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Optimization of Water Distribution Networks Using a Deterministic Approach

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Abstract

Water Distribution Networks (WDN) are the main component of industrial and urban water distribution systems and are currently formed by pipes, nodes, and loops. In the present paper a deterministic Mathematical Programming approach is proposed, aiming to minimize the cost of looped WDN, considering known pipe lengths and a discrete set of available commercial diameters. The optimization model constraints are mass balances in nodes, energy balances in loops and hydraulic equations, in such a way that no additional software is needed to find the appropriated pressure drops and water velocities. Generalized Disjunctive Programming is used to reformulate the discrete optimization problem to a Mixed Integer Non-Linear Programming (MINLP) problem. GAMS (General Algebraic Modeling System) environment is used to solve the problem. Four cases were studied to test the model applicability and the results show compatibility with the literature.

Keywords: Water Distribution Networks, Optimization, Generalized Disjunctive Programming, MINLP, GAMS.

1. Introduction

Water is a natural resource of fundamental importance for the life in our planet and water supply services are part of the basic necessities in the contemporary urban centers and industries, in the industrial society. Water supply systems involve water acquisition, treatment, and distribution to the final users. Pumping stations are responsible by the water network pressurization with the aid of elevated reservoirs. In general, these systems are complex and must consider the topology heterogeneity in the zones to be supplied and must attend simultaneously adequate water flow rates, pressures and velocities along the network with proper quality, considering structures, piping and equipment design.

A water distribution network (WDN) is composed of a set of special equipment intended to transport water to the demand points in a continuous and safe way. The main used devices are pipes, pumps, valves, reservoirs, meters, among other accessory destined to do this job.

Looped WDN can be represented by a set of connected nodes and branches. Nodes can represent reservoirs or demand points and branches represent pipes, valves or pumps. It is important to consider the flow direction and if loops exist or not. It can be situated at different levels if the city relief is rugged. Generally main conducts have large diameters to feed secondary branches and to achieve the demand points. To the water distribution network (WDN) optimization problem, two different approaches can be used. The first one considers a unique diameter per pipe and is known as Single pipe. The second approach, known as Split-pipe, considers one or more diameters in the network.

Most of the published papers in WDN optimization, like the ones that will be cited in this section, consider the minimization of the pipe diameters cost, subject to mass balances in the network nodes, energy balances in the network loops and pressure and velocity limits. In general, the diameters are chosen among a set of commercial ones. The hydraulic problem of finding the adequate node pressures and pipe velocities is a complex non-linear optimization problem, mainly due to the complex correlations available in the literature to calculate the pressure drop in pipes.

In the present work, the main objective is to achieve the optimal piping commercial diameters configuration for the WDN. The Single pipe approach is used to avoid solutions with a large number of diameters in the length pipe. Besides, additional pressure drop can exist in the pipe junctions and must be significative if a large number of different diameters exist in the pipe length. An MINLP optimization model is proposed and GAMS (General Algebraic Modeling System) environment is used to solve the problem, without the use of hydraulic simulators in order to calculate pressure drops and velocities. GAMS is a high-level modeling system, very appropriated in solving mathematical programming and optimization problems. It consists of a language compiler and a set of high-performance solvers. In GAMS it is possible to model linear, nonlinear and mixed-integer optimization problems.

Four case studies with different levels of complexity were used to test the model applicability.

Research development in Water Distribution Networks (WDN) is increasing and in the last decades, several approaches have been published in this important field of research. WDN design can be formulated as an optimization problem and involves discrete variables if a set of available commercial diameters and pipe lengths is considered. Also, the problem is nonlinear and nonconvex and global optimization techniques are not commonly able to solve it. So, the development of new models is, certainly, a substantial contribution to solving this type of problem.

WDNs are a vital part of the water supply systems and represent one of the largest infrastructure assets. Simulation of hydraulic behavior within a pressurized and looped pipe network is not a trivial task, once it means effectively solving a system of non-linear equations. Hydraulic simulators have been used to solve the problem and the most used is EPANET (Rossman, 2000).

Stochastic and deterministic approaches have been used to solve the WDN optimization problem. In deterministic optimization approaches the output is determined by the parameter values and initial conditions. The same parameters and initial values will always provide the same results. In stochastic approaches, the output is the result of a random search procedure and the same set of parameters can lead to different results. Linear Programming (LP), Nonlinear Programming (NLP), Mixed Integer Linear Programming (MILP) and Mixed-Integer Nonlinear Programming (MINLP) formulations were used in solving WDN optimization problems. The Nonlinear Programming formulations are more representative for the real WDN optimization problems, according to the existing hydraulic calculation equations. In the majority of the studied cases, the problems are also nonconvex. In these cases, it is not possible to ensure optimal global solutions.

The pioneering works focusing on deterministic approaches used Linear Programming. Alperovits and Shamir (1977) applied the method known as Linear Programming Gradient (LPG) to the WDN optimization. They presented a well-known case study named Two Loop WDN, one of the most used benchmark problems in this area. Morsi *et al.* (2012) proposed a MILP model to solve WDN systems and a Branch and Bound approach was used. With good upper and lower bounds their method is able to find good solutions.

Watanatada (1973) proposed an NLP formulation to solve the WDN design problem and equality constraints corresponding to the mass and energy conservation equations and inequality constraints corresponding to physical limits were used in the model development. Bragalli *et al.* (2008), Bragalli et al. (2012) and D'Ambrosio *et al.* (2014) proposed MINLP models to solve the WDN optimization problem and the solver Bonmin was used, in the environment AMPL - A Mathematical Programming Language (Fourer et al., 2003), in the first work and spatial branch and bound and piecewise linear relaxations was used in the second work.

Caballero and Ravagnani (2019) presented an MINLP model to the optimal design of WDN when the flow directions are unknown. A convex hull reformulation was used in the model and the global optimization solver BARON was used to solve the problem. Two case studies were used to test the model and results showed compatibility with the literature.

