# Sum-Rate Maximization Based Relay Selection for Cooperative NOMA Over Nakagami-m Fading

Yan Li, Tao Li, Yongzhao Li, Senior Member, IEEE, Qiang Ni, Senior Member, IEEE and Charilaos Zarakovitis

Abstract—This paper proposes a new sum-rate maximization based relay selection (RS) scheme for cooperative non-orthogonal multiple access (NOMA) networks over Nakagami-*m* fading, where one base station communicates with two mobile users by means of multiple relays. The outage probability of the proposed scheme is derived in a closed-form expression, and the diversity order is also obtained. Simulation results are shown to compare the outage performance of the proposed scheme with that of the existing RS schemes.

*Index Terms*—Sum-rate maximization, cooperative NOMA, Nakagami-*m* fading, outage probability, relay selection.

### I. INTRODUCTION

**R** ECENTLY, non-orthogonal multiple access (NOMA) has been considered as a promising candidate technology to improve the spectral efficiency (SE) for the fifth generation (5G) wireless networks [1], [2]. To reduce the effect of fading and to harvest spatial degrees of freedom, cooperative communications have been applied into NOMA, forming cooperative NOMA [3].

Cooperative NOMA was initially proposed in [4], where users with stronger channel conditions acted as relays to assist other users. Afterwards, the works in [5], [6] established the communications between source nodes and destination nodes by virtue of a dedicated relay. Relay selection (RS) is an effective approach for exploiting space diversity and for increasing SE when multiple relays are deployed in cooperative NOMA, and several RS schemes have been proposed in the open literature [7]–[12]. In [7], a two-stage max-min RS scheme based on fixed power allocation (PA) was investigated, where a relay was selected to serve the high-rate user by adopting the max-min scheme while satisfying the quality of service (QoS) requirements of the low-rate user. The work in [8] redesigned the RS scheme in [7] to further improve the system outage performance, where dynamic PA using the

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instantaneous channel state information (ICSI) was considered. Nevertheless, the users in [7] and [8] were all ordered by their QoS requirements, which leads to the suboptimal outage performance. Therefore, in [9], [10], two types of RS schemes adopting the ICSI based user ordering strategy were proposed to achieve better outage performance. Moreover, in [11], a partial relay selection scheme based on the ICSI of the first steps was designed, where the full-duplex (FD) amplify-andforward relays were considered. In [12], a buffer-aided relay selection scheme for cooperative NOMA networks in the Internet of Things was studied. It is worth noting that all these RS schemes were designed based on the fixed individual target rates of different users. However, in some practical situations, each user may have multiple target rates related to various services. For example, one user can at most have five categories of data flow regarding different QoS in IEEE 802.16/WiMAX, and for each data flow there exists a minimum reserved traffic rate (MRTR) to specify the minimum information rate [13]. In this case, RS schemes involving NOMA can be designed to satisfy all paired users' MRTR for basic real-time service firstly, while trying to provide the users having better ICSI with additional data transmission for non-real-time service as much as possible to improve the overall SE. Moreover, to our best knowledge, none of the existing RS schemes for cooperative NOMA has been proposed to maximize the sum rate of the considered networks, which motivates the study of this work.

In this paper, we propose a new sum-rate maximization based RS scheme for cooperative NOMA networks, where each user has two alternative target rates. Specifically, the proposed scheme consists of two stages, where the first stage is to find a subset of relays that can decode both users' signals correctly, and the second stage is to select from the subset an optimal relay that maximizes the sum rate of the considered networks with the constraint that the weak user's basic target rate is satisfied. Both users' adopted target rates are associated with their ICSI, and the user ordering and PA strategy are also based on the ICSI to achieve the goals of RS. In contrast to the existing works [7]–[10], Nakagami-*m* fading rather than Rayleigh fading is considered in the proposed scheme to depict a wider class of fading environments.<sup>1</sup> We derive a closed-form expression for the outage probability and approximate it in the high SNR region to obtain the diversity order. Simulation results are presented to compare the performance of the proposed RS scheme with that of the existing ones [7]–[10].

<sup>1</sup>We note that there exist other fading models (i.e., correlated Nakagami-m fading considered in [14]) providing a more general or practical system setting, which are worth studying in our future work.

