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Decision Support

Large-scale group decision-making with non-cooperative behaviors and heterogeneous preferences: An application in financial inclusion☆

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ABSTRACT

Non-cooperative behavior is a common situation in large-scale group decision-making (LSGDM) problems. In addition, decision makers in LSGDM often use different preference formats to express their opinions, due to their educational backgrounds, knowledge, and experiences. Heterogeneous preference information and non-cooperative behaviors bring challenges to LSGDM. This study develops a consensus reaching model to address heterogeneous LSGDM with non-cooperative behaviors and discuss its application in financial inclusion. Specifically, the cosine similarity degree is introduced to build a distance measure for different preference structures. Clustering analysis is employed to divide large-scale groups and handle non-cooperative behaviors in LSGDM. A consensus degree and a weighting process are proposed to decrease the influence of non-cooperative behaviors and facilitate the consensus reaching process. The convergence of the proposed approach is proven by theoretical and simulation analyses. Experimental studies are carried out to compare the performances of the proposed approach with existing methods. Finally, a real-life example from the "targeted poverty reduction project" in China is presented to validate the proposed approach. The selection of beneficiaries in finance inclusion is difficult due to the lack of credit history, the large number of participants, and the conflicting views of participants. The results showed that the proposed consensus model can integrate opinions of participants using diverse preference formats and reach an agreement efficiently.

1. Introduction

Group decision-making (GDM) is carried out by groups of experts and aims to integrate collective intelligence to make decisions about alternatives (Eklund, Rusinowska & De Swart, 2007; Liu, Zhou, Ding, Palomares & Herrera, 2019; Woolley, Chabris, Pentland, Hashmi & Malone, 2010). Large-scale group decision-making (LSGDM), which involves more than 20 decision makers (DMs), has become an important topic in decision science (Liu et al., 2019; Liu, Shen, Zhang, Chen & Wang, 2015; Wu, Zhang, Liu & Cao, 2019; Zhang, Dong & Herrera-Viedma, 2018) and has a wide range of applications in areas like e-democracy (Efremov, Insua & Lotov, 2009; Kim, 2008), social networks (Ding, Wang, Shang & Herrera, 2019; Friedkin, Proskurnikov, Tempo & Parsegov, 2016; Liu et al., 2019; Wu et al., 2019), emergency decision-making (Xu, Du & Chen, 2015), and urban resettlement (Bai, Shi & Liu, 2014).

Consensus reaching is an important topic in GDM (Coch & French, 1948; French, 1956). Traditional consensus means a full and unanimous agreement in a group, which is virtually impossible in real-world settings (Kacprzyk & Fedrizzi, 1986). Thus, consensus typically means reach a consent, not necessarily the agreement of all group participants (Herrera-Viedma, Cabrerizo, Kacprzyk & Pedrycz, 2014). The consensus reaching procedure is based on the psychological behavior of a group, such as groupthink and collective choice (Arrow, 1963), individual utility reaching (Neumann & Morgenstern, 1944), and interactive behavior (Simon, 1955). The satisfaction of decision makers (DMs) increases when their opinions are reconsidered to achieve an acceptable level of consensus (Bergstrom & Bak-Coleman, 2019; Ding et al., 2019; Dong, Zhao, Zhang, Chiclana & Herrera-Viedma, 2018; Fu, Chang, Xue &

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Yang, 2019; Zhang, Dong, Chiclana & Yu, 2019). Over the past 20 years, many consensus reaching mechanisms have been proposed to improve the consent state in GDM (Dong et al., 2014; Herrera, Herrera-Viedma & Chiclana, 2001; Herrera-Viedma, Herrera & Chiclana, 2002; Xu et al., 2015; Wu et al., 2019; Zhang, Kou & Peng, 2019; Liu, Xu & Herrera, 2019; Kou et al., 2014).

However, the consensus reaching process is particularly challenging for LSGDM due to the complexity and uncertainty caused by large groups of participants. One challenge is that there exist non-cooperative behaviors or minority opinions in LSGDM. Another challenge is the heterogeneous preference formats used in LSGDM, such as preference vectors and pairwise comparison matrixes.

Non-cooperative behavior means that DMs are unwilling to modify their preferences in order to reach group consensus, and might even undermine the consensus reaching. Since the presence of non-cooperative behaviors will not only bias the consensus-reaching process, but also hinder and prolong the consensus-reaching process, they need to be identified and properly handled. Palomares, Martinez and Herrera (2014) used a fuzzy clustering algorithm to divide a large number of DMs into small groups and detect non-cooperative behaviors using a set distance among preferences. Xu et al. (2015) defined non-cooperation degrees to detect the opinion modification process of DMs. Dong, Zhang and Herrera-Viedma (2016) proposed multi-attribute mutual evaluation matrixes to manage non-cooperative behaviors and reach consensus.

Heterogeneous preference formats are common in large-scale groups (Chen, Zhang & Dong, 2015), in which DMs with diversified educational backgrounds, knowledge, experiences, and decision habits utilize different formats to express their individual preferences. For example, pairwise comparisons (Saaty, 1980) can be used to evaluate alternatives using multi-criteria judgments. However, it is unreasonable to ask a DM who does not know AHP (Saaty, 1980) to provide their preferences and keep its consistency using pairwise comparisons. It is more intuitive for them to provide a simple ranking. Various methods have been proposed to integrate heterogeneous preference formats, such as transformation function methods (Herrera et al., 2001; Herrera-Viedma et al., 2002), optimization-based method (Quesada, Palomares & Martínez, 2015), and feedback adjustment method (Dong & Zhang, 2014). Zhang et al. (2018) studied LSGDM with heterogeneous preference relations by considering individual concern and satisfaction, but they did not consider non-cooperative behaviors.

Though there have been some works on LSGDM and heterogeneous preference formats, very few studies address consensus building in LSGDM with heterogonous preference formats and non-cooperative behaviors. In addition, existing works only consider the additive preference relation or multiplicative preference relation (Palomares et al., 2014; Quesada et al., 2015; Xu et al., 2015). Clustering algorithms cannot be used to detect non-cooperative behavior and improve decision efficiency due to the lack of similarity measure to handle the heterogeneous preference information in LSGDM.

Based on these observations, this study proposes a consensus reaching model for LSGDM with heterogonous preference formats and non-cooperative behaviors. The proposed consensus reaching process includes the following steps: (1) a similarity measure is established for heterogeneous preference formats. Based on this measure, clustering is used to deal with non-cooperative behaviors in LSGDM. (2) An optimization model is proposed to integrate the heterogeneous preference information provided by DMs and reduce the computational cost. (3) A consensus measure and a weighting process for LSGDM are proposed, and the convergence of the consensus process is proved using theoretical and simulation analyses. An experimental study is conducted to compare the performance of the proposed approach with some existing measures. (4) Finally, the proposed consensus reaching model is applied to the beneficiary evaluation in a real-life financial inclusion project. The results showed that the proposed consensus model can integrate opinions of participants using diverse preference formats and reach an agreement efficiently.

The remainder of this paper is organized as follows. Section 2 introduces the similarity relations and establishes a similarity measure for heterogeneous preference formats. Section 3 illustrates a consensus reaching framework and the detailed consensus building process. Section 4 uses a real-life poverty reduction project in the Qinghai-Tibet plateau to validate the effectiveness of the proposed model. The convergence of the proposed approach is discussed and compared with different consensus reaching methods in Section 5. Section 6 concludes the study.

2. Related works and preliminaries

This section reviews basic concepts related to consensus reaching, heterogeneous preference structures, the properties of the heterogeneous preferences relations and their priority vector, and establishes a similarity relation among heterogeneous preferences.

2.1. General consensus reaching process

Consensus in GDM means achieving a collective opinion by negotiation and opinion evolution. The purpose of a consensus reaching process is to make as many DMs as possible satisfied with the decision result and achieve a high consensus level. A typical consensus reaching process in GDM is guided by a supervisor or moderator, who collects individuals' preferences, evaluates the consensus degree, and decides whether to continue or stop the decision process (Herrera et al., 2001; Herrera-Viedma et al., 2002). A general consensus-reaching process has three steps: aggregation of individual preferences, consensus measure, and consensus improvement. Fig. 1 describes the steps in a general GDM process. First, a set of alternatives is presented to a group of DMs. Then, DMs provide their opinions about these alternatives. All individual preferences are integrated into a collective opinion. Next, if the consensus degree reaches a pre-defined value, the GDM continues to the next step, which selects a final alternative. Otherwise, a moderator returns the collective information to the DMs, asks them to modify their preferences, and aggregates preferences again until a consensus is reached. Compared to traditional GDM, LSGDM needs to develop methods to deal with non-cooperative DMs and split large DMs into small groups to improve decision efficiency.

A consensus degree is the total deviation among individual preferences, or the distance between individual preferences and the collective opinion (Dong & Zhang, 2014; Palomares et al., 2014). Based on the consensus degree, various consensus-reaching models have been proposed to decrease the deviations and achieve a consensus. However, these measures are established based on the Euclidean distance, which cannot be directly used to address heterogeneous preferences. For example, the Euclidean distance between a vector and a matrix cannot be defined. Thus, new measurements are needed to deal with the heterogonous preference formats. Section 2.2 introduces four common preference structures, Sections 2.3 and 2.4 describe similarity measures and relations between different preference structures. Section 2.5 proposes similarity measures for different preference formats, which are the basis of our proposed consensus-building model.

2.2. Heterogeneous preference structures

For a given GDM question, several preference formats are used to represent DMs' preferences for a set of alternatives. Assume that $X = \{x_1, ..., x_j, ..., x_n\}$ is a set of feasible alternatives.

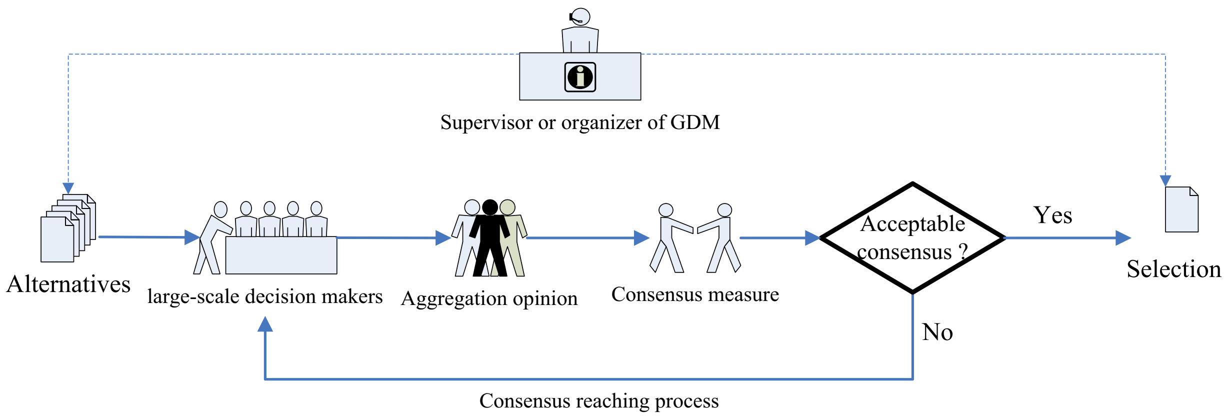

Fig. 1. Basic steps in GDM.

Utility Value. Assume that $u = \{u_1, u_2, ..., u_n\}$ is the set of utility values provided by one of the DMs. $u_i \in [0, 1], i = 1, 2, ..., n$ represents the utility values of alternative x_i. Generally, the higher the utility value, the more important the alternative (Chiclana, Herrera & Herrera-Viedma, 2001; Herrera-Viedma et al., 2002; Xu, Cai & Liu, 2011). For example, for four alternatives $\{x_1, .x_2, x_3, x_4\}$, DM's utility values are {0.4, 0.6, 0.2, 0.8}, respectively. This means that x_4 is the most preferred alternative, followed by x_2, x_1, and x_3.

Preference ordering. Let $o = \{o_1, o_2, ..., o_n\}$ be a preference ordering set. This set is the permutation function over the set $\{1, 2, ..., n\}$. o_i denotes the positional order of alternative x_i in$X = \{x_1, ..., x_j, ..., x_n\}$. For example, the corresponding preference ordering of the utility value {0.4, 0.6, 0.2, 0.8} in the above example is $\{3, 2, 4, 1\}$ for the four alternatives $\{x_1, .x_2, x_3, x_4\}$ (the corresponding order of the alternatives is$x_4 \succ x_2 \succ x_1 \succ x_3$). The preference order is a priority sequence of the alternatives. Therefore, x_4 is the most preferred alternative, followed by x_2, x_1, and x_3 (Chiclana et al., 2001; Herrera-Viedma et al., 2002; Xu et al., 2011)

Multiplicative preference relation (Kou & Lin, 2014; Saaty, 1980; Brunelli, 2019). For a given set of alternatives, the multiplicative preference relation is represented by a pairwise comparison matrix (PCM). It contains n^2 preference elements that belong to [1/9, 9], whose entries represent the preference degree for the two alternatives. Assume that the matrix $A = (a_{ij})_{n\times n}, i, j = 1, 2, ..., n$ is a PCM provided by DMs. The entry a_{ij} of the PCM represents the degree of preference for alternative x_i over x_j. In this setting, "9" means that the alternative completely dominates the others, while "1" indicates that two alternatives are equal. The PCM satisfies $a_{ij}a_{ji} = 1$ and $a_{ij} > 0$. For example, a comprehensive plan for a residence project considers four criteria: construction cost, environmental standards, designing style, and residential function. If we want to assess their relative importance, we can invite experts to construct a multiplicative preference relation. Assume one of the relations is as follows:

$$\begin{pmatrix} 1 & 4 & 2 & 8 \\ \frac{1}{4} & 1 & \frac{1}{2} & 2 \\ \frac{1}{2} & 2 & 1 & 4 \\ \frac{1}{8} & \frac{1}{2} & \frac{1}{4} & 1 \end{pmatrix}$$

This expert considers the construction cost as the most important factor, which is strongly preferred to the residential function ($a_{14} = 8$), more preferred to the environmental standards ($a_{12} = 4$), and a little preferred to the design style ($a_{13} = 2$).

Additive preference relation (Mikhailov, 2003; Xu, Patnayakuni & Wang, 2013). An additive preference relation is also determined by a PCM, and each element indicates the degree that an alternative is preferred to another. Let $B = (b_{ij})_{n\times n}, i, j = 1, 2, ..., n$ be an additive preference relation. In contrast with the multiplicative preference relation, the values of an additive PCM range from zero to one and satisfy$b_{ij} + b_{ji} = 1$. If b_{ij} is equal to 0.5, it indicates indifference between$x_i$and x_j. If the value of b_{ij} is one, it indicates that x_i is unanimously preferred to x_j. The following additive preference relation is transformed from the same example of the multiplicative preference relation using a transformation function$b_{ij} = \frac{1}{2}(1 + \log_9 a_{ij})$, which was adopted from Chiclana et al. (2001)), and these two preferences can obtain the same ordering of the alternatives:

$$\begin{pmatrix} 0.5 & 0.82 & 0.66 & 0.97 \\ 0.18 & 0.5 & 0.34 & 0.66 \\ 0.34 & 0.66 & 0.5 & 0.82 \\ 0.03 & 0.34 & 0.18 & 0.5 \end{pmatrix}$$

2.3. Similarity measures

Similarity measures have been used to derive priority vectors of the multiplicative and additive preference relations (Kou and Lin, 2014; Chao, Kou, Li & Peng, 2018). This subsection describes similarity measure and cosine similarity measure.

Similarity measure. For two n-vectors $\vec{v}_1 = (a_1, a_2, ..., a_n)$ and $\vec{v}_2 = (b_1, b_2, ..., b_n)$, the similarity measure $SM(\vec{v}_1, \vec{v}_2)$ between them in the n dimensional vector space $V$is a mapping from $V \times V$ to the interval [0, 1].

The similarity measure has the following characteristics (Salton & Mcgill, 1983):

(1) for $\forall \vec{v}_i \in V$, $SM(\vec{v}_i, \vec{v}_i) = 1$;
(2) for $\forall \vec{v}_i, \vec{v}_j \in V$, if $SM(\vec{v}_i, \vec{v}_j) = 0$, then, $\vec{v}_i$ and $\vec{v}_j$ are not similar at all;
(3) for $\forall \vec{v}_i, \vec{v}_j, \vec{v}_k \in V$, if $SM(\vec{v}_i, \vec{v}_j) < SM(\vec{v}_i, \vec{v}_k)$, then, $\vec{v}_i$ is more similar to$\vec{v}_k$than to $\vec{v}_j$.

Cosine similarity measure (Salton & Mcgill, 1983)**.** The cosine similarity value of two non-negative n-vectors,$\vec{v}_1 = (a_1, a_2, ..., a_n)$ and $\vec{v}_2 = (b_1, b_2, ..., b_n)$, is denoted as follows:

$$\langle \vec{v}_1, \vec{v}_2 \rangle = \frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} (a_i)^2} \sqrt{\sum_{i=1}^{n} (b_i)^2}}. \quad (1)$$

The similarity measure is not regarded as a distance in mathematics because it does not satisfy the triangle inequality. When the n-vectors are transformed into a normalized vector by $\frac{\vec{v}_1}{\|\vec{v}_1\|}$ and $\frac{\vec{v}_2}{\|\vec{v}_2\|}$, the cosine similarity measure$\langle \frac{\vec{v}_1}{\|\vec{v}_1\|}, \frac{\vec{v}_2}{\|\vec{v}_2\|} \rangle$ becomes an inner product. In that case, the cosine similarity is a distance measure that satisfies the reflexivity, symmetry, and triangular inequality.

