Chapter

Evolutions of Growing Waves in Complex Plasma Medium

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Abstract

The purpose of this chapter to discuss the waves and turbulence (instabilities) supported by dusty plasma. Plasmas support many growing modes and instabilities. Wave phenomena are important in heating plasmas, instabilities, diagnostics, etc. Waves in dusty plasma are governed by the dynamics of electrons, ions and dust particles. Disturbances in solar wind, shocks and magnetospheres are the sources of generation of plasma waves. The strong interest in complex plasma provides us better understanding of physics of dusty universe, solar winds, shocks, magnetospheres, dust control in plasma processing units and surface modifications of materials. The theory of linearization of fluid equation for small oscillation has been introduced. The concept of fine particles in complex plasma and its importance is also explained. The expressions for the growth rate of the instabilities in turbulence plasma have been derived.

Keywords: plasma oscillations, dispersion, turbulence, instabilities, dusty plasma, fine particles, Hall thrusters, resistive plasma, growth rate

1. Introduction to dusty plasma

The presence of fine particles of mass 10^{-10} to 10^{-15} kg and size 1 to 50 micrometer in an electron-ion plasma is called dusty plasma. Dusty plasma also termed complex plasma, plasma crystals, colloidal crystals, fine particle plasma, coulomb crystal or aerosol plasma and has been found in naturally in solar system, planetary rings, interplanetary space, interstellar medium, molecular clouds, circum-stellar clouds, comets, Earth's environments, etc. Manmade plasmas are ordinary flames, dust in fusion devices, rocket exhaust, thermonuclear fusion, Hall thruster, atmospheric aerosols etc. [1–7]. The detail of existence of dusty plasma is given in **Table 1**. Earlier works shows that dust in plasmas has been considered as unwanted constituents and researchers had tried various methods to eliminate dust particles from plasma-processing units. For the moment, Positive aspects of dusty plasmas emerged, dust particles are playing various positive roles in plasma processing devices. The dust particles experience different forces in plasma. The Gravitational force, drag force, electromagnetic forces, polarization force and radiation pressure [1–8].

2. Physical processes in dusty plasma

There are many circumstances when astrophysical plasma and dust particles are found to coexist together. The study of dusty plasmas systems has an exciting

Cosmic dusty plasmas	Dusty plasmas in the solar system	Dusty plasmas on the earth	Man-made dusty plasmas
Solar nebulae	Cometary tails and comae	Ordinary flames	Rocket exhaust
Planetary nebulae	Planetary ring Saturn's rings	Atmospheric aerosols	Dust on surfaces of space vehicle
Supernova shells	Dust streams ejected from Jupiter	charged snow	Microelectronic fabrication
Interplanetary medium	Zodiacal light	lightning on volcanoes	Dust in fusion devices
Molecular clouds	Cometary tails and comae		Thermonuclear fireballs
Circumsolar rings			Dust precipitators used to remove pollution from
Asteroids			

Table 1.
Classification of dusty plasmas.

properties which has attracted researchers over the world. These fine particles acquire some charges from the electrons to get charged. Moreover, in ordinary plasma, the charge considered to be constant on each particle, whereas, the charge on the dust particle varies with time and position [9, 10]. The charge on the dust particle generally depends on the type of dust grain, the surface properties of dust grain, the dust dynamics, the temperature, density of plasma and the wave motion in the medium. The plasma environments around these particles determine the nature of the charge (positive or negative) of these dusty plasmas. Although, most of the cases, these charged dust particles are negatively charged through different charging process. Their electric charge is determined by the size and composition of the grains [9, 10]. The fact that the frequencies associated with dust particles are smaller than those with electrons -ions and presences of fine particles modifies the dynamics of plasma motions and give rise to new types of propagating modes.
For Dust acoustic waves, where ions and electrons are supposed to be inertia less pressure as gradient is balanced by the electric force, leading to Boltzmann electron and ion number density perturbations, whereas the mass of the dust play an important role in dust dynamics. In the dust acoustic wave the inertia is provided by the massive dust particles and the electrons and ions provide the restoring force. The effect of dust is to increase the phase velocity of the ion acoustic waves. This can be interpreted formally as an increase in the effective electron temperature which has important consequences for wave excitation. The dusty plasma also has a trend to oscillate at its plasma frequency [9, 10].

3. Parameters of dusty plasma

Dusty plasma and ordinary (electron-ion) plasma are different from each other due to the charge to mass ratio difference.

3.1 Dust plasma frequency

The electron and ion plasma frequency is much greater than the dust plasma frequency and it is defined as

$$\omega_{pd} = \sqrt{\frac{Z^2 e^2 n_{d0}}{\varepsilon_o m_d}} < < \omega_{pi}, \omega_{pe} \tag{1}$$

3.2 Gyro frequency in dusty plasma

When charged dust grain/particle executes a spiral motion about the magnetic lines of force, then dust particle moves perpendicular to the magnetic field with Gyro frequency of the dust particle. The centrifugal force is balanced by the Lorentz force. In mathematically,

$$\frac{m_d v^2}{r_d} = eZvB \tag{2}$$

The radius of gyration is

$$r_d = \frac{m_d v}{eZB} = \frac{v}{\Omega_{cd}} \tag{3}$$

where, $\Omega_{cd} = \frac{eZB}{m_d}$, is called the dust cyclotron frequency.

3.3 Macroscopic neutrality

The quasi-neutrality condition is obtained for the negatively charged dusty plasma by $n_{e0} = n_{i0} + Zn_{d0}$, here n_{d0} is equilibrium dust particle density and Z is the electric charge number on the dust particles.

