AN ADAPTIVE SCHEME FOR THE DERIVATION OF HARMONIC IMPEDANCE CONTOURS

by

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INTRODUCTION

The harmonic voltage levels at busbars where the connection of non-linear plant is being considered, are directly related to the harmonic impedances of the a.c. network at that point. Such information is also needed for the economic design of local filters, should the distortion levels exceed those permitted by legislation.

Information on network equivalent impedances is normally presented in the form of impedance/frequency loci[1], obtained either from measurements or from lengthy computer calculations at many discrete frequencies[2][3]. Considering the variety of network configurations and loading conditions, the number of impedance loci involved and therefore the computing burden soon becomes unmanageable.

This paper describes two important concepts that can make the calculation of harmonic impedances more accurate and efficient. One is the use of individual harmonic impedance contours, that take into account the expected range of fundamental frequency variation. The second concept is the use of an adaptive sampling technique that minimizes the number of discrete frequencies required for accurate derivation of the harmonic impedances.

HARMONIC IMPEDANCE LOCI

Detailed studies of the a.c. transmission scheme[4], modelling all lines, generators, transformers and loads are needed to produce the impedance/frequency locus. Moreover, line outage conditions and varying load patterns must be considered which generate large amounts of data.

In the past, an impedance circle encompassing all evaluated impedance loci was used for all frequencies together with computer search techniques which maximised the harmonic voltage distortion. This approach leads to unduly pessimistic filter designs, particularly at low order harmonics. Besides, such an approach requires considerable computing and engineering time which is often not available at the tendering stage.

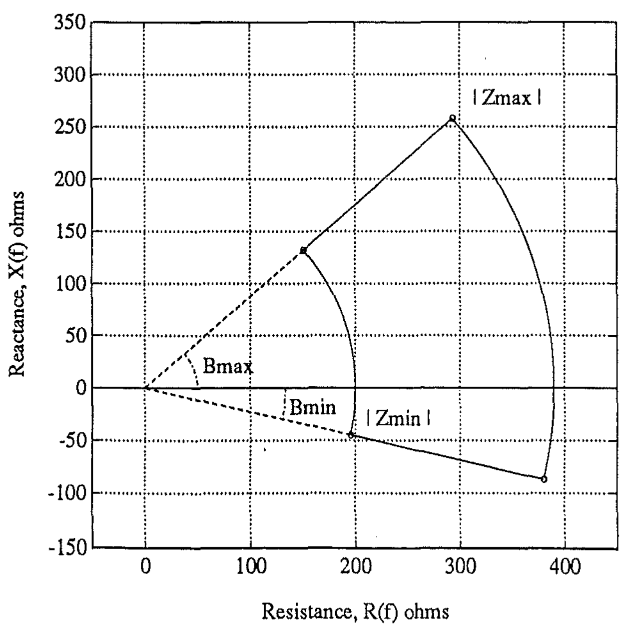

Figure 1: Annular Sectors.

An alternative to the impedance/frequency locus is the Discrete Harmonic Impedance concept, which is only concerned with harmonic information. Two methods in common use are Annular Sectors and Discrete Polygons.

Annular sectors

The Annular Sector concept, illustrated in Fig. 1, restricts the geometric area applicable to each harmonic by setting up upper and lower limits to the magnitude and phase of each harmonic impedance. Taking into account all the relevant operating conditions, a comprehensive scatter plot is produced for each harmonic on the impedance plane; all these points are then encompassed by two circles and a sector and the resulting values of |Zmax|, |Zmin| , Bmax and Bmin are tabulated.

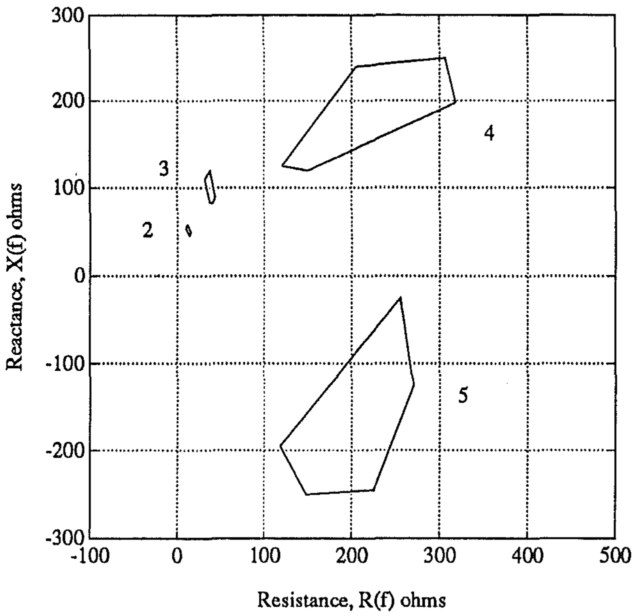

Figure 2: Harmonic impedances, harmonic order 2 to 5.

