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Impact of social influence in English proficiency and performance in English examinations of mathematics students from a Sino-US undergraduate education program

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Abstract. This study examines the influence of certain academic and demographic variables upon the academic performance of Chinese students enrolled in a cooperative Bachelor's degree program in Pure and Applied Mathematics. The program is English taught and jointly organised by Jiangsu University, China and Arcadia University, USA. Data from a sample of 166 students is processed using inferential and path analysis, as well as mathematical modelling. As evidenced by the inferential and path analysis, no steady improvement in the English proficiency of students has been observed, while the latter has been found to be influenced by gender and to strongly influence academic performance in Mathematics courses. The effects of negative social influences are assessed via a qualitative analysis of the mathematical model. Threshold quantities similar to the basic reproduction number of mathematical epidemiology have been found to be stability triggers. Possible interventional measures are discussed based on these findings.

Keywords: English proficiency, academic performance, social influence, Chinese Mathematics students, differential equations, threshold parameter.

1 Introduction

English language education is viewed by the Chinese leadership as having a vital role to play in national modernization and development [1]. English is now used as a major

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international language of communication because it is the language of science, technology, education and international travel [8, 9]. As a result, a variety of programs taught in English are now opened for students of Chinese universities. As of June, 2018, there were about 1090 active Sino-foreign cooperative institutions and projects at the undergraduate level and above. The trend since 2008 is that all undergraduate programs, with few exceptions, are for dual degree awards, which allows for strict supervision by the authorities on degree quality.

Many research studies have examined the relationship between the English proficiency of students and their academic performance in content based courses. Proficiency in English prepares students for meaningful instruction and academic performance in subjects taught using the English language [20]. A significant positive correlation has been found between English language proficiency and high GPA scores in different disciplines at the university level [23, 25, 26]. However, other studies have found no relationship between the English proficiency of students and their GPA at the graduate level [16]. Consequently, the association between English language proficiency and performance in content based courses is worth further investigation.

Empirical studies that examine gender difference in English proficiency of undergraduate students from the cooperative education programs are rare. An international study by [29] showed that girls tend to engage better in communication and social interaction, hence they progress faster than boys in language learning, which reflects in their English language proficiency test scores. This study also revealed that numerous socio-cultural and economic factors contribute to gender differences with regards to English language proficiency, leading to stronger female English language ability in all language skills. However, countries where the difference is greatest are also societies with known gender equity issues [29]. In contrast, a study by [11] in Abu Dhabi University found that gender is not significantly associated with English language proficiency.

Since most Chinese students from the same program will be lodged in several dormitories on the same floor of the same residence hall, which is the traditional on-campus Chinese culture, the social interactions of undergraduate students with their peers play a key role in individual learning [14]. This implies that students with bad study habits can easily have a bad influence on their roommates and close classmates with a low internal locus of control. It is expected that the "contact" rate, which takes into account the probability of bad influence being passed on in a social contact between a student who has to take make-up examinations or has to resit courses and a student who pass all core courses, acts as a mediator in predicting the relationship between social influence and academic performance, including College English examination scores, via mathematical modeling.

1.1 Background

Jiangsu University (JU), China, in accordance with the regulations of the People's Republic of China on Sino-foreign cooperation between schools, has signed a memorandum of understanding (MOU) with Arcadia University (AU) in the United States of America

Table 1. Five-point GPA scale.

Percent grade	5.0 scale	Remark
90–100	4.0–5.0	Excellent
80–89	3.0–3.9	Good
70–79	2.0–2.9	Fair
60–69	1.0–1.9	Satisfactory
0–59	0.0–0.9	Fail

Table 2. Example.

Course	Grade	5.0 scale (x)	Course credit (y)
Course I	89	3.9	8
Course II	85	3.5	6
Course III	94	4.4	4

$$\mathrm{GPA} = \frac{\sum xy}{\sum y} = \frac{(3.9\times 8)+(3.5\times 6)+(4.4\times 4)}{(8+6+4)} = 3.88$$

to establish a cooperative, English taught Bachelors degree program in Pure and Applied Mathematics.

The length of the program is four years. However, based on the MOU between the two universities, students can spend three years in JU and one year in AU. Before students can proceed to AU to complete the program, they should successfully complete the first three years of the jointly recognized curriculum in JU with a grade point average (GPA) of 3.0 (overall average score of 80) or above in Mathematics courses. The GPA is the average of the product between course credits and the scaled grade a student receives for the course (Table 1). The value of the course credit depends on the significance that is attributed to the course. For instance, all English language courses of the first two academic years have a total of 32 credits altogether. Additionally, students who want to complete the program in AU must establish their English proficiency by obtaining qualified scores in Test of English as a Foreign Language (TOEFL $\geqslant$ 73), International English Language Testing System (IELTS $\geqslant 6.0$) or International Test of English Proficiency (ITEP $\geqslant 5.0$). TOEFL, IELTS and ITEP are internationally recognized examinations used to test the English proficiency of students who study English as a foreign language (EFL). Students who do not meet these requirements or cannot proceed to AU due to personal reasons or to other circumstances will enter the fourth year in JU to complete the course, which leads to the award of Bachelor of Science degree in China.

Curriculum reforms have already been made to the Pure and Applied Mathematics degree program based on the MOU. In order to improve the English proficiency of students, the language of instruction for the courses is English. Also, all main textbooks and teaching materials are in English and Chinese textbooks serve only as supplementary materials. Apart from the required Pure and Applied Mathematics courses, students are expected to sit through English language courses as well. They are expected to take College English (I) and College English (II) in the first academic year and also College English (III) and College English (IV) in the second academic year. Since many qualified students need enough time to prepare the Graduate Record Examination (GRE) and improve their level of academic ability, to avoid violating the regulations of the Ministry of Education of the People's Republic of China, the students would now prefer to go to AU as one-year exchange students after two-year study in JU, owing to the updated MOU between the two universities.