Because of the great difficulties in solving MINLP models for WDN optimization problems with large scale using deterministic methods, the majority of published papers in this area use metaheuristic methods. Genetic Algorithms (GA), in Savic and Walters (1997) and Kadu *et al.* (2008), Ant Colony Optimization (ACO) in Zecchin *et al.* (2006), Honey Bee Mating Optimization (HBMO), in Mohan and Babu (2009), Harmony Search (HS), in Geem (2006), Particle Swarm Optimization (PSO), in Ezzeldin *et al.* (2014) and Surco *et al.* (2017) and Simulated Annealing (SA), in Cunha and Sousa (1999), among others methods have been used.

De Corte and Sorensen (2016) presented an overview of the metaheuristic techniques developed for the WDN design optimization problem. However, most of the published papers using metaheuristic techniques use also a hydraulic simulator to solve pressure drop and velocity equations, like EPANET (Rassman, 2000).

2. Material and method

The design of a WDN must consider the optimal tube diameters to carry water from one or more reservoirs to the demand points at adequate pressures and velocities. In the present paper, an optimization model is proposed, based on the papers of Surco *et al.* (2017) and Surco *et al.* (2018) to solve the problem.

2.1 Optimization model

The optimization model can be formulated as the minimization of the network cost considering a set of pipe commercial diameters, constrained to mass balances in the nodes and energy balances in the loops of the WDN. Each element of this set has a cost per length unity and a specific rugosity.

The following sets, parameters, and variables are defined:

Sets:

Diameters	Commercially available diameters
Pipes	WDN pipes
Nodes	WDN nodes
FI_k	Pipes with flow entering node k
FO_k	Pipes with flow going out node k
Loops	WDN loops
PPD_γ	Positive pressure drops
NPD_γ	Negative pressure drops
Pumps	WDN pumps
τ_k	Pipes in which a flow path is determined, beginning in the reservoir and finishing in node k

Parameters:

L_j	Pipes length (m)
$dmd(j)$	Nodes demand (m^3/h)
$E_P^{\eta}(\gamma)$	Pump energy (m)
C_j	Hazen-Williams rugosity coefficient
α, β and ω	Hazen-Williams equation constants
$pr_{min}(k)$	Minimum allowed pressure (m)
$elv(k)$	Node elevation k (m)
$elv(re)$	Reservoir elevation (m)
v_{min}, v_{max}	Minimum and maximum velocities (m/s)
n_d	Number of available diameters
n_n	Number of nodes
D_i	Diameter (m)
$Cost(D_i)$	Cost ($/m)
R_i	Pipe rugosity

Variables:

x_j	Pipe diameter (m)
$Cost(x_j)$	Pipe cost ($/m)
C_T	Total cost ($)
q_j	Volumetric flowrate (m^3/s)
$h_f(j)$	Pressure loss (m)
$pr(k)$	Pressure (m)
v_j	Water velocity (m/s)
$Y_{i,j}$	Boolean variable
λ_j	Cost ($)
σ_j	Pipe j rugosity coefficient
$y_{i,j}$	Binary variable

In the model, $Diameters = \{D_1, D_2, \ldots, D_{nd}\}$, and the following inequality must be respected: $D_{min} = D_1 < \cdots < D_{nd} = D_{max}$.

The objective function must consider the sum of all tube diameters and its costs:

(1)

$$C_T = \sum_{j \in Pipes} L_j Cost(x_j)$$

where C_T is the total installation cost, L_j is the j pipe length, x_j is the j tube diameter, *Pipes* is the set of links between nodes and $Cost\ (x_j)$ is the j pipe cost per length.

The network pressurized design problem consists in solving simultaneously the continuity and the pressure drop equations. Consider *Nodes* the set of nodes and k = 1,..., n_n the elements of the set, being n_n the number of nodes in the WDN. The model constraints are:

1) Material balance:

The difference between the node inlet flow rate and the node outlet flow rate must be equal to the node demand. Consider FI_k and FO_k the sets in which the elements are the branches corresponding to the inlet and outlet flow rates in the node k, respectively, in the flow direction:

$$FI_k = \{ j \mid q_j\, flow\ rate\ that\ enters\ node\ k \}$$

$$FO_k = \{ j \mid q_j\, flow\ rate\ that\ leaves\ node\ k \}$$

$$\sum_{j \in FI_k} q_j - \sum_{j \in FO_k} q_j = dmd(k), \qquad \forall\, k \in Nodes \tag{2}$$

where q_j is the branch j flow rate and $dmd(k)$ is the node k demand.

2) Energy balance:

Consider *Loops* the set of loops γ in the WDN and PPD_γ and NPD_γ, the sets where the pressure drops are positive and negative in the loop γ, respecting the flow rate direction in each loop:

$$PPD\gamma \ = \ \{j \mid is\ the\ positive\ pressure\ drop\ hf(j)\ belonging\ to\ the\ loop\ \gamma\}, \forall\ j \in Pipes$$

$$NPD\gamma \ = \ \{j \mid is\ the\ negative\ pressure\ drop\ hf(j)\ belonging\ to\ the\ loop\ \gamma\}, \forall\ j \in Pipes.$$

Consider, also, that *Pumps* is the set of η pumps in the network if they exist.

The sum of pressure drops in the branches belonging to a loop must be equal to the energy liberated by a pump, in case it exists:

$$\sum_{j \epsilon PPD_\gamma} h_f(j) - \sum_{j \epsilon NPD_\gamma} h_f(j) = \sum_{\eta \epsilon Pumps} E_P^\eta(\gamma)\,, \qquad \forall\, \gamma \in Loops \tag{3}$$

where $h_f(j)$ is the pressure drop in the pipe j and $E_P^\eta(\gamma)$ is the energy of the pump η in the loop γ.

The pressure in any point in the piping network must attend a minimum limit:

$$pr_{min} \ \leq \ pr(k), \qquad \forall\ k \ \in Nodes \tag{4}$$

where $pr_{min}(k)$ is the minimum pressure limit in the node k and $pr(k)$ is the pressure in the node k.