2.4. Similarity relation between different preference structures and their priority vector

The priority vector of preference relations is a normalized vector that determines the relative merit of a set of alternatives. It has been proved that a cosine similarity relation exists between different preference structures and their priority vectors (Kou and Lin, 2014; Chao et al., 2018. This subsection reviews the relations between different preference structures and their priority vectors. The preference relations and vectors are nonnegative in real-world management issues.

Let $w = (w_1, ..., w_n)^T$ be the priority vector of different preferences. The cosine similarity relation between their column vectors (in the PCM), or vectors, and their priority vectors are as follows:

Utility Values. Generally, u_i represents the utility value of the alternatives, which indicates the relative importance of the alternative. In the case of a single utility value, the priority vector is $w_i = u_i/\sum_{k=1}^n u_k, i = 1, 2, ..., n$ if it is consistent with the utility values. It follows that $w_i/w_j = u_i/u_j$(Chiclana et al., 2001; Herrera-Viedma et al., 2002). The cosine similarity measure between the utility values and the priority vector is:

$$\left\langle \vec{u}_j, w \right\rangle = \frac{\sum_{i=1}^n \frac{u_i w_i}{u_j}}{\sqrt{\sum_{i=1}^n \left(\frac{u_i}{u_j}\right)^2} \sqrt{\sum_{i=1}^n w_i^2}} = \frac{\sum_{i=1}^n \frac{w_i}{w_j} w_i}{\sqrt{\sum_{i=1}^n \left(\frac{w_i}{w_j}\right)^2} \sqrt{\sum_{i=1}^n w_i^2}} = 1, \tag{2}$$

where the vector$\vec{u}_j = (\frac{u_1}{u_j}, \frac{u_2}{u_j}, ..., \frac{u_n}{u_j})^T, j = 1, 2, ..., n.$

Preference orderings. The preference order is the permutation function over the set $\{1, 2, ..., n\}$. The smallest ordering value corresponds to the largest utility value. The utility value u_i associated with alternative x_i depends on the value of its position, o_i, in such a way that the bigger the value of $n - o_i$, the bigger the utility value of u_i. This implies that u_i is a function f with respect to $n - o_i$. In other words, $u_i = f(n - o_i)$, where f is a non-decreasing function. Herrera et al. (2001) proposed that a typical example of this function is $u_i = (n - o_i)/(n - 1)$. The maximum utility value corresponds to the first alternative and the minimum utility value corresponds to the last alternative in the preference order. In this context, we obtain a normalized set of n utility values, that is:

$$Max_i\{u_i\} - Min_i\{u_i\} \leq 1.$$

As a result, we obtain utility values based on a different scale. We cite the function transformed the ordering values to utilities (Herrera et al., 2001) as follows:

$$u_i = \frac{n - o_i}{n - 1}, i = 1, 2, ..., n. \tag{3}$$

Therefore, the priority vector w_i is equal to $u_i/\sum_{k=1}^n u_k, i = 1, 2, ..., n$ in the preference order. In other words:

$$w_i = \frac{n - o_i}{n - 1} \Big/ \sum_{k=1}^n \frac{n - o_k}{n - 1}, i = 1, 2, ..., n. \tag{4}$$

The following condition must be satisfied (based on Eq. (4)):

$$\frac{w_i}{w_j} = \frac{n - o_i}{n - o_j}, \tag{5}$$

and the cosine similarity measure is equal to 1 in this case:

$$\left\langle \vec{o}_j, w \right\rangle = \frac{\sum_{i=1}^n \frac{(n-o_i)w_i}{n-o_j}}{\sqrt{\sum_{i=1}^n \left(\frac{n-o_i}{n-o_j}\right)^2} \sqrt{\sum_{i=1}^n w_i^2}} = \frac{\sum_{i=1}^n \frac{w_i}{w_j} w_i}{\sqrt{\sum_{i=1}^n \left(\frac{w_i}{w_j}\right)^2 \sum_{i=1}^n w_i^2}} = 1, \tag{6}$$

where the vector $\vec{o}_j = (\frac{n-o_1}{n-o_j}, \frac{n-o_2}{n-o_j}, ..., \frac{n-o_n}{n-o_j})^T, j = 1, 2, ..., n.$

Multiplicative preference relation (Kou, Ergu, Lin & Chen, 2016**).** In contrast with the utility values and preference ordering, the multiplicative preference relation is always represented by a PCM. Let $A = (a_{ij})_{n\times n}$be a multiplicative preference relation. It is entirely consistent if the condition $a_{ij}a_{jk} = a_{ik}, i, j, k = 1, 2, ..., n$ is satisfied (Saaty, 1980). In this case, the priority vector has the following property:

$$a_{ij} = \frac{w_i}{w_j}, i.j = 1, 2, ..., n. \tag{7}$$

Therefore, Eq. (8) holds when the PCM is entirely consistent. If $\vec{a}_j = (a_{1j}, a_{2j}, ..., a_{nj})^T, j = 1, 2, ..., n$ is a column vector of a PCM, then:

$$\langle \vec{a}_j, w \rangle = 1, \tag{8}$$

and this condition was used to derive the priority vector in the analytic hierarchy process (Kou & Lin, 2014).

Additive preference relation (Mikhailov, 2003; Xu et al., 2013). An additive preference relation is similar to a multiplicative preference relation, and it is also determined by a PCM. The difference is that an additive preference relation uses fuzzy judgments rather than crisp (exact) values. Let $B = (b_{ij})_{n\times n}$be an additive preference relation. The consistent additive preference relation of the priority vector w is defined as in Eqs. (9) and (10) (Xu et al., 2011):

$$b_{ij} = \frac{w_i}{w_i + w_j}; i, j = 1, 2, ..., n. \tag{9}$$

$$b_{ij}b_{jk}b_{ki} = b_{ji}b_{kj}b_{ik}, i, j, k = 1, 2, ..., n. \tag{10}$$

We can obtain the cosine similarity relation under the following transformation for an additive preference relation:

$$p_{ij} = \frac{b_{ij}}{1 - b_{ij}}. \tag{11}$$

Let $\vec{p}_j = (p_{1j}, p_{2j}, ..., p_{nj})^T$be a column vector of an additive PCM. Then, the following condition is satisfied (Chao et al., 2018):

$$\left\langle \vec{p}_j, w \right\rangle = 1.. \tag{12}$$

For multiplicative and additive preference relations, we utilize the PCM as follows:

$$\bar{A} = (\bar{a}_{ij})_{n\times n} = \begin{pmatrix} \frac{a_{11}}{\sqrt{\sum_{i=1}^n (a_{i1})^2}} & \frac{a_{12}}{\sqrt{\sum_{i=1}^n (a_{i2})^2}} & \cdots & \frac{a_{1n}}{\sqrt{\sum_{i=1}^n (a_{in})^2}} \\ \frac{a_{21}}{\sqrt{\sum_{i=1}^n (a_{i1})^2}} & \frac{a_{22}}{\sqrt{\sum_{i=1}^n (a_{i2})^2}} & \cdots & \frac{a_{2n}}{\sqrt{\sum_{i=1}^n (a_{in})^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{a_{n1}}{\sqrt{\sum_{i=1}^n (a_{i1})^2}} & \frac{a_{n2}}{\sqrt{\sum_{i=1}^n (a_{i2})^2}} & \cdots & \frac{a_{nn}}{\sqrt{\sum_{i=1}^n (a_{nn})^2}} \end{pmatrix}, \tag{13}$$

and

$$\bar{B} = (\bar{p}_{ij})_{n\times n} = \begin{pmatrix} \frac{p_{11}}{\sqrt{\sum_{i=1}^n (p_{i1})^2}} & \frac{p_{12}}{\sqrt{\sum_{i=1}^n (p_{i2})^2}} & \cdots & \frac{p_{1n}}{\sqrt{\sum_{i=1}^n (p_{in})^2}} \\ \frac{p_{21}}{\sqrt{\sum_{i=1}^n (p_{i1})^2}} & \frac{p_{22}}{\sqrt{\sum_{i=1}^n (p_{i2})^2}} & \cdots & \frac{p_{2n}}{\sqrt{\sum_{i=1}^n (p_{in})^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p_{n1}}{\sqrt{\sum_{i=1}^n (p_{i1})^2}} & \frac{p_{n2}}{\sqrt{\sum_{i=1}^n (p_{i2})^2}} & \cdots & \frac{p_{nn}}{\sqrt{\sum_{i=1}^n (p_{nn})^2}} \end{pmatrix}. \tag{14}$$

The preference ordering and utility value can be expressed in a matrix format, and the following transforming matrix is proposed to obtain a matrix from preferences with the vector format.

Set $U = (\bar{u}_{ij})_{n\times n} = ({}^{u_i}/_{u_j})_{n\times n}$ and obtain the unitized matrix $\bar{U}$ of the utility value:

$$\bar{U} = \left(\bar{u}_{ij}\right)_{n\times n} = \begin{pmatrix} \frac{u_{11}}{\sqrt{\sum_{i=1}^{n}(u_{i1})^2}} & \frac{u_{12}}{\sqrt{\sum_{i=1}^{n}(u_{i2})^2}} & \cdots & \frac{u_{1n}}{\sqrt{\sum_{i=1}^{n}(u_{in})^2}} \\ \frac{u_{21}}{\sqrt{\sum_{i=1}^{n}(u_{i1})^2}} & \frac{u_{22}}{\sqrt{\sum_{i=1}^{n}(u_{i2})^2}} & \cdots & \frac{u_{2n}}{\sqrt{\sum_{i=1}^{n}(u_{in})^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{u_{n1}}{\sqrt{\sum_{i=1}^{n}(u_{i1})^2}} & \frac{u_{n2}}{\sqrt{\sum_{i=1}^{n}(u_{i2})^2}} & \cdots & \frac{u_{nn}}{\sqrt{\sum_{i=1}^{n}(u_{in})^2}} \end{pmatrix}. \quad (15)$$

Being similar to utility values, preference orderings are set as $O = (o_{ij})_{n\times n} = (\frac{n-o_i}{n-o_j})_{n\times n}$, and we utilize the proposed matrix as follows:

$$\bar{O} = (\bar{o}_{ij})_{n\times n} = \begin{pmatrix} \frac{o_{11}}{\sqrt{\sum_{i=1}^{n}(o_{i1})^2}} & \frac{o_{12}}{\sqrt{\sum_{i=1}^{n}(o_{i2})^2}} & \cdots & \frac{o_{1n}}{\sqrt{\sum_{i=1}^{n}(o_{in})^2}} \\ \frac{o_{21}}{\sqrt{\sum_{i=1}^{n}(o_{i1})^2}} & \frac{o_{22}}{\sqrt{\sum_{i=1}^{n}(o_{i2})^2}} & \cdots & \frac{o_{2n}}{\sqrt{\sum_{i=1}^{n}(o_{in})^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{o_{n1}}{\sqrt{\sum_{i=1}^{n}(o_{i1})^2}} & \frac{o_{n2}}{\sqrt{\sum_{i=1}^{n}(o_{i2})^2}} & \cdots & \frac{o_{nn}}{\sqrt{\sum_{i=1}^{n}(o_{in})^2}} \end{pmatrix}. \quad (16)$$

2.5. Proposed similarity measures for heterogeneous preference structures

To deal with heterogeneous structures, new similarity measures need to be constructed for the heterogeneous preference formats so that the measure can be used to establish efficient clustering methods in LSGDM.

In this study, we define three different similarity measures: between vectors, between a vector and a matrix, and between matrices. The principle is based on Eqs. (2), (6), (8), and (12): the preferences are highly similar if they have similar priority vectors.

Based on Eqs. (13)–(15), we define the inner product among different preference relations and introduce a cluster analysis into LSGDM. The proposed similarity measures based on the inner product for different preference relations are computed as follows:

Case 1: The inner product between utility values and preference ordering is:

$$D(u,o) = \frac{1}{n}\sum_{j=1}^{n} < \vec{u}_j, \vec{o}_j > = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\bar{u}_{hj}\bar{o}_{hj} = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\frac{u_{hj}}{\sqrt{\sum_{i=1}^{n}\left(u_{ij}\right)^2}}\frac{o_{hj}}{\sqrt{\sum_{i=1}^{n}\left(o_{ij}\right)^2}} \quad (17)$$

where$\vec{u}_j$,$\vec{o}_j$ are column vectors of the matrixes$\bar{U} = (\bar{u}_{ij})_{n\times n}, \bar{O} = (\bar{o}_{ij})_{n\times n}$.(in Eqs. (15) and 16)

Case 2: The inner product between multiplicative and additive preference relations is:

$$D(A,B) = \frac{1}{n}\sum_{j=1}^{n} < \vec{a}_j, \vec{p}_j > = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\bar{a}_{hj}\bar{p}_{hj} = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\frac{a_{hj}}{\sqrt{\sum_{i=1}^{n}\left(a_{ij}\right)^2}}\frac{p_{hj}}{\sqrt{\sum_{i=1}^{n}\left(p_{ij}\right)^2}} \quad (18)$$

where$\vec{a}_j$,$\vec{p}_j$ are column vectors of the matrixes$\bar{A} = (\bar{a}_{ij})_{n\times n}, \bar{B} = (\bar{p}_{ij})_{n\times n}$(in Eqs. (13) and 14), respectively.

Case 3: The inner product between utility values or preference orderings and multiplicative or additive preference relations is the comparison of two matrices and defined as:

$$D(u,A) = \frac{1}{n}\sum_{j=1}^{n} < \vec{u}_j, \vec{a}_j > = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\bar{u}_{hj}\bar{a}_{hj} = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\frac{u_{hj}}{\sqrt{\sum_{i=1}^{n}\left(u_{ij}\right)^2}}\frac{a_{hj}}{\sqrt{\sum_{i=1}^{n}\left(a_{ij}\right)^2}}$$

$$D(o,A) = \frac{1}{n}\sum_{j=1}^{n} < \vec{o}_j, \vec{a}_j > = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\bar{o}_{hj}\bar{a}_{hj} = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\frac{o_{hj}}{\sqrt{\sum_{i=1}^{n}\left(o_{ij}\right)^2}}\frac{a_{hj}}{\sqrt{\sum_{i=1}^{n}\left(a_{ij}\right)^2}}$$

$$D(u,B) = \frac{1}{n}\sum_{j=1}^{n} < \vec{u}_j, \vec{p}_j > = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\bar{u}_{hj}\bar{p}_{hj} = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\frac{u_{hj}}{\sqrt{\sum_{i=1}^{n}\left(u_{ij}\right)^2}}\frac{p_{hj}}{\sqrt{\sum_{i=1}^{n}\left(p_{ij}\right)^2}}$$

$$D(o,B) = \frac{1}{n}\sum_{j=1}^{n} < \vec{o}_j, \vec{p}_j > = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\bar{o}_{hj}\bar{p}_{hj} = \frac{1}{n}\sum_{j=1}^{n}\sum_{h=1}^{n}\frac{o_{hj}}{\sqrt{\sum_{i=1}^{n}\left(o_{ij}\right)^2}}\frac{p_{hj}}{\sqrt{\sum_{i=1}^{n}\left(p_{ij}\right)^2}} \quad (19)$$

where$\vec{u}_j$,$\vec{o}_j$,$\vec{a}_j$,$\vec{p}_j$ are column vectors of the matrixes$\bar{U} = (\bar{u}_{ij})_{n\times n}, \bar{O} = (\bar{o}_{ij})_{n\times n}, \bar{A} = (\bar{a}_{ij})_{n\times n}, \bar{B} = (\bar{p}_{ij})_{n\times n}$, respectively.

Table 1 summarizes the symbols used in this paper.

3. Consensus reaching model

This section outlines a consensus-reaching model that considers non-cooperative behaviors and heterogeneous preference structures in LSGDM.

Fig. 2 outlines the flowchart of the proposed consensus-reaching model. We propose two approaches to deal with heterogeneous preference information. First, we construct an integrating optimization model to aggregate individual preferences to obtain a collective opinion. Since this approach avoids deriving individual priority vectors, it is more efficient than the order weight average-based operator (OWA). Second, we divide a large group into small subgroups and detect non-cooperative behaviors using a clustering algorithm. We employ a similarity measure (Section 2.5) to handle the heterogeneous preference information in LSGDM.

3.1. Aggregation of individual preferences

In GDM, individual preferences should be aggregated into a collective opinion using operators. Then, the collective opinion can be used to assess whether the group has reached a satisfying consensus degree. This subsection develops a similarity-based optimization model to integrate heterogeneous preferences.

Let$w$be a collective opinion/group opinion and $\vec{\cdot}$ be a column vector of a matrix, as in matrixes $\bar{U} = (\bar{u}_{ij})_{n\times n}, \bar{O} = (\bar{o}_{ij})_{n\times n}, \bar{A} = (\bar{a}_{ij})_{n\times n}, \bar{B} = (\bar{p}_{ij})_{n\times n}$ (listed in Table 1). We assume that $\Lambda = \{\sigma_1, \sigma_2, ..., \sigma_m\}$is a finite set of weights of the DMs and it is often determined by a moderator or organizer to reflect the importance or influence of DMs in a GDM problem. The most used methods to build the weights include expert interview, analytic hierarchy process, and entropy weight method. Let $\cdot^{(k)}$ indicates the kth DM's normalized PCM and let $\Omega = \{\Omega_u, \Omega_o, \Omega_A, \Omega_B\}$ be a set of DMs with utility values, preference ordering, and multiplicative and additive preference relations, respectively.

Table 1
Summary of the symbols used in this study.