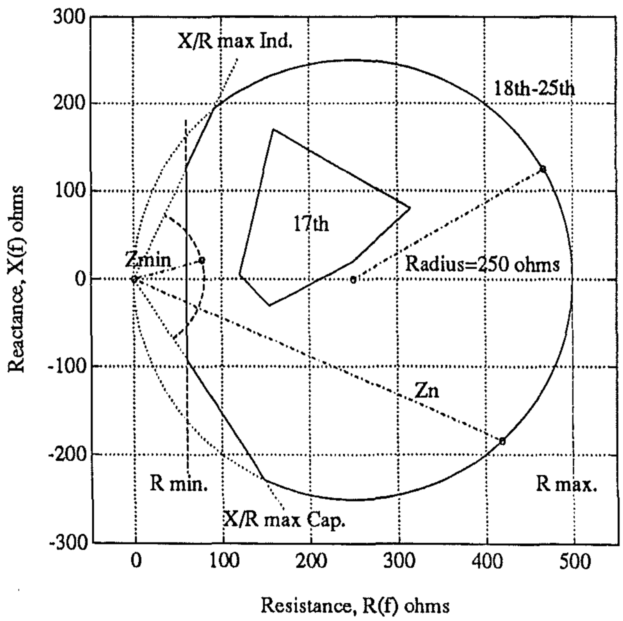

Figure 3: Harmonic impedances for harmonic order 17, and envelope of impedance loci 18 to 25.

Discrete Polygons

In this case a distinction is made between low and high harmonic orders. At the lower harmonics discrete points are obtained for the different operating conditions as for the Annular Sector. Encompassing these points by a polygon results in a set of polygons for the harmonics of interest; an example is shown in Fig. 2 for harmonic order 2 to 5. It is often assumed that at high harmonic frequencies, e.g. above the 17th, the scatter of the R±jX values and hence the boundary of the encompassing polygons becomes increasingly large. From detailed information of the particular system involved it is possible to decide on the use of a realistic outer boundary with a single geometrical shape without introducing an unacceptable degree of pessimism into the filter design studies. This is illustrated in Fig. 3, which also displays the largest discrete polygon considered (i.e. the 17th). A computer technique is then used to search each polygon in turn to evaluate the system impedance which maximises voltage distortion at, or current injection into, the point of common coupling.

TEST SYSTEM

·Fig. 4 represents part of the Primary Transmission System of the south island of New Zealand and a large converter plant is connected at the Tiwai bus. Each component of the system has been simulated using the state of the art models contained in current commercial packages. Discrete frequency impedances observed from this bus and a continuous curve (the impedance/frequency locus) are shown in Figs 5 and 6, respectively; these have been calculated very accurately at 2Hz intervals using harmonic penetration algorithms described in reference [3] for a range of frequencies between 50 and 1250Hz. The results will be used as a reference for the curves to be obtained in the next section.

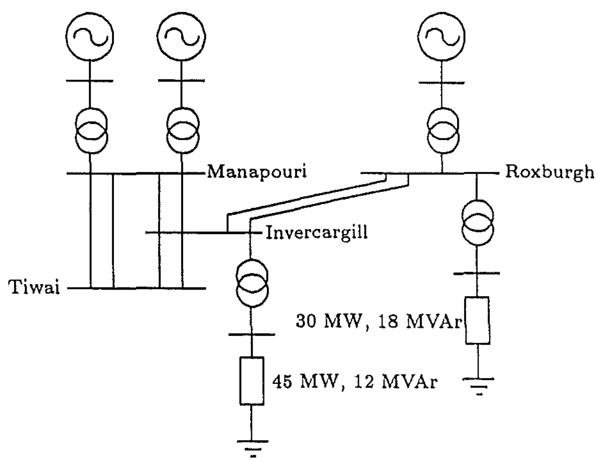

Figure 4: Test system including load and generation.

ADAPTIVE SAMPLING

It should be clear from inspection of Fig. 5 that it is wasteful to use such a small frequency increment between impedance evaluations over the whole range. The locus can be summarised much more efficiently by using small frequency increments where needed and much larger increments elsewhere. Because some loops may involve significant impedance variations it is important to be able to display them accurately, and the danger in using a larger frequency increment is that some loops may be missed completely. Of course, the real difficulty is that the positions of the loops are not known until after the curve has been plotted. The problem is illustrated in Fig. 5, where the regions from 50 to 150, 300 to 800 and 1,100 to 1,250Hz have too many points, while the samples are much more spread in the region around 846Hz. This effect is more obvious in Figs 7 and 8 where the sampling interval is 20Hz. Clearly, there are still excessive samples in some regions and insufficient information in others.

