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Ranks, Subdegrees and Suborbital graphs of the product action of Affine Groups

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Abstract

The action of affine groups on Galois field has been studied. For instance, [3] studied the action of $Aff\ (q\ )$ on Galois field $GF\ (q)$ for q a power of prime p. In this paper, the rank and subdegree of the direct product of affine groups over Galois field acting on the cartesian product of Galois field is determined. The application of the definition of the product action is used to achieve this. The ranks and subdegrees are used in determination of suborbital graph, the non-trivial suborbital graphs that correspond to this action have been constructed using Sims procedure and were found to have a girth of 0, 3, 4 and 6.

Keywords: Ranks, Subdegrees, Transitivity and Suborbital graphs

1. Introduction

Affine group $Aff\ (q\ )$ over Galois field $GF\ (q)$ is a group of all transformations of the form $ax+b$, where $a, b\ \in GF(q)$ and $a \neq 0$, these elements can be viewed as $\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$. Let a group G act transitively on a set Ω and G_a the stabilizer of a in G for $a \in \Omega$, then $Orb_G(a)$ is the Orbit of a in G. The G_a-orbits are called suborbits of G denoted by Δ_i and the number of them is the rank while the length of the suborbits is called the subdegrees of G on Ω .

The concept of suborbital graphs of non-trivial suborbits of a permutation group G was introduced by Sims. [2] Investigated some properties of the action of the stabilizer of ∞ in Γ(modulargroup) acting on the set of integers is transitive and imprimitive. Moreover, if $x, y\ \in Z$ with $x\ \neq\ y$ then the suborbital $O(o, x)$ and $O(o, y)$ are disjoint. The suborbital graph $\Gamma(0, x)$ has $|x|$ components and it is paired with $\Gamma(x, 0)$. [4] studied the action of $PSL(2, q)$ on the cosets of its cyclic subgroup $C_{\frac{(q-1)}{k}}$, where $k = (q-1,2)$. The action was found to be imprimitive and the number of the self-paired suborbital is $q\ +\ 2, q\ +\ 3$ and $q\ +\ 1$, for $p\ =\ 2, q\ \cong\ 1mod4$ and $q\ \cong\ 1mod4$ respectively.

2. Preliminary Notes

Definitions 2.1 A group G is said to be transitive on a set Ω if for each pair of points $a, b \in \Omega$, there correspond $g \in G$ such that $ga = b$. In other words the action is transitive if it has only one orbit.

Definition 2.2 Let G act transitively on Ω. Then G also acts on $\Omega \times \Omega$ by, $g(\alpha, \gamma)\ =\ (g\alpha, g\gamma), g\ \in\ G_\alpha, \gamma \in \Omega$. The G_a- orbits resulting from this action are referred to as suborbital of G on Ω denoted by $O(\alpha, \gamma)$.

Definition 2.3 A graph $\Gamma(V, E)$ is said to be bipartite if V can be partitioned into two subsets V_1 and V_2 such that the edges join the two vertices from different subsets and no edge joining vertices in the same subsets.

Theorem 2.3 [2] Let $G_1, G_2, . . . , G_n$ be groups with respect to the binary operations $(\star_1, \star_2, . . . , \star_n\ )$. Then the external direct product $G_1\ \times\ G_2\ \times\ . . .\ \times G_n$ is defined as:

$G_1 \times G_2 \times . . . \times G_n = \{(g_1, g_2, g_3, \ldots, g_n) :\ g_i \in G_i\}$ and $(g_1, g_2, \ldots, g_n) \star (h_1, h_2, \ldots, h_n) =$

$(g_1 \star_1 h_1, g_2 \star_2 h_2, \ldots, g_n \star_n h_n)$. The order of the group $G_1 \times G_2 \times G_3, \ldots, \times G_n$ is given by $|G_1||G_2||G_3| . . . |G_n|$. Suppose G_1 acts on Ω_1, G_2 acts on Ω_2 and G_3 acts on Ω_3. Then the product action of $G_1 \times G_2 \times G_3$ on $\Omega_1\ \times\ \Omega_2\ \times\ \Omega_3$ is given by the rule,