The flow velocities must also attend minimum and maximum limits:

$$v_{min} \ \leq \ v_j \ \leq \ v_{max}, \qquad \forall\, j \ \in Pipes \tag{5}$$

where v_{min} and v_{max} are the minima and maximum velocities allowed in the WDN and v_j is the water velocity in the pipe j.

The most used equation to the pressure drop calculations is the Hazen-Williams equation:

$$h_{f(j)} = \frac{\omega q_j^\alpha L_j}{C_j^\alpha D^\beta}, \qquad \forall\ j \ \in Pipes \tag{6}$$

where C is the Hazen-Williams rugosity coefficient and is non-dimensional. The parameters ω, α and β depend on the unities system being used and can vary in the literature. Savic and Walters (1977) presented a series of different equations and coefficients, with the most used unity systems used in this type of problem.

To the pressure drop calculations, it must be considered the existing pressure in each node. Consider τ_k a set in which is defined a flow path, initiating in the reservoir and finishing in the node k, $k \in Nodes$. If the node $k = k_r$ corresponds to the reservoir, then its pressure is:

$$pr(k_r) = elv(k_r) = elv(re) \tag{7}$$

$$else,$$

$$pr(k) = -\sum_{j\in\tau_k} h_f(j) + [elv(re) - elv(k)], \qquad \forall\, k \in Nodes, k \neq kr$$

where $elv(k)$ is the altimetric quota of each node k, i.e., the elevation of each node and $elv\ (re)$ is the reservoir elevation.

The flow velocities can be calculated by:

$$v_j = \frac{4q_j}{\pi D_j^2}, \quad \forall\, j \in Pipes. \tag{9}$$

The optimization model can be described as:

$$\min C_T = \sum_{j\in Pipes} L_j Cost(x_j), \qquad \forall\, x_j \in Diameters$$

$$s.t. \qquad \sum_{j\in FI_k} q_j - \sum_{j\in FO_k} q_j = dmd(k), \qquad \forall\, k \in Nodes \tag{10}$$

$$\sum_{j\in PPD_\gamma} h_f(j) - \sum_{j\in NPD_\gamma} h_f(j) = \sum_{\eta\epsilon Pumps} E_P^\eta(\gamma), \qquad \forall\, k \in Nodes$$

$$pr_{min}(k) \leq pr(k), \qquad \forall\, k \in Nodes$$

$$v_{min} \leq v_j \leq v_{max}, \forall\, j \in Pipes$$

2.2 MINLP reformulation

Given $j \in Pipes$, the pipes sequence and y_i^j the binary variables associated to the Boolean variable $Y_{i,j}$ for the pipe j and diameter D_i. Given, also, λ_j and σ_j, are the cost and the rugosity associated to the same diameter, in such a way that:

$$\begin{aligned} y_i^j &\in \{0,1\}, \quad \forall\, i \in Diameters\ and\ \forall\, j \in Pipes \\ \lambda_j &= L_j\, Cost(D_i), \quad \forall\, i \in Diameters\ and\ j \in Pipes \\ \sigma_j &= R_i, \quad \forall\ i \in Diameters\ and\ j \in Pipes \end{aligned} \tag{11}$$

$$\underset{i \in \{1,\dots,n_d\}}{\vee} \begin{bmatrix} Y_{i,j} \\ x_j = D_i \\ \lambda_j = L_j Cost(D_i) \\ \sigma_j = R_i \end{bmatrix}$$

The hull reformulation of the previous disjunction, according to Trespalacios and Grossmann (2015) yields the following equations:

$$\begin{aligned} x_j &= \textstyle\sum_{i \in Diameters} D_i\, y_{i,j}\ \forall\, i \in Diameters\ and\ j \in Pipes \\ \lambda_j &= L_j \sum_{i \in Diameters} Cost(D_i)\, y_{i,j}, \forall\, i \in Diameters\ and\ j \in Pipes \\ \sigma_j &= \sum_{i \in Diameters} R_i\, y_{i,j}, \forall\, i \in Diameters\ and\ j \in Pipes \end{aligned} \tag{12}$$

So, the reformulated MINLP is:

$$\min C_T = \sum_{j \in Pipes} \lambda_j$$

$$s.t. \qquad \sum_{j \in FI_k} q_j - \sum_{j \in FO_k} q_j = dmd(k), \qquad \forall\, k \in Nodes \tag{13}$$

$$\sum_{j \in PPD_\gamma} h_f(j) - \sum_{j \in NPD_\gamma} h_f(j) = \sum_{\eta \epsilon Pumps} E_P^\eta(\gamma), \qquad \forall\, \gamma \in Loops$$

$$pr_{min}(k) \leq pr(k), \qquad \forall\, k \in Nodes$$

$$v_{min} \leq v_j \leq v_{max}, \forall\, j \in Pipes$$

$$x_j \in Diameters = \{D_1, \ldots, D_{nd}\}$$

$$x_j = \sum_{i \epsilon Diameters} D_i y_{i,j}, \forall\, j \in Pipes$$

$$\lambda_j = L_j \sum_{i \epsilon Diameters} Cost(D_i) y_{i,j}, \forall\, j \in Pipes$$

$$\sigma_j = \sum_{i \epsilon Diameters} R_i y_{i,j}, \forall\, j \in Pipes$$

Figure 1 presents a block diagram to better understand the optimization model and a simultaneous approach to solve the problem.

2.3 Case studies

To test the model applicability, four case studies from the literature with different levels of complexity were used. In this section, the cases will be introduced and the results and discussions will be presented in the next section.

2.3.1 Case Study 1

Case study 1 is the well-known benchmark problem, named *Two Loop* WDN. This network can be classified as a small case and was originally proposed by Alperovits and Shamir (1977). Figure 2 presents the WDN topology with flow directions. This WDN has two loops, 1 reservoir, 8 links (1,000 m length each one) and 7 nodes. The minimum pressure required in each node is 30 water column meters and this value can be considered as the lower bound for the pressure in the

WDN nodes. Velocity limits are 0.3 m/s and 3 m/s, respectively. Hazen-Williams coefficients are ω = 10.667, α = 4.871 and β = 1.852 and the dimensionless roughness coefficient C is 130 for all links. The diameters set is composed by 14 types of diameters (mm): D = {25.4, 50.8, 76.2, 101.6, 152.4, 203.2, 254.0, 304.8, 355.6, 406.4, 457.2, 508.0, 558.8, 609.6}. Table 1 presents costs for the specified diameters and Table 2 presents the nodes elevation and demands.

