

**Water Allocation Under Uncertainty –
Potential Gains from Optimisation
and Market Mechanisms**

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Abstract

This thesis first develops a range of wholesale water market design options, based on an optimisation approach to market-clearing, as in electricity markets, focusing on the extent to which uncertainty is accounted for in bidding, market-clearing and contract formation. We conclude that the most promising option is bidding for, and trading, a combination of fixed and proportionally scaled contract volumes, which are based on optimised outputs. Other options include those which are based on a post-clearing fit (e.g. regression) to the natural optimised outputs, or constraining the optimisation such that cleared allocations are in the contractual form required by participants. Alternatively, participants could rely on financial markets to trade instruments, but informed by a centralised market-clearing simulation.

We then describe a computational modelling system, using Stochastic Constructive Dynamic Programming (CDDP), and use it to assess the importance of modelling uncertainty, and correlations, in reservoir optimisation and/or market-clearing, under a wide range of physical and economic assumptions, with or without a market. We discuss a number of bases of comparison, but focus on the benefit gain achieved as a proportion of the perfectly competitive market value (price times quantity), calculated using the market clearing price from Markov Chain optimisation. With inflow and demand completely out of phase, high inflow seasonality and volatility, and a constant elasticity of -0.5 , the greatest contribution of stochastic (Markov) optimisation, as a proportion of market value was 29%, when storage capacity was only 25% of mean monthly inflow, and with effectively unlimited release capacity. This proportional gain fell only slowly for higher storage capacities, but nearly halved for lower release capacities, around the mean monthly inflow, mainly because highly constrained systems produce high prices, and hence raise market value. The highest absolute gain was actually when release capacity was only 75% of mean monthly inflow. On average, over a storage capacity range from 2% to 1200%, and release capacity range from 100% to 400%, times the mean monthly inflow, the gains from using Markov Chain and Stochastic Independent optimisation, rather than deterministic optimisation, were 18% and 13% of market value, respectively.

As expected, the gains from stochastic optimisation rose rapidly for lower elasticities, and when vertical steps were added to the demand curve. But they became nearly negligible when (the absolute value of) elasticity rose to 0.75 and beyond, inflow was in-phase with demand, or the range of either seasonal variation or intra-month variability reduced to $\pm 50\%$ of the mean monthly inflow. Still, our results indicate that there are a wide range of reservoir and economic systems where accounting for uncertainty directly in the water allocation process could result in significant gains, whether in a centrally controlled or market context. Price and price risk, which affect individual participants, were significantly more sensitive. Our hope is that this work helps inform parties who are considering enhancing their water allocation practices with improved stochastic optimisation, and potentially market based mechanisms.

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Notation, Acronyms and Abbreviations

Symbol	Definition	Chapter
ANSD	Negative Semi-Deviation of Annual measure	C5
APSD	Positive Semi-Deviation of Annual measure	C5
ARC	Set of Connected Arcs	C3
AVWAP	Mean of annual VWAP for each sampled year	C5
B	Intra-Period Benefit function	C2
b	Net demand indices	C3
B'	Marginal benefit function	C3
BAPMV	Benefit Achievable as a Proportion of Market Value	C5
BOP	Beginning of Period	C2
C	Convex cost function	C3
C'	Marginal cost function	C3
CDCR	Conditional Demand Curve for Release	C2
CDCS	Conditional Demand Curve for Storage	C5
CDCW	Conditional Demand Curve for Water	C2
CDDP	Constructive Dual Dynamic Programming	C2
CDF	Cumulative Distribution Function	C5
CfD	Contracts for Difference	C2
CK	Contract Strike (Marginal) Price	C2
CKV	Contract Strike Value	C2
CORS	Conditional Optimum Release Schedule	C5
CPDFW	Conditional Probability Distribution Function for Water	C5
CQ	Contract Quantity	C2
d	Release Deviation	C5
DC	Deterministic	C1
DCR	Demand Curve for Release	C2
DCS	Demand Curve for Storage	C2
DCW	Demand Curve for Water	C5
DF	Deviation Factor	C5
DP	Dynamic Programming	C2
DV	Decremental DRC Marginal Price Vector	C5
e	Price Point Elasticity	C5
E	Elasticity Factor	C5
EB	Expected Benefit	C5
EOH	End of Horizon	C2
EOHS	End of Horizon Value function for Storage	C3
EOP	End of Period	C4
ESP	Ensemble Streamflow Prediction	C2
F	Intra-Period Inflow	C2
FMAX	Maximum Inflow	C5

Notation, Acronyms and Abbreviations

FMEAN	Mean Inflow per period/stage	C5
FS	Flow Shifted	C5
FT	Tributary Flow	C3
FTR	Financial Transmission Right	C2
GNSD	Negative Semi-Deviation of Global measure	C5
GPSD	Positive Semi-Deviation of Global measure	C5
GUB	Global Upper Bound	C5
h	Current Hydrology (inflow) State	C2
H	Number of Hydrology States / (Weather) Index Level	C2/C4
h'	Prior Hydrology (inflow) state	C2
H'	New Index	C4
HD	Hydrology Data (Module)	C5
HMAX	End of Index Range / Maximum Index Value	C2/C4
HMIN	Start of Index Range / Minimum Index Value	C2/C4
H/M/L	High/Medium/Low Hydrology States	C4
Hrange	Index Range	C4
i	participant index	C3
I	Total set of market participants	C3
ID	Set of Consumers	C3
IDF	Index Dependency Factor	C4
IDPSA	Incremental DP with Successive Approximations	C2
IS	Set of Suppliers	C3
ISO	Independent System Operator	C2
IV	Incremental DRC Marginal Price Vector	C5
j	supply indices	C3
k	demand indices	C3
LBR	Lower bound of total Benefit with no Release	C5
LBS	Lower bound of total Benefit with no Storage	C5
LDC	Load Duration Curve	C5
LMP	Locational Marginal Pricing	C2
LP	Linear Programming	C1
M	Marginal Price for net bid	C2
MAXDEM	Maximum market demand bid	C3
MAXSUP	Maximum aggregate supply offered	C3
MBENEFIT	Marginal Benefit	C3
MBID	Marginal (demand) bid	C3
MBO	Maximum Benefit from Optimisation	C5
MBRR	Maximum Benefit Range achievable by increasing Release capacity	C5
MBRS	Maximum Benefit Range achievable by increasing Storage capacity	C5
MC	Markov Chain	C1
MCE	Market Clearing Engine	C2
MCOST	Marginal cost	C3

Notation, Acronyms and Abbreviations

MCP	Market Clearing Price	C1
MCPR	Market Clearing Price for Release	C5
MCPS	Market Clearing Price for Storage	C5
MD	Market Data (Module)	C5
MDP	Markov Decision Process	C2
MILP	Mixed Integer Linear Programming	C2
MIP	Mixed Integer Programming	C2
MM	Monday Morning	C5
MM_DCS	Monday Morning Demand Curve for Storage	C2
MOFFER	Marginal (supply) offer	C3
MR	Market Reporting (Module)	C5
MREF	Marginal Price Reference Point for Demand	C5
MRV	Marginal Release Value	C2
MS	Merge Sort	C5
MV	Market Value	C5
MWV	Marginal Water Value	C2
n	Number of Reservoirs / Origin node / sample	C2/C3/C5
N	Set of network nodes / Number of Monte Carlo Samples	C3/C5
n'	Destination node	C3
NEM	National Electricity Market	C2
NEMS	The National Electricity Market of Singapore	C2
NSD	Negative Semi-Deviation	C5
OR	Operations Research	C2
OTC	Over The Counter	C2
P	Inter-period Probability of occurrence/ Nodal Probabilities	C2/C4
p	scenario path	C3
p'	preceding node on scenario path	C3
\check{P}	Conditional Probability	C3
PDF	Probability Distribution Function	C5
PDFS	Probability Distribution Function for Storage	C5
PDFW	Probability Distribution Function for Water	C5
PRPBAC	The Proportion of Remaining Potential Benefit Achievable by investing in Capacity	A7.1
PRPBAO	Proportion of Remaining Potential Benefit Achievable by investing in improved Optimisation	A7.1
PRPBAS	Proportion of Remaining Potential Benefit Achievable by increasing Storage capacity	C5
PSD	Positive Semi-Deviation	C5
PTPBAR	Proportion of Total Potential Benefit Achievable by increasing Release capacity	C5
PTPBAS	Proportion of Total Potential Benefit Achievable by increasing Storage capacity	C5
PWAP	Probability Weighted Average Price	C5
q	Allocated quantity	C2
Q	Quantity for net bid	C2

Notation, Acronyms and Abbreviations

q_d	aggregate demand variable	C3
QDRY	Minimum net bid limit	C4
QR	Rainfall	C8
Qrange	Range of release	C2
QREF	Quantity Reference Point for Demand	C5
q_s	aggregate supply variable	C3
QWET	Maximum net bid limit	C4
r	Utilisable reservoir release	C3
RBRS	Remaining Benefit Range achievable by increasing Storage capacity	C5
RC	Reservoir discrete Convolution simulation (Module)	C5
RCAP	Release capacity	C3
RD	Reservoir Data (Module)	C5
RO	Reservoir Optimisation (Module)	C5
ROR	Run of River	C5
RS	Reservoir Monte Carlo Simulation (Module)	C5
SCAP	Storage Capacity	C2
SCDDP	Stochastic Constructive Dual Dynamic Programming	C1
SD	Standard Deviation	C5
SDDP	Stochastic Dual Dynamic Programming	C2
SDP	Stochastic Dynamic Programming	C1
SI	Stochastic Independent	C1
SINITIAL	Initial Storage Level	C3
SLP	Stochastic Linear Programming	C1
SN	Sunday Night	C5
SN_DCS	Sunday Night Demand Curve for Storage	C2
SOH	Start of Horizon	C2
Srange	Range of storage levels	C2
s^t	Storage value at end of period	C2
t	time period / stage	C2
T	Total number of periods / stages	C2
TB	Total Net Demand / Total Benefit	C3/C5
TCALLP	Total Call option Pay-out	C2
TCFC	Total Cost of the Financial Contract	C2
TCP	Total Contract Pay-out	C2
TCPC	Total Cost of the Physical Contract	C2
TCQ	Total Contract Quantity	C4
TCV	Total Contract Value	C4
TD	Total Demand	C3
TF	Total Flow	C5
TFS	Truncated Flow Shifted	C5
TMS	Truncated Merge Sorted	C5
TPM	Transitional Probability Matrix	C5

Notation, Acronyms and Abbreviations

TQ	Total Demand Quantity	C5
TS	Total Supply	C3
TUBUR	Tight Upper Bound with Unconstrained Release	C5
TUBUS	Tight Upper Bound with Unconstrained Storage	C5
TWAP	Time Weighted Average Price	C5
TWAPR	TWAP for Release	C5
TWAPS	TWAP for Storage	C5
V	Value Function	C2
v	Tributary spill variable	C3
VAR	Variance	C5
VWAP	Volume Weighted Average Price	C5
VWPSD	(Global) Volume Weighted Positive Semi-Deviation	C5
w	Spill to sink	C3
WAP	Weighted Average Price	C5
WAPMV	Welfare as a Proportion of Market Value	C5
W_F	Seasonal Weighting Factor	C5
W_L	Lag Weighting	C5
x	Decision	C2
y	Random Variable	C2
z	State variable	C2
α	Contract Index Dependency Factor	C4
β	Participant Index Dependency Factor	C4
η	Market Clearing Quantity Shadow Price	C4
θ	Shadow Price on a Particular Bid	C4
λ	Market Clearing (Shadow) Price	C2
ρ	Release Capacity Shadow Price	C4
σ	Storage Capacity Shadow Price	C4
v	Spill from Tributary Flow Shadow Price	C4
ψ	Marginal Water Value Shadow Price	C4
ω	Spill from Reservoir Shadow Price	C4

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1. Introduction and Thesis Outline

1.1. Introduction

Water supply and demand is commonly uncertain, varying by location and time of year. Historically, water has been priced well below the environmental and opportunity costs. While demands for fresh water are increasing, there is a reduction in access to clean, abundant and easily harvestable, treatable and transferable water supplies. Markets could connect willing trading parties to maximise benefit to society. Markets could also provide price (and hence value) information for each unit of water available to trade.

Basic water markets do exist in the real world. In the literature, water markets, and their future potential, have been heavily discussed. Some water market designs have also been proposed. One key question any water market design must address is: How do we decide how much water should be used today, and how much should be saved until tomorrow? Some designs have ignored this storage issue for trading water in subsequent periods. Others have modelled inter-temporal connections, but assumed water can be traded as if the future were deterministic.

The main research goal of this thesis is to investigate the performance of various wholesale multi-use water market design options, for a system where storage is important, under uncertainty. To address these issues we propose a three step approach:

- First, we develop a range of water market design options, discussing the merits of some of the more promising options.
- Second, we discuss the key features of our simplified computational modelling system, used to assess the performance of some of the design options.
- Third, and finally, we perform some experimental investigations, running some of the design options on the computational modelling system.

Many of our water market design options will, within the market clearing, account for storage and uncertainty as it evolves over time. Some options assume supplies are uncertain, while others also assume demand is uncertain. All the identified design options are initially reviewed in terms of practical relevance for potential real-world implementation. Having short listed some preferred water market design options we then use experimental testing to

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provide some key insights into how various options might perform when trading under uncertainty. This assessment includes highlighting the consequences, if any, of not accounting for the uncertainty. Our hope is that this work helps inform parties who are considering enhancing their uncertain water allocation practices with market based mechanisms.

While the wholesale water industry has many unique characteristics, it shares many similar traits to wholesale electricity and natural gas industries. All three industries began various phases of deregulation some twenty years ago. Within specific countries, and indeed regions within those countries, these industries each followed their own deregulatory path to meet those local situations. In many OECD countries, electricity and natural gas industry reform has followed the common theme of market decentralisation.

These commodities are now actively traded, according to the constraint based (primal/dual) pricing theory, using deterministic optimisation techniques (typically Linear Programming, LP) to clear markets, while typically ignoring (long-term) inter-temporal linkages. This same theory and approach is used here to develop our water market design options. But many of our water market options must directly account for both inter-temporal storage, and stochasticity, within the market clearing formulation. So we must use stochastic mathematical programming techniques, specifically Stochastic Linear Programming (SLP) and Stochastic Dynamic Programming (SDP), to formulate and investigate pricing relationships.

1.2. Thesis Outline

The mathematical notation, acronyms and abbreviations used throughout this thesis are listed at the start of this thesis, prior to the contents. In Chapter 2 we review literature relating to three key areas we deem relevant to this thesis: the role of water in socio-economic systems, related markets to trade resources with associated issues, and existing modelling practices. First, we review existing water issues, including scarcity, valuation, costing and pricing experiences, and discuss water as an economic good.¹ Second, we investigate existing markets, including water, environmental, and model-based markets. The latter discussion

¹ These initial discussions are in Appendix 2.

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includes reviewing natural gas and electricity markets. In discussing the third issue of modelling practices we explore existing models for reservoir systems, electricity systems, and general market trading systems.

Having undertaken a literature review in Chapter 2, Chapter 3 outlines a formulation of our most complex centralised water trading model. Here a central authority is assumed to have marginal cost (offer) and marginal benefit (bid) information about (or from) parties (participants) wishing to trade water, from which they independently allocate the water to (and from) those parties (participants). Water is stored in reservoirs and within the transfer network. We envisage a system where water supplies (and also potentially demands) are uncertain, and differentiated temporally and locationally. Hence our most general model formulation takes account of these three key dimensions: time, space and uncertainty.

Demand for water is discussed as four parts. First, is a description of a simple optimal water exchange between two parties. Second, a multi-party (multiple suppliers and consumers) optimal water exchange is discussed. Third, water exchanges with wider sets of buyers and sellers of water are represented in terms of a net demand function. Fourth, net demand is treated as potentially uncertain, and subject to various hydrological factors.

Chapter 3 also discusses central optimisation issues, including pricing penalties.² This is followed by two stochastic centralised water trading formulations. The first is a multi-node model, formulated with explicit suppliers and demanders. The second is a simplified single-node (reservoir) model with a net demand bid function.

Chapters 4 through 8 explore how we should account for stochasticity, of both supply and demand, when designing and testing various water allocation models. These chapters envisage that the parties seeking water exchanges are participants who actively trade water in a market setting. For simplicity, to understand the basic interactions under uncertainty, these chapters only focus on the stochastic single-node model. All the model variants discussed here use net demand functions, and are re-formulated so as to apply in a decentralised market trading setting. Of course we still assume the need for a central body, the market manager, to independently co-ordinate the market on behalf of market participants.

² Penalty issues are discussed in Appendix 3.

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Chapter 4 develops a conceptual framework which outlines how water could be traded in a wholesale market setting. It begins by discussing the allocation of decision making roles between the market participants and the market manager. Next we look at two main types of options, outlining how market interactions could be co-ordinated to facilitate managing a reservoir system under uncertainty. In the first set of options, each participant owns a virtual share of the reservoir system. In these options participants make their own storage and release decisions about their system share. In the second set of options, market participants provide marginal price and quantity data, using ‘offers’ of some form. Market participants are then allocated water, subject to availability, through some market clearing process. In this group of options the stochasticity is managed explicitly within the market.

The remainder of Chapter 4 then focuses on developing centrally co-ordinated stochastic market options. We consider three key inter-related issues:

1. Modelling Uncertainties: Incorporating stochasticity in the centralised market-clearing model.
2. Interacting with Participants: Organising market interaction via participant net bids.
3. Trading Products: The nature of the products actually traded by participants.

If the market is to evolve to increasingly account for the assumed underlying stochasticity then the market manager and participants also need to manage increasingly complex and accurate forms of information. So this section looks at the inter-play between complexity, precision, and simplicity.

In Chapters 3 and 4 we use SLP formulations, to outline our framework, which is then analysed at an implicit level. From this there follows an initial assessment of the pros and cons of the outlined water trading options. After this we then employ SDP, to explicitly test option performance. This more extensive second stage of assessment focuses on investigating market pricing relationships via computational testing. Chapter 5 outlines the computational design we developed to undertake these experimental investigations. An outline of the modelling system is discussed, and this generally consists of input data and the following three main computing modules:

1. A Stochastic Constructive Dual Dynamic Programming (SCDDP) Optimisation Module.
2. A Simulation Module, comprising two main sub-modules:
 - Implicit limited sampling of the state space via a Monte Carlo simulation,

Chapter 1: Introduction and Thesis Outline

- Explicit enumeration of the full state space via a convolution algorithm.
3. A Market Reporting Module.

Chapter 5 initially discusses the input data requirements. These include reservoir data, hydrology data and market data. The reservoir data relates to the relativities of the modelling systems storage and release capacities. The hydrology data discussion focuses on net inflow and probability data pairs. Finally, the market data concentrates on participant demand. A key experimental design decision is to generally assume constant elasticity of demand curves to represent aggregate market demand. Although Chapter 8 explores alternative demand curve forms. We then discuss the key features of each of the three main modules in more detail.

The final part of Chapter 5 focuses on market reporting measures. First, is a discussion on net welfare and its associated market value. Benefits range from the run to river (only) flows, to the ideal unconstrained reservoir system. These upper and lower bound benefits are used in conjunction with the benefits from the reservoir storage system being modelled. By combining these benefits in various ways we propose three main proportional benefit measures. Each of these measures looks at the value of more complex market optimisation, from a number of different viewpoints. Second, is a discussion on price reporting measures. Here we discuss the types of prices generated by our market clearing process. This is followed by how these optimised prices can be averaged to report time (probability) and volume weighted average pricing measures. Third, is a discussion on volatility, risk, and variability, for both benefits and prices. Fourth, and finally, ‘spill’ and ‘empty’ issues are discussed, including what to report when the reservoir modelling system hits its bounds.

Chapters 6 through 8 then undertake a series of simplified experiments on the SCDDP computational modelling system. In these chapters we assume a relatively coarse monthly approximation, and investigate the effect of optimising the system under different levels of uncertainty. Results are presented for three increasingly complex levels of uncertainty. The simplest being Deterministic (DC), then Stochastic Independent (SI), and the most complex level of uncertainty we model is a lag-one Markov Chain (MC) process. We simulate market performance by assuming the real-world is also a lag-one MC process. Result analysis is focused on market efficiency, emphasising only aggregate market performance.

Chapter 1: Introduction and Thesis Outline

The experimental results part of the thesis is showcased with some modelling results. Chapter 6 illustrates various monthly reporting measures, but also averages each of these into equivalent single annual measures. This simple set of experiments fixes various physical and economic parameters. Reasonably relaxed storage and release capacities are used. These result in a relatively unconstrained reservoir system. Surprisingly there is still a marked change in market outcomes, especially in terms of price.

Details of the experimental parameter settings are provided first. Following these results are presented in two sub-sections. First, outputs from the optimisation. Second, reporting measures output from the simulation. The latter focuses on reservoir storage levels (including empty and spill values at the storage bounds), Market Clearing Prices (MCPs), and finally market value (benefits) from the run of river and storage system.

Chapter 7 explores physical system sensitivities. Each sub-section is tackled in the same fashion. Parameter settings for each experiment are stated, followed by results and analysis. Finally discussions and conclusions are presented for each experiment. The main physical parameters we explore are:

1. Storage and Release Capacities,
2. Inflow Variance,
3. Inflow Seasonality, and
4. Phasing of peak market demand to inflow.

Chapter 8 is the final experimental chapter, and it is laid out in a similar fashion to Chapter 7. The experimental sub-sections focus on varying the following economic parameters:

1. Varying Constant Elasticity Demand Curves,
2. Varying the Demand Curve Shape, including:
 - Inserting a horizontal flat into a Constant Elasticity Demand Curve, and
 - Inserting a vertical step into a Constant Elasticity Demand Curve.
3. State dependent bids.

The thesis concludes by summarising the main lessons, in terms of market design from each of these chapters, and suggesting some areas where there is potential for future exploration, by others.

2. Literature Review

Water is a precious resource, which has many issues that need to be considered when looking to allocate it. We start this review by directly discussing market based issues. But, in Appendix 2 we also discuss wider water issues.

2.1 Market Issues

2.1.1 Existing Water Markets

There are many water markets, of one kind or another, which are already in operation. Easter et al. (1998) conclude (p.289) that “There are both formal and informal water markets at work today”. Water is also indirectly traded for example when generating hydroelectric power, as noted by Dinar & Subramanian (1997).³ We now discuss more formal water markets, which are predominantly for agricultural uses, unless stated otherwise.

In the US, rural water trading occurs with both temporary spot and option trades, Easter et al. (1998). They also note that users see these arrangements as risky and would prefer permanent allocations. Trading occurs, for example, in California and the Rio Grande Valley, Texas. California is discussed in detail by Hanak (2002), and Texas by Griffin & Characklis (2002). Easter et al. (1998, p. 119) discuss how many existing water markets are based around simple spot or option rights, and suggest that these should be encouraged as they “can provide a basis for comparison with more sophisticated market structures in other resource-based commodities”.

South Australia, New South Wales and Victoria have been trading permanent rural water transfers since 1983, 1989 and 1991, respectively, T. Anderson & Snyder (1997). Interstate trading commenced in 1992. T. Anderson et al. (2012) report that Australian trading volumes and numbers of trades increased from an average of 146,000 acre-ft over 468 trades in the late 80's early 90's to 3.3 million acre-ft over 25,000 trades in 2009/2010. The Murray-Darling Basin accounts for 93% of all Australian water trading. Many farming communities

³ There are informal water markets in South Asia and Mexico, Thobani (1997). There is a report a ground water market in Pakistan, Jacoby et al. (2001). There is an indirect water market for water in France, where 60% of all abstracted water is used to generate hydroelectric power, but most is returned to the watercourse, as discussed by Dinar & Subramanian (1997). They also state that (pp. 61) Israel allocates some agricultural water through markets which operate at the margin, and Tsur (2009) confirms the Israeli water markets. In New Zealand the government sells wholesale water to agricultural and urban users in a market Dinar & Subramanian (1997, p. 81).

Chapter 2: Literature Review

in Australia seem to have embraced water trading, viewing water as “just another” farming input that, depending on market conditions, can be bought, sold or substituted, Bjornlund & McKay (1998), Bjornlund (2003). Zaman et al. (2009) remark that the growing Australian rural water market uses paper based trades.

There is also an online market to trade temporary and permanent water allocations, across major irrigation regions in Australia, Waterfind (2013). Waterfind provides weekly summary information in a consolidated one page report. The information is designed for water holders and includes rainfall, storage levels, allocation announcements, and policy changes.

An account of the water markets in Chile is given by Schleyer (1992). The economic and social policy shift in Chile towards water market-allocation has been a huge success, and created great social benefits and redistribution of wealth. Since market implementation Chile has become one of the largest fruit exporters, as farmers could obtain the water required to move to higher value crop production. Cities have the right to buy rural water and potable water coverage rose to 99/94% for urban/rural areas from 63/27% respectively from 1970/1992.

Chilean water trading between 1986-1993 has been studied in detail by Hearne & Easter (1995). They report significant gains from trade within these regional water markets. Many of the rights that were transferred were not used prior to their sale. Most right transfers were inter-sectoral and there was much less transfer within agriculture. They studied four valleys which employed different technologies and conclude that having the technology and ability to control flows significantly improves trading. Public investment in storage and delivery systems helped reduce transaction costs, but the cost of adjusting fixed stream flows could be prohibitive.

Other authors, including Easter et al. (1998), report that Chile had regional issues in implementing the reform. T. Anderson et al. (2012, p. 158) comments that uncertainty in some regions of Chile arose from a 1981 Water Code recognizing all pre-existing rights regardless of how they were acquired. These authors discuss how the definition of rights was a major issue constraining effective water trading in some regions of Chile.

Assuming a clear framework of rights exists, parties are free to trade without concern for repercussion. Other markets trading environmentally related products are discussed next.

2.1.2 Environmental Markets

Markets exist to trade permits in SO₂, CFCs, NO_x, and water quality, Stavins (1998), Burtraw & Szambelan (2009). In the US informal sulphur dioxide (SO₂) markets were operating from the 70's and this was formalised in 1990 with the introduction of the Clean Air Act, which also kick-started the development towards NO_x markets. The first NO_x market appeared in the US from 1994, the second in 1999.

The EU implemented the Emissions Trading Scheme (EU ETS) in 2005 and this impacted oil, gas, coal and power markets, James & Fusaro (2006). The introduction of carbon dioxide (CO₂) emissions trading via carbon credits (Kyoto units) also impacted these markets. The European trading scheme has markets for spot, futures and option. The main EU markets are discussed by Daskalakis et al. (2009). While in 2007 the EU carbon market was regulated, more trades were happening voluntary in the likes of the USA, and New South Wales (in Australia), Hamilton et al. (2007). Voluntary carbon trade volumes had more than doubled in the following year, 2008, Hamilton et al. (2009).

Woodward et al. (2002) report that Water Quality Trading markets were rapidly developing in the US, in 2002. Four main types of market structure that have evolved in the US to facilitate such trades: Exchanges, Bilateral Negotiations, Clearinghouses, and Sole-source Offsets. Water pollution and quality trading in the US occurs at "various watersheds," Ribaudo et al. (2009). Poor performance is reported if non-point sources are not included under the cap. Indeed Ribaudo & Gottlieb (2011) discuss the lack of progress in the US of non-point source trading programmes for nutrients, such as phosphorous and nitrates. They quote research and program design as key issues for developing workable markets.

Air pollution, water resources management, including water pollution control and fisheries schemes are reviewed by Tietenberg (2002). He noted that tradable permit systems will not work unless market conditions are right, citing market power, transaction costs and insufficient monitoring and enforcement as issues to be addresses. He notes that schemes usually consist of two types, credit or cap-and-trade. He discusses the various options

associated with the method of initial allocation: Random Access (Lotteries), First-come, First-Served, Administrative rules based upon eligibility criteria, and Auctions. Whichever is used, the initial allocation problem is a key and contentious issue. Clear transferability rules are also important. Tietenberg then discusses evaluation criteria and economic effects of such schemes, noting that the air pollution schemes seem to have been most successful, while water users and fisheries can benefit (no doubt in the long-term) the most from protection of their resources. Finally Tietenberg emphasises the temporal aspect of the schemes, and how the size of the change can evolve relative to the policy implemented. More stringent limits are often accepted given lower costs and greater flexibility of meeting the limit.

2.2 Model-Based Markets

In this section we mainly focus on electricity markets, but we also briefly review gas and water market issues. First, we discuss a brief history of electricity market design including lessons from reform and institutional design. We illustrate the key components required in designing and operating an electricity market, using the National Electricity Market of Singapore (NEMS) as an example. Second, we discuss the key aspects of spot market design. These include bilateral versus pool trading, and the type of market interfaces and market price clearing mechanisms. We discuss aspects of grid operation, with regard to ensuring physical delivery, and this is followed by inter-locational, ancillary service, and inter-temporal modelling issue discussions. We finish that section with a discussion of the NEMS LP formulation. Third, we discuss forward markets. Discussions include the key differences between physical versus financial products, but highlight the need for physical modelling in both cases. We discuss details of a few derivative products, including: Call Options, Financial Transmission Rights (FTRs), and Weather Derivatives. We also discuss the handling of stochasticity. Fourth, we discuss gas markets, and fifth and finally, we discuss water market proposals.

2.2.1 History

McCabe et al. (1989) describe model-based markets as “smart markets”, while Bichler et al. (2010) describe them as “optimisation markets”. The smart markets we are interested in are those which are configured as auctions with side constraints, and cleared using a formulation, based on a mathematical optimisation. Clearing of such markets must therefore be automated, with participants trading well-defined resources.

McCabe et al. (1989) discuss the theoretical application of model based markets. Gallien & Wein (2000, p. 6) discuss the history of smart markets for many products, services, and industries. Bichler et al. (2010) states that model based markets are widely tried and tested in practice. While most models discussed here have one or more real markets in mind, research is also undertaken in the field of experimental economics. McCabe et al. (1989) developed a key model based around a market for natural gas. This work was later widened to include electricity markets by McCabe et al. (1991), and then water markets by Dinar et al. (1998), Murphy et al. (2000), Murphy et al. (2006), and Murphy et al. (2009). Plott (1994) discusses institutional design issues related to experiments, while Rassenti et al. (2002) explain how these experiments could help inform the privatisation / deregulation process, which is still progressing in many countries. It is worth noting that in all instances the central bidding process is similar. Key differences across electricity, natural gas and water models are in the type of constraints which are modelled, or indeed ignored, in the market clearing formulation, to represent the associated transportation system.

2.2.1.1 Why we focus heavily on Electricity Market Literature

There are model-based markets for natural gas and electricity, and similar markets have been proposed for water. Easter et al. (1998, p. 120) state that: “Like other public commodities (natural gas and electricity) subject to uncertainty in supply, transportation, and value, water markets can develop a full set of spot, option, and stock price markets within the constraints of the physical mobility and transaction costs.” Moreover, all three industries feature a centrally co-ordinated transport system of pipes, or in the case of electricity, cables. To date, though, most activity and application of model-based markets has been in the electricity industry, and in its literature. So this review focuses heavily on what has been done by the electricity industry, the areas where it continues to make improvements, and the lessons learned.

2.2.1.2 Lessons from Reform Process

So we know model-based markets work, but what has been learned from all that experience, and what issues are important for this thesis? At a general level is the importance of the local political –regulatory environment Gilbert & Kahn (1996) discuss regulatory reform from an international perspective. Newberry (2002) discusses the privatisation and restructuring of network utilities. Woo et al. (2003) discusses reform failures in electricity markets. Jamasb

& Pollitt (2005) discuss electricity market reform in the EU, while Zhang et al. (2008) discuss reform progress in developing countries. Correlje & De Vries (2008) discuss different patterns of reform. The general message is that reform is a unique process to a country, and indeed often regions within a country. There is no one size fits all model. Joskow (2008) lists many general lessons learned from over a decade of electricity market liberalisation. General points include carrying out the textbook reforms fully, having strong political commitment, and seeing reform as an ongoing process. Most reform has been possible in places where they were willing to follow the text book model. Correlje & De Vries (2008) say that one key issue is the reform starting position. Places with initial public (as opposed to private) ownership tended to reform more. And places where the prices charged for products and services at least covered the efficient economic cost also reformed more. Joskow (2008) states that a key aspect of reform is having the flexibility to adapt, by seeing the reforms as an ongoing process.

2.2.1.3 Institutional Design

The UK is widely cited as one of the first countries to undertake widespread reform across a range of industries. Armstrong et al. (1994) discuss institutional design and reform in electricity, gas, and water industries, amongst others. A common issue with utility industries, which tend to be natural monopolies, is the degree of regulation versus liberalization. Each industry in the UK was treated differently based on its unique challenges. Even within a single industry, such as the electricity industry, there is no “one size fits all” solution, as discussed by Correlje & De Vries (2008). There are different degrees of vertical integration and separation, with regard to generators and retailers. There are also different degrees of state versus private ownership.

A common theme with electricity markets is that a spot market trades (at least some of) the physical resource, and Hogan (1995) states that this must be coordinated by an Independent System Operator (ISO). The independence allows for third party access to the transport system (the power grid) to create competition, where possible, while the ISO also ensures the system is operated securely, and within regulated limits. Ruff (1999) states that where a market is appropriate, there must be a supportive political, legal, and social framework. There should be well defined property rights, and commercial rules, such as for prudential management, dispute resolution, etc.

The detail of such rules might be developed and enforced by a regulator, and/or established by law. But it might equally well be agreed by participants, under the loose supervision of a regulator. Such choices may have a very significant bearing on market outcomes, but they are likely to depend as much on the institutional history of a particular society, and the politics of the day, as on any technical analysis. So, while Section 4.4.2 briefly discusses the roles that various parties might play in the market, our main focus is on market design, not institutional design.

For a good example of the way a market could be set-up and operated see the Singapore EMC website, SEMA (2014b). The website provides a comprehensive set of information, relating to the ongoing operation of the system, and the market, including:

- Detailed price information and market reports;
- The status of current proposals with respect to rule changes, including participant submissions, and final rulings;
- Resolution of disputes; and
- Upcoming events.

It also provides a great deal of standing information, all of which is required for operating a market, including:

- Corporate, public, financial, and tendering information;
- Training and consultancy contact details;
- Processes, for consultation, rule changes, and dispute resolution;
- Performance standards for system operation, including ancillary services; and
- A set of market rules, including technical requirements for registering participants and generating facilities.

This last document, or set of documents, is central to the market, and the shape and bulk of it (well over 500 pages) gives some idea of the complexities involved in establishing a real market. The chapters are laid out as follows (where the total number of pages is given in parenthesis):⁴

1. Introduction and Interpretation of the Rules (46)
2. Participation(59)

⁴ This list excludes the detailed Appendices, and their number of pages.

3. Administration, Supervision & Enforcement (102)
4. Connection and Metering (4)
5. System Operation (76)
6. Market Operation (48)
7. Settlement (56)
8. Definitions (29)

Central to our concerns is Chapter 6, which describes the operation of the market. This is discussed in more detail in Section 2.2.2.8 below.

2.2.2 Electricity Spot Markets

Many detailed market design issues have generally been addressed in conjunction with reform. Wolak (2000) discusses the different electricity market designs that initially emerged. Zhou et al. (2003) say that the early experimental markets provided a rich set of lessons from the failings of the first UK and California markets, and the successes of the New Zealand (NZ) and NordPool (Norwegian and Swedish) markets. Cramton (2003) provides a comprehensive list of the good, the bad, and the ugly electricity market design features. He states that the basic design features should include voluntary spot markets for energy and ancillary (reserve) services, but that the market should also accommodate (physical) bilateral contracts and self-scheduling of generation. We focus first on “real-time” spot market arrangements.

2.2.2.1 Bilateral versus Pool Trading

Zhou et al. (2003) discuss how many of the US markets had begun to converge towards common design features. Correlje & De Vries (2008) discuss the two main market designs which generally emerged: First, a decentralised market, with bilateral contracting or voluntary power exchanges and; second, an integrated pool market.

Traditionally, bilateral trading has occurred across a physical boundary, typically between two adjacent vertically integrated power companies. In a bilateral market, though, suppliers and demanders may have physical contracts, and may request delivery of power across a centrally co-ordinated grid. Typically, now, an independent system operator manages connection and use of system for both generation and consumption, and deals with balancing,

losses, ancillary services, and congestion. Dettmer (2002) describes the revised England and Wales market design. This market is designed around bilateral trading, with a balancing spot market to deal with security and imbalance issues. Lucia & Schwartz (2002) say that in NordPool only a modest percentage of energy trades on the spot market, quoting a value of approximately 20% in 1999.

A pool market is where supply (and demand) is traded into and out of a central pool, rather than directly between participants. In the electricity industry, a pool market place is where offers from generators and bids from consumers (or their representative retailers) are centrally co-ordinated by an independent market and/or system operator. Onaiwu (2009) states that an electricity pool facilitates competition between generators, producing the price paid by buyers for electricity. There can be voluntary and compulsory pools. A compulsory (or mandatory) is typically a “gross pool” where all generation is sold through the spot market. Correlje & De Vries (2008) say that a mandatory pool market is preferred when the physical and economic aspects of trade are strongly connected. In a net pool, participants declare their contract positions, and trade deviations from those positions. This allows for self-scheduling, if participants do not wish to trade. If contract declaration is made voluntary we have a hybrid “open trade” model, which allows bilateral contracts to exist in parallel with the spot market. Participants can buy their power needs in the spot market, or have private agreements. In all cases, though, power delivered through the centralised grid must still be scheduled with/by the system operator, and transportation costs must be paid for.

While we later discuss bilateral trading in longer term contexts, we will focus on pure mandatory pool spot markets models, which are most commonly employed for real-time trading in the electricity sector. Here we first discuss experience with market interfaces and then with market clearing.

2.2.2.2 Market Interfaces

Pools may require either simple or complex bids, which may be based on audited costs, but are generally set freely by participants. Market participants submit bids, offers, and other required information through a participant interface. General lessons for designing such interfaces are that they should be simple enough to easily interact with, while ensuring they meet the underlying needs of the system. At a basic level, bids and offers are translated into

stepped demand/supply curves, where the bids/offers are ordered from highest/lowest marginal price/cost, respectively. See Section 3.6.2 for a more detailed example. Bidding is often more complex in real markets, though, because dispatch must account for the actual characteristics of real generation units. See Sections 2.2.2.6 and 2.2.3.1, for details on markets which offer more than one product, or service.

Even within the US markets there can be significant differences regarding how market participants interact, and what flexibility they have to bid and take electricity. Helman et al. (2008) compares many differences between the PJM and New York electricity markets, to illustrate design choices and trade-offs. For example, PJM requires a single energy offer, with multiple prices to be submitted for the full day, while New York allows offers to change hour by hour. New York also allows start-up offers to change hour by hour while PJM only allows start-up offers to change once every six months. The New York market design was based on setting prices for as many market features as possible, while PJM initially tried to keep physical scheduling facilities consistent with prior utility practice. Hence PJM allows non-priced bilateral schedules, and/or self-schedules.

2.2.2.3 Market Clearing

Power can be traded ahead of time, and/or in a real-time “spot” market. Either way, supply and demand must physically balance in real time to ensure secure operation of the grid. And the efficient economic outcome is where accepted supply offers and demand bids balance, thus maximising producer and consumer surpluses. Most markets for energy (and reserve) clear with all units priced at this common, or “uniform” optimum clearing price, where supply and demand balance. Two examples are the Australian and New Zealand NEMs’ (National Electricity Markets’).

By way of contrast, Dettmer (2002) discuss the UK spot market, in which participants pay/receive the prices they bid, a “pay-as-bid” concept. Oren (2004) argues that the non-homogeneous nature of the products in the reserve market, and market incompleteness give a high degree of product fragmentation and a pay as bid settlement might be promising. Helman et al. (2008) note that the England and Wales spot market requires trading participants to pay as bid. But Cramton & Stoft (2007) say there is a strong case for keeping uniform price for the energy spot market, and we will adopt that approach.

Many electricity markets are cleared using LP, while some have evolved to be cleared using Mixed Integer Linear Programming (MILP), given the on/off nature of generation units. The models take the participant submitted offers and bids and clear the market, producing (primal) dispatch instructions and (dual) prices at which the electricity is traded. These LP models incorporate, to a greater or lesser extent, the topological and electrical realities of the transportation system, and sometimes inter-temporal aspects, too.

2.2.2.4 Grid operation and physical delivery

Even with a private contract for electricity supply, there is no guarantee electricity can be delivered from the supply (“injection”) point(s), to demand (“withdrawal”) point(s), unless the trading parties have their own delivery grid. Otherwise power needs to be delivered across a centrally co-ordinated grid, controlled by an ISO (in the US, or a Transmission System Operator in Europe), who manages the network constraints, and often also the balancing / spot market.

2.2.2.5 Inter-Locational Modelling

Ventosa et al. (2005) emphasise that electricity cannot be stored, and that the transportation of power requires a physical link, namely transmission lines. Unlike water and natural gas, electricity moves “instantaneously” through its transport system, in which physical line losses are present. These attributes give rise to the need for near real-time co-ordination, hence electricity markets developed with a very strong emphasis on the representation of physical and economic connections. This has often led to the incorporation of a Locational Marginal Pricing (LMP) methodology, within the market clearing process. Hogan (2008, p. 1) describes this as the “Marriage of Engineering and Economics”.

LMP markets developed based on the seminal work of spot pricing electricity principles developed by Schweppe et al. (1988), and Hogan (1992). The theory of (deterministic) locational spot pricing of electricity is built around reflecting the shadow prices of constraints on an optimisation of power system operation, accounting for transmission constraints and security constraints. Hogan’s often quoted criterion is “bid-based, security-constrained, economic dispatch with nodal prices”. A guide to how mathematical programming could be used for electricity spot pricing is presented by Hogan et al. (1996). A dispatch based pricing

model for the New Zealand decentralised power system was presented by Ring & Read (1996), based on the thesis by Ring (1995).

See Alvey et al. (1998) for a discussion of the inter-locational market design aspects of the New Zealand (NZ) market, while Lu & Gan (2005) describe the Singapore market. In the US, Ott (2003) and Cheung (2004) discuss the PJM Interconnection and New England markets, respectively. Nodal pricing principles have also been applied to several other US electricity markets, including Midwest US, California, ERCOT (Texas) and New York. Wolak (2000) states that the US LMP market clearing formulations are not as detailed as the NZ LMP market clearing formulation. But many of the individual US markets are much larger than the New Zealand market, in terms of geographic area and population density.

Helman et al. (2008) discuss LMP (transmission) charges, including marginal congestion and marginal losses. Charges are computed based on total real-time dispatch, but settled against deviations from the day-ahead market. Uplift can be added to ensure generators are compensated for. They also discuss how LMPs can be averaged on an inter-locational basis. The classification of physical zones is usually based, in part, on system function. For example, supply zones can be set where there is a large concentration of generators who are injecting into the grid. Demand zones can be set where there is a load concentration, such as a city, or an industrial estate.

Other electricity spot markets could be described as hub-based. These feature several zones, or regions, where the zonal prices are the same unless there is a constraint between zones. As discussed at the Energy Efficiency Exchange, see eex.gov.au (2014), the National Electricity Market (NEM) of (Eastern) Australia has five trading regions, or hubs (New South Wales and the Australian Capital Territory, Victoria, Queensland, South Australia and Tasmania). Harris (2006) describes this as a postage stamp system with market splitting. Other examples are the European EPEX SPOT, the Nordic Nord Pool Spot, and the UK electricity market.

2.2.2.6 Ancillary Services

The system operator always manages ancillary services, which can be used to control frequency (often via three types of reserve: Primary, secondary, and tertiary), reactive power

(voltage regulation), and black-starts.⁵ There is often a hierarchy of reserves, some which have interchangeable properties. Each takes a different amount of time to come online. Helman et al. (2008) table the different times it takes for the ancillary service products as bid for in the US markets, from seconds to half an hour.

The co-optimisation of energy and ancillary service (reserves) markets is discussed by E. Read (2010). He discusses the history of co-optimisation formulations dating back to Ring et al. (1993), for a simplified model, and E. Read & Ring (1995), for a full AC nodal pricing model. Read states that these multi-commodity markets were first developed in New Zealand (see Alvey et al. (1998) and E. Read et al. (1998)), then Australia, and Singapore. Oren (2004) states that, by then, New York and California also co-optimised energy and reserve, and E. Read (2010) notes that this is becoming increasingly common.

There can be different limits on energy, and reserve bids, and different requirements on the way participants are asked to submit offers, and how the Market Clearing Engine (MCE) translates that into something that can be solved. E. Read et al. (1998) use an LP to co-optimise energy and reserve, so participants must submit both generation and ancillary service (reserve) offers. The same generation capacities cannot be used for both purposes, at the same time. So an integrated joint offer must be made, in which a generator offers blocks of a unit's capacity to supply energy and/or reserve, up to that unit's physical capacity. Different prices can be applied to the blocks, for example to reflect running a unit at different levels of efficiency. And each generator specifies caps reflecting how much their unit can ramp up, in each required reserve timeframe (e.g. 6 and 60 seconds).

2.2.2.7 Inter-Temporal Modelling

Helman et al. (2008, p. 181) discuss how the real-time and day-ahead LMP markets often include different forms of look-ahead within the auction clearing process. But that does not mean that they all employ inter-temporal optimisation. In fact some electricity markets only employ single-period models. Examples include New Zealand, (Alvey et al. (1998)), Singapore (Lu & Gan (2005)), and Australia (Wolak (2000)).

⁵ Re-energising the system by starting a power station after a system wide shut-down.

All of these market models feature ramping-rate constraints, to ensure feasibility by making the (real-time or simulated) end position from the last period's dispatch, the start position for the next period's dispatch. But this does not link the market clearings into a single inter-temporal optimisation. Nor does it account for the fact that it costs money, and can take time to safely start and stop thermal units, so that, once started, they are effectively committed across a number of future clearing periods.

In those markets participants are left to make their own commitment decisions, based on their own assessment of information provided in future market simulations. But Sioshansi, Oren, et al. (2008) say that in most US markets, suppliers are (or are moving towards being) centrally committed. Thus the PJM and New York markets, for example, model inter-temporal aspects, over a relatively short (daily) timeframe, to account for costs of, and restrictions on, starting and stopping power plants. Sioshansi, O'Neill, et al. (2008) say that the combination of central commitment and the look-ahead to start up generating units has generally meant the US day-ahead LMP markets are now cleared via Mixed Integer Programming (MIP), as opposed to LP. MIP turns units on and off, based on the expected value of their contribution over many periods.

The integerisation means that pricing is not consistent, since turning on/off a generating unit will commonly over/under supply the market, while also producing higher/lower than optimal price incentives for buyers. Various compensation arrangements must therefore be employed to ensure that participants committed by the model are not commercially disadvantaged. But the key point to note is that (prices in) these electricity markets are not only spatially connected, but also temporally connected. It also means that participants must specify more than just a price and quantity for energy, in their bids and offers.

2.2.2.8 Example of a spot Market LP Formulation

As discussed in Section 2.1.1.3, Chapter 6 of the NEMS documentation has several appendices. These generally relate to inputs, outputs, and dispatch data from the Market Clearing Engine (MCE), a detailed formulation. But the most substantial appendix (6D) details the market clearing formulation. In this section numbers in parenthesis relate to the approximate number of items involved in the NEMS MCE formulation. The first part of the appendix contains sections defining the index sets (58 types), parameters (94 types), variable

(58 primal and 3 dual types), and functions (40 types) used in the formulation. Of course many of these definitions are indexed by one or more index, leading to a very large number of variables in the final LP model. Details are provided on the pre-processing steps which are required to get the data into the required form. There is also a detailed prescription of how constraints are to be configured to deal with multi-unit generators, generator ramping constraints, operating and regulatory reserves, etc. The form of bids and offers for each product is stipulated, as are processes for dealing with tie-breaking issues.

The second part of Appendix 6D contains the LP formulation itself, including an objective function, to maximise benefit. There are also detailed sets of constraints for:

- Energy generation and purchases (5 types)
- Transmission system modelling (Node balance (4 types), Line flow (4 types), line losses (3 types), Line relaxation (5 types))
- Risk and operating reserve (Risk(7 types), Reserve(12 types), Matching requirements and availability (7 types))
- Regulation (12 types)
- Ramping (8 types)
- Generic and Multi-unit restrictions (2 types)
- Tie-Breaking (4 types)
- Violation Penalties (13 types)

Finally, there are sections on post-processing, price formulation and additional output details. We will not discuss the detail, and while most markets do not publish such explicit details of their market clearing formulations, a similar (less detailed) formulation can be found in Alvey et al. (1998). Our next focus is to review forward markets.

2.2.3 Forward Markets for Electricity

Electricity spot markets co-ordinate the market-clearing and dispatch, while respecting the physical transport constraints of delivering electricity in real time. But, while this may provide reasonable assurance of physical supply, it does not provide any guarantee at all with respect to future prices. Even in some pool markets, most power is actually traded in the day-ahead contract market. More importantly, Cramton (2003) states that participants should

secure long-term physical and financial contracts, where possible, to avoid volatile short-term spot prices.

For example, Green (1999) describes how prices in the original England and Wales Pool market (where all energy was traded on the spot market), were largely hedged via Contracts for Differences (CfDs). Likewise, Lucia & Schwartz (2002), discuss the Norwegian electricity market, NordPool, which supports bilateral trading of longer term contracts. They report that, in 1999, the financial futures market traded approximately a three times the volume as the physical spot market, while nine times as much was traded via physical “Over-The-Counter” (OTC)⁶ bilateral trades.

In fact, all jurisdictions allow participants to secure, and privately trade longer term (e.g. multi-year) contracts, of some form. But the “products” employed vary widely, as do the institutional arrangements for creating and trading them:

- The products sold for future periods can be “physical” or “financial”;
- And they can be tailored to consumers’ particular needs, and traded bilaterally (over-the-counter, with varying formats), or standardised and traded through organised markets, such as power exchanges, or futures markets.

In the next sub-section we discuss the relative merits of financial versus physical contractual “products” which could be traded to achieve that goal, followed by a discussion of more detailed products: call options and Financial Transmission Right (FTR) markets. Finally we discuss the handling of uncertainty, in such markets.

2.2.3.1 Financial versus Physical Products

In many markets, participants who contract (bilaterally, or through a pool or an exchange) physically are expected to take delivery in real time. In electricity markets that may not be practical. Indeed, given the situations which are likely to arise in the transmission system, it is often difficult to define exactly what is meant by a physical delivery of particular contracts in an electricity network, let alone control it to precise levels. Penalties may apply but, at a minimum, deviations from that schedule might be bought/sold on the spot market.

⁶ Which actually means privately between two parties.

Still, we could imagine a participant having a forward physical contract to buy CQ^t units at a price of CK^t per unit, then buying (or selling) $q^t - CQ^t$ units on the spot market, at the spot Market Clearing Price (MCP), λ . Hence the Total Cost of the Physical Contract, TCPC, to the participant would be:

$$TCPC = CK^t \times CQ^t + \lambda^t \times (q^t - CQ^t) \quad (2.1)$$

As an alternative to buying the physical energy there are a range of financial products that can be bought. Unlike a physical product, where the seller and buyer must supply/consume the physical energy, with a financial product there is no physical delivery of energy, merely a cash settlement in line with the terms of the contract. But a financial product/instrument still enables market participants to hedge their position in the market; that is they offset their perceived risk of losses/gains due to unfavourably high/low (for bids/offers respectively) spot prices. Thus Stoft et al. (1998) note that financial derivatives provide socially beneficial hedging, while Deng & Oren (2006) say that, when well understood, futures and derivatives can share and control undesired risks, via structured hedging strategies.

A good tutorial on the products which are traded in electricity markets is provided by Pérez-Arriaga (2010). Stoft et al. (1998) and Deng & Oren (2006) discuss various electricity futures and derivatives. They note that products are often detailed by time of day, such as on-peak, off-peak and round the clock. They go on to outline the type of financial products traded, including:

- Forwards, to buy/sell a fixed volume at a pre-specified, forward (contract) price.
- Futures, similar to forwards but based on more standardised contracts.
- Financial Swaps, where the buyer pays a fixed price for electricity, which can be seen as a strip of forwards with identical prices but multiple settlement dates.
- Options, based on a pre-specified strike price and volume. These include Contracts for Difference (CfDs), calls and puts, which could depend on timing, delivery location, and more exotic variable volume “swing” options and fuel related “spark spread” options.

The most common options are of the CfD form, and these may also be seen as a form of swap. They provide a Total Contract Pay-out, TCP, determined by the difference between the final (spot) market clearing price in the actual market, λ^t and the agreed “strike price”, CK^t , for the agreed contractual quantity, CQ^t . That is:

$$\text{TCP}(\text{CQ}^t, \text{CK}^t) = \text{CQ}^t \times (\lambda^t - \text{CK}^t) \quad (2.2)$$

The payout for a CfD is shown in Figure 2.1 below.

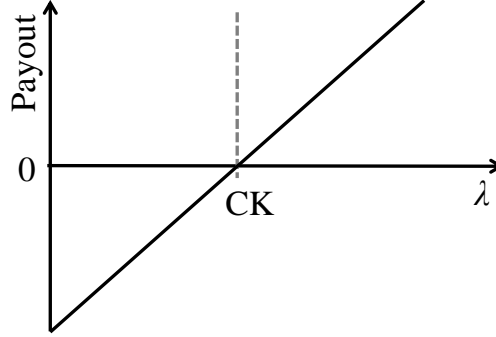


Figure 2.1: Illustration of a CfD pay-out

If the participant holding a CfD actually buys a total of q^t units, at the spot market clearing price of λ^t per unit, then the Total Cost of the Financial Contract, TCFC, to the participant is:

$$\text{TCFC} = (\text{CK}^t - \lambda^t) \times \text{CQ}^t + \lambda^t \times q^t \quad (2.3)$$

But note that, if the Lambda terms are collected on the RHS of Equation (2.3) we get:

$$\text{TCFC} = \text{CK}^t \times \text{CQ}^t + \lambda^t \times q^t - \lambda^t \times \text{CQ}^t = \text{CK}^t \times \text{CQ}^t + \lambda^t (q^t - \text{CQ}^t) = \text{TCPC} \quad (2.4)$$

Thus there is an equivalence between the financial and physical contract payouts in Equations (2.1) and (2.3). So, economic theory says that if participants' bids appropriately reflect the value of energy to them, they should be indifferent whether they receive the energy or the money.

Although economically equivalent, these financial products do not deliver the physical energy. This could be seen as an advantage and a disadvantage to different market participants. While engineers, or politicians, might have the mind-set that they want delivery of physical energy, the main advantage of the financial product is simplicity. Specifics of the market, such as the physical transmission system, do not feature in the way they are defined, and that makes them easy to understand, value, and settle. Thus they can be traded by traders who know little about the details of the electricity system, and that adds to market liquidity. This is good, because the greater the liquidity, the more likely participants are to arbitrage away excessive economic rents, thus moving the futures price closer to its optimum market clearing price.

2.2.3.2 Other Derivatives

Apart from CfDs there are more complex (also called “exotic”) options. See Wystup (2007) for a comprehensive guide to options and other structured products. In this section we focus on Calls and FTRs, which are commonly traded in electricity markets, and weather derivatives, which are closely analogous to the index dependent products we will propose in Chapter 4.

Call Options

As an alternative to a CfD, participants could agree on a call option for a quantity, CQ^t , at a strike price, CK^t . A financial call option is where a buyer is paid the profit made on the specified number of units when the price goes above the specified strike price, in the specified period.⁷ Ignoring the initial cost, the buyer always receives a non-negative pay-out when the market is cleared:

$$TCALLP = CQ^t \times \max\{\lambda^t - CK^t, 0\} \quad (2.5)$$

Compared to a CfD, the main advantage of a call option is that the holder is insured against the downside risk, i.e. the risk of the market clearing price rising above the agreed strike price, without taking on any obligation should the market clearing price fall below the agreed strike price. Such options attract a higher premium, though, because the buyer is not taking on any obligation to pay out when prices are low. The payout for a call is shown in Figure 2.2 below.

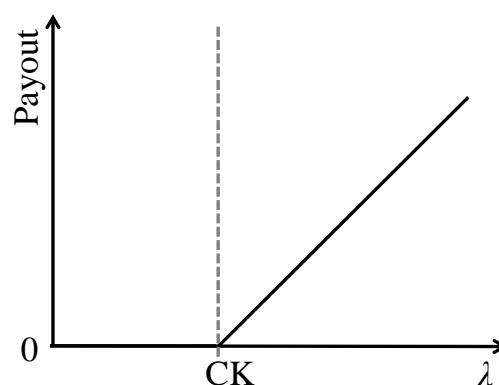


Figure 2.2: Illustration of a Call Option pay-out

⁷ This is a “European” option. We will not concern ourselves with the more complex “American” form.

FTRs

Physical constraints produce locational market clearing prices, from which physical and financial trading occurs, as explained by Hogan (2002). Network interactions, such as loop flow, prevent physical transmission rights being feasible. But Hogan (1992) defines transmission prices as the difference in LMPs at buses, where the LMP summarizes all the information about the interactions in the network.

Kumar et al. (2005) provide a comprehensive literature review of a wide range of congestion management instruments. The most common of these is an FTR, which gives the holder the financial right to claim the difference between two defined LMPs. FTRs are effectively locational swaps for hedging price risk between two different locations. FTRs can also be seen as options where if the price difference is positive/negative the holder receives/pays the difference in the LMPs. In principle, that price difference would include loss effects, as in the New Zealand FTR market, discussed by E. Read & Jackson (2013). But Hogan (2012) notes that FTRs typically only account for the congestion component of the LMP differences. FTRs are now routinely traded in many markets around the world, and Helman et al. (2008) provide a detailed explanation and examples.

Kristiansen (2004) discusses the various ways that FTRs can be issued to participants. He discusses how holders of FTRs can sell these in re-configuration auctions, which can enhance price discovery. Ma et al. (2003) discuss re-configuration auctions in the PJM market. These monthly auctions allow FTR holders, and other market participants, the opportunity to sell, and buy, residual FTR capability.

Weather Derivatives

A weather derivative is a form of insurance against unforeseen meteorological events. As discussed by Zeng (2000), just like other derivatives, there is a contract between two parties which specifies how payments are exchanged, based on a well-defined weather index, H , and a strike value, CKV^t . Just like CfDs and calls, there is a contract period, and a payment per-unit, CM^t , often termed a “tick”. In fact as Zeng states “There are three commonly used forms of weather derivatives: call, put, and swap”.

Alaton et al. (2002) say that different types of options can be combined to cap the maximum payout. They illustrate a common form of payout, which is a truncated linear payout

function, with maximum and minimum payout limits. This option is constructed by buying one call option with a low strike value H_{MIN} , and selling a second call option with a higher strike value, H_{MAX} , as in Figure 2.3.

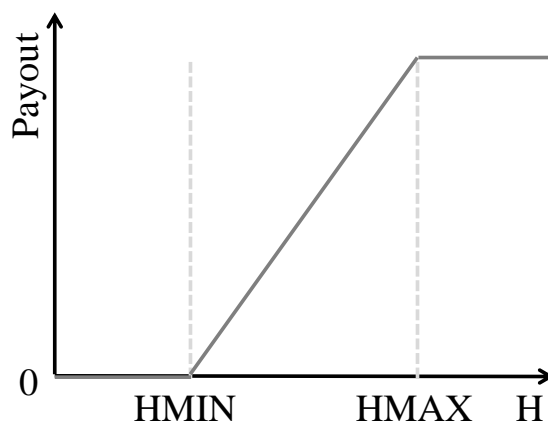


Figure 2.3: Illustration of a Weather Derivative with a Truncated Linear Pay-out

Benth & Benth (2013) say that weather derivatives were first traded OTC in 1996, based on a contract for selling electricity between two energy companies. By 1999 weather derivatives were standardised, and traded on the Chicago Mercantile Exchange (CME). They say, that according to the Weather Risk Management Association, the “notional value” of the market for weather derivatives in 2011 was USD 11.8 billion. Ruegg (2013) presents a good overview of the variety of industries that are using weather derivatives to manage their risk exposure to weather related issues. Y. Lee & Oren (2009) also note that weather derivatives markets are expanding rapidly as diverse industries seek to manage their exposure for weather risks, while Y. Lee & Oren (2010) note that the energy industry is one of the main buyers of weather derivatives. They also say (page According to Y. Lee and Oren (2009, page 703), “The notional value of CME weather products in 2004 was \$2.2 B, and grew ten-fold to \$22 B through September 2005, with volume surpassing 630,000 contracts traded (CME, 2005). In 2006 the value of traded weather instruments rose to \$45 B.”

Thus weather derivatives are well defined financial instruments which have been trading for over fifteen years. These types of derivatives, based on one or more underlying indices are clearly attractive to traders, and this is no doubt partly due to their often linear payouts, which are easy to understand and value. This experience suggests the viability of a similar product for water trading, as proposed in Section 4.7.2.

2.2.3.3 The Role of Physical Modelling

Physical models clearly play a critical role in clearing electricity spot markets, because the market must ensure physical feasibility, in real time. The role of physical modelling is less clear with respect to futures trading, though. It should be recognised that most financial contract markets operate quite well by simply matching bids and offers for simple standardised products, without any need for physical modelling to support market clearing. This is not always the case in electricity markets, though, and the same may be true in water markets.

First, physical feasibility becomes more important as real time approaches. Helman et al. (2008) say that while many forward markets do not have to be concerned about the intricacies of the transmission network, as real-time approaches the risk of non-delivery needs to be considered. Thus, in pools where power is traded in the day-ahead contract market, market-clearing models are used to generate physical and/or financial future contracts.

Singh (2008) note that many US markets feature day-ahead markets. Helman et al. (2008) note that both the day-ahead and real-time US markets tend to feature “security constrained unit commitment” auctions. In this setting the market scheduling/clearing software decides which generators should run for each hour of the subsequent day. Decisions are based on start-up and energy offers, but also based on other financial and physical parameters, and network constraints. At the same time, short term forward (financial) contracts are created to match the planned delivery of physical power. Thus, in this instance, financial contracts are formed on the basis of the results from a detailed system model which simultaneously clears the market, and determines physical dispatch.

Second, physical modelling plays a critical role in FTR markets, because network constraints and losses not only cause the LMP price differences, but create “transmission rents”, which provide the revenue to fund FTR payments. These are generated naturally when the market clears, because the prices paid by loads account for losses and congestion, and so exceed the payments made to generators, resulting in a spot market surplus. Hogan (1992) showed that, ignoring losses, if the powerflow corresponding to a set of FTRs, can be supported by the physical transmission system capability, then the rents generated by market-clearing are sufficient to support the FTRs themselves. Hence, an FTR market must be a model-based

market, because the physical and financial equivalence means having to run a physical model to determine “revenue adequacy”.

Similarly, the point of a “re-configuration” auction is that the pattern of FTRs resulting from the auction may be superficially quite different from the pattern of FTRs submitted to the auction. Each reconfiguration auction must ensure that the residually awarded FTRs are simultaneous feasible with the previously awarded FTRs, such that they can be supported within the physical transmission system capability. Once more, the award of financial contracts is based on solutions from a physical system model.

In a similar vein, Raffensperger et al. (2009) discuss the need for physical modelling to support a groundwater allocation market, to ensure the simultaneous feasibility of a proposed pattern of trades between various locations and times. That proposal is discussed in Section 2.2.5 below. But the key point is that detailed physical models of the type discussed here may well need to be solved in order to support the trading of (even) financial contracts for future water. And, given the importance of uncertainty in water systems, the treatment of stochasticity is a potentially important issue in such models.

Handling Stochasticity

Many electricity markets feature forward simulation, by re-running the market-clearing under a number of scenarios to produce forward trial clearings. These runs do simulate future market-clearings, but future uncertainty is left to participants to manage by adjusting offers, and/or trading financial instruments in whatever market they may choose. Those markets have no direct connection to the spot market, and are normally operated quite independently of the (spot) market operator, often as part of a general futures market, and not necessarily even in the same jurisdiction.⁸ Critically, these markets are typically organised in such a way that inter-temporal storage is owned and managed by individual participants. So the management of that storage capacity does not need to be modelled in the market-clearing process.

An example of where the spot and futures markets are collectively managed by the same organisation is NordPool. Marckhoff & Wimschulte (2009) say that CfDs, for most network

⁸ Ironically, historical circumstances have led to the New Zealand /Australian electricity sectors trading electricity futures in markets operated by each other’s exchanges, the ASX and NZX, respectively.

regions, are listed on NordPool. CfDs are used to hedge price spreads, and these can be combined with futures or forwards on the system price, to hedge against price changes over time. Botterud et al. (2010) say that the NordPool futures and forwards are cleared on the Eltermin market, and this market also includes the OTC market, which predominantly trades forward contracts and options. Even so, NordPool appears to clear deterministically, taking no direct account of any underlying uncertainties in the market-clearing. This is consistent with storage capacity not being owned by the pool and so not needing to be modelled in the market-clearing process.

NordPool is heavily hydro dominated, though, and there have been numerous stochastic models in the literature. Some models focus on constructing bidding curves, such as those of S-E Fleten & Lemming (2003) and S-E Fleten & Pettersen (2005). A model to optimise bidding strategies is proposed by S-E Fleten & Kristoffersen (2007). They describe a Stochastic MILP model, which they use to compare stochastic versus deterministic optimisation approaches. They note that the stochastic approach is less sensitive to data changes than the deterministic approach. S-E Fleten & Kristoffersen (2008) test a Stochastic MILP on the NordPool market, from the perspective of short-term hydropower production planning. They conclude that the expectation-based objective function criterion results in risk neutral production planning, although most power producers are actually risk averse. So, to account for this, they suggest adding a risk function into the objective function. They note that while prices and inflow are uncorrelated in the short run they are usually negatively correlated in a longer time span i.e. as inflows decrease prices tend to rise and vice-versa. So, uncertainty is clearly a major issue, and the central focus of forecasting models. Davison et al. (2002) refer to two main modelling approaches for estimating future market clearing prices under uncertainty: Top-Down and Bottom-Up, being based on “data driven” and “detail driven” modelling methods, respectively. S-E Fleten & Lemming (2003) describe these two approaches as “Financial” and “Bottom-up”, and discuss an integrated approach to estimating “forward curves”, by combining information from these two approaches. Tipping & Read (2010) and Möst & Keles (2010) review various stochastic models in both the top-down and bottom-up groups, and report hybrid modelling approaches, with utilise both approaches. As noted by Tipping and Read, the top-down approach typically involves time series modelling, but the literature in this area is vast, and outside our area of interest here.

Chapter 2: Literature Review

Davison et al. (2002) states that bottom-up models are typically found in the engineering literature, because they take account of the physical system. Many are also found in the Operations Research literature. Again, as for top-down models, the literature in this area is vast. Wallace & Fleten (2003) provide a comprehensive literature review of stochastic optimisation models in both regulated and deregulated electricity (and wider energy) markets, but most of those models are not directly relevant here. In Section 2.3 we have discussed reservoir management models, which we use in our experiments. Our focus here, though, is on the possible use of bottom-up models to clear a forward market under uncertainty, and we have not found any papers on that topic. So this is one area where this thesis will attempt to add to the literature.

Unlike most of the ancillary services co-optimisation models discussed in Section 2.2.2.6, Bouffard et al. (2005), Garcia-Gonzalez et al. (2008), Bouffard & Galiana (2008) and Ruiz et al. (2009) do discuss a somewhat related set of models. These models all propose clearing electricity spot markets under uncertainty, where the emphasis is on managing the short-term issue of system security. Rather than simply specifying ancillary service requirements, these models explicitly aim to ensure that there is enough reserve, and a contingency response plan and resulting powerflow, to allow the system to operate without disruption, within some pre-specified control limits, under a range of different scenarios. These types of stochastic models can be used to produce prices for all scenarios, and these prices could, in principle, be used to buy an amount of reserve for each scenario. More realistically, these models could form the basis from which the system operator procures ancillary services contracts, under which participants would be contracted to provide contingency responses which varied, not according to the contingency, *per se*, but depending on the frequency drop, as discussed in Section 2.2.2.6.

There are analogies here with the kind of water market developments we discuss in Chapter 4, albeit water markets tend to operate over much longer timeframes. Just as in the “ancillary service” market, the objective there is to establish contracts defining the way in which system resources will be allocated to meet participant demands, over the range of uncertainties that can occur, and impact on system operation. So, in an electricity market an ancillary services provider could be contracted to provide a contingent ancillary service response as a linear function of system frequency. Similarly, in a water market, a water supplier could be contracted to provide water as a linear function of a hydrology index. The water system

operator is actually saying that they do not know exactly how much water they are going to supply, but the amount they will supply is well defined, for each situation that will occur. This provides participants who possess such contingent contracts with the kind of assurance provided by ancillary services contracts in electricity markets.

In our case, the uncertainties involve both the natural supply of water to the system, and the demand for water drawn from the system. But the approach we have taken requires modelling of system storage within the market-clearing process, and intra-system storage is not a significant feature of electricity system. It is, though, an important aspect within gas markets, as now discussed.

2.2.4 Gas Markets

Unlike electricity, natural gas can be stored within the transportation system. Susmel & Thompson (1997) note that gas storage can be seen as a substitute for transportation, and this has helped reduce price volatility in the US gas markets. Thus storage becomes a major physical feature of a gas transportation system. Bushnell (1998) discusses natural gas systems, and emphasises that the gas is stored within the networks pipes and tanks. Gas also takes time to move between two locations. This means that, in some regards, there is less need for real-time co-ordination, compared to electricity.

Thus the degree of sophistication surrounding the models, and associated spot and futures markets, has generally been limited to operate a hub based approach. Examples are the Henry Hub in the US, the National Balancing Point in the UK, and the Title Transfer Facility in the Netherlands. The US hub is a physical delivery point, whereas the other two European hubs are virtual delivery points. On the East-Coast of Australia there is an intra-state Short Term Trading Model with trading hubs in Sydney, Adelaide, and Brisbane.

With hubs, the gas is generally delivered between injection and withdrawal points by a (common or private) carrier. Carriers enter into agreements with one or more shippers. These agreements are generally classed as contract carriage or common carriage. Common carriage gives open access to any party wishing to ship gas, and this can be on a first-come, first-serve basis, or based on paying access fees. Contract carriage can be more restrictive and the carrier can pick and choose shippers.

KEMA (2013) explains the concepts of entry-exit and point-to-point systems, including explaining how they could coincide. An entry-exit system defines the price to sell, and buy volumes of gas across the transport system. The price differences can reflect congestion. A point-to-point system has a defined flow path.

There are many gas models. O'Neill et al. (1979) discuss a regulated gas market model. Equilibrium models to maximise social welfare are reported by Gabriel et al. (2003) and Brooks (2003), the Gas System Analysis Model and GRIDNET model, respectively. Gabriel et al. (2005) discuss the GRIDNET model, and in their review of other models they note a peak-load pricing model for Great Britain by Tzoannos (1977) and an “important recent example” being the Market-Clearing Engine for the state of Victoria in Australia, this was reported at a conference by Pepper & Lo (1999). Zheng et al. (2010) discuss many different types of gas models, focusing on models implemented for the US market, including the Department of Energy’s “National Energy Modeling System”, within which the Natural Gas Transmission and Distribution Module is an important sub-module.

E. Read et al. (2012) discuss a primal and dual LP nodal formulation which was somewhat considered for market implementation in Victoria. This model takes explicit account of the inter-locational, and inter-temporal utility network, and they note that this model is similar to LMP models for electricity. At the time (in the late 1990’s) this type of detailed LMP model was employed in electricity markets, for example in New Zealand and Australia.

Pepper et al. (2012) discuss the implementation challenges associated with this model. Prior to implementation a key design objective of the proposed market was to review the potential for close integration with the existing regional electricity market. This was on the basis that both energy markets were closely associated; given the growing importance of gas generators in the Victorian electricity market.

It was ultimately decided to trade inter-temporal variation but on a much simpler basis, ignoring nodal (LMP) constraints, but incorporating storage. A single tank system, cleared with a single daily gas price was initially implemented, as there were only minor gas transmission losses⁹ and a lack of congestion for the transport system. The success of this gas market meant that, over time the system was used much more and the risk of greater

⁹ Due to the “light” nature of the gas and thus relatively cheap transportation costs, to compress the gas through the network. In comparison to relatively “heavy” water being pumped through an equivalent water network.

congestion led to a market re-design. A major change was that the market clearing frequency was revised to being re-cleared once a day to several times within a given day. Inter-locational linkages were able to be ignored by simply accounting for the inter-temporal linkages on a finer timescale. The transportation system was also developed to reduce congestion.

2.2.5 Water Market Proposals

Up to now we have discussed models which have been implemented. There are other proposed model-based markets, which have yet to be considered for implementation. A number of smart markets have been proposed by the Water/Energy Management Research Groups at the University of Canterbury, in New Zealand. Water markets have been proposed which take explicit account of the inter-locational groundwater network, which incorporates storage, Raffensperger et al. (2009). In fact a wide range of market clearing models were developed which allocate their resource consents via their unique ‘transportation systems’, including models for: Groundwater,¹⁰ Raffensperger et al. (2009), Nitrate Removal from Water, Prabodanie & Raffensperger (2007), Water Pollution, Prabodanie et al. (2009). and Sediment Discharge, Pinto et al. (2012). Most of these models are formulated as deterministic LPs except for the latter which considers uncertainty within the modelling framework.

2.3 Reservoir System Models

2.3.1 Introduction

There is a vast literature on reservoir system models, which have been applied to many theoretical and practical situations. We are not attempting to conduct a comprehensive literature review of such models, which are not the primary focus of our research. We are merely looking to use an existing modelling method, as a research tool to investigate various water market design options.

Many reservoir models in the literature were developed to optimise hydroelectric power generation systems. The electricity sector is a large scale industry which used “high” technology from an early date, requiring real-time, and accurate, data, allowing for

¹⁰ Where the modelling complexity is inter-location trading, under a capacity constrained catchment.

widespread use of Operations Research (OR) techniques. Interest was further stimulated by the fact that the electricity sector is also environmentally sensitive, an essential service to society, and a growth engine of the economy. Utility planners' needs for optimisation models have changed over time to account for environmental concerns, increased competition, and growing uncertainty.

General modelling methods and techniques that have been applied to reservoir management are described in Simonovic (2012), Loucks et al. (1981), and Loucks et al. (2005). A major survey, across a wide range of optimisation approaches, is given by Labadie (2004). Optimisation and simulation modelling approaches are surveyed by Rani & Moreira (2010), Wurbs (1993) and Yeh (1985).¹¹ Dynamic Programming (DP) applications and limits are discussed by Nandalal & Bogardi (2007). Many of the early DP models are surveyed by Yakowitz (1982), and later by Stedinger (1998). We focus on the two most popular optimisation techniques: LP and DP. See Dantzig (1955) and Bellman (1957), for the seminal works on each respective technique.

Our water market models will compare the implications of using deterministic versus stochastic optimisation to clear water markets. We do not intend to solve models for the optimisation of particular systems, but instead wish to present generalised insights and results, which are somewhat non-system specific. Thus we need to select SLP and/or SDP techniques to conduct both qualitative and quantitative research. First, we wish to formulate some theoretical market model(s). Second, we wish to present various water market design option assessments. Third, and finally, we wish to conduct a series of computational experiments.

2.3.2 Simulation versus Optimisation

Our computational experiments will involve simulation of real-world outcomes, assuming the output of optimisation models. In terms of the implementation methodology, Labadie (2004) concludes that it is important to link optimisation models with simulation models. The operating rules which are produced by the optimisation are input into the simulation, to analyse how well the optimised system would actually perform. The policy can also be re-optimised after decisions are tested within the simulation. Tejada-Guibert et al. (1993) say

¹¹ This author presents a more mathematical review (including formulations) of many of the types of models.

this can lead to better system performance. With regard to combined simulation and optimisation modelling, Rani & Moreira (2010) note that many reservoir systems are modelled by LP and DP optimisations which are combined with simulation, and discuss a study by Lund & Guzman (1999) who simulated many operating policies for series and parallel reservoir operations. They say that in most cases simulation modelling is the standard by which operating rules are refined and tested. Our water market models will be comprised of a modelling suite, including optimisation and simulation modules. We will report the results of “real-world” simulation using the optimisation outputs.

2.3.3 Deterministic versus Stochastic Returns

An LP can easily clear a deterministic water market. But what about situations where the modelling solution has ignored the underlying uncertainty associated with river flows? Wallace & Fleten (2003) discuss how Massé & Boutteville (1946) walked readers through the arguments as to why deterministic models are often not good enough, and how “looking at a deterministic future is far too optimistic”. E. Read & Boshier (1989), and more recently Philpott (2013), report that deterministic models can introduce significant bias when the real system naturally features underlying uncertainties. Philbrick Jr. & Kitanidis (1999) and Wallace & Fleten (2003) discuss further issues associated with deterministic models, and how these underestimate the true cost (and risk) of running out of, or spilling, water.

General stochastic programming techniques are discussed by Kall & Wallace (1994). SLP with recourse and SDP are the two modelling methods we review further.

2.3.4 SLP

SLP was developed by Dantzig (1955). Yeh (1985) discusses a wide range of SLP techniques for water resource management. Kall & Wallace (1994) provide a more recent overview of SLP. Clearly a key restriction of any SLP model is that all the constraints must be linear. Thus, for water systems with underlying non-linearities, these must be approximated into an acceptable linear form. In an SLP the underlying stochastic process is modelled as a scenario tree. Høyland & Wallace (2001) discuss the modelling of scenario trees for multistage SLPs. A scenario tree defines how the uncertainty evolves over time. The tree always starts from (the root) a single known point, or node. A node represents a modelled state, while arcs represent realisations of the uncertain variables. They also note that “A path through the tree is called a scenario and consists of realizations of all random

variables in all time periods.” The scenario tree structure enforces non-anticipativity conditions on the decisions. That is, decisions cannot be based upon the outcomes of uncertainties that have not been realised when the decision is made. Higle (2005) states this is to “impose the condition that scenarios that share the same history (of information) until a particular decision epoch should also make the same decisions”.

Wallace & Fleten (2003) note that, while a “deterministic multi-period optimisation yields decisions for all periods, a stochastic approach only yields policies or strategies”. Even so, an SLP must be run from a unique starting point (the root node), i.e. a known storage level, from which the scenario solutions gradually diverge. Starting from a single storage level is ideal for a specific real-world system, but we wish to undertake an academic exploration which highlights generalised insights on a non-system specific model. Specifically, for our experiments we want a technique which will allow a wide range of simulations, quickly, to determine the impact of policies which have been optimised (and are equally optimal) over the whole range of possible state space values. Rani & Moreira (2010) note that often a lot of computational power is required to solve operational multi-reservoir system models, especially models which account for uncertainty.

2.3.5 SDP

SDP was developed by Bellman (1957), and Bellman & Dreyfus (1962). Since its development, SDP has been applied in many industries, to model many issues within those industries, including general water resource problems. DP constructs an optimal solution policy by breaking the problem into a number of simple (independent) sub-problems, as follows.

A DP problem is broken into sequential decision stages (typically periods), $t=1, \dots, T$. Within each period DP assumes the existence of a “state space”, which defines the range of possible values that can be taken by a state variable, z^t . A state variable is bounded, $0 \leq z^t \leq Z^t$, and generally discretised. There is an optimal decision x , in each stage and state, to be chosen from a set of feasible decisions for that stage, that might depend on the state, $x^t \in X^t(z^t)$. There is an objective function which, for simplicity, we assume to be maximising the sum of intra-period benefits over the horizon.

So, as input for the DP, there is an intra-period benefit function B , which is defined as a function of each stage, *state*, and decision, $B^t(x^t, z^t)$.¹² There is also an EOH (End of Horizon) value function, at $T+1$, which is defined as a function of each state, $V^{T+1}(z^{T+1})$. Taking a decision x^t , at state z^t in period t , means that that state transitions to a new state z^{t+1} in period $t+1$, according to a “transition function”, $z^{t+1} = f^t(x^t, z^t)$.

The goal of DP optimisation is to find the optimal decision, $x^{*t}(z^t)$, for each state, and stage.¹³ A value function, $V^t(z^t)$, is defined as a function of the state variable, for each stage. That value function defines the optimal return of the system, if it were to be in that state, at that stage, and a sequence of optimal decisions was followed from that state on, to the end of horizon. Thus it can only be determined once we know the optimal decision for each future state, and stage. To determine these optimal decisions, and value functions, the DP algorithm works backwards from the EOH, where the value function, V^{T+1} , is assumed to have been specified exogenously, as above. In general, once the value function $V^{t+1}(z^{t+1})$ has been established for the set of states (z^{t+1}), the algorithm can step back to the prior stage, t .

For each state z^t , the immediate benefits $B^t(x^t, z^t)$ can be computed for the set of all feasible intra-period decisions from that state, $x^t \in X^t(z^t)$.¹⁴ The corresponding future returns are specified by the value function for the state transitioned to $V^{t+1}(z^{t+1}) = V^{t+1}(f^t(x^t, z^t))$. The value function for z^t , $V^t(z^t)$, is then determined by the decision $x^{*t}(z^t)$ which maximises the sum of immediate and future benefits, i.e.:

$$V^t(z^t) = \max_{x^t} \{B^t(x^t, z^t) + V^{t+1}(f^t(x^t, z^t))\} \quad (2.6)$$

The algorithm stops when it reaches stage 0, the Start of Horizon (SOH). If we know the initial state, there is a single optimal path across all stages, defining a unique optimal state in each stage. Otherwise, there is an operating policy, defining an optimal decision for each state, and stage. There is also a corresponding value function, for each state and stage, representing the remaining value to the end of the horizon, for following the optimal operating policy.

¹² More generally this could be a return, reward, or cost function.

¹³ Note that, implicitly, the (vector of) state space variable(s), z^t , is thus assumed to define all the necessary, and available, information to enable computation of the optimal decision for that state and stage. For simplicity we assume there are not multiple optima, so there is only one optimal decision in each state, and stage.

¹⁴ In reality, an efficient algorithm is likely to exploit properties of the specific functions involved (e.g. monotonicity, convexity or differentiability) to minimise the number of decision options for which benefits need to be computed. But here we are describing a basic algorithm that will work for functions of any form.

So far, we have given a general description of a deterministic DP implementation. With an SDP the cost/benefit is uncertain, as is the next state. SDP only produces a policy, which is used to guide decisions under uncertainty. We do not know how the optimal policy will actually perform until it is tested via a simulated reality, or through actual practical application.

With SDP the objective is to find the best expected outcome. We now assume a random variable, y^t , at each stage. This models the underlying stochastic process. The transition function becomes $z^{t+1} = f^t(x^t, z^t, y^t)$, $t = 0, \dots, T$. The intra-period benefit function becomes, $B^t(x^t, z^t, y^t)$. At each stage the problem becomes to find an optimal decision $x^{*t}(z^t)$, for each state, and thus determine the expected value function:

$$V^t(z^t) = \max_{x^t} E_{y^t} \{B^t(x^t, z^t, y^t) + V^{t+1}(f^t(x^t, z^t, y^t))\} \quad t=T, \dots, 0 \quad (2.7)$$

The output of this process is again an operating policy, still with a single optimal decision for each state, and stage. If we know the starting state, we get one decision in the first stage, but subsequent decisions depend on what happened in prior stages. So, unlike the single trajectory we get from deterministic DP, we have an expanding network of possible paths. But that policy, and/or solution network, is still only optimal for the assumed EOH value function.

Alternatively, instead of running the algorithm from the last stage to the start of the first stage, it could be run to equilibrium. This effectively converts the SDP into a Markov Decision Process (MDP), assuming an infinite time horizon. One way to find the equilibrium is to set the EOH value function to match the SOH value function, and the algorithm is iterated, until the SOH and EOH value functions are approximately equal. In that case, under some mild assumptions on the underlying stochastic process, the process produces the long-run equilibrium, or steady state, operating policy.

This basic SDP algorithm has been adapted by many authors over the years, mainly in an effort to overcome the computational difficulties implied by the “curse of dimensionality” discussed below. Recent trends in the research focus of the SDP and SLP communities are compared and contrasted by Powell (2012).

2.3.6 Reservoir Applications

The earliest English language SDP paper was by Little (1955), and SDP has been widely applied to such problems since. Early SDP developments and applications for reservoir management were surveyed by Yakowitz (1982), including discrete, differential (both constrained and unconstrained) and incremental DP, under Markov and independent inflow assumptions. Yeh (1985) surveys other models, some the same techniques as Yakowitz, and some based on other techniques, including Incremental DP with Successive Approximations (IDPSA), reliability-constrained DP, and algorithms based on the principle of progressive optimality.

Lamond & Boukhtouta (1996) discuss SDP inflow assumption models, labelling these as MDP. They discuss MDP relative to discrete DP models, and note a further technique, sampling SDP. They also discuss techniques using MDP and approximate future value functions, these include parameter iteration, gradient and spline DP, and dual DP. In dual DP they discuss two main variants. The first, Stochastic Dual DP (SDDP) as developed by Pereira & Pinto (1991), is described as a mix of LP and DP which employs Benders decomposition. The second, as developed by E. Read (1989), is described as using duality and parametric LP, applied in a SDP framework.

Labadie (2004) discusses other more recent models based on the same techniques, noting various issues with some of them. Labadie also notes sampling SDP saying it is a scenario-based method, similar to SLP, but using an SDP algorithm. Even more recently, Rani & Moreira (2010) discuss other SDP techniques, including aggregation–disaggregation and decomposition methods, and neurodynamic DP. But many of the techniques they discuss focus on modelling of future streamflows, including sampling, sampling with Ensemble Streamflow Prediction (ESP), Bayesian, demand driven, and fuzzy-state SDP.

Stedinger et al. (2013) also discuss recent developments in sampling, and sampling ESP SDP techniques. These models focus on streamflow persistence and forecasting better hydrologic information, driven by individual streamflow scenarios, rather than just a Markov description of streamflow probabilities. This allows ESP forecast series to be employed directly, taking full advantage of the description of streamflow variability, and of temporal and spatial correlations captured within the series.

Still, we are not concerned, here, with developing reservoir optimisation models for practical application. Thus we are not concerned with maximising computational efficiency, or with details of streamflow prediction, and a relatively simple Markov chain model will suffice for our purposes. When applied to reservoir management, the key characteristics of SDP are as follows.

The main SDP state variable is the reservoir storage level. So, at each stage there is a water value function, which is defined as a function of storage. There is an intra-period benefit function which is defined as a function of release. For each storage state, the economic value of present stage release decisions must be balanced against that of future stage release decisions. The SDP algorithm described above applies, with the random vector being reservoir inflow.

With a stochastic DP implementation, we could assume that only the prior period's inflow is known, and available, when the release decision is made, with the present period's inflow only affecting end-of-period storage. Under this "decision-first" assumption, looking forwards from a start of period storage level, there is a single release decision, followed by multiple (uncertain) inflow realisations, each producing a different end of period storage level (state). The intra-period inflow may also have an impact on intra-period benefits. In Section 3.7.2, though, we introduce a simplified "conservative" variant of this decision-first approach, in which the intra-period inflows do not have an impact on intra-period benefits, only affecting end-of-period storage.

Alternatively, the present period's inflow could be assumed known at the start of the period, and available to fill up the reservoir, or released to meet demand within the period. Obviously, it would be better to use that information, if it would actually be available to the real-life decision-maker. So, under this "observation-first" assumption, looking forwards from a start of period storage level, there will be multiple (uncertain) inflow realisations, each of which is followed by a potentially unique release decision, again producing a different end of stage storage level for each realisation. In Section 3.7.1 we refer to this observation-first approach as an "informed" inflow allocation policy.

In the basic SDP reservoir model, the "transition function", $z^{t+1} = f^t(x^t, z^t, y^t)$, is provided by the continuity (mass balance) equation, which describes how the storage state, $z^t = s^t$, changes

from one stage to another due to a combination of inflow, $y^t=F^{t,h}$, and release, $x^t=q^{t,h}$ (or q^t for the decision-first formulation). Note that we use h to be the current hydrology state. In a classic DP formulation the storage state would be represented by a grid of discrete storage levels. In principle, though, storage is a continuous variable, and we later (from Chapter 5 onwards) use a very fine storage grid where the increment size is 1. So, in our description here we will also assume storage is a variable.

With an observation-first inflow allocation policy, there is a distinct decision, $q^{t,h}$, for each h . So the continuity equation for period t , can be expressed, for any beginning of period storage state, s^{t-1} and the current inflow state h , as:

$$s^t = s^{t-1} + F^{t,h} - q^{t,h} \quad \forall t, h \quad (2.8)$$

With a decision-first inflow allocation policy, there is a common decision, q^t , applying for all h . So the continuity equation for period t , can be expressed, for any beginning of period storage state, s^{t-1} , and the current inflow state h , as:

$$s^t = s^{t-1} + F^{t,h} - q^t \quad \forall t, h \quad (2.9)$$

Either way, if we can ignore inter-period correlation of inflows, each inflow state has an associated probability of occurrence, $P^{t,h}$. So the end-of period water values can be simply weighted independently, to account for the underlying uncertainty. And we can construct a lag-zero SDP model, using only storage as a state variable, because no other information needs to be carried over from one period to the next. So the optimal release decision becomes a function of the state variable, i.e. $q^{*t} = q^{*t}(s^{t-1})$, or includes the current inflow, if known, i.e. $q^{*t,h} = q^{*t,h}(s^{t-1})$. That is, for the observation-first policy, solving the following optimisation determines both the set of optimal $q^{*t,h}$ values, and the corresponding Beginning of Period (BOP) value function:

$$V^{t-1}(s^{t-1}) = \sum_h P^{t,h} \times \left[\max_{q^{t,h}} \left\{ B^t(s^{t-1}, F^{t,h}, q^{t,h}) + V^t(s^{t-1} + F^{t,h} - q^{t,h}) \right\} \right] \quad t=T, \dots, 0 \quad (2.10)$$

And for the decision-first policy, solving the following optimisation determines both a single optimal q^{*t} value, and the corresponding BOP value function:

$$V^{t-1}(s^{t-1}) = \max_{q^t} \left\{ \sum_h P^{t,h} \times \left[B^t(s^{t-1}, F^{t,h}, q^t) + V^t(s^{t-1} + F^{t,h} - q^t) \right] \right\} \quad t=T, \dots, 0 \quad (2.11)$$

Implicitly, using storage as the state variable amounts to assuming that the amount of water in storage summarises all the information we need, or can have, about the past history of the system in order to compute the optimal decision for this stage. But, if other information is available, and relevant, it must also be carried forward by a continuity equation, or some other form of inter-temporal linkage. And adding extra inter-temporal linkages creates extra dimensions in the state space.

In SDP, a Markov lag-one model allows for a simple form of temporally-connected modelling, linking the prior (F^{t-1}) and present (F^t) period's inflows. In this case, though, the extra information carried forward is the prior period's inflow. That does not absolutely determine the next period's inflow, via a deterministic equation, but it does determine the probability distribution of inflows for the next period. The conditional probability of transitioning from state, $h(t-1)=h'$, in stage $t-1$, to state, $h(t)=h$, in stage t is denoted, $P^{t,h(t-1),h(t)} = P^{t,h',h}$.

We generalise the state z^t to be a vector, (s^t, F^t) .¹⁵ The expected value function, $V^{t,h}$, now becomes conditional on the inflow state. With an observation-first inflow allocation policy, we still make one decision for each initial storage level, and inflow state in the current period. So the continuity equation in Equation (2.8) is still valid. All that changes is that the probabilities in (2.10) become conditional on the prior observed state. So, for the observation-first policy and the set of optimal releases $q^{*t,h}(s^t)$, the BOP value function is determined by solving:

$$V^{t-1,h'}(s^{t-1}) = \sum_h P^{t,h',h} \times \left[\max_{q^{t,h}} \left\{ B^{t,h}(s^{t-1}, F^{t,h}, q^{t,h}) + V^{t,h}(s^{t-1} + F^{t,h} - q^{t,h}) \right\} \right] \quad (2.12)$$

$t=T, \dots, 0; h'=1, \dots, H^{t-1}$

With a decision-first inflow allocation policy, though, the optimal release decision for t now becomes a function of the beginning of period storage state, s^{t-1} , and the prior inflow state, h' . So the continuity equation is:

$$s^t = s^{t-1} + F^{t,h} - q^{t,h'} \quad \forall t, h', h \quad (2.13)$$

And so, for the decision-first policy, the set of optimal releases $q^{*t,h'}(s^t)$, and BOP value function, are determined by solving:

¹⁵ We will again treat storage as a variable, but use a discrete index, h , for flows.

$$V^{t-1,h'}(s^{t-1}) = \max_{q^{t,h'}} \left\{ \sum_h P^{t,h',h} \times [B^{t,h}(s^{t-1}, F^{t,h}, q^{t,h'}) + V^{t,h}(s^{t-1} + F^{t,h} - q^{t,h'})] \right\} \quad (2.14)$$

$t=T, \dots, 0; h'=1, \dots, H^{t-1}$

Either way, the output of the SDP process is a conditional operating policy, still with a single optimal decision for each state, and stage.

Finally, another form of inter-period linkage would be linking bids. As discussed in Section 4.6.3.5, participants may wish to limit variation from month to month, or specify a total requirement to be taken over several months. SLP can model this by assigning a bid parameter, and the associated allocation variable, to each node in the tree, with constraints to ensure that (say) the total requirement is met by the total of the allocation, over the relevant periods, for each participant.

SDP can also model such inter-temporal linkages, but the extra link (like the continuity and Markov lag-one inflow links) creates an extra dimension to the problem, thus increasing computational requirements. So, an SDP is more limited than an SLP, in terms of the type of water market bidding models that could be experimented on. For our experiments, though, we will ignore linked bids.

2.3.7 Computational Issues

The size of the state space required often proves to be a significant computational barrier for SDP.¹⁶ A reservoir system is often comprised of multiple reservoirs. Unlike SLP, which has a scenario tree which expands over time, SDP has the same (possibly large) number of states and stages in each period. For example, imagine an n reservoir problem, with ZNUM=10 storage states (levels), in each reservoir. The computational problem expands with each additional reservoir, with 10, 100, 1000, ..., ZNUM ^{n} release decisions having to be determined, and each requiring an n -dimensional optimisation.¹⁷

¹⁷ Incorporating aspects of the stochastic “real-world” into a model, often means a high-dimensional state space, and assuming temporally and spatially correlated hydrologic inflows, Lee & Labadie (2007).

This situation is aggravated if we also have to consider H (typically n -dimensional) inflow state vectors in each period, and particularly if we model inflow correlations. With H inflow states in each period, we have $T \times H \times ZNUM^n$ decisions to optimise.¹⁸

In fact, this so-called “curse of dimensionality”, is the single biggest issue limiting SDP applications. In this case, it limits the number of reservoirs that can be modelled.

By way of comparison, SLP will also have storage and release variables for each reservoir. Although it computes values from a known starting storage point, it also has an exponentially expanding tree, where variables are defined at each node. So, as the number of future stages grows, so does the size of the problem. For example, for the same n reservoir problem, with the same H and T parameters, the SLP could, theoretically, expand into a problem with H^T nodes, each having n dimensional variables for storage, release, and inflow. In practice, though SLP size can be limited by just selecting a representative set of nodes to form a much more limited scenario tree. Although that potentially implies significantly less accuracy in the optimisation, particularly in later periods, the degree of inaccuracy can be controlled by techniques such as a multi-stage Stochastic Decomposition algorithm of Hingle & Sen (1996), or iterative refinement, as in the SDDP algorithm of Pereira & Pinto (1991).

Rani & Moreira (2010) discuss decomposition models, which can also be combined with aggregation-disaggregation approaches. Archibald et al. (1997) propose a predominantly quantity decomposition method, where a reservoir system is split into subsystems and associated sub-problems. A more general quantity decomposition approach was later developed by Archibald et al. (2006). There are also price decomposition approaches, such as those by E. Read (1989), and more recently R. Read (2014).

2.3.8 SLP versus SDP

Thus SLP and SDP are both applicable to the reservoir optimisation problem, but they have different strengths and weaknesses, and it is not immediately clear which will be best suited to our purposes here.

¹⁸ If we also had a participant wanting a constant consumption of water over several months, we could have Y states representing alternative consumption rates, giving levels $Y \times T \times H \times ZNUM^n$ size problem.

First, while an SLP, by definition, can only handle linear system characteristics, an SDP is slightly more flexible, in that it can model non-linearities, such as losses (from evaporation, for example) or cost and benefit functions. Since an SDP is divided into separate states and stages, the benefit function must also be separable, by state and stage. But, unlike a direct application of SLP,¹⁹ the intra-period optimisation problem need not be solved using the same optimisation technique as the inter-period optimisation within which it is embedded. Thus an SDP model can employ any kind of intra-period sub-model that generates a well-behaved benefit function. For example, Scott & Read (1996) employ a Cournot gaming intra-period model, while Mahakalanda et al. (2013) employ a CDDP intra-period model which pre-computes intra-period demand functions, for a multi-use reservoir system.

Second, an SLP model requires an EOH value function, at least defining the value of water in storage for each terminal node of the scenario tree. But we always have a single starting storage level, inherited from the present reality, or chosen to model the system from an assumed starting reality. Hence, SLP only computes the optimal solution for a single initial reservoir storage state (or level).

SDP and SLP require an EOH value function, and that will need to be comprehensive, covering all possible EOH state values. Later, in Chapter 5 we want to generate an equilibrium version of the EOH value function, because we think this is a reasonable thing to do in practice. But an SDP algorithm works backwards from all possible ending positions and can produce solutions for all reservoir storage states that are also optimal, given that EOH value function. Further, the SDP is better suited to find the EOH, as described above, by iterating to an equilibrium, in which the EOH value function is equal to the SOH value function. Thus eliminating the dependence on an assumed EOH value function.

So, SDP is better suited to finding the equilibrium as it automatically generates the optimal policy for all possible starting states, while the SLP needs to be re-optimized a vast number of times (for every possible final stage ‘state’) to achieve the same thing.

Third, while an SDP is ideal for Markov Chain modelling, an SLP has an entire path defining the information available to make decisions, up to any point on that path. So the random

¹⁹ An SLP could be decomposed to achieve this.

variables at a given node in the scenario tree can implicitly depend on all the outcomes of any prior random variables on that same path, up to that point. Thus an SLP can model inter-temporal interactions in a more generalised stochastic tree structure, and this could be useful in clearing real markets.

Fourth, the computational speed of an SDP depends on the number of states and stages which expands exponentially for each additional reservoir. For water systems with multiple reservoirs, SDP models must be approximated into one, or at most a few reservoirs. However, SDP does compute a policy for all reservoir storage levels, and that policy is equally optimal for all states and stages.

On the other hand, an SLP readily computes the optimal solution for multiple reservoir systems, but only for the assumed scenario tree structure. An SLP solution only tells us what to do if real flows turn out to follow that tree structure. Thus, unlike an SDP, whose policy can be simply looked up at the beginning of each period, the SLP should really be re-optimised, starting from the new storage/flow situation reached at the beginning of that period.

In order to explore the impact of uncertainty modelling, though, we wish to simulate market performance over a very large number of hydrological sequences. So re-solving an SLP for each stage in the simulation process would be impractical. Conversely, provided we avoid the curse of dimensionality by focussing on a single reservoir, a basic SDP implementation will allow us to model correlated inflows, using one additional state dimension.

2.3.9 Interim Conclusions

In light of the above, then, our general strategy will be to use a mix of LP and DP approaches. SLP is (in some ways) more general than SDP, and thus allows us to consider the depth and breadth of potentially feasible modelling options. From a primal SLP formulation, we can also readily construct its dual, and then discuss pricing implications. So Chapters 3 and 4 will formulate our conceptual water market models, and discuss trading options, in terms of SLP.

From that point on, though, we focus on quantitative results, computationally exploring market options and outcomes for a single tank system, under uncertainty with lag-one correlation, and SDP is well suited to that. These simplifications allow us to use an SDP approach, which can efficiently optimise the kind of policies whose impacts we want to simulate. And since we wish to explore the performance of our market arrangements over the whole state space, SDP more readily fits into our experimental strategy. We generally assume an observation -first (informed) inflow allocation policy, but also discuss an alternative “conservative” decision-first policy. We will also vary the optimisation complexity from Deterministic (DC), to Stochastic Independent (SI) (lag-zero), to Markov lag-one (Markov Chain: MC), to see the effect on market outcomes.

2.3.10 Primal versus Dual SDP

If we are to use SDP, though, we still have to decide whether to use a primal, or dual, technique. The market clearing approach means that we would prefer a technique which uses economic concepts, produces prices, and if possible works on the dual solution, and dual allocation implementation strategies.

Using water for electricity generation is non-consumptive, and historically, in single use systems, the water was often viewed as a “free” marginal cost resource. But with other generating units, such as thermal (coal and gas) plants, the costs saved by not running each plant must be considered, and this implies an actual marginal value for water, in that system. The marginal value for water was first discussed (in French) by Massé & Boutteville (1946), as summarised by Wallace & Fleten (2003).²⁰

The marginal water value is set so as to balance reservoir storage and release decisions, in each decision period. The objective is to optimise system performance over the modelled planning horizon, assuming an EOH Marginal Water Value (MWV) function. This ensures water is not given away for free at the end of the “water year”, and that enough is carried over into the subsequent water year. More generally, the expected marginal water value is a function of time, reservoir level, and (if flows are correlated) observed inflow. It is, in fact, the derivative of the SDP value function discussed above, as discussed by E. Read (1979).

²⁰ Masse was concerned about whether to maximise expected profit or expected utility, concerned with well-known paradoxes about expected profit, referring to Borel for examples. Masse came very close to defining the efficiency frontier with expected profit versus probability of shortage of water on each axes. Masse argued that the reservoir operator has agreements with demanders, and that agreement was conditional, such that in dry years the contract to deliver water cannot be fully fulfilled.

Thus the MWV function could be produced by solving an SDP, using backward recursion, then differentiating the SDP value function. More efficient dual techniques can be used, though, if the benefit and value functions are differentiable, which is the case for reservoir systems as E. Read (1979) proved.

Stage & Larsson (1961) describe an iterative process to compute the “incremental cost of water” used to generate power, or as we might say the “marginal value of water”, in terms of being able to reduce future thermal/shortage costs. Hveding (1968) reports extensive use of a model based on this method in the Norwegian electricity system, but concluded that the process could easily be applied to irrigation. Boshier et al. (1983), and later E. Read & Boshier (1989), describe the application of the method to the New Zealand electricity system. They discuss several alternative methods which were tested, but conclude that this marginalistic iterative variant of SDP produces “near-optimal” solutions.

This work leads on to the development of the Constructive Dual DP (CDDP) technique, which we use later in Chapter 5, and now focus on discussing. First we discuss its main line of development history for reservoir management in the NZ electricity system, second we give a brief history of other CDDP developments, third we compare CDDP and SDDP,²¹ fourth, and finally we give a brief description of the CDDP procedure. The “constructive” label relates to how the MWV surface is explicitly and systematically “constructed” for the entire state space, unlike the SDDP approach in which an approximation to that surface is an implicit by-product of an iterative algorithm.

E. Read & Hindsberger (2010) provide a summary history of CDDP and SCDDP development. SCDDP was developed around 1984 for reservoir management, in the two reservoir PRISM model, of the New Zealand (NZ) electricity system. At that time, Read prepared a series of four reports [E. Read (1984, (1985a, (1985b, (1985c))] for the then NZ Ministry of Energy, relating to the development of a modelling suite, and options associated with the computational design and interfaces. The first report detailed the simulation model, the second report detailed the pre-processing of demand data, the third report detailed the SCDDP model used to optimise reservoir operating rules, and the fourth report detailed how to interpret outputs from the modelling system in making investment decisions with respect

²¹ Note that earlier papers referred to CDDP simply as “Stochastic Dual DP”, with the CDDP terminology being introduced later to distinguish it from the better known SDDP.

to future plant. The key aspects of the modelling suite were later published by E. Read et al. (1987), which focuses mainly on the simulation model. E. Read (1989) later summarised the key features of a single, then a two, reservoir SCDDP model. Culy et al. (1990) discuss how the PRISM model was developed into the subsequent SPECTRA model. Both models featured the same computational module, RESOP, which contains the SCDDP algorithm.

Those models were all developed while the New Zealand power system was owned and operated by a single entity; first the Ministry of Energy, then the Electricity Corporation of New Zealand (ECNZ). But the system was eventually dis-aggregated, and coordinated in a competitive market setting. Thus subsequent research on SCDDP models focused on gaming, and subsequently risk aversion. Scott & Read (1996) and Batstone & Scott (1998) discuss SCDDP gaming applications, and Kerr et al. (1998) discuss an SCDDP application including risk aversion. Craddock et al. (1999) discuss a SCDDP model for a risk-averse two reservoir RAGE model of the New Zealand electricity system. More recently, Stewart et al. (2004) and E. Read et al. (2006) discuss gaming by suppliers with inter-temporal constraints. E. Read & Hindsberger (2010) report that SCDDP has also been used in the ECON BID model and applied particularly to model the Nordic electricity sector (Denmark, Norway, and Sweden) since around 2005/2006. Boland et al. (2011) note that participants provide the bids as inputs to the LP, and as such they suggest potential problems, from gaming, are exacerbated when there are too few players in the market, or, possibly, too much vertical integration of the players. They note that more work needs to be done in this area.

Casseboom & Read (1987) discuss a potential application to coal stockpiling, while E. Read & George (1990) discuss CDDP for linear production and inventory systems. Essentially the same technique was developed independently, for a linear system with storage, by Bannister & Kaye (1991). Those models were deterministic and (as noted by Lamond & Boukhtouta (1996)) could be described as the iterative application of parametric LP, to obtain a MWV as a piece-wise linear function of storage level. But a stochastic variant, incorporating a discrete Markov Chain, was discussed by Macgregor (1991). That approach is developed further in Chapter 5 of this thesis, and has previously been reported by and Dye et al. (2012) and Starkey et al. (2012). An alternative CDDP approach to modelling correlations, using continuous variables, was reported by Yang & Read (1999).

Fundamentally, the maximisation in classical SDP is replaced by the optimality conditions of that problem. Thus the allocation of water is balanced across the planning horizon such that, to the extent feasible, the marginal value of intra-period water usage equals the expected marginal value of End of Period (EOP) water stored for future use. And that implies that the BOP MWV also equals both.

Various notation and terminology of CDDP has developed over the years. This notation is discussed by E. Read (1985b), E. Read (1989), E. Read & George (1990), Scott & Read (1996), E. Read & Hindsberger (2010) and Dye et al. (2012). Here we use the terminology of Dye et al. (2012). For simplicity we assume a single reservoir problem, in which case the MWV is a function of storage level, for each decision period, and may equivalently be called a “Demand Curve for Storage”, (DCS). There is also an intra-period Marginal Release Value (MRV), defined as a function of release (q), which may equivalently be called a “Demand Curve for Release”, (DCR). Following the convention of Dye et al. (2012), the demand curves $DCS(s)$ and $DCR(q)$ are defined in the form where marginal price is a function of quantity, $M(Q)$.²²

Using that notation, in a deterministic setting, we could say that, for a given state (s^{t-1}) at the beginning-of-period t /end-of-period $t-1$ we must choose $q^{*t}(s^{t-1})$ such that:

$$DCS^{t-1}(s^{t-1}) = DCR^t(q^t) \quad \forall t \quad (2.15)$$

$$s^t = s^{t-1} + F^t - q^t \quad \forall t \quad (2.16)$$

$$DCR^t(q^t) = DCS^t(s^t) \quad \forall t \quad (2.17)$$

But note that, rather than searching to find the optimal q corresponding to each beginning-of-period storage level, Equation (2.17) can be used to directly identify the range of all releases, q^{*t} , that could optimally correspond to any end-of-period storage level, s^t . i.e.:

$$Qrange^t(s^t) = [q^{*t} : DCR^t(q^{*t}) = DCS^t(s^t)] \quad \forall t \quad (2.18)$$

Further, with that range of releases defined, Equation (2.16) can be re-arranged to identify the range of all beginning-of-period storage levels, from which we could optimally reach end-of-period storage level, s^t . i.e.:

²² Note that Read and Hindsberger follow the opposite convention.

$$S_{range}^{t-1}(s^t) = \left[s^{*t-1} : s^{*t-1} = s^t - F^t + q^{*t}(s^t) \right] \quad \forall t \quad (2.19)$$

And, for any beginning-of-period storage level in that range, Equation (2.15) must hold.

For many EOP storage levels, there will be a unique EOP MWV, and a unique optimal release, and the corresponding BOP range will be a single storage level, with the same MWV. For others, though, there will be a unique EOP MWV, but a range of optimal intra-period releases that could be optimal, each starting from a different BOP storage level, in the range defined by Equation (2.18) above. By construction, though the MRV for all those release levels must be the same, and the BOP MWV must equal that MRV, for all levels in the range defined by Equation (2.19) above.²³ In practice, such ranges are called “flats”, and each represents the range of BOP storage levels over which the loading of a particular thermal station should decrease linearly, as a function of increasing storage level, so as to target the corresponding EOP storage level.

As originally described by E. Read (1989), the SCDDP algorithm exploited these relationships directly. It works in terms of “guidelines” which are the contours of the MWV surfaces corresponding to a specific grid of dual (marginal cost) values at which the nature of the release decision function is known to change, e.g. to start increasing generation from a particular thermal power station. The algorithm then ensures that the above conditions hold using a backward recursion, divided into two phases:

- First, if the EOP MWV function is known, as a function of EOP storage, then an “uncertainty adjustment” phase can be employed to calculate the expected MWV, as a function of target EOP storage, forming the expected DCS. Guidelines for each thermal station can then be identified as contours of that surface, as above. That is, the guideline represents the storage level below which MWV exceeds the marginal cost of operating that station, implying that it should be operated to conserve storage.
- Second, the decision phase of a classical primal SDP is replaced by a “guideline augmentation” phase, by which each EOP guideline is translated back to become a pair of BOP guidelines, with a “flat” in between, as discussed above. These guidelines then define the BOP MWV surface, and the process steps back another period.

²³ If the EOP MWV is not unique, several such BOP ranges may be associated with the same EOP storage level, but that case is handled implicitly by the CDDP algorithms below.

Later papers, though, described the process in terms of “adding demand curves”. For ease of exposition, we will assume, here, that each DCS and DCR is continuous and strictly monotone, so that their inverses also exist, in each state and stage.²⁴ Their inverses, $DCR^{-1}(MWV)$ and $DCS^{-1}(MRV)$, are thus defined in the form where quantity is a function of price, i.e. $Q(M)$.

E. Read & George (1990), E. Read & Hindsberger (2010), and Dye et al. (2012) all describe the backward adjustment procedure as employing two types of curve adding:

- “Horizontal” curve addition means that we add two (or more) physical Q values corresponding to the same price level, M , and allows us to describe the “guideline augmentation process” above as adding an intra-period DCR to an EOP DCS. But this requires DCR and DCS to be represented in the $Q(M)$ form.
- “Vertical” curve addition, means that we add two (or more) price values corresponding to the same physical level, and this allows us to describe the “uncertainty adjustment process” above as adding several shifted copies of the EOP DCS to form a new expected DCS. But this requires DCR and DCS to be represented in the $M(Q)$ form.

Following Macgregor (1991), Dye et al. (2012) describe the implementation of these processes, using equivalent list sorting algorithms, and/or simple spreadsheet functions. We will develop our own version of that algorithm in Chapter 5. But note that the order in which these processes occur depends on whether an observation-first, or decision-first, allocation policy is being employed.