3.4 Strongly vs. weakly coupled dusty plasma (Coulomb correlation parameter)

The property of dust particles in the plasma is expressed by coupling parameter Γ. It is the ratio of the interparticle Coulomb potential energy to the thermal energy of the particles. When the value of Γ exceeds unity, the species are termed to be strongly coupled otherwise weakly coupled dusty plasma. if r_d is the average interparticle separation between particles, then coupling parameter

$$\Gamma = \frac{e^2 Z^2}{4\pi\varepsilon_0 r_d k_B T_d} \tag{4}$$

The interparticle separation can be found out by the relation $r_d = \left(\frac{4\pi n_d}{3}\right)^{-\frac{1}{3}}$. For the typical values of $Ze = 5000e$, $k_B T_d = 0.05$ eV, $n_d = 10^{10}$ m^{-3}, the coupling parameter comes out to be 1500. It is also experienced that, when $\Gamma \sim 170$, the dust particles are found in arranged fashion and said to be Coulomb crystals.

4. Applications of ordinary plasma

Plasma technology is safe, less costly and playing important roles in every fields of daily life. Some of these applications are discussed in **Table 2**.

Fields	Applications
Telecommunication	The Global Positioning System (GPS) use ionosphere's plasma layer to reflect the signal transmitted by GPS satellite for further communication usage.
Sterilization	To sterilize the surgical equipments, which are directly connected with patient's immune system, where cleanliness is difficult
Medical treatment	Plasma treatment is contact-free, painless hardly damage tissue.
In dentistry	Plasmas treatment are used inside the root canal to kill the bacteria
Pollution controlling	Plasma technology is used to control gaseous and solid pollutions.
Water Purification	for destroying viruses and bacteria in a water, Ozone (O3) generated by plasma technology is more effective and less costly at large scale than existing chlorination method
Etching and cleaning of materials	To removes contaminants and thin layers of the substratum by bombarding with the plasma species which break the covalent bonds. It is also used to control the weight of the exposed substrate.
fusion research	Plasma is used to achieve high temperature to run the controlled thermonuclear fusion reactors
nanotechnology	Plasma discharges are helpful in growing the nanoparticles for nano world.

Table 2.
Applications of plasma in different fields.

4.1 Applications of dusty plasma

The presence of dust particles in a system also has positive impacts and has many applications in nanotechnology to synthesize the desired shape and size of the particles by controlling the dynamics of charged dust grains. Surface properties of the exposed materials could be improved by coating with plasma enhanced chemical vapor deposition method. Methane plasma is used to synthesize productive Carbon nanostructures which have like high hardness and chemical inertia. Dusty plasmas are also used for the fabrication of semiconductor chips, solar cells and flat panel displays.

5. Current status of the research

As we know that plasma support electrostatic as well as electromagnetic waves because of the motions of the charged particle. Studies of these waves provide the useful information about the state of the system. The resonance frequencies of plasmas waves can be used as diagnostics tool to characterize the plasma parameters. Plasma waves are generated for acceleration of energetic particles and heating plasmas. The exponential growing waves and modes in plasma removes the free energy from the system and permit the system to become unstable. The study of dusty plasma has gained interest in the last few decades due to its observations [1–10] and applications in the space and laboratory [1–10]. Many authors studied the linear and nonlinear electrostatic wave in the presence and absence of the external magnetic field [11–13]. Sharma and Sugawa studied the effect of ion beam in dusty plasma on ion cyclotron wave instability [13]. The presence of the charged dust grains in the plasma modifies the collective behavior of a plasma and excites the new modes [12–14].

The present charged dust particles introduces dust acoustic and dust ion acoustic waves in the plasma after altering the dynamics of electrostatic and electromagnetic waves of ordinary plasma [11–14]. The charged dust grain also introduces growing and damped modes. Tribeche and Zerguini studied the dust ion-acoustic waves in collisional dusty plasma [15]. Rao et al. [16] predicted the existence of dust-acoustic wave in an unmagnetized plasma that has inertial dust and Maxwellian distributed electrons and ions. Shukla and Silin [17] showed the existence of dust-ion acoustic wave in a plasma. Barkan experimentally investigated that negatively charged dust grains enhances the growth rate of the electrostatic ion cyclotron instability [18]. Akhtar et al. [19] studied the dust-acoustic solitary waves in the presence of hot and cold dust grains. The existence of dust-acoustic wave and dust ion acoustic wave has been confirmed by many investigators in a laboratory experiments [18–20]. Ali [21] reported the electrostatic potential due to a test-charge particle in a positive dusty plasma. Bhukhari et al. derived generalized dielectric response function for twisted electrostatic waves in unmagnetized dusty plasmas [22]. Mendonça et al. showed that a modified Jeans instability lead to the formation of photonbubble in a dusty plasma which in turn form two different kinds of dust density perturbations [23]. Pandey and Vranjes predicted that growth rate of the instability is proportional to the whistler frequency in a magnetized dusty plasma [24].

6. Plasma model and basic equations

Phase and group velocity can be calculated by finding the relation between ω and k. This relation $\omega = \omega(k)$, is called the dispersion relation and contains all the physical parameters of the given medium in which wave propagates. If the frequency has an imaginary part, that indicates an instability. Plasma instability involves some growing modes, whose amplitude increases exponentially. In other words instability represents the ability of the plasma to escape from a configuration of fields [8, 9].

6.1 Electrostatic and electromagnetic waves in ordinary (electron-ion) plasma

Charged particles in a plasmas couples to electromagnetic field. Because of this effect various kinds of waves are formed in plasmas. Plasma waves are electrostatic or electromagnetic based on perturbed (oscillated) magnetic field. If there is a perturbed magnetic field ($\vec{B}_1 \neq 0$), plasma support electromagnetic waves. If the oscillating magnetic field associated with the wave is absent ($\vec{B}_1 = 0$), then only electrostatic waves are supported by plasma. In addition, Electrostatic waves may have longitudinal and transverse component depending on the direction of propagation with the perturbed electric field [8, 9].

