Neutrino lasing in the Sun

R.A. Malaney
Canadian Institute for Theoretical Astrophysics, University of Toronto, Toronto, Ontario, Canada M5S 1A7
G.D. Starkman
Canadian Institute for Advanced Research, Cosmology Program and Canadian Institute for Theoretical Astrophysics, University of Toronto, Toronto, Ontario, Canada M5S 1A7
M.N. Butler
Department of Physics, Saint Mary's University, Halifax, Nova Scotia, Canada B3H 3C3
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Applying the phenomenon of neutrino lasing in the solar interior we show how the rate for the generic neutrino decay process $\nu \to$ fermion + boson can in principle be enhanced by many orders of magnitude over its normal decay rate. Such a large enhancement could be of import to neutrino-decay models invoked in response to the apparent deficit of electron neutrinos observed from the Sun. The significance of this result to such models depends on the specific form of the neutrino decay, and the particle model within which it is embedded.

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The solar neutrino problem (SNP) is well established. Neutrino fluxes predicted by standard solar models [1] are in disagreement with the fluxes observed by current experiments. In terms of the ratio of observed-to-predicted neutrino fluxes η, the Cl-Ar experiment [2] reports $\eta_{\rm Cl} = 0.28 \pm 0.03$, whereas the Kamiokande Collaboration [3] reports $\eta_K = 0.51 \pm 0.07$. These two experiments are mostly sensitive to the high-energy ^8B neutrinos. In order to measure the lower-energy pp neutrinos, two Ga-Ge experiments have been attempted. The SAGE Collaboration [4] reports $\eta_{\rm SAGE} = 0.53 \pm 0.19$, whereas the GALLEX Collaboration [5] reports $\eta_{\rm GALL} = 0.62 \pm 0.15$. The errors are the 1σ statistical and systematic errors reported by the different groups added in quadrature; theoretical uncertainties from the standard solar model are not included.

Neutrino decay is often invoked as a possible remedy to the above discrepancies [6,7]. Although ν_e decay as a solution to the SNP would appear to be in contradiction with the observed $\tilde{\nu}_e$ pulse from SN 1987A, a viable solution remains where the mass eigenstates ν_1, ν_2 are a substantial mixture of the weak eigenstates ν_e, ν_x $(x = \mu, \tau)$ [8]. Matter-induced effects can also require some additional interpretation of the supernova limits [9]. The generic neutrino decay can be described as $\nu_2 \to \nu_1 + B$, where B is some unknown boson. The most favored identification of the boson is with some type of Majoron particle ϕ. For example, the two decay modes $\nu_2 \to \nu_1 + \phi$ and $\nu_2 \to \tilde{\nu}_1 + \phi$, have been previously analyzed in detail, and regions of the lifetime and mixing-angle phase space consistent with current observations determined [7]. Fast Majoron decay into sterile neutrino states has also been discussed with regard to the SNP [10].

The main purpose of this report is to point out that the phenomenon of *neutrino lasing* may be of import with regard to the generic neutrino-decay paradigm. Specifically, we will find that the bosons emitted in relativistic neutrino decays can stimulate the decay to proceed orders of magnitude faster than that described by the normal decay rate. Regions in the allowed parameter space consistent with the observed neutrino fluxes are therefore subject to revision.

The phenomenon of neutrino lasing in the context of the early Universe and the dark-matter problem has previously been discussed [11]. Neutrino lasing can best be described as a process in which the decay of a relativistic neutrino proceeds by stimulated emission of bosons, thereby dramatically increasing its decay rate. We wish to see how solar neutrinos can have their decay rates affected by this phenomenon. Considering the generic decay (and its inverse) of a heavy neutrino H into a lighter neutrino F and a boson B, it can be shown that [11] the evolution of the occupation number distributions f_B is described by the Boltzmann equation

$$\dot{f}_B(E_B) = \frac{m_H^2 \Gamma_0}{m_0 E_B p_B} \int_{E_H^-(E_B)}^{E_H^+(E_B)} dE_H [f_H(1-f_F)(1+f_B) - f_B f_F (1-f_H)] \,, \tag{1}$$

where p_B is the boson three-momentum, Γ_0 is the free decay rate for a H at rest, and $m_0/2$ is the three-momentum of the decay products in the H rest frame. Here we have assumed isotropic decay in the H rest frame. In the massless boson limit we have