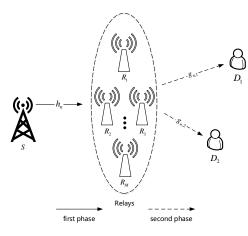


Fig. 1: The illustration of the considered downlink cooperative NOMA network.

# II. SYSTEM MODEL

We consider a downlink cooperative NOMA network with one base station (S), two users  $(D_1, D_2)$  and M halfduplex decode-and-forward (DF) relays<sup>2</sup> ( $R_n, n = 1, ..., M$ ), as illustrated in Fig. 1. Each node is equipped with a single antenna. It is assumed that there is no direct link between the source and users [7]-[10]. All wireless channels experience independent quasi-static Nakagami-m fading and additive white Gaussian noise (AWGN). Furthermore, we assume that M relays are clustered closely together, and hence the channel gains between a certain node and the relays are independent and identically distributed (i.i.d.). Specifically, the channel coefficient between S and  $R_n$  is denoted by  $h_n$  with fading parameter  $m_0$  and  $E[|h_n|^2] = \Omega_0$ , and the channel coefficient between  $R_n$  and  $D_k$  (k = 1, 2) is denoted by  $g_{n,k}$  with fading parameter  $m_k$  and  $E[|g_{n,k}|^2] = \Omega_k$ . Both users are ordered according to their ICSI, and it is assumed that the weak user is denoted by  $D_{n,f}$  with channel gain  $|g_{n,f}|^2 =$  $\min\{|g_{n,1}|^2, |g_{n,2}|^2\}$ , and the strong user is denoted by  $D_{n,c}$ with channel gain  $|g_{n,c}|^2 = \max\{|g_{n,1}|^2, |g_{n,2}|^2\}$ . Moreover, the alternative target rates for both two users are denoted by  $R_f$  and  $R_c$ .  $R_f$  will be adopted by  $D_{n,f}$ , and  $R_c$  will be adopted by  $D_{n,c}$ . Without loss of generality,  $R_f \leq R_c$  is assumed. It is worth noting that perfect successive interference cancellation (SIC) is employed as in [7]-[10], our future work will relax this assumption.

There are two separate phases involved to achieve data transmission. Assume that  $R_n$   $(1 \le n \le M)$  is selected as the optimal relay. In the first phase, S transmits a superposed signal,  $(\sqrt{\gamma_{n,f}}x_f + \sqrt{\gamma_{n,c}}x_c)$ , to  $R_n$ , where  $x_i$  (i = f, c) denotes  $D_{n,i}$ 's message;  $\gamma_{n,i}$  denotes the PA coefficient for message  $x_i$  satisfying  $\gamma_{n,i} > 0$  and  $\gamma_{n,f} + \gamma_{n,c} = 1$ . Therefore,  $R_n$  receives

$$y_n^r = \sqrt{P_s} h_n \left( \sqrt{\gamma_{n,f}} x_f + \sqrt{\gamma_{n,c}} x_c \right) + w_n^r, \tag{1}$$

where  $P_s$  denotes the transmit power of S;  $w_n^r$  denotes the AWGN with zero mean and variance  $\sigma^2$  at  $R_n$ .

<sup>2</sup>FD DF relay can also be adopted in our system model, while the impact of residual loop self-interference should be carefully considered in the analysis.

Based on the basic principle of NOMA, the achievable rates for  $R_n$  to decode the messages  $x_f$  and  $x_c$  can be given by

$$R_{f}^{n} = \frac{1}{2} \log_{2} \left( 1 + \frac{\gamma_{n,f} \rho |h_{n}|^{2}}{\gamma_{n,c} \rho |h_{n}|^{2} + 1} \right),$$
(2)

$$R_{c}^{n} = \frac{1}{2} \log_{2} \left( 1 + \gamma_{n,c} \rho |h_{n}|^{2} \right), \qquad (3)$$

respectively, where  $\rho = P_s/\sigma^2$  denotes the transmit signal-tonoise ratio (SNR).