A thermal unmagnetized plasma support many modes as discussed by Tonks and Langmuir in 1929. One is transverse waves in a plasma have dielectric constant $\varepsilon_r(\omega) = \frac{\varepsilon(\omega)}{\varepsilon_0} = \left(1 - \frac{{\omega_{pe}}^2}{\omega^2}\right)$. In case of a lower frequency wave ($\omega < \omega_{pe}$), the dielectric constant would be negative. It turn out that if refractive index become imaginary, then waves cannot propagate but are damped (absorbed). Therefore plasma behaves like a waveguide with propagation and cut-off regions depending on the range of frequencies [8, 9].

The electron plasma wave and ion acoustic wave are the examples of electrostatic longitudinal modes, that is particle oscillate parallel to the direction of wave propagation. Ion acoustic waves are electrostatic waves, when both ions and electrons are allowed to oscillate in the wave-field. IA waves are low frequency longitudinal wave and we can use the plasma approximation, $n_{e1} \approx n_{i1} \approx n_0$.

The electron plasma wave (Langmuir mode) satisfy the dispersion relation $\omega(k) = \sqrt{{\omega_{pe}}^2 + 3k^2 \frac{k_B T_e}{m_e}}$, whereas the ion acoustic mode satisfy the dispersion relation $\omega(k) = \frac{k}{\sqrt{1+{\lambda_{De}}^2 k^2}} \frac{k_B T_e}{m_i}$, here λ_{De} is the Debye length of electron [8, 9]. The propagation of electromagnetic waves in the unmagnetized plasma yield the dispersion relation $\omega(k) = \sqrt{{\omega_{pe}}^2 + k^2 c^2}$.

7. Theoretical formulation for the studies of waves in dusty plasma

We consider a unmagnetized collisionless plasma consisting of electrons, ions and dust particles. Here we use the fluid equations and Maxwell's equations to derive the dispersion relations in dusty plasma corresponding to ordinary electron –ion plasma. We denote $\vec{v}_\alpha$ and n_α are the plasma velocity and density of the different species $(\alpha = e, i, d)$ having mass m_α, temperature T_α in electron-volt. We write the equations of continuity and equation of motion of particles to derive the dispersion relation. Then the equations of motion governing the plasma can be written as

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla} \cdot \left(\vec{v}_\alpha n_\alpha\right) = 0 \tag{5}$$

$$\frac{d\vec{v}_\alpha}{dt} = \left\{\frac{\partial}{\partial t} + \left(\vec{v}_\alpha \cdot \vec{\nabla}\right)\right\}\vec{v}_\alpha = \frac{Q}{m_\alpha}\left(\vec{E} + \vec{v}_\alpha \times \vec{B}\right) - \frac{T_\alpha \vec{\nabla} n_\alpha}{m_\alpha n_\alpha} \tag{6}$$

If we define thermal velocities $V_{T\alpha} = \sqrt{\frac{T_\alpha}{m_\alpha}}$.

In the above equation, the derivative $\frac{d\vec{v}_\alpha}{dt}$ is called the convective derivative. $\frac{d\vec{v}_\alpha}{dt}$ can be viewed as the time derivative of $\vec{v}_\alpha$ taken in a "fluid" frame of reference moving with a velocity of $\vec{v}_\alpha$ relative to a rest frame. $\frac{\partial \vec{v}_\alpha}{\partial t}$ represents the rate of change of $\vec{v}_\alpha$ at a fixed point in space and $\left(\vec{v}_\alpha \cdot \vec{\nabla}\right)\vec{v}_\alpha$ represents the change of $\vec{v}_\alpha$ measured by an observer moving in the fluid frame into a region where $\vec{v}_\alpha$ is inhomogeneous.

7.1 Linearization of fluid equations

We consider the perturbed density $n_{\alpha 1}$ and velocity $\vec{v}_{\alpha 1}$ indicated by subscript 1 along with their unperturbed density $n_{\alpha 0}$ and velocity $u_{\alpha 0}$. The unperturbed electric field (magnetic field) as $\vec{E}_0$ ($\vec{B}_0$) and the perturbed value of the electric field (magnetic field) is taken as $\vec{E}_1$ ($\vec{B}_1$). To linearize all the equations, let us write $n_\alpha = n_{\alpha 0} + n_{\alpha 1}$, $\vec{v}_\alpha = \vec{v}_{\alpha 1} + \vec{v}_{\alpha 0}$ and $\vec{E} = \vec{E}_1 + \vec{E}_0$. If the amplitude is chosen to be much smaller than the wavelength of the instability, the equations of motion can be linearized. The perturbed quantities $f_{\alpha 1}$ are much smaller than their unperturbed values $f_{\alpha 0}$, that is $f_{\alpha 1} < < f_{\alpha 0}$. If $\vec{v}_{\alpha 0}$ and $n_{\alpha 0}$ are constant, the terms $\left(\vec{v}_{\alpha 0} \cdot \vec{\nabla}\right) n_{\alpha 0}$, $n_{\alpha 0}\left(\vec{\nabla} \cdot \vec{v}_{\alpha 0}\right)$ and $n_{\alpha 1}\left(\vec{\nabla} \cdot \vec{v}_{\alpha 0}\right)$ are equal to be zero. Further the terms $\left(\vec{v}_{\alpha 1} \cdot \vec{\nabla}\right) n_{\alpha 1}$, and $n_{\alpha 1}\left(\vec{\nabla} \cdot \vec{v}_{\alpha 1}\right)$ are neglected as they are quadratic in perturbation. The linearized form of the fluid equations can be written as

$$\frac{\partial n_{\alpha 1}}{\partial t} + n_{\alpha 0}\vec{\nabla}\cdot(v_{\alpha 1}) + v_{\alpha 0}.\vec{\nabla} n_{\alpha 1} = 0 \tag{7}$$

$$\frac{\partial v_{\alpha_1}}{\partial t} + v_{\alpha 0}\left(\vec{\nabla}\cdot v_{\alpha 1}\right) + \frac{V^2{}_{th\alpha}}{n_{\alpha_0}}\vec{\nabla} n_{\alpha 1} = \frac{Q}{m_\alpha}\left(-\vec{\nabla}\varphi_1 + \vec{v}_{\alpha 1}\times\vec{B}_0\right) \tag{8}$$

$$\varepsilon_0\nabla^2\varphi_1 = \rho = e(n_{e1} + Zn_{d1} - n_{i1}) \tag{9}$$

Let us define $\delta = \frac{n_{d0}}{n_{i0}}$ is the relative dust density, then quasi-neutrality condition follow

$$\frac{n_{e0}}{n_{i0}} = Z\delta + 1 \tag{10}$$

Thus, the assumptions of small oscillation give a set of linear equations.