1.2 Relevant examination policies

Before registering to a course, the students are entitled to have full knowledge regarding the evaluations procedures, policies and standards of that course. For most courses, the final examination is taken at the end of the semester and the students are to be informed of their scores before the last day of that semester. If a student disagrees with the obtained score, an appeal can be lodged with the Secretary of Academic Affairs for examination re-marking. Then, the Vice Dean of Academic Affairs will nominate an *ad-hoc* Administrative Board for re-marking. After re-marking, the student will receive a formal reply from the Secretary of Academic Affairs communicating the new score, which remains final. To be admitted to the subsequent academic year, students have to complete a certain number of credits. If the total number of credits completed by the student after the first, second and third academic year is less than 15%, 35% and 60% of total amount of credits necessary for obtaining the bachelor degree, respectively, then the student is not admitted to the subsequent academic year.

A student who is subject to compelling circumstances such as serious illness or injury may submit an application for leave of absence before the final examination takes place. This application must be accompanied by appropriate supporting evidence. If this application is approved, then the student is granted a leave of absence. If the leave of absence is not granted and the student is still absent, the final examination is considered as being failed.

A makeup examination, held at the beginning of the new semester, is scheduled for the students who failed the final examination. For students with leave of absence, this examination will be considered, for all procedural purposes, as a final examination, not as a makeup examination. The students who fail the makeup examination are required to resit the course, no further makeup examinations being scheduled.

To distinguish between credits obtains through different procedures, the Ministry of Education of P.R. China has recently promulgated revised administrative regulations concerning students in colleges and universities (Order No. 41/2017), which require that the academic credits obtained through makeup and resit examinations be marked as such on the issued transcript. Hopefully, this measure will make students prepare more comprehensively for the initial examination.

1.3 Statement of the problems

The introduction of cooperative programs in Pure and Applied Mathematics comes with its merits and challenges. Even though it provides students who qualify with an opportunity to have international exposure, little is known about the influence of this opportunity on the level of students' English proficiency and performance in Mathematics. While it is often assumed that study abroad programs offer the best opportunities and motivation for language learning [2], the fact that this kind of programs is the best way to learn a foreign language [5, 18] is still an unconfirmed myth. It is therefore important to examine the extent to which the cooperative program in Pure and Applied Mathematics is achieving its goals by investigating the English proficiency and performance of students who are taking this program.

To improve the academic performance of undergraduate students in the collaborative program, the present study seeks therefore to investigate and clarify the following points:

- The relationship among demographics, entrance English examination score and English proficiency.

- The relationship between English proficiency and the academic performance of first and second-year Pure and Applied Mathematics students.
- The effect of social influence on the academic performance of students, including College English examination scores.

2 Methods and materials

2.1 Design

In regard to the first two points, this study uses correlational research with path modelling. Correlational research involves collecting data to determine the extent to which relationships exists between two or more quantifiable variables [13]. We use Pearson's correlation coefficient (r) to measure the extent to which the variables under study relate. However, to find out the effect of one variable on the other, path modelling is used. To clarify the third point, we formulate a mathematical model and then establish its basic well-posedness, stability and backward bifurcation properties. A threshold quantity similar in its scope to the basic reproduction number relying on the contact rate, used in mathematical epidemiology, is found to be a stability trigger of the system. The instability of the system would lead to an increasing number of students with poor academic performance in the collaborative program.

2.2 Participants

The study uses data from a sample of 166 undergraduate students from the Department of Financial Mathematics in Jiangsu University, China. The necessary data on students demographics, performance in English courses and GPAs is collected from students information database for the study. The demographic data collected is the gender and locality of students. The locality of students refers to whether the student is coming from a town or rural area in Jiangsu Province based on Chinese residence classification. For students' performance in English, the scores in the English entrance examination, College English I, II, III and IV are collected. Students have to take College English I and II in the first academic year and College English III and IV in the second academic year. Student performance in Pure and Applied Mathematics courses in first and second years is measured using their grade point averages in the first year (GPA I) and the second year (GPA II). The data was entered into IBM SPSS version 23 for analysis.

2.3 Data analysis

In the data analysis correlation, inferential and path modelling analysis are conducted on the data in order to test our research hypothesis. For the inferential analysis, t-test and analysis of variance (ANOVA) are used for hypothesis testing. Using IBM SPSS AMOS version 23, maximum likelihood estimation is used in the path modelling. Chi square test (χ^2), comparative fit index (CFI) and root mean square error of approximation (RMSEA) are used to test the overall goodness of fit of the model. A good fit model requires nonsignificant chi-square test, CFI values > 0.95 and RMSEA values < 0.06, [10, 17, 19]. In the path model, the direct effect of independent variables on dependent variables are discussed using path coefficients (beta weights β). The path coefficients, which are the same as beta weights in the linear regression, are used to determine the strength of direct effects of independent variable on the dependent variables. Additionally, the coefficient of determination (R^2) is used to explain the proportion of variance in the dependent variables that can be explained by the independent variables. R^2 is usually expressed as a percentage for easy understanding.

3 Results

Among students, 73.5% are male and 26.5% are female. The majority (81.9%) of them are from towns and 18.1% are from rural localities. Male students are more in both town (75%) and rural (67%) localities. Only 25% of students from towns and 33% from rural regions are female.

3.1 Correlation analysis

The overall student performance in university English is the calculated averaging scores of male and female students in College English I, II, III, and IV. We compare the performance of male and female students using the t-test in order to assess the hypothesized differences between the university English performance of male and female students. Comparing the average English scores of male and female students in the four semesters, the average score of female students ($M = 80.7614$, $SD = 3.89$) is higher than that of male students ($M = 75.2264$, $SD = 10.86406$). As shown in Table 3, this difference is statistically significant ($t = -3.299$, $p = 0.000 < 0.05$). Therefore, we reject the null hypothesis and conclude that there is a difference between the performance of male and female students in College English courses.

Table 4 shows the descriptives of student performance in various courses. Comparing to the entrance English score, there is a drop in the average score of students in College English I ($M = 75.679$, $SD = 11.476$). However, the performance of students in College English II increases ($M = 77.938$, $SD = 8.246$) but drops again in College English III ($M = 75.488$, $SD = 11.295$). The performance of students in College English IV increases ($M = 77.668$, $SD = 13.668$) when compared to College English III. The performance improvement in College English IV may be due to the fact that the students were motivated to meet the English requirement for travelling to the Arcadia University in USA to complete their program.