2.3.2 Case Study 2

The second case study considered in the present paper was presented originally by Gomes *et al.* (2009). In the network there exists 72 pipes, 61 demand nodes, 1 reservoir, and 11 loops. Figure 3 presents the WDN topology.

Table 3 presents diameters, Hazen-Williams roughness coefficient and costs. The diameters set is composed by 10 elements. Table 4 presents demands, elevations and minimum pressures in each one of the nodes. Table 5 presents tube lengths.

2.3.3 Case Study 3

In this case study it is considered a WDN with two reservoirs, known as Two Source WDN. The problem was originally proposed by Kadu *et al.* (2008) and the network has 26 nodes, 34 links and 9 loops. Figure 4 presents the network topology. A Hazzen-Williams rugosity coefficient of 130 is supposed for all pipes. Table 6 presents the diameters set, with the respective costs and lengths. Table 7 presents nodes demand and the minimum allowed pressure. Table 8 presents link lengths.

2.3.4 *Case Study* 4

This case study was first published by Carvalho (2007) and is known as Bessa WDN. Figure 5 presents the network topology, with flowrate directions, tubes length and nodes elevation and demands. The reservoir is at 54 m. The set with 10 available commercial diameters, with the correspondent costs and rugosity coefficients, is presented in Table 9. Velocities are between v_{min} = 0,3 m/s and v_{max} = 3 m/s and the minimum nodes pressure is 25 m. Hazzen-Williams coefficients are ω = 10.674, α = 4.871 and β = 1.852.

3 Results and Discussion

Case Study 1 was solved in GAMS using the global optimization solver BARON, and Table 10 presents the results for velocities and pressure drops for each pipe. Table 11 presents a comparison of the achieved results for the optimal diameters and the total WDN cost with some literature works using a single pipe approach. With the exception of the work of Suribabu (2012), whose optimal value was $ 420,000, the global optimum solution of $ 419,000 was achieved. This solution is exactly the same obtained by other researchers, even with different values of *w*, like Surco *et al.* (2017), Zhuo *et al.* (2016), Eryigit (2015), Ezzeldin *et al.* (2014) and Geem (2006), among others authors. In the present paper, a single pipe approach is used, different from the Alperovits and Shamir (1977), who used a split pipe approach.

In Case Study 2, the model was solved in GAMS, using the solver SBB. Table 12 presents the pressures in the nodes and Table 13 presents a comparison with the final results and some literature papers. As it can be observed in Table 13, the result achieved in the present paper is the best, among the published two ones. Other important information is that no additional software was used in finding the best solution, contrary to the papers used in order to compare the results, Gomes *et al.* (2009) and Surco *et al.* (2017), which used EPANET2 as a hydraulic simulator. As SBB is not a global optimization solver, it is not possible to ensure that this is the global optimum.

In Case Study 3, the model was solved in GAMS and the solver SBB was used. Table 14 presents the results obtained and a comparison with other solutions in the literature. In order to compare our results with the literature, two distinct values for the Hazzen-Williams ω parameter were used, $\omega = 10.667$ and $\omega = 10.6744$. In both cases, the solutions achieved in the current paper are better than published in the literature. Table 15 presents calculated nodes pressures when compared to the values reported by Kadu *et al.* (2008). In the papers of Suribabu (2012) and Ezzeldin *et al.* (2014) pressure values are not presented. It is interesting to relate that Kadu *et al.* (2008) used the values $\omega = 2.234\text{x}10^{12}$, $\alpha = 1.85$ and $\beta = 4.87$ (q in m^3/min and D in mm) and obtained a WDN cost of R$ 123,268,864. Suribabu (2012) used $\omega = 10.667$, $\alpha = 1.852$ and $\beta = 4.871$ (q in m^3/min and D in mm) and the WDN cost obtained was R$ 140,177,210. However, the author reported in his paper that the result of Kadu *et al.* (2008) exhibits a deficit in hydraulic gradient level at node 26 when analyzed under EPANET software. This point was also reported by Ezzeldin *et al.* (2012), who recalculated the results of Kadu et al. (2008) and, considering the pressure violation, reported a new cost of R$ 126,368,865.

In Case Study 4 the problem was solved using the solver SBB in GAMS. Table 16 presents the results obtained and a comparison with the work of Carvalho (2007). As can be observed, the results achieved with the current approach were better than the ones presented in the literature.

4 Conclusions

In the present paper, the synthesis of WDN was formulated as an optimization problem. An MINLP model was proposed and solved using a deterministic Mathematical Programming approach. Four case studies were used to test the model applicability. Two of them are benchmark problems, one with a unique reservoir and two loops (Two Loop WDN) and other considering the existence of two reservoirs (Two Source WDN). The other two cases study belong to real cases, in small cities in Brazil (R9 WDN and Bessa WDN).

In all cases, the mathematical model was solved in GAMS. For the first (Two Loop WDN) problem, the solution corresponds to the global optimum, being the same value found by other authors. The result was achieved using the global optimization solver BARON. The other three cases were solved using the solver SBB and have distinct degrees of complexity. The solutions for all problems are the best found in the literature.

The great novelty and the main contribution of this paper is that the model developed is able to calculate pressures and velocities, without the use of additional software. In all case studies results were compared with literature, with papers that used additional software to the hydraulic calculus and results are compatible, in the case study 1 and better, in cases 2, 3 and 4. Besides, this independence of extra software was combined with the best results found when compared to the papers published in the literature. Also, the model includes only integer and continuous variables and no approximations are used to find the diameter values in the discrete set of available commercial diameters.