In the second phase,  $R_n$  broadcasts the superposed signal,  $(\sqrt{\alpha_{n,f}}x_f + \sqrt{\alpha_{n,c}}x_c)$ , to the users, where  $\alpha_{n,i}$  (i = f, c) denotes the PA coefficient for  $D_{n,i}$  satisfying  $\alpha_{n,i} > 0$  and  $\alpha_{n,f} + \alpha_{n,c} = 1$ . Therefore,  $D_{n,i}$  observes

$$y_{n,i}^d = \sqrt{P_r} g_{n,i} \left( \sqrt{\alpha_{n,f}} x_f + \sqrt{\alpha_{n,c}} x_c \right) + w_{n,i}^d, \quad (4)$$

where  $P_r$  denotes the transmit power of  $R_n$ , to simplify the analysis,  $P_s = P_r = P$  is assumed;  $w_{n,i}^d$  denotes the AWGN with zero mean and variance  $\sigma^2$  at  $D_{n,i}$ .

According to (2) and (3), the achievable rate for  $D_{n,f}$  to decode its own message  $x_f$  is expressed as

$$\hat{R}_{f}^{n} = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha_{n,f} \rho |g_{n,f}|^2}{\alpha_{n,c} \rho |g_{n,f}|^2 + 1} \right), \tag{5}$$

and the achievable rates for  $D_{n,c}$  to decode both users' messages  $x_f$  and  $x_c$  are respectively expressed as

$$\hat{R}_{c \to f}^{n} = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha_{n,f} \rho |g_{n,c}|^2}{\alpha_{n,c} \rho |g_{n,c}|^2 + 1} \right), \tag{6}$$

$$\hat{R}_{c}^{n} = \frac{1}{2} \log_2 \left( 1 + \alpha_{n,c} \rho |g_{n,c}|^2 \right).$$
(7)

# III. RELAY SELECTION SCHEME

Recall that the proposed RS scheme is to maximize the sum rate of considered NOMA systems, therefore the following theorem can be applied.

Theorem 1: In the case of using a specific relay  $R_n$ , the maximal sum rate of considered NOMA networks is  $R_{\max}^{o,n} = \min\{R_{\max,1}^{o,n}, R_{\max,2}^{o,n}\}$ , where  $R_{\max,1}^{o,n}$  and  $R_{\max,2}^{o,n}$  are the maximal sum rate of each phase, respectively.

Proof: The sum rate of the first phase is  $R_{n,1}^o = R_f^n + R_c^n$ , and that of the second phase is  $R_{n,2}^o = \hat{R}_f^n + \hat{R}_c^n$ . According to the character of DF protocol, the sum rate of NOMA systems is  $R_n^o = \min\{R_f^n, \hat{R}_f^n\} + \min\{R_c^n, \hat{R}_c^n\}$ . It is observed that  $R_n^o \le \min\{R_{n,1}^o, R_{n,2}^o\}$ , thus we can derive  $R_{\max}^{o,n} = \min\{R_{\max,1}^{o,n}, R_{\max,2}^{o,n}\}$ , where  $R_{\max,1}^{o,n}, R_{\max,1}^{o,n}$  and  $R_{\max,2}^{o,n}$  are the maximal  $R_n^o, R_{n,1}^o$  and  $R_{n,2}^o$ , respectively.

Based on Theorem 1, we can now focus on the realization of  $R_{\max,1}^{o,n}$  and  $R_{\max,2}^{o,n}$  to reach  $R_{\max}^{o,n}$ . It is deduced that  $R_{n,1}^{o} = \frac{1}{2} \log_2(1 + \rho |h_n|^2)$ , which is irrelevant to the PA coefficients of both users; while  $R_{n,2}^{o}$  is proved to decrease with  $\alpha_{n,f}$ . As such, for  $R_{n,2}^{o}$ , we can just allocate opportune power to  $D_{n,f}$  so as to satisfy its minimum rate constraint  $R_f$  ( $\hat{R}_f^n = R_f$ ) and then to achieve  $R_{\max,2}^{o,n}$ , leading to  $\alpha_{n,c} = \max\left\{0, \frac{\rho |g_{n,f}|^2 - \varepsilon_f}{\rho |g_{n,f}|^2(1+\varepsilon_f)}\right\}$ , where  $\varepsilon_f = 2^{2R_f} - 1$ ; while for  $R_{n,1}^{o}$ , the constraint  $R_f^n = R_f$  is also applied to ensure that  $R_n^o = \min\{R_{\max,1}^{o,n}, R_{\max,2}^{o,n}\}$  can be realized in each transmission. Similarly,  $\gamma_{n,c} = \max\left\{0, \frac{\rho |h_n|^2 - \varepsilon_f}{\rho |h_n|^2(1+\varepsilon_f)}\right\}$  is derived.