$\{(\ g_1,\ g_2,\ g_3\ )\ (a, b, c)\ =\ (g_1(a), g_2(b), g_2(c)).\}$

Theorem 2.4 [1] The $G_1 \times G_2 \times G_3 -$ orbit containing $(a, b, c,) \in\ \Omega_1\ \times\ \Omega_2\ \times\ \Omega_3$ is given by $Orb_{G_1}(a) \times Orb_{G_2}(b) \times Orb_{G_3}(c)$ and the stabilizer of (a, b, c) in $G_1 \times G_2 \times G_3$ is given by $Stab_{G_1}(a) \times\ Stab_{G_2}(b) \times Stab_{G_3}(c)$.

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Theorem 2.5

Let G be group acting on set Ω. Then,

$|Orb_G(a)| = \frac{|G|}{|G_a|}$, for all $a \in \Omega$

Theorem 2.6 [3] The suborbits of G are of the form $\Delta_0 = \{0\}$, $\Delta_1 = \{a \in \Omega | a \neq 0\}$.

Corollary 2.7 [3] Let G act on Ω. The subdegrees are of the form 1 and $(q-1)$; thus the rank of G is two.

Theorem 2.8 Let G act transitively on Ω and let suborbit $\Delta_i, (i = 0, 1, \ldots, s-1)$ correspond to suborbital O_i. Then the suborbital graph Γ_i is undirected if Δ_i is self-paired and directed if Δ_i is not self-paired.

Theorem 2.9 A transitive action is said to be primitive if and only if each of the non-trivial suborbital graph is connected.

Theorem 2.10 The chromatic number of a bipartite graph is 2.

Theorem 2.11 Let Γ be a connected undirected graph. Then Γ is Eurelian if and only if every vertex is of even degree.

Theorem 2.12 [5] Let Γ be a suborbital graph of a transitive action. Then all disconnected components of Γ are isomorphic.

3. Main Results

Lemma 3.1 Let the group $G = Aff(q)$ act on the set $\Omega = (0, \alpha, \alpha^2, \alpha^3, \ldots, \alpha^{(q_1-1)})$. Then, $Stab_G(0)$ is a cyclic group $C_{(q-1)}$.

Proof.

$$\text{Let } Stab_G(0) = \{ax + b : a(0) + b = 0, a \neq 0, a, b \in G(q)\}$$
$$= \{(ax + b : b = 0, a \neq 0), \forall a, b \in G(q)\}$$
$$= \{(ax : a \in GF(q)^*)\} = C_{(q-1)}. \tag{2.1}$$

This can also be proved by a method applied by [3].

Proposition 3.2 Let $Aff(q_1) \times Aff(q_2)$ acts on $\Omega = \Omega_1 \times \Omega_2 = GF(q_1) \times GF(q_2)$. Then, $Stab_G(0,0)$ is isomorphic to $C_{(q_1-1)} \times C_{(q_2-1)}$.

Proof. By Lemma 3.1, $Stab_{G_1}(0)$ and $Stab_{G_2}(0)$ are cyclic groups $C_{(q_1-1)}$ and $C_{(q_2-1)}$. It follows from theorem [1] that,

$$Stab_G(0) = Stab_{G_1}(0) \times Stab_{G_2}(0) = C_{(q_1-1)} \times C_{(q_2-1)}. \tag{2.2}$$

Theorem 3.3 Suppose that $G = G_1 \times G_2 = Aff(q_1) \times Aff(q_2)$ acts on $\Omega = \Omega_1 \times \Omega_2 = GF(q_1) \times GF(q_2)$. Then the rank is 4 and subdegree are 1, $(q_1 - 1)$, $(q_2 - 1)$ and $(q_1 - 1)(q_2 - 1)$.

