# MULTISUPPLIER PROCUREMENT UNDER UNCERTAINTV IN INDUSTRIAL FISHING ENVIRONMENTS 

A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF SAINT MARY'S UNIVERSITY<br>BY<br>Melvina Marius

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FOR THE DEGREE OF

MASTER OF SCIENCE

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# Multisupplier Procurement Under Uncertainty in Industrial Fishing Environments 

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#### Abstract

In this paper we address the issue of multi supplier sourcing as a tool for hedging against supply yield uncertainty. Our work was motivated by the problems in the fishing industry whereby fish processing firms are constantly faced with the problems of random supply yields. We formulated a mathematical programming model that can be used to determine the quantities to be ordered from two or more suppliers so as to minimize annual expected procurement cost while attempting to satisfy demand requirements and operating constraints. The cost included are purchasing cost, inventory related cost and ordering cost. We assume that at the beginning of a planning horizon comprised of 12 periods a firm enters into minimum contractual agreement with two suppliers, and in return each supplier offers a discounted price schedule.

In our numerical analysis we solved the model for both the 2 -supplier case and the single supplier case and compared the cost of using a single supplier versus two suppliers under varying levels of yield variability. We compared deterministic solutions for the single and two-supplier case and use Monte Carlo simulation to assess the robustness of the solutions under varying levels of yield uncertainty. Results show that as the variability of the yield rate increases it becomes cost effective to use two suppliers as a means for hedging against uncertainty. We compared the results from our model to that of a heuristic procedure proposed by Parlar and Wang, an alternative approach for solving the 2 -supplier inventory problem. The results indicated that our model provides superior solutions to that of the heuristic procedure.


## CHAPTER 1

## Introduction

### 1.1 Introduction

Purchasing decisions are becoming increasingly strategic for many organizations. Many are now looking to their suppliers to help them attain a strong competitive market position. Selecting the most appropriate suppliers is an important strategic management decision that may impact all areas of an organization (Jayaraman et al 1999). A large percentage of the total cost for many organizations is from purchases, thus the reduction of purchasing cost is the major concern of managers.

A major decision faced by purchasing managers is determining the configuration of the supply base. For example, working with a few suppliers enables a firm to enter into long-term contractual relationships. On the other hand purchasing managers may want to split their orders when faced with the need to reduce risk in the conditions characterized by uncertainty in demand and supply yields and as a means of maintaining competition among a set of suppliers.

Faced with a dramatic decline in the ground fish resource in Atlantic Canada, fish processing industry firms are forced to obtain fish resources from external suppliers. Because of the nature of the fishing industry, fish harvesters experience less than perfect
yields. For this reason, a supplier's ability to meet a firm's demand for raw fish is uncertain. This can create periodic shortages, which may prove detrimental to the buyers. As such techniques for handling supply uncertainty is critical to the competitiveness of fish processing firms. Therefore, firms must determine an effective strategy that would enable them to determine the best ordering policies, to maximize total yield and minimize average annual cost associated with procurement.

The supplier selection and allocation decisions made may incorporate minimum commitment contracts. Many researchers have shown the benefits of commitment contracts (Anupindi and Bassok (1999), Serel et al. (2001), Larviere (1998)). By committing to purchasing a minimum quantity, the buyer can negotiate a better price, and the supplier will be provided with the guarantee that his/her fish will be sold. In return for the buyer's commitment, the supplier provides a price discount.

Purchasing fish from more than one supplier is necessary to sustain a desirable service level and to reduce the total system cost incurred when acquisition lead-time and order quantities are uncertain. In a multi-supplier system, deliveries from all suppliers do not take place at the same time and are distributed over different intervals over a period of time. Thus when supply yield is uncertain the chance of shortages can be reduced. That is to say that multi-supplier sourcing can facilitate splitting an order to consider the variability in arrival time and the quantity of fish delivered.

### 1.2 Objectives and Scope of this Research

There are few models that address the issue of yield uncertainty in industrial fishing environments. For this reason our paper is based on the following objectives;

1. To gain insight into the deterministic representation of the random yield problem

2 To compare the cost of using two suppliers to the cost associated with a single supplier under supply uncertainty

3 To use discrete simulation to compare the cost of two supplier sourcing versus single supplier sourcing under varying levels of supply yield rates
3. To ascertain the effectiveness of multi-supplier sourcing as a strategy for hedging against the effect of supply yield uncertainty

This research presents a formulation and solution methodology for the multi-supplier lotsizing problem under conditions of uncertainty. The problem is not modeled as a stochastic problem but rather as a deterministic problem based on the mean values for random yield rates. The model is formulated as a non-linear mathematical program with quantity discounts and minimum commitment. It will be solved using a commercial nonlinear solver called "What'sBest" developed by LINDO Systems INC.

### 1.3 Organization of the Thesis

The next chapter presents the background to the problem and cites the relevant literature. Chapter three describes the mathematical formulation of the model and the solution procedure. The computational study and reports on the computational results are presented in chapter four. Finally, chapter five concludes with a brief summary and discussion of future research possibilities.

## CHAPTER 2

## Literature Review of Inventory Lot-sizing Problems

### 2.1 Introduction

The lot-sizing procurement problem is to determine when to order and how much to order given the demand of a product so as to minimize total procurement cost with demand being either stochastic or deterministic.

The earliest solution to the lot-sizing problem was the Economic Order Quantity Model (EOQ) developed by Harris (1913). The EOQ model is a continuous time model that seeks to minimize total inventory cost by making optimal order quantities under certain conditions. It assumes that the demand for a single product is constant and deterministic with a known fixed set up cost. Backlogging and shortages are not allowed. There is no capacity constraint and delivery is instantaneous. This means that there is no delay between placing an order and receiving that order. With the EOQ it is always optimal to place an order when the inventory level is at zero. The EOQ can be easily applied to other inventory situations and provides good starting solutions for more complex models. For this reason it has been used as the basis for a number of heuristic solutions. Examples of this approach can be found in Mazzola et al. (1987), Silver (1976), and Parlar and Berkin (1991).

Maintaining most of the assumptions of the classical EOQ Wagner and Whitin (1985) developed an algorithm for solving the dynamic lot-sizing problem. They based their model on the property that under an optimal lot-sizing policy there exists an optimal plan such that the inventory carried out from a previous period $t$ to period $t+1$ will be zero or the production quantity in period $t+1$ will be zero. Like the EOQ the Wagner and Whitin algorithm is being used by many researchers as the basis for solving dynamic lot sizing inventory problems. See Britran et al. (1984), Wagleman (1992) and Aggarwal and Park (1993).