*Remark 1:* The adopted dynamic PA strategy in the first phase can also result in the same optimal outage performance as that caused by the coding strategy in the first time slot in [10]. Due to limited space, the proof is omitted herein.

As a result,  $R_{\max}^{o,n}$  can be achieved by using the proposed PA strategy above, and the optimal relay is the one which can obtain the maximal  $R_{\max}^{o,n}$  among all the relays.

Due to the fact that the achievable rate for the weak user is always  $R_f$ , the optimal relay can be thus determined by the following two stages.

1) The first stage is to build a subset of relays by concentrating on correctly decoding both users' messages, which can be expressed as

$$S_n = \left\{ n : 1 \le n \le M, R_f^n \ge R_f, R_c^n \ge R_c \right\}.$$
(8)

2) The second stage is to select from  $S_n$  the optimal relay that maximizes  $D_{n,c}$ 's rate, which can be given by

$$n^* = \underset{n}{\arg\max} \left\{ \min\left\{ R_c^n, \hat{R}_c^n \right\}, n \in S_n, \hat{R}_f^n \ge R_f \right\}.$$
(9)

As a consequence, the maximal sum rate of considered NOMA networks after RS can be expressed as

$$R_{\max}^{o} = R_{f} + \min\left\{R_{c}^{n^{*}}, \hat{R}_{c}^{n^{*}}\right\}.$$
 (10)

### IV. PERFORMANCE ANALYSIS

In this section, the outage probability and diversity order of the proposed RS scheme will be characterized. Before calculating the overall outage probability, we will firstly discuss the outage probability which occurs in the second stage for a specific relay  $R_n$   $(n \in S_n)$ .

The coverage probability of  $R_n$  in the second stage can be written as

$$P_{c}^{n} = \Pr\left\{\hat{R}_{c}^{n} \geq R_{c}, |g_{n,c}|^{2} \geq |g_{n,f}|^{2}, \frac{\rho|g_{n,f}|^{2} - \varepsilon_{f}}{\rho|g_{n,f}|^{2} (1 + \varepsilon_{f})} \geq 0\right\}$$
$$= \Pr\left\{|g_{n,c}|^{2} \geq |g_{n,f}|^{2} \geq \frac{\varepsilon_{f}|g_{n,c}|^{2}}{\rho|g_{n,c}|^{2} - \tau_{1}}, |g_{n,c}|^{2} > \frac{\tau_{2}}{\rho}\right\}, (11)$$

where  $\varepsilon_c = 2^{2R_c} - 1$ ,  $\tau_1 = \varepsilon_c (\varepsilon_f + 1)$ ,  $\tau_2 = \tau_1 + \varepsilon_f$ . Since  $g_{n,k}$  (k = 1, 2) follows Nakagami-*m* distribution

with fading parameter  $m_k$  and  $E[|g_{n,k}|^2] = \Omega_k$ , the channel gain  $|g_{n,k}|^2$  will follow a Gamma distribution with probability density function (PDF) [15]

$$f_{|g_{n,k}|^2}(y) = \frac{m_k^{m_k} y^{m_k - 1}}{\Omega_k^{m_k} \Gamma(m_k)} e^{-\frac{m_k y}{\Omega_k}},$$
(12)

where  $m_k$  can take any real value greater than or equal to 0.5 (Rayleigh fading is a special case for  $m_k = 1$ ),  $\Gamma(\beta)$  denotes the Gamma function, and  $\Gamma(\beta) = (\beta - 1)!$  when  $\beta$  takes integer values. In this paper,  $m_k$  is constrained to take integer values by following the same assumption as in [15], as a result, the corresponding cumulative distribution function (CDF) of  $g_{n,k}$  can be expressed as

$$F_{|g_{n,k}|^2}(y) = 1 - e^{-\frac{ym_k}{\Omega_k}} \sum_{s=0}^{m_k-1} \frac{\left(\frac{ym_k}{\Omega_k}\right)^s}{s!}.$$
 (13)