7.2 Dust-acoustic waves (DAW)

The DAW is an electrostatic wave generated in dusty plasma, where inertia is provided by the dust grains. It is same to the ion-acoustic wave in general plasma, where inertia is provided by the ions. The frequency ω_{pd} of dust acoustic wave is very low due to the high dust mass than the ion (electron) plasma frequency $(\omega_{pi}, \omega_{pe})$. That why dusty plasma supports low frequencies waves. In mathematically, $\omega_{pd} = \sqrt{\frac{e^2Z^2n_{d0}}{\varepsilon_o m_d}} < < \omega_{pi}, \omega_{pe}$, where n_{d0} is the equilibrium dust density.

Let us consider a situation, when dust density is get disturbed. This change will alter the charge on the dust particles and results to an enhancing negative space charge due to the process of negative dust charging. This total space charge density of dust $\rho_d = eZn_{d0}$ is shielded by the surrounding plasma ions and electrons. Therefore an electric field is generated due to the space charge by the fluctuations of dust charge density. This oscillating electric field imparts the force on the dust particle, which further pushes the fluctuations in the direction of the electric field and thus the wave propagates.

We consider that fluctuations are plane wave, which propagating inside the dusty plasma having the form $f = f_o \exp\left\{i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right\}$. Them the time derivative $(\partial/\partial t)$ can be replaced by $-i\omega$ and the gradient $\vec{\nabla}$ by ik. Here $f_1 \equiv n_{\alpha 1}$, $\vec{v}_{\alpha 1}$, $\vec{E}_1$, $\vec{B}_1$. The electrons and ions are assumed to inertia less as compared with mass of the dust grains and should have a Boltzmann distribution, namely

$$n_e = n_{e0}\exp\left(\frac{e\phi_1}{T_e}\right) \cong n_{e0}\left(1 + \frac{e\phi_1}{T_e}\right) \tag{11}$$

$$n_i = n_{i0}\exp\left(-\frac{e\phi_1}{T_i}\right) \cong n_{i0}\left(1 - \frac{e\phi_1}{T_i}\right) \tag{12}$$

Here, n_{e0} and n_{i0} denote the unperturbed values of the electron and ion density respectively. Let us limit that, all the unperturbed velocities are zero, then equation of continuity and equations of motion follows.

The above three equations can be written as

$$-i\omega\, n_{d1} + ikn_{d0}v_{d1} = 0 \tag{13}$$

$$-i\omega\, v_{d1} + ik\frac{V^2{}_{thd}}{n_{d0}}\, n_{d1} = \frac{Ze}{m_d} ik\phi_1 \tag{14}$$

$$-k^2\phi_1 = \frac{e}{\varepsilon_0}(n_{e0} + n_{e1} + Zn_{d0} + Zn_{d1} - n_{i0} - n_{i1}) \tag{15}$$

Eqs. (13) and (14) gives

$$n_{d1} = \frac{k^2 n_{d0}\phi_1 Ze}{m_d\left(k^2 {V^2}_{thd} - \omega^2\right)} \tag{16}$$

After substituting into Poisson's equation, we obtain

$$-k^2\phi_1 = \frac{e}{\varepsilon_0}(n_{e0} + Zn_{d0} - n_{i0}) + \frac{e}{\varepsilon_0}\left\{n_{e0}\left(1 + \frac{e\phi_1}{T_e}\right) - n_{i0}\left(1 - \frac{e\phi_1}{T_i}\right)\right\} + Z\frac{e}{\varepsilon_0}n_{d1} \tag{17}$$

The first term reduces to zero under the quasi-neutrality condition ($n_{i0} = n_{e0} + Zn_{d0}$). Let, the relative dust density is defined by $\delta = n_{d0}/n_{i0}$, then we get

$$-k^2\phi_1 = \frac{e^2 n_{i0}}{\varepsilon_0 T_{i0}}\left(1 + \frac{T_i}{T_e}(1 - \delta Z_d)\right)\phi_1 + Z\frac{e}{\varepsilon_0} \times \frac{k^2 n_{d0}\phi_1 Ze}{m_d\left(k^2 {V^2}_{thd} - \omega^2\right)} \tag{18}$$

After simplification for the nontrivial solution, we readily obtain

$$\omega^2 = k^2 {V^2}_{thd} + \frac{{\omega_{pd}}^2}{1 + \frac{1}{{\lambda_{Di}}^2 k^2}\left(1 + \frac{T_i}{T_e}(1 - \delta Z_d)\right)} \tag{19}$$

the dispersion relation of the DAW shows depends on dust density, temperature of electron, ion and dust. It also shows depends on the inertia of electron and ion.

7.2.1 Limiting cases

In the limit of cold dust ($T_d = 0$) and cold ions ($T_i < < T_e$). Then, the dispersion relation simplifies into the dispersion relation of ion-acoustic wave in an ordinary plasma

$$\omega = \frac{k\lambda_{Di}\omega_{pd}}{\sqrt{1 + {\lambda_{Di}}^2 k^2}}, \tag{20}$$

which is the same as for the ion-acoustic wave in classical plasma.