$$m_0 = m_H - \frac{m_F^2}{m_H} \,. \tag{2}$$

The integration is over the energy-conserving plane $E_H = E_F + E_B$, with limiting values

$$E_H^+ = \infty \,, \qquad E_H^- = \frac{m_0 m_H}{2} \left(\frac{2E_B}{m_0^2} + \frac{1}{2E_B} \right) , \tag{3a}$$

and

$$E_{B,F}^{\pm} = \frac{m_0}{2m_H}(E_H \pm p_H) \,. \tag{3b}$$

Considering first only the initial growth of $\dot{f}_B$, our Boltzmann equation becomes

$$\dot{f}_B \approx \frac{E_H^2}{m_H^3} f_B \Gamma_0 \int dE f_H , \tag{4}$$

where we have taken the growth rate to be maximized at E_B^- ($\approx m_H^2/E_H$). We can approximate

$$\int dE f_H \sim \frac{n_\nu}{E_H^3} E_H . \tag{5}$$

Therefore,

$$\dot{f}_B \approx \frac{n_\nu}{E_H^3}\left(\frac{E_H}{m_H}\right)^3 \Gamma_0 f_B . \tag{6}$$

For significant decay of particles we require $p_B^3 f_B \sim p_H^3 f_H$; that is, we need

$$f_B \sim \left(\frac{E_H^3 n_\nu}{m_H^6}\right) . \tag{7}$$

Therefore from Eqs. (6) and (7) we find that the time required for the number of bosons to equal the number of neutrinos at some point in the Sun is given by

$$t \sim \Gamma_0^{-1}\left[\frac{E_H^3}{n_\nu}\left(\frac{m_H}{E_H}\right)^3 \ln\left(\frac{E_H^3 n_\nu}{m_H^6}\right)\right] , \tag{8}$$

which typically is much less than the age of the Sun.

Using $\Gamma_0 = (E_H/m_H)\Gamma_\odot$, where $\Gamma_\odot$ is the decay rate in the solar frame, Eq. (6) allows us to define a new effective decay rate in the presence of lasing as

$$\Gamma_{\rm lase} \sim \frac{n_\nu}{E_H^3}\left(\frac{E_H}{m_H}\right)^4 \Gamma_\odot . \tag{9}$$

As an example of the effect, consider the decay of the *pp* neutrinos as they pass through the point $R = 0.1R_\odot$. From the *pp* neutrino density at $0.1R_\odot$, we find $\frac{n_\nu}{E_H^3} \sim 10^{-25}$, and, therefore, for $E_H/m_H \gtrsim 10^7$, the decay rate is enhanced. For example, if the mass of the electron neutrino is $m_H \sim 0.01$ eV, the lasing rate is a factor $\sim 10^6$ larger than $\Gamma_\odot$. Clearly, for even smaller values of the neutrino mass the effect on the decay rate will become more significant. This would be the case as long as $E_B^- \gtrsim 1/R$, since if this bound is not satisfied the above analysis would break down as the wavelength of the emitted boson would be larger than the size of the region producing it. For practical purposes, we can impose the limit $m_H > 10^{-14}$ eV as the mass above which our analysis would be valid. Another limitation of our analysis is that we have neglected transport terms in the Boltzmann equation. This is a good approximation as long as $\Gamma_{\rm lase} \gg 1/R$, since then f_b will be able to grow. Finally, note from the f terms of Eq. (1) that the lasing effect "switches off" when the abundance of decaying neutrinos equals that of the decay-product neutrinos.

As mentioned earlier, we have assumed in the above analysis that decays in the rest frame of the decaying neutrino are isotropic. What form does Eq. (9) take if we drop this assumption? In the more general case, the Boltzmann equation Eq. (1) takes the form [12]

$$\dot{f}_B = \frac{m_H^3 \Gamma_0}{m_0 E_B p_B}\int_{E_H^-(E_B)}^{E_H^+(E_B)} dE_H [f_H(1-f_F)(1+f_B) - f_B f_F(1-f_H)]\left[\frac{1}{m_H} + \alpha \frac{E_B^+ + E_B^- - 2E_B}{p_H m_o}\right] , \tag{10}$$