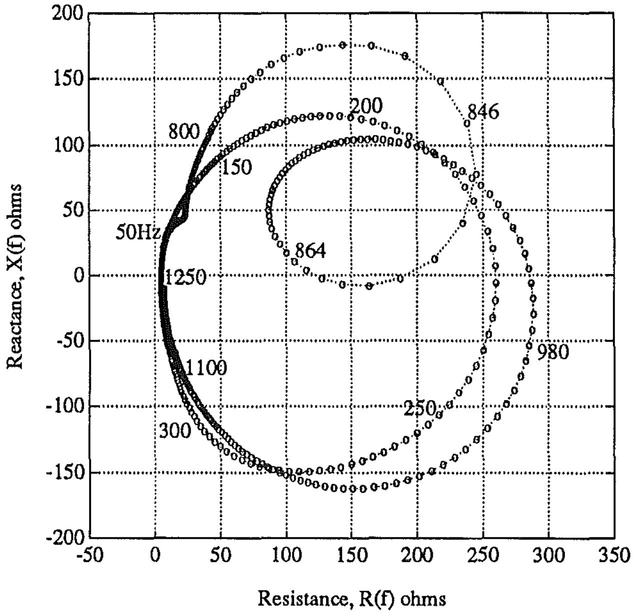

Figure 5: Impedance locus, 2 Hz sampling, 601 Points.

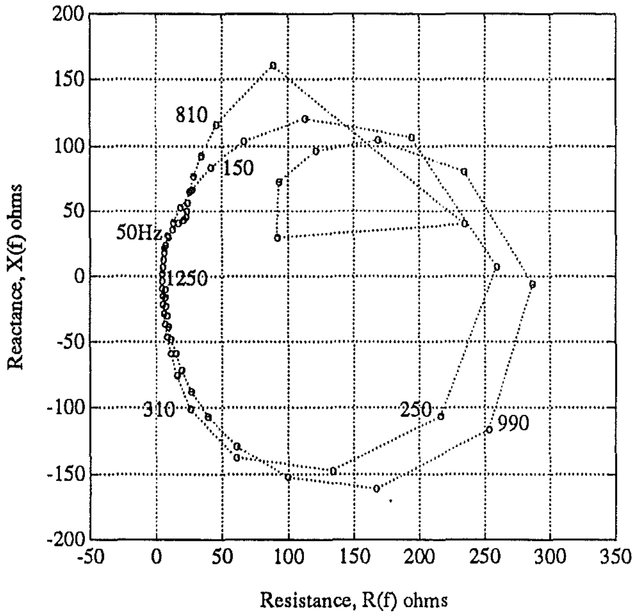

Figure 7: Impedance locus, 20 Hz sampling, 61 Points.

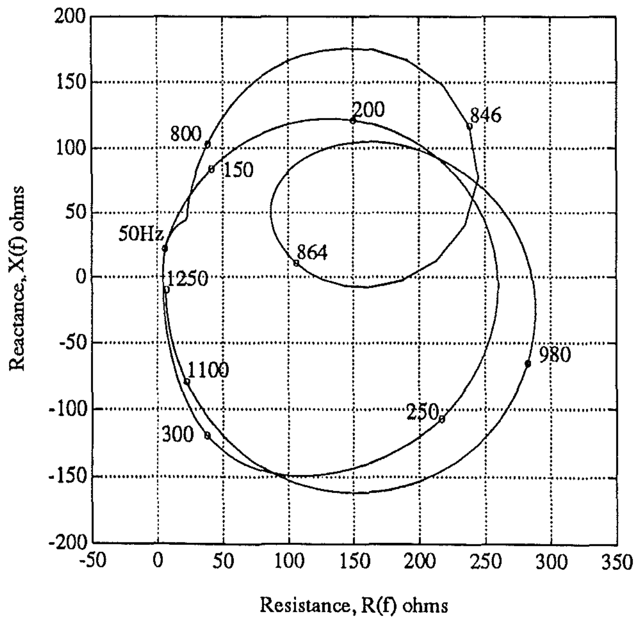

Figure 6: Impedance locus, 2 Hz sampling, 601 Points.