With an observation-first policy, in a lag-one Markov setting, the first step in determining the set of optimal releases $q^{*t,h}(s^t)$, and set of BOP DCS's, is thus to “horizontally add” the intra-period release and EOP Sunday Night Demand Curve for Storage (SN_DCS) to form a Conditional intra-period Demand Curve for Water (CDCW^{*t,h*}), for each h , in $Q(M)$ form:

$$\left[CDCW^{t,h} \right]^{-1} (MWV) = \left[CDCR^{t,h} \right]^{-1} (MRV) + \left[SN_DCS^{t,h} \right]^{-1} (MWV) - F^{t,h} \quad (2.20)$$

t,h and $\forall MWV$

Note that in Equation (2.20), the DCR is also now a Conditional DCR (CDCR). In order to account for the intra-period inflows need to convert the CDCW into its $M(Q)$ form, where

²⁴ If each DCR is monotone, along with DCST, then it can be shown that DCS will be monotone for all t . This will be the case for our problem, but clearly was not for the original CDDP development, where RESOP explicitly exploited the “flats” in the DCR.

$M(Q) = [Q(M)]^{-1}$. In this case the inverse function becomes $CDCW(s^{t-1})$ where s^{t-1} is the storage, $s^{t-1} = s^t - F^{t,h} + q^{t,h}$, between the bounds 0 and the Storage Capacity (SCAP).²⁵ . Then, working backwards, we account for the Probability Distribution Function (PDF) of intra-period inflows, weighting and aggregating these Conditional DCW's to form:

$$SN_DCS^{t-1,h'}(s^{t-1}) = \sum_h P^{t,h',h} \times CDCW^{t,h}(s^{t-1}) \quad \text{for } t \text{ and } \forall s^{t-1}; h' = 1, \dots, H^{t-1} \quad (2.21)$$

With a decision-first policy, in a lag-one Markov setting, the first step in determining the set of optimal releases $q^{*,h}(s^t)$, and set of BOP DCS's. In order to account for the intra-period inflows we need to have the SN_DCS in its $M(Q)$ form. Then, working backwards, we account for the PDF of intra-period inflows, weighting and aggregating these conditional DCW's to form a Monday Morning Demand Curve for Storage (MM_DCS):

$$MM_DCS^{t,h'}(s^{t-1}) = \sum_h P^{t,h',h} \times SN_DCS^{t,h}(s^{t-1} + F^{t,h}) \quad \text{for } t \text{ and } \forall s^{t-1}; h' = 1, \dots, H^{t-1} \quad (2.22)$$

Then we “horizontally add” the intra-period release and the MM_DCS to form a BOP Conditional Demand Curve for Water ($DCW^{t,h'}$), for each h' , in $Q(M)$ form:

$$\left[CDCW^{t,h'} \right]^{-1} (MWV) = \left[CDCR^{t,h'} \right]^{-1} (MRV) + \left[MM_DCS^{t,h'} \right]^{-1} (MWV) \quad \text{for } t, h' \text{ and } \forall MWV \quad (2.23)$$

Where CDCW is then truncated to the range $[0, SCAP]$ to form SN_DCS.

Finally, this discussion assumes that an intra-period DCR function exists. So long as it is monotone non-increasing, that DCR function can be produced by any intra-period sub-model. For example, E. Read & Hindsberger (2010) refer to Scott & Read (1996), Batstone & Scott (1998), Kerr et al. (1998), Stewart et al. (2004), and E. Read et al. (2006). All of which use the kind of “demand curve adding” approach, which we employ here. They also note that the CDDP technique itself can be used to pre-compute intra-period DCR functions. Mahakalanda et al. (2012) and Mahakalanda et al. (2013) describe two such intra-period DCR computation procedures, applied to a multi-nodal catchment, for a mixed use (hydro and irrigation) catchment, for CDDP and then SCDDP, respectively. For our experiments, though we assume intra-period DCRs already exist, and these represent the aggregate markets requirements to take water, in each state and stage. But more details of our implementation may be found in Chapter 5.

²⁵ In chapter 5, we will refer to this truncated version of $CDCW(s^{t-1})$ as $MM_DCS(s^{t-1})$.

2.4 Summary

In this chapter we noted that water supplies are becoming scarcer, while demand is increasing. We noted that water has been historically priced well below its true value, often without the incorporation of environmental and opportunity costs. But, in some places, as demand to access (often increasingly) constrained water supplies increases, pricing measures have been accepted as an appropriate water resource management tool. We identified many water, and wider environmental, markets operating at this time. Many model-based (“smart”) markets trade other similar commodities, electricity and natural gas. Model-based markets involve formulating an auction with side constraints, via mathematical optimisation. This was the technique we decided to follow, and hence we surveyed the literature for reservoir, electricity, and market models. While many model-based water markets have also been proposed, we found that there was a gap in the literature relating to trading water under uncertainty, while accounting for the inter-spatial and inter-temporal storage issues.

3. System Description

3.1 Chapter Introduction

In this chapter we discuss the type of system within which we envisage our water allocation models operating, including its natural hydrology and topology, the built water systems, and economics of water usage. Figure 3.1 shows a schematic of a water network for Melbourne CBD and surrounding regions, in the state of Victoria, Australia. This regional schematic outlines the major storage reservoirs and transfer pipelines for both water and wastewater systems. Although wastewater is an important component in the overall water cycle, this thesis is only concerned with “clean” water.

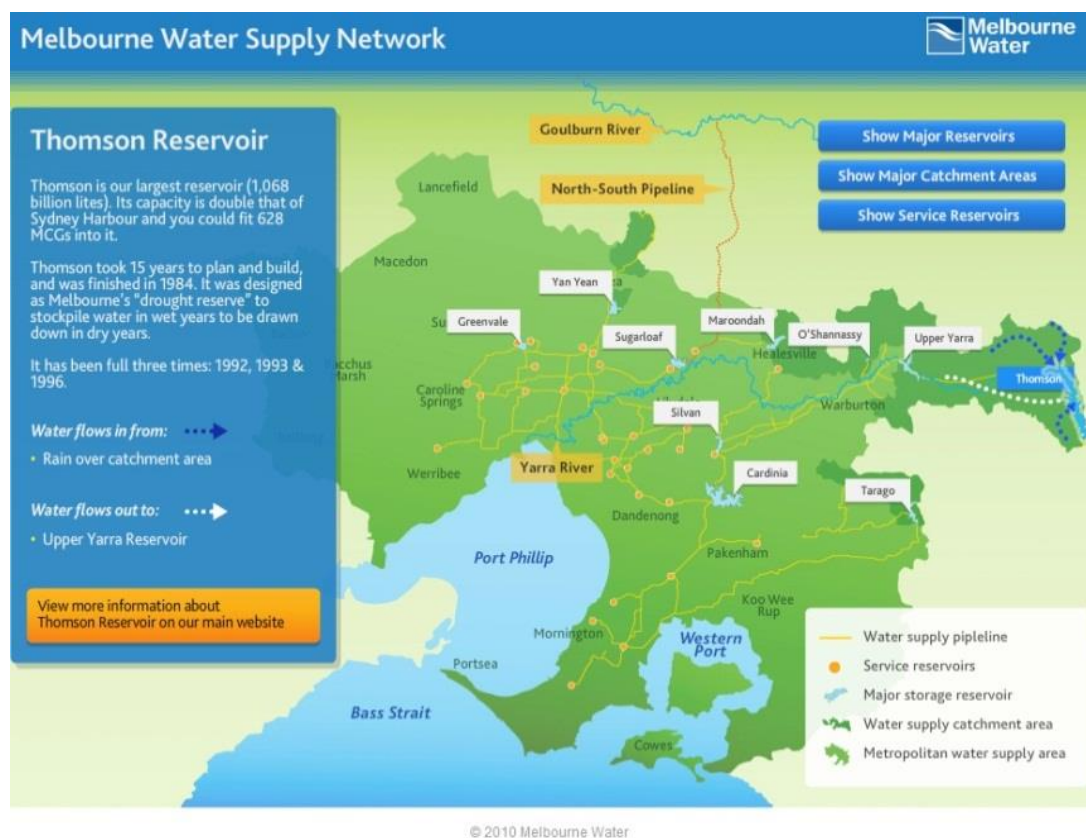


Figure 3.1: Schematic of main water network assets for Melbourne Water (2014)

Figure 3.1 illustrates the complexity and interconnectedness of a real world water network. This thesis investigates issues related to simplified network models. Figure 3.2 below shows one such simplification.

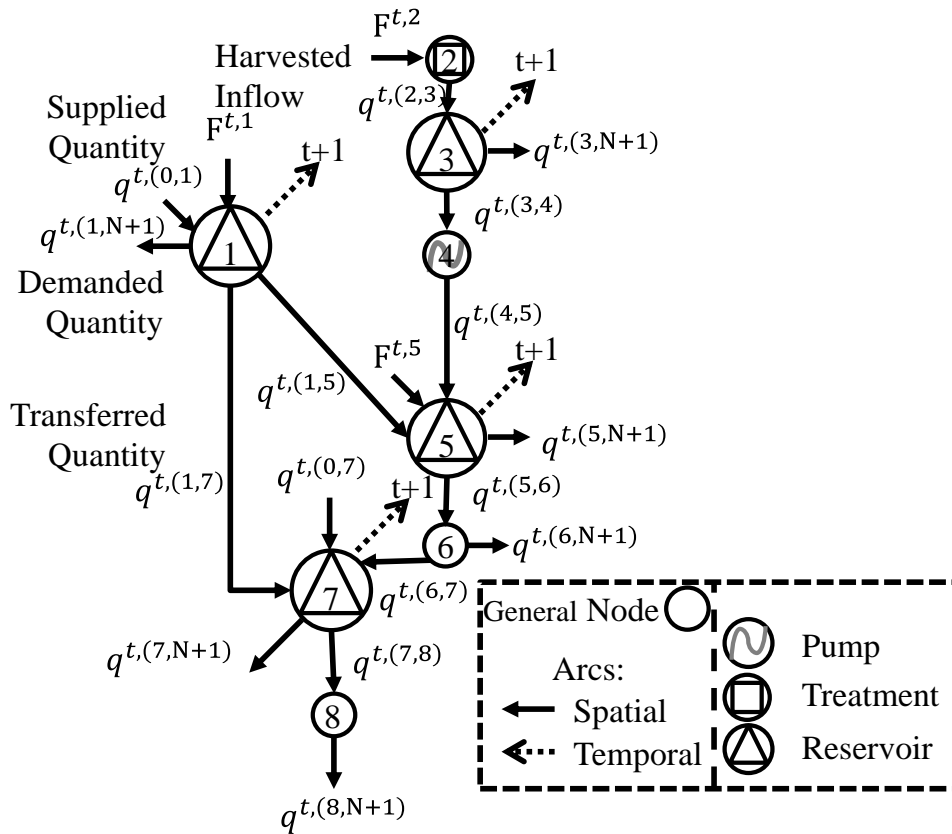


Figure 3.2: Illustrating a simplified water network

3.2 Locational Aspects

Figure 3.2 depicts the infrastructure network at a macro level, namely the wholesale bulk storage and transfer of water, for a single time period, t . We do not consider the retail level, at which individual instantaneous customer demands are met, at a given point on the pressurised water network (grid). A water network takes time to transport the water from source to destination. Assigning specific water units to specific users, in an interconnected network with a diverse set of customer needs, is not practical. In a reservoir, water is constantly moving and mixing. In directed pipes, water will take the path of least resistance when faced with multiple paths (a tee junction).

Different water sources have different mineral compositions, and may contain various organic or inorganic contaminants. The water quality of a single source varies over time. Within our system, we assume all water is within some acceptable quality range and is interchangeable, although potentially subject to treatment, to ensure it is within the acceptable quality range.

Once water has been treated to its required (potable) quality, it is usually transported in closed pipes. These pipes are full in normal operation, and thus pressurised. The tendency of the water to naturally flow (due to gravity) from origin to destination depends on the installed pipeline levels and thus the overall net pressure difference. Where natural flow from one place to another is not feasible, but required, the water is pumped.

As shown in Figure 3.2 above, at a given time t within our modelling system, we account for flows and stored quantities at nodes and, later, we define prices at nodes. Network nodes include pumpsets to transport the water, treatment facilities to adjust the water quality to within acceptable limits, and other node types. We define a set of nodes, $n \in \{0, 1, \dots, N, N+1\}$, where this set includes a source node, '0', and a sink node, $N+1$. All network flows (through pipes or channels) are represented on arcs, which have upper and lower flow limits. In our models, water moves between origin node n and destination node n' . We describe this using the pairwise notation (n, n') . Hence, we then use this notation to describe a flow variable $q^{(n,n')}$. These can be sourced from outside the network, $q^{t,(0,n)}$, or, consumed or wasted outside the network, $q^{t,(n,N+1)}$. These q^t variables represent the total accumulated flows during period t . We define a set of connecting arcs as follows:

$$\text{ARC} \subseteq \{0, 1, \dots, N\} \times \{1, 2, \dots, N+1\} \quad (3.1)$$

Note that the origin nodes include the source node, '0', while the destination nodes include the sink node, $N+1$.

In practice, non-utilisable water it may have some value (for example, recreational activities not directly accounted for within the system) or create a cost (for example, too much causing flooding). But, in our modelling system anything going to waste is assumed to be non-utilisable and having no economic and/or environmental value, positive or negative.²⁶ This is opposed to utilisable water which has some positive economic value, and/or some positive or negative environmental value. This does not restrict the model as 'wasted' water generating value or incurring a cost can be modelled as utilisable.

We could assume that water is only utilised when it is consumed and exits the system via the sink. In this setting there would be no exogenous parameters valuing storage and flows.

²⁶ But, spills to the sink (for example) could have environmental value, positive or negative.

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Water in store, or flowing downstream, would have an opportunity value via an endogenously computed MWV based on the ultimate consumer utilisation.

Alternatively, a more general system could also allow for non-consumptive uses, such as hydropower generators. In some respects these users could be competing with consumptive users, such as agricultural consumers, but they also complement one another, adding more value to the water as it moves down the river chain to be ultimately consumed. In this setting, water could be assigned general benefit (and penalty) values, represented by endogenous variables which control different types of system utilisation. In such a system there is explicit parameter valuation of units in storage and flowing through the system.

See Mahakalanda et al. (2013) for details of one such mixed use model which features multiple trading nodes. In such a model, participants can provide benefit/bid and cost/offer parameters, but there can also be wider (environmental) benefit and penalty parameters. These set penalties (or rewards) for violating (or meeting) a specified soft constraint. Hence, soft limits can be set higher or lower than the true (physical) lower or upper bounds, respectively, of (as examples) the reservoir or release channel. An example for each is: to preserve the reservoir freshwater ecosystem by having a higher lower bound, and to allow for a flood buffer by having a lower upper bound. Each marginal penalty/reward has an associated cost/benefit, which is applied in the form of a marginal value for each unit violating/meeting the specified target. Appendix 3 discusses environmental costs and penalty pricing issues in more detail, but here we only model “hard limits”, representing physical capacity limits. In our multi-nodal model we assume that there are no non-consumptive uses, to simplify the modelling. It is relatively straightforward to allow for non-consumptive uses, for example, to model a mixed use system comprising of hydro generators and farmers, as in Mahakalanda et al. (2013).

From Figure 3.2, the model receives exogenous flows, which we cannot control, at node n , $F^{t,n}$. We assume that this type of flow only occurs at nodes, and not on arcs. At a node with positive storage capacity, these “net inflows” are either stored or released. But, there could be non-storage nodes, such as to account for two rivers joining. A non-storage node is easily modelled by setting the upper and lower storage variable bounds to zero.

In our modelling system constants are inputs and are thus exogenous. For example, the exogenous flows, but also the supply and demand data, see Section 3.6 below. Variables are endogenous and are output from the model. So apart from F , we assume that we can “control” all the other flows in our model. These include supply and demand/spill flows feeding in to, and out of, nodes. We use q variables to represent controlled flows in our models.

In Figure 3.2, spill from a reservoir node is implied as part of the flow represented by a q variable. Later, in Section 3.5, we define reservoir spills separately, using the variable w . Either way, there are potentially two types of spill. If a reservoir is full, spills exit via some form of spillway, or channel. A spillway could direct spills downstream, until that path reaches its upper flow capacity limit. Alternatively, and/or in the event of exceptional floods, spills would exit to the sink, via the spillway, a flood conveyance channel, or other means. Such spills, going to waste, would be represented in the model as arcs of the form $(n, N+1)$. We assume spills going downstream, on arc (n, n') , are potentially utilisable, up to the channel’s capacity, and can have economic and/or environmental value, which could be positive or negative. We assume that total spill is unbounded.

3.3 Temporal Aspects

Our allocation models relate to the operation of the network in the short to medium term. Long term capital development is outside the thesis scope. For simplicity, we will assume an annual time horizon, T , for all our models and experiments. We state formulations here on an annual basis, assuming that an EOH water value function, $EOHS(s^T)$, has been specified, where s^T is a given storage level at the end of the water year. Otherwise, in the last bidding period of the current water year, the optimisation may give away all the water in store.

In our computational experiments from Chapter 5 onward, we aim to compare performance in long run equilibrium. In this setting, the computational model is run for multiple years replacing the EOH with the water value function from the first period, after each run. The model is run until the initially specified $EOHS(s^T)$ converges to the equilibrium value function. Thus we can compare different market design models in terms of their annual financial performance in equilibrium.

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With an annual wholesale water market, the “decision period” length (or “market-clearing interval”) would be case specific. A key factor would be how much intra-period change results from uncertainty in the local environment. Operationally, major water treatment facilities tend to have a weekly agenda and it can take time to transfer water. So a weekly decision-period length seems reasonable, with respect to releases from significant storage reservoirs, but longer periods may prove adequate. Electricity futures markets, for example, often deal with monthly options.

Further, parties often wish the freedom to take their total allocation over an allotted period of time, according to their own variable withdrawal patterns. Parties may demand different flows in different periods. For example, they may take more/less during summer/winter. Also, they may wish to withdraw their scheduled periodic allocation at a certain point within a period. For example, for a weekly allocation period, a party may wish to take their total allocation on Monday morning. But generally, demands to take water develop in parallel to the practical withdraw limits of the local system. And, in reality, there is often a more detailed allocation schedule (broken down into finer time steps) which is generally agreed between parties and the system operator. But, in our simplified models, we assume that participant bid quantities, and resulting allocation quantities, only represent the total allocation from each time period. We assume that inflow, release, and spill, are continuously realised across the specified decision period at a constant rate. So, from the start of period t until the start of period $t+1$, release and inflow draw down and fill up the reservoir at a constant rate, respectively. Otherwise there could be one or more instances within a period, when the system runs over one, or both, of its storage bounds, creating shortage and/or spillage, with the marginal value of water varying over the period. To capture these effects would require creation of an intra-period model but, if that seemed desirable, we might just shorten the decision period length.

Generally the further into the future we look, the more uncertainty exists. So really, once we add time into consideration, we naturally add uncertainty. Storage exists to reserve supplies received today to meet shortfalls at a later date. Storage also enables us to manage unforeseen fluctuations in both supply and demand that may eventuate. Stochasticity and storage quantities are two key factors to consider when managing a water network. But, before dealing with uncertainty, we consider storage, and then demand, based on simplified deterministic assumptions.

3.4 Multi-nodal Deterministic Flow & Storage Aspects

Before developing a simplified single reservoir formulation, we first discuss some wider network flow issues. Most water systems are not as simple as a single reservoir; they comprise a wider network as indicated in Figures 3.1 and 3.2. Networks commonly have various transporting media, not just pipes but also channels, rivers, and even water tankers.

If we know the water flow (speed) and the cross section of the transport medium (pipe or channel), we can compute the total volumetric flow rate at which the transporting medium moves water between two given (supply and delivery) points in the network. Whenever we discuss water flow rates, we talk about an average measure. But what about the delay in transporting water between two given points, in a network system?

Water, for our purposes, is classed as a Newtonian fluid and is incompressible.²⁷ So, in normal operation, and assuming no leaks, closed pipelines appear to move water over large distances very quickly. In normal operation, water is stored in the transporting pipes and, while there may be minor delays when flow rates change, it is reasonable to assume that if you inject one unit into the supply end, another unit is automatically ejected out of the delivery end. Transport is also virtually instantaneous across a body of still water, such as a lake. There will be significant delays, though, when water is flowing in open channels, such as rivers or canals, and this means that there can be significant volumes of water “stored” in those channels that is not required for operational purposes, and will eventually become available for consumption downstream. In our models we ignore any (potential) open channel delays. So, while water takes time to move through its network (unlike electricity within its network), given the network allows for storage it will be assumed to be instantaneously transported, just like electricity. This assumption is valid if the water is traded over a long enough period of time, such as a day or a week, and where there were not significant transfer delay constraints.

The physical system consists of a real network, with near real time flow data which is transmitted over a communication network, such as a SCADA system. The allocation system deals with longer timeframes. Our discussions assume that all periods are of equal length. Thus flow rate and quantity are implicitly interchangeable terms. So, for the physical and

²⁷ By way of contrast, natural gas pipeline systems transfer a compressible product.

allocation systems to align, flows would have to be normalised to their correct relative quantities over a given decision period length. (e.g. cumecs to cumec-weeks, or whatever).

We specify a quantity to be moved from one node to another over the decision period assuming that the physical system operator will operate the system so as to achieve the required transfers efficiently. For example, they will want to pump water from a downstream to an upstream reservoir when electricity is at off-peak rates, so as to pump at the most efficient time. For the purposes of this thesis, we only need to consider a simplified single reservoir model.

At each reservoir node flows balance by accounting for the reservoir inflow parameter, F , and upstream and downstream flow variables, q , where the q implicitly includes spills downstream and to the sink. Hence, the generalised deterministic storage, s , and flow balance equation is as follows:

$$s^{t,n} = s^{t-1,n} + F^{t,n} - \sum_{(n,n') \in \text{ARC}} q^{t,(n,n')} + \sum_{(n',n) \in \text{ARC}} q^{t,(n',n)} \quad \forall t, n \quad (3.2)$$

Equation (3.2) is the continuity equation for a multi-nodal, and inter-temporal, system model, balancing storage and release decisions for each node, n . Here, $s^{t,n}$ is interpreted as the storage for node n , at the end of period t . This depends on the storage at the end of the prior period $t-1$, this reservoir's inflow, aggregate flows released downstream and to the sink, and the aggregate flows from upstream reservoirs, within period t . Storage and flow variables are subject to capacity constraints, but we only define these explicitly in the single-node case, in Section 3.5 below.

We note, though, that such multi-nodal models can be quite complex, with “trading” potentially occurring at every node. In particular, bids and offers made at one node are not interchangeable with those made at other nodes, and they can not be netted off against one another. Thus, such models have a great many parameters determining the extent to which inter-nodal trading (water flow) is possible, and all of those parameters could be varied, along with the offer/bid patterns at each node. For the purposes of this thesis, though, we wish to exclude variables and variations that would introduce extraneous noise into our experiments on the impact of basic market design choices on economic efficiency. So, for experimental purposes, we will only consider a simplified model.

3.5 Single-Node Deterministic Flow & Storage Aspects

This simple system assumes a single reservoir node and a single trading node. Our single reservoir formulation focuses on net inflow, and the sets of controllable flows which are supplied into the system, and consumed from the system. We assume that these supplies are “downstream” of the reservoir, meaning that they can be directly allocated to demand, at a common trading node. This configuration also allows us to readily construct a net demand formulation. For details see Section 3.6.3 below. We also assume that all spills are non-utilisable and have a zero marginal value.

In the single node setting we can drop the ‘ n ’ notation. This also implies dropping the sink and source notation. In this setting all outflows go to the sink, and we re-name some of the flow variables to be explicit about their purpose. The utilisable release from the reservoir is r , and the associated spill to the sink is w . The assumed basic (dynamic) flow interaction for a single reservoir system is shown in Figure 3.3 below. Between the reservoir node and the trading node is a capacity constrained reservoir release channel.

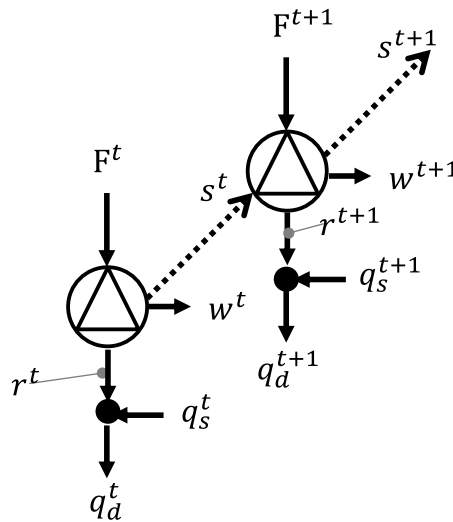


Figure 3.3: Deterministic Single Reservoir Illustration

A non-negative lower release capacity means that water can not flow back into the reservoir. The utilisable release r^t may also have some physical bounds, thus:

$$0 \leq r^t \leq \text{RCAP} \quad \forall t \tag{3.3}$$

There might be ‘real’ sellers of water, who are willing to supply water, at a cost. We introduce aggregate supply and demand variables, q_s^t , and q_d^t , respectively. At any given

time, the actual net quantity consumed by the market is the net total of utilisable flows released from the reservoir and supplied, as follows:

$$r^t = q_d^t - q_s^t \quad \forall t \quad (3.4)$$

The quantity released is traditionally determined by the reservoir operator, seeking to balance immediate and future demands. In some of our later formulations we assume a market can determine storage and release quantities. With a single reservoir, utilisable water releases from the reservoir exit the system to the trading node, and then the sink. The storage balance equation is simply:

$$s^t = s^{t-1} + F^t - r^t - w^t \quad \forall t \quad (3.5)$$

Storage limits can be represented by the following set of constraints, or bounds:

$$0 \leq s^t \leq \text{SCAP} \quad \forall t \quad (3.6)$$

Having discussed our simplified physical model, with its single trading node, we discuss how we model supplies and demands, at the one trading node.

3.6 Water Demand

Society can have a wide range of demands for fresh water, at various times and places. While many demands involve consuming the water, others are non-consumptive. But, for simplicity, we ignore such distinctions and assume that all water demands are consumptive, so that water is not returned to the system after use. We assume that each party who seeks water can explicitly order their preferences, from most to least important, and assign a marginal dollar value to each consecutive unit of water used. In other words we can assume a convex benefit function, $B(q)$, for each party, as illustrated below in Figure 3.4.

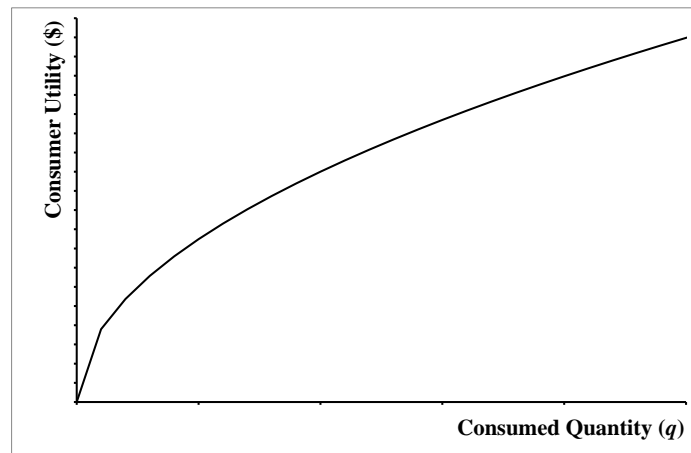


Figure 3.4: Example of a Convex Benefit Function

3.6.1 A Two Party Optimal Water Exchange

There may also be suppliers, and we will assume that each supplier has a convex cost function, $C(q)$, ignoring fixed costs and economies of scale, as illustrated below in Figure 3.5.

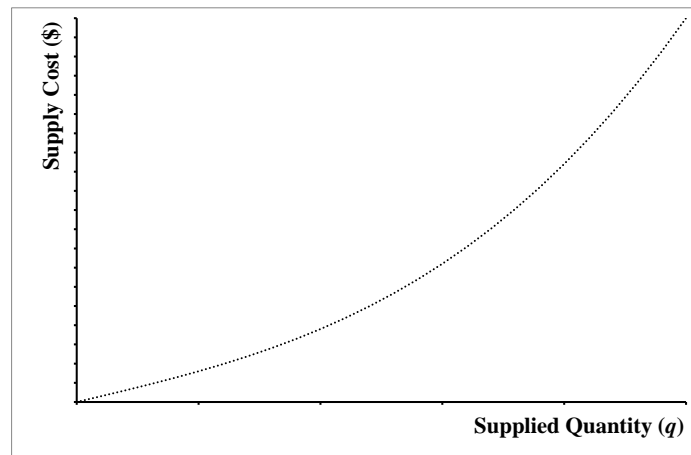


Figure 3.5: Example of a Convex Supply Function

If we centrally allocate the water between supplier and consumer, we would optimise the quantity of water exchanged to q^* . This is the quantity where there is the maximum difference between the curves (point 1 to 2) as shown in Figure 3.6 below.

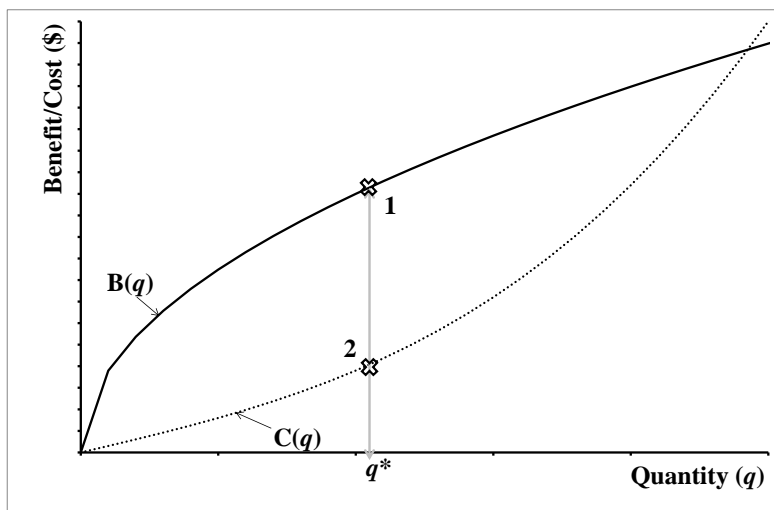


Figure 3.6: Benefit and Cost, Optimum Quantity Illustration

This optimum quantity q^* is also where the derivatives (slopes) of each curve are equal i.e. $B'(q) = C'(q)$, at points 1 and 2, respectively. Figure 3.7 shows the two derivative curves intersecting at point 3, q^* . It also identifies the optimal marginal price, λ^* at which both parties would happily exchange the associated optimal q^* units of water. This alternative

representation, in terms of intersecting derivative curves, is important because it corresponds to the standard economic concept of finding the market equilibrium by intersecting supply and demand curves.

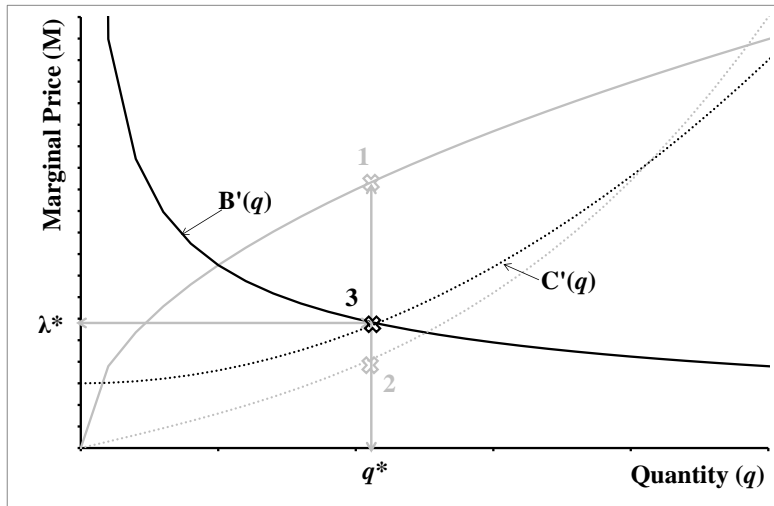


Figure 3.7: Marginal Benefit and Marginal Cost Curves, Optimum Price and Quantity Illustration

Often demand units can be grouped into blocks, with essentially the same marginal use value. Thus we can approximate the continuous benefit and cost curves as piecewise linear. Equivalently, we can represent the marginal cost and benefit curves using a set of steps. In a market model, these stepped curves would be offers to sell, and bids to buy, from various participants. This is the standard representation employed, for example in most electricity market clearing models, and we will use it here.

The welfare maximisation problem is equivalent to clearing a perfectly competitive market. In both instances, parties who wish to consume or supply known quantities of water can each assign an associated marginal benefit or marginal cost, respectively, to each unit of water. In the welfare maximisation problem the marginal price information is commonly estimated by the central co-ordinating body. In the market clearing problem the marginal price information is estimated by the market participants. In the market setting, consumers and producers provide bids and offers. In a perfectly competitive market those bids and offers are expected to reflect the marginal benefit of consumption and marginal cost of supply, to each participant.

We now have a choice of notation. We can use marginal benefit (MBENEFIT) and cost (MCOST) and discuss welfare implications for the economy as a whole, and hence by implication for parties who are subject to central allocation. Or we can use bid (MBID) and offer (MOFFER) and discuss trade implications, for market participants. Given that this thesis is about market design we opt to adopt the latter, at this point, on the understanding that these ‘bids’ and ‘offers’ could be estimated by a central planner, in a centrally planned regime.

3.6.2 Optimum Allocation with Multiple Market Participants

In some contexts we follow common practice, using the term ‘bid’ in a loose sense, to refer to supply, demand, or net demand (see Section 3.6.3 below) bids. More precisely, though, we use the terms bid and offer to refer to a single price and quantity ‘bid’ pair.²⁸ For supply and demand we define monotone non-decreasing and non-increasing offer/bid lists (or ‘stacks’) indexed by $j \in TS$, and $k \in TD$, respectively. Each bid has an associated maximum quantity (Q_k) that the bidding party is willing to take, up to the associated price level (MBID_k). Each offer has an associated maximum quantity (Q_j) that the offering party is willing to supply, down to the associated price level (MOFFER_j).

The optimisation has supply and demand quantity variables q_j and q_k , respectively. And these variables are related to the offer and bid quantity bid bounds, as follows:

$$0 \leq q_j \leq Q_j \quad \forall j \in TS \tag{3.7}$$

$$0 \leq q_k \leq Q_k \quad \forall k \in TD \tag{3.8}$$

From Section 3.5 above, the aggregate supply and demand variables, in a given period, are related as follows:

$$q_s = \sum_{j \in TS} q_j \tag{3.9}$$

$$q_d = \sum_{k \in TD} q_k \tag{3.10}$$

As in Section 3.5 and Figure 3.3, consumers ‘at’ the single reservoir node take supplies downstream from the reservoir outlet, while supplementary suppliers meet demands at that same point. Hence these supplies and demands are all collectively constrained by a common

²⁸ Sometimes referred to as offer/bid ‘tranches’.

limit on utilisable release capacity, as in Equation (3.3). We can define the set of active parties, I , at the reservoir to include a set of suppliers IS and consumers ID , such that:

$$I = IS \cup ID \quad (3.11)$$

Each individual supplier or consumer (i) will provide its own set of offers $TS(i)$, or bids $TD(i)$. Individual supply and demand sets can be aggregated into total supply (TS) and demand (TD) sets at this point in the network, such that:

$$TS = \bigcup_{i \in IS} TS(i) \quad (3.12)$$

$$TD = \bigcup_{i \in ID} TD(i) \quad (3.13)$$

These two expressions respectively define a single aggregated marginal supply and demand curve. If we were concerned with individual participants, we would use an additional index to track their positions. Adding this additional index would not change the optimisation, or market clearing, though, just the individual accounting. We use the participant index, i , for offer/bid lists for discussions in Chapter 4. But from Chapter 5 onwards, our computational models are concerned with the system operating as a whole, and with the total net benefit delivered to all market participants, not with the individual positions of specific participants. For simplicity, until Section 3.8 below, we ignore indexing individual participant bids.

In Figure 3.8 below, the LHS and RHS side illustrations show demand and supply stacks, respectively, where each unique shade represents a different water consumer and each unique striped-shading represents a different water supplier. Demand bids are stacked in monotone non-increasing price order, to form a classic “demand curve”. Supply offers are stacked in monotone non-decreasing order, to form a classic “supply curve”.

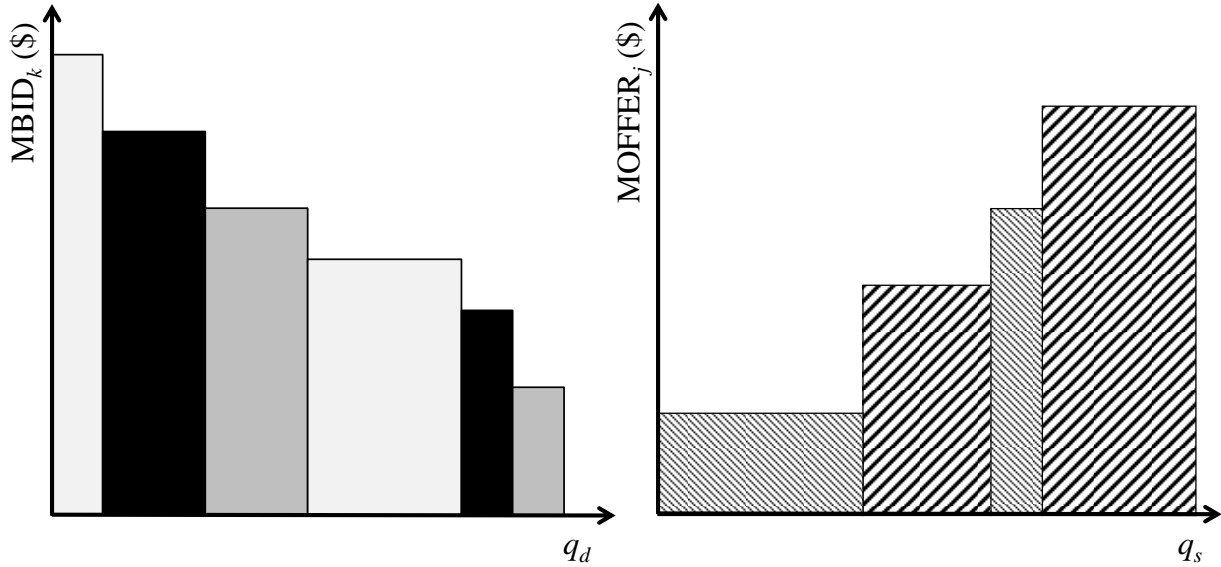


Figure 3.8: Illustration of Demand/Supply Marginal Bid/Offer Stacks

Figure 3.9 below is the discrete version of Figure 3.7, above. Figure 3.9 highlights the optimum allocation point, (q^*, λ^*) which produces the maximum net social welfare for all participants wishing to supply and consume water. The shaded region indicates the maximum welfare. This consists of the consumer surplus above λ^* (black shading) and the producer surplus below λ^* (grey shading). The consumer surplus is the gain to allocated consumers from being able to purchase the water at the market price, which is less than they are willing to pay. The producer surplus is the gain to allocated suppliers from being able to sell the water at the market price, which is more than they are willing to sell for.

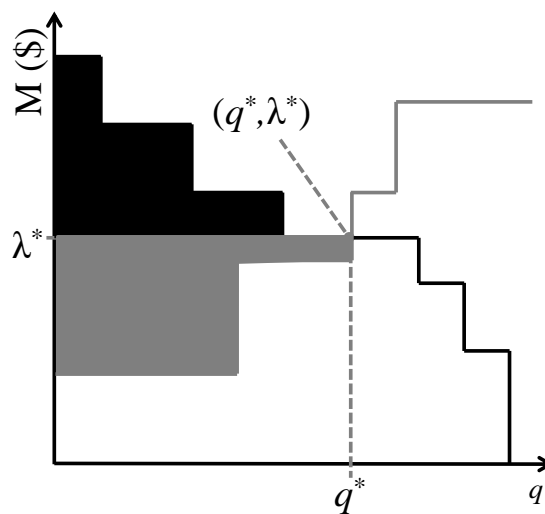


Figure 3.9: BID and OFFER Stacks, Optimum Price and Quantity Illustration

The solution illustrated in Figure 3.9 is maximising the total economic welfare, in a particular period, ignoring any inflows, release, or storage interactions. In other words, it is equivalent to finding the solution of the following optimisation problem:

$$\max \left[\sum_{k \in \text{TD}} \text{MBID}_k q_k - \sum_{j \in \text{TS}} \text{MOFFER}_j q_j \right] \quad (3.14)$$

In Section 3.3 we defined an End of Horizon water value function, $\text{EOHS}(s^T)$. For simplicity, until we define our formulation in Section 3.8, we will ignore this complication in our objective function discussions.

With Equation (3.14) the constraints (3.7) through (3.10) above must be satisfied, and supply and demand must also be equal, so:

$$q_d - q_s = 0 \quad (3.15)$$

Note that, since Equation (3.4) equates the LHS of Equation (3.15) to the reservoir release, r , this condition corresponds to clearing the downstream market without any reservoir release, or tributary flow. But the treatment of both is discussed in the next section, where, instead of separate supply and demand curves, we deal with a single “net demand curve”.

3.6.3 Creating a Net Demand Curve

The separate supply and demand curves, discussed in Section 3.6.2, can be concatenated to produce an equivalent net demand curve. This represents the net demand for water released from the reservoir. To form this, we could imagine an auctioneer first announcing a price so high that all possible units are supplied, and all discretionary demand curtailed. This represents the maximum net supply, or minimum net demand. Then, as the price is lowered, net demand increases either by reducing supply or increasing demand, as each offer/bid price is reached.

We can define a single set of net demand bids, TB, where we index that joint set by $b \in \text{TB}$. Supply and demand sets (TS and TD) could be combined such that $\text{TB} = \text{TS} \cup \text{TD}$. Alternatively, we choose to define TB as a separate set, and use the notation $j(b) \in \text{TS}$ or $k(b) \in \text{TD}$, to indicate the original supply or demand bid corresponding to net bid b . The procedure we use to process bid and offer data into defined inputs in an LP for a net demand formulation is fairly trivial.

From Equation (3.14) the price bids (from consumers) are converted into net demand bid prices, via:

$$M_b = \text{MBID}_{k(b)} \quad \forall b \text{ such that } k(b) \in \text{TD} \quad (3.16)$$

The associated maximum demand bid quantities (from consumers) are converted into maximum net demand quantities, via:

$$Q_b = Q_{k(b)} \quad \forall b \text{ such that } k(b) \in \text{TD} \quad (3.17)$$

For demand bids the net demand quantity is that part of the demand bid that is accepted, as follows:

$$q_b = q_{k(b)} \quad \forall b \text{ such that } k(b) \in \text{TD} \quad (3.18)$$

The offer prices (from suppliers) are converted into net demand bid prices, via:

$$M_b = \text{MOFFER}_{j(b)} \quad \forall b \text{ such that } j(b) \in \text{TS} \quad (3.19)$$

And the associated maximum supply bids quantities (from producers) are converted into maximum net demand quantities, via:

$$Q_b = Q_{j(b)} \quad \forall b \text{ such that } j(b) \in \text{TS} \quad (3.20)$$

For supply offers the net demand quantity is that part of the supply offer that is rejected. So, we can use the associated maximum supply quantities from producers to adjust the net allocation variable, as follows:

$$q_b = Q_{j(b)} - q_{j(b)} \quad \forall b \text{ such that } j(b) \in \text{TS} \quad (3.21)$$

Now we can limit the output quantity variables by their input bounds. Rather than Equations (3.7) and (3.8), the net demanded quantity can be bound by:

$$0 \leq q_b \leq Q_b \quad \forall b \in \text{TB} \quad (3.22)$$

And then the optimisation problem becomes:

$$\max \left[\sum_{b \in \text{TB}} M_b q_b \right] \quad (3.23)$$

Where constraint (3.22) above must be satisfied, and supply and demand are equal:

$$\sum_{b: k(b) \in \text{TD}} q_b - \left[\sum_{b: j(b) \in \text{TS}} Q_b - \sum_{b: j(b) \in \text{TS}} q_b \right] = 0 \quad (3.24)$$

Here the second term on the LHS is the maximum aggregate supply offered to the market, MAXSUP:

$$\text{MAXSUP} = \sum_{b: j(b) \in \text{TS}} Q_b \quad (3.25)$$

We can also define the maximum market demand bid to the market, MAXDEM, as:

$$\text{MAXDEM} = \sum_{b \in \text{TB}} Q_b \quad (3.26)$$

MAXSUP is a constant which will have no impact on the optimisation, but it impacts on the representation of the effective net DCR, as illustrated in Figure 3.11 below. This DCR is worth understanding, because it will play a critical role in the model employed in all of our experiments, in Chapters 5-8.

Taking the supply and demand curves from Figure 3.8 above, we reproduce them as a net demand curve in Figure 3.10. The marginal price M axis is effectively re-positioned to represent the point where supply equals demand at the equilibrium price, λ^* . Note that demands with prices greater than λ^* are to the left of the M axis, while supplies with prices less than λ^* are to the right of the M axis. The optimisation, or market-clearing, thus amounts to accepting all the demand bids (shaded) to the left of the M axis, and all supply offers (striped) to the right of the price axis. So, the point $q = 0$ represents the point where the sum of the allocated net demand supplied quantities ($b \in \text{TS}$) balances the sum of the allocated net demand demanded quantities ($b \in \text{TD}$), maximising net welfare.

Note that accepting a supply offer amounts to rejecting the corresponding net demand bid, as discussed above, in defining Equation (3.23). So, moving from left to right on this diagram corresponds to accepting net demand bids, or equivalently to accepting demand bids but rejecting supply offers, as prices fall. Thus:

- The leftmost point on this curve corresponds to allocating all supply offers and no demand bids, and is defined by $q = -\text{MAXSUP}$,
- The point at which $q = 0$ corresponds to the optimum market clearing,
- The rightmost point corresponds to allocating all demand bids and no supply offers, and is defined by $q = \text{MAXDEM} - \text{MAXSUP}$.

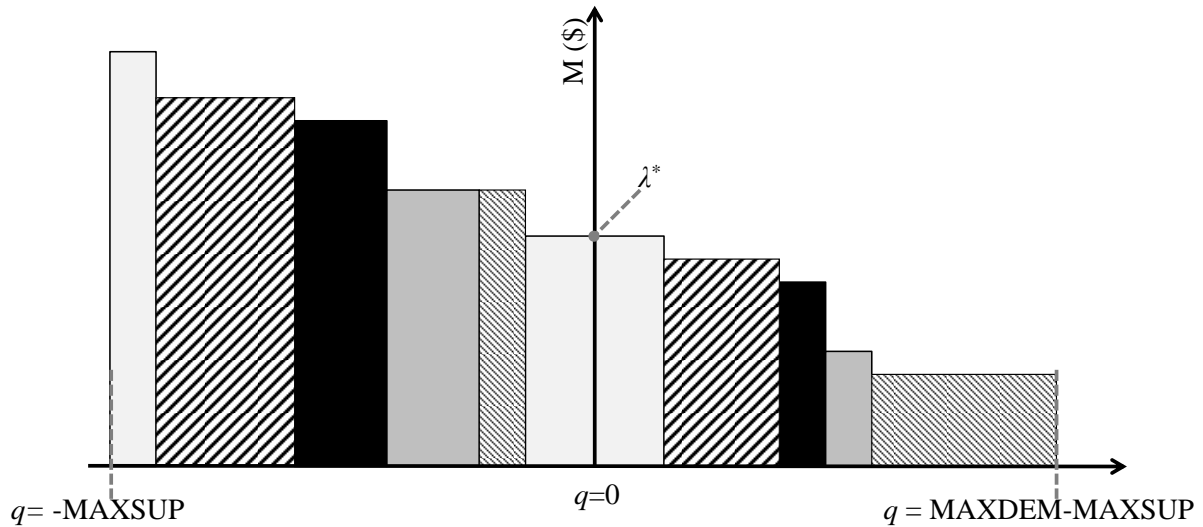


Figure 3.10: Net Benefit with Marginal Price axis on Optimum zero Net Quantity

Figure 3.10 represents the solution of the nodal market equilibrium problem for a single node market, in a single period, assuming no reservoir release or tributary flows. Once tributary flows are accounted for, this sets the starting position from which “inter-temporal” (or indeed “inter-nodal”) trading can proceed, indirectly, by varying reservoir release, as in the next section

3.6.4 Accounting for Tributary and Release Flows

Harvested net inflows (F) are received in the reservoir “free of charge”, and we treat these as zero cost supplies. But, there could also be Tributary Flows (FT s) received downstream of the reservoir. We assume these to be received in the market “free of charge”, and thus equivalent to zero cost supply offers. If tributary flows are large they could meet all market demand, and “spill” downstream of the market. We use a tributary spill variable v to represent unutilised tributary flows.

Tributary flows can be explicitly accounted for by adjusting Equation (3.4), as follows:

$$r^t = q_d^t - q_s^t - FT^t + v^t \quad \forall t \quad (3.27)$$

In Figure 3.3 above, the FT units would be illustrated as entering the system at the trading node, beneath the reservoir node. This is shown in the inset box of Figure 3.11 below. In that figure we assume $FT = 4$ units, and these are shown on the RHS of the graph given that they are zero cost. Accepting these zero cost ‘supply offers’, though, is equivalent to

rejecting the corresponding net demand bids, thus increasing the maximum aggregate supply at this node to $FT+MAXSUP$. This effectively shifts the whole net demand curve to the left, as in Figure 3.11.

Note that any additional units of tributary flow would shift the net demand curve further to the left and, potentially, change the market-clearing price. The same effect occurs for reservoir release flows. In this way we can trace out the market-clearing price by incrementally adjusting the reservoir release from 0 to RCAP. These prices correspond exactly to the part of the net demand curve to the right of the vertical axis for r^t between 0 and RCAP (or $MAXDEM - MAXSUP - FT$, if this is less). In this way we can form a DCR from the reservoir by truncating the net-demand curve to the usable release bounds. This is the DCR we use for our experiments later in this thesis. The DCR is used to balance the value of releasing water to the trading node in one period with the value of storing the water for subsequent release in a later period.

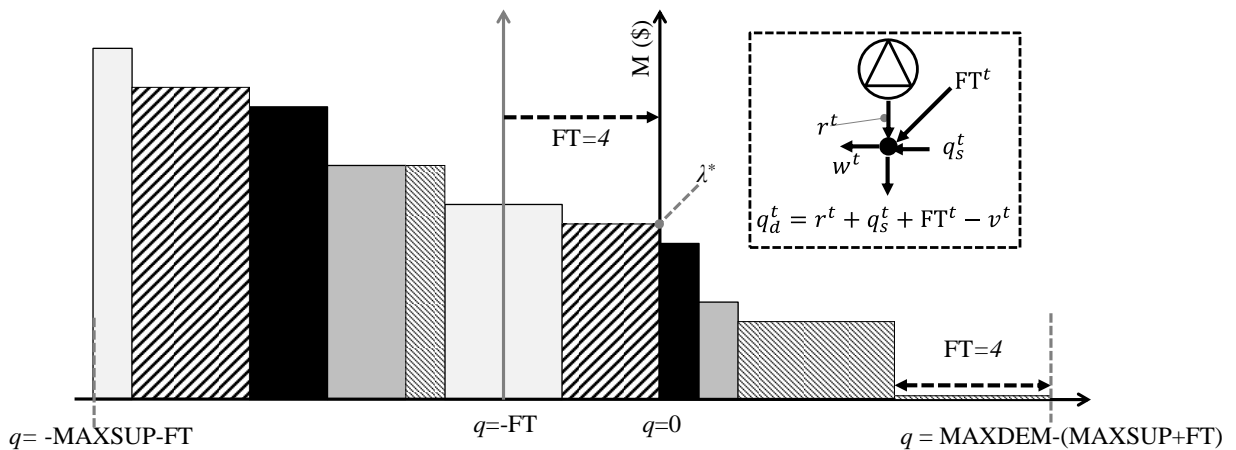


Figure 3.11: Example Illustration of Net Demand Curve Adjustment for Tributary Flow

With a net demand function Equation (3.27) can be transformed, using Equations (3.25) and (3.26), as follows:

$$r^t = \sum_{b \in TB} q_b^t - FT^t - MAXSUP + v^t \quad \forall t \quad (3.28)$$

Of course we have not yet embedded our single node/period optimisation in the context of the inter-temporal optimisation described earlier. We do that in the next section, where we

recognise that harvested flows are typically uncertain, and so the optimisation needs to consider stochasticity.

3.7 Uncertainty Aspects

The model allows uncertainty in inflow and net demand. An independent body tasked with optimally allocating the water on behalf of various parties also has a number of key challenges, some of which we discuss in the following sub-sections.²⁹ In Section 3.7.1 we discuss the inflow element of the hydrology. In Section 3.7.2 we discuss notation used to define a general SLP event tree. In Section 3.7.3 we relate the general event tree notation to the hydrological index. Finally, in Section 3.7.4 we discuss demand uncertainty.

3.7.1 Exogenous Net Inflow

We denote F as a parameter because it is an output of the hydrological process, and not a variable that we control within our models. Net inflow represents the balance between inflows and evaporation/leakage. Both may depend on how full the reservoir is, and hence its surface area, but we will ignore that correlation here, making F independent of our modelling solutions.

Inflows can vary depending on macro conditions, for example El Niño or La Niña years. For our experiments we assume inflows are stable from year to year. Inflows typically exhibit seasonal patterns over a year, and inflows may vary significantly around their expected seasonal levels. Our market design makes no assumptions about the form of the inflow distribution. Our experiments, though, typically assume a discrete symmetric distribution, thus simplifying some comparisons, as explained in Chapter 5. The probability distribution of inflows in any month is more likely to depend on what has arrived in the last few months, and also on other observations such as weather patterns and snow-pack. Some of our models will ignore such serial correlations, and assume inflows to be independent between periods. Others will assume a lag-one Markov process.

²⁹ If we were to centrally optimise this system on behalf of the trading parties, we will look to satisfy each demand bid, Q , by issuing equivalent quantity allocation variables q . But the amount of water available in store can also depend on environmental conditions, resulting in: $q'_k = q'_k(h')$.

A key issue is the relative timing of inflow, within a given period, and release to consumers. Under uncertainty, this issue incorporates the information available to decision makers, at the time when release decisions must be made.

Our optimisation models assume information is gained and acted on instantaneously at each discrete decision period. In reality, between each adjacent decision period, inflows are continually being revealed. Given the discrete nature of our models, we model two extreme policies, which we term “conservative” and “informed”:

- A conservative policy assumes that release decisions are made without knowing the intra-period inflow. So intra-period inflow is either allocated to future storage, or spilled. Hence the conservative policy can not run the reservoir as close to its lower bound.
- With an informed inflow allocation policy, inflow is assumed to be known at the beginning of the period, before the release decision is made, and to arrive in a constant stream over the period. An informed flow allocation policy allows intra-period inflow to be available for both future storage and present release decisions.

We note that both of these extreme policies lead to approximation errors compared to the real-world situation. The difference between these two policies affects the continuity equation from the SLP formulation, as discussed in Section 2.3.6. The conservative and informed inflow allocation policy implications are shown in the next section, in Equations (3.29) and (3.30) below, respectively. But we generally assume an informed inflow allocation policy.

3.7.2 General Stochastic Process

Accounting for uncertainty in the model means developing some form of underlying stochastic process. For a stochastic recourse formulation, we construct an equivalent event (or scenario) tree over a fixed time horizon. Scenario trees were first discussed in Section 2.3.4. As discussed there, the size of the tree does not depend on the number of reservoirs. The tree defines linkages between modelled states. Each event node represents a modelled state, while arcs represent realisations of the uncertain variables. The number of realisations can change by period. Conditional probabilities are defined on each arc, from one state and stage, to all possible states in the subsequent stage. From a known, single root node, the

branches of the tree represent unique paths, or scenarios, of the stochastic process evolving over time.

With a tree structure, the entire prior history is present within the tree, and potentially relevant to decision-making. We label event nodes by an ordered pair $(t,p) \in E$, following Gassmann & Ireland (1995) with each node labelled according to a specific scenario path p which passes through the node in period t . With an event tree there are information and structuring issues. For example, there are multiple paths passing through the same node.

Figure 3.12 shows an example of an event tree. The tree represents a stochastic process which has four quarterly periods within a fixed annual time horizon, where there are two states for all t . Each event node is indexed by an ordered pair $(t,p) \in \{(1,1), (2,1), (3,1), \dots, (4,8)\}$. Each scenario path is indexed numerically $p \in \{1, 2, \dots, 8\}$.

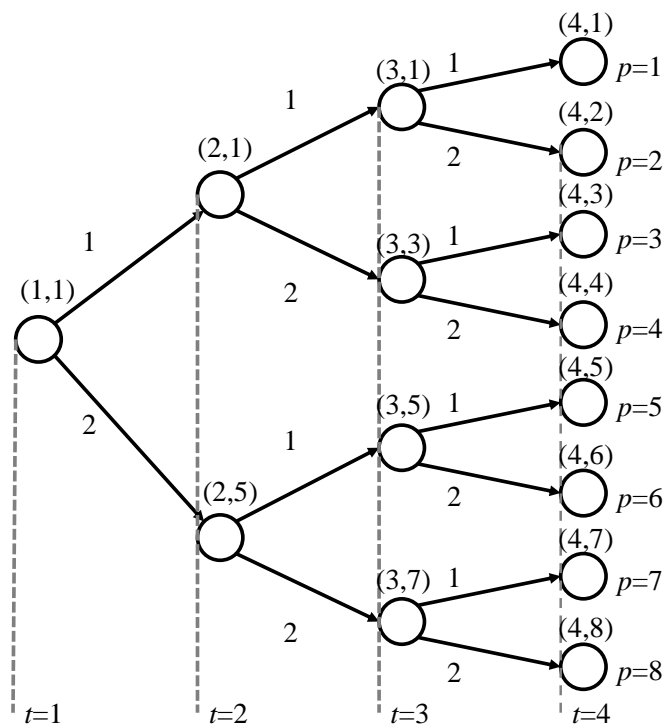


Figure 3.12: Example Event Tree Illustrating the Event Node Labelling

From Figure 3.12 above, for $p = 1$, the scenario path goes from $(1,1)$ to $(4,1)$. At each subsequent stage, the number of paths doubles, so the p attached to each node tends to

change. For example, scenario path 8 passes through event nodes (1,1), (2,5), (3,7) and (4,8). So, we cannot just use $(t-1,p)$ to define a previous node as the p might be different.

We use a similar notation to Gassmann and Ireland (1995) to refer to the preceding node by defining $prev(t,p) = (t-1, p')$ when (t,p) is a child node of $(t-1,p')$. For example $prev(3,7) = (2,5)$.

With a conservative inflow allocation policy Equation (3.5), the storage balance equation, will become:

$$s^{t,p} = s^{prev(t,p)} + F^{t,p} - r^{prev(t,p)} - w^{t,p} \quad \forall (t,p) \in E \quad (3.29)$$

In Equation (3.29), the conservative approach implies that we do not know the inflow for the coming period and the same release should be made for any inflow. To achieve that in the SLP the release variable is associated with the previous event node and is interpreted as the release for period $t-1$. Notice that since the uncertain inflow could be more than can be stored, the spill can depend on the inflow.

Alternatively, with an informed inflow allocation policy Equation (3.5) will become:

$$s^{t,p} = s^{prev(t,p)} + F^{t,p} - r^{t,p} - w^{t,p} \quad \forall (t,p) \in E \quad (3.30)$$

In Equation (3.30), the informed approach implies that we do know the inflow for the coming period and can adjust the release accordingly. In this case the release variable is associated with the current event node and interpreted as the release for period t given the random variable realisation for the current event node.

Each arc has an associated conditional probability. The conditional probability gives the chance of each realisation given the particular history of previous realisations. The conditional probability of the arc ending at event node (t, p) is denoted $\check{P}^{t,p}$, where the P has a ‘‘breve’’ accent. For example, from Figure 3.13, the conditional probability of moving from node (2,1) to node (3,3) is $\check{P}^{3,3}$. The sum of conditional probabilities for all arcs leaving the same node is one.

Path probabilities give the probability the system will pass through the state represented by an event node. They are defined as the product of all prior conditional probabilities on the path

terminating at the event node. Path probabilities are distinguished from conditional probabilities by not having a “breve” accent, $P^{t,p}$. For example, from Figure 3.13, the path probability at node (3,3) is $P^{3,3} = \check{P}^{1,1} \times \check{P}^{2,1} \times \check{P}^{3,3}$. Given $P^{1,1}$ always equals 1, this simplifies to $P^{3,3} = \check{P}^{2,1} \times \check{P}^{3,3}$. The sum of all path probabilities with the same t is one.

At each node on the event tree, there needs to be information (data) relating to that specific situation. For example, parties often only require water at various times and for certain conditions relevant to their unique situation, including the past weather conditions. To model this uncertainty, some central co-ordinating body may need to collect or estimate party requirements and marginal costs/benefits to inject/remove water for every event node. The information requirements could be prohibitive if the various parties are expected to supply data for every node in the scenario tree.

3.7.3 Modelling Uncertainty via a Hydrological Index

To simplify this data issue (which later becomes a market design issue), the requirements and marginal costs/benefits (offers/bids and contractual allocations) may be defined as solely a function of an index, h , and the period. In principle this index could be calculated from a variety of factors affecting both water supply and demand. If F and G represent two factors, say net inflow and average temperature measured at various points i in the catchment, we could have:

$$h^t = \hat{h}^t(F_1^t, F_2^t, \dots, F_n^t, G_1^t, \dots, \text{etc.}) \quad \forall t \in T \quad (3.31)$$

Other examples of factors include evaporation, rainfall, soil moisture, and ambient temperature. The hydrological index is representative of the general situation, and we wish to construct an objective index that is well correlated with measures of water supply and/or demand. Modelling the relationships between the underlying factors well may require many states. Alternatively, a coarse index with only a few states could be used. For example, a simple three state approximation could be qualitatively described as dry, medium, and wet.

Practically, we need to be able to associate an index value with each event node in our scenario tree. The hydrological state observed at event node (t,p) is denoted by $h(t,p)$. Based

on an underlying continuous hydrology state, our stochastic process uses a varying number of discrete hydrological states H^t , for each period t .

The h is the hydrological state at a given node. Different nodes can have the same hydrological state. For example, from Figure 3.12, all nodes are defined to have a hydrology state of 1 or 2. Parameters and variables are associated with each event node. The history of hydrological states for all of the preceding periods is unique for each event node. The histories provide unique labels for the event nodes, as do the (t,p) node labels. Both types of labels are equivalent. The history contains all h 's from all prior nodes, on the present path, up to and including the current node.

Figure 3.13 below, shows the same scenario tree as Figure 3.12 above, with $h(t,p)$ shown for each node. In this example we associate a hydrology state of 1 with the root node. The unique hydrological history is indicated by the data in the square brackets “[#]”. For example, event node (3,5) has hydrology index $h(3,5)=1$, with a history of [1,2,1].

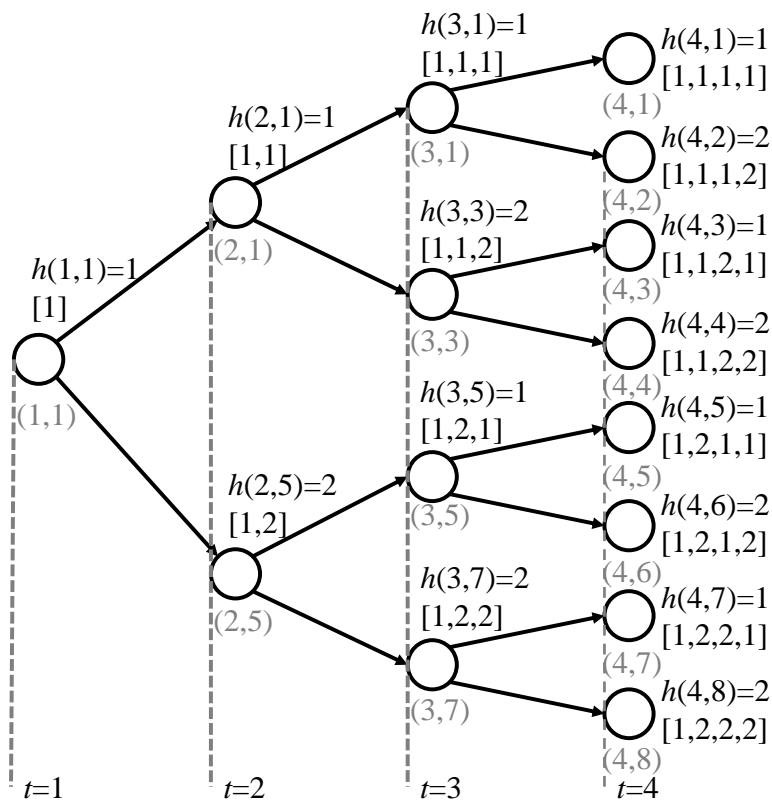


Figure 3.13: Example Event Tree with Hydrological Node Addressing

The hydrological state defines the net inflow for each event node. This means we can replace $F^{(t,p)}$ with $F^{t,h}$ where the hydrological state for event node (t,p) is h . This ensures that, for period t , any event node with the same hydrological state will have the same inflow. For example, for the event tree shown above in Figure 3.13, event nodes (3,1) and (3,5) both have hydrology states of 1, hence $F^{(3,1)}=F^{(3,5)}=F^{3,1}$.

Using the hydrological state notation, the storage balance constraint Equation (3.30), with an informed inflow policy, would become:

$$s^{t,p} = s^{prev(t,p)} + F^{t,h(t,p)} - r^{t,p} - w^{t,p} \quad \forall (t,p) \in E \quad (3.32)$$

As well as the hydrological parameters (probability and inflow), there are also the demand parameters to consider.

3.7.4 Demand Uncertainty

Some parties may have a relatively constant demand pattern for water all year round, but demand is typically quite seasonal. Various hydrological factors may also affect many party's preferences, at any given time and place. We allow a party's demand to be correlated with the hydrology index. For example demand may be inversely correlated with the hydrological index, requiring more water during a drought and wishing for less water during a flood. Or simply, if it rains and the crop's water needs are fulfilled then there is no need to irrigate in the short-term.

If the formulation used supports demand quantities that are "state-dependent" upon a hydrological index, $h(t,p)$, we denote each net demanded bid tranche, b , quantity as:

$$Q_b^{(t,p)} = Q_b^{t,h(t,p)} = Q_b^t(h(t,p)) \quad (3.33)$$

Marginal bid prices could also be state-dependent, as follows:

$$M_b^{(t,p)} = M_b^{t,h(t,p)} = M_b^t(h(t,p)) \quad (3.34)$$

In the general event node form, demand could be specified as a different net demand bid for each event node: $(M^{t,p}, Q^{t,p})$.

Under uncertainty, in the most general form we model, the optimisation problem in Equation (3.23) becomes:

$$\max \left\{ \sum_{(t,p) \in E} P^{t,p} \left[\sum_{b \in TB} M_b^{t,p} q_b^{t,p} \right] \right\} \quad (3.35)$$

In Equation (3.35) we have allowed the marginal bid prices, M 's, to vary by event node.

3.8 Optimisation Models

Bringing together the discussion in the previous sections, we conclude this chapter by outlining two stochastic LP reservoir models, in a form that might be used by a central planner responsible for wholesale water allocation. Firstly, we show a multi-nodal model under uncertainty, then a stochastic single node model. We assume an informed inflow allocation in both models. The multi-nodal model explicitly considers demands and supplies, while the single node model uses a net demand formulation. The models shown allow for state-dependent bidding, correlated to the system hydrology.

Recall, in Section 3.7 we label the event tree nodes as ordered pairs (t,p) given by the stage t , and representative scenario path p . Not all ordered pairs (t,p) are represented in the event tree so we adopted the notation $prev(t,p)$ to refer to the event node on the same path as (t,p) in the previous period, $(t-1)$. To set the initial storage we define $prev(1,1)=(0,0)$, and then the inherited initial storage position at the beginning of trading is set by the initial storage variable/parameter, as:

$$s^{prev(1,1),n} = s^{0,0,n} = \text{SINITIAL}^n \quad (3.36)$$

The objective function includes the End of Horizon marginal storage value, EOHS, as discussed in Section 3.3 above. In theory the EOHS values could be bid for explicitly. The bids could be generated by market participants and/or estimated via the market manager. Alternatively the EOHS values could be the long run equilibrium values. We assume the EOHS is set by the market manager. This is piecewise linear concave function of the storage variable.

3.8.1 Stochastic Multi-Node Formulation

Our most general formulation expands the objective in Equation (3.14) to maximise net welfare temporally, spatially and under uncertainty.

$$\max_{q, r, s, v, w} \left\{ \begin{array}{l} \sum_{(t,p) \in E, n \in N} P^{(t,p),n} \left[\sum_{i \in I} \left(\sum_{k \in TD} MBID_{i,k}^{t,h(t,p),n} q_{i,k}^{t,p,(n,N+1)} - \sum_{j \in TS} MOFFER_{i,j}^{t,h(t,p),n} q_{i,j}^{t,p,(0,n)} \right) \right] \\ + \sum_{(T,p) \in E, n \in N} P^{(T,p),n} \left[EOHS^{T,h(T,p),n}(s^{T,p,n}) \right] \end{array} \right\} \quad (3.37)$$

This maximisation is subject to constraints setting the following:

Bid Quantity Constraints for:

Meeting Demand, where feasible, up to Max Stack Width:

$$0 \leq q_{i,k}^{t,p,(n,N+1)} \leq Q_{i,k}^{t,h(t,p),(n,N+1)} \quad \forall i \in I(n); k \in TD(i,n); (n, N+1) \in ARC; (t, p) \in E \quad (3.38)$$

Scheduling Supply, where feasible, up to Max Stack Width:

$$0 \leq q_{i,j}^{t,p,(n,N+1)} \leq Q_{i,j}^{t,h(t,p),(n,N+1)} \quad \forall i \in I(n); j \in TS(i,n); (0,n) \in ARC; (t, p) \in E \quad (3.39)^{30}$$

Total Demand from Reservoir n to Sink, on the event tree:

$$q^{t,p,(n,N+1)} \geq \sum_{i \in I, k \in TD(i)} q_{i,k}^{t,p,(n,N+1)} \quad \forall (n, N+1) \in ARC; (t, p) \in E \quad (3.40)$$

Total Supply to Reservoir n from Source, on the event tree:

$$q^{t,p,(0,n)} = \sum_{i \in I, j \in TS(i)} q_{i,j}^{t,p,(0,n)} \quad \forall (0,n) \in ARC; (t, p) \in E \quad (3.41)$$

Total release at each event node and reservoir:

$$r^{t,p,n} = \sum_{(n,n') \in ARC} q^{t,p,(n,n')} - \sum_{(n',n) \in ARC} q^{t,p,(n',n)} + FT^{t,p} \quad \forall n \in N; (t, p) \in E \quad (3.42)$$

$$s^{t,p,n} - s^{prev(t,p),n} + r^{t,p,n} = F^{t,h(t,p),n} \quad \forall n \in N \text{ for } (t,p) \in E \quad (3.43)$$

$$s^{0,0,n} = SINITIAL^n \quad \forall n \in N \quad (3.44)$$

³⁰ As discussed in Section 3.2, Equations (3.39) and (3.38) allow for supplies and demands from node 0 and to node N+1, respectively.

Capacity Constraints:

$$0 \leq r^{t,p,n} \leq \text{RCAP}^n \quad \forall n \in \mathbf{N}; (t, p) \in \mathbf{E} \quad (3.45)$$

$$0 \leq s^{t,p,n} \leq \text{SCAP}^n \quad \forall n \in \mathbf{N}; (t, p) \in \mathbf{E} \quad (3.46)$$

A given infrastructure network often consists of a large number of reservoirs and interconnecting links. A multi-nodal stochastic formulation is complex and challenging to analyse. Every constraint that might bind in any part of the system in any period of every conceivable future scenario defines a “resource” that could be priced and allocated at that time, or in prospect. Further, every decision that might be made with respect to any part of the system in any period of every conceivable future scenario will imply a potential cost, in terms of resource use, and that should, in principle, affect current prices. There could also be multiple cross correlations between the modelled hydrology state, relative to different reservoirs and participants, trading at different points in the catchment.

At this stage this model appears too complex to analyse the direct effect of stochasticity with so many interacting system elements. The size of the required scenario tree can make such a model, if formulated as an LP, difficult to solve, particularly with many stages. Also, to analyse equilibrium behaviour of such a model requires repeated solution with adjusted starting and ending conditions.

The focus of Chapters 4-8 is to understand the impact of allocating water where parties can trade various levels of stochasticity in a market setting. A stochastic market design, with an associated set of experiments, will have enough complexity for us to analyse. As a first step to understanding the impact of uncertainty is to consider a single tank model for our stochastic experiments. This allows us to use CDDP as a solution method, which provides other benefits for generating equilibrium results. So we now simplify the stochastic multi-nodal formulation to a single tank formulation.

3.8.2 Stochastic Single “Reservoir” Formulation

For our single tank model we use a Net Demand representation of supply and demand as discussed above in Section 3.6.3. Also as this is a single reservoir formulation we can eliminate locational node indexing.

$$\max_{q, r, s, v, w} \left\{ \sum_{(t,p) \in E} P^{(t,p)} \left[\sum_{i \in I, b \in TB(i)} M_{i,b}^{t,h(t,p)} q_{i,b}^{t,p} \right] + \sum_{(T,p) \in E} P^{(T,p)} \left[\text{EOHS}^{T,h(T,p)}(s^{T,p}) \right] \right\} \quad (3.47)$$

This maximisation is subject to constraints setting the following:

The bid quantity limit associated with the bid tranches for each participant, at each event node:

$$0 \leq q_{i,b}^{t,p} \leq Q_{i,b}^{t,h(t,p)} \quad \forall i \in I; b \in TB(i); (t,p) \in E \quad (3.48)$$

The total allocation for each individual participant, at each event node:

$$q_i^{t,p} = \sum_{b \in TB(i)} q_{i,b}^{t,p} \quad \forall i \in I; (t,p) \in E \quad (3.49)$$

The total release at each event node, within period t :

$$q^{t,p} = \sum_{i \in I, b \in TB(i)} q_{i,b}^{t,p} \quad \forall (t,p) \in E \quad (3.50)$$

The net reservoir release at an event node:

$$r^{t,p} = q^{t,p} - \text{FT}^{t,p} - \text{MAXSUP} + v^{t,p} \quad \forall (t,p) \in E \quad (3.51)$$

The continuity equation for each event node, where we define $s^{t,p}$ to be the storage at the end of period t :

$$s^{t,p} - s^{\text{prev}(t,p)} + r^{t,p} + w^{t,p} = F^{t,h(t,p)} \quad \text{for } (t,p) \in E \quad (3.52)$$

The inherited storage position of the reservoir, at the start of the water year:

$$s^{0,0} = \text{SINITIAL} \quad (3.53)$$

Physical Reservoir Bound Storage Constraints for each event node, for what can be released to the market:

$$0 \leq r^{t,p} \leq \text{RCAP} \quad \forall (t,p) \in E \quad (3.54)$$

And for what can be stored for the future:

$$0 \leq s^{t,p} \leq \text{SCAP} \quad \forall (t,p) \in E \quad (3.55)$$

Non-negativity constraints:

$$0 \leq w^{t,p} \quad (3.56)$$

$$0 \leq v^{t,p} \quad (3.57)$$

3.9 Summary

This chapter outlines the underlying models and set notation used throughout the remainder of this thesis. Our models assume exogenous net inflow which is harvested from an uncertain environment. We defined a general wholesale water network, where water supplies and demands are differentiated temporally and locationally. We noted that water is stored in reservoirs, and within the transfer network, and that supply and demand can be uncertain.

We assume parties would receive some value from giving or taking water, at various times and places, and we also assume that party's preferences may depend on the underlying uncertainty which we model via some hydrological index. We discussed how we could centrally co-ordinate these parties to maximise net welfare.

The optimisations aggregate the demand curves, as defined by the associated maximum quantities that each party would be willing to supply or consume, at particular marginal cost/benefit levels. Essentially the same formulations can be used for central planning or market clearing. We assume that a central authority allocates the water under uncertainty, based on marginal offers and marginal bids from participants wishing to trade water through the centrally co-ordinated market.

A general market clearing formulation was developed to account for locational, temporal and hydrological uncertainty aspects. But many models used in practice aggregate reservoirs and market locations, for example the Victorian Gas Market discussed by Pepper et al. (2012). In addition, a multi-locational, single reservoir market induces a demand curve for release at the reservoir. So we drop the locational issues now, while Chapters 4-8 explore various options and issues that could affect the design of a market where there is underlying stochasticity. Thus, only the single reservoir, single demand/market node, and consumptive use (only) formulation is used in those chapters. We recognise this is not ideal, but given the relative complexity of these issues, our investigations cover only some initial exploration work on this topic. And even then we will call on many existing ideas from electricity and natural gas industries, which have developed markets in tradable commodities, in similar situations.

4. Water Trading Conceptual Framework

4.1 Chapter Introduction

In this chapter we assume we are in a market trading environment, in which “parties” compete for water as “participants”. Participants may wish to actually trade a common water resource, over a prolonged period of time. This resource may be subject to uncertainty from a range of exogenous factors affecting both supply and demand. The resource may also be limited, and with a constrained uncertain resource then some rationing of allocation will occur. So while each participant may want certainty about their future allocation, in reality allocations cannot be made certain for all participants. Hence participants need to be able manage variations over time and hedge against unfavourable future situations.

For this chapter only we relax the perfect competition assumption. The water trading framework discussed does not require it. Where we present formulations to illustrate concepts these may assume perfect competition to simplify the exposition.

In a trade setting the marginal utility curve is related to the demand curve by the marginal price that each participant is willing to pay for each consecutive unit of that good. Hence here participants may explicitly provide price information to express their desires to trade water. We commonly refer to supply offers and consumer bids. With a net demand function we refer to these as defining simply bid prices for water.³¹ And the marginal value each participant places on using each unit of water is now central to the market allocation process.

Bids can be notional, in situations where participants do not provide full information for all scenarios in all periods, so that likely future bids must be estimated by the market manager. Generally, though, the bids can be real, in as much as participants really provide the price, and quantity, information for each bid tranche. Real bids can be indicative or binding. With binding bids the market requires participants to complete corresponding transactions based on some agreed set of market clearing rules; whereas indicative bids are only used to perform indicative market-clearings, and thus simulate future prices. This is common practice in many electricity markets, for example, in order to facilitate iterative price discovery.

³¹ With each piece of price information participants provide a maximum quantity that they are willing to trade at the associated marginal price.

Chapter 4: Water Trading Conceptual Framework

While participants' utility functions may be expressed in continuous form, solution of the market clearing model requires the bids to be discretised at various points, in the computational process. This continuous versus discrete issue also occurs at the market interface where participants submit their bids. Offers and bids can either be discrete or continuous. Or they could be some mix, combining a basic discrete bid with a continuous adjustment. For example, they could firmly allocate a basic amount, based on the minimum net inflow state, and this basic allocation could subsequently be adjusted up, in response to the observed net inflow state. These adjustments can be determined by coefficients.

In this chapter we are investigating the allocation of an uncertain net inflow which feeds into, and out of, a single reservoir system, where storage and release decisions are made over a varying decision period length, within an annual time horizon. We first discuss the key issue of decision making roles, which would require careful consideration when developing a water trading system in practice.

The main focus of this chapter is to discuss options by which "water" trading is organised through a more conventional centrally coordinated market, in a stochastic environment. In all these options water can be traded in the present, on the "spot market". But this leads us to ask how much water to save for the future? And, how to value future water demands in an uncertain environment?

In the first group of options, participants do not trade water, per se, but each owns a tradeable "share" of the system, for which they make their own release and/or storage decisions. In the second group of market design options, forward market simulation may be driven by participant bids, but only spot water rights are physically traded, although participants may trade financial futures instruments. In the third group of market design options, future water rights are also traded, with future quantity allocations being determined, with varying degrees of sophistication, on the basis of solutions determined by the market-clearing mechanism.

4.2 Decision Making Roles

4.2.1 Right and Responsibilities: Social, Economic, Legal and Political Considerations

From a market design perspective, the first key issue is to determine what infrastructure “the market” will “operate”, and control for its purposes? Who decides aggregated storage and release patterns over time and the allocation of releases to particular users? Who is responsible for management of (which) contingencies, compliance with environmental constraints (and who pays), prudential management, settling trading disputes, and creating and disseminating information about current and past bidding patterns and the range of future outcomes?

Firstly, there is no point giving decision-making “responsibility” for a resource to a party that cannot legally or physically control it, or will not have the required information at the right time. Thus roles need to be clearly defined and the demarcation must respect limits imposed by existing ownership, and the agreed level of regulation, while accounting for lines of communication, available information, and realistic control.

But economic and policy objectives are also critical. Will reservoir storage and release management, for example, be determined by the collective “wisdom of the market”, or by a centralised storage policy planner, or by private parties who own reservoirs? Or will it be determined by a hybrid regime, such as: The wisdom of the market, except with respect to privately owned reservoirs, and unless there is some form of declared crisis?

Strong arguments can, and will, often be made for more, or less, reliance on market forces from economic and societal perspectives. Economists typically favour “more market” solutions, while engineering tradition has often favoured the opposite. The “correct” answer for each of these questions depends on the inter-play within the given environment; the scope of which lies outside this thesis. That is, we will focus on the broad question of how a market could be made to work, in general, not whether a particular market structure should actually be implemented, in any particular situation.

Either way, there are decisions to be made periodically, from the present into the future, to balance water needs which are also periodic. Decisions can be made directly or indirectly,

and for them to be effective market participants must have clearly defined rights to implement a set of actions. Likewise, a co-ordinating body must have the authority to: manage the system of rights, to moderate infeasible requests, and to potentially penalise violations which go beyond allocated rights.

Thus practical market management raises a number of legal and prudential issues, and is likely to involve individual and common liabilities for both the market manager and participants. Barton (2004) discuss legal issues in energy markets, giving an international, and country by country perspective. We do not discuss these issues for water markets. But, in Section 4.4.2 below, we do discuss general operational requirements, for the market manager and participants.

4.2.2 Market Allocation: Decisions and Options with Constrained Resources

With a constrained resource some participants will not receive an allocation. Ultimately decisions must be made about how much of the water presently in storage should be released now. This decision is taken periodically, and any decision about future releases will not be certain. The reservoir, and potentially participant demand, is subject to the hydrological condition which eventuates. Future storage and release amounts are not actually decided until the realised situation is revealed.

In the first instance we will discuss a set of options where each participant is individually responsible for a sub-set of the physical storage and/or release decisions. But the bulk of this chapter focuses on quite a different set of options by assuming a central body is responsible for the total set of storage and/or release decisions, based ((in)directly) on the aggregate bids for water from the participants.³²

In the centralised market responsibility to manage the underlying uncertainty lies with the market. The market manager must operate in accordance with the market clearing rules to clear bids, where these bids express the aggregate requirements of the trading participants. In certain situations of shortage or excess water, the price would naturally become abnormally high or low, respectively. So in general, in this set of options the market responds to account

³² Participants provide economic information relating to the value they place on water at various times, and potentially due to varying levels of uncertainty.

for the modelled hydrology. These adjustments then feature in the on-going decision making processes, directly signalling what to allocate to whom, and when,³³ depending on the hydrological conditions which could eventuate.

We now discuss an overview of the key issues for the participant share of system options.

4.3 Share of System Options

4.3.1 Overview

Here we assume a set of participants all own an initial right to a “virtual share”,³⁴ or slice, of the reservoir, and in each decision period they decide how much of their water to release now, and how much to store for the future. A participant’s share of the system would include their proportional share of net inflow, and of release/transport channel capacity. Spill is a function of participants’ decisions. Physical constraint limits apply to aggregate storage and release decisions, and spills may incur a range of penalties.

This type of arrangement specifically avoids centralised optimisation as participants are responsible for their own storage and release decisions; with the sum of the individual participant decisions determining the collective way forward. In this arrangement, participants directly benefit from their share of releases and net inflows, while also forfeiting their share of potential spills. There is no explicit information on how each participant values the water and the market manager merely facilitates the participants’ wishes. In principle, the manager merely collates and actions participants’ storage and release decisions for their respective shares of the system. In reality the market manager must ensure that the participants’ instructions are feasible, in aggregate. This is achieved by inputting participant storage and release decisions into a reservoir status model. The decisions are either verified or adjusted, as required, while ensuring feasibility and system security. The model may also highlight opportunities, where some participants hold under-utilised storage/release capacity that would be valuable to others.

³³ With a multi-tank system “where” it is allocated also becomes an issue. A system with different risk characteristics and hydrology indices, which are inter-correlated in some fashion, has further issues.

³⁴ Potentially allocated via some separate tendered process, or based on historical rights.

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Thus the market manager may also administer rules that allow one participant to “borrow” some capacity from another, or facilitate short or long term trading of system/capacity shares between participants. This is one way of reconciling short-term schedules while respecting aggregate constraints, given a fixed share. Hence, participants could make separate transactions outside of the market, but they still need to decide what to explicitly release now or store for later. Their initial storage position (share of system) nets out their separate storage and release capacity decisions. There would be a capacity limit on the total aggregate virtual release capacity.

Hughes et al. (2013) discuss some “virtual share” models of this type which already exist in Australia, and others have been proposed, as discussed by Barroso et al. (2012). Starkey et al. (2011) suggested the possibility that this decentralised approach could be taken instead of the “Centralised Market” options discussed below. That remains an option, but we will not pursue the concept further, here.

4.3.2 Option Discussion

These options assume all parties already “own” proportional shares of the available system capacity. Participants would have positions in the market, both before, and after trading. The market manager facilitates trading shares. We assume the market manager’s objective is to maximise gains from purchases, while ensuring shares are not over allocated. Post-trading storage shares are updated with the result from aggregate participant decisions. Regardless of which market option is chosen, the system share options all have a valuable commodity to be bid for, given post-trading storage is subject to net inflow (and possibly spill) adjustments. That is, unless net inflow is auctioned separately.

If participants bid for a revised share at the beginning of a period then the storage and release decision is taken based on the shares traded. And, of course, this could include spill and net inflow adjustments. But there could be a range of alternative constructions depending on the interpretation of the system shares. Inflow and spill adjustments could be unaffected by the present periods trading, but this might be inconsistent. Alternatively inflow and spill

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adjustments could be based on end of period shares.³⁵ Another alternative is to trade end of period shares for each hydrology condition, but this starts to become too complex.

If participants bid for only virtual storage capacity they decide in the present how much of their present share to store and release now. But there will also be a capacity limit on release. If in total the participants wish to release more than the release capacity, the release amounts will also need to be rationed. Release capacity could be allocated to participants in proportion to their storage capacity share. But this type of release constraint is likely to be too restrictive as desired storage and release for individual participants might be in very different proportion to the capacities. Those participants wanting to release less than their proportion of the release capacity will artificially restrict the total amount available for release. So, participants will probably also need to trade virtual release capacity.³⁶

Since a unit released is a unit not stored, we could construct bids for swapping storage and release capacity. So we could assume participants' willingness to pay for storage capacity is implicit in their release capacity bids. Further, participants may as well just bid what they are prepared to pay to "release" rather than for "release capacity" as it amounts to the same thing.

Trading system shares is effectively the same as collecting and reallocating tolls as follows: With spare release capacity the release price would be zero and no money would change hands. When total demand is above the release capacity there will be a toll collected for all water released, reducing demand to exactly match capacity. This toll creates a net surplus. It is the proportional redistribution of the toll, to participants, which creates the equivalent trading system.

Channel capacity is unlikely to be sufficient to allow all participants to extract their stored water simultaneously, or at their preferred times. Thus release capacity will often have to be allocated according to some schedule, and may become a critical constraint. So far we have only discussed rationing release capacity in a single period, with no information about (or for) future periods. While the discussed options should ensure aggregate feasibility, the

³⁵ Here the purchase variable now multiplies the spill, which would have implementation issues if deployed through a LP.

³⁶ Actually a detailed issue is that participants could buy limited storage capacity but buy all the release capacity and hold other participants to ransom, to release their water. But this could potentially be avoided by bidding for release rather than release capacity.

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release schedules for particular participants could be quite variable. Without any other information about release capacities, these could be constrained in proportion to current storage, across all scenarios. Going further would require participants to express what they are prepared to pay, indicatively, or for real, for stored water and/or release capacity in future periods. And previous discussions apply to future period trades.

In these slice of system markets, each individual participant must take into account that this stored water could be sold in the future, at the future spot price. Better market outcomes are likely if participants have better information about future water values, perhaps from a forward simulation. Some forms of forward market prices are explored more directly in the next section, on centralised market options.

4.3.3 Summary

The concept of participants simply trading off storage versus release for their own storage share is conceptually appealing and the option of implicitly valuing spot storage and release capacities appears to be the most promising, at this stage of the assessment. These types of models generally involve only a one period optimisation and no central body trades off current versus future usage. The trade-off is made by participants who can hedge their risk through their proportional ownership of “system shares”. Hence participants do not have to explicitly contract for release in any particular period, and we get no explicit simulation of future conditions. These conceptual options could be developed further, including extensions into multi-period models. But this is left for others, given we now focus on an overview of the key issues for centrally co-ordinated options.

4.4 Centralised Market Options Overview

4.4.1 Outline

In this group of options the hydrological uncertainty is accounted for in the market. All water in store is explicitly valued, perhaps using forward simulation of outcomes in increasingly uncertain environments. Present and expected future demand information is centrally co-ordinated. Both hydrology and demand information is used in the decision making process, to optimise water allocations and potentially to simulate future situations that could develop.

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In all cases, we envisage the market manager employs a MCE to optimise allocations based on some assumed distribution of future inflows, for some present bids received (or perhaps generated from historical observation). The market manager clears the spot market allocating temporary rights to take water to trading participants, based on the present hydrological situation.

Importantly, we assume that participants will actually take their allocations, so that the amount of water in the system can be assumed to balance as planned, and remains in sync with the market modelling. We note that this assumption may not be true in real systems, where market managers might have to make a range of remedial adjustments, particularly in order to deal with extreme events such as spill, flood, and shortage. Conversely, a model would not really be able to provide a realistic “simulation” of how that regime would operate, unless a rule could be specified to describe the manager's activities in this regard. We do not explore that option, but in Section 4.7 we explore options under which participants specify rules describing how their actual take will vary from contracted base levels, as hydrological conditions vary.

4.4.2 Market Operation

In this section we discuss how trading participants could generally interface with the market manager, including what information these parties exchange. Market participants need to be able to express their willingness to trade. In general in these options each bid informs the market manager of a participants’ willingness to accept a ceiling marginal price M for an associated maximum number of quantity units Q . Participants may want water both now and later. The market manager optimises allocations accounting for some assumed distribution of future inflows, for the M/Q bids received.

We now discuss the general tasks the participants and manager would perform in the market, at various times.

Ahead of time, market participants would provide multiple tranche M/Q bids to the market manager for:

- The next spot period, for which bids are deterministic and binding, thus forming real contracts; and

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- Potentially for future periods, for which bids could be conditional, either indicative or binding.

Binding bids commit the participant to buy something, if cleared, although, as we will see, what the participant buys might not be exactly what they bid for. Giving participants the opportunity to iteratively re-bid, and presenting resulting market information back to participants should help the market converge towards a solution, as real time approaches.

Also ahead of time, the market manager processes participant demand. They:

- Collect binding M/Q bids for the present period;
- Potentially collect real indicative and/or binding bids for future periods, where these could be conditional;
- Potentially generate notional indicative bids for future periods, which could also be conditional;
- Run the MCE which optimises allocations for all participants;
- Ensure security of system concerns are accounted for;
- Verify/adjust to ensure operational feasibility;
- Confirm the spot market clearing solution for the present period; and
- Issue spot orders to participants.

Optionally, still ahead of time, the market manager could also:

- Run a reservoir simulation model, based on some assumed distribution of future inflows and notional and/or binding and/or indicative future bids, to demonstrate the potential performance of the market clearing solution over a (wider) of range of possible future hydrological conditions;
- Generate a set of forward M/Q scenarios³⁷ (for Stochastic Programming), and/or operating rules (for Dynamic Programming);³⁸
- Publish forward clearing/simulation model results to participants; and

³⁷ Estimated M/Q give/take bid information, based on notional (historical) data, or indicative participant data.

³⁸ "M/Q scenarios" refers to using the right M/Q bid stacks in each scenario tree event node. "M/Q operating rules" applies to using the right M/Q bid stacks for the various combinations of 'state' and 'stage' in the DP.

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- Create a firm allocation of potentially conditional forward M/Q contracts, in the sense that the quantities involved may be subject to some form of adjustment depending on what the future hydrological condition turns out to be.

In the given trading period participants would take their subsequent physical water (Q) allocations, as required, and within the terms of their allocated temporary right. Also, during the trade period the market manager would liaise with the reservoir operator to verify the quantities that participants actually take.

After the given trading period the market manager could:

- Run a post-settlement clearing model to analyse actual takes; and
- Penalise participants who deviated from their allocated rights by assigning the associated knock-on costs.

Also after the given trading period participants would:

- Settle their obligations regarding taking and/or paying monies, at the market price and their cleared unit rights, for both current and realised future contracts, where applicable;³⁹ and
- Pay violation premiums in certain situations, for example in a flood event,⁴⁰ or say if they violate their temporary right by taking more, or less, than they are allocated.

Having discussed some general market operations we now discuss general market options.

³⁹ These could include violations or penalties, in line with the market rules and participants rights.

⁴⁰ For example: With a centrally co-ordinated market if they deviate too far from their take allocation causing a spill and a post-settlement re-clearing. With a virtual share option the premium is based on their share of ownership.

4.4.3 Spectrum of Market Options⁴¹

To develop a range of centrally co-ordinated market options, we consider three key issues as follows:

1. Modelling Uncertainties: Incorporating stochasticity in the centralised market-clearing model;
2. Interacting with Participants: Organising market interaction via participant bids; and
3. Trading Products: The nature of the products actually traded by participants.

With respect to each key issue, there is a spectrum of options available, involving varying degrees of detail, and hence complexity. Deciding on a market design will involve deciding on the level of detail with which each of these aspects needs to be addressed, in order to achieve an acceptable trade-off between precision and simplicity. We visualise an overview of three issues, and a coarse spectrum of the options and their interactions as shown below in Figure 4.1.

1.1 Present (single) Period Valuation to Release now, plus:						1. MODELLING UNCERTAINTIES
1.2 General Estimate to Store Water for later	1.3 Deterministic Optimisation	1.4 Deterministic Optimisation with Simulation	1.5 Forward Stochastic Optimisation:			
			1.5.1 Stochastic Independent	1.5.2 Lag-one Markov	1.5.3 Scenario Trees	
One future period	One or more future periods					
2.1 Spot Bidding at each Decision Period, plus:						2. INTERACTING WITH PARTICIPANTS
2.2 Notional Future Estimated:		2.3 Real Participant Future Bids:				
2.2.1 Demand (Historic Info)	2.2.2 Bids	2.3.1 Indicative:		2.3.2 Binding:		
		2.3.*.1 Deterministic Release	2.3.*.2 Conditional Release:			
			2.3.*.2.1 Parameter Bidding	2.3.*.2.2 H/M/L Bidding	2.3.*.2.3 Scenario Bidding	
			Varying Price (M), Quantity (Q), or both			
3.1 Firm Spot Release Quantities at Market Price, plus Forward Contracts of type:						3. TRADING PRODUCTS
3.2 None	3.3 Primary Markets: Physical and/or Financial			3.4 Secondary Markets: Financial and/or Physical, and with or without Forward Market Simulation		
	3.3.1 Naturally Adjusted Form	3.3.2 Constrained MCE Form	3.3.3 Constrained MCE with Allowable Deviations Form			
	Fixed Contracts		Proportional Contracts			

Figure 4.1: Centralised Market Options: A Spectrum of Key Issue Interactions

⁴¹ In the earlier “share of system” options, participants dealt with uncertainty themselves.

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So the form of the model, of participant bid interactions, and of traded products, can take on increasingly complex forms. In particular, we must decide the extent to which the MCE will explicitly account for the underlying uncertainty.

The more complex the MCE becomes, the more we may seek participant involvement in providing information about the economic value of water to them, in those alternative situations the market deems could occur. Participant information could be in the form of indicative or real bids which, within the MCE, lead to the creation of information about the potential price and/or physical availability of water in future periods. This future information could simply lead individual participants to pursue alternate water sourcing strategies outside of the market, or for them to obtain external financial insurance, such as CfDs. Or it could lead to the creation of real forward contracts traded internally within the market; and these contracts could be either physical and/or financial.

If the market was to centrally trade real forward contracts the products must necessarily be simple enough for participants to understand, and value. Also, participants will not become involved if the market interfaces are too complex. Nor should we expect participants to provide information in a form more complex than that which can be handled by the MCE.

Thus we see the above diagram as representing a hierarchy, in which the form of information provided by participants via market interfaces will be no more complex than the data structure implicit in the MCE, and no less complex than that required to define the instruments being traded, if any. Importantly though, the level of detail adopted can only decrease as we move up this hierarchy. For example:

- There is no point asking participants to specify parameters in their bids that will not be represented in the market-clearing model. For example, there is no point asking participants for conditional bids if the market clearing cannot accept conditional bids.
- Conversely, if participants are to trade instruments based on market clearing solutions, they will at least want to specify how much more or less they are prepared to pay in order to vary the parameters defining those instruments which may ultimately vary

their bids. In other words, we expect the instruments traded to be differentiated by a set of parameters which is no more complex than those specified in participant bids.⁴²

The next three main sub-sections now discuss further details of these three key issues as just outlined above in Figure 4.1. First we focus on the complexity of the MCE, and specifically on the various ways it can model the uncertainty.

4.5 Modelling Uncertainties

This section discusses the various options for modelling uncertainties as outlined in Section 1 of Figure 4.1. We propose optimising and dispatching water based on its highest economic value and subject to water availability. Availability and demand could both depend on increasingly complex levels of hydrological uncertainty. Models can be run for various purposes: For simulation and/or trading, and over various decision periods, with various levels of (un)certainty, generally deterministic or stochastic.

A primal stochastic single tank formulation is given in Section 3.8.2. The objective is to maximise the total aggregate market benefit. In that formulation q is indexed by event node, (t,p) and M and Q are indexed by period, t , and hydrology state, h . This means we can have conditional releases, based on conditional bids as in Item 2.3.*.2 in Figure 4.1 above.

Note that, when dealing with forward allocations, discounting issues must be considered. If participants must settle all trades now, they will need to borrow, or forego interest on, the value of future allocations. This will diminish the present value of future allocations to them, to an increasing extent the further in the future the allocation is for. In that case we would expect these lower present values to be reflected in their bids. But, because participants will have implicitly discounted future values themselves, we do not need any explicit discounting in the primal objective function, as shown below.

Since no discount factors were included in the primal objective function, no discounting is applied to the prices produced by the dual formulation, either. This is consistent with the interpretation that all of these prices apply to transactions which are to be settled at the present point of time. As noted above, though, we expect that participants would discount

⁴² That does not apply to a parameter, such as the degree of scaling, that may be applied to achieve matching with hydrological states, if that parameter will apply equally to all instruments traded. In other words the parameter does not imply any differentiation between instruments.

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their bids, under this assumption. So that the market-clearing prices reported by the MCE for allocations in future periods would be similarly discounted. This does not mean that we expect prices to fall over time, though. If the system is in a stable equilibrium cycle, we expect prices will be too. It is just that the prices reported by the MCE are discounted present values of the prices ultimately expected to apply in real time. As real time approaches, and the discounting period gets shorter, the prices in successive MCE solutions will gradually rise to the equilibrium level for each period.⁴³

The primal SLP formulation is a nodal formulation, which implicitly assumes nodal probabilities, $P^{t,p}$. We define the node probability as the product of the conditional probabilities leading to that node. In developing the dual SLP formulation we need to scale the dual variables such that they make sense in term of the probability distribution in the scenario tree. This is equivalent to multiplying through each primal constraint by the associated nodal probability. We briefly re-state the primal, each primal constraint is stated with its corresponding dual price and nodal probability, and we then develop the associated dual formulation.

$$\max_{q, r, s, v, w} \left\{ \sum_{(t,p) \in E} P^{(t,p)} \left[\sum_{i \in I, b \in TB} M_{i,b}^{t,h(t,p)} q_{i,b}^{t,p} \right] + \sum_{(T,p) \in E} P^{(T,p)} \left[\text{EOHS}^{T,h(T,p)}(s^{T,p}) \right] \right\} \quad (3.47)$$

This maximisation is subject to constraints setting the following:

$$0 \leq q_{i,b}^{t,p} \leq Q_{i,b}^{t,h(t,p)} \quad \forall i \in I; b \in TB(i); (t, p) \in E \quad : P^{t,p} \theta_{i,b}^{t,p} \quad (3.48)$$

$$\sum_{i \in I; b \in TB(i)} q_{i,b}^{t,p} - q^{t,p} = 0 \quad \forall (t, p) \in E \quad : P^{t,p} \eta^{t,p} \quad (3.50)^{44}$$

$$q^{t,p} + v^{t,p} - r^{t,p} = \text{FT}^{t,p} + \text{MAXSUP} \quad \forall (t, p) \in E \quad : P^{t,p} \lambda^{t,p} \quad (3.51)^{45}$$

$$s^{t,p} - s^{\text{prev}(t,p)} + r^{t,p} + w^{t,p} = F^{t,h(t,p)} \quad \text{for } (t,p) \in E \quad : P^{t,p} \psi^{t,p} \quad (3.52)$$

$$s^{0,0} = \text{SINITIAL} \quad : P^{0,0} \psi^{0,0} \quad (3.53)$$

$$0 \leq r^{t,p} \leq \text{RCAP} \quad \forall (t, p) \in E \quad : P^{t,p} \rho^{t,p} \quad (3.54)$$

$$0 \leq s^{t,p} \leq \text{SCAP} \quad \forall (t, p) \in E \quad : P^{t,p} \sigma^{t,p} \quad (3.55)$$

⁴³ By way of contrast, for a depletable resource, Hotelling's rule would require prices to be constant, in present value terms, so that real-time prices rose over time, at the discount rate.

⁴⁴ The MCP at each event node, for all quantities allocated to participants.

⁴⁵ In Section 3.6.3 we introduced the MAXSUP notation to account for the possibility of priced supplies.

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$$0 \leq w^{t,p} \quad : P^{t,p} \omega^{-t,p} \quad (3.56)$$

$$0 \leq v^{t,p} \quad : P^{t,p} \upsilon^{-t,p} \quad (3.57)$$

Equations (3.48), (3.54), and (3.55) show a pair of primal constraints. Hence there are two individual dual variables for each primal bound pair. We assign a superscript – and + to the (non-positive) shadow price on the lower bound, and the (non-negative) shadow price on the upper bound, respectively. We then sum these two variables to form an unrestricted “composite” dual variable. For example, from Equation (3.54), $r^{t,p} \geq 0$ and $r^{t,p} \leq \text{RCAP}$ have the corresponding (respective) dual variables $\rho^{-t,p}$ and $\rho^{+t,p}$, where $\rho^{t,p} = \rho^{+t,p} + \rho^{-t,p}$. We could have done the same thing for w and v , but we assume no upper bound on spills, so only the lower bound is shown. But, to be consistent with the other lower bounds w and v have been given (non-positive) shadow prices $\omega^{-t,p}$ and $\upsilon^{-t,p}$. Note that since we are treating these lower bounds explicitly, the variables themselves are treated as being unbounded in the primal thus producing equality constraints in the dual. The other dual variables θ , η , λ , ψ , ρ , and σ are all unrestricted in sign.

The **Dual** of this optimisation has the objective to minimise the total aggregate value of resources corresponding to the physical allocation constraints:

$$\min_{\theta, \eta, \lambda, \psi, \rho, \sigma, \omega, \eta} \left\{ (\text{SINITIAL})\psi^0 + \sum_{(t,p) \in E} P^{t,p} \times \left[\sum_{i \in I, b \in \text{TB}(i)} (Q_{i,b}^{t,h(t,p)}) \theta_{i,b}^{t,p} + (\text{FT}^{t,p} + \text{MAXSUP}) \lambda^{t,p} + (\text{F}^{t,h(t,p)}) \psi^{t,p} + (\text{RCAP}) \rho^{t,p} + (\text{SCAP}) \sigma^{t,p} \right] \right\} \quad (4.1)$$

The dual constraint corresponding to initial storage is:

$$\psi^{0,0} - P^{1,1} \psi^{1,1} = 0 \quad : s^{0,0} \quad (4.2)$$

Here $\psi^{0,0}$ is the MWV of the inherited initial storage position at the beginning of trading is the expected MWV for period 1. As discussed in Section 3.8, we assume only one decision for the first period. Hence, Equation (4.2) merely assigns the same per unit value to the initial stock as the stock at the end of the first period. This reflects the fact that Equation (3.53) exists merely for convenience and could be substituted into the first period instance of Equation (3.52).

In this section, for simplicity, we introduce a local convention similar to the “ $\text{prev}(t, p)$ ” notation introduced in Section 3.7.2. We refer to the set of subsequent nodes as $\text{next}(t, p)$,

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where $(t+1, p') \in next(t, p)$ is a child node of (t, p) . That is, if $(t+1, p') \in next(t, p)$, then $prev(t+1, p') = (t, p)$. With this convention, the more general dual constraint corresponding to storage is:

$$P^{t,p} (\psi^{t,p} + \sigma^{t,p}) - \sum_{(t+1,p') \in next(t,P)} P^{t+1,p'} \psi^{t+1,p'} = 0 \quad \forall (t, p) \in E; 1 \leq t \leq T \quad : s^{t,p} \quad (4.3)$$

Re-arranging Equation (4.3) in terms of the MWV we get:

$$\psi^{t,p} = \sum_{(t+1,p') \in next(t,P)} \frac{P^{t+1,p'}}{P^{t,p}} \psi^{t+1,p'} - \sigma^{t,p} \quad \forall (t, p) \in E \quad : s^{t,p} \quad (4.4)^{46}$$

When storage is strictly between its bounds then $s^{t,p} < SCAP$ and $s^{t,p} > 0$, so $\sigma^{t,p} = 0$. In this case the storage limits are not binding at event node (t, p) , and Equation (4.4) states that the MWV of storage at event node (t, p) is equal to the average MWV across its child nodes for the subsequent period. Since the $P^{t,p}$ are nodal probabilities the ratio $P^{t+1,p'}/P^{t,p}$ gives the conditional probability that event node $(t+1, p')$ is visited given event node (t, p) is visited.

If the upper limit on storage is binding, i.e. if $s^{t,p} = SCAP$, then the shadow price $\sigma^{+,t,p}$ is greater than or equal to zero⁴⁷ and $\sigma^{-t,p}$ is zero. Thus $\sigma^{t,p} = \sigma^{+,t,p}$ and on average ψ increases, increasing the average MWV in period $t+1$, to signal that there is no longer a need to be concerned about the possibility of spilling water in period t .⁴⁸ If the lower limit on storage is binding, i.e. if $s^{t,p} = 0$, then the shadow price $\sigma^{-t,p}$ is less than zero and $\sigma^{+,t,p}$ is zero. Thus $\sigma^{t,p} = \sigma^{-t,p}$ and on average ψ decreases, decreasing the average MWV in period $t+1$ to signal that there is no longer a need to be concerned about the possibility of running short of water in period t .

The dual constraint corresponding to carrying water over at the end of the water year is:

$$\psi^{T,p} + \sigma^{T,p} = \frac{d}{ds} \left[EOHS^{T,h(T,p)}(s^{T,p}) \right] \quad \forall (t, p) \in E \quad : s^{T,p} \quad (4.5)$$

As with Equation (4.4), when the storage is between its bounds $\sigma^{T,p} = 0$, in Equation (4.5). So unless the final storage level is at either bound, $\psi^{T,p}$ is the marginal value of carrying water

⁴⁶ For the nodal probabilities: $P^{t,p} = \sum_{(t+1,p') \in next(t,P)} P^{t+1,p'} \quad \forall (t, p) \in E$.

⁴⁷ This is required to account for the degenerate case.

⁴⁸ When working backwards a positive ψ decreases the MWV to account for the threat of spilling. Working forwards the MWV rises progressively as we pass through the "spill danger zone", and falls progressively as we pass through the "shortage danger zone". In reality, actual spill, or shortage, is often quite rare.

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over at the end of the year, and this is equal to the derivative of the (convex differentiable) End of Horizon Benefit for Storage function, EOHS.

In Equation (4.5) we dropped the common factor $P^{T,p}$. In the subsequent dual constraints we also drop the common factor $P^{t,p}$.

The dual constraint corresponding to the market clearing quantity is:

$$-\eta^{t,p} + \lambda^{t,p} = 0 \quad \forall (t, p) \in E \quad : q^{t,p} \quad (4.6)$$

In words, the MCP at node (t,p) , $\lambda^{t,p}$,⁴⁹ is equal to the price paid, $\eta^{t,p}$,⁵⁰ for each unit of allocated water, in that period and for that event node.

The dual constraint corresponding to a particular bid, for event node (t, p) is:

$$\eta^{t,p} + \theta_{i,b}^{t,p} = M_{i,b}^{t,h(t,p)} \quad \forall i \in I; b \in TB(i); (t, p) \in E \quad : q_{i,b}^{t,p} \quad (4.7)$$

Most tranches will be fully allocated, or not allocated, but, there will typically be some “marginal” tranche which is partially allocated, so that its θ is zero.⁵¹ Then Equation (4.7) effectively sets the market price in that period to the bid price of that “marginal” tranche. For any other tranche, the primal variable is either zero (no allocation), or at its upper bound (fully allocated). Participants are paying the same $\eta^{t,p}$ for the water they are allocated at node (t,p) , for each of their allocated bid tranches. The θ variable ‘explains’ the difference between the market price and the bid price. If the bid price M is higher than η the bid is allocated and θ is positive. If the bid price M is lower than η the bid is not allocated and θ is negative.

The marginal per unit cost of release from the reservoir is the MWV i.e. shadow price on the continuity equation, ψ , as above. The marginal per unit value of release to the market is λ . So the release capacity limit has a shadow price, ρ , which can explain the difference between λ and ψ via:

$$-\lambda^{t,p} + \psi^{t,p} + \rho^{t,p} = 0 \quad \forall (t, p) \in E \quad : r^{t,p} \quad (4.8)$$

⁴⁹ Since Equation (3.50) equates supply and demand.

⁵⁰ Since Equation (4.7) equates η to the marginal bid price, M .

⁵¹ Ignoring the possibility of two marginal tranches having the same bid price.

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That is, ρ represents the gain that could be made by increasing release capacity by one unit, so as to allow one more unit of stored water to reach the market. If the lower bound binds, ρ is a negative because even with minimum release the tributary flows flood the market, to such an extent that water is worth less downstream than in the reservoir. This reflects the fact that, if possible, we would like a negative release bound allowing water to flow from downstream back into the reservoir. When the release capacity is not at its bound then $\rho=0$, and $\psi=\lambda$.

The dual constraint corresponding to reservoir spill is:

$$\psi^{t,p} + \omega^{-t,p} = 0 \quad \forall (t, p) \in E \quad : w^{t,p} \quad (4.9)$$

Equation (4.9) merely says that ω equals $-\psi$, that is that the marginal value that would be lost if spill were to occur is the MWV.⁵² If the lower spill bound is not binding, its shadow price, ω , equals zero, implying that the MWV is also zero, reflecting the fact that we would not choose to spill water if it had any positive value. So Equation (4.9) says that the MWV must also be zero if spilling. If the lower spill bound is binding the shadow price on the lower spill bound will be negative, in which case Equation (4.9) simply says that the MWV is positive. If there were an upper bound on spill, we could have ω positive, making MWV negative. This would reflect the possibility that we could be prepared to pay to offload water now in order to avoid the possibility of the spill limit being breached in a future flood.