Proof. Using Theorem[1] we have $Stab_G(0,0) = Stab_{G_1}(0) \times Stab_{G_2}(0) = (q_1 - 1) \times (q_2 - 1)$. By Theorem 2.5, we have,

$|Orb_G(0,0)| = \frac{|G|}{|Stab_G(0,0)|} = \frac{q_1 (q_1-1) \times q_2(q_2-1).}{= (q_1-1) \times (q_2-1).} = q_1 \times q_2. = \Omega$. Thus transitive.

Suppose that A = $Stab_{G_1}(0)$, $B = Stab_{G_2}(0)$, α be a primitive element in Ω_1 and β be a primitive element in Ω_2. Then A −orbits in Ω_1 are $\{0\}$ and $\{\alpha\}$ while B −orbits in Ω_2 are $\{0\}$ and $\{\beta\}$, using Theorem [5] the suborbits of G on Ω are:

$$\Delta_0 = Orb_G(0,0) = \{0\} \times \{0\} = \{(0,0)\}. \tag{3.1}$$

Therefore, $|\Delta_0| = 1$.

$\Delta_1 = Orb_G(0,\beta) = \{0\} \times \{\beta, \beta^2, \beta^3, \ldots, \beta^{(q_2-1)}\}$

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$$= \{(0,\beta),(0,\beta^2),(0,\beta^3),\ldots,(0,\beta^{(q_2-1)}\}. \quad (3.2)$$

Therefore, $|\Delta_1| = (q_2-1)$.

$$\Delta_2 = Orb_G(\alpha,0) = \{\alpha, \alpha^2, \alpha^3, \ldots, \alpha^{(q_1-1)})\} \times \{0\}$$

$$= \{(\alpha,0),(\alpha^2,0),(\alpha^3,0), \ldots, (\alpha^{(q_1-1)},0)\}. \quad (3.3)$$

Therefore, $|\Delta_2| = (q_1-1)$.

$$\Delta_3 = Orb_G(\alpha,\beta) = \{\alpha, \alpha^2, \alpha^3, \ldots, \alpha^{(q_1-1)})\} \times \{\beta, \beta^2, \beta^3, \ldots, \beta^{(q_2-1)})\} \quad (3.4)$$

Therefore, $|\Delta_3| = (q_1-1)(q_2-1)$.

Hence rank is 4 and subdegree are $1, (q_1-1), (q_2-1)$ and $(q_1-1)(q_2-1)$.

Suppose that $Stab_{G_1\times G_2\times G3}(a,b,c)$ acts on $\Omega_1 \times \Omega_2 \times \Omega_3$ and Δ is an orbit of $Stab_{G_1\times G_2\times G3}(a,b,c)$ on $\Omega_1 \times \Omega_2 \times \Omega_3$. The suborbital O corresponding to Δ is given by,

$$O = \{(g1,g2,g3)(a,b,c),(g1,g2,g3)(\alpha,\beta,\gamma) : (\alpha,\beta,\gamma) \in \Delta, (g1,g2,g3) \in G_1 \times G_2 \times G_3\}$$

$$= \{(g1(a),g2(b),g3(c)),(g1(\alpha),g2(\beta),g3(\gamma)) : (\alpha,\beta,\gamma) \in \Delta, (g1,g2,g3) \in G_1 \times G_2 \times G_3\}. \quad (3.5)$$

The suborbital graph Γ corresponding to suborbital O is formed by taking $\Omega_1 \times \Omega_2 \times \Omega_3$ as the vertex set and directed edges from (p,q,r) to (s,t,u), if and only if $((p,q,r),(s,t,u)) \in O$. The suborbital graph corresponding to Δ_0 is the null graph.