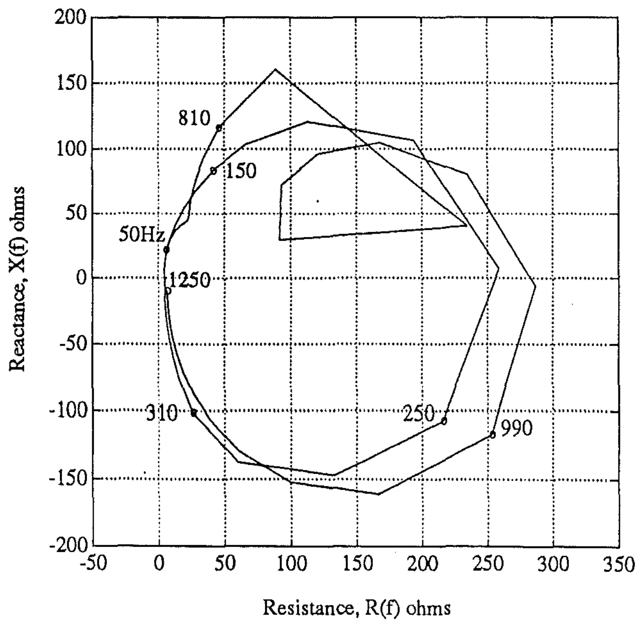

Figure 8: Impedance locus, 20 Hz sampling, 61 Points.

Equal winding angle criterion

What is required is an adaptive scheme, capable of automatically adjusting the frequency increment according to the local geometry of the curve. When the curvature is large the impedance should be sampled at small frequency intervals and when the curvature is small it should be sufficient to sample at intervals which are larger, provided that the interval is not so large that complete loops are "smoothed away".

Ideally, the impedances should be calculated at frequencies $\{f_k, k = 1, 2, \ldots\}$ such that $\psi_{k+1} - \psi_k = \alpha$, where $\psi_k = \psi(f_k)$ is the angle between the tangent to the locus at the frequency f_k and the R-axis (or some other fixed line) and α represents a constant angle.

This approach would guarantee that small loops are given as much attention as large loops. For example, if the true locus were a circle and if α were fixed at 45°, then the locus would be summarised by eight equally spaced points around the circumference, irrespective of the radius of the circle. The change in angle, $\Delta\psi_k = \psi_{k+1} - \psi_k$, as the frequency changes from f_k to f_{k+1} is referred to as the winding angle.

Estimation of winding angle

The diagram in Fig. 9 represents a magnified view of a small portion of the impedance locus. Suppose that the impedance $Z(f) = R(f) + jX(f)$ has already been calculated at each of two frequencies f_k, f_{k+1}. Then the angle through which the tangent to the curve winds as the locus is traversed from $Z_k = Z(f_k)$ to $Z_{k+1} = Z(f_{k+1})$ can be estimated by evaluating the impedance $Z(\tau)$ at an intermediate point τ in the interval $[f_k, f_{k+1}]$.

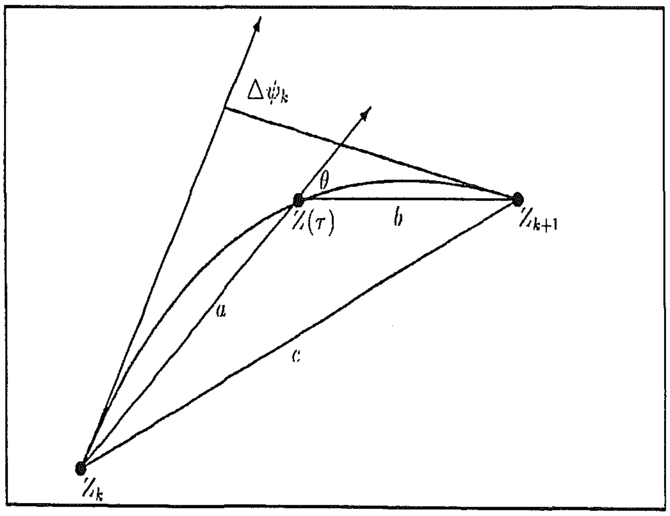

Figure 9: Estimating the winding angle, $\Delta\psi_k$, using the chord angle θ

If θ denotes the angle between the chords $Z(\tau) - Z_k$ and $Z_{k+1} - Z(\tau)$ and if the locus has constant curvature over the interval $f_k \leq f \leq f_{k+1}$, (i.e. it is a circular arc,) then some simple geometry shows that the tangent winds through an angle $\Delta\psi_k = 2\theta$. Thus it can be hoped that the winding angle is estimated well by 2θ, if the curvature does not vary too much over the interval in question.