The dual constraint corresponding to tributary spill, i.e. water which cannot be used by the market because it is saturated, is:

$$\lambda^{t,p} + v^{-t,p} = 0 \quad \forall (t, p) \in E \quad : v^{t,p} \quad (4.10)$$

Equation (4.10) merely says that λ equals $-v$, i.e. that the marginal value that would be lost by not using the water to meet demand, is the MCP. If the lower spill bound is not binding, its shadow price, v , equals zero, implying that the MCP is also zero, reflecting the fact that we would not choose to spill this water if it had any positive value to the market. If the lower tributary spill bound is binding (i.e. if we are utilising all possible tributary flows), its shadow price will be negative, and MCP therefore positive.

⁵² Strictly, this shadow price reflects the value that would be added if we could reduce the lower spill bound by one unit, to -1, thus allowing water to be added to the reservoir, for free.

This formulation is the most complex general formulation we consider. With this in mind, we now discuss increasingly complex optimisations building up to this level of complexity.

4.5.1 Single-Period Deterministic Optimisation

A single-period deterministic optimisation model would optimise “spot” water allocation without any consideration of its future value. Present net demand would be met by releasing all available quantity units from the reservoir. With only one period, and one certain hydrological history, the above equations primal formulation hydrology parameters simplify to $F^{t,h(t,p)} = F$ and $P^{t,p} = 1$. The key demand variable becomes $q^{t,p} = q$. For this model the EOHS(s^T) which is used to define the value of future stored water is zero, so at the optimal solution the final storage variable is $s^{T,p} = 0$.

This optimisation strategy does not make use of the reservoir’s storage capacity, which exists to generate more “value” over time. So the single-period allocation strategy is short-sighted, and unlikely to be optimal, thus we do not consider this for implementation.

All of our subsequent options incorporate some model of spot water trading, but also model balancing future water trading, as highlighted in Item 1.1 of Figure 4.1 above.

4.5.2 Deterministic Optimisation with General Estimate to Store Water

A deterministic optimisation model would optimise water allocation balancing net demand both now and later. The model would operate as a pseudo “two-period” optimisation model which only trades present water. The model would be run again to trade each subsequent future period, as those periods arose. Each time the model uses some exogenous future bid estimate, presumably determined by the market/reservoir operator. For this model $T = 1$, and EOHS(s^1) is used to define the value of future stored water (corresponding to Item 1.2 in Figure 4.1). That estimate might, or might not, be determined by running a multi-period model, of one of the types discussed below, but that modelling would not form part of the market, per se.

For this case, the hydrology parameters in the formulation above also simplify to $F^{t,h(t,p)} = F$ and $P^{t,p} = 1$.

4.5.3 Multi-Period Deterministic Optimisation

The model described in this section corresponds to Item 1.3 from Figure 4.1, above. A multi-period deterministic optimisation would optimise water allocation over time, balancing demand now and later. To do so it will require bids for each future period, t , either from bid data or some other estimate.

Here there are multiple decision periods and one ‘certain’ hydrological history. From the primal formulation equations the hydrology parameters simplify to F^t and P^t , while the key variables become s^t , and q^t for all t .⁵³

From this point forward, for the subsequent water year, each decision period could be modelled at the present time. The future period release schedules could be indicative, or they could be firm, in the sense that they determine participants’ allocations of some actual traded product, such as a firm right to take water, as discussed in Section 4.7 below.

Either way, the optimisation model would be re-run to clear the (then) spot market at each subsequent period (point) in time, and participants would be free to change their bids to match their actual (expected) situation, as it becomes clear. The market manager would also update for the then actual reservoir storage level, and then revise future hydrological forecasts.

In the presence of hydrological uncertainty, if future period release schedules are firm then the allocation is probably not optimal. In the real world, the presence of stochasticity could even lead to infeasibility when trying to exactly match both release and demand quantities. New information continually, and gradually, reveals the actual situation at each subsequent point in the future. So participants will desire to change their position with regard to their future allocations, or the market manager may wish to revise allocations for participants. Both instances might warrant adjusting the storage and/or release schedules which were estimated based on the then (earlier) expected hydrology.

⁵³ In this instance the scenario tree could be visualised as collapsing to a single, certain, path. Alternatively we could view that a stochastic models tree is populated with exactly the same information for all forward paths nodes, where within period alternate paths arcs have zero weights on those branches.

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The output from the deterministic MCE would notionally allocate water now, to meet “expected” future demand needs. But future price and quantity data, as yet unknown, will determine how the market price, and allocations, actually change, in each subsequent modelled decision period. A range of possible future data could be processed through a simulation model to demonstrate the range of future outcomes that could arise from applying the MCE’s recommendations with respect to future price and quantity data in informing potential future market clearings. An extension with a forward simulation is now discussed.

4.5.4 Multi-Period Deterministic Optimisation with Forward Simulation

The model described in this section corresponds to Item 1.4 from Figure 4.1, above. Market participants are likely to want to see the results of a forward simulation of market outcomes, for scenarios other than the one assumed in the deterministic optimisation. For example the Australian electricity market provides participants, each half hour, with a wide range of simulated market-clearing outcomes for the next 1-2 days, under different demand conditions. For each modelled decision period and conditional state, the market manager could produce PDFs for prices and release allocations. Possibilities would include performing alternative deterministic market-clearings for each scenario, assuming that it was known with perfect foresight, or simulating the implications of applying the policy recommended by the market-clearing performed for expected conditions, under each scenario. The form of the stochastic model, which can be used for simulation and/or optimisation, is discussed in the next section.

4.5.5 Stochastic Optimisation

In a real world environment, there will be uncertainty about future water availability, and probably to some extent demand requirements. The market could directly account for uncertainty in the pricing structure, and hence the allocation process. In this instance the market manager would have to run a forward stochastic optimisation, assuming some stochastic model of hydrology, and possibly also demand data. This stochastic data could take on a variety of forms, as discussed below.

If the stochasticity can reasonably be represented as having some simplified form, such as inter-period independence, or lag-one Markov correlation, the MCE may be simpler to solve

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and have simpler data requirements. At least market outcomes will be simpler to simulate, as is done in Chapter 5. Such structures also imply that the demand to store water can be represented in a simplified and intuitive form, as a monotone function of storage level. The primal formulation at the start of Section 4.5 above assumes our most general form of stochasticity, described by a general scenario tree based on the hydrology conditions, with no particular structure. This matches the model described in Item 1.5.3 from Figure 4.1, above.

Participants are not restricted to provide discrete stochastic bids, these could be of a simpler form, and those forms can be added into the optimisation. Such forms are discussed in Section 4.6.3 below when each participant bid form is discussed. Assuming the underlying continuous random variable which represents the hydrology condition can be discretised, we can populate an SLP scenario (event) tree. We could assume some discrete hydrology conditions of the simple stochastic independent form.

4.5.5.1 Multi-Stage Stochastic Independent SLP Model

The model described in this section corresponds to Item 1.5.1 from Figure 4.1, above. With stochastic independence for each future decision period there are H^t discrete hydrology conditions, in each period t , each with a conditional probability dependent solely on the period and hydrology condition.⁵⁴ The underlying scenario tree and data are structured to match the stochastic independence.

At each node in the scenario tree the bid data form can, at most, be stochastic independent: $(Q_{i,b}^{t,h}, M_{i,b}^{t,h})$. That is, the bid sets can only depend on the period and hydrology condition combination.

Running the MCE with a stochastic MCE and deterministic equivalent bid constraints would generate a single solution value which maximises expected value across all equivalent scenarios. Of course, if the hydrology can vary significantly over $1, \dots, h(t)$ then deterministic equivalent linking constraints may incur large costs. The bid price (M) may outweigh the cost making it feasible to allocate that bid; else the allocation variable (q) may be reduced,

⁵⁴ It may be more densely populated nearer real time. Or in a given stochastic environment in those seasons where more diverse reactions are possible.

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potentially all the way to 0. This means that across the range of hydrology conditions a participant's outcome for a bid might vary from fully allocated to not allocated.

For some systems a better model would be a Markov Chain, as now discussed for a simple 1-lag stochastic dependent MCE.

4.5.5.2 Lag-1 Markov

The model described in this section corresponds to Item 1.5.2 from Figure 4.1, above. Assuming a 1-lag Markov chain to connect hydrology conditions across each subsequent discrete decision period allows conditions to more explicitly evolve, and not only over time but also from one event to another. Hence through the probability weights we can design in the rate of change of transition from wetter to drier periods, and vice versa. This could be designed to be gradual, or at the other extreme immediate. Hence, where the local real world situation is well modelled by a Markov Chain this modelling tool better aligns the market with reality.

Here there are multiple periods and stochastic dependent hydrological histories. This level of complexity has been implemented in the model described later in Chapter 5. There we reformulate our model as a SCDDP. We could also model wider stochastic inter-connections but there are many market design implications even just developing the modelling system to a lag-1 Markov Chain level of complexity.

4.5.6 Summary of Modelling Uncertainties

In this section we simplified the stochastic single tank formulation from Section 3.8.2 to be only dependent on the hydrology condition. We discussed options ranging from a single decision period and a single deterministic hydrological history to a multi-period model where the optimisation is stochastic lag-one Markov model. The more complex models use more sophisticated hydrology data sets to more accurately represent real world uncertainty as it evolves over time. The most complex of these formulations allows for the bid set to be precisely matched to the stochastic process, via "scenario bidding".

In order to perform these increasingly complex optimisations, the "bid" information would also need to be represented in increasingly complex forms. Such "bids" might be just

hypothesised by the market manager. Otherwise, though, populating these increasingly sophisticated models with equally increasingly sophisticated sets of demand data would require more intense and complex forms of market interaction between manager and participant. The practical market design issue is whether that interaction can be still be designed in a way to make the process manageable, from a participant perspective. This is discussed in the next section.

4.6 Participant Interaction

This section discusses the range of options shown in Section 2 of Figure 4.1, with respect to the interaction of participants with the market. In Section 4.4.2 above, we discuss how the market manager and participants could interact with one another to provide bids and allocate water. It was envisaged that the market manager would determine how much water to release now, and store for later, predominantly based on participant bid information, which could be either binding or indicative.

In principle, with a stochastic MCE, participants could submit bids which are conditioned on the history, as described by the scenario tree, or a sub-set of that tree. That would give the MCE more degrees of freedom and, if bids moved in the same direction as the underlying hydrological conditions, also make it easier to find satisfactory solutions. But often participants would want more water as conditions get drier, and vice versa. If participant bids are hydrology dependent, the drier it gets, the larger the bid price would generally need to be, to stand a chance of being allocated in a competitive constrained system.

The participant sourced parameters required in the model are $Q_{i,b}^{t,p}$, and $M_{i,b}^{t,p}$ from Equations (3.47) and (3.48). This data must either come directly from participant bids each time the market is cleared, or be generated by the market manager based on some estimate of what those bids might turn out to be (potentially from bids or other information provided by the participants for other periods.).

Bids are at least required for the spot market, providing data for the first event node in the scenario tree. In the following sub-sections the most complex bid form we recommend participants provide is index dependent. We start with discussing spot bidding, then ask whether future (Q, M) bid estimates, should be provided by the market manager, or market

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participants. We then consider how sophisticated bids might need to become, in order to account for stochasticity.

In theory, the Section 2 options could be mixed. For example, binding High/Medium/low (H/M/L) bids could apply for the first three months, indicative deterministic bids for the next six months, and notional bids for the last three months (of a water year). But we do not address such mixed options in this thesis.

4.6.1 Spot Bidding with Present Valuations

In all cases, we assume participants provide real, binding bids into the spot market, that is for the current period. This is highlighted as Item 2.1 in Figure 4.1. For the spot market there is only the one (present) period and one (certain) hydrology condition. Thus each participant would only need to provide one set of bid tranches for the present period. The bid data requirements are:

$$\{(Q_{i,b}, M_{i,b}) : i \in I, b \in BT\}$$

To save water until the next spot market allocation, the market needs to balance release now against storage for later use. Hence we must have some way of estimating the value of each unit of water that can be stored in the reservoir. The market manager could estimate the MWV of storing water, based on historical demand information, perhaps with the objective of maximising net welfare. But that would be equivalent to maximising value in a “pseudo-market”, based on some form of bid estimate. We now discuss different future bid estimates in more detail, starting with notional estimates.

4.6.2 Notional Estimates for Future Valuations

This section corresponds to Item 2.2 in Figure 4.1. Here the market manager is estimating how much water each participant would demand, at specified prices, at defined points in the future. Participants are told how many units to take, in the present, and the cost per unit (the market price), from the spot market. Indicative market prices for future periods would reflect the value participants say they expect to associate with given water quantities, in the future.

Spot market bids, current and historical, are key pieces of demand information. The market manager could base future demand estimates on historical demand data, and correlations with

hydrology data (Item 2.2.1 in Figure 4.1). Likely changes might also be identified, be reference to trends in economic and sectorial development, and thereby building micro-economic sub-models of particular water-consuming activities (e.g. farms).

4.6.3 Real Participant Bids for Future Periods

Here participants provide their own bids (Item 2.3 in Figure 4.1 above) for some or all future periods modelled. These could be simply indicative (Item 2.3.1), as in many electricity markets, for example in Australia and Singapore. In this case the bids are just used to estimate prices and demand quantities, and no rights are transferred. Alternatively, they could be binding (Item 2.3.2) to buy actual products, as discussed in Section 4.7 below. Winning a binding bid transfers the associated right to the participant once payment is made. Either way, the bidding/optimisation process remains the same, and the issue becomes the specific bid form, which itself partly depends on the complexity of the MCE.

4.6.3.1 Deterministic

In this case, each participant would provide one set of bid tranches for each future period. With deterministic future bids (Item 2.3.*.1 in Figure 4.1) the bidding requirements are:

$$\{(Q'_{i,b}, M'_{i,b}) : i \in I, b \in BT\} \forall t$$

As just discussed, real participant future “bids” could be indicative or binding. While such an “expected” projection could take account of many factors it would normally only consist of a single set of bids, for each future period. The MCE could be a deterministic optimisation, with exactly the above bid data requirements. But these participant bids could be used in a stochastic MCE, using the same bids for all event nodes in each period. As the inflows differ across the event nodes of each period, different allocations and market prices will arise for each node, leading to a different storage trajectory for each path. But, as discussed next, participants might want to achieve greater control over the way in which allocations adjust under different scenarios by specifying how the bids themselves are to be adjusted relative to some observable factors.

4.6.3.2 Conditional Bids for Future Periods

Instead of a single set of bids for each future period, whether binding or indicative, the natural extension is for the bids to be made conditional on key parameters that might vary in

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future periods. This would create a range of future bid scenarios, requiring the MCE to solve a stochastic optimisation problem. These options correspond to Items 2.3.1.2 and 2.3.2.2 in Figure 4.1, for indicative and real bids, respectively.

The market manager could perhaps perform such bid adjustments, using assumed correlation factors. But correlations may vary by participant and, if bids are to be binding, then participants will definitely want to specify how their future requirements are likely to vary, because this will determine what they are allocated by the market. Also, that information will be valuable to the market as a whole, even if bids are only indicative.

Hence, participants can indicate, to varying levels of complexity, how they wish their bids to be adapted to match alternative hydrology conditions, in future periods. These bids would support the formation of at least equivalently complex optimisations for spot market clearing. The challenge is to create a regime which allows participants to convey that information to the market, without creating an unduly complex market interface.

In principle, the bids (and subsequent contracts) can depend on a wide range of indices. For example, the regional water price could depend on the price of electricity, if water was manufactured (e.g. desalination) and/or pumped into the demand region. Hence, the following concepts about an index can be seen as quite general. But, often we discuss examples where the index is “hydrology” dependent.

Participants could generate conditional bids for the future which are either:

- A single base value point which is either deterministic, or specified for a set of discrete states representing the hydrological condition; or
- A single base value point which is fixed, and a secondary variable value point which is adjusted as a function of a continuous index, representing the hydrology condition; or
- A set of fixed value points for each discrete hydrology condition in each given period.

Hence there are various ways in which participants could specify how their bids adjust to account for hydrological uncertainty. The next three sub-sections cover the conditional release options in Item 2.3.*.2, in reverse order, starting with the most general option, and working towards simplification

4.6.3.3 Scenario Bidding

Participants could present bids for every node in the tree. This corresponds to Item 2.3.*.2.3 in Figure 4.1. This is the most flexible arrangement, which would allow participants to specify different sets of bid pairs for each node, (t, p) , in the scenario tree. In this case, each participant must provide one set of bid tranches for each event node. That is:

$$\left\{ \left(Q_{i,b}^{t,p}, M_{i,b}^{t,p} \right) : i \in I, b \in \text{BT}(i, t, p) \right\} \forall (t, p) \in E$$

In Section 3.7 we developed a scenario tree and noted there that the exponential growth of the tree will quickly lead to information overload. In our market environment, requiring discrete bids from each participant for each event node is probably unmanageable. For example, on a monthly level over an annual time horizon, with three inflow states at each period participants would be required to submit trading data for the $\frac{1}{2}(3^{12}-1)$ (= 265,720) event nodes in the scenario tree, every month.

So, in our view, the data requirements for participants would be excessive and the approach is not practical. Accordingly, we focus on regimes under which participants would be allowed, or required, to provide less complex bid forms. Participant input bids are then processed into the required SLP form. In particular, we focus on “index dependent” bids, where the index would most likely represent a measure of the hydrological state in the period.

4.6.3.4 High / Medium / Low Bidding

The data requirements from participants could be significantly reduced if participants only provide sets of bids based on a discrete state classification, h , such as High, Medium and Low (H/M/L) hydrology conditions for each period, regardless of preceding history. So, for each discrete hydrology condition, participants could provide a set of bid tranches, $(Q_{i,b}^{t,h}, M_{i,b}^{t,h})$. The discrete bid data would then be used to populate the SLP scenario tree, based on some mapping between the bid states and the index calculated for each event node. The MCE could use a stochastic independent, or Markov, level of complexity, as in Item 2.3.*.2.2 in Figure 4.1.

The number of discrete states could be expanded, but the greater the number of states, the more difficult it is for participants to differentiate between them, and the approach may become unmanageable. Accordingly, we next explore some ways of simplifying the task

faced by participants by allowing them to specify a set of contingent bids in a more compact way, using continuous bid functions.

4.6.3.5 Parameter Bidding

Participants may find it convenient to specify their bids as continuous functions of some index, leaving the bid interface to produce discrete bids for each scenario node, on their behalf. Such parameter bidding corresponds to Item 2.3.*.2.1 in Figure 4.1. A bid parameter (e.g. Q or M) can be mathematically represented as a continuous function of the underlying index. Examples include: Constant, Linear, Affine, Truncated Linear, Piecewise Linear, Quadratic, Cubic, or higher order functions as shown in Figure 4.2 below. For ease of illustration, the participant is assumed to take and pay more as the situation gets wetter. Of course this assumption might be unrealistic in a real market.

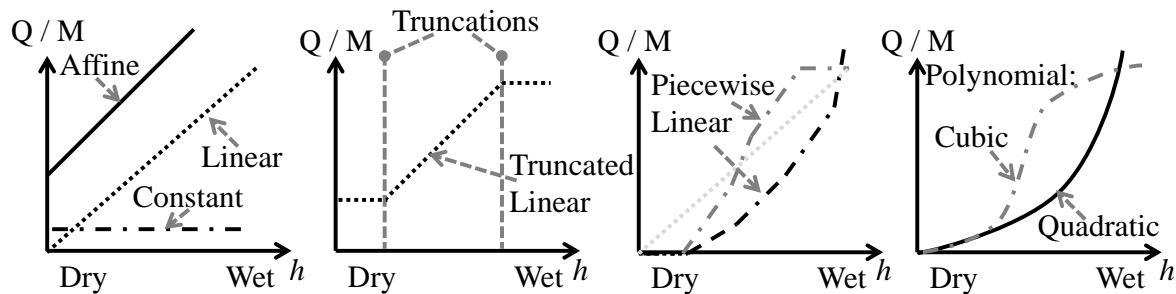


Figure 4.2: Increasingly Complex Continuous Bid Parameter Adjustment Forms

Given that these adjustment rules are applied to form scenario offers before any optimisation commences, they can be of any degree of complexity participants' desire. But the emphasis is on creating a simple bid interface, and we envisage higher order polynomial bids, quadratic or cubic, as requiring too much information from participants and hence being too complex to understand and value. Constant bidding is equivalent to a fixed/deterministic bid discussed in Section 4.6.3.1. Generally piecewise linear bidding is equivalent to multiple sets of truncated linear bids, where each piecewise set is ordered. But it is also somewhat equivalent to H/M/L bidding, and has at least the data requirements of this option.

This leaves linear, affine, and truncated linear. We imagine truncated linear could be more realistic than (strictly) linear, especially where participants are willing to take in proportion to available hydrology but only within certain limits. For example, when it is too wet participants may wish to take no water, and they may demand some minimum when it is dry.

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Consider a “watermelon farmer” who requires a minimum amount of water for his crop to mature, while above some limit, more water will cause the crop to rot. Generally, at a high level of supply each participant is eventually saturated, while at a low supply level no demand can be satisfied due to lack of availability.⁵⁵

So, a truncated linear form could be used, in which bids vary linearly over the range between some minimum and maximum index levels. Outside those limits, fixed bids might apply, but subject to drought/flood rules respectively.

Bids do not need to be restricted to be only dependent on a one-dimensional index. There could be multiple indices, such as cumulative catchment inflows, or an evapo-transpiration index, for example. And an SLP formulation would allow us to model dependence on any aspect of the system history represented by the scenario path.

For simplicity, though, all subsequent discussion in this sub-section assumes a single-dimensional index, H^t .⁵⁶ We, further, assume the reservoir hydrology states are conveniently ordered from drier to wetter conditions, and only consider bidding regimes that allow participants to specify fixed and index-dependent bid components, with the index-dependent component applying between some minimum and maximum index levels, HMIN and HMAX. This is also possible with the truncated linear example, discussed above. We could define a new index $H'=(H-HMIN)$, and define the “fixed” bid as applying for $H'=0$, and the conditional bid over H' in the range $Hrange = (0, HMAX - HMIN)$. In that context, we also note that, while the fixed bid effectively specifies participants’ exposure at the HMIN start of the index range, they may also want to limit their exposure at the HMAX end of the index range. They could do that by specifying bid limits of the form:

$$Q_i^{t,p} = QDRY^t \quad \forall p \text{ such that } H^{t,p} \leq HMIN^t \text{ and for each } t \text{ and } i \quad (4.11)$$

$$Q_i^{t,p} = QWET^t \quad \forall p \text{ such that } H^{t,p} \geq HMAX^t \text{ and for each } t \text{ and } i \quad (4.12)$$

⁵⁵ In any case, water availability and water allocation are both limited below by zero, and may also have a distinct upper limit. So any “linear” bidding or allocation rule must ultimately be truncated at those limits.

⁵⁶In principle, participants could be allowed to apply different bid adjustment rules to each tranche, giving them great flexibility to reflect whatever issues they may face. Since our aim is simplicity, though, we assume that each participant only specifies one adjustment parameter, applying to all tranches, whatever form of adjustment is implemented.

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Where the bidding regime requires continuity, these might be set implicitly by other parameter choices.

In all regimes under consideration, the way in which the bid is adjusted to match the index, depends on a user-specified “Index Dependency Factor” (IDF), β_i^t . Figure 4.3 shows how participants can select β_i^t to vary their bid quantity with the index.

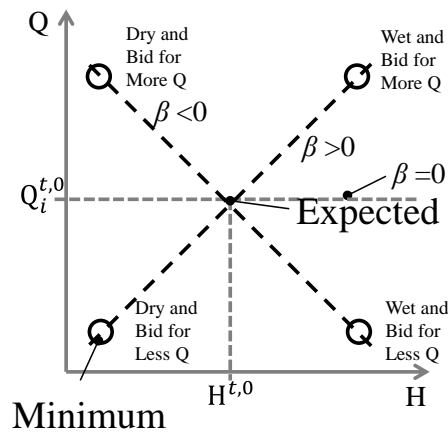


Figure 4.3: Impact of Index Dependency Factor on bids.

As in Figure 4.3 above the β values have different interpretations. If $\beta = 0$ then the participant is effectively providing a fixed bid. When $\beta < 0$ participants are signalling to buy higher/lower quantities (at the same price) as the index decreases/increases. For example, a farmer buying water to irrigate their crop, is likely to require less if it rains. When $\beta > 0$ participants are signalling a willingness to buy lower/higher quantities (at the same price) as the index decreases/increases. For example, a desalination plant might be willing to make/supply more water when it is at a relatively drier than average, for a given time of year.

Focussing first on quantity rather than price adjustment, we let $Q_i^t(H^t)$ be the quantity bid corresponding to the index level H^t . We assume a base bid is stated for a reference point (for example, the expected hydrology value), $H^{t,0}$, and let participant i provide, with their bid, an associated IDF, $\beta_{Q_i}^t$. In Section 4.7.2 below we will assume that the adjustment mechanisms discussed here are merely ways of producing a set of independent bid stacks, each applying in a particular scenario, and cleared independently in solutions for that scenario. In Section 4.7.3 though, we briefly discuss an alternative approach, in which constraints are used to link bid stacks across scenarios, so as to force solutions to match bids in the way that the allocated

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quantities vary by scenario. In that case, the set of bid tranche quantities for a particular bid price may be interpreted as forming a single bid, for a product allocating that set of conditional quantities, at that price.

First, if $\beta_{Qi}^t = 0$, then participant i is simply providing a fixed bid for all tranches, $BID_{i,b}^{t,0} = (Q_{i,b}^{t,0}, M_{i,b}^{t,0})$, there would be no adjustment for uncertainty. That might be thought to imply the same allocation regardless of the hydrology but, in reality, the participant's allocation, in the model, would be ramped back as the hydrology gets drier, and prices rise.⁵⁷

Second, the IDF parameter could be used to specify a “shift” in the bid curve, as discussed further in Section 8.4.2, or to scale it in various ways, with adjustments being made both above and below some reference index level, as in Figure 4.3 above. One form of linear adjustment in response to the realised index $H^{t,p}$ would be to “stretch” the reference bid curve by scaling the bid quantity associated with each price level $M_{i,b}^{t,0}$ using:

$$Q_{i,b}^{t,p} = Q_{i,b}^{t,0} \left(1 + \beta_{Qi}^t (H^{t,p} - H^{t,0}) \right) \quad \forall i, b, (t, p) \quad (4.13)$$

Third, in the context of Section 4.7.2 below, the participant could provide separate bids for two distinctly different products, one “fixed” (or deterministic) and one “conditional” (or “index dependent”). Or they could just specify two distinct bid components. In either case we would have two sets of quantity tranches, each of which could have its own set of price levels. Rather than purchase, or specify a bid for, a quantity q of a product that allocates β water units for each unit increase in the index, the participant may as well bid for, or be allocated, $q*\beta$ units of a standard product that allocates 1 water unit for each unit increase in the index (i.e. for which $\beta = 1$). Thus we can create markets in two simple products that all can trade. The fixed quantities, Q^F , would be defined as for Q in the base formulation, but we can re-set the reference index to zero, using:⁵⁸

$$Q_{i,b}^{F,t} = Q_{i,b}^{t,0} (1 - \beta_{Qi}^t H^{t,0}) \quad \forall i, b, (t, p) \quad (4.14)$$

Then the conditional quantities, Q^C , would be defined in proportion to a base quantity, $Q_{i,b}^{C,t,0}$, by:

$$Q_{i,b}^{C,t,p} = Q_{i,b}^{C,t,0} H^{t,p} \quad \forall i, b, (t, p) \quad (4.15)$$

⁵⁷ β could vary by tranche, but in that case it would also depend on price.

⁵⁸ This assumes a linear contract form, but truncated or piece-wise linear forms could also be specified.

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If participants choose to use the same price levels in each stack, they could link their fixed and proportional bids in some way. One example would be setting a constraint limiting the aggregate purchase across both product types. Another corresponds to a special case of Equation (4.13), in which the reference index is set to $H^{t,0}=0$, and the base for proportional component, $Q_{i,b}^{C,t,0}$, is itself set to be proportional to the fixed bid, so that:

$$Q_{i,b}^{C,t,0} = \beta_{Qi}^t Q_{i,b}^{F,t} \quad \forall i,b,(t,p) \quad (4.16)$$

Thus:

$$Q_{i,b}^{t,p} = Q_{i,b}^{F,t,0} + Q_{i,b}^{C,t,p} = Q_{i,b}^{F,t,0} + Q_{i,b}^{C,t,0} H^{t,p} = Q_{i,b}^{F,t,0} (1 + \beta_{Qi}^t H^{t,p}) \quad \forall i,b,(t,p) \quad (4.17)$$

Either way, the possible patterns of participant preferences are shown in Figure 4.4 below. Here the fixed (base) bids are shown on the vertical Q axis, corresponding to the index, $H^{t,0}=0$. A negative fixed bid quantity indicates a desire to sell water at the bid price for this step, in this driest possible state, whereas a positive bid quantity indicates a desire to buy water. A participant's beta signals how their bid varies with the index, with positive beta signalling a desire to buy more water (or sell less water) as conditions get wetter, zero beta signalling no impact, and negative beta signalling a desire to buy less water (or sell more water) as conditions get wetter.

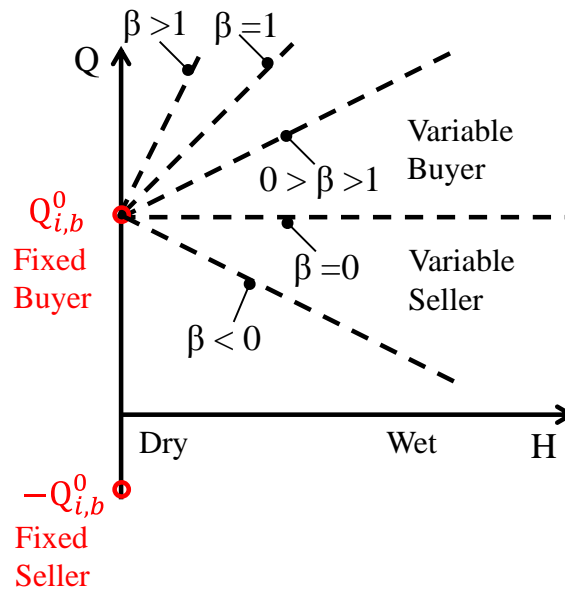


Figure 4.4: Impact of Index Dependency Factor on bids with Linear Adjustment from Base Bid.

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Thus negative β corresponds to selling, rather than buying the proportional product. That would be logical for a supplier who can provide water, and is prepared to provide more when conditions are wet. But it would also be logical for a net consumer to “buy fixed and sell proportional”, meaning that they will buy enough to cover their needs in dry conditions, but plan to reduce their market off-take as conditions become wetter. Most likely because those wet conditions also increase their own (non-market) water supply. And the market could operate in a mixed mode, with all participants providing bids for both product types, but only some participants pre-specifying linkages. The greater the variety of participant preferences, the better chance the market has of finding a solution that is acceptable to all.

Finally, note that the index relates to the amount of water available, and that will also impact on the market price. For a participant to be allocated a fixed quantity in certain situations, for example a drought, that participant might actually have to bid extremely high prices to secure adequate supply. If a participant really wants to exercise control over the quantities they get allocated under different hydrological conditions, they might prefer to adjust bid price and quantity simultaneously. So, there could also be a $\beta_{Mi,b}^t$ to adjust the price bid steps, as follows:⁵⁹

$$M_{i,b}^t = M_{i,b}^{t,0} \left(1 - \beta_{Mi,b}^t (H^{t,p} - H^{t,0}) \right) \quad \forall i, b, t \quad (4.18)^{60}$$

Participants would soon learn what kind of $\beta_{Qi,b}^t$ and $\beta_{Mi,b}^t$ combinations delivered the kind of outcome they prefer. Indeed the interface could communicate back to participants the corresponding M and Q combinations, for the market scenarios.

We conclude that some form of index-dependent bidding could reasonably be used to provide the inputs required to model uncertainty in the way discussed by Section 4.5 above. But just specifying bid adjustments does not directly produce corresponding allocation adjustments. The resultant MCE solutions must somehow be interpreted to determine how participants will trade one or more products, as we now discuss in more detail.

⁵⁹ Note that the signs in this equation have been chosen so that a positive beta shifts the bid curve in the same general direction as for the quantity shift above. That is, positive beta implies an increasing desire to take water, as conditions become wetter, either because the desired quantity increases, at the same price, or the price they are prepared to pay increases, at the same quantity.

⁶⁰ We think the scaling of physical quantities seems more intuitive, in practice.

4.7 Arrangements for Trading Future Water

In Figure 4.1, Item 3.1 shows that all our options assume that physical water is traded in the spot market, based on solutions from the MCE. The real-world interpretation and application of market clearing solutions for future periods is somewhat problematic, though.

In this section our goal is to start to understand what types of products could be designed to allow trading of “future water” in a constrained water market facing uncertain supply, and demand. Before discussing the detail of future trading options, though, there are some key issues to be considered when creating products which can be traded.

As discussed in Section 2.2.1.3, there should be a set of market rules outlining market participants’ rights, and responsibilities in relation to each product being traded. For example, prudential management issues are an important consideration, particularly with derivative contracts, and especially where short selling is allowed. The market manager needs to ensure that each participant has provided enough security to cover existing forward liabilities.⁶¹

As noted in Section 2.2.3.1, the products being traded should reasonably match both the economic and engineering characteristics of the system. And they should have a clear and robust interpretation in terms of property rights as they apply now, and in any future circumstances. Most importantly, participants should be able to readily understand and value these products, easily create net bids and offers for them, and reasonably match them to their needs.

Market “Products” can be customised, or commoditised. Customised products are specialised, and tailored to an individual participant’s requirements. These may be sold by the market manager, but are unlikely to be traded between participants. Alternatively, participants might want homogeneous products, based on standard contracts, so that they can be on-sold, or traded, at any point in time up until the time they are ultimately realised. Examples include transferable fishing quotas, energy derivatives and, financial market

⁶¹ For example, with financial contracts, a prudential management measure could be ensuring participants deposit money into a margin account to cover their presently estimated future obligations. Future obligations for a participant would be the sum of the perceived settlement losses, as presently estimated for all future contracts held by that participant.

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products such as CfDs and options. Trading such instruments makes for a more liquid market, iterative price discovery, and adjustment of positions in response to underlying uncertainty.

In all cases, the product definition would be stated by a contract, defining the rights and obligations of the parties entering into that contract. But all contracts are, to some extent, provisional on certain outcomes which may eventuate, and circumstances may occur in which, “all bets are off”, and it is likely one or both parties will lose some value. For example, contracts commonly specify *force majeure* clauses. Here, though, we focus on instruments that specify quantities (and/or perhaps prices) that vary as a function of water availability, and hence subject to hydrological uncertainty. Such products could be seen as implicitly assigning the right to receive a “bundle” of constrained resources, in some defined proportions, over a specific range of possible future circumstances. Participants would initially agree to buy a fixed quantity of such a bundled contract, as a “product”, at an agreed price, with the market manager ultimately allocating the contractually specified quantity, or payment, to each participant, in the future period, when the circumstances will be apparent.

As discussed in Section 2.2.3.2 products whose payout depends on an underlying index are already commonly traded as “weather derivatives”. The index can be based on a wide range of underlying parameters such as temperature, rainfall, wind, frost and snowfall. The index aggregates the weather variable, and the payoff can be constructed to be linear for every index point above the contractually agreed strike value. Such products are already traded by a range of industries, such as: Agriculture, utilities, renewable energy, and wholesale and retail. This makes the idea not only conceptually, but also practically appealing.

In part this development is motivated by the observation that it will generally be impossible to assign all the expected water to participants in the form of 100% firm allocations, or to meet all participant demands, in that form. It may not even be easy if all participant requirements vary in a similar fashion with hydrology, and particularly if they are anti-correlated with water availability. But a diverse set of rights and trading options gives the MCE more degrees of freedom, to meet a hopefully diverse set of participant needs. Multiple trading options also help to create a more liquid trading environment, and that makes the market much more likely to work, overall.

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Conversely, we note that the MCE makes a very important assumption: That participants actually take what has been allocated, for the current periods, and also what has been notionally allocated, for future periods. In some markets, it might be thought more realistic to assume that participants could purchase options, giving them the right, but not the obligation, to take water. But that would mean that any realistic forward simulation, or MCE clearing, would need to account for a probability that the option would not be (fully) exercised. Otherwise, the amount of water in the system will not balance as planned, and real system outcomes could become increasingly out of sync with MCE solutions, forcing the market manager to make remedial adjustments in order to avoid spill. The conditional contracting regimes discussed here provide an alternative, in which the degree to which options are exercised is implicitly defined, as a function of hydrology. The physical quantities being allocated should thus be much more closely matched to each participant's circumstances. So we can more reasonably assume that the buyer would be obliged to trade their way into a position from which they can physically take the allocated quantity, or trade any differences through the spot market, before real time.

Here, though, we will ignore the practical details of establishing and operating this kind of market, and focus only on the mathematics of product definition, allocation, and pricing. In Section 4.5 we discussed how the MCE determines what to release now, and what to store for the future. In Section 4.6 we discussed how participants could interact with the market by specifying bids, which could be index-dependent. For simplicity, we will assume that bid parameters are either defined for discrete cases, or defined by linear adjustment parameters, and then discretised to match the scenarios for which the market manager solves the MCE.

The MCE will produce a set of optimal spot prices and quantities for all nodes in the scenario tree. The question is, then, what use the market manager might make of that information, in forming “products” to be sold to, and possibly traded by, participants. We want to form index-based contracts and, just as for bidding, the most attractive forms of indexing would seem to be discrete and (possibly truncated) linear. As we will show in Section 4.7.2, even if bids are expressed in, say, a linear form, there is no guarantee that the discrete scenario solutions produced by the MCE will be naturally consistent with contracts of that form. So the issue is how the manager can define and allocate contracts which have the desired form, and at least approximately match the discrete set of stochastic / conditional “market-clearing

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solutions”, once it has been found. As identified in Section 3 of Figure 4.1, four broad options seem possible, for the market manager:

- A. The market manager may decline to form, or allocate any future contracts at all, thus leaving participants to make their own trading arrangements, in light of the information provided.
- B. The MCE could be solved in its natural form, with no extra “contractual form:” constraints, and the manager could attempt to form and allocate contracts of the defined form in such a way as to reasonably approximate the discrete set of outcomes produced by the MCE.
- C. Alternatively, the manager could impose constraints in the MCE to ensure that discrete solutions produced by the MCE are naturally consistent with a contract of the defined form. The bids could then be interpreted as determining the prices participants are prepared to pay for particular quantities of actual contracts they are willing to accept. For example, the contracts (and bids) could be restricted to be affine functions of the index.
- D. Finally, the MCE could not only model the trading of contracts, but the treatment of variations from contracts. We will explore that option using a formulation in which the same underlying bids are interpreted as determining the prices participants are prepared to pay for contracts (on a weighted average basis), and also what they will ultimately be prepared to pay for deviations from their contractual allocations, subject to penalty costs.

The remaining sections of this chapter discuss these options, A-D, in order, noting that, in each case, the instruments to be traded could be either:

- Physical contracts to take a defined water volume at an agreed price; or
- Financial contracts to take/pay the difference between a spot price and an agreed “strike” price, for some defined volume.

In Option A we only discuss financial contracts in secondary markets. In Options B through D we discuss contracts in a general sense, where these could be physical or financial. That is, the contract quantities and prices could be defined and determined in the same way. The key difference with a physical contract is participants would not only need to buy/sell deviations

at the spot price, but penalties would probably also apply for taking more/less than the contractual amount. These might result in effective aggregate penalties for deviation being much higher than those implied by the spot market price.

4.7.1 Option A: No Centralised Future Traded Products

Participants may only wish to trade water in the spot market (Item 3.1, in Figure 4.1), meaning they will be exposed to the volatility of spot market prices (Item 3.2, in Figure 4.1). The market manager could just publish the overall allocated price-quantity information for the whole scenario tree, and participants could use this information to reduce their own risk (Item 3.4 in Figure 4.1), by making their own side, or out of market arrangements via trading physical or financial futures.

4.7.1.1 Spot Market Trading Only

We start with the first option, which corresponds to Item 3.1 in Figure 4.1. The spot market trades physical water in the present period. The market manager trades off current versus future requirements in some fashion, based on their own judgement, and without reference to any centralised market for future water. Participants provide binding spot bids to take a maximum number of water units up to an associated marginal price for each tranche. The market manager decides the aggregate quantity available for the current period, and clears the spot market, declaring price and quantity allocations for that period.

The advantage of this option is that it is simple. But the disadvantage is that it only creates a market for the current period, and so does not allow participants to determine the trade-off between buying water now, or later. Nor does it give participants any certainty about future allocations.

4.7.1.2 Indicative Future Market Clearing

With only spot market trades there are no future products traded in the centralised market, and hence no binding future participant bids. This option corresponds to Item 3.2, in Figure 4.1. In some situations, water supply and prices might be so stable that participants are willing to buy all their water needs in the spot market. But, in environments with significant underlying uncertainties, participants may at least demand market simulation to forecast the potential distribution of future price outcomes over some discrete scenarios chosen.

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For example, as discussed by E. Anderson et al. (2007), the Australian NEM accepts participant offers for each of the 48 half-hour periods, of the next trading day. These offers are matched with forecast load, and indicative market-clearing solutions published. There is rebidding, and the pre-dispatch updated each half hour until the real-time dispatch is finalised. The Singapore market operates similarly. On their website (at SEMA (2014a)) they publish price information for the last, the present, and the next 70 indicative future half-hour periods. Price information includes a breakdown of various energy and ancillary service prices.

There is no short term forward market in either case, though.⁶² So these offers are purely indicative, although market rules constrain the bidding to be realistic, and to converge in a stable fashion as real time approaches.⁶³ Our stochastic water market simulations can extend over months, not days, so prices could be even more uncertain, as they are further in the future. Still, the advantage of this option is that it is relatively simple. The market only trades water on the spot, but the market, rather than the manager is, allowed to trade off current versus future requirements via the indicative future market clearings. Importantly, participants have much more information about the range of possible future choices, and can make better informed bidding choices in the present. So this should be expected to produce more stable spot market prices.

A disadvantage is that participants still cannot guarantee future water, physically or financially, from this market. And participants might not be greatly motivated to provide honest and accurate future bids, if they are only indicative, and may even distort them in order to mislead other participants.

4.7.1.3 Trading Future Products in Secondary Markets

In this situation, participants who are averse to future price fluctuations in the spot market might wish to hedge their risk. Participants could use the indicative future market clearings to negotiate off-market contracts. This option corresponds to Item 3.4, in Figure 4.1. This type of commercial agreement is assumed to be “settled” externally to the centralised market.

⁶² Although there is a secondary market, the Sydney Futures Exchange, where derivative products are sold.

⁶³ By way of contrast, most North American electricity markets allow for “virtual” (financial only) bids and offers, and sell financial instruments such as FTRs (e.g. PJM). These are used to cover existing positions, hedge forward supply and demand projections, and to arbitrage prices towards real-time.

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So the market manager does not need to know about them, unless they directly determine physical flows in, or from, the system they control.

For example, a third party supplier could guarantee future physical water delivery through channels outside the centralised network. The terms of such bilateral physical agreements are likely to be customised, confidential, and not readily analysed. And the contracts themselves will not be tradable. So we ignore the possibility of physical contracting, and focus only on secondary markets for financial instruments, as are common in electricity markets, for example.

Participants could use the future simulated “market clearing solutions”, and/or other information, to inform trading in off-market (private) financial instruments. These contracts could be unique, and tailored to an individual’s needs. Or they could be tradable, in so much as the whole contract, or part of it, could be sold on to other participants. As discussed in Section 2.2.3.2, traders commonly use CfDs and call options to speculate on, or hedge against, commodity price movements, as happens in electricity markets in Australia, New Zealand, and elsewhere.

The advantage of this type of market design is that it puts all the risk firmly onto the participants to manage. The advantage of a secondary financial contract is that they can happen without the market knowing about them. The market manager merely publishes the simulated future clearings from the centralised market. There is no requirement for the manager to deliver, or participants to take, water to exactly match those solutions, and financial contracts mean that only money, not water, needs to change hands between participants. Thus you can decouple from the physical situation of what is going on and just settle the financial contract. On the other hand many participants may consider this to be unsatisfactory, because they still want some guarantee of future physical water. Also, if the spot market clearing is based on water values determined by future bids that are only indicative, we may be concerned about market integrity. As with the last option, participants might not provide accurate future bids, either because they do not care, or because they do care and they are looking to play games in the market. Either way, indicative future bids could change dramatically by real-time, and that makes it hard to identify, and take action on, participants who are using gaming strategies to influence spot prices with their future indicative bids.

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Much of the problem here arises because participants have been provided with a mechanism that allows them to mis-inform the market by signalling their “intentions” costlessly, because no financial commitment is required. Participants can, for example, over-state future requirements to drive indicative prices up, thus creating a surplus; while still being in a position to take advantage of low prices when that surplus eventuates. In general the best incentive for participants to take bidding seriously, and to provide honest future bidding, is for those bids to be binding, resulting in the allocation of firm long-term contracts. Participants can then only take advantage of future price changes with respect to variations from those contract volumes, which might also be penalised when they are closed out.

More parties might be willing to speculate, and trade, financial instruments. And, in turn, this might bring prices down. If sellers of the financial contracts have physical capacity they can sell that capacity on the spot market, and use the money to settle the obligations of the financial instruments they sold. Without physical capacity sellers of financial contracts are taking risks. In this setting there would be prudential issues, possibly requiring insurances, or a margin account, such issues would be managed by the secondary market.⁶⁴

Even if contracts are just financial, there might need to be a physical market-clearing model. For example, FTRs are financial instruments which are commonly traded in electricity markets. But FTRs interact with each other, because the ability of the system to provide sufficient revenue to support them depends on the ability of the physical system to support the corresponding physical power flow. So they can only be traded in the context of an MCE that explicitly models the physical system interactions. As discussed in Section 2.2.3.2, in markets like PJM, short term CfDs are also routinely traded on the basis of day-ahead MCE solutions, which link them to the “commitment” of the corresponding physical generation units.

While the water market we actually simulate in later chapters is extremely simplified, physical interactions of a similar nature would need to be accounted for, if trading future water in real markets operating in the complex environments discussed by Raffensperger et al. (2009), or Mahakalanda et al. (2013). In any case, regulators, market/system operators and water market participants are all likely to want to be assured that the contracts being

⁶⁴ Alternatively, with physical forward contracts where sellers do not have the required capacity to meet their commitments, they may have to buy water on the spot market to cover their obligations. And this may mean making secondary payments to their clients, if they cannot deliver to the same standard, as outlined in their contract.

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traded correspond to water allocations that can actually be delivered, at the specified time and place, given the specified index. Accordingly, the remainder of this chapter examines the possibility of using centralised market-clearing solutions to define and trade forward contract “products” that participants would use to directly hedge future risks.

4.7.2 Option B: Basing Contracts on Unconstrained SLP Outputs

Participants may wish to trade water in the spot market (Item 3.1, in Figure 4.1), and in future periods (Item 3.3, in Figure 4.1). The market manager could clear the MCE in its natural form, and then allocate contracts of an acceptable defined form which reasonably approximates the MCE outputs, (Item 3.3.1, in Figure 4.1).

4.7.2.1 Bids

This option corresponds to Item 3.3.1 in Figure 4.1. Commercially binding bids for future periods are input into the MCE as either deterministic, or conditional. Participants have the option to trade physical water in future periods. The market manager issues contracts, based on the outputs from a stochastic MCE.

4.7.2.2 Approximating the MCE Solution

Contracts would be naturally conditionally based on discrete MCE outputs, but those are most likely not smooth curves across all feasible hydrology paths, in a given period. For example, in Figure 4.5 below we show two different stochastic market clearings, for a given period. The stochastic market clearings are represented by the intersections between a set of conditional DCRs with two alternative end of period MWV Demand Curves for Storage (DCSs). The DCS concept is introduced in Section 2.3.10, while Section 5.4.5 discusses how such demand curves may be formed, and Sections 6.2 and 6.3 show some examples.

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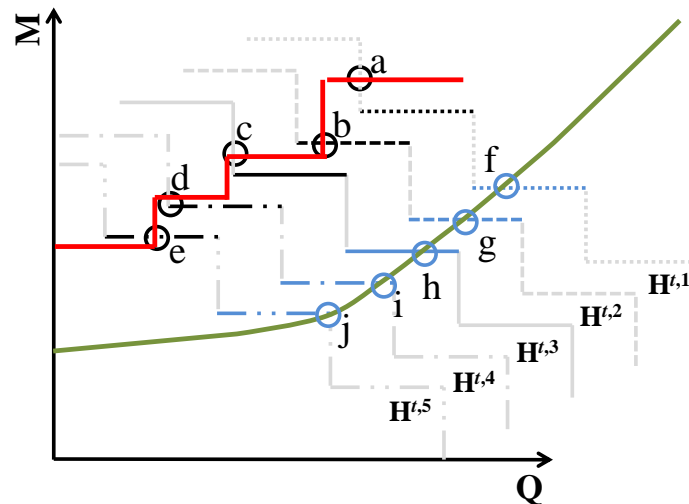


Figure 4.5: Example Stochastic Market clearing / allocation for Aggregate Market DCRs

In Figure 4.5 there are five aggregate DCRs shown, one for each scenario. The driest and wettest scenarios are $H^{t,1}$ and $H^{t,5}$, respectively. The DCRs shift right because participants are prepared to bid higher prices to buy water in the drier scenarios. Suppose the second tranche in each scenario, which is coloured black, represents the bid of one participant. We consider each DCR and DCS intersection from the driest to wettest scenario.

Consider the intersection of $DCR(H^{t,1})$ with the red stepped DCS curve. Note that the participant gets no allocation, as point ‘a’ clears above the second step in $DCR(H^{t,1})$. In the second scenario ($H^{t,2}$) the participant gets a small allocation to the left of point ‘b’. The participant gets no allocation in the third ($H^{t,3}$) and fourth ($H^{t,4}$) scenarios, because the market clears above the participant’s bid. See points ‘c’ and ‘d’. Finally, in the fifth ($H^{t,5}$) scenario the participant gets approximately half their maximum allocation, as represented by the offer step to the left of the intersection at point ‘e’. So that particular participant’s allocation is clearly non-linear, and in fact non-monotone, as a function of hydrology.

This kind of “bumpy” DCS could occur in a system where the storage is small relative to the release capacity. But the smoother (green) DCS is perhaps more plausible in the case where storage capacity is larger relative to annual average inflow. In that setting, the DCS would be quite smooth, since it represents an expected value over a wide range of future values. (See

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Figure 6.2 (Section 6.3.2) later, where over a wide range of storage values the MWV is nearly flat, except at the reservoir bounds (shortage and spill).)

In that case the allocation to our first participant is now constant, at the maximum. Intersections with the third tranche in each scenario (points f through j), which is coloured blue, and represents the bid of another participant, also produce a much smoother allocation, increasing monotonically from the driest ($H^{t,1}$) to the wettest ($H^{t,5}$) scenarios.

Still, in general, we must ask how the market manager could interpret non-linear MCE outputs to form contracts of acceptable form? This could have to be done via some form of curve, or line fitting.

If the market manager fits a linear regression line, we may think of them as estimating the parameters in Equation (4.17) above. And those parameters could be used to define a contract for each participant, with a fixed component, $CQ_i^{F,t}$, and a proportional component, $CQ_i^{C,t}$, which is scaled in proportion to $H^{t,p}$.

In Figure 4.6 below, the greyscale example illustrates a case where the net demand for water reduces with increasing hydrology, which would be typical of an agriculturally dominated region. Here the dashed grey line fit is quite linear. Here, the MCE results for participant i would be approximated well if they were buying a positive fixed allocation, where $CQ_i^{F,t} > 0$, indicated on the vertical axis, offset by selling a negative proportional allocation, $CQ_i^{C,t} \times H^t$ (where $CQ_i^{C,t} < 0$), as the index increases on the horizontal axis. This type of product is likely to be desirable by a farmer who buys a fixed quantity of water to irrigate their crop, and reduces the quantity of water they actually take as rainfall increases.

The two Figure 4.5 examples (above) have also been superimposed onto Figure 4.6, as have their respective (approximate) linear contract solution lines of best fit. These two examples are likely to be desirable for participants who, given the higher prices expected in drier scenarios, are relatively willing to “go with the flow”, and accept an allocation that is positively correlated with the index. (Note, by the way, that the reason why the blue participant’s allocations are falling as it gets drier, is not because they are bidding for less, but

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because they are being outbid by other participants who are willing to pay higher prices for an allocation which applies in those drier scenarios.)

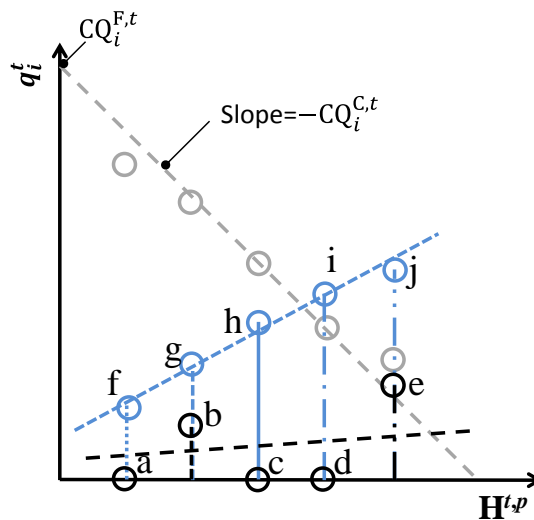


Figure 4.6: Example Stochastic Market clearing / allocation for Aggregate Market DCRs

This linear contract form represents a substantial improvement over the basic “fixed” constant form. But discrepancies are still likely, particularly in very dry and wet situations where net allocations may be lower and higher than the market results indicate the participant might wish. In some cases such discrepancies may be large enough to justify fitting, say, a truncated linear curve, and allocating contracts of that form.

In Figure 4.6 we assumed that a proportional contract is specified over the entire range of the index. But, just like the bids as discussed in Section 4.6.3.5, a conditional contract could be defined linearly, over a more constrained range, between minimum and maximum index limits: HMIN and HMAX, respectively. If all participants are happy with contracts defined between the same index limits, then the contract allocation can still be achieved using a combination of just two contract types, each of which can be traded independently, and has its own market-clearing price, applicable to all participants. Alternatively, if participants wish to use different index limits, the market could still trade standardised instruments, but there would be more types, resulting in market segmentation. These truncated linear contract forms are similar to those employed for weather derivatives, as illustrated in 2.2.3.2. Brockett et al. (2005) note that, while most weather derivatives are still privately negotiated

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over the counter, standardised weather contracts traded on at least two exchanges, the Chicago Mercantile Exchange (CME) and the London International Financial Futures and Options Exchange (LIFFE).

When it comes to pricing, each contract might be subject to some premium, and participants could have the right to refuse contracts if they are deemed to be too expensive. As a baseline, though, the price for the contract allocation to participant i should depend on the quantities it promises to deliver to that, participant. If we ignore index range limits and assume two contract types, fixed and proportional, that is:

$$TCQ_i^t = CQ_i^{F,t} + CQ_i^{C,t} H^{t,p} \quad \forall i, t \quad (4.19)$$

The per unit price for the fixed contract component, CQ^F , is just:

$$CM^{F,t} = \sum_p P^{t,p} \lambda^{t,p} \quad \forall i, t \quad (4.20)$$

And the per unit price for the proportional contract component, CQ^C , is just:

$$CM^{C,t} = \sum_p P^{t,p} \lambda^{t,p} H^{t,p} \quad \forall i, t \quad (4.21)$$

Thus, ignoring any premium, the aggregate expected value of the contracts allocated to i is:

$$TCV_i^t = CM^{F,t} \times CQ_i^{F,t} + CM^{C,t} \times CQ_i^{C,t} \quad \forall i, t \quad (4.22)$$

4.7.2.3 Products based on Multiple Indices

In the above discussions we imagined two products: one fixed and the other proportional, with the latter being defined in proportion to some single hydrology index. But, on reflection, the single index might not adequately capture the incentives of participants. For example, the weather derivatives discussed in Section 2.2.3.3 can often be based on max/min temperature, precipitation, rainfall and snowfall over predefined time periods. And there is no reason why, with an SLP implementation, the index cannot be based on wider indices. So, the regression could be further refined using, say, both Short and Long Term Indices. The Short Term Index could be a function of inflow in the current or immediate past period, whereas the Long Term Index could be a function of cumulative inflows.⁶⁵ It seems reasonable to suppose that agricultural demand is based in part on water stored in plants/soil (accumulated over the long-term) as well as the amount of rainfall they just received (short-

⁶⁵ This approach would be most effective if similar indexing was also applied to the bids.

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term). We could then imagine defining and allocating individualised contracts defined in terms of these two indices. More practically, we could define two “proportional” contract types: a Stochastic Short Term Indexed Product, and a Stochastic Long Term Indexed Product. Each could be traded independently, and has its own market-clearing price, applicable to all participants, as above.

Participants could buy/sell any combination of fixed and proportional Short/Long Term Indexed Products. The actual allocation of water will then depend on the respective indices, in the defined contract period.

Extending Equation (4.19) for a two-index case, we have:

$$TCQ_i^t = CQ_i^{F,t} + CQ_i^{L,t} H^{L,t,p} + CQ_i^{S,t} H^{S,t,p} \quad \forall i, t \quad (4.23)$$

An example total allocation is illustrated in Figure 4.7 below.

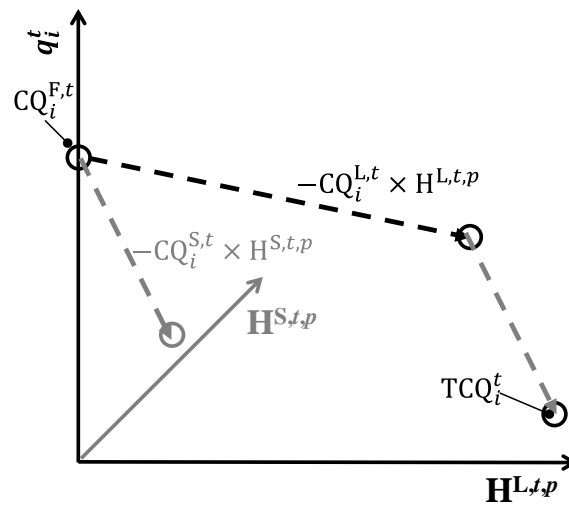


Figure 4.7: Example Multi-Indexed Fixed and Proportional Product Allocation

In Figure 4.7, participant i buys a fixed contract, where $CQ_i^{F,t} > 0$, indicated on the vertical axis. They sell a proportional Long Term Indexed contract, $CQ_i^{L,t}$ (where $CQ_i^{L,t} < 0$), thus reducing their allocation as the index $\times H^{L,t}$ increases on the horizontal axis. They sell a proportional Short Term Indexed contract, $CQ_i^{S,t}$ (where $CQ_i^{S,t} < 0$), thus reducing their allocation as the $H^{S,t}$ index increases on the axis going into the page. This could be a suitable strategy for a farmer buying a fixed quantity of water to irrigate their crop over the year, and

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selling the long-term and short-term components, for example if they find themselves in a wetter than average season/year and if rainfall increases in a given period, respectively.

An OTC contract of this form could be created and traded for any particular participant. But there is no reason why each product can not be traded separately, and combined to produce this result, as above. Critically, by buying this product combination in advance, the participant gives the market manager advanced warning of their intended pattern of water consumption, while greatly reducing their own need to adjust their contract position by constantly trading as real time approaches.

4.7.3 Options C and D

Post-clearing curve fitting might be accepted by many participants as adequate, or at least the best available option. In fact, as discussed in Section 4.7.2.2 above, if the DCS is fairly smooth and not too curved then this method may produce near optimal results. Still, the approximation could turn out not to be close, and the curve fitting approach could lead to arguments amongst participants who are adversely affected. So we should consider the possibility of constraining the MCE such that it directly creates instruments of the desired form.

Options C and D correspond to Items 3.3.2 and 3.3.3, respectively, in Figure 4.1. Both of these options could result in Fixed or Proportional (e.g. linear as above) contracts. In either case participants could provide fixed or conditional bids, but a stochastic MCE solution is required if stochastic contracts are to be produced. Whilst Option C only produces contractual allocations, Option D also simulates future spot trading around that contractual allocation. While that refinement may seem unnecessary, it would produce more realistic projections of future prices, and hence inform contract prices more appropriately, too. While the technical details for these two options are outside the main scope of this thesis, these are presently being developed by E. Read et al. (2015).

4.8 Conclusions

Here we have considered various market design options, with a view to determining the main directions for exploration and quantification. The system share options allow participants the freedom to make their own trade-offs between storage and release. These options seem conceptually appealing, but have not been examined in detail. We decided to focus our investigative effort on markets in which water stored in a reservoir is traded through a market, and stored in a reservoir system managed by the market manager. Thus the treatment of stochasticity must be directly accounted for within the market clearing system.

Section 4.4 lays out a framework covering many variants of that centrally co-ordinated approach. Section 4.5 describes an increasingly complex modelling system, which handles different forms of stochasticity within the market clearing. Section 4.6 then outlines various ways in which participants can interact with the market, and modelling system, at the bid interface. In principle, participants could provide indicative and/or binding deterministic and/or conditional discrete (scenario specific) bids, for various future periods. But, to simplify the data input requirements, we suggest that participants could provide conditional bids which vary as a continuous function of some underlying index. Affine bids seem most plausible, where bids are comprised of “fixed” and “proportional” components. The latter specifies bid quantities and/or prices that are linear in some index.

We note that these two components can also readily be interpreted as forming bids for two separate products. The first product has a fixed contract volume, irrespective of the index. The second product has a proportional contract volume, which is scaled linearly with one or more indices. Then, in Section 4.7, we considered four broad options with respect to the way contracts are defined and traded.

In Option A, financial instruments could be traded in one or more secondary markets. Participants could indicate their future preferences using indicative bids, which are employed in a notional market-clearing process to generate scenario information about the future, but MCE outputs would only be used, by participants and others, to inform their trading in financial futures contracts.

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In Option B, participants trade a mix of fixed and proportional “futures” products, as cleared by the centralised market. The allocation and pricing of those products would be determined, after the stochastic MCE has run, by the market manager fitting a regression line to the MCE outputs. Because the contracts are formed from the results produced by a “naturally unconstrained” MCE we get the right market clearing prices, both from a contract pricing and market simulation perspective.

In Option C, the stochastic MCE has additional constraints imposed such that outputs from the MCE are consistent with the contractual form requested by participants. The problem is, though, that the prices forecast by this “artificially constrained” MCE may not be very realistic. Thus, we suggest a final Option, D which models the way in which participants may ultimately trade around the contracts formed by the initial market clearing, to deviate from those allocations, at more realistic (projected) spot prices. But Options C and D lie outside the scope of this thesis, and are only briefly discussed.

Several of these options seem conceptually appealing, and each could be appropriate in a different environment. In fact, in a real market, having a variety of trading options gives different participants a range of choices, and that should be seen as a good thing. But, the main focus of the remainder of this thesis is to understand how much impact the underlying uncertainty could have on a market, and the degree of sophistication that may be warranted in accounting for that uncertainty within the MCE.

5. Computational Stochastic Market Design

5.1 Chapter Introduction

In the last chapter we developed a range of market design options, proposing various trading mechanisms to clear and settle the market, under increasing levels of uncertainty. The centrally co-ordinated set of options directly manages hydrological uncertainty within the MCE. But, in a given trading environment, in the presence of real world hydrological uncertainty, there are some key questions:

- How much impact can the stochasticity have on a given reservoir system?
- In what type of systems is a deterministic MCE adequate, and
- When is there potential value from accounting for some level of stochasticity within the MCE? Where the benefit could be for:
 - Participants' buying and selling water, and/or
 - The system operator managing the water system.

To answer these questions we now design and develop a stochastic computational modelling system. From this point forward this thesis conducts a series of high level experiments without concern for the individual position, and wealth, of specific trading participants. We look to design markets which optimally allocate the water resources to generate the greatest net benefit for the local economy. Given this focus we also assume perfect competition, meaning each participant's net bid tranche exactly matches that participant's (estimated) benefit valuation. We also assume that all participants are risk neutral. Thus participants do not need to create contracts in order to manage risk, because they are indifferent to price or quantity changes in the spot market. For example, the participants could be a small number of local authorities, or state owned enterprises – risk neutral but too few to assume no market power. Alternatively, the participants could be a large number of farmers – possibly not risk neutral but an ample number to assume no market power. With these two assumptions, we can ignore contract markets, and assume that participants only trade water in the spot market and are price takers, taking what they bid for, if allocated.

In Chapter 3, we proposed a series of SLP formulations. In Chapter 4 we discussed allocation of water resources by market trading; focussing on the Dual (economic) interpretation. To understand the implications of uncertainty on a multi-reservoir system we

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first need to understand its implications on a simplified single reservoir system. Accordingly, we have selected (Stochastic) CDDP to implement our computer modelling system, since it is computationally efficient for a single reservoir system, directly generating the dual information we are interested in, and producing results in a convenient form for simulation.

In Section 5.2 we briefly discuss the merits of, and assumptions underlying, our chosen implementation strategy, for our single reservoir stochastic modelling system. In Section 5.4 we then overview each main computational module in the modelling suite. We start by describing the reservoir, hydrology, and market data modules. Then we outline the SCDDP optimisation module. Next we discuss the two simulation sub-modules. These are a discrete convolution, and a Monte Carlo simulation.

The rest of this chapter is then dedicated to the average, and finally volatility, reporting measures. These reporting measures are used to gauge the market performance of different market design choices, across different case runs. In Sections 5.5, and then 5.6, we discuss the average reporting measures for benefit, and then price, respectively. Section 5.7 we overview volatility reporting measures which can be applied to the benefit and price average reporting measures. Finally, in Section 5.8, we briefly discuss other miscellaneous average reporting measures. These include the probability of running out of water, and/or spilling water, at different times of the year.

5.2 Design Assumptions

In Chapters 6 through 9, we compare stochastic market design options, for a set of physical, hydrological and economic case data, by running the market clearing optimisation using different levels of uncertainty modelling. Obviously there is no point designing, or optimising, an allocation system to deal with more complexity than we assume in simulating the world. Thus the simulation model used to evaluate the market is always run with at least as complex a level of uncertainty modelling as the market clearing model.

We assume that our water models operate over an annual time horizon and that we have some known underlying hydrology distribution. We discretise the decision stages, $\{1, \dots, T\}$, and hydrology states, $\{1, \dots, H\}$. We use the same assumptions in both the optimisation and

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simulation models, although transition probabilities can change. For ease of implementation, modelled water units are fixed onto a regular integer grid.

We assume that the market clearing model optimises a release decision for each modelling stage, with no opportunity for intra-period recourse. We are thus forced to approximate the gradual revelation of uncertainty (inflows) over the period, and any subsequent intra-period release decision adjustments relative to the actual disclosed inflow. For our experiments we assume one of two extreme intra-period flow allocation policies, labelled “informed” and “conservative”, as discussed in Section 3.7.1.

In Chapter 3 we formulated our models using SLP, and in Chapter 4 we went on to carry out high level simplified investigations on a hypothetical single reservoir system. Here we adapt the SLP model from Chapter 3 to an SDP as it allows computationally efficient optimisation, and subsequent simulation, for a single reservoir system. Instead of commencing from a single initial reservoir starting level, SDP efficiently computes optimal solution values for all integer storage/hydrology states in the reservoir system. This allows an efficient simulation with only a single run of the optimisation. Using an SLP implementation the simulation would require the SLP model to be re-optimised each period for each simulated year, with multiple years simulated. Thus the SDP implementation allows us to explore and analyse the whole reservoir state space, and analyse how a given reservoir system configuration copes with different inputs (net inflow) and outputs (market demands).

Marginal Water Values (MWVs) are a key driver of the market. For our model methodology we have decided to use SCDDP, a particular form of SDP which works directly with the MWVs. CDDP models are discussed in Section 2.3.10.

For a single (reservoir) state space, even with a relatively dense grid, the solution times are reasonable. Hence SCDDP is practical for experimental purposes. Solution times range from several minutes for multi-year storage capacities, down to a few seconds with capacities less than half annual storage capacity.⁶⁶

⁶⁶ This is on an Intel® Core™2 Duo CPU T9400 @ 2.53GHz with 4GB of RAM and a 32-bit Operating System.

5.3 SDP Notation

In Section 3.8.2 we outlined a node based notation, assuming an informed inflow allocation policy, for a general tree structure, in an SLP framework. We now assume a lag-1 Markov process, in an SDP framework, where the conditional probabilities only depend on the prior hydrology state, and not the entire history of states. In this section we link our SLP model from Chapters 3 and 4, to our SDP model, which we use from this point forward. In an SDP setting a period is often termed a stage. We discuss notation in terms of the transition (continuity) equation, followed by the transition probabilities.

The hydrology state information is implicit in the continuity Equation in (3.52), $s^{t,p} - s^{prev(t,p)} + r^{t,p} + w^{t,p} = F^{t,h(t,p)}$.⁶⁷ All the variables (and parameters) are defined at nodes which were indexed by period and path. With the SLP formulation the path contains information relating to the whole event tree.

With an SDP formulation, and a Markov lag-one process, the hydrology state only depends on the previous stage's state. In stage t , F^t only depends on the present hydrology state. It is easy to show that both r and w depend on the prior period's storage and present hydrology states in that present stage. But the storage position still depends on the whole path, thus the storage variables still contains the path notation for the hydrology state history, \hat{h}^t and \hat{h}^{t-1} . Hence:⁶⁸

$$s^t(\hat{h}^t) - s^{t-1}(\hat{h}^{t-1}) + r^t(s^{t-1}, h^t) + w^t(s^{t-1}, h^t) = F^t(h^t) \quad (5.1)$$

where, comparing Equation (5.1) to Equation (3.52) the variables (and parameters) are still indexed by period (stage), but in an SDP setting we make the various 'state' dependencies clear, by making them explicitly dependent on the hydrological state information. Releases and spill also depend on the prior storage state information. We term Equation (5.1) the SDP transition equation.⁶⁹

⁶⁷ Under uncertainty, the multiple net inflows create different pathways and this increases the number of release decisions in a given period.

⁶⁸ Note that, because the decision maker is assumed to know the current hydrology state, no inference needs to be made from the previous state. Consequently this has exactly the same form as for the stochastic independent case. The probabilities themselves will be different, though, and hence also s , q and u .

⁶⁹ It is not strictly the transition equation. For that, and to get to the SDP, we have to admit that we do not know the correct $s^{t-1}(h^{t-1})$. In that setting we make $s^{t-1}(h^{t-1})$ a state variable, and then try all possible values to form optimal sub-policies, and consequently the optimal solution.

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Alternatively, with a conservative flow allocation policy, release and spill are now a function of the prior known hydrology state, and spill can still depend on the present hydrology state, as follows:

$$s^t(\hat{h}^t) - s^{t-1}(\hat{h}^{t-1}) + r^t(s^{t-1}, h^{t-1}) + w^t(s^{t-1}, h^{t-1}, h^t) = F^t(h^t) \quad (5.2)$$

With a Stochastic Independent (lag-0 Markov process) formulation and an informed flow allocation policy release is still a function of the present hydrology state. The equivalent conservative flow allocation policy notation, for a Stochastic Independent formulation, is:

$$s^t(\hat{h}^t) - s^{t-1}(\hat{h}^{t-1}) + r^t(s^{t-1}) + w^t(s^{t-1}, h^t) = F^t(h^t) \quad (5.3)$$

With a deterministic formulation we have perfect foresight and can not be uninformed.

At the start of stage t , the hydrology state for period $t-1$, h^{t-1} , transitions to the next stage's hydrology state, h^t with some probability. The conditional probability of being in state h^t , starting from state h^{t-1} is denoted $\tilde{P}^t(h^t : h^{t-1})$. Using the same example in Figure 3.12, if we assume two probability transitions in each stage and we assume a Markov chain, then the whole tree can be replaced with the below Figure 5.1.

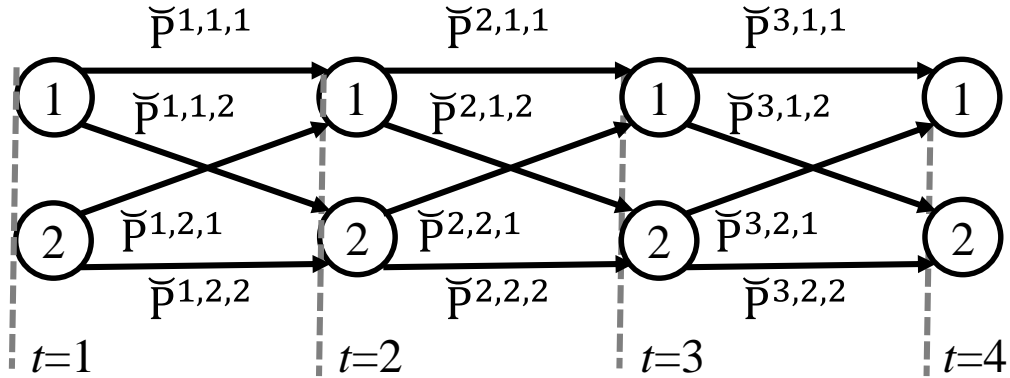


Figure 5.1: Example Conditional Probability and Event Node Labelling for Markov Chain

In Figure 5.1 these are the same probabilities as used in the scenario tree, in Figure 3.12, namely:

$$\tilde{p}^{t,p} = \tilde{P}^{t,h(\text{prev}(t,p)),h(t,p)} = \tilde{P}^t(h(t,p)|h(\text{prev}(t,p))) \quad (5.4)$$

In an SDP setting we write the transition probability as:

$$\tilde{\mathbf{P}}^{t,h^{t-1},h^t} = \tilde{\mathbf{P}}^t(h^t | h^{t-1}) \tag{5.5}$$

But the LHS has an unnecessary notational complexity. So we can simplify the notational form of the transition probability by letting $h' = h^{t-1}$ and $h = h^t$, thus:

$$\tilde{\mathbf{P}}^{t,h^{t-1},h^t} = \tilde{\mathbf{P}}^t(h^t | h^{t-1}) = \tilde{\mathbf{P}}^t(h|h') \tag{5.6}$$

where the LHS of Equation (5.6) is the notation used in Figure 5.1 above. For some analyses we further assume the Markov chain is stationary year-to-year, with transition from the hydrology states in the last stage to those in the first stage.

This formulates a multi-state Transitional Probability Matrix, the TPM. The TPM, is viewed as a sparsely populated matrix of mainly zeroes, apart from the offset block diagonal which contains each intra-stage TPM, the TPM^t . TPM^t is a sub-matrix for each t , and these can be different in different stages. The LHS illustration in Figure 5.2 below shows this for the same example as in Figure 5.1 above. The RHS illustration highlights the transition from the stage 2 “columns” to the stage 3 “rows”.

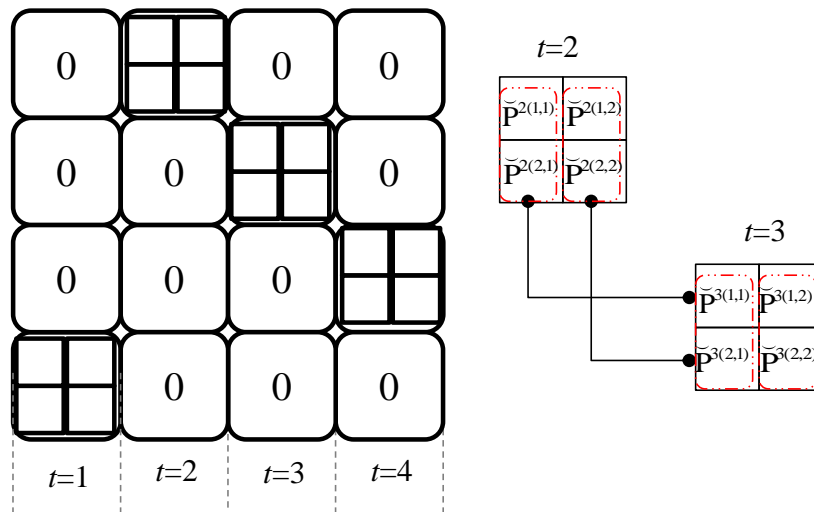


Figure 5.2: Example Transitional Probability Matrix Interaction across Time

From Figure 5.2, TPM^t is a square matrix of size $H \times H$, while TPM is of total size $(H \times T) \times (H \times T)$.

The steady state probabilities, $\tilde{\mathbf{P}}^{t,h}$, assume a stationary year-on-year stochastic process, and those probabilities give the probability of being in any hydrology state at any stage.

Daellenbach et al. (1983) illustrates one method for computing these values. These are useful for defining Stochastic Independent equivalent probabilities, as discussed in Section 5.4.3 below.

5.4 Modelling System Details

5.4.1 Overview

Our modelling system can be configured to analyse a given stochastic market design option. The option can be analysed over a wide range of reservoir, hydrology and economic (market) parameters. A given market design option run, with a given fixed set of reservoir system parameters, is called a case. The modelling system is designed to run batches of cases. Output data files are catalogued to allow relatively fast post-processing and facilitate quick high-level analysis across multiple case runs.

The modelling system has a modular block structure, developed using Matlab,⁷⁰ a modular programming language. We create functions which interact with one another to compute the main system outputs. There are three main interface levels: Input, Computation and Output. Each of these three levels has three main components, as outlined in Figure 5.3 below.

⁷⁰ Our computational strategy involved two levels of programming. First we developed general simplified models in Excel to test our ideas of how to mechanically determine various factors. This provided a good foundation to allow us to develop equivalent, more fluid models, over a wider range of variables and parameters in Matlab. We chose Matlab initially because its matrix based approach to structuring data complements Excel's, including importing and exporting data between the two packages. Importantly Matlab's modular nature allows us to create separate functions, allowing the central modules to be utilised for a wider research programme.

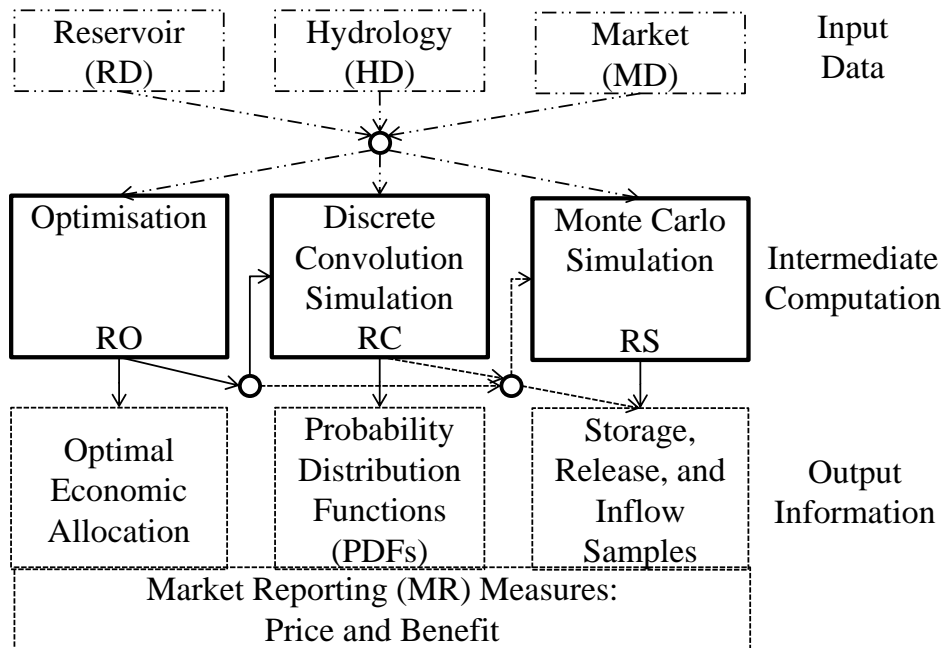


Figure 5.3: Overview of Computational Modelling System Key Interfaces

At the first level, case data is fed into the main computational modules. Key system parameters include the reservoir storage and release capacities, net inflows and water demand curves. Hence the key data modules comprise: Reservoir Data (RD), Hydrology Data (HD), and Market Data (MD). The RD module sets the high level physical and financial system parameters for the given case run,⁷¹ while the HD and MD modules pre-process net inflow and net demand data sets for a given case.

The core of the system comprises: the Reservoir Optimisation (RO), Reservoir (Discrete) Convolution (RC), and Reservoir (Monte Carlo) Simulation (RS) modules.

First, the RO module uses SCDDP to compute the optimum economic utilisation schedules by constructing various optimum M_{WV} surfaces for the entire reservoir discrete state space. At the same time as generating the M_{WV} surfaces, the RO module generates the Conditional Optimum Release Schedule (CORS), and also the optimum storage schedule which is easily generated from the CORS. The CORS is a key input into any subsequent simulation module, and combined with the input data, we implemented two main simulation sub-modules:

- The RC module uses a discrete convolution to compute the full set of Probability Distribution Functions (PDF) for the entire reservoir storage state space, and / or

⁷¹ Such as RCAP, SCAP, elasticity, and flow allocation policy.

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- The RS module uses a Monte Carlo simulation to compute limited samples (for inflow, release and storage levels) from the underlying hydrological probability distribution.

The exact details of these simulation options are discussed later, in Sections 5.4.6 and 5.4.7. And while both of these are used to generate market performance reporting measures, in general we report Monte Carlo results which are backed up with discrete convolution results, where necessary.

Outputs from the RO, RC and RS modules can then be passed through a series of Market Reporting (MR) modules to produce key performance measures, as required. Key measures are market price relative to volume traded, and net welfare reported in terms of three “market value” measures. These measures provide a high level analysis of how well a given market design, and associated case, performs in a stochastic trading environment.

Given that our SCDDP system is designed to run at a lag-1 Markov Chain (MC) level of stochastic complexity, then we require input data, and the resulting computations, in a consistent form. The datasets are also constructed in such a way that we can consistently model the simpler Stochastic Independent (SI) and Deterministic (DC) cases.

The rest of this section discusses the key details of the first two levels from Figure 5.3 above, namely input and computation. Starting with the three data modules (RD, HD, then MD), we move on to outline details of the optimisation (RO) module, and then the two simulation modules (RC and RS).

5.4.2 Reservoir Data

The key reservoir system parameters are the storage and release capacities. These are deterministic. In a given case run we assume that these capacities are fixed to the same value for all periods, thus $SCAP^t = SCAP$ and $RCAP^t = RCAP$. These capacities are set as positive integer values, only. For our experiments we assume that the minimum storage and release capacities are set to zero. Implementation details of the relative sizing of SCAP and RCAP are covered in Chapter 7.

The discretisation of decision stages within the annual time horizon ($t = 1, \dots, T$) can be as fine as desired.

5.4.3 Hydrology Data

We assume there are no tributary flows, which could be used to offset reservoir releases. For our stochastic single reservoir experiments this is a reasonable to just focus on uncertain reservoir inflows, and potentially uncertain net market demand (which by definition may already have accounted for some tributary flows). As discussed in Section 3.7.3, a generalised “hydrology” state might encapsulate other uncertain aspects of the system, such as temperature. For our experiments we assume that net inflow wholly represents the hydrology state. We assume that environmental needs, such as ensuring a minimum river flow, and any losses, have been accounted for before producing the net inflow value. The modelling system can accept any discrete number of hydrology states, and symmetric or asymmetric hydrology data. We assume that the number of discrete hydrology states ($h=1, \dots, H$) is the same for each decision stage.

For experimental design reasons, we generally assume an odd number of total hydrology states where the inflow for the middle value, \bar{h} , matches the average inflow, with drier and wetter states either side. The average inflow is also used in the deterministic model.

The hydrology data consists of net inflow, F , and conditional probability, \check{P} , data pairs, (F, \check{P}) . The model has the facility to accept different \check{P} and F data, for each computational module.⁷² For our experiments if the optimisation and both simulation models are run at the same level of stochastic complexity they have the same hydrology distributions.

For consistency, the F values used in the optimisation (FO) and simulation (FS) are equal, $FO = FS$, when RO is run as MC or SI. If RO is run as DC then FO uses the mean inflow from FS. To eliminate experimental noise, the \check{P} values used in the optimisation ($\check{P}O$) and simulation ($\check{P}S$) are equal, if both modules are run at the same level of market design complexity (MC, SI, or DC), $\check{P}O = \check{P}S$. When testing the market as a MC, then the simulation is always loaded with a MC probability distribution. For consistency, and as a choice within in our experimental design, if we run the optimisation at a SI level of complexity then we use the long-run steady state probabilities from the MC probability distribution, to eliminate experimental noise.

⁷² Hence RO has hydrology data pairs (FO,PO), RC and RS have data pairs (FS,PS).

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In Section 3.7.2 conditional probabilities are implicitly defined in a tree structure, where they could be different at each node, and hence dependent on the whole path to that node. But the conditional probabilities could also have a very specific structure. For our experiments we describe the hydrological parameters using a Markov Process and the model inputs probabilities as Markov transition probabilities.

With a lag-one Markov process there are $H \times H$ total transition probabilities for each stage. The conditional probability of moving from hydrology state h' to state h , in stage t , is expressed as $\check{P}^{t,h',h}$. For a stochastic independent process this is simplified to $\check{P}^{t,h',h} = \check{P}^{t,h}$.

Our modelling system assumes that net inflow values must be non-negative. From the perspective of our experimental design, we also ensure that net inflow is large enough to avoid shortage cost. Hence we assume that there is at least one unit of inflow available in every modelling period. Net inflow is correlated between decision stages, t , and there are $T \times H$ discrete net inflow values. Inflow in period t is denoted by $F^{t,h}$, assuming a lag-1 Markov process. In most places where a market might be considered we envisage some annual seasonal inflow pattern. We simplify the scaling of our experiments by assuming that the pattern of discrete inflow values is symmetric over the year. While not realistic, this has been done for experimental design purposes. It is expected that many of the insights also apply when seasonal demand is not symmetric.

Specifically, we assume that there is only one full ‘sinusoidal cycle’ within the water year, and we also assume that the water year commences at the start of the wet season.⁷³ This crude seasonal approximation results in two seasons: The first (second) half of the water year is wetter (drier) than average. The mean (expected) periodic net inflow, FMEAN, represents the average condition over the whole water year. A Weighting Factor, W_F , scales the amplitude of the sine curve between 0 and 1. This is multiplied by FMEAN, and gives:

$$F^{t,h} = \text{FMEAN} \times \left[1 + W_F \times \sin\left(\frac{2\pi \times t}{T}\right) \right] \quad (5.7)$$

In Equation (5.7) above, changing W_F stretches the vertical peaks and troughs of the periodic net inflow ‘curve’. If $W_F = 0$ then $F^{t,h}$ is FMEAN for all t , so effectively there is no seasonality modelled. Generally $W_F < 1$ and the actual value is chosen such that lowest

⁷³ Although the model is run to equilibrium, so this should not matter.

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inflow is at least one unit. So as to reduce the noise in our experiments, non-integer values are carefully rounded to ensure consistent symmetry about the mean.

Non-mean hydrology states, where $h \neq \bar{h}$, also need to be set. To do this we introduce a deviation parameter, the Deviation Factor (DF). To create a symmetric distribution $DF(h)$ must be set to the same positive and negative step increment values, either side of the mean inflow, for each stage. That is $DF(h') = -DF(h)$ when $\bar{h} - h = h' - \bar{h}$. This ensures the probability weighted sum of $DF(h)$ equals zero, to ensure the middle hydrology state is the mean.

We adapt Equation (5.7) as follows:

$$F^{t,h} = (FMEAN + DF(h)) \times \left[1 + W_F \times \sin\left(\frac{2\pi \times t}{T}\right) \right] \quad \text{where } h \neq \bar{h} \quad (5.8)$$

In Equation (5.8) above, the addition of $DF(h)$, vertically shifts the periodic net inflow “curve”. W_F is set such that it is always less than 1, and $DF(h)$ is set relative to this, such that the inflow in any stage and state is at least one unit. For seasonality experiments we fix $DF(h)$ and vary W_F . For seasonal variance experiments we vary $DF(h)$ and fix W_F .

Implementation details, and exact inflow values chosen are given in Chapters 6 through 8.

5.4.4 Market Data

While the modelling system has the facility to allow for priced supply sources, in all our experiments we assume that there are only net bids to consume water. We also assume that participant net bids can be aggregated to form a net DCR, in each decision stage, and potential hydrology state. The DCR terminology was first introduced by Scott & Read (1996). Participant bid value pairs (Q, M) are pre-processed and aggregated to construct a net DCR. In this section we generally discuss continuous aggregate net Conditional DCR (CDCR) functions.

Over the water year, we assume that the average periodic inflow, $FMEAN$, fed into the reservoir, equals the average periodic demand Quantity Reference point, $QREF'$, drawn from the reservoir.

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$$\text{FMEAN} = \frac{1}{T} \sum_t \text{QREF}^t \quad (5.9)$$

Equation (5.9) is intended to model a catchment with a single reservoir source in which supply and demand balance on average, over time. Generally we assume non-state dependent demand. Hence, the same data is used for all hydrology states. See Chapter 8, Section 8.4, for state-dependent demand experiments.

We assume that QREF^t follows a sinusoidal curve similar to $F^{t,\bar{h}}$. Similarly, this is for experimental design purposes rather than realism. We also simplify the scaling of our experiments by assuming that the pattern of discrete QREF^t values is also symmetric over the year. In the real world we would expect peak inflow and peak demand not to be in phase. For example, demand might be highest during the dry season, when very little inflow is actually available. We introduce a Lag Weighting (W_L) so that demand can lead (or lag) inflow, as well as simply being synchronised with inflow if $W_L = 0$. We also re-use the Weighting Factor, W_F , for consistency and simplicity in our experiments we assume W_F is the same one as in the inflow Equations (5.7) and (5.8). We use an equation similar to (5.7) to model QREF^t as follows:

$$\text{QREF}^t = \text{FMEAN} \times \left[1 + W_F \times \sin \left(\frac{2\pi \times t}{T} + W_L \right) \right] \quad (5.10)$$

Equation (5.10) creates just one point on each intra-period DCR. To generate the entire net DCR data, we restricted its form to only 1st quadrant, non-negative bid values, only. The modelling system accepts any set of demand values (DCR shape), as long as they are ordered monotone non-increasing, by price. The market (bid) data consists of Quantity (Q) and Marginal Value (M) data pairs, (Q, M), for all units which can be utilised (i.e. released to the market). For the experiments we assume unit bid steps: $Q=1$.

A common measure of demand responsiveness is elasticity. For most of our experiments we assume that a fixed constant elasticity curve represents demand. This is a standard modelling choice, an alternative being a linear demand curve. This assumption is relaxed somewhat in experiments in Chapter 8. The elasticity value represents the aggregate market price sensitivity, relative to the cumulative availability quantity of water. Point-price elasticity can be given by:

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$$e = -\frac{M}{q} \times \frac{\Delta Q}{\Delta M} \tag{5.11}$$

From equation (5.11), $\Delta Q / \Delta M$ is the marginal change in the Q and M values, at any given point on the demand curve. This derivative influences the shape, and hence the rate of change, of the DCR. Without loss of generality, we design our DCRs such that over the water year, on average, each curve passes through the point where the standard demand quantity reference point QREF, has a Marginal price Reference point MREF. Hence our constant elasticity curves have the form:

$$M(q) = MREF \left(\frac{q}{QREF} \right)^{-1/e} \tag{5.12}$$

We use an Elasticity Factor (E) to form a normalised constant elasticity curve. If E is 1, the DCR curve is unitary, or perfectly elastic. We express the DCR via the following equation:

$$DCR(q,t) = MREF \left(\frac{q}{QREF^t} \right)^{-1/E} \tag{5.13}$$

As our implementation discretizes demand, we let the demand quantity take on each integer value up to the feasible release capacity ($q = 1, \dots, RCAP$) of the reservoir outlet pipe, or channel. The “pinning” of the DCR, at $q = QREF$ and MREF, is illustrated in Figure 5.4 below.

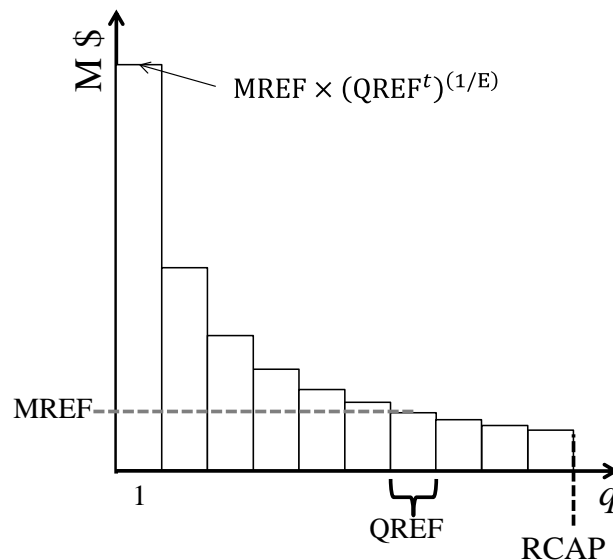


Figure 5.4: Constant Elasticity Curve for integer discrete bid steps

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As illustrated in Figure 5.4 above, choosing MREF effectively, scales the ‘price units’, and the choice of MREF is arbitrary. We arbitrarily set MREF to be 1. So, as a consequence of our design choice we can compare directly across different cases, with different shaped DCR curves, as Equation (5.13) ensures that all the curves are pinned, go through the same reference point $(Q,M) = (Q_{REF}, M_{REF}=1)$, on average, over all decision stages, and hydrology states. Hence, with an unconstrained (optimal) system, this design feature should always produce an average market clearing price of one, both at a monthly and annual level. This issue is discussed further in Section 5.5.4 below.

The next three chapters use these constant elasticity curves as inputs, when producing various case results. We generally use a constant elasticity value of 0.5, which would create inelastic demand, and resulting asymmetric benefit returns. This assumption means that there is a large amount of benefit returned from just being able to release a small amount of water to the market. This is not unreasonable, given its life giving properties. Participants (and wider interested groups, such as politicians) would not expect to see such extreme prices in the market very often, if at all. In practice we would expect water restrictions, legislation, and (the likes of) desalination plants built as means of avoiding these extreme prices. However, in the model these extreme prices are likely to appear with low probability and we can view them as approximating the economic cost of avoiding the possibility of shortfall. The choice of constant elasticity value is discussed in Chapter 6, Section 6.2. In Chapter 8 we experiment with different constant elasticity curve shapes by changing E , over a wide range of values.

Industry DCR curves are commonly not shaped like constant elasticity curves. For example, in electricity markets DCRs are often represented by Load Duration Curves (LDCs), which generally do not have a constant elasticity structure. In Chapter 8 we experiment with different shaped DCRs, see Section 8.3 for details.

5.4.5 Reservoir Optimisation

The RO module maximises net welfare producing the optimal economic allocation price and quantity schedules. RO obtains net bid data from the MD module, the inflow distribution from the HD module, and reservoir capacity limits from the RD module. The RO outputs the

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optimum price and quantity schedules. A general description of the SCDDP algorithm is provided by in Dye et al. (2012).

We describe the procedure using “Demand Curve” terminology, similar to that discussed by Dye et al. But we note that this could just as easily be described in equivalent list form, as they also discuss. The procedure we describe here is for a generic stage from $t+1$ to t .

For simplicity, here we assume that the stages are weeks, and we break each stage into two sub-stages with the start of a stage labelled Monday Morning (MM) and the end of a stage labelled Sunday Night (SN). For a Markov implementation, a set of Conditional Demand Curves for Storage (CDCS) are constructed for each stage via a series of intermediate adjustment steps. The process always starts with multiple CDCSs’ for the beginning of the next period, $CDCS^{t+1,h}$.⁷⁴ These are adjusted to produce multiple Conditional intra-period Demand Curves for Water (CDCW), and then multiple CDCSs’ at the beginning of this period, $CDCS^{t,h}$. The key adjustment steps are:

- Weight and Sum,
- Merge Sort,
- Flow Shift, and
- List Truncation to within utilisable bounds.

The exact procedure depends on the flow allocation policy, and the level of uncertainty being assumed in the Market Clearing module. We first describe the process for an informed inflow allocation policy under Markov assumptions, and then later discuss how the process can be simplified to deal with the conservative policy, and/or Stochastic Independent (SI), and Deterministic (DC) inflow assumptions.

We assume that inflow and release happen continuously over the week. The actual inflow and release values are accounted for between MM and SN. With an informed inflow allocation policy the reservoir manager observes the inflow at the start of the week, on the MM. Of course the SCDDP algorithm works backward through the stages. Hence accounting for inflow and release happens towards the end of the procedure for each stage, which we now describe in detail.

⁷⁴ If this was deterministic there would be only one DCS curve per stage.

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Figure 5.5 below shows the first few parts of an SCDDP iteration process for period t , for three discrete hydrology states, getting progressively drier, from $h=1$ a charcoal grey, to $h=2$ a grey, and finally to $h=3$ a light grey. Each part is described as follows:

1. Shows what we already know, namely the Conditional DCS's. These give the value of water left in storage next Monday Morning, for each hydrology state we could observe then, for the next week. The notation for these curves is $MM_DCS^{t+1, h}$, where h is the hydrology state for MM in period $t+1$.
2. Shows how, for each hydrology state in the current week, where h' is the hydrology state for SN in period t , we weight each $MM_DCS^{t+1, h}$ curve by the appropriate inter-period Markov transition probability $\check{P}^{t, h', h}$
3. Illustrates the formation of the $SN_DCS^{t, h'}$, the Conditional "Sunday Night" DCS for each hydrology state in the current week, by summing the probability weighted $MM_DCS^{t+1, h}$ curves.⁷⁵ On MM there is a 'perfect' forecast of the hydrology state for period $t+1$, but SN is before this forecast has been made.

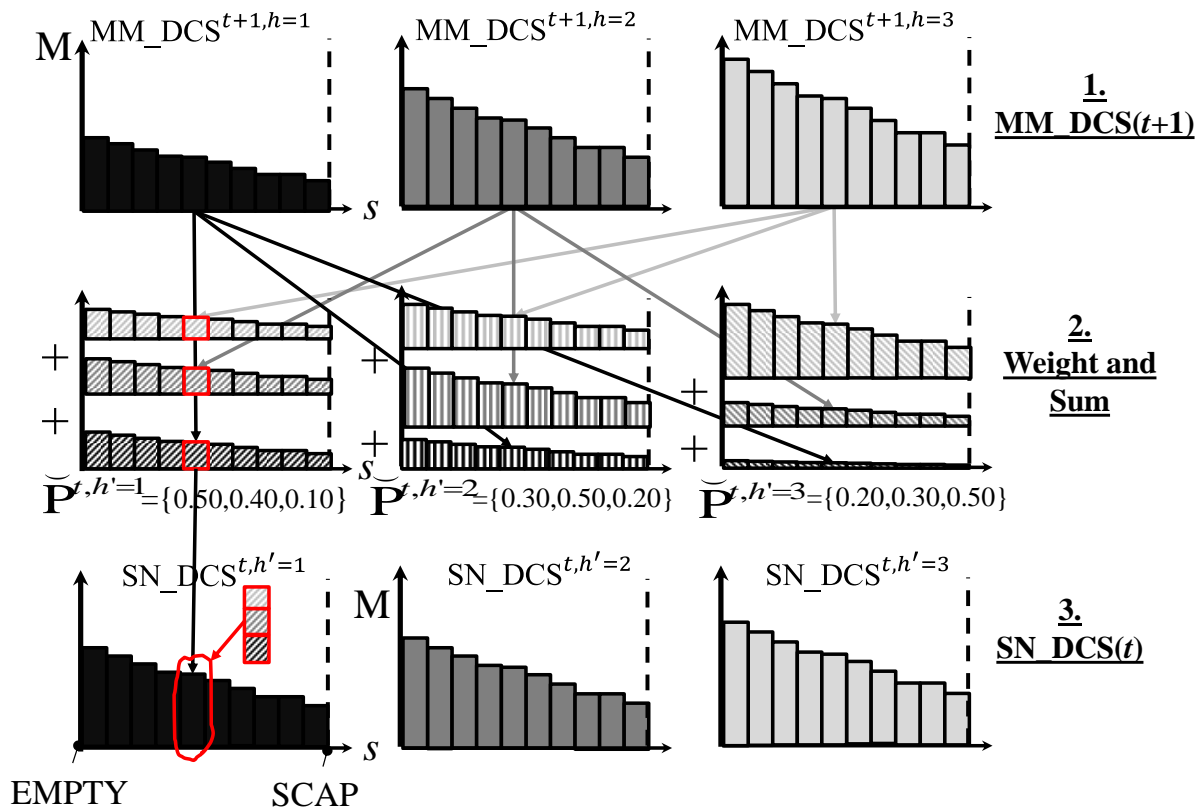


Figure 5.5: Illustration of SCDDP iteration: Weight and Sum Adjustments

⁷⁵ The beginning-of-period curves for $t+1$ are "conditional", because there is one for each state that might be observed in that period. But these end-of-period curves for t are also "conditional", because there is one for each state that might be observed in the period t .

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From Figure 5.5 above, the end of period Sunday Night DCS (SN_DCS) represents the marginal expected value of water stored at the end of the current period, for any hydrology state and feasible reservoir storage level, from empty to SCAP.

Figure 5.6 below illustrates the MS (Merge Sort) adjustment step. This creates the overall demand curve for water available at the beginning of the period, for each intra-period hydrology state. This is achieved by merge sorting the end-of-period (SN) DCS for that hydrology state, with the intra-period DCR for the same state. The processing for Figure 5.6 occurs left to right, and the hydrology states are represented from top to bottom, for progressively drier hydrology states.

- The LHS shows what is already known for this step: The $SN_DCS^{t,h'}$ produced above, and the Conditional Demand Curves for Release for this period, $CDCR^{t,h'}$. Recall that the SN_DCS contains marginal water values for storage values from empty to SCAP. The CDCR is shown to the right of SCAP, and contains the marginal value of release up to the utilisable release capacity, RCAP. In this example the CDCRs are state-conditional, where the top (bottom) left illustration represents the driest (wettest) state, so water is valued least (most) when it is most (least) available. Taller bars indicate higher marginal values.
- The RHS shows what is created by this step. For each hydrology state h' there is a Merge Sorted (MS) intra-period Conditional Demand Curve for Water ($CDCW_{MS}$), and an associated CORS is constructed. In this informed policy example each $CDCW_{MS}$ can utilise water (u) from 0, to $SCAP+RCAP$. Note that beneath each $CDCW_{MS}$ is its associated cumulative CORS.

The “Merge Sort” process, introduced above, ensures that all demands to store and release water have been considered, and ordered economically, from most to least valuable use. The CORS is easily calculated by assigning each unit stored a zero and each unit released a one. The zeros and ones are sorted with the corresponding value. The CORS is then the cumulative sum of these zeroes and ones.

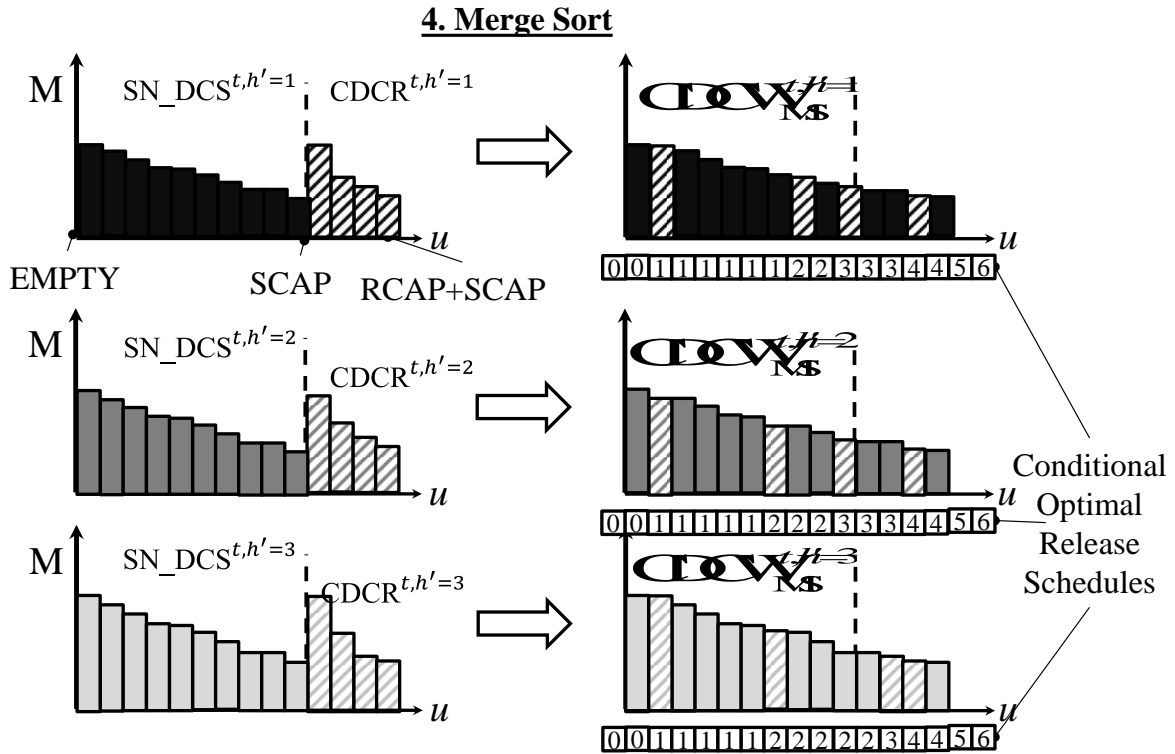


Figure 5.6: Illustration of SCDDP iteration: Merge Sort Adjustment.

In Figure 5.6 when units 5 and 6 are released they are valued at a marginal price of zero, given these units cannot be utilised, but are also not penalised.

Following on from Figure 5.6, Figure 5.7 below illustrates the final steps within the adjustment cycle. We interpret the CDCW as the combined demand curve for any available water. In step 5 we account for the inflow, by shifting each intra-period $CDCW_{MS}$ to the left, by its corresponding conditional net inflow $F^{t, h(t)}$ to create a Flow Shifted (FS) $CDCW_{FS}$. This flow shifting represents using the inflow first to meet the combined demand. $CDCW_{FS}$ is then truncated to the storage capacity bounds to only record the values for feasible storage levels, as shown below.⁷⁶ This forms MM_DCS . These two steps adjust the total utilisable water, u , to $SCAP+F$.

The resulting MM_DCS represents the value of water in storage at the start of a week.

⁷⁶ We actually record $SCAP+2$ MWV values (“slots”), from 0 up to $SCAP+1$. While we trialled averaging values for the increments to the left and right, we did not finally implement this, given the specific structure of our experiments (as per the discussions in Sections 5.4.3 and 5.4.4 above). We actually observe storage levels up to $SCAP+1$ ($SCAP+2$ “slots”), this “informed” policy also allows us to utilise incoming flows, so the combined $CDCW$ curve typically includes valid non-zero MWVs for aggregate water units above $SCAP+2$ (“slots”, or $SCAP + 1$ level i.e. spilling), and the MWV for water unit $SCAP+1$ (“slots”), is needed to form a valid MWV at a storage level of $SCAP$.

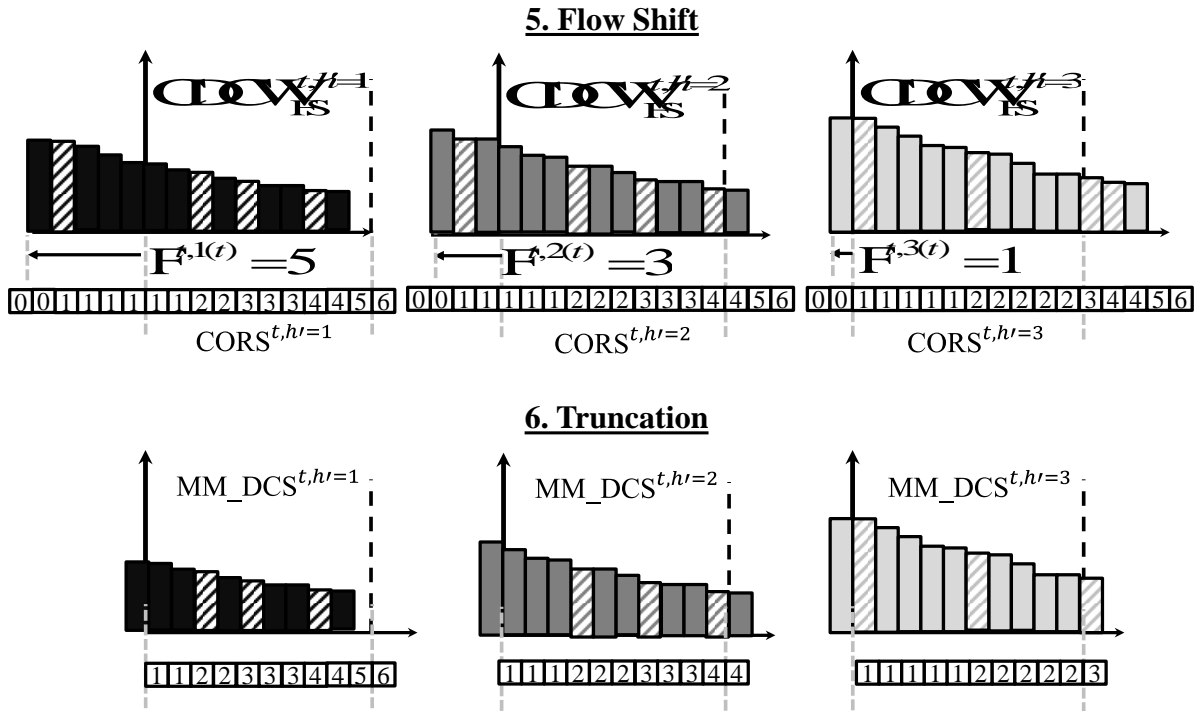
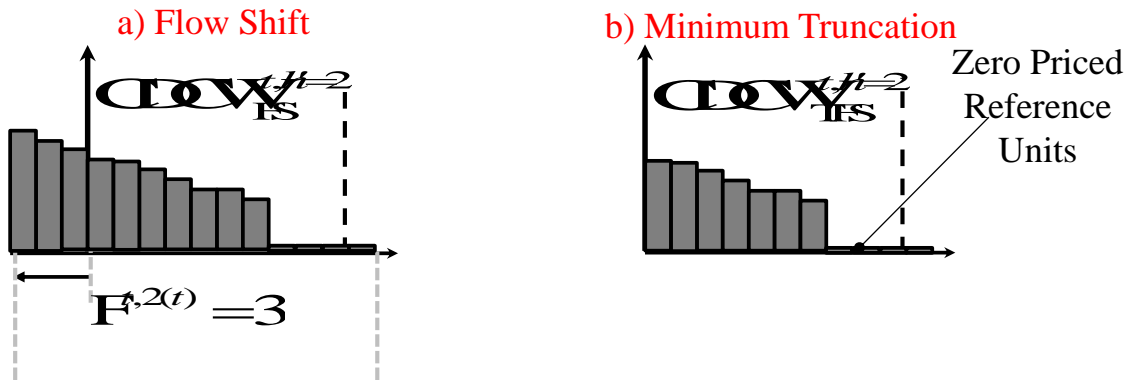


Figure 5.7: Illustration of SCDDP iteration: Flow Shift Adjustment.