The construction of suborbital graphs corresponding to the action of $Aff(q_1) \times Aff(q_2)$ on $GF(q_1) \times GF(q_2)$ is as follows:

By Equations (3.1), (3.2), (3.3) and (3.4), it follows that $(0,0) \in \Delta_0, (0,\beta) \in \Delta_1, (\alpha,0) \in \Delta_2$ and $(\alpha,\beta) \in \Delta_3$. By application of Equation (3.5), we obtain,

$$O_1 = \{((g1,g2)(0,0),(g1,g2)(0,\beta) : g1,g2 \in G \times G)\} = \{((g1(0),g2(0)),(g1(0),g2(\beta))\}$$

$$= \{((p,q),(s,t)) : p = s, q \neq t\}. \quad (3.6)$$

$$O_2 = \{((g1,g2)(0,0),(g1,g2)(\alpha,0))\} = \{((g1(0),g2(0)),(g1(\alpha),g2(0)))\}$$

$$= \{((p,q),(s,t)) : p \neq s, q = t\}. \quad (3.7)$$

$$O_3 = \{((g1,g2)(0,0),(g1,g2)(\alpha,\beta))\} = \{((g1(0),g2(0)),(g1(\alpha),g2(\beta)))\}$$

$$= \{((p,q),(s,t)) : p \neq s, q \neq t\}. \quad (3.8)$$

Theorem 3.4 All the non-trivial graphs corresponding to this action are undirected.

Proof

By Equations (3.6) to (3.8), if $((p,q),(s,t)) \in O_i$, $(i = 1,2,3)$ then $(s,t),(p,q)) \in O_i$.

Theorem 3.5 The suborbital graph Γ_3 is Eulerian if q is odd.

Proof.

If q is odd, $|\Delta_3| = q-1$ and therefore by Theorem 2.11, Γ_3 is Eulerian.

Lemma 3.6 Let $Comp_{\Gamma_i}(0,0), (i = 1,2,3)$ be the set of all vertices in the component containing $(0,0)$ in the suborbital graph Γ_i. Then,

$$Comp_{\Gamma_1}(0,0) = \{(0,x) : x \in GF(q_2)\}, \quad (3.9)$$

$$Comp_{\Gamma_2}(0,0) = \{(x,0) : x \in GF(q_1)\} \text{ and} \quad (3.10)$$

$$Comp_{\Gamma_3}(0,0) = \{GF(q_1) \times GF(q_2)\}. \quad (3.11)$$

Proof

Let $(a,b) \in Comp\Gamma_1\,(0,0)$. Then there exist a path$(0,0) \to (a_1,b_1) \to \ldots \to (a_i - 1, b_i - 1) = (a,b)$. It follows from Equation (3.6) that, $0 = a_1 = a_2 = \ldots = a$. Therefore,

$(a,b) \in \{(0,x) : x \in GF(q_2)\}$. This implies that,

$$\{Comp_{\Gamma_1}(0,0) \subseteq (0,x) : x \in GF(q_2)\} \tag{3.12}$$

Let $(0,b) \in \{(0,x) : x \in GF(q_2)\}$. *If* $b = 0$, then $(0,b) = (0,0) \in Comp_{\Gamma_1}(0,0)$. If $b \neq 0$, then by Equation (3.6) there exist an edge $(0,0) \to (0,b) \in \Gamma_1$. Therefore, $(0,b) \in Comp_{\Gamma_1}(0,0)$.This implies that, $\{(0,x) : x \in GF(q_2) \subseteq Comp_{\Gamma_1}(0,0)\}$. (3.13)

From Equations (3.12) and (3.13) we get Equation(3.9). Equation (3.10) is proved in a similar way as Equation(3.9).

Let $(x,y) \in GF(q_1) \times GF(q_2)$. Consider the vertex (a,b), where $x \neq a \in GF(q)^{*}\setminus\{a\}$ and $y \neq b \in GF(q)^{*}\setminus\{b\}$. By Equation (3.8), (a,b) is adjacent to both $(0,0)$ and (x,y). It follows that $(x,y) \in Comp_{\Gamma_3}(0,0)$, hence $GF(q_1) \times GF(q_2) \subseteq Comp_{\Gamma_3}(0,0)$. But $GF(q_1) \times GF(q_2)$ is the set of all the vertices in Γ_3, thus $Comp_{\Gamma_3}(0,0) \subseteq GF(q_1) \times GF(q_2)$. Thus we get Equation (3.12).