Moreover, for a smooth locus (at least differentiable) the mean value theorem can be invoked twice to show that the winding angle must be at least θ on an interval that contains τ and which is a strict subset of the interval $[f_k, f_{k+1}]$. In any event if the angle θ is not small then clearly more points are required if the locus is to be summarised well.

The diagram in Fig. 9 is appropriate only if $\theta < 90°$. This situation is easily detected by calculating the chord lengths a, b and c. If $a^2 + b^2 < c^2$ then the angle θ is acute and $\Delta\psi_k$ can be estimated by 2θ; the smaller this angle the more accurate is the approximation. If $a^2 + b^2 \geq c^2$ then it is not even worth calculating the angle θ, clearly the points $Z_k, Z(\tau)$ and Z_{k+1} are spaced too far apart. In this case each of the sub-intervals $[f_k, \tau]$ and $[\tau, f_{k+1}]$ can be processed separately by evaluating the impedance again at an intermediate frequency and estimating the winding angles on each new sub-interval.

The calculation of the chord lengths is not wasted when $\theta < 90°$, because the lengths a, b and c can be used to get *estimates* of the winding angles $\Delta\psi_{k,1}$ and $\Delta\psi_{k,2}$ on each of the sub-intervals. Again, some simple geometry provides the estimates:

$$\Delta\psi_{k,1} = \arcsin(\frac{a}{c}\sin\theta), \quad \Delta\psi_{k,2} = \arcsin(\frac{b}{c}\sin\theta),$$

and it is straightforward to verify that these estimates satisfy $\Delta\psi_{k,1} + \Delta\psi_{k,2} = 2\theta$, as expected. In fact the angle θ can also be calculated from the chord lengths since it satisfies the equation:

$$\cos|\theta| = \frac{c^2 - a^2 - b^2}{2ab},$$

but it is preferable to calculate θ in terms of the arctan function in order to preserve the orientation of the winding angle.

The advantage in obtaining these estimates is that they can be used to get error estimates whenever the interval is subdivided and the winding angle subsequently recalculated. If two successive estimates agree within a pre-specified tolerance and if the best estimate indicates that the winding angle is sufficiently small, (again to within a pre-specified tolerance) then there should be no need to subdivide further. This is the basis of the adaptive scheme.

Recursive algorithm

The impedance is evaluated initially at the two end points of the interval in question before the first call. The length of the chord between these two initial points is also required together with an estimate of the winding angle over the entire length of the locus. Any value exceeding 180° will suffice initially since this value is re-estimated by the routine. A tolerance parameter is then required to be set before entry to the subroutine. Each subsequent call of the subroutine causes the impedance to be evaluated at the mid-point of the frequency interval. This effectively produces two sub-intervals. The chord lengths corresponding to each of these intervals are calculated and if the angle between them is not acute or if this angle is positive (corresponding to an anti-clockwise rotation) then the subroutine is called again immediately on each of the sub-intervals.