For a conservative inflow allocation policy the inflow occurs after the release decision. This means steps 4 and 5 would be reversed. Under this policy the inflow (flow shift) is only applied to storage (solid bar values) at the end of the period, $SN_DCS^{t,h(t)}$. Hence release decisions are made based on beginning of period storage only. There are a few minor complexities compared to the informed inflow allocation policy. In the conservative case the two ends of the intra-period CDCWs are truncated at different times. The revised steps 4 and 5, namely 4' and 5', are illustrated in Figure 5.8 below. The example only shows the procedure for the middle ($h=2$) hydrology state.

4': Truncated Flow Shift



5': Truncated Merge Sort

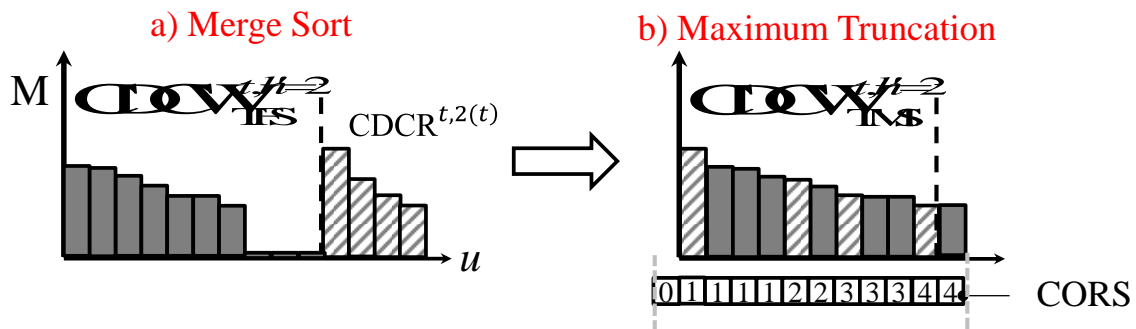


Figure 5.8: Illustration of SCDDP iteration: Conservative Flow Shift and Merge Sort Adjustments.

So, from Figure 5.8 above, the top-left-hand-side step 4'a illustrates flow shifting the $SN_DCS^{t,2(t)}$ curve to produce the corresponding $CDCW_{FS}$ curve. The top-right-hand-side graphic truncates the lower bound after this “left shift”. This creates a Truncated Flow Shifted (TFS) $CDCW_{TFS}$ curve, and this is step 4'b of the adjustment procedure. In other words, the $CDCW_{FS}$ values less than zero storage capacity are met by inflow at that point in the intra-period. Only now can step 5' be applied to merge sort the CDCR for this period, with the associated $CDCW_{TFS}$. This is shown in the bottom-left hand side graphic, step 5'a. The bottom-right-hand-side graphic illustrates step 5'b of the adjustment procedure. Here, after the merge sort, the resulting curve is truncated to within the upper storage capacity limit. Together, the level 5' adjustments produce the Truncated Merge Sorted (TMS), $CDCW_{TMS}$, where the truncation step only applies to the upper storage bound, SCAP.

For a SI case, the process is identical, but the intra-period Markov Probability weights $P^{t+1, h(t)}$ in step 2 are now the same for all intra-period hydrology states. A DC algorithm could be

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stated, and implemented, much more simply. This is not done here. All cases (MC, SI, and DC) were run with the same algorithm, using the appropriate probability weight data and the same inflow data, see discussion in Section 5.4.3 for more details.

Above we described the process for a generic period t . The algorithm is initiated from the End of Horizon (EOH) (T+1) MWV curve (or list). The backward passing RO algorithm always starts from some EOH MWV curve, working backward to construct the optimal DCS for all stages, and hydrology states. The EOH values we use come from running our model to equilibrium. Hence case runs are compared consistently in their steady-state.⁷⁷ Through experimentation we found that initially setting the EOH values to 1 (the reference price, MREF), generally led to much quicker convergence to equilibrium, especially with relatively unconstrained systems.

In this section we described a modelling system which constructs intra-period MWV lists (DCWs), and start and end of period DCSs', for each consecutive unit of water stored in the system up to SCAP.⁷⁸ This is made possible by combining the final period DCR, with the EOH MWV function. Then for all prior periods, the resulting DCS is combined with the prior period DCR. When the CDCS and CDCR are combined a corresponding CORS is also generated, for each possible utilisable water unit in the system. CORS, the Conditional Optimum Release Schedules, are an important input into the RC simulation module.

5.4.6 Discrete Convolution Simulation

Assuming stationary demand, and inflow distributions, the RC module forms a Markov chain, with a very large state-space. The forward passing RC algorithm computes the steady state probabilities of this Markov chain to allow cases to be directly compared in their long-run. It commences with a Start of Horizon (SOH) probability distribution. The algorithm iterates forwards, constructing the PDF for Storage, the PDFS, for all stages and hydrology states. The PDFS defines the likelihood of the system being in each storage/ hydrology state, at each stage. The RC algorithm is iterated year-on-year to find the steady state probabilities

⁷⁷ Assuming the start of the next water year has the same set of values. This assumption aligns with running those sets of demand data equations to their equilibrium.

⁷⁸ Available at the start of each decision period, and state, for storage and release decisions, within that period.

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for each hydrology and storage state, in all stages. This iterative approach to finding the steady state is just one method for computing the long-run probabilities.

Our description of the algorithm focuses on a general iteration from period t to $t+1$. As discussed in Section 5.4.3 above, the inflow probability distribution data used by RC could differ from the distribution used for RO. But, for these experiments we assume that the distributions are consistent. The RC module iteratively constructs the PDFS using the same hydrology data, hence the same set of inflows and, if run at the same level of stochastic complexity as the optimisation, the same probability values. It also utilises the CORS output from the RO module.

We describe the PDFS construction for a Markov chain level of uncertainty. PDFS^{t+1} is constructed from PDFS^t using the hydrological state realisations and the associated release decisions. The order in which the inflow probability distribution and release decisions are accounted for depends on the inflow allocation policy. First we discuss the informed, then the conservative, inflow allocation policies, respectively.

From the RO module we have the Conditional Optimum Release Schedule, $\text{CORS}^{t,h}(s^{t-1,h})$. At the beginning of period t , starting from any feasible storage level (state), s^{t-1} , we probabilistically experience any of the inflow states (going from hydrology state h' to h). Together, t,h and s^{t-1} imply an associated optimum release, and this implies a transition to a new storage state, s^t , for the end of period t , such that we define:

$$s^{t,h} = s^{t-1,h'} + F^{t,h} - \text{CORS}^{t,h}(s^{t-1,h}) \quad (5.14)$$

The RO module has already determined CORS so as to ensure that the storage state stays within feasible bounds. The CORS is adjusted to ensure a minimum storage of 0, and a maximum of SMAX. As such the end of period storage level, for any cases involving spill, will at most be the maximum SCAP.

We define PDFS to mean the probability that storage is in state s^t and the hydrology is in state h . The probability that we start in state $s^{t-1,h'}$, and transition to $s^{t,h}$ as given by Equation (5.14) is given by:

$$\text{PDFS}^{t-1,h'}(s^{t-1,h'}) \times \tilde{\text{P}}^{t-1,h',h} \quad (5.15)$$

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These joint probabilities are accumulated to the transitioned state. We write $s^{t-1,h'} \rightarrow s^{t,h}$ to indicate that storage state $s^{t-1,h'}$ transitions to state $s^{t,h}$ via (5.14). Then, the aggregate probability that we end in hydrology state h is:

$$\text{PDFS}^{t,h}(s^{t,h}) = \sum_{h', s^{t-1,h'} \rightarrow s^{t,h}} \text{PDFS}^{t-1,h'}(s^{t,h} - F^{t,h} + \text{CORS}^{t,h}(s^{t-1,h'})) \times \check{\text{P}}^{t-1,h',h} \quad (5.16)$$

Equation (5.16) defines PDFS in the next period (t) at the starting storage level (s^t).

In the first period the forward convolution commences with any set of SOH PDFS probabilities, which sum to 1 across all conditional hydrology states. The algorithm then iterates forward applying equation (5.15) to create joint probability elements which are collected to form the PDFS for the next period, according to equation (5.16).

The forward passing algorithm is easily iterated to the steady state probability. Once the PDFS has iterated to the modelled EOH those Conditional EOH probability weights are used as the respective Conditional SOH probability weights. This process is iterated (looped) until there is some agreed margin of error between EOH and SOH probability values. An example is given in Appendix 5.

When a storage state finds itself transitioning into a situation requiring spill, via Equation (5.14), the probability of that spill is recorded via Equation (5.16). Thus one of the “storage states” is reserved for spill.

The DC case is just a simplification of the SI case, where there is one inflow value with a weight of 1. Hence there is no PDFS and there is no recombining, just a single trajectory with a single inflow and release value. The SI case is also a simplification of the MC case.

The Conditional PDFS output from the RC module can be time consuming to run for cases with a large storage state space. The PDFS cannot be readily used to determine all outputs of interest. For example, it cannot easily be used to determine the distribution of annual returns as different cumulative returns may have been generated on different ‘paths’ to the same storage state in any period. A key use of the RC PDFS output is as a checking, and configuration, tool to ensure that the RS module is performing accurately.

5.4.7 Monte Carlo Simulation

The RS module employs a Monte Carlo simulation. The algorithm increments for each sample and decision stage simulating an intra-period inflow. A sampled inflow is used to look up an associated release, as discussed below. The sampled inflow and release are used to update the storage level for each consecutive decision period, within that given sample.

To initiate the algorithm in the first period, of the first sample, the simulation has to pick a starting storage level, and this can be achieved in many ways. The alternatives we explored were either sampling the level from:

1. The cumulative equilibrium PDFS⁷⁹ for $t = 1$, as output from the RC module described above, or
2. A random number between 0-1 which is aligned (and rounded, to snap to a grid point) with the discrete storage grid for a given case run,⁸⁰ or
3. A consistent, known, starting storage level each time, for instance from the tank: Empty, Half-Full, or Full.

Implementing (1) means running the simulation for a single start-up year, and this sample is then discarded to eliminate any minor bias. Implementing (2) or (3) means running the simulation for a number of start-up sample years, and these samples are then discarded to eliminate bias. After trialling all these options we opted for (3), and also found starting the tank from empty was adequate.

We also had to determine a starting hydrology state (h'), and given we discarded the first samples, and we use a symmetric hydrology distribution with equal probabilities away from the mean value, we decided to start in the mean hydrology state.

At the start of each decision period, the intra-period random hydrology state value is generated and the corresponding inflow state inferred, $F^{t,h}$. With an informed inflow allocation policy the intra-period hydrology state (h), which is transitioned to from the prior hydrology state (h'), determines which of the CORS lists to look at, i.e. the h^{th} .

⁷⁹ Using pseudo random numbers for a PDFS with states with extremely small probabilities could cause issues. Using infinite precision arithmetic, and genuinely random numbers, the PDFS method should produce unbiased starting states. Finite precision pseudo random numbers may mean some states are not sampled often enough.

⁸⁰ We tried this with the pre-designed PDFS and with equally likely storage states.

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Alternatively, with a conservative inflow allocation policy the h' list is used. With either inflow allocation policy the optimisation has already incorporated the correct decision into generating the CORS. Thus the Monte Carlo simulation only needs to use the sampled start of period storage level to “look-up” the associated sampled intra-period release value, from the corresponding CORS ^{t,h} ($s^{t-1,h}$).

At the end of each decision period the algorithm updates the storage level to account for inflow and release, while also truncating for the upper and lower bounds, and in those instances recording empty and spill events, as required. The algorithm then iterates the sample for each consecutive decision period, until the algorithm reaches the EOH.

At the EOH the next sample is taken. At this point, the run samples the EOH final period finishing storage level, and transitions into a new hydrology state. Both are used to initiate the next sample’s SOH starting storage level and hydrology state. Using the finishing state for the previous sample as the starting state for current/next sample, in this way, could lead to unexpected correlation biases.

After extensive testing a range of values between 100 to 1,000,000 samples, we found 10,000 samples, gave reasonably efficiently computed results, comparable to both increasing the number of samples taken, and also when compared to the RC outputs.

During extensive trialling we also took advantage of Matlab’s random seed function. This ensured that DC, SI and MC case runs used the same random seed for all case runs, and this also further reduced experimental noise.

We found further issues with benefit results, when experimenting with correlated inflow and (state-dependent) demand, as discussed in Section 8.4. In part due to the random seed selected, we noticed that the Monte-Carlo samples were slightly bias with the drier than average hydrology states being sampled less frequently than otherwise expected (compared to the long-run equilibrium values). To reduce experimental noise further we used antithetic sampling of the intra-period hydrology state, to record two values (derived from symmetric random numbers, a and $1 - a$), and thus produce two 10,000 case samples. Antithetic sampling, introduced by Hammersley & Morton (1956), can dramatically reduce the standard error from sampling, improving the estimates produced. The sampled hydrology, storage and

release states were then used to look-up two corresponding 10,000 price and benefit samples, from which two lots of other sets of measures, such as volatility measures, were computed. Finally, we averaged the resulting two sets of antithetic price and benefit outputs, prior to final reporting.

We use subscript RS to define an output from the RS module, and superscript n when it is indexed by sample. The output sample values can be used to generate market reporting measures, and we start this discussion by focusing on benefits.

5.5 Average Benefit Reporting

The purpose of this section is to define a range of benefit measures that we report for subsequent experiments. We discuss how to transform outputs from the main computational modelling system into total benefit measures. This is followed by discussing the minimum, and then maximum, limiting total benefits, for various case options. After this, we bring all these ideas together to discuss relative case performance. And the output of this is a description of three key benefit reports, all of which are proportional, relative to various total benefits, for various case runs.

5.5.1 Computing Benefits for the Reservoir Storage System

In our reservoir modelling system, all the inflow passes through the capacity-constrained reservoir, and is released to meet demand (or spilled). Benefits accumulate from releasing water. Where feasible, the reservoir manager attempts to maximise benefit over time, by storing the inflow until times when water is most valued. In general, the first few units of water are highly valuable. The total net welfare is theoretically infinite. With a constant elasticity curve when shortage occurs the price goes to infinity. However, we avoid this issue by only pricing tradable steps. Hence, the benefit values we compute are always truncated net welfare values.

In Chapter 3, we discuss maximising net welfare as the difference between consumer benefits and supply costs. However, in these simplified experiments we use a net demand function, with freely harvested net inflow as the only source of supply. As in Section 3.6, we denote the benefit function by B . In Section 3.6 we also defined the derivative of this B curve as B' . B' is the marginal price (M) element of the DCR, in our market based allocation system. The

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bid pairs $(M_k^{t,h}, Q_k^{t,h})$ form the DCR, for each stage and state, t and h . Benefit, B , is computed as total net welfare, where the optimisation simply maximises:

$$B^{t,h}(q) = \sum_k M_k^{t,h} q_k^{t,h} \quad \text{for all } t, h \quad (5.17)$$

Subject to:

$$q_k^{t,h} \leq Q_k^{t,h} \quad \text{for all } t, h \text{ and } k \quad (5.18)$$

In our modelling system, each k bid tranche has a width of 1. So, in theory, each quantity q in (5.18) is between 0 and 1. In our experiments, though, we set up the data in such a way that each quantity step is either wholly met, or not met at all. Hence, $B^{t,h}(q)$ is generated by accumulating each consecutive marginal price (M) step in the DCR ^{t,h} (q), for any t and h . Having defined $B^{t,h}(q)$, for each t and h , we can sum those values to calculate the Total Benefit, TB, over all t and h . The exact procedure for computing TB depends on which module's outputs we employ.

Indeed, the RC module did not only compute the PDF for Storage (PDFS), see Section 5.4.6, and Equation (5.16) for further details. Within this module we also record a TB (CORST ^{t,h}) value. These values are accumulated for each storage level, similar to PDFS. As a pseudo-equation this is implemented as:

$$TB^{t,h}(s) \rightarrow TB^{t,h}(s) + B^{t,h} \left(\text{CORST}^{t,h}(s') \times \text{PDFS}^{t,h'}(s'') \times \check{P}^{t,h',h} \right) \quad (5.19)$$

In ‘‘Equation’’ (5.19) each state (s, t, h) within the state space, could probabilistically be reached from more or one starting point. Various beginning of period starting storage levels (s'), combined with an intra-period hydrology state, h , could accumulate TB to the resulting point (s, t, h) . Hence the ‘‘TB ^{t,h} (s)’’ term features on the LHS and RHS, to allow each ‘‘bin’’ to be updated, to accrue further benefits, as required.

The computation for the (Expected) TB from the RC module uses intermediate calculations by storage level and stage. In a given state, (s,t) , the quantity released is given by the CORS ($q = \text{CORST}^{t,h}(s)$). We also use the PDF for Storage to weight that conditional states benefit (PDFS ^{t,h} (s)) accordingly. The following equation computes the RC TB, for a particular storage level and period:

$$TB_{\text{RC}}^t(s) = \sum_h \text{PDFS}^{t,h}(s) \times B^{t,h}(\text{CORST}^{t,h}(s)) \quad \text{for all } s \text{ and } t \quad (5.20)$$

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Using Equation (5.20) will produce a $T \times \text{SCAP}$ matrix of Convolution Expected values, given $t = 1, \dots, T$, and $s = 0, \dots, \text{SCAP}$. In effect it weights all h states by their associated probability of occurrence.

With $\text{TB}_{\text{RC}}^t(s)$, we aggregate by s to produce TB values for each t , via:

$$\text{TB}_{\text{RC}}^t = \sum_s \text{TB}_{\text{RC}}^t(s) \quad (5.21)$$

Further aggregating TB_{RC}^t by t results in a single annual TB_{RC} , as estimated by the RC module, via:

$$\text{TB}_{\text{RC}} = \sum_t \text{TB}_{\text{RC}}^t \quad (5.22)$$

In general though, we compute our reporting measures using outputs from the RS module, because it is more computationally efficient. In this setting, we compute reporting measures from the RS Module, and back this up with RC module outputs, and associated reports, to validate our initial findings, where necessary.

The RS module discretely samples the entire s and h state space via Monte-Carlo sampling. This produces a vector of $n = 1$ to N samples, for each t .⁸¹ We compute benefits for the intra-period sampled hydrology state, $h(t,n)$, and sampled release, $q^{t,n}$, via the (Sampled) TB:

$$\text{TB}_{\text{RS}}^{t,n} = \mathbf{B}^{t,h(t,n)}(q^{t,n}) \quad (5.23)$$

Using Equation (5.23) produces an $N \times T$ matrix of values. These produce a TB for each t via:

$$\text{TB}_{\text{RS}}^t = \frac{\sum \text{TB}_{\text{RS}}^{t,n}}{N} \quad (5.24)$$

Equation (5.24) gives total benefits per stage as estimated by the RS module, TB_{RS}^t , and these can be compared to Equation (5.20), which is the estimated total benefits per stage from the RC Module, TB_{RC}^t .

Aggregating Equation (5.24) by t produces a single annual TB, as estimated by the RS module, via:

$$\text{TB}_{\text{RS}} = \sum_t \text{TB}_{\text{RS}}^t \quad (5.25)$$

⁸¹ Generally $n=10,000$.

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Equation (5.25) results in a single total annual benefit measure as estimated by the RS Module outputs, TB_{RS} , and this can be compared to Equation (5.22) as estimated by the RC Module outputs, TB_{RC} .

Alternatively, summing $TB_{RS}^{t,n}$ from Equation (5.23) by t , gives an unsorted list of annually sampled total benefits, via:

$$TB_{RS}^n = \sum_t TB_{RS}^{t,n} \quad (5.26)$$

Sorting this list and plotting against the cumulative probability of each sample produces an annual benefit Cumulative Distribution Function (CDF).

In addition, several of these TB_{RS}^n measures, and their aggregations, are used to compute volatility measures, as discussed in Section 5.7 below.

5.5.2 Bounds on Benefits

Figure 5.9 gives a 2 dimensional representation of various bounds on the benefits available from our system, as used in the remainder of this thesis, and how they vary with varying RCAP and SCAP.

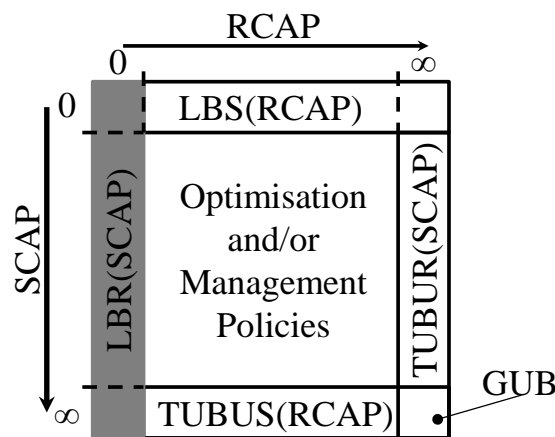


Figure 5.9: Illustration of Storage System Benefit Bounds

With $RCAP = 0$ there is no market, and the Lower bound of total Benefit with no Release LBR, irrespective of SCAP, is always 0. With $SCAP = 0$ there is no reservoir to store water, but there is still the Run-Of-River (ROR) flows which can be immediately used by the

market. This ROR benefit provides the Lower bound of Total Benefit with no Storage LBS, as a function of RCAP. With $RCAP = \infty$, then any size of inflow can be released to the market.⁸² This allows us to determine a Tight Upper Bound with Unconstrained Release, TUBUR, as a function of SCAP. With $SCAP = \infty$, the reservoir can store an infinite amount of water, and long-run equilibrium benefits are maximised, for a given level of RCAP.⁸³ This allows us to determine a Tight Upper Bound with Unconstrained Storage, TUBUS, as a function of RCAP. With $RCAP = \infty$, and $SCAP = \infty$, we can store and release an infinite amount of water, this allows us to compute the Global Upper Bound (GUB) on benefits, as shown in the bottom right corner of the figure. Between all these SCAP and RCAP extremes, we try to maximise benefits via optimisation.

5.5.3 Run-of-River Lower Bound Benefit

There is a large benefit from simply trading the net inflow, as it arrives. The ROR case assumes no storage reservoir ($SCAP = 0$), and it provides an LBS for our system. This is not really a reporting measure by itself, but it is a lower bound which we use as a baseline to form other measures.

We could assume that the river has an unrestricted release capacity ($RCAP = \infty$), but that is not helpful, because it does not provide a lower bound on benefits. The ROR case can, at best, only release up to the inflow in the river. But to provide a comparable measure, the amount of river flow that can be diverted to the market, must be constrained to the release capacity of the reservoir, RCAP, in the case being reported. This also implies that ROR benefits are a function of RCAP.

Without running any optimisation or simulation we could use the flow data, $F^{t,h}$, output from the HD module (see Equation 5.7), to compute the discrete ROR benefits for each h and t . The ROR lower bound has the release as the minimum of the release capacity or the river flow, hence $q = F^{t,h}$, or $q = RCAP$, whichever is lower. Then we multiply each resulting TB value by its associated steady-state long-run probability ($\tilde{P}^{t,h}$), as discussed in Section 5.3. Hence, the ROR LBS is:

⁸² But we can determine a finite RCAP level which does not constrain any desirable release level. In our case this is the maximum of the observed inflow distribution, FMAX.

⁸³ And we can determine a finite SCAP level which does not constrain any desirable storage pattern, and thus benefits from the market are maximised for that system (i.e. patterns of inflow and demand).

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$$\text{LBS}'(\text{RCAP}) = \sum_h \tilde{\text{P}}^{t,h} \times \text{B}^{t,h}(\min\{\text{RCAP}, \text{F}^{t,h}\}) \quad (5.27)$$

Equation (5.27) is similar to Equation (5.35) discussed below in Section 5.5.4, where the release capacity can be incremented up to the maximum monthly average inflow, across all h and t . Equation (5.27) produces a t vector of values, and these can be simply aggregated to produce an average annual total expected benefit function, which is also the LBS, for increasing RCAP in the ROR case.

The problem with this measure, though, is that we can not readily form a corresponding annual CDF. Before multiplying by $\tilde{\text{P}}^{t,h}$ and aggregating by h , looking-up the extreme $\text{B}^{t,h}(\text{F}^{t,h})$ values produces bounds on the total ROR benefit CDF. More exactly, combining the driest hydrology (and inflow) states, for all t , produces a lower bound on total ROR benefit CDF. Combining the wettest hydrology (and inflow) states, for all t , produces an upper bound on total ROR benefit CDF. However, the intermediate distribution of value is not known in this case, without computing all paths. With a monthly time step, and 7 hydrology states, we did not to compute the resultant 12^7 paths.

Since we employ optimisation and simulation, though, we can also compute benefits for the ROR system by a process similar to that in Equation (5.23) above. However, here we replace the sampled release with the sampled inflow value. To do this, the intra-period hydrology state h is aligned with its associated inflow $\text{F}^{t,h}$, where $h = h(t,n)$. Hence, the ROR TB, is:

$$\text{TB}_{\text{ROR}}^{t,n} = \text{B}^{t,h(t,n)}(\text{F}^{t,h(t,n)}) \quad (5.28)$$

Equation (5.28) can be treated in exactly the same manner as (5.23); in so much as it can be transformed into an annual CDF, via Equation (5.26). Or the ROR TB can be processed via Equations (5.24) and (5.25), for monthly and annual values, as shown below.

$$\text{TB}_{\text{ROR}}^t = \frac{\sum \text{TB}_{\text{ROR}}^{t,n}}{N} \quad (5.29)$$

$$\text{TB}_{\text{ROR}} = \sum_t \text{TB}_{\text{ROR}}^t \quad (5.30)$$

The annual TB_{ROR} is a function of release capacity, and with a sufficiently large sample it is the LBS.

5.5.4 Unconstrained Storage Upper Bound Benefit

This thesis is concerned with assessing the value of stochastic optimisation, and that value is entirely driven by the benefit derived by improving the management of stored water. The optimisation employs the continuity equation to manage storage and release over time, such that the value of trade is maximised. Release capacity is not the driving force in the optimisation, but does imply a constraint on the value of trade. In this section we are looking for TUBUS, which is the total benefit that could be derived from storage, if infinitely expanded and optimally managed, for a given level of release capacity. We call this TUBUS(RCAP), but we also define a weaker GUB, which applies over all storage and release capacity combinations.

Assuming an infinite storage capacity means that the pattern of inflow is not important. Infinite storage means we can always move water arriving according to any pattern of inflows to meet any pattern of demand. This is, of course, assuming that the average total annual inflow (TF) is equal to the average total annual release (TQ), as follows:

$$TF = TQ \tag{5.31}$$

We also let q^* be the release for an optimal equilibrium solution. In this setting, we examine the change in TUBUS when increasing RCAP, and thus the impact on maximising benefits for a given set of market DCRs. Since the (truncated) curves we use to represent demand have bounded integrals there will be a GUB on total benefits, irrespective of RCAP. We define GUB as the limit of TUBUS as RCAP increases. As RCAP increases, TUBUS(RCAP) rises until it eventually reaches GUB, and RCAP reaches some level, $RCAP^*$. There is no point in RCAP going beyond $RCAP^*$ because, by construction, $RCAP^*$ is the level beyond which a release capacity constraint will not be binding across all $t \times h$ DCR lists. At $RCAP^*$ the optimal solution will have the property that this is the maximum of all total benefits that can be returned for this system, with a given hydrological distribution, a given set of market demand curves, and the fact that in equilibrium Equation (5.31) must hold.

By construction of the demand, computing GUB is somewhat trivial, compared to TUBUS. The GUB case has a truly unrestricted storage and release capacity. These capacities need to be large enough that the optimisation is able to store, and release, water without ever being constrained.

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The “conditional” GUB, for each t and h , is stated in a similar form to the ROR case, via:

$$\text{GUB}^{t,h} = \text{B}^{t,h}(q^{*,t,h}) \quad (5.32)$$

Of course the process of finding $q^{*,t,h}$ in our experiments is specific, and simple, given that we construct the DCRs such that they go through a price point of \$1.00, on average, (as discussed in Section 5.4.4), hence $q^{*,t,h} = \text{QREF}^t$ where $\text{DCR}^{t,h}(\text{QREF}^t) = 1$.⁸⁴

It is straightforward to ‘prove’ our assertion. First using $q^{*,t,h} = \text{QREF}^t$ meets the assumption in Equation (5.32) due to Equation (5.9), $\frac{1}{T} \sum_t \text{QREF}^t = \text{FMEAN}$. This means that using 1 unit more in one stage means using 1 unit less in another stage. In the stage where release is reduced by 1 the benefit reduces by exactly 1. But for the other stage $\text{DCR}(\text{QREF}+1) < 1$ as the marginal price is decreasing (see Figure 5.4 above), so there will be less benefit and the new solution will be worse.

Thus for each t and h DCR list we simply find the corresponding marginal price point of 1. We then use this set of $q^{*,t,h}$ s’ to look up the cumulative benefits, in each h and t , from their corresponding $\text{B}^{t,h}$ lists.⁸⁵ The conditional GUB in (5.32) needs to be turned into an expected GUB. To do this, we multiply each $\text{GUB}^{t,h}$ value by its associated steady-state long-run probability ($\tilde{\text{P}}^{t,h}$). Hence, the GUB, for each t , is:

$$\text{GUB}^t = \sum_h \tilde{\text{P}}^{t,h} \times \text{GUB}^{t,h} \quad (5.33)$$

But we are really more interested in the annual GUB, and this is just the aggregate of GUB^t , over all t , as follows:

$$\text{GUB} = \sum_t \text{GUB}^t \quad (5.34)$$

Having outlined the process to compute the weak GUB, we now discuss how we compute the entire set of TUBUSs, up until the point we find the corresponding maximum TUBUS^U , $\text{TUBUS}^U = \text{GUB}$. First, we describe the algorithm for the simplest case, where $\text{RCAP} \leq \text{FMEAN}$.

⁸⁴ There are on average 84 units per month, and 1008, per annum.

⁸⁵ A simple function traverses each DCR step, over all t and h , recording all the q points when $M = 1$. Then we use $\text{B}^{t,h}(q)$ for all the recorded q points to accrue the associated TBs. With non-state dependent demand, we only need to look up the average h state values, over all t . However, with state dependent demand, we need to process all t and h . In this case, the resulting values in each h , and over all t , need to be weighted by their associated steady state probability values.

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If the release capacity is less than, or equal to, the monthly average inflow, in each decision stage, the best choice is to release up to that capacity, in all t and h , spilling all the rest. So we compute the first lower bound $TUBUS^L$, via:

$$TUBUS^L(RCAP \leq FMEAN) = \sum_{h,t} \tilde{P}^{t,h} \times B^{t,h}(RCAP) \quad (5.35)$$

With Equation (5.35), we can generate the total benefit function for increasing RCAP by simply iterating from 1 to FMEAN and collecting the total benefits from the cumulative DCR.

From the point where $RCAP = FMEAN$, increasing release in one period requires us to decrease release in another. We now gradually increase RCAP by one unit at a time. With each unit increase in RCAP, we search for all possible arbitrage opportunities: That is for profitable de-allocation/ re-allocation pairs. So, extending RCAP beyond FMEAN means we need to adopt a different computational strategy.

For simplicity of explanation, we now assume that demand is non-state dependent, and hence the same prices appear in each h list. With no storage restrictions, therefore, the same release decision will be optimal in each period regardless of the inflow (and hydrology state). So we can equivalently compute TUBUS for a single h . But the process could be extended to the general non-state dependent case, and it may be possible to adapt for the state-dependent case.

Using a bottom-up type approach, the t DCR lists represent a grid of marginal values, from the first (most valuable) potentially tradable step, to the last (least valuable) potentially tradable step. We also have a release index, for each t , indicating the current release schedule, such that:

$$1 \leq r(t) \leq RCAP \quad (5.36)$$

The absolute largest RCAP can get while increasing TUBUS is TF. This is because there might be one stage with a minimum marginal price which is greater than the largest marginal price in all other stages.

For $RCAP = FMEAN + 1$ we begin with $r(t)=FMEAN$. Then, with t DCR lists, for each consecutive additional unit of release capacity, we compare the maximum incremental

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marginal price to the minimum (feasible) decremental marginal price and adjust the release indexes accordingly.

In the first increment price vectors are taken from the corresponding Incremental and Decremental DCR marginal price Vectors: $IV(t) = DCR^t(t, r(t) + 1)$ and $DV(t) = DCR^t(t, r(t))$, respectively. And at most, there are T items in each of those vectors. Now we identify profitable swaps which increase the total benefit. An arbitrage opportunity exists if $\min\{DV(t)\} < \max\{IV(t)\}$. If this is the case, we allocate release to the maximal period, t^{inc} , and reduce release from the minimal period, t^{dec} . Note that $t^{inc} \neq t^{dec}$, as the DCR is non-decreasing and $IV(t^{inc}) > DV(t^{dec})$. At this point, updates to IV and DV are as follows:

- For the allocated step, $\max\{IV(t)\}$, the associated $r(t^{inc})$ is incremented by one, $r(t^{inc}) = r(t^{inc}) + 1$, then the marginal price point is also updated, $IV(t^{inc}) = DCR^t(t, r(t^{inc}))$, using the new $r(t^{inc})$. But until the release capacity unit increments forward a further unit, period t^{inc} cannot be compared for subsequent arbitrage opportunities at this release capacity level. We use a flag to ensure that there is only one incremental reallocation in each period.⁸⁶ We also update the same stage's decremental marginal price with the old IV(t) value: $DV(t^{inc}) = DCR^t(t, r(t^{inc}) - 1)$, using the new $r(t^{inc})$ given it has already been updated.
- For the de-allocated step, $\min\{DV(t)\}$, the associated $r(t^{dec})$ is decremented by one, $r(t^{dec}) = r(t^{dec}) - 1$, and then its marginal price point is also updated, $DV(t) = DCR^t(t, r(t^{dec}))$, using the new $r(t^{dec})$. Unlike IV, DV updates can be compared for subsequent arbitrage opportunities at this release capacity level. This is because the new price may well be lower than prices in the other periods. So, the same DV(t^{dec}) price vector might be updated multiple times within a given RCAP iteration.^{87,88} We also update the same stage's Incremental marginal price with the old DV(t) value: $IV(t^{dec}) = DCR^t(t, r(t^{dec}) + 1)$, using the new $r(t^{dec})$ given it has already been updated.

⁸⁶ $Flag(t) = 1$ when not yet potentially allocated and 0 when allocated. Thus $\max\{IV(t^{inc}) \times Flag(t)\}$ more correctly ensures we only use the IV(t^{inc}) step once for each iteration of $r(t^{inc})$, at which point $Flag(t)$ is reset to 1.

⁸⁷ So we can apply some logic here, such as if $r(t^{dec}) = 0$, the marginal price is $\max\{DCR\}$, this ensures that this step is not further de-allocated.

⁸⁸ And, if a de-allocated step goes below $r(t^{dec}) = 1$, then that whole period (and this is relevant maybe at the height of the wet season) is of very low economic value, no one really wants any water here, relative to the other periods.

Thus we can now repeat the process of comparing the maximum and minimum marginal price values until no more opportunities exist, $\min\{DV(t)\} \geq \max\{IV(t)\}$. At this point we increment forward by another unit of release capacity, starting with the $r(t)$ as for the previous RCAP level and repeating the above process. The whole process is repeated until increasing RCAP offers no more arbitrage opportunities, at which point the upper bound no longer increases as RCAP increases, with $TUBUS^U = GUB$.

GUB is a function of both the DCR and the inflow distribution.

5.5.5 Benefit Range

The DCR and hydrology data used in Chapter 6 is now used to show the sets of TUBUS, and the associated GUBs, for the storage system and the ROR LBS. Figure 5.10 below shows the total benefit and increases in release capacity on the horizontal and vertical axes, respectively.

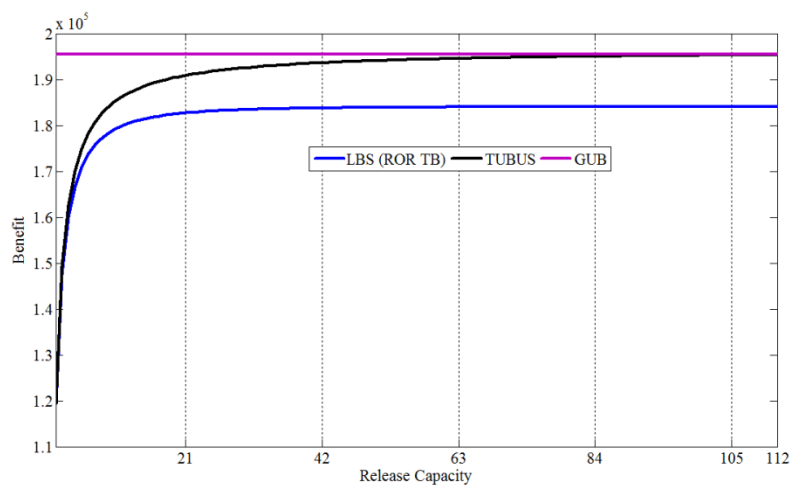


Figure 5.10: Illustration of Total Benefit Upper Bound Functions for Increasing Release Capacity for Storage and Run-of-River Systems

The blue LBS line generally returns much less benefit than the black TUBUS line, over the total RCAP range. With SCAP=0, benefits increase along the blue line as RCAP increases. With reasonably unconstrained storage capacity, and “perfect” optimisation, benefits increase along the black line as RCAP increases. Up to the average monthly inflow, $FMEAN = 84$, both the controlled (storage system) and uncontrolled (river) systems visibly return increasing TUBUS total benefits, relative to release capacity. The storage system hits its maximum,

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TUBUS^U, just short of twice FMEAN (RCAP = 160, which is also the maximum mean inflow for this case i.e. $\max\{F^{t,h}\}$). From this point onwards increases in release capacity provide constant benefit returns where the TB = 195443. The corresponding Time Weighted Average Benefit (TWAB, as discussed in Section 5.5.10 below) is 16287, and we generally report TWABs values this point forward. The ROR case continues to provide (minor) benefit improvements all the way to its maximum inflow value (309). More generally, as discussed in Chapter 7, once $RCAP \geq \max\{F^{t,h}\}$ the system can release any pattern of inflow to meet demand i.e. the system is unconstrained in terms of RCAP.

Finally, removing the expected ROR benefit from the TB produces the minimum “net” benefit from storage value.

Now we are in a position to talk about the differences between the LBS and GUB relative to cases being optimised within the Market Clearing Engine (MCE).

5.5.6 Relative Performance Measures

Our modelling system can run a case for any size of reservoir, namely any integer SCAP and RCAP values, where the system is optimised under a spectrum of increasingly sophisticated assumptions about complexity, these range from: Deterministic (DC), Stochastic Independent (SI), Markov Chain (MC), and ultimately MC with State dependent bids. The last is somewhat stand-alone, and investigated in Chapter 8.

The key question is, what is the impact on benefits from modelling the underlying uncertainty? Assuming the inflow distribution is Markov, the ROR and MC optimisation case can be used to define the Maximum Benefit from Optimisation (MBO). The MBO is achievable for a given (SCAP,RCAP) pair, via:

$$MBO(SCAP, RCAP) = MC(SCAP, RCAP) - LBS(RCAP) \quad (5.37)$$

The LBS and TUBUS generate bounds on the benefit, and the difference between these is the Maximum Benefit Range achievable by increasing Storage capacity (MBRS), namely:

$$MBRS(RCAP) = TUBUS(RCAP) - LBS(RCAP) \quad (5.38)$$

Similarly, using the LBR and TUBUR, we have the Maximum Benefit Range achievable by increasing Release capacity (MBRR), via:

$$\text{MBRR}(\text{SCAP}) = \text{TUBUR}(\text{SCAP}) - \text{LBR}(\text{SCAP}) \quad (5.39)$$

Hence, we expect to see the more complex market designs runs producing greater returns, on average, as the underlying uncertainty is more closely matched, between the MBRS limits. Figure 5.11 below shows an example TB CDF for a single SCAP and RCAP case. The data is taken from the Chapter 6 example, see Section 6.4.5. The case is where SCAP = 12 and RCAP = 1.33, times FMEAN, respectively. The optimisation is run for DC, SI and MC levels of complexity, and the GUB and LBS CDFs are shown.

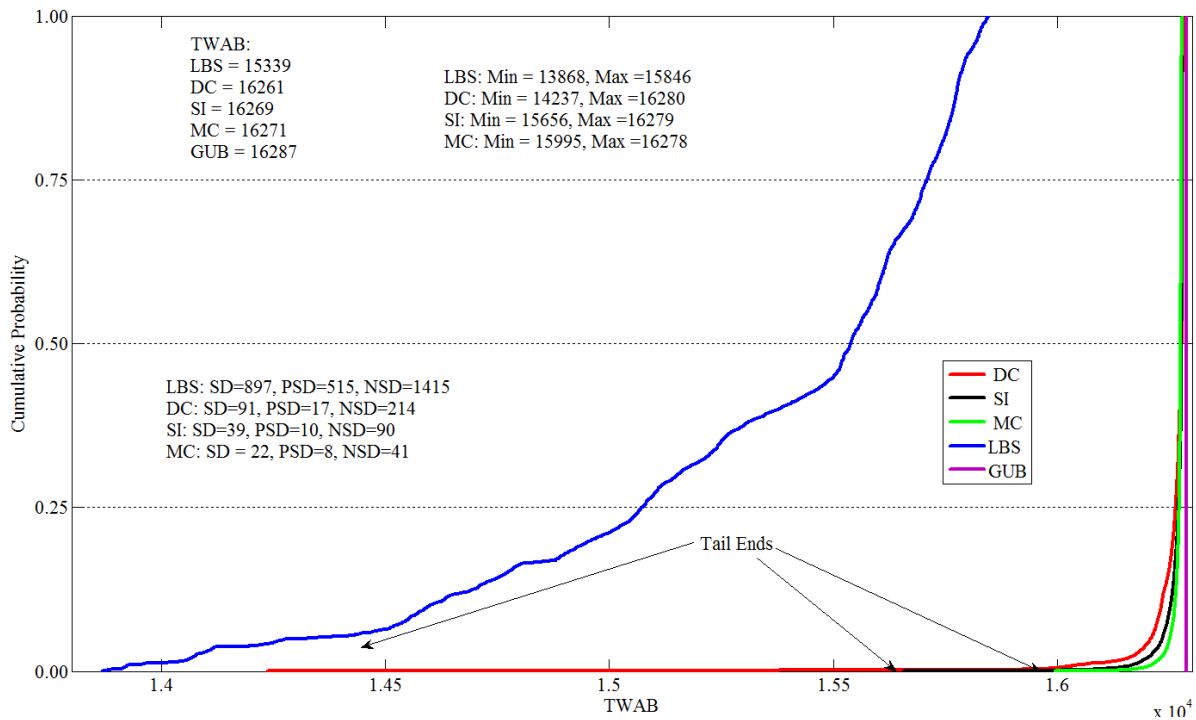


Figure 5.11: Illustration of Total Benefit CDF for Case Example showing range of returns under different uncertainty assumptions⁸⁹

Figure 5.11 shows the range of returns that can be expected. The general points to take from this illustration are the general relationships between the bounds (LBS and GUB). That is that the DC, SI, and MC CDF's lie between the LBS and GUB. The expected value of the DC CDF will be less than the expected value of the SI CDF, which in-turn will be less than the expected value of the MC CDF.

More specifically, for this particular reservoir case run, the ROR CDF case has a chance of producing lesser returns than the DC CDF case, which in turn has a chance of producing

⁸⁹ These results are predominantly from Monte-Carlo sampling, via the RS module.

lesser returns than the SI CDF case, which in turn has a chance of producing lesser returns than the MC CDF case. All of the storage system optimisations stand a (relatively small) chance of getting close to the GUB. However, the ROR case does not. And, nor can the ROR case be expected to return total benefits anything like those from the storage system, as we would come to expect. The GUB was computed directly, via Equation (5.34), and via the TUBUS method, discussed in Section 5.5.4 above.

The rest of this Section 5.5 is devoted to outlining the three different ways we report benefits. We report these in the next three experimental chapters. To highlight issues, and key points, we use the example given in Figure 5.11 above.

5.5.7 Proportion of Total Potential Benefit Achieved

For a given case, with an expected TB, the benefit improvement from building a reservoir and using a given market structure is compared to the total range of benefit improvement available. The MBRS is the total improvement available, and this is the denominator when reporting ‘case’ performance, with given SCAP and RCAP, as the Proportion of Total Potential Benefit Achievable by increasing Storage capacity (PTPBAS), via:

$$PTPBAS^{case}(SCAP, RCAP) = \frac{TB^{case}(SCAP, RCAP) - LBS(SCAP, RCAP)}{MBRS(SCAP, RCAP)} \quad (5.40)$$

Equation (5.40) will tell us the proportion of the MBRS that is achieved, for any given case run. We can also compare across different market configurations. For example, from Figure 5.11 above, the average annual TB for the LBS, and the GUB cases, are 15339, and 16287, respectively. The difference between the GUB and LBS is the MBRS, and this is 948.

The benefit improvement from building a reservoir, and employing the most basic market design we model (a deterministic market, the DC case) is thus 97.3%.⁹⁰ So, having a reservoir with SCAP and RCAP capacities set to 12 and 1.33 times FMEAN, and managing the given system deterministically, is expected to deliver just over 97% of the total ideal benefit that could be achieved. The equivalent MC case has a PTPBAS of 98.3%.⁹¹ Hence, an additional expected benefit of 1% is gained from matching the underlying stochasticity within the market. Thus, the gain from improved optimisation is not that high when compared to the

⁹⁰ $(16261 - 15339 / 948) \times 100$

⁹¹ $(16271 - 15339 / 948) \times 100$

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gain from building the reservoir with the most basic optimisation allocation policy, which is just over 97%. However, the cost of improved optimisation (or improved market design) is also probably many orders of magnitude less than the cost of constructing the reservoir system.

We can also use the MBRR, to define the Proportion of Total Potential Benefit Achievable by increasing Release capacity (PTPBAR), via:

$$PTPBAR^{case}(SCAP, RCAP) = \frac{TB^{case}(SCAP, RCAP)}{MBRR(SCAP, RCAP)} \quad (5.41)^{92}$$

Equation (5.41) is used in an example in Section 7.2.2.

5.5.8 Proportion of Remaining Potential Benefit Achieved

If we assume the reservoir exists as a sunk cost, then we are just comparing gains from the GUB case to the case where we have the reservoir, with the most basic form of market design. For our experiments, we generally assume the most basic optimisation is the DC case.

For our system, we generally compare any two cases from DC, SI, and MC. Hence, the Proportion of Remaining Potential Benefit Achievable by increasing Storage capacity (PRPBAS) compares any MORE, versus LESS complex optimisation run. A key difference to PTPBAS, is that the denominator of PRPBAS does depend on the case's actual reservoir size.

The remaining improvement available is the difference between the GUB and the least complex (DC) optimisation run case. This difference we name the Remaining Benefit Range achievable by increasing Storage capacity (RBRS), and this is defined as:

$$RBRS(SCAP, RCAP) = GUB - TB^{DC}(SCAP, RCAP) \quad (5.42)$$

The RBRS is the denominator in PRPBAS, which is defined as:

$$PRPBAS^{case}(SCAP, RCAP) = \frac{TB^{case}(SCAP, RCAP) - TB^{DC}(SCAP, RCAP)}{RBRS(SCAP, RCAP)} \quad (5.43)$$

⁹² Where LBR(SCAP) is not explicitly stated because it is always 0.

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Equation (5.43) will tell us the proportion of the RBRS that is achieved, when comparing against the simplest market design (deterministic) option, for any given case run. In other words, for a given case the PRPBAS tells us the proportion of remaining gains achieved from building an infinitely sized reservoir system and managing it optimally.

For example, from Figure 5.11 above, comparing the MC and DC cases gives a PRPBAS of 38.5%.⁹³ Alternatively, comparing the SI and DC cases gives a PRPBAS of 30.8%.⁹⁴

Thus, modelling stochasticity simplistically is expected to deliver approximately 31% of the remaining gains that could be made from building an infinite (unconstrained) reservoir and managing it optimally. Approximately an additional 8% gain is expected from matching the underlying (Markov) stochasticity, optimally. To get any more expected gains than this, we would need to build a bigger reservoir.

There is an alternative measure that just looks at the overall value of the market.

5.5.9 Proportional Benefit Achieved Relative to Market Value

What is the value of a water market in, and to, a local economy? Scarily, but potentially true, is that it is the entire value of that economy. For without water there is no life, and thus no activity we can value, economically, or otherwise. But no one really wants to pay that price. So the value then becomes the price that market participants are willing to pay. Market participants might be very interested in a market value measure, one which combines the average price paid relative to the average volume traded. So what is the associated value, for the total average units which are traded? Generally, market participants set that macro-economic value of a market through their micro-economic bids, of their willingness to pay. To potential water traders, this macro-economic value is their perception of how much money is actually on the table, and thus how much is potentially up for grabs.

The DCR data has been constructed such that all the TB values are subject to a large underlying constant value, given the use of constant elasticity curves as discussed in Section 5.4.4. At a monthly (or annual) level net welfare can be expressed as a proportion of the

⁹³ $[(16271-16261)/(16287-16261)] \times 100$

⁹⁴ $[(16269-16261)/(16287-16261)] \times 100$

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underlying Market Value (MV). And MV can be computed using an optimum MCP and an optimum quantity (volume) traded.

The DCRs have been designed to go through a price point of \$1.00, using the average demand, as shown in Section 5.5.4.⁹⁵ We also carefully ensure that the average periodic inflow FMEAN is equal to the average periodic reference demand QREF, on average over the entire water year. This is achieved by simply dividing the TB by the average number of units traded in the market, at a marginal price of \$1. The average quantity traded in each t , is multiplied by the unconstrained case's optimum marginal price of \$1/unit of water. This is the "Ideal" Market Value (MV^{*t}), for each t , and this is given as follows:

$$MV^{*t} = 1 \times F^{t,\bar{h}} \quad (5.44)$$

From Equation (5.9) the mean annual demand quantity QREF is:

$$QREF = \sum_t QREF^t = T \times FMEAN \quad (5.46)$$

All T decision stage average values have simply been aggregated to create the constant underlying annual IMV value, where each unit is traded at optimal price (MREF) of \$1/unit. Hence, the annual IMV is:

$$MV^* = 1 \times QREF \quad (5.47)$$

Alternatively, the "Actual" MV could be computed using the MCP produced by the MC optimisation i.e. the optimum optimisation with a MC simulated "real world". Both the Ideal and Actual MV options are tried on some experiments in Section 7.4.2.

But generally, the associated annual Welfare as a Proportion of Market Value (WAPMV) is:

$$WAPMV(SCAP, RCAP) = \frac{TB(SCAP, RCAP)}{MV} \quad (5.48)$$

WAPMV reports the market value from optimising under different levels of increasing complexity within the Market Clearing Engine. But perhaps more usefully, we can compare the value of less versus more complex market configurations. For example, we can compare the MC versus DC annual TB values within the WAPMV framework.

The gain from the most complex optimisation we describe as the Benefit Achievable as a Proportion of Market Value (BAPMV), via:

⁹⁵ There are on average 84 units per month, and 1008, per annum.

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$$\text{BAPMV}^{case}(\text{SCAP}, \text{RCAP}) = \frac{\text{TB}^{case}(\text{SCAP}, \text{RCAP}) - \text{TB}^{\text{DC}}(\text{SCAP}, \text{RCAP})}{\text{MV}} \quad (5.49)$$

For example, from Figure 5.11 above, comparing the MC and DC cases gives a BAPMV of 11.9%,⁹⁶ and comparing the SI and DC cases gives a BAPMV of 9.5%.⁹⁷

Thus, accounting for some uncertainty in the market clearing engine will deliver a minimum gain of at least 10%, and nearly another 2% for matching the underlying stochasticity. With a highly active and fluid market, participants would probably wish to benefit from some of those additional gains, even if it comes as the expense of higher “joining” and/or transaction fees, to implement changes to the allocation and management system.

5.5.10 Time Weighted Average Benefit

For consistency with price reporting (discussed in Section 5.6), we generally report annual Time Weighted Average Benefits (TWABs), which are the total annual benefits, TB, averaged by the number of decision periods, via:

$$\text{TWAB} = \frac{\text{TB}}{\text{T}} \quad (5.50)$$

As well as benefits, the prices which the market is cleared at are also important, especially politically. Market participants and regulators, alike, are always concerned with the price paid for goods in such markets.

5.6 Price Reporting

In this section we review the types of price measures we can generate from our modelling system. The modelling system operates at a Markov lag-one level of complexity, hence marginal price lists exist for each stage, t , and hydrology state h . There are two key conditional price lists: for Release, the DCR is input into the SCDDP optimisation, and for Storage, the DCS is output from the SCDDP optimisation. Each conditional list has an associated PDF: the PDFR for release, and the PDFS for storage. Those PDFs can be computed via discrete convolution, and we briefly discuss how to use the PDFS to compute probability weighted average prices. But, most of our results are reported from our Monte Carlo Simulation, which does not produce an explicit PDF, but a discrete set of values, from

⁹⁶ (16271–16261)/(84)

⁹⁷ (16269–16261)/(84)

which we produce our key price reporting measures, including volume weighted average prices.

5.6.1 Time Weighted Average Prices (TWAPs)

Strictly speaking the price implied by any DCR or DCS is always ambiguous because, in our discretised model, a bid step is either wholly met, or not, as discussed in Section 5.4.4. Theoretically, the MCP could be the price of the last increment cleared, or that of the next increment to be cleared, or anything in between. A balanced response might take the simple average of the two prices, and the RO module has the facility do this simple price averaging. But, to be consistent with the CDDP assumptions and the way we have set-up our example dataset in Sections 5.4.3-5 above, we normally take the price of the last increment cleared from the DCS. So, for example, the Market Reporting (MR) module takes the MCP for Storage, MCPS, for each simulation period t and sample n , from the DCS determined by the RO for that period and the simulated hydrology state:⁹⁸

$$\text{MCPS}^{t,n} = \text{DCS}^{t,h(t,n)}(s^{t-1,n}) \quad (5.51)$$

So, when the storage is full, the price is naturally set to that of the very last increment in the DCS, which is potentially non-zero.⁹⁹ This corresponds to assuming that unstorable spill can be disposed of in each period, without collapsing the price for the volume that can be stored for the next period. If the storage level is empty, there is a DCS value for the “empty” step, see Section 5.4.5 above.

TWAP, the Time Weighted Average Price is a well-known concept, and is commonly used as a reporting measure in markets. With our Markov system any MCP for storage, or release, is a conditional price $\text{MCP}^{t,h}$, for the simulated hydrology state. Averaging by both t and n will produce an expected TWAP, but we also average across sample points, for each period, to produce an average which is not strictly time weighted. And perhaps this is better defined as a Probability Weighted Average Price PWAP, even so we refer to this as TWAP. So we define the TWAP for Storage in period t , to be:

⁹⁸ Here $s^{t-1,h}$ is the start of period storage level and, with an informed inflow allocation policy, h is the intra-period hydrology state. With a conservative inflow allocation policy we would use the start-of-period hydrology state, h' , to look up the market clearing price, $\text{DCS}^{t,h'}(s^t)$.

⁹⁹ Over time we are always looking forward, therefore the MWV does not drop to zero now, just because spill is happening. Of course we are not modelling penalties, and any flooding of the market.

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$$\text{TWAPS}^t = \frac{\sum \text{MCPS}^{t,n}}{N} \quad (5.52)$$

But this is an average price for a “storage market” which really only exists, in our model, to service the intra-period release market. Units are released from the reservoir storage system to satisfy demand, so the prices we clear the market (trade) at are represented in the DCR. While our model tries to equate these two prices, they can differ because the release capacity constraint can bind, as shown in Equation (4.8) in Section 4.5. So, while we refer to the storage prices when discussing the consistency of the solutions in Chapter 6, we mainly focus on release prices. The MR module also uses the RO/RS outputs to report an MCP for Release, MCPR, for each t and n :

$$\text{MCPR}^{t,n} = \text{DCR}^{t,h(t,n)}(q^{t,n}) \quad (5.53)$$

As for the DCS above, we normally take the MCP to be the price of the last unit cleared. So, even if the sampled release is equal to the release capacity (RCAP), and spill possibly occurring, the market price is not the spill value of zero, but is taken from the last utilisable increment in the $\text{DCR}^{t,h}(q^{t,n}=\text{RCAP})$. This corresponds to assuming that unutilisable spill can be disposed of, through other channels, without collapsing the price for the volume that can be utilised by the market in the current period. If there is no release ($q^{t,n} = 0$) then, to maintain consistency with the CDDP, the market price is set to the first utilisable price on the DCR, for that t and h . In other words, the first priced step is used to signal the minimum possible cost of shortage, since there is no “shortage cost”, as such, in these experiments. As above, we define the Time Weighted Average Price for Release TWAPR in period t , to be:

$$\text{TWAPR}^t = \frac{\sum \text{MCPR}^{t,n}}{N} \quad (5.54)$$

TWAPS^t and TWAPR^t can also be averaged to an annual level by taking their mean over the water year. One issue with TWAP is that it puts as much emphasis on prices in periods where a low volume is traded as it does on periods where a high volume is traded. This may be appropriate for participants who only wish to trade a small volume, and particularly for virtual traders who never physically buy water, but may buy and sell CfD type hedging instruments with the same volume in all periods. But, on average, physical participants will want to know the average price for the water they actually buy. Thus, our preferred price reporting measure is one which accounts for trade volumes.

5.6.2 Volume Weighted Average Price (VWAP)

The Volume Weighted Average Price, VWAP, accounts for the volume of water traded at each price. Although we could compute a VWAP for Storage, we only discuss VWAP in terms of Release. VWAP will place no weight on periods in which shortage occurred, because no water is traded at that price. However, spill could be included, or excluded, from the VWAP calculation, and this leads to two VWAP comparison options: Total VWAP (with spill), or Utilisable VWAP (without spill). As noted above, we have taken the MCP to be the DCS price at SCAP, or the DCR price at RCAP, when spilling. But we clearly would not want to value the spill itself at that price. If the price of the spilled volume was set to zero, though, those periods would also add nothing to the weighted sum of prices in Equation (5.55) below. But the spill volume itself would increase the bottom line, thus decreasing the reported price.

In instances with a lot of spill, this can significantly distort VWAPs. Hence we decided to report utilisable VWAP and report spill separately, as discussed in Section 5.8 below. The annual utilisable global VWAP is defined as follows:

$$\text{VWAP} = \frac{\sum_{t,n} \lambda^{t,n} \times q^{t,n}}{\sum_{t,n} q^{t,n}} \quad (5.55)$$

This global VWAP is our primary price reporting measure. But we can also produce an annual VWAP for each sampled year, averaging across periods within that year, via:

$$\text{AVWAP}^n = \frac{\sum_t \lambda^{t,n} \times q^{t,n}}{\sum_t q^{t,n}} \quad (5.56)$$

This measure, AVWAP^n , is used in computing the probability weighted variance of the annual VWAP, as discussed in Section 5.7. We also compute the mean of the annual VWAP^n , via:

$$\text{AVWAP} = \frac{\sum_n \text{AVWAP}^n}{N} \quad (5.57)$$

We can also define VWAP^t for each period, which is used in computing the volume weighted variance of the price of a unit of water in Section 5.7, by averaging across samples for that period:

$$\text{VWAP}^t = \frac{\sum_n \lambda^{t,n} \times q^{t,n}}{\sum_n q^{t,n}} \quad (5.58)$$

For any price measure, though, the critical issue is to understand whether better optimisation should be expected to produce higher or lower prices, as discussed next.

5.6.3 Expected Price Relationships

It is often asserted, particularly by political reformers, that an efficient, competitive market will both maximise benefits and minimise the prices faced by consumers. There are many reasons why the proposition may tend to be true, in real life. A competitive market creates incentives to drive producers to bid at marginal cost (allocative efficiency), to find more efficient ways of producing their goods, thus reducing their marginal costs (productive efficiency), and to invest and innovate to lower future production costs (dynamic efficiency).

But none of those effects are relevant in the stylised setting assumed here. An optimisation will look to maximise the producer and consumer surpluses to maximise the net welfare to society. A low price is an advantage to consumers, but a disadvantage to producers, and there is nothing inherently optimal about lower prices. In fact, in later chapters we find cases where improved optimisation delivers higher benefits, as expected, but also produces higher market clearing prices. In this section we briefly outline two reasons why this might occur.

5.6.3.1 Inter-temporal trade-offs in a deterministic resource allocation problem

As discussed in Section 2.3.6, the fundamental driver of reservoir optimisation is towards equalising MCPs over the modelling horizon. Thus, in an unconstrained market, prices would be equal in all periods. But, since the optimality result applies to marginal units, it effectively relates to TWAP, not VWAP. Nor does it necessarily mean that optimisation will minimise even TWAP. Consider the following counter-example, in which we are allocating a fixed water quantity between periods.

We might imagine two aggregate DCRs in two adjacent monthly markets as shown in Figure 5.12 below. In this asymmetric representation, the demand curve for period 2 is effectively the supply curve for period 1. We let q , varying between 0 and 1, be the proportion of the

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available water sold at $t=1$, so the proportion sold at $t=2$ is $(1-q)$. We let λ_1 and λ_2 represent the prices paid in periods $t=1$ and $t=2$, respectively, and we let them be determined by the following linear demand/supply curves:

$$\lambda_1 = a_1 + b_1 q \tag{5.59}$$

and

$$\lambda_2 = a_2 + b_2 q \tag{5.60}$$

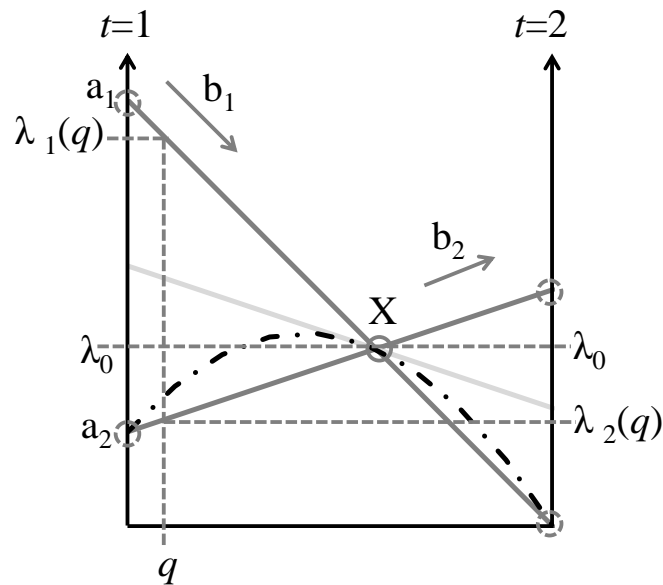


Figure 5.12: Example of inter-temporal allocation between periods

If the first month has a relatively abundant supply and the second month has a relative shortage then, without storage, $\lambda_1 < \lambda_2$, while in the opposite scenario $\lambda_1 > \lambda_2$. But, if we have enough storage capacity, we would release enough water in each month such that the marginal prices are equal, thus producing the equilibrium price, $\lambda_1 = \lambda_2 = \lambda_0$, shown by the point ‘X’. Equating (5.59) and (5.60) we see that:

$$q(X) = \frac{a_1 - a_2}{b_2 - b_1} \tag{5.61}$$

Thus:

$$\lambda_0 = a_1 + b_1 \frac{a_1 - a_2}{b_2 - b_1} \tag{5.62}$$

Since the price is equal in both periods, all units are sold at that price, which must be both TWAP and VWAP. But now consider a solution in which, say, $\lambda_1 > \lambda_2$, either because

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solution has not been fully optimised, or because storage limits intervene. As shown by the light grey line in Figure 5.12, we can define TWAP as the average of the two prices:

$$\text{TWAP} = (\lambda_1 + \lambda_2) / 2 \quad (5.63)$$

Substituting Equations (5.59) and (5.60) into (5.63), and re-arranging gives:

$$\text{TWAP} = \frac{a_1 + a_2}{2} + \frac{q(b_1 + b_2)}{2} \quad (5.64)$$

Therefore, for the linear case, the TWAP corresponding to a sub-optimal q can be greater or less than the optimal price λ_0 , corresponding to the optimal q . In our experiments we actually employ constant elasticity curves, which are not linear but strongly convex, and the effects discussed in the next section may dominate in our reported results, particularly under uncertainty. However, the effect becomes more relevant as elasticity increases (see Section 8.2). And, in any case, we report most of our results in terms of VWAP, for which the effect may be stronger than for TWAP.

For the same example, the dash-dot curve in Figure 5.12 represents VWAP. VWAP must equal λ_1 when all water is sold in the first period, or λ_2 when all water is sold in the second period. And VWAP must equal λ_0 when water is sold in the optimal proportions, at X . Mathematically, we have:

$$\lambda_{\text{VWAP}} = q \lambda_1(q) + (1 - q) \lambda_2(q) \quad (5.65)$$

Substituting Equations (5.59) and (5.60) into (5.65), and re-arranging gives:

$$\lambda_{\text{VWAP}} = a_2 + (a_1 - a_2 + b_2) \times q + (b_1 - b_2) \times (q)^2 \quad (5.66)$$

This quadratic goes through the three points mentioned above, but its maximum must be greater than λ_0 . Taking its derivative, and setting that to zero, the maximum value of VWAP occurs at:

$$q' = \frac{a_1 - a_2 + b_2}{2(b_2 - b_1)} \quad (5.67)$$

As can be seen in Figure 5.12 the VWAP associated with a sub-optimal q could be greater than λ_0 , but it is quite likely to be less.

5.6.3.2 Concave vs. convex marginal benefit functions in a stochastic optimisation

We will show that, in situations with concave marginal benefit functions in a stochastic environment, average market clearing prices may be higher for optimal solutions than for sub-optimal ones. E. Read (1982) discussed the opposite of this effect, showing that, with a convex marginal cost curve function a deterministic optimisation will always under-estimate marginal costs, relative to the average prices corresponding to the true stochastic optimum. Conversely, because this policy produced by the deterministic optimisation is necessarily sub-optimal in its management of stochasticity, simulated application of that policy produces greater than optimal variation in marginal costs, prices which are, on average, higher than those for the true stochastic optimum. This effect was demonstrated by E. Read & Boshier (1989), for a model of the New Zealand system.

First, though, we discuss a much better known result, which is fundamental to stochastic optimisation. We imagine a water market with an expected inflow $E(F)$, in which benefits are convex function of release, $B(R)$. Thus the possible benefit from releasing the expected inflow is $B(E[F])$. With an uncertain (or varying) release distribution, though, the benefits would be $B(F)$, giving an expected benefit of $E(B(F))$. The well-known Jensen's inequality states that, for a convex benefit function:

$$E[B(F)] < B(E[F]) \tag{5.68}$$

More generally, we see that, the greater the release variance the lower the expected Benefit and, importantly, that we can improve the expected benefits by narrowing the release distribution. For example, Figure 5.13 below shows two uncertain releases with symmetric deviations from $E(F)$ of d_1 and d_2 , respectively, where $d_1 > d_2$. Here $B(E[F]) > EB(d_2) > EB(d_1)$, where $EB(d)$ represents the Expected Benefit for a given release deviation, d .

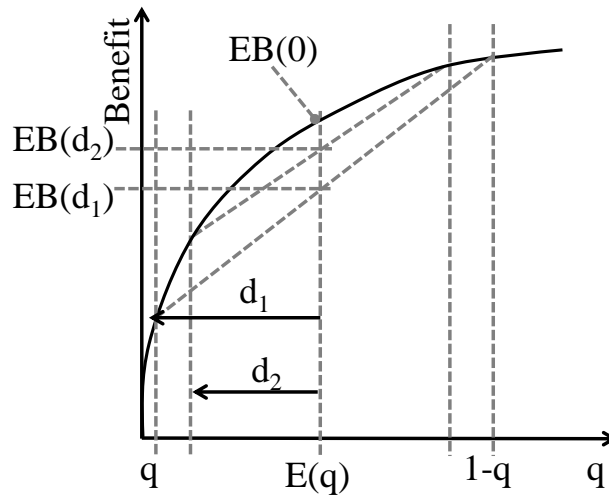


Figure 5.13: Example of Jensen's Inequality on Market Benefits

If the benefit function were the same in all periods, we would thus expect stochastic optimisation to systematically reduce release variation, between periods, and between scenarios. More generally, with benefit functions varying from period to period, stochastic optimisation systematically reduces variation in marginal cost/benefit, both between periods, and between scenarios, as discussed in Section 2.3.3. But we are concerned here with prices, not benefits, so we need to consider the application of this concept to **marginal** benefit functions, as in E. Read & Boshier (1989). In our context the marginal benefits functions are the DCRs, which typically have a convex shape, as in the region between points 'c' and 'g' in Figure 5.14 below. In that region, Jensen's Inequality implies that, by narrowing the range of prices, improved optimisation will lower the expected price. Hence:

$$E[M(q)] > M(E[q]) \tag{5.69}$$

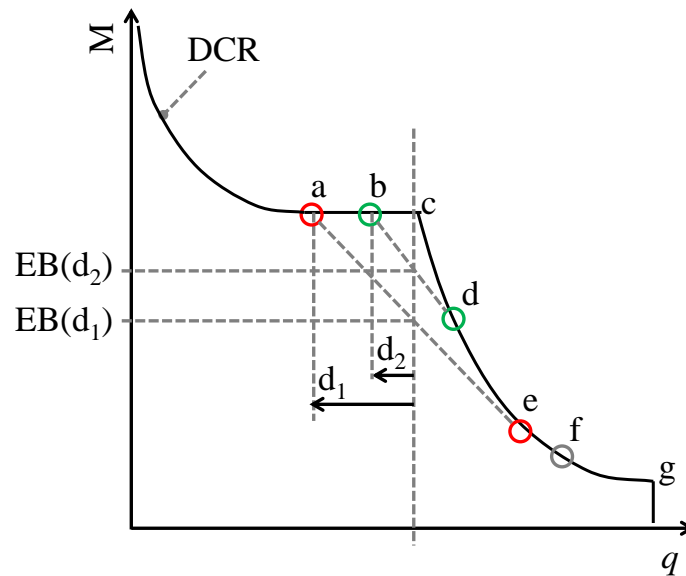


Figure 5.14: Example of Impact on Optimisation Complexity on Market Prices

But the opposite effect is also possible. An improved optimisation can produce a higher expected benefit and higher expected price, if the system optimally operates in a region where the DCR curves the other way, as it does when the MWV suddenly falls to zero at point g , and also around the end of the “flat”, at point c . For simplicity, we only show simple two point distributions, a & e and b & d , of the outcomes from the simulated application of deterministic (DC) and stochastic (e.g. MC) optimisation models, respectively. In this case, the tighter distribution will not lower prices, but raise them.

Simulation effectively samples across the DCR to produce an expected DCR, as a function of expected release. This might be represented by the green or red DCRs in Figure 5.15 below, which “cuts corners” off the underlying black DCR. The better the optimisation the tighter the price distribution, and the more closely the expected DCR will follow the underlying DCR i.e. the green DCR. Conversely, any optimisation will tend to produce an expected DCR lying very close to the original DCR, if simulated outcomes typically lie in regions where that DCR is flat, or even linear. Thus the difference between average prices, or price variances, may be quite low for cases in which operation occurs around some regions of the curve, and quite high in regions on either side, producing behaviour which may seem erratic, but reflects the shape of the underlying DCR, as discussed in Section 8.3.

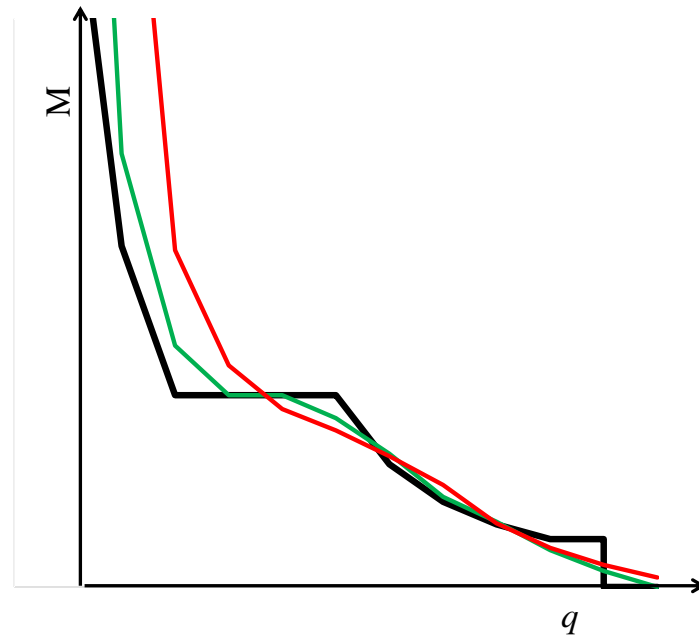


Figure 5.15: Example of Optimisation Complexity on Simulated Market Prices

Finally, a better optimisation might move the mean release in one direction, say raising the price. But, at the same time, it might tighten the distribution, and hence lower the price. These effects are particularly noticeable at the monthly level, as discussed in Section 6.4.4. At that level, optimisation may lower benefits in some periods so that benefits can be higher in others, while the offsetting effects discussed here may move average prices for a particular period in either direction.

5.7 Volatility Reporting

As noted at the start of this chapter, we assume risk neutral participants. And, in these high-level investigations, we are not concerned with the individual positions of participants. Still, in the real world, the price participants pay for goods in markets is of economic, social and, ultimately, high political concern. Low prices are generally considered a good thing, while excessively high prices might even lead to a change in government. So price volatility is an important aspect of aggregate market performance.

We might think that the volatility of higher/lower than average prices/benefits would be associated with a system which is managed sub-optimally. But our optimisation does not try to minimise volatility, as such. So, in cases where the underlying benefit functions are variable, as in Section 8.4, we should expect that the optimal solutions will have higher

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benefit variability, than sub-optimal solutions. But we do generally expect that optimisation will minimise TWAP price volatility, because optimisation always strives to align the marginal value of the last increment of stored or released water in each period, and therefore align the probability weighted average prices, across periods and scenarios, as discussed in Section 2.3.6. As noted in Section 5.6.3.1 above, optimisation does not necessarily affect VWAP in quite the same way as TWAP, though, and so while we focus on VWAP we also sometimes refer to TWAP reporting measures.

While variability (e.g. results due to seasonal patterns) is not the same as risk and/or volatility (e.g. results due to inflow variance), we do not readily distinguish between these. But we tend to discuss risk in terms of participants' perspective of market transactions, and variability in terms of the more general market measures. And we just compute simple volatility measures: Standard Deviation (SD), semi-variance, and semi-deviation, for both benefits and TWAP/VWAP prices. We generally report the semi-deviation, and are most interested in the Negative Semi-Deviation, NSD, as a measure of the downside volatility of low benefit returns, and the Positive Semi-Deviation, PSD, as a measure of the upside volatility of high market prices.

Our primary focus is on reporting measures produced by the Monte Carlo simulation, where we have N samples, in each period t . Although we have defined various means above, such as TWAP, VWAP etc., here we use general formulae for any Value function, V , (e.g. market prices or benefits), and first compute the periodic \bar{V}^t , annual \bar{V}^n , and global $\bar{\bar{V}}$ means. First to compute a periodic expected value, for each period, we use:

$$\bar{V}^t = \frac{\sum V^{t,n}}{N} \quad \forall t \quad (5.70)$$

To compute an annual expected value, for a particular simulation year n , we use:

$$\bar{V}^n = \frac{\sum V^{t,n}}{T} \quad \forall n \quad (5.71)$$

To compute the global expected value, over all periods and simulation years, we use:

$$\bar{\bar{V}} = \frac{\sum V^{t,n}}{N \times T} \quad (5.72)$$

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where Equation (5.72) is the mean of both Equations (5.70) and (5.71).

Then the periodic VAR^t, annual AVAR, and global GVAR variances are:

$$\text{VAR}^t = \frac{\sum (V^{t,n} - \bar{V}^t)^2}{N} \quad \forall t \quad (5.73)$$

$$\text{AVAR} = \frac{\sum (\bar{V}^n - \bar{\bar{V}})^2}{N} \quad \forall n \quad (5.74)^{100}$$

$$\text{GVAR} = \frac{\sum_{t,n} (V^{t,n} - \bar{\bar{V}})^2}{N \times T} \quad (5.75)$$

Note that the “Annual” variance defined here is the variation in the annual average value.

The PSD and NSD for each period are:

$$\text{PSD}^t = \sqrt{\frac{\sum_{n:V^{t,n} > \bar{V}^t} (V^{t,n} - \bar{V}^t)^2}{\sum_{n:V^{t,n} > \bar{V}^t} n}} \quad \forall t \quad (5.76)$$

$$\text{NSD}^t = \sqrt{\frac{\sum_{n:V^{t,n} < \bar{V}^t} (V^{t,n} - \bar{V}^t)^2}{\sum_{n:V^{t,n} < \bar{V}^t} n}} \quad \forall t \quad (5.77)$$

The APSD and ANSD of the annual averages are:

$$\text{APSD} = \sqrt{\frac{\sum_{n:\bar{V}^n > \bar{\bar{V}}} (\bar{V}^n - \bar{\bar{V}})^2}{\sum_{n:\bar{V}^n > \bar{\bar{V}}} n}} \quad \forall n \quad (5.78)$$

$$\text{ANSD} = \sqrt{\frac{\sum_{n:\bar{V}^n < \bar{\bar{V}}} (\bar{V}^n - \bar{\bar{V}})^2}{\sum_{n:\bar{V}^n < \bar{\bar{V}}} n}} \quad \forall n \quad (5.79)$$

And the “Global” GPSD and GNSD are:

¹⁰⁰ Note that this is not the variation within a year (i.e. $\text{VAR}^n = \frac{\sum (V^{t,n} - \bar{V}^n)^2}{T}$). It is the variance of the distribution of annual means.

$$\text{GPSD} = \sqrt{\frac{\sum_{t,n:V^{t,n} > \bar{V}} (V^{t,n} - \bar{V})^2}{\sum_{t,n:V^{t,n} > \bar{V}} n}} \quad (5.80)$$

$$\text{GNSD} = \sqrt{\frac{\sum_{t,n:V^{t,n} < \bar{V}} (V^{t,n} - \bar{V})^2}{\sum_{t,n:V^{t,n} < \bar{V}} n}} \quad (5.81)$$

Equations (5.76), (5.78) and (5.80) can be used as measures of the variance associated with higher than average prices. Equations (5.77), (5.79) and (5.81) can be used as measures of the variance associated with lower than average benefits.

The above volatility measures could all be applied to benefits, and to both TWAP and VWAP price measures. But we only calculate the PSD, as defined here, for TWAP, and refer to this as the TWAP PSD. When considering variation in VWAP, though, we are more interested in a volatility measure that accounts for the volumes traded at each price. When computing the variance of the price of a unit of water, we weight each price value by its corresponding traded volume, via the group frequency weighted variance formula. So the Volume Weighted Positive Semi-Deviation (VWPSD) of the price of a unit of water in period t is:

$$\text{VWPSD}^t = \sqrt{\frac{\sum_{n:\text{MCP}^{t,n} > \text{VWAP}^t} q^{t,n} (\text{MCP}^{t,n} - \text{VWAP}^t)^2}{\sum_{n:\text{MCP}^{t,n} > \text{VWAP}^t} q^{t,n}}} \quad (5.82)$$

The corresponding global measure is:

$$\text{VWPSD} = \sqrt{\frac{\sum_{n,t:\text{MCP}^{t,n} > \text{VWAP}} q^{t,n} (\text{MCP}^{t,n} - \text{VWAP})^2}{\sum_{n,t:\text{MCP}^{t,n} > \text{VWAP}} q^{t,n}}} \quad (5.83)$$

For analysing price volatility we mainly use VWPSD but, where appropriate, we also refer to the TWAP PSD, as defined above. Alternatively, we can report benefits and prices from the convolution in the RC module, as discussed in Section 5.4.6. Unlike the Monte Carlo simulation, which produces N discrete values for each variable in each period, this produces a PDF for storage, $\text{PDF}^{t,h}(s)$. We use these convolved outputs to check our limited sampling results from the Monte Carlo simulation, at the period level. Rather than compute variances,

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we turn the PDFs into CDFs to inspect their tails, and compare CDFs for different optimisation run complexities. But, given that V (= market prices or benefits) is computed for all storage states (levels), stages, and hydrology states (s, t, h) we can compute the expected value of any V , for period t , as:

$$\bar{V}^t = \sum_{s,h} (V^{t,h}(s) \times \text{PDF}^{t,h}(s)) \quad \forall t \quad (5.84)$$

For computing release prices over the PDF for storage we use the optimum release schedule from the SCDDP, $\text{CORS}^{t,h}(s)$, and look up the corresponding MCP for Release (MCPR) in $\text{DCR}^{t,h}$ i.e. $V^{t,h}(s) = \text{MCPR}^{t,h}(s) = \text{DCR}^{t,h}(q = \text{CORS}^{t,h}(s))$.

5.8 Reporting of Spill, Empty and Shortage

Finally, we can compute the expected value of spill from both the RC and RS modules. These are reported by period and annually as a proportion of the average inflow quantity. These simplistic experiments have been designed so that the system should never face shortage. But the reporting module does check this, and with a different set of data would report shortage instances. Our worst case situation is that the reservoir is empty, and receives no water, thus staying empty. So we report instances where the system runs empty in both the RC (the PDFS weight in the first row) and RS modules.

5.9 Conclusions

In this chapter we outlined our modelling and testing system, while discussing various assumptions and modelling choices. The chapter initially discusses a number of computational design assumptions, while also providing notation connecting the single-tank Stochastic Linear Programming (SLP) formulation to the Stochastic Dynamic Programming (SDP) implementation. We assumed the single hydrology index is based wholly on exogenous river flow. Results are focused on market efficiency, emphasising only aggregate market performance. The modelling system was developed in Matlab, and comprises a series of modules.

First, a series of input parameter modules: Reservoir Data (RD), Hydrology Data (HD), and Market Data (MD). These allow us to parameterise cases to run various physical and economic experiments. The overall data management system allows us to batch process

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thousands of cases, quickly and efficiently, to compare across cases. In order to facilitate precise across case comparisons we matched reservoir storage and release capacities to the hydrology net inflow and probability data, and matched the average demand quantities (market data) to the average net inflow quantities, at a reference price of 1.

Second, at the heart of the system, is a Stochastic Constructive Dual Dynamic Programming (SCDDP) Reservoir Optimisation (RO) module. This module creates a set of policies which define release decisions, and Marginal Water Values (MWVs) over the entire state space, under Deterministic (DC), Stochastic Independent (SI), or Markov (MC) assumptions. Constructive Dual Dynamic Programming (CDDP) was the preferred choice given it is a dual approach, which works directly with the Demand Curves for Release (DCRs) defined by participant bids, to calculate MWVs, stored in a convenient form. Also, for a single reservoir system, SCDDP readily constructs long-run (steady-state) equilibrium operating policies for the entire state-space. This makes it ideal as a research tool, because we can quickly explore the entire solution space, to look at system behaviour, especially near its bounds. And it allows for a consistent comparison of system performance across various market designs.

Third, a series of simulation modules allow us to test various optimisations, with the two main sub-modules being a Monte Carlo Reservoir Simulation (RS), and a fully enumerated Reservoir Convolution (RC). While we generally use the RS module, the RC module allows us to validate those findings, while also looking at extreme outcome events.

Fourth, and finally, the Market Reporting (MR) Module allows us to generate various price, benefit, and other storage level, reporting measures. With regard to benefits, the key reporting measure for across case comparisons is the Time Weighted Average of monthly Benefits (TWAB), which is calculated for, and averaged over all sample years. Related measures which allowed for across case comparisons were developed based on:

- The absolute gain from building a reservoir and employing a given level of stochastic optimisation, i.e. the Proportion of Total Potential Benefit Achieved (PTPBA),
- The remaining gain from employing a more complex level of stochastic optimisation, i.e. the Proportion of Remaining Potential Benefit Achieved (PRPBA), and

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- The value of the market based on the average volume traded at the actual optimum Market Clearing Price (MCP) via Markov Chain (MC) optimisation, i.e. the Benefit Achieved as a Proportion of Market Value (BAPMV).

For price, the model generates Time Weighted Average Prices (TWAPs) and Volume Weighted Average Prices (VWAPs). The VWAP of water released to the market, (VWAPR), is the key price reporting measure we use, because it takes account of the volumes traded within the market. For volatility measures the modelling system computes the periodic, annual, and global standard deviations, and their corresponding Positive and Negative Semi-Deviations (PSDs and NSDs), for the TWAB, TWAP, and VWAP. With our constant elasticity DCRs the downside benefit and upside price variability will be much more significant than the respective upside benefit and downside price variability. So the key volatility reporting measures are the ANSD, the Negative Semi-Deviation of the annual average of monthly benefits, TWAB, and the global Volume Weighted Positive Semi-Deviation (VWPSD) of the price of a unit of water.

With a more sophisticated optimisation, which more closely matches the underlying uncertainty, we should always expect larger average benefits. But, under uncertainty, we may well find that benefit variability is not always minimised. In fact if a system features variable patterns of inflow and/or demand, then higher benefit variability can be a sign of optimality, because improved optimisation better matches that inherent variability. Similarly, for the reasons discussed in Section 5.6.3.1, volume weighted average prices, in particular, may not always be minimised under uncertainty, especially if marginal cost/benefit curves are not convex. And, while we might expect price variability to be minimised with a more sophisticated optimisation, which more effectively balances MWVs over the state space, this might not always be the case with non-convex marginal cost curves.

The remainder of this thesis discusses experiments performed with this system to determine the impact that modelling stochasticity might have on market performance, in terms of these reporting measures. In the next chapter we showcase some of the results we can produce with our lag-one Markov modelling system, applying DC, SI, and MC optimisation to a base case example. Later chapters go on to explore system performance, under various degrees of optimisation, while changing various physical and economic parameters.

6. Modelling Results

6.1 Chapter Introduction

In the last chapter we described the key design features of a lag-one Markov (stochastic) computational modelling system, which we implemented. Starting in this chapter, for the next three chapters, we run a series of high level experiments to investigate market performance under an assumed real-world lag-one Markov Chain level of uncertainty, in the simulation module. In the optimisation module, we vary the market being modelled at varying levels of complexity, namely: lag-one Markov Chain (MC), Stochastic Independent (SI), and Deterministic (DC).

The optimisation objective is to maximise net welfare, and the underlying stochastic process is modelled as Markov lag-one. Hence, we expect an optimisation model which is designed to operate at a Markov lag-one level of complexity (the MC optimised runs) will perform best. But it is not clear, a priori, how much better it will perform, and for what kind of reservoir system that performance difference will be significant enough to make it worthwhile to model stochasticity in the market clearing process. Thus we aim to determine when is it important to represent uncertainty in the optimisation, and/or market clearing, depending on various system parameters.

While our experiments use simplified (hypothetical) data, and hence the observations and resulting conclusions may not apply precisely to any particular real system, we believe they do indicate the range of system parameters over which incorporating stochasticity into the market clearing process is likely to add most value to the market.

Our system can be run over a wide range of physical and economic parameters, and can produce outputs in many varying forms. For each main experiment, we fix some reservoir modelling system parameters while varying others. We do this to review the impact of each of these parameter changes on market performance. In this chapter we showcase the wide range of optimisation and simulation reporting measures that the modelling system can generate, for a run with fixed physical and economic parameters i.e. results in this chapter are based on a single case. In Chapters 7 and 8 we explore varying the physical and economic parameter settings, respectively, to review their impact on market performance, for

often many cases at a time. Many experiments in Chapters 7 and 8 assume that various parameters are the same as those described in this chapter.

In the first sub-section of this chapter we give an overview of the parameters which are fixed for this set of experiments, and act as a basis for future experiments. In the second sub-section we present, in varying forms, the optimisation outputs from the system. In the third sub-section we discuss detailed simulation outputs, and associated reporting measures. Finally, we present some conclusions.

6.2 Experimental Model and Parameter Settings

In all experiments we assume an informed inflow allocation policy. We also generally assume non-state dependent demand, apart from in Section 8.4. In our experiments we generally report annual measures using the RS module, with the valuation of performance for each case employing a Monte Carlo simulation with 10,000 samples. The RC module is used to back-up the limited sampling results, as required.

Compared with a simple DC optimisation, the MC optimisation produces a much more dynamic policy, in which the marginal value of water depends strongly on the current hydrology state. In part this is because we are assuming an “informed policy”, in which the decision maker is assumed to know the inflow state for the coming month, at the beginning of that month. Since we make this assumption in our simulation (later), it seems reasonable to assume that a decision maker would interpret and adapt the results from a DC optimisation in a similar manner. So we have adapted our CDDP so that it can also produce intra-period release decisions that depend on the state observed at the beginning of each period, while still assuming a deterministic model of future periods.

To do this all three optimisation run complexity assumptions (DC, SI and MC) use the full range of conditional inflow values, to generate conditional hydrology state lists (DCSs). As such, even a DC optimisation uses all 7 inflow states in the computation of its associated DCS. But, crucially, the DC optimisation only applies a probability weight of 1 to the middle inflow state. The SI optimisation applies the steady-state (long run) probabilities from the corresponding MC Probability Distribution.

In all experiments we assume twelve monthly time steps, within the annual water market. We assume seven hydrology states. And, without loss of generality hydrology states go from driest ($h=1$) to wettest ($h=7$) about the mean/middle ($h = 4$) state value. The Deterministic hydrology state is effectively the mean. We use the same probability values in the optimisation and simulation, and for each period. Hydrology states are made “sticky”, by applying a high (50%) weight to the leading diagonal. This assumption is reasonable, in many places when it is drier or wetter than average it is likely to stay in that condition, due to seasonal and wider events, for example the El Niño-Southern Oscillation (ENSO) cycle. The probability distribution used, however, is not based on actual data. Within a single stage, the MC optimisation can transition from any hydrology state to another except for going directly from the driest to wettest states. The SI optimisation uses the corresponding long-run steady state probability distribution values from the Markov distribution, in each stage and state. The DC optimisation applies a weight of one on the middle (mean) hydrology state. See Appendix 6, Table 6A.1, for the explicit probability distribution values used.

Our experiments are carefully constructed such that we match average water quantities input and output from our model, F and Q respectively, see discussions in Section 5.4. So, deciding on the average inflow means also making decisions about average demand. Further, there is also some relativity between the average inflow, demand, and the physical storage and release capacities, $SCAP$ and $RCAP$ respectively, of the reservoir. We decided to fix the monthly average inflow, $FMEAN$, and scale everything from this point.

The number assigned to $FMEAN$ is somewhat arbitrary. For convenience we decided to use a highly divisible number, which simplifies for the construction of “strictly comparable” experiments, especially for scaling the reservoir capacities in Chapter 7. As such, we set $FMEAN = 84$.

In our experiments we are actively seeking differences due to uncertainty in both supply and demand. Hence, we assume that inflow varies by state and stage, otherwise there would be no supply difference and potentially a very simple situation eventuates. So, we carefully construct our base case, used in Chapters 6 through 8, such that there is a high amount of seasonality and a high amount of inflow variance.¹⁰¹ We term this the “Extreme” case. Of

¹⁰¹ Inflow variance and seasonality are varied in experiments in Sections 7.5.1 and 7.5.2, respectively.

course, many other real-world systems might be much more extreme, especially in more recent years with the growing awareness of the impact of climate change. On the other hand, many other reservoir systems might be much more moderate, and easy to manage.

Using FMEAN and Equation (5.8) we fix the seasonal weighting factor to approximately 90% ($W_F = 0.9048$) and fix the deviation factor DF ($\pm 78, 54, 28$) to non-zero values for the non-mean hydrology state. We select these values such that in the driest inflow state, at the driest time of the year, we have an inflow value greater than zero. Our extreme inflow pattern varies from 309 units the wettest month and state ($h=7$ and $t=3$), down to 1 in the driest month and state ($h=1$ and $t=9$). Thus the total inflow range is approximately 3.67 times FMEAN. See Appendix 6, Table 6A.2, for the explicit inflow distribution values used.

We generally assume that demand is out of phase with inflow, in the sense that peak demand occurs in the opposite season (the driest) from peak inflow. But this is varied in Section 7.5.3. Hence we use a lag factor of $W_L = \pi$ in Equation (5.10) to generate corresponding QREF' points.

We generally assume constant elasticity curves to represent our DCRs, as discussed in Section 5.4.4, and apart from in Section 8.3 where we explore the impact of different DCR shapes. The issue is now: Which W_E factor to use to generate our constant elasticity curves from Equation (5.13)?

There is various literature on price elasticity of demand for water, but few numerical estimates. Nevertheless one article quotes price elasticity of demand for municipal water, in Tucson, Arizona for over a 25 year period, Young (1973). Price elasticity was estimated at about -0.63 during 1946-1964 and -0.41 during 1965-1971. A more recent paper on price elasticity of demand for irrigation in the Goulburn–Murray Irrigation District, in Australia, is provided by Wheeler et al. (2008). This provides a good literature review of similar studies and quotes their short and long run demand elasticities as between -0.52 and -0.81, respectively for real water trading. Schoengold et al. (2006) quoted a similar figure near the upper bound of this at -0.79. For urban water Worthington & Hoffman (2008) quote short run price elasticities of between 0 and -0.5.

Chapter 6: Modelling Results

Of course water can be required for many purposes, each with its own price elasticity. For example, electricity consumers tend to have lower price elasticities, than those discussed above, and this may imply similarly low elasticities for water used by hydro-generation, in a hydro dominated power system. For this review, we assume water consumers could be competing with non-consumptive users, such as hydro-generators. In that setting, those associated elasticities would also need to be considered.

Dale et al. (2009) compare electricity and water and natural gas elasticities in California. They summarise that the average values for all three commodities ranged between -0.2 and -0.5. But they note that values could take on a wider range, depending on the estimation method involved. Fan & Hyndman (2011) discuss price elasticity values for electricity in South Australia, quoting historical annual values ranging from -0.363 and -0.428. But they note these can vary significantly based on time of day and time of year. Bernstein & Griffin (2006) conduct a survey of US national short-run elasticities quoting residential and commercial values of -0.24 and -0.21, respectively. They also review regional, state, and utility company short-run elasticities. They note that elasticities vary in each particular instance. Dahl (2011) conducts a global survey from over 60 countries, mostly in the OECD, quoting average and median constant elasticity values of -0.21 and -0.14, respectively.

For market operation we are only concerned with short-run elasticity values, although there are also real-time price elasticities. Lijesen (2007) reports real-time price elasticities, based on Dutch electricity prices in 2003. He concludes that it depends on what proportion of electricity is traded in the spot market (in the Dutch instance this was 15% on average). He reports log linear and linear constant elasticity values of -0.029 and -0.009, respectively.

As expected, because electricity cannot be stored, real-time elasticities are close to zero. The short-run elasticity values in the electricity papers we surveyed ranged -0.14 to -0.5, while the earlier short-run elasticity values, in the water papers we surveyed, ranged 0 and -0.81. We presented early results in Starkey et al. (2012) using elasticity values of -0.25, and these generally produced a much larger price variation. Here, for our base case we generally assume a more moderate elasticity of -0.5, and then vary this, and the type of demand curve, in Chapter 8.

Chapter 6: Modelling Results

As discussed in Section 5.4.4 we assume a price reference point, $MREF = 1$. Using this, $E = 0.5$, and the computed $QREF^t$ in Equation (5.10) which produce twelve DCRs, one for each month of the water year. Values for the first ten most highly valued release units are shown in Table 6.1. Note that periods 1 and 2 are the same as 5 and 4, respectively, while 6, 7 and 8 are the same as 12, 11 and 10, respectively. Thus this repetitive data has been omitted. The cumulative value achievable in each period is given in the final row.

q ↓ / t	3	4	5	6	7	8	9
1	64	324	2116	7056	14884	22500	25600
2	16	81	529	1764	3721	5625	6400
3	7	36	235	784	1654	2500	2844
4	4	20	132	441	930	1406	1600
5	3	13	85	282	595	900	1024
6	2	9	59	196	413	625	711
7	1	7	43	144	304	459	522
8	1	5	33	110	233	352	400
9	1	4	26	87	184	278	316
10	1	3	21	71	149	225	256
DCR Total	105	530	3462	11544	24351	36811	41883

Table 6.1: First Release values from a DCR with a -0.5 Constant Elasticity curve.¹⁰²

The lowest, average, and highest Marginal Benefit Stair Plots are shown in Figure 6.1. Given the large difference in values the Marginal Bid Price (vertical axes) is shown on a log-scale.

¹⁰² With values approximated to 1.d.p.

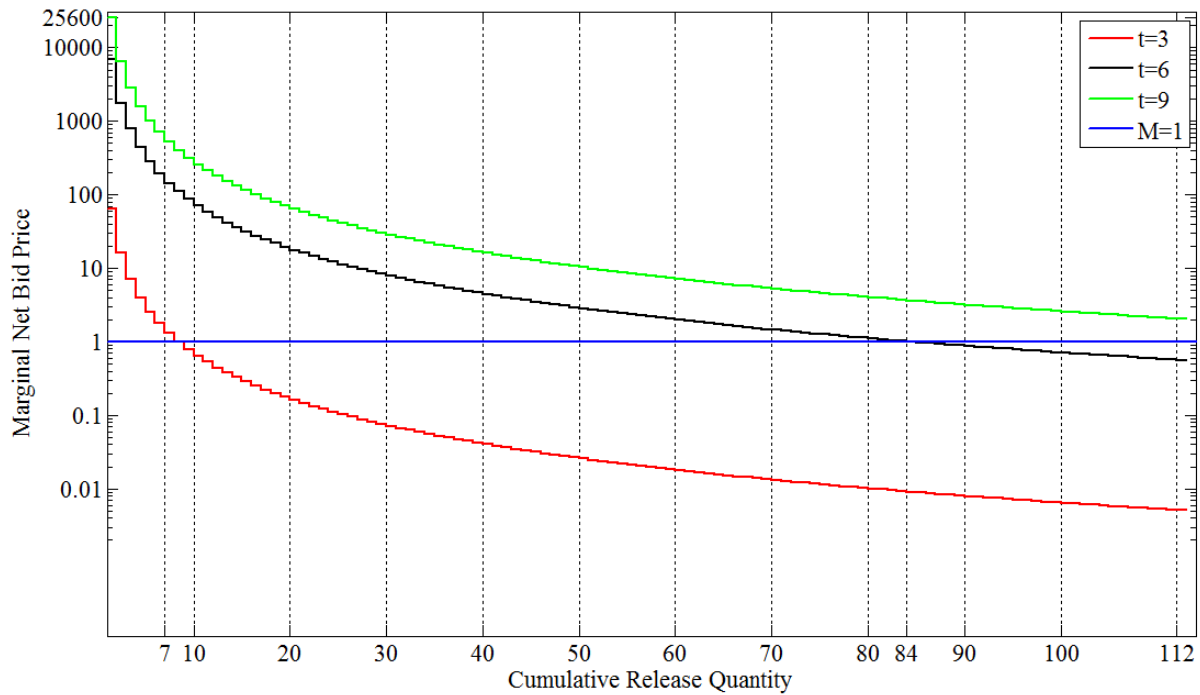


Figure 6.1: DCR Data for the Least (Wettest), Average, and Most (Driest) Months

From Figure 6.1 clearly in this experiment water is valued a lot more in the dry season than the wet season, 400 times more from peak ($t=9$) to trough ($t=3$). The reference price (MREF) of 1 cuts in for months 3 and 6 at steps (QREF) 8 and 84, respectively.

The reservoir storage and release capacities are configured as a multiple of FMEAN. For these experiments the storage capacity (SCAP) is fixed at the average annual inflow value, i.e. $SCAP=12FMEAN=1008$ units, while the release capacity is $RCAP=1.33FMEAN=112$ units.

The hydrology and demand data are used to create optimisation outputs, as now discussed.

6.3 Optimisation Outputs

In this section we discuss outputs from the Reservoir Optimisation (RO) module. We can combine optimisation inputs (DCR) and outputs in various ways, to generate simplistic storage, release, empty and spill data. The main optimisation outputs consist of market clearing prices, in the DCS, and the optimal allocation quantities, in the Conditional Optimal Release Schedule, CORS.

The optimisation generates its price and release value (DCS and CORS) surfaces, or operating policy, for the entire reservoir system state space. This space comprises the conditional hydrology states, the storage state, and each monthly decision period. In this section we report (simplistic) average value comparisons for the DC, SI, and MC levels of optimisation run complexity. These reports give monthly and annual average values. To achieve this we multiply the conditional storage grids (t, s, h) by their corresponding steady-state (long run) probabilities to produce a single “expected” benefit grid, (t, s) . The mean (expected) value for each decision period is then computed. Results for the DC, SI and MC optimisation runs, are now discussed in more detail.

6.3.1 Outputs Using Steady-State (long run) Probabilities

One of the key optimal outputs from the RO module is the CORS. We can combine this with the steady-state (long run) probabilities $(\tilde{P}^{t,h})$, to compute the expected Total Benefit (TB), and the expected storage release, and spill levels. We could also apply them to the MWV CDSCS curves to compute an Expected DCS (EDCS) MWV.

Having computed these we can report that the DC and MC cases return the highest and lowest expected benefits, from the corresponding highest and lowest expected release levels, respectively. The DC and MC cases keep the reservoir level lowest and highest on average, respectively. And DC, SI, and MC cases produce the same amount of spill. The DC and MC runs produce the lowest and highest EDCS MWVs, over the entire water year.

But, using the steady-state probabilities to apply weights to each conditional hydrology state assumes that all storage states are equally likely, and in reality we do not know which storage states will be visited. So using the steady-state probabilities to apply to storage states is an overly simplistic assessment, as we will show later in Section 6.4 with the simulated outputs. But the steady-state probabilities can be applied to the MWV curves, as discussed in the next section.

6.3.2 Marginal Water Values

Given the hydrology and bid data, the SCDDP optimisation module computes the MWV surface, the DCS. It computes DCS values for MC, SI and DC assumptions. For each optimised run, DCS grids (storage versus decision period) are output for each conditional hydrology state.

Given the SCDDP adaption discussed in Section 6.2 the DC CDDP also produces a range of state dependent DCS curves, as follows. Inspecting the DCS data for the DC optimised run, they are all shifted-down versions of the same MWV lists,¹⁰³ as shown in Table 6.2. This table shows the DCS MWVs for the first 20 units in storage, from the driest to wettest hydrology states, for the CDCS in period 9, the driest and most valuable time of the year. The grey shaded cells show the MWV of \$64.0 being traced for the seven corresponding inflow values of 1, 3, 5, 8, 11, 13 and 15.

s ↓ / h	1	2	3	4	5	6	7
1	25600.00	2844.44	1024.00	400.00	211.57	151.48	113.78
2	6400.00	1600.00	711.11	316.05	177.78	130.61	100.00
3	2844.44	1024.00	522.45	256.00	151.48	113.78	88.58
4	1600.00	711.11	400.00	211.57	130.61	100.00	79.01
5	1024.00	522.45	316.05	177.78	113.78	88.58	70.91
6	711.11	400.00	256.00	151.48	100.00	79.01	64.00
7	522.45	316.05	211.57	130.61	88.58	70.91	62.33
8	400.00	256.00	177.78	113.78	79.01	64.00	58.05
9	316.05	211.57	151.48	100.00	70.91	62.33	56.25
10	256.00	177.78	130.61	88.58	64.00	58.05	52.89
11	211.57	151.48	113.78	79.01	62.33	56.25	51.02
12	177.78	130.61	100.00	70.91	58.05	52.89	48.39
13	151.48	113.78	88.58	64.00	56.25	51.02	46.49
14	130.61	100.00	79.01	62.33	52.89	48.39	44.44
15	113.78	88.58	70.91	58.05	51.02	46.49	42.53
16	100.00	79.01	64.00	56.25	48.39	44.44	40.96
17	88.58	70.91	62.33	52.89	46.49	42.53	39.06
18	79.01	64.00	58.05	51.02	44.44	40.96	37.87
19	70.91	62.33	56.25	48.39	42.53	39.06	36.00
20	64.00	58.05	52.89	46.49	40.96	37.87	35.12

Table 6.2: CDCS MWVs for first few units in store, in the driest month.

Looking at Table 6.2, in these experiments the MWV for the driest ($h=1$) state, in the driest month of the year ($t = 9$) is \$25,600. Comparing this to the highest value in Table 6.1, from the DCR, means that the DC run keeps the highest price from the market, as a shortage cost, in equilibrium. But, without running a simulation we do not know if that MWV is in a “forbidden zone”, and the optimal policy actually never visits that storage state in practice. In that situation (stage and state) the market actually receives one unit of inflow, and thus should always be able to avoid running short.

In Figure 6.2 we graph the DC DCS Stair plots for the wettest, driest, and average hydrology states in months 3 and 9. Note that given the large differences in both MWV and across the

¹⁰³ Or horizontally-shifted curves.

Storage Space (1010 units, including ‘empty’ and ‘spill’ states) then log scales are used on the MWV (vertical) axes. Purple and pink indicates the driest ($t=9$) and wettest ($t=3$) period values. In Figure 6.2 the top and bottom illustrations are identical apart from the top illustration uses a log scale for the storage axis. From top to bottom, in either period (and plot) shown, the curves are ordered such that the MWV is highest in the driest hydrology state, and lowest in the wettest hydrology state. The 4th hydrology state, is actually the “true” state for a DC optimised run, in a DC world the other states would have a weighting of zero. The first unit is actually the reservoir running empty, thus it is the MWV of shortage.

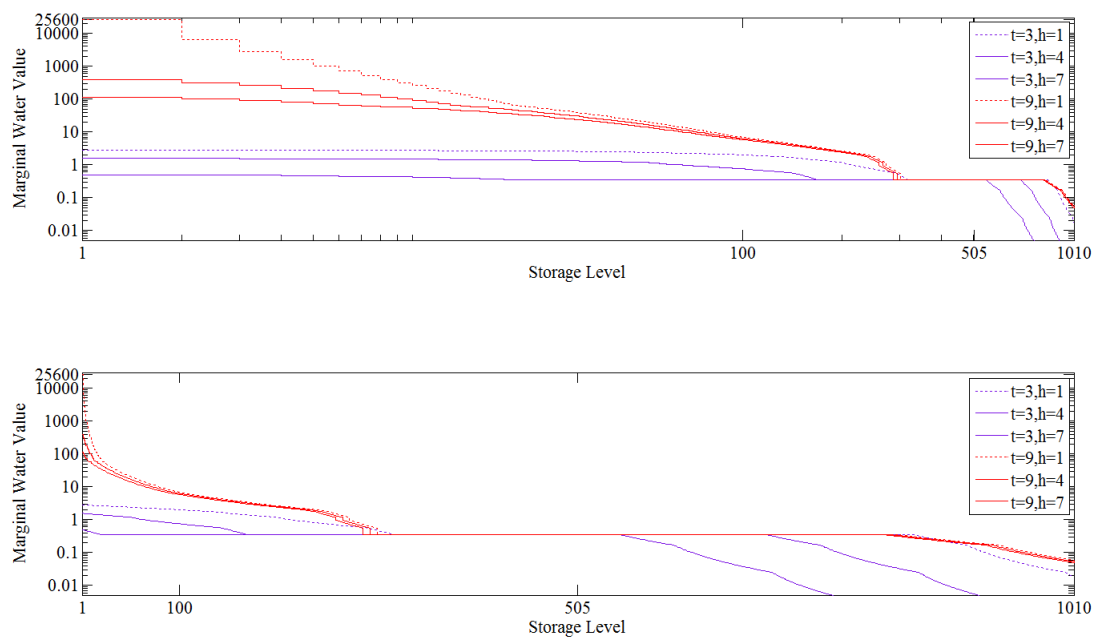


Figure 6.2: Conditional DCS Stairs Plot for Deterministic CDDP, for the Wettest and Driest Months

For the wettest month, $t = 3$, the MWVs range from approximately \$2.87 to \$0.50 from driest to wettest states. Generally all the DCS prices for this run are below 1 when the storage system is approximately 30% full. Most of the MWV states in the wettest period approach zero by the time the storage system is full, while in the driest period the MWV states appear more tightly bunched, but this is because of the log scaling, the MWVs are actually higher (larger) and wider ranging. The flat in both periods is present for a large proportion of the storage reservoir, and this is clearer in the bottom illustration with an equally spaced storage axis. This “flat” indicates a region where the conditional storage prices have stabilised, the single “flat” price is to be expected for a DC optimisation as it can only take the average decision in all states.

The corresponding MC run is shown in Figure 6.3. The driest and wettest states from the DC run in Figure 6.2 above are also shown, these act as guides to visually compare the two cases.

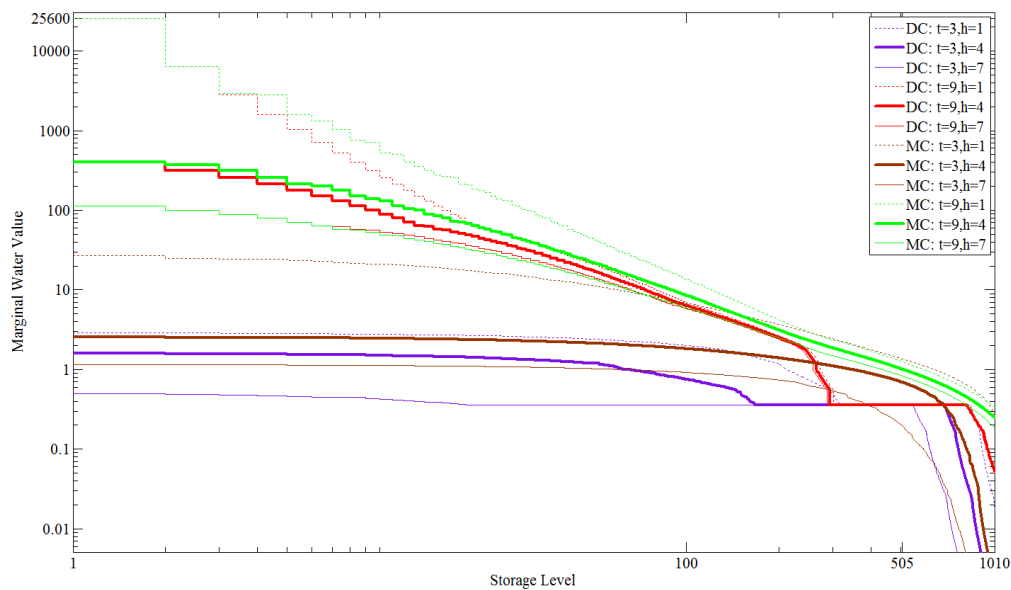


Figure 6.3: Conditional DCS Stairs Plot for Markov Chain CDDP, for the Wettest and Driest Months

From Figure 6.3, prices are highest during the driest times of the year, and in either period prices reduce with increasingly wetter hydrology states. Like the DC run, the MC run is still unable to remove the highest price from the market, in equilibrium (when $t = 9$ and $h = 1$, the MWV is \$25,600). At the driest time of the year ($t = 9$, and the green values) the driest and wettest states in the MC run still have the same value as the DC run (the red guidelines) for the first few units in store. In fact, it is only the intermediate wetter than average states (5 and 6) where maximum MWVs for the MC optimised run are approximately \$44 and \$27 higher than the corresponding DC run. For the wettest month ($t=3$, brown lines), the MC MWVs range from approximately \$27.21 to \$1.15 from driest to wettest hydrology states. These are significantly higher than the DC optimised run, as indicated by the DC purple guidelines. There is a noticeable shift upwards in the red curves, in Figure 6.3, compared to those in Figure 6.2.

Comparing with the DC DCS in Figure 6.3, the MC DCS sees a gradual drop in prices across the entire storage range, as opposed to a sudden collapse in price in the DC run. The flat that is present for the DC run is not present in the MC run. This indicates that the DC run always follows the single, average inflow trajectory, believing in its real optimum deterministic

“real-world”, prior to any simulation. Over the storage range the MC run produces a wider band of MWVs across the hydrology states, compared to the DC optimised run.