7.2.2 Behavior at low wave length

For small wave numbers $k^2\lambda_{D,i}^2 < < 1$ the wave is acoustic $\omega = kC_{DAW}$ with the dust-acoustic wave speed

$$C_{DAW} = \sqrt{\varepsilon Z_d^2 \frac{kT_i}{m_d}} \tag{21}$$

7.2.3 Behavior at high wave length

For large wave numbers $k^2\lambda_{D,i}^2 > > 1$, the wave is not propagating and just oscillates at the dust plasma frequency.

7.3 Dust ion acoustic wave (DIAW)

In the previous expression of ion-acoustic wave, the wave speed depends on ion temperature and on the mass of the dust. In the DIAW, the dust particles are supposed to immobile. We write the equation of motion, continuity and Poisson's equation

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (v_i n_i) = 0 \tag{22}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{e}{m_i} \frac{\partial \phi}{\partial x} \tag{23}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{e}{\varepsilon_0}(n_i - n_e) \tag{24}$$

The Poisson's equation contains the perturbed electron and ion densities. The electrons are treated as Boltzmann distributed as follow

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{kT_e}\right) \tag{25}$$

The only place, where the dust properties enter is the quasi-neutrality condition

$$n_{i0} = n_{e0} + Z_d n_{d0} \tag{26}$$

Eqs. (24), (25) and (26) gives the dispersion relation for the DIAW as

$$\omega^2 = \frac{\omega_{pi}^2 k^2 \lambda_{D,e}^2}{1 + k^2 \lambda_{D,e}^2} = \left(\frac{n_{i0}}{n_{e0}}\right) \frac{kT_e}{m_i} \frac{k^2}{1 + k^2 \lambda_{D,i}^2} \tag{27}$$

Using Eq. (25)

$$\omega^2 = \frac{\omega_{pi}^2 k^2 \lambda_{D,e}^2}{1 + k^2 \lambda_{D,e}^2} = \left(1 + \frac{Z n_{d0}}{n_{e0}}\right) \frac{kT_e}{m_i} \frac{k^2}{1 + k^2 \lambda_{D,i}^2} \tag{28}$$

It is clear from Eq. (27), that phase speed of the DIAW is increase as dust charge density increases. But the electron density has opposite effect on the speed of the DIAW.

8. Dissipative turbulence/instabilities in Hall thruster plasma

The section is devoted to the existing instabilities in a Hall thruster plasma. The principle of thrusters is the ionization of a Noble gas (propellant) in a crossed filed discharge channel. The accelerated heavy ions of inert gas are used to generate a thrust by the use of electrostatic forces. Xenon is used as an ion thruster propellant because of its low reactivity with the chamber and high molecular weight [25–33]. These types of devices support many waves and instabilities because of the turbulence nature of the plasma. These instabilities affect the performance and the efficiency of the device. In order to control these instabilities and further consequences, it has become necessary to study the growth rate of these instabilities. In a Hall thruster, the electrons experiences force along the azimuthal direction because of $\vec{E} \times \vec{B}$ drift. The collision momentum transfer frequency (v) between the electrons and neutral atoms are also taken into account to see the resistive effects in the plasma. Since ions do not feel magnetic field because of their larger larmor radius compared to length of the device. There equation of motion for ions can be written as

$$M\left\{\frac{\partial}{\partial t}+\left(\vec{v}_i \cdot \vec{\nabla}\right)\right\}\vec{v}_i = e\vec{E} \tag{29}$$

Motion of electrons under the electric and magnetic fields

$$mn_e\left\{\frac{\partial}{\partial t}+\left(\vec{v}_e \cdot \vec{\nabla}\right)+v\right\}\vec{v}_e = -en_e\left(\vec{E}+\vec{v}_e \times \vec{B}\right)-\vec{\nabla}p_e \tag{30}$$

8.1 Linearization of fluid equations

Let us denote the perturbed densities for ions and electrons as n_{i1} and n_{e1} velocities as $\vec{v}_{i1}$ and $\vec{v}_{e1}$ respectively. The unperturbed velocities v_0 and u_0 are taken in the x- and y-direction respectively. The amplitude of oscillations of the perturbed densities are taken small enough. The linearized form of Eq. (29) and Eq. (30) are written as

$$M\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)\vec{v}_{i1} = -e\vec{\nabla}\phi_1 \tag{31}$$

$$m\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial y}+v\right)\vec{v}_{e1} = e\left(\vec{\nabla}\phi_1-\vec{v}_{e1}\times\vec{B}_0\right)-\frac{T_e\vec{\nabla}n_{e1}}{n_0} \tag{32}$$

The continuity equations of electrons and ions can be linearized as below

$$\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)n_{i1}+n_0\left(\vec{\nabla}\cdot\vec{v}_{i1}\right)=0 \tag{33}$$

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial y}\right)n_{e1}+n_0\left(\vec{\nabla}\cdot\vec{v}_{e1}\right)=0 \tag{34}$$

Fourier analysis: We seek the sinusoidal solution of the above equations, therefore the perturbed quantities are taken as $f_1 \sim f_0 \exp\left(i\omega t - i\vec{k}\cdot\vec{r}\right)$. Them the time derivative $(\partial/\partial t)$ can be replaced by $i\omega$ and the gradient $\vec{\nabla}$ by ik, here $f_1 \equiv n_{i1},\ n_{e1},\ \phi_1,\ \vec{v}_{i1},\ \vec{v}_{e1},\ \vec{E}_1$ together with ω as the frequency of oscillations and $\vec{k}$ as the propagation vector.

By using Fourier analysis from Eq. (31)–(34), the perturbed ion and electron densities are given as follows,

$$n_{i1}=\frac{ek^2n_0\phi_1}{M(\omega-k_xv_0)^2} \tag{35}$$

$$n_{e1}=\frac{en_0(\omega-k_yu_0-iv)k^2\phi_1}{m\Omega^2(\omega-k_yu_0)+m(\omega-k_yu_0-iv)k^2{V_{th}}^2} \tag{36}$$

The expression for the electron density n_{e1} is derived under the assumptions that $\Omega >> \omega$, k_yu_0 and v in view of the oscillations observed in Hall thrusters.