Symbols	Meaning
$E = \{e_1, ..., e_i, ..., e_m\}$	Set of DMs, e_i is the ith DM of m DMs.
$X = \{x_1, ..., x_j, ..., x_n\}$	Set of alternatives, x_j is the jth alternative.
$\vec{v}_i = (a_1, a_2, ..., a_n)$	n-vectors.
$w = \{w_1, ..., w_i, ..., w_n\}$	Priority vector obtained from the preference relation.
$\sigma = \{\sigma_1, ..., \sigma_i, ..., \sigma_n\}$	The corresponding weights of DMs.
$u = \{u_1, u_2, ..., u_n\}$	Set of utility values provided by one of the DMs.
$u_i, i = 1, 2, ..., n$	Utility values corresponding to alternative x_i.
$U = (u_{ij})_{n\times n}$	$u_{ij} = u_i/u_j$ in utility value matrix
$o = \{o_1, o_2, ..., o_n\}$	Preference ordering set.
o_i	Order of alternative x_i.
$O = (o_{ij})_{n\times n}$	$o_{ij} = (n - o_i)/(n - o_j)$ in preference ordering matrix
$A = (a_{ij})_{n\times n}, i, j = 1, 2, ..., n$	A PCM provided by DMs.
a_{ij}	The degree of preference for alternative x_i overx_j.
$B = (b_{ij})_{n\times n}, i, j = 1, 2, ..., n$	An additive PCM.
b_{ij}	The fuzzy degree of preference for alternative x_i over x_j.
$D(A,B)$	The similarity measure between two preference structures based on cosine similarity.
$\Omega = \{\Omega_u, \Omega_o, \Omega_A, \Omega_B\}$	A set of DMs with utility values, preference ordering, and multiplicative and additive preference relations, respectively.
${u_i}^{(k)}, {o_i}^{(k)}, {a_{ij}}^{(k)}, {b_{ij}}^{(k)}$	The preference of kth DMs.
$\vec{u}_j^{(k)}, \vec{o}_j^{(k)}, \vec{a}_j^{(k)}, \vec{p}_j^{(k)}$	Column vectors of matrixes $U, O, A, B$of the kth decision makers.
$\bar{u}_{ij}^{(k)}, \bar{o}_{ij}^{(k)}, \bar{a}_{ij}^{(k)}, \bar{p}_{ij}^{(k)}$	Normalized entries of matrixes (13)–(16) of the kth decision makers.
m	The number of DMs.
cm	The threshold whose preference needs modification.
cl	The pre-defined threshold for the consensus-reaching degree.
${P_i}^{(t)}$	The individual preference of the ith DM in the tth iteration.
${P_C}^{(t)}$	The collective preference of all DMs in the tth iteration.
${C_i}^{(t)}$	The center of ith clusters in the tth iteration.

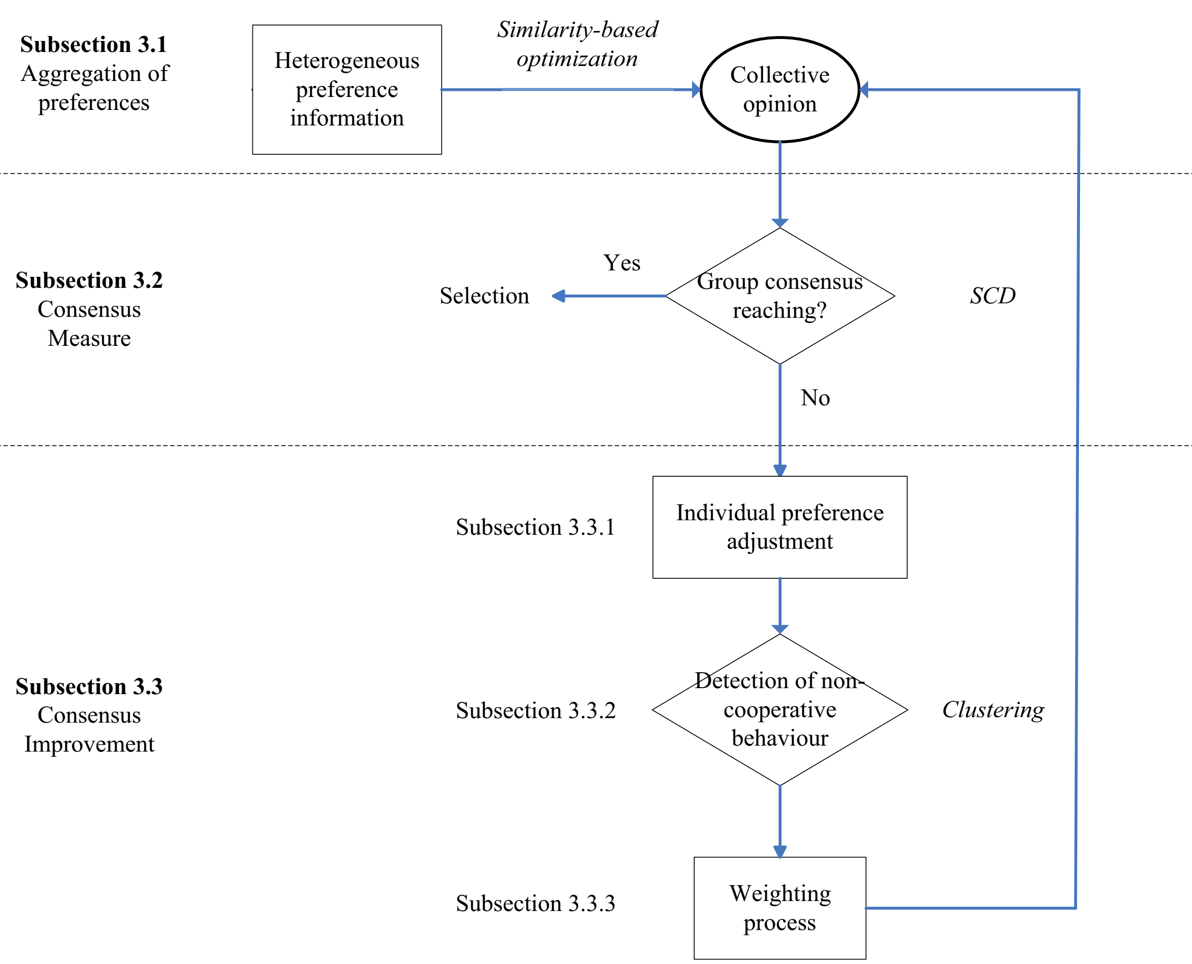

Fig. 2. Consensus-reaching model.

Section 2.4 shows that the cosine similarity is equal to 1 if the priority vector is entirely consistent. However, opinion conflicts do exist in real-life GDM. The perfect consistency cannot always be achieved. Instead, the priority vector should maintain the maximum possible similarity degree. To aggregate all preference relations, the collective opinion should be close to each preference relation, which means that the deviation between the collective opinion and each preference relation simultaneously holds the maximum similarity value. From the perspective of each DM, his/her own objective is that the collective opinion should be more adjacent to his/her preference, so that the collective opinion is more representative of his/her preference, that is, $Maximize \sum_{k\in\Omega_u, j=1}^{n} \sigma_k \langle \vec{u}_j^{(k)}, w\rangle$, $Maximize \sum_{k\in\Omega_o, j=1}^{n} \sigma_k \langle \vec{o}_j^{(k)}, w\rangle$, $Maximize\sigma \sum_{k\in\Omega_A, j=1}^{n} \sigma_k \langle \vec{a}_j^{(k)}, w\rangle$, $Maximize \sum_{k\in\Omega_B, j=1}^{n} \sigma_k \langle \vec{p}_j^{(k)}, w\rangle$ where $\vec{u}_j^{(k)}$, $\vec{o}_j^{(k)}$, $\vec{a}_j^{(k)}$, $\vec{p}_j^{(k)}$ are column vectors of the matrixes $\bar{U} = (\bar{u}_{ij})_{n\times n}, \bar{O} = (\bar{o}_{ij})_{n\times n}, \bar{A} = (\bar{a}_{ij})_{n\times n}, \bar{B} = (\bar{p}_{ij})_{n\times n}$of the k$^{\text{th}}$ DM. The mathematical notations are listed in Table 1. The greater the similarity degree between the collective opinion and an individual preference, the closer the collective opinion is to the individual preference according to Eqs. (2), (6), (8) and (12).

To obtain a solution of the proposed multi-objective optimization question, it can be transformed into a single objective optimization (20) using a linear weighted sum (also called linear scalarization) of multiple objective functions. The optimal solution of the model (20) is a solution of the proposed multi-objective optimization question (Arora, 2015), which can be used as a compromise solution for all DMs from Eq. (20).

$$Max \sum_{k\in\Omega_u} \sum_{j=1}^{n} \sigma_k \left\langle \vec{u}_j^{(k)}, w\right\rangle + \sum_{k\in\Omega_o} \sum_{j=1}^{n} \sigma_k \left\langle \vec{o}_j^{(k)}, w\right\rangle + \sum_{k\in\Omega_A} \sum_{j=1}^{n} \sigma_k \left\langle \vec{a}_j^{(k)}, w\right\rangle + \sum_{k\in\Omega_B} \sum_{j=1}^{n} \sigma_k \left\langle \vec{p}_j^{(k)}, w\right\rangle$$
$$Subject\ to \begin{cases} \sum_{i=1}^{n} w_i = 1; \\ 0 \le w_i \le 1. \end{cases} \tag{20}$$

In this case, this model shows that the total deviation between the priority vector and each column of the normalized matrixes (13), (14), (15), and (16) is the smallest (i.e., the similarity degree is the largest). The optimal solution of the model has the most substantial similarity to each of the DMs. Let $\bar{w} = (\bar{w}_1, \bar{w}_2, ..., \bar{w}_n) = (\frac{w_1}{\sqrt{\sum_{s=1}^{n} w_s^2}}, \frac{w_2}{\sqrt{\sum_{s=1}^{n} w_s^2}}, ..., \frac{w_n}{\sqrt{\sum_{s=1}^{n} w_s^2}})^T$ be the normalized vector of the collective opinion$w$. We rewrite the optimization model (20) into following inner product form (21):

$$Maximize \quad C = \sum_{k\in\Omega_u} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_k \bar{w}_i \bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_k \bar{w}_i \bar{o}_{ij}^{(k)} + \sum_{k\in\Omega_A} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_k \bar{w}_i \bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_k \bar{w}_i \bar{p}_{ij}^{(k)}$$
$$Subjectto \begin{cases} \sum_{i=1}^{n} \bar{w}_i^2 = 1; \\ 0 \le \bar{w}_i \le 1. \end{cases} \tag{21}$$

where $\bar{u}_{ij}^{(k)}, \bar{o}_{ij}^{(k)}, \bar{a}_{ij}^{(k)}$ and $\bar{p}_{ij}^{(k)}$ are the entries of matrixes (13), (14), (15), and (16), respectively. σ_k is the weight of each DM.

The solution is stated in the following Theorem 1.

Theorem 1. *The solution of optimization model (21) exists and is unique. The optimal solution is:*

$$w_i = \frac{\sum_{k\in\Omega_u} \sum_{j=1}^{n} \sigma_k \bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o} \sum_{j=1}^{n} \sigma_k \bar{o}_{ij}^{(k)} + \sum_{k\in\Omega_A} \sum_{j=1}^{n} \sigma_k \bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B} \sum_{j=1}^{n} \sigma_k \bar{p}_{ij}^{(k)}}{\sum_{t=1}^{n} \left(\sum_{k\in\Omega u} \sum_{j=1}^{n} \sigma_k \bar{u}_{tj}^{(k)} + \sum_{k\in\Omega_o} \sum_{j=1}^{n} \sigma_k \bar{o}_{tj}^{(k)} + \sum_{k\in\Omega_A} \sum_{j=1}^{n} \sigma_k \bar{a}_{tj}^{(k)} + \sum_{k\in\Omega_B} \sum_{j=1}^{n} \sigma_k \bar{p}_{tj}^{(k)}\right)}, i = 1, 2, ..., n \quad . \tag{22}$$

Proof. in Appendix A.

In the proposed consensus model, each DM $e_i \in E$ expresses her/his preference about alternative set$X$ in terms of judgment and relative interest, such as the utility value, preference orderings, and multiplicative and additive preference relations. For example, $(x_i, x_j) \in X \times X$ are two alternatives selected from a set of alternatives, and DMs can evaluate them using$\{u_i = 0.4, u_j = 0.2\}$,$\{o_i = 1, o_j = 2\}$,$a_{ij} = 2$, and $b_{ij} = 0.6$, which are expressed using the utility value, preference order, and pairwise comparison, respectively. Then, DMs normalize different preference relations according to matrixes (13), (14), (15), and (16) to obtain the collective preference P_C.

3.2. Similarity-based consensus measure

Consensus measure in GDM is used to assess the consensus degree of DMs, which is the proximity between the collective opinion and individual preferences (del Moral, Chiclana, Tapia & Herrera-Viedma, 2018). Consensus measures usually use "soft" measures (Chiclana, GarcíA, del Moral & Herrera-Viedma, 2013; González-Arteaga, de Andrés Calle & Chiclana, 2016; Herrera-Viedma et al., 2014) based on total distances or deviations between the group opinion and individual preferences. It is a fuzzy judgment of the consensus degree. This subsection proposes a similarity-based consensus measure.

A similarity measure based on the cosine similarity can be used to assess the consensus degree. In this study, we define a similarity consensus degree (*SCD*) to measure the deviation between individual opinions and a group opinion, which is the total cosine similarity degree of the individual preferences and the collective opinion. It is the mean similarity degree that can be computed using the optimal value of the objective function of (21) divided bymn.:

$$SCD = \frac{1}{mn}\left(\sum_{k\in\Omega_u} \sum_{j=1}^{n} \sigma_k \left\langle \vec{u}_j^{(k)}, w\right\rangle + \sum_{k\in\Omega_o} \sum_{j=1}^{n} \sigma_k \left\langle \vec{o}_j^{(k)}, w\right\rangle + \sum_{k\in\Omega_A} \sum_{j=1}^{n} \sigma_k \left\langle \vec{a}_j^{(k)}, w\right\rangle + \sum_{k\in\Omega_B} \sum_{j=1}^{n} \sigma_k \left\langle \vec{p}_j^{(k)}, w\right\rangle\right), \tag{23}$$

where m is the number of DMs, n is the number of alternatives, $w$is the collective opinion computed by Eq.(22), σ_k is the weight of each DM and $\vec{u}_j^{(k)}$, $\vec{o}_j^{(k)}$, $\vec{a}_j^{(k)}$, $\vec{p}_j^{(k)}$ are column vectors of matrixes (13)–(16).

The similarity measure is established at a normalized column vector in matrixes (13)–(16). It converts cosine similarity to an inner product and satisfies all the properties of the distance measure. The *SCD* expresses the total similarity degree between the individual preferences and the collective opinion, and is used to assess the total deviations. A larger *SCD* indicates a higher consensus degree in the group decision-making process. GDM will obtain unanimous agreement and full consensus when the *SCD* equals 1. A detailed comparison with another existing distance measures will be presented in Section 5.2.

A consensus measure is often used to judge the agreement among DMs and determine whether to proceed to the next stage in GDM. In this step, the consensus level of GDM, which is based on the *SCD*, is computed. Such consensus represents the similarity between each P_i andP_C. The overall consensus degree is a "soft" border, which can be regarded as an acceptable level of consent. For a given degree cl, if$SCD < cl$, consensus deepening is needed. Otherwise, the decision-making process is completed, and DMs can move on to the selection process. The pre-set consensus degree cl is determined by management experience and decision expectation. A higher threshold always causes multiple rounds of negotiation, longer decision time, and higher cost.

3.3. Consensus deepening and consensus reaching

A consensus-based solution can increase the satisfaction of DMs since their opinions are reconsidered to reach an acceptable consensus level. To deepen the consensus, the initial aggregated opinion is provided to DMs, who are asked to modify their preferences by considering the group opinion. The next step will detect non-cooperative behaviors.

3.3.1. Feedback adjustment

If a group cannot reach the pre-defined consensus degree at the initial step, the GDM will turn to feedback adjustment. The goal of feedback adjustment is to allow DMs to modify their preferences by considering the group opinion.

Since the group opinion is a vector composed of the weights of alternatives, and the individual preference relation is a matrix composed of a pair-wise comparison value or a vector of utility and ordering, DMs cannot directly compare individual preference relations and the group opinion. Therefore, the group opinion is first transformed into the same format as the individual preference relation. Specifically, we assume that the group opinion is $w^{(t)} = ({w_1}^{(t)}, {w_2}^{(t)}, ..., {w_n}^{(t)})^T$at the tth iteration.

For utility value, if a DM takes the ith utility value as ${u_i}^{(G,t)} = {w_i}^{(t)} \sum_{j=1}^{n} {u_j}^{(k,t)}$, then his/her individual preference is consistent with the group opinion. If a DM takes the ith value in a preference ordering as ${o_i}^{(G,t)} = s$, where o_i is an order permutation function of the alternatives and s is the sth ranked value in descending order in the group opinion, then this order is consistent with the group opinion.

For a multiplicative or additive preference relation, if a DM takes the value in the ith row and jth column of preference relation as ${a_{ij}}^{(G,t)} = \frac{{w_i}^{(t)}}{{w_j}^{(t)}}$ or ${b_{ij}}^{(G,t)} = \frac{{w_i}^{(t)}}{{w_i}^{(t)}+{w_j}^{(t)}}$, then the ranking of the alternatives provided by this DM is consistent with the group opinion. Through the above transformation, the group opinion is converted to the same format as the individual preference relation.

In the second step of the feedback adjustment, the converted group opinion and each DM's own preference value form a numerical interval, ranging between the converted group opinion and the individual preference value. DMs can re-evaluate their preferences using this reference information.