The driest time of the year ($t=9$), where water is most valued, is where we now focus our across optimisation run comparisons. First we compare the corresponding DC (shown in red) versus SI (shown in black) versus MC (shown in green) DCS stair plots, for the wettest, driest, and average hydrology states, as shown in Figure 6.4.

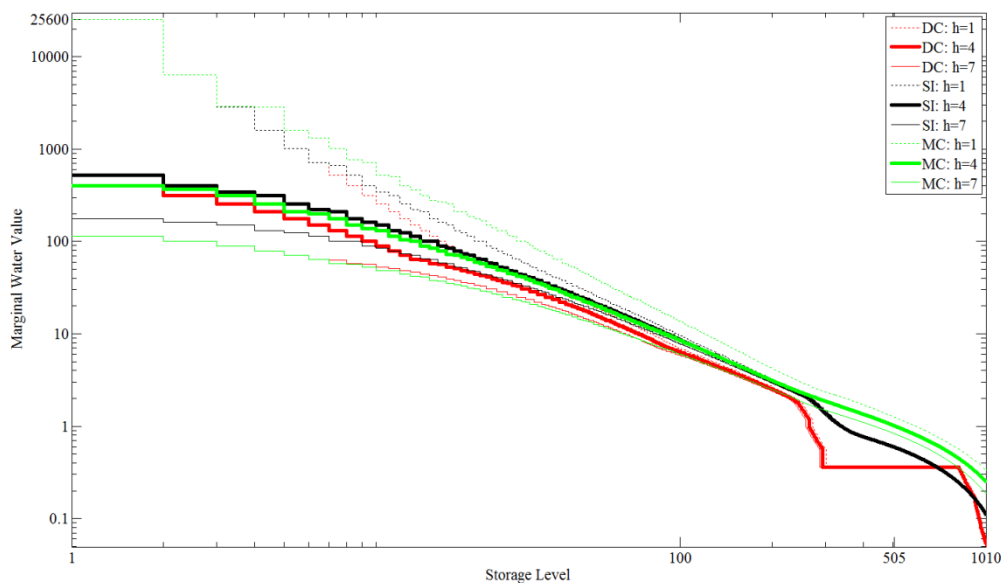


Figure 6.4: Conditional DCS Stairs Plot for all run complexities, for the wettest, average, and driest hydrology states, for the Driest Month.

From Figure 6.4, for the first few units in store, for the driest state (the highest line plots) all the optimisation run complexities result in the same MWVs. For the wettest state (the lowest line plots), the DC and MC optimisation run complexities result in the same MWVs, for the first few units in store. In the wettest and driest states the MC optimisation run produces the highest and lowest average MWVs for approximately the first 10% of units in store. By 30% of the storage capacity there has been a dramatic fall in MWV, in the DC optimised run, for all hydrology states. At this point there is less dramatic fall in MWV for the SI optimised run, yet still significant compared to the MC optimised run, for all hydrology states.

The SI optimisation run maintains the highest prices, for the first few hundred units in store, for the wettest and average hydrology states. But, generally the SI optimisation produces an

intermediate result, reflecting the fact that it can account for the range of possible inflow states, but not for their correlation, as with the MC optimised run.

Over all hydrology states the DC DCS curves take the narrowest range of MWVs, reflecting the fact that the DC SCDDP MCE does not account for a range of possible future hydrology states. The SI DCSs also take a narrower range of MWVs, when compared to the MC DCSs. This implies that the SI SCDDP MCE assigns less value to the water, in its own optimal world, compared to the corresponding MC optimised run.

We now review a similar graphic for the wettest time of the year, where water is least valued, as shown in Figure 6.5 below. This figure shows that, for the three hydrology states reported, the MC and DC optimisation runs value water the most and least, respectively, for the first 40% of SCAP. The vertical axes is a log scale (purely so it is easier to see the detail), and the MC and DC optimisations produce the widest and narrowest, respectively, range of prices over the modelled hydrology states. In the driest hydrology state, the MC optimisation has much higher MWVs than the SI and DC optimised runs. In the wettest hydrology state, the DC optimisation generally has much lower MWVs than the SI and MC optimised runs. But, between approximately 40 and 60 % of the range of storage capacity the MC and SI cases have the lowest MWVs, respectively. As the reservoir becomes fuller, each optimised run (MC, SI, and finally DC) signals that there is more available water by reducing the MWV. The DC optimised run is unable to fully experience the hydrology which is arriving, given it cannot weight the non-average hydrology state MWVs. Thus the DC optimisation policy continues forward, until the average hydrology state eventually signals that the reservoir might become too full, relative to future demands, in other periods.

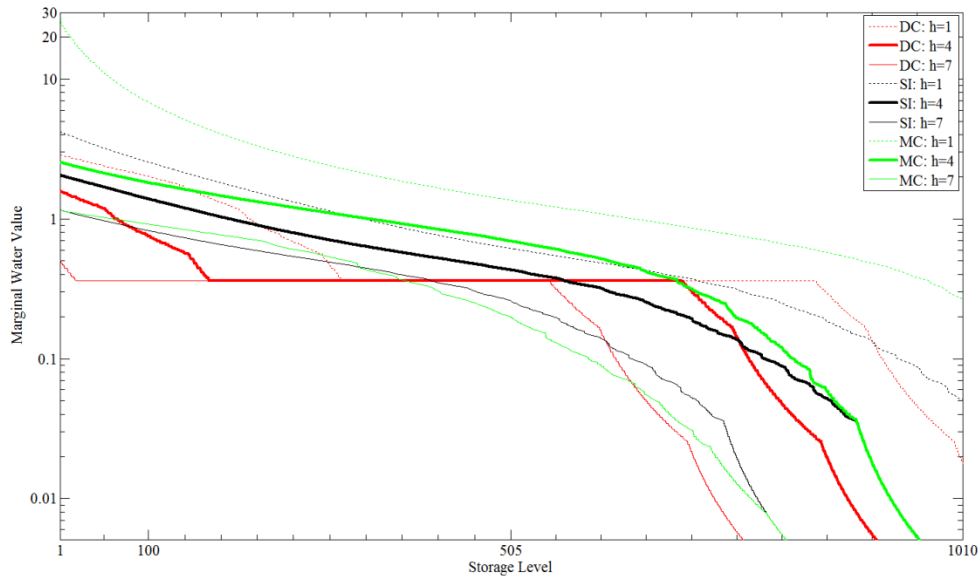


Figure 6.5: Conditional DCS Stairs Plot for all run complexities, for the wettest, average, and driest hydrology states, for the Wettest Month.

In order to facilitate average comparisons, the Conditional DCS grids can be multiplied by their corresponding steady state probability to produce a single Expected DCS grid, via:

$$EDCS'_{RO}(s) = \sum_h \tilde{P}^{t,h} \times DCS^{t,h}(s) \quad \text{for all } t \quad (6.1)$$

Employing Equation (6.1) gives the results in Table 6.3. These Expected DCS results are for DC, SI and MC optimised runs, in that order, for the first several reservoir storage states, and the last two reservoir storage states. The table shows values for all 12 monthly periods and the last row for each optimisation run complexity, shows the total MWV for all steps of the DCS (from 0 to SMAX+1).

DC		Stage											
Storage Level	1	2	3	4	5	6	7	8	9	10	11	12	
0	3.02	0.85	1.60	3.24	8.08	33.33	204.16	2389.77	3235.00	2349.14	175.79	21.11	
1	2.53	0.80	1.59	3.23	7.55	27.77	131.99	706.21	1093.46	662.98	103.57	15.88	
2	2.16	0.77	1.58	3.21	7.13	24.38	97.46	375.15	611.01	335.81	69.67	12.49	
3	1.88	0.74	1.57	3.20	6.83	21.97	78.64	260.32	408.80	213.72	50.93	10.14	
4	1.66	0.73	1.56	3.17	6.75	20.24	67.44	197.34	299.64	152.96	39.38	8.45	
5	1.49	0.72	1.55	3.17	6.69	18.81	58.46	165.98	232.10	117.39	31.71	7.18	
6	1.34	0.72	1.54	3.15	6.65	17.73	53.46	138.70	187.03	94.29	26.32	6.21	
...													
SMAX	0.00	0.00	0.00	0.04	0.14	0.26	0.29	0.18	0.05	0.01	0.00	0.00	
SMAX +1	0.00	0.00	0.00	0.04	0.14	0.21	0.29	0.18	0.05	0.01	0.00	0.00	
TOTAL	251	260	400	746	1348	2361	4107	8518	9252	5351	1059	384	
SI		Stage											
Storage Level	1	2	3	4	5	6	7	8	9	10	11	12	
0	3.50	1.44	2.27	4.44	10.75	38.72	225.68	2476.05	3290.75	2354.68	176.60	21.41	
1	3.01	1.39	2.26	4.41	10.66	33.95	151.09	775.16	1139.30	668.27	104.41	16.22	
2	2.65	1.39	2.25	4.38	10.47	32.94	133.80	452.37	677.74	341.71	70.46	12.83	
3	2.37	1.38	2.24	4.34	10.16	31.56	113.74	317.83	475.45	219.96	51.71	10.48	
4	2.16	1.38	2.23	4.31	10.04	29.24	97.83	279.37	373.85	160.30	40.18	8.78	
5	1.99	1.37	2.22	4.28	9.82	28.60	92.11	239.23	298.89	124.13	32.59	7.52	
6	1.85	1.37	2.21	4.26	9.68	27.18	80.35	206.96	267.54	101.01	27.16	6.55	
...													
SMAX	0.01	0.01	0.01	0.04	0.11	0.20	0.23	0.17	0.11	0.06	0.03	0.01	
SMAX +1	0.01	0.01	0.01	0.04	0.10	0.16	0.22	0.17	0.11	0.06	0.03	0.01	
DCS <i>t</i> Total	361	404	602	1020	1766	3038	5287	10450	11113	5937	1262	498	
MC		Stage											
Storage Level	1	2	3	4	5	6	7	8	9	10	11	12	
0	4.28	3.27	5.53	11.24	29.41	101.06	379.20	2482.71	3246.22	2349.64	176.02	21.67	
1	3.78	3.24	5.30	11.15	28.77	81.01	273.57	812.56	1112.07	663.72	103.80	16.49	
2	3.41	3.22	5.23	10.81	27.37	77.38	235.43	597.95	641.04	336.56	69.90	13.10	
3	3.13	3.20	5.19	10.65	25.99	74.88	207.33	443.88	600.87	215.26	51.27	10.76	
4	2.91	3.17	5.07	10.54	25.41	68.83	167.47	358.93	412.77	154.37	39.73	9.07	
5	2.73	3.14	5.04	9.78	23.39	66.80	158.10	327.93	337.34	119.95	32.08	7.82	
6	2.59	3.10	4.92	9.59	23.15	57.26	153.19	291.75	294.39	98.82	26.68	6.85	
...													
SMAX	0.05	0.03	0.03	0.07	0.19	0.38	0.43	0.35	0.26	0.17	0.11	0.07	
SMAX +1	0.05	0.03	0.03	0.07	0.19	0.32	0.42	0.35	0.26	0.17	0.11	0.07	
DCS <i>t</i> Total	661	756	1053	1624	2622	4313	7073	11933	11721	6294	1570	791	

Table 6.3 MWVs from Expected DCS for all Optimisation Runs

From Table 6.3, for the first half of the year, and at the end of the water year ($t=12$), the MWVs are minimal, as are the associated MWV price surfaces that could be constructed from this table. The DC and MC optimised runs have the lowest and highest MWVs, over the entire water year. The expected annual total MWV for the entire surface for the DC, SI, and MC optimised runs are \$34035, \$41740, and \$50411, respectively. The maximum MWV is in the MC optimised run, approximately \$3246 in period 9, for the first unit in storage. Still, this is much less than the potential value of the first unit released at the same time of year, some \$25,600 (see Table 6.1 above). Throughout the wetter half of the water year (periods 1 through 6) the MC optimised run appears to signal conserving water until the dry

season, at least more directly than the SI, and DC corresponding optimisation runs. The MC optimised run does this by having higher average MWV prices in the DCS.

Figure 6.6 shows the Expected DCS MWV stair plots for periods 7 through 9. Here the CDDP results are presented for all three optimisation run complexities. Note that leading up to the critical period, months 7 and 8, the MC optimised run increases the MWV more than the SI optimised run, and the SI optimised run increases the MWV more than the DC optimised run. (The simulated monthly MCPs are shown in Section 6.4.4 below).

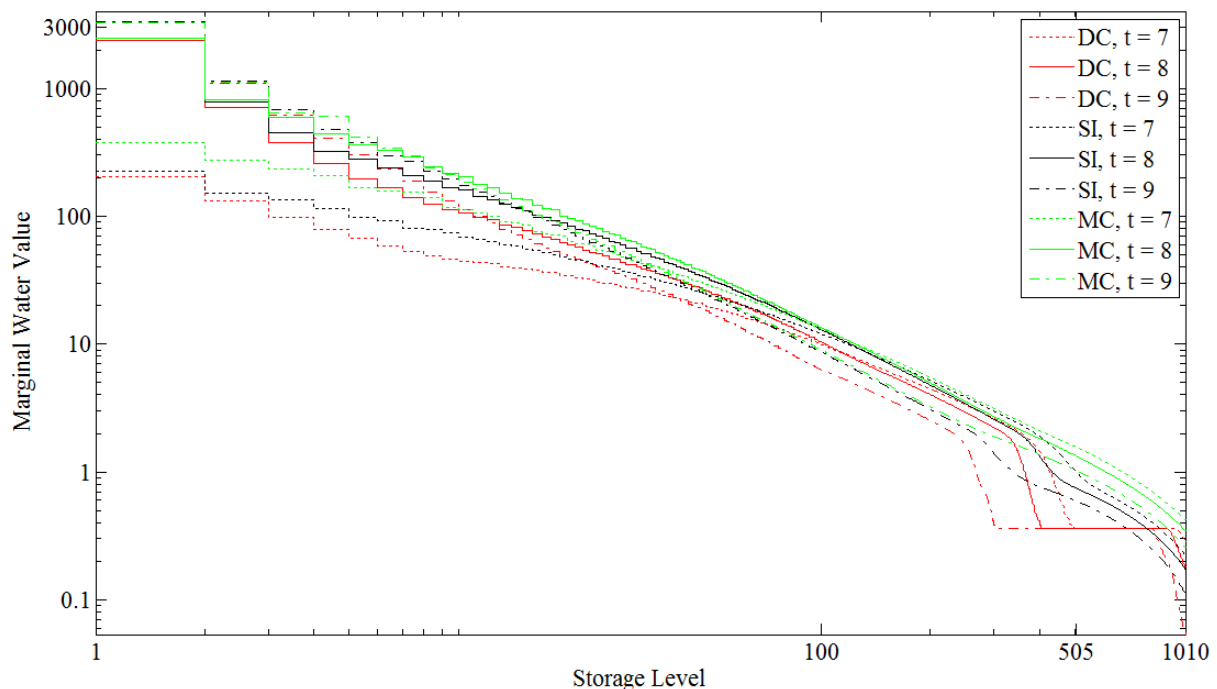


Figure 6.6: Expected DCS Stairs Plot for DC, SI and MC CDDP, for months leading to the peak of the dry season¹⁰⁴

The actual difference between optimisation runs depends on how full the reservoir is. When the reservoir is between approximately 10 and 30% of capacity there appears to be no great difference in MWV for any CDDP optimised run complexity. But, the MWVs are shown on a log scale, so differences are bigger than they appear. On closer inspection, the MC optimisation keeps MWVs higher than the SI optimisation, and SI optimisation keeps MWVs higher than the DC optimisation. There is a sharp reduction in MWV at about 30% of

¹⁰⁴ While on average there are 84 inflow units per month, in the dry season the average number of inflow units is a lot less.

capacity for both the DC, and SI optimised runs, and this is not present in the MC optimised run.

Interestingly, at the upper storage bound the MC optimisation seems to have a consistently higher MWV than the SI optimisation. And although it also appears that the MC and DC optimisations produce the highest and lowest MWVs (in periods 7 and 9, respectively), there is no clear pattern. At the lower bound, in period 7 and 9 the MC and DC optimised runs have the lowest and highest MWVs, while they both have the highest and lowest MWVs near the upper bound. This could be in part due to the fact that water is more valuable going into the peak of the wet season, than after the peak, when the value of water begins to fall.

Figure 6.7 below shows the annualised Expected DCS MWV stair plot for DC, SI, and MC optimised runs. Here all the monthly values, after being weighted and aggregated by their steady-state (long run) probabilities, they have simply been averaged over the year, at each storage level.

From Figure 6.7 the MC and DC cases generally always have the highest and lowest average MWVs over the water year. At the lower storage bound, for the first few units in store, all three optimised runs generate similar MWVs. At the upper storage bound the SI and DC cases have very similar MWVs, which are proportionally much less than in the MC optimised run. This indicates that, on average over the water year, the MC optimisation is always managing to keep a higher MWV in the storage market. We might interpret these differences as ‘risk premiums.’ Of course, more water stored in one period, results in more water available for demand in the next period. And given there is no actual storage market, and only demand market, then this should be seen as a good thing.

Another view might be that over the year the MC optimisation should eventually reduce MWVs as it manages to clear all the high value demand bids, which subsequently do not appear in the storage market, but this does not appear to be the case.

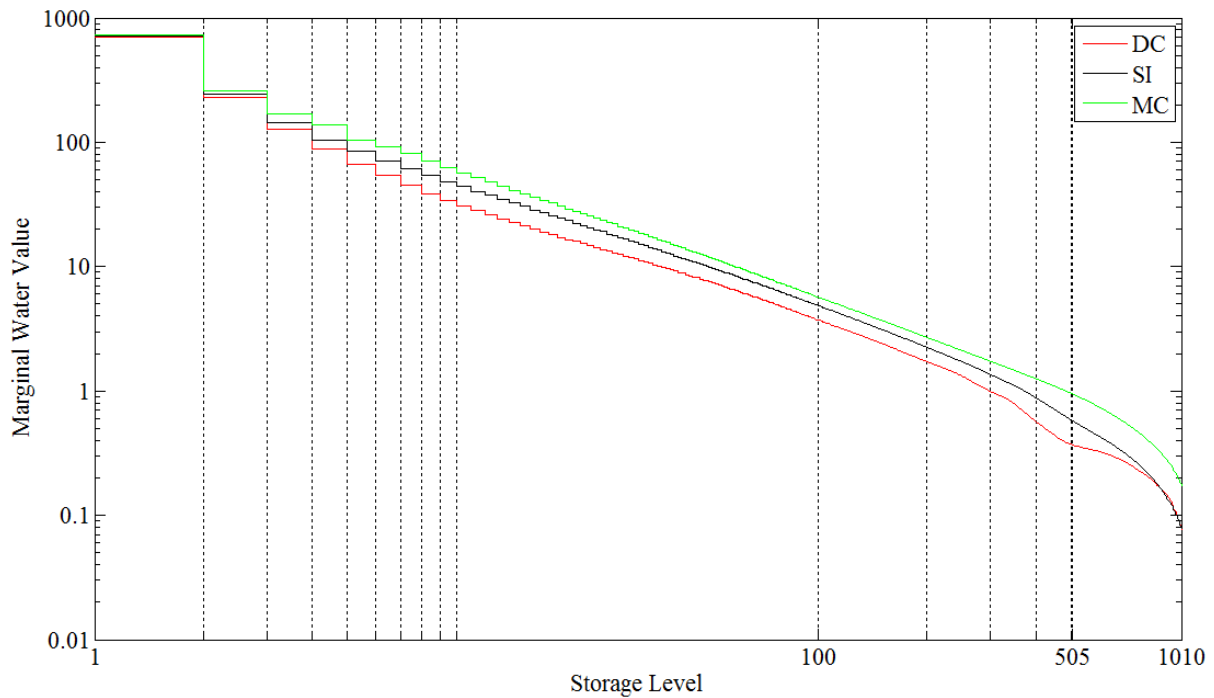


Figure 6.7: Expected Annual DCS Stairs Plot for DC, SI and MC CDDP

6.3.3 Summary

The main points surrounding the outputs from the optimisation module are that the DC and MC optimisation runs produce the lowest and highest MWVs in all months. We might also expect a more sophisticated optimisation to have a pervasive impact on the average marginal water value level, hopefully setting values that do a better job of steering solutions away from the spill and/or shortage bounds. This can be investigated by inspecting the probabilities of being in a given state. So, many of these assessments are far too simplistic. We need to simulate outcomes over the entire state space.

6.4 Simulation Outputs

We test the DC, SI, and MC optimisations against an assumed, and simulated, MC level of real-world uncertainty. In Chapter 5 we discussed two main modules which we use to simulate performance of each of our optimised runs. First, a full simulation is performed by a Markov Chain equilibrium calculation. Second, the sampled simulation is performed by a Monte-Carlo calculation. Simulated outputs are now discussed.

6.4.1 Storage

As well as these Expected Demand Curve surfaces, the Reservoir Convolution (RC) module produces corresponding hydrology indexed and non-indexed PDFs. Figure 6.8 shows a non-indexed PDF for the storage grid.

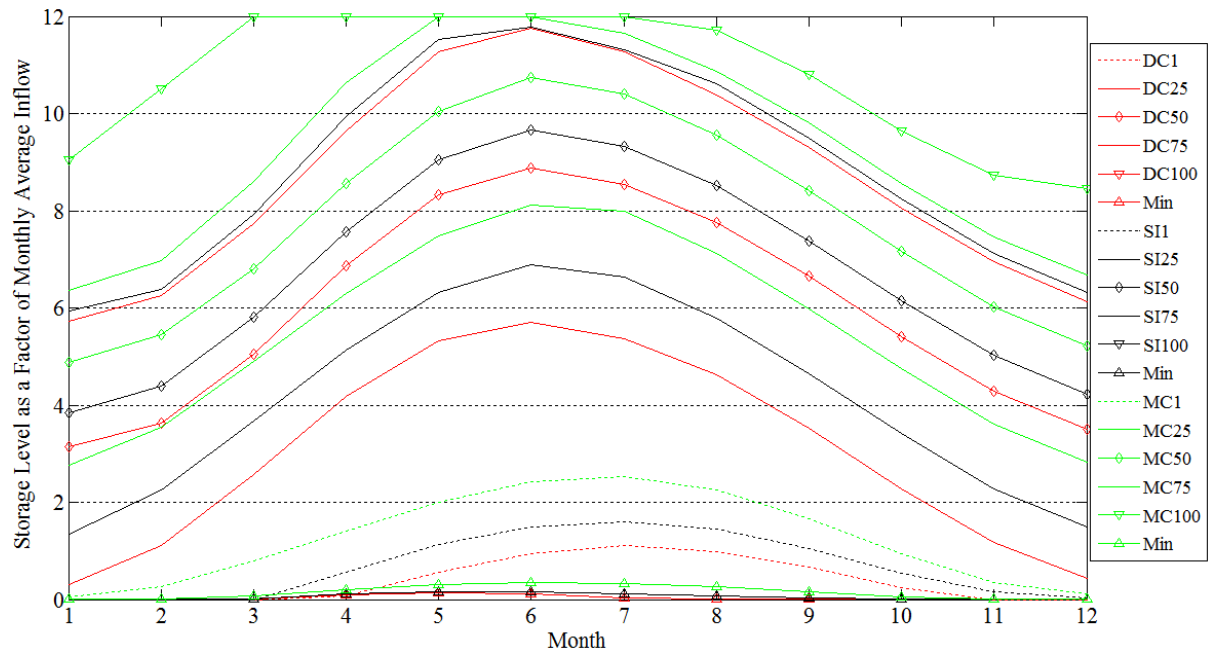


Figure 6.8: Non-indexed PDF of Storage Contour Plots, for all Optimisation Run Complexities

Figure 6.8 is created from the corresponding hydrology indexed PDF for storage grids,¹⁰⁵ and the DC, SI and MC optimisation run complexities are shown by Red, Black and Green lines, respectively. The upward arrow markers indicate the first non-zero probability; the dashed lines indicate the 1 percentile, while the downward arrow markers indicate the 100 percentile.¹⁰⁶ The 50 percentile (median) is indicated by Diamond markers. The other two intermediate whole lines are the 25 and 75 percentiles. All similar lines are ordered Red, Black, and then Green. Thus the DC optimisation keeps the reservoir least full, while the MC optimisation keeps the reservoir most full.

¹⁰⁵ The simulation module is always run at a Markov level of complexity, thus the probability weights in each conditional state naturally sum to a total state value of one. More precisely each Conditional PDF has grid points (q, t, h) . Summing for all h results in an Expected PDF with grid points (q, t) , and probability weights in each cell. Summing down the quantity vector, q , on the Expected PDF produces a total of 1 for each t . Alternatively, cumulatively summing down that same vector produces the Cumulative Expected PDF for each t .

¹⁰⁶ Note that DC100 and SI100 lie directly behind MC100.

Generally Figure 6.8 shows that near the lower storage bound (minimum non-zero probability values and the 1 percentile) the DC optimisation produces the least cautious optimal policy, by reducing the quickest, and thus indicating a higher average release. The SI optimised run is more cautious, whereas the MC optimised run is the most cautious. Apart from at the 100 percentile line, where all three optimisation runs sit on top of one another, at other storage levels we can see that the MC and DC optimisations are highest and lowest in all other shown percentile lines.

An interesting aspect is how the PDF for storage peak is not in-phase with either inflow (month 3 being the wettest) or demand (month 9 being the highest demand month). All optimisation policies generate peaks and troughs in the same months. And the median storage level is not symmetric, as evident from Table 6.4, which reports the median storage levels from Figure 6.8 above.

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
DC	265.00	306.00	424.00	578.00	700.00	747.00	718.00	653.00	559.00	455.00	361.00	294.00
SI	323.00	370.00	488.00	637.00	760.00	812.00	783.00	716.00	621.00	517.00	422.00	355.00
MC	411.00	458.00	573.00	719.00	844.00	904.00	875.00	803.00	707.00	602.00	507.00	438.00

Table 6.4: Monthly and Annual Median Storage Levels for all Optimisation Run Complexities

So, within the year the MC and DC optimisations are maintaining the highest and lowest median storage level in the reservoir. Another assessment of the difference between the DC, SI and MC Storage PDF functions is achieved by plotting their Annual CDF values; which are given in Figure 6.9.

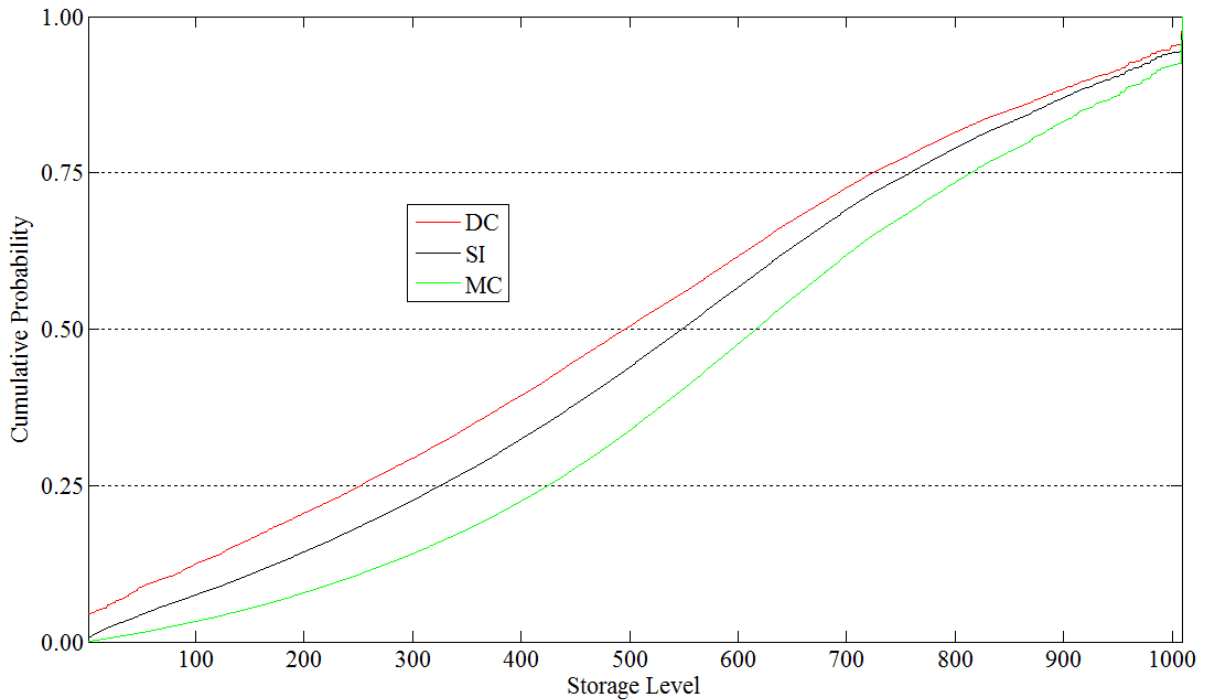


Figure 6.9: Annual Storage CDF for all Optimisation Run Complexities

In Figure 6.9, the Red, Black and Green lines are the DC, SI and MC optimisation run levels, respectively. The DC annual CDF has approximately a 5% probability of running empty, while the SI annual CDF has approximately a 1% probability of running empty. The MC annual CDF never runs empty. The MC and DC optimisations show the highest and least likelihood of the reservoir being full.

The net storage levels can be multiplied by their corresponding PDFS weights, and those conditional state weighted values added together to create an (s,t) grid. Summing those values for all storage levels creates the mean monthly storage level, as shown in Table 6.5.

	Month												Monthly Average
	1	2	3	4	5	6	7	8	9	10	11	12	
DC	276.34	321.62	435.53	571.87	671.37	707.50	687.98	629.33	539.96	439.91	350.96	293.97	493.9
SI	317.17	367.63	485.60	623.22	718.99	755.11	736.27	676.22	585.83	484.84	394.60	334.31	540.0
MC	383.84	441.13	559.84	694.77	784.26	820.63	804.35	743.12	651.21	548.83	456.94	395.52	607.0

Table 6.5: RC Monthly and Annual Mean Storage Levels for all Optimisation Run Complexities

Table 6.5 shows that the MC and DC optimisations keep the average storage level in the reservoir the highest and lowest, respectively. These mean results can be compared to the

medians in Table 6.4. The means are generally all lower than the medians, apart from the first three months in the DC optimised run. The differences are also generally smallest in the DC optimised run and highest in the MC optimised run. Both medians (6.4) and means (6.5) peak in period 6, at the transition from dry to wet season, and trough in period 1, the start of the water year.

The Reservoir Simulation (RS: via Monte Carlo simulation) records the corresponding sampled storage levels, as shown in Table 6.6.

	Month												Monthly Average
	1	2	3	4	5	6	7	8	9	10	11	12	
DC	276.65	321.99	435.97	572.54	671.98	708.09	688.65	629.93	540.49	440.36	351.35	294.32	494.4
SI	317.59	368.18	486.19	624.06	719.71	755.84	737.08	676.95	586.50	485.45	395.14	334.76	540.6
MC	384.74	442.08	560.79	695.97	785.32	821.73	805.49	744.19	652.26	549.87	457.95	396.44	608.1

Table 6.6: RS Monthly and Annual Start of Period Storage Levels for all Optimisation Run Complexities

Comparing Table 6.6 with Table 6.5, it can be seen that the RS results (6.6) are all a fraction higher than the RC results (6.5). Lowest differences are in the DC optimised run, the lowest 0.31 (0.03% of SCAP), and the highest in the MC optimised run, the highest 1.20 (0.12% of SCAP). The DC, SI, and MC RS results are on average 0.5, 0.61, and 1.03 units higher, and differences are slightly higher in the middle of the year (periods 4 through 9), when levels are also higher. Clearly the error is minimal, but minor differences are to be expected from a Monte-Carlo sample with only runs 10,000 samples over a grid size of 1008 storable units. Further, the Convolution is also run to an approximate equilibrium, where the starting and ending storage levels are within a 1×10^{-10} margin of error.

6.4.2 Release

In this section we focus on the average release. The CORS array is multiplied by the PDFS array and the resulting values are summed by hydrology states, and then storage state, to produce an average release value for each month of the water year. These values are given in Table 6.7.

	Month												Total	Monthly Average
	1	2	3	4	5	6	7	8	9	10	11	12		
DC	76.39	36.70	22.98	41.77	74.28	99.28	104.34	107.25	108.01	106.80	102.49	100.78	981.1	81.8
SI	71.32	32.70	20.97	42.17	72.26	98.43	105.80	108.31	108.95	108.17	106.02	100.61	975.7	81.3
MC	64.93	32.35	23.48	45.16	69.65	94.93	106.92	109.80	110.30	109.76	107.21	95.18	969.7	80.8

Table 6.7: Expected Monthly Releases from CORS and PDFS outputs, for all Optimisation Run Complexities

As expected, for all optimisation run complexities, Table 6.7 shows the lowest release at the peak of the low valued wet season, and highest release at the peak of the high valued dry season. The DC optimised run is utilising releasing the most, on average over the water year, while the MC optimised run is utilising releasing the least, on average over the water year. But, while the DC optimised run is releasing the most, on average, it is releasing it at less profitable times of the year, than the MC optimised run. The MC optimised run generally releases a few units less, an average of approximately 4, in the wet season (periods 1 through 5). Yet again, the SI optimised run is producing an intermediate result.

The Monte Carlo simulation records 10,000 sampled release levels for all periods. These can be averaged, and the results are given in Table 6.8.

	Month												Total	Monthly Average
	1	2	3	4	5	6	7	8	9	10	11	12		
DC	76.66	35.95	21.43	40.63	73.76	99.88	104.72	107.44	108.13	107.01	103.03	101.67	980.3	81.7
SI	71.41	31.91	19.35	40.86	71.87	98.95	106.13	108.45	109.05	108.31	106.38	101.18	973.9	81.2
MC	64.66	31.18	21.47	43.42	68.43	95.45	107.30	109.93	110.39	109.91	107.51	95.71	965.4	80.4

Table 6.8: Expected Monthly Releases from the Monte Carlo Simulation, for all Optimisation Run Complexities

Table 6.8 results are comparable to those in the combined optimisation and convolution in the immediate prior Table 6.7 above. It can be seen that the RS results (6.8) are generally a fraction lower than the RC results (6.7) in periods 1 through 5, and they are higher in periods 6 through 12. Lowest (-2) and highest (0.5) differences are in the MC optimised run, and the DC, SI, and MC RS results are on average 0.06, 0.15, and 0.36 units lower in the RS (6.8) case. The difference in values is relatively minor.

6.4.3 Reservoir Bounds: Spill and Empty

In this section we briefly report spill and empty probabilities from the RS module. We have compared these results to the RC module, and we can confirm that any differences are negligibly small. RS generated spill probabilities are given in Table 6.9. For this case, spills are only possible in periods 2 through 6, thus the non-event results are omitted.

	Month				
	2	3	4	5	6
DC	0.26%	2.99%	12.05%	17.44%	10.12%
SI	0.29%	4.13%	14.70%	21.07%	11.09%
MC	0.39%	5.18%	18.86%	26.35%	13.84%

Table 6.9 Expected Spill Probabilities from the RS module, for all Optimisation Runs

Table 6.9, shows that period 5 is most likely to have spill. This is the start of the transition from the wet to dry season. This is to be expected, the reservoir is filling up in preparation for the dry season, when it is trying to target highly valued demand. This also matches the contour plot in Figure 6.8.

Table 6.10 shows the corresponding RS generated probabilities for the system running empty, again non-events are omitted. Generally, the DC and MC optimisations are simulated to run empty the most and least, respectively. The DC optimisation is approximately 59 and 7 times as likely to run empty compared to the MC and SI optimisations.

	Month							
	1	2	3	8	9	10	11	12
DC	20.52%	7.72%	2.28%	0.05%	0.05%	0.18%	2.53%	19.41%
SI	3.85%	2.35%	1.10%	0.00%	0.00%	0.01%	0.07%	0.58%
MC	0.47%	0.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.08%

Table 6.10 Expected Empty Probabilities from the RS module, for all Optimisation Run Complexities

If spilling or running empty had a large negative impact in the real world, for example flooding land downstream or killing all aquatic life in the reservoir, then these would need to be taken into account.

6.4.4 Prices

Here we discuss the Weighted Average Prices (WAPs) which could be used to trade in the market. As discussed in Section 5.6, we use Time Weighted Average Prices (TWAPs) and Volume Weighted Average Prices (VWAPs). The TWAPs are weighted by the number of samples (and periods) for which the market price occurs. The VWAPs are weighted by the number of units traded, at that price over all simulations. The WAPs we report are generated from the MCPs from the RS module.

We expect to observe that, in a Markov Chain Real World, a model which more closely represents the underlying uncertainty should more effectively move stored water from periods of low value, to periods of high value. Also, an efficient market should flatten the actual price paid for water over the year.

6.4.4.1 TWAP for Storage (TWAPS) and TWAP for Release (TWAPR)

In this section we briefly review TWAPS and TWAPR, and some of their risk measures. Table 6.11 shows the TWAPS monthly and annual averages, for the MC, SI, and DC optimisation run complexities. Generally the MC optimisation produces the flattest prices over the water year. In the second half of the water year (periods 7 through 12), the DC and MC optimisations produce the highest and lowest corresponding TWAPSs. In the first half of the year, generally, the DC and MC optimisations tend to produce the lowest and highest corresponding prices. And, the SI optimisation generally produces an intermediate result. This type of pattern is expected, given that the first half of the year is the wet season, where water has low value, and the second half of the year is the dry season, where water has high value.

	Month												Annual Average
	1	2	3	4	5	6	7	8	9	10	11	12	
DC	1.21	0.44	0.51	0.58	0.60	0.78	1.68	8.27	15.01	20.29	8.94	6.26	5.38
SI	0.96	0.57	0.62	0.59	0.59	0.77	1.06	1.15	1.44	2.00	1.51	1.59	1.07
MC	0.89	0.84	0.83	0.71	0.67	0.81	0.98	0.98	0.99	1.00	1.00	0.99	0.89

Table 6.11: Monthly and Annual TWAPS, for all Optimisation Run Complexities

Table 6.12 shows the corresponding TWAPR values.

	Month												Annual Average
	1	2	3	4	5	6	7	8	9	10	11	12	
DC	1.22	0.44	0.57	0.66	0.82	1.18	2.25	8.18	12.08	15.80	9.61	6.27	4.92
SI	0.96	0.59	0.72	0.74	0.86	1.11	1.68	2.25	2.62	2.75	2.26	1.74	1.52
MC	0.91	0.90	1.07	0.93	0.94	1.05	1.44	1.96	2.20	1.99	1.47	1.07	1.33

Table 6.12: Monthly and Annual TWAPRs, for all Optimisation Run Complexities

Comparing Table 6.12 (TWAPR) to Table 6.11 (TWAPS), they produce a similar pattern of results. The maximum prices are generally in period 10, just after the peak of the wet season. The minimum prices are generally in period 2, just before the “peak” of the dry season. Generally TWAPR is larger than TWAPS, apart from in the DC optimised run in periods 8 through 10. The differences are lowest in the wet season, and highest in the dry season. The overall average differences for the DC, SI, and MC optimised runs are -0.46, 0.45, and 0.44, respectively. So, TWAPRs are generally higher, on average, as we would come to expect.

An efficient market should reduce the price variation over the water year. We are generally more interested in the risk of higher than average prices. But, in this first instance, when comparing TWAPS and TWAPR we report the Annual Standard Deviation (ASD), Annual Positive Semi-Deviation (APSD), and Annual Negative Semi-Deviation (ANSD) of both TWAPs. Variance measures for TWAPS are given in Table 6.13, in the same format as the two prior tables. There are three sets of grouped rows, for ASD, APSD, and then ANSD results. The corresponding annual variation values are given in the far right column.

Looking at Table 6.13, across all three measures, the DC and MC optimisations consistently have the highest and lowest values in periods 6 through 12, and in period 1. The DC and MC optimisations generally have the lowest and highest values in periods 2 through 5. The average APSD to ANSD ratio of annual values in the DC, SI, and MC optimised runs are 104, 22, and 5, respectively. Evidently, the DC optimisation has significantly higher TWAPS risk.

		Month												Annual Variation
		1	2	3	4	5	6	7	8	9	10	11	12	
ASD	DC	4.58	0.43	0.56	0.76	0.91	1.64	23.04	390.03	574.09	638.78	104.77	32.12	112.36
	SI	3.18	0.48	0.65	0.71	0.95	1.66	4.14	7.38	29.01	41.29	24.53	10.64	6.50
	MC	1.48	1.12	1.23	1.07	1.19	1.26	1.56	1.56	1.83	2.48	3.25	3.08	1.31
APSD	DC	23.90	0.95	1.14	1.55	1.95	3.68	57.48	2859.31	6152.98	6806.29	516.52	121.84	496.37
	SI	8.37	0.67	0.98	1.13	1.71	3.39	9.37	17.31	74.40	122.92	68.02	39.54	15.09
	MC	2.54	1.74	1.90	1.62	1.89	2.18	2.87	2.87	3.41	4.74	6.26	5.92	2.24
ANSD	DC	0.87	0.18	0.26	0.34	0.36	0.47	1.29	7.54	14.17	19.40	8.22	5.79	4.77
	SI	0.57	0.32	0.38	0.39	0.40	0.47	0.67	0.75	1.01	1.53	1.07	1.13	0.69
	MC	0.51	0.54	0.57	0.53	0.50	0.44	0.46	0.46	0.48	0.49	0.51	0.53	0.46

Table 6.13: Monthly and Annual ASD, APSD, and ANSD of TWAPS measures, for all Optimisation Run Complexities

The corresponding measures for TWAPR are given in Table 6.14.

		Month												Annual Variation
		1	2	3	4	5	6	7	8	9	10	11	12	
ASD	DC	4.55	0.43	0.71	0.98	1.44	2.36	20.93	354.11	468.04	514.63	105.09	31.62	91.01
	SI	3.16	0.50	0.89	1.04	1.61	2.43	4.03	5.32	16.31	25.92	27.36	10.87	5.49
	MC	1.55	1.25	2.95	2.24	2.17	1.84	1.44	1.28	1.44	1.92	2.83	2.89	1.48
APSD	DC	23.89	0.97	1.49	1.99	3.07	5.29	63.19	2694.40	4423.00	5006.07	523.62	122.51	380.76
	SI	8.18	0.69	1.44	1.80	3.25	5.67	11.16	17.62	63.17	105.38	97.53	43.08	14.55
	MC	2.69	1.99	5.51	3.86	3.91	3.60	3.79	4.61	5.81	7.04	7.74	5.97	2.92
ANSD	DC	0.86	0.18	0.31	0.41	0.51	0.60	1.03	6.22	9.87	13.77	8.17	5.60	3.84
	SI	0.56	0.33	0.45	0.48	0.52	0.51	0.48	0.45	0.57	0.93	1.05	1.10	0.57
	MC	0.51	0.58	0.74	0.66	0.57	0.42	0.25	0.17	0.16	0.19	0.28	0.43	0.37

Table 6.14: Monthly and Annual ASD, APSD, and ANSD of TWAPR measures, for all Optimisation Run Complexities

Comparing the conditional formatting on Table 6.14 (TWAPR risk) with Table 6.13 (TWAPS risk), they show similar patterns. The highest values are in the dry season, and especially in the DC optimised run. The DC and SI optimised runs generally both have maximum and minimum values in periods 10 and 2, respectively.¹⁰⁷ Looking at the risk differences, i.e. TWAPR minus TWAPS for all risk measures, these are generally small and positive in the wet season. In the dry season, and the peak periods (8 through 10), the differences are smallest (negative) and largest in the DC and MC optimised runs, respectively.

The absolute TWAP (Tables 6.11 and 6.12), and TWAP variances (Tables 6.13 and 6.14), averages show that the SI and MC prices, and risks, are both lower and more tightly bunched,

¹⁰⁷ Apart from the SI NSD of TWAP from release maximum which is period 12

than the DC prices, and risks. Having briefly reviewed TWAPS and TWAPR, and their risk measures, we now focus on VWAP.

6.4.4.2 VWAP

In this section we only focus on the global VWAP from release (VWAPR), and the VWPSD of VWAPR (see Equations (5.55) and (5.83), for each measure’s general definition, respectively). But we also briefly compare this to the annual VWAP for each sampled year, AVWAPRⁿ, and its mean, AVWAPR (see Equations (5.56) and (5.57), for each measures general definition, respectively). We show DC, SI, and then MC VWAPR for Release values, and we compare the relative size of each result, for each optimisation run complexity by taking differences: i.e. DC minus SI, DC minus MC, and SI minus MC. We then look at the upside VWAPR variation.

Table 6.15 shows the VWAPR monthly average prices, for the MC, SI, and DC optimised runs. The last two columns show the global VWAPR, and AVWAPR.

	VWAPR by Month												VWAPR	AVWAPR
	1	2	3	4	5	6	7	8	9	10	11	12		
DC	0.45	0.31	0.25	0.30	0.48	0.80	1.46	2.09	2.36	2.21	1.78	1.03	1.36	1.79
SI	0.50	0.40	0.30	0.32	0.51	0.80	1.39	1.97	2.21	1.99	1.41	0.79	1.26	1.48
MC	0.61	0.47	0.31	0.32	0.57	0.84	1.33	1.88	2.12	1.89	1.32	0.83	1.25	1.37
DC - MC	-0.16	-0.17	-0.06	-0.02	-0.09	-0.04	0.13	0.20	0.24	0.32	0.46	0.20	0.11	0.42
DC - SI	-0.05	-0.09	-0.06	-0.02	-0.03	0.00	0.07	0.11	0.15	0.21	0.38	0.24	0.10	0.31
SI - MC	-0.11	-0.08	0.00	0.00	-0.06	-0.04	0.06	0.09	0.09	0.11	0.08	-0.04	0.01	0.11

Table 6.15: Monthly and Mean VWAPR for Release, for all Optimisation Run Complexities

Comparing Table 6.15 with the TWAPR values in Table 6.12 (above) shows that for all three optimised runs the VWAPR is always less than the corresponding TWAPR, but the same general pattern is evident i.e. low/high prices in the wet/dry seasons. This is to be expected, given the prices would be weighted by the volume traded. Within the VWAPR table the global VWAPRs are always smaller than the AVWAPR, as are their differences. The global VWAPRs in the SI and MC optimised runs are nearly identical. The SI – MC differences are at least about half the DC – MC differences in the wet season.

The DC optimised run releases too much water in the wet season, which really should have been saved and released in the dry season. The lowest prices in the wet season, and the

highest prices in the dry season, indicate that the DC optimisation is unable to manage the storage system in an efficient manner. With a constant elasticity curve, set at an elasticity of -0.5 , this behaviour has been observed over many hundreds of different cases. As such this repetitive behaviour confirms our observation that, even in a relatively unconstrained system, the DC optimised run struggles to deal with the correlated hydrological uncertainty, thus leading to some degree of market inefficiency. The MC optimisation is generally more efficient in managing price fluctuations in the market, over the water year.

Figure 6.10 gives results for the DC optimised run. The figure reports the AVWAPR CDF and its mean (purple) and the global VWAPR CDF and mean (red). We also show the ATWAPR CDF and its mean (blue). As with Table 6.15 (above) the mean AVWAPR value is less than the mean ATWAPR values. While the Global VWAPR CDF has a much wider spread than the AVWAPR and ATWAPR CDFs, as we would expect. But the mean of the AVWAPR is still higher in this case.

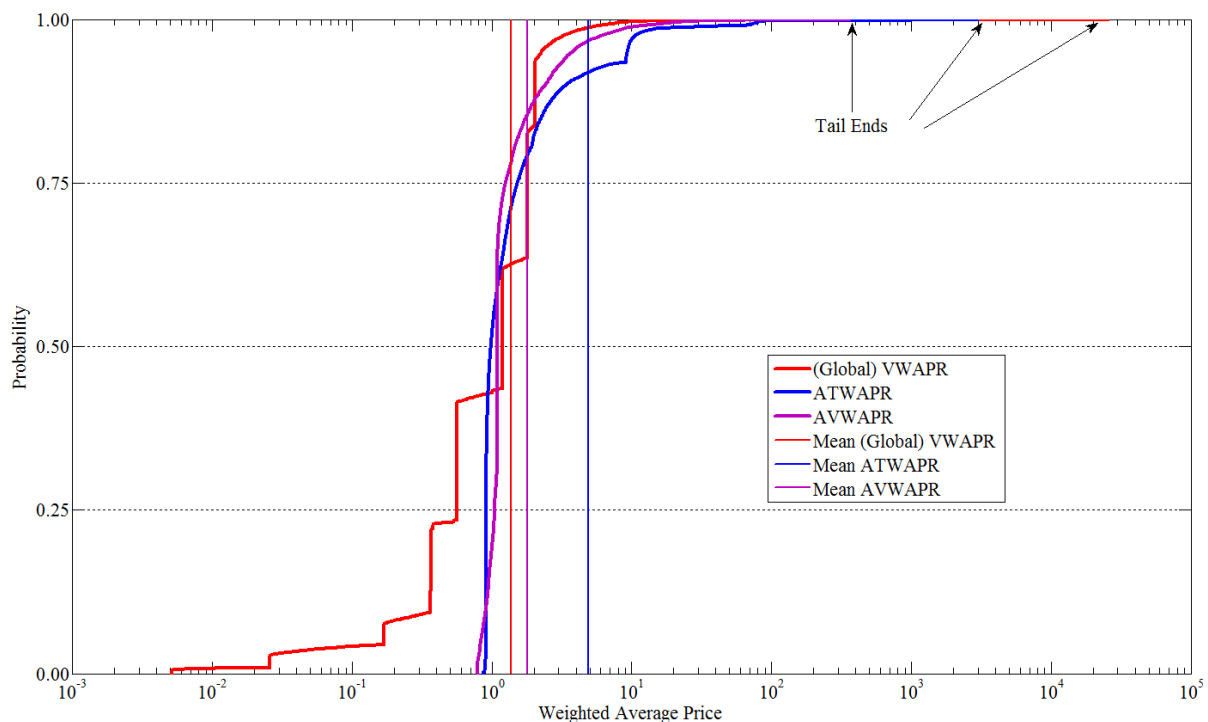


Figure 6.10: Global and Annual VWAPR CDFs and Means, and Annual TWAPR and Mean, for DC Optimisation

In Figure 6.11 we show the DC, SI, and MC Global VWAP CDFs. The DC (red) CDF had the longest tail, while the MC (green) CDF has the shortest tail. The SI (black) CDF has a

tail approximately half way between the two. All three CDFs cross over one another, a number of times, showing no clear stochastic dominance. The expected value lines are also shown, but the mean SI line sits directly behind the mean MC line, at this scale.

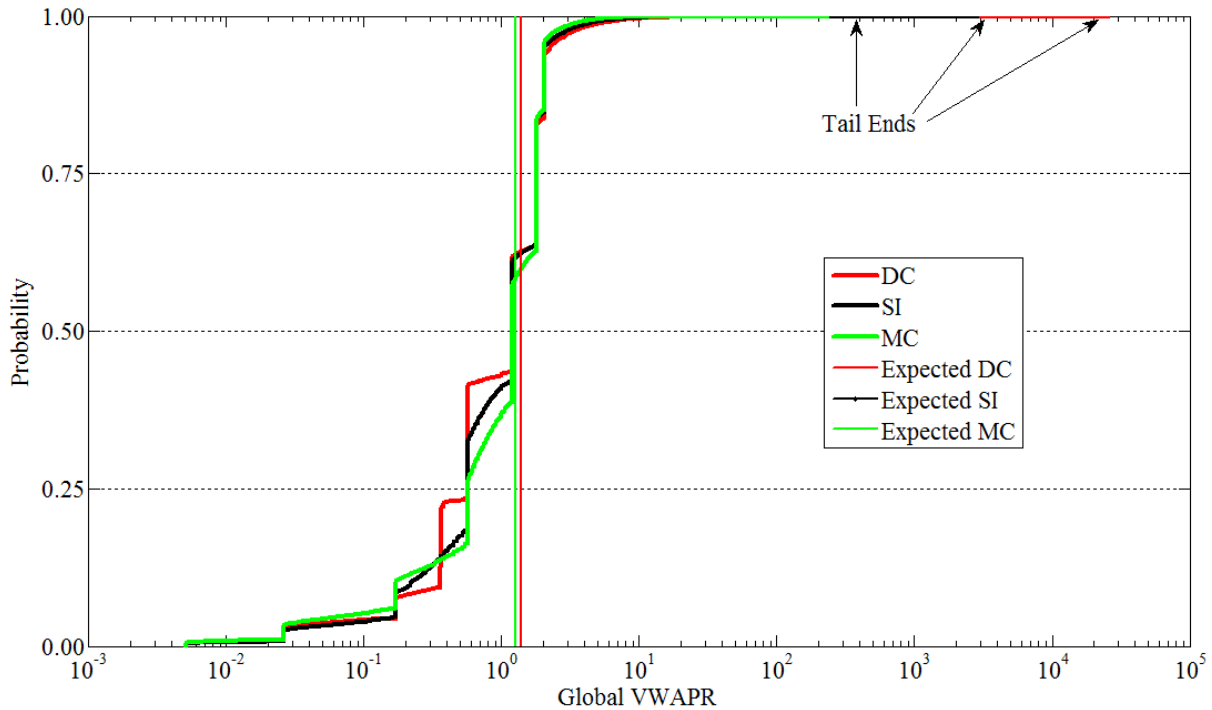


Figure 6.11: Global VWAPR CDFs for all Optimisation Run Complexities

Comparing the AVWAPR CDFs, they show a similar pattern. In the tail of the distribution (last 1% illustrated), the DC optimised run shows much higher price variance than the SI optimised run, which in-turn produces much higher price variance than the MC optimised run. This has been observed over many cases, with asymmetric constant elasticity curves, the lesser the optimisation run complexity the greater the risk of much higher than average prices, in some of the sampled instances.

Table 6.16 shows the corresponding monthly and annual average VWPSD of VWAPR results (see Equations (5.82) and (5.83), for each measure's general definition, respectively), and the APSD value (as generally defined by Equation (5.78)). We show DC, SI, and then MC values, and then DC minus MC, DC minus SI, and SI minus MC differences.

	VWPSD by Month												APSD	VWPSD
	1	2	3	4	5	6	7	8	9	10	11	12		
DC	5.73	0.34	0.58	0.73	2.12	3.12	13.45	126.94	205.00	220.28	63.09	36.85	18.87	50.76
SI	2.09	0.55	0.83	0.92	1.43	2.62	4.86	7.66	19.55	26.21	14.52	6.76	5.63	5.93
MC	1.23	1.15	1.54	1.42	1.50	1.63	2.15	2.93	3.76	4.00	3.18	2.10	2.04	1.49
DC - MC	4.49	-0.81	-0.97	-0.69	0.62	1.49	11.31	124.01	201.23	216.28	59.90	34.75	16.83	49.27
DC - SI	3.63	-0.21	-0.25	-0.19	0.69	0.51	8.60	119.28	185.45	194.07	48.56	30.09	13.23	44.83
SI - MC	0.86	-0.60	-0.72	-0.50	-0.07	0.99	2.71	4.73	15.79	22.21	11.34	4.66	3.60	4.43

Table 6.16: Monthly and Annual VWPSD of VWAPR, and APSD of VWAPR, for all Optimisation Run Complexities

Generally, apart from around the peak of the wet season, the DC and MC optimised runs have the highest and lowest PSDs, both in each month and on average over the year. In the dry season the MC optimised run is massively outperforming the DC optimised run, in managing PSDs. The MC optimised run is also outperforming the SI optimised run, although differences are much more moderate.

The APSD and VWPSD show that, on average over the year, the DC and SI optimised runs are between approximately 9-34 and 3-4 times as high as the MC optimised run, respectively, depending on which measure is used. We tend to use the global VWPSD measure, the higher of the two in each range, as this is a truer measure of the prices that could be experienced in the market.

The monthly price pattern for the DC optimised run tends to match the peak inflow and peak demand characteristics of the system. The DC optimised run reports low prices when there is a lot of inflow and low valued demand, and high prices when there is very little inflow and high valued demand. Clearly the DC optimisation is struggling to manage prices. While the closer the uncertainty modelled in the optimisation matches the modelled real-world Markov-Chain process, then the more effective the MCE at providing price signals to conserve water to later periods when there is higher valued demand.

Of course the MC optimisation “knows” the correct probability distribution used in the simulation. The above results imply that most of the reduced risk is gained from modelling the uncertainty reasonably well. This is to be expected, and is essentially a well-known general result. In many instances it could be reasonable to assume a SI level of uncertainty within the optimisation. Of course individual situations will need to be investigated to better

understand the risks, and rewards of increased (from DC to SI) or reduced (from MC to SI) levels of optimisation run complexity.

6.4.5 Benefits

Having storage allows us to move water from one time of the year to another; it also allows us to absorb uncertainty fluctuations. In this section we generally report annual Time Weighted Average Benefits (TWABs), as discussed in Section 5.5.10. We now investigate the value of the system without storage, by looking at the TWAB from ROR only flows.

The ROR case has no storage system, as discussed in Section 5.5.3. Here we discuss the RS Monte-Carlo simulated values. This is shown by the blue line in Figure 5.11 (in Section 5.5.6), which shows the average TWAB from the ROR is at about 30% of the samples, and the downside tail showing the risk of lower than average TWAB is much longer than the upside tail showing higher than average TWAB. With our asymmetric constant elasticity curve this is to be expected. The NSD is 16992, which is approximately 1.7 times larger than the PSD of 6285. These ROR values can be used as the base comparison when comparing the Monte-Carlo Net Welfare values which include the storage system.

We use the CORS to look-up the cumulative returns, as discussed in Section 5.5.1: Equation (5.20). But, we can then apply those state space benefits to their associated PDFS values. Conditionally formatted results are given in Table 6.17.

	TWAB by Month												TWAB
	1	2	3	4	5	6	7	8	9	10	11	12	
DC	287	44	8	43	287	961	2028	3066	3488	3065	2025	959	16261
SI	287	43	8	43	287	961	2028	3067	3489	3066	2028	961	16269
MC	287	43	8	43	287	961	2028	3067	3490	3067	2028	961	16271

Table 6.17 Benefit Values for all Optimisation Runs, using RC and RO Outputs

From Table 6.17 the pattern shows lowest and highest benefits in the wet and dry seasons, respectively. On average, comparing across optimisations, the benefit differences reduce with increased run complexity. Comparing annual averages across optimisation runs, by taking case differences as a proportion of the MC result, moving from MC to SI reduces benefits by 0.02%, while moving from SI to DC reduces it by a further 0.05% of the MC value. Having run a number of test cases, with increasingly large sample sizes, we can

confirm that the differences do not seem to be within the margin of error, but they are symptomatic of the constant elasticity demand curves we used, i.e. constant elasticity curves place a large value on the first few units released, and increasingly less value on each subsequent unit released.

The Monte-Carlo equivalent benefits table follows exactly the same pattern as in Table 6.17. The maximum average percentage difference, compared to the MC difference is a trivial 0.06%. But if taking the monthly sampled CDFs from the Monte-Carlo outputs, these can be represented as an equivalent benefit contour plot, somewhat similar to the storage level contour plot in Figure 6.8 above. Figure 6.12 below shows the monthly benefit contour plot for all three optimisation run levels, in the usual colours. For all three the minimum benefit value, and the 1 and 100 percentiles are shown. The storage system values are shown by unbroken lines, to differentiate from the broken lines for ROR benefits, which are also shown in blue. The ROR benefits are shown for the minimum value, the 50 and 100 percentiles.

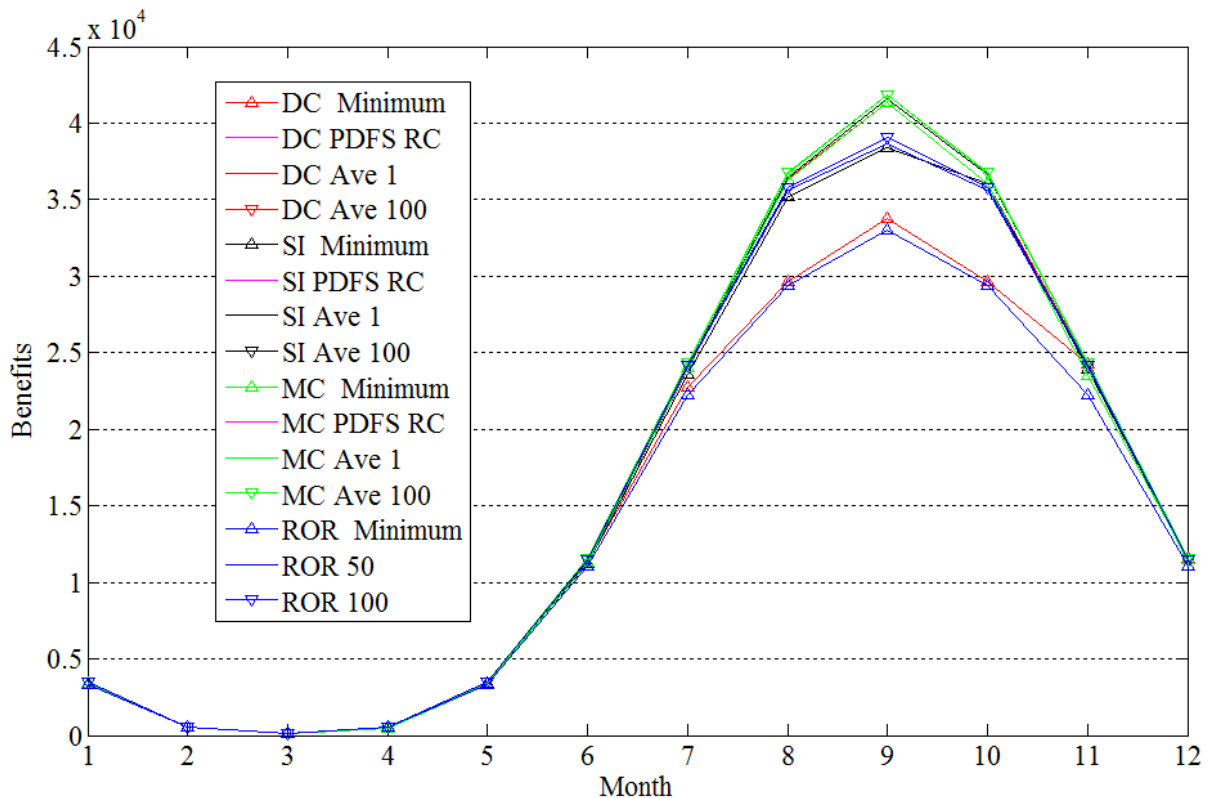


Figure 6.12: Monte-Carlo Simulation: Monthly Benefit Contour Plot, for all Optimisation Run Complexities

Figure 6.12 shows an interesting facet of TWAB. That is that there is generally very little difference between the benefits that can be returned over the water year. Most of the benefit lines are sitting near enough exactly on top of one another, from the 1 percentile to the 100 percentile. This is because there is such a large underlying net welfare value, even from just the ROR values. It is only during the highly valued dry season, and especially around its peak, periods 8 through 10, where the lesser complex optimisation runs have a very small probability of retuning much less value than the MC equivalent run. And in those instances the SI fares better than the DC run. In fact, from the graph the DC run is seen to have a very small probability of return approximately the same benefits as the river, in period 7 onwards. The 10,000 monthly samples can be added together, and sorted, to give an annual TWAB CDF. Figure 5.11 shows the annual TWAB CDFs, for DC (Red), SI (Black) and MC (Green) optimisation runs. And for further comparison we include the ROR (Blue) CDF and the Global Upper Bound (GUB) line, the ideal benefit from this case, as discussed in Section 5.5.4.

From Figure 5.11, perhaps surprisingly, in this experiment the TWAB values are still very close for all three optimisation run levels. But, as we expected, the MC optimisation performs the best. The SI optimisation run and MC optimisation run TWAB values are also closer to one another, compared to the DC optimisation run values.

But in general, the relative closeness of overall results is because there is sufficient storage to manage the benefit fluctuations between optimising the reservoir under different uncertainty assumptions. With less storage capacity there are greater gains from optimising the system at MC, rather than DC level of run complexity, as discussed in Section 7.2.

The CDF graphs also appear to indicate a fixed benefit return for a large proportion of the samples taken. But when we zoom in on the DC, SI, and MC CDFs we see that they criss-cross over one another. Bunn (1984) discusses screening prospects by dominance¹⁰⁸, and in that regard no one run dominates by First-Degree Stochastic Dominance (FSD), given the “criss-crosses”. Next we looked for Second-Degree Stochastic Dominance (SSD) across the optimisation run complexities. We subtract probabilities, for each fixed level of TWAB, and

¹⁰⁸ Pages 69 and 70

then accumulate the areas on that curve. Figure 6.13 below shows SSD for the MC optimisation over the DI optimisation, given area differences are all ≥ 0 .

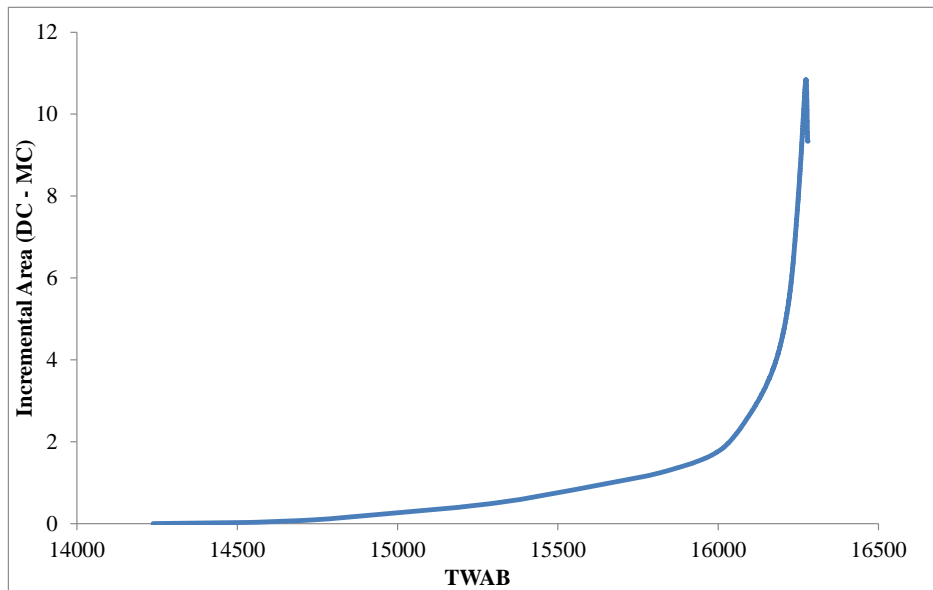


Figure 6.13: Plot Check for Second-Order Stochastic Dominance

Given the optimisation’s objective is to maximise expected benefit, and we never explicitly included risk aversion in the optimisation, having SSD is a somewhat “lucky side-effect”. We should expect the probability weighted average price variability (time weighted, as described in Section 5.6.1) to reduce, but it is not clear that the benefit variability will. But we have seen this pattern in Figure 5.11 for hundreds of runs. So evidently, an optimisation with a run complexity which better matches the underlying uncertainty (lag-1 Markov) clearly manages the lower tail of the benefit distribution, and much better than benefits in the upper end of the distribution.

The monthly TWAB SD, NSD, and PSD are given in Table 6.18 below.

		Month												Annual Variation
		1	2	3	4	5	6	7	8	9	10	11	12	
ASD	DC	34.95	4.37	2.68	7.51	21.55	42.61	92.34	242.97	320.54	359.53	285.62	167.79	90.50
	SI	26.03	5.20	3.01	7.86	21.39	39.13	55.45	66.97	84.55	107.49	95.80	71.97	38.53
	MC	19.29	7.93	4.08	9.42	21.38	31.10	34.20	34.37	36.10	39.28	39.78	33.64	21.72
APSD	DC	8.14	2.87	1.79	4.54	10.33	16.32	18.92	20.53	23.14	29.50	46.00	37.81	17.46
	SI	10.80	4.66	2.59	6.29	12.29	14.67	13.97	12.60	12.44	14.56	15.85	14.94	9.93
	MC	13.15	6.48	3.49	8.24	14.77	14.58	9.15	6.20	5.58	6.50	9.06	13.94	7.57
ANSD	DC	118.07	8.39	4.74	13.32	40.70	88.60	222.18	711.91	1024.23	1080.13	781.48	483.03	214.19
	SI	40.65	5.75	3.46	9.48	32.29	78.55	140.53	201.56	281.28	327.64	263.29	169.83	89.73
	MC	25.36	9.33	4.60	10.39	27.68	51.04	83.22	118.98	140.17	136.60	99.70	56.40	40.77

Table 6.18: Monthly and Annual SD, PSD, and NSD of TWAB values, for all Optimisation Run Complexities

Generally the DC optimisation run has the highest SD of TWAB in each month (apart from $t = 2$ to 4) and annually. The SI and MC SDs are much lower, and closer together. The PSD and NSD are both generally highest in the DC optimisation run, and lowest in the MC optimisation run. The NSDs are also generally much larger than the PSDs. And the downside risk is generally much larger in the dry season, when water is scarce and most valued. The least and most complex optimisation runs generally produce the highest and lowest downside to upside risk ratios, and definitely on average over the water year.

Clearly the DC optimisation run is unlikely to return much more than average TWAB, but much more likely, compared to the SI and MC optimisation runs, to return much less than average TWAB. In a real market, NSDs of the above orders of magnitude could equate to millions of dollars.

6.4.5.1 Simulated Proportional Value Measures from the Reservoir System

As discussed in Section 5.5.2, we can use the ROR as the Lower bound of Total Benefit with no Storage (LBS), and the Global Upper Bound (GUB) to undertake some proportional comparisons.

- First, the Proportion of Total Potential Benefit Achievable by increasing Storage capacity (PTPBAS, as discussed in Section 5.5.7): The total benefit from building the reservoir and employing a given level of optimisation complexity in proportion to the total range of benefit improvement available.
- Second, the Proportion of Remaining Potential Benefit Achievable by increasing Storage capacity (PRPBAS, as discussed in Section 5.5.8): The remaining benefit from

assuming an infinitely sized reservoir and employing a given level of optimisation complexity, in proportion to the remaining range of benefit improvement available, after optimising the reservoir system at the most basic level of run complexity, DC.

- Third, and finally, the Benefit Achievable as a Proportion of Market Value (BAPMV, as discussed in Section 5.5.9): The gain in market value compared to optimising the reservoir system at the most basic level of run complexity, DC, in proportion to the underlying market value, as measured by the average annual inflow at a price of 1, and the MC VWAPR of 1.25 (from Table 6.15 above).

Results are provided in Table 6.19 below. The first column gives SI and MC results in each instance. The second column reports the DC equivalent value from the first measure the PTPBAS. The 3rd to 6th columns report the % increases in the SI and MC cases, relative to the DC case, for the above three proportional value measures. The last two columns are for the Ideal and Actual optimum MCPs.

RO	DC Base PTPBAS	% Increase from DC Base Case			
		PTPBAS	PRPBAS	BAPMV	
				Ideal	MC VWAPR
SI	97.29%	0.78%	28.7%	8.78%	7.03%
MC		0.98%	36.3%	11.12%	8.90%

Table 6.19: Annual Proportional Value Measures for all Optimisation Runs

From Table 6.19 above, the benefit improvement for the case, from building a reservoir, and employing the most basic form of optimisation we model (managing the system deterministically) is 97.3% of the total ideal benefit that could be achieved, when using different simulated outputs.¹⁰⁹ As discussed in Section 5.5.5, while the gain from increased optimisation complexity is not high, the cost of improved optimisation complexity, compared to the cost of building the reservoir, is probably many orders of magnitude less.

The second measure, the PRPBA, compared to the DC optimisation, optimising the system under simple (SI) and matching (MC) uncertainty assumptions, those cases are expected to deliver between approximately 29% and 36% the remaining gains possible from building an infinite reservoir and managing it optimally. The RS module outputs show the most gains,

¹⁰⁹ An additional expected benefit of approximately 0.78% and 0.99% is gained from including for some basic and matching the underlying stochasticity, respectively.

but this is probably the most inaccurate, compared to the fully convoluted outputs. Either way, matching the underlying MC uncertainty produces the largest improvements.

Finally, the BAPMV, compared to the DC optimisation, optimising the system under simple (SI) and matching (MC) uncertainty assumptions, those cases are expected to deliver approximately 9 and 11% gain compared to the ideal annual market value. Compared to the actual MC VWAPR annual market value it only delivers 7% and 9% gains, for the same optimisation runs.

Opposite to the PRPBA, the RS module outputs show the least gains compared to the fully convoluted outputs. And BAPMV returns are modest compared to the PRPBA measure. Still, the BAPMV shows the MC case giving greater improvements than the SI case. But, on average including for some uncertainty in the Market Clearing Mechanism can give about a 10% gain in market value, over the water year. Depending on the local economy, this could be worth billions of dollars.

6.5 Conclusions

In this chapter we showed a sample of the range of outputs that can be produced by the computational modelling system described in Chapter 5. Our testing showed that the Stochastic Constructive Dual Dynamic Programming (SCDDP) is producing consistent solutions for the Deterministic (DC), Stochastic Independent (SI), and Markov Chain (MC)) cases,. But, although the stand-alone optimisation outputs are each optimal for their own set of assumptions, their performance must be analysed in a “real-world” (simulated) setting. Our testing has showed that the Monte-Carlo simulation, using antithetic sampling over 10,000 years, provides adequate accuracy for general reporting, backed up by discrete convolution results, as required. In some cases it can also be useful to examine the seasonal pattern of average release and storage levels and the probability of spill, empty, and shortage. As expected, the DC optimisation allows the reservoir system to fall to its lower storage bound more often than it should, thus resulting in a greater probability of relative shortage, and hence very high prices, thus producing lower overall benefits, despite there being fewer spills.

But, having verified the effectiveness of the modelling system, the focus from here on will be on indicators of the market efficiency being achieved under various parameter settings. In

this chapter, annual reporting has been backed up with monthly reporting, where needed, to add further insight into how a case is actually behaving in a given setting. Henceforth, though, we will only report the key measures introduced in the previous chapter. The results in this chapter establish results for a base case involving a high degree of inflow seasonality, a high degree of inflow variance, a high degree of persistence, a reasonably constrained release capacity, and a reasonably unconstrained storage capacity. And, with those parameter settings:

- Moving from MC optimisation, which is truly optimal for this system, to Stochastic Independent (SI) optimisation only reduces TWAB by 0.01%, increases its downside Semi-Deviation (ANSD) by 120%. But this largely reflects the fact that we employed constant elasticity curves, where most of the value is realised from releasing the first few units of water. Still, it only increases VWAPR by 1%, although it increases its upside Semi-Deviation (VWPSD) by 297%.
- Moving from MC optimisation to Deterministic (DC) optimisation still only reduces TWAB by 0.06%, but increases its downside Semi-Deviation by 425%. It increases VWAPR by 9%, though, and increases its upside Semi-Deviation by 3304%.

To put these results into perspective, the % gains in TWAB seem very small because there is a very large underlying value from the first few units released in our constant elasticity demand curves. The benefit gain achieved as a proportion of market value (BAPMV), calculated using the market clearing price from Markov Chain optimisation, is approximately 7% and 9% for the SI and MC optimisation runs, respectively, and that is likely to seem quite significant to market participants. Put another way, the run of river solution alone, with this release capacity but no storage capacity, and hence no optimisation, already delivers 94% of the possible economic benefit (GUB) from this system. Building a reservoir of this size, and employing DC optimisation, raises this to 99.8% of the potential, thus delivering 97% of the benefit potentially available from building storage and optimising its management (PTPBAS). Going to MC stochastic optimisation, without any further storage expansion, delivers a further 1%, or approximately 36% of the remaining potential benefit achievable from storage expansion and/or optimisation (PRPBA). The real world significance of that gain depends on the cost of alternatives, but it should be recognised that the cost of improving optimisation is likely to be very much lower than the cost of building additional

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physical capacity. A 9% reduction in average prices, is also likely to be considered significant from a participant perspective.

The next two chapters now explore the sensitivity of our results to variation in physical, and then economic, parameters.

7. Impact of Physical Parameters

7.1 Chapter Introduction

In this chapter we ask how much impact accounting for stochasticity in the market clearing/optimisation has on system performance, over a wide range of possible system parameters. We address that question by performing experiments while varying different physical reservoir parameters, assuming a given set of demand curves. In the next chapter, we will consider the impact of varying the pattern of demand.

In Section 7.2 we establish upper and lower bounds on the system performance that could be expected from adopting our most sophisticated optimisation. System performance is reviewed over a range of storage and release capacities, and measured in terms of benefits, prices, benefit volatility, and price volatility. In the next two sections we explore the impact of optimisation methodology over the same capacity range. In Section 7.3 we review the differences on benefits, while in Section 7.4 we look at the differences on prices. In Section 7.5, we consider the impact of varying inflow. First we consider the impact of inflow variation. Second, we vary inflow seasonality. Third, and finally, we explore the impact of peak demand being in or out of phase with peak inflow, over the annual cycle. In Section 7.6 we present conclusions.

Apart from in Section 7.5.3, we assume that peak inflow and demand are out of phase by six months, as in Chapter 6. Apart from in Section 7.5, inflow values are the same as those used in Chapter 6. We assume that aggregate market demand is non-state dependent, and represented by constant elasticity curves with an elasticity of 0.5. Probability values are the same as those used in Chapter 6.

7.2 Bounds on Performance

In this section we determine the upper and lower bounds on benefits, while also discussing modelling performance of the truly optimal system, the MC optimisation case results. This gives us a benchmark, for the range of benefits we can expect from exactly matching the underlying uncertainty. This is useful for comparing MC, SI, and DC optimisation case runs,

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where results are presented for a wide range of storage and release capacities: SCAP and RCAP, respectively.

Another way of benchmarking is to analyse the trade-off between infrastructure investment and expenditure on improved optimisation. Reporting measures such as the Proportion of Remaining Potential Benefit Achievable by increasing Storage capacity (see Section 5.5.8) have been designed to allow such analysis. We have not pursued that analysis, in general, but a short example is given in Appendix 7.1.

7.2.1 Limiting Values

Firstly, as expected, spill reduces with increased RCAP and SCAP. But, we are more concerned with the benefits delivered to the market. Table 7.1 shows expected annual “net” Time Weighted Average Benefit values from having a reservoir system, using MC optimisation,¹¹⁰ for given SCAP and RCAP values. Both storage and release capacity values are shown as multiples of the mean monthly inflow, FMEAN. In this table we report net TWABs for a 32×24 grid of 768 individual cases, where bright red indicates the lowest values and bright green indicates the highest values. Note that the grid spacing is irregular.¹¹¹ Each data point in the table has been produced by running the optimisation and simulation modelling suite described in Chapter 5 on variations of the base case example discussed in Chapter 6.

While the storage and release capacity ranges we experimented on might seem excessive in many real life instances, they might be informative in others. These over simplified theoretical experiments are meant to act as a basic guide to the types of systems which might be susceptible to (in)significant gains from improved optimisation and water trading.

¹¹⁰ In other words, we have computed the Time Weighted Average of Monthly Benefits (TWAB) for each sample year, then averaged these over all sample years.

¹¹¹ While not explicitly reported here, we complemented this set of irregularly spaced discrete results with continuous plots, in order to confirm that the results from the RS module (with limited sampling) are monotone non-decreasing with increases in each respective capacity, and that those rates of change are consistent. We have also compared the limited sampling RS results with those from the RC module, to ensure both the consistency and accuracy of our results.

		RCAP (\times FMEAN) \rightarrow																									
		0.02	0.04	0.07	0.11	0.14	0.19	0.25	0.38	0.50	0.63	0.75	0.88	1.00	1.17	1.33	1.42	1.50	1.71	2.00	2.25	2.50	2.75	3.00	4.00		
\leftarrow SCAP (\times FMEAN)	0.00	2344	3385	4525	4898	5082	5201	5273	5342	5362	5374	5380	5384	5386	5387	5388	5388	5389	5389	5389	5389	5389	5389	5389	5389	5389	
	0.02	2477	3527	4696	5076	5259	5380	5453	5520	5541	5553	5558	5562	5564	5566	5566	5567	5567	5567	5568	5568	5568	5568	5568	5568	5568	5568
	0.04	2488	3560	4743	5130	5315	5437	5510	5577	5598	5610	5615	5619	5621	5622	5623	5624	5624	5624	5624	5625	5625	5625	5625	5625	5625	5625
	0.07	2488	3594	4818	5224	5416	5543	5619	5685	5706	5717	5723	5727	5729	5730	5731	5731	5731	5732	5732	5732	5732	5732	5732	5732	5732	5732
	0.11	2488	3594	4855	5277	5477	5610	5688	5755	5776	5787	5793	5797	5798	5800	5801	5801	5801	5802	5802	5802	5802	5802	5802	5802	5802	5802
	0.14	2488	3594	4875	5311	5517	5656	5737	5805	5826	5837	5843	5847	5848	5850	5851	5851	5851	5852	5852	5852	5852	5852	5852	5852	5852	5852
	0.19	2488	3594	4887	5338	5552	5698	5784	5854	5875	5887	5892	5896	5898	5899	5900	5900	5900	5901	5901	5901	5901	5901	5901	5901	5901	5901
	0.25	2488	3594	4890	5356	5581	5735	5826	5900	5922	5933	5938	5942	5944	5945	5946	5946	5946	5947	5947	5947	5947	5947	5947	5947	5947	5947
	0.38	2488	3594	4890	5370	5610	5780	5883	5968	5990	6002	6007	6011	6013	6014	6015	6015	6015	6016	6016	6016	6016	6016	6016	6016	6016	6016
	0.50	2488	3594	4890	5372	5619	5799	5913	6008	6033	6044	6050	6053	6055	6057	6057	6058	6058	6058	6058	6058	6058	6059	6059	6059	6059	6059
	0.63	2488	3594	4890	5372	5622	5808	5933	6040	6067	6079	6085	6088	6090	6091	6092	6092	6092	6092	6093	6093	6093	6093	6093	6093	6093	6093
	0.75	2488	3594	4890	5372	5623	5813	5942	6060	6091	6104	6109	6113	6114	6116	6117	6117	6117	6117	6117	6117	6118	6118	6118	6118	6118	6118
	0.88	2488	3594	4890	5372	5623	5814	5949	6076	6112	6125	6131	6135	6136	6138	6138	6139	6139	6139	6139	6139	6139	6139	6139	6139	6139	6139
	1.00	2488	3594	4890	5372	5623	5815	5952	6086	6127	6142	6148	6151	6153	6154	6155	6155	6155	6156	6156	6156	6156	6156	6156	6156	6156	6156
	1.33	2488	3594	4890	5372	5623	5815	5955	6102	6154	6175	6182	6186	6187	6189	6189	6190	6190	6190	6190	6190	6190	6190	6190	6190	6190	6190
	1.50	2488	3594	4890	5372	5623	5815	5955	6106	6162	6187	6195	6199	6201	6202	6203	6203	6203	6203	6203	6203	6203	6203	6203	6203	6203	6203
	1.71	2488	3594	4890	5372	5623	5815	5955	6109	6170	6199	6209	6213	6215	6216	6217	6217	6217	6217	6217	6217	6217	6217	6217	6217	6217	6217
	2.00	2488	3594	4890	5372	5623	5815	5956	6110	6176	6210	6222	6228	6230	6231	6232	6232	6232	6232	6232	6232	6233	6233	6233	6233	6233	6233
	2.25	2488	3594	4890	5372	5623	5815	5956	6111	6179	6216	6231	6238	6241	6242	6243	6243	6243	6243	6243	6243	6243	6243	6243	6243	6243	6243
	2.50	2488	3594	4890	5372	5623	5815	5956	6112	6181	6220	6238	6247	6249	6251	6252	6252	6252	6252	6252	6252	6252	6252	6252	6252	6252	6252
	2.75	2488	3594	4890	5372	5623	5815	5956	6112	6182	6224	6243	6253	6257	6259	6259	6259	6260	6260	6260	6260	6260	6260	6260	6260	6260	6260
	3.00	2488	3594	4890	5372	5623	5815	5956	6112	6182	6226	6247	6259	6263	6265	6266	6266	6266	6266	6266	6266	6266	6266	6267	6267	6267	6267
	4.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6230	6255	6271	6279	6283	6285	6285	6285	6285	6285	6285	6285	6285	6285	6285	6285	6285
	6.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6259	6279	6291	6299	6303	6304	6304	6305	6305	6305	6305	6305	6305	6305	6305	6305
12.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6261	6284	6298	6311	6319	6321	6322	6324	6325	6325	6325	6325	6325	6325	6325	6325	
24.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6300	6315	6324	6327	6329	6331	6332	6332	6332	6332	6332	6332	6332	6332	
36.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6300	6316	6325	6328	6330	6333	6333	6334	6334	6334	6334	6334	6334	6334	
48.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6301	6316	6326	6329	6331	6334	6334	6334	6334	6334	6334	6334	6334	6334	
60.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6301	6316	6326	6329	6331	6334	6334	6334	6334	6334	6334	6334	6334	6334	
72.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6301	6316	6326	6329	6332	6334	6335	6335	6335	6335	6335	6335	6335	6335	
96.00	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6301	6317	6327	6330	6332	6334	6335	6335	6335	6335	6335	6335	6335	6335	
∞	2488	3594	4890	5372	5623	5815	5956	6112	6184	6232	6262	6285	6301	6317	6327	6330	6332	6335	6335	6335	6335	6335	6335	6335	6335	6335	

Table 7.1: TWAB with MC optimisation.

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In Table 7.1, RCAP and SCAP are reported horizontally (left to right) and vertically (top to bottom), respectively. The top left corner case value is for the most constrained system we report on here, with RCAP and SCAP at 2 units and 0 units, that is 0.02 and 0 times FMEAN, respectively. The bottom right result is for the least constrained reservoir system reported here, with RCAP and SCAP at 4 (336 units) and 96 (8064 units) times FMEAN, respectively.

At one extreme, with no release capacity, we can only store and spill water. Since we assume that spills are not delivered to consumers, no release capacity results in no market benefits. If we had reported this in Table 7.1 this would mean adding a new first column, where RCAP = 0, and absolute benefits all 0. Even with a single unit of RCAP, the minimum TWAB from the ROR case (with no SCAP) is significant: 9952 (119,424 total annual). As discussed in Section 5.5.9, this value is large because we model demand using constant elasticity curves, which assign a very high value to the first unit consumed. But we do not include a column for RCAP=1 in Table 7.1, and subtract this minimum benefit value from all reported values in Table 7.1, to form “minimum net” benefit values, as discussed in Section 5.5.5.

At the other extreme, with infinite release capacity, any amount of water in store, or arriving as inflow, could be released to meet any pattern of demand. Considering storage, for any RCAP, the ROR case, as discussed in Section 5.5.3, provides the Lower bound of total Benefit with no Storage LBS, LBS(RCAP), for each RCAP value.

With SCAP = 0 and RCAP = ∞ , then all inflow can, and must, be passed directly onto the market, in the period when it arrives. With SCAP = ∞ and RCAP = ∞ , then the reservoir system can accept any amount of inflow, and store it until it is most valuable, without spilling, to deliver maximum benefits to the market, for any pattern of demand. In Section 5.5.4 we discussed forming the Global Upper Bound (GUB) assuming non-constraining RCAP, and the Tight Upper Bound with Unconstrained Storage (TUBUS) on Total Benefits as a function of RCAP, both while assuming a non-constraining SCAP. Practically, we only wish to perform experiments for RCAP values up to the point where RCAP is no longer constraining the solution, so that TUBUS(RCAP) = GUB.

In the ROR case (SCAP = 0), RCAP will only constrain the solution if it is less than the maximum inflow, which in this case is between 3 and 4 times FMEAN. For our experiments, the same is also true at the opposite extreme (SCAP = ∞), because we have constructed our

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experiments to align inflow ($F^{t,h}$) and demand ($QREF^{t,h}$) at the optimal non-constraining market clearing price of 1, and chosen the $QREF^{t,h}$ points from the set of $F^{t,h}$ reference points. As a result, the maximum $QREF^{t,h}$ value is equal to the maximum $F^{t,h}$ value, and so also between 3 and 4 times FMEAN. Thus, given that this is a linear system, we do not expect to see any value from increasing RCAP above $4 \times FMEAN$, and this was confirmed by running cases up to $5 \times FMEAN$.

The first row in Table 7.1 (where $SCAP = 0$) corresponds to the ROR LBS(RCAP) cases. For this inflow and demand case, the net LBS value varies between 2344 and 5389. The final row (where $SCAP = \infty$) shows TUBUS(RCAP) cases. Similarly, the net TUBUS value varies between 2488 and 6335. This latter value equals the net GUB value computed by the methodology explained in Section 5.5.4.

As discussed in Section 5.5.6, the difference between the LBS and the TUBUS is the Maximum Benefit from Storage Range (MBSR). Thus the MBRS varies as a function of RCAP, ranging from 144 ($2488 - 2344$) for $RCAP = 0.02 \times FMEAN$, up to 946 ($6335 - 5389$), for $RCAP 4 \times FMEAN$. But, even with a moderate level of release capacity, say FMEAN, the MBRS of 915 ($6301 - 5621$) is modest compared to the total benefits from the ideal ROR case ($TWAB(ROR) = 15339$). This reflects the fact that most of the benefits are returned by managing to deliver at least a small proportion of the water to the market.

7.2.2 Maximum Achievable Benefit from MC Optimisation

The body of the Table 7.1 reports the net benefits achieved by our MC optimisation model, for each RCAP and SCAP value. This represents the maximum achievable benefit for each case, because the optimisation and simulation models use the same inflow and demand probability distributions. Thus, while the MC case is not a realistic test of the real-world performance of the MC optimisation, it provides a consistent test of how well a given optimisation performs on the same set of data, with no extraneous “noise”.

7.2.2.1 The impact of Release capacity

To illustrate the impact of increasing release capacity on the achievable benefit, Figure 7.1 shows plots of rows 1, 10, 14, 25, and 32 (the last) from Table 7.1 above. For each discrete RCAP case modelled, the LBS(RCAP) results are shown with ‘^’ markers, the TUBUS(RCAP) results are shown with ‘v’ markers. Both these sets of markers have solid black lines, and the distance between these lines is the MBRs(RCAP). Between the LBS and TUBUS case results we have also shown three intermediate SCAP cases, which all have grey lines. These are where SCAP is half, equal to, and 12×FMEAN (i.e. able to store two week’s, one month’s, and one year’s worth of average inflow).

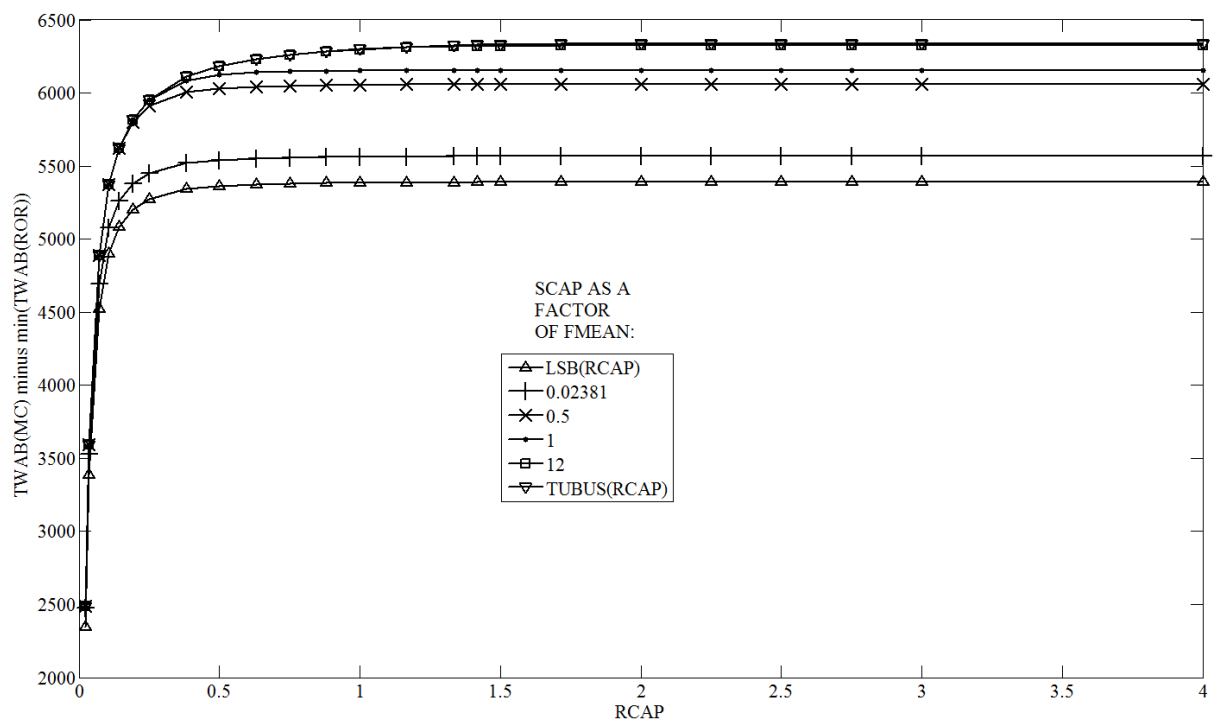


Figure 7.1: TWAB, for a range of capacity values.

As expected, the system returns monotonically increasing benefits as RCAP is progressively increased, until the point when RCAP becomes non-constraining. Figure 7.1 indicates that, net benefits quickly increase for a small increase in RCAP. Once RCAP is at least one half of FMEAN, benefits are already approaching a maximum. Practically, a “sensible” minimum level for RCAP would be FMEAN, which is central in our table. To the right of this line net benefit returns are all relatively high, and stable. This is especially true when combined with a sensible minimum level of SCAP, which is perhaps around 0.5×FMEAN (i.e. two weeks storage).

Once RCAP is approximately $1.5 \times \text{FMEAN}$, benefits have effectively reached their maximum possible level. Beyond this additional benefits are negligible, and not illustrated. In fact, the Table suggests that there is no net benefit increase after RCAP reaches $2 \times \text{FMEAN}$. This is because values are rounded to the nearest whole number, and the benefit differences are very small.

7.2.2.2 The impact of Storage Capacity

To illustrate the impact of increasing storage capacity on the achievable benefit, Figure 7.2 shows plots of columns 1, 9, 13, and 24 (the last) from Table 7.1 above. Obviously, when $\text{RCAP} = 0$ there are no net benefits, as discussed above. The lowest non-zero total benefit case results for each SCAP case modelled, when $\text{RCAP} = 2$ units) are shown with ‘^’ markers, while the TUBUR(SCAP) results are shown with ‘v’ markers. The Maximum Benefit Range achievable by increasing Release capacity, $\text{MBRR}(\text{SCAP})$ is the distance from the horizontal axis to the TUBUR(SCAP), given that when $\text{RCAP} = 0$ there are no benefits, as discussed above. Two intermediate RCAP cases have been shown, where RCAP is half of, and equal to FMEAN (i.e. able to store half, or one month’s worth of average inflow). Cases with higher RCAP are not shown, because they are indistinguishable from the ones illustrated.

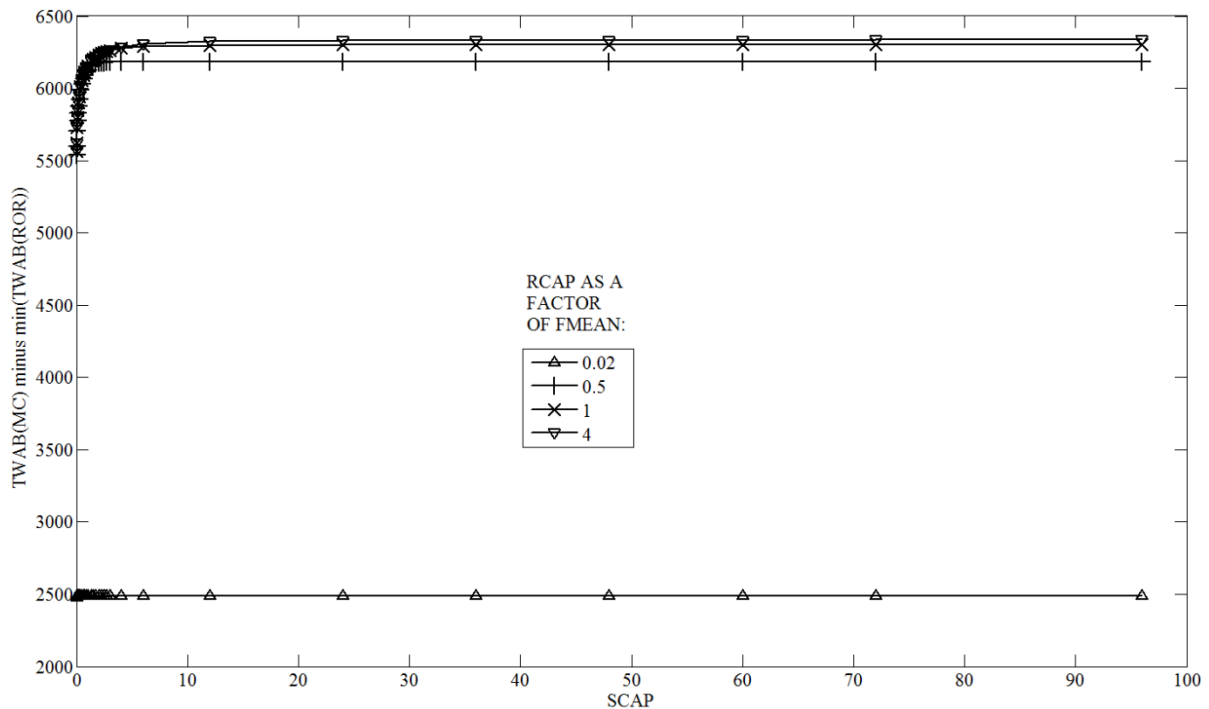


Figure 7.2: Net benefits, for a range of capacity values.

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As expected, the system returns monotonically increasing benefits as SCAP is progressively increased, until the point when SCAP becomes non-constraining. From Table 7.1, for RCAP values greater than FMEAN, SCAP is non-constraining only in the final row TUBUS results (when SCAP is infinite). Figure 7.2 shows that, even with no storage, net benefits are reasonably high, for cases with a non-zero RCAP, and especially when $RCAP \geq 0.5 \times FMEAN$. There are significant benefit gains as SCAP increases from 0 to $1 \times FMEAN$. The RCAP lines are all close together, as we would expect, given that increasing RCAP adds little benefit, in this region, as discussed in Section 7.2.2.1, above. On the other hand, the benefits continue to rise slowly, with increasing SCAP.

With a “sensible” minimum level for RCAP being FMEAN, as above, Table 7.1 shows that increasing SCAP increases benefits by much more than increasing RCAP by the same amount. The maximum benefit gained in increasing SCAP from FMEAN to ∞ ranges from 148 to 179. By way of contrast, the maximum benefit gained by increasing RCAP from FMEAN, to $4 \times FMEAN$ (or effectively ∞) ranges from just 3 for SCAP = 0, to 34 for SCAP = ∞ . Thus system value is increased mainly by being able to store inflow until periods when demand is more valuable, once RCAP is sufficient to realise the high valued benefits arising from at least being able to release the mean inflow.

7.2.2.3 Relative versus Absolute Impacts

If we can take away the minimum (non-zero) ROR benefit value, $LBS(RCAP)$, from the absolute benefit value for a given (SCAP,RCAP) pair, we get a simple measure of the benefit from building that storage capacity, SCAP, assuming MC optimisation. Since the inflow distribution here actually is Markov, this represents the Maximum Benefit achievable for that (SCAP,RCAP) pair, using any form of Optimisation (MBO); that is as per Equation (5.37):

$$MBO(SCAP, RCAP) = MC(SCAP, RCAP) - LBS(RCAP)$$

Table 7.2 shows these MBO values, for a range of SCAP and RCAP values. Table 7.2 is presented in the same general format as Table 7.1, except that it has been condensed by reporting only a selection of rows and columns. The first row contains all zero values, as there is no “benefit from optimisation” (or storage) when there is no storage capacity to optimise. The first column shows cases with only two units of RCAP. In those cases, increasing SCAP beyond $0.07 \times FMEAN$ adds no more value. The next few columns show

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that, as RCAP is increased, there is value in adding additional SCAP, up to and somewhat above RCAP. For example, in the second (third) column, where $RCAP = 0.07$ (0.14), $SCAP = 0.25$ (0.75) times FMEAN, before benefits reach their maximum level. We have observed that increasing SCAP without increasing RCAP quickly creates a situation in which no more water can be profitably released to the market. So TWAB (Table 7.1 above) and the ANSD of TWAB (Table 7.6 below) stabilise at their highest levels, while VWAPR (Table 7.4 below) and VWPSD of VWAPR (Table 7.7 below) stabilise at their lowest levels, with further increases in SCAP having no impact. We term this phenomenon the “leading diagonal” effect.

		RCAP (\times FMEAN) \rightarrow														
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00			
\leftarrow SCAP (\times FMEAN)	0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0.02	133	172	178	180	179	179	178	178	178	178	178	178	178	178	178
	0.07	144	293	334	345	344	343	343	343	343	343	343	343	343	343	343
	0.14	144	350	435	464	464	463	463	463	463	463	463	463	463	463	463
	0.25	144	365	499	552	559	559	558	558	558	558	558	558	558	558	558
	0.50	144	365	537	640	670	670	670	669	669	669	669	669	669	669	669
	0.75	144	365	541	669	729	729	729	729	729	728	728	728	728	728	728
	1.00	144	365	541	678	764	768	767	767	767	766	766	766	766	766	766
	1.50	144	365	541	682	800	815	815	814	814	814	814	814	814	814	814
	2.00	144	365	541	682	814	842	844	844	844	843	843	843	843	843	843
	3.00	144	365	541	682	820	867	877	878	878	877	877	877	877	877	877
	4.00	144	365	541	682	821	875	893	897	897	896	896	896	896	896	896
	6.00	144	365	541	682	822	880	905	915	916	916	916	916	916	916	916
	12.00	144	365	541	682	822	882	912	930	934	935	935	935	935	935	935
	24.00	144	365	541	682	822	882	914	936	940	943	943	943	943	943	943
	36.00	144	365	541	682	822	882	915	937	942	944	944	944	944	944	944
48.00	144	365	541	682	822	882	915	938	943	945	945	945	945	945	945	
60.00	144	365	541	682	822	882	915	938	943	945	945	945	945	945	945	
72.00	144	365	541	682	822	882	915	938	943	945	945	945	945	945	945	
∞	144	365	541	682	822	882	915	939	944	946	946	946	946	946	946	

Table 7.2: Maximum Benefit (MBO) with MC Optimisation.

The green region in Table 7.2, showing the region of highest benefits, commences once RCAP and SCAP are at least approximately $0.5 \times FMEAN$. We illustrate the impact of increasing storage capacity on the benefits from MC optimisation in Figure 7.3. This plots all the columns from Table 7.2, up to and including column 8, (i.e. RCAP values up to 1.33), after which the plots become indistinguishable from TUBUR(SCAP) (when $RCAP = 4 \times FMEAN$, as discussed above). It may be seen that the curve for each fixed RCAP value follows the same pattern, as SCAP increases. Benefits initially increase quickly with a small

increase in SCAP, but eventually approach a maximum benefit level. The maximum benefit level is itself (very nearly) maximised, at TUBUR(SCAP), once $RCAP \geq 1.5 \times FMEAN$.

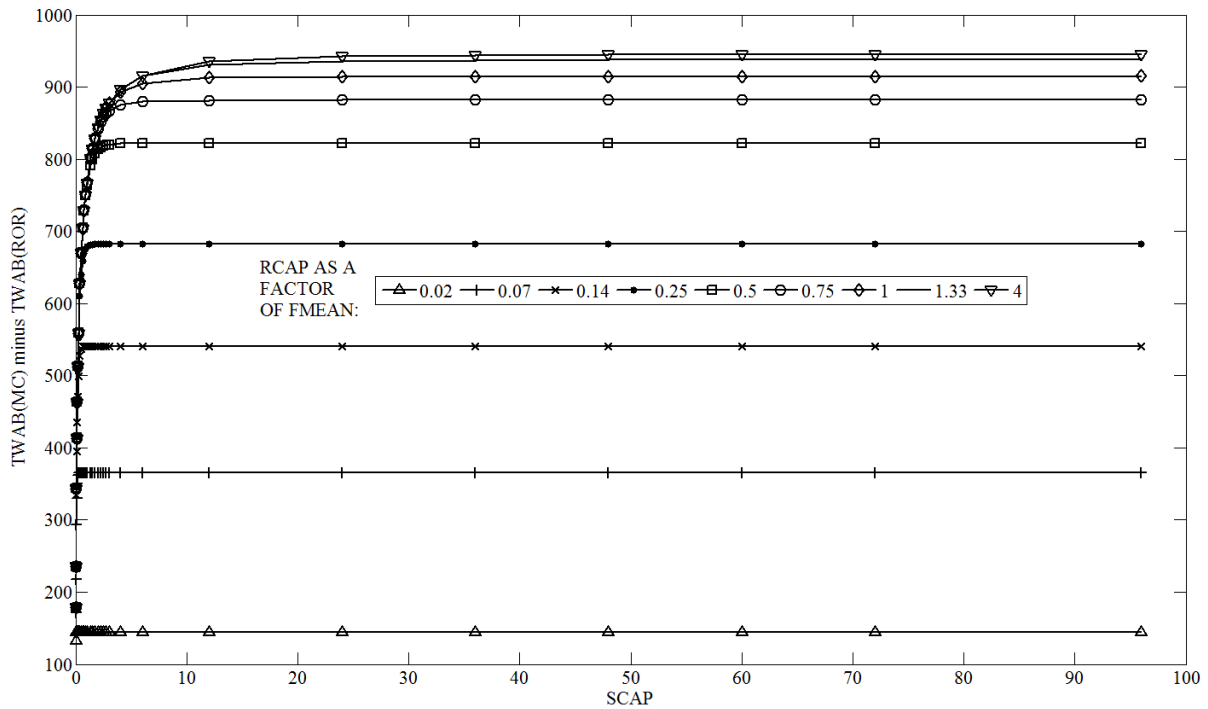


Figure 7.3: MC benefits from optimisation, for a range of capacity values.

Since our experiments use theoretical inflow and demand data, the absolute benefit numbers mean very little. Hence, for this analysis we tend to use proportional benefit measures.

We can transform the MBO values in Table 7.2 above into a simple proportional (percentage) value measure. As discussed in Section 5.5.6, we can compute the Maximum Benefit Range achievable by increasing Storage capacity, MBRS, for a particular RCAP, via Equation (5.38):

$$MBRS(RCAP) = TUBUS(RCAP) - LBS(RCAP)$$

Then we simply divide $MBO(RCAP, SCAP)$ by $MBRS(RCAP)$ to get the Proportion of Total Potential Benefit Achievable by increasing Storage capacity, $PTPBAS(RCAP, SCAP)$, as discussed in Section 5.5.7. $PTPBAS$ values are shown in Table 7.3, which is formatted similar to Table 7.2.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.02	0.921	0.469	0.329	0.263	0.218	0.202	0.195	0.190	0.189	0.189	0.189	0.189
	0.07	1.000	0.802	0.618	0.506	0.418	0.389	0.375	0.365	0.363	0.362	0.362	0.362
	0.14	1.000	0.957	0.804	0.680	0.564	0.525	0.506	0.493	0.490	0.489	0.489	0.489
	0.25	1.000	1.000	0.923	0.810	0.680	0.633	0.610	0.594	0.591	0.590	0.590	0.590
	0.50	1.000	1.000	0.994	0.938	0.816	0.760	0.732	0.713	0.709	0.708	0.708	0.708
	0.75	1.000	1.000	1.000	0.981	0.887	0.827	0.797	0.776	0.772	0.770	0.770	0.770
	1.00	1.000	1.000	1.000	0.994	0.930	0.871	0.838	0.817	0.813	0.811	0.811	0.811
	1.50	1.000	1.000	1.000	1.000	0.973	0.924	0.891	0.868	0.863	0.861	0.861	0.861
	2.00	1.000	1.000	1.000	1.000	0.990	0.955	0.923	0.899	0.894	0.892	0.892	0.892
	3.00	1.000	1.000	1.000	1.000	0.998	0.983	0.959	0.935	0.930	0.928	0.928	0.928
	4.00	1.000	1.000	1.000	1.000	0.999	0.992	0.976	0.955	0.950	0.948	0.948	0.948
	6.00	1.000	1.000	1.000	1.000	1.000	0.997	0.989	0.975	0.971	0.968	0.968	0.968
	12.00	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.991	0.990	0.989	0.989	0.989
	24.00	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.996	0.997	0.997	0.997
	36.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.998	0.999	0.999	0.999
	48.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999
60.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999	
72.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	
∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

Table 7.3: MC PTPBAS values, for a range of capacity values.

The bottom of Table 7.3 shows that loosely constrained SCAP cases (i.e. with at least a year’s worth of storage) achieve approximately a ratio of 1 (100%) of the possible value from adding optimally managed storage, across a wide range of RCAP values. Interestingly, this seems to be true for all cases below what, on the table, looks like a diagonal line, along which SCAP equals RCAP plus some factor. This appearance is a little misleading, because the table is not evenly spaced. But it is true that there will be a storage capacity level beyond which further expansion yields little benefit.

Figure 7.4 provides an alternative visualisation of PTPBAS, by plotting all the columns from Table 7.3, up to and including column 8, (RCAP values up to 1.33), after which the plots become indistinguishable from the final column, which is also plotted. The figure shows PTPBAS varying as a function of SCAP, for various fixed RCAP values. The SCAP axis has been constrained to $6 \times \text{FMEAN}$, given that values beyond this are all approximately 1 (0.968-1.000 from Table 7.3). Each curve shows a similar pattern, except that the maximum potential is lower, and hence reached more quickly, for lower RCAP values.

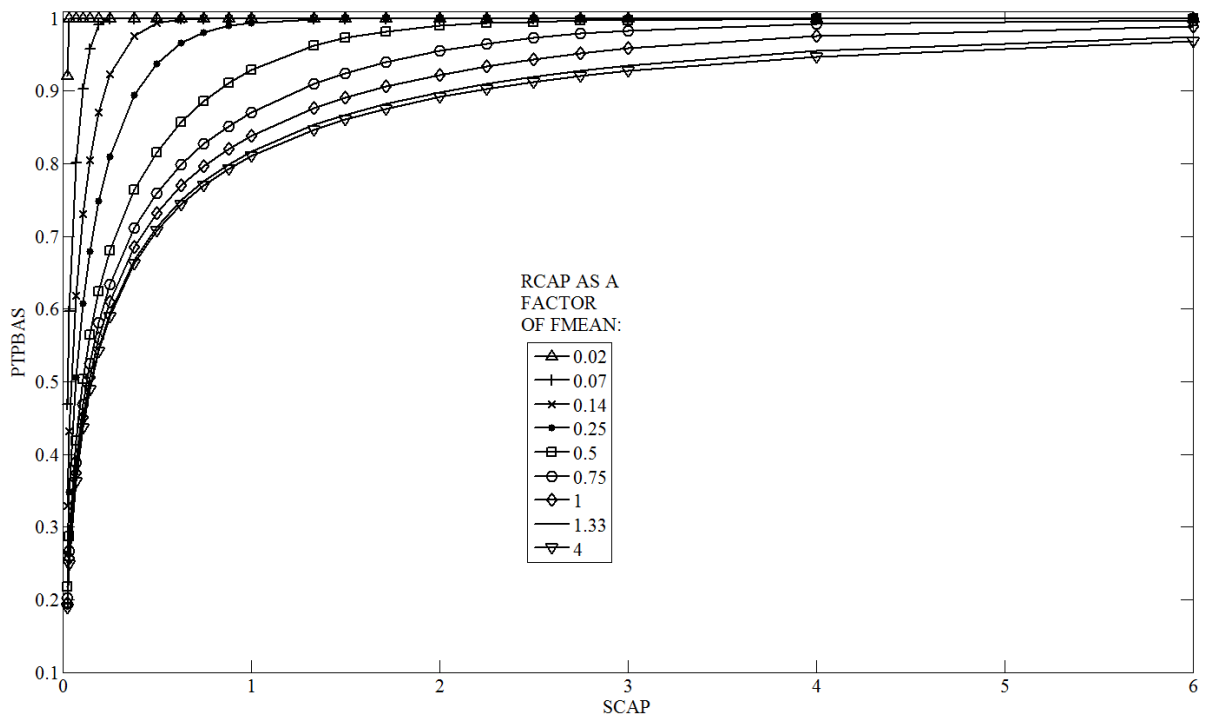


Figure 7.4: MC PTPBAS values, for a range of capacity values.