### 2.2 Yield Uncertainty in Inventory Lot-Sizing

Both the EOQ and the Wagner-Whitin algorithm are based on the assumption that product delivery is immediate and the amount ordered is the amount received. However in real life situations many firms are faced with yield randomness. For this reason researchers have seen the need to incorporate yield randomness into inventory problems.

Yield uncertainty is viewed in two different ways in inventory lot-sizing. It can be viewed as uncertain lead-time where delivery is not immediate and as uncertain delivery where the quantity delivered is a fraction of the quantity requested.

The problem has been addressed in various forms by many authors such as Ehrhardt and Taube (1987), Gerchark et al. (1986), Gerchak and Wang (1994, Amihud and Medelson (1993), Kelle and Silver (1990), Ilan and Yardin (1885), Nahmias and Moinzaden (1997)
and Parlar (1997). An extensive survey of literature on the concept can be found in Yano and Lee (1995), who presented a survey on quantitative oriented approaches to solving the random yield lot-sizing problem.

### 2.3 Survey Of Multi- Supplier Lot-Sizing Problems

Research on multi-supplier inventory systems began in 1981, by Sculli and Wu. They considered an inventory item with two suppliers where the lead times are normally distributed and the reorder level is the same for both suppliers. Since then many other researchers have considered such systems.

Hayya et al. (1987) reiterated Sculli and Wus' model using simulation and Sculli and Shum (1990) extend their results to the case of $n>2$ suppliers. Gerchak and Parlar (1990) considered the diversification strategy when two independent suppliers have different yield rates. They examined the problem of determining the optimal lot sizes to be ordered simultaneously from the suppliers to meet demand and minimize cost. Yano (1991) extend this model to investigate the issue when quality is reflected in the yield rate distribution, and where two suppliers are used for strategic reasons. Yano (1991) modeled the case where the customer alternately orders from the two suppliers.

Parlar and Wang (1993) extended the results found in Gerchak and Parlar (1990) by making the assumption that the prices charged by the two suppliers and the unit holding
cost incurred for the items purchased from the two suppliers are different. They developed a convex total cost expression function of the order quantities from each supplier.

Anupindi and Akella (1993) addressed the operational issue of quantity allocation between two uncertain suppliers and its effects on the inventory policies of the buyer. They assumed that demand is stochastic and continuously distributed with a known distribution and developed three models for supply processes.

Lau and Zhoa (1993) developed a procedure that determines the order policy that optimizes the inventory system cost when the daily demand and suppliers' lead-time are all stochastic. Lau and Zhoa (1994) presented an easily solvable version of the procedure where there existed no restrictions on lead- time distribution and order split proportion.

These papers generally studied two-supplier systems. Nevertheless, other researchers have considered multiple-supplier systems. Among these are Tempelmeier (2001), Millar (2000 a) and Millar (2000 b), who developed a model for assessing multi-supplier versus single supplier sourcing under deterministic conditions and varying supply. Sedarage et al. (1999) considered a general n-supplier single item inventory system where the item acquisition lead times of suppliers and demand arrival is random. They developed an optimization model to determine the reorder level and order split quantities for n suppliers.

### 2.4 Survey of Lot-Sizing Problems with Supplier Selection and Quantity Discounts

Solutions to lot-sizing problems under considerations of quantity discounts have been on going for some time. Benton and Park (1996) presented a paper, which classified and discussed some of the significant literature on lot-sizing under several types of discount schemes. They observed that most of the studies thus far have investigated single buyer and single supplier situations with a single or a small number of price breaks. Examples of papers in this area are by Chaundry et al (1993), Kasilingam and Lee (1996), Jayayam et al (1999) and Geneshan (1999) who all studied the single period problem. The multiperiod problem was considered by Gaballa (1974), Buffa and Jackson (1983), Pikul and Aras (1995), Sharma et al. (1989) and Benton (1991).

With the emphasis on supply chain management many firms see the need to enter into contractual agreements with their suppliers. Consequently there has been an increasing amount of research in the area of supply chain contracts. Most recent literature in this area of research has considered the issue of commitments by the buyer to purchase certain minimum quantities. These commitments are usually referred to as Minimum Quantity Commitment Contracts whereby a buyer at the beginning of a horizon period agrees to purchase a minimum quantity during the entire period. The buyer has the flexibility to order any amount in any period as long as at the end of the horizon the
specified minimum quantity is purchased. In return the supplier may offer discount prices.

Several researchers have investigated this problem. Moinzadeh and Nahmias (1997) and Anupindi and Akella (1993) presented models that assume a constraint on every period's purchase, while Bassok (1997) and Millar (2000 a) and Millar (2000 b) considered an agreement where the constraint is applied to the cumulative purchase over a given planning horizon or N periods.

### 2.5 Solution Approaches

Table 2.1: Classification of Lot-Sizing Literature According to Solution Procedure

| Myopic Heuristics | Mathematical Programming Based Heuristics |
| :---: | :---: |
| - Bollapragada and Morton [1999] <br> - Morton and Pentico [1995] <br> - Ciarallo, Akella, and Morton [1994] <br> - Heyman and Sobel [1984] <br> - Gerchak and Wang [1994] <br> - Nandakumar and Morton [1993] <br> - Gavirneni and Morton [1999] | - Noori and keller [1986] <br> - Federguuen and Heching [1999] <br> - Mazzola, MaCoy and Wagner [1987] <br> - Sliver [1976] <br> - Syam and Shetty [1996] <br> - Sedrage, Fujiwara, and Luong [1999] <br> - Tempelmeier [2001] <br> - Millar [2000.a] <br> - Parlar and Wang [1993] <br> - Bassok and Anupindi [1997] <br> - Anupindi and Akella [1993 |

Table 2.1 provides a summary of solution approaches used in solving procurement problems in supply chain systems. The table is by no means complete, however we note that a wide range heuristics have been applied to solving random yield inventory lot-
sizing problems. The heuristic methods have been classified in two groups, namely myopic heuristics known as "simple rules" and mathematical programming based heuristics. Myopics are based on the knowledge of the system, whilst mathematically programming based heuristics attempt to solve problems as mathematical programming problems. No one method is better than the other as they all work well under different circumstances. The choice of solution procedure will depend on the application.

### 2.6.1 Myopic Heuristic Procedures

Most researchers have provided evidence that myopic policies provide optimal or close to optimal solutions to the general periodic review stochastic inventory problem. Myopic rules involve the solution of problems iteratively. It begins with a partial solution to the problem, which is improved upon by selecting one of a number of available options.