The feedback information for the four preference formats based on the collective opinion are as follows (in line with Dong et al., 2014).

Case 1: For$P_i, i \in \Omega_u$, we can obtain feedback information from the collective opinion (Eq. (22)). Let ${u_i}^{(k,t+1)}$ be the $t+1$ round modification, the derived weights of the alternatives is $w_i = \frac{u_i}{\sum_{j=1}^{n} u_j}, i = 1, 2, ..., n$. Thus, the group opinion can be converted into each entry ${u_i}^{(G,t)} = {w_i}^{(t)} \sum_{j=1}^{n} {u_j}^{(k,t)}$of the utility value. DMs can refer to the following interval when making modifications:

$$
{u_i}^{(k,t+1)} \in \left[\min\left\{{u_i}^{(G,t)}, {u_i}^{(k,t)}\right\}, \max\left\{{u_i}^{(G,t)}, {u_i}^{(k,t)}\right\}\right]. \tag{24}
$$

Case 2: For $P_i, i \in \Omega_o$, the collective opinion needs to be transformed into preference orderings. Let ${o_i}^{(k,t+1)}$ be the $t+1$ round modification, the relationship between preference ordering and the group opinion is $\frac{w_i}{w_j} = \frac{n-o_i}{n-o_j}$. Thus, the group opinion can be transformed into the preference ordering using ${o_i}^{(G,t)} = s$ (where o_i is an order permutation function of the alternatives and s is the sth ranked value in descending order in the group opinion) for each position. DMs can refer to the following interval when making modifications:

$$
{o_i}^{(k,t+1)} \in \left[\min\left\{{o_i}^{(G,t)}, {o_i}^{(k,t)}\right\}, \max\left\{{o_i}^{(G,t)}, {o_i}^{(k,t)}\right\}\right]. \tag{25}
$$

Case 3: For $P_i, i \in \Omega_A$, the derived weights of the alternatives satisfy $a_{ij} = \frac{w_i}{w_j}, i.j = 1, 2, ..., n$. The group opinion can be transformed into a multiplicative preference relation, which value in the ith row and jth column is ${a_{ij}}^{(G,t)} = \frac{{w_i}^{(t)}}{{w_j}^{(t)}}$. DMs can select the modification value from the following interval:

$$
{a_{ij}}^{(k,t+1)} \in \left[\min\left\{{a_{ij}}^{(G,t)}, {a_{ij}}^{(k,t)}\right\}, \max\left\{{a_{ij}}^{(G,t)}, {a_{ij}}^{(k,t)}\right\}\right], i > j \quad . \tag{26}
$$

The different values in a matrix of multiplicative preference relations can be easily ascertained through the above condition.

Case 4: For $P_i, i \in \Omega_B$, the derived weights of the alternatives satisfy $b_{ij} = \frac{w_i}{w_i+w_j}; i, j = 1, 2, ..., n$. Thus the group opinion can be transformed into an additive preference relation, in which the ith row and jth column is ${b_{ij}}^{(G,t)} = \frac{{w_i}^{(t)}}{{w_i}^{(t)}+{w_j}^{(t)}}$. DMs can refer to the following interval when making modifications:

$$
{b_{ij}}^{(k,t+1)} \in \left[\min\left\{{b_{ij}}^{(G,t)}, {b_{ij}}^{(k,t)}\right\}, \max\left\{{b_{ij}}^{(G,t)}, {b_{ij}}^{(k,t)}\right\}\right], i > j \quad . \tag{27}
$$

When $\cdot^{(k,t+1)} = \cdot^{(k,t)}$, a DM does not need to update his/her preference throughout the decision process. DMs may refuse to modify their preferences using these intervals. It is also not clear whether they will modify or how they will modify their preferences. If their preferences do not belong to these intervals, the consensus degree can be hardly improved and will not reach the pre-set threshold. This complication will inevitably lead to an increase in decision-making costs and complexity. Preferences that are far from the collective opinion reduce the consensus degree. If the DMs with these preferences refuse to modify their individual preferences, they are identified as non-cooperative, and moderators or organizers can reduce the weights of non-cooperative DMs to achieve collective opinions and generate higher satisfaction.

3.3.2. Non-cooperative detection

The existence of non-cooperative behaviors increases the difficulties of the consensus reaching in LSGDM, and may result in a failure to reach a group consensus. In this study, a weighting process is used to decrease the influence of non-cooperative DMs by weight adjustments and guarantee a higher consensus degree. Clustering analysis is used to detect non-cooperative behaviors in LSGDM. The similarity changes at the center of different clusters can be used to judge the preference modification trend. In addition, clustering analysis can divide a large-scale group into smaller subgroups with similar preferences. Instead of detecting non-cooperative DMs and adjusting weights at an individual level, organizers of LSGDM can act at a subgroup level and improve the efficiency of the whole process.

Clustering based on individual preferences: Clustering analysis groups data into clusters so that data in the same cluster are more similar to each other than those in other clusters. It is used to reduce the complexity of a decision-making process caused by the large number of DMs and detection of non-cooperative behaviors. This study uses the most well-known clustering algorithm, K-means, to identify clusters.

In this study, data points may take different preferences, $P_i, i = 1, 2, ..., n$, which are the four preference formats represented by column vectors and matrixes. The collective opinion (Eq. (22)) is used as the first initial center in the clustering process. Based on this procedure, non-cooperative behaviors can be identified and managed using a weight penalty strategy. The final group opinion is calculated after the last iterative clustering is stabilized.

The steps of the clustering procedure are as follows:

Step 1: initial centers. In the first round, initial centers are assigned. The number of centers is determined by the management experience or the number of data points. The collective preference in the current round ${P_C}^{(t)}$ is treated as the first center ${C_1}^{(t)}$of the cluster, and the second center is decided using the farthest preference from ${C_1}^{(t)}$, that is, the ${P_i}^{(t)}$ with the smallest cosine similarity measure with respect to ${C_1}^{(t)}$. Next, we compute the similarity measure of the remainder of preference ${P_i}^{(t)}$ with respect to ${C_1}^{(t)}$

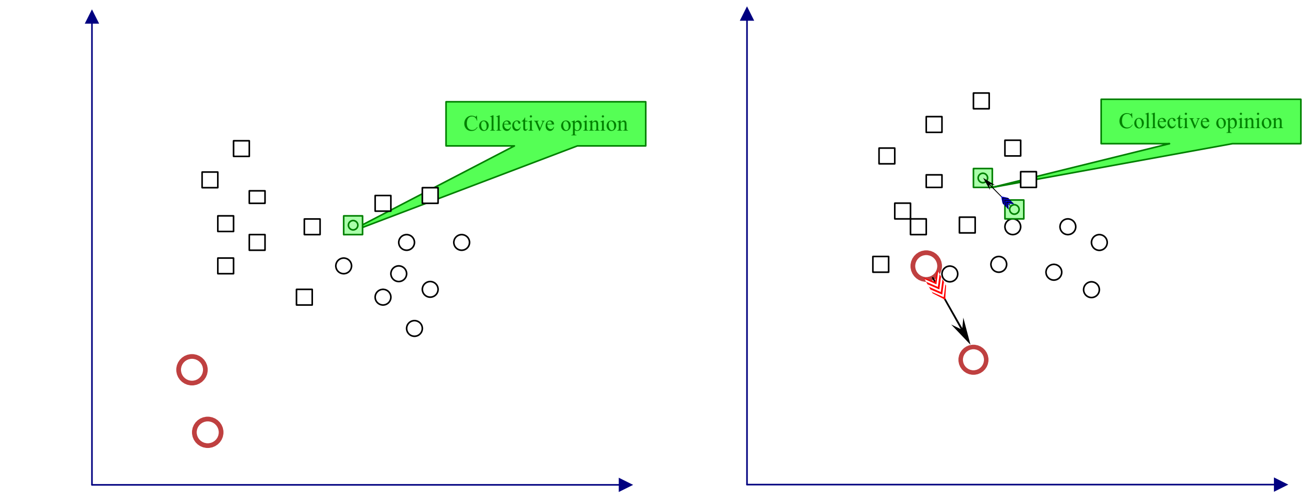

Fig. 3. Non-cooperative behavior (red circles). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and ${C_2}^{(t)}$, respectively, and select a preference ${C_3}^{(t)}$ among those whose similarity is the farthest from ${C_1}^{(t)}$and${C_2}^{(t)}$, that is,${C_3}^{(t)} = \min\{{P_h}^{(t)}|D({P_h}^{(t)}, {C_i}^{(t)}); i = 1, 2\}$. Repeat the above process until all cluster centers are initialized.

Step 2: update centers. In the proposed approach, cluster centers are updated using K-means until the stopping criterion is reached. The distance function in the K-means clustering is the similarity measure defined in Section 2.5.

Step 3: stopping criterion. A clustering iteration stops when it reaches *Maxround* or centers do not change anymore. In this model, we set the stopping criterion based on $\xi \to 0$ as follows:

$$\frac{\sum_{i=1}^{m}\sum_{h=1}^{N}|D({P_i}^{(t)}, {C_h}^{(t-1)}) - D({P_i}^{(t)}, {C_h}^{(t)})|}{mN} \le \xi. \tag{28}$$

When $D({C_i}^{(t)}, {C_1}^{(t)}) < cm$, DMs belonging to ${C_i}^{(t)}$ will be given advice to modify their preferences using the result from Section 3.3.1.

Remark 1: the cm is a preference modification threshold. It identifies DMs who need to modify their preferences. The range of this parameter is [0, 1], in which 0 indicates that no DM modifies her/his preference and 1 means that each DM needs to reconsider their preferences. The DMs who are far away from the collective opinion (less than the threshold cm) are asked to reconsider his/her preference.

Definition of non-cooperative behavior: This study defines non-cooperative behavior in two cases (Fig. 3):

Case 1: Far away from the collective opinion. Considering a preference P_i and $D({P_i}^{(t)}, {P_C}^{(t)}) < cm$ in round t. If e_i modifies the preference to ${P_i}^{(t+1)}$ after round t, and$D({P_i}^{(t+1)}, {P_C}^{(t)}) < cm$ holds, this behavior is regarded as non-cooperative. A non-cooperative subgroup detection approach is proposed to identify members with similar non-cooperative behaviors, and improve the efficiency of consensus reaching. Subgroups that satisfy the following two conditions are non-cooperative: first, the subgroup has stable members, which means that DMs in the subgroup are similar and stick to their preferences; second, the center ${C_i}^{(t)}$of the subgroup is far away from the collective opinion, which means that$D({C_i}^{(t+1)}, {P_C}^{(t)}) < D({C_i}^{(t)}, {P_C}^{(t)})$. If everyone stubbornly stays at their own preference, we can detect each non-cooperative DM one by one. At the extreme situation, if every DM is non-cooperative, the GDM process stops and the group opinion is directly integrated. The consensus degree is low and unsatisfied since the level of consensus degree cannot be improved.

Case 2: Unchanged preference. There are two subcases in the unchanged preference scenario. First, DMs are unwilling to modify their preferences based on the feedbacks derived from the collective opinion. Second, hesitant DMs may ponder on their preferences or randomly provide a preference to avoid revealing their true intentions. In this subcase, in addition to$D({P_i}^{(t)}, {P_C}^{(t)}) < cm$, they will modify their preference with $D({P_i}^{(t+1)}, {P_C}^{(t+1)}) > D({P_i}^{(t)}, {P_C}^{(t)})$ in the next round, but in the following step $D({P_i}^{(t+2)}, {P_C}^{(t+2)}) < D({P_i}^{(t+1)}, {P_C}^{(t+1)})$, where the t is round of preference iteration.

Detection of non-cooperative behaviors: The preference relations provided by DMs are divided into subgroups using a clustering algorithm based on the similarity measure. Interactions always exist in different subgroups. Therefore, DMs with more experience and knowledge may influence other DMs in a subgroup. We propose to combine clustering algorithms and feedback adjustments to detect the two types of non-cooperative behaviors. The relative changes of the similarity measure in each modification are computed to identify non-cooperative behaviors. The detailed process is as follows:

Case 1: Individuals' non-cooperative behavior. In real-life management activities, minority opinions are an essential component of GDM and are the focus of non-cooperative behavior. There are two rules to identify minority opinions. First, a preference ${P_i}^{(t)}$ does not belong to any cluster, which means that $D({P_i}^{(t)}, {C_i}^{(t)}) < \zeta$ ($\zeta \in [0, 1]$are pre-set values), and it is considered to be an outlier in clustering. Second, $D({P_i}^{(t)}, {P_C}^{(t)}) < D({P_i}^{(t-1)}, {P_C}^{(t-1)})$ and $D({P_i}^{(t)}, {P_C}^{(t)}) < \varsigma$ are hold which means this DM is far away the collective opinion, where $\varsigma = \min\{cm, \sum_{i=1}^{m} D({P_i}^{(t)}, {C_1}^{(t)})/m\}$.

Case 2: Subgroup's non-cooperative behavior. Similar to a non-cooperative individual, the similarity degree between the center of a non-cooperative subgroup ${C_i}^{(t)}$ and the collective opinion $P_C^{(t)}$ will gradually decrease with each iteration. This implies that$D({C_i}^{(t)}, {P_C}^{(t)}) < D({C_i}^{(t-1)}, {P_C}^{(t-1)})$ and $D({C_i}^{(t)}, {P_C}^{(t)}) < \tau$.$\tau$is a threshold to judge similarity changes, and $\tau = \min\{cm, \min\{D({C_i}^{(t-1)}, {P_C}^{(t-1)})\}, i = 1, 2, ..., N$, N is the number of clusters.

Another critical task is to identify stabilized subgroups and decide whether a subgroup comprises DMs with similar preferences. For a given thresholdυ, we consider that two clusters are similar if their similarity deviation $d({C_h}^{(t)}, {C_k}^{(t-1)}) < \upsilon, \upsilon \to 0$, where $d({C_h}^{(t)}, {C_k}^{(t-1)}) = \frac{\sum_{i=1}^{m}|D({P_i}^{(t)}, {C_h}^{(t)}) - D({P_i}^{(t-1)}, {C_k}^{(t-1)})|}{m}$.

Algorithms 1 detects non-cooperative behaviors in LSGDM.

3.3.3. Weighting process

Several studies have proposed weighting processes to manage non-cooperative behaviors in GDM. The basic idea of these methods is to decrease the influence of non-cooperative DMs by updating weights, which are assigned to an aggregation model (Eq. (21)). As a result, the collective opinion includes more preference information and holds a higher satisfaction degree for the majority of the DMs.

This study modifies the weighting process using partial weight penalty strategy to identify non-cooperative behaviors. For $P_i^{(t)}$ that is found to be non-cooperative, the weights σ_i will be updated using the following formula:

$$\sigma_i^{(t+1)} = \sigma_i^{(t)} \frac{D(P_i^{(t)}, P_C^{(t)})}{\sum_{j=1}^{Q_h} D(P_j^{(t)}, P_C^{(t)})}, \ t = 3, 4, \ldots, Maxround \quad (29)$$

where Q_h is the total number of non-cooperative DMs. Since non-cooperative detection has to wait until at least the third round, it is unnecessary to compute $\sigma^{(2)}$.

Remark 2**:** In our method, the impact of each DM is considered in the group opinion rather than decreasing to 0 when he/she is non-cooperative, which is different from Palomares et al. (2014).

The detailed consensus-reaching process is summarized in Algorithm 2. The model has five key steps, and each step is described in the previous subsections.

4. Application in China's targeted poverty reduction project

Financial inclusion, as proposed by the United Nations, aims to provide financial services at an affordable cost to low-income groups in need (United Nations, 2005). Even though access to sustainable financial services (such as savings, credit, and insurance) is essential for economic development and inclusive growth, around 2 billion adults still have no chance to obtain formal financial services. Research has shown that financial inclusion plays a vital role in ending poverty (Chibba, 2009; Manji, 2010; Sarma & Pais, 2011), improving the level of education (Chiapa, Prina & Parker, 2016), and promoting gender equality (Swamy, 2014). Many countries, especially developing countries, have made substantial efforts to promote financial inclusion and have achieved noticeable progress. For example, the Chinese government has encouraged banks to set up inclusive finance divisions to increase loans for money-starved small firms and poverty relief (Xinhuanet, 2017). Financial inclusion attracted growing interest in the field of business economics and public administration, including academics, professionals, and policy-makers (Fig. 4).

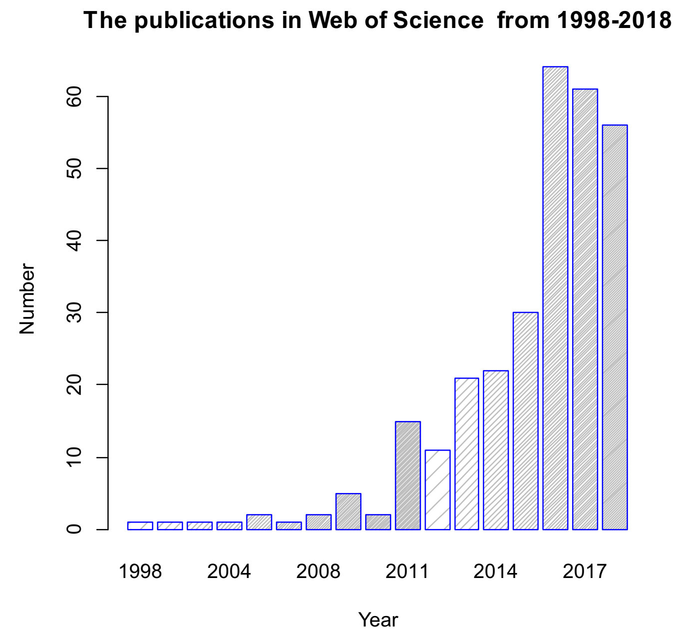

Fig. 4. The publications on financial inclusion from 1998–2018.