We can also calculate the Maximum Benefit Range achievable by increasing Release capacity (MBRR), as discussed in Section 5.5.6, via Equation (5.39):

$$MBRR(SCAP) = TUBUR(SCAP) - LBR(SCAP)$$

Dividing the benefit for the case, TWAB(MC), by MBRR(SCAP) gives the Proportion of Total Potential Benefit Achievable by increasing Release capacity, PTPBAR(RCAP,SCAP), as discussed in Section 5.5.7. We could report the full range of values, as in Table 7.3. But with $RCAP \geq FMEAN$, nearly all cases have a PTPBAR of 1 (100%), and with $RCAP \geq 0.5 \times FMEAN$ all cases have a PTPBAR of at least 0.99 (99%). So, instead of the table, Figure 7.5 shows PTPBAR varying as a function of RCAP, for various fixed SCAP values.

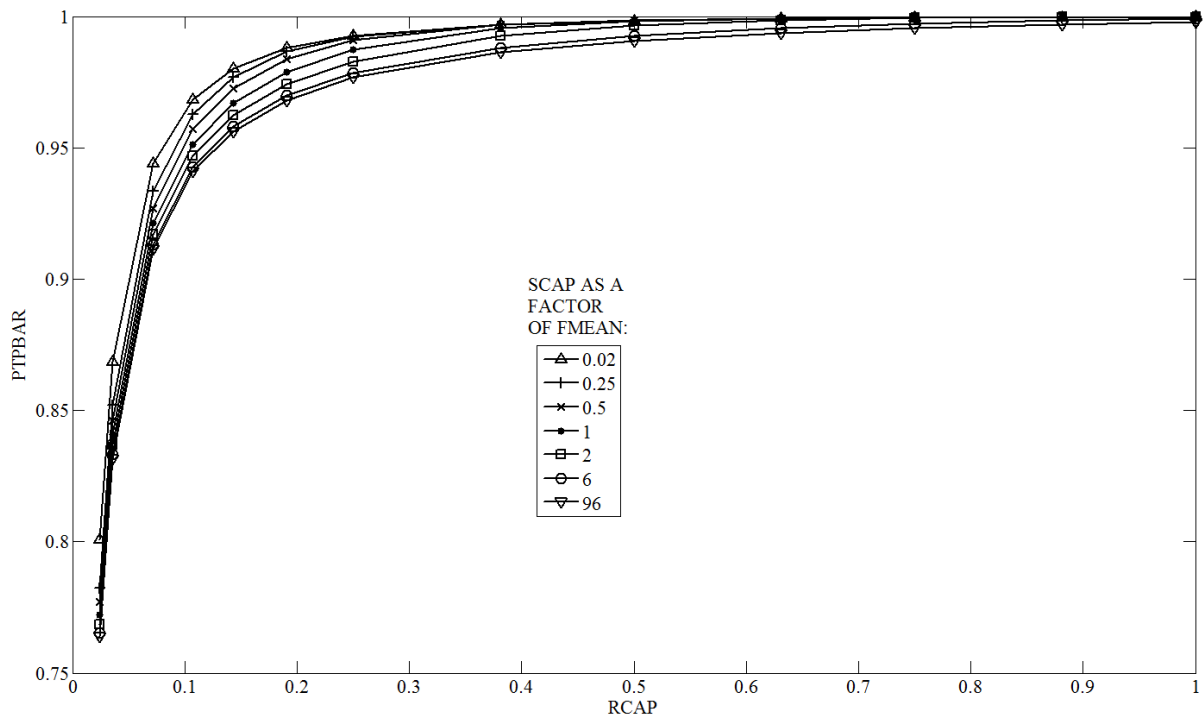


Figure 7.5: MC PTPBAR values, for a range of capacity values.

Figure 7.5 is laid out in the same general format as Figure 7.4 above. Directly comparing those two figures, we can see that the PTPBAR(SCAP) curves are much more tightly bunched than PTPBAS(RCAP). Also, PTPBAR(SCAP) reaches a 75% level with only two units of release capacity (approximately 2% of FMEAN), and virtually 100% by the time RCAP is equal to FMEAN. This suggests that (nearly) 100% of the possible benefits can be realised from building a reservoir with little or no release capacity above mean inflows (FMEAN). This result may seem surprising, but reflects the fact that we are using constant elasticity DCRs which assign extremely high values to the first few units released and low values to releases above the mean inflow. It may also be unrealistic, because of the need to build a factor of safety to allow more flows to be released in flood events etc.

7.2.3 Absolute Volume Weighted Average Prices

The average market clearing price is also an important consideration. Our main price reporting measure is the Volume Weighted Average Price for Release (VWAPR), as discussed in Section 5.6.2. Table 7.4 shows (annual expected) VWAPR values, for a reservoir system which uses MC optimisation, as a function of SCAP and RCAP. The first and final rows (where SCAP = 0 and ∞), have been left blank, because the ROR and

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TUBUS(RCAP) VWAPR values are not computed using the modelling suite, and require separate computation.

Inspecting each row and column, the VWAPR values are monotone non-increasing with increases in RCAP and/or SCAP, as expected. In the first row, with negligible storage capacity, VWAPR starts at a very high level, being approximately \$2500/unit, and then falls rapidly with increased RCAP. But, for an RCAP of $4 \times \text{FMEAN}$, it is still nearly 12, that is approximately 12 times the ideal unconstrained MCP. This reflects the impact of a highly constraining SCAP, even with a non-constraining RCAP. Across the last row (where SCAP has 6 years' worth of capacity), VWAPR starts at a very similar level, but drops off more rapidly, being approximately 1.4 when $\text{RCAP} = \text{FMEAN}$, and approximately (to 2 decimal places) reaches the optimal unconstrained price of \$1/unit once $\text{RCAP} > 1.5$. So, as with benefits discussed in Section 7.2.2, there is an SCAP level beyond which further expansion yields no price improvement.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
← SCAP (\times FMEAN)	0.02	2502.01	331.95	117.26	60.84	31.68	23.05	18.83	15.85	14.90	13.06	11.75	11.58
	0.07	2488.00	296.12	95.23	47.08	24.13	17.54	14.33	12.07	11.35	9.97	8.98	8.85
	0.14	2488.00	280.62	82.68	38.31	19.28	14.01	11.46	9.68	9.10	8.01	7.23	7.14
	0.25	2488.00	276.44	74.82	32.10	15.60	11.34	9.30	7.87	7.41	6.54	5.92	5.85
	0.50	2488.00	276.44	69.87	26.01	11.42	8.30	6.85	5.83	5.51	4.89	4.46	4.41
	0.75	2488.00	276.44	69.20	23.93	9.26	6.71	5.57	4.77	4.52	4.04	3.70	3.66
	1.00	2488.00	276.44	69.13	23.20	7.97	5.70	4.75	4.09	3.88	3.49	3.22	3.19
	1.50	2488.00	276.44	69.11	22.76	6.69	4.45	3.73	3.25	3.10	2.82	2.62	2.60
	2.00	2488.00	276.44	69.11	22.61	6.18	3.74	3.11	2.74	2.62	2.41	2.26	2.24
	3.00	2488.00	276.44	69.11	22.57	5.88	3.11	2.41	2.15	2.08	1.94	1.84	1.83
	4.00	2488.00	276.44	69.11	22.57	5.76	2.89	2.09	1.82	1.77	1.68	1.61	1.60
	6.00	2488.00	276.44	69.11	22.57	5.67	2.72	1.84	1.51	1.46	1.40	1.37	1.36
	12.00	2488.00	276.44	69.11	22.57	5.64	2.55	1.63	1.25	1.18	1.13	1.12	1.12
	24.00	2488.00	276.44	69.11	22.57	5.64	2.51	1.52	1.15	1.08	1.04	1.04	1.04
48.00	2488.00	276.44	69.11	22.57	5.64	2.51	1.46	1.11	1.05	1.01	1.01	1.01	
72.00	2488.00	276.44	69.11	22.57	5.64	2.51	1.44	1.10	1.04	1.00	1.00	1.00	

Table 7.4: VWAPR with MC optimisation.

7.2.4 Volatility Measures

With respect to volatility, we are most concerned with reporting the downside variance measures, because they are more important to participants, and the economy, and they are

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also much higher than the upside volatility measures, in all the experiments we have conducted.¹¹²

In particular, we focus on Annual Negative Semi-Deviation (ANSD) as a measure of lower than average benefits and the global Volume Weighted Positive Semi-Deviation (VWPSD) as a measure of higher than average prices. For cases where RCAP is at least the monthly average inflow, the ANSD of TWAB is approximately 5, 4, and 3 times the corresponding APSD of TWAB in the DC, SI, and MC optimisation run cases, respectively.¹¹³ For prices, the global VWPSD is approximately 82, 55, and 25 times the corresponding global VWNSD in the DC, SI, and MC optimisation run cases.

Table 7.5 shows the ANSD of TWAB values, for a reservoir system using MC optimisation, as a function of SCAP and RCAP. The pattern in Table 7.5 tends to reflect the proportional benefit pattern shown in Table 7.3, with the best performance, in this case low semi-deviation, naturally occurring in regions where SCAP is high, at least relative to RCAP. Generally, as benefits rise to 100% of their potential, the variability to those benefits tends to fall to zero, as expected.

The ANSD of TWAB is monotone non-increasing with increases in SCAP, as expected. Although the $SCAP = \infty$ row is excluded, we expect there to be no variance, when the system is large enough to deal with any pattern of inflow and demand. Evidently our experiments do not quite achieve this ideal, with some residual variance still visible with a $72 \times FMEAN$ reservoir,¹¹⁴ and $RCAP \geq 1$. This may reflect some experimental error, but is consistent with Table 7.2, which shows that the optimal unconstrained benefit has not quite been achieved in those cases, either, indicating that the storage bounds are still occasionally constraining.

Variability is limited for low RCAP because higher inflow states can not be fully utilised, irrespective of SCAP. (In the limit “variability” would be zero, for the $RCAP=0$ column, if it were shown.) But, while not quite monotone, variability rises rapidly with increasing RCAP, until it approaches a limit when RCAP is approximately $1.5 \times FMEAN$. In loosely constrained

¹¹² The hydrology data used in our experiments is symmetric (both inflows and probabilities), but our constant elasticity curves produce asymmetric returns.

¹¹³ The equivalent global average negative semi-deviations are all approximately 0.7.

¹¹⁴ And the $92 \times FMEAN$ in the wider data set, not illustrated in the Table here.

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cases, with both SCAP and RCAP high, the variability is becoming increasingly small, as the reservoir system can absorb fluctuations and thus reduce the volatility.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
←SCAP (\times FMEAN)	0.02	457.29	882.52	979.95	963.54	972.32	973.66	975.32	976.04	976.30	976.46	976.33	976.33
	0.07	0.00	421.66	583.27	620.44	639.65	643.28	644.49	644.76	644.89	644.86	644.88	644.88
	0.14	0.00	259.79	389.23	430.69	446.42	450.10	450.97	451.22	451.29	451.19	451.27	451.27
	0.25	0.00	0.00	251.48	302.55	315.19	319.36	320.38	320.62	320.74	320.75	320.77	320.77
	0.50	0.00	0.00	125.85	167.52	186.68	190.70	192.04	192.84	192.85	193.01	193.07	193.07
	0.75	0.00	0.00	9.02	98.55	135.01	139.81	140.95	141.84	141.97	142.06	142.03	142.04
	1.00	0.00	0.00	1.33	52.07	108.48	114.25	115.90	116.94	117.15	117.38	117.42	117.41
	1.50	0.00	0.00	0.00	18.36	79.71	88.76	90.70	92.24	92.52	92.87	92.92	92.92
	2.00	0.00	0.00	0.00	8.08	62.75	79.07	81.17	82.66	83.05	83.23	83.25	83.25
	3.00	0.00	0.00	0.00	0.27	34.83	73.28	80.96	81.99	82.25	82.22	82.25	82.24
	4.00	0.00	0.00	0.00	0.00	17.19	59.63	84.08	87.59	87.69	87.66	87.62	87.62
	6.00	0.00	0.00	0.00	0.00	5.91	29.93	59.73	80.99	83.95	85.07	85.04	85.04
	12.00	0.00	0.00	0.00	0.00	0.24	11.13	23.97	40.77	43.96	44.38	44.31	44.31
	24.00	0.00	0.00	0.00	0.00	0.00	3.84	10.39	19.96	22.60	22.25	21.87	21.87
36.00	0.00	0.00	0.00	0.00	0.00	0.17	3.77	12.18	13.68	14.48	14.22	14.21	
48.00	0.00	0.00	0.00	0.00	0.00	0.00	1.55	7.64	9.09	9.94	9.71	9.71	
60.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	5.40	6.76	7.62	7.45	7.45	
72.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	4.19	5.39	6.20	6.08	6.08	

Table 7.5: ANSD of TWAB with MC optimisation.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
←SCAP (\times FMEAN)	0.02	9703	11410	11831	11967	12020	12028	12030	12030	12030	12030	12030	12030
	0.07	9714	11523	11975	12120	12172	12180	12182	12182	12182	12182	12181	12181
	0.14	9714	11576	12069	12229	12284	12291	12293	12293	12293	12293	12293	12293
	0.25	9714	11590	12126	12310	12371	12379	12381	12381	12381	12381	12381	12381
	0.50	9714	11590	12160	12389	12473	12482	12483	12483	12483	12483	12483	12483
	0.75	9714	11590	12162	12414	12527	12536	12538	12538	12538	12538	12538	12538
	1.00	9714	11590	12162	12420	12559	12571	12573	12573	12573	12573	12573	12573
	1.50	9714	11590	12162	12422	12589	12614	12616	12616	12616	12616	12616	12616
	2.00	9714	11590	12162	12422	12598	12638	12643	12643	12643	12643	12643	12643
	3.00	9714	11590	12162	12422	12601	12657	12673	12674	12674	12674	12674	12674
	4.00	9714	11590	12162	12422	12601	12661	12685	12692	12692	12692	12692	12692
	6.00	9714	11590	12162	12422	12600	12661	12691	12708	12710	12710	12710	12710
	12.00	9714	11590	12162	12422	12600	12661	12692	12716	12722	12727	12727	12727
	24.00	9714	11590	12162	12422	12600	12661	12692	12717	12725	12733	12733	12733
36.00	9714	11590	12162	12422	12600	12661	12691	12718	12726	12734	12735	12735	
48.00	9714	11590	12162	12422	12600	12661	12691	12718	12727	12735	12735	12735	
60.00	9714	11590	12162	12422	12600	12661	12691	12718	12727	12735	12735	12735	
72.00	9714	11590	12162	12422	12600	12661	12691	12717	12727	12735	12735	12735	

Table 7.6: Global NSD of TWAB with MC optimisation.

Table 7.6 shows the global (time and probability weighted) NSD of benefit values, for a reservoir system using MC optimisation, as a function of SCAP and RCAP. This is much

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larger than the ANSD of TWABs in Table 7.5, as we would expect, since the global measure covers the underlying variability over all periods and scenarios within each year.

In particular, where the annual measure falls to zero in the lower LHS corner of the table, the global measure maintains a high and constant value. This is because the release capacity is constraining the solution such that the case is always releasing at full capacity, RCAP. Thus the benefit and price values cleared, are maximised and minimised, respectively, for the given situation. In those situations there is an inherent, and unavoidable, amount of variability from the inflow distribution. This variability is not really risk, as such, although we sometimes intermix the terms variability and risk.

Moving away from that corner, the global NSD, measuring the downside variability of annual TWAB, is monotone increasing with increased RCAP, up until $RCAP = FMEAN$, then falls a little for $SCAP > 12 \times FMEAN$. Up until $SCAP = 12 \times FMEAN$ the variability is monotone increasing with increased SCAP, then occasionally falls a little. The non-monotonicities are small and probably can be attributed to sampling error.

The pattern in Table 7.6 is similar to the MBO benefit pattern shown in Table 7.2, with lowest variability in the most constrained case and highest variability in the least constrained case. Thus, as average benefit rises, so does the global NSD of benefit. This relationship between annual and global benefit volatility measures has been observed for many cases, but we expect participants to be more interested in variations in annual profit than in monthly returns in what is, after all, a highly seasonal sector. So, to save repetition, we generally only report the ANSD of TWAB, as the benefit volatility measure, in the remainder of our experiments.

Table 7.7 shows that the pattern of the global VWPSD of VWAPR (that is the VWPSD of the price of a unit of water) which tends to reflect the absolute price pattern in Table 7.4. Low price variability is associated with low prices, and high price variability is associated with high prices. The top left corner shows the highest price variability, for the most constrained SCAP and RCAP case reported, as might be expected given the very high expected VWAPR shown in Table 7.4. From this point, increasing SCAP without increasing RCAP reduces prices very little, due to the leading diagonal effect discussed in Section 7.2.2.3 above.

Conversely, increasing RCAP, without increasing SCAP, reduces the price variability quickly at first, but it approaches a limit when RCAP is approximately $2 \times \text{FMEAN}$. When $\text{RCAP} \geq 0.25 \times \text{FMEAN}$, though, the price variability occasionally becomes non-monotone in RCAP, around the diagonal on which $\text{RCAP} = \text{SCAP}$. This effect, which is not really evident in the normal two-sided variance results (not shown here) may largely reflect the fact that the number of points from which the VWPSD is calculated increases in discrete steps from zero, to 1 to 2 etc., across that range.¹¹⁵

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	10096.1	4521.9	4298.6	4640.8	3789.7	3354.6	3355.3	3130.0	2883.3	2866.9	2867.8	2867.9
	0.07	9553.1	1674.7	1317.7	1407.0	1468.6	1354.6	1228.1	1230.1	1230.8	1231.8	1232.8	1233.0
	0.14	9553.1	1171.6	684.1	769.5	777.8	685.7	688.7	690.9	691.7	692.3	648.3	648.4
	0.25	9553.1	1061.5	411.1	360.3	440.5	406.2	409.3	411.1	393.9	361.2	361.3	361.4
	0.50	9553.1	1061.5	274.6	154.8	179.7	185.0	171.3	167.4	168.0	169.1	169.7	169.8
	0.75	9553.1	1061.5	265.1	101.0	106.7	104.7	101.9	103.4	103.9	104.7	100.6	100.7
	1.00	9553.1	1061.5	265.3	89.5	62.0	74.0	72.9	74.1	72.5	71.7	72.0	72.0
	1.50	9553.1	1061.5	265.4	86.2	35.3	43.1	43.4	43.7	43.9	43.9	44.1	44.2
	2.00	9553.1	1061.5	265.4	86.5	26.3	28.7	29.9	30.5	30.6	30.1	30.2	30.2
	3.00	9553.1	1061.5	265.4	86.6	21.8	16.2	19.7	19.8	19.8	19.8	19.5	19.5
	4.00	9553.1	1061.5	265.4	86.6	21.4	12.2	13.5	16.1	16.0	15.9	15.8	15.7
	6.00	9553.1	1061.5	265.4	86.6	21.6	9.8	8.0	10.3	11.9	12.1	11.9	11.8
	12.00	9553.1	1061.5	265.4	86.6	21.7	9.6	5.4	5.2	4.9	6.2	6.1	6.1
	24.00	9553.1	1061.5	265.4	86.6	21.7	9.6	5.2	2.8	2.8	2.8	2.8	2.8
	36.00	9553.1	1061.5	265.4	86.6	21.7	9.6	5.2	2.3	2.1	1.8	1.7	1.7
48.00	9553.1	1061.5	265.4	86.6	21.7	9.6	5.2	2.1	1.7	1.0	0.9	0.9	
60.00	9553.1	1061.5	265.4	86.6	21.7	9.6	5.3	2.1	1.6	0.7	0.7	0.7	
72.00	9553.1	1061.5	265.4	86.6	21.7	9.6	5.3	2.1	1.6	0.5	0.5	0.5	

Table 7.7: Global VWPSD of VWAPR with MC optimisation.

7.3 Economic Perspective on the Value of Stochastic Optimisation

Having carried out a brief review of how the MC optimisation performs, we now explore the relative performance of less sophisticated optimisation approaches/market designs (SI and DC) over a wide range of release/storage capacities. We focus on the maximum differences

¹¹⁵ In these cases there may be so little spare capacity that almost all of the 10,000 Monte Carlo samples produce the same pattern of releases (at RCAP), and hence the same average VWAPR, over the year. So the mean must lie slightly above that TWAB, with only a few extreme outliers, where release is less than RCAP, producing prices above the mean. This can cause the PSD to vary erratically. Suppose the PSD for one case was determined by only one point, and the situation was then relaxed by a small enough fraction to allow another point to rise just above the mean. Then the top line of the PSD calculation would change very little, but the bottom line would double, nearly halving the PSD, even though the PDF is essentially the same.

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i.e. between MC and DC optimisation runs, but briefly discuss SI versus DC, and SI versus MC differences, at various points. In this section we do not report LBS ($SCAP = 0$) or TUBUS ($SCAP = \infty$) case results in the tables. As expected, with no storage to optimise, the differences between “optimisation runs” are all zero. Similarly, with infinite storage, it does not matter how badly the optimisation performs, it can always get out of trouble, and hence the differences between optimisation runs are again all expected to be zero.

At one extreme, we might imagine reservoir systems so large that they can absorb all the uncertainty, regardless of how we manage our ‘optimal’ decision making. Conversely, at the other extreme, some reservoir systems may be so highly constrained (e.g. with more water available than can be profitably released) that their performance can not be improved, no matter how good the optimisation is. In either case, better optimisation may add little or no value. Between these two extremes we would expect to find a range of systems where there is significant value in accounting for uncertainty, and that value will depend on both reservoir storage and release capacities. Thus we have computed SI and DC results for the same 744 cases used in producing the MC result Table 7.1, above. And we plot, graph, and quote average values, from the full set of case data. But we present results in a condensed tabular form, similar to that of Tables 7.2 and 7.3, with only 216 cases, in a modest 16×12 grid.

7.3.1 Absolute Benefit Differences

The SI and DC absolute benefit tables are not shown here, but reveal a very similar benefit pattern to those in the MC case in Table 7.1. While the SI and DC results are believed to be optimal for their assumptions, they are suboptimal for the situation simulated, in which the stochastic process exactly matches the one used to optimise the MC case, as discussed in Section 7.2 above. Thus, we should not be surprised that the SI optimisation yields consistently lower benefit than the MC optimisation, and consistently higher benefit than the DC optimisation. Also, while spill is not reported here, each MC case typically has more spill than the corresponding DC case. This is because the MC optimisation keeps the reservoir fuller to avoid shortage, and to maximise benefits across the water year. Shortage is avoided due to the lost income it represents, rather than any penalty.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	0.0	33.2	62.0	65.4	69.0	69.0	68.6	68.9	68.0	68.1	68.1	68.1
	0.07	0.0	24.9	113.2	125.9	131.2	131.2	130.1	130.5	128.8	128.8	128.8	128.8
	0.14	0.0	4.3	89.3	133.8	144.1	149.7	142.8	144.0	139.9	139.7	139.8	139.8
	0.25	0.0	0.0	44.4	128.8	142.7	152.0	143.9	142.3	140.6	140.1	140.3	140.3
	0.50	0.0	0.0	2.7	61.5	96.8	102.1	98.7	95.6	90.8	89.6	90.0	90.1
	0.75	0.0	0.0	0.2	23.1	66.1	76.8	72.6	68.0	65.1	64.3	65.3	65.3
	1.00	0.0	0.0	0.0	8.7	47.7	63.3	58.0	54.9	52.6	52.8	54.0	54.0
	1.50	0.0	0.0	0.0	1.7	32.7	50.2	50.1	47.9	47.7	48.9	49.5	49.5
	2.00	0.0	0.0	0.0	0.3	20.6	42.2	43.2	44.5	44.8	45.7	46.3	46.3
	3.00	0.0	0.0	0.0	0.0	9.8	30.7	29.3	30.0	29.8	30.9	31.5	31.5
	4.00	0.0	0.0	0.0	0.0	4.0	22.7	25.9	21.9	22.3	23.4	23.5	23.5
	6.00	0.0	0.0	0.0	0.0	0.5	10.9	23.1	16.2	16.9	17.6	17.6	17.6
	12.00	0.0	0.0	0.0	0.0	0.0	1.4	15.0	9.3	8.2	7.9	7.9	7.9
	24.00	0.0	0.0	0.0	0.0	0.0	0.0	8.7	6.0	5.4	5.5	5.6	5.6
48.00	0.0	0.0	0.0	0.0	0.0	0.0	4.5	3.4	3.1	3.1	3.2	3.2	
72.00	0.0	0.0	0.0	0.0	0.0	0.0	3.2	2.5	2.2	2.2	2.2	2.2	

Table 7.8: MC minus DC TWAB Differences.

Table 7.8 shows MC minus DC absolute benefit differences, for a range of SCAP and RCAP values. Table 7.8 is presented in the same general format as Table 7.2. With a small release capacity ($RCAP < FMEAN$), the system can release very little to the market, even with a high storage capacity and optimal management. So the DC and MC cases again return the same benefits, and there is no improvement from optimising the system with a more sophisticated model, hence the zeros in the bottom-left corner of Table 7.8.

In contrast to this, the red zone in the table shows that the greatest benefit increases from using a more sophisticated optimisation occur when there is quite a modest SCAP, between 0.02 and 0.75 times FMEAN. The maximum occurs at $SCAP = 0.25 \times FMEAN$, with $RCAP = 0.75 \times FMEAN$, but similar benefit levels occur across all reasonable RCAP values (i.e. except when RCAP is significantly below FMEAN).

It may seem surprising that the maximum gain occurs at such low SCAP levels, but it seems that the results are showing that the greatest value of stochastic optimisation is where storage is small, and needs to be effectively managed, rather than in cases where storage is large and potentially offering greater opportunities over the wider water year.

With low SCAP, water can not be stored in the wet season to be used in the dry season. So the issue becomes one of managing the storage week to week, or month to month given our

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discretisation of the stages. Storage levels must always be close to their bounds, and DC optimisation does not perform well when the reservoir is close to its storage bounds, as discussed in Section 6.4. In particular DC optimisation tends to run small reservoirs empty more often than MC optimisation. The DC optimisation does this on the assumption that it will observe (and have available) the average inflow state in the subsequent stage. But clearly the DC optimisation is often caught out, having no water to meet highly valued demand. By comparison, the MC optimisation takes a more cautious approach. It can see the whole distribution of inflows, and their correlations. With this observation the MC optimisation takes measures to ensure that spill/shortage is avoided. In particular, the MC optimisation tends to store more, when water levels are low, to offset the variability of realising a much drier than average inflow state.

By way of contrast, the DC optimisation does quite well when reservoirs are larger. Presumably that is because, although it pushes the reservoir system closer to its bounds than an MC optimisation, the DC optimisation still finds a long run equilibrium which transfers inflows between seasons appropriately, with release equalling inflow on average. The average monthly storage level is typically much lower in the DC case, though, implying a higher probability of shortage, should inflows be less than expected by the DC optimisation. But, with a large enough storage capacity, the DC case will hit the lower and upper storage bounds infrequently, implying reasonable performance, most of the time.

We note that these results may be partly an artefact of the Markov Chain which we have modelled with only one month of correlation. With longer term correlations, a wider storage capacity range would be required to deal with uncertainty, in which case the maximum benefit of more sophisticated stochastic optimisation might become evident at higher SCAP levels. Still, we have shown that more sophisticated optimisation can yield significant benefit gains. The real issue is, though, how significant these gains might be relative to the maximum gain achievable, and to market value.

7.3.2 Proportional Benefit Differences

In Chapter 5 we outlined various proportional value added measures. In each case, the numerator term is the benefit difference between more and less complex optimisation runs. But, by changing the denominator term, each measure gives a different perspective on the

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improvement achieved. In Section 7.4 we discuss another value measure, focused on market participants. Here we focus on the PTPBAS, as in Section 7.2.2.3 above.

Table 7.9 shows MC versus DC PTPBAS differences, in the same format as Table 7.3, and shows a very similar pattern to that of the absolute benefit difference values in Table 7.8. That is, there is no difference in performance when SCAP is either very low, or very high, or at least higher than RCAP. The greatest proportional benefit differences from using a more sophisticated optimisation are when SCAP is less than FMEAN, except when RCAP is significantly below FMEAN.

The maximum MC versus DC % differences are approximately 20% and 15%, for RCAP values of 0.25 and 4 times FMEAN, respectively. In the $RCAP = 0.25 \times FMEAN$ case these reduce to approximately 1% once $SCAP \geq FMEAN$. This is consistent with observations made in Section 7.3.1 about the relative sizing of RCAP and SCAP. From Table 7.9, taking the cases with RCAP at least equal to the monthly average inflow (FMEAN), which seems more likely for many real-world systems, we sum all the non-zero values,¹¹⁶ and then dividing by the total number of non-zero cases,¹¹⁷ we conclude that the average reduction in expected benefit from using a DC optimisation which totally ignores the underlying stochasticity is 6.3% of PTPBAS.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
← SCAP (\times FMEAN)	0.02	0.000	0.091	0.115	0.096	0.084	0.078	0.075	0.073	0.072	0.072	0.072	0.072
	0.07	0.000	0.068	0.209	0.185	0.160	0.149	0.142	0.139	0.137	0.136	0.136	0.136
	0.14	0.000	0.012	0.165	0.196	0.175	0.170	0.156	0.153	0.148	0.148	0.148	0.148
	0.25	0.000	0.000	0.082	0.189	0.174	0.172	0.157	0.152	0.149	0.148	0.148	0.148
	0.50	0.000	0.000	0.005	0.090	0.118	0.116	0.108	0.102	0.096	0.095	0.095	0.095
	0.75	0.000	0.000	0.000	0.034	0.080	0.087	0.079	0.072	0.069	0.068	0.069	0.069
	1.00	0.000	0.000	0.000	0.013	0.058	0.072	0.063	0.058	0.056	0.056	0.057	0.057
	1.50	0.000	0.000	0.000	0.002	0.040	0.057	0.055	0.051	0.051	0.052	0.052	0.052
	2.00	0.000	0.000	0.000	0.000	0.025	0.048	0.047	0.047	0.047	0.048	0.049	0.049
	3.00	0.000	0.000	0.000	0.000	0.012	0.035	0.032	0.032	0.032	0.033	0.033	0.033
	4.00	0.000	0.000	0.000	0.000	0.005	0.026	0.028	0.023	0.024	0.025	0.025	0.025
	6.00	0.000	0.000	0.000	0.000	0.001	0.012	0.025	0.017	0.018	0.019	0.019	0.019
	12.00	0.000	0.000	0.000	0.000	0.000	0.002	0.016	0.010	0.009	0.008	0.008	0.008
	24.00	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.006	0.006	0.006	0.006	0.006
48.00	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.004	0.003	0.003	0.003	0.003	
72.00	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.003	0.002	0.002	0.002	0.002	

Table 7.9: MC versus DC PTPBAS Differences.

¹¹⁶ By comparison, the average reduction in expected benefit from ignoring inflow correlations is 4.4% (SI versus DC case). Hence the reduction is 1.9%, when comparing SI versus MC.

¹¹⁷ Including the non-condensed row and column case results, but excluding the LBS (first row) and TUBUS (last row) results.

To put these gains into perspective, Figure 7.6, parallels Figure 7.4, but now for the DC (red), SI (black), and MC (green) optimisations, but with only RCAP columns 0.25, 1, and 4 times FMEAN shown. The curves in the bottom-left corner show the benefit differences. The figure shows that just building increased storage capacity, even with DC optimisation, delivers most of the potential benefit. But significant further gains can be made by optimising the system with a simple model of the underlying uncertainty (SI), and modest extra gains from modelling correlation using the MC model.

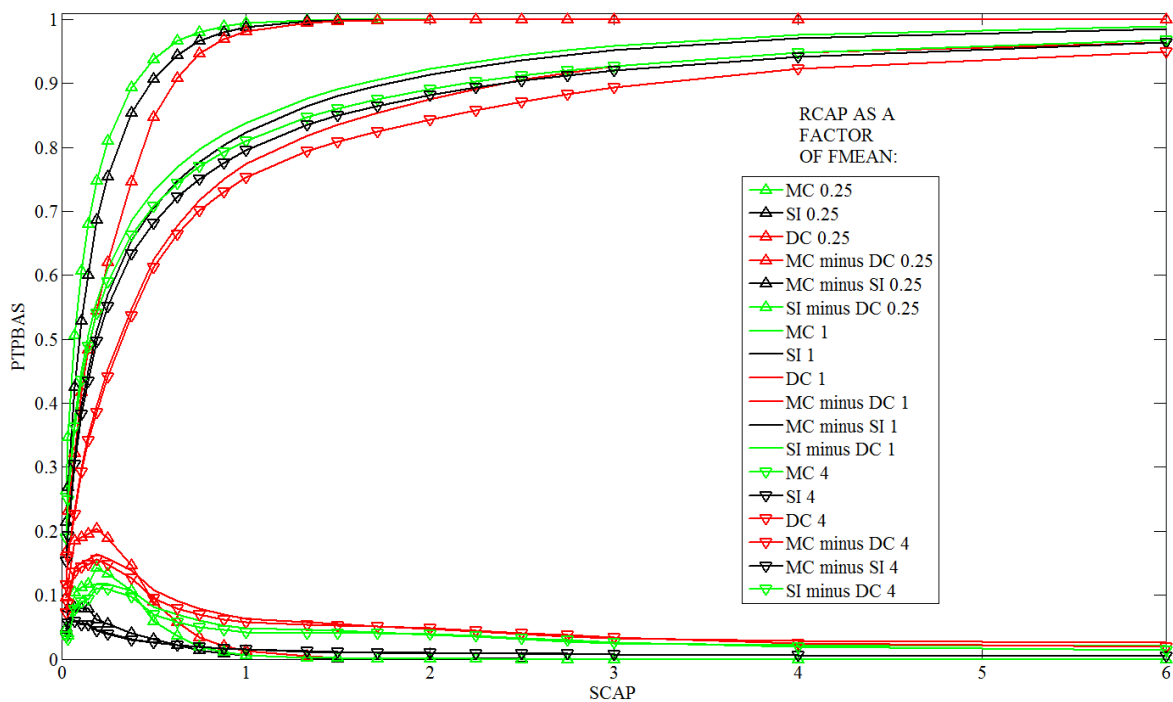


Figure 7.6: PTPBAS values, for a range of modelled SCAP and RCAP values and all optimisation run complexities.¹¹⁸

7.3.3 Benefit Variability Differences

In this section we briefly report downside variability measure differences between the most (MC) and least (DC) complex optimisations we model. Table 7.10 is formed by taking the DC optimisation ANSD of TWAB results away from the corresponding values for the MC optimisation (in Table 7.5).

¹¹⁸ All the graphs start from the SCAP = 0 case which, as discussed earlier, is always zero.

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Table 7.10 is in a similar format as Table 7.5. In Table 7.10 there is no difference in benefit variability when there is a high SCAP, at least relative to a low RCAP. This is similar to Table 7.5, which shows absolute MC negative semi-deviation of benefits as zero in this same region. There are a few unusually high differences just above that diagonal, and a few unusually low (negative) differences just below that diagonal. These are probably due to the effect discussed in Footnote 117.

As SCAP increases the ANSD of TWAB differences reduce, but there are still some reasonable differences. In Section 6.4.5, and the earlier Figure 5.11 (in Section 5.5.5) alludes to the reason why, there for a case when SCAP = 12 and RCAP = 1.33 times FMEAN. The benefit CDF in the DC optimisation has a very long tail compared with the MC optimisation, and we have seen this in higher capacity cases too.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
← SCAP (\times FMEAN)	0.02	0.000	293.376	307.888	360.015	364.345	364.648	361.807	361.125	359.507	359.508	359.777	359.777
	0.07	0.000	490.866	752.314	722.776	702.248	698.052	694.482	694.390	691.988	692.440	692.193	692.193
	0.14	0.000	276.491	808.559	749.491	743.360	753.435	736.758	740.698	727.516	727.632	727.456	727.499
	0.25	0.000	0.000	686.851	803.962	774.989	790.843	769.087	766.231	763.256	761.618	762.417	762.559
	0.50	0.000	0.000	356.437	696.601	685.015	673.146	660.212	648.484	618.007	613.068	615.256	615.790
	0.75	0.000	0.000	345.447	603.599	545.472	577.733	560.383	518.039	506.703	500.017	507.474	508.129
	1.00	0.000	0.000	-1.330	550.125	428.597	501.681	456.738	437.959	426.833	429.047	438.239	438.343
	1.50	0.000	0.000	0.000	702.084	406.099	392.004	391.879	380.768	388.901	400.238	404.128	404.128
	2.00	0.000	0.000	0.000	451.142	477.204	361.223	337.501	363.766	370.692	377.223	381.304	381.304
	3.00	0.000	0.000	0.000	-0.270	523.555	383.688	303.162	297.521	295.676	307.244	312.409	312.420
	4.00	0.000	0.000	0.000	0.000	401.708	395.823	298.313	236.195	237.824	246.361	246.336	246.335
	6.00	0.000	0.000	0.000	0.000	244.462	271.440	271.916	208.876	196.764	188.733	188.682	188.682
	12.00	0.000	0.000	0.000	0.000	-0.243	185.797	235.464	173.415	153.742	137.735	137.752	137.756
	24.00	0.000	0.000	0.000	0.000	0.000	127.564	259.386	209.739	189.021	168.584	168.973	168.973
48.00	0.000	0.000	0.000	0.000	0.000	0.000	234.716	195.545	183.180	173.872	174.105	174.105	
72.00	0.000	0.000	0.000	0.000	0.000	0.000	235.240	193.492	181.091	172.752	172.751	172.751	

Table 7.10: DC minus MC for the ANSD of TWAB.

For cases where RCAP is at least the monthly average inflow, a quick comparison across optimisation runs is achieved by just taking case differences as a proportion of the base case (ANSD of TWAB for MC optimisation). Moving from MC to SI increases variability by 77%,¹¹⁹ while moving from SI to DC increases it by a further 125%¹²⁰ of the MC base case value.

¹¹⁹ (370.5-209.7)/209.7

¹²⁰ (632.5-370.5)/209.7

7.4 Market Participant Perspective on value of optimisation

Consumers are most interested in the price they pay for water. So, price is a secondary measure of market performance. In this section we focus first on the price differences between MC and DC optimisation runs, but also briefly discuss SI versus DC, and SI versus MC differences. Participant perspectives on benefit differences will also be affected by price levels, so we also discuss the Benefit Achievable as a Proportion of Market Value (BAPMV), as discussed in Section 5.5.9. First we assume the Market Value (MV) is based on the “ideal” Market Value (MV^*), and $BAPMV^*$. Then we divide the $BAPMV^*$ results by their corresponding MC VWAPR values to get the Benefit Achievable as a Proportion of the “Actual optimal” Market Value, $BAPMV(MC\ VWAPR)$. Finally, we discuss the downside price volatility.

7.4.1 Participant Perspective on Price Comparison

The SI and DC absolute VWAPR tables are not shown here, but reveal a very similar benefit pattern to those in the MC case in Table 7.4. Table 7.11 shows MC minus DC VWAPR differences (i.e. a positive result indicates the DC VWAPR is higher than the MC VWAPR), for a range of SCAP and RCAP values, using a condensed format, as in Table 7.7. From the full data set, the maximum and minimum differences are 21.19 and -0.13. This latter result does not seem to be attributable to sampling error. Although we generally expect that maximising welfare means minimising the prices that consumers pay, this is not always the case, for the reasons discussed in Section 5.6.3. These negative price differences emerge along the roughly diagonal line first noted in Table 7.3, where RCAP is less than the average inflow, and SCAP is somewhat above RCAP. We believe this reflects the setup of our particular experiments, as discussed in Section 5.4.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	0.00	12.69	11.33	7.81	4.65	3.43	2.82	2.38	2.23	1.97	1.77	1.75
	0.07	0.00	8.52	18.38	12.79	7.67	5.68	4.67	3.95	3.71	3.27	2.95	2.92
	0.14	0.00	1.61	12.82	11.70	7.37	5.68	4.54	3.87	3.57	3.15	2.86	2.84
	0.25	0.00	0.00	5.78	10.15	6.66	5.22	4.16	3.52	3.29	2.92	2.67	2.66
	0.50	0.00	0.00	0.05	4.09	3.81	3.04	2.45	2.05	1.84	1.64	1.54	1.55
	0.75	0.00	0.00	-0.05	1.27	2.27	2.03	1.63	1.31	1.20	1.07	1.05	1.06
	1.00	0.00	0.00	-0.02	0.29	1.45	1.53	1.18	0.98	0.89	0.82	0.83	0.85
	1.50	0.00	0.00	0.00	-0.04	0.86	1.06	0.92	0.77	0.75	0.72	0.74	0.75
	2.00	0.00	0.00	0.00	-0.02	0.48	0.82	0.73	0.68	0.67	0.65	0.68	0.69
	3.00	0.00	0.00	0.00	0.00	0.16	0.51	0.44	0.41	0.40	0.42	0.46	0.46
	4.00	0.00	0.00	0.00	0.00	0.03	0.32	0.34	0.28	0.29	0.32	0.34	0.34
	6.00	0.00	0.00	0.00	0.00	-0.01	0.09	0.25	0.20	0.22	0.25	0.25	0.25
	12.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.11	0.11	0.11	0.11	0.11	0.11
	24.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.08	0.08	0.08	0.08	0.08
48.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.04	0.04	0.04	0.04	
72.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.04	0.03	0.03	0.03	0.03	

Table 7.11: DC minus MC differences in VWAPR.

Below that line, the DC and MC cases produce the same prices, and hence the zeroes in the bottom-left corner of Table 7.11, as is also the case for benefit differences in Table 7.8 and ANSD of TWAB differences in 7.11. As in Table 7.11, the top-left corner value is zero, because both optimisations badly manage this highly constrained situation. In both tables, the highest differences are in the top-left quadrant, where RCAP is low, and SCAP is at least that low, while differences in the bottom-right quadrant are tending to zero. Comparison with Table 7.8 shows that price differences fall off more quickly than benefit differences, with increasing RCAP.

Since the prices are relative to an arbitrary ‘ideal’ price of 1, participants might be more interested in VWAPR differences as a percentage of the actual VWAPR they might expect to experience under MC optimisation. MC versus DC differences for this measure are given in Table 7.12 below. This table is formed by subtracting each MC VWAPR value (in Table 7.4) from the corresponding DC VWAPR value, and then dividing the resulting difference by the MC VWAPR value.

Clearly the same region of zeroes is evident as in Table 7.11. There are large differences when SCAP is low, and they remain at a similar level for increasing RCAP. And, even when SCAP is FMEAN or larger, there are still double digit percentage gains (inferred) from clearing the market using the MC optimisation, rather than using a DC optimisation.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	0.00	0.04	0.10	0.13	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	0.07	0.00	0.03	0.19	0.27	0.32	0.32	0.33	0.33	0.33	0.33	0.33	0.33
	0.14	0.00	0.01	0.16	0.31	0.38	0.41	0.40	0.40	0.39	0.39	0.39	0.40
	0.25	0.00	0.00	0.08	0.32	0.43	0.46	0.45	0.45	0.44	0.45	0.45	0.46
	0.50	0.00	0.00	0.00	0.16	0.33	0.37	0.36	0.35	0.33	0.33	0.34	0.35
	0.75	0.00	0.00	0.00	0.05	0.24	0.30	0.29	0.28	0.27	0.27	0.28	0.29
	1.00	0.00	0.00	0.00	0.01	0.18	0.27	0.25	0.24	0.23	0.24	0.26	0.27
	1.50	0.00	0.00	0.00	0.00	0.13	0.24	0.25	0.24	0.24	0.26	0.28	0.29
	2.00	0.00	0.00	0.00	0.00	0.08	0.22	0.23	0.25	0.25	0.27	0.30	0.31
	3.00	0.00	0.00	0.00	0.00	0.03	0.16	0.18	0.19	0.19	0.22	0.25	0.25
	4.00	0.00	0.00	0.00	0.00	0.01	0.11	0.16	0.15	0.16	0.19	0.21	0.21
	6.00	0.00	0.00	0.00	0.00	0.00	0.03	0.13	0.14	0.15	0.17	0.19	0.19
	12.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.09	0.09	0.10	0.10	0.10
	24.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.07	0.07	0.08	0.07	0.07
48.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.04	0.04	0.04	0.04	
72.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.03	0.03	0.03	0.03	

Table 7.12: DC minus MC differences in VWAPR, as a proportion of MC VWAPR.

For cases where RCAP is at least the monthly average inflow, comparing across optimisation runs by taking optimisation run differences as a proportion of the base case (VWAPR for MC optimisation), then moving from MC to SI increases VWAPR by 8.9%,¹²¹ while moving from SI to DC increases it by a further 19.4%¹²² of the actual prices which might be expected under MC optimisation. While taking the same differences as a proportion of the ideal unconstrained MCP of 1, gives a 39.4% increase when moving from MC to SI, and a further 86.2% when moving from SI to DC.

7.4.2 Participant Perspective on Benefit Differences

While economists might be concerned with maximising total benefit and the theoretical benefits derived from the first few units of water consumed, as per the previous section, participants are more likely to be concerned with benefits and prices experienced in the real world. In order to review benefit differences in terms of market clearing prices, we could form $BAPMV^*$, as discussed in Section 5.5.9, by dividing the benefit differences in Table 7.8 by the annual average ideal unconstrained market value traded i.e. a quantity of 1008, at a price of 1 (or 10.08 for a % measure, or 84 and 0.84 given we generally report the TWAB measure). Without displaying the table, we can see that the pattern of results will be exactly the same as the results in Table 7.8. Restricting attention to cases with an RCAP of at least

¹²¹ $(4.83-4.43)/4.43$

¹²² $(5.69-4.83)/4.43$

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FMEAN, then the difference between the MC and DC BAPMV* ranges from 3% for high SCAP values to 178% for low SCAP values, and averages 71%.

Again, though, participants might be more interested in a measure based on the differences as a proportion of the actual market clearing price they might expect to experience under MC optimisation, the MC VWAPR. This is the BAPMV(MC VWAPR) measure, as discussed in Section 5.5.9.

So, Table 7.13 reports the MC minus DC BAPMV(MC VWAPR) results, by dividing the values in Table 7.8 by the corresponding value of the market at optimal MC prices, as determined by multiplying the MC VWAPR values in Table 7.4 by the mean monthly volume traded of 84.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.04	0.05	0.05	0.06	0.06
	0.07	0.00	0.00	0.01	0.03	0.05	0.07	0.08	0.10	0.10	0.12	0.13	0.13
	0.14	0.00	0.00	0.01	0.03	0.06	0.09	0.11	0.13	0.13	0.15	0.16	0.17
	0.25	0.00	0.00	0.01	0.04	0.08	0.11	0.13	0.15	0.16	0.18	0.19	0.20
	0.50	0.00	0.00	0.00	0.02	0.08	0.11	0.13	0.14	0.15	0.16	0.18	0.18
	0.75	0.00	0.00	0.00	0.01	0.07	0.10	0.12	0.13	0.14	0.15	0.16	0.16
	1.00	0.00	0.00	0.00	0.00	0.06	0.10	0.12	0.13	0.13	0.15	0.16	0.16
	1.50	0.00	0.00	0.00	0.00	0.05	0.11	0.13	0.14	0.15	0.16	0.18	0.18
	2.00	0.00	0.00	0.00	0.00	0.04	0.11	0.13	0.16	0.16	0.18	0.19	0.19
	3.00	0.00	0.00	0.00	0.00	0.02	0.10	0.12	0.14	0.14	0.16	0.16	0.16
	4.00	0.00	0.00	0.00	0.00	0.01	0.08	0.13	0.12	0.13	0.14	0.14	0.14
	6.00	0.00	0.00	0.00	0.00	0.00	0.05	0.13	0.11	0.12	0.13	0.13	0.13
	12.00	0.00	0.00	0.00	0.00	0.00	0.01	0.10	0.08	0.08	0.08	0.08	0.08
	24.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.06	0.06	0.06	0.06	0.06
48.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.04	0.03	0.04	0.04	0.04	
72.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.02	0.03	0.03	0.03	

Table 7.13: MC minus DC BAPMV(MC VWAPR).

Obviously, zeros appear in the same areas as in Table 7.8. But the high benefit differences evident in Table 7.8, seem less significant when seen as a proportion of the high market value/ prices for the MC case shown in Table 7.4, particularly when $RCAP < 0.5 \times FMEAN$. These proportional benefits increase with RCAP, where the absolute benefits tend to be flat in Table 7.8. On the other hand, proportional benefits are quite stable, at around 15%, over a wide range of SCAP values, rather than peaking sharply at low SCAP values, as in Table 7.8.

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Given the high VWAPR values when RCAP is constrained, the region where DC optimisation costs the most is when RCAP is at least FMEAN. Over that region, it loses approximately 12% of the market value expected under MC optimisation. As we have seen, it increases prices by around 28.3% in the same region. Using SI optimisation loses approximately 3% of the benefit, and adds 8.9% to the price. Thus, from this perspective, participants might be quite interested in the MCE being able to clear the market in a more sophisticated manner.

7.4.3 Price Volatility Differences

In this section we briefly report the differences in the downside volatility measure for price (VWAPR), focusing on the differences between the MC and DC optimisations. Table 7.14 is formed by taking the difference between the value for the DC optimisation and the corresponding value for the MC optimisation (in Table 7.7).

In Table 7.14, the top-left corner value is zero, for the most constrained case reported, because the MC and DC optimisations do an equally bad job at managing this situation, producing the same high price variability in both the MC and DC cases. There is also no difference in price variability between the MC and DC optimisation, when there is a high SCAP, at least relative to a low RCAP (as per VWAPR differences in Table 7.11). Neither optimisation can do much about the bad situation, because they cannot release any more water to the market. There are also unusually high differences just above that diagonal.

The pattern in Table 7.14 is similar to that in Table 7.10, for the DC minus MC differences for ANSD of TWAB, in that differences generally reduce with increased SCAP. The global VWPSD of VWAPR differences appear more well behaved near the leading diagonal, than those in Table 7.10.

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	0.0	1886.3	2623.1	3177.4	2639.4	3068.7	2343.0	2184.1	2391.1	1986.8	1986.6	1986.6
	0.07	0.0	2241.0	4870.4	5540.7	4299.3	3809.1	3917.0	3580.8	3237.6	3236.1	3235.5	3235.5
	0.14	0.0	791.4	2871.2	3475.7	3877.0	3597.5	3324.7	2993.1	2951.4	2949.4	2984.6	2984.5
	0.25	0.0	0.0	1906.1	3203.6	3220.1	3069.2	2701.8	2653.0	2649.3	2680.7	2666.9	2666.5
	0.50	0.0	0.0	250.8	1467.8	1785.7	1714.6	1634.4	1611.8	1503.8	1367.6	1369.4	1369.5
	0.75	0.0	0.0	24.0	877.1	1082.8	1187.7	1117.6	971.8	953.9	944.5	958.9	960.6
	1.00	0.0	0.0	0.1	511.8	772.1	899.1	775.7	733.3	724.0	725.0	728.1	728.1
	1.50	0.0	0.0	0.0	214.1	499.8	559.3	563.1	547.7	536.9	548.0	550.3	550.3
	2.00	0.0	0.0	0.0	6.4	450.4	488.8	449.4	462.4	471.1	477.2	484.7	484.7
	3.00	0.0	0.0	0.0	0.0	339.2	381.4	367.6	371.9	358.4	372.9	381.3	381.1
	4.00	0.0	0.0	0.0	0.0	186.3	336.8	366.2	323.8	314.3	309.0	297.6	297.6
	6.00	0.0	0.0	0.0	0.0	59.9	173.0	297.7	247.3	381.5	369.1	360.8	360.9
	12.00	0.0	0.0	0.0	0.0	0.0	19.3	185.0	170.7	155.2	265.9	262.2	262.2
	24.00	0.0	0.0	0.0	0.0	0.0	1.6	143.5	157.0	141.5	328.8	328.9	327.1
48.00	0.0	0.0	0.0	0.0	0.0	0.0	102.6	90.4	109.3	333.8	333.5	333.5	
72.00	0.0	0.0	0.0	0.0	0.0	0.0	76.3	67.6	82.3	285.5	285.5	285.5	

Table 7.14: DC minus MC differences in Global VWPSD of VWAPR

For cases where RCAP is at least the monthly average inflow, we compare across optimisation runs by taking case differences as a proportion of the base case (VWPSD of VWAPR for MC optimisation). Moving from MC to SI increases global VWPSD of VWAPR variability by 138%,¹²³ while moving from SI to DC increases it by a further 187%¹²⁴ of the MC base case value. Note that these increases are more significant than the benefit increases reported in Section 7.3.3. Going from MC to DC optimisations increases benefit variability by 202%, while price variability increases by 324%.

7.5 Varying Inflow and Demand Characteristics

The base case used in the experiments above was constructed to have a high level of seasonality and inflow variance, with peak demand exactly six months out of phase with peak inflow. In this section we briefly report the impact of reducing inflow variance, and then seasonality, and finally the extent to which demand is out of phase with inflow to peak demand has on our MC results, and on MC versus DC differences. In this section we focus purely on MC and DC optimisation results, and do not report SI optimisation results. In all

¹²³ $(826.24 - 347.68) / 347.68$

¹²⁴ $(1475.10 - 826.24) / 347.68$

the cases we have inspected, they lie between the MC and DC results, and closer to the MC results.

We still set the reference demand level, for the ideal unconstrained price of 1, to match inflow levels, on average. We model a range of RCAP levels where $RCAP \geq FMEAN$, but only model SCAP up to $12 \times FMEAN$. So, our findings are based on an underlying 24×24 grid, or 576 cases for DC and MC optimisation assumptions i.e. 1152 total cases, for each inflow to demand parameter setting. Rather than provide detailed result tables, we only summarize the overall picture, reporting average numerical results, and graphing deviations from the base case.

7.5.1 Reduced Inflow Variance

7.5.1.1 Parameters

In this section we evaluate different levels of inflow variance, using Equation (5.8):

$$F^{t,h} = (FMEAN + DF(h)) \times \left[1 + W_F \times \sin\left(\frac{2\pi \times t}{T}\right) \right] \quad \text{where } h \neq \bar{h}$$

We fix W_F to approximately 0.9,¹²⁵ and model four levels of inflow variance, with maximum and minimum flow ranges of:

- Extreme (E), F ranges between 1 and 309 (where $DF(h) = \pm 78, \pm 54, \text{ and } \pm 28$),
- High (H), F ranges between 2 and 282 (where $DF(h) = \pm 64, \pm 43, \text{ and } \pm 22$),
- Medium (M), F ranges between 4 and 240 (where $DF(h) = \pm 42, \pm 28, \text{ and } \pm 14$), and
- Low (L), F ranges between 7 and 189 (where $DF(h) = \pm 15, \pm 10, \text{ and } \pm 5$).

The exact inflow data for these experiments is in Appendix 7.2. In the mean hydrology (\bar{h}) state $DF(h) = 0$. So, all four levels of inflow variance have the same set of average inflow points: $F^{t,\bar{h}} = \{122, 150, 160, 150, 122, 84, 46, 18, 8, 18, 46, 84\}$. As before we assume that the DCRs vary with a similar pattern, via Equation (5.10):

$$QREF^t = FMEAN \times \left[1 + W_F \times \sin\left(\frac{2\pi \times t}{T} + W_L\right) \right]$$

¹²⁵ $W_F = 1 - 8/84$. The value was chosen to ensure the smallest F are close to but greater than 0.

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We use the same W_F as above (0.9), and a lag weight (W_L) of π . This gives us the same set of QREF^t points as the set of $F^{t,\bar{h}}$, just six months out of phase. Hence the situation has not changed from the base case. These inflow variance experiments still have the same average inflow, and average demand, opportunities.

7.5.1.2 MC Optimisation Results

For the MC optimisation, we report results for varying SCAP and RCAP, while the inflow variance reduces from E to L. Our key reporting measures are TWAB, the ANSD of TWAB, VWAPR, and the global VWPSD of VWAPR, for the SCAP and RCAP values given above. Figure 7.7 shows the result tables for the E to L cases, in a similar format to all other tables in this chapter (e.g. Table 7.1 for benefits), but condensed so that only the conditional formatting can be seen, with green being “good” i.e. a high benefit, low price, and low volatility,¹²⁶ and red not so good. The conditional formatting is for each set of columns. As may be seen, the TWAB and VWAPR patterns are similar. TWAB always increases, and the VWAPR always reduces as SCAP increases. But, as expected, the MC optimisation produces higher benefits and lower prices with lower inflow variability i.e. the red (in the first half of a block of RCAP/SCAP cells) fades to orange, while the green area increases, when looking from top to bottom.

The ANSD of TWAB always reduces as SCAP increases but, as expected, performance improves significantly across the whole SCAP range as inflow variability reduces. The global VWPSD of VWAPR also reduces for increased SCAP, and improvement is much quicker than for the ANSD of TWAB.

¹²⁶ Low price variability is good thing. But as discussed in Section 5.7 benefit risk may just be inherent within the situation. In that case it is not clear if low/high benefit variance is just a product of the particular system. So, while we recognise that benefit variance can be a sign of optimality, and potentially more benefit variability is not, we generally look at low variability as a good thing.

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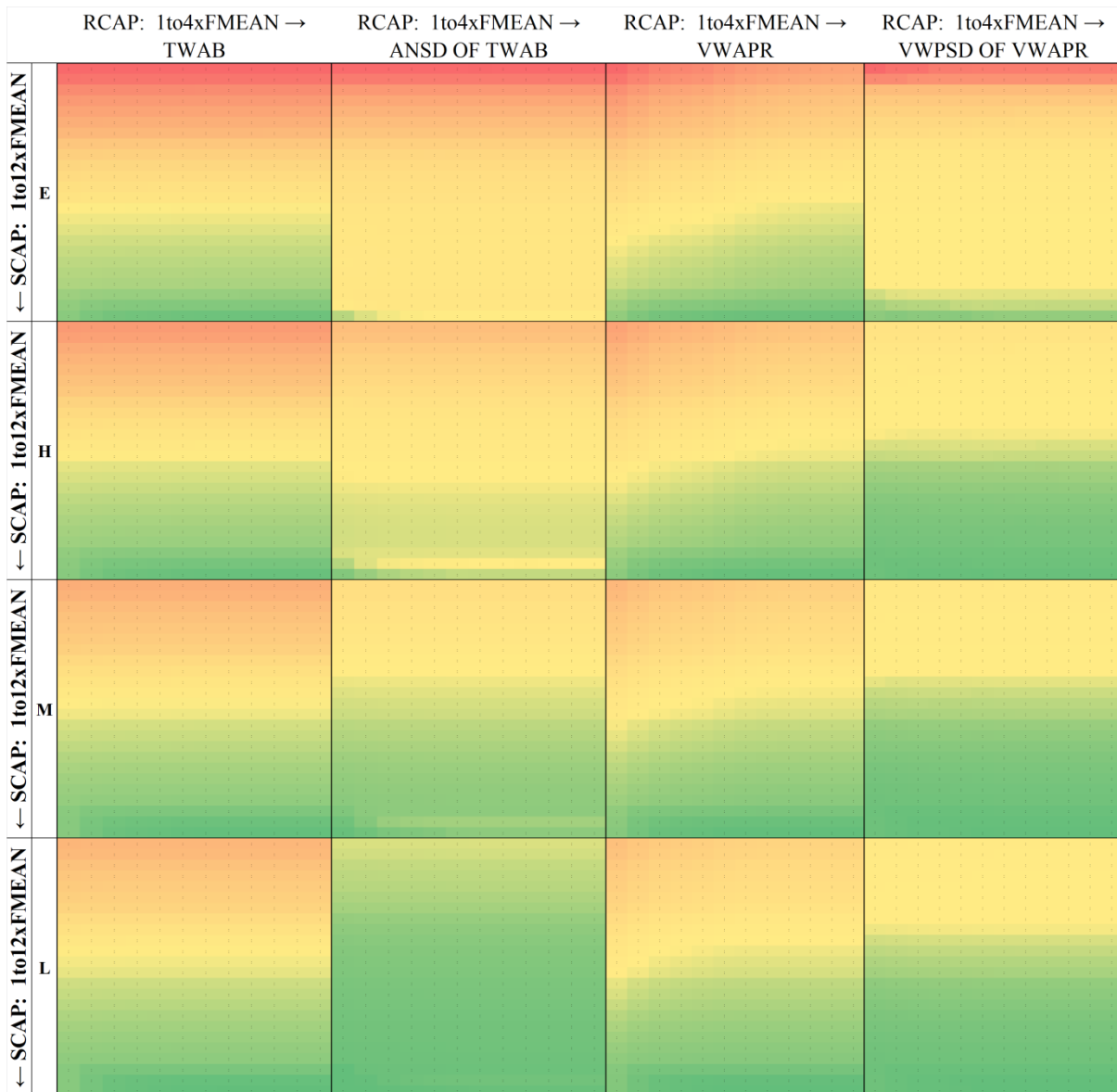


Figure 7.7: MC optimisation results, as inflow variance is reduced.

Table 7.15 below gives the average, maximum and minimum values of each measure, for the MC optimisation case runs. The average annual TWAB value in the “extreme” base case is 16026, with the H, M, and L cases having slightly higher benefits by 16086, 16115, and 16128 units, respectively, on average. So less inflow variance means higher benefits, as expected. The average VWAPR in the extreme base case is 5.13. The H, M, and L cases have lower prices, on average, by 3.95, 3.39, and 3.13 units, respectively. So less inflow variance means lower prices, as expected. The ANSD of annual TWAB starts from a level of 251 reducing down to 12 with a low level of inflow variance. The global VWPSD of VWAPR starts quite high in the extreme case, 420, but reduces to only 20 with low inflow variance. So less inflow variance means lower benefit and price variability, as expected, with

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the impact on (upside) price variability being higher than on the (downside) benefit variability, also as expected.

	TWAB	ANSD of TWAB	VWAPR	VWPSD of VWAPR
E Ave	16026	251.19	5.13	419.68
E Max	16277	976.46	18.83	3355.28
E Min	15516	23.97	1.12	4.60
H Ave	16086	110.61	3.95	45.22
H Max	16282	367.83	11.90	231.81
H Min	15755	9.24	1.06	0.67
M Ave	16115	47.75	3.39	27.20
M Max	16285	143.57	9.32	132.70
M Min	15850	1.51	1.02	0.29
L Ave	16128	12.25	3.13	20.31
L Max	16287	33.27	8.33	109.30
L Min	15888	0.03	1.00	0.07

Table 7.15: Max, min, and average values for MC optimisation, as inflow variance changes.

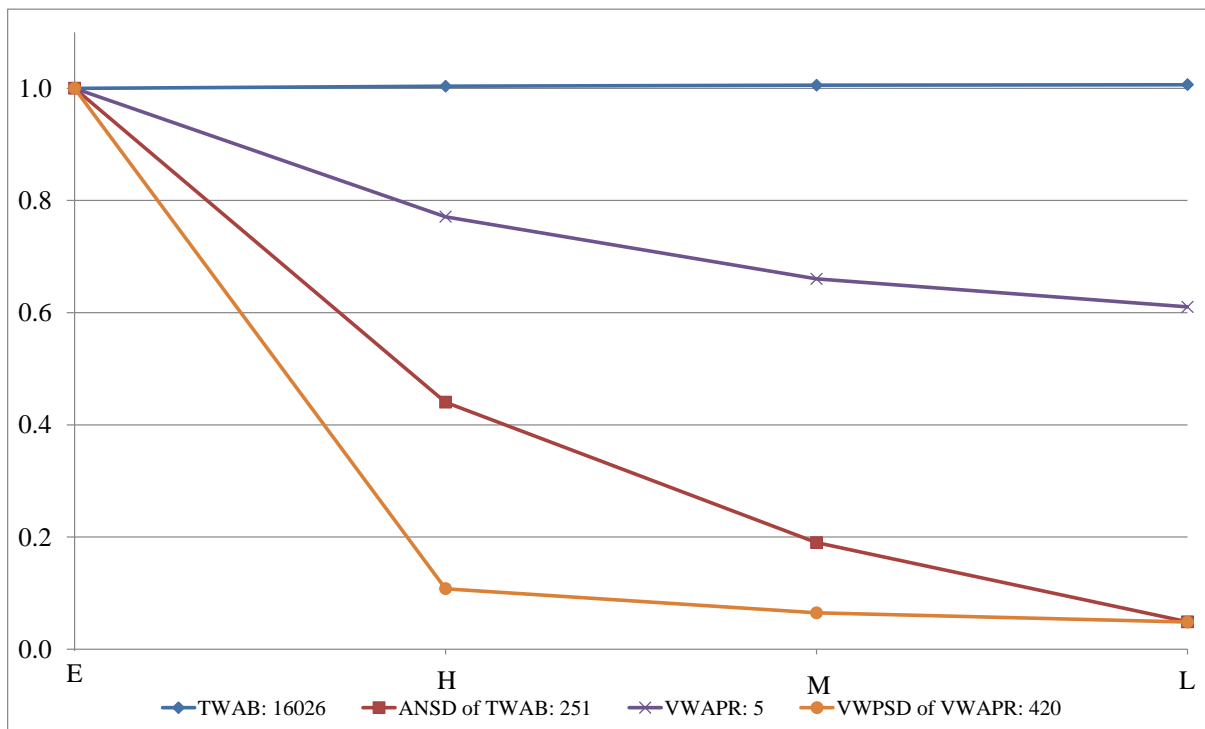


Figure 7.8: Proportional Average values for MC optimisation, as inflow variance changes.

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Another way of showing the impact of inflow variability is to divide each of the four measures by the base case (extreme) values in the first row of Table.7.15, as in Figure 7.8. Average benefits appear relatively stable as inflow variance falls, because there is a large underlying benefit implied by the first few release units in our constant elasticity curves. But the average price reduces steadily, while the average benefit and price variances quickly reduce to nearly zero, as one might expect.

7.5.1.3 MC versus DC Optimisation

We now briefly compare MC versus DC results, across those same four measures. We take differences which we normally expect to be positive. Hence we take MC optimisation TWAB values away from the equivalent DC optimisation values, while for VWAPR we take the DC values from the equivalent MC values i.e. $TWAB(MC) - TWAB(DC)$ and $VWAPR(DC) - VWAPR(MC)$. We take the MC volatility measure from the DC volatility measure in both cases, i.e. for the ANSD of annual TWAB and global VWPSD of VWAPR. Figure 7.8 is displayed in exactly the same format as Figure 7.9 above. Here, the conditional formatting is set such that green represents a difference of zero, or less. In fact the underlying table does contain some negative differences (see Table 8.16 below), as can be expected from the discussion in Section 5.6.3.

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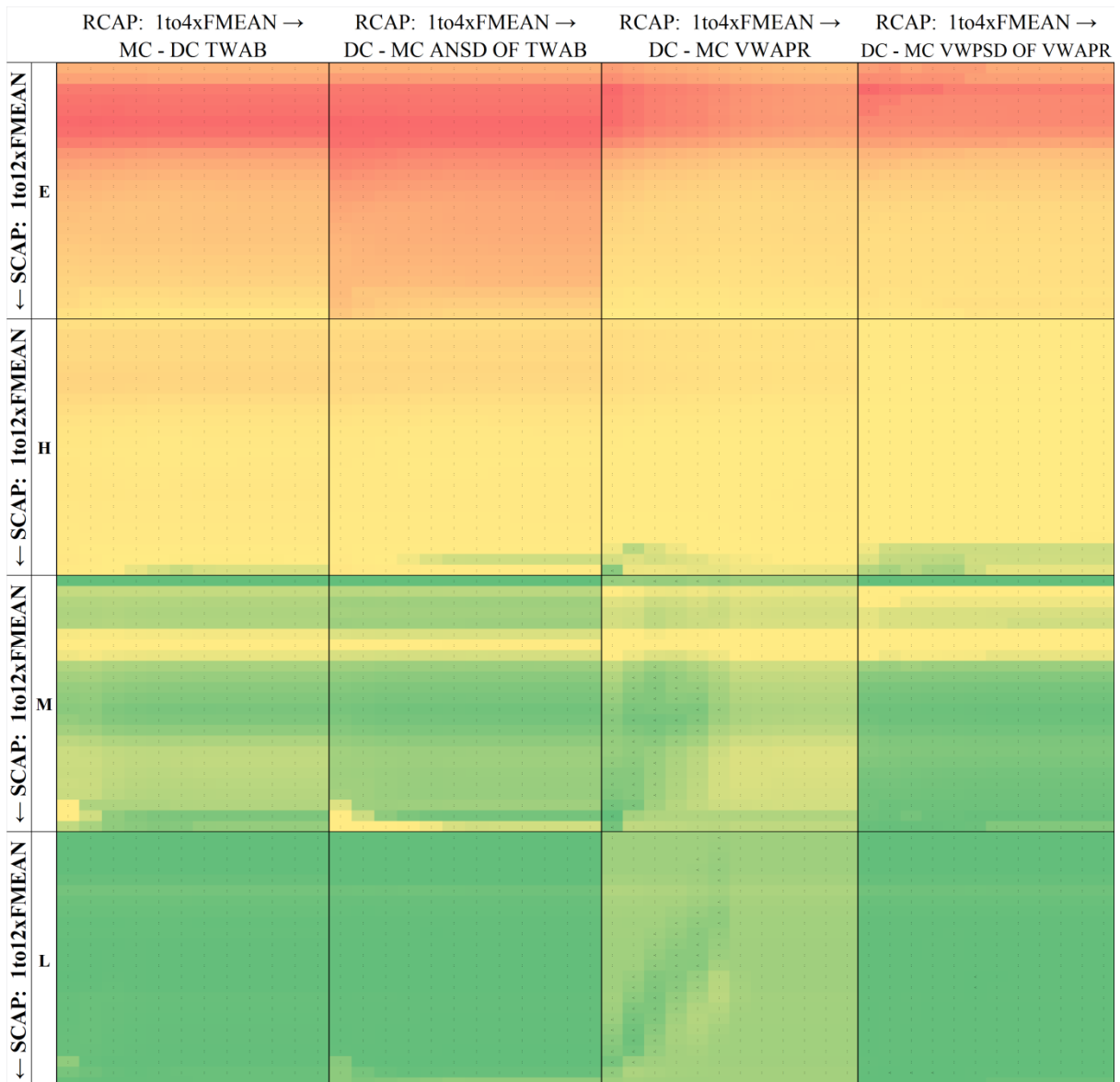


Figure 7.9: MC versus DC differences, as inflow variance is reduced.

As inflow variance is reduced, we see the differences reduce significantly. All four measures generally show similar patterns and trends to one another. Generally, with a medium (M) level of inflow variance the MC and DC optimisations appear to be performing quite similarly. But to investigate this further, as in Table 7.15 above, Table 7.16 reports average, maximum, and minimum values for the (MC – DC) differences in TWAB and (DC – MC) differences in VWAPR, the ANSD of TWAB, and the global VWPSD of VWAPR. These averages confirm the patterns discussed above. Once the medium inflow variance case is reached, there is little further change to any of the four measures. Figure 7.10 (where we reproduce Figure 7.8 but comparing MC versus DC differences) reinforces the conclusion that, below a medium level of inflow variance, the MC and DC optimisations are performing

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“near enough” equally well. The VWPSD of VWAPR falls off the fastest, and TWAB the slowest, while the other two measures fall off at a more moderate rate.

	TWAB	ANSD of TWAB	VWAPR	VWPSD of VWAPR
E Ave	70.96	472.31	1.51	1319.77
E Max	149.92	782.91	4.67	3916.96
E Min	7.71	137.72	0.11	152.80
H Ave	12.75	60.22	0.19	28.27
H Max	28.56	135.68	0.59	108.50
H Min	1.45	6.86	-0.01	0.73
M Ave	1.09	3.89	0.01	0.73
M Max	2.73	14.27	0.04	3.07
M Min	0.00	0.00	-0.02	0.00
L Ave	0.07	0.27	0.00	0.02
L Max	0.65	3.15	0.01	0.16
L Min	0.00	0.00	-0.01	-0.01

Table 7.16: MC versus DC differences in max, min, and average reporting measure values, for different inflow variances.

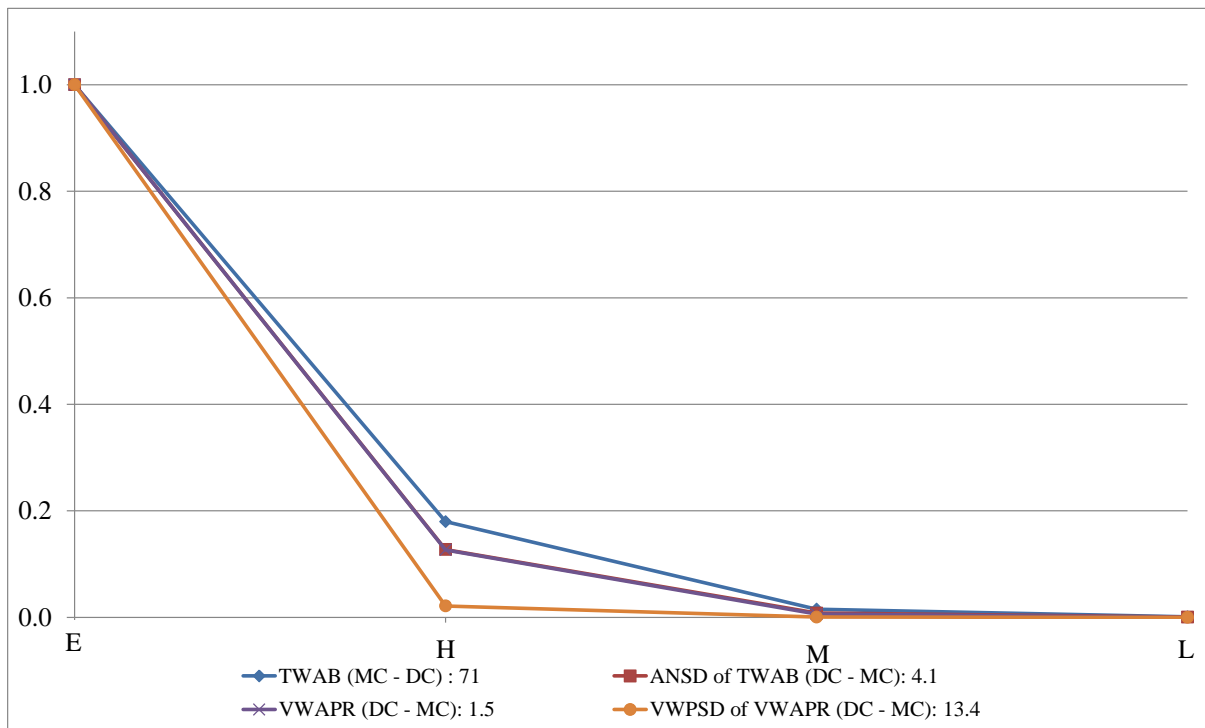


Figure 7.10: MC versus DC proportional average differences, for different inflow variances.

7.5.2 Reduced Seasonality

In this section we evaluate the impact of different levels of seasonality, again using Equation (5.2), this time fixing DF (to ± 18 , ± 12 , and ± 6), and varying W_F . Including the base case (E), the five e levels of seasonality, and their maximum and minimum flow ranges, are:

- Extreme (E), F ranges between 1 & 309 (where $W_F \approx 0.90$ and $DF(h) = \pm 78, \pm 54, \& \pm 28$).
- High (H), F ranges between 2 and 166 (where $W_F = 0.76$),
- Medium (M), F ranges between 24 and 144 (where $W_F = 0.50$),
- Low (L), F ranges between 46 and 122 (where $W_F = 0.24$), and
- Very Low (VL), F ranges between 62 and 106 (where $W_F = 0.05$).

While varying inflow seasonality, in this set of experiments we fix our DCRs for each period, such that they consistently use the same QREF^t points as the base case. This was done to allow for a fair comparison, because varying the set of available DCRs can vary the total benefits available to the optimisation, as discussed in Section 8.4 See Appendix 7.2 for seasonal inflow data, and Appendix 6 for a reminder of the Extreme base case data.

7.5.2.1 MC Optimisation Results

Results are reported in the same three forms as in Section 7.5.1 above. First, Figure 7.11 shows TWAB, ANSD of TWAB, VWAPR, and VWPSD of VWAPR, for our grid of SCAP and RCAP values, for the E to VL seasonality cases, in a similar format to Figure 7.7 above. Again, green is “good” i.e. a high benefit, low price, and low volatility, and red not so good.

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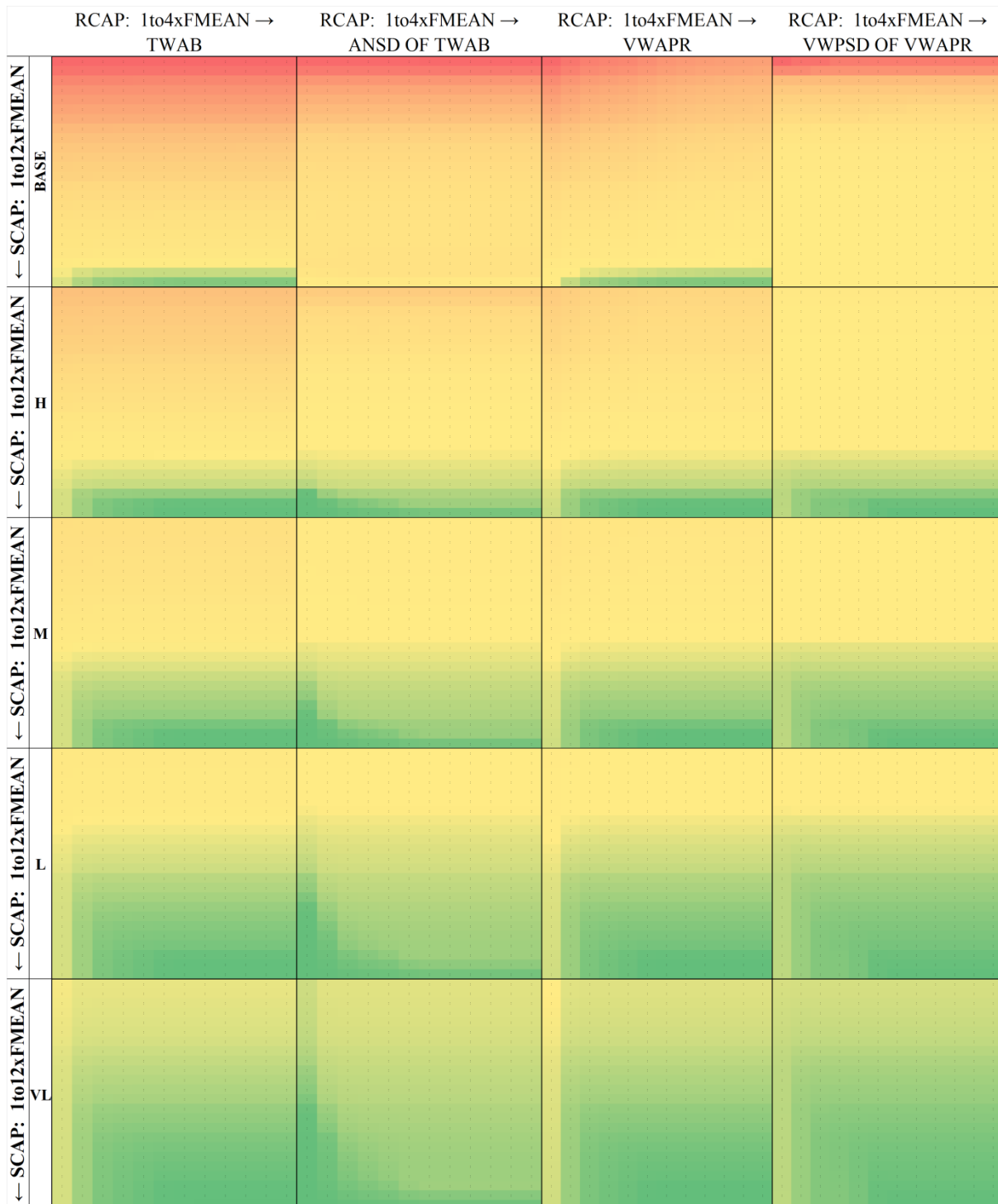


Figure 7.11: MC optimisation results, as seasonality varies.

As may be seen, the four seasonal measures all show very similar patterns. As expected, reduced seasonality reduces prices and increases benefits, while at the same time reducing their corresponding upside/downside variances. With extreme seasonality, a green band can only be seen in the TWAB and VWAPR columns for an SCAP of 6 or 12 times FMEAN, i.e. the least constrained cases. But for all four measures, the green band grows larger for each

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subsequent reduction in seasonality from H to VL. Thus reducing seasonality reduces the need for reservoir storage capacity.

As in Table 7.16 above, Table 7.17 below gives the average, maximum and minimum values of our four measures (TWAB, ANSD of TWAB, VWAPR, and VWPSD of VWAPR), for the 5 sets of cases (E to VL). Figure 7.12 gives the proportional change in each of these measures, as a function of seasonality. The average annual TWABs only rise slightly (at most 1.5%), again reflecting the large benefit inherent in the first few units released, but all are higher than for the base case. The VWAPR falls sharply from the extreme base case, on average, and the minimum values show that it can get very close to the unconstrained optimal value of 1, for some cases, with lower levels of seasonality. The VWPSD of VWAPR falls to near zero with moderate seasonality, while the ANSD of annual TWAB levels off at around 3%, of the base case level. Presumably this reflects the natural variability inherent in the DCRs, as discussed in Section 5.7.

	TWAB	ANSD of TWAB	VWAPR	VWPSD of VWAPR
E Ave	16026	251.19	5.13	419.68
E Max	16277	976.46	18.83	3355.28
E Min	15516	23.97	1.12	4.60
H Ave	16172	59.27	2.50	17.06
H Max	16287	245.99	5.89	104.68
H Min	16009	0.06	1.00	0.07
M Ave	16230	18.37	1.71	3.62
M Max	16287	36.89	2.77	8.67
M Min	16178	0.05	1.00	0.07
L Ave	16254	10.46	1.40	1.68
L Max	16287	17.51	1.89	3.37
L Min	16226	0.05	1.00	0.06
VL Ave	16264	7.79	1.27	1.10
VL Max	16287	12.26	1.62	1.98
VL Min	16243	0.05	1.00	0.06

Table 7.17: Max, min, and average values for MC optimisation, as seasonality varies.

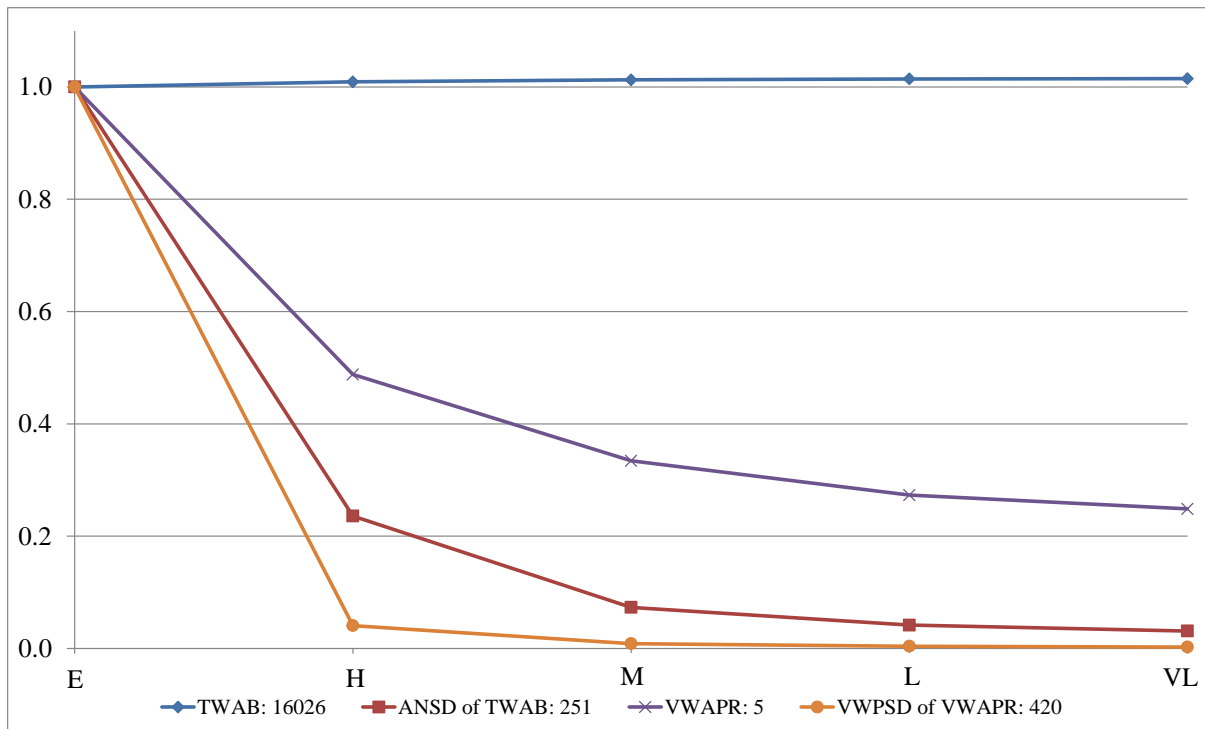


Figure 7.12: Proportional average values for MC optimisation, as seasonality varies.

7.5.2.2 MC versus DC Optimisation

MC versus DC differences are presented for the same four measures, as in Section 7.5.1.2 above. Figure 7.13 is displayed in exactly the same format as Figure 7.11 above. And, as seasonality reduces, the differences in all measures reduce significantly. With high (H) and medium (M) seasonality, the relative performance of the DC optimisation improves, over all four measures, when SCAP is larger, and thus more able to handle fluctuations. But, with low and very low levels of seasonality, the DC optimisation compares favourably with the MC optimisation over the full range of SCAP values.

In the base case, the leading diagonal effect discussed in Section 7.2.2.3 was only really visible for cases where RCAP was less than FMEAN. Since we only report cases here where RCAP exceeds FMEAN, we do not see that effect in our base case table. But, as seasonality is reduced to the H level, the leading diagonal effect starts to show for cases where SCAP exceeds FMEAN, and it becomes more prominent for lower levels of seasonality. In those cases, the optimisations are both releasing as much as they can, and thus there is less scope for improvement when comparing across optimisation runs.

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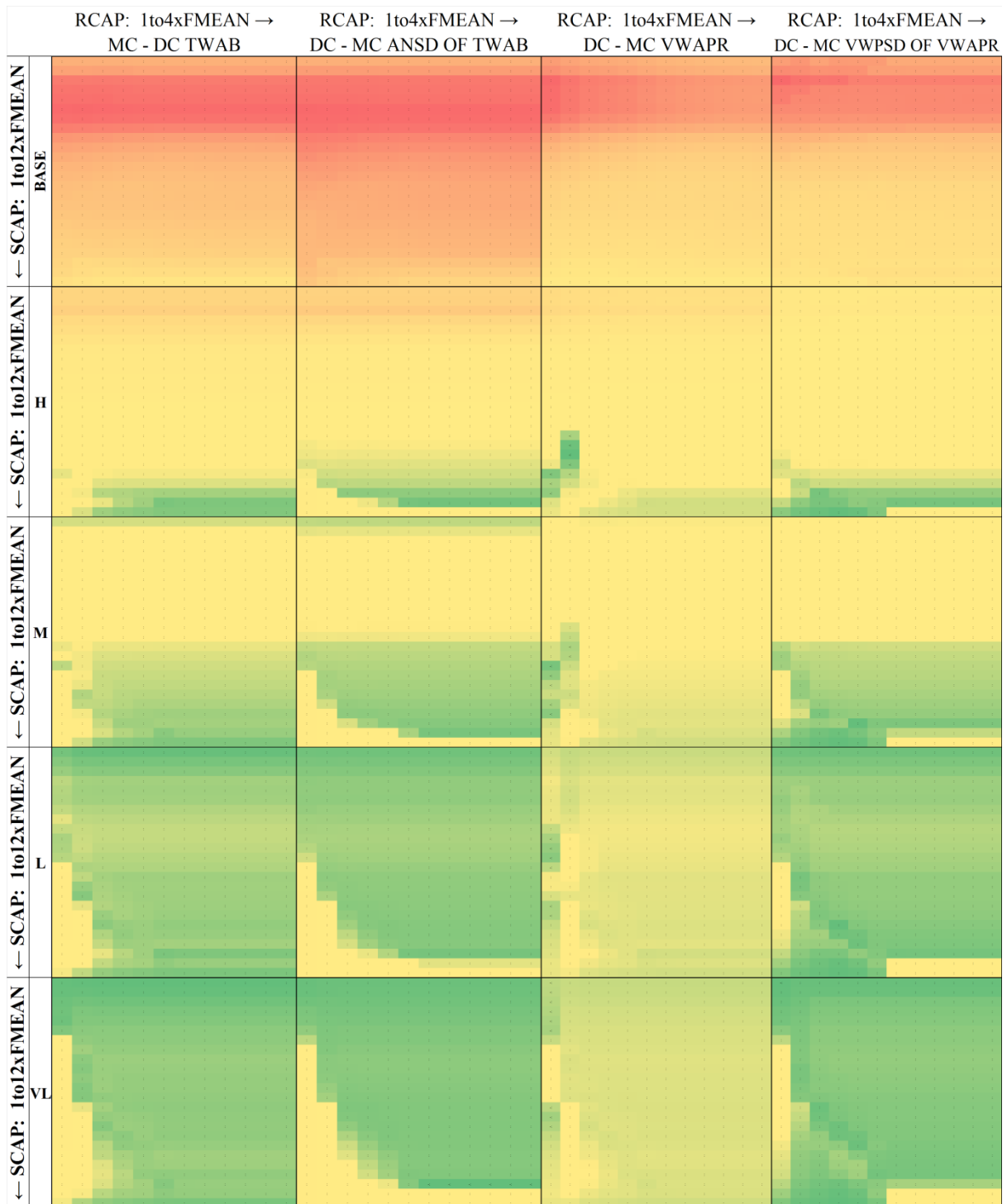


Figure 7.13: MC versus DC differences, as seasonality is reduced

Table 7.18 reports the average, maximum, and minimum MC versus DC differences from the 20 blocks of SCAP/RCAP cells in Figure 7.13. All reduced seasonality cases show significantly smaller differences than the base case. In fact, on average, for medium or lower seasonality, the DC optimisation is generally performing as well as the MC optimisation. We see that absolute TWAB and VWAPR differences fall to near zero for medium and lower

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levels of seasonality, while the volatility drops from high to medium seasonality. The ANSD of TWAB then rises slightly, presumably reflecting the fact that the optimal unconstrained solution involves significant variation in TWAB, for the reasons discussed in Section 5.6.3

These trends are further highlighted in Figure 7.14, where the average values are shown as a proportion of those for the Extreme (E) base case. Because the falls are so dramatic, from the extreme base case, the range on the vertical axis has been restricted to a maximum of 0.1, even though all lines go up to 1, for the base case. The differences in VWPSD of VWAPR fall off the quickest, which is consistently followed by price, then ANSD of TWAB and finally TWAB.

	TWAB	ANSD of TWAB	VWAPR	VWPSD of VWAPR
E Ave	70.96	472.31	1.51	1319.77
E Max	149.92	782.91	4.67	3916.96
E Min	7.71	137.72	0.11	152.80
H Ave	6.75	37.81	0.11	21.19
H Max	32.57	203.85	0.68	148.24
H Min	0.08	0.15	-0.01	0.00
M Ave	0.45	1.90	0.01	0.15
M Max	2.05	10.70	0.01	0.67
M Min	0.09	0.22	0.00	0.00
L Ave	0.23	1.16	0.00	0.04
L Max	2.11	10.80	0.01	0.19
L Min	0.03	0.13	0.00	0.00
VL Ave	0.23	1.20	0.00	0.03
VL Max	2.39	10.50	0.01	0.19
VL Min	0.02	-0.04	0.00	0.00

Table 7.18: MC versus DC max, min, and average values, as seasonality is reduced.

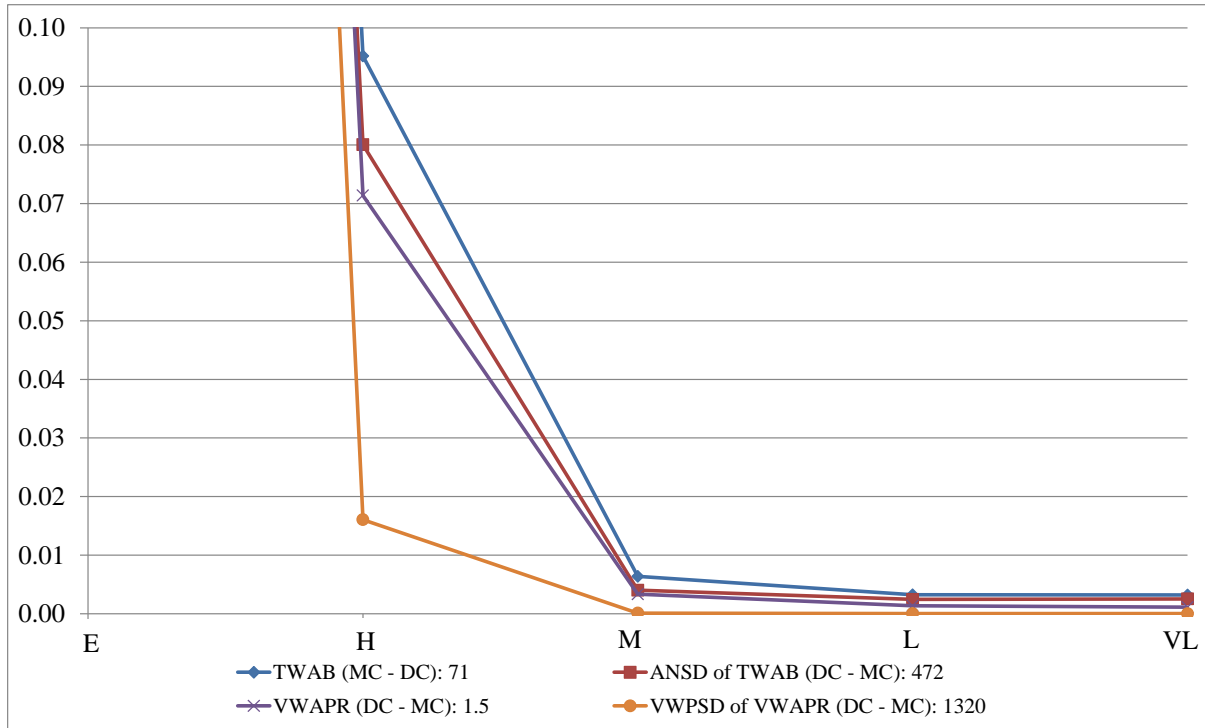


Figure 7.14: MC versus DC proportional average differences, as seasonality is reduced.

7.5.3 Phasing of Peak Inflow to Peak Demand

In this section we evaluate the impact of four different levels of phasing of peak inflow to peak demand. As discussed in Section 5.4.4, this could be achieved by changing inflow, or demand. But we chose to change demand. So far, all of our experiments have set demand such that it peaks when it is “Out of Phase” with peak inflow, via Equation (5.10):

$$QREF' = FMEAN \times \left[1 + W_F \times \sin \left(\frac{2\pi \times t}{T} + W_L \right) \right]$$

We use Equation (5.10) in this section, and set $W_L = \{0, 0.5\pi, \pi, 1.5\pi\}$ to model different phasing of peak inflow to peak demand. We term this relative phasing as follows:

- OUT - Out of Phase, $W_L = \pi$,
- LAG - Demand Lag, $W_L = 1.5\pi$,
- IN - In Phase, $W_L = 0$, and
- LEAD - Demand Lead, $W_L = 0.5\pi$.

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The pattern is cyclic so, while results are initially presented in the same way as in the inflow variance and seasonality section, the proportional graphs are plotted using a radar plot, not a line plot. QREF^f points for these experiments are given in Appendix 7.2.

7.5.3.1 MC Optimisation Results

Conditionally formatted TWAB, VWAPR, ANSD of annual TWAB, and VWPSD of VWAPR tables are displayed in Figure 7.15, where green is again “good” i.e. a high benefit, low price, or low volatility, and red not so good.

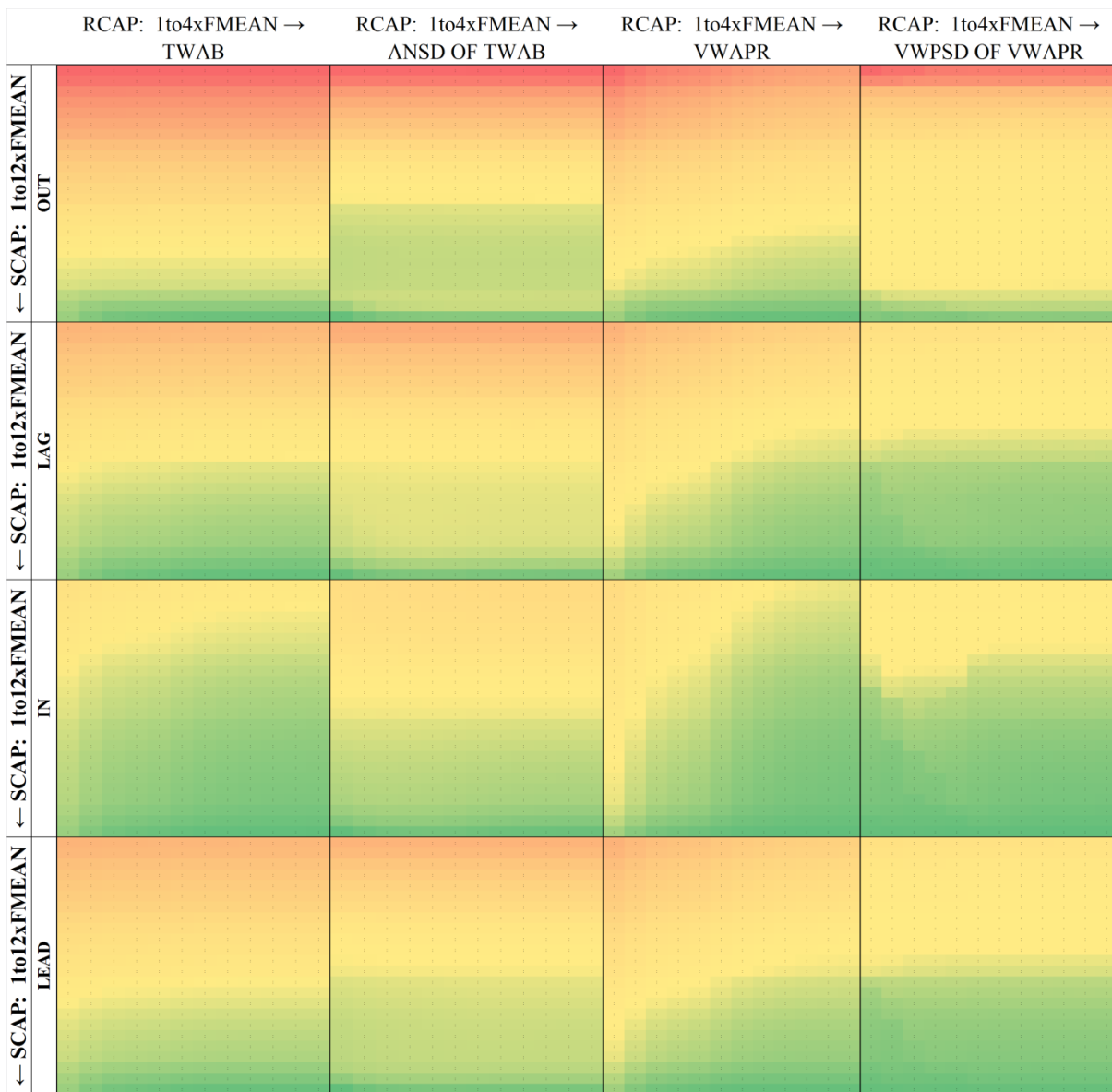


Figure 7.15: MC optimisation results, as inflow and demand phasing varies.

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The first set of SCAP rows are for the out of phase (base) case, and on average these show the lowest benefits, highest prices, and highest benefit and price variability. The third set of SCAP rows are for the in phase case, and these show the opposite i.e. the highest benefits, lowest prices, and lowest benefit and price variability. The second and fourth sets of rows show intermediate results, as expected, for the other two cases. These trends are confirmed by Table 7.19 which gives the average, maximum and minimum values from Figure 7.15.

	TWAB	ANSD of TWAB	VWAPR	VWPSD of VWAPR
OUT Ave	16026	251.19	5.13	419.68
OUT Max	16277	976.46	18.83	3355.28
OUT Min	15516	23.97	1.12	4.60
LAG Ave	16153	195.36	3.25	53.54
LAG Max	16280	537.43	9.69	316.13
LAG Min	15916	24.99	1.08	1.28
IN Ave	16229	122.84	2.06	16.86
IN Max	16282	227.08	4.49	57.51
IN Min	16146	22.24	1.06	1.13
LEAD Ave	16142	169.23	3.32	48.83
LEAD Max	16279	485.42	9.76	288.17
LEAD Min	15910	21.69	1.10	1.59

Table 7.19: Max, min, and average values for MC optimisation, as inflow and demand phasing varies.

In the table, the demand lag case gives slightly better benefits and prices, but also higher variability than the demand lead case. Figure 7.16 shows a radar plot of the average values, in proportion to the out of phase case. Proportionally, absolute benefits are all very similar, staying near 1, as expected, while the other three measures are all significantly lower in the in phase case, as expected. While VWAPR looks similar in the lead and lag cases, the lag case has slightly higher and lower ANSD of TWAB and VWPSD of TWAB, respectively, compared to the lead case. It is interesting to note, though, how little difference there is between these cases, despite the fact that the lag case requires peak inflows to be held for 9 months to meet peak demand requirements, and the lead case only 3. This may be interpreted as implying that the MC optimisation, unlike the DC optimisation below, does a similarly good job of managing the inflow peak, irrespective of whether it leads or lags the demand peak.

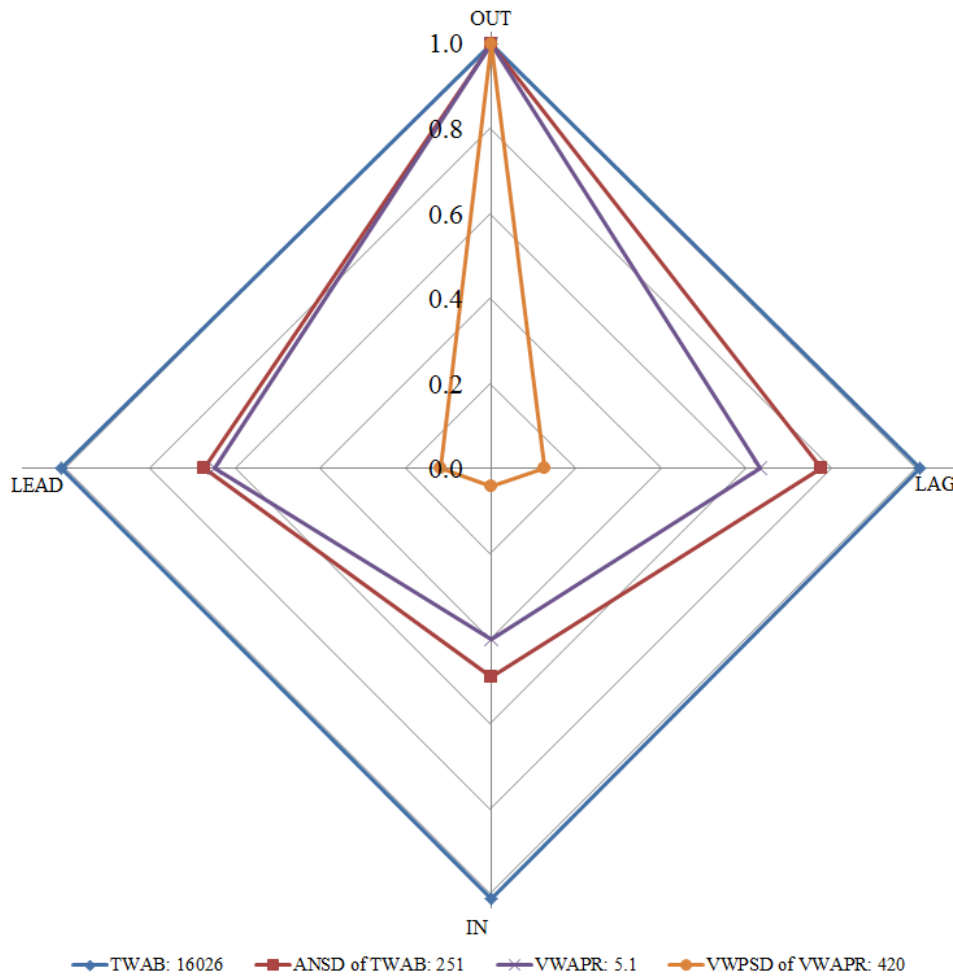


Figure 7.16: Proportional values for MC optimisation, as inflow and demand phasing varies.

7.5.3.2 MC versus DC Optimisation

We now briefly discuss MC versus DC differences, across the same four measures, in the same way as in Sections 7.5.2.1 & 7.5.2.1 above. Figure 7.17 is displayed in the now familiar format for this section. The first and third set of RCAP columns are the absolute TWAB and VWAPR differences and these show that, other than in the in phase case (third set of rows), the DC optimisation struggles to manage benefit and price compared to the MC optimisation, unless SCAP is quite unconstraining. As expected, a larger SCAP reduces the advantage of the MC optimisation in managing price variability, in both the LAG and LEAD cases, and benefit variability in the LEAD case.

The second and fourth set of RCAP columns, for the ANSD of TWAB and VWPSD of VWAPR difference measures, compared to the MC optimisation, the DC optimisation

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struggles the most to manage (downside) benefit and (upside) price variability in the out of phase case (first set of rows), while it compares most favourably in the in phase case (third set of rows).

Unlike the MC optimisation, the DC optimisation is worse at managing price and benefit variability when faced with demand lag, than demand lead. This is discussed further below.

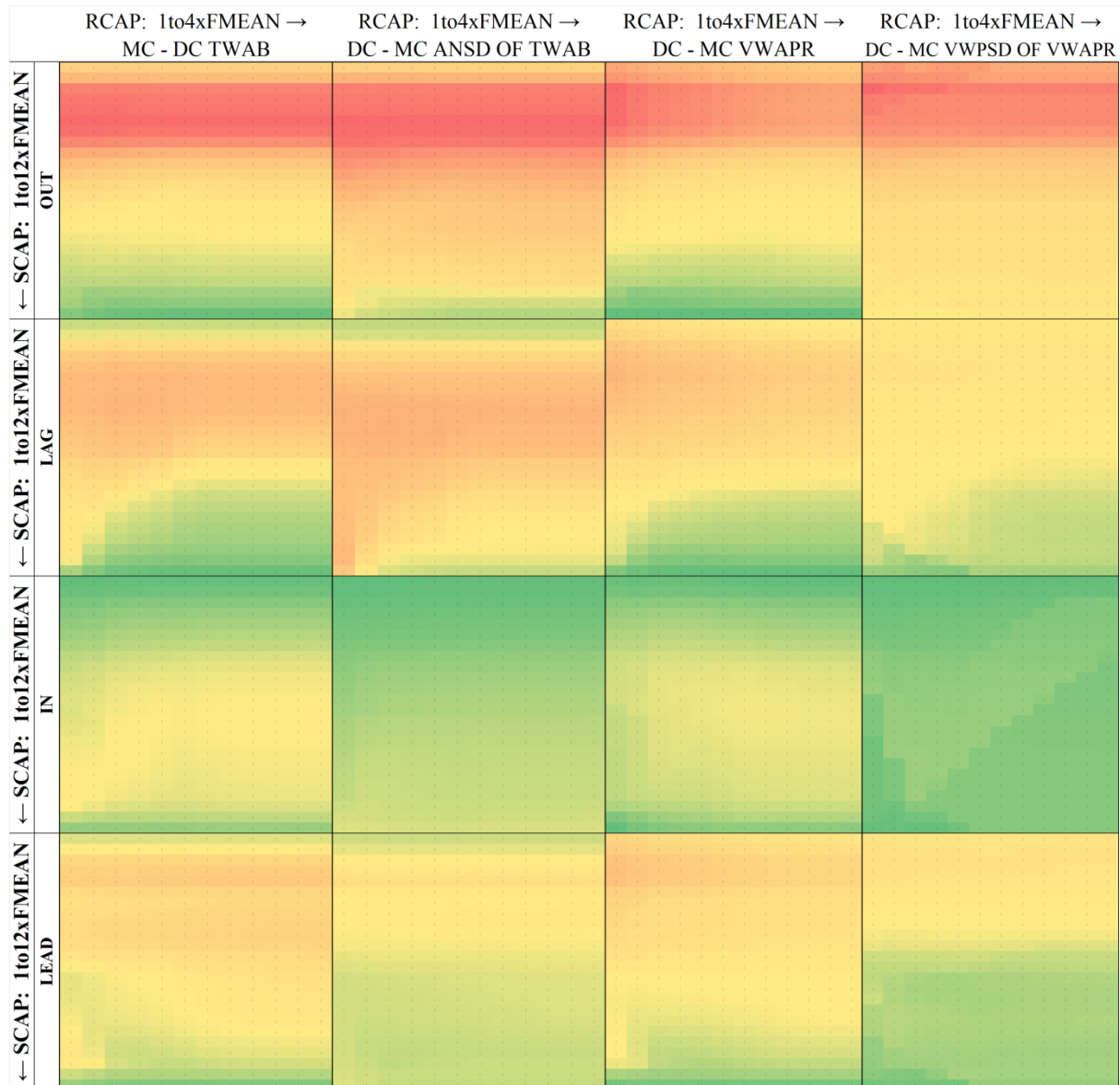


Figure 7.17: MC versus DC differences, as inflow and demand phasing varies.

As with Table 7.19 above, Table 7.20 gives the average, maximum and minimum values from Figure 7.17. In the table, differences between average values are highest and lowest in the out of phase (OUT) and in phase (IN) cases. The ratio of these average values to those

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for the out of phase base case is shown in Figure 7.18, which reproduces Figure 7.16, but for MC versus DC differences.

The IN case still shows significant differences between the MC and DC optimisations, implying that the DC optimisation can still have trouble managing inflow variability, even when the average inflow distribution is immediately sought by the market, with an informed inflow allocation policy. This figure shows that the DC optimisation is performing quite poorly, compared to the MC optimisation, in the out of phase and, to a lesser extent, the demand lag/lead cases. But the demand lag (LAG) case has higher differences for all four measures, compared to the demand lead (LEAD) case. So the DC optimisation generally performs worse in the LAG than LEAD case. From the QREF^f data in Table 7A.8 of Appendix 7A.2, the LEAD case demand peaks 3 months after the peak inflows arrive, whereas in the LAG case this is 9 months after they arrive. The DC optimisation only “sees” the average set of inflows, and it determines the periodic storage and release decisions on that basis. With a significant period of time between peak inflow and peak demand, the DC optimisation has plenty of time, and thus many more chances, to get its allocation policy wrong. The DC optimisation performs better in the LAG case than in the OUT case, because it has high valued demand in the first three months of the year, when there is an abundance of inflow, whereas the OUT case has all high valued demand in the dry season.