The differences in the average scores of students in the College English courses are tested using one way ANOVA. No significant difference ($F = 3.129$, $P > 0.05$) is found between the average scores of students in all English courses (see Table 5). Therefore the null hypothesis is not rejected and we conclude that there is no significant difference in the performance of students in the College English courses. This means that there is no steady improvement in the performance of students in the various English courses compared to their entrance performance. The relationship between the English proficiency of students and their academic performance is explored using a bivariate

Table 3. t-test for differences in College English performance according to gender ($df = 164$, $P = 0$).

Gender	N	M	SD	t
Male	122	75.2264	10.86406	−3.299
Female	44	80.7614	3.89	−3.299

Table 4. Descriptive statistics of students scores in English courses.

	Min	Max	M	SD
	Total score of 120			
Entrance English	61.00	99.00	78.878	8.901
	Total score of 100			
College English I	33.00	90.00	75.679	11.476
College English II	55.00	90.00	77.938	8.246
College English III	30.00	97.00	75.488	11.295
College English IV	7.00	96.00	77.668	13.668

Table 5. One way ANOVA for differences in the average English scores of students ($F = 3.129$, $p = 0.19$).

	Sum of Squares	df	Mean Square
Between Groups	1487.980	4	371.995
Within Groups	98087.365	825	118.883
Total	99566.345	829	

Table 6. Bivariate correlation and descriptive statistics.

Variable	1	2	3	4	5	Mean	SD
1. Entrance English score	–	0.247*	0.134	−0.002	0.140	78.818	8.901
2. First year English score		–	0.812**	0.749**	0.689**	76.809	9.317
3. Second year English score			–	0.615**	0.819**	76.578	11.376
4. First year GPA				–	0.641**	2.854	0.702
5. Second year GPA					–	2.771	0.792

**Correlation is significant at the 0.01 level (2-tailed).
*Correlation is significant at the 0.05 level (2-tailed).

correlation analysis (Table 6). A strong positive relationship is found between the first year English score of students and their first year GPA ($r = 0.749$, $p < 0.01$). The same happens for their second year English score and their second year GPA ($r = 0.819$, $p < 0.01$). This is expected because all the main learning materials from the Pure and Applied Mathematics courses are in English. Also, the core courses are taught in English, which means that students who are proficient in English have an advantage towards getting a high GPA. The entrance English score of students correlated only with first year English score but the correlation was weak ($r = 0.247$, $p < 0.05$). This means that a weak relationship exists between the entrance English score of students and their performance in first year English examinations but a high score in English entrance alone has no consequence on students overall performance in the program. However, a strong positive correlation is found between the first year and second year English scores ($r = 0.812$, $p < 0.01$) as expected. The second year GPA scores of students relate positively with their first year scores ($r = 0.689$, $p < 0.01$) and with second year English scores ($r = 0.819$, $p < 0.01$). A positive relationship ($r = 0.641$, $p < 0.01$) is also found between the first and second year GPAs of students.

3.2 Path analysis

In this context, it is important to know more about the directed dependencies among a set of variables, which consist of independent variables including gender and locality, and dependent variables including entrance English score, grade point average in first year, grade point average in second year, average English score in first year and average English score in second year. This study is guided by the hypothesis that the locality, gender and entrance English score of the students affect their grade point averages and performance in English during their undergraduate studies within the framework of the cooperative program.

Using IBM SPSS AMOS version 23, a path analysis is conducted to investigate the effects among the variables under study. The model fit indices reveal a non significant chi-square statistics ($\chi^2_{(12)} = 16.080$; $p < 0.01$) with CFI $= 0.985$ and RMSEA $= 0.054$ (see Fig. 1). This indicates that the model fits the data and it adequately accounts for the input covariance matrix reflecting the relationships among the variables in the model [19].

The entrance English score of students is weakly affected by their locality ($\beta = 0.07$, $P < 0.05$). The positive beta weight means that students from town localities have a better change of performing well in the entrance English examination than students from rural localities. However, in recent years, both urban and rural schools have gotten almost the same support from the local government and relevant education administration. Also, the parents from rural regions have the same awareness for the significance of English education as the urban parents. Consequently, the gap of English education between urban and rural regions has been narrowing.

The average performance of students in first year English is directly affected by their gender ($\beta = 0.25$, $P < 0.05$), English entrance score ($\beta = 0.23$, $P < 0.05$) and locality ($\beta = 0.25$, $P < 0.05$). These factors contribute 18% of the variance in the first year English performance of students. It is seen that male students from town localities

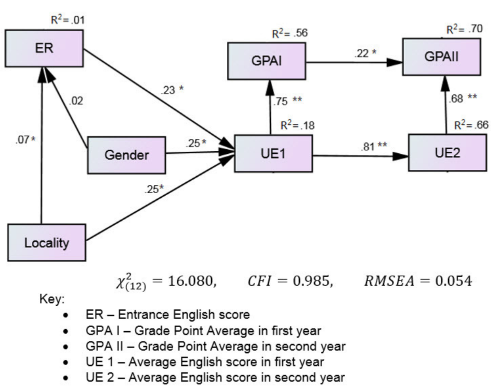

Figure 1. Path diagram for final model. Asterisks indicate level of significance ($^*p < 0.05$, $^{**}p < 0.01$) of path coefficients. The squared multiple correlation (R^2) represents the proportion of variance in each dependent variable explained by the independent variables that directly influence it.

Table 7. Granger causality Wald tests (two lags).

Equation	Excluded	F	df	df_r	Prob $> F$
UE	GPA	2.0845	2	657	0.1252
UE	ALL	2.0845	2	657	0.1252
GPA	UE	4.7016	2	657	0.0094
GPA	ALL	4.7016	2	657	0.0094

Table 8. Granger causality Wald tests (three lags).