Researchers such as Heyman and Sobel (1984), Morton and Pentico (1995), Nandakumar and Morton (1993), Clarello et al (1994), Gerchak and Wang (1994) and Bollapragada and Morton (1999) have investigated conditions under which myopic rules provide optimal solutions to random yield lot sizing problems. In particular Bollapragada and Morton (1999) demonstrated that the random yield problem is similar to the newsvendor problem and that myopic policies provides a fairly good approximation to the optimal policy under fairly general conditions. Their solution method involved the use of several heuristics, one of which is an alteration of the newsvendor heuristic based on the
stationary approximation of the random yield problem. A second heuristic ignores the variability of the yield and merely attempts to correct the mean of the yield. With this heuristic the random yield problem is first solved using perfect yield and then the order quantity is expanded and changed by dividing it by the mean yield. It was further improved upon by assuming a linear ordering function with the safety stock dependent on both the demand and the supply variance. The closed-form expression for the safety stock was constructed using a myopic approximation.

### 2.6.2 Mathematical Programming Based Heuristics

Solution in this category employs integer and dynamic programming to solve lot-sizing problems. The development time of such solution techniques can be time consuming. However, the resulting algorithm tends to give optimal or near optimal solutions in relatively short time. For simplicity and to reduce computational time they are usually combined with local search techniques that obtain an initial solution from a simple rule, which can be improved upon by other simple heuristics.

Dynamic programming heuristics are often based on the algorithm developed by Wagner and Whitin (1958). Although the Wagner and Whitin algorithm (WW) applies specifically to the single supplier problem, literature evidence has shown it can easily be applied to the multi-supplier inventory problems. For this purpose, only the solution
where there can be only one supplier for a particular product in any one given period will be considered.

Some researchers have argued that managers find the (WW) algorithm difficult to understand and time consuming to solve. For this reason a number of researchers such as Sliver and Meal (1973), Evans (1985) and Jacobs and Khumawala (1987), have contributed faster heuristics to solve the algorithm. They focused on improving the performance of the algorithm by developing efficient rules to reduce the search time, which lead to a reduction in the computational time. More recently, Heady and Zhu (1994) reduced the run time by making the WW algorithm linear in each period.

Many multi supplier inventory problems have been formulated as integer or dynamic programs. These include the work of Sedrarage et al (1999), Benton et al. (1999) and Jayaraman et al. (1999).

Most multi supplier mathematical programming heuristics are mostly based on search strategies involving two phases namely the construction phase and the improvement phase. The construction phase sometimes referred to as the equal order quantity heuristic, aims at assigning order quantities to suppliers thereby arriving at an initial solution to the problem. In the improvement phase the solution is approved upon leading to an optimal or near optimal solution. This method is quick and efficient, as in most cases the heuristic in the construction phase forces the problem to become a single supplier problem which can be easily solved using simple known heuristics such as the Wagner-Whitin algorithm
or the Silver -Meal heuristic. A good example of this procedure can be found in a paper written by Tempelmeier (2001).

Syam and Shetty (1996) employed slightly different solution method. In that they developed a heuristic based on a sub gradient procedure. They used Lagrangean Relaxation method to detect a lower bond on the optimal value of the model. This was done by dualizing certain complicating constraints into the objective function with the use of multipliers.

Another category of problem typically solved by mathematically programming methods is lot-sizing problems with quantity discounts and planning horizons. Examples of this can be found in Benton and park (1996), Chung et al (1996), Chaudhry et al (1993), Abad (1988), Benton and Whybark (1982) and Chaug et al (1987.

Lagrangian techniques have been used to solve quantity discount problems. Pirkul and Aras (1985) and Benton (1991) are two authors who formulated the problem as a nonlinear program, which they solved via a heuristic procedure using Lagrangian relaxation and simulation.

## Chapter 3

## Motivation, Formulation and Solution Methodology

### 3.1 Introduction

Our work was motivated by a problem confronted by most fish processing companies. In the face of random yield they have to decide how to manage procurement as cost effectively as possible. When using lot-sizing models purchasing managers must select an appropriate model with which to determine order quantities. Many authors have developed methods for determining lot sizes under stochastic demand and yield variability. Others have examined supplier selection with discount schedules while others have researched supply contracts and commitment. Few models so far deal with random yields supplier selection with price break quantities and commitment contracts with flexibility agreement.

Firms are beginning to realize that significant savings can be achieved throughout a supply chain if both parties work together. Companies are now requesting all unit quantity discounts from their suppliers while offering commitment contracts. To keep a competitive edge on the market, suppliers are now willing to do whatever it takes to maintain long lasting relationships with their buyers. Hence a fish-processing firm for example will be offered price discount schedules from one or more suppliers. It is now the purchasing manager's responsibility to decide how much to order and how many suppliers to source from whilst keeping procurement cost at a minimum and satisfying demand.

### 3.2 Definition of the Problem and Notation

The problem deals with lot-sizing faced by a fish processing company sourcing from 2 suppliers with uncertain supply yield rates. The objective is to determine order quantities that minimize expected annual total procurement cost consisting of purchasing cost, ordering cost and holding cost.

The model is based on the assumption that the firm has known periodic demand $d_{t}$ for raw fish over a fixed planning horizon of length $T$ periods. To satisfy demand in each period the buyer commits to buying a minimum quantity over the entire horizon from one or more suppliers. Each supplier offers a discounted price schedule, has a fixed ordering cost per period and has specific minimum and maximum order sizes. For each supplier quantities above or at the minimum quantity are paid for at the non-discounted price. The buyer however, can purchase up to a fixed amount above the minimum commitment at the non-discounted price. It is also assumed that inventory level at the beginning of the horizon is at zero, and backlogging is not allowed. A carrying cost is charged for each period of ending inventory and a shortage cost is charged when demand is not met. All costs are non-negative.

Supply is always available but yield is random such that the amount received is a fraction of the quantity ordered. This forces the buyer to order larger quantities to compensate for uncertainties.

### 3.2.1 Notation

D - forecasted annual demand;
$d_{t} \quad$-demand in period $t$;
J - a set of suppliers with index $\mathrm{j}, \mathrm{j}=1 \ldots \ldots \mathrm{~J}$;
T - the set of periods in the planning horizon with index $t, t=1 \ldots T$;
$\mathrm{S}_{\mathrm{jt}} \quad$ - ordering cost for supplier j in period t ;
$Z_{j} \quad$ - minimum commitment for supplier $j$;
$\mathrm{P}_{\mathrm{j}}\left(\mathrm{Z}_{\mathrm{j}}\right)$ - unit price for supplier j as a function of the commitment level $\mathrm{k}_{\mathrm{j}}$;
$h_{4 k} \quad-$ the cost of ordering one unit in period $t$ for use in period $k$. Note if $k<t$ we have backorders;

$$
\begin{aligned}
& h_{4 k}=I(k-t) \text { for } k \geq t ; \text { carrying cost } \\
& h_{4 k}=B(t-k) \text { for } k \leq t ; \text { backorder cost }
\end{aligned}
$$

where I is the unit carrying cost and, B the unit backorder cost
$P_{j}^{0} \quad$ - undiscounted price for supplier $j$
$\gamma_{j} \quad$ - flexibility factor for supplier j ;
$c_{t} \quad-$ the maximum amount that can be ordered in period $t ;$
$u b_{j}$ - an upper bound on the amount that can be purchased from supplier $j$;
¢jitk - the amount received from supplier j in period t for use in period k ;
$\mathrm{y}_{\mathrm{jt}} \quad-$ is set to 1 if an order is placed with supplier j in period t and 0 otherwise
$\sigma_{j}^{2} \quad$ - the variance of the yield rate for supplier $j$