Note that  $|g_{n,c}|^2$  and  $|g_{n,f}|^2$  have been ordered as  $|g_{n,f}|^2 \le |g_{n,c}|^2$ , hence the joint PDF of  $|g_{n,c}|^2$  and  $|g_{n,f}|^2$  can be given by [16]

$$f_{\left|g_{n,f}\right|^{2},\left|g_{n,c}\right|^{2}}\left(x,y\right) = \sum_{i_{1},i_{2} \in \{1,2\}} f_{\left|g_{n,i_{1}}\right|^{2}}\left(x\right) f_{\left|g_{n,i_{2}}\right|^{2}}\left(y\right), \quad (14)$$

where  $0 \le x \le y$ ,  $i_1, i_2 \in \{1, 2\}$  means  $i_1 = 1$ ,  $i_2 = 2$  or  $i_1 = 2$ ,  $i_2 = 1$ . Based on (12) (14),  $\mathbb{P}^n$  can be further written as

Based on (12)-(14),  $P_c^n$  can be further written as

$$P_{c}^{n} = \sum_{i_{1}, i_{2} \in \{1, 2\}} \int_{\frac{\tau_{2}}{\rho}}^{\infty} f_{|g_{n, i_{2}}|^{2}}(y) \left(F_{|g_{n, i_{1}}|^{2}}(y) - F_{|g_{n, i_{1}}|^{2}}\left(\frac{\varepsilon_{f}y}{\rho y - \tau_{1}}\right)\right) dy$$
$$= \sum_{i_{1}, i_{2} \in \{1, 2\}} \int_{\frac{\tau_{2}}{\rho}}^{\infty} f_{|g_{n, i_{2}}|^{2}}(y) \left(Q_{1} - Q_{2}\right) dy, \tag{15}$$

where  $Q_1$  and  $Q_2$  are respectively expressed as

$$Q_{1} = e^{-\frac{m_{i_{1}}\varepsilon_{f}y}{\Omega_{i_{1}}(\rho y - \tau_{1})}} \sum_{r=0}^{m_{i_{1}}-1} \frac{\left(\frac{r\varepsilon_{f}y}{\Omega_{i_{1}}(\rho y - \tau_{1})}\right)^{r}}{r!}, \qquad (16)$$

$$Q_2 = e^{-\frac{m_{i_1}y}{\Omega_{i_1}}} \sum_{s=0}^{m_{i_1}-1} \frac{\left(\frac{m_{i_1}y}{\Omega_{i_1}}\right)^s}{s!}.$$
 (17)

From (15), we can firstly calculate the partial integral result  $R_1$ , which can be defined as

$$R_1 = \int_{\frac{\tau_2}{\rho}}^{\infty} f_{|g_{n,i_2}|^2}(y) Q_1 \, dy.$$
(18)

Let  $t = \frac{\varepsilon_f}{(\rho y - \tau_1)}$ , then the above  $R_1$  can be evaluated as

$$R_{1} = \sum_{r=0}^{m_{i_{1}}-1} \frac{(m_{i_{1}}/(\Omega_{i_{1}}\rho))^{r}}{r!} \frac{m_{i_{2}}m_{i_{2}}\varepsilon_{f}}{\Omega_{i_{2}}^{m_{i_{2}}}(m_{i_{2}}-1)!\rho^{m_{i_{2}}}}$$
$$\times \int_{0}^{1} e^{-\frac{(\varepsilon_{f}+\tau_{1}t)(\Omega_{i_{1}}m_{i_{2}}+\Omega_{i_{2}}m_{i_{1}}t)}{\Omega_{i_{1}}\Omega_{i_{2}}t\rho}} \frac{(\varepsilon_{f}+\tau_{1}t)^{m_{i_{2}}+r-1}}{t^{m_{i_{2}}+1}}dt.$$
(19)

Define the integral in (19) as  $I_1$ . It is difficult to find an exact expression for  $I_1$ , therefore Gauss-Chebyshev (G-C) quadrature [10] is applied to derive an approximation

$$I_{1} \approx \sum_{j=1}^{N} \varphi \ e^{-\frac{\left(\varepsilon_{f} + \tau_{1} t_{j}\right) \left(\Omega_{i_{1}} m_{i_{2}} + \Omega_{i_{2}} m_{i_{1}} t_{j}\right)}{\Omega_{i_{1}} \Omega_{i_{2}} t_{j} \rho}} \frac{\left(\varepsilon_{f} + \tau_{1} t_{j}\right)^{m_{i_{2}} + r - 1}}{t_{j}^{m_{i_{2}} + 1}},$$
(20)

where  $\varphi = \frac{\pi}{2N} \left| \sin \frac{2i-1}{2N} \pi \right|$ ,  $t_j = \frac{1}{2} \left( 1 + \cos \frac{2j-1}{2N} \pi \right)$ , and N is a complexity-accuracy tradeoff parameter.