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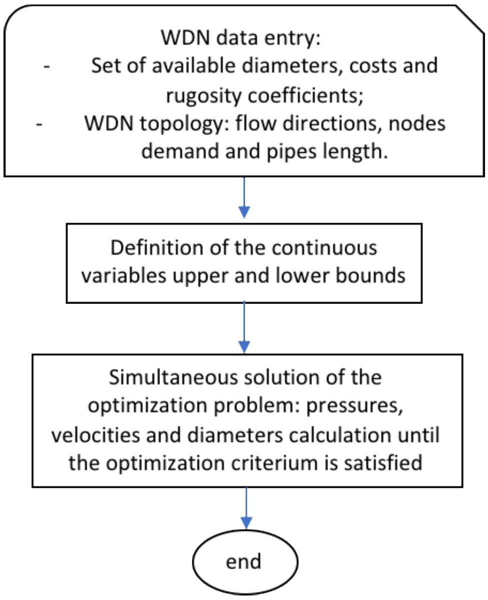

Figure 1 – Optimization problem block diagram

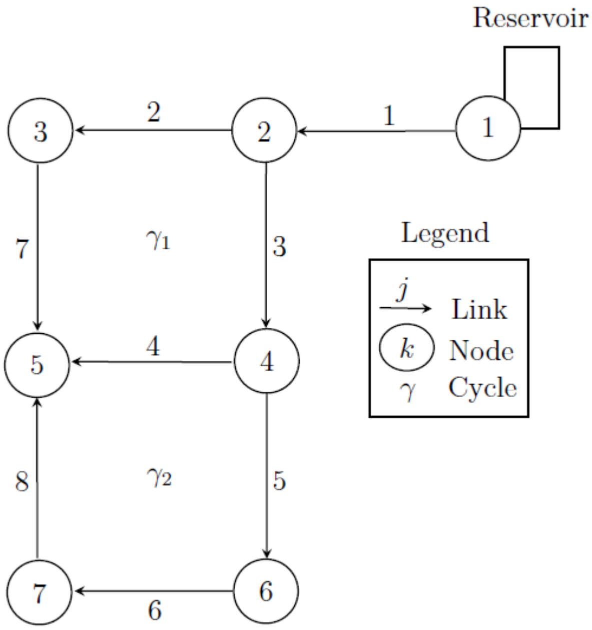

Figure 2 – Two Loop WDN

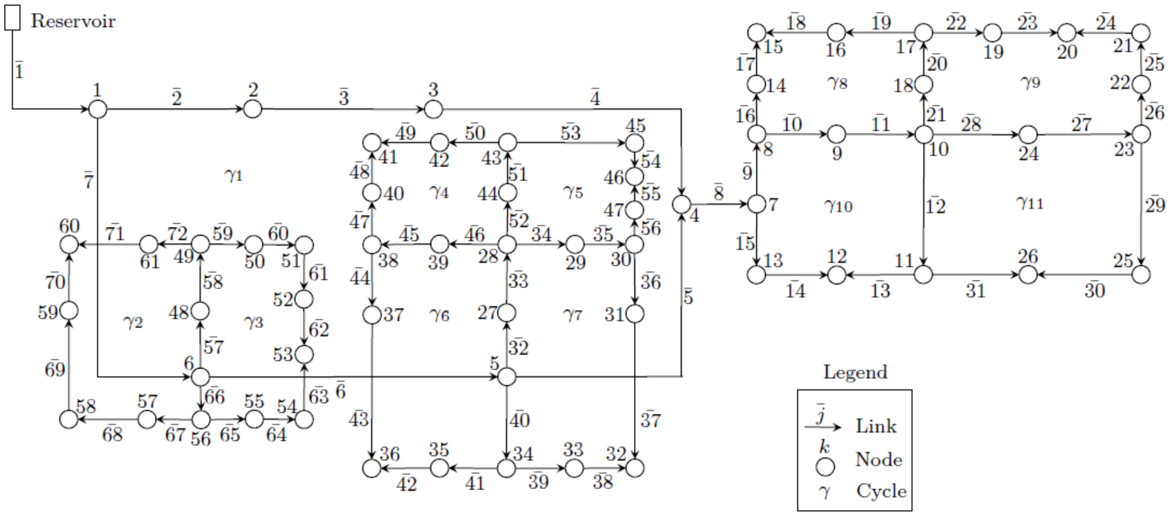

Figure 3 – R9 WDN

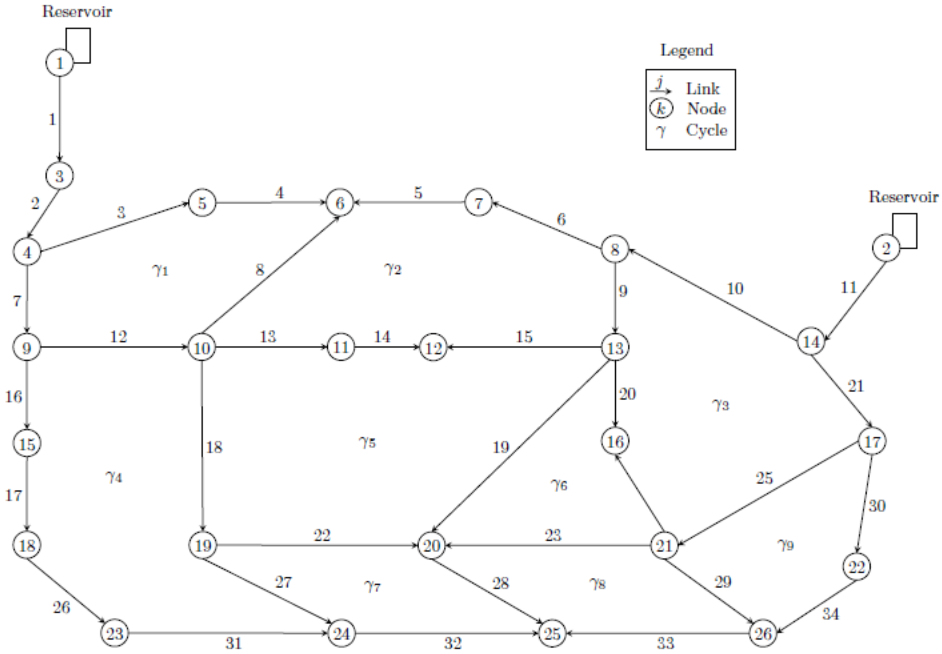

Figure 4 – R9 network topology

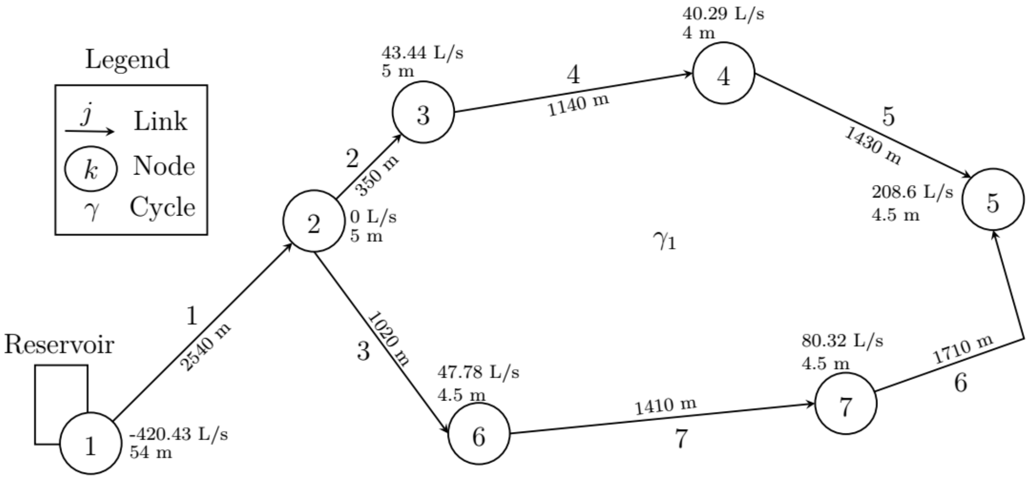

Figure 5 –Bessa WDN

Table 1 – Costs for the commercial available diameters set

Diameter (*mm*)	Cost ($/*m*)
25.4	2
50.8	5
76.2	8
101.6	11
152.4	16
203.2	23
254.0	32
304.8	50
355.6	60
406.4	90
457.2	130
508.0	170
558.8	300
609,6	550

Table 2 – Nodes characteristic for the Two Loop WDN

Node	Elevation (*m*)	Demand (m^3/h)
1	210	-1120
2	150	100
3	160	100
4	155	120
5	150	270
6	165	330
7	200	160

Table 3 – Available diameters, Hazen-Williams roughness coefficients and costs for the Gomes *et al.* (2009) WDN

Diameter (*mm*)	Hazzen-Williams roughness coefficient C	Cost (*US*$/*m*)
100	145	17.98
150	145	44.75
200	145	63.68
250	145	85.19
300	130	101.95
350	130	121.55
400	130	136.83
450	130	171.09
500	130	195.21
600	130	255.32

Table 4 – Nodes demands, elevations and minimum pressures in the for the Gomes *et al.* (2009) WDN

Node	Demand (*L/s*)	Elevation (*m*)	Minimum Pressure (*m*)	Node	Demand (*L/s*)	Elevation (*m*)	Minimum Pressure (*m*)
1	2.51	5.0	25	31	4.94	3.5	15
2	44.07	5.0	25	32	4.09	4.5	15
3	41.24	4.0	25	33	3.68	5.0	15
4	1.04	4.5	25	34	4.04	5.0	15
5	0.86	4.5	25	35	3.22	6.0	15
6	1.32	4.5	25	36	2.53	4.5	15
7	1.35	4.5	15	37	2.31	4.5	15
8	8.59	5.0	15	38	2.50	4.0	15
9	6.40	4.5	15	39	2.89	4.0	15
10	6.07	5.0	15	40	2.48	4.0	15
11	4.95	3.5	15	41	4.61	4.0	15