In the field of financial inclusion, financial services are still considered as a business activity rather than financial aids. One of the fundamental issues in inclusive finance is to select the proper recipients and maximize the utility of funding and resources (Ghosh, 2013; Lopez & Winkler, 2018; Schwittay, 2011; Yousaf, Ali & Hassan, 2019) since financial resources are limited. In contrast with traditional credits and loans, financial inclusion products have some unique features. First, due to high credit risk and low returns, the development of inclusive finance must be supported and guided by governments or humanitarian organizations (Ghosh, 2013; Marshall, 2004; Cobb, Wry & Zhao, 2016; Chen & Jin, 2017; Gupta & Mahakud, 2019; Misati, Kamau & Nassir, 2019; Ergün, & Doruk, 2020). This makes the decision-making process more complex than commercial loans. Besides credit risk assessment, equality is another issue that must be considered in inclusive finance due to the shortage of funds (Myers, Cato & Jones, 2012). Second, credit information is normally missing in this decision procedure. Experts or managers from local financial institutions and governments can use pairwise comparison and consider multi-criteria non-financial information, such as labor ability, social relations, and credit motivation. In addition, soft information is used to complement the evaluation when traditional business principles do not apply to the beneficiary selection (Allen, Demirguc-Kunt, Klapper & Peria, 2016; Schotten & Morais, 2019). The opinions of representatives from beneficiary groups or rural poor people can also be used to reach a more comprehensive understanding of the actual financial demand, daily behavioral habits, and moral characteristics.

The beneficiary selection for financial inclusion is a LSGDM process because it involves a large number of participants, including governments, local financial institutions, humanitarian organizations, and representatives of potential beneficiaries. LSGDM can improve equality and democracy by considering risk management and conflicting preferences. It represents a fusion of different substitute information to obtain more accurate measurements and choose suitable and reasonable beneficiaries for financial institutions.

4.1. The intrinsic data features

The data used in this section were collected from the "targeted poverty reduction project" carried out in the Qinghai-Tibet plateau in China. This project was designed to provide interest-free microcredit (small loans) to people with a better repayment ability in this region whose annual income is lower than 212 US dollars. These data are accessible and reported by Chao (2017).

Five major parties were involved in this project: officers from the People's Bank of China (PBOC, the central bank of China), rural credit cooperatives, local government representatives, village committees, and delegates for low-income people. Participants in this project can be seen as an LSGDM with non-cooperative behaviors and heterogeneous preference relations due to the following reasons:

(i) The number of participants is large (generally more than 20). The final decision is the integrated opinion of all parties.
(ii) The diverse educational backgrounds and experiences of DMs lead to diverse preference formats.
(iii) Preference conflicts exist among different parties. For instance, the central bank and local governments prefer offering grant loans to the poorest people, while rural credit cooperatives consider risk minimization as their priority. Thus, non-cooperative behaviors characterize the decision-making process. DMs need to evaluate five alternatives from poor

Table 2
Members of the DMs.

Organizations	Members	Preference formats		Total preference Members
Central bank officials	4	Multiplicative or additive preferences	Preference ordering	10
Project management representatives	5	Multiplicative or additive preferences	Utility value	8
Local government officials	7	Multiplicative or additive preferences	Additive preference	14
Local small and medium financial institutions	11	Multiplicative or additive preferences	Multiplicative preference	20
Poor representatives	17	Preference ordering or utility value	Total DMs	52
Village self-government organizations	8	Additive preferences		

villagers, and they will rank the alternatives considering the opinions of all participants. The characteristics of DMs are summarized in Table 2.

The alternatives in this project are listed as follows:

Alternative 1: The potential beneficiary is 55 years old. There are 3 laborers in his family. The purpose of the loan is to build a field free-range chicken farm.

Alternative 2: The potential beneficiary is 51 years old single male. The purpose of the loan is to get a living support when he is out-migration for work.

Alternative 3: The potential beneficiary is 42 years old. There are 2 laborers in his family. His family income is agricultural products trading. The purpose of the loan is to get circulating capital for his business.

Alternative 4: The potential beneficiary is 45 years old. His family has 4 laborers and owns 8 cows and 24 sheep. His purpose of the loan is to buy a tractor to improve agricultural production.

Alternative 5: The potential beneficiary is 42 years old and divorced. He needs to take care of his father at home and has a daughter attending high school. The purpose of the loan is to rebuild his house collapsed in heavy rain.

The five alternatives were provided to the 52 DMs. If they choose a multiplicative relation, they use ratio scale [1/9, 9] to indicate their subjective preferences about the relative important degree of two alternatives (Saaty, 1980). For example, the scale "9" means that the alternative is extremely important than the others, and "1" indicates that two alternatives are equal. The importance of "8, 6, 4, 2" lie between two adjacent odd numbers. In addition, the preference value a_{ij}(between$x_i$andx_j) and the preference value a_{ji}(between$x_j$andx_i) is reciprocal.

If they prefer additive preference relation, they use a value between 0 and 1 to indicate their subjective preferences about the relative importance of two alternatives. The closer the preference value to 1, the better this alterative compare to the other one.

If they use preference orderings, they sort the alternatives to indicate their subjective preferences about the importance of the alternatives. The most important alternative is listed first, and the other alternatives are listed in order of importance.

If they select utility value, they provide a utility ratio of the alternatives by comprehensively evaluate the proportion of each alternative in the entire project. The ratio is a number in the range of 0 to 1, which represents the utility of different alternatives. The larger the utility ratio of an alternative, the more important it is.

In the following feedback adjustment step, the DMs can update their preferences using the same principles. The difference is that they can refer a feedback adjustment mechanism, which was described in Section 3.3.1. During this step, the DMs' preference adjustments were made within a smaller range of values following Eq. (24)–(27).

4.2. Parameters in the experiment

The consensus-reaching process for each iteration is shown in Table 3. In the initial step, all DMs are given equal importance, that is$\sigma_i = 1, i = 1, 2, ..., 52$. The consensus degree progressively increases until it reaches the threshold $cl = 0.93$. In our example, the value of cl was determined by data analysis and management experience. Firstly, we tested the acceptance degree and support level of the collective opinion. Since we needed a consensus deepening process and sought higher satisfaction degree, we conducted several rounds of discussion and negotiations. At the 6th preference iteration, there were still 3 noncooperative DMs who did not change their preferences and 94.2% of the DMs accepted the results. The moderator, who comes from the sub branch of the central bank, determined the consensus degree was high enough and finalized the threshold. The consensus degree SCD was slightly greater than 0.93 at this time. Secondly, we consulted the robust results of the collective opinion in each iteration. In this procedure, a collective opinion is robust when the components of the collective opinion no longer change at the 6th preference iteration. This can be observed in Fig. 7.

The potential beneficiaries can be divided into three classes. The first class comprises those who have some assets and are more likely to be selected by local banks. The second class includes groups with better productivity capacities, which can improve the rural economic development supported by the local government and PBOC. The last class comprises poverty-stricken groups, including disabled people, lonely seniors, and people with serious diseases. They are more concerned by village committees and delegates for low-income people.

In addition, data clustering analysis (Fig. 5) shows that the data distribution of cluster 3 (green star) in the three cluster setting and cluster 3 (green star) and 4 (black "+") in the four cluster setting are similar. Thus, we set three clusters to describe the structure of the DMs.

In the iteration process, DMs whose similarity measure is less than $cm = 0.86$ need to modify their opinions. The threshold of the change in the distance of two cluster centers is set to $\xi, \upsilon = 0.15$, which measures the similarity between two clusters. In addition, ζ is set to 0.90, which is the deviation of an outlier with respect to each cluster. The values of ς and τ are determined by cm and the average deviation of the preference data. The consensus degree is reached when DMs move towards collective opinion using a penalty coefficient (Section 3.3.3).

Table 4 summarizes the definitions, selected values, and suggestions of the parameters that need to preset in the proposed approach. They can be classified into three categories: consensus degree, clustering convergence parameters, and non-cooperative identification. The values of consensus degree and non-cooperative identification need to be determined by management experiences, and data analysis can be used to determine clustering convergence

Table 3
Consensus-reaching process.

Iteration	Number of centers	Cluster including Collective preference	Non-cooperative behavior	weights	SCD(*cr=0.9300*)
$t = 0$	3	1	–	Initial	0.9092
$t = 1$	3	1	–	Initial	0.9181
$t = 2$	3	3	–	Initial	0.9216
$t = 3$	3	3	Detected	Adjusted	0.9229
$t = 4$	3	3	Detected	Adjusted	0.9249
$t = 5$	3	2	Detected	Adjusted	0.9271
$t = 6$	3	2	Detected	Adjusted	0.9302

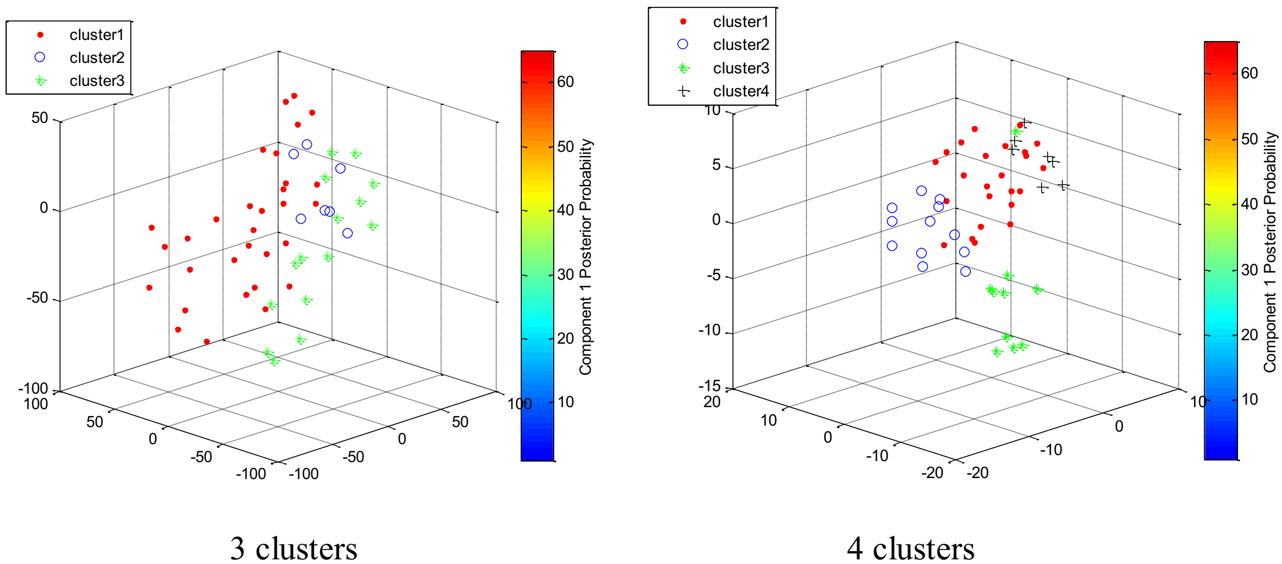

Fig. 5. Comparison of the pre-set 3 clusters and 4 clusters.

parameters. Data analysis can improve the accuracy of the noncooperative behavior detection and decrease decision time.

The determination of the value of the first class, including one parameter, depends on the initial consensus degree and the specific decision goal of a given GDM question. The expected consensus degrees for different GDM problems vary. In our financial inclusion example, loans are interest-free and provided by the central bank. To ensure fairness, the central bank requires high consensus degrees among the decision makers (such as representatives of the poor, rural credit cooperatives, and local governments) on the selection of beneficiaries. Otherwise, the central bank will not issue loans to this area. In this example, a high consensus degree is necessary to obtain the financial inclusion loans.

The second class is clustering convergence parameters, which is a stop condition of the iterations in clustering algorithm and has two parameters. Their values can be decided by the preference data analysis using data mining software, such as Matlab (pdist, linkage and cophenetic functions) and Weka (visible cluster). These data mining software can be used to test the convergence and analyze the control process.

The last class is noncooperative identification, including two parameters, which is used to identify DMs who need to modify their preferences and detect noncooperative behavior. In our example, the mean deviation among individual preferences to the collective opinion was 0.9 at the first preference iteration. According to the previous experiences of similar financial inclusion projects, the non-cooperative decision makers accounted for about 20% of DMs. We set the $cm = 0.86$to persuade more DMs to modify their preferences because data analysis shows that 23.1% of the DMs were included in the non-cooperative group and only 15.4% of the DMs were in the group when this value was set as 0.90. The noncooperative DMs will slow down the consensus-reaching process. Generally, a higher consensus degree, *SCD,* needs more rounds of feedback modification to build a satisfied group consensus. In sum, the different thresholds affect the time cost of the iterative preference modification.

The convergence and clustering tests in our method need to meet some technical conditions. Table 5 lists these conditions, analyzes the limitations, and provides suggestions on how to deal with the technical conditions in other applications.

4.3. Main results

This subsection summarizes the results of this project using the proposed model. Firstly, the group opinion is converted back to one of the corresponding four preference relations, and a reference adjustment interval of each preference value (formulas (24)–(27) in Section 3.3.1) is given to the DMs. Then, DMs can reconsider whether adjustments are needed and select new values in the interval if they consider that the initial preference values should be modified. This adjustment guarantees that a group consensus can be reached because the mechanism assures that the individual preferences converge to the group opinion. Mathematically, a group opinion $w^{(t)} = ({w_1}^{(t)}, {w_2}^{(t)}, ..., {w_n}^{(t)})$ is converted into each entry in ${u_i}^{(G,t)} = {w_i}^{(t)} \sum_{j=1}^{n} {u_j}^{(k,t)}$ to get the utility values. If the preference ordering is used to represent DMs' preferences, the group opinion is transformed into the preference ordering using${o_i}^{(G,t)} = s$, where s is the sth ranked value in descending order in the group opinion. If the multiplicative preference relation is used to represent DMs' preferences, the group opinion is transformed into${a_{ij}}^{(G,t)} = \frac{{w_i}^{(t)}}{{w_j}^{(t)}}$. If the additive preference relation is used to represent DMs' preferences, the group opinion is transformed into${b_{ij}}^{(G,t)} = \frac{{w_i}^{(t)}}{{w_i}^{(t)}+{w_j}^{(t)}}$. Decision

Table 4
The parameters selections.

Class	Parameters	Values used in the paper	Suggestions
Consensus metric	Consensus degree parameter (cl): it is a consensus metric based on a mean "distance" between individual preference and the collective opinion. The upper bound of this parameter is 1, which indicates a unanimous consensus in GDM. A higher consensus degree needs more rounds of discussion and negotiation, which leads to a slower consensus convergence process.	The financial inclusion loan requires a high consensus degree to avoid opinion conflicts in the selection process. This value was set to 0.93 according to the experimental result, which indicated that enough DMs accepted the results and the collective opinion was robust at this value.	The value can be determined by the actual requirement of a decision problem. For example, it can be set to 2/3 according to the "minority obeying majority" principle in a democratic decision.
Noncooperative identification	Noncooperative behavior detection parameter (ζ): it measures the isolated individuals whose "distance" is far away from a collective opinion. The value range of this parameter is [0, 1]. Generally, a smaller detection threshold (corresponding a larger cosine similarity measure) slows down the convergence speed of the consensus process (in Fig. 11).	We need to detect noncooperative behavior to obtain a higher consensus degree. The value was set as 0.9 according to the data analysis. It is the average distance of an individual preference and the collective opinion.	ζ indicates the distance from individual preferences to a collective opinion. This threshold can be set as the mean deviation of individual preferences to a collective opinion.
	Preference modification (cm): it identifies DMs who need to modify their preferences. The range of this parameter is [0,1], in which 0 indicates that no DM modifies her/his preference and 1 means that each DM needs to reconsider their preferences. More DMs modify their preferences can accelerate the consensus process.	We set the value of this parameter to 0.86 to persuade more DMs to modify their preferences based on the results of data analysis (for detailed explanation, refer to Section 3.3.2).	It can be set equal to or slightly lower than the initial consensus degree.
Clustering convergence parameters	Subgroup stability parameter (υ): it is used to judge whether two subgroups can be grouped into one cluster in each iteration. The lower bound is 0, which means that two subgroups are the same. A higher value will produce more noncooperative subgroups and increase complexity in the GDM.	v was set to 0.15 in this paper because the result showed that when v was 0.15, the similarity of DMs in any of the two subgroups been merged is high in our example.	This parameter can be determined by data analysis using a data mining software.
	Clusters convergence parameter (ξ): it is a stop condition of the clustering algorithm. The lower bound is 0. If the value is 0, the center of each cluster must remain the same at each iteration. The smaller this value, the longer the clustering process takes because it will increase the number of iteration rounds.	We experimented several values and tested the clustering convergence results. When the value is 0.15, the clustering process can be stopped since the process converges and the clusters do not change any more.	This parameter can be determined by the convergence test of the clustering process using real data for a specific decision-making question.

makers adjust individual preferences to approach the group opinion $w^{(t)} = ({w_1}^{(t)}, {w_2}^{(t)}, ..., {w_n}^{(t)})$, so the range of DMs' preference adjustment is between the above feedback information (value) of the group opinion at last round and the individual opinion of the previous round iteration. That is, the ith entry of the utility value is ${u_i}^{(k,t+1)} \in [\min\{{u_i}^{(G,t)}, {u_i}^{(k,t)}\}, \max\{{u_i}^{(G,t)}, {u_i}^{(k,t)}\}]$ at the $t+1$th iteration; the ith entry of the preference ordering is ${o_i}^{(k,t+1)} \in [\min\{{o_i}^{(G,t)}, {o_i}^{(k,t)}\}, \max\{{o_i}^{(G,t)}, {o_i}^{(k,t)}\}]$ at the $t+1$th iteration; the entry at the ith row and the jth column of the multiplicative preference relation is ${a_{ij}}^{(k,t+1)} \in [\min\{{a_{ij}}^{(G,t)}, {a_{ij}}^{(k,t)}\}, \max\{{a_{ij}}^{(G,t)}, {a_{ij}}^{(k,t)}\}], i > j$ at the $t+1$th iteration; and the entry at the ith row and the jth column of the additive preference relation is ${b_{ij}}^{(k,t+1)} \in [\min\{{b_{ij}}^{(G,t)}, {b_{ij}}^{(k,t)}\}, \max\{{b_{ij}}^{(G,t)}, {b_{ij}}^{(k,t)}\}], i > j$ at the $t+1$th iteration. The cluster results and iterative process are summarized in Table 3. The data distribution is described in Fig. 6.