Equation	Excluded	F	df	df_r	Prob $> F$
UE	GPA	1.8187	3	654	0.1424
UE	ALL	1.8187	3	654	0.1424
GPA	UE	6.0576	3	654	0.0005
GPA	ALL	4.7016	2	654	0.0005

performed better in first year English courses. The performance of students in English has a strong direct effect on the overall GPA both in first year ($\beta = 0.75$, $P < 0.01$) and second year ($\beta = 0.68$, $P < 0.01$). This finding underscores the importance of the English proficiency of students in Pure and Applied Mathematics courses. The first year GPA of students also directly affects their second year GPA ($\beta = 0.22$, $P < 0.05$), likewise their English performance ($\beta = 0.81$, $P < 0.01$). These findings point out the impact of the past performance and experience of students on their future performance. Therefore, we reveal the strong direct effect of students performance in English on their GPA and the fact that the average performance of students in English is directly affected by their gender, entrance English score and locality.

3.3 Granger causality tests

In the above, we have used a path analysis to sketch the directed dependencies. A conceptually related investigation can be performed by means of employing Granger causality as a measure of dependency instead, using Stata 15.1 for our analysis.

As shown in Tables 7 and 8, since 0.1252 and 0.1424 are both greater than 0.05, we cannot reject the null hypothesis that "lagged GPA" does not cause "UE". However, 0.0094 and 0.0005 are both lower than 0.05, which implies that we can reject the null hypothesis that "lagged UE" does not cause "GPA". It means that we do accept the alternative hypothesis that "lagged UE" does cause "GPA", which is in agreement to the conclusions of the path analysis. Similar causality investigations can be performed with respect to the other directed dependencies outlined in Fig. 1.

3.4 Social interactions

To discuss the possible effects of social interactions upon the academic performance of students, we adhere to the paradigm that the outcomes associated with various sociological and behavioral processes chiefly depend on interactions between the individuals or populations involved [27]. For instance, several aspects of improving the proficiency in a foreign language rely on adequate interactions between those who have already mastered the language and those who have yet to do that. Specifically concerning our

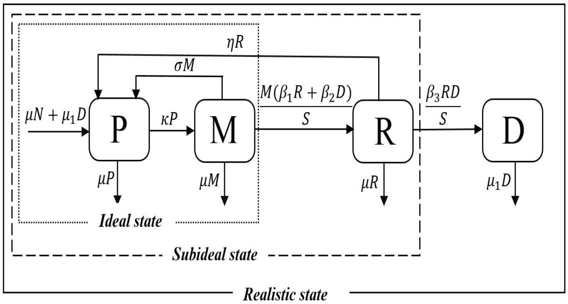

Figure 2. A schematic diagram for our model.

investigation, it is to be noted that, as peers gain importance in students' lives, they also gain influence over shaping their behaviors in positive or negative ways, including altering their attitudes towards pursuing academic achievements.

On a more general level, the dynamics of many sociological and behavioral processes are shaped by possibly nonlinear interactions between individuals or populations, which are in different states as far as these processes are concerned, these interactions often leading to changes of state (for instance, proficiency, habit or attitude) for the individuals or populations involved, which can be thought as transmission of state. This is the rationale behind using the compartmental models of Mathematical Epidemiology, where the main aim is to describe the spread of infectious diseases via suitable contacts, *in vivo* or within a given population, distinguishing between various infectious and noninfectious states, to describe the dynamics of such processes.

In our model, the total student population (S) is divided into two sub-populations: the enrolled undergraduate student population, whose size is denoted by $N(t)$ at any time t, and the dropout population, whose size is denoted by $D(t)$. The enrolled undergraduate student population is further divided into three compartments: students who pass all courses, whose size is $P(t)$, students who fail at least one course and have to take make-up examinations, whose size is $M(t)$ and students who fail make-up examinations and have to resit courses, whose size is $R(t)$. As shown in Fig. 2, newly enrolled students enter the pass compartment at a rate μ, $1/\mu$ representing the average number of years spent in college by an undergraduate student. The rate of movement from the pass population to the make-up population is denoted by κ, understood as being the rate at which students fail courses and subsequently attend make-up examinations.

Our model considers multiple mechanisms of (negative) social influence between make-up, resit and dropout students (typified by the terms $M(\beta_1 R + \beta_2 D)/S$) and between resit and dropout students (typified by the term $\beta_3 DR/S$). Make-up students re-enter the pass population at a rate σ upon passing all their make-up examinations, as a result of their high internal locus of control. Similarly, resit students re-enter the pass population at a rate η, upon successfully resitting all their failed courses, as a result of their self-efficacy.

Assuming that all students pass all examinations, ever, is highly unrealistic. Understandably, some of them will have to rely on make-up examinations, which they can be

Table 9. The meaning of the parameters appearing in model (1).

Parameter	Description
$1/\mu$	average number of years spent in college by a typical student
μ_1	dropout rate
κ	rate of movement from from the passing compartment to the make-up compartment
σ	rate of movement from the make-up compartment to the passing compartment
η	rate of movement from the resit compartment to the passing compartment
β_1	average negative influence of resit students on make-up students
β_2	average negative influence of dropout students on make-up students
β_3	average negative influence of dropout students on resit students

reasonably expected to pass. The state in which all students belong to the pass and make-up compartments will therefore be referred as the *ideal state*. Furthermore, the *subideal state* is meant to be the situation in which no student is expelled or drops out from the university before graduation. In a more realistic scenario, while there will be students passing all their examinations, others have to take make-up examinations, resit courses or even dropout. Hence, we refer to the state in which all compartment sizes are nonzero as the *realistic state*.

In view of the above-mentioned assumptions, our model can be stated as follows:

$$\begin{aligned}
\dot{P} &= \mu N + \mu_1 D + \eta R + \sigma M - \kappa P - \mu P, \\
\dot{M} &= \kappa P - \frac{M(\beta_1 R + \beta_2 D)}{S} - \sigma M - \mu M, \\
\dot{R} &= \frac{M(\beta_1 R + \beta_2 D)}{S} - \eta R - \frac{\beta_3 RD}{S} - \mu R, \\
\dot{D} &= \frac{\beta_3 RD}{S} - \mu_1 D.
\end{aligned} \tag{1}$$

The model will be subject to the initial conditions $P(0) = P_0 > 0$, $M(0) = M_0 > 0$, $R(0) = R_0 \geqslant 0$, $D(0) = D_0 \geqslant 0$. All model parameters summarized in Table 9 are assumed to be positive.