8.2 Dispersion equation and growth rate of electrostatic oscillations

Finally, we use the expressions for the perturbed ion density n_{i1} and electron density n_{e1} in the Poisson's equation $\varepsilon_0 \nabla^2 \phi = e(n_{e1} - n_{i1})$ in order to obtain

$$-k^2\phi_1 = \frac{{\omega_e}^2 \hat{\omega} k^2 \phi_1}{\Omega^2(\omega - k_y u_0) + \hat{\omega} k^2 {V_{th}}^2} - \frac{{\omega_i}^2 k^2 \phi_1}{(\omega - k_x v_0)^2} \tag{37}$$

For the nontrivial solution of the above equation, the perturbed potential $\phi_1 \neq 0$, we have from Eq. (37)

$$\frac{{\omega_e}^2 \hat{\omega}}{\Omega^2(\omega - k_y u_0) + \hat{\omega} k^2 {V_{thE}}^2} + \frac{(\omega - k_x v_0)^2 - {\omega_i}^2}{(\omega - k_x v_0)^2} = 0 \tag{38}$$

This is the dispersion relation that governs the electrostatic waves in the Hall thruster's channel. In the above equations, we introduced parameter $\omega_{e(i)} = \sqrt{\frac{e^2 n_0}{m(M)\varepsilon_0}}$, $\Omega = \frac{eB_0}{m}$ and $V_{th} = \sqrt{\frac{Y_e T_e}{m}}$.

After simplification of Eq. (38) we obtain

$$\begin{aligned}
&\omega^3\left({\omega_e}^2 + \Omega^2 + k^2 {V_{th}}^2\right) - \omega^2\left[\left({\omega_e}^2 + \Omega^2 + k^2 {V_{th}}^2\right)\left(k_y u_0 + 2k_x v_0\right) + iv\left({\omega_e}^2 + k^2 {V_{th}}^2\right)\right] \\
&+\omega\left[{k_x}^2 {v_0}^2 {\omega_e}^2 + 2k_x v_0\left(k_y u_0 {\omega_e}^2 + iv{\omega_e}^2 + ivk^2 {V_{th}}^2\right) + \left(k^2 {V_{th}}^2 + \Omega^2\right)\left(2k_x v_0 k_y u_0 + {k_x}^2 {v_0}^2 - {\omega_i}^2\right)\right] \\
&-\left({k_x}^2 {v_0}^2 - {\omega_i}^2\right)\left[k_y u_0\left(\Omega^2 + k^2 {V_{th}}^2\right) + ivk^2 {V_{th}}^2\right] - {\omega_e}^2 {k_x}^2 {v_0}^2\left(k_y u_0 + iv\right) = 0
\end{aligned} \tag{39}$$

This is the dispersion equation that governs the electrostatic waves in the Hall thruster's channel. It is clear from the above equation that Hall thruster support different waves and instabilities which satisfies the dispersion relation (39).

8.3 Results and discussion

To estimate the growth rates of the instability, we numerically solve Eq. (39) by giving typical values of all parameters used for the thruster [25–32]. Therefore, for investigating the growths of the waves, we plot the negative imaginary parts of the complex roots (correspond to the instabilities) in **Figures 1**–**3**.

It is found that the growth of the wave is enhanced for the larger values of the wave number of the oscillations as shown in **Figure 1**. In others words, the oscillations of larger wavelengths are stable. The findings are consistent to the results predicted by Kapulkin et al. [34]. In **Figure 2**, the variation of growth of the wave with the momentum transfer collision frequency is shown and it has been depicted that instability grow at faster rates in the presence of more electron collisions. This is mainly due to the resistive coupling with electrons' drift's in the presence of more collisions. The similar results are also predicted by Fernandez et al. [35] in the simulation studies of dissipative instability which shows that the growth of the instability is directly proportional to the square root of the collision frequency.

In **Figure 3** it is noted that the wave grows faster if the electrons carry higher temperature. The present finding matched with the same results observed by other investigators [36, 37] of dissipative instability in an plasma.

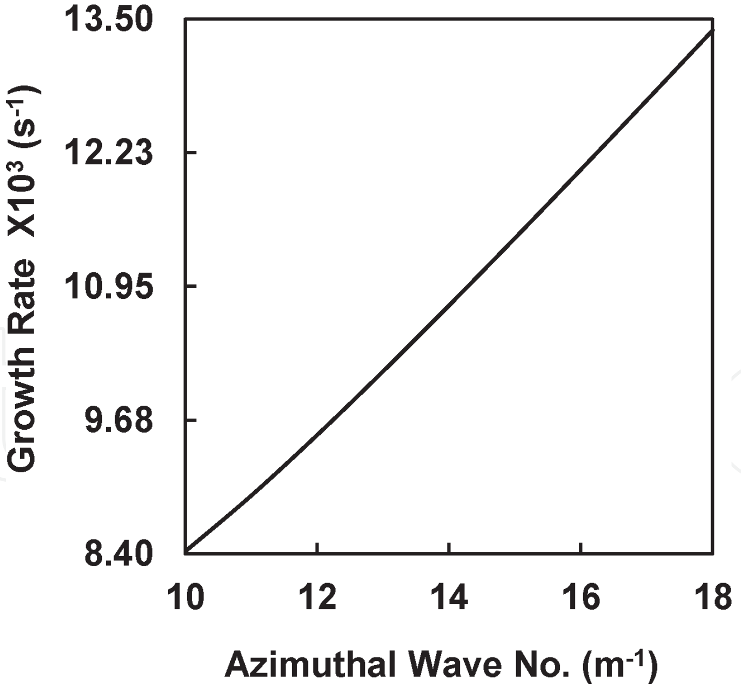

Figure 1.
Growth rate versus azimuthal wave number, with parameters of Hall plasma thrusters as $B_0 \sim 100 - 200$ *G,* $n_0 \sim 5 \times 10^{17}$ 10^{18} m^3, $T_e = 10 - 15$ *eV,* $u_0 \sim 10^6$ *m/s,* $v \sim 10^6$ */s and* $v_0 \sim 2 \times 10^4 - 5 \times 10^4$ *m/s.*