where α is defined through the probability distribution $P(\theta)$ given by

$$P(\theta) = \tfrac{1}{2}(1 - \alpha\cos\theta) , \tag{11}$$

and where θ is the angle in the rest frame between the decay-product velocity and the parent velocity in the solar frame. The isotropic case discussed previously corresponds to $\alpha = 0$, and is valid (up to factors of ~ 2) for all $\alpha > -1$. For $\alpha \simeq -1$, the emission of low-momentum bosons is suppressed. Utilizing Eq. (10), and applying similar arguments as those used in the isotropic case, we find the equivalent equation to Eq. (9) is

$$\Gamma_{\rm lase} \sim \frac{n_\nu}{E_H^3}\left(\frac{E_H}{m_H}\right)^2 \Gamma_\odot . \tag{12}$$

From comparison of Eq. (9) with (12) it can be seen that $\alpha = -1$ results in a suppression factor $(m_H/E_H)^2$, relative to the case of isotropic decays.

Let us consider the *pp* neutrinos produced by the Sun, and focus on the decay of these neutrinos into a τ or muon neutrino ($\nu_e \to \nu_x + B$). In order to accurately assess the importance of neutrino lasing we must average the effect over the varying matter and neutrino densities in the Sun. Let us assume that matter effects dominate the neutrino masses. In this case we can replace m_H in Eqs. (8) and (12) with $\sqrt{2EV}$, where E is the ν_e energy and the effective potential V is given by

$$V = \sqrt{2} G_F \rho m_n^{-1} Y_e . \tag{13}$$

Here, G_F is the Fermi constant, ρ is the matter density, m_n is the nucleon mass, and Y_e is the number of electrons per nucleon.

In the solar frame, the decay rate for some process described by some dimensionless coupling constant g can be written

$$\Gamma_\odot = \frac{g^2 m_H^2}{16\pi E} \equiv \frac{g^2 V}{16\pi} . \tag{14}$$

Neglecting lasing effects, in terms of the electron-neutrino flux Φ_e^0 at some origin, the flux $\Phi_e(r)$ at distance r from the origin is given by [13]

$$\Phi_e(r) = \Phi_e^0 \exp\left[-\frac{1}{8d_0}\sum_x g^2 d^{\rm eff}\right] , \tag{15}$$

where the effective matter width $d^{\rm eff}$ for the decay is given by

$$d^{\rm eff} = \int_0^r \rho(r) Y_e(r) dr , \tag{16}$$

and where the refraction width d_0 is given by $d_0 = \sqrt{2}\pi G_F^{-1} m_n$.

The reduction in the neutrino flux when lasing effects are included can be approximated by

$$\Phi_e(r)^{\text{lase}} = \Phi_e^0 \exp\left[-\frac{1}{8d_0}\sum_x g^2 D^{\text{eff}}\right] , \tag{17}$$

where

$$D^{\text{eff}} \approx \int_0^r \chi(r)\rho(r)Y_e(r)dr , \tag{18}$$

and where

$$\chi(r) = \frac{n_\nu}{EV^2(r)} . \tag{19}$$

We apply Eq. (17) to a solar model [14], and determine the ratio $\Phi_e(r)/\Phi_e(r)^{\text{lase}}$. Our calculations show that, for a decay such as $\nu_e \to \nu_x + B$ where the chirality of the neutrinos remains unchanged, $\Phi_e(r)/\Phi_e(r)^{\text{lase}}$ can be $\gg 1$ for both *pp* and ^8B neutrinos. This means that any such decay applied to the solar neutrinos must take into account this effect. For processes where $\alpha = -1$, for example chirality-flipping decay with spin-zero boson emission, Eq. (19) becomes

$$\chi(r) = \frac{n_\nu}{E^2 V(r)} , \tag{20}$$

and there is a negligible effect due to the spin suppression effects discussed earlier.

In order to see the effects of lasing let us assume a chirality-conserving interaction and apply the current laboratory bounds on the chirality-conserving decay $\nu_e \to \nu_x + \phi$, namely, g as defined in Eq. (14) is $< 7.0 \times 10^{-3}$ [15]. Remembering that the lasing switches off when $f_{\nu_e} = f_{\nu_x}$, we find that half the *pp* neutrinos could in fact undergo decay in the solar interior. This conclusion remains valid for $g \gtrsim 10^{-6}$. Decays of ν_e to a left-handed antineutrino through Majoron emission give a similar result (though a slight modification to the potential V is required in this case). Decay into a right-handed antineutrino would require that the chirality be flipped, and thus is unimportant due to spin suppression effects.