12	8.38	3.5	15	42	3.47	4.0	15
13	11.70	3.5	15	43	3.61	4.0	15
14	5.63	5.0	15	44	5.17	4.0	15
15	5.57	6.0	15	45	6.48	4.0	15
16	6.30	6.0	15	46	4.91	4.5	15
17	3.26	6.0	15	47	6.50	4.0	15
18	3.60	6.0	15	48	4.97	4.5	15
19	4.83	6.0	15	49	2.97	3.0	15
20	4.50	6.0	15	50	1.80	5.0	15
21	2.80	5.0	15	51	2.96	4.0	15
22	5.46	3.0	15	52	4.66	3.0	15
23	62.45	3.5	15	53	4.54	4.5	15
24	8.19	6.0	15	54	8.80	4.5	15
25	58.87	3.5	15	55	4.26	4.5	15
26	3.26	3.5	15	56	2.98	5.0	15
27	4.36	4.3	15	57	3.91	5.0	15
28	4.25	4.0	15	58	3.70	4.7	15
29	4.56	2.5	15	59	1.86	5.0	15
30	8.32	2.5	15	60	3.12	5.0	15
				61	3.52	4.5	15

Table 5 – Pipe Lengths for the Gomes *et al.* (2009) WDN

Pipe	Length (*m*)	Origin	Destination	Pipe	Length (*m*)	Origin	Destination
1	2540	---	1	37	285	31	32
2	350	1	2	38	210	38	33
3	1140	2	3	39	240	34	33
4	1430	3	4	40	250	5	34
5	1020	5	4	41	340	34	35
6	1430	6	5	42	270	35	36
7	1710	1	6	43	240	37	36
8	220	4	7	44	160	38	37
9	190	7	8	45	260	39	38
10	295	8	9	46	250	28	39
11	390	9	10	47	330	38	40
12	370	10	11	48	230	40	41
13	190	11	12	49	385	42	41
14	310	13	12	50	160	43	42
15	205	7	13	51	330	44	43
16	305	8	14	52	210	28	44
17	295	14	15	53	150	43	45
18	300	16	15	54	255	45	46
19	290	17	16	55	260	47	46
20	180	18	17	56	230	30	47
21	315	10	18	57	115	6	48
22	300	17	19	58	180	48	49
23	295	19	20	59	140	49	50
24	215	21	20	60	215	50	51
25	140	22	25	61	175	51	52
26	220	23	22	62	180	52	53
27	220	24	23	63	260	54	53
28	285	10	24	64	205	55	54
29	300	23	25	65	255	56	55
30	315	25	26	66	260	6	56
31	170	11	26	67	275	56	57
32	110	5	27	68	315	57	58
33	280	27	28	69	200	58	59
34	225	28	29	70	175	59	60
35	200	29	30	71	300	61	60
36	190	30	31	72	250	49	61