An investigation of high school student dropout, which accounts for parental involvement, demographic factors and peer pressure and categorizes students into passing, vulnerable and failing was done in [3], also using an SIR-inspired model. The primary focus of [3] is, however, on discussing the effects of parental involvement. The possible outcomes of negative social influences on the dynamics of student performance have also been investigated in [21] for a model, which keeps track of passing, resit and makeup students, with a view towards establishing results concerning the stability and bifurcation of equilibria as quantifiers. Due to their predictive power and to the possibility of drawing upon an extensive body of work, disease propagation models have also been successfully tailored to discuss other social processes such as spread of alcoholism abuse [6, 22, 27], dynamics of eating disorders [15], the curtailment of the smoking habit [28, 31] and the influence of correctional centers upon limiting gang violence [24].

Since the total population size is constant, we adimensionalize the state variables and then rewrite the system into proportions of the total student population, denoted with the corresponding small letters. We are therefore led to considering the following system:

$$\begin{aligned}
\dot{p} &= \mu + (\mu_1 - \mu)d + \eta r + \sigma m - \kappa p - \mu p,\\
\dot{m} &= \kappa p - m(\beta_1 r + \beta_2 d) - \sigma m - \mu m,\\
\dot{r} &= m(\beta_1 r + \beta_2 d) - \eta r - \beta_3 r d - \mu r,\\
\dot{d} &= \beta_3 r d - \mu_1 d
\end{aligned} \tag{2}$$

accompanied by the initial conditions $p(0) = p_0 > 0$, $m(0) = m_0 > 0$, $r(0) = r_0 \geqslant 0$, $d(0) = d_0 \geqslant 0$. We shall then be concerned with this system for the rest of our paper.

As previously mentioned, our model (2) is loosely based on the standard SIR (susceptible-infective-recovered) model of mathematical epidemiology. With a view to the standard techniques employed therein, particularly to the next generation method, we shall find in what follows two stability thresholds, not unlike the basic reproduction number, which govern the stability of the equilibria.

The well-posedness of system (2)

Let $s(t) = p(t) + m(t) + r(t) + d(t)$. We study the system (2) in the feasible region $\Gamma = \{(p(t), m(t), r(t), d(t)) \in (0,1)^2 \times [0,1)^2 \mid s(t) = 1\}$, which is positively invariant with respect to system (2). This means that our system is well posed there and all solutions of system (2) starting with (p_0, m_0, r_0, d_0) in Γ remain in Γ for all $t > 0$.

The stability of the ideal steady state

The model has a resit and dropout-free equilibrium (understood as the "ideal" equilibrium) given by

$$I_0 = (p_0, m_0, r_0, d_0) = \left(\frac{\mu + \sigma}{\kappa + \mu + \sigma}, \frac{\kappa}{\kappa + \mu + \sigma}, 0, 0\right).$$

To analyze the stability of the resit and dropout-free equilibrium, we shall introduce *ad hoc* an adequate threshold parameter, hereinafter called the resit reproduction number of the model and denoted by $\mathcal{R}_{0R}$, defined as the average number of make-up students that a single member of the resit group will influence to resit a course. By similarity with the basic reproduction number often employed in mathematical epidemiology and described in detail in [12], it is expected that $\mathcal{R}_{0R} < 1$ will describe a situation in which the resit student population will vanish in the long term, whereas $\mathcal{R}_{0R} > 1$ will imply that resit students will persist in the academic community.

The explicit expression of $\mathcal{R}_{0R}$ is obtained using the next generation matrix approach presented in [12]. Considerations similar to those in [12] using the make-up, resit and dropout compartments in place of the infective compartments give

$$F = \begin{pmatrix} -\beta_1 r - \beta_2 d & -\beta_1 m & -m\beta_2 \\ \beta_1 r + \beta_2 d & \beta_1 m & m\beta_2 \\ 0 & 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} \sigma + \mu & 0 & 0 \\ 0 & \beta_3 d + \eta + \mu & \beta_3 r \\ 0 & -\beta_3 d & -\beta_3 r + \mu_1 \end{pmatrix}.$$

The evaluation of FV^{-1} at the resit and dropout-free equilibrium I_0 gives

$$FV^{-1} = \begin{pmatrix} 0 & -\frac{\kappa\beta_1}{(\kappa+\sigma+\mu)(\eta+\mu)} & -\frac{\kappa\beta_2}{\mu_1(\kappa+\sigma+\mu)} \\ 0 & \frac{\kappa\beta_1}{(\kappa+\sigma+\mu)(\eta+\mu)} & \frac{\kappa\beta_2}{\mu_1(\kappa+\sigma+\mu)} \\ 0 & 0 & 0 \end{pmatrix},$$

its eigenvalues being $\lambda_1 = 0$, $\lambda_2 = 0$ and $\lambda_3 = \beta_1\kappa/((\kappa + \mu + \sigma)(\eta + \mu))$. Therefore, the resit reproduction number $\mathcal{R}_{0R}$ is given by $\mathcal{R}_{0R} = \beta_1\kappa/((\kappa + \mu + \sigma)(\eta + \mu))$. As expected, it is seen from a quick analysis of the eigenvalues that the resit reproduction number $\mathcal{R}_{0R}$ is a threshold parameter as far as the stability of the resit and dropout-free equilibrium I_0 is concerned.