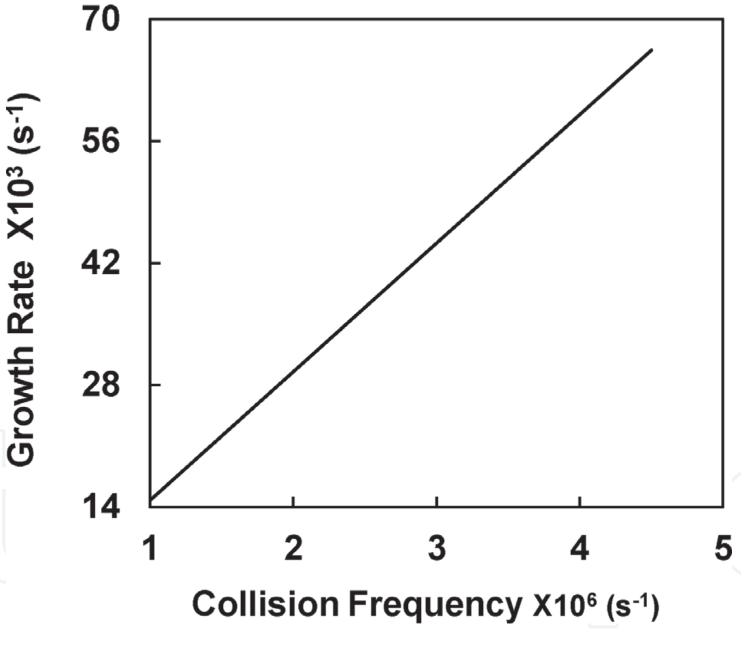

Figure 2.
Growth rate versus collision frequency.

8.4 Conclusion

In summary, we can say that dusty plasma physics has vital role in novel material processing and diagnostics tools. The nanoparticles of desired shape can be synthesized by controlling the dynamics of charged particles in the semiconductor industry. For example, the rotation of the dust particles can extract the electron flux in the magnetron sputtering unit. Thus, dusty plasma is a remarkable field in all areas of natural sciences. Though, the elimination of dust particles in the semiconductor industry is still a main alarm. The advanced development tools to learning

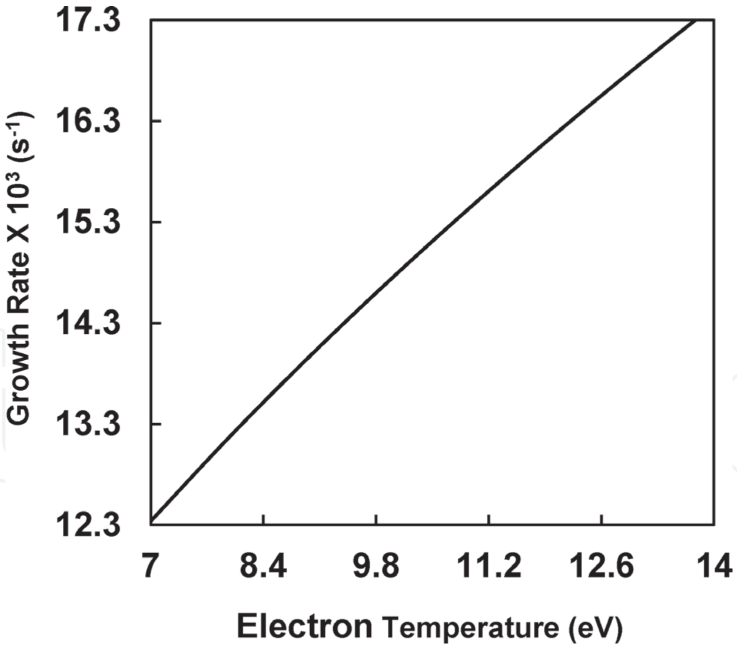

Figure 3.
Variation of growth rate with electron temperature.

dusty plasma will lead to additional discoveries about the astrophysical events and new scientific findings. The different dispersion relations are derived to see the behavior of the wave with wave number in complex plasma. The theory of linearization has been used for the smaller amplitude of the oscillations to derive the perturbed quantities. In the last section, the growth rate of dissipative instability has been depicted in a Hall thruster turbulence plasma. The instability grow faster with the collision frequency, azimuthal wave number and the electron temperature.

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References

[1] Shukla PK, Mamun AA. Series in Plasma Physics: Introduction to Dusty Plasma Physics. Bristol: Taylor & Francis Group, CRC Press; 2001

[2] Kersten H, Thieme G, Fröhlich M, Bojic D, Tung D H, Quaas M, Wulff H, Hippler R. Complex (dusty) plasmas: Examples for applications and observation of magnetron induced phenomena 2005 Pure and Applied Chemistry. 77 415. DOI: 10.1351/pac200577020415

[3] Merlino RL. Dusty plasmas and applications in space and industry. Plasma Physics Applied. 2006. ISBN: 81-7895-230-0

[4] Bonitz M, Henning C, Block D. Complex plasmas: A laboratory for strong correlations. Reports on Progress in Physics 2010;**73**(6):066501. DOI: 10.1088/0034-4885/73/6/066501

[5] Dusty Plasmas in the Laboratory, Industry, and space physics. Today. 2004;**57**(7):32. DOI: 10.1063/1.1784300

[6] Kersten H, Wolter M. Complex (dusty) plasmas: Application in material processing and tools for plasma diagnostics. Introduction to Complex Plasmas. 2010 (pp. 395–442). Springer, Berlin, Heidelberg.