The collective opinion of the initial preferences provided by the 52 DMs is {0.2209, 0.1962, 0.2030, 0.1867, 0.1933} using the optimization rule from Eq. (22), and the ranking is $x_1 \succ x_3 \succ x_2 \succ x_5 \succ x_4$. The initial consensus degree SCD is 0.9092, and the degree is not higher than the preset thresholds. Thus, the feedback information is returned to the DMs, who are asked to modify their preferences toward the collective opinion. The feedback information and their modification process are as follows:

For a multiplicative preference relation, the feedback information is $({a_{ij}}^{(G,t)})_{5\times5}, {a_{ij}}^{(G,t)} = {}^{w_i}/_{w_j}$, that is:

$$\left({a_{ij}}^{(G,t)}\right)_{5\times5} = \begin{pmatrix} 1 & 1.1262 & 1.0882 & 1.1834 & 1.1431 \\ 0.8880 & 1 & 0.9663 & 1.0508 & 1.0151 \\ 0.9190 & 1.0349 & 1 & 1.0875 & 1.0505 \\ 0.8450 & 0.9516 & 0.9196 & 1 & 0.9660 \\ 0.8748 & 0.9852 & 0.9520 & 1.0352 & 1 \end{pmatrix}.$$

For the additive preference relation, the feedback information is $({b_{ij}}^{(G,t)})_{5\times5}, {b_{ij}}^{(G,t)} = w_i/(w_i + w_j)$, that is:

$$\left({b_{ij}}^{(G,t)}\right)_{5\times5} = \begin{pmatrix} 0.5 & 0.5297 & 0.5211 & 0.5420 & 0.5334 \\ 0.4703 & 0.5 & 0.4914 & 0.5124 & 0.5037 \\ 0.4789 & 0.5086 & 0.5 & 0.5210 & 0.5123 \\ 0.4580 & 0.4876 & 0.4790 & 0.5 & 0.4913 \\ 0.4666 & 0.4963 & 0.4877 & 0.5087 & 0.5 \end{pmatrix}.$$

Table 5
The technical conditions in our methods.

Technical conditions	Parameters	Constraints and hypotheses	Limitations	Application suggestions
Noncooperative identification	Noncooperative behavior detection parameter (ζ)	Non-cooperative DMs' preferences are independent and far from most DMs.	Unable to separate independent non-cooperative individuals.	If there are no independent individuals, then DMs whose preferences are away from the group opinion can be regarded as non-cooperative.
	Preference modification (cm)	Most DMs' preferences are similar and concentrated in a small range.	This value can lead to uncertainty about the number of DMs who need to adjust their preferences.	If the condition is not satisfied, a certain percentage of DMs can be determined to modify preferences according to the decision task. Sort the distance between individual preferences and the group opinion in ascending order, and select enough people to consider their preference modification.
Clustering convergence	Subgroup stability parameter (υ)	DMs have similar preferences can be regarded as a subgroup. Technically, the distance between individual preferences in the same subgroup is less than a certain threshold.	The data distribution needs to be analyzed in advance to determine parameter values. If individual preferences differ greatly, they cannot be grouped together.	Visualization techniques, such as t-SNE, can determine the parameter of similarity degree between different subgroups. If the subgroup conditions are not met, each individual will be performed as non-cooperative detection and weight management (in Section 3.3.2).
	Clusters convergence parameter (ξ)	DMs can be divided into different clusters. Technically, with each preference adjustment, the sum of the distances between the cluster center and their members gradually converges.	In some decision problems, the clustering of preference relations does not converge quickly.	In a specific decision-making question, if there is non-convergent clustering, it can set the maximum round of iterations to replace this pre-set parameter and then analyze the clustering results.

Thus, the DMs can compare their preference relations to the above transformed collective opinion, and modify their judgments. For example, the eighth DM from the local government provided the modified preference relation as follows:

$$DM^{(8,0)} = \begin{pmatrix} 1 & 2.50 & 0.67 & 3.00 & 0.50 \\ 0.40 & 1 & 0.20 & 1.80 & 0.67 \\ 1.50 & 5.00 & 1 & 5.00 & 1.67 \\ 0.33 & 0.56 & 0.20 & 1 & 0.44 \\ 2.00 & 1.50 & 0.60 & 2.25 & 1 \end{pmatrix}.$$

The priority vector of this DM is {0.1973, 0.1019, 0.4026, 0.0709, 0.2273}, and the alternative ranking is $x_3 \succ x_5 \succ x_1 \succ x_2 \succ x_4$. In this case, the preference is different from the collective opinion$x_1 \succ x_3 \succ x_2 \succ x_5 \succ x_4$. The preference modification interval for each pairwise comparison is expressed by the following matrix:

$$a_{ij}{}^{(8,1)} \in \begin{pmatrix} 1 & [1.1, 2.5] & [0.67, 1.1] & [1.2, 3] & [0.5, 1.1] \\ & 1 & [0.2, 1] & [1.1, 1.8] & [0.67, 1] \\ & & 1 & [1.1, 5] & [1.1, 1.67] \\ & & & 1 & [0.44, 1] \\ & & & & 1 \end{pmatrix}.$$

The DM modified the comparison value at two positions, a_{23} and a_{34}, and the new preference relation is as follows:

$$DM^{(8,1)} = \begin{pmatrix} 1 & 2.50 & 0.67 & 3.00 & 0.50 \\ 0.40 & 1 & 0.40 & 1.80 & 0.67 \\ 1.50 & 2.50 & 1 & 4.00 & 1.67 \\ 0.33 & 0.56 & 0.25 & 1 & 0.44 \\ 2.00 & 1.50 & 0.60 & 2.25 & 1 \end{pmatrix}.$$

As another example, the preference relation provided by a banker from local credit union is expressed by the following matrix:

$$DM^{(9,0)} = \begin{pmatrix} 0.5 & 0.91 & 0.55 & 0.66 & 0.57 \\ 0.09 & 0.5 & 0.41 & 0.34 & 0.37 \\ 0.45 & 0.59 & 0.5 & 0.79 & 0.32 \\ 0.34 & 0.66 & 0.21 & 0.5 & 0.46 \\ 0.43 & 0.63 & 0.68 & 0.54 & 0.5 \end{pmatrix}.$$

The following matrix expresses the modification interval for each pairwise comparison:

$$b_{ij}{}^{(9,1)} = \begin{pmatrix} 0.5 & [0.53, 0.91] & [0.52, 0.55] & [0.54, 0.66] & [0.53, 0.57] \\ & 0.5 & [0.41, 0.41] & [0.34, 0.51] & [0.37, 0.50] \\ & & 0.5 & [0.52, 0.79] & [0.32, 0.51] \\ & & & 0.5 & [0.46, 0.59] \\ & & & & 0.5 \end{pmatrix}.$$

This DM only modified $b_{12}{}^{(9,0)} = 0.91$ $(b_{21}{}^{(9,0)} = 0.09)$ into $b_{12}{}^{(9,1)} = 0.71(b_{21}{}^{(9,1)} = 0.29)$.

In addition, the 35th DM from villagers' representatives provided the preference order {1, 2, 4, 5, 3}, referred to the collective opinion {1, 3, 2, 5, 4}, and modified preference $DM^{(35,1)} = \{1, 2, 5, 4, 3\}$. The 49th DM from villagers' representatives provided the preference {0.76, 0.74, 0.39, 0.66, 0.17} and feedback information from the collective opinion is {0.60, 0.53, 0.55, 0.51, 0.53}. The DM modified the preferences in the following interval [0.60,0.76, 0.53,0.74, 0.39, 0.55, 0.17,0.53], and the preference was finally modified to $DM^{(49,1)} = \{0.76, 0.74, 0.45, 0.66, 0.35\}$.

The reasons for DMs to modify their judgments vary. In the financial inclusion example, decision makers agree to modify their preferences because they share a common motivation in decision making. DMs from local governments are willing to adjust their preferences to reach a consensus because financial inclusion projects can reduce local poverty and enhance regional economic performance. From their perspective, the important thing is to reach a group consensus so that the financial inclusion projects can be implemented. The DMs from the central bank hope to maximize the effectiveness of inclusive finance's funds through a fair beneficiary selection process. They are willing to adjust their preferences by referring to the group opinion because a higher consensus degree in the group increases the chance of a successful financial inclusion implementation. The villagers hope to carry out agriculture production through interest-free loans and raise their income levels. Since the central bank requires the group consensus to reach a certain level in order to start the implementation of the interest-free loans, the villagers are willing to adjust their preferences to promote the development of the entire village. Thus,

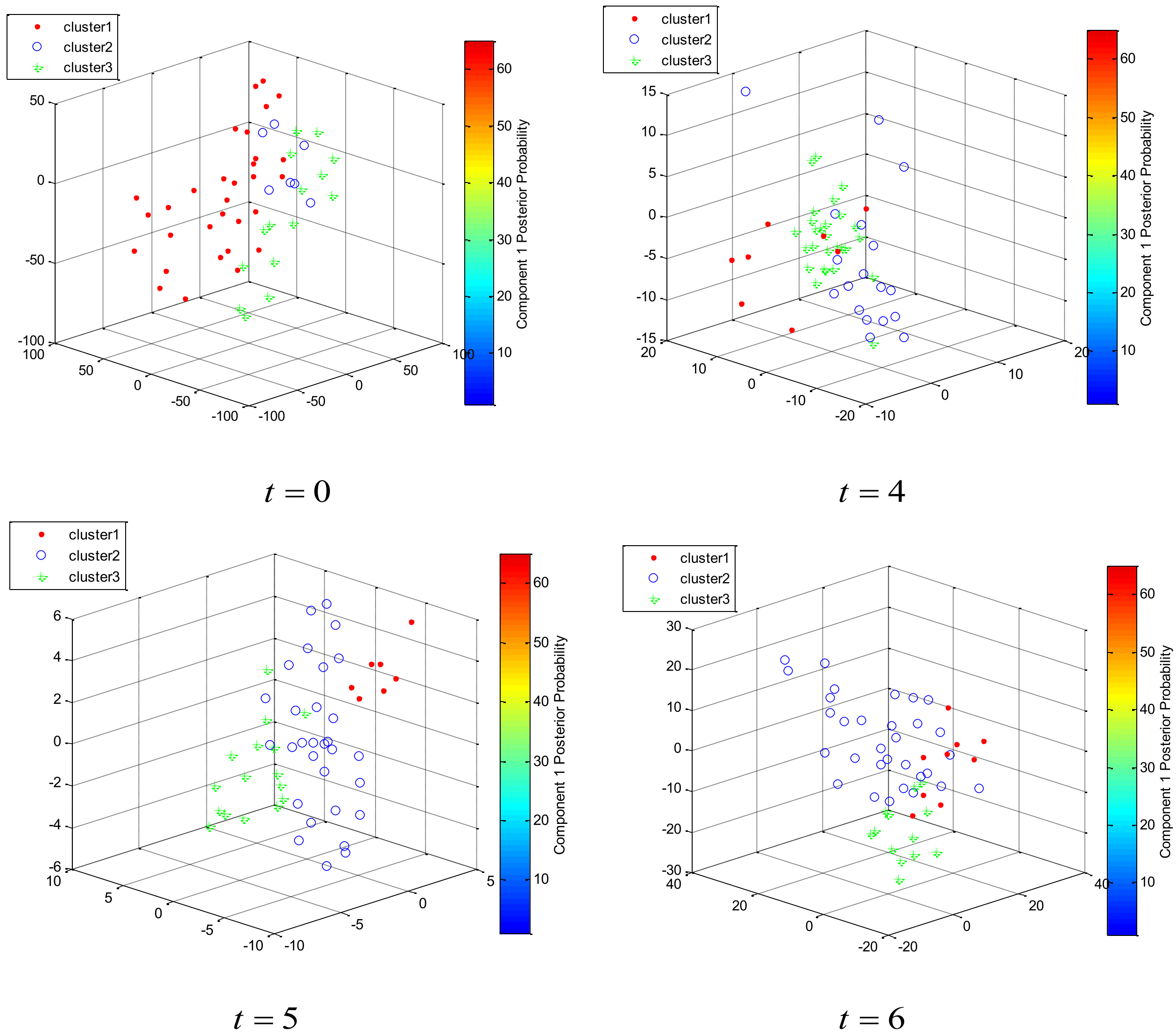

Fig. 6. The consensus-reaching process in iteration $t = 0$, $t = 4$, $t = 5$and $t = 6$.

the decision makers will follow the collective opinion in order to reach a consensus. Mathematically, the individual preference vector will move towards the group opinion, which means that the similarity degree between the individual preference vector and the group opinion will gradually increase (the convergence is provided in lemma (Page 34)).

In the following step, the non-cooperative behavior is detected, and the corresponding weighting process is implemented. The main results are shown in Table 5. After the individual preference modification, the final collective opinion at round 6 is {0.2508, 0.1741, 0.2051, 0.1799, 0.1900} and the ranking is $x_1 \succ x_3 \succ x_5 \succ x_4 \succ x_2$when the *SCD* reaches 0.9302.

The dynamic visual process is summarized in Fig. 6 and is implemented by decreasing dimensions with t-SNE algorithm (Maaten & Hinton, 2008). The multiplicative preference relations and additive preference relations are handled as a vector synthesis of each column vector. Due to the non-cooperative behavior in GDM, some DMs in the consensus-reaching process are far away from the collective opinion all the time (Fig. 6). However, most participants are willing to modify their preferences and obtain a satisfied final collective opinion during iterations.

As the consensus degree deepens, the value of the first position in the collective opinion, which is the weight of alternative 1, increases. The trend of alternative 2 is the opposite of alternative 1. The trends of alternative 3, 4 and 5 are not monotonic throughout the process. As the consensus reaching deepens, the value of each alternative in the collective opinion tends to stable. As Fig. 7 shows, the components (weights of the alternatives) become stable after the 4th preference iteration.

Remark 3**:** This real-life financial inclusion project was implemented through GDM. During the implementation of this project, the proposed approach was simplified in the detection of non-cooperative behavior. Specifically, no participants from the central bank were tested for non-cooperative DMs because the central bank is the policymaker and fund provider in the financial inclusion project. All other participants in this project, including local small and medium financial institutions (the rural credit union in China), project management representatives, local government officials, poor representatives, and village self-government organizations were tested for non-cooperative behaviors. If each DM is non-cooperative in an extreme case, the collective opinion will be aggregated by a weighted sum of each preference relation and the consensus degree will be low. This extreme case means that there's no negotiation or consensus process in a GDM, which is not a situation considered in this study.

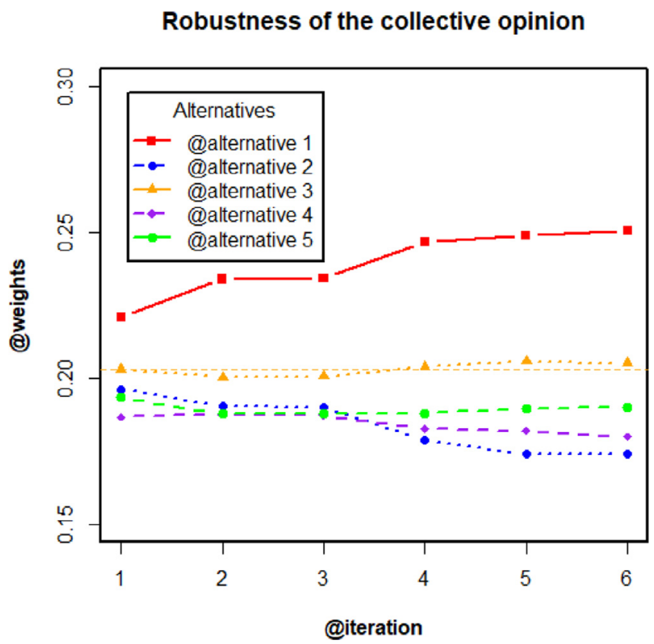

Fig. 7. The trends of the collective opinions as the consensus reaching deepens.

Remark 4: Interactive method can be used to explore the Pareto frontier. Theoretically, the Pareto optimal frontier can be obtained in multi-objective optimization by different combinations of weights of the objective functions in the weighted sum approach (Arora, 2015). The proposed interactive method includes the weight modifications in the weighting process (Eq. (29)). In real life applications, the interactive method may promote the collective opinion toward the Pareto frontier. The consensus reaching can lead to a rational choice. However, since the payoff function of the Pareto is uncertain in a GDM problem, it is impossible to determine whether the GDM problem is a Pareto question.

5. Discussion

In this section, we compare the proposed consensus-reaching model with existing methods. We also address the convergence of the proposed consensus-reaching model by both theoretical and simulation analyses.