Theorem 1. *The resit and dropout-free equilibrium I_0 is locally asymptotically stable if $\mathcal{R}_{0R} < 1$ and unstable if $\mathcal{R}_{0R} > 1$.*

Proof. In fact, the Jacobian matrix evaluated at I_0 is

$$J(I_0) = \begin{pmatrix} -\kappa-\mu & \sigma & \eta & \mu_1-\mu \\ \kappa & -\mu-\sigma & -\frac{\kappa\beta_1}{(\kappa+\mu+\sigma)} & -\frac{\kappa\beta_2}{(\kappa+\mu+\sigma)} \\ 0 & 0 & \frac{\kappa\beta_1}{(\kappa+\mu+\sigma)} - \eta - \mu & \frac{\kappa\beta_2}{(\kappa+\mu+\sigma)} \\ 0 & 0 & 0 & -\mu_1 \end{pmatrix}.$$

The eigenvalues of $J(I_0)$ are $\lambda_1 = -\mu_1$, $\lambda_2 = -\mu$, $\lambda_3 = -(\kappa + \mu + \sigma)$ and $\lambda_4 = (\kappa\beta_1-(\eta+\mu)(\kappa+\mu+\sigma))/(\kappa+\mu+\sigma)$. Here, λ_4 can be written as $\lambda_4 = (\eta+\mu)(\mathcal{R}_{0R}-1)$.

We note that if $\mathcal{R}_{0R} < 1$, then λ_4 is negative, which means that in this situation all eigenvalues are negative. One can conclude that the resit and dropout-free equilibrium I_0 is locally asymptotically stable if $\mathcal{R}_{0R} < 1$.

However, if $\mathcal{R}_{0R} > 1$, then $\lambda_4 > 0$ and consequently the resit and dropout-free equilibrium I_0 is unstable. This completes the proof. □

The stability of the subideal steady state

Model (2) has a dropout-free equilibrium (understood as the "subideal" equilibrium) given by

$$I_\vartheta = \left(\frac{(\mu+\eta)(\sigma+\beta_1-\eta)}{\beta_1(\mu+\kappa+\eta)}, \frac{\eta+\mu}{\beta_1}, \frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{\beta_1(\mu+\kappa+\eta)}, 0\right).$$

Obviously, $\mathcal{R}_{0R} > 1$ implies the existence of the dropout-free equilibrium. To investigate its stability, we shall perform an analysis, which is parallel to the one done above for the resit and dropout-free equilibrium.

The dropout reproduction number $\mathcal{R}_{0D}$ of the model is defined as the average number of resit students that a single member of the dropout group will influence to leave the university. By employing again the next generation method (this time using the dropout compartment only in place of the infective compartments), one has $F = (\beta_3 r - \mu_1)$,

$V = (\mu_1)$. Evaluating FV^{-1} for the dropout-free equilibrium I_ϑ gives the explicit form of the dropout-free reproductive number $\mathcal{R}_{0D}$ as

$$\mathcal{R}_{0D} = \frac{\beta_3(\kappa\beta_1 - (\eta+\mu)(\mu+\kappa+\sigma))}{\mu_1\beta_1(\eta+\kappa+\mu)} = \frac{\beta_3(\mu+\kappa+\sigma)(\eta+\mu)}{\mu_1\beta_1(\eta+\kappa+\mu)}(\mathcal{R}_{0R}-1).$$

Similarly, it is seen that the dropout reproduction number $\mathcal{R}_{0D}$ is a threshold parameter as far as the stability of the dropout-free equilibrium I_ϑ is concerned.

Theorem 2. *The dropout-free equilibrium I_ϑ is locally asymptotically stable if $\mathcal{R}_{0D} < 1$ and unstable if $\mathcal{R}_{0D} > 1$.*

Proof. The Jacobian matrix $J(I_\vartheta)$ evaluated at I_ϑ is

$$\begin{pmatrix} -\kappa-\mu & \sigma & \eta & \mu_1-\mu \\ \kappa & -\left(\frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{(\mu+\kappa+\eta)}\right)-\mu-\sigma & -(\eta+\mu) & -\frac{\beta_2(\eta+\mu)}{\beta_1} \\ 0 & \left(\frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{(\mu+\kappa+\eta)}\right) & 0 & \frac{\beta_2(\eta+\mu)}{\beta_1} - \frac{\beta_3(\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma)}{\beta_1(\mu+\kappa+\eta)} \\ 0 & 0 & 0 & \frac{\beta_3(\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma))}{\beta_1(\mu+\kappa+\eta)}-\mu_1 \end{pmatrix}.$$

One of the eigenvalues of this Jacobian matrix is

$$\lambda_1 = \frac{\beta_3(\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma))}{\beta_1(\mu+\kappa+\eta)} - \mu_1,$$

which can be expressed as $\lambda_1 = \mu_1(\mathcal{R}_{0D}-1)$. The remaining eigenvalues are the roots of the equation

$$\begin{vmatrix} (-\kappa-\mu)-\lambda & \sigma & \eta \\ \kappa & [-(\frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{(\mu+\kappa+\eta)})-\mu-\sigma]-\lambda & -(\eta+\mu) \\ 0 & (\frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{(\mu+\kappa+\eta)}) & -\lambda \end{vmatrix} = 0. \tag{3}$$

We denote

$$\begin{aligned} &A_1 = -(\kappa+\mu), \quad A_2 = \sigma, \quad A_3 = \eta, \quad A_4 = \kappa, \\ &A_5 = -\left(\frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{(\mu+\kappa+\eta)} + (\mu+\sigma)\right) = -\left(\frac{\mu_1\beta_1}{\beta_3}\mathcal{R}_{0D}+\mu+\sigma\right), \\ &A_6 = -(\eta+\mu), \\ &A_7 = \frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{(\mu+\kappa+\eta)} = \frac{\mu_1\beta_1}{\beta_3}\mathcal{R}_{0D}. \end{aligned}$$