	TWAB	ANSD of TWAB	VWAPR	VWPSD of VWAPR
OUT Ave	71	472.31	1.51	1319.77
OUT Max	150	782.91	4.67	3916.96
OUT Min	8	137.72	0.11	152.80
LAG Ave	54	356.82	1.02	209.52
LAG Max	89	494.91	2.32	401.71
LAG Min	8	157.99	0.12	37.87
IN Ave	34	111.59	0.46	35.86
IN Max	49	205.13	0.64	57.88
IN Min	6	4.03	0.10	6.53
LEAD Ave	51	221.23	0.88	162.29
LEAD Max	73	304.93	1.98	459.53
LEAD Min	15	160.60	0.18	24.99

Table 7.20: MC versus DC differences in max, min, and average values, as inflow and demand phasing varies.

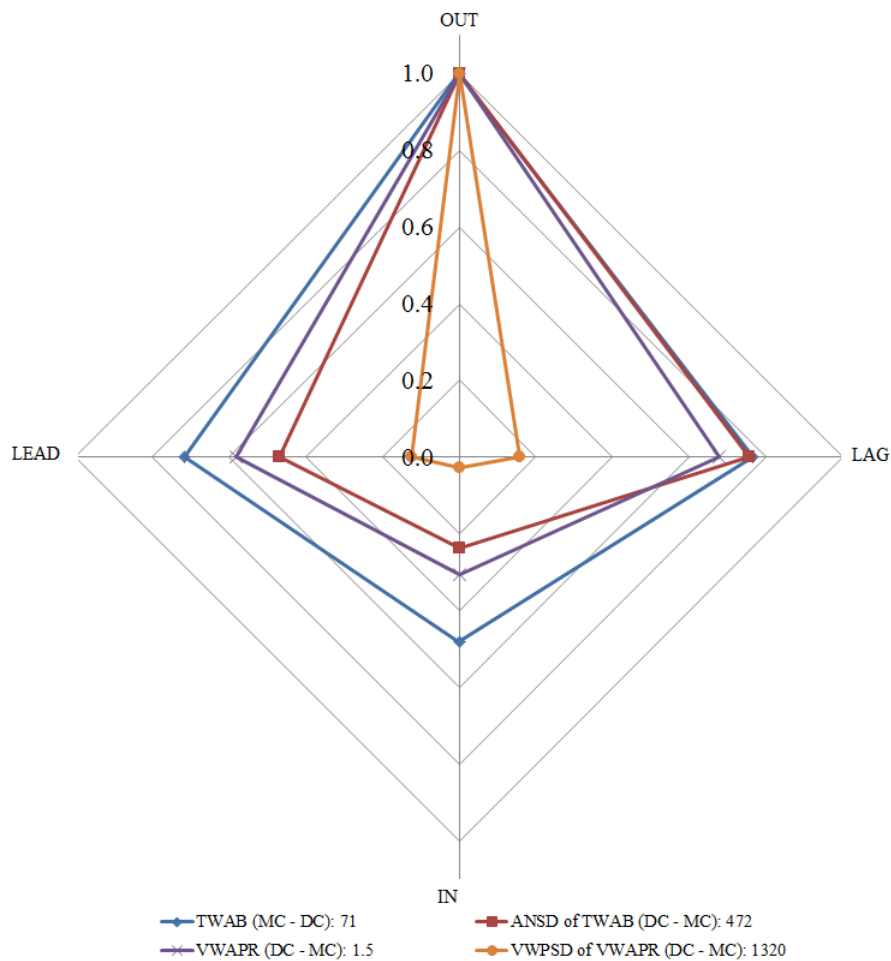


Figure 7.18: MC versus DC proportional average differences, as inflow and demand phasing varies.

7.6 Conclusions

In this chapter some limited experiments were undertaken to explore the impact that varying physical reservoir parameters has on the results from Markov Chain (MC) optimisation, and on its performance relative to Deterministic (DC) and Stochastic Independent (SI) approximations, in a simulated MC world. Our base case parameters involve high inflow seasonality, high inflow variability, and a high degree of persistence i.e. if the system is in a wetter or drier than average state it tends to stay in that state. We also assume that the peak inflow and peak demand seasons are perfectly out of synch, within the annual water cycle.

First, we focussed on varying the storage and release capacities. Clearly, with no storage or release capacity there is no ability to optimise, so optimisation level makes no difference to average benefits or price, with the corresponding benefit and price variability then wholly

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depending on the variability of the inflows. However, increasing storage and/or release capacity increases the ability of the system to manage inflows to match demand patterns. In that setting, employing a more accurate model to optimise the system does allow us to more fully realise those opportunities.

For this case, we find that the maximum gains from MC optimisation are when the release and storage capacities were about 75% and 25% of the mean monthly inflow, respectively, which is perhaps surprisingly low. At this point, the benefit gain achieved with MC optimisation is only 1% of the total benefit. But, this is approximately 16% (or 13% for the SI run) as a proportion of market value calculated using the market clearing price from Markov Chain (MC) optimisation. That is likely to seem quite significant to market participants, but the real world significance of the gain from improving optimisation depends on the cost of alternatives, like building additional physical capacity.

From this point, the benefit from MC optimisation reduces slowly for increased storage capacity (SCAP) and increases very little for increased release capacity (RCAP). On average, over the entire range of SCAP from 0.02 to 12 times the mean monthly, and RCAP from 1 to 4 times the mean monthly inflow, the gains from using MC/SI optimisation rather than DC were 18%/13% respectively, as a proportion of market value calculated using the market clearing price from MC optimisation. But this partly reflects the fact that average prices, and hence market value, drop significantly as storage increases. The actual gain from MC optimisation drops off steadily as storage capacity increases. Eventually, we might expect there to be no need to use stochastic optimisation, because the DC optimisation has enough storage that it can never get into trouble at either storage bound. We found that the gains fell to 3% of the market value by the time storage capacity reached 6 years' worth of inflow, and unlimited release capacity. Over the same SCAP and RCAP range, DC optimisation increased prices by 29%, benefit variability by 188% and price variability by 314% of the MC base case values.

We also tested the impact of varying inflow seasonality, inflow variance, and the phasing of peak inflow to peak demand, over this same more limited range of storage capacities, and release capacities. As expected, reducing inflow seasonality reduces the need for stochastic optimisation, as there is less need to balance prices over time. Averaging over all the cases tested, we found that the benefits from MC optimisation reduced from 18% of the market

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value, down to 0.3%, when the range of monthly average inflows reduced from +/- 90% of the mean monthly inflow, down to +/- 50%. At that point, the differences between MC and DC optimisations, in terms of absolute benefit, price, and their volatility measures, have all become negligible.

Reducing inflow variance within each period also means less need to balance prices over hydrology states. Averaging over all the cases tested, we found that the benefits from MC optimisation reduced from 18% of the market value, down to 0.5%, when the range of inflows was reduced from (approximately) +/-90% of the average for the month, down to between +/-50%. Again, the differences between MC and DC optimisations, in terms of absolute benefit, price, and their volatility measures, have all become negligible at that point.

Finally, shifting the peak market demand closer to the peak inflow reduced the need for, and hence value of MC optimisation, in both leading and lagging cases. While the “in-phase” case might not be a realistic situation (anywhere?), it gives us a hypothetical measure of how good outcomes might get, given changes to other parameters. The differences were lowest for the “in-phase” case, where what comes in goes out, on average, in the same period, but they were still significantly larger than in the low variance or seasonality cases discussed above.

- The benefits from MC optimisation were 18% of the market value in the out-of-phase case and 23% in the in-phase case. This is because the market clearing price, VWAPR, was much lower in the in-phase case than in the out-of-phase case. The absolute differences in TWAB dropped by approximately 52%. The MC vs DC differences in price dropped by 70%, benefit variability by 76%, and price variability by 97%.
- For the “lead” case, where peak inflows occur 3 months before peak loads, the benefits from MC optimisation were similar, at 22% of the market value. The absolute differences in TWAB fell by about 29%. The MC vs DC differences in price dropped by 42%, benefit variability by 53%, and price variability by 88%.
- For the “lag” case, where peak inflows occur 3 months after peak loads, the benefits from MC optimisation were slightly less, at 21% of the market value. The absolute differences in TWAB fell by about 24%. The MC vs DC differences in price dropped by 32%, benefit variability by 24%, and price variability by 84%.

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So, based on these results we can see that the gains from MC optimisation are consistently lower in the lead case than in the lag case. And this is because the peak inflow and peak benefits are 9 months apart, thus giving the DC optimisation plenty of time to get into trouble. As expected the gains are lowest in the in-phase case, and highest in the out-of-phase case, which we assume for most of our experiments in this chapter, and the next.

Finally, though, it should be recognised that all of the results in this chapter assume that demand can be described by a set of constant elasticity curves, all with a fairly moderate elasticity of -0.5 , and no correlation between demand and inflow states. The next chapter explores the sensitivity of our results to variations in those assumptions about the economics of the situation.

8. Impact of Economic Parameters

8.1 Chapter Introduction

In this Chapter we investigate changes to the demand curve shape and structure. To do this we use of the same system and model configuration, and most of the same input parameters as in Chapters 5 through 7. We assume the “extreme inflow variance and seasonality” inflow distribution defined in Section 6.2, with demand and inflow out of phase by half a year, as in Chapter 6, and most of Chapter 7. This inflow distribution also exhibits a high degree of “persistence”, as discussed in Section 6.2. Rather than varying storage and release capacities over a wide range, we focus on a limited set of cases. We fix RCAP at $2 \times \text{FMEAN}$, because this not too constraining, and fairly realistic, but still produces near maximum benefit and minimum price results in the base case experiments of Chapter 7. We vary SCAP where relevant.

Given this system and model configuration, we ran the modelling system to explore the impact of adjusting some key economic parameters. With a net demand curve, and freely supplied inflows, market performance will clearly depend on the value that market participants place on water extracted from the system. We explore this issue by performing experiments over a range of Demand Curve for Release (DCR) shapes.

In this chapter we only report MC and DC optimisation run results, and exclude reference to SI results. But, having inspected those results, we can confirm that the SI optimisation produces results between those of MC and DC optimisation. And, when the gains from modelling the underlying MC uncertainty are significant, the SI optimisation results are closer to the MC, than the DC optimisation results.

In Section 8.2 we outline our general DCR variation strategy, using constant elasticity curves. We then vary the elasticity values of constant elasticity demand curves. In Section 8.3 we experiment with a reservoir system case that is relatively constrained, for the assumed degree of inflow variation, with storage/release capacities at four/two times the average monthly inflow/release value. In this section first we insert horizontal flats in the DCR, which could simulate a situation such as having manufactured water supplied at a fixed cost. Next we insert vertical steps in the DCR, which could simulate a situation where a group of market

participants are willing to pay more/less for water. In Section 8.5 we explore the implications of allowing participants to submit state-dependent bids.

As in Chapter 7, we report four outputs from the model, namely: The expected TWAB; The ANSD of TWAB; The expected VWAPR; and the VWPSD of VWAPR. We discuss the impact which each type of DCR shape variation, has on each performance measure under MC optimisation, and then on the extent to which those performance measures might be compromised by only using DC optimisation.

8.2 Varying Demand Elasticity

A demand curve describes the marginal price (M) that consumers are prepared to pay, as a function of net quantity consumed (Q): Normally, these demand curves would be intersected with supply curves to determine a market equilibrium. In our case, though, we are assuming that the demand and supply curves have already been merged to form a net “Demand Curve for Release” (DCR), which represents the prices to be expected from clearing the intra-period market, as a function of the net quantity made available in that period. And we have been assuming that each DCR is a constant elasticity curve, of like Equation (5.12) i.e.:

$$M = M(Q) = MREF \times \left[\frac{Q}{QREF} \right]^{(-1/E)} \quad (8.1)$$

Alternatively, we can express Q as a function of M:

$$Q = Q(M) = QREF \times \left[\frac{M}{MREF} \right]^{-E} \quad (8.2)$$

In Equations (8.1) and (8.2) E is the constant elasticity, while (QREF, MREF) is an (essentially) arbitrary reference point on the curve.

As in previous chapters, we assume that MREF = 1, representing an arbitrary reference price. Conversely, QREF varies by period, but is set so as to match the monthly inflow level, on average over an annual cycle, as discussed in Section 5.4.4. This experimental design feature ensures that supply and demand quantities in the market are consistently scaled such that, in a perfectly managed unconstrained system, with constant elasticity curves, the price would always equal the reference price of 1.

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So, varying E gives a whole family of curves, with different elasticities, all going through the same reference point. If $E = 1$ the resulting curve is perfectly elastic, and if $E < 1$ then the resulting curve is inelastic. In the last two chapters we assumed that the elasticity value was fixed at $E = 0.5$, as a reasonable average value across urban and/or rural water consumers. Based on the review in Section 6.2, though, we will now test the sensitivity of our results to this assumption, exploring a wider range of constant elasticity values. For the rest of this section we use E values of 2.00, 1.00, 0.75, 0.50, 0.375, 0.25, 0.175, and 0.15. Table 8.1 shows the first ten unit MRV price steps for the DCR in the average inflow months. The QREF row and a row either side are also shown. The last price step value at RCAP is also shown, as is the next step of 0's indicating a spill of zero value. The final row shows the total cumulative benefit available for a given constant elasticity DCR. As may be seen, low elasticities result in a DCR with very high prices for the first few units released, and this implies a very high total “consumer surplus” value.

Release Level	Elasticity								
	-2.000	-1.000	-0.750	-0.500	-0.375	-0.250	-0.175	-0.150	-0.100
1	9.17E+00	8.40E+01	3.68E+02	7.06E+03	1.35E+05	4.98E+07	9.91E+10	6.74E+12	1.75E+19
2	6.48E+00	4.20E+01	1.46E+02	1.76E+03	2.13E+04	3.11E+06	1.89E+09	6.63E+10	1.71E+16
3	5.29E+00	2.80E+01	8.50E+01	7.84E+02	7.23E+03	6.15E+05	1.86E+08	4.44E+09	2.96E+14
4	4.58E+00	2.10E+01	5.79E+01	4.41E+02	3.36E+03	1.94E+05	3.59E+07	6.53E+08	1.67E+13
5	4.10E+00	1.68E+01	4.30E+01	2.82E+02	1.85E+03	7.97E+04	1.00E+07	1.47E+08	1.79E+12
6	3.74E+00	1.40E+01	3.37E+01	1.96E+02	1.14E+03	3.84E+04	3.54E+06	4.37E+07	2.89E+11
7	3.46E+00	1.20E+01	2.75E+01	1.44E+02	7.55E+02	2.07E+04	1.47E+06	1.57E+07	6.19E+10
8	3.24E+00	1.05E+01	2.30E+01	1.10E+02	5.29E+02	1.22E+04	6.84E+05	6.43E+06	1.63E+10
9	3.06E+00	9.33E+00	1.97E+01	8.71E+01	3.86E+02	7.59E+03	3.49E+05	2.93E+06	5.02E+09
10	2.90E+00	8.40E+00	1.71E+01	7.06E+01	2.92E+02	4.98E+03	1.91E+05	1.45E+06	1.75E+09
...									
83	1.01E+00	1.01E+00	1.02E+00	1.02E+00	1.03E+00	1.05E+00	1.07E+00	1.08E+00	1.13E+00
84	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
85	9.94E-01	9.88E-01	9.84E-01	9.77E-01	9.69E-01	9.54E-01	9.35E-01	9.24E-01	8.88E-01
...									
168	7.07E-01	5.00E-01	3.97E-01	2.50E-01	1.57E-01	6.25E-02	1.90E-02	9.84E-03	9.77E-04
RCAP +1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
TOTAL	2.25E+02	4.79E+02	1.12E+03	1.16E+04	1.74E+05	5.39E+07	1.01E+11	6.81E+12	1.75E+19

Table 8.1: Average Inflow DCR Values for various elasticities

We initially included an even lower elasticity value of 0.1 in our experiments. Some of the results from these initial experiments turned out to be quite extreme, though, particularly with storage capacity set to only four times the average monthly inflow. We carried out a thorough investigation (not reported here) of the detailed outputs from the RO, RS, and the Reservoir Convolution (RC) modules to verify that the results are, in fact, correct for the relatively tightly constrained system we had assumed.

In summary, with an elasticity of 0.1 and SCAP of $4 \times \text{FMEAN}$, the absolute benefit is approximately 2×10^{22} for all optimisation runs. But 99.9% of the total benefit comes from the first 3 units of water in the three driest months, and this is achieved by all optimisations. Still, the MC versus DC benefit difference is approximately 2×10^{16} , while the negative semi-deviation benefit difference is approximately 1×10^{19} . The VWAPR difference is 2×10^{16} , while the positive semi-deviation difference is approximately 6×10^{18} . Thus, with this extreme elasticity, the DC optimisation performs extremely badly at managing the uncertainty. But it also became evident that results were being affected by numerical accuracy issues within Matlab. Accordingly, we limit our detailed reporting to runs with elasticities higher than 0.15.

8.2.1 Results for MC optimisation

In light of these initial investigations, in our main batch of experiments we vary the elasticities as above, for SCAP values of 4, 6, 12, 24, 36, and 48 times FMEAN. We keep RCAP at $2 \times \text{FMEAN}$ in all cases.

We use the Matlab modelling suite to produce results using DC and MC optimisation, for each elasticity. We take the difference, and/or ratio between MC versus DC depending on what is best to report. But we present the MC versus DC results graphically, as a set of curves corresponding to various SCAP values (as multiples of FMEAN), with elasticity varying along the horizontal axis. These base results are given in the various tables throughout Chapter 7 (i.e. for the chosen SCAP values in the $\text{RCAP} = 2 \times \text{FMEAN}$ column). Thus, in this section, figures report measures for 8×6 (Elasticity \times SCAP) grid of 48 individual cases. In general, we use a log scale on the vertical axes.

In Figure 8.1, we show the annual Time Weighted Average Benefit (TWAB). This shows that the benefits increase rapidly, by many factors of 10, when elasticities are smaller than 0.5, and decrease much more slowly for elasticities larger than 0.5. All six curves are closely bunched because storage does not have a visible impact at the scale shown (although there are differences when inspecting the data). This is because of the large value being placed on the first units of release in the DCR, and hence the large underlying benefit value, which is mostly met with the RCAP chosen.

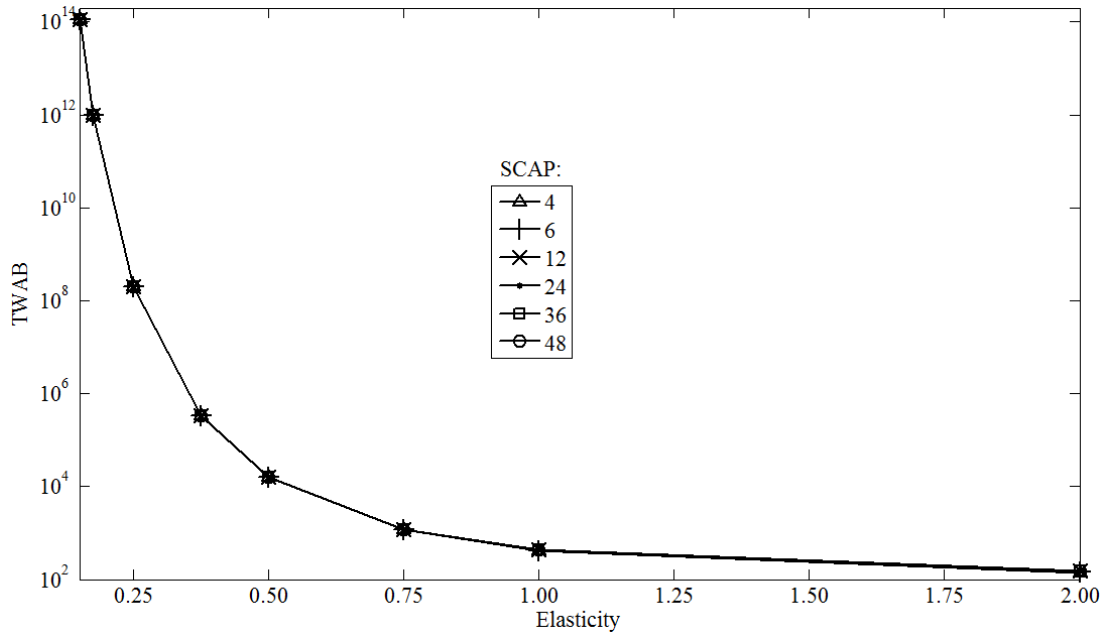


Figure 8.1: TWAB with MC optimisation, as constant elasticity and SCAP varies.

In Figure 8.2 we show the VWAPR values, with MC optimisation. Again, the prices increase rapidly when elasticities are smaller than 0.5. But note that the maximum price is “only” about 150, while the maximum benefit value is about 10^{14} . With an elasticity of 2, the VWAPR values actually fall to less than 1: These range between 0.924 and 0.999, with SCAP values of 4 and 48 times FMEAN, respectively.

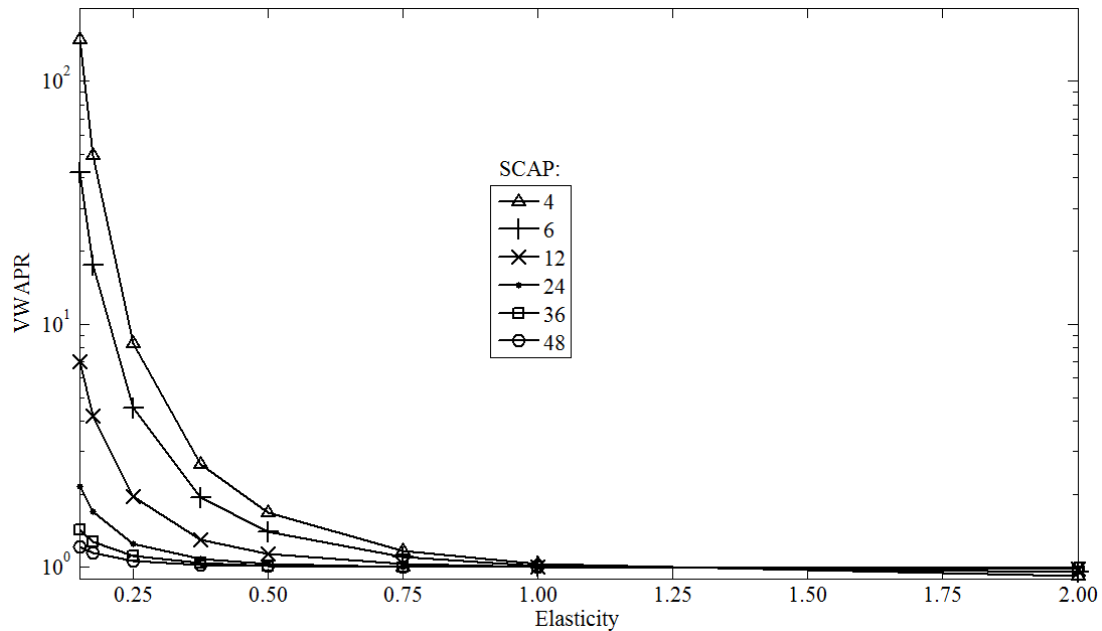


Figure 8.2: VWAPR with MC optimisation, as constant elasticity and SCAP varies.

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Figure 8.3 shows the ANSD of TWAB, with MC optimisation, while Figure 8.4 shows the VWPSD of VWAPR. Both elasticity and storage capacity affect the VWAPR and TWAB downside variability. As expected, lower elasticities (<0.5) result in higher downside variability, while higher elasticities (> 0.5) result in lower downside variability. The scales are similar, and varying elasticity seems to affect the VWAPR variability more than the TWAB variability.

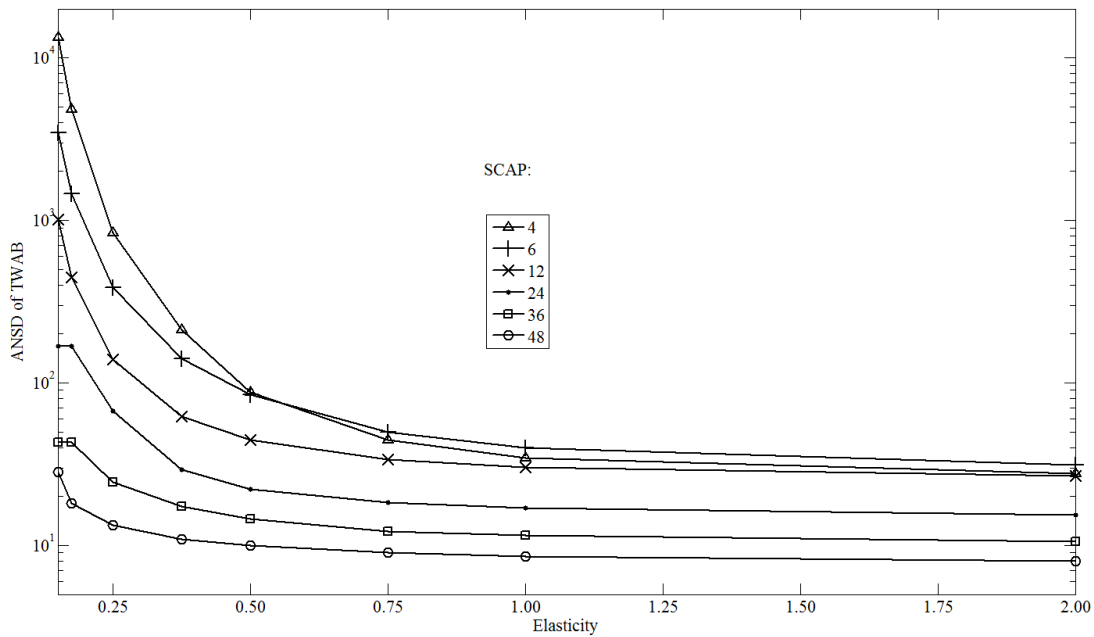


Figure 8.3: ANSD of TWAB with MC optimisation, as constant elasticity and SCAP varies.

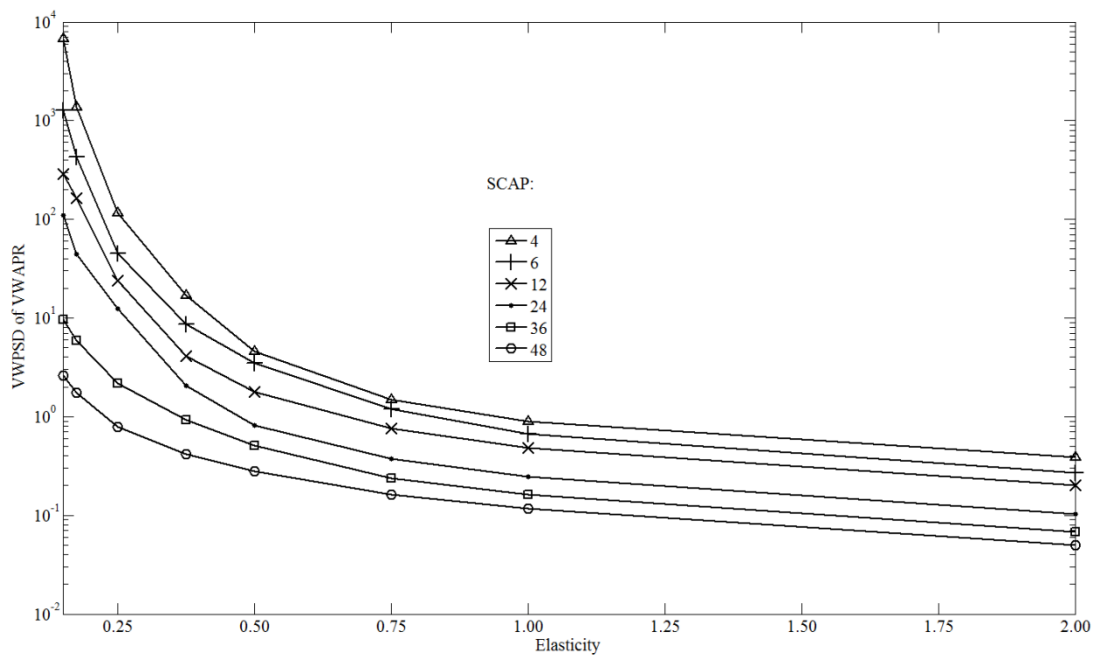


Figure 8.4: VWPSD of VWAPR with MC optimisation, as constant elasticity and SCAP varies.

8.2.2 Value of MC versus DC optimisation

In this section, the figures report the differences and/or ratios between DC optimisation and MC optimisation with respect to the measures reported above, over the same 8×6 grid of results. Thus another 48 cases were run to produce DC optimisation results.

Figure 8.5 shows the MC minus DC TWAB for each case. With low elasticity and low SCAP the differences are higher, than with a high SCAP (≥ 3 years' worth of average inflow). Benefit differences generally reduce with increased SCAP, because increased storage capacity means that the simulated trajectory for DC optimisation is less likely to hit a bound.

Figure 8.6 shows the corresponding DC to MC ratio of TWAB. The ratios are always less than one. The higher the elasticity (>1), the worse the relative performance of DC optimisation. Figure 8.6 can be compared with Figure 8.5, which shows MC minus DC TWAB differences reducing with higher elasticity. Constant elasticity curves with lower elasticities imply higher DCR prices for the first few units released, and this results in a larger underlying benefit in all cases.

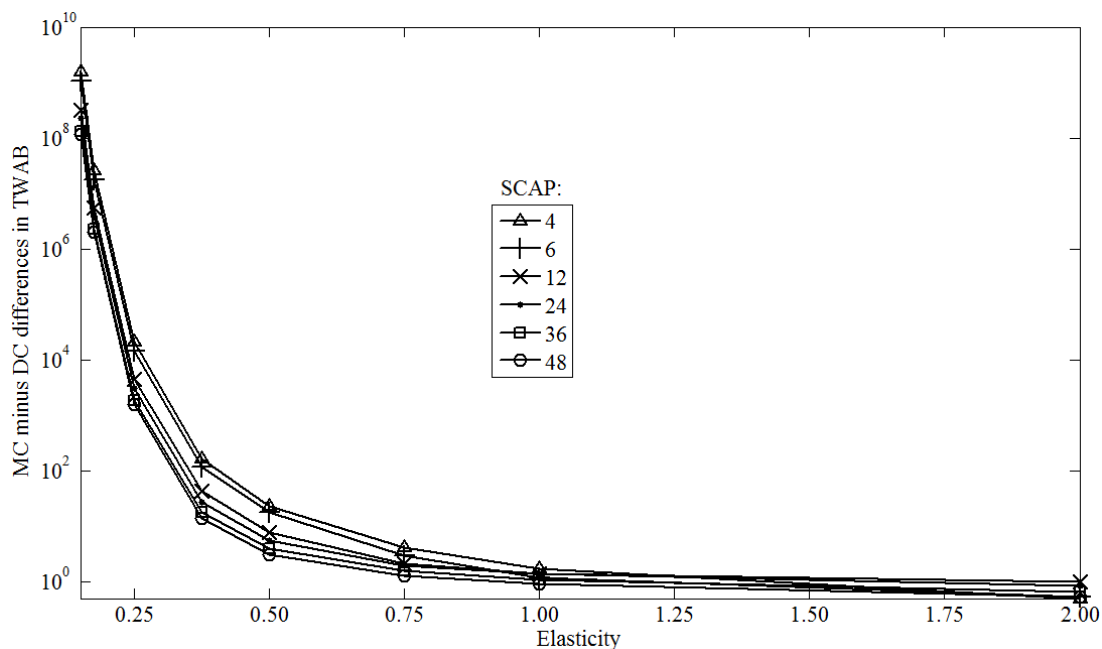


Figure 8.5: MC minus DC differences in TWAB, as constant elasticity varies.

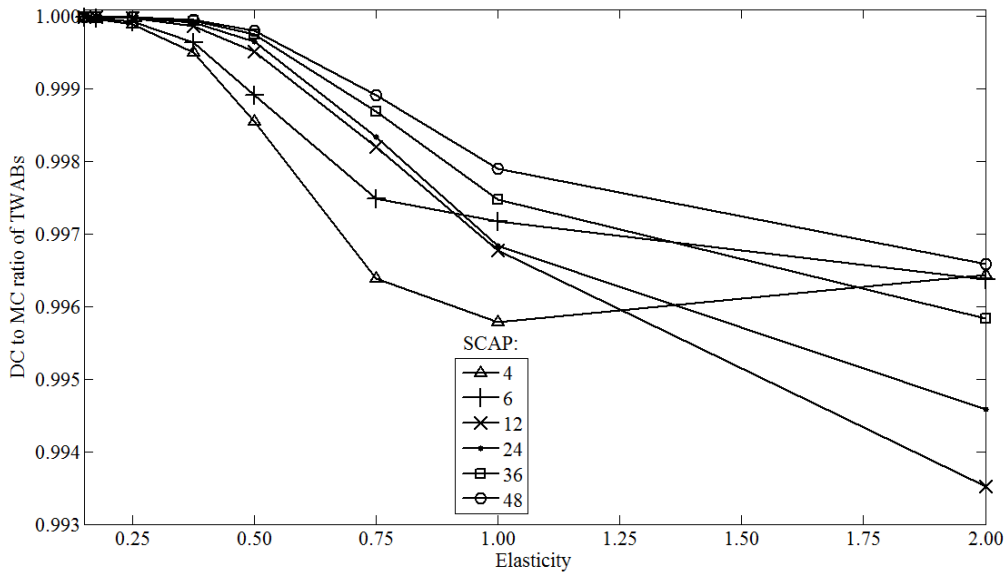


Figure 8.6: DC to MC ratio of TWAB, as constant elasticity varies.

In Figure 8.7 below, we show the DC to MC ratio of VWAPR. With a low elasticity the VWAPR ratio is very high, of the order 10^7 . With high elasticity, the DC to MC VWAPR ratio falls rapidly, to unity, and even slightly lower. So, whereas MC minus DC TWAB differences are always positive (as we would expect from superior optimisation), thus implying a ratio of less than 1, DC minus MC VWAPR differences can be negative, implying a DC to MC price ratio less than 1. The negative differences occur at a high elasticity (2) and are very small (of the order 1×10^{-3}). This could occur for various reasons discussed in Section 5.6.3, but in this case the issue seems to be due to volume weighting, since the DCRs are all convex, and TWAP differences (not displayed here) are all positive.

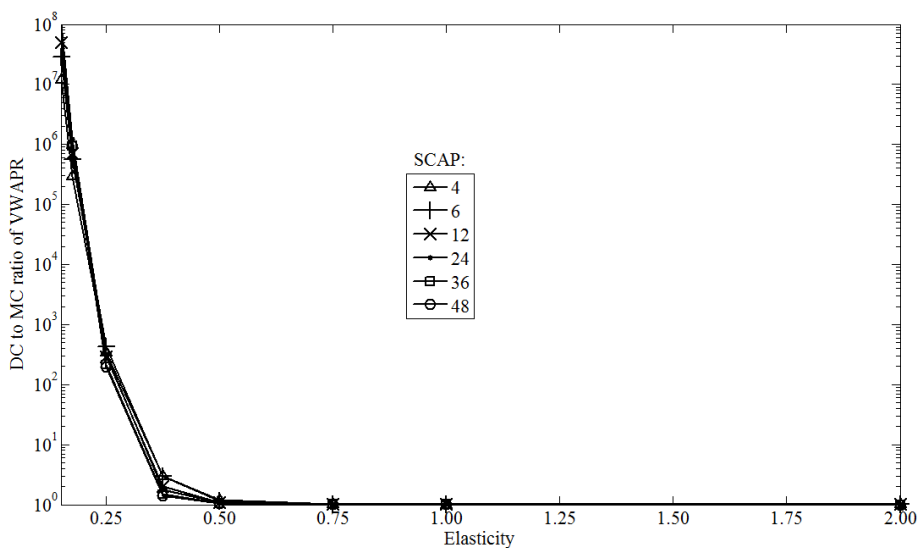


Figure 8.7: DC to MC ratio of VWAPR, as constant elasticity varies.

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Figure 8.8 shows the DC to MC ratio of the ANSD of annual TWAB, while Figure 8.9 shows the DC to MC ratio of the VWPSD of VWAPR. The downside variability for TWAB (8.8) and VWAPR (8.9) show very similar patterns. From the graphs we can deduce that with low elasticities we will get large variability differences between MC and DC optimisation runs. This presumably reflects the fact that the DC optimisation does not save enough water for the driest periods in drier than average years. For the reasonably unconstrained case reported in Chapter 6, we compared stochastic (SI and MC) and deterministic optimisations. The results show that the DC VWAPR and TWAB CDFs have much longer tails.

With higher elasticities ($E > 0.5$), the DC optimisation performs relatively well, with the ratios being much closer to one. With an elasticity of $E = 2$, for high SCAP values, the ratio of VWAPR is actually less than one, approximately 0.998. So, the VWAPR differences are actually negative in those cases (approximately -0.002), and thus the VWAPR variability is lower in the DC optimisation. These differences might be small, but we believe they are not sampling error. In Chapter 5 we noted that the optimisation will tend to reduce variability of TWAP more strongly than VWAP. And inspection of the corresponding TWAP variability shows that these are in fact always higher for the DC optimisation than for the MC optimisation, in these constant elasticity cases.

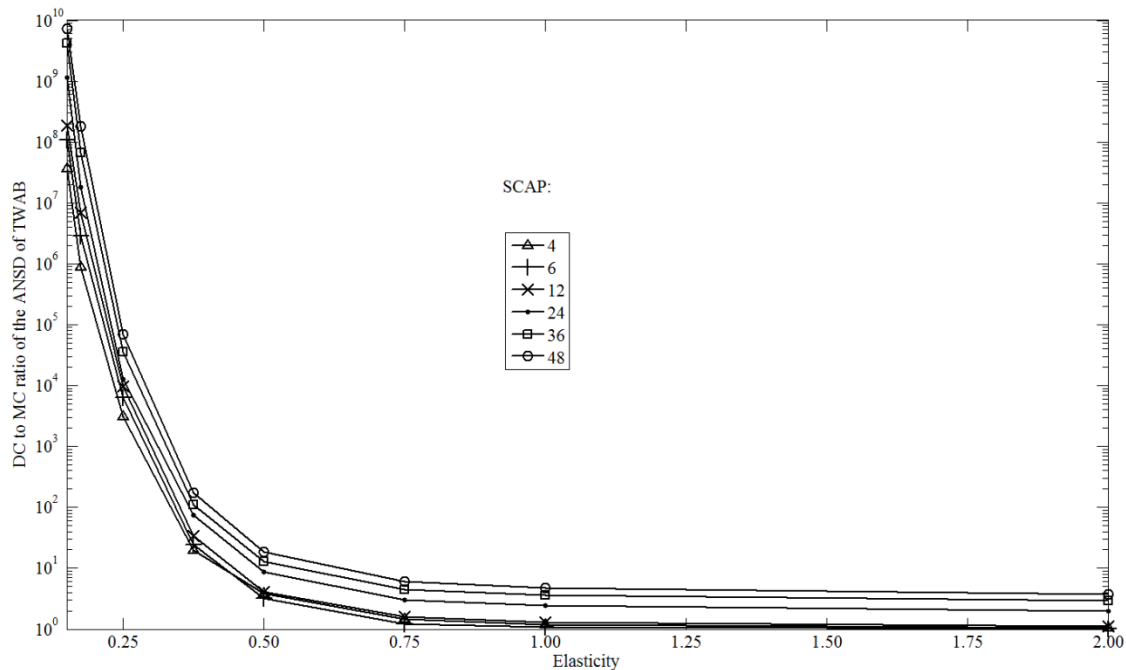


Figure 8.8: DC to MC ratio of the ANSD of TWAB, as constant elasticity varies.

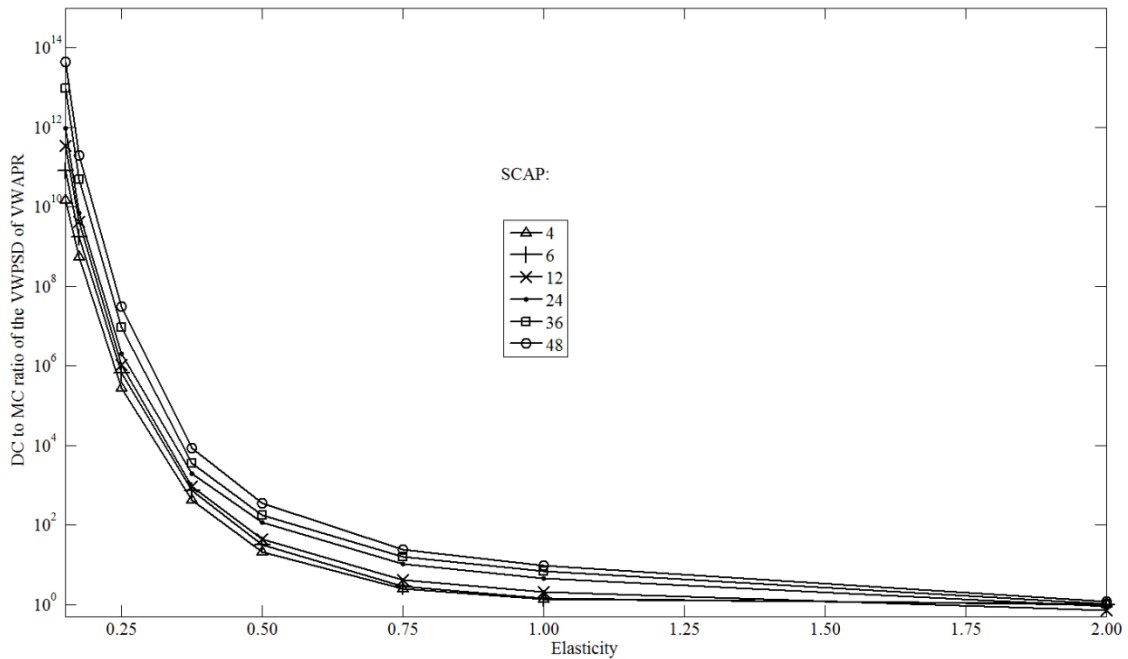


Figure 8.9: DC to MC ratio of the VWPSD of VWAPR, as constant elasticity varies.

With a lower (<0.5) elasticity, the DC optimised market produces (slightly) lower benefits, significantly higher prices, and significantly higher variability. With a higher (>0.5) elasticity, the proportional benefit loss from a DC market is higher, but the prices are similar to those in an MC optimised market. Finally, while the price and benefit variability are higher for the DC optimisation, they are not dramatically higher.

8.3 Varying the DCR Shape

Up to now our experiments have used constant elasticity curves, to model the DCR used to clear the market. In this section we explore the impact of uncertainty when demand is modelled by different DCR shapes, especially cases where the DCR is more “lumpy”, which might be the case in many real situations. With irregular curves, the “pinning” of the DCRs at (QREF, MREF) cannot always be guaranteed to produce any particular expected value, either in price or benefit terms, because this represents a fundamentally different supply and demand situation. So, in this section results cannot be expected to be close to the base case, in terms of total net benefit and the market clearing price.

We hypothesise that using a linear DCR will reduce the value of modelling stochasticity, compared to a constant elasticity DCR that passes through the same MREF, QREF point, with the same slope (and therefore elasticity) at that point. With a linear DCR the ratio of M/Q

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gets smaller as Q increases, so demand is more/less elastic at high/low prices. But we do not investigate linear curves here.

We also hypothesise that adding horizontal flats into the DCR will reduce the value of modelling stochasticity, as they increase the effective elasticity. Such flats could represent a situation in which there was demand for, or supply of, water at a constant price. In the limit, a large flat around the unconstrained Market Clearing Price (MCP) of 1 may eliminate the need to model stochasticity entirely, because it represents a situation where you can entirely manage any inflow uncertainty, at a fixed marginal cost, within a period, because there is no need to store water for higher value periods.

Conversely, we hypothesise that adding vertical steps into the DCR will increase the value of modelling stochasticity, as they decrease the effective elasticity. In the limit, a large step around the expected mean quantity traded effectively creates the situation where demand is perfectly inelastic. In the limit, this situation might be impossible to manage, even with a stochastic model. The model provides no way of choosing between poor (infeasible) solutions, and thus deterministic and stochastic approaches could be seen to be ‘just as bad’. But, as we approach the limit, and the situation gets closer and closer to perfectly inelastic we might expect a deterministic optimisation to get worse and worse relative to a stochastic optimisation. Although this may be physically unrealistic, if it were possible it would imply the same benefit being delivered irrespective of optimisation approach, but price volatility might produce extreme prices.

8.3.1 General Parameter Settings

For all these experiments we fix some key parameters at the levels assumed for earlier experiments. We use the “extreme” inflow seasonality hydrology dataset, with bids out of phase by six months, as above. As a base case, we still assume a constant elasticity DCR with $E = 0.5$, pinned at a reference price (MREF) of 1, with QREF changing monthly, but matching monthly inflows, on average. Mainly we study the case where SCAP is four times FMEAN, with RCAP twice FMEAN as above. But we also range SCAP in the horizontal flat section, to highlight a point about price convergence. Although this case produced extreme results for some lower elasticities in the previous section, it produced moderate results in Chapter 7, for the moderate elasticity assumed there.

8.3.2 Inserting a Horizontal Flat

In this section we look at inserting a flat in to our base-case constant elasticity net DCR. This flat is designed to model a fixed cost supply, such as from “manufactured water”, such as desalination or recycled stormwater, which is assumed to supply water at a fixed marginal offer. Thus we model a situation where market demand is met by a mix of freely harvested (stochastic) and priced manufactured (certain) water. The latter assumption might be a more realistic situation, where the freely harvested water would generally be at a lower price, but we test a wider range of circumstances. But these results would also apply to situations where demand is perfectly elastic for the length of the inserted flat.

The DCR reduces in a monotone fashion until it reaches the flat, where the price remains constant until the flat is fully allocated. At that point, the price could drop down to the original DCR level for that quantity, but that would create a large vertical step in the curve, which does not correspond to any likely supply technology. Instead we assume that the original curve continues from the end of the flat, as shown in Figure 8.10. In order to maintain the (QREF, MREF) average (ideal) market clearing point, the horizontal flat must be inserted by shifting the DCR as shown in the figure. When the flat is inserted at a price above the reference price, it represents a high priced supply offer which is not accepted at the reference price. Therefore, the net demand still matches the average supply in the market, at the reference price. So that point is still the unconstrained optimum. Conversely, if a flat is inserted at a price below the reference price, it represents a low priced supply offer which we assume to have been accepted in forming the base case DCR. So again the reference point is still the unconstrained optimum.

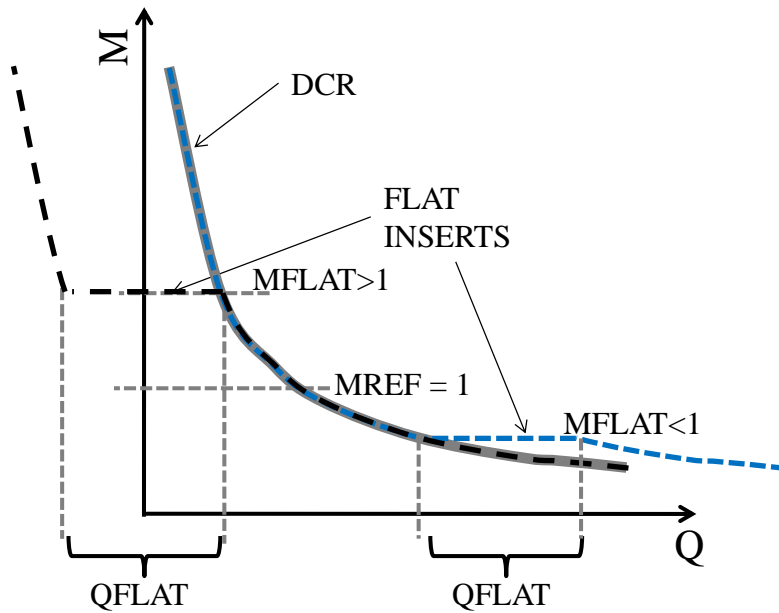


Figure 8.10: Example of DCR with Horizontal Flat Insertion.

In Figure 8.10, the grey thick line represents a constant elasticity DCR. The black dashed line shows a horizontal flat of length QFLAT being inserted into the constant elasticity DCR at a marginal price, $MFLAT > MREF$. The top of the DCR shifts QFLAT units to the left of the axis and is discarded, representing the fact that those highly valued demand units are met by increased supply from the supplier. If QFLAT is large enough it will meet (shift to the left of the axis) all higher valued demand and cap the MCP (as shown).

Similarly the blue dashed line shows a horizontal flat of length QFLAT being inserted into the constant elasticity DCR at a marginal price, $MFLAT < MREF$. The bottom of the DCR shifts QFLAT units to the right of the axis, representing the fact that those lower valued units are either displaced by a consumer with a higher value on water, or not met by a supplier as the case may be.

Given that this is a theoretical exercise, we decided to coarsely sample various arbitrary MFLAT prices, at various arbitrary QFLAT widths, as in Table 8.2 below. Constant elasticity curves become rather flat below our chosen MREF of 1, as shown in the MWV DCSs in Section 6.3.2. And so far we have not seen VWAPR results much less than 1. So,

while we discuss inserting flats and steps at MREF, and below MREF, we only model them being inserted above MREF.¹²⁷

MFLAT	1.1		1.5		2		5		10		50		250	
QFLAT	0	2	4	8	16	24	32	40	48	56	64	72	84	100

Table 8.2: Price and Quantity Reference points for inserting Horizontal Flats

Here, QFLAT = 0 creates the constant elasticity base-case, irrespective of MFLAT. But we model the 13 non-zero QFLAT values for each of the 7 MFLAT values, and this produces 91 case runs, or 182 when running DC and MC levels of optimisation run complexity.

8.3.2.1 Results, Analysis and Interpretation

Results with MC optimisation

Prior to discussing differences between different optimisation runs, we show the general pattern of results. In all figures, the top-left corner value is near enough the base case result. In Figure 8.11 below we show the annual TWAB. The QFLAT values from 0 to 100 are shown on the horizontal axis. The lines correspond to MFLAT values from 1 to 250, that is for all MFLAT prices at or above MREF, with MC optimisation.

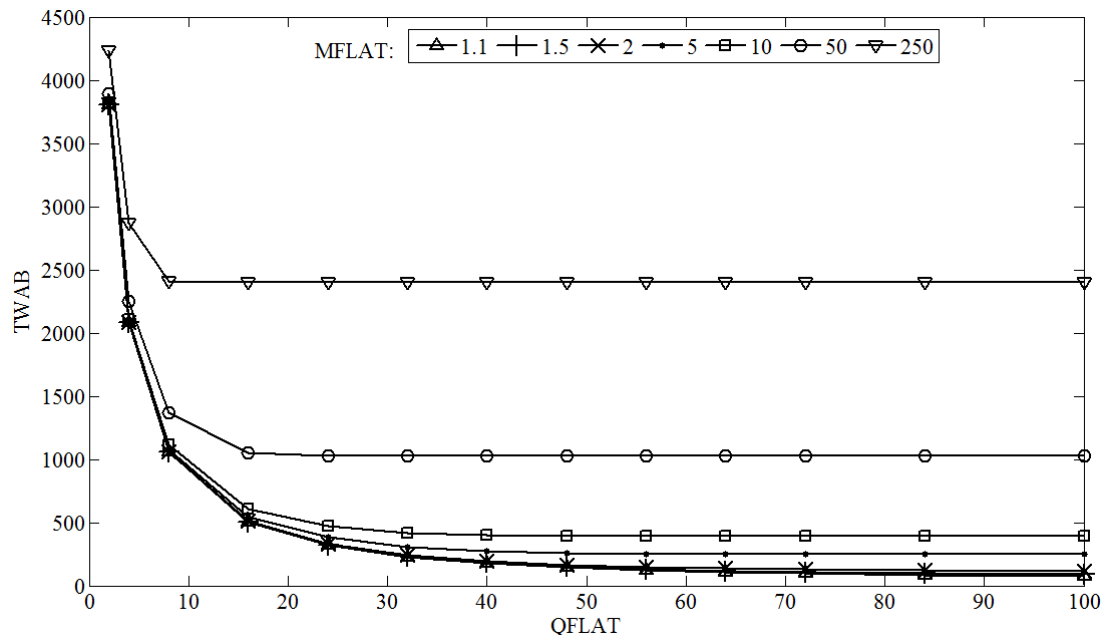


Figure 8.11: TWAB from MC optimisation for various MFLAT, with horizontal flats.

¹²⁷ At an MREF of 1, inserting a flat should lower both price and benefit. Below an MREF of 1, inserting a flat should raise both price and benefit.

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In Figure 8.12 we show the same data as in Figure 8.11 but use MFLAT values on the horizontal axis, and the lines correspond to some of the QFLAT values.

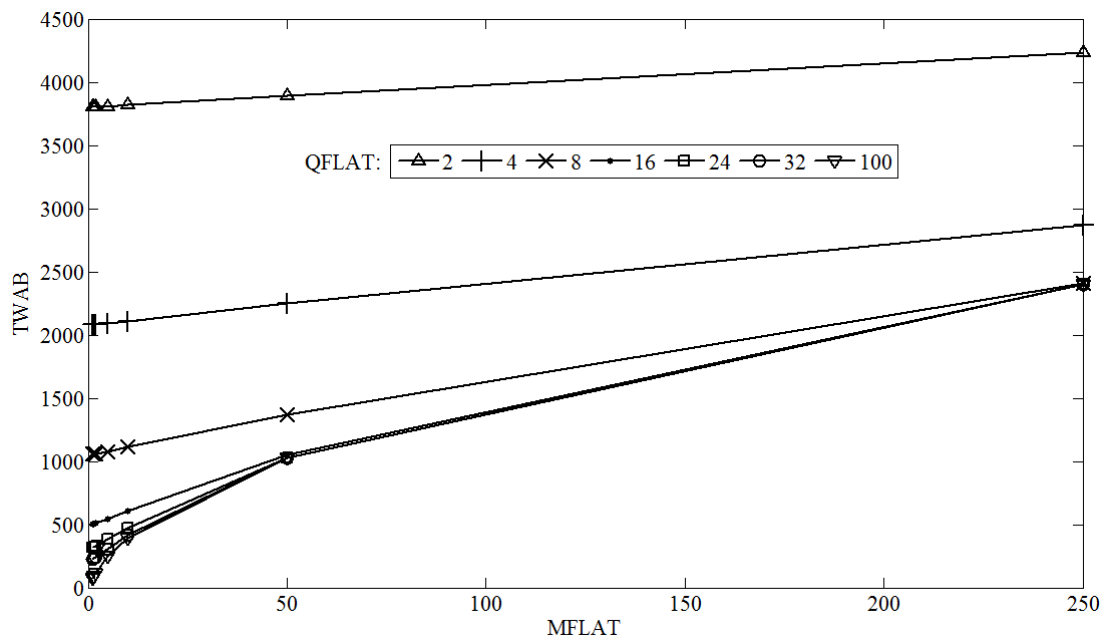


Figure 8.12: TWAB from MC optimisation for various QFLAT, with horizontal flats.

As QFLAT increases the net benefits decrease. This does not mean that the total benefits to society are lower, as a result of there being an alternative supply of cheap water. But it does reduce the need for, and hence value of, the harvested water supply, as represented by our net DCR. As MFLAT increases, our modelled benefits increase, because this represents the situation where the alternative supply of water becomes more costly. And so the need for, and hence value of, harvested water increases. As MFLAT rises, the QFLAT supply becomes less relevant because it is less likely to be utilised, and benefits eventually revert to the base case levels, irrespective of QFLAT.

In Figure 8.13 below we show VWAPR in the same format as Figure 8.11 above.

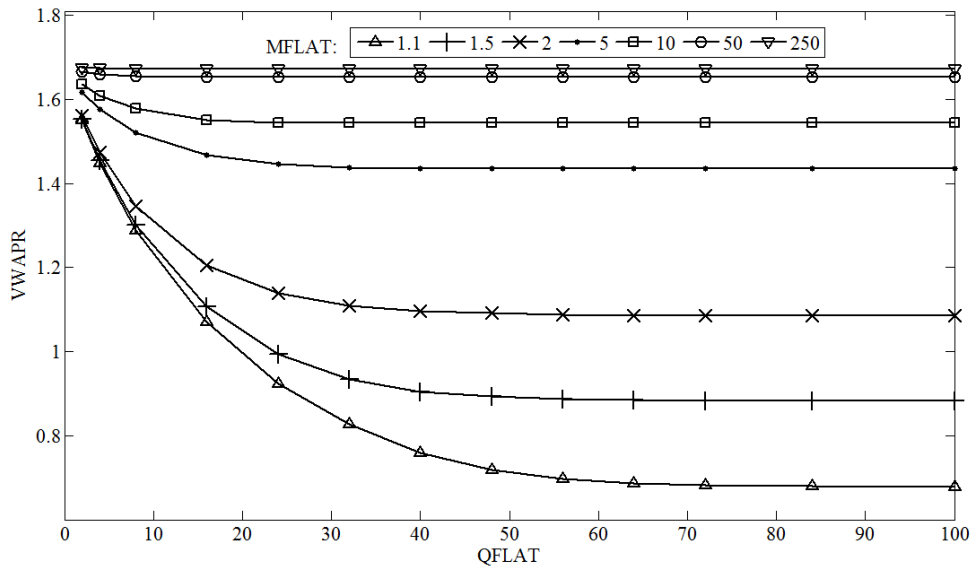


Figure 8.13: VWAPR from MC optimisation, with horizontal flats.

The VWAPR pattern in Figure 8.13 above follows the TWAB pattern in Figure 8.11, as expected. Longer low priced flats reduce the annual MCP, as cheap supplies meet the higher valued net demand. A similar pattern to Figure 8.12 emerges, if VWAPR is plotted as a function of MFLAT, for various QFLAT values. Higher QFLAT implies lower prices, and prices increase for moderate increases in MFLAT. In Figure 8.14 we show the ANSD of TWAB, with MC optimisation, and this shows a very similar pattern to the absolute TWAB in Figure 8.11 above.

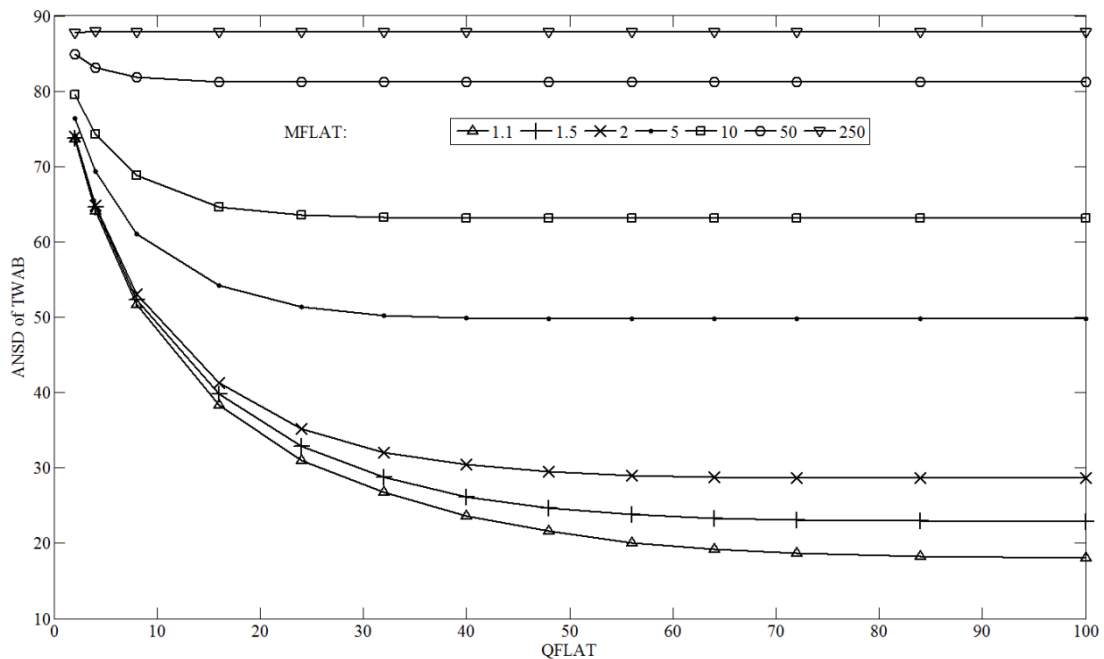


Figure 8.14: ANSD of TWAB from MC optimisation, with horizontal flats.

In Figure 8.15 we show the VWPSD of VWAPR, with MC optimisation, which shows a very similar pattern to the absolute VWAPR in Figure 8.13 above.

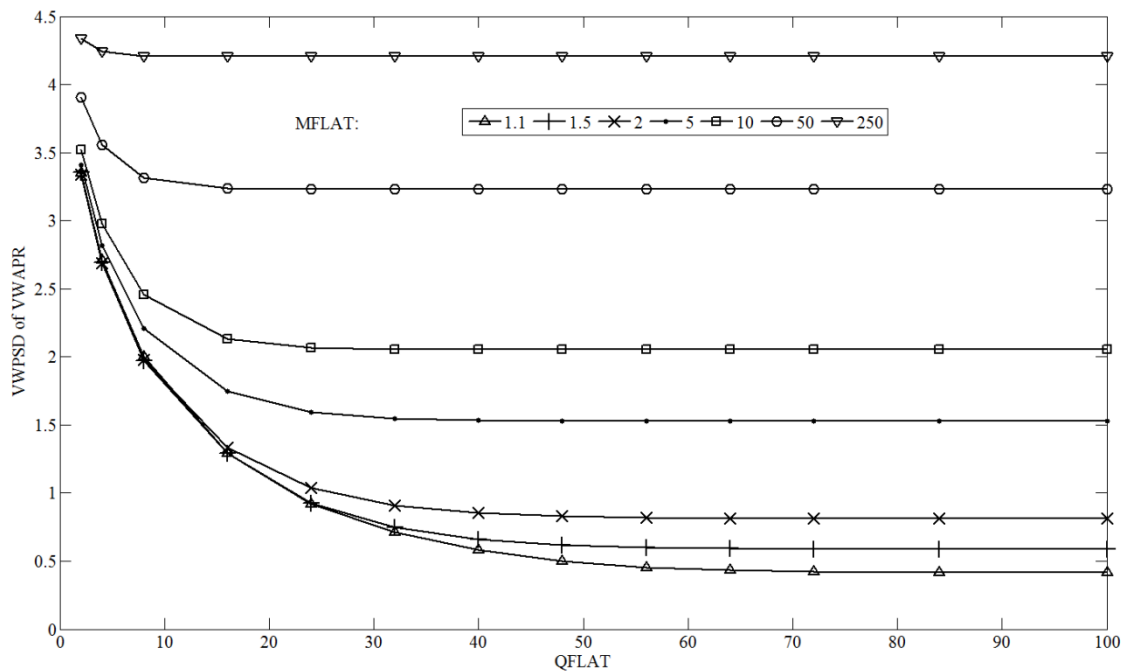


Figure 8.15: VWPSD of VWAPR from MC optimisation, with horizontal flats.

All the above runs were performed with a fixed SCAP of $4 \times \text{FMEAN}$. In Figure 8.16 below we show a similar graphic for a fixed MFLAT of 1.1, just larger than the ideal unconstrained MCP of 1, but varying SCAP. Note that VWAPR are at their highest levels for the lowest modelled SCAP, and with a minimal flat inserted. VWAPR are reduced to their lowest levels for the same SCAP but with a high QFLAT. This reflects the fact that, with a price flat set at a low level, and with a short enough flat, the DCR prices above the inserted MFLAT will still be frequently sampled. But, in the case where the price flat is wider, the DCR prices above the price flat (if they exist) will be seldom visited. Also, with our net demand formulation, visiting the price flat could imply utilising the cheap supply, in that situation we can potentially allocate more water to meet net demand in other periods. So we are potentially visiting prices less than MFLAT more often. For a large enough SCAP, though, the optimisation does always produce the ideal unconstrained MCP of 1, thus confirming our hypothesis with respect to the way in which these DCRs were constructed i.e. that with an unconstrained system the optimisation will return the ideal unconstrained market clearing price of 1.

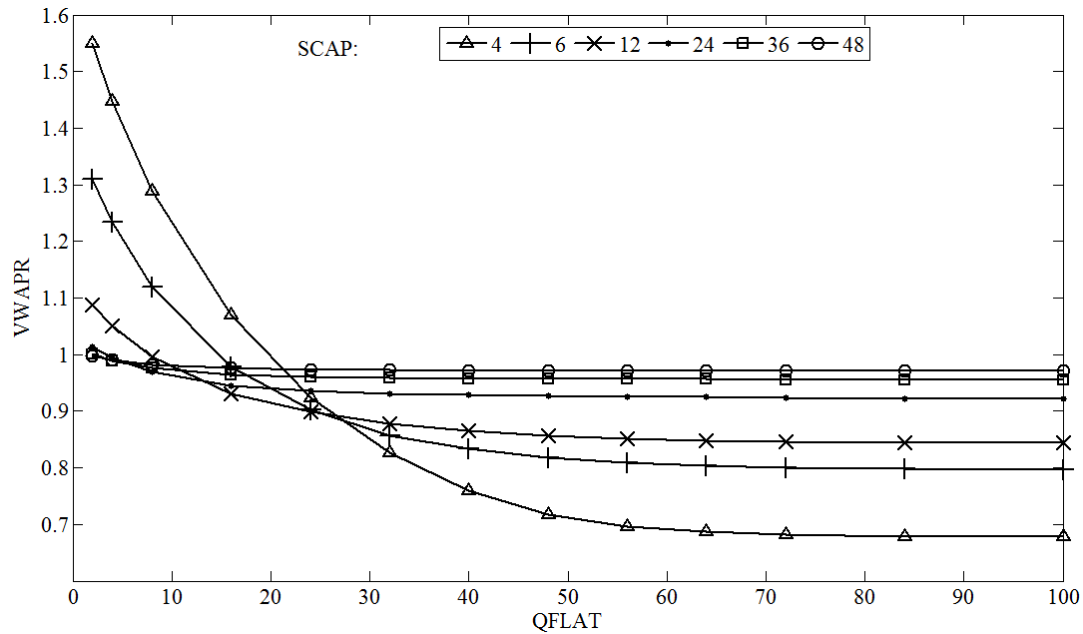


Figure 8.16: VWAPR from MC optimisation when varying SCAP, with horizontal flats.

Impact of MC versus DC optimisation

In Figure 8.17 we show the difference (MC – DC) between the MC optimisation TWAB in Figure 8.11 and the corresponding DC optimisation TWAB, in the same format as the two prior figures. As expected, the effect of a lower price and/or wider flat is to reduce the value of modelling stochasticity, as manufactured water gives us a physical way of handling stochasticity, at moderate (short run) cost. As MFLAT increases, so do the differences. With an infinite MFLAT price, we return to the original constant elasticity DCR, and thus the differences in the top-left corner represent the base case differences. As QFLAT increases the differences decrease, until the point where QFLAT is greater than approximately half the monthly average inflow (42 units), after which the benefit differences level off to their minimum levels, for a given MFLAT.

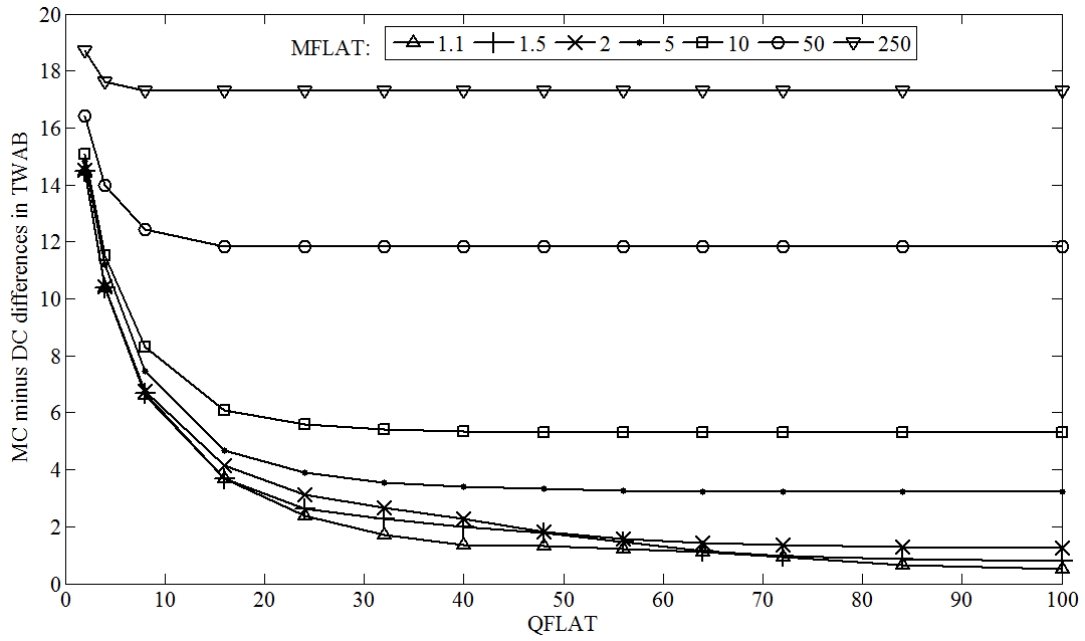


Figure 8.17: MC minus DC differences in TWAB, with horizontal flats.

Figure 8.18 shows the corresponding DC to MC ratio of TWAB. The ratio is always less than one, but unlike the differences in Figure 8.17 above, the ratios are not showing smooth monotone decreasing curves, with MFLAT decreasing and QFLAT increasing. With the price flat positioned at $MFLAT \leq 2$ the DC and MC optimisations are sampling those flat points more frequently, and those interactions are visible here. Section 5.6.3.2 discusses this issue in more detail.

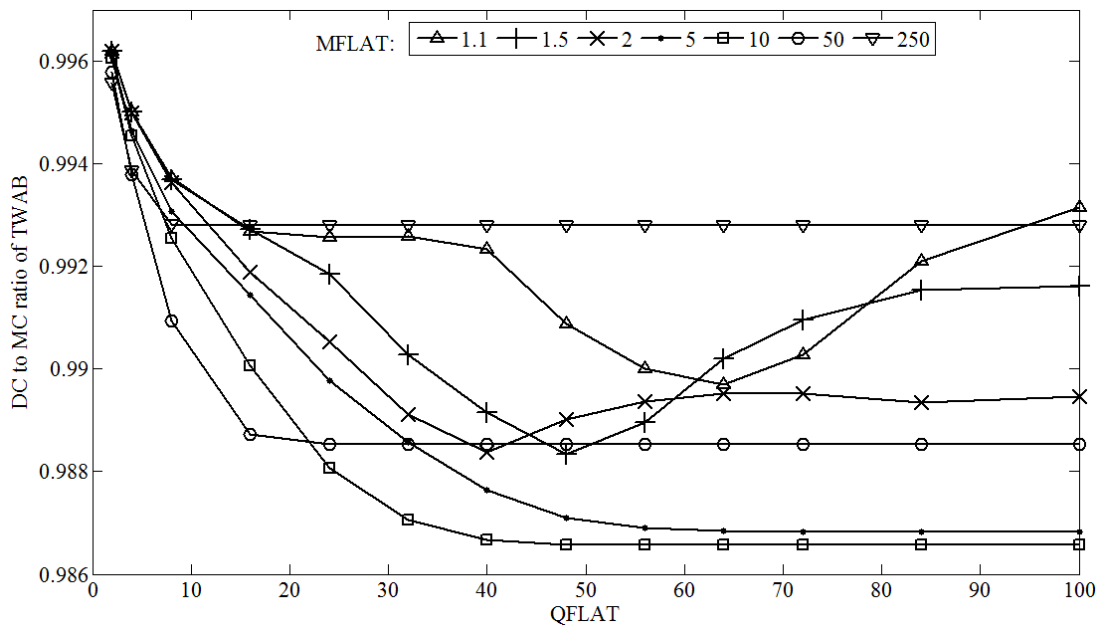


Figure 8.18: DC to MC ratio of TWAB, with horizontal flats.

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In Figure 8.19 we illustrate the corresponding VWAPR differences, in the same format. While Figures 8.17 and 8.18 show that the MC optimisation is correctly generating the lowest TWAB, we see here that it is not always generating the lowest VWAPR. For the two highest MFLAT prices, 50 and 250, at the top of the graph, ratios are more than one, and vary smoothly, as we might expect. For lower MFLAT prices we see ratios of less than one, indicating that the less complex optimisation produces higher prices. Possible reasons are discussed in Section 5.6.3.2. But in general, a long enough flat, inserted at an intermediate price level tends to reduce the value of a stochastic market, in terms of prices.

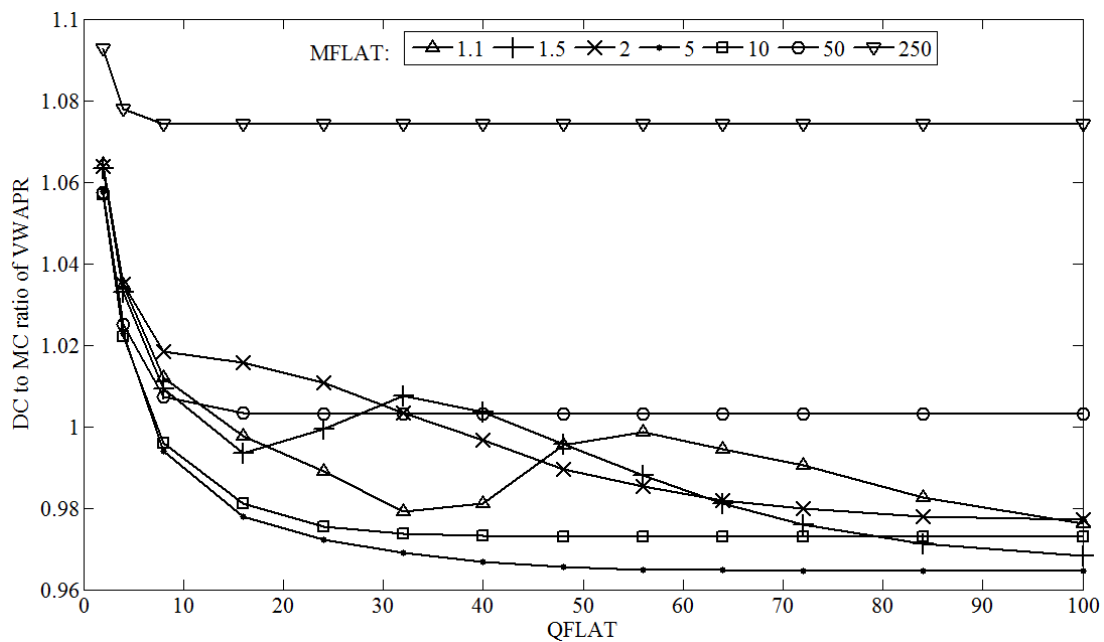


Figure 8.19: DC to MC ratio of VWAPR, with horizontal flats.

In Figure 8.20 we show the DC to MC ratio of the ANSD of TWAB. This figure shows a very similar pattern to the absolute ANSD of TWAB under MC optimisation in Figure 8.14 above.

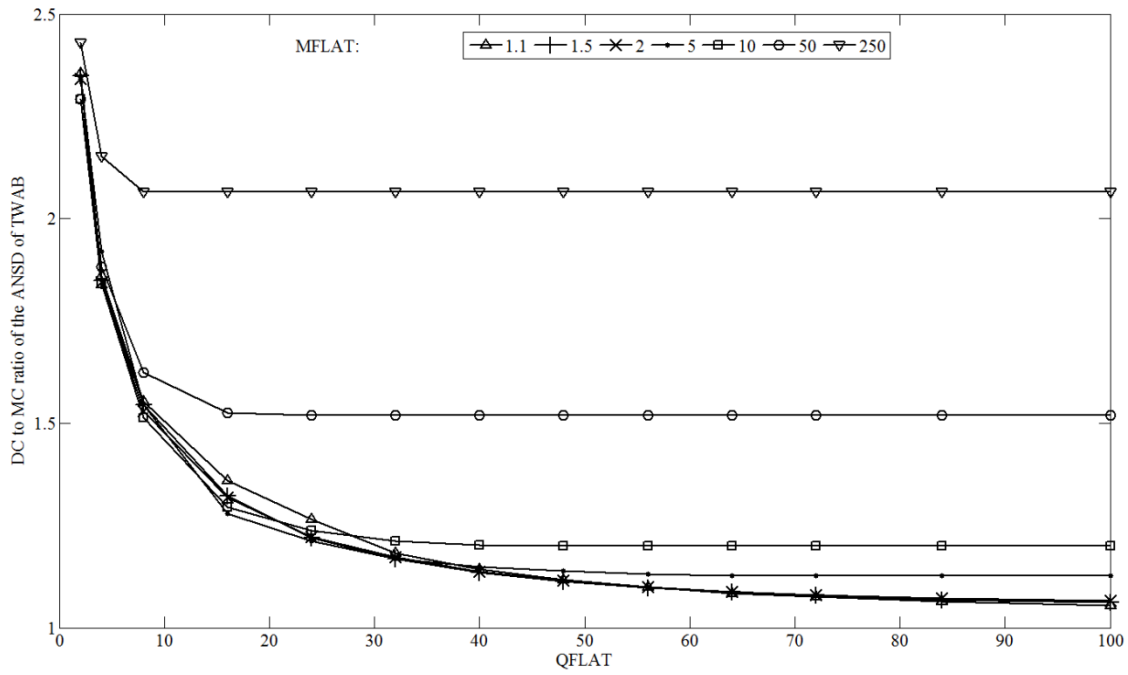


Figure 8.20: DC to MC ratio of the ANSD of TWAB, with horizontal flats.

In Figure 8.21 we show the DC to MC ratio of the VWPSD of VWAPR, with a similar pattern to the DC to MC ratio of VWAPR in Figure 8.19 above.

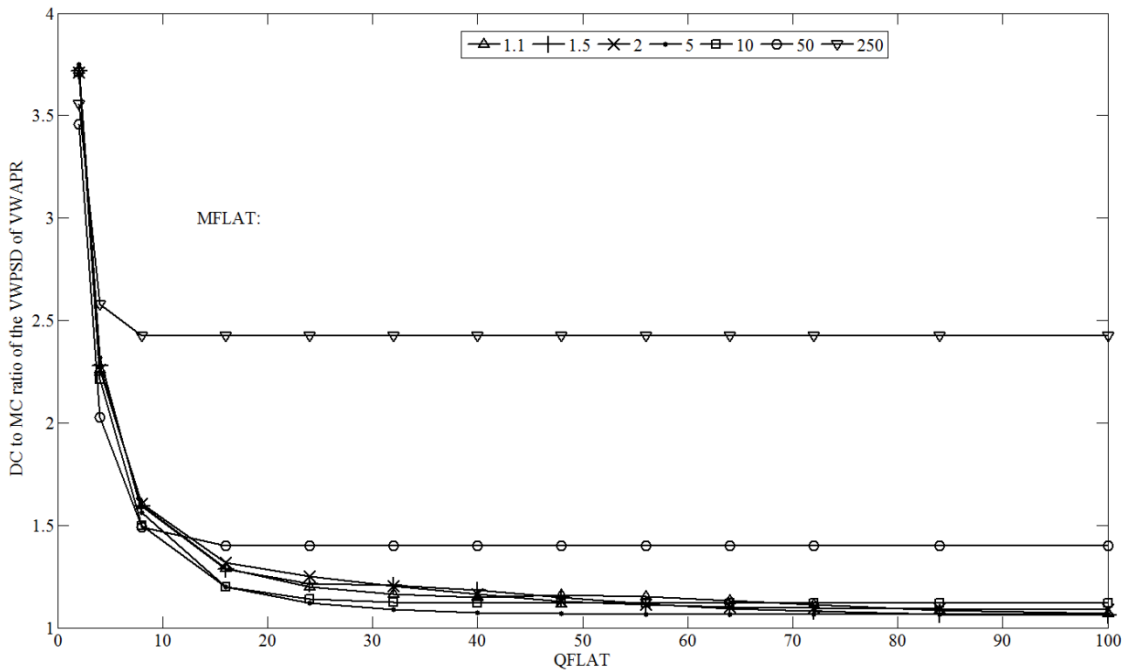


Figure 8.21: DC to MC ratio of the VWPSD of VWAPR, with horizontal flats.

Using DCRs with horizontal flats inserted tends to reduce the value of a stochastic MCE over using a deterministic MCE.

8.3.3 Inserting a Vertical Step

In this section we look at inserting a step in to our DCR. This step is designed to model the situation where one class of users (e.g. Urban drinking water) is prepared to pay much more than another (e.g. Agricultural or Environmental). This assumption is realistic, and is even the case in a purely rural system, such as the Goulburn-Murray Water Irrigation District. In that system the “environment” is allocated a large quantity of low cost water. And farmers can bid for the remaining water, generally at a much higher cost. This means that the aggregate DCR could be effectively seen as a composite of two constant elasticity curves, as shown in Figure 8.22.

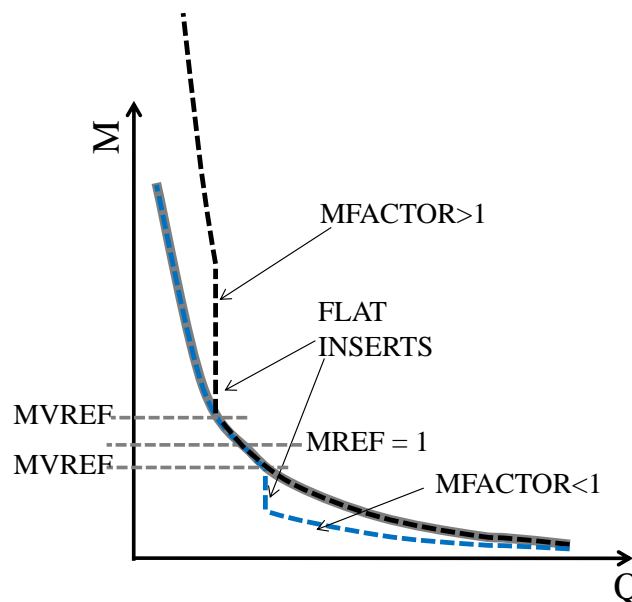


Figure 8.22: Example of Vertical Flats being inserted into a Constant Elasticity DCR

In Figure 8.22, the grey thick line represents an underlying constant elasticity DCR. The black DCR, follows that DCR up to a marginal price of $MVREF > MREF$, after which the original curve is scaled up by $MFACTOR > 1$ i.e. $MREF$ is effectively replaced by $(MREF * MFACTOR)$, but the constant elasticity is still the same on both parts of the curve. This creates a vertical step, and represents the possibility of a participant group being prepared to pay a significantly higher price for supplies, but only up to a certain limit, which might be set by the capacity of an offtake channel, industrial process etc. The blue DCR, follows the underlying DCR down to a marginal price of $MVREF < MREF$, after which the original curve is scaled down by $MFACTOR < 1$. This creates a vertical step, to the right of this step the high value demand is saturated and only the low value demand is still increasing

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in response to falling prices. This maintains the (MREF, QREF) reference point for all cases, and this is still the ideal unconstrained market equilibrium, as in the base case.

When the vertical step occurs, at a price above the reference price, the higher valued demand is saturated at any price below that, including the reference price. Therefore, the net demand still matches the average supply in the market, at that price, and this point is still the unconstrained optimum. Conversely, by inserting a vertical step at a price below the reference price, we are assuming that the higher priced demand had not been fully accepted, at the reference price, when forming the base case DCR. So again the reference point is still the unconstrained optimum. And it creates a set of curves for which all prices and quantities are positive.

Given that this is a theoretical exercise, we did not try to create curves to model any specific circumstances, but coarsely sample various arbitrary MVREF prices, at various arbitrary MFACTORS, as in Table 8.3 below. Again, we only model MVREF values greater than one.¹²⁸

MVREF	1.1		1.5		2		3.5		5		7.5		10	
MFACTORS	0	2	4	8	16	24	32	40	48	56	64	72	84	100

Table 8.3: Price and Quantity Reference points for inserting Horizontal Flats

8.3.3.1 Results, Analysis and Interpretation

MC optimisation results

As with horizontal flats, first we show MC optimisation results then the differences between different optimisation runs, focusing on MC versus DC differences. All the figures are laid out in the same form as in the horizontal flat section. We insert the vertical steps at the same price levels in the DCR as with the horizontal flats i.e. $\{MFLAT\} = \{MVR\}$. But, the QFLAT is replaced by MFACTOR, where the portion of the DCR to the left of MVREF has its marginal prices increased as a multiple of MFACTOR. This creates a vertical jump in the DCR, as discussed above.

¹²⁸ Inserting a step at an MREF of 1 should raise both price and benefit. Inserting a step below an MREF of 1 should lower both price and benefit.

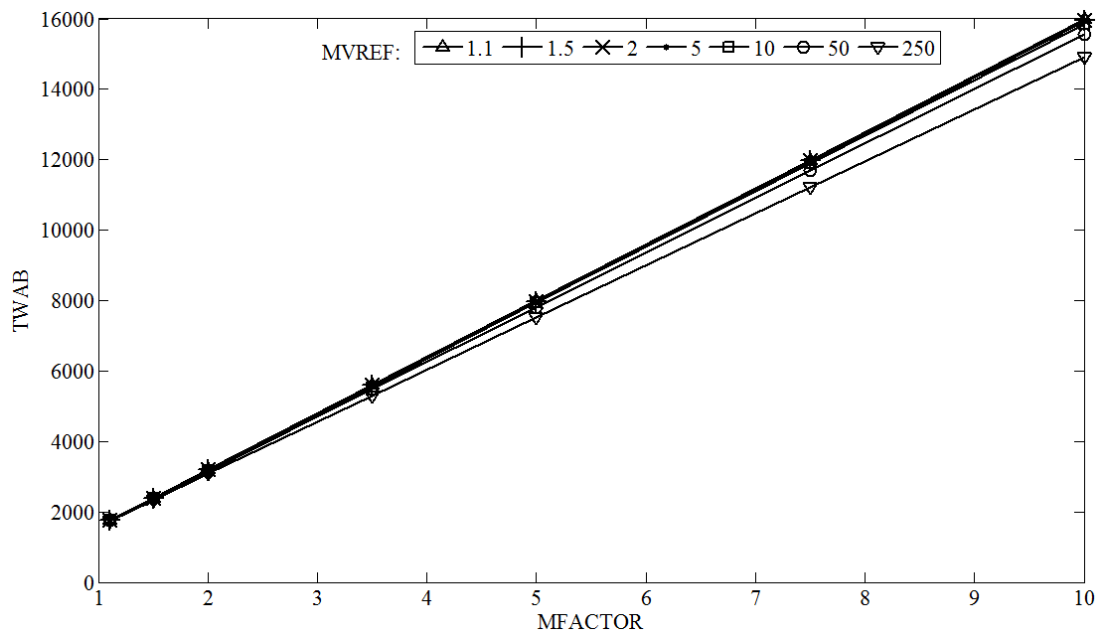


Figure 8.23: TWAB from MC optimisation, with vertical steps.

In Figure 8.23 we show the TWAB from MC optimisation. Benefits are not only monotone increasing, but virtually linear with increasing MFACTOR. This is not surprising given we have a group of aggregate market participants who are willing to pay higher and higher prices to be allocated water, and thus benefits rise quickly with a larger MFACTOR. That effect largely dominates any impact from the optimisation adjusting allocations, reflecting the fact that water will be allocated, as far as possible, to the high value zone in all cases. Thus a similar linear relationship is evident in all figures in this section. Still, benefits are also monotone decreasing with increasing MVREF. As MVREF increases this pushes the vertical step higher up the DCR, reducing the consumer surplus added by participants who are willing to pay higher prices.

The VWAPR pattern in Figure 8.24 above is also nearly linear and monotone increasing, but prices vary more significantly, with MVREF, than with benefits. The average MCP is higher when a vertical step is inserted nearer the optimal unconstrained clearing price of 1, i.e. when $MVREF \leq 2$. This is because the simulation visits that region of the DCR, with the inflated marginal prices, more often.

The ANSD of TWAB, reveals a very similar pattern to the absolute TWAB in Figure 8.23 above, changing linearly with increased MFACTOR. The benefit variability ranges from about 2 when $MFACTOR=1$ to 1.8×10^5 when $MFACTOR=10$.

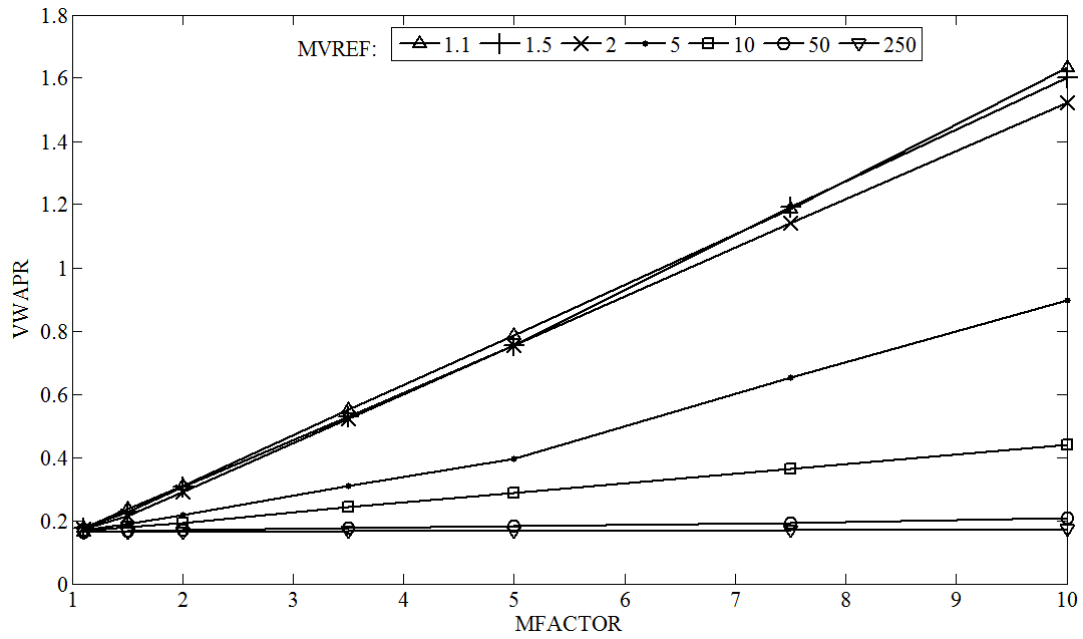


Figure 8.24: VWAP from MC optimisation, with vertical steps.

In Figure 8.25 we show the MC VWPSD of VWAPR, also revealing a very similar pattern to the absolute VWAPR in Figure 8.24 above.

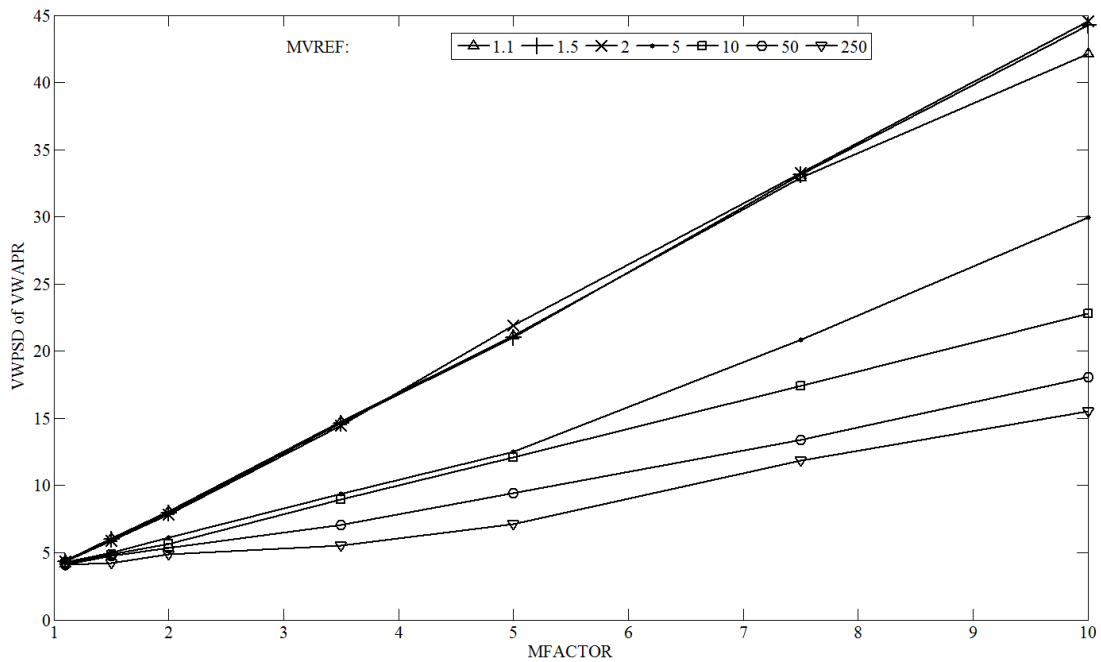


Figure 8.25: VWPSD of VWAPR from MC optimisation, with vertical steps.

MC versus DC optimisation

In Figure 8.26 we show the ratio of the DC optimisation benefits to the corresponding MC optimisation benefits in Figure 8.23. As can be seen, the DC optimisation only achieves about 73-77% of the benefits of the MC optimisation with the worst performance for the lower MVREF values. With higher MVREF values the DC optimisation performs worse with lower MFACTOR's.

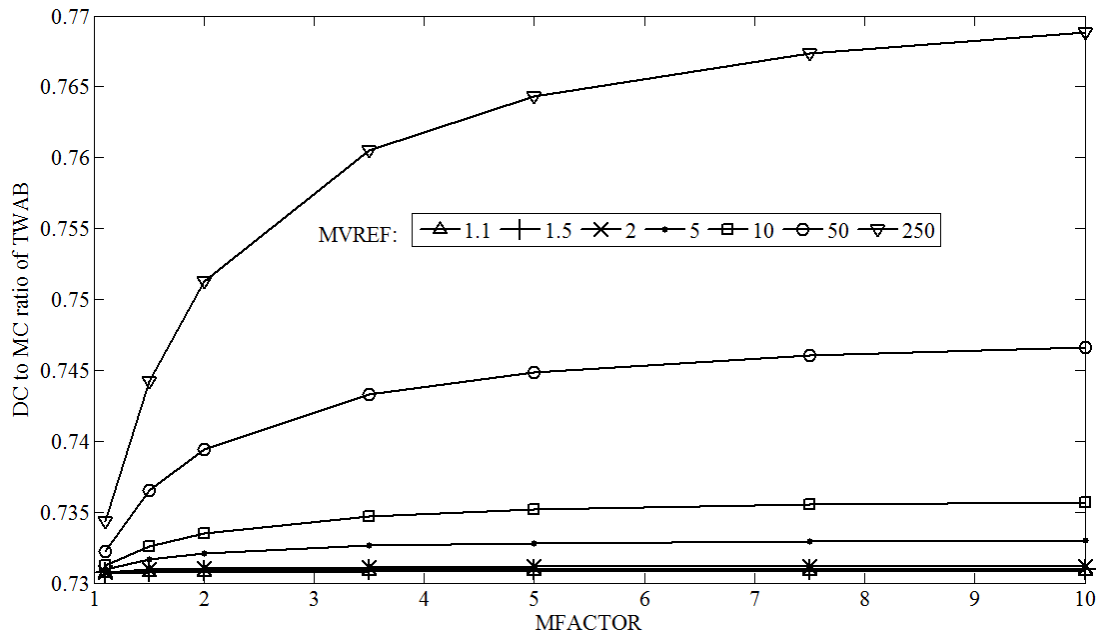


Figure 8.26: DC to MC ratio of TWAB, with vertical steps.

In Figure 8.27 we show the DC to MC ratio of VWAPR, which shows a similar pattern as for the absolute VWAPR in the MC optimisation runs in Figure 8.24. Most noticeably the VWAPR ratios are large, and they get even larger with increasing MVREF and increasing MFACTOR. A large DC to MC ratio means that prices in the DC optimisation are larger than those in the MC optimisation. The ratios are generally monotone increasing with increasing MFACTOR and increasing MVREF, but not for low MVREF values (1.1, 1.5, and 2). With those parameters, a large proportion of market participants are demanding water at quite high prices, creating a non-convex region in the DCR, as discussed in Section 5.6.3.2. And that region is close to the actual MCP, and so visited often within the simulation.

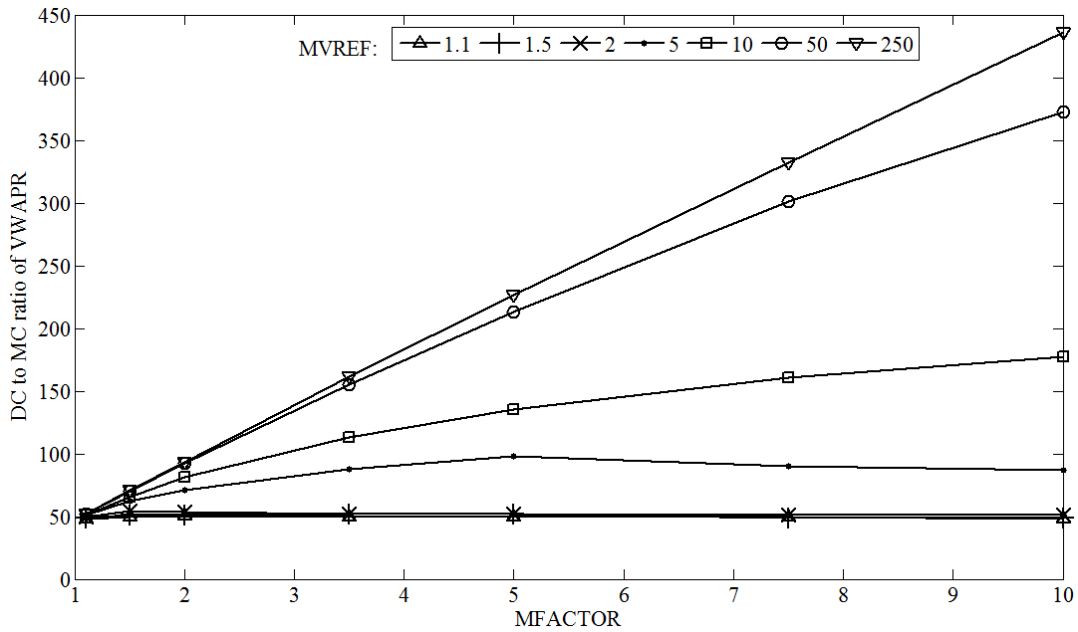


Figure 8.27: DC to MC ratio of VWAPR, with vertical steps.

The MC minus DC differences in the ANSD of TWAB does show a linear response similar to Figure 8.23, but the MC variability values are higher than the DC variability values. In Figure 8.28 we show the DC to MC ratio of the ANSD of TWAB. This figure shows that the ratio of benefit variability has the same relationship as the ratio of benefits in Figure 8.26. In fact they both have approximately the same average ratio, approximately 0.75. Variability is actually higher in the corresponding MC optimisation results. This is because there is inherent variability in the benefits, and MC optimisation produces solutions which better match that underlying variability, as discussed in Section 5.7. Figure 8.29 shows the DC to MC ratio of the VWPSD of VWAPR, revealing a very similar linear pattern to the DC to MC ratio of VWAPR in Figure 8.27 above. There we noted that the VWAPR ratios were generally quite large, and here the VWPSD of VWAPR ratios are even larger. This indicates that the DC optimisation is doing a poor job of managing prices over the water year.

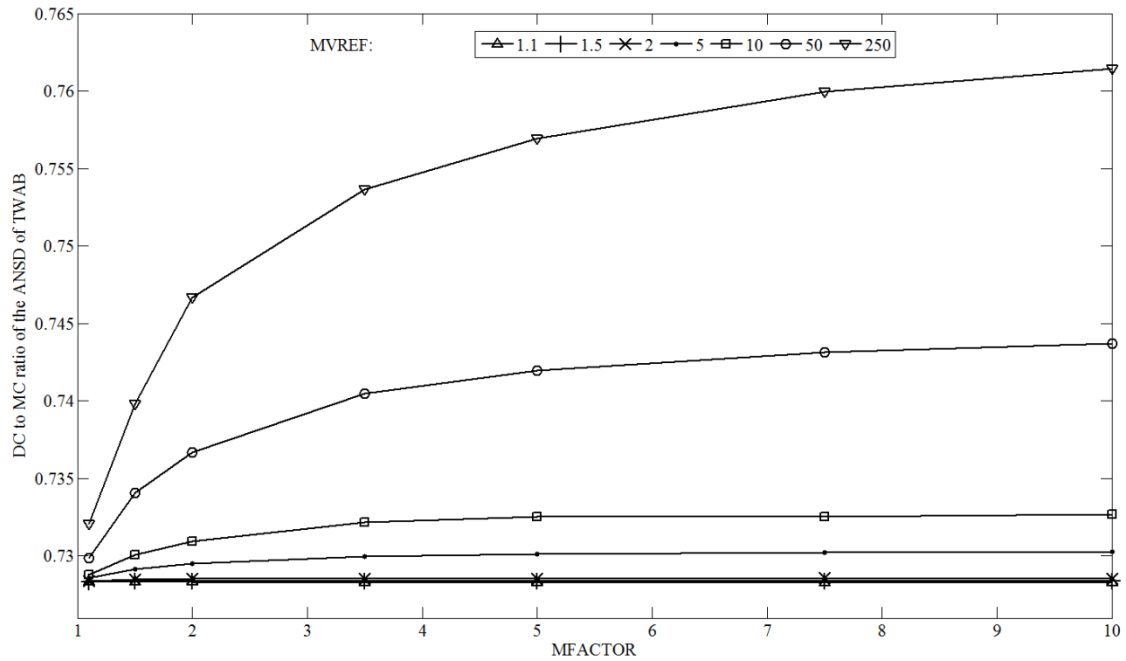


Figure 8.28: DC to MC ratio of the ANSD of TWAB, with vertical steps.

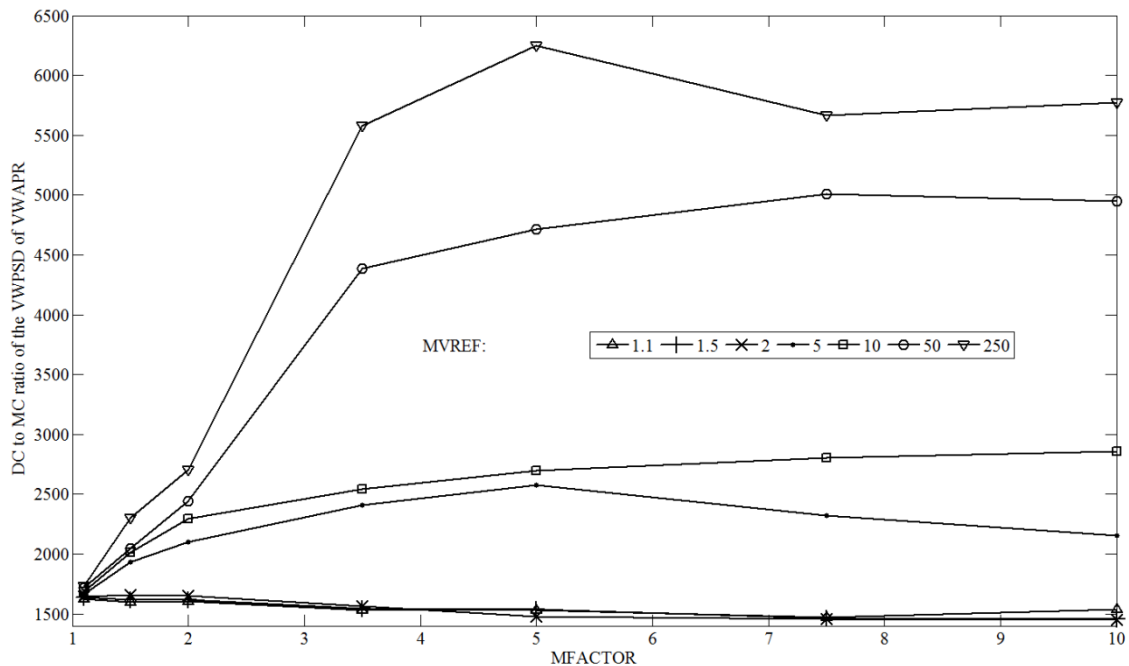


Figure 8.29: DC to MC ratio of the VWPSD of VWAPR, with vertical steps.

Having a vertical step in the demand curve, such as created by a group of aggregate market participants who are willing to pay potentially significantly more for water, greatly improves the value of stochastic optimisation, over clearing the market with a deterministic optimisation. In Section 8.2 we saw similar improvements for decreasing elasticity, and these results are consistent with those.

8.4 State Dependent Bids

8.4.1 Basic Concept

Up to now, non-state dependent aggregate DCRs have been directly formed by aligning QREF with the average inflow values, subject to some bid to inflow phasing. We do not want to adjust the way we form the average DCR. But to form state dependent DCRs, we need to explore how we can adjust (“shift”) the average DCRs to account for (say) rain falling on farms. Rain and reservoir inflow may well be correlated, meaning demand for water may reduce when it is naturally wet. This means that the state dependent DCR could be adjusted by a rain to inflow correlation factor. More generally, as discussed in Chapter 4 we can have an Index Dependency Factor (IDF) where the index can depend on wider factors.

In Chapter 4 we outlined a framework which included individual participants potentially providing continuous or discrete state-dependent bids. In Section 4.6.3.5 we discussed the potential (and merits) of “affine bids”. We described how affine bids would have fixed and variable bid components. The base bid is fixed at some reference point, probably the expected hydrology value. The variable bid component is adjusted linearly in some proportion β to the modelled hydrology state, where $\beta \neq 0$ implies some form of state-dependence. In this section we explore the potential of this concept, but for our experiments we will adjust the entire aggregate DCRs, and thus all market participants’ bids.

Thus the adjustment parameters may be thought of as weighted averages of some set of individual adjustment parameters. We consider cases in which the net aggregate demand of market participants may increase when it is drier and decrease when it is wetter. That is probably realistic for systems where demand is strongly correlated with reservoir hydrology. But we also consider cases in which the net aggregate demand of market participants may decrease when it is drier and increase when it is wetter.

This latter case could model a situation in which supplies were available from desalination or neighbouring catchment areas, or where some agricultural activities are abandoned in dry conditions. But, in Section 8.3.2 above, we considered the case where a particular participant was prepared to reduce net demand (i.e. increase supply), as an explicit response to rising prices. And we can imagine that such participants might also be prepared to trade water

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using contracts under which their supply increased as the hydrology index fell, causing prices to rise, in expectation, as discussed in Chapter 4. Evidence from other markets suggests that, although that might not be the ideal form of offer for them, such instruments will be traded if consumers demand them. In electricity markets a moderate/high-priced thermal station would be analogous to the desalination plant discussed in Section 8.3.2 above. Such plant will offer in the spot market to create flats. Strictly speaking the owner of such a plant would like to hedge their risk by selling a call option. In reality participants prefer to buy simpler hedging instruments, such as time (e.g. day-time) of use CfDs. Thermal generators are prepared to sell such CfDs since they expect the prices in the spot market are likely to be high enough to justify them operating, at the times when those CfDs apply.

So, results in Section 8.3.2 and here could be seen as complementary. Those in Section 8.3.2 could apply to the spot market, while the results from this section could apply to a contracts market, where participants offer bids which vary by hydrology. In fact, both bidding strategies could apply to the same market environment, where in the short-term participants bid via fixed marginal prices, and in the medium to long-term they provide state-dependent bids. Even in the same market it may well be attractive to have demand represented using index dependent bids, and supply with explicit fixed capacity offers. While it seems unlikely that net aggregate demand for the entire market will vary in strict proportion to the hydrology index, results for the full set of correlations are presented, partly for completeness and because they provide an easily verifiable test of the optimal results, as discussed below.

Note that, in this section we are seeing what impact modelling stochasticity has while assuming state-dependent bids, i.e. optimising and simulating while assuming state dependency. Earlier, we investigated the impact of ignoring serial correlations by simulating the performance of an SI optimisation, where there were actually correlations. By analogy, it would be desirable to test the impact of ignoring demand / inflow correlation, by optimising a release policy assuming non-state dependent bids, and then simulating the performance of that policy with demand correlation. But this development has not been pursued here.

8.4.2 Curve Shifting

For direct comparison across our experiments, we need to design our state-dependent DCRs such that their weighted average matches the DCR used in our earlier experiments. This is important, as it means that, in a perfectly managed and unconstrained system, the equilibrium price will always equal the reference price of 1, as before. We accomplish this by creating a symmetric pair of shifted DCRs, for each symmetric pair of MC states. Implicitly, they will then be weighted by the symmetric equilibrium probabilities of the MC.

We first considered simple curve translations, where adding/subtracting a constant value C within (or upon) the function shifts it in the desired direction. The curve can be shifted left/right, where $C >/< 0$ respectively, via:

$$Q(M,C) = Q(M)+C \tag{8.3}$$

With multiple curves, one for each hydrology state, and with symmetric ($\pm C$) shifting, then using Equation (8.3) we are effectively taking the expected value of $Q(M)$ (i.e. quantity) by averaging the curves horizontally, for each price M . And this average will equal the original $Q^0(M)$ value if the $Q(M)$ curves were all shifted left/right and weighted symmetrically, as the $+/-C$ in wetter and drier hydrology states cancel. That is:

$$E(Q) = \sum_{h \in H} Q^h P^h \tag{8.4}$$

$$E[Q(M)] = Q^0(M) \tag{8.5}$$

Alternatively, the curves could be shifted up/down, where $C </> 0$ respectively, via:

$$M(Q,C) = M(Q)+C \tag{8.6}$$

In this case we are effectively taking the expected value of $M(Q)$ (i.e. price) by averaging the curves vertically, for each quantity Q . And this average will equal the original $M^0(Q)$ value if the $M(Q)$ curves were all shifted up/down and weighted symmetrically. That is:

$$E(M) = \sum_{h \in H} M^h P^h \tag{8.7}$$

$$E[M(Q)] = M^0(Q) \tag{8.8}$$

Either type of shift raises a problem in that, either the quantity will become negative (with horizontal shifting), or the price will become negative (with vertical shifting). So there is a problem if the shift is both symmetric and uniform. But we could make the shift symmetric and non-uniform, so that the shift tapers out to nothing near the relevant axis. For example, for vertical shifting, we can specify the original “reference curve” in form $M(Q,0) = M^0(Q)$,

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and produce state-dependent curves at proportional distances above/below the reference curve, $M(Q, \pm \text{IDF})$, if we replace (8.6) by:

$$M(Q, \pm \text{IDF}) = (1 \pm \text{IDF}) * M^0(Q) \quad (8.9)$$

So long as $0 \leq \text{IDF} \leq 1$, we can add any pair of +/- IDF we like, with equal probability weights, and we still get $E[M(Q)] = M^0(Q)$, without violating any price bounds (with constant elasticity curves going to infinity at $Q = 0$). Similarly, for horizontal shifting the original “reference curve” is of the form $Q(M, 0) = Q^0(M)$, and we can produce state-dependent curves at proportional distances to the left/right of the reference curve, $Q(M, \pm \text{IDF})$, if we replace Equation (8.3) by:

$$Q(M, \pm \text{IDF}) = (1 \pm \text{IDF}) * Q^0(M) \quad (8.10)$$

where we can add any pair of +/- IDF we like, with equal probability weights, and we still get $E[Q(M)] = Q^0(M)$. And again $0 \leq \text{IDF} \leq 1$, so as not to violate the constant elasticity curve going to a price of infinity at $Q = 0$. Either way, when the Q value drops to near 0 the P value can be quite high. But we have another problem, because the horizontal shift in Equation (8.3), or (8.10) does not give us a set of curves that can also be averaged vertically to match the original. That is $E[M(Q)] \neq M^0(Q)$. Similarly, the vertically shifted curves produced by (8.6) or (8.9) can not be averaged horizontally to yield the original reference curve. That is $E[Q(M)] \neq Q^0(M)$.