5.1. Comparison of aggregation operators

This subsection compares the proposed approach with three existing aggregation operators (Ma, Fan, Jiang & Mao, 2006; Chiclana et al., 2001; Xu et al., 2011) using an example. The proposed approach can decrease the number of consensus steps and directly obtain a collective opinion. The results show that the collective opinion generated by our approach is closer to each individual's preference than the other three methods.

Example: Integrating four different preference structures. This example was used by Chiclana et al. (2001)), Ma et al., 2006, Xu et al. (2011). Assume $\Pi = \{DM_1, DM_2, ..., DM_K\}$ represents K DMs and each DM has the same importance degree. The different preference formats are as follows.

The first two DMs provide utility values with the format DM_1 and DM_2, where:

$$DM_1 = \{u_i | i = 1, 2, 3, 4\} = \{0.5, 0.7, 1.0, 0.1\},$$

$$DM_2 = \{u_i | i = 1, 2, 3, 4\} = \{0.7, 0.9, 0.6, 0.3\},$$

Table 6
Comparative result with other methods.

Approaches	Priority vector and ranking of alternatives	*SCD*
Chiclana et al. (2001))	0.5651,0.7826,0.6619,0.4973$A_2 \succ A_3 \succ A_1 \succ A_4$	0.8243
Ma et al. (2006)	0.2210, 0.3426, 0.2755, 0.1159 $A_2 \succ A_3 \succ A_1 \succ A_4$	0.8274
Xu et al.(2011)	0.2210, 0.3426, 0.2827, 0.1537 $A_2 \succ A_3 \succ A_1 \succ A_4$	0.8289
Our model	0.2303, 0.3588, 0.2563, 0.1547 $A_2 \succ A_3 \succ A_1 \succ A_4$	***0.8306***

Remark 5:The ranking of alternatives in Chiclana et al. (2001)) was computed using a selection operator (OWA). We normalized the vector and then calculated the cosine similarity measure.

Their rankings of alternatives are $A_3 \succ A_2 \succ A_1 \succ A_4$ and $A_2 \succ A_1 \succ A_3 \succ A_4$, respectively.

The third and fourth DMs provide preference orderings, with the preference structures DM_3 and DM_4, where:

$$DM_3 = \{o_i | i = 1, 2, 3, 4\} = \{3, 1, 4, 2\},$$

$$DM_4 = \{o_i | i = 1, 2, 3, 4\} = \{2, 3, 1, 4\}.$$

The fifth and sixth DMs express their preference information in terms of a multiplicative preference relation as DM_5 and DM_6,where:

$$DM_5 = \begin{pmatrix} 1 & 1/7 & 1/3 & 1/5 \\ 7 & 1 & 3 & 2 \\ 3 & 1/3 & 1 & 1/2 \\ 5 & 1/2 & 2 & 1 \end{pmatrix},$$

$$DM_6 = \begin{pmatrix} 1 & 3 & 1/4 & 5 \\ 1/3 & 1 & 2 & 1/3 \\ 4 & 1/2 & 1 & 2 \\ 1/5 & 3 & 1/2 & 1 \end{pmatrix}.$$

The last two DMs provide their preference formats using fuzzy preference relation. The fuzzy PCM are DM_7 and DM_8, respectively:

$$DM_7 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.5 \end{pmatrix} DM_8 = \begin{pmatrix} 0.5 & 0.5 & 0.7 & 1 \\ 0.5 & 0.5 & 0.8 & 0.6 \\ 0.3 & 0.2 & 0.5 & 0.8 \\ 0 & 0.4 & 0.2 & 0.5 \end{pmatrix}.$$

The ranking of alternatives obtained by all methods is the same $A_2 \succ A_3 \succ A_1 \succ A_4$. However, our result has higher similarity value than the other methods. This indicates that our method guarantees higher consistency for each DM than the other models (as shown in Table 6). The *SCD* measure also shows that our method achieves the highest consensus degree.

Table 7 compares several representative methods by qualitative analysis. The four methods reported in Table 7 must derive individual priority vectors, and may cause more deviations in this step. Therefore, the complexity of consensus-reaching increases. Our method does not need to derive a priority vector and thus decreases the decision-making complexity.

5.2. Consensus measurement analysis

The existing consensus measures in GDM can be divided into two classes (Chiclana et al., 2013; González-Arteaga et al., 2016; Herrera-Viedma et al., 2014; Zhang et al., 2018). The first establishes a total deviation among individual preference relations. The computation has three steps: the individual deviation, weighted similarity, and total consensus degree. However, the deviation among heterogeneous preference relations cannot be computed,

Table 7
Comparisons of the representative consensus building methods.

Measurements	Handle heterogeneous preference relation	Transformation into unified preference structure	Derive individual priority vector	Analytical solution in optimization	Iterative consensus reaching
Herrera-Viedma et al., 2002	√	√	√	×	√
Dong & Zhang, 2014	√	×	√	×	√
Zhang & Guo, 2014	√	×	√	×	×
Cheng, Zhou, Cheng & Wang, 2018	×	×	√	×	×
Our method	√	×	×	√	√

and the weight of two preference relations is also hard to be determined. The other class of models calculates the weighted deviation from each preference relation to the collective opinion. This approach derives a priority vector from the individual preference relation as a first step and then obtains the total deviation between individual preferences and the collective opinion. Thus, it is more complicated.

In this study, we establish a similarity consensus measure,*SCD*, and introduce a new definition of similarity measure based on the cosine similarity between heterogeneous preference relations and a collective opinion. This approach can reduce the number of computational steps since it does not need to derive each preference vector.

To compare the proposed consensus degree measurement and existing indexes, we introduce an ordinal consensus measure based on the same principle as ours.

We assume that the order of alternatives in the derived priority vector of DM_i is $h_i = (h_1, h_2, ..., h_n)$, and the order of alternatives in the collective opinion is $p_c = (p_1, p_2, ..., p_n)$. The ordinal consensus degree (*OCD*) (Dong et al., 2014; Herrera-Viedma et al., 2002) is:

$$OCD(DM_i) = \frac{1}{n^2}\sum_{j=1}^{n}\left|h_j - p_j\right|,$$

and the consensus degree of GDM is:

$$OCD = \frac{1}{m}\sum_{v=1}^{m} OCD(DM_v),$$

where $m$is the total number of DMs.

Let the priority vector of DM_i be $\omega^{(i)} = ({\omega_1}^{(i)}, {\omega_2}^{(i)}, ..., {\omega_n}^{(i)})$, and the order of the alternatives in the collective opinion be $w^C = (w_1^C, w_2^C, ..., w_n^C)$. The cardinal consensus degree (*CCD*) (Dong et al., 2016) is:

$$CCD(DM_i) = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left({\omega_k}^{(i)} - w_k^C\right)^2},$$

and the consensus degree of GDM is:

$$CCD = \frac{1}{m}\sum_{v=1}^{m} CCD(DM_v),$$

where m is the total number of DMs.

The Kendall rank correlation coefficient (KDC) (Kendall, 1970) is another standard measure. It corresponds to a rank correlation coefficient and is used to measure the ordinal association between two measured quantities. The Kendall rank correlation coefficient is defined as follows:

$$KDC(DM_i) = \tfrac{1}{n(n-1)}\sum_{k<j}\mathrm{sgn}\left(\omega_k^{(i)} - \omega_j^{(i)}\right)\mathrm{sgn}\left(w_k^C - w_j^C\right),$$

$$KDC = \tfrac{1}{m}\sum_{v=1}^{m} KDC(DM_v),$$

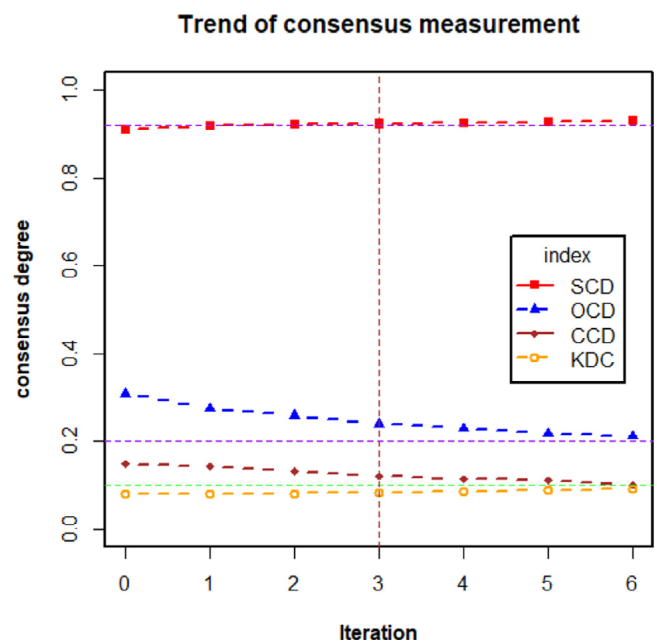

Fig. 8. Trend of the *SCD*, *OCD*, *CCD*, and *KDC*.

where the sign function is $\mathrm{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0. & x = 0 \\ -1, & x < 0 \end{cases}$.

Fig. 8 shows the trend of two consensus degrees as the iteration changes. At the initial situation, the consensus degrees for *SCD*, *OCD*, and *CCD* are 0.9092, 0.3077, and 0.1475, respectively. Moreover, the growth or decline rate of *SCD* and *CCD* is smaller than that of *OCD*. The Kendall rank correlation coefficient at the first and last iterations is 0.0785 and 0.0905, respectively, and the corresponding ordinal consensus degrees are 0.3077 and 0.2113, respectively

A qualitative comparison with the existing consensus measures is also provided (Table 8). The existing distance measures cannot be directly applied to GDM with heterogeneous preference information. The reasons are that the Euclidean distance cannot be used to measure it, and the priority vector from each preference relation needs to be derived as a first step. Followed by integration of the collective opinion and computation of the consensus degree. The *SCD* in our consensus-reaching model is measured by the similarity between the individual preferences and the collective opinion.

5.3. Convergence of the proposed model

The convergence of the consensus-reaching model aims to assure that the consensuses degree can be improved up to a pre-set threshold in the decision process. The convergence of the proposed consensus-reaching model is obtained in two steps: first, the individual preference modification described in Eqs. (24)–(27) assures

Table 8
Comparisons of the representative consensus measurements.

Distance Measurements	Methods	Drawbacks	Derive priority vector of individual preference	Use to heterogeneous preference information
Ordinal consensus degree (Dong & Zhang, 2014),	Order deviation between individual preference and collective opinion	Must derive priority vector from incomplete preference matrix	√	√
Cardinal consensus degree (Dong & Zhang, 2014),	Weights deviation between individual preference and collective opinion		√	√
CR (Palomares et al., 2014; Pérez, Cabrerizo, Alonso & Herrera-Viedma, 2014	Distance among individual preference relations	Only used for homogeneous preference matrix	×	×
Geometric Compatibility Index (Escobar, Aguarón & Moreno-Jiménez, 2015)	Logarithm deviation between individual preference value and the collective opinion	Only used for multiplicative preference matrix	×	×
SCD	Similarity between individual preference and collective opinion		×	√

Algorithm 1
Detection of non-cooperative behavior.

1. **For** iterations that do not reach the preset max round **do**
2. Judge whether the two cluster centers in two iterations belong to the same class; // whether the two clusters have stable members (stabilized subgroups).
3. **If** D(cluster center| t time, collective opinion| t time)<D(cluster center| t-1 time, collective opinion | t-1 time) **do**
4. Assign subgroup;
5. Continue
6. **End**
7. **Else if** D (cluster center| t-1 time, collective opinion| t-1 time)< D (cluster center| t-2 time, collective opinion | t-2 time) **do**
8. Assign subgroup;
9. Continue
10. **End**
11. **End**

Algorithm 2
Consensus-reaching model.

1. **While** consensus measure is not reached, **do**
2. **For** each iteration **do**
3. Compute collective opinion; // by Cosine maximization-based model;
4. Clustering by K-means; // define measure using cosine similarity; K number from real-world problem and evaluation.
5. **If** consensus degree is less than a given threshold **do**
6. **If** individual and subgroup are beyond the preset threshold **do**
7. Modify preference value by the given guidance; // feedback adjustments
8. **End if**
9. Detect a non-cooperative behavior; // whether an individual and subgroup move against collective preference in each iteration;
10. Update weights; // non-cooperative DMs;
11. Continue;
12. **End if**
13. **End for**
14. **End while**

that the preference relation moves towards the group opinion. Second, the consensus degree is monotonic with respect to repeated iterations. The following Lemma 1 proves the first condition, and the second condition is verified by a numerical simulation.

Lemma 1. *: the individual preference relation moves towards the group opinion using the preference modification method described in Equations (24–27) if at least 1 DM agrees to modify his/her preference according to the feedback adjustment interval.*

Proof. : Let $\alpha = (a_1, a_{2,}..., a_n)^T$and $\beta = (b_1, b_{2,}..., b_n)^T$be two given unit vectors, that is, an individual preference vector and the collective preference opinion. Assume that α rotates towards β, and $\gamma = (r_1, r_2, ..., r_n)^T$ is the modified vector fromα. The modification interval is$r_i \in [\min\{a_i, b_i\}, \max\{a_i, b_i\}], i = 1, 2, ..., n$.

Firstly, we assume that a DM modifies one position in his/her preference relation and the modified preference is $\gamma^{(1)}$, then, the modified preference vector $\gamma^{(1)}$ is located "between" the two vector α and β.

Suppose the $r_i, i \in \{1, 2, ..., n\}$ is the first modified value. Then, $0 \le |r_i - b_i| \le |a_i - b_i|and0 \le |r_i - a_i| \le |a_i - b_i|$.

Since the remainder positions are not changed, thus

$$\sum_{t\neq i} |r_t - b_t| b_t = \sum_{t\neq i} |a_t - b_t| b_t$$

For this ith position, without loss of generality, we assume $a_i \le b_i$ for the i. That is:

$$-\sum_{t\neq i} (r_t - b_t)b_t - (r_i - b_i)b_i \le -\sum_{t\neq i} (a_t - b_t)b_t - (a_i - b_i)b_i$$

and

$$\sum_{t\neq i} (r_t - b_t)b_t + (r_i - b_i)b_i \ge \sum_{t\neq i} (a_t - b_t)b_t + (a_i - b_i)b_i$$

Then,

$$\sum_{t=1}^{n} (r_t - b_t)b_t \ge \sum_{t=1}^{n} (a_t - b_t)b_t$$

That is$\langle \gamma - \beta, \beta \rangle \ge \langle \alpha - \beta, \beta \rangle$.

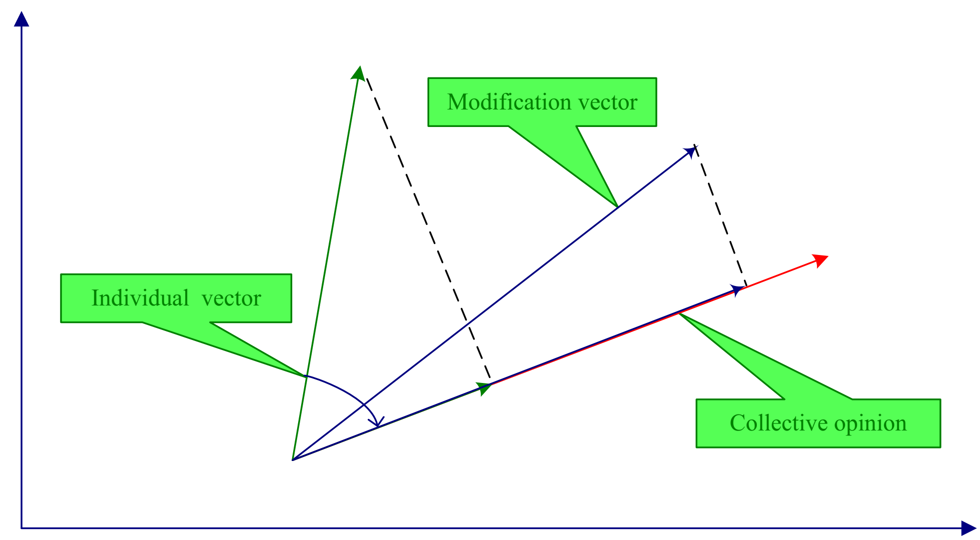

Fig. 9. Modification principle of feedback information.

The above vector equation can be transformed into $\langle \alpha - \gamma, \beta \rangle \leq 0$, and we obtain $\langle \alpha, \beta \rangle \leq \langle \gamma, \beta \rangle$. This indicates that the angle between α and β is less than that between γ and β.

Secondly, based on the same reason, we can also prove that the modified vector α moves towards β after multiple modifications in proper order. Thus, if a γ is the modified vector from α where $r_i \in [\min\{a_i, b_i\}, \max\{a_i, b_i\}], i = 1, 2, ..., n$. Then the γ moves toward β.

Finally, we prove the convergence. Let$\gamma^{(k)}$be the kth preference modification and $C^{(k)} = \langle \gamma^{(k)}, \beta \rangle$. For each$k$, the sequence $\{C^{(k)}\}$ is monotonically increasing and has a least upper bound. Then,

$$\lim_k \left\{C^{(k)}\right\} = Sup\left\{C^{(k)}\right\}$$

Let $\lim_{k\to\infty}\{\gamma^{(k)}\} = \gamma^{\infty}$, then $\langle \gamma^{\infty}, \beta \rangle = Sup\{\langle \gamma^{(k)}, \beta \rangle\}$.

