\overline{x}_j \ge \underline{x}_j^*, j = 1,2,\ldots,k_1 \tag{21}$$

$$\underline{x}_j \le \overline{x}_j^*, j = k_1+1, k_1+2,\ldots,n \tag{22}$$

V. NUMERICAL EXAMPLE

In this section we solve a numerical example to illustrate the application of GMO-TCTM in real problems. Suppose that a project consists of eight activities based on data is illustrated in table 2.

TABLE II. PROJECTS DATA

Activity	Predecessor	D_n	D_c	C_{ij}	C_n
A	-	[3.5,4.5]	[2.25,3.5]	[2.3,3.5]	[95,110]
B	A	[4.5,5.5]	[3,4.5]	[4.5,5.75]	[145,157]
C	A	[2.5,3.5]	[2.75,3.5]	[0,0.25]	[480,515]
D	B	[5.5,6.5]	[4.5,5.75]	[6.4,7.3]	[70,86]
E	C,D	[3.5,4.5]	[2.5,3.25]	[9.25,10.3]	[87,99]
F	C,D	[1.5,2.5]	[0.75,2]	[1.5,2.3]	[110,127]
G	F	[2.5,3.5]	[0.75,2]	[0.75,1.6]	[95,108]
H	E,G	[4.5,5.5]	[3.5,4.75]	[7.25,8.5]	[62,75]

The project's graph is illustrated in figure 1.

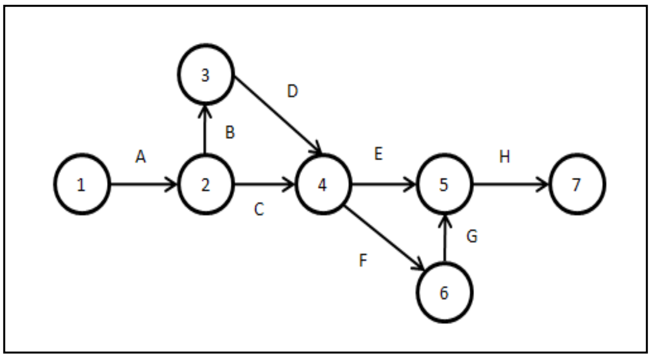

Figure 1. Project's graph

The G-TCTM for this example can be formulated as follows:

$$Min\, Z = [12,20]\otimes T_7 + [-20,-12]\otimes T_1 + [-3.5,-2.3]\otimes X_{12} + [-5.75,-4.5]\otimes X_{23} + [-0.25,0]\otimes X_{24} + [-7.3,-6.4]\otimes X_{34} + [-10.3,-9.25]\otimes X_{45} + [-2.3,-1.5]\otimes X_{46} + [-1.6,-0.75]\otimes X_{65} + [-8.5,-7.25]\otimes X_{57} + [1214.625,1402.15]$$

$S.T:$

$$\begin{cases} \otimes T_7 - \otimes T_1 \le [21,24] \\ \otimes T_1 + \otimes X_{12} \le \otimes T_2 \\ \otimes T_2 + \otimes X_{23} \le \otimes T_3 \\ \otimes T_2 + \otimes X_{24} \le \otimes T_4 \\ \otimes T_3 + \otimes X_{34} \le \otimes T_4 \\ \otimes T_4 + \otimes X_{45} \le \otimes T_5 \\ \otimes T_4 + \otimes X_{46} \le \otimes T_6 \\ \otimes T_6 + \otimes X_{65} \le \otimes T_5 \\ \otimes T_5 + \otimes X_{57} \le \otimes T_7 \end{cases}$$

$$\begin{cases} \otimes X_{12} \le [3.5,4.5] \\ \otimes X_{23} \le [4.5,5.5] \\ \otimes X_{24} \le [2.5,3.5] \\ \otimes X_{34} \le [5.5,6.5] \\ \otimes X_{45} \le [3.5,4.5] \\ \otimes X_{46} \le [1.5,2.5] \\ \otimes X_{65} \le [2.5,3.5] \\ \otimes X_{57} \le [4.5,5.5] \end{cases} \quad \begin{cases} \otimes X_{12} \ge [2.25,3] \\ \otimes X_{23} \ge [3,4] \\ \otimes X_{24} \ge [2,2.75] \\ \otimes X_{34} \ge [4.5,5.5] \\ \otimes X_{45} \ge [2.5,3.25] \\ \otimes X_{46} \ge [0.75,1.5] \\ \otimes X_{65} \ge [0.75,2] \\ \otimes X_{57} \ge [3.5,4.5] \end{cases}$$