### 3.3 Mathematical Programming Formulation

$$
\begin{align*}
& \operatorname{Min} \sum_{j=1}^{2} \sum_{t \in T} S_{j t} y_{j t}+\sum_{j=1}^{2} \sum_{t \in T} \sum_{k \in T} P\left(Z_{j}\right) \phi_{j t k}+\sum_{j=1}^{2} \sum_{t \in T} \sum_{k \in T} h_{t k} \phi_{j t k} \\
& \quad+\sum_{j=1}^{2} \operatorname{Max}\left\{0,\left(p_{j}^{0}-p\left(\mathbb{Z}_{j}\right)\left\{\sum_{t \in T} \sum_{k \in T} \phi_{j t k}-\gamma_{j} Z_{j}\right]\right\}\right. \tag{1}
\end{align*}
$$

subject to:

$$
\begin{array}{ll}
\sum_{j=1}^{2} \sum_{k \in T} \phi_{j t k} \geq d_{k} & \forall_{k} \\
\sum_{j=1}^{2} \sum_{k \in T} \phi_{j t k} \leq c_{t} & \forall_{t} \\
\phi_{j t k}-d_{k} y_{j t} \leq 0 & \forall_{j i k} \\
\beta_{j} Z_{j} \leq \sum_{t \in T} \sum_{k \in T} \phi_{j k} \leq \beta_{j} u b_{j} & \forall_{j} \\
y_{j t} \in\{0,1\} & \forall_{j t} \\
\phi_{j t k}, k_{j}, q_{j t k} \geq 0 & \forall_{j k t}
\end{array}
$$

The objective function seeks to determine order quantities that minimize the sum of purchase cost, ordering cost, the holding cost for remaining inventory and incremental cost for purchases above the flexibility limit at which the discount price applies. Constraint (2) requires that demand be met in each period. Constraint (3) is a capacity constraint, which, places a limit on the total amount that can be received in any given period. Constraint (4) is an inventory balance constraint. Constraint (5) sets upper and lower bounds on the amount that can be received for a given supplier in any given period. Constraint (6) is a binary constraint and constraint (7) are non negativity constraints.

The model presented minimizes the total procurement cost involved. It permits the orders to be split between unreliable suppliers characterized by random supply yield distributions. Each supplier has a specific price schedule and the buyer makes a commitment prior to purchases. All purchases received are accepted.

### 3.4 Solution Methodology

The model presented is a non-linear program with linear constraints. This type of program is unique in nature and can be classified as a separable program whereby the objective function can be written as the sum of $n$ functions (Wagner, 1969). The main techniques that have been proposed for solving such problems are reduced gradient methods, sequential linear and quadratic programming methods and methods based on Lagrangian relaxation. Most of these techniques, if not all are the foundation of most
commercial codes for mathematical programming software packages. One such software is What'sBest, which is used to solve the program.

In our approach we restricted ourselves to two suppliers. First we solve the problem assuming a singe supplier thereby obtaining independent solutions for each supplier. In the second case, we consider the suppliers jointly and we use WhatBest's to find an "optimal" procurement schedule. Because the problem is non-linear the optimal solutions may be a local optimum.

An alternative approach to solving the problem of multi supplier sourcing versus single supplier sourcing in the presence of random supply yield is by using a ratio based on EOQ principles proposed by Gerchak and Parlar (1990). In their paper they compared the cost of multi sourcing versus single supplier sourcing in the presence of random yields. Under EOQ conditions and assuming that the ordering cost from the two facilities are the same but different yield distribution, they propose that if a producer diversifies, then the ratio of the order quantities from each supplier conforms to the following relationship:

$$
\frac{Q_{1}}{Q_{2}}=\frac{\mu_{1} \sigma_{2}^{2}}{\mu_{2} \sigma_{1}^{2}}
$$

where $Q_{i}$ is the order quantity from supplier $i, \mu_{i}$ the mean yield rate of supplier $i$ and $\sigma_{i}$ the standard deviation of supplier $i$ for $i=1$ to 2

Based on this assumption, Millar (2000.a) developed the following heuristic for solving the 2 -Supplier problem under random yields. First solve the deterministic case of the
single supplier problem. Notation for the parameters and variables used in the approach are as follows:
$Q_{1}=$ the quantity received ordered in period $t$ for the single supplier solution;
$\sigma_{j}^{2}=$ the variance of the yield for supplier $j ;$
$\beta_{j}=$ the expected yield rate for supplier j ;
$X_{j}=$ a set of price breaks for the minimum buyer commitment schedule of supplier $\mathrm{j}, X_{j}=\left[x_{1} \ldots \ldots \ldots . . x_{m}\right]$;

### 3.4.1 Summary of the Heuristic

Step 1: Determine the order quantities for the two suppliers using the following formula;

$$
\begin{aligned}
& q_{1 t}=Q_{t}^{*} *\left[\frac{\beta_{1} \sigma_{2}^{2}}{\beta_{1} \sigma_{2}^{2}+\beta_{2} \sigma_{1}^{2}}\right] \\
& q_{2 \ell}=Q_{i}^{*} *\left[\frac{\beta_{2} \omega_{1}^{2}}{\beta_{1} \sigma_{2}^{2}+\beta_{2} \sigma_{1}^{2}}\right]
\end{aligned}
$$

Step 2: Set the final quantities by dividing the split amounts by the actual yield ratios.

Step 3: Use the following formula to calculate the unit purchase $\operatorname{cost} \mathrm{P}\left(\mathrm{Z}_{\mathrm{j}}\right)$ for each supplier.

$$
Z_{j}^{*}=\left\{\begin{array}{l}
x_{m}, x_{m} \in X^{j} \mid Q_{j} \geq x_{m} \\
x_{i},\left.x_{i} \in X^{j}\right|_{x_{i}} \leq Q_{j}<x_{i+1}, i=1, \ldots, m-1
\end{array}\right.
$$

We used this heuristic procedure to solve both the single supplier problem and the two supplier problem and then compared the solutions to the solutions we obtained from What'sBest.