We suppose that

$$\langle \gamma^{\infty}, \beta \rangle < Sup\left\{\left\langle \gamma^{(k)}, \beta \right\rangle\right\}$$

By applying the above preference modification, we can obtain a new preference$\bar{\gamma}^{\infty}$, which is a vector adjusted at least one position ofγ^{∞}according to the reference interval. It is clear that the adjusted preference relation $\bar{\gamma}^{\infty}$. Obviously, we have

$$\langle \bar{\gamma}^{\infty}, \beta \rangle > \langle \gamma^{\infty}, \beta \rangle$$

Thus,

$$\langle \bar{\gamma}^{\infty}, \beta \rangle > Sup\left\{C^{(k)}\right\}$$

which contradicts the definition of$Sup\{C^{(k)}\}$.

This completes the proof of the Lemma. ■

Thus, based on Eqs. (24)–(27), the modification vector will move towards the collective opinion compared to the original vector. It is clear that the $Sup\{C^{(k)}\} = 1$ since the maximum cosine similarity measure is 1, and γ^{∞} and β coincide.

Based on geometry visualization, the modified column vector must be located in the "middle" of the original preference and the collective opinion (Fig. 9). Further, this lemma can be used as a theoretical proof of the preference modification method proposed byDong et al. (2014).

Next, the consensus degree can be divided into two classes. One is cooperative DMs. The similarity between their preferences and the collective opinion must be substantial due to the above proof. The other one is non-cooperative DMs, and the similarity between the non-cooperative preference relations and the collective opinion is less than a threshold, as shown in Section 3.3.2 (i.e., the external circle in Fig. 10). The weighting process (Eq. (29)) assures that the influence decreases in the consensus measure *SCD* (Eq. (23)). The principle is shown in Fig. 10.

Finally, we conduct a numerical simulation to show the convergence of the proposed consensus process. In this simulation, we set the number of DMs as 80, 100, 120, and 150 units, respectively. The parameters are pre-set. The simulated preference adjustment is randomly selected in the given adjustment interval, and then the average consensus degree is calculated. We set the max round of iteration to 11 and add 15% noisy data as non-cooperative DMs, which correspond to outliers that do not need to change their preferences in each preference modification (for simplification). The trends are reported in Fig. 11.

Fig. 11 indicates that the proposed consensus-reaching model can help DMs achieve consensus when setting different parameter values. The management of non-cooperative behavior can accelerate the convergence (begin to detect from the vertical line). The stricter the parameters (the latter case), which means that more DMs are asked to modify their preferences, the quicker the convergence. The latter case reaches the same threshold at the fourth iteration compared to the sixth iteration in the former case.

Finally, we simulate the impact of non-cooperative behavior on LSGDM. The ratio of noisy data is set to 0%, 10%, 20%, and 30%, respectively. We select 100 DMs and $cm = 0.75$. In the simulation, we assume that each DM will modify the preference in terms of feedback information, and the non-cooperative behavior is the noisy data.

Fig. 12 shows the convergence process with a different number of non-cooperative DMs. The improvement of the consensus degree gradually decelerates along with the increase in non-cooperative preferences, but this trend decreases after multiple iterations.

5.4. The consensus in GDM and equilibrium in a game

The consensus deepening mechanism proposed in this paper was motivated by our real-world experience with the financial inclusion project, which is different from traditional financial loans in that it requires a much higher consensus level among the participants.

The financial inclusion loan was launched by the credit union in the grass roots and supported by the Agriculture-supporting Relending and the Retrocession of the central bank of China. Since this type of loan is interest-free, the central bank of China stip-

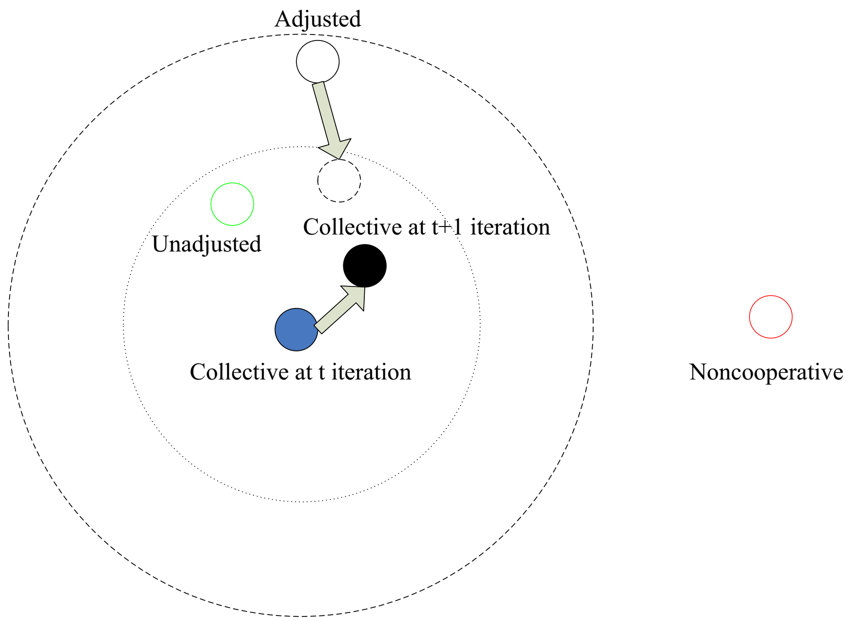

Fig. 10. Preference modification at two iterations.

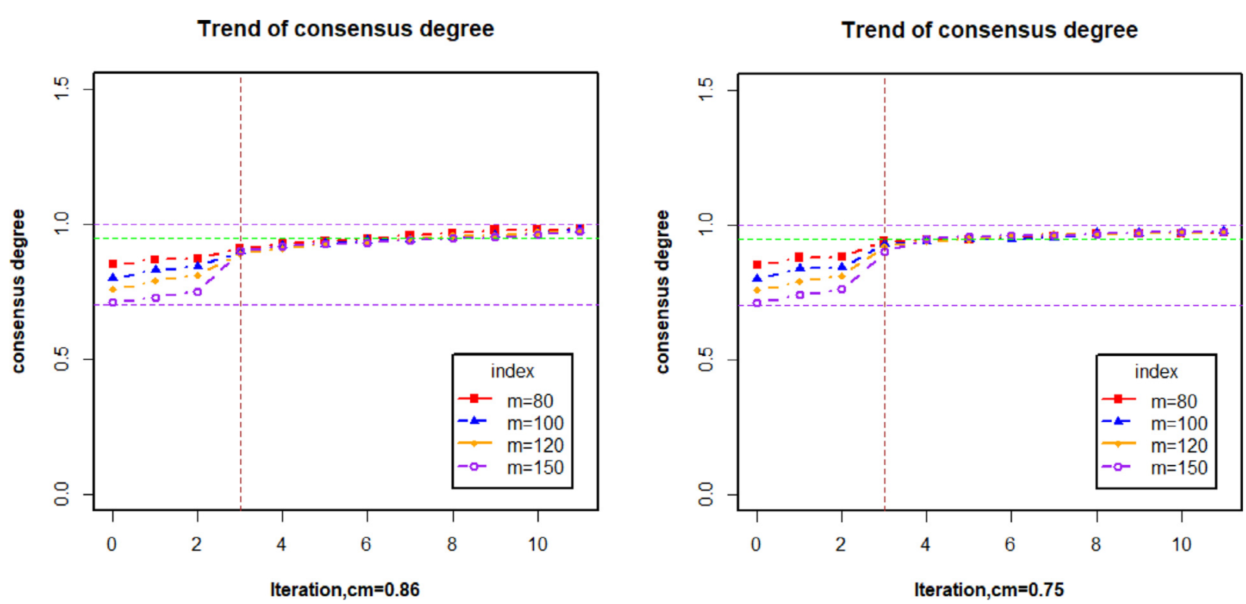

Fig. 11. Trend of consensus degree with different parameters.

ulates high consensus degrees on the selection of beneficiaries among the decision makers (including representatives of the poor, rural credit cooperatives, and local officials) to ensure fairness. Otherwise, the central bank of China will not issue loans to this area. Potential beneficiaries of the financial inclusion loans (alternatives) should satisfy two conditions: they are financially poor, which are assessed by the local government officials and village self-government organizations; they are creditable, which are evaluated by the local credit union and the central bank of China.

In this real-world financial inclusion project, the government promotes interest-free loans to families in need to improve the economic situation in poverty-stricken areas and eradicate poverty. The villagers hope to carry out agriculture production through interest-free loans and raise their income levels. The local credit union launches the "Agriculture-supporting Re-lending" from the central bank to beneficiary in targeted areas to increase its liquidity. The local government wants to use the funds provided by the central bank to improve local poverty and enhance regional economic performance.

Because all participants share a common goal, they are willing to follow the consensus deepening mechanism in order to reach a high-level consensus and enable the implementation of the interest-free loans.

In addition, the beneficiary selection for financial inclusion loans is a decision-making problem in social planning, rather than a game. A game in group decision making is an equilibrium with the maximal individual utility (or a trade-off of individual benefits) and it has different decision conditions (Azam, Zhang & Yao, 2017; da Silva Rocha & Salomão, 2019; Kellner & Schneiderbauer, 2019; Le Cadre, Mezghani & Papavasiliou, 2019). Take transportation de-

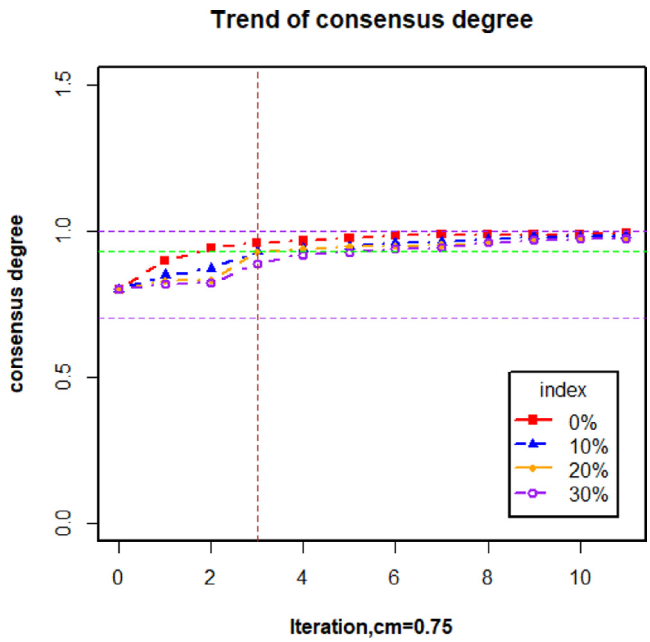

Fig. 12. Trend of census degree with non-cooperative behavior.

cision as an example. The equilibrium is a group path flow combined with optimal individual paths, but the consensus is one of the paths which can be accepted by the majority DMs. In a social planning decision problem, a consensus is frequently used, such as water resource allocation, urban resettlement, and other public affairs and government management. During a consensus reaching process, the goal of DMs is not maximizing individual interests. Rather, it is a process of evaluating and selecting alternatives while pooling the wisdom of all participants and taking into account the opinions of all participants. The goal of the consensus reaching process is to obtain a collective opinion with the highest possible acceptance or satisfactory degree.

Indeed, the consensus deepening process, which is only one step of the consensus reaching in GDM, and a cooperative game have similarities. For example, in the cooperative game, it includes a penalty mechanism for non-cooperative behavior, and the game is based on collective rationality. The preference modification of the consensus deepening also depends on the collective rationality since each DM has the potential to benefit from the results, and weights penalty mechanism exists in our GDM problem as well. It is difficult for both cooperative game and aggregative game to solve an equilibrium question when the number of DMs is large (Nisan et al., 2007). In real-life management applications, we need an operational mechanism to facilitate the consensus reaching in GDM problems.

Remark 6: To the best of our knowledge, this is the first time that a unified similarity measurement for heterogeneous preference formats in LSGDM is proposed. Zhang et al. (2018) developed methods for heterogeneous LSGDM. However, in their model, the consensus-reaching process is based on individual concerns and satisfaction without considering non-cooperative behaviors. Our approach is based on a real-world application, which does not consider individual concerns. Thus, the two methods are not comparable.

6. Conclusions

Heterogeneous preferences and non-cooperative behaviors are common concerns in LSGDM. However, only a few studies addressed consensus deepening when GDM includes large-scale heterogeneous preference information and non-cooperative behaviors. This study proposed a novel consensus-reaching model based on the cosine similarity measure. In this approach, a similarity measure was defined for heterogeneous preferences, and subgroup clustering was applied for non-cooperative detection based on the measure. Finally, we proposed a weighting process and a consensus degree to manage the different subgroups. Compared to the existing methods, the proposed approach directly integrates the collective opinion from individual preference relations. Thus, it avoided the derivation of a priority vector for each DM and transformation of heterogeneous preferences into a uniform structure. We also proved the convergence of the proposed consensus-reaching process using a geometric insight and compared our results with different methods using qualitative and quantitative analyses.

To show the advantages of the proposed consensus-reaching model, it was applied to a real-life financial inclusion project in China. The selection of beneficiaries in inclusive finance is a difficult task. Not only because low-income groups lack credit history, but also because of the large number of decision-making participants and their conflicting views. The results of this application showed that the proposed consensus model can integrate opinions representing various strata of the population and efficiently reach an agreement in LSGDM.

The proposed approach is suitable for GDM problems with these characteristics: large-scale, heterogeneous preference formats, and non-cooperative DMs. One of our future research directions is to apply the proposed approach to more empirical applications, such as urban resettlement projects, which have the similar features to the problems addressed by this study. The development of web-based or mobile device-based decision support system for LSGDM based on the proposed method is another research direction.

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Appendix A

Proof: The solution of the model (21–2) must exist, since the feasible region is bounded, and the objective function has an upper bound.

We take the partial derivatives of the Lagrangian function of the objective function. It follows that

$$L(C,\lambda) = \sum_{k\in\Omega_u}\sum_{j=1}^{n}\sum_{i=1}^{n}\sigma_k\bar{w}_i\bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sum_{i=1}^{n}\sigma_k\bar{w}_i\bar{o}_{ij}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sum_{i=1}^{n}\sigma_k\bar{w}_i\bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sum_{i=1}^{n}\sigma_k\bar{w}_i\bar{p}_{ij}^{(k)} - \lambda\left(\sum_{i=1}^{n}\bar{w}_i^2 - 1\right) \quad (A1)$$

$$\frac{\partial L(C,\lambda)}{\partial \bar{w}_i} = \sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{ij}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{ij}^{(k)} - 2\lambda\bar{w}_i \tag{A2}$$

Set the partial derivative is equal to 0. That is

$$\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{ij}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{ij}^{(k)} - 2\lambda\bar{w}_i = 0 \tag{A3}$$

We can obtain the $\bar{w}_i$ as follows:

$$\bar{w}_i = \frac{\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{ij}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{ij}^{(k)}}{2\lambda},\quad i = 1,2,\ldots,n. \tag{A4}$$

By the constraint condition $\sum_{i=1}^{n}\bar{w}_i^2 = 1$, (A4) can be changed into

$$\sum_{t=1}^{n}\left(\frac{\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{tj}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{tj}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{tj}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{tj}^{(k)}}{2\lambda}\right)^2 = 1. \tag{A5}$$

From (A5), the Lagrangian multiplicator λ can be obtained as following (A6):

$$2\lambda = \sqrt{\sum_{t=1}^{n}\left(\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{tj}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{tj}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{tj}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{tj}^{(k)}\right)^2} \tag{A6}$$

Therefore, the solution can be obtained by (A4), that is

$$\bar{w}_i = \frac{\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{ij}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{ij}^{(k)}}{\sqrt{\sum_{t=1}^{n}\left(\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{tj}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{tj}^{(k)} + \sum_{k=\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{tj}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{tj}^{(k)}\right)^2}},\quad i = 1,2,\ldots,n. \tag{A7}$$

We can show the uniqueness of the solution as follows. Since the above solution is the normalized priority vector, it is obvious that

$$\bar{w}_i = \frac{w_i}{\sqrt{\sum_{s=1}^{n} w_s^2}},\quad i = 1,2,\ldots,n \tag{A8}$$

Then, we can get the following equation from $\sum_{i=1}^{n} w_i = 1$.

$$\sum_{t=1}^{n} w_t = \left(\sum_{t=1}^{n}\bar{w}_t\right)\sqrt{\sum_{s=1}^{n} {w_s}^2} = 1 \tag{A9}$$

From Eq. (A9), the following formula is hold:

$$\sqrt{\sum_{s=1}^{n} {w_s}^2} = \frac{1}{\sum_{t=1}^{n}\bar{w}_t} \tag{A10}$$

Therefore, the solution of the priority vector can be obtained from (A8)

$$w_i = \bar{w}_i\sqrt{\sum_{s=1}^{n} w_s^2} = \frac{\bar{w}_i}{\sum_{t=1}^{n}\bar{w}_t},\quad i = 1,2,\ldots,n \tag{A11}$$

and we can get the solution from (A7):

$$w_i = \frac{\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{ij}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{ij}^{(k)} + \sum_{k\in\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{ij}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{ij}^{(k)}}{\sum_{t=1}^{n}\left(\sum_{k\in\Omega_u}\sum_{j=1}^{n}\sigma_k\bar{u}_{tj}^{(k)} + \sum_{k\in\Omega_o}\sum_{j=1}^{n}\sigma_k\bar{o}_{tj}^{(k)} + \sum_{k\in\Omega_A}\sum_{j=1}^{n}\sigma_k\bar{a}_{tj}^{(k)} + \sum_{k\in\Omega_B}\sum_{j=1}^{n}\sigma_k\bar{p}_{tj}^{(k)}\right)},\quad i = 1,2,\ldots,n. \tag{A12}$$

This completes the proof. ■

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