A question remains as to whether there is a well-defined model which couples neutrinos to a boson in such a way that we will see lasing. As stated above, the most obvious candidate to consider is the singlet Majoron model [16]. Consider a scalar potential of the form

$$V[\Phi] = \lambda(\Phi^2 - v^2)^2 . \tag{21}$$

Clearly the vacuum corresponds to $|\langle\Phi\rangle| = v$, but we have a number of ways of describing excitations. If we consider the linear realization of the symmetry breaking, then we write

$$\Phi = v + \rho + i\phi \tag{22}$$

and find that ρ is a massive scalar field, and that ϕ is a massless Goldstone boson—the Majoron. Including matter effects, we can determine that part of the Lagrangian which involves the neutrinos and the linear Majoron coupling:

$$\mathcal{L} = -[\bar{N}_R M_D N_L + \tfrac{1}{2}\bar{N}_R M_R (N_R)^c + \text{H.c.}] + \frac{i}{2v}\phi \bar{N}_R M_R (N_R)^c - \frac{1}{\sqrt{2}}\zeta_u \bar{N}_L \gamma^\mu N_L , \tag{23}$$

where $N_{L(R)}$ are the weak-lepton-doublet (singlet) neutrinos, $\zeta_u = G_F \rho m_n^{-1} Y_n v_\mu$ (where v_μ is the collective nucleon four-velocity), and the mass matrices M_D and M_R are assumed real. Considering only two generations and assuming $M_R \gg M_D$, diagonalization of the mass matrix determines the physical fields $\nu_{1,2}$. The coupling of these neutrino fields to ϕ in this case is direct, and has the form

$$ig\phi\bar{\nu}_2\gamma_5\nu_1 , \tag{24}$$

meaning that the local operator for the transition $\nu_2 \to \nu_1 + \phi$ necessarily involves a chirality flip. As seen earlier, the spin suppression associated with this chirality flip renders lasing in the Sun unimportant. Only within environments where the neutrino density is much larger than solar neutrino densities (e.g., supernova explosions), would the coupling of Eq. (24) be important.

It is possible to have chirality-conserving decays from other processes. In the linear expansion this occurs when mass insertions are added to the external neutrino lines. However, in the most simple models each mass insertion introduces a suppression factor of order $(m/E)^2$. If the exponential expansion $\Phi = (v + \rho)\exp(i\phi/v)$ is used, the chirality-conserving decays naively involve derivative couplings which suppress the emission of low-momentum bosons, and hence the lasing rate also. The actual result must be independent of which expansion is utilized, but only after all relevant diagrams at each order are included would the exact suppression factor be determined. Such suppression would clearly be model dependent. Indeed, we note that it is possible to construct more complicated Majoron models which exhibit no suppression of the chirality-conserving decays [17].

Another hopeful possibility is that the neutrino decays through a new spin-1 field. The coupling would be between neutrinos of the same chirality state, and the rate would not be suppressed by any chirality or momentum effects. Massive vector fields were speculated on as possible sources of lepton number violation through local symmetry breaking [18], but were superseded by Majoron models and global symmetry breaking, which seemed to arise more naturally in grand unified theories (GUT's).

In summary, we have seen how neutrino lasing may play a role in neutrino-decay processes occurring in the solar interior. We have highlighted how the spin nature of the particles involved in the decay process plays an important role. In particular, we noted that within the context of the simplest singlet Majoron models, no significant neutrino lasing in the Sun would proceed. More complicated extensions to the standard model need to be invoked if lasing is to be viable in the solar interior. Since the energy spectrum of the decaying electron neutrinos is not degraded by the lasing phenomenon, future detectors should be able to distinguish between the process described here and that anticipated from the usual decay paradigm.

A similar analysis of neutrino lasing applied to supernova explosions may provide some additional constraints. We note again, however, that since bosons are preferentially produced at low momentum the energetics of such explosions should not be significantly altered by excess energy losses, unless the decay-product neutrino is sterile.

Neutrino lasing has the potential to be an important phenomenon. In addition to its impact on the dark-matter problem, it may also have interesting implications for Majoron models of baryogenesis due to enhanced decays of sterile neutrinos. Any experimental information on the viability of neutrino lasing would therefore be very valuable.

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