Theorem 3.7 The graphs Γ_1, Γ_2 and Γ_3 has q_1 , q_2, and 1 components respectively, where $q_1, q_2 > 2$.

Proof.

By Theorem [5], all the components in a suborbital graph are isomorphic. Applying Lemma 3.6, $|\,Comp_{\Gamma_1}(0,0)\,| = q_1, |\,Comp_{\Gamma_2}(0,0)\,| = q_2, |\,Comp_{\Gamma_3}(0,0)\,| = q_1q_2$. Since each of the graphs has q_1q_2 vertices, there are q_1 components for Γ_1, q_2 components for Γ_2 and 1 component for Γ_3.

Lemma 3.8 For the action of $Aff(2) \times Aff(q_2)$ on $GF(2) \times GF(q_2)$, where $q_2 > 3$, Γ_3 is bipartite.

Proof.

Let $V\,(\Gamma_3)$ be the vertices corresponding to Γ_3, then the vertices of G can be partitioned into two subsets V_1 and V_2 such that, $V_1 = \{(0,b) : b \in GF(q_2)\}$ and $V_2 = \{(1,b) : b \in GF(q_2)\}$. Thus from Equation (3.8), the edges move from a vertex in V_1 to a vertex in V_2 and no edge in Γ_3 joining vertices in the same subset. Hence from Definition 2.3, Γ_3 is bipartite.

Corollary 3.9 The chromatic number of a connected suborbital graph Γ_3 of $Aff(2) \times Aff(q_2)$ acting on $GF(2) \times GF(q_2)$, where $q_2 \geq 3$ is 2.

Proof.

By Lemma 3.8, Γ_3 is bipartite and using Theorem 2.10, chromatic number of a bipartite graph is 2.

Theorem 3.10 The suborbital graph Γ_3 corresponding to the action *of* $Aff(2) \times Aff(q_2)$ on $GF(2) \times GF(q_2)$, where $q_2 > 3$, has a girth of either 4 or 6.

Proof.

Let Γ_3 be bipartite and a be a primitive element in $GF(2)$, If $q_2 > 3$, $(0,0) \to (1,a) \to (0,a^2) \to (1,a^3) \to (0,0)$ and $(0,0) \to (1,a^2) \to (0,a^2) \to (1,a^3) \to (0,a^3) \to (1,a^4) \to (0,0)$, are all circuits of even length by Equation(3.8). Thus the girth is 4 and 6 respectively since 4 and 6 are the smallest length of a cycle respectively. Note that if $q_2 = 3$, the results is as shown in Fig 3.6

Theorem 3.11 The suborbital graph Γ_1 and Γ_2 corresponding to the action of $Aff(q_1) \times Aff(q_2)$ on $GF(q_1) \times GF(q_2)$, has a girth of 0 when $q_1 = q_2 = 2$ and a girth of 3 when $q_1 = q_2 > 2$.

Proof.

Note that when $q_1 = q_2 = 2$,the results follows from Fig.1, Fig 2 and Fig 3. If $q_1 = q_2 > 2$,

$(0,0) \to (0,b) \to (0,b^2) \to (0,0) \in \Gamma_1$,

$(0,0) \to (a,0) \to (a^2,0) \to (0,0) \in \Gamma_2$,

$(0,0) \to (\alpha,\beta) \to (\alpha^2,\beta^2) \to (0,0) \in \Gamma_3$, by Equation (3.6) to (3.8). Thus the girth is *3* since *3* is the smallest length of a cycle.

Corollary 3.12 The action of $Aff(q_1) \times Aff(q_2)$ on $GF(q_1) \times GF(q_2)$ is imprimitive.

Proof.

From the Theorem *3.7*, there are some non-trivial suborbital graphs which are disconnected. Thus from Theorem 2.9 the action is imprimitive.