Table 6 – Diameters cost for the Two Source WDN

Diameter (*m*)	Cost (*Indian Rupias/m*)
.15	1,115
.20	1,600
.25	2,154
.30	2,780
.35	3,475
.40	4,255
.45	5,172
.50	6,092
.60	8,189
.70	10,670
.75	11,874
.80	13,261
.90	16,151
1.00	19,395

Table 7 – Nodes demand and minimum required pressure for the Two Source WDN

Node	Demand (m^3/min)	Minimum pressure (m)
1	---	100
2	---	95
3	18.4	85
4	4.5	85
5	6.5	85
6	4.2	85
7	3.1	82
8	6.2	82
9	8.5	85
10	11.5	85
11	8.2	85
12	13.6	85
13	14.8	82
14	10.6	82
15	10.5	85
16	9.0	82
17	6.8	82
18	3.4	85
19	4.6	82
20	10.6	82
21	12.6	82
22	5.4	80
23	2.0	82
24	4.5	80
25	3.5	80
26	2.2	80

Table 8 – Links length for the Two Source WDN

Link	From node to node	Length (*m*)
1	1 to 3	300
2	3 to 4	820
3	4 to 5	940
4	5 to 6	730
5	6 to 7	1,620
6	7 to 8	600
7	4 to 9	800
8	6 to 10	1,400
9	8 to 13	1,175
10	8 to 14	750
11	2 to 14	210
12	9 to 10	700
13	10 to 11	310
14	11 to 12	500
15	12 to 13	1,960
16	9 to 15	900
17	15 to 18	850
18	10 to 19	650
19	13 to 20	760

Table 9 – Available diameters for the Bessa WDN

Diameter (m)	Cost ($/m)	Rugosity coefficient	Diameter (m)	Cost ($/m)	Rugosity coefficient
0,10	1.629	145	0,35	11.012	130
0,15	4.054	145	0,40	12.397	130
0,20	5.769	145	0,45	15.501	130
0,25	7.769	145	0,50	17.696	130
0,30	9.237	130	0,60	23.132	130

Table 10 – Velocities and pressure losses for the Two Loop WDN

Pipe	Velocity (*m/s*)	Pressure drops (*m*)
1	1.9	6.76
2	1.85	12.79
3	1.46	4.80
4	1.12	14.65
5	1.14	3.00
6	1.1	4.90
7	1.3	6.66
8	0.31	6.75

Table 11 – Optimal diameters (mm) comparison for the Two Loop WDN

Pipe	Geem (2006) *w* = 10.4973 *w* = 10.5879	Suribabu (2012) *w* = 10.667	Ezzeldin et al. (2014) *w* = 10.6744	Eryigit (2015) *w* = 10.4973 *w* = 10.5088 *w* = 10.667	Zhou et al. (2016) *w* = 10.5088 *w* = 10.6744	Surco et al. (2017) *w* = 10.667	Present work *w* = 10.667
1	457.2	508	457.2	457.2	457.2	457.2	457.2
2	254	254	254	254	254	254	254
3	406.4	406.4	406.4	406.4	406.4	406.4	406.4
4	101.6	25.4	101.6	101.6	101.6	101.6	101.6
5	406.4	355.6	406.4	406.4	406.4	406.4	406.4
6	254	254	254	254	254	254	254
7	254	254	254	254	254	254	254
8	25.4	25.4	25.4	25.4	25.4	25.4	25.4
Cost ($)	419,000	420,000	419,000	419,000	419,000	419,000	419,000

Table 12 – Nodes Pressure and demand for the Gomes *et al.* (2009) WDN

Node	Demand (*L/s*)	Elevation (*m*)	Minimum Pressure (*m*)	Node	Demand (*L/s*)	Elevation (*m*)	Minimum Pressure (*m*)
1	2.51	5.0	39.72	31	4.94	3.5	18.09
2	44.07	5.0	38.24	32	4.09	4.5	16.85
3	41.24	4.0	35.62	33	3.68	5.0	16.51
4	1.04	4.5	25.32	34	4.04	5.0	17.82
5	0.86	4.5	26.69	35	3.22	6.0	15.12
6	1.32	4.5	34.64	36	2.53	4.5	16.37
7	1.35	4.5	24.80	37	2.31	4.5	16.38
8	8.59	5.0	23.19	38	2.50	4.0	17.09
9	6.40	4.5	22.24	39	2.89	4.0	18.88
10	6.07	5.0	19.96	40	2.48	4.0	16.54
11	4.95	3.5	15.36	41	4.61	4.0	16.52
12	8.38	3.5	15.10	42	3.47	4.0	17.66
13	11.70	3.5	16.85	43	3.61	4.0	19.14
14	5.63	5.0	17.66	44	5.17	4.0	21.91
15	5.57	6.0	15.27	45	6.48	4.0	16.95
16	6.30	6.0	15.28	46	4.91	4.5	15.98
17	3.26	6.0	17.40	47	6.50	4.0	16.38
18	3.60	6.0	18.36	48	4.97	4.5	26.76
19	4.83	6.0	15.38	49	2.97	3.0	20.54
20	4.50	6.0	15.24	50	1.80	5.0	16.05
21	2.80	5.0	16.59	51	2.96	4.0	16.68
22	5.46	3.0	19.37	52	4.66	3.0	16.63
23	62.45	3.5	19.47	53	4.54	4.5	15.06
24	8.19	6.0	18.10	54	8.80	4.5	15.56
25	58.87	3.5	15.14	55	4.26	4.5	20.05
26	3.26	3.5	15.12	56	2.98	5.0	29.35
27	4.36	4.3	26.09	57	3.91	5.0	22.84
28	4.25	4.0	24.57	58	3.70	4.7	19.41
29	4.56	2.5	22.76	59	1.86	5.0	15
30	8.32	2.5	20.64	60	3.12	5.0	15
				61	3.52	4.5	15