Substituting (20) into (19), a closed-form approximate expression for  $R_1$  can be obtained.

To proceed, another partial integral  $R_2$  from (15) can be calculated, which is defined as

$$R_{2} = \int_{\frac{\tau_{2}}{\rho}}^{\infty} f_{|g_{n,i_{2}}|^{2}}(y) Q_{2} dy$$
  
= 
$$\sum_{s=0}^{m_{i_{1}}-1} \frac{m_{i_{2}}^{m_{i_{2}}} m_{i_{1}}^{s}}{\Omega_{i_{2}}^{m_{i_{2}}}(m_{i_{2}}-1)! \Omega_{i_{1}}^{s} s!} \int_{\frac{\tau_{2}}{\rho}}^{\infty} e^{-\omega y} y^{m_{i_{2}}+s-1} dy, \quad (21)$$

where  $\omega = \frac{m_{i_1}\Omega_{i_2} + m_{i_2}\Omega_{i_1}}{\Omega_{i_1}\Omega_{i_2}}$ . Denote the integral in (21) as  $I_2$ , with the aid of (eq. 3.381.3) and (eq. 8.352.7) in [17],  $I_2$  can be further expressed as

$$I_2 = (m_{i_2} + s - 1)! \,\omega^{-(m_{i_2} + s)} e^{-\frac{\tau_2 \omega}{\rho}} \sum_{q=0}^{m_{i_2} + s - 1} \frac{(\tau_2 \omega/\rho)^q}{q!}.$$
(22)

Substituting (22) into (21), a closed-form analytical expression for  $R_2$  is obtained.

Based on (15), (19) and (21), a closed-form expression for  $P_c^n$  can be derived as

$$\mathbf{P}_{c}^{n} = \sum_{i_{1}, i_{2} \in \{1, 2\}} R_{1} - R_{2}.$$
 (23)

On the other hand, the probability that  $\lambda$  relays are in  $S_n$ for the first stage can be calculated as

$$\Pr\left\{|S_n| = \lambda\right\} = \binom{M}{\lambda} \left( e^{-\frac{\tau_2 m_0}{\Omega_0 \rho}} \sum_{u=0}^{m_0-1} \frac{\left(\frac{\tau_2 m_0}{\Omega_0 \rho}\right)^u}{u!} \right)^{\lambda} \times \left( 1 - e^{-\frac{\tau_2 m_0}{\Omega_0 \rho}} \sum_{u=0}^{m_0-1} \frac{\left(\frac{\tau_2 m_0}{\Omega_0 \rho}\right)^u}{u!} \right)^{M-\lambda}, \quad (24)$$

where the similar CDF of  $h_n$  to (13) has been used.

Consequently, the outage probability expression of the proposed scheme can be summarized in the following theorem.

Theorem 2: The closed-form expression for the outage probability of the proposed RS scheme is given by

$$P_{\text{out}} = \sum_{\lambda=0}^{M} \left(1 - P_{\text{c}}^{n}\right)^{\lambda} \Pr\left\{\left|S_{n}\right| = \lambda\right\}.$$
 (25)

Proof: Note that the overall outage probability of considered NOMA networks can be written as

$$P_{\text{out}} = \sum_{\lambda=0}^{M} \underbrace{\Pr\left\{\min\left\{R_{c}^{n^{*}}, \hat{R}_{c}^{n^{*}}\right\} < R_{c} \mid |S_{n}| = \lambda\right\}}_{W} \Pr\left\{|S_{n}| = \lambda\right\},$$
(26)

where W can be further expressed as

$$W = \left(1 - \Pr\left\{R_c^n \ge R_c \mid |S_n| = \lambda\right\} \Pr\left\{\hat{R}_c^n \ge R_c \mid |S_n| = \lambda\right\}\right)^{\lambda}$$
$$= \left(1 - \Pr\left\{\hat{R}_c^n \ge R_c \mid |S_n| = \lambda\right\}\right)^{\lambda} = (1 - \Pr_c^n)^{\lambda}, \quad (27)$$

substituting (27) into (26), the proof is thus completed.