## CHAPTER 4

## COMPUTATIONAL STUDY

### 4.1 Introduction

In this chapter we analyze the quality of our formulation and compare our results to that obtained from the heuristic proposed by Millar (2000). To conduct this analysis we first solve the model for both the single supplier case and the two-supplier case using What'sBest. We then use the results from the single supplier case to perform the heuristic for the two-supplier case. The solutions from both scenarios are then analyzed using Monte Carlo simulation in Microsoft Excel. All experiments were performed on an IBM PC, Intel P4, 2.4 GHz, 256MB RAM, Windows Professional.

### 4.2 Numerical Analysis

To perform the numerical analysis demand was generated from a random generator with normal probability distribution and a mean of 200 tons. Table 4.1 shows the resulting demand. Annual demand is set at 2391 tons of raw fish. The planning horizon is comprised of 12 periods where demand is known in each period.

Table 4.1: Periodic Demands.

| Period | 1 | 2 | 3 | 4. | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Demanal | S9 | 201 | 203 | 03 | 98 | 208 | 88 | 205 | On | 56 |  | 201 |

The global inputs and supplier specific inputs are contained in table 4.2 below.

Table 4.2: Summary of Inputs


As indicated in table 4.2 ordering costs are fixed and remain the same for both suppliers. The two suppliers have different upper bounds primarily due to the discount schedules proposed by each supplier (refer to table 4.3 for the structure of the price breaks).

In the numerical analysis capacity constraints and backorders were not considered. As such we only considered the case in the formulation where $k \geq t, \forall_{k}$ and for $t \in T$. As a result a unit shortage cost would be incurred whenever shortages occur. Since inventory can be carried a linear unit price will also be charged for each unit of inventory carried. Holding and shortage costs are fixed throughout the horizon and they are the same for both suppliers

It is worth noting that if orders are placed in the same period for the two suppliers a single ordering cost is incurred. We assumed that the marginal cost of placing an order to additional suppliers is zero.

Table 4.3 shows the price breaks for each supplier. The unit purchase price is a function of the minimum buyer commitment. For example if a buyer commits to purchasing 500 tons of fish from Supplier 1 he would pay 27 units per pound. Likewise if he commits to purchasing 1200 pounds form Supplier 2 he would pay 26 units per pound.

The two suppliers are assumed to have the same price structure with Supplier 2 offering one more incremental discount making it the cheaper supplier. This allows us to focus on the variability of the cost.

Table 4.3: Price-Break Schedules for Supplier 1 and Supplier 2

| Price Break Quantities |  |  |  |
| :---: | :---: | :---: | :---: |
| S Suppier 1 |  | Suppriert |  |
| Amouint | Price | Arroumt | Price |
| 0 | 28 |  |  |
| 500 | 27 | 500. | 7 |
| 1000 | 26 | 000 | 26 |
| 1500 | 5 | 15000: | 5 |
|  |  |  |  |

As a main experimental factor we considered the variability of the yield rate. Two cases of yield variability were considered, a high yield rate of $95 \%$ and a low yield rate of $50 \%$.

In performing the numerical analysis the following scenarios were considered:

## A What'sBest Solution-Single Supplier Case

Case 1 Supplier 1 - yield $95 \%$
Case 2 Supplier 2 - yield $95 \%$
Case 3 Supplier 1 - yield $50 \%$
Case 4 Supplier 2 - yield $50 \%$

B What'sBest Solution-2-Supplier Case
Case 1 Supplier 1-yield $958 \%$, Supplier 2 - yield $95 \%$
Case 2 Supplier 1 - yield 95\%, Supplier 2 - yield 50\%
Case 3 Supplier 1 - yield $50 \%$, Supplier 2 - yield $95 \%$
Case 4 Supplier 1 - yield $50 \%$, Supplier 2 - yield $50 \%$

C Heuristic Solution
For this case we first solve the single supplier problem for Supplier 1 using What'sBest and a yield rate of $100 \%$. Then we applied the heuristic formulas mentioned in Chapter 3 to the resulting order quantities thereby solving the problem for the 2-Supplier case. In the solution process for the 2-Supplier case the following cases of yield variability were examined.

Case 1 Supplier 1- yield 95\%, Supplier 2 - yield 95\%
Case 2 Supplier 1 - yield $95 \%$, Supplier 2 - yield $50 \%$
Case 3 Supplier 1 - yield $50 \%$, Supplier 2 - yield $95 \%$
Case 4 Supplier 1 - yield $50 \%$, Supplier 2 - yield 50\%

### 4.3 Computational Results

The results for each of the scenarios are presented in the tables below. They are categorized according to solution methodology.

### 4.3.1 Solution Obtained From What'sBest.

The following two tables presents results for the various combinations of yield variability.

Table 4.4 Summary of What'sBest Results for the Single Supplier Case

| Yield Rate | $95 \%$ |  | $50 \%$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Supplier 1 | Supplier 2 | Supplier 1 | Supplier 2 |
| Amount Ordered | 2518 | 2518 | 4782 | 4782 |
| Amount Received | 2391 | 2391 | 2391 | 2391 |
| Inventory Carrying Cost | 1223 | 1223 | 1214 | 1214 |
| Ordering Cost | 1734 | 1734 | 1734 | 1734 |
| Purchase Cost | 59775 | 57384 | 59775 | 57384 |
| Incremental Cost | 0 | 0 | 0 | 0 |
| Total Cost | 62723 | 60332 | 62723 | 60332 |

Table 4.5 Summary of What'sBest Results for the 2-Supplier Case

|  | 2-Suppliers <br> $(95 \%, 95 \%)$ | 2-Suppliers <br> $95 \%, 50 \%$ | 2-Suppliers <br> $(50 \%, 95 \%)$ | 2-Suppliers <br> $(50 \%, 50 \%)$ |
| :--- | :---: | :---: | :---: | :---: |
| Yied Rate | 517 | 1465 | 982 | 1790 |
| Amount Ordered from Supplier 1 | 2000 | 2000 | 2000 | 2992 |
| Amount Ordered from Supplier 2 | 2517 | 3465 | 2982 | 4782 |
| Total Amount Ordered | 491 | 1391 | 491 | 895 |
| Amount Received from Supplier 1 | 1900 | 1000 | 1900 | 1496 |
| Amount Received from Supplier 2 | 2391 | 2391 | 2391 | 2391 |
| Total Amount Received | 24 | 126 | 123 | 300 |
| Inventory Carrying cost | 3468 | 3468 | 3468 | 3468 |
| Ordering Cost | 58861 | 60185 | 58857 | 58279 |
| Purchase Cost | 0 | 0 | 0 | 0 |
| Hncremental Cost | $\mathbf{0 2 3 5 3}$ | $\mathbf{6 3 7 7 9}$ | $\mathbf{6 2 4 4 8}$ | $\mathbf{6 2 0 4 7}$ |

If we focus on table 4.4 we will observe that for both Supplier 1 and Supplier 2, the total cost in the presence a high yield rate and a low yield rate are the same. This may not necessarily be the case in a real life setting. Meaning that the solution presented here did not take into consideration the effect of varying supply yield on expected procurement cost since we only considered the deterministic case. For example as indicated in table 4.4, in the presence of an average low yield rate of $50 \%$ the buyer placed an order for 4782 tons of fish from Supplier 1. Being that the variance of the yield rate is 0.067 the buyer may receive as much as 2677 tons or as little as 2104 tons resulting in a large volume of on hand inventory or shortages. However, with a yield rate of $95 \%$ and the same variance indicated above, if the buyer were to order 2518 tons as indicated in the table, the maximum amount that the buyer would receive is 2560 tons. The result would be lower purchase cost and lower inventory levels thereby making expected procurement cost cheaper in the presence of high yield rates.