Equation (3) can now be written as

$$\lambda^3 - (A_1+A_5)\lambda^2 - (A_2A_4+A_6A_7-A_1A_5)\lambda + A_3A_4A_7 - A_1A_6A_7 = 0$$

or, with the notations

$$B_1 = -(A_1 + A_5) = \frac{1}{\beta_3}\big(\mu_1\beta_1\mathcal{R}_{0D} + \beta_3(\kappa + 2\mu + \sigma)\big),$$
$$B_2 = -(A_2A_4 + A_6A_7 - A_1A_5) = \frac{1}{\beta_3}\big(\mu_1\beta_1(\eta + 2\mu + \kappa)\mathcal{R}_{0D} + \mu\beta_3(\kappa + \mu + \sigma)\big),$$
$$B_3 = A_1A_6A_7 - A_3A_4A_7 = \frac{1}{\beta_3}\big(\mu\mu_1\beta_1(\kappa + \eta + \mu)\mathcal{R}_{0D}\big),$$

as the cubic equation

$$\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0. \tag{4}$$

Let us denote

$$H_1 = (B_1), \qquad H_2 = \begin{pmatrix} B_1 & 1 \\ 0 & B_2 \end{pmatrix}, \qquad H_3 = \begin{pmatrix} B_1 & 1 & 0 \\ B_3 & B_2 & B_1 \\ 0 & 0 & B_3 \end{pmatrix}.$$

To apply the Routh–Hurwitz criterion to Eq. (4), we need to verify that $\det H_1 = B_1 > 0$, $\det H_2 = B_1B_2 > 0$, $\det H_3 = B_3(B_1B_2 - B_3) > 0$. To this purpose, note that

$$\begin{aligned}
\det H_1 &= \frac{1}{\beta_3}\big(\mu_1\beta_1\mathcal{R}_{0D} + \beta_3(\kappa + 2\mu + \sigma)\big) > 0,\\
\det H_2 &= \frac{1}{\beta_3^2}\big(\mu\beta_3(\kappa + \mu + \sigma) + \mu_1\beta_1\mathcal{R}_{0D}(\eta + 2\mu + \kappa)(\beta_3(\kappa + 2\mu + \sigma)\\
&\quad + \mu_1\beta_1\mathcal{R}_{0D}\big) > 0,\\
\det H_3 &= \frac{1}{\beta_3^3}\big(\mu\mu_1\beta_1\mathcal{R}_{0D}(\kappa + \eta + \mu)\big(\beta_3(\kappa + \mu + \sigma) + \mu_1\beta_1\mathcal{R}_{0D}\big)\\
&\quad \times \big(\mu\beta_3(\kappa + 2\mu + \sigma) + \mu_1\beta_1\mathcal{R}_{0D}(\eta + 2\mu + \kappa)\big)\big) > 0.
\end{aligned}$$

Since all determinants of the Hurwitz matrices are positive, all eigenvalues of the Jacobian $J(I_\vartheta)$ have negative real parts for $\mathcal{R}_{0D} < 1$. Therefore, the dropout-free equilibrium I_ϑ is stable provided that $\mathcal{R}_{0D} < 1$.

However, if $\mathcal{R}_{0D} > 1$, then $\lambda_1 > 0$, and, consequently, I_ϑ is unstable. This completes the proof. □

The existence of the realistic equilibrium (*RDPE*)

We now characterize the existence of the resit and dropout-persistent equilibrium (shortened from now on as RDPE and understood as the "realistic" equilibrium)

$$I^* = (p^*, m^*, r^*, d^*).$$

Note first that its coordinates need to satisfy the following equilibrium relations:

$$\begin{aligned}
&\mu + (\mu_1 - \mu)d^* + \eta r^* + \sigma m^* - \kappa p^* - \mu p^* = 0,\\
&\kappa p^* - m^*(\beta_1 r^* + \beta_2 d^*) - \sigma m^* - \mu m^* = 0,\\
&m^*(\beta_1 r^* + \beta_2 d^*) - \eta r^* - \beta_3 r^* d^* - \mu r^* = 0,\\
&\beta_3 r^* d^* - \mu_1 d^* = 0.
\end{aligned} \tag{5}$$

From the last equation of (5) one sees that

$$r^* = \frac{\mu_1}{\beta_3}. \tag{6}$$

Substituting (6) into the third and second equation of (5), respectively, leads to

$$m^* = \frac{\mu_1(\eta + \mu + \beta_3 d^*)}{\beta_1\mu_1 + \beta_2\beta_3 d^*} \tag{7}$$

and

$$p^* = \frac{\mu_1(\eta + \mu + \beta_3 d^*)(\beta_3(\sigma + \mu + \beta_2 d^*) + \beta_1\mu_1)}{\kappa\beta_3(\beta_1\mu_1 + \beta_2\beta_3 d^*)}. \tag{8}$$

Substituting (7) and (8) into the first equation of (5) leads to the following quadratic equation in d^*:

$$Ad^{*2} + Bd^* + C = 0$$

in which

$$\begin{aligned}
A &= \beta_2\beta_3^2(\kappa + \mu_1),\\
B &= \beta_3\big[\beta_1\mu_1(\kappa + \mu_1) + \beta_2\mu_1(\mu + \eta + \kappa) + \beta_3\mu_1(\mu + \kappa + \sigma) - \kappa\beta_2\beta_3\big],\\
C &= \mu_1^2\beta_1(\mu + \eta + \kappa)(1 - \mathcal{R}_{0D}).
\end{aligned}$$

For the RDPE to exist, the solutions of (5) must be real and positive. One notes that

(i) $A > 0$.
(ii) $C < 0$ if $\mathcal{R}_{0D} > 1$, and $C > 0$ if $\mathcal{R}_{0D} < 1$.

Let us denote

$$\begin{aligned}
\mathcal{R}_{0D}^* &= 1 - \frac{\big(\beta_1\mu_1(\kappa + \mu_1) + \beta_2\mu_1(\mu + \eta + \kappa) + \beta_3\mu_1(\mu + \kappa + \sigma) - \kappa\beta_2\beta_3\big)^2}{4\mu_1^2\beta_1\beta_2(\kappa + \mu_1)(\mu + \eta + \kappa)},\\
D &= \kappa\beta_1 + \beta_3(\mu + \kappa + \sigma) + \beta_2(\mu + \eta + \kappa),\\
\mu_1^* &= \frac{-D + \sqrt{D^2 + 4\kappa\beta_1\beta_2\beta_3}}{2\beta_1}
\end{aligned}$$

and observe that, whenever the RDPE I^* exists, its coordinate r^* is given by (6), while the coordinates m^* and p^* can de obtained from d^* via the use of (7) and (8), respectively. We are now in position to fully characterize the existence of the RDPE of the model (2).