It can be found that the statistics of all channels and two target rates have an impact on the outage probability of the proposed scheme. Afterwards, based on Theorem 2, the diversity order of the proposed RS scheme can be obtained by assuming  $\rho \to \infty$ , which is given in the following theorem.

Theorem 3: The diversity order of the proposed RS scheme is  $d = -\lim_{\rho \to \infty} (\log P_{out}) / (\log \rho) = M \min \{m_0, m_1, m_2\}.$ *Proof:* When  $\rho \to \infty$ , the following approximate expressions can be obtained:

$$R_{1} \approx \sum_{r=0}^{m_{i_{1}}-1} \frac{\left(\frac{m_{i_{1}}\varepsilon_{f}}{\Omega_{i_{1}}\rho}\right)^{r}}{r!} \frac{m_{i_{2}}m_{i_{2}}e^{-\frac{m_{i_{1}}\varepsilon_{f}}{\Omega_{i_{1}}\rho}}}{\Omega_{i_{2}}m_{i_{2}}(m_{i_{2}}-1)!} \int_{\frac{\tau_{2}}{\rho}}^{\infty} e^{-\frac{m_{i_{2}}y}{\Omega_{i_{2}}}} y^{m_{i_{2}}-1} dy$$

$$\stackrel{a}{\approx} 1 - \frac{\left(\frac{m_{i_{1}}\varepsilon_{f}}{\Omega_{i_{1}}\rho}\right)^{m_{i_{1}}}}{m_{i_{1}}!} - \frac{\left(\frac{m_{i_{2}}\tau_{2}}{\Omega_{i_{2}}\rho}\right)^{m_{i_{2}}}}{m_{i_{2}}!} + \frac{\left(\frac{m_{i_{1}}\varepsilon_{f}}{\Omega_{i_{1}}\rho}\right)^{m_{i_{1}}}\left(\frac{m_{i_{2}}\tau_{2}}{\Omega_{i_{2}}\rho}\right)^{m_{i_{2}}}}{m_{i_{1}}!m_{i_{2}}!},$$
(28)

$$R_2 \approx \sum_{s=0}^{m_{i_1}-1} \frac{m_{i_2}m_{i_1}s(m_{i_2}+s-1)!}{\Omega_{i_2}m_{i_2}(m_{i_2}-1)!\Omega_{i_1}s!} \omega^{-(m_{i_2}+s)}, \quad (29)$$

$$\Pr\left\{|S_n| = \lambda\right\} \approx \begin{pmatrix} M \\ \lambda \end{pmatrix} \left(\frac{\left(\frac{\tau_2 m_0}{\Omega_0 \rho}\right)^{m_0}}{m_0!}\right)^{M-\lambda}, \quad (30)$$

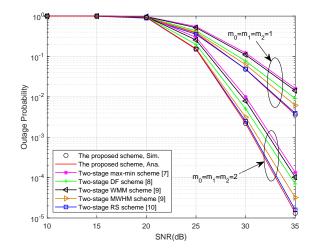


Fig. 2: Outage probability versus the transmit SNR (M = 2, $d_0 = d_2 = 1$  meter (m),  $d_1 = 2$  m,  $R_f = 1$  bit per channel use (BPCU),  $R_c = 2.5$  BPCU, and different  $m_0, m_1, m_2$ ).

where step a is obtained with the aid of (eq. 3.381.3) in [17], and  $e^{-x} \approx 1 - x$  has been used when  $x \to 0$ . Substituting (28)-(30) into (25), We have

$$P_{\text{out}} \propto \frac{1}{\rho^{m_0(M-\lambda)+\min\{m_{i_1},m_{i_2}\}\lambda}} \propto \frac{1}{\rho^{M\min\{m_0,m_1,m_2\}}}.$$
(31)  
s a result Theorem 3 is proved

As a result, Theorem 3 is proved.

Remark 2: One can find that maximal diversity order can be achieved by the proposed scheme, which is determined by the number of relays and the minimum value of all channels' fading parameters. Despite that the diversity orders of other RS schemes [7]–[10] were not discussed under Nakagami-m fading, it is observed that the same full diversity order can be achieved by these schemes through simulations. However, the proposed RS scheme reaches lower outage probability than the ones in [7]–[9], and outperforms the RS scheme in [10] for some cases, which will be verified in Section V.