The results from table 4.5 indicate that for the 2 -supplier case the cheapest solution was achieved when both suppliers had average low yield rates of $50 \%$. When we modeled the case of one supplier having a high yield rate and the other a low yield rate we observed that the total cost was at its highest.

On comparing the total cost for the single supplier case to the 2 -supplier case we noticed that in the presence of high yield rates the buyer does not get the cheapest price by splitting orders. However when the yield rate is low the total cost for Supplier 2 is lower than the total cost for the 2-supplier case, but the total cost for Supplier 1 is higher than the total cost for the 2 -supplier case. One reason for this is because Supplier 2 is the
cheapest supplier. Also in the 2 -supplier case there is an upper bound placed on the amount that can be ordered from each supplier. As can be observed from table 4.5 , the maximum amount is always ordered from the cheapest supplier. The second more expensive supplier is then used to satisfy remaining demand. If both suppliers were to offer the same price schedules then the purchase cost in the 2 -supplier case would be less or would be the same as the supplier case. The differences in cost would be in the ordering cost and inventory related cost. From both tables 4.4 and 4.5 , it can be observed that the 2 -supplier solution has a lower level of carrying inventory but a higher level of ordering cost.

### 4.3.2 Results from Heuristic Procedure

The results attained from What'sBest for Supplier 1 with a yield rate of $100 \%$ is presented in table 4.6 below.

Table 4.6 What'sBest Solution for the Single Supplier Problem With 100\% Yield Rate

| Period | 1. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Amount Ordered | 400 | 0 | 406 | 0 | 407 | 0 | 398 | 0 | 386 | 0 | 394 | 0 | 3291 |
| Inventory Carrying Cosi 1214 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Ordering Cost 1734 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Purchase Cost <br> Incremental Cost |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Toial Cost |  |  |  |  |  |  |  |  |  |  |  |  | 60332 |

For the 2 -supplier problem we model the case where yield rate is a random variable and solve it by splitting the orders obtained in table 4.6 in accordance with the ratios discussed earlier. The solutions for each situation are presented in the table below.

Table 4.7 Summary of the Heuristic Solution to the 2-Supplier Case

|  | 2-Suppliers <br> $(95 \%, 95 \%)$ | 2-Suppliers <br> $95 \%, 50 \%$ | 2-Suppliers <br> $(50 \%, 95 \%)$ | 2-Suppliers <br> $(50 \%, 50 \%)$ |
| :--- | :---: | :---: | :---: | :---: |
| Yield Rate | 1258 | 1648 | 1648 | 2391 |
| Amount Ordered from Supplier 1 | 1258 | 1648 | 1648 | 2391 |
| Amount Ordered from Supplier 2 | 2516 | 3296 | 3296 | 4782 |
| Total Amount Ordered | 1196 | 1567 | 824 | 895 |
| Amount Received from Supplier 1 | 1196 | 824 | 1567 | 1496 |
| Amount Received from Supplier 2 | 2392 | 2391 | 2391 | 2391 |
| Total Amount Received | 1214 | 1214 | 1214 | 1214 |
| Inventory Carrying Cost | 1734 | 1734 | 1734 | 1734 |
| Ordering Cost | 62116 | 58950 | 58208 | 58279 |
| Purchase Cosi | 0 | 0 | 0 | 0 |
| Incremental Cosi | 65114 | 61898 | 61156 | 61527 |
| Total Cost |  |  |  |  |

The heuristic results again shows that the buyer does not get the cheaper price by splitting the orders. It should be noted that since the yield rate for the single supplier case is $100 \%$, then any shortage cost incurred would be minimal. In the two-supplier case savings from improved yield would counterbalance this cost.

Figure 4.1 Total Cost for Each Supplier Scenario Under Varying Levels of Yield Rates

Sup. 1 What'sBest Sup. 2 What'sBest
-2-Sup What'sBest $\square 2$-Sup. Heuristic

Figure 4.1 shows a comparison of the solutions obtained from each supplier scenario under varying levels of yield rates. On observation it can be noticed that under both levels of yield rates the cheapest solution was obtained from a Supplier 1. As indicated earlier these cost structures only considered the deterministic case and may not be so if the stochastic case were examined.

In the presence of a high yield rate the worst solution was obtained from the heuristic procedure, however in the presence of a low yield rate the heuristic performed slightly better than What'sBest. The reason for this is because the heuristic solution only 6 orders were placed during the planning horizon, compared to 12 orders with What'sBest. Therefore a higher ordering cost was incurred with the What'sBest solution resulting in a higher procurement cost.

### 4.4 Sensitivity Analysis

In this section we study the sensitivity of the total cost function with respect to the input data, in particular the yield rate using computer simulation. The purpose of this simulation is to test the robustness of our solutions and to see how the deterministic case applies to the stochastic case. It should be noted that the simulation being performed is not a real time period-by-period simulation where the buyer has the opportunity to adjust the orders. In other words, the real time policy is to keep the order quantities fixed over the planning horizon.

We perform a Monte Carlo simulation using a spreadsheet simulation modeling software called @Risk developed my Palisade Corporation. We used the following algorithm proposed by Law and Kelton (1991) to determine the number of simulation runs.

Let

$$
\begin{aligned}
& \mathrm{n}=\text { the number of replications; } \\
& \bar{X}(n)=\text { the sample mean; } \\
& S^{2}(n)=\text { the sample variance } \\
& Y=\text { the relative error of } \bar{X}=0.1 ;
\end{aligned}
$$

Choose an initial number of replications $n \geq 2$ and compute the following

$$
\bar{X}(n) \pm t_{n-1,1-\alpha} \sqrt{\frac{S^{2}(n)}{n}}
$$

where $t_{n-1,1-\alpha} \sqrt{\frac{S^{2}(n)}{n}}$ is the confidence interval half length (CIHL).
If $\sqrt{\frac{C I H L}{\bar{X}}} \leq \gamma$ then stop and set the simulation runs to $n$ times else increase n to $\mathrm{n}+1$ and repeat procedure.