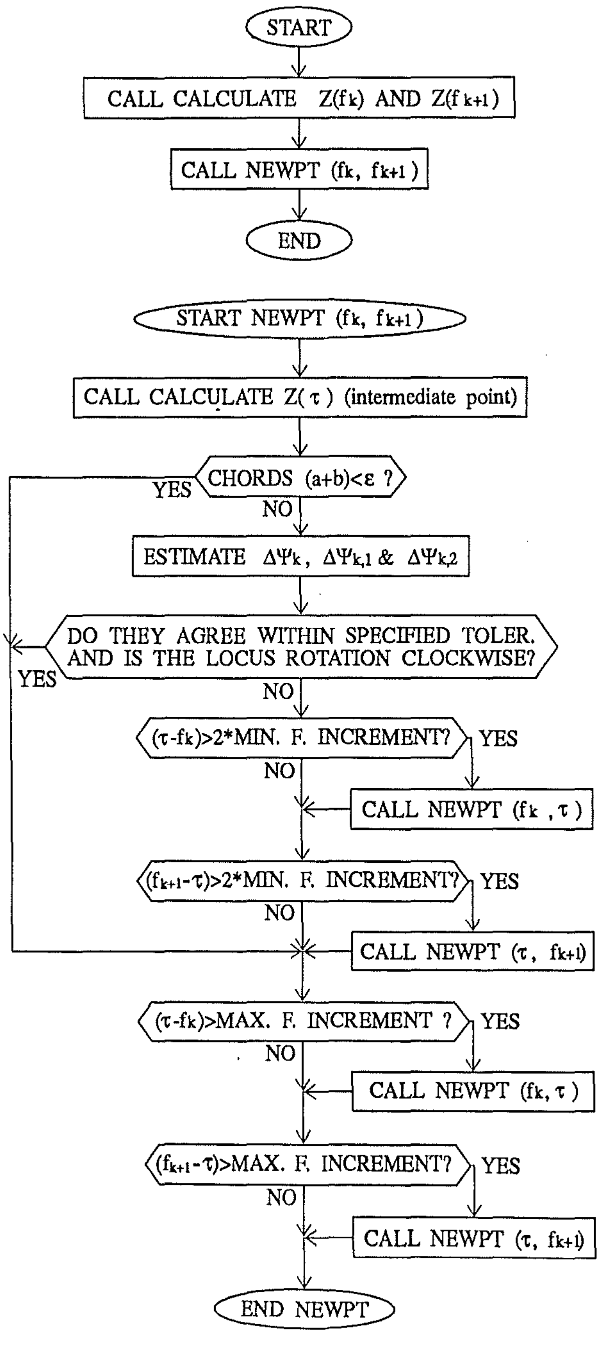

Figure 10: Flow chart of Adaptive Sampling technique.

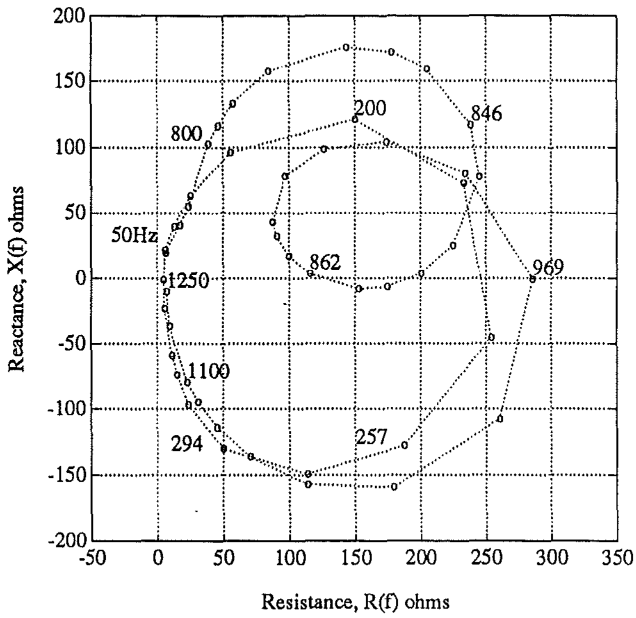

Figure 11: Adaptive sampling, 49 Points.

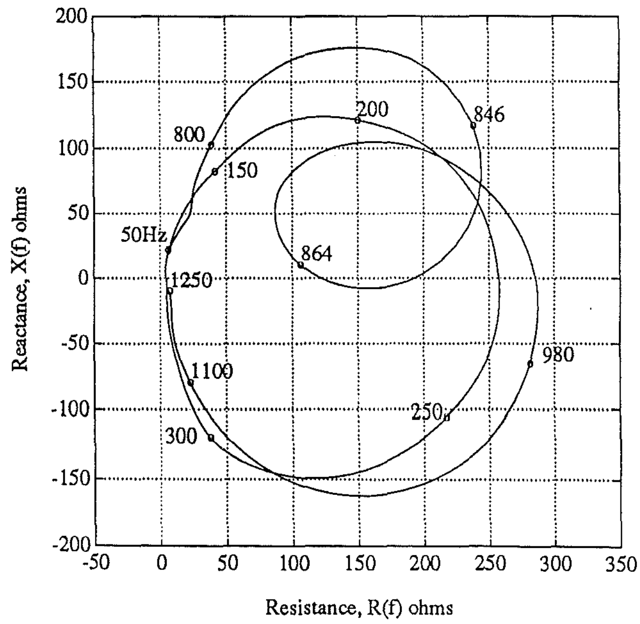

Figure 12: Parametric Cubic Spline on 49 Points.

The subroutine is also called again if either the initial estimate of the winding angle (passed in the parameter list) or the new estimate of the winding angle calculated by the subroutine exceeds the given tolerance. To prevent a possibly infinite recursion when the true locus is not differentiable the subroutine is not called if the range of a frequency interval is too small. Moreover, as an extra safeguard, the subroutine is also called automatically if the range is considered to be too large.

In practice, the user provides values for minimum and maximum frequency increments (the latter not normally exceeding the fundamental frequency) and the algorithm effectively varies the actual increment between these bounds in a way that ultimately causes the winding angle between adjacent points to be approximately constant. In practice this criterion automatically produces every loop of the loci regardless of its size.