Example 3.1 Let $G = Aff(2) \times Aff(2)$ act on $\Omega = GF(2) \times GF(2)$. The suborbital O_1, O_2 and O_3 corresponding to the suborbit Δ_1, Δ_2 and Δ_3 is,

$$O_1 = \{((g_1,g_2)(0,0),(g_1,g_2)(0,1)) : (g_1,g_2) \in G_1 \times G_2\}$$
$$= \{((g_1(0),g_2(0)),(g_1(0),g_2(1)) : (g_1,g_2) \in G_1 \times G_2\}$$
$$= (p,q),(s,t) : p = s, q \neq t). \tag{3.13}$$

$$O_2 = \{((g_1,g_2)(0,0),(g_1,g_2)(1,0)) : (g_1,g_2) \in G_1 \times G_2\}$$
$$= \{((g_1(0),g_2(0)),(g_1(1),g_2(0)) : (g_1,g_2) \in G_1 \times G_2\}$$
$$= (p,q),(s,t): p \neq s, q = t). \tag{3.14}$$

$$O_3 = \{((g_1,g_2)(0,0),(g_1,g_2)(1,1)) : (g_1,g_2) \in G_1 \times G_2\}$$
$$= \{((g_1(0),g_2(0)),(g_1(1),g_2(1)) : (g_1,g_2) \in G_1 \times G_2\}$$
$$= (p,q),(s,t) : p \neq s, q \neq t). \tag{3.15}$$

Using Equations (*3.13*), (*3.14*) and (*3.15*), the suborbital graphs Γ_1, Γ_2 and $\Gamma_3$will be constructed as in Fig. *1*, Fig. *2* and Fig. *3* respectively.

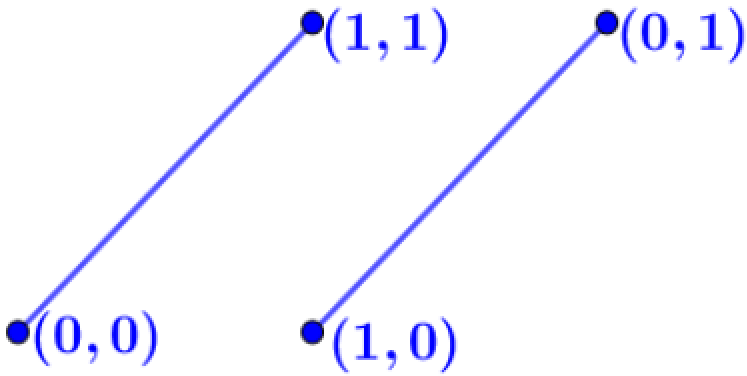

Figure *3.1*: Suborbital graph Γ_1 corresponding to the action of $Aff(2) \times Aff(2)$ on $GF(2) \times GF(2)$

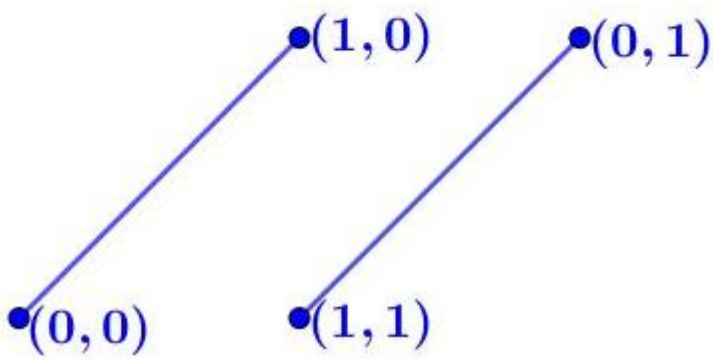

Figure *3.2*: Suborbital graph Γ_2 corresponding to the action of $Aff(2) \times Aff(2)$ on $GF(2) \times GF(2)$

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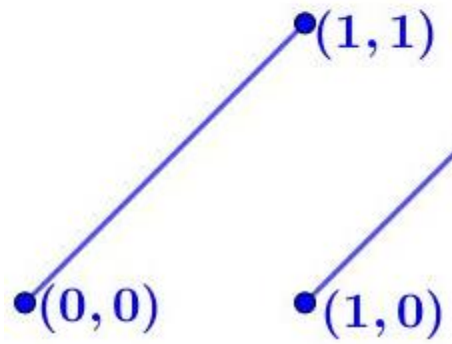