Using a confidence interval of $90 \%$ we solved the algorithm and set the number of simulation runs to 100 .

### 4.4.1 Experimental Design

In performing the analysis we considered three levels of variability in the yield rate; a low level with a coefficient variation (cv) of $10 \%$, a medium level with a cv of $25 \%$ and a high level with a cv of $50 \%$. The coefficient of variation is assumed to be constant over all periods.

## A Optimal Solution - Single Supplier Case

Case 1 Supplier 1 - yield $95 \%$

Case 2 Supplier 2 - yield $95 \%$
Case 3 Supplier $1-$ yield $50 \%$

## B Optimal Solution - Two-Supplier Case

Case 1 Supplier 1 - yield $95 \%$, Supplier 2 - yield $95 \%$
Case 2 Supplier 1 - yield $95 \%$, Supplier 2 - yield $50 \%$
Case 3 Supplier 1 - yield $50 \%$, Supplier 2 - yield $95 \%$
Case 4 Supplier 1 - yield $90 \%$, Supplier 2 - yield $50 \%$

C Heuristic Solution
Case 1 Supplier 1-yield $95 \%$, Supplier 2 - yield $95 \%$
Case 2 Supplier 1 - yield 95\%, Supplier 2 - yield 50\%
Case 3 Supplier 1 - yield $50 \%$, Supplier 2 - yield $95 \%$
Case 4 Supplier 1 - yield 50\%, Supplier 2 - yield 50\%

### 4.4.2 Simulation Results

Tables 4.8, 4.9 and 4.10 presents the simulation results for supplier sourcing under the various combinations of yield variability.

Table 4.8 Simulation Results of the What'sBest Solutions to the Single Supplier Case

| 10\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yield Rate | 95\% |  |  |  | 50\% |  |  |  |
|  | Supplier 1 |  | Supplier 2 |  | Supplier 1 |  | Supplier 2 |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Sid |
| Total Cost | 63670 | 2785 | 57526 | 2479 | 63475 | 2456 | 61247 | 2719 |
| Amount Ordered | 2546 | 0 | 2516 | 0 | 2516 | 0 | 4782 | 0 |
| Orders Received | 2404 | 100 | 2398 | 93 | 2398 | 95 | 2404 | 102 |
| Shortage Cost | 359 | 473 | 387 | 522 | 406 | 542 | 405 | 481 |
| Inventory Cost | 1467 | 645 | 1447 | 584 | 1379 | 529 | 1430 | 630 |
| 25\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| Yield Rate | 95\% |  |  |  | 50\% |  |  |  |
|  | Supplier 1 |  | Supplier 2 |  | Supplier 1 |  | Supplier 2 |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| Total Cost | 64776 | 5983 | 64887 | 62930 | 63850 | 5793 | 61616 | 5270 |
| Amount Ordered | 2416 | 0 | 2415 | 2516 | 4782 | 0 | 4782 | 0 |
| Orders Received | 2399 | 240 | 2425 | 254 | 2365 | 235 | 2369 | 229 |
| Shortage Cosi | 1299 | 1830 | 912 | 1483 | 1442 | 2052 | 1323 | 2021 |
| Inventory Cost | 1765 | 1401 | 1923 | 1348 | 1540 | 1296 | 1699 | 1236 |
| 50\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| Yield Rate | 95\% |  |  |  | 50\% |  |  |  |
|  | Supplier 1 |  | Supplier 2 |  | Supplier 1 |  | Supplier 2 |  |
|  | Mean | Std | Mean | Sid | Mean | Std | Mean | Std |
| Total Cost | 67813 | 11303 | 65071 | 10621 | 66865 | 11971 | 64067 | 10241 |
| Amount Ordered | 2516 | 0 | 2516 | 0 | 4782 | 0 | 4782 | 0 |
| Orders Received | 2421 | 493 | 2390 | 507 | 2367 | 530 | 2369 | 459 |
| Shortage Cost | 2980 | 4535 | 3442 | 5294 | 3412 | 5241 | 3109 | 4450 |
| Inventory Cost | 2572 | 2300 | 2518 | 2309 | 2523 | 2535 | 2362 | 2312 |

Table 4.9 Simulation Results of the What'sBest Solutions to the 2-Supplier Case

| 10\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yield Rate | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  |
|  | (95\%, 95\%) |  | (95\%, 50\%) |  | (50\%, 95\%) |  | (50\%, 50\%) |  |
|  | Mean | Std | Mean | Stid | Mean | Sid | Mean | Sid |
| Total Cost | 70778 | 1712 | 65471 | 1579 | 64974 | 1597 | 67263 | 1927 |
| Amount Ordered | 2918 | 0 | 3500 | 0 | 3500 | 0 | 5184 | 0 |
| Orders Received | 2602 | 78 | 2427 | 63 | 2644 | 66 | 2575 | 63 |
| Shorrage Cost | 2657 | 717 | 310 | 522 | 190 | 255 | 186 | 243 |
| Inventory Cost | 434 | 268 | 565 | 369 | 1092 | 381 | 956 | 421 |
| 25\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| Yrield Rate | $\begin{gathered} \text { 2-Supplier } \\ (95 \%, 95 \%) \end{gathered}$ |  | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  |
|  |  |  | (95\%, 50\%) |  | (50\%, 95\%) |  | (50\%, 50\%) |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| Total Cost | 71166 | 3867 | 66938 | 3994 | 65831 | 4685 | 67760 | 4306 |
| Amount Ordered | 2918 | 0 | 3500 | 0 | 3500 | 0 | 5184 | 0 |
| Orders Received | 2584 | 181 | 2441 | 170 | 2679 | 2586 | 2586 | 143 |
| Shorlage Cost | 3338 | 1988 | 1083 | 1710 | 684 | 1020 | 606 | 919 |
| Lnventory Cosi | 597 | 619 | 928 | 686 | 1501 | 1069 | 1211 | 889 |
| 50\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| Yield Rate | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  |
|  | (95\%, 95\%) |  | (95\%, 50\%) |  | (50\%, 95\%) |  | (50\%, 50\%) |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| Total Cost | 73670 | 8879 | 69159 | 6681 | 72663 | 7416 | 69785 | 8958 |
| Amount Ordered | 2918 | 0 | 3500 | 0 | 3500 | 0 | 5184 | 0 |
| Orders Received | 2667 | 374 | 2442 | 313 | 2673 | 322 | 2642 | 307 |
| Shortage Cost | 2733 | 3320 | 2669 | 3989 | 1783 | 2764 | 1621 | 2266 |
| Inventory Cost | 1715 | 1845 | 1547 | 1638 | 1761 | 1477 | 1904 | 1722 |