### V. SIMULATION RESULTS

In this section, we conduct simulations to evaluate the outage performance of several RS schemes in cooperative NOMA networks under Nakagami-m fading channels. The two-stage max-min RS scheme in [7], two-stage DF RS scheme in [8], two-stage weighted-max-min (WMM) RS scheme and max-weighted-harmonic-mean (MWHM) RS scheme in [9], and two-stage RS scheme in [10] are taken for comparisons with the proposed RS scheme. Specifically, it is assumed that  $\Omega_i = d_i^{-\alpha}, i = 0, 1, 2$ , where  $d_i$  denotes the distance of corresponding link,  $\alpha$  denotes the path loss exponent and  $\alpha = 3$  is set here. Moreover, the PA coefficients for fixed PA based RS schemes are set as  $\gamma_{n,f} = \alpha_{n,f} = 0.8$ , and G-C complexity-accuracy tradeoff parameter is set as N = 100. Without loss of generality, for the RS schemes in [7]–[10],  $R_f$  is adopted by  $D_1$  and  $R_c$  is adopted by  $D_2$ , respectively.

Fig. 2 plots the outage probability versus the transmit SNR in different fading parameters. It is shown that the proposed RS scheme outperforms the RS schemes in [7]-[9], and achieves the similar performance to that in [10]. This is due to the

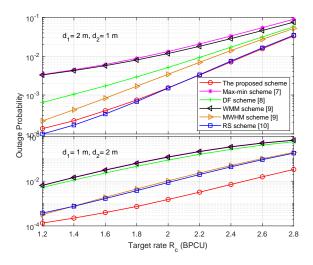


Fig. 3: Outage probability versus the target rate  $R_c$  (M = 3,  $m_0 = m_1 = m_2 = 1$ ,  $R_f = 1$  BPCU,  $\rho = 30$  dB,  $d_0 = 1$  m and different  $d_1, d_2$ ).

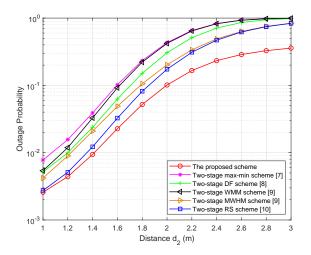


Fig. 4: Outage probability versus the distance  $d_2$  (M = 3,  $m_0 = 1$ ,  $m_1 = m_2 = 2$ ,  $R_f = 1$  BPCU,  $R_c = 2.5$  BPCU,  $d_0 = 1$  m,  $d_1 = 2$  m, and  $\rho = 30$  dB).

reason that in order to maximize the sum rate, the ICSI has been used in the joint design of user ordering and PA strategy for the proposed scheme, which benefits the outage performance, as concluded in [10]. Moreover, It is observed that all the curves under the identical  $m_0$ ,  $m_1$  and  $m_2$  have the same slope, which means that the same diversity order can be achieved. In addition, the system under Rayleigh fading ( $m_k = 1$ , k = 0, 1, 2) exhibits a poorer performance than that under Nakagami-m fading, which demonstrates the necessity of investigating Nakagami-m fading.

Fig. 3 shows the outage probability versus the target rate  $R_c$  in different  $d_1$  and  $d_2$ . It can be seen that the outage probability increases with  $R_c$ . Furthermore, It is observed that the situation that  $d_2$  is larger than  $d_1$  leads to the better outage performance of the proposed RS scheme than the RS scheme in [10]. The reason for this performance gain is that in the proposed scheme, the weak user is always associated with the lower target rate constraint, thus more remaining

power can be allocated to the strong user to meet the higher target rate requirement; while in [10], the weak user may be associated with the higher target rate constraint, which leads to a noteworthy power loss for the strong user to meet another target rate requirement. The outage probability versus the distance  $(d_2)$  can be illustrated more intuitively in Fig. 4. One can find that the outage probability increases with  $d_2$ , and the performance gap between the proposed scheme and the RS scheme in [10] enlarges with  $d_2$  as expected.

# VI. CONCLUSIONS

We have proposed a new sum-rate maximization based RS scheme for cooperative NOMA networks under Nakagami-m fading in this paper, where each user has two alternative target rates. A closed-form outage probability expression and the diversity order of the proposed scheme have been derived to characterize the performance. Moreover, simulation results have been presented to compare the proposed RS scheme with other RS schemes in terms of outage probability.

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