Figure 3.3: Suborbital graph Γ_3 corresponding to the action of $Aff(2) \times Aff(2)$ on $GF(2) \times GF(2)$

The suborbital graph Γ_1, Γ_2 and Γ_3 are all undirected, regular of degree 1, has a girth of 0 and they are disconnected each with 2 components and each component has diameter 1

Example 3.2 Let $G = Aff(2) \times Aff(3)$ act on $\Omega = GF(2) \times GF(3)$. The suborbital O_1, O_2 and O_3 corresponding to the suborbit Δ_1, Δ_2 and Δ_3 is,

$$O_1 = \{((g_1, g_2)(0,0), (g_1, g_2)(0,1)) : (g_1, g_2) \in G_1 \times G_2\}$$
$$= \{((g_1(0), g_2(0)), (g_1(0), g_2(1)) : (g_1, g_2) \in G_1 \times G_2\}$$
$$= (p,q),(s,t) : p = s, q \neq t). \tag{3.16}$$

$$O_2 = \{((g_1, g_2)(0,0), (g_1, g_2)(1,0)) : (g_1, g_2) \in G_1 \times G_2\}$$
$$= \{((g_1(0), g_2(0)), (g_1(1), g_2(0)) : (g_1, g_2) \in G_1 \times G_2\}$$
$$= (p,q),(s,t): p \neq s, q = t). \tag{3.17}$$

$$O_3 = \{((g_1, g_2)(0,0), (g_1, g_2)(1,1)) : (g_1, g_2) \in G_1 \times G_2\}$$
$$= \{((g_1(0), g_2(0)), (g_1(1), g_2(1)) : (g_1, g_2) \in G_1 \times G_2\}$$
$$= (p,q),(s,t) : p \neq s, q \neq t). \tag{3.18}$$

Using Equations (3.16), (3.17) and (3.18), the suborbital graphs Γ_1, Γ_2 and Γ_3 will be constructed as in Fig. 4, Fig. 5 and Fig. 6 respectively.

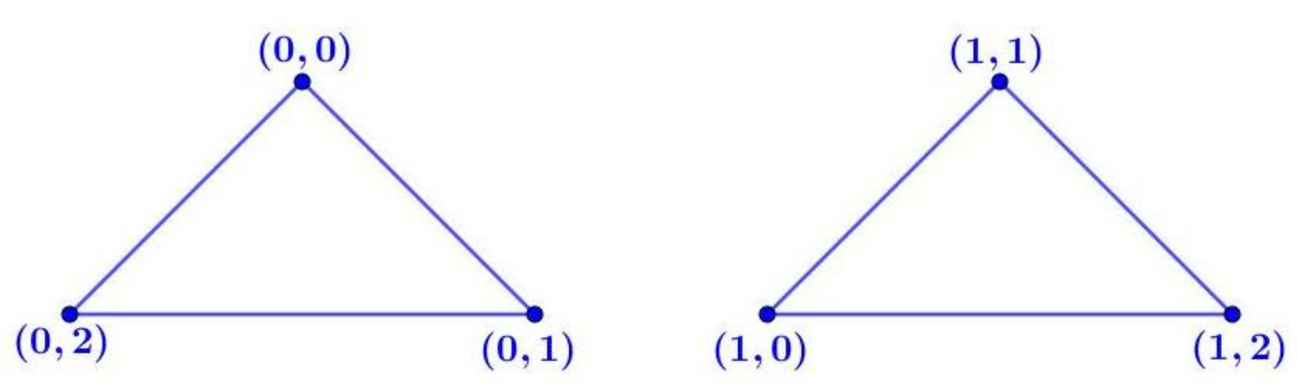

Figure 3.4: Suborbital graph Γ_1 corresponding to the action of $Aff(2) \times Aff(3)$ on $GF(2) \times GF(3)$

The suborbital graph Γ_1 is undirected, regular of degree 2, has a girth of 3 and it is disconnected. It has 2 connected components each with a diameter of 1.