Modified criterion

Although the adaptive scheme, as described, provides perfect results, the equal winding angle criterion can be relaxed in regions where the impedance increment between samples is small (e.g. the case of very small loops).

An impedance tolerance (ε) index can be used as an escape condition in such cases, i.e. if $a+b$ (in Fig. 9) is smaller than ε the equal winding angle criterion is abandoned (i.e. the presence of very small loops is eliminated, as they provide no useful practical information).

A flow chart of the modified algorithm is shown in Fig. 10 and the effect is illustrated in Figs 11 and 12. The latter is the impedance locus after applying a parametric cubic spline [5] to the discrete samples. The resulting locus is practically the same as that of Fig. 6 even though the number of sample frequencies used in its derivation has been reduced from 601 to 49.

HARMONIC IMPEDANCE CONTOURS

The annular sector and discrete polygons techniques are very efficient if the harmonic impedances are only calculated with reference to a nominal fundamental frequency.

It is not the purpose of this contribution to decide on the frequency variation to be considered, but rather to provide an economical way of determining the harmonic impedances for any specified frequency range.

To illustrate the effect of the fundamental frequency, a mere 1Hz reduction will make the 50th harmonic impedance equal to that of the nominal 49th and a 2Hz increase will advance the 13th harmonic halfway towards the 14th. To quantify the effect in a practical case, a 3Hz frequency range (say 48 - 51) is considered in the test system of Fig. 4. The result of such variation on the 17th harmonic, displayed in Fig. 13 (continuous trace), is a Harmonic Impedance Contour. The derivation of Harmonic Impedance Contours requires the following steps:

- (i) Specification of the fundamental frequency range, alternative system configurations and operating conditions.
- (ii) For each selected configuration and individual harmonic two discrete frequencies are initially selected, i.e. fmax.h and fmin.h. This will provide a number of initial samples equal to 2.nh (where nh is the number of harmonics to be considered); however overlapping impedances from previous contours can be used in the derivation of the next harmonic contour.
- (iii) Starting with the samples derived in (ii), apply an adaptive sampling scheme to decide which other (if any) intermediate samples are needed for accurate derivation of the impedance/frequency locus.
- (iv) From the impedance/frequency loci derived in (iii), the harmonic impedance contours are extracted, the beginning and end of each contour being as defined in step (ii).
- (v) If required, the above procedure can be done for each independent phase

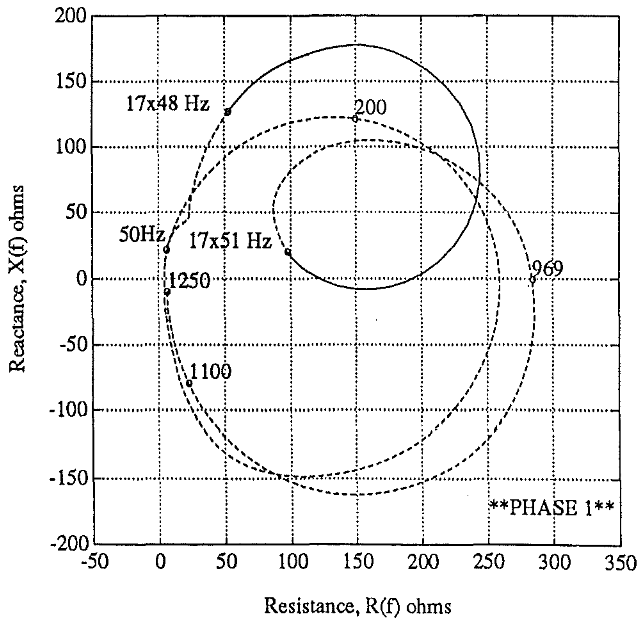

Figure 13: 17th harmonic for frequency variations.

Illustrative example

To illustrate the harmonic impedance contour method, the test system of Fig. 4 was used under four different line outage conditions. For each configuration the fundamental frequency was permitted to vary from 48 to 51Hz, and the load from 100% to 10%. The contours for the 17th harmonic impedance in phases 1 and 3 are