So there is no obvious transformation which will have $E[Q(M)] = Q^0(M)$ and also $E[M(Q)] = M^0(Q)$. Thus we have to choose which we want, and the choice is not obvious. Vertical shifting is easier, in our experimental set-up, because the discretisation of the DCR to integer bid quantities means that we have to round prices to integer values with horizontal shifts, but this is not an issue with vertical shifts.

Conceptually, though, horizontal shifting seems more appropriate, if we think of a farmer having an overall demand curve for water, which may be partially met by water falling on their land, as rainfall (QR). Then, if $QR(h)$ is received directly in hydrology state h , the demand curve for controlled release from the irrigation system (i.e. DCR) may be expected to shift “horizontally”, by the quantity $QR(h)$. Thus, an equivalent volume in the DCR, which is valued at the farmer’s highest marginal return, can be replaced with the rain which was received for free. So, we do horizontal shifting. This does mean though, that the economic benefit derived from the system can not be the same, on average, across

experiments, because the area under our constant elasticity DCRs depends on the horizontal scaling, as shown in Figure 8.30, in the next section.

8.4.3 Experimental Parameter Settings

We allow the inflow states, $F^{t,h}$, to be correlated with a corresponding set of demand reference points, $QREF^{t,h}$. Without loss of generality, for these experiments we assume the expected monthly demand is out of phase with inflow by six months, in the base case, $QBASE^{t,\bar{h}}$. We also assume the following convention: An $IDF = 1$ means that there is an increasingly lower bid (i.e. a lower price for the same quantity or a lower quantity for the same price) the drier it gets, and an increasingly higher bid the wetter it gets. So, when $IDF = -1$ we get the reverse situation i.e. an increasingly higher bid the drier it gets, and an increasingly lower bid the wetter it gets. When $IDF = 0$ we get the base case situation, with non-state dependent bids, like those in earlier experiments. With this convention we can define $QREF^{t,h}$, in terms of $QBASE^{t,\bar{h}}$ and IDF , as follows:

$$QREF^{t,h}(IDF) = QBASE^{t,\bar{h}} + IDF \times (QBASE^{t,\bar{h}} - QBASE^{t,h}) \quad (8.11)$$

In our experiments, care has been taken when using Equation (8.11) to round $QREF^{t,h}$ values into matched pairs, above and below the mean hydrology state, so as to ensure the same average value, a in both correlated and anti-correlated cases (i.e. $\pm IDF$). For example, with our seven hydrology states ordered dry to wet i.e. $h = \{1, \dots, 7\}$, and the extreme inflow pattern in the base case, then we have the conditional inflows in stage 9 of $F^{9,h} = \{1, 3, 5, 8, 11, 13, 15\}$. So, in stage 3 (given the six month phasing), in the three corresponding IDF cases we would have $QBASE^{3,h}(IDF=-1) = \{1, 3, 5, 8, 11, 13, 15\}$, $QBASE^{3,h}(IDF=0) = \{8, 8, 8, 8, 8, 8, 8\}$, and $QBASE^{3,h}(IDF=1) = \{15, 13, 11, 8, 5, 3, 1\}$. Taking the latter case, $QBASE^{3,h}(IDF=1)$, and a constant elasticity of 0.5, we get $QREF^{3,h}(IDF = 1) = \{225, 169, 121, 64, 25, 9, 1\}$. The resulting seven $QREF^{3,h}$ curves are shown in Figure 8.30 below.

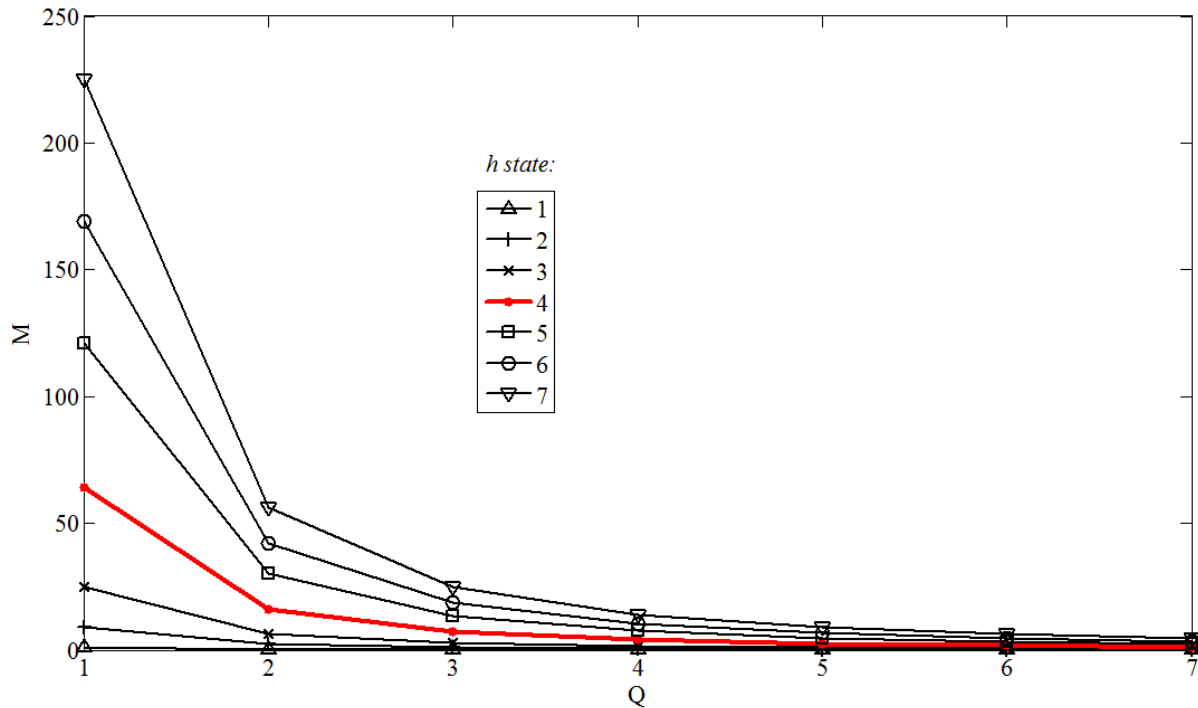


Figure 8.30: Example of a State-Dependent DCR with Demand and Inflow in Sync.

Figure 8.30 shows the seven conditional hydrology state DCRs, where the highest and lowest curves represent demand in the driest and wettest hydrology states, respectively. If $IDF = -1$ we have the same set of curves but the highest and lowest DCRs are in the wettest and driest hydrology states i.e. $QREF^{3,h}(IDF=-1) = \{1,9,25,64,121,169,225\}$. So, an IDF of 1 represents the best natural fit (however unlikely), in that high flow states occur when the demand for water is highest. An IDF of -1 represents the worst natural fit, where the DCRs are signalling that participants most need water when flows are low. Unfortunately this situation is much more likely. An IDF of 0 means the market is non-state dependent, as per all experiments we have run, prior to this point. In that case we have seven identical curves at a QREF of 64, i.e. the average inflow state, $h = \bar{h} = 4$, the red curve in the figure.

The area under each DCR is a conditional benefit which represents the average value that the market is willing to pay for water, in those given situations. Comparing the range of DCRs shown in the Figure with the base-case DCR (for $h=4$, or equivalently $IDF \neq 0$), we see that the increase in value in the higher demand states is clearly greater than the decrease in lower demand states, i.e. $E[M(Q)] \neq M^0(Q)$ as noted above. Thus, since we place equal weight on the upper and lower curves in each pair, there is more (probability weighted) area under the DCRs, in aggregate, when Equation (8.11) correlates demand with inflow.

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As discussed in Section 8.4.1, the key issue we hope to address in our experiments is how the level of optimisation complexity affects the value delivered by our system for a given physical storage capacity. And the higher DCRs represent opportunities to fulfil high value needs, in particular states. So, if the physical water system allows water to be stored for use in those states, an optimisation that recognises those conditional opportunities can create even more value from the water system, than would be possible with non-state dependent DCRs. With a different IDF, we get a different set of DCRs, and this means that we cannot readily compare benefits across cases. But note that the set of DCRs is actually just the same for each member of a +/-IDF pair. Thus the opportunities to maximise value, and the maximum value attainable (i.e. GUB as discussed in Section 5.5.4), are the same for +/-IDF, increasing as the absolute value of IDF increases, from 0 to 1. So, to compare cases, we can ask what proportion of the optimum allocation, the GUB, is delivered.

In our experiments we run our modelling system over a range of IDF and SCAP values. An IDF of 0 is the base case, and we experiment with non-zero IDF values of $\pm \{1.00, 0.75, 0.50, 0.25, 0.10\}$. SCAP is set as a factor of FMEAN. For these experiments we use values of $\{0.02, 1, 2, 3, 4, 12, 36\}$ times FMEAN. As in earlier experiments, we use the “extreme” inflow seasonality hydrology dataset, and constant elasticity DCRs with $E = -0.5$. We fix RCAP to 2 times FMEAN. Hence, we produce 77 case runs each, for DC and MC levels of optimisation run complexity. First we present the MC results, followed by the MC versus DC differences.

8.4.4 Results, Analysis, and Interpretation

8.4.4.1 Results for MC optimisation

In Figure 8.31 below we show the TWAB values. The IDF values are shown on the horizontal axis. The lines correspond to SCAP values where the key indicates SCAP as a multiple of FMEAN.

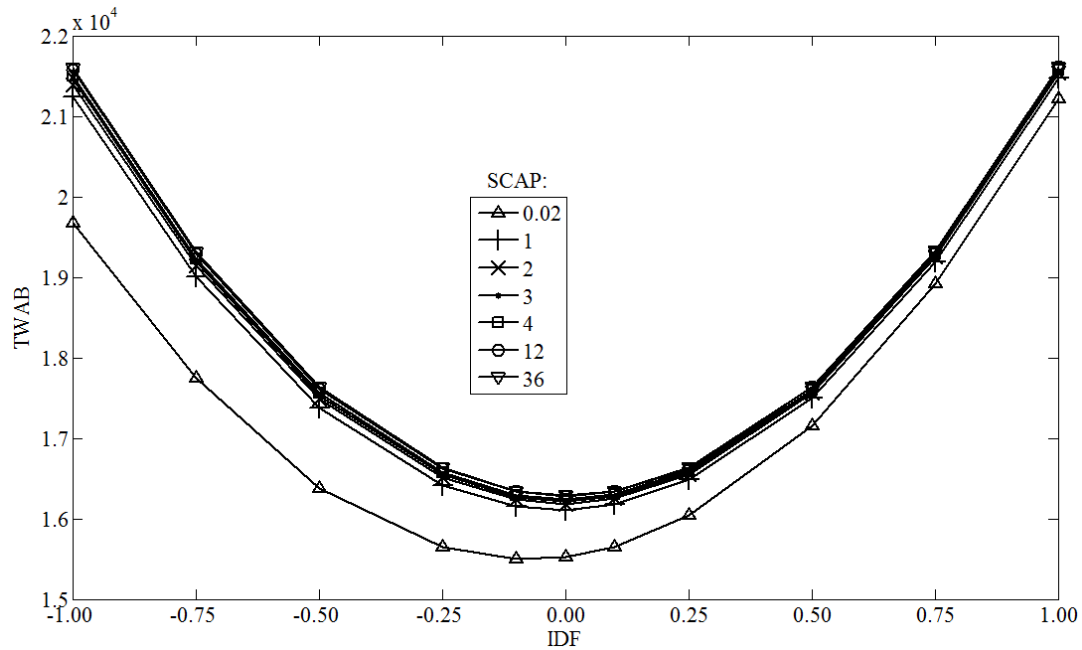


Figure 8.31: TWAB from MC optimisation, with state-dependent demand.

The highest benefits are in the $IDF = 1$ case, on the RHS of the figure, because demand is low when it is dry and demand is high when it is wet. Thus the system does not have to do much work to maximise MC benefits. In fact, if RCAP was not constraining, the ROR situation ($SCAP = 0$) would actually be the optimum solution, given that the optimal unconstrained price points of 1 have been constructed to correspond to the inflow points. Thus we would obtain the same benefit for all SCAP cases. In reality, with $RCAP = 2 \times FMEAN$, we cannot deliver inflows above that RCAP level to the market, and thus the unconstrained optimum (GUB) can not be attained, particularly when SCAP is very low.

As the IDF reduces from 1 the benefits also reduce, until the benefits generally reach their minimum level at an IDF of 0. An IDF of 0 is the base case result reported earlier in this chapter, and in Chapters 6 and 7. Benefits are lowest at this point because the non-state dependent constant elasticity DCRs offer the least opportunities, as discussed in Section 8.4.3 above.

If the IDF continues to fall below zero, the bids get gradually higher when it is drier and lower when it is wetter. This increasing anti-correlation with inflow increases the opportunities for value maximisation, as above, but only if the system has enough storage capacity to store water to be used in the driest hydrology states, when they occur. So with

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virtually no storage (in the $SCAP = 0.02 \times FMEAN$ case), and with an $IDF = -1$ then the MC optimisation is unable to generate the same level of benefits as in the $IDF = 1$ case, because SCAP is constraining water being moved from the low demand (wet) states in the period's when it arrives, to high demand (dry) states in later periods. But a moderate increase in SCAP allows the system to shift water across hydrology states to return benefits approximately on par with the corresponding $IDF = 1$ cases.

An alternative view of the annual TWAB is to view it as a proportion of the relevant GUB, as shown in Figure 8.32 below. The figure shows that even with a very small SCAP ($0.02 \times FMEAN$) then 91% of the optimum benefits are achievable, even when $IDF = -1$. Increasing to 98% with $IDF = +1$. Increasing SCAP, even to $FMEAN$, returns a minimum of 98% of GUB, in all IDF cases.

Figure 8.33 shows the corresponding MC VWAPR values. The lowest prices are in the $IDF = 1$ case, on the RHS of the figure. As discussed for the MC benefits immediately above, this is when demand is going with the flow, and bids are lower/higher when it is drier/wetter. Thus, not only are benefits maximised at this point, but we expect the lowest average MCPs, as shown. In the limit, if SCAP is high enough, these approach the ideal unconstrained price which, given the way we have constructed our DCR set, is still 1.

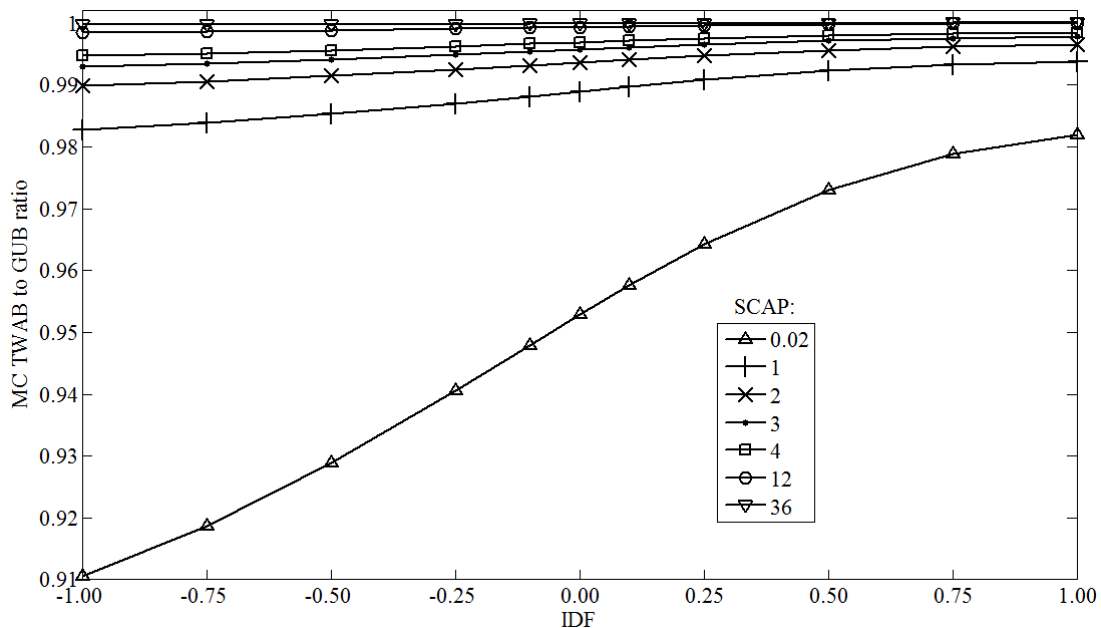


Figure 8.32: TWAB to GUB ratio with MC optimisation, with state-dependent demand.

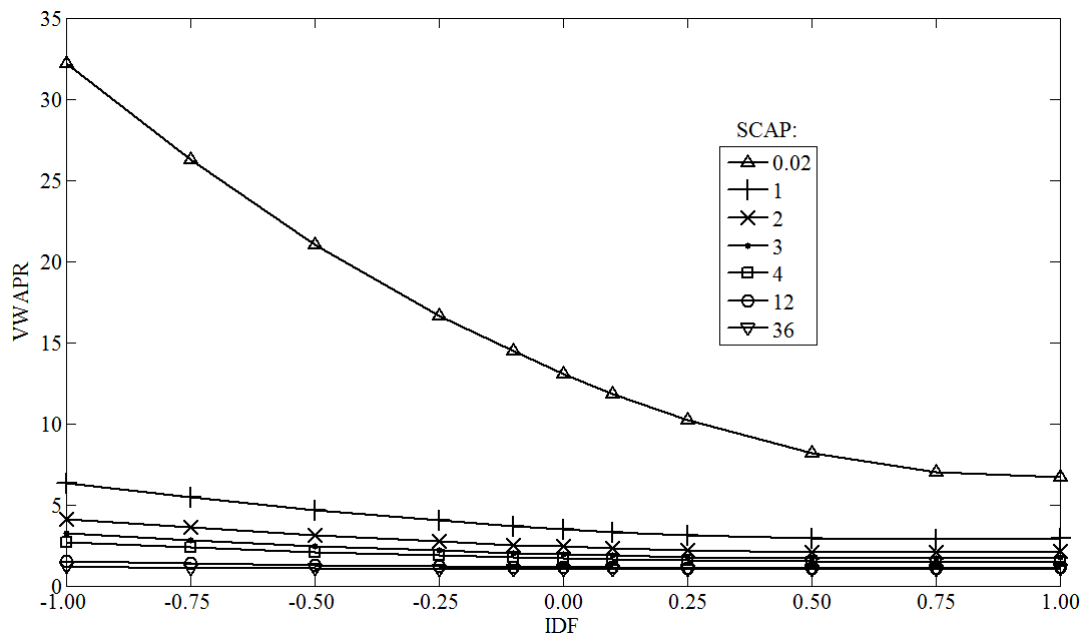


Figure 8.33: VWAPR from MC optimisation, with state-dependent demand.

Reducing the IDF from 1 to -1 results in demand going totally against the flow, and prices rise steadily to match that anti-correlation. The line showing the highest VWAPR for all IDFs is for the most constrained SCAP modelled here (0.02 times FMEAN). In this case the MC optimisation, with limited storage capacity, is beginning to struggle to equate prices over the water year, and between hydrology states, and thus to obtain the ideal unconstrained price of 1, on average. Increased SCAP allows the MC optimisation more flexibility to shift water around to meet the anti-correlated demands, and thus reduces the average MCP, towards the ideal unconstrained value of 1.

In Figure 8.34 we show the ANSD of TWAB for MC optimisation, which has a somewhat similar pattern to the absolute benefits in Figure 8.31 above. The benefit variability is highest with an IDF = 1, and somewhat less with an IDF = -1, particularly for the heavily constrained SCAP (0.02×FMEAN). Also, the benefit variability is generally minimised at an IDF of 0, or very close to it (IDF = -0.1, for SCAP = 0.02×FMEAN). But, unlike benefit, the benefit variability curve shows a sharp change around the minimum point of IDF = 0. This is because, as the absolute value of IDF increases, the DCR curves are moving further apart, and this creates more conditional opportunities, but also more underlying variability in benefit, irrespective of the optimisation. This effect, which is more or less linear in the absolute value of IDF, dominates that of the optimisation, thus making the curves virtually indistinguishable.

Still, the benefit variability is lower with a negative IDF than with a positive IDF. And, the equivalent DC results show a much more rounded ‘v’, around the IDF = 0 point, as shown by the red lines in the insert in the top centre of Figure 8.34. This is because the DC optimisation does a poorer job of matching variable inflows to the varying benefit opportunities. Away from the IDF=0 point, both the MC and DC results give a linear response to variability, and become indistinguishable.

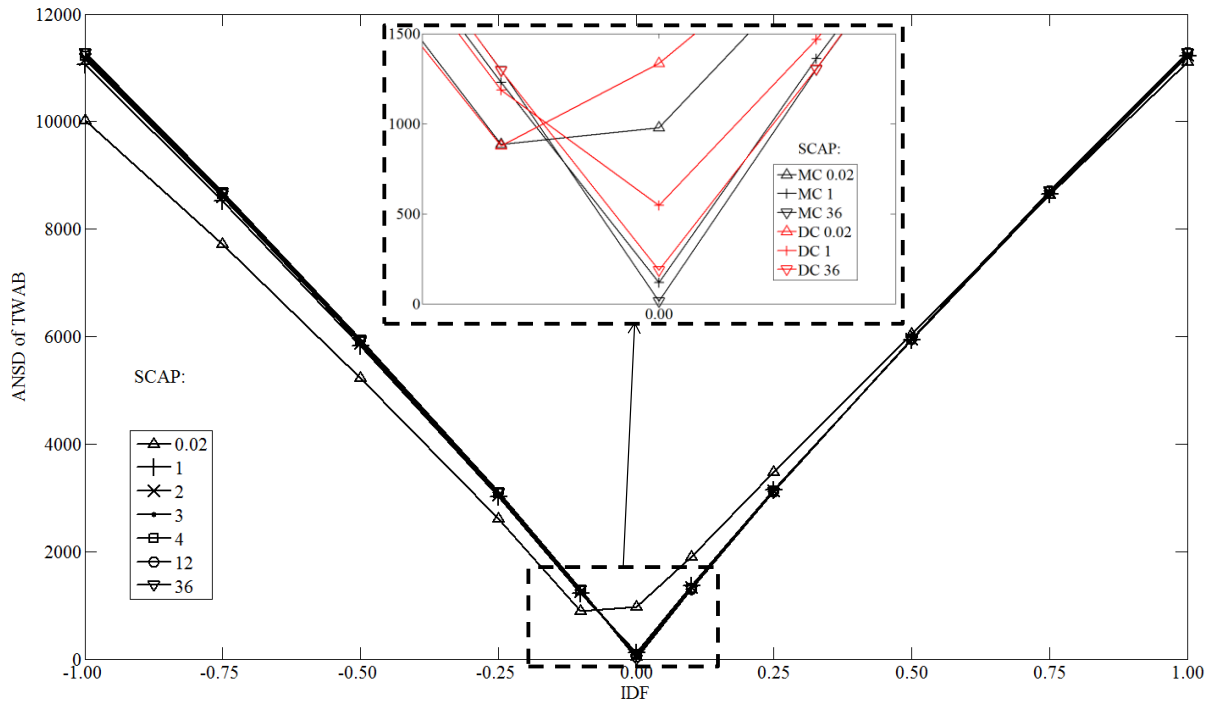


Figure 8.34: ANSD of TWAB from MC (and DC) optimisation, with state-dependent demand.

On a linear scale, the VWPSD of VWAPR, for MC optimisation, shows a very similar pattern to the VWAPR in Figure 8.33 above. For closer inspection of how price variability varies with high SCAP, in Figure 8.35 we plot the Positive Semi-deviation of MC global VWAPR using a log scale. We see that MC price variability is monotone decreasing with increasing SCAP, and generally also when moving from an IDF of -1 to 1. The slight rise in variability when SCAP is large, and IDF near +1, we thought was wholly due to the fact that we are reporting the (asymmetric) VWPSD of VWAP, not the (symmetric) standard deviation of TWAP, as discussed in Section 5.6.3.1. The global SD of TWAP is monotone decreasing in all cases except when $SCAP = 36 \times FMEAN$ and with an IDF going from 0.5 to 0.75 and from 0.75 to 1 (differences of -0.06 and -0.12). The Annual SD of TWAP is similar, only having an issue with the same SCAP when going from 0.75 to 1 (differences of -0.03). As of yet, we

are still unable to explain the upturn in SD, for a high SCAP when the situation moves into its ideal starting position i.e. going with the flow (IDF = 1).

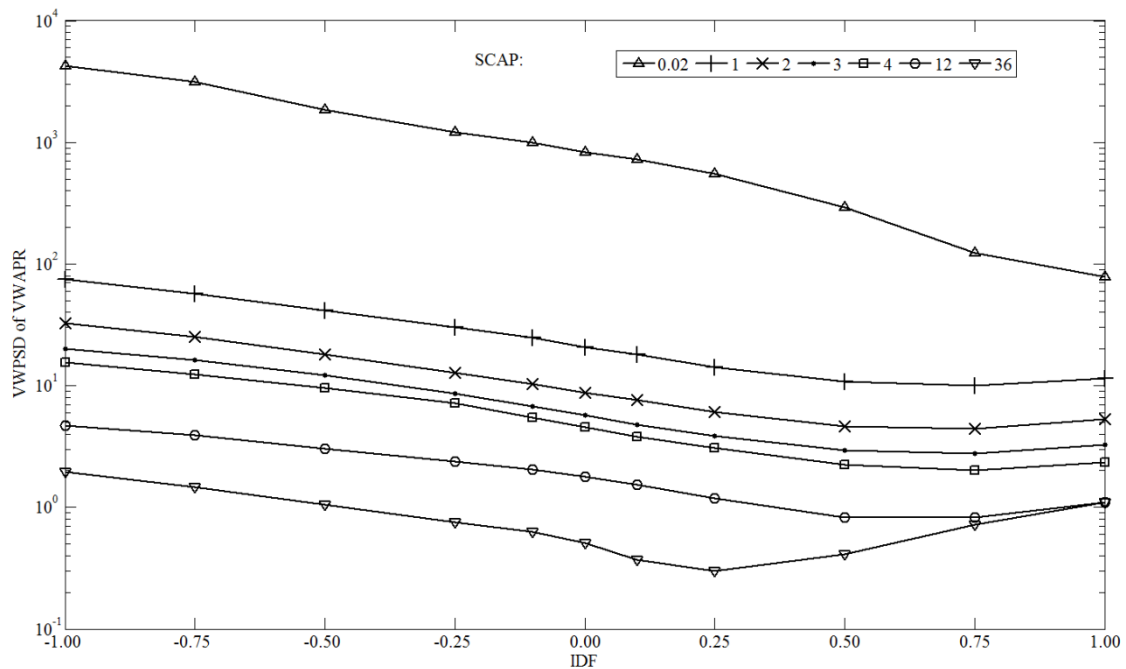


Figure 8.35: VWPSD of VWAPR with MC optimisation and state-dependent demand.

8.4.4.2 MC versus DC optimisation

In Figure 8.36 we show the ratio between the annual TWAB from MC optimisation in Figure 8.31, and the corresponding annual TWAB from DC optimisation. Basically, the MC optimisation consistently returns higher benefits than the DC optimisation, especially when the aggregate market demand is anti-correlated to inflow. The rising ratio implies that benefit differences are monotone increasing as IDF changes from -1 to +1. They are generally also monotone decreasing with increasing SCAP. The one exception occurs when SCAP is very low, and this reflects the fact that, as SCAP tends to zero, results from the MC and DC optimisations tend to the same ROR values, because neither optimisation can improve the situation.

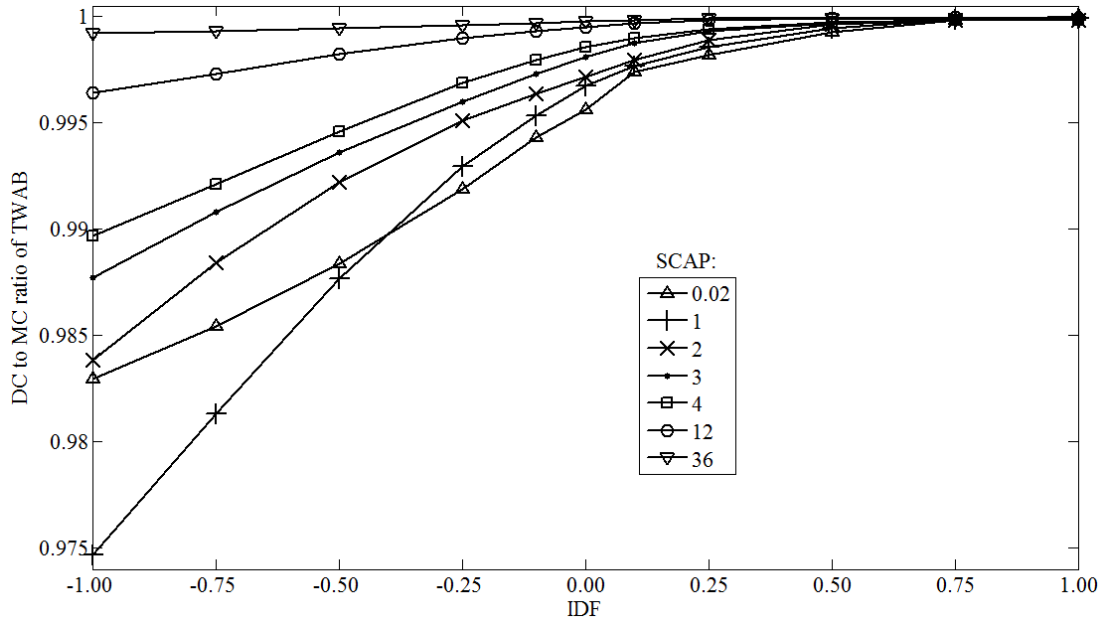


Figure 8.36: DC to MC ratio of TWAB, with state-dependent demand.

In Figure 8.37 we show the DC to MC ratio of VWAPR, which shows the reverse of the pattern for benefit ratios in Figure 8.36, in that the DC optimisation consistently produces higher prices (not lower benefits) than the MC optimisation. Price differences are generally monotone decreasing with IDF moving from -1 to 1, and monotone increasing with reduced SCAP. As discussed for the prior figure, with heavily constrained SCAP, the MC optimisation cannot improve as it otherwise would, hence the low SCAP lines cross over.

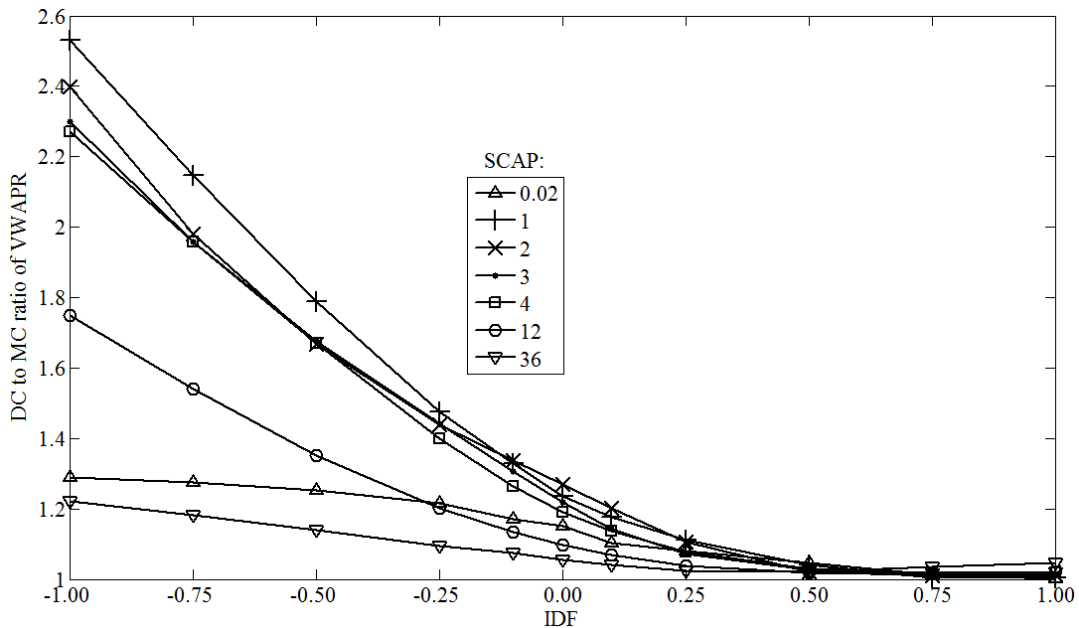


Figure 8.37: DC to MC ratio of VWAPR, with state-dependent demand.

In Figure 8.38 we show the DC to MC ratio of the ANSD of TWAB. The pattern shows very similar ratios for \pm IDF, except at $IDF = 0$. As discussed with respect to the ANSD of TWAB in the MC case shown in Figure 8.34 above, this is because irrespective of optimisation, the underlying variability grows linearly, as function of the absolute value of IDF. The insert of the DC curves in Figure 8.34 shows that the DC variability increases in the region near $IDF=0$. This is reflected in Figure 8.38, where the variability is much larger in the DC case, by a factor of over 10, for cases where storage is not particularly constraining.

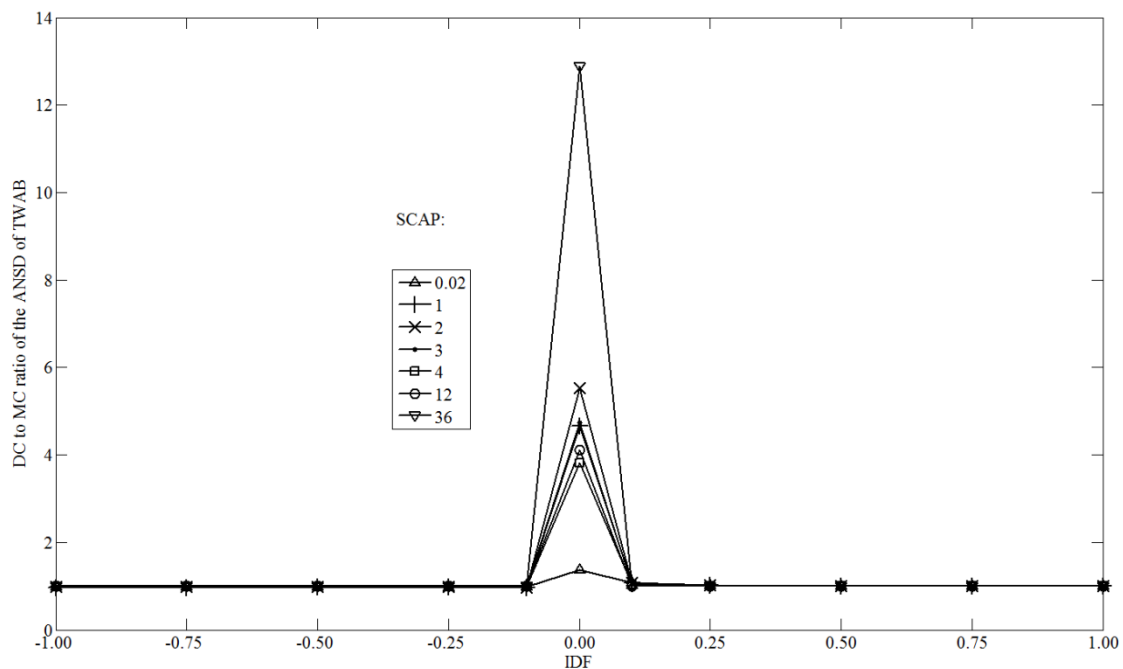


Figure 8.38: DC to MC ratio of the ANSD of TWAB, with state-dependent demand.

In Figure 8.39 we show the DC minus MC differences in the VWPSD of VWAPR. This figure shows a quite similar pattern to the VWAPR for MC optimisation in Figure 8.33 above. When inflow and demand are anti-correlated ($IDF = -1$), the DC optimisation manages the VWPSD of VWAPR much worse than the MC optimisation, with large differences. As correlation increase, relative performance of the DC optimisation improves, until, when inflow and demand are perfectly correlated ($IDF = 1$) the DC optimisation is managing the VWPSD of VWAPR as well as the MC optimisation.

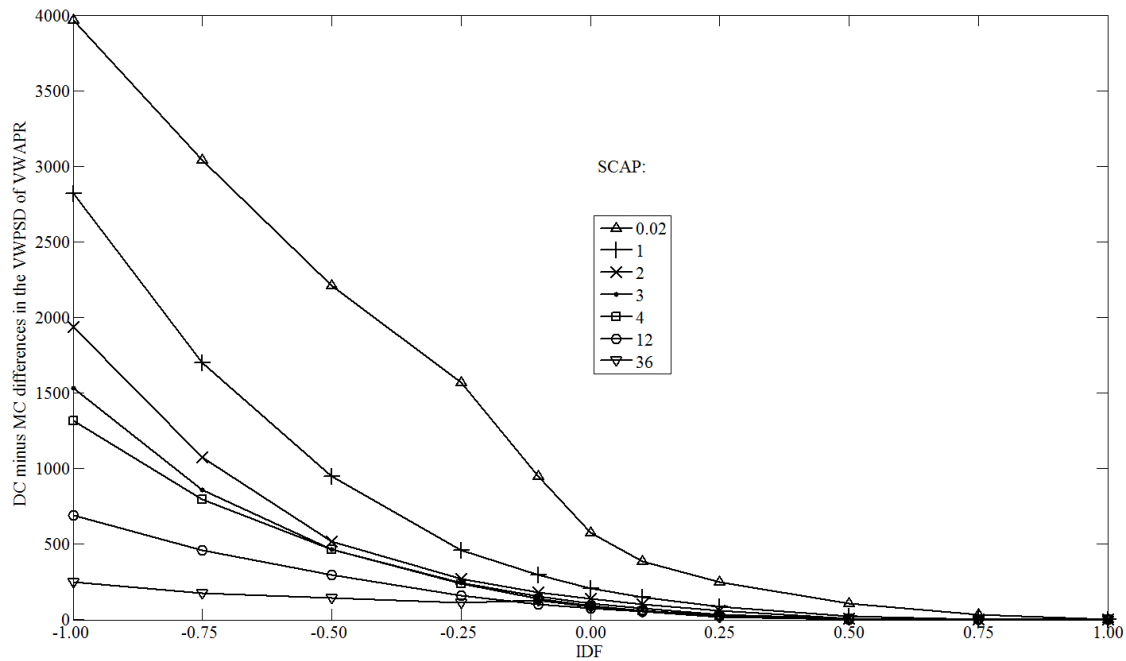


Figure 8.39: DC minus MC differences in the Global VWPSD of VWAPR, with state-dependent demand.

8.5 Conclusions

Our experiments in the previous chapter showed that accounting for stochasticity when clearing the market might make little difference in many cases, but can have a significant impact on benefit and price outcomes when inflow variation and volatility were significant and/or system storage/release capacity tight. But all of the results in that chapter assumed that demand could be described by a set of constant elasticity curves, all with a fairly moderate elasticity of -0.5 , and no correlation between demand and inflow states. In this chapter we undertook some limited experiments to explore the impact of varying the shape of the DCRs by varying the constant elasticity, inserting horizontal flats or vertical steps, and using state-dependent demand curves to model correlation with inflows. As expected, varying those economic parameters can have a very significant impact on results. Specifically:

- A lower constant elasticity (<0.5) greatly increases benefits, prices, (downside) benefit volatility, and (upside) price volatility. In that setting a stochastic optimisation can also greatly improve market outcomes, as measured by all four measures, compared to a deterministic optimisation.

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- Conversely, a higher constant elasticity (>0.5) reduces benefits, prices, (downside) benefit volatility, and (upside) price volatility. In that setting a stochastic optimisation makes little difference to market outcomes, compared to a deterministic optimisation.
- Inserting a flat into a DCR (e.g. representing an alternative supply of water) effectively raises the elasticity, and hence reduces the need to use a stochastic optimisation. In the limit, a long enough flat inserted near the optimum MCP (representing a cheap, abundant supply of water) completely removes the need for modelling uncertainty.
- Inserting a vertical step into a DCR (e.g. representing a group of market participants who are willing to pay significantly higher marginal prices, up to a volume limit) effectively lowers the elasticity, and hence significantly increases the value of using stochastic optimisation.
- If aggregate demand is negatively correlated with inflow, as seems likely, it becomes significantly more important to model the stochasticity, to maximise benefits and minimise market prices, particularly if storage capacity is limited.
- If aggregate demand is positively correlated with inflow, there is significantly less need to model stochasticity. At the extreme, our results confirm that, if the market demand exactly matched inflow, there would be no need to build any reservoir capacity, or optimise reservoir management.

Finally, in the course of these experiments, we have observed two somewhat counter-intuitive effects that were predicted by earlier discussions.

- First, anti-correlated demand can mean more benefit variability under MC optimisation than under DC optimisation, because there is more inherent variability in the market demand curves, and the MC optimisation still produces solutions which better match that underlying variability. See discussion in Section 5.7.
- Second, we see that MC optimisation does not always reduce (volume weighted) average prices, or price variability, particularly in situations where non-convexities are present in the market demand curves. See discussion in Section 5.6.3.

9. Conclusions

The major contribution of this thesis is the conceptual framework, which outlines various potential water market designs. The other main contribution is our improved understanding of the various physical and economic characteristics of water markets where stochastic optimisation can add the most value, and where such complexities are not required.

The market model classification system demarcates three dimensions of the market design: Uncertainty modelling, participant interaction and products traded. Our framework investigates the inter-play between complexity, precision, and simplicity over these three dimensions. Each dimension is characterised by an increasingly complex form. With market-clearing the complexity ranges from clearing for a single period only to a scenario tree (SLP) type model. Participants could interact with the market by providing bids ranging from spot bids to (fixed and proportional) bids dependent on the hydrological index. The bids can facilitate market trading ranging from firm spot releases only, to products which are adjusted based upon the hydrological index, to secondary market products. Alternatively, participants could rely on financial markets, to trade instruments outside of the centralised market.

In terms of the added value of stochastic optimisation, we found that modelling stochasticity within the water market can be very beneficial but does not always provide a significant benefit. The physical characteristics where the most difference was found were for a range of moderate storage capacities, centred on 25% of mean monthly inflow. Increasing storage capacity can improve market outcomes but for these cases modelling stochasticity within the market provided an even greater benefit. Such storage capacities are likely for systems with restricted storage capacity. The economic characteristics where accounting for stochasticity contributed the most are where the underlying demand curves are inelastic and where alternative supplies are not readily available. This is likely for inland regions with single resource systems, or in coastal regions where they do not have the ability to construct desalination plants.

These main contributions are supported by more thorough conclusions. We structure this discussion around the main structure of this thesis, concluding with some suggestions for future research.

At the start of this thesis we asserted that water markets could maximise benefits to society by connecting willing parties. So, in Chapter 2 we surveyed reservoir management models, and reviewed model-based (“smart”) markets trading similar commodities, using mathematical optimisation to clear an auction with side constraints. We observed a gap in the literature in the area of wholesale multi-use water market design, where participants trade water under uncertainty while also accounting for the inter-spatial and inter-temporal storage issues.

In Chapter 3 we used Stochastic Linear Programming (SLP) to outline a general formulation to optimise water allocation over time and space, under uncertainty. We noted many detailed multi-nodal water network modelling issues which could be considered, first in a deterministic framework. A primal and dual LP formulation which explicitly accounts for network restrictions, delays, etc., for a corresponding gas market paper is discussed by E. Read et al. (2012). That work featured natural gas compressors moving a lighter and compressible fluid, natural gas. So one development being pursued is the modelling of the transportation costs for water, and this is done by modelling the large friction losses, and net reservoir head effects, within the nodal continuity equations¹²⁹. But, rather than repeatedly solving such a complex formulation, we proposed a simplified single reservoir formulation that could more readily be used to create a modelling system in which extensive simulations could be performed to study the impact that using stochastic optimisation to clear the market might have on system performance under uncertainty.

In Chapter 4 we proposed a conceptual framework outlining various water market design options, assuming the single reservoir formulation. First, there were the virtual share of the reservoir system options proposed by Starkey et al. (2011), and these could be developed further. Indeed some similar ideas have already been explored by Barroso et al. (2012), who note a number of schemes have implemented in South America (Brazil, Argentina, and Uruguay), and in Canada, with proposals in New Zealand and Australia. Hughes et al. (2013) also discuss a range of such schemes already in operation in the Murray-Darling Basin in Australia. The lack of central co-ordination in these schemes has been noted by Barroso et al. as a potential issue. But they conclude that the resulting allocations are not deficient, merely novel, and that it is better to have a range of solutions to a problem, than no solution,

especially when central optimisation will not necessarily be accepted by incumbent parties. The scope for development in these options appears to be in enhancing the contractual mechanisms themselves, to allow the benefits of competition to be more readily realised.

But the main focus of our discussion in that chapter, and in this thesis, was on centrally co-ordinated market, in which an independent system operator co-ordinates a centralised market for trading participants. In this context we investigated the inter-play between complexity, precision, and simplicity while exploring the following three inter-related issues:

1. Modelling Stochasticity: How much uncertainty should be incorporated into the market-clearing model?
2. Participant Interaction: How should participants interact with the market, in presenting their bids, when the market is organised to account for various levels of uncertainty?
3. Trading Products: What type of products can be traded by participants who interact with the centrally co-ordinated, and potentially uncertain, market-clearing model?

With an SLP implementation, the modelling system can be as complex as desired, to take account of the key factors which affect the real system. At the same time, though, the modelling interface should be easy for participants to interact with, both in terms of submitting bids and in terms of understanding what products they are potentially buying. A key issue is that participants do not know what their future demand will actually be at each point in the future. Nor can the system manager predict exactly how much supply will actually be available at each future point in time. And supply and demand are often correlated, with demand typically being higher than normal when flows are lower than normal. Also, while we might imagine participants submitting discrete bids for every future scenario, this would quickly become impractical because participants would have to bid in a potentially infinite number of discrete future scenarios. In this situation the parameter bidding concept would allow participants to bid in proportion to an underlying index, which we think is an easy concept to understand and interact with.

We discussed a range of physical and financial products, and noted that, in principle, participants would be free to trade such products outside of the water market itself. This is common in electricity markets, for example, with the physical market typically only clearing on a spot basis. But, unlike electricity, water storage must be managed over time, and might also involve complex physical interactions across distribution networks, or natural systems,

such as the underground aquifers modelled by Raffensperger et al. (2009). So, water markets might need to operate more like FTR markets, which are always cleared using physical models of network flows in future periods.¹³⁰

Thus we suggested that traders might at least want to be informed by indicative future market clearing solutions, produced by an inter-temporal optimisation. In that context bids would be indicative, which may be an issue because, not being binding, participants might not take the trouble to bid realistically, or conversely might manipulate these “costless” bids for gaming purposes. Still that option deserves careful consideration, because it would allow financial markets to operate independently of the physical market, which has significant advantages from an organisational/institutional perspective.

But we focussed on markets in which physical or financial products would actually be traded, based on firm bids, and resulting in binding, contracts being formed. We discussed supplementing fixed contracts with a range of proportional contracts based on one, or more, underlying indices, and thus adjusted by the market manager (in the delivery period) to account for what is actually available. We suggested that such contracts could be allocated by fitting contracts of the required form to solutions produced by an optimisation that produced the market-clearing solutions for all nodes in a scenario tree. But option C in Section 4.7.3 involved a formulation in which proportional contracts were allocated endogenously within the market clearing process itself. Option D went further, by also modelling future trading around those allocations. This was expected to produce more realistic future price projections, reflecting the way in which participants would update their bids, as previously estimated future positions become clearer. These options are presently being developed by E. Read et al. (2015), using SLP formulations.

We consider that these options have significant potential for further research and development, as a way to allow participants to match their requirements to what can be delivered by a tightly constrained and geographically limited physical system, in an environment where both supply and demand are subject to significant uncertainty. But, before developing any of these options in more detail, we considered that it was important to

¹³⁰ In a multi-nodal context, there is also the potential to develop FTR-like products to help manage the risks involved in moving, or storing, water under uncertainty. But this development was ignored here because it is being pursued by Mahakalanda (2015 (In Progress)).

understand the potential impact that accounting for uncertainty in the optimisation could actually have. So Chapter 5 outlined the Stochastic Constructive Dual Dynamic Programming (SCDDP) based modelling system we developed to test various market design options for a single reservoir system. It can perform optimisation under lag-one Markov Chain (MC), Stochastic Independent (SI), and Deterministic (DC) assumptions, and simulate performance under lag-one Markov uncertainty. The key reporting measures were expected market benefits, expected market prices, downside volatility of market benefits, and upside volatility of market prices.

In Chapter 6 we showcased the computational modelling system, simulating the policies produced by the MC, SI and DC optimisations, in a world with MC inflow variability. We constructed our base case to include relatively extreme inflow variance and seasonality, with a high degree of persistence in wetter and drier than average states, and inflow and demand perfectly out of sync. But we did use a reasonably unconstraining storage capacity, and a moderate release capacity. We summarised outputs in terms of our market reporting measures, and found that with constant elasticity curves there was very little difference in benefits between DC and MC optimisation runs, but a significant difference in prices. As expected, the DC optimisation naturally allows the reservoir system to fall to its lower storage bound more often than it should, thus resulting in there being less spillage. Also as expected with constant elasticity curves, the downside benefit and upside price variability was much more significant than their respective upside benefit and downside price variability.

In Chapter 7 we explored physical system sensitivities by varying the reservoir capacities, and the inflow and demand distributions. Where there are capacity constraints, or significant inflow variations over time, or between hydrology states, it can be valuable to account for uncertainty. In that setting, optimising the system with a stochastic model can significantly improve market outcomes, compared to deterministic optimisation. The largest differences in absolute benefits (and benefit variability) were around 29% of the market value, for a reasonably small reservoir, with storage capacity of only approximately 25% of the monthly average inflow. Once the release capacity was at a moderate level, at least the monthly average inflow, it had very little further impact across all measures. Increasing storage capacity saw the absolute benefits rise moderately while absolute price, and price and benefit variability, reduced by much larger amounts. Naturally, though, the larger the reservoir, the

less need for stochastic optimisation. So we did find a wide range of storage and release capacities for which stochastic optimisation made very little difference to benefit outcomes, although prices were more sensitive. The same was true of systems where there was no significant inflow variance, and especially in systems that lacked inflow seasonality, or where market demand and inflows were perfectly in phase.

Analysis of the type reported in Appendix 7.1 provides another perspective on the significance of the benefits achievable by adopting a more sophisticated optimisation approach. The results of that single point analysis, using the Chapter 6 base case, shows that using MC optimisation we can achieve the same level of benefits as we could using DC optimisation, on a system with approximately twice the storage capacity. Investing in enhanced optimisation is likely to be much more cost effective than expanding physical reservoir storage and/or release capacity. And expanding the physical reservoir might not even be possible, if site conditions or environmental restrictions do not allow for an increase in storage and/or release capacity.

In Chapter 8 we explored various economic system sensitivities by varying the aggregate market demand curve shape. As expected, lowering the elasticity of constant elasticity demand curves) resulted in much higher welfare and market clearing prices, under any optimisation approach, but also much greater differences between stochastic and deterministic optimisations across all benefit, price and volatility measures, making it more important to model stochasticity in the market clearing. Raising the elasticity naturally had the opposite effect, but the impact on welfare, price and volatility was smaller, as were the differences between deterministic and stochastic optimisations. Inserting a horizontal flat, e.g. to represent a supply of manufactured water, effectively raises the elasticity. The wider the flat, and the closer the price to the market average, the less is the need to model the underlying uncertainty. Inserting a vertical step, e.g. to represent a group of market participants who are willing to pay significantly more for water up to a volume limit, effectively lowers the elasticity, and increases the need to model uncertainty. Inserting flats and steps, of reasonable size and near the market clearing price, each significantly reduces and increases the need for stochastic optimisation, respectively.

We also studied the impact of state-dependent demand. As expected, if inflow and demand are perfectly correlated, and/or storage release capacity is large enough, there is no need to

model uncertainty at all. But if inflow and demand are anti-correlated, and/or the reservoir/release capacities are constraining, then there is an increased need to model uncertainty. Perfectly correlated and anti-correlated state dependent demands significantly reduce and increase the need for stochastic optimisation, respectively. This is as we would expect. In analysing the results, though, some counter-intuitive results were noticed. It might be thought that maximising aggregate market benefits should also minimise the average market clearing price and minimise the variability of both prices and benefits. But some of our results confirmed the prediction, in Chapter 5, that both the time and volume weighted average prices could actually be higher, in a more efficient market, especially when the demand curves are non-convex, and uncertainty is significant. Similarly, when demand is variable, then benefit variability can also be a sign of optimality, because the optimisation is adjusting the solution to better match varying inflow and demand conditions.

From a market design perspective, the experiments in these two chapters confirmed that sophisticated market arrangements of the type discussed in Chapter 4 may well be justified in tightly constrained situations subject to high inflow variability. That conclusion may be partially justified on the basis of benefit maximisation but, from a participant perspective, it is also important to note that the level of optimisation run complexity had a much larger impact on price levels, and on price volatility, than it did on benefits. But these experiments also showed that there are probably a wide range of less constrained situations in which a deterministic approach to contracting, and market-clearing optimisation would suffice.

To put this into perspective, it should be recognised that, while it is obviously true that stochastic optimisation is not important if reservoirs are “large enough” to easily cope with natural inflow variations the creation of such large reservoirs can often be expensive, if not impossible. We do not contemplate, or recommend, the development of water markets in situations where the water management situation is easy, because demand is well matched to natural supply. Water markets are only likely to develop where the demand/supply balance is tight, and hard to manage. In that context, the critical question is really the trade-off between the incremental cost of market development and/or more sophisticated optimisation, and the cost of infrastructure development. In terms of our reporting measures, the proportion of total potential benefit achieved from storage can be employed to analyse this trade-off.

Also, the importance of modelling uncertainty has not yet been fully explored, because the assumptions made in our experimental setup have obviously impacted results. For example, we used constant elasticity curves which have a large underlying value from releasing the first few units of water, and we used symmetric hydrology distributions, so as to simplify the matching of market demand, at the reference price, to the average flows. Hence, there is wide scope for future development.

First, there is a much wider set of experiments that have been considered, and could be undertaken using our market modelling system, but which we have not had the time (or space) to report here. Obviously, potential gains from stochastic optimisation will be system specific we could use datasets for real systems, to analyse and calibrate modelling outcomes. For example:

- Using different types of demand curves, for example linear demand curves. The results in Chapter 8 indicate that using demand curves with less extreme benefits will reduce the gains from stochastic optimisation. So, a linear demand curve passing through the same reference point as a constant elasticity curve, with the same slope (and hence elasticity) at that point, would have a significantly smaller total benefit, and imply much lower prices for low supply levels. Thus we could expect results more like those for higher effective elasticity curves, as in Section 8.2. But real demand distributions are lumpy, and in Chapter 8 we have already conducted some limited experiments with inserting flats or steps. So we expect the effect would be similar to inserting some combination of steps and flats, which may make the demand curve more non-convex, and perhaps lead to higher volatility in prices and benefits. But the opposite may be true for many systems.
- Experimenting with different levels of serial correlation. Since our existing experiments assume a relatively high degree of serial correlation, the gains from MC vs SI optimisation will reduce at lower levels of correlation.
- Using asymmetric hydrology distributions, to study the impact of more extreme weather patterns. Our physical experiments assumed symmetrical distributions because our experimental design required this to easily keep the same inflow value, on average, while scaling discrete inflow distributions. But we expect that distributions skewed towards droughts will enhance the gains from stochastic optimisation. Distributions

skewed towards floods will reduce the gains from stochastic optimisation, in the present system where spill is not restricted or penalised in any way.

- Penalising floods and/or restricting spill could also be explored in our modelling system, though, although that would create the possibility of negative marginal water values. More generally both positive and negative valuation of environmental flows could be modelled if the CDDP sub-models discussed by Mahakalanda et al. (2012) were embedded in our SCDDP model, as discussed below. In this setting, the more complex stochastic optimisation will become important as a means of avoiding the bounds and taking a more conservative set of decisions, and hence a narrower set of storage trajectories.
- Finally, all of our experiments could be re-run, simulating different hydrology or demand distributions from those assumed in optimising the policies. Clearly MC optimisation will no longer be optimal, for the situation simulated, but its advantage over less complex optimisations may increase in some situations, and decrease in others. In cases with higher level correlations (e.g. due to el Niño effects) we will presumably see increased “stickiness” in wetter or drier than average states, and this will lead to higher price and benefit volatility.

A second set of experiments could be undertaken using our market modelling system, by varying more market design choices. For example:

- Adjusting the inflow allocation policy from informed to conservative, or perhaps some other intermediate (and potentially more realistic) policy. As discussed in Sections 5.4.5 and 6.2, we assume that, even under deterministic optimisation, the model “knows” the inflow state for the coming month, at the beginning of that month. So our SCDDP produces intra-period release decisions which depend on the state observed at the beginning of each period, while still assuming a deterministic model of future periods. We expect that adopting a conservative allocation policy, under both optimisation regimes, would reduce the gains from stochastic optimisation because even the stochastic optimisation is assumed not to know the present period’s inflow. A range of other, potentially more realistic, inflow allocation policies could also be tested. For example, a conservative decision could be made assuming the lowest likely inflow state, followed by a mid-period adjustment once the realised inflow state has been observed. Clearly, this should produce better decision-making than the “conservative”

approach, and more realistic decision-making than the “informed” approach. But similar improvements might also result from just using shorter decision periods.

- Adjusting the decision period length from monthly to, for example, weekly or quarterly. This is easily accommodated in our experimental design, and modelling system, and has significant implications for market design. We expect that a finer decision period will reduce the gains from stochastic optimisation because a deterministic optimisation can re-evaluate its decisions more frequently, and thus potentially adjust course before hitting a storage bound. Conversely, we expect that a coarser decision period will reduce the gains from stochastic optimisation because a deterministic optimisation will get less chance to adjust its decisions before hitting one storage bound or the other.
- Finally, the experiments started in Section 8.4 should be completed by modelling the impact of restricting participants to non-state dependent bids, in a world where supply/demand correlations actually exist. We can do this by ignoring real-world correlations in the optimisation, and then simulating that policy’s performance with actual demand correlation. Obviously, restricting participants to non-state dependent bids will make no difference when their demand actually is not state dependent. And ignoring state dependence will probably not matter much when demand is positively correlated with inflows, because the system will naturally deliver less extreme outcomes than had been expected in the optimisation. On the other hand, outcomes could be significantly impacted in the more likely case where demand actually is negatively correlated with inflows. This highlights the potential importance of further developing options under which participants can deal with this issue by submitting index-dependent bids and/or trading index-dependent contracts.

Finally, as well as experimenting on the existing modelling system, the modelling system itself could be developed further. One obvious direction of development is to explicitly model the position of individual agents by modelling, and tracking, individual bids. Our modelling system has actually been designed to allow this, although that facility was not employed in our experiments. And that leads on naturally to modelling the behaviour of individual agents, using the kind of Agent Based Methods discussed by Newberry (2012). Within that field, techniques such as reinforcement learning, as discussed by Roth & Erev (1995), or Q-learning, as discussed by J.-H. Lee & Labadie (2007) and Tellidou & Bakirtzis (2007), might be applied in conjunction with our CDDP/simulation modelling framework.

Such a modelling system would use our CDDP optimisation to clear the market using a trial set of bids, then simulate market performance as we have, but reporting the positions of each participant. Agent modules would then need to be developed to assess that performance, from a participant perspective, and adjust bids for the next iteration on that basis. Alternatively, a similar process could be employed using live “agents”, in an experimental economics setup of the type discussed by Dinar et al. (1998). A modelling system of either type could be used to study gaming/market power issues, risk aversion, etc.

Multi-reservoir SCDDP developments could also be considered, and such systems have already been implemented for electricity markets, as discussed by E. Read (1989) and Craddock et al. (1999), and E. Read & Hindsberger (2010). Simplified and generalised variants of this approach, suitable for integration with our modelling system, have recently been developed by R. Read (2012) and R. Read (2014).

Another worthwhile path of development is to take account of more intra-period detail. This development could be targeted to maximise computational efficiency, by pre-computing the intra-period Demand Curve for Release (DCR). This has already been explored for a single reservoir multi-nodal catchment involving consumptive and non-consumptive uses, by Mahakalanda et al. (2012). And this has been further extended by Mahakalanda et al. (2013) to accommodate parallel paths and mixing constraints in catchments with two long term storage reservoirs. To embed the results from such models into our SCDDP model, a set of (perhaps conditional) periodic DCRs for release from the long term storage reservoir(s) would be pre-computed, and then processed through the lag-one SCDDP modelling system, to determine the release policy for the reservoir.

Alternatively, a multi-nodal SLP model based on the formulation in Chapter 3 could be developed. Computationally, SCDDP is easily solved, for a single reservoir, and has the major advantage, for our experimental purposes, that it develops operating policies for the entire state space. But a market always starts from a known position, clearing the present spot market. Hence there is no need to generate optimum policies for all starting positions, only those which we could potentially end up in. Thus a real water market would most likely develop an SLP modelling system, because SLP is more versatile, as discussed in Section 2.3.8. In particular, the major advantage of SLP modelling is that it does not suffer from the

same “curse of dimensionality” as SDP, or SCDDP. Thus multi-reservoir problems can be readily optimised, even though MWV surfaces are not readily computable.

Also, at any node in the scenario tree, an SLP has an entire path (up to that point) defining the information available to make decisions. So the random variables at a given node can implicitly depend on all the outcomes of any random variables on that same path, up to that point. In Section 4.7 we concluded that it could be desirable to give participants choices of how to submit bids at the market interface, and to give participants a range of trading options. In particular, options C suggested creating conditional contractual allocations for future clearings, while option D also modelled trading around those allocations to further hedge demand and supply uncertainty. This might be beneficial to participants, as it gives them the facility to update their bids, as previously estimated future positions becomes more clear. These options are presently being developed by E. Read et al. (2015), using SLP formulations. Thus SLP can model interactions which might be useful in clearing real markets, and deal with bids that are conditional on a wide variety of factors apart from inflow. For example, an SLP market-clearing model could be developed which handled bids which correlated various hydrology factors (e.g. reservoir inflow with catchment rainfall, and/or temperature), or correlated demand with wider economic factors, or linked fixed and conditional bids for future products, as discussed in Section 4.7. Care should be taken, though, not to create a market that is too complex for participants to understand.

In conclusion, then, we consider that water markets of the type discussed here have potential to increase economic benefits, by allowing parties to trade water in a centrally coordinated fashion. In some cases it seems likely that such markets could operate effectively without explicitly modelling inflow uncertainty in the market-clearing processes. But in others where constraints are tight and/or there is a high degree of natural supply and/or demand uncertainty, stochastic optimisation, and conditional contracting could be important. Thus further work on these concepts seems justified, particularly in the area of SLP market design using proportional contracts to trade in a stochastic market, while also potentially allowing trading around those positions.

Appendix 2: Water Issues

This Appendix provides an overview of wider water issues that would need to be accounted for, if considering implementing a water market with price based mechanisms.

2.1 Scarcity

Consumptive fresh water use for human endeavours is roughly 70% agricultural, 20% industrial, and 10% domestic, but of course this varies between catchments and countries. Historically many water systems which supply urban and rural areas were sourced from plentiful easily harvestable rain fed catchments. As such, much investment in surface water reservoirs and ground water extraction systems was undertaken to secure long term supplies through intercepting and controlling point releases in the natural water cycle. But having exploited all the easy options there is growing global concern about the scarcity of clean water sources (Rosegrant et al. (2002), Smakhtin et al. (2004), and Oki & Kanae (2006)), which is generally attributed to climate change and population growth (Hardin (1968), Vorosmarty et al. (2000), and DSE (2008)). Significant changes in precipitation patterns, which feed the catchments, are occurring more frequently, as discussed by Howe et al. (2005). In many dry regions average precipitation has been drastically lower than the historical average, as noted by Dore (2005). As such, the long term sustainability of these natural water environments to purify and provide a continuous cyclic resource is in question. Dinar & Subramanian (1997) say that a common feature across countries is the decline in water availability index, defined as available water resources divided by population.

In response to growing water scarcity some countries began developing water legislation to tackle these problems. Smakhtin et al. (2004, p. 20) list Australia, New Zealand, South Africa and the UK as examples where reform is progressing. Institutional water reforms and global progress are also discussed by Saleth & Dinar (2005). They note that Chile began reform early in the 1980's but many issues subsequently slowed progress. Most reform has been seen in Chile, South Africa and Australia. They also discuss how this is a social-political process with short-term and long-term changes in perception.

Water management reform is being actively pursued by many of the so-called developed countries. The OECD (1999) discusses the benefits of using price mechanisms in response to

scarcity. They discuss how it was recognised at the UN Rio+5 1997a Summit, where article 34.e states (p. 33):

Consideration should be given to the gradual implementation of pricing policies that are geared towards cost recovery and the equitable and efficient allocation of water, including the promotion of water conservation.

Using pricing as a management mechanism means valuing water, and this is complex and highly political as now discussed.

2.2 Water Valuation

The OECD report quoted at the end of the last section also discusses human welfare, OECD (1999, p. 32):

Although recognising that water is a foundation of life itself, and should therefore be provided at least in the minimum quantities necessary to meet basic requirements, this Statement also recognises that individual users should not have the right to limitless quantities of cheap water. There remains a need to allocate water resources efficiently, and to run water services cost-effectively, while still taking account of the "social good" character of water. More specifically, Principle 4 of the Dublin Statement asserted that: water has an economic value in all its competing uses and should be recognised as an economic good.

Valuing and pricing water economically frequently raises social issues relating to people's basic water needs, and water's public good features. This is noted by Tsur (2009) who actually discusses economically valuing water, and concludes that the evolution of markets should at least address these questions, even if it means specifically excluding certain groups from that competitive (economic) system. For example, vulnerable individuals may source water sustenance needs via other means, accounted for in welfare payments, or as Graff Zivin & Zilberman (2002) discuss, subsidised bottled water.

Water's value can be based on both ecological and economic concepts. Farber et al. (2002) study extensive literature on these concepts and conclude that there is still much work to be done on these definitions. Economically, water has a monetary value. It often takes time, energy, and resources to harvest, treat, store, and transport clean water to users. Water which is used but not consumed is commonly termed sewage, or "waste water". For waste water, there are often costs to transport from dischargers, to treat and to dispose of the various resulting products, such as sludge and dirty water.

The environmental cost of removing the clean water and returning the waste water from/to their respective locations could also be valued. Within this there are other costs. Water and waste water treatment often produce by-products which are returned to the environment and can have a negative impact. For example, by-products could be the various chemicals used in water treatment or post desalination return brines. Also treatment, storage, and transport often involve exchanging energy into other forms. For example the energy costs involved with pumping water.

Often, the value associated with water, and the price users pay for that water, are well below total costs. The actual total cost is frequently discounted, to implicitly or explicitly exclude certain elements, such as the environment, Rogers et al. (2002). It is recognised that general costs exist and that users should pay "the cost". OECD (1999) formally adopted the User Pays Principle (UPP) in their Recommendation on Water Resource Management policies in 1989. They also suggest that the Polluter Pays Principle (PPP) should be encouraged, as specifically referenced in the 1992 Maastricht Treaty. At a conference in Paris in 1998 (p. 33) countries' head delegates committed themselves to national frameworks to deal with their countries' specific issues.¹³¹

A significant subset of OECD countries is within the EU. The EU parliament debated their water issues and produced a common text, the EU Water Framework Directive. Kaika (2003) describes how that process involved parties with very different views on how to value the water, given their own local social-political issues relative to their abundance or lack of clean/waste water. The political process became a negotiation (p. 310), "in order to produce the joint text, both sides made significant compromises". The final wording in the directive

¹³¹ Attendees also recognised this was only possible through the mobilisation of financial resources from both the public and private sectors to enhance the effective use of available resources.

(p. 311) states “full cost recovery should be ‘taken into account’ (European Parliament, 2001b)”. This was followed by the next statement: “member states can opt out of environmental cost recovery”. Unnerstall (2007) states that, from 2010 onwards, full cost recovery of water services within the EU is expected with binding force, as opposed to the prior ‘taken into account’ view.

Within this EU mandated system, the UK is one example where that further subset of countries (England, Wales, Scotland and Northern Ireland) had to prioritise their water issues. Bakker (2005) reviews the progress of UK water privatization, noting that a key driver was gaining investment to meet EU environmental standards, (p. 550):

Some of the great gains in human welfare during the twentieth century associated with the state hydraulic paradigm were made at the expense of the environment with the state temporarily devolving costs onto the environment in what might be termed an ecological fix (Bakker 2004).

In summary, the valuation of water relative to scarcity and the knock on effects of the cost of both producing clean water, and disposing of waste water, depends on resolving a range of focused local issues, addressed at varying political and social levels, in some prioritised rationale.

The human welfare issue at the start of this section is also important, but as discussed in Section 2.1.1, there are various case studies where water markets have been shown to add value. For example, in Chile, where over a wide demographic, both urban and rural access to water has improved significantly, while also redistributing wealth, and reducing poverty, Schleyer (1996). Prior to this, we discuss more general water costing and pricing experiences.

2.3 Water Costing and Pricing Experiences

There can be no doubting that water is an economic good, Rogers et al. (2002), can be valued as such, Young (2005), and is a sub-commodity of producing many other commodities, Hoekstra & Chapagain (2007).

Historically, there has been a huge disparity between the cost of water resources and the prices users pay for access to those resources, and for the systems used to deliver water services. Rogers et al. (2002) state that originally private water companies thrived throughout Europe and the US until the 19th century “sanitation revolution”. Social political movements demanded public ownership and management of water companies in the name of public health, both in the US, the UK, and elsewhere. With the exception of France, until the late 1980s water was not treated as an economic good, but as a public-good and this led to heavily subsidized public systems while, in most places in the world, utilities provided water almost free of charge. There are exceptions, Dinar & Subramanian (1997) mention that, prior to World War II, farmers in India and Pakistan fully paid for their water supplies.

Price subsidisation often historically meant that governments covered the costs, by obtaining the money from other indirect revenue sources. But water shortages, demand growth, supply reinforcement costs, environmental protection costs, and government debt from borrowing has led to policy changes which are focused on adopting more direct cost and pricing incentives to water users. International water pricing experiences are discussed by Dinar & Subramanian (1997). They note how budget deficits have led to a reduction in subsidisation in countries such as Spain, the United States and Australia.

In the UK prior to 1974 the state directly subsidized water bills. Bakker (2005, p. 548) quotes her own earlier work on this topic: “Water pricing was based on a concept of social equity: household supply was not metered, and bills were linked to property value, supported through cross-subsidies between consumers and, in some instances, between regions and level of governments (Bakker 2001).”. Reform began in the late 1970s and this led to a policy switch from national equalization to regional efficiency maximization. Water charges were de-coupled from rateable property values when public sector water utility companies were instructed to earn rates-of-return similar to those made in the private sector. This policy switch eventually led to full privatization in the UK. Counsell (2003) discusses the issue of ownership of water services by territorial authorities, and how this leads to problems with accountability and commercial performance.

UK privatisation was driven by a large national debt, a highly inefficient water industry, and major issues with water pollution. The latter was required by the EU to meet water pollution legislation. Hence the UK prioritised water quality issues, predominantly wastewater

discharge into water courses. The cost to put this right was over 30 billion GBP over ten years (1990-2000) Bakker (2005). Those costs were increasingly passed on to consumers. Price increases were so high, especially for the limited number of parties on meters, that this led the UK regulator, OFWAT, to equalize rates between metered and non-metered households. Five years after privatisation, price caps were introduced by the government to politically and socially make privatisation look successful, Saal & Parker (2001).

The UK also has regional scarcity issues, Dinar & Subramanian (1997). At privatisation, the UK government hoped the quantity issue would also be addressed by installing universal meters by the year 2000. But, Bakker (2005) says that less than 20% were metered by 2000, and by 2005 the UK still did not have meters installed for most water users. Severe water stress (droughts) in the South East of England in 2007 finally led to a change in the Water Resources Management Plan which paved the way for volumetric (metering) charges in sustained drought areas, Division (2007).¹³² That region is having compulsory meters fitted. But, even with meters and volumetric pricing, UK water prices are still heavily regulated.

Water costing and pricing issues can evolve nationally and/or locally. Dinar & Subramanian (1997) discuss a Water Pricing Progress Index which measured how far along countries were with reforms to price water more in line with costs. The index consists of three main measures Gross National Product, Budget Deficit, and Water Availability. Countries with a high Gross National Product and a high Budget Deficit had made significant progress. Countries which appear to have abundant Water Availability on a country level, but showed serious regional shortages, had also made progress. Australia and New Zealand are given as two such examples, and they state that (p. 10): “perhaps pricing reforms began at a regional level and were later scaled up”. But there are often many contributing factors.

Those two countries, Dinar & Subramanian (1997, p. 9) “Australia and New Zealand, which introduced water charging systems only in the early 1990s, has (sic.) adopted some quite radical and advanced policies”, to suit their local issues. Counsell (2003) discussed the 1994 water crisis in Auckland, New Zealand, leading to all users being metered, and charged on a

¹³² For example, in the UK the water industry runs on a 5 year cycle termed the Asset Management Plan (AMP). Each water company submits their projected 5 year investment plan to the water regulator. These plans detail investment and supply security, as well as compliance with any UK and wider EU legislation. OFWAT review these submissions and fix water and wastewater prices for the AMP while trying to strike a balance with keeping bills low, securing supplier and managing the impacts on climate change.

fixed and variable tariff structure. Counsell also states meters are widespread in Australia. Presently there are five regions in New Zealand which charge users on a metered basis Powley & Bilby (2013).

Wider international pricing practices and trends are also discussed by Dinar & Subramanian (1997), but they also point out exceptions to the rule, highlighting countries as examples when they go against the norm. They quotes results from two studies for water utilities services in major US cities where residential water prices increased nearly 10% above inflation between 1988 and 1994. About 37% of those utilities charged a uniform price regardless of quantity consumed, 22% and 38% use rising and declining block rates, respectively; and the other 3% a mixture of schemes. Dinar & Subramanian note that US water utility services were becoming financially self-sufficient and funding operations with customer charges.

From these examples, it can be seen that user water prices have evolved to recover some of the costs through the price users pay. But this price is a social and political “hot potato”, generally everywhere. Prices are often set through consultation with regulators, and remain fixed for a certain period of time. The price should account for an “acceptable portion of the cost” of water: treatment, storage and transportation; and, for wastewater: collection, treatment and disposal. Having looked at historical aspects of costing and pricing we now briefly review pricing structures.

OECD (1999) state in their Recommendation on Water Resource Management policies, that the resource price should cover the opportunity costs of water services including: operation, maintenance, environmental and capital costs. With regard to capital costs, Dinar & Subramanian (1997) noted that pricing of the capital cost component was starting to gain momentum but that, at that time, only a few countries attempted to recover capital costs, with only France pricing capital development costs and a proportion being recovered in Brazil and Australia. Unnerstall (2007) discusses recovery costs with respect to environmental and water resource needs and services. But cost assessments can include consideration of sunk costs, rates of return, depreciation, re-investment, compliance with legislation, and wider operation and maintenance costs.

Appendix 2: Water Issues

Some costs are fixed per unit but often costs vary as more units are demanded. Water prices are rising all over the world Clark (2007). Generally, for urban water supplies, flat fees are replaced with a two-part tariff structure with fixed and variable (volumetric) parts, as discussed by Dinar & Subramanian (1997) and OECD (1999). The volumetric component is typically at least three-quarters of the total water bill. The fixed charge gives the service provider a reliable income stream. Often the first few cubic meters (“lifeline”) are free, or priced well below the cost of recovery, for social and political reasons. Further, there is a (p 2) “shift toward the reduction (or even abolition) of large minimum free allowances. For example, Australia and South Korea have both recently made significant strides in this direction.”. For the variable component, most industrial countries had increasing block rate schemes. On the other hand, agricultural water prices are generally developed by averaging the service cost across the area irrigated, perhaps adjusted for season or, for example, for gravity versus pumping.

Dinar & Saleth (2005) discuss the price of water having two roles, the first is financial to recover costs, the second is economic to signal the scarcity value and opportunity cost. Some water companies are adding both these dimensions into their pricing, such as South East Water, in Melbourne, Australia, Water (2012) in their 2013 to 2018 water plan. Consumers often pay monthly or quarterly water bills, but commonly see prices rise annually. This is also generally the case with urban consumers of electricity and natural gas. Electricity and natural gas can be traded multiple times within a day at a wholesale level. But these “near real-time” prices are not passed onto urban consumers. Consumer prices are averaged into their annual contract with specified usage tariffs. So consumers are generally protected from these price fluctuations. An extreme example is the UK water industry where given price caps water prices are fixed for five years, Lobina & Hall (2001). Saal & Parker (2001) discuss the negative impact of the price cap on UK industry performance.

There are numerous concerns when politics becomes involved in setting prices, as highlighted by Dinar & Saleth (2005). Olmstead & Stavins (2007) discuss price versus non-price (for example, hose pipe bans) water conservation programs. They found that only mandatory, well enforced, non-price programs tend to work. Based on hundreds of studies since the 1960s, in the US a 10% marginal rise in price saw urban demand diminish between 3 to 4 %. So while periodic price setting can send direct signals to consumers, the need for certainty and the formation of medium, to long-term contracts usually limit the effects that

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shorter term shocks due to higher (flood) or lower (drought) than average inflow years. This means that in increasingly negative situations developing in the period before the next regulatory price setting period, the risk is borne by the seller of the contract.

As populations grow and needs expand generally so must the water systems to meet those demands. Developing water systems to meet growing demand incurs escalating costs which must be financed, accounted for, and recovered somehow. So water is valued, and in many places that have security of supply issues, their water management policies have developed to tackle their local problems. When system congestion constraints bind, shortage occurs, and/or rationing becomes a local issue, then meters can be employed to measure takes, and prices used to incentivise users to conserve and adequately value their water takes. The price of water is dynamic, varying depending on the relativities of supply and demand, at any given time and place. Isolated parties can be cross-connected with adjacent parties to trade between one another, at times when there is sufficient supply to demand mismatch between both entities.

Marginal cost pricing has been cited as an efficient mechanism for water allocation by Dinar & Subramanian (1997), with a spot market alleviating disadvantages of public misallocation. Easter et al. (1998, p. 127)) note various authors advocate options and spot prices for water. Water challenges and reform progress in a few selected countries is also discussed by Saleth & Dinar (2005). Although there seems little progress from there earlier work, Saleth & Dinar (1999), where there they noted how some water markets had started to develop, as discussed in Section 2.1.1.

Appendix 3: Modelling Penalties

This Appendix provides an overview of penalties that could be used in the formulations in Chapter 3.

In real-world implementations, pricing penalties causes issues with revenue neutrality, and this can cause problems for markets, which is the focus of this thesis. We avoid these issues in our simplified market design experiments. Our experiments assume no soft bounds, or associated penalties. We choose not to potentially force water onto consumers who do not want it, and we do not model negative prices, which are theoretically possible, in certain situations. But, in this appendix we briefly discuss the modelling of socio-environmental values, and discuss how they could be accommodated in a market framework, and the optimisation.

In an optimisation framework, water will only be released if the value of releasing water is greater than the expected value of storing it. Generally water prices will rise/fall near empty/full to signal storing/releasing more water, accordingly. In a market based setting, the market could look to participants to help absorb issues, up to a point. For example, decreasing/increasing demand during a potential drought/flood with much higher/lower than average prices. In reality natural disasters occur in many places. In those instances the market probably ceases trading.

Constraints could be implemented in the LP, such that when they bind, the additional revenue collected by the market operator (if markets are used) could go toward averting the bad outcomes, by penalises ‘responsible’ market participants, and compensating affected parties.

Penalties could also be designed into the pricing system to affect storage and release decisions, to account for environmental and/or recreation limits: For example, requiring that there is enough water to ensure sustainability of the associated aquatic ecosystems in reservoirs, lakes, rivers, etc. A real-life example is the need for environmental flows in the Murray-Darling Basin, in Australia. Violating such bounds could incur significant cost, especially if the eco-system collapses. Such requirements would have to be expressed as “uses”, and given a high “economic” value, in order to ensure that they are always met. Non-essential, but desirable, “uses” could be valued at a level designed to achieve a trade-off

Appendix 3: Modelling Penalties

between higher downstream flows and economic uses. The actual values would depend on the local situation, including any affected parties, economic conditions, and participants' willingness to pay.

Many penalties could be thought of as "soft limits". Multiple soft limits can be thought of as buffer zones, with increased penalty costs for passing each limit.¹³³ For example, a reservoir may have upper and lower soft limits, and these could vary over time.¹³⁴ The lower and upper bound requirements could change for fish breeding and flood control purposes, respectively, at different times of the year. Pipes and rivers can also have minimum and maximum flow requirements.

Storage and flow variables are explicit decisions, and each could have a storage and flow violation variable, where each marginal breach could carry some associated marginal price penalties, respectively. These are functions of their corresponding violation variables, respectively. Including such terms in the objective would reduce the total benefit from trade.¹³⁵

Our stochastic models use a scenario tree. If soft limits were implemented we could find ourselves in extreme event situations, where we still need to violate the soft limits.

If the system is interconnected then the aggregate effects may activate various penalties. Penalties should be tailored to target actions which directly cause system violations. Including penalty prices in the optimisation will actively price the impact of hitting those designed violation constraints. This happens because the continuity equation links and balances all water units in the reservoir system. Hence, if implemented, bounded marginal penalty values will reflect back through the pricing system.

¹³³ Similar to engineering manufacturing systems when products on the production line start to breach an upper or lower confidence limit

¹³⁴ To ensure feasible recourse reservoirs often also have induced bounds, in the form of upper and/or lower buffer zones. Although we do not explicitly allow for buffer zones, the CDDP algorithm determines these buffers endogenously.

¹³⁵ $s^t + w^t \geq W^{-t}$; $s^t - w^t \leq W^{+t}$; $q^t - v^t \leq V^{+t}$, and $q^t + v^t \geq V^{-t}$. $WPEN^t(w^t)$ and $VPEN^t(v^t)$.

Appendix 5: Convolution Example

This Appendix provides an example for the convolution described in Section 5.4.6.

For an MC run level of uncertainty modelling, with an informed inflow allocation policy the inflows are observed first, and release decisions are taken second. This is accounted for in the process as illustrated in Figure 5A.1 below, for a case with SCAP = 10, FMAX= 5 and RCAP = 4. Here the LHS represents PDFS at time $t-1$, and the RHS shows PDFS at time t . PDFS values are stored for all storage levels (from 0 to SCAP).

The process creates a Conditional Probability Distribution Function for Water (CPDFW) which signals water availability during the week, from $s=0$ up to SCAP + FMAX. Assuming the informed policy, all of these water units (current storage level + net inflow) are available for both storage and release decisions. Since the maximum intra-period release utilisation is RCAP, water will be spilled if $W > SCAP + RCAP$. Otherwise release will be as determined by RO.

In Figure 5A.1 below we indicate the same example from Figures 5.3 to 5.6, so hydrology states go from wettest to driest, from top to bottom. The transitions from PDFS ^{$t-1, h'$} ($s^{t-1, h'}$) to PDFS ^{t, h} ($s^{t, h}$) are shown from left to right, as follows:

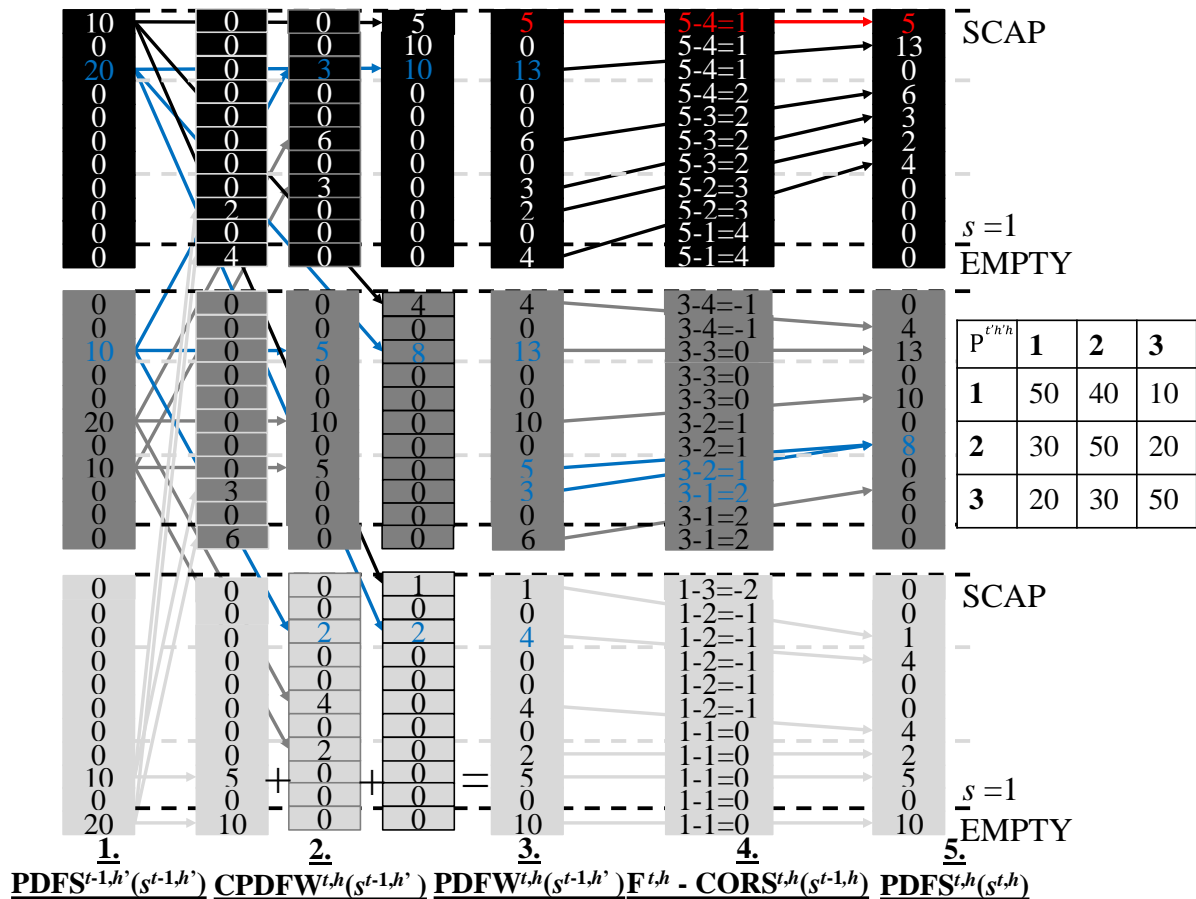
1. Shows the end of last period, start of this period, starting conditional probabilities. In this example the wettest/intermediate/driest states are set with total conditional probabilities of 30/40/30.¹³⁶ The wettest/driest states show a 10/20 percent chance of starting full/empty respectively.
2. Applying equation 5.11: Shows how, for each hydrology state at the start of the week, we weight each PDFS ^{$t-1, h'$} ($s^{t-1, h'}$) vector by the appropriate inter-period Markov transition probability $P^{t, h', h}$ to form the inter-period Conditional PDFs for Water, all nine CPDFW vectors are in the Figure.
3. Is where the same hydrology state CPDFWs are summed together to produce a PDFW for water for each hydrology state.
4. Applying equation 5.12: Shows the net transition from a given conditional starting storage level, at the start of the period, to the corresponding conditional ending storage

¹³⁶ Which are not the long-run, steady-state, values for the probability matrix shown in Figure 5.7.

Appendix 5: Convolution Example

level, at the end of the period. Hence it is the net change in the corresponding conditional F and CORS terms. The resulting net transition points to the end of period final storage level.

- Shows how, for each hydrology state, the inter-period CPDFW vectors individual probability weighted cells move up / down the storage state to form the end of period PDFS^{t,h}(s^t) vectors.



**Figure 5A.1: Illustration of PDF for Storage Construction for a Markov Chain
Uncertainty Modelling Run with an Informed Inflow Allocation Policy**

In Figure 5A.1 above the red arrow and text indicates that in the wettest hydrology state there is a probability of starting full and then experiencing a net change in level that would produce a spill. But in the PDFS all spill probabilities are aggregated to the SCAP conditional storage state. We class spill as a function of release, not storage. The blue arrows and text indicates where probabilities from different initial conditional storage states (PDFS^{t-1, h'}(s^{t-1,h'})) aggregate. Probabilities recombine (aggregate) from the 1 to 2 transition because there are

Appendix 5: Convolution Example

conditional probabilities which are located at the same storage level, at the start of the period. Thus the intra-period bins for each resulting intra-period CPFW (at 3) hold aggregated probability values. Note that their sum, $13+13+4=30$, is equal to their starting totals, $20+10=30$.¹³⁷ Probabilities recombine (aggregate) from the 3 to 4 transition because there are conditional probabilities in a given hydrology state, that given their associated net changes in level, point to the same storage level at the end of the period.

¹³⁷ This is always the case; else the model would be losing probability.

Appendix 6: Base Case Hydrology Data

This Appendix provides the explicit inflow and probability data used throughout the majority of Chapters 6 through 8.

Table 6A.1 below gives the monthly probability data, which is the same for all months, for all seven hydrology states and for all optimisation run complexities, DC, SI, and MC.

		Hydrology State (End of Period)						
		1	2	3	4	5	6	7
Hydrology State (Start of Period)	MC 1	0.5000	0.1500	0.1250	0.1000	0.0750	0.0500	0.0000
	MC 2	0.1250	0.5000	0.1250	0.1000	0.0750	0.0500	0.0250
	MC 3	0.0750	0.1250	0.5000	0.1250	0.0750	0.0650	0.0350
	MC 4	0.0500	0.0750	0.1250	0.5000	0.1250	0.0750	0.0500
	MC 5	0.0350	0.0650	0.0750	0.1250	0.5000	0.1250	0.0750
	MC 6	0.0250	0.0500	0.0750	0.1000	0.1250	0.5000	0.1250
	MC 7	0.0000	0.0500	0.0750	0.1000	0.1250	0.1500	0.5000
	SI 1-7	0.0981	0.1443	0.1672	0.1806	0.1672	0.1443	0.0981
	DC 1-7	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000

Table 6A.1: Probability Data for Base Case

Table 6A.2 below gives the monthly inflow data for all seven hydrology states.

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	9	11	11	11	9	6	3	1	1	1	3	6
	2	44	54	57	54	44	30	16	6	3	6	16	30
	3	81	100	107	100	81	56	31	12	5	12	31	56
	4	122	150	160	150	122	84	46	18	8	18	46	84
	5	163	200	213	200	163	112	61	24	11	24	61	112
	6	200	246	263	246	200	138	76	30	13	30	76	138
	7	235	289	309	289	235	162	89	35	15	35	89	162

Table 6A.2: Inflow Data for Base Case

Appendix 7.1: Gains from Optimisation vs Capacity Investment

This Appendix provides a short example of how the reporting measures discussed in Section 5.5, and the benefit tables reported in Chapter 7, can be re-processed to compare investing in either improved optimisation or increased physical storage and/or release capacity.

Starting from a base case in which Deterministic (DC) optimisation is applied to a given system. This analysis compares how the benefits increase as either: The storage and/or release capacities are increased, while still using DC optimisation; or the optimisation is enhanced to a Markov Chain (MC) level. This type of analysis gives decision-makers a tool to assess the relative economic merits of investing in improved optimisation software or investing in storage and/or release capacity.

As discussed in Section 5.5.8, for a given storage and release capacity, the Remaining Benefit Range achievable (RBR), is the difference between the GUB benefit and the benefit with the least complex (DC) optimisation, for that (SCAP, RCAP). With the RBR on the bottom line, we can compute the Proportion of Remaining Potential Benefit Achievable by investing in improved Optimisation (PRPBAO), via:

$$\text{PRPBAO}(\text{SCAP}, \text{RCAP}) = \frac{\text{TB}^{\text{MC}}(\text{SCAP}, \text{RCAP}) - \text{TB}^{\text{DC}}(\text{SCAP}, \text{RCAP})}{\text{GUB} - \text{TB}^{\text{DC}}(\text{SCAP}, \text{RCAP})} \quad (7A.1.1)$$

Applying formula 7A.1.1 to our batch of computational experiments yields Table 7A.1.1. For example, taking the Chapter 6 base case reference point, i.e. RCAP = 1.33 and SCAP = 12 × FMEAN, we achieve 36% of the maximum potential improvement by just going from DC to MC optimisation.

Appendix 7.1: Gains from Optimisation vs Capacity Investment

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrow SCAP (\times FMEAN)	0.02	0.00	0.02	0.05	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
	0.07	0.00	0.02	0.11	0.15	0.17	0.18	0.18	0.18	0.18	0.18	0.18	0.18
	0.14	0.00	0.00	0.10	0.18	0.22	0.23	0.23	0.23	0.22	0.22	0.22	0.22
	0.25	0.00	0.00	0.06	0.20	0.26	0.28	0.27	0.27	0.27	0.27	0.27	0.27
	0.50	0.00	0.00	0.00	0.13	0.24	0.26	0.26	0.26	0.25	0.24	0.25	0.25
	0.75	0.00	0.00	0.00	0.06	0.21	0.25	0.25	0.24	0.23	0.23	0.23	0.23
	1.00	0.00	0.00	0.00	0.02	0.19	0.25	0.24	0.23	0.23	0.23	0.23	0.23
	1.50	0.00	0.00	0.00	0.00	0.16	0.26	0.27	0.27	0.27	0.27	0.27	0.27
	2.00	0.00	0.00	0.00	0.00	0.11	0.27	0.29	0.30	0.30	0.31	0.31	0.31
	3.00	0.00	0.00	0.00	0.00	0.06	0.26	0.29	0.30	0.30	0.31	0.32	0.32
	4.00	0.00	0.00	0.00	0.00	0.03	0.22	0.32	0.30	0.31	0.32	0.32	0.32
	6.00	0.00	0.00	0.00	0.00	0.00	0.13	0.34	0.34	0.36	0.37	0.37	0.37
	12.00	0.00	0.00	0.00	0.00	0.00	0.02	0.29	0.36	0.39	0.43	0.44	0.44
	24.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.36	0.47	0.65	0.66	0.66
	48.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.28	0.45	0.80	0.81	0.81
72.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.23	0.40	0.86	0.87	0.87	

Table 7A.1.1: MC versus DC PRPBAO Differences

Table 7A.1.1 shows that, over a wide range of storage capacities, there is little to no benefit value from enhancing the optimisation methodology when RCAP is significantly smaller than SCAP, presumably because the optimisation has little leeway to work with. If the storage capacity is very small, enhancing the optimisation methodology will unlock a relatively small proportion of the remaining potential benefits, because what the system really needs is more storage capacity. As storage capacity increases, and provided release capacity is sufficient, the optimisation unlocks a significant, and steadily increasing, proportion of the remaining potential value: Rising from 8% to 32%, with four months' worth of storage capacity. Beyond this level of storage capacity, the ratio appears to rise sharply, but this is misleading because the scale is nonlinear. In reality the rate of increase, per unit of storage volume, is actually dropping off. But the ratio does keep rising, because the remaining benefit from system expansion drops off significantly, so that optimisation is able to achieve an ever larger proportion of the remaining value.

Going across the table we observe a "leading diagonal effect", as discussed in Section 7.2.2.3. Generally, as release capacity increases the optimisation starts to yield a higher proportion of the remaining benefit, until this levels off. As expected, that levelling off occurs sooner for small storage capacities than larger storage capacities. Unsurprisingly, upgrading the optimisation methodology can unlock most of the remaining benefit if both storage and release capacities are already large, so the ratio keeps rising. But these high

Appendix 7.1: Gains from Optimisation vs Capacity Investment

ratios need to be kept in perspective. In the limit, there may be little or no benefit from enhancing the optimisation methodology because we believe that better storage management is not needed when the storage capacity is infinite. But the remaining benefit achievable by expansion also drops to zero, so the ratio actually tends to infinity.

Next, we can create a related measure, the potential gains from physical enhancement for a reservoir system with Storage and Release capacities ($SCAP_{ref}$, $RCAP_{ref}$). Starting from that reference point, if we were to adjust capacities to ($SCAP$, $RCAP$), while retaining DC optimisation, we would achieve the Proportion of Remaining Potential Benefit Achievable by investing in Capacity ($PRPBAC_{ref}$), defined by:

$$PRPBAC_{ref}(SCAP, RCAP) = \frac{TB^{DC}(SCAP, RCAP) - TB^{DC}(SCAP_{ref}, RCAP_{ref})}{GUB - TB^{DC}(SCAP_{ref}, RCAP_{ref})} \quad (7A.1.2)$$

We consider both positive and negative adjustments in capacity. Even though a plant owner is unlikely to willingly “invest” in reduced capacity, it might be necessary, for example, to accommodate environmental concerns or reservoir maintenance issues. And, in that situation, the plant owner would be interested in determining the benefits lost by that reduction in capacity.

At the reference point, the $PRPBAC_{ref}$ improvement is zero, as shown in Table 7A.1.2. With infinite release/storage capacity 100% of the potential improvement is achieved, as we are at the optimum benefit level, irrespective of optimisation complexity. If we double SCAP from $12 \times FMEAN$ to $24 \times FMEAN$ we can get an improvement of 34% of the maximum potential, while still retaining DC optimisation. This almost matches the improvement offered by a move from DC to MC optimisation, i.e. 36% of the maximum potential available, but would most likely be vastly more expensive. No matter how much we spend to increase release capacity, we can still only achieve 30% of the potential, though, so long as we stick with DC optimisation. The negative numbers reflect the value that would be lost by reducing storage and/or release capacity.

Thus we conclude that, for a wide range of reservoir systems, more accurate accounting for the underlying uncertainty, via a stochastic optimisation, is likely to significantly improve outcomes at a small fraction of the cost of enhancing storage and/or release capacity to achieve the same benefit.

Appendix 7.1: Gains from Optimisation vs Capacity Investment

		RCAP (\times FMEAN) \rightarrow											
		0.02	0.07	0.14	0.25	0.50	0.75	1.00	1.33	1.50	2.00	3.00	4.00
\leftarrowSCAP (\times FMEAN)	0.02	-149.2	-64.1	-43.3	-35.9	-32.6	-31.9	-31.67	-31.59	-31.53	-31.51	-31.50	-31.50
	0.07	-148.7	-59.0	-39.2	-31.8	-28.6	-27.9	-27.66	-27.59	-27.51	-27.48	-27.48	-27.48
	0.14	-148.7	-56.0	-34.3	-27.5	-24.4	-24.0	-23.49	-23.45	-23.27	-23.24	-23.23	-23.24
	0.25	-148.7	-55.2	-30.1	-23.8	-20.6	-20.3	-19.82	-19.67	-19.59	-19.54	-19.55	-19.55
	0.50	-148.7	-55.2	-27.0	-17.8	-14.5	-14.1	-13.72	-13.52	-13.31	-13.25	-13.26	-13.26
	0.75	-148.7	-55.2	-26.7	-15.2	-11.1	-10.8	-10.40	-10.14	-10.01	-9.96	-10.00	-10.00
	1.00	-148.7	-55.2	-26.7	-14.2	-9.0	-8.8	-8.35	-8.14	-8.04	-8.03	-8.07	-8.07
	1.50	-148.7	-55.2	-26.7	-13.8	-7.0	-6.4	-6.18	-6.02	-6.00	-6.02	-6.04	-6.04
	2.00	-148.7	-55.2	-26.7	-13.8	-6.0	-5.0	-4.77	-4.75	-4.74	-4.76	-4.79	-4.79
	3.00	-148.7	-55.2	-26.7	-13.8	-5.3	-3.6	-2.94	-2.86	-2.83	-2.87	-2.89	-2.89
	4.00	-148.7	-55.2	-26.7	-13.8	-5.0	-3.0	-2.18	-1.80	-1.81	-1.84	-1.84	-1.84
	6.00	-148.7	-55.2	-26.7	-13.8	-4.9	-2.4	-1.62	-0.86	-0.84	-0.85	-0.85	-0.85
	12.00	-148.7	-55.2	-26.7	-13.8	-4.9	-1.9	-1.01	0.00	0.19	0.30	0.30	0.30
	24.00	-148.7	-55.2	-26.7	-13.8	-4.9	-1.9	-0.69	0.34	0.55	0.67	0.67	0.67
	48.00	-148.7	-55.2	-26.7	-13.8	-4.9	-1.8	-0.51	0.52	0.73	0.85	0.85	0.85
72.00	-148.7	-55.2	-26.7	-13.8	-4.9	-1.8	-0.46	0.57	0.78	0.90	0.90	0.90	
∞	-148.7	-55.2	-26.7	-13.8	-4.9	-1.8	-0.33	0.68	0.89	1.00	1.00	1.00	

Table 7A.1.2: DC PRPBAS_{ref} Differences

Appendix 7.2: Case Hydrology & Bid Data

This Appendix provides the explicit data used in Section 7.5.

Tables 7A.2.1, 7A.2.2, and 7A.2.3 below give the monthly inflow data for the high, medium, and low inflow variance experiments in Section 7.5.1, for all seven hydrology states.

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	29	36	38	36	29	20	11	4	2	4	11	20
	2	60	73	78	73	60	41	22	9	4	9	22	41
	3	90	111	118	111	90	62	34	13	6	13	34	62
	4	122	150	160	150	122	84	46	18	8	18	46	84
	5	154	189	202	189	154	106	58	23	10	23	58	106
	6	184	227	242	227	184	127	70	27	12	27	70	127
	7	215	264	282	264	215	148	81	32	14	32	81	148

Table 7A.2.1: Inflow Data for High Inflow Variance Case

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	61	75	80	75	61	42	23	9	4	9	23	42
	2	81	100	107	100	81	56	31	12	5	12	31	56
	3	102	125	133	125	102	70	38	15	7	15	38	70
	4	122	150	160	150	122	84	46	18	8	18	46	84
	5	142	175	187	175	142	98	54	21	9	21	54	98
	6	163	200	213	200	163	112	61	24	11	24	61	112
	7	183	225	240	225	183	126	69	27	12	27	69	126

Table 7A.2.2: Inflow Data for Medium Inflow Variance Case

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	100	123	131	123	100	69	38	15	7	15	38	69
	2	107	132	141	132	107	74	41	16	7	16	41	74
	3	115	141	150	141	115	79	43	17	8	17	43	79
	4	122	150	160	150	122	84	46	18	8	18	46	84
	5	129	159	170	159	129	89	49	19	8	19	49	89
	6	137	168	179	168	137	94	51	20	9	20	51	94
	7	144	177	189	177	144	99	54	21	9	21	54	99

Table 7A.2.3: Inflow Data for Low Inflow Variance Case

Appendix 7.2: Case Hydrology & Bid Data

Tables 7A.2.4, 7A.2.5, 7A.2.6, and 7A.2.7 below give the monthly inflow data for the high, medium, low, and very low seasonality experiments in Section 7.5.2, for all seven hydrology states.

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	98	121	130	121	98	66	34	11	2	11	34	66
	2	104	127	136	127	104	72	40	17	8	17	40	72
	3	110	133	142	133	110	78	46	23	14	23	46	78
	4	116	139	148	139	116	84	52	29	20	29	52	84
	5	122	145	154	145	122	90	58	35	26	35	58	90
	6	128	151	160	151	128	96	64	41	32	41	64	96
	7	134	157	166	157	134	102	70	47	38	47	70	102

Table 7A.2.4: Inflow Data for High Seasonality Case

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	87	102	108	102	87	66	45	30	24	30	45	66
	2	93	108	114	108	93	72	51	36	30	36	51	72
	3	99	114	120	114	99	78	57	42	36	42	57	78
	4	105	120	126	120	105	84	63	48	42	48	63	84
	5	111	126	132	126	111	90	69	54	48	54	69	90
	6	117	132	138	132	117	96	75	60	54	60	75	96
	7	123	138	144	138	123	102	81	66	60	66	81	102

Table 7A.2.5: Inflow Data for Medium Seasonality Case

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	76	83	86	83	76	66	56	49	46	49	56	66
	2	82	89	92	89	82	72	62	55	52	55	62	72
	3	88	95	98	95	88	78	68	61	58	61	68	78
	4	94	101	104	101	94	84	74	67	64	67	74	84
	5	100	107	110	107	100	90	80	73	70	73	80	90
	6	106	113	116	113	106	96	86	79	76	79	86	96
	7	112	119	122	119	112	102	92	85	82	85	92	102

Table 7A.2.6: Inflow Data for Low Seasonality Case

Appendix 7.2: Case Hydrology & Bid Data

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Hydrology State	1	68	70	70	70	68	66	64	62	62	62	64	66
	2	74	76	76	76	74	72	70	68	68	68	70	72
	3	80	82	82	82	80	78	76	74	74	74	76	78
	4	86	88	88	88	86	84	82	80	80	80	82	84
	5	92	94	94	94	92	90	88	86	86	86	88	90
	6	98	100	100	100	98	96	94	92	92	92	94	96
	7	104	106	106	106	104	102	100	98	98	98	100	102

Table 7A.2.7: Inflow Data for Very Low Seasonality Case

Table 7A.2.8 below gives the monthly QREF data for the peak inflow to peak demand experiments in Section 7.5.3, these apply to all seven hydrology states given non-state dependent bids.

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
Lag Weight (WL)	0	122	150	160	150	122	84	46	18	8	18	46	84
	0.5π	150	122	84	46	18	8	18	46	84	122	150	160
	π	46	18	8	18	46	84	122	150	160	150	122	84
	1.5π	18	46	84	122	150	160	150	122	84	46	18	8

Table 7A.2.8: QREF^f Data for Phasing of Peak Inflow to Peak Demand Cases

The upper and lower inflow variance ranges for all twelve months are shown below in Figure 7A.2.1, where only the minimum, maximum and average states are shown. The average state $F(h, t)$ is the same for all inflow variance experiments. With a total of seven hydrology states there are two other states between each upper and lower limiting states and the mean state, for each inflow variance test. Each interim state is approximately equally spaced.

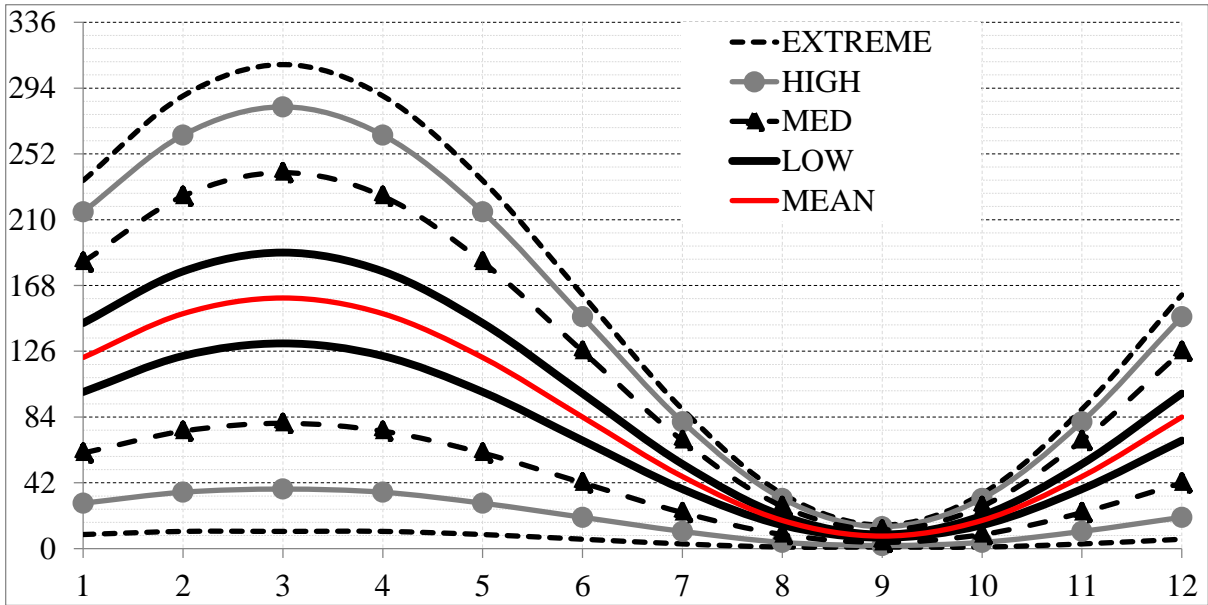


Figure 7A.2.1: Net Inflow Limits for Seasonal Variance Experiments

The upper and lower seasonal flow ranges for all twelve months are shown below in Figure 7.A.1.2, this is laid out the same as in Figure 7A.2.1, and the mean inflow values for the Extreme base case have been included here.

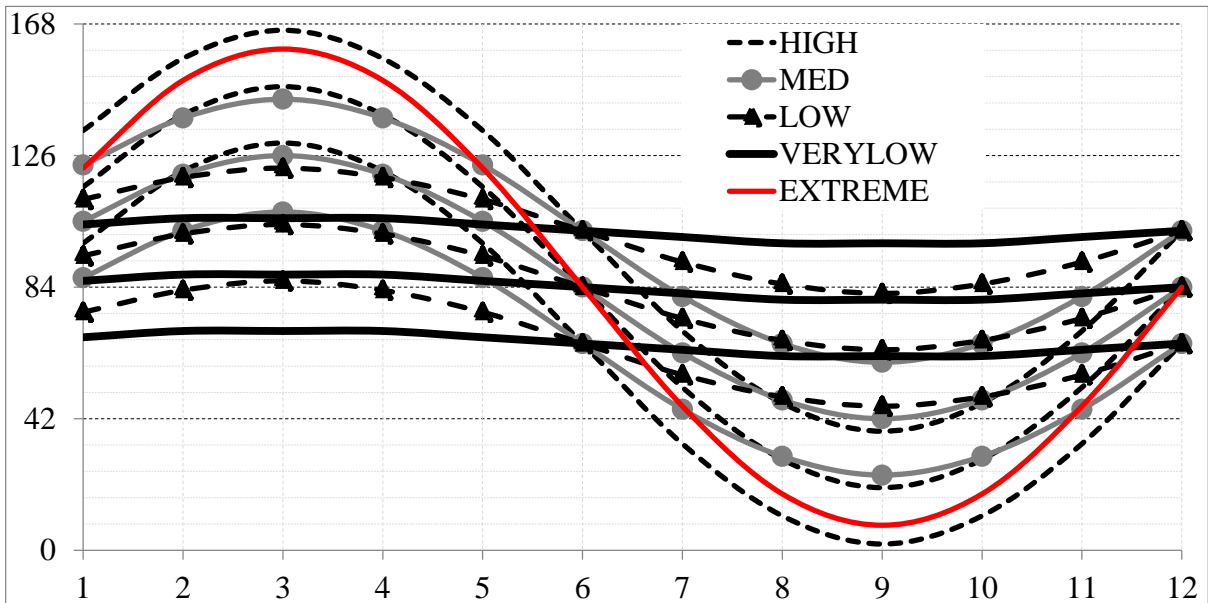


Figure 7A.2.2: Relative Net Inflow Limits for Seasonality Experiments

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