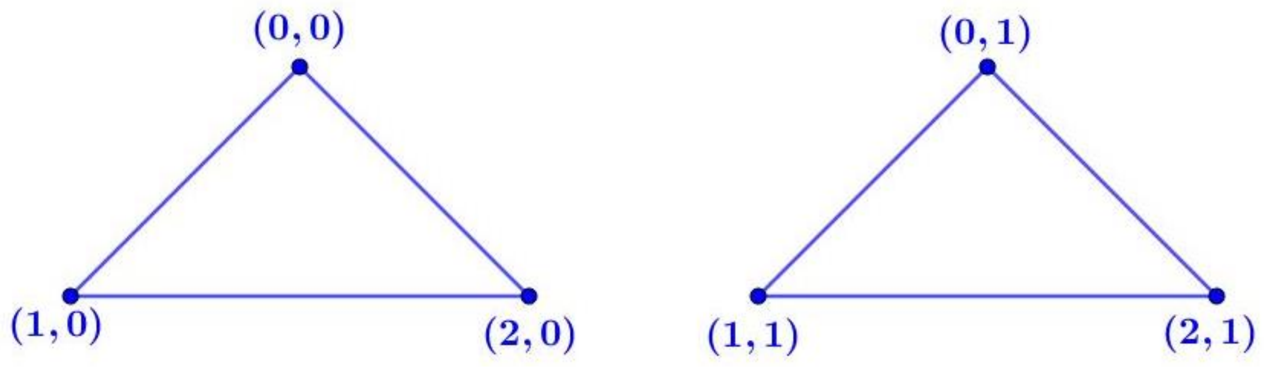

Figure 3.5: Suborbital graph Γ_2 corresponding to the action of $Aff(2) \times Aff(3)$ on $GF(2) \times GF(3)$

The suborbital graph Γ_1 is undirected, regular of degree *2*, has a girth of *3* and it is disconnected. It has *2* connected components each with a diameter of *1*.

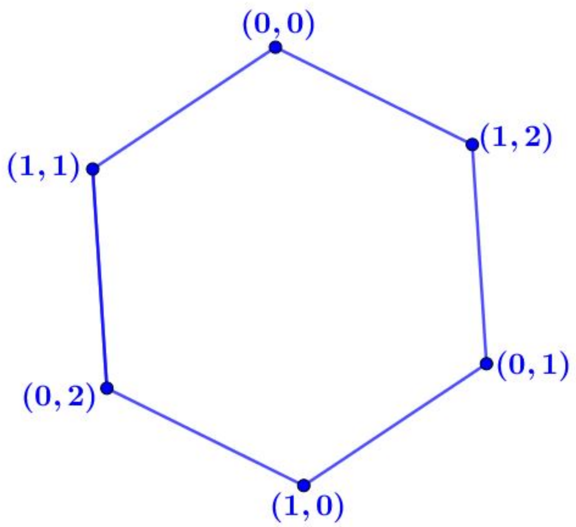

Figure 3.6: Suborbital graph Γ_3 corresponding to the action of $Aff(2) \times Aff(3)$ on $GF(2) \times GF(3)$

The suborbital graph Γ_3 is undirected, connected, regular of degree *2* hence Eurelian, has a girth of *6* and a diameter of *2*.

Conclusion

The subdegrees of the product action of affine groups $Aff\ (q_1) \times Aff\ (q_2)$ acting on $GF\ (q_1) \times GF\ (q_2)$.

is $1, (q_1 - 1), (q_2 - 1)$ and $(q_1 - 1)(q_2 - 1)$. The number of elements in all the four G −orbit is,

$1 + (q_1 - 1) + (q_2 - 1) + (q_1 - 1)(q_2 - 1) = q_1 q_2 = |\Omega|$. Thus the rank is 4.

Author Biographies

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