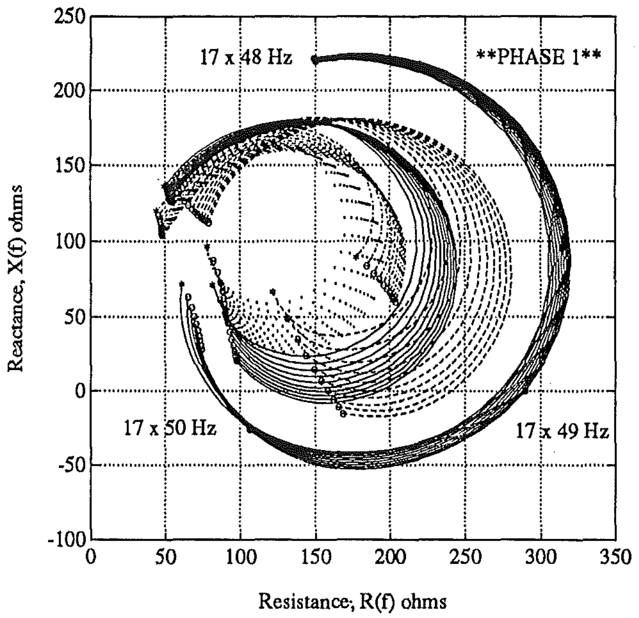

Figure 14: Impedance contours, 17 th harmonic for frequency and load variations, four line outage conditions.

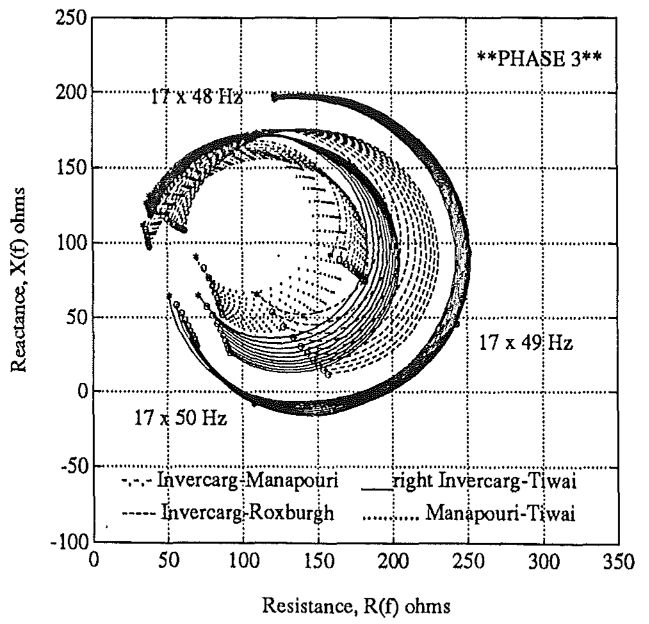

Figure 15: Impedance contours, 17 th harmonic for frequency and load variations, four line outage conditions.

displayed in Figs 14 and 15, respectively, and the following observations can be made with reference to these results.

- The fundamental frequency variation has an enormous influence on the harmonic impedance. This is best appreciated in Fig. 14 where a mere 1Hz increment (between the 49 and 50Hz) causes almost a 300% impedance reduction.
- The network configuration does not change in a continuous manner with the load and as the configuration changes (e.g. by connecting or disconnecting transmission lines) the harmonic impedances vary in an irregular and discrete manner.
- A comparison of Figs 14 and 15 indicates that the harmonic impedances are far from symmetrical and that, in general, the studies should be three-phase.

CONCLUSIONS

An adaptive technique has been developed for the derivation of impedance/frequency loci which minimises the number of discrete samples without reducing the accuracy of the results. The specified fundamental frequency range is used to decide the discrete frequency boundaries for each harmonic and the adaptive technique is then used to extract accurate harmonic impedance contours.

For each harmonic, a three-phase set of impedance contours is collected covering all the specified configurations and operating conditions. This information is stored in memory and can be presented graphically; if required the discrete information can also be converted into encompassing annular sectors or discrete polygons. A structure diagram of Harmonic Impedance Contours and required input data is illustrated in Fig. 16.

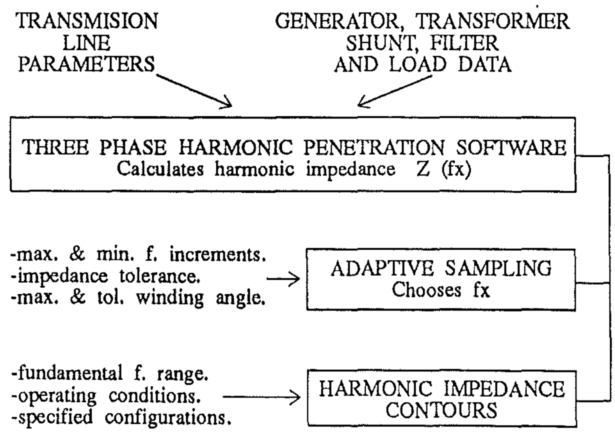

Figure 16: Structure diagram of Harmonic Impedance Contours and required input data.

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