Table 13 – Results comparison for the Gomes *et al.* (2009) WDN ($\omega = 10.674$)

Cost (*US*$)	Gomes *et al.* (2009)	Surco *et al.* (2017)	present paper
	2,200,784.60	2,123,238.50	2,121,618.90

Diameters (mm)

Pipe	Gomes *et al.* (2009)	Surco *et al.* (2017)	present paper	Pipe	Gomes *et al.* (2009)	Surco *et al.* (2017)	present paper
1	600	600	600	37	100	100	100
2	600	500	500	38	100	100	100
3	600	450	500	39	100	100	100
4	500	450	400	40	150	100	100
5	350	100	100	41	100	100	100
6	100	300	300	42	100	100	100
7	250	400	400	43	100	100	100
8	400	400	500	44	100	100	100
9	200	400	400	45	100	100	100
10	100	400	400	46	100	100	100
11	100	400	400	47	100	100	100
12	100	100	100	48	100	100	100
13	400	100	100	49	100	100	100
14	400	100	100	50	100	100	100
15	400	100	100	51	150	150	150
16	150	100	100	52	150	150	150
17	150	100	100	53	100	100	100
18	100	100	100	54	100	100	100
19	100	100	100	55	100	100	100
20	100	150	150	56	150	100	100
21	100	200	200	57	150	150	100
22	100	100	100	58	150	100	100
23	100	100	100	59	100	100	100
24	150	100	100	60	100	100	150
25	150	100	100	61	100	100	100
26	150	150	150	62	100	100	100
27	100	350	350	63	100	100	100
28	100	350	400	64	100	100	100
29	300	200	200	65	150	100	100
30	350	100	100	66	150	150	150
31	350	100	100	67	100	100	100
32	250	250	250	68	100	100	100
33	250	250	250	69	100	100	100
34	150	150	150	70	100	100	100

35	150	150	150	71	100	100	100
36	100	100	100	72	100	100	100

Table 14 – Diameters (mm) for the Two Source WDN

	Kadu *et al.* (2008) $\omega = 2.234 \times 10^{12}$	Ezzeldin *et al.* (2014) $\omega = 10.6744$	Present work $\omega = 10.6744$	Suribabu (2012) $\omega = 10.667$	Ezzeldin *et al.* (2014) $\omega = 10.667$	Present work $\omega = 10.667$
1	1000	900	900	1000	900	900
2	900	900	900	1000	900	900
3	350	400	350	400	350	350
4	250	250	300	200	300	300
5	150	150	150	150	150	150
6	250	200	250	250	250	250
7	800	800	800	1000	800	800
8	150	150	150	150	150	150
9	600	400	450	450	450	500
10	700	500	600	600	500	600
11	900	900	900	1000	800	900
12	700	700	700	800	700	700
13	500	600	500	500	600	500
14	450	450	300	350	450	300
15	150	150	150	150	150	150
16	450	500	500	500	500	500
17	350	350	350	300	350	350
18	400	350	350	450	400	400
19	450	200	150	150	150	250
20	150	150	200	150	150	150
21	600	700	750	900	700	700
22	150	150	150	150	150	150
23	150	500	450	450	450	400
24	400	350	350	300	350	350
25	500	700	600	750	700	600
26	200	250	250	150	250	250
27	350	300	300	300	250	300
28	250	300	250	250	300	250
29	150	200	250	150	200	250
30	300	250	300	300	300	300
31	150	150	150	150	150	150
32	150	150	150	150	150	150
33	150	150	150	150	150	150
34	200	150	150	150	150	150
Cost (R$)	123,268,864[a]	125,843,995	124,986,030	140,177,210	125,501,130	125,136,870

[a] value corrected by Ezzeldin *et al.* (2014): R$ 126,368,865.

Table 15 – Nodes calculated pressure for the Two Source WDN

Nodes	Kadu *et al.* (2008)	Present work $\omega = 10.667$	Present work $\omega = 10.6744$
3	98.98	98.31	98.31
4	95.76	95.14	95.14
5	88.79	87.88	87.80
6	85.28	86.20	86.07
7	88.01	87.84	88.40
8	91.64	91.09	91.52
9	91.84	91.31	91.31
10	88.89	88.55	88.58
11	87.11	86.88	86.89
12	85.15	85.21	85.16
13	86.81	85.58	84.18
14	94.13	94.14	94.14
15	87.12	88.26	88.20
16	82.10	82.08	82.26
17	90.26	91.26	91.88
18	85.25	85.85	85.67
19	85.97	85.95	83.90
20	83.89	82.22	82.49
21	84.03	85.81	85.70
22	84.23	86.80	87.38
23	82.20	83.74	83.27
24	83.70	82.41	80.69
25	80.64	80.10	80.23
26	80.16	82.05	82.07

Table 16 – Calculated diameters, velocities, pressures and final cost for the Bessa WDN

Pipe/Node	Diameter (m) Carvalho (2007)	Diameter (m) Present work	Velocity (m/s) Present work	Pressure (m) Carvalho (2007)	Pressure (m) Present work
1	0,6	0,6	1,49	Reservoir	
2	0,5	0,45	1,82	41,03	41,02
3	0,35	0,35	1,37	39,69	38,8
4	0,45	0,45	1,54	35,33	34,45
5	0,4	0,4	1,63	26,26	25,4
6	0,1	0,1	0,48	36,39	36,36
7	0,3	0,3	1,19	29,79	29,69
Cost ($)	127.574.470	126.806.220			