Table 4.10 Simulation Results of the Heuristic Solutions to the 2-Supplier Case

| Yield Rate | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (95\%. 95\%) |  | (95\%, 50\%) |  | (50\%, 95\%) |  | (50\%, 50\%) |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| Total Cost | 63722 | 1338 | 60713 | 1504 | 59712 | 1300 | 60202 | 1462 |
| Amount Ordered | 2516 | 0 | 3296 | 0 | 3296 | 0 | 4782 | 0 |
| Orders Received | 2390 | 65 | 2401 | 73 | 2388 | 67 | 2393 | 71 |
| Shortage Cost | 359 | 489 | 291 | 430 | 357 | 400 | 349 | 402 |
| Inventory Cost | 1317 | 368 | 1445 | 462 | 1329 | 392 | 1363 | 438 |
| 25\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| Yield Rate | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  |
|  | (95\%.95\%) |  | ( $95 \%$, 50\%) |  | ( $50 \%, 95 \%$ ) |  | ( $50 \%, 50 \%$ ) |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| Total Cost | 64224 | 3598 | 621245 | 2840 | 60823 | 3198 | 61081 | 3017 |
| Amount Ordered | 2516 | 0 | 3292 | 0 | 3296 | 0 | 4782 | 0 |
| Orders Received | 2382 | 174 | 2387 | 155 | 2404 | 173 | 2409 | 158 |
| Shortage Cost | 1058 | 1375 | 1202 | 1435 | 1065 | 1497 | 742 | 1188 |
| Inventory Cost | 1515 | 996 | 1413 | 1023 | 1555 | 1020 | 1602 | 956 |
| 50\% Coefficient of Variation |  |  |  |  |  |  |  |  |
| Yield Rate | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  | 2-Supplier |  |
|  | (95\%. $95 \%$ ) |  | (95\%, 50\%) |  | (50\%, 95\%) |  | (50\%, 50\%) |  |
|  | Mean | Std | Mean | Std | Mean | Std | Mean | Sid |
| Total Cost | 65706 | 6512 | 64461 | 5960 | 62812 | 5951 | 62645 | 6466 |
| Amount Ordered | 2516 | 0 | 3292 | 0 | 3296 | 0 | 4782 | 0 |
| Orders Received | 2413 | 306 | 2038 | 366 | 2426 | 352 | 2412 | 367 |
| Shortage Cost | 1730 | 2406 | 3345 | 4448 | 2516 | 4124 | 2333 | 3593 |
| Inventory Cost | 2079 | 1687 | 1713 | 1729 | 2233 | 1914 | 2250 | 1899 |

Again, it is clear from the results in tables 4.114 .12 and 4.13 that as the coefficient of variation increase so do the expected cost of procurement. This means that varying yields do have an effect on the total expected cost of procurement.

Figures 4.2 and 4.3 show a comparison of mean inventory related cost, the most important variable impacted by yield uncertainty, for each supplier scenario and solution methodology. It can be seen from these figures that as the coefficient of variation of the
yield rate increases so does the inventory related cost. Also in the presence of a high yield rate, as the variance of the yield rate increases the inventory related cost realized from sourcing from 2 suppliers decreases and becomes less than that of single supplier sourcing. In the presence of low yield rates inventory related cost for the 2 -supplier sourcing is always lower than that of single supplier sourcing. Therefore one can conclude that as the variance of the yield rate increases there is much savings to be achieved by multi-sourcing as oppose to single sourcing.

Figure 4.2 Inventory Related Cost for Each Supplier Scenario Under High Yield Rates


Figure 4.3 Inventory Related Cost for Each Supplier Scenario Under Low Yield Rates


## CHAPTER 5

## SUMMARY AND CONCLUSION

### 5.1 General Results Obtained From the Model

The computational study performed on the model indicated that the algorithm is computational efficient. Locally optimal solutions were obtained in an average CPU time of 23 seconds.

The results from both the What'sBest approach and the heuristic method indicated that when supply yield is uncertain a second supplier can act as a hedge against uncertainties. From our numerical analysis we observe that when a buyer sources from two suppliers with varying supply yields, different unit purchase cost and with upper bounds placed on the amount that can be purchased for each supplier, the optimal solution is to purchase the maximum amount form the cheaper supplier and use the second more expensive supplier to satisfy the remaining demand. In that case the solution for single supplier may be better than dual sourcing. However, when order costs are equal it may be optimal to source from two suppliers.

In the sensitivity analysis we noticed that for both the optimal approach and the heuristic procedure, mean inventory and mean shortage levels were highly impacted by the
uncertainty of the yield rate. In both cases mean inventory levels generally increased as the yield variability increased. On comparing the single supplier model and the 2 -supplier model we observe that the 2 - supplier model has a lower level of both inventory and shortages and as the variability increases, the level of inventory increases. The highest carrying inventory occurs when the yield rate is low with a coefficient variation of $50 \%$. Thus from our numerical analysis we can conclude that for any given mean yield rate as the variability increases it becomes cost effective to split orders.

### 5.2 Conclusion

In this paper we have provided an analysis of single supplier sourcing versus dual supplier sourcing when yield is random under minimum commitment contracts with flexibility agreement. We have obtained solutions for order quantities from two different approaches: the formulation of the problem in this and a heuristic procedure proposed by Millar (2000 a) and Millar (2000 b). We assumed a 2-supplier problem with a planning horizon of 12 periods, where each supplier offers a quantity-discounted schedule and where upper bounds are placed on the amount that can be sourced from each supplier.

We provided computational results and compared the results obtained from our formulation to that of the heuristic procedure. The results indicated that our formulation performs better in the presence of varying levels of low yield rates. We also compared the
results obtained from dual sourcing to that of a single supplier. We concluded that under conditions of random yield it is cost efficient to split orders between suppliers. Sensitivity analysis performed on the solutions indicated that as the variance in the yield rate increased so does the total procurement cost.

### 5.3 Future Research

So far in the model we assumed that procurement lead-time is zero. A logical extension of our model would be to formulate the problem as a lead-time problem. In our analysis we examined the impact of yield rate on the total procurement cost. It would be interesting to observe the effect of the commitment contracts and price schedules on total expected cost. We can also extend the analysis to examine the effect of setup cost, by allowing each supplier to incur a different setup cost. In our solution methodology capacity constraints were relaxed. The problems should be examined where capacity constraints are imposed and also where shortages are allowed and can be backordered.

Another issue is to consider the multi-product multi-supplier case where each supplier has different yield rate distributions for each product. A further issue is the impact of real time procurement polices on the expected cost. Instead of Monte Carlo simulation we could conduct a discrete event simulation, which allows for order updates based on realized demands. The input of various orders updating strategies could be studied.

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