$$[2.3,3.5]\otimes X_{12} + [4.5,5.75]\otimes X_{23} + [0,0.25]\otimes X_{24} + [6.4,7.3]\otimes X_{34} + [9.25,10.3]\otimes X_{45} + [1.5,2.3]\otimes X_{46} + [0.75,1.6]\otimes X_{65} + [7.25,8.5]\otimes X_{57} \le [197.85,285.375]$$

$$\otimes T_1, \otimes T_2, \ldots, \otimes T_7 \ge 0$$

$$\otimes X_{ij} \ge 0, i = 1,2,\ldots,6;\ j = 1,2,\ldots,7 \tag{23}$$

Using the proposed 2 phase approach, the solution of (23) will be as table 3.

TABLE III. GREY VARIABLES OPTIMAL VALUES

Variables	Lower bound	Upper bound
Z	1489.725	1508.975.5
$\otimes T_1$	0	0
$\otimes T_2$	3	4
$\otimes T_3$	7	8
$\otimes T_4$	12	13
$\otimes T_5$	16.5	16.5
$\otimes T_6$	14.5	14.5
$\otimes T_7$	21	21
$\otimes X_{12}$	3	3
$\otimes X_{23}$	4	4
$\otimes X_{24}$	2.5	3.5
$\otimes X_{34}$	5	5
$\otimes X_{45}$	3.5	4.5
$\otimes X_{46}$	1.5	2.5
$\otimes X_{65}$	2	2
$\otimes X_{57}$	4.5	4.5

As can be seen from table 3, based on our grey model's solution, the crashed time of each activity can take a value between its lower and upper bound. Under the scheme for $\underline{Z}$, all values of $\otimes T_i$, except for $i = 1$, take their lower bound values and all $\otimes x_{ij}$ the upper bound. While under the scheme for $\overline{Z}$, $\otimes T_1$ and all $\otimes x_{ij}$ take their lower bound values, but $\otimes T_i, i \ne 1$ take their upper bound. Thus the final decision for $\otimes T_i$ and $\otimes x_{ij}$ values can be determined from the created alternatives according to the projected applicable conditions. For example, a lower $\otimes T_i, i = 2,3,\ldots,7$ and a higher $\otimes T_1$ and $\otimes x_{ij}, \forall i, j$ within their solution intervals could be chosen for a lower $\otimes Z$.

VI. CONCLUSION

If we define a project as a temporary endeavor undertaken to produce a unique product, service or result, then we can envisage all of the tasks of an organization as a project. One of the important problems in project management is time and cost tradeoffs which determine the best combination of activities and time in order to complete a project in a given time and with a given budget. There are various methods and models to time and cost tradeoffs in project management. Almost all of these methods are based on crisp data in which the approximation of activity's required time and cost are certain and distinctive. When we apply a time and cost tradeoffs model, we need an approximation of normal time and cost, mid theirs crashed time and cost. In such a situation, using crisp data as approximated values of activities' time and cost is a restriction to model which limited their applications. Indeed, our approximations about time and cost characteristics on a project's activity always attend uncertainty and qualm. In response to such a situation, we need to bring uncertainty and ambiguousness in time and cost tradeoffs models. In this paper, we cope with uncertainty in time and cost tradeoffs by introducing G-TCTM in which we use grey numbers as our approximation of project activity time and cost in normal and crashed form. This model allows us to nose the ill defined data in real projects with time and cost tradeoffs problems. In this paper, it is assumed that the relationship between time and cost is a linear function, while it is possible that the relationship takes a different form. Also, the continuity of activities' times in a normal-crashed interval is another shortcoming of proposed method which can be solved through multi mode time and cost tradeoff model. As a direction for future research, it would be interesting to apply the other forms of uncertain data, such as fuzzy sets and interval valued fuzzy sets in time and cost tradeoffs. Moreover, another clue for future research is development of project tradeoffs model by appending quality and risk aspects to time and cost under non crispness, by applying grey numbers.

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