Theorem 3. *The following statements regarding the existence and multiplicity of the RDPE of model* (2) *hold*:

(i) *If* $\mathcal{R}_{0D} > 1$, *then there is a unique RDPE of* (2) *given by* $I^* = (p^*, m^*, r^*, d^*)$, *where* $d^* = (-B + \sqrt{B^2 - 4AC})/(2A)$.
(ii) *If* $\mathcal{R}_{0D} = 1$ *and* $\mu_1 < \mu_1^*$, *then there is a unique RDPE of* (2) *given by* $I^* = (p^*, m^*, r^*, d^*)$, *where* $d^* = -B/A$.

(iii) *If* $\mathcal{R}_{0D} = \mathcal{R}_{0D}^*$ *and* $\mu_1 < \mu_1^*$*, then there is a unique RDPE of* (2) *given by* $I^* = (p^*, m^*, r^*, d^*)$*, where* $d^* = -B/(2A)$*.*

(iv) *If* $\mathcal{R}_{0D}^* < \mathcal{R}_{0D} < 1$ *and* $\mu_1 < \mu_1^*$*, then there are two RDPEs of* (2) *given by* $I_1^* = (p_1^*, m_1^*, r_1^*, d_1^*)$ *and* $I_1^* = (p_2^*, m_2^*, r_2^*, d_2^*)$*, where* $d_{1,2}^* = (-B \pm \sqrt{B^2 - 4AC})/(2A)$*.*

(v) *If* $\mathcal{R}_{0D} \leqslant 1$ *and* $\mu_1 \geqslant \mu_1^*$*, then there is no RDPE of* (2)*.*

(vi) *If* $\mathcal{R}_{0D} < \mathcal{R}_{0D}^*$*, then there is no RDPE of* (2)*.*

4 Discussion

The findings of this study provide empirical evidence for the existence of differences between the performance of male and female students in College English courses. In that regard, female students performed better than male students. This is consistent with [29], which shows that girls tend to progress faster than boys in language learning since they engage more in communication and social interactions. No significant differences were found between the performance of students from rural and town localities. This means that student performance in English entrance examination is weakly affected by locality and, from that viewpoint, the gap between urban and rural regions has been narrowing.

Interestingly, there was no steady improvement in the performance of students in College English courses compared to their entrance performance. Student performance in College English courses has not improved significantly, despite the fact that they were taking Mathematics courses taught in English with the prospect of travelling abroad. This agrees with the finding of other studies, which indicates that study abroad programs do not necessarily help students achieve greater language proficiency gains [5, 7, 18]. Therefore, there is a need to introduce innovative ways of helping students to be proficient in English as part of the cooperative program.

In accordance with the findings of other studies [23, 25, 26], it has been found that the performance of students in English has a significant positive correlation with their GPA in Mathematics courses in both first and second year. Additionally, the path analysis revealed a strong direct effect of students' performance in English on their GPA contributing to 56% and 70% of the variances in students GPA in first year and second year, respectively. This underpins the importance of English proficiency for students attending English taught courses. The average performance of students in first year English is directly affected by their gender, entrance English score and locality. These factors contributes 18% of the variances in students first year English performance. This means that students' demographics and entry qualifications have a consistent effect on their university performance. However, since entrance English score only correlated with first year English score, it can be concluded that entry qualification only relates to student performance at the university in first year but does not necessarily shows how students will perform as they progress in the university. We also found a direct effect of students GPA scores in first year on their GPA in second year, likewise on their English performance, which reinforces the importance of students' past performance and experiences on their future performance.

Finally, we developed a bare-bones mathematical model, which investigates the effects of negative social influences on the academic performance of students. By similarity to the basic reproduction number of mathematical epidemiology, we defined the resit reproduction number of the model and denoted by $\mathcal{R}_{0R}$, defined as the average number of make-up students that a single member of the resit group will influence to resit a course. Similarly, the dropout reproduction number $\mathcal{R}_{0D}$ of the model is defined as the average number of resit students that a single member of the dropout group will influence to leave the university. Both reproduction numbers were determined to be stability triggers. In this setting, an effective way of eliminating resit is to adopt new strategies for motivating resit students to study harder and attend lectures.

5 Conclusion and implications

The findings of the present study have implications for assessing and improving the English proficiency and academic performance of Chinese undergraduate students. Gender differences are determined to exist in the English proficiency of students, further measures being needed to improve the English proficiency of male students. Since no steady improvement in was found in the performance of students in College English courses, it can be inferred that they do not have much access or chance to speak or use English. To learn English effectively, students should be encouraged to interact and communicate with foreign students on campus. Interacting with competent speakers of English, regardless of whether they are native or nonnative, is a prerequisite for developing high proficiency in English [30]. We therefore recommend that collaborative social groups be formed between Chinese and foreign students to help them develop English proficiency skills, which include speaking, listening, reading and writing.

This study has certain limitations, which must be recognized when drawing conclusions. Cause and effect links cannot be asserted in this study due to its design. Since the data was collected from a specific cohort of students in a particular cooperative program, the research outcomes may differ for different cohorts. Consequently, the findings of this study are context specific. Further studies are therefore required in order to investigate the extent of difference in English proficiency and performance of students who travel abroad compared with students who remain in China.

As observed by one of the referees, the findings of this paper, apart from being tied to the particulars of this cooperative program, may also be discipline-specific. The study of language issues in relation to other types of programs and courses (in international relations, business, economics, political sciences, psychology, for instance) can presumably lead to different conclusions as the different circumstances of those courses can be more encouraging to learn a foreign language, mathematics being by contrast perhaps less communication-reliant and more technique-reliant. Further, the effects of peer pressure (and generally of social selection dynamics) can also be modeled via game theory (see, for instance, [4] and the references therein) rather than via "pure" ODE modelling. However, we have not pursued this avenue here.

Also, the influence of teachers' knowledge of English is not investigated here. It would be interesting to see whether or not the students who have had professors with good linguistic skills (for instance, who studied or stayed for other academic reasons abroad) would perform better. However, our paper is concerned only with student-related matters and variables, and such an investigation is beyond the scope of our manuscript.

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