[7] Shukla PK. Dusty Plasmas: Physics, Chemistry and Technological Impacts in Plasma Processing. In: Bouchoule A. Wiley, New York. 2000

[8] Bellan PM. Fundamentals of Plasma Physics. 1st ed. UK: Cambridge University Press; 2008

[9] Chen FF. Introduction to Plasma Physics and Controlled Fusion. 2nd ed. New York: Springer-Verlag. 2006; p. 200

[10] Vladimir EF, Gregor EM. Complex and Dusty Plasmas: From Laboratory to Space. Bosa Roca, United States: Taylor & Francis Inc, CRC Press Inc.; 2010

[11] Varma RK, Shukla PK, Krishan V. Electrostatic oscillations in the presence of grain-charge perturbations in dusty plasmas. Physical Review E. 1993;**47**(5): 3612. DOI: 10.1103/PhysRevE.47.3612

[12] Cui C, Goree J. Fluctuations of the charge on a dust grain in a plasma. IEEE Transactions on Plasma Science. 1994; **22**(2):151–158. DOI: 10.1109/27.279018

[13] Sharma SC, Sugawa M. The effect of dust charge fluctuations on ion cyclotron wave instability in the presence of an ion beam in a plasma cylinder. Physics of Plasmas. 1999;**6**(2):444–448 (1999).

[14] Merlino RL. Current-driven dust ion-acoustic instability in a collisional dusty plasma. IEEE Transactions on Plasma Science. 1997;**25**(1):60–65. DOI: 10.1109/27.557486

[15] Tribeche M, Zerguini TH. Current-driven dust ion-acoustic instability in a collisional dusty plasma with charge fluctuations. Physics of Plasmas. 2001;**8** (2):394–398. DOI: 10.1063/1.1335586

[16] Rao NN, Shukla PK, Yu MY. Dust-acoustic waves in dusty plasmas. Planetary and Space Science. 1990;**38** (4):543–546. DOI: 10.1016/0032-0633 (90)90147-I

[17] Shukla PK, Silin VP. Dust ion-acoustic wave. Physica Scripta. 1992;**45**: 508. DOI: 10.1088/0031-8949/45/5/015

[18] Barkan A, Merlino RL, D'angelo N. Laboratory observation of the dust-acoustic wave mode. Physics of Plasmas. 1995;**2**(10):3563–3565. DOI: 10.1063/1.871121

[19] Morfill GE, Thomas H. Plasma crystal. Journal of Vacuum Science & Technology, A: Vacuum, Surfaces, and

Films. 1996;**14**(2):490–495. DOI: 10.1116/1.580113

[20] Barkan A, D'angelo N, Merlino RL. Experiments on ion-acoustic waves in dusty plasmas. Planetary and Space Science. 1996;**44**(3):239–242. DOI: 10.1016/0032-0633(95)00109-3

[21] Ali S. Potential distribution around a test charge in a positive dust-electron plasma. Frontiers of Physics. 2016;**11**(3): 115201. DOI: 10.1007/s11467-015-0545-2

[22] Bukhari SS. Ali, Rafique M, Mendonca JT. Twisted electrostatic waves in a self-gravitating dusty plasma. Contributions to Plasma Physics. 2017; **57**:404–413. DOI: 10.1002/ctpp.201700063

[23] Mendonça JT, Guerreiro A, Ali S. Photon bubbles in a self-gravitating dust gas: Collective dust interactions. The Astrophysical Journal. 2019;**142**(6pp): 872. DOI: 10.3847/1538-4357/aafe7e

[24] Pandey BP, Vranjes J. Physics of the dusty Hall plasmas. Physics of Plasmas. 2006;**13**(12):122106. DOI: 10.1063/1.2402148

[25] Kaufman HR. Technology of closed drift thrusters. AIAA Journal. 2012;**23** (1):78–86. DOI: 10.2514/3.8874

[26] Goebel DM, Katz I. Fundamentals of Electric Propulsion: Ion and Hall Thrusters. New York: Wiley; 2008

[27] Jahn RG. Physics of Electric Propulsion. New York: McGraw-Hill; 1968.

[28] Singh S, Malik HK, Nishida Y. High frequency electromagnetic resistive instability in a Hall thruster under the effect of ionization. Physics of Plasmas 2013;**20**:102–109 (1–7).

[29] Singh S, Malik HK. Growth of low frequency electrostatic and electromagnetic instabilities in a Hall thruster. IEEE Transactions on Plasma Science. 2011;**39**:1910–1918

[30] Singh S, Malik HK. Resistive instabilities in a Hall thruster under the presence of collisions and thermal motion of electrons. The Open Plasma Physics Journal. 2011;**4**:16–23

[31] Malik HK, Singh S. Resistive instability in a Hall plasma discharge under ionization effect. Physics of Plasmas. 2013;**20**:052115(1–8)

[32] Singh S, Malik HK. Role of ionization and electron drift velocity profile to Rayleigh instability in a Hall thruster plasma: Cutoff frequency of oscillations. Journal of Applied Physics. 2012;**112**:013307 (1–7)

[33] Malik HK, Singh S. Conditions and growth rate of Rayleigh instability in a Hall thruster under the effect of ion temperature. Physical Review E. 2011; **83**:036406 (1–8)

[34] Kapulkin A, Kogan A and Guelman M. Noncontact emergency diagnostics of SPT in flight. Acta Astronautica. 2004;**55**:109–119

[35] Fernandez E, Scharfe MK, Thomas CA, Gascon N, and Cappelli MA. Growth of resistive instabilities in $\vec{E} \times \vec{B}$ plasma discharge simulations. Physics of Plasmas. 2008;**15**:012102(1-10)

[36] Alcock M W and Keen B E. Experimental observation of the drift dissipative instability in an afterglow plasma. Physical Review A. 1971;**3**:1087–1096

[37] A. Kapulkin and M. M. Guelman. Low-frequency instability in near-anode region of Hall thruster. IEEE Transactions on Plasma Science. 2008; **36**:2082–2087