SIMULTANEITY AND TEST-THEORIES OF RELATIVITY

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Abstract

Two intertwined issues in special relativity—clock synchronisation and the experimental verification of special relativity—are investigated, and novel results are given in both areas. The validity of the conventionality of distant simultaneity is supported, and the special theory of relativity is recast in a general synchrony "gauge" to reveal the operational significance of synchronisation in measurement and prediction within special relativity. For similar reasons, the Mansouri-Sexl test-theory is extended to allow arbitrary synchrony to be properly taken into account in the verification of relativistic theories. The generalised test-theory is used to analyse recent experiments and to demonstrate that there is no basis to claims that simultaneity relations are empirically definable.

Synchrony considerations are extended to the case of a non-inertial observer exhibiting arbitrary motion within the context of any metric theory: a local co-ordinate system is developed using differential geometric techniques and a generalisation of the Frenet frame.

The analysis used for the accelerated observer is adapted to produce a test-theory of local Lorentz invariance in a space of arbitrary curvature. The test-theory incorporates the conventionality of distant simultaneity which, combined with the geometric approach, illuminates the role of synchrony in test-theories. The Sagnac effect is investigated within this new test-theory, enabling the use of a precision ring laser, such as the Canterbury Ring Laser, to bound parameters of the theory and thus to test local Lorentz invariance.

Chapter 1

Introduction

The re-analysis of the concepts of time and simultaneity formed one of the crucial and distinguishing elements of Einstein's Special Theory of Relativity of 1905 [22]. Instead of an absolute time associated with the rest frame of the aether, actual time for an observer was that of clocks attached to the inertial frame of the observer. Whether or not events were simultaneous was determined by the readings of clocks at the place of the events. Such clocks were synchronised by an operational procedure using light signals. The uniqueness of Einstein's understanding of time may be seen by comparing it to that of Lorentz. In 1904, and unknown to Einstein, Lorentz had presented a transformation equation for the time coordinate that was mathematically equivalent to that obtained by Einstein [64]. Lorentz's concept of time was that of a "local time," a concept which he had originally introduced in 1895 [63] when establishing Maxwell's equations in a frame in motion with respect to the aether frame. In this 1895 paper, Lorentz expressed the local time, t_L, for a frame moving at a speed v with respect to the aether frame, in terms of the spatial and temporal co-ordinates, x and t, ofthe aether frame

$$t_L = t - vx \tag{1.1}$$

Later, in 1904, Lorentz used the same concept of local time, but with a transformation equation equivalent to what is now known as the time component for the Lorentz transformation (equation (2.14)). With this transformation for time, and the "Lorentz-FitzGerald" contraction factor for transforming the spatial coordinate, Lorentz was able to obtain the proper transformation equations for Maxwell's equations, although only for the case of electrostatics [64]. For Lorentz, t_L was merely a mathematical time co-ordinate without physical significance; the true time remained the absolute Galilean time. For Einstein, however, Lorentz's local time became the real physical time for a moving observer. Einstein made a comment on the significance of this transformation in a review article published in 1907[23]. Noting the difficulties of Lorentz's theory he remarked:

> Surprisingly, however, it turned out that a sufficiently sharpened conception of time was all that was needed to overcome the difficulty discussed. One had only to realise that an auxiliary quantity introduced by H. A. Lorentz and named by him "local time" could be defined as "time" in general. If one adheres to this definition of time, the basic equations of Lorentz's theory correspond to the principle of relativity, provided the above transformation equations [the spatial Galilean transformations] are replaced by ones that correspond to the new conception of time. [26, p. 253]

Furthermore, Lorentz himself, when comparing Einstein's theory to his own, remarked in 1915 on the significance of the same change in understanding the nature of time within a co-ordinate system: "The chief cause of my failure was my clinging to the idea that the variable t [the time of the aether frame] only can be considered as the true time and that my local time t' must be regarded as no more than an auxiliary mathematical quantity" [65, p. 321].

The standard case when light propagation is assumed isotropic is known as "Einstein synchronisation," since this method was proposed by Einstein in the kinematic section of his 1905 paper along the following lines. A signal, from a clock at position A, is sent to a distant clock at position B and then reflected back to the clock at A. If t_1 is the time of departure of the light signal from the clock at position A to the clock at position B and t_3 the time of arrival of the light returning back to A, the light's time of arrival at position B is defined to be the mean of the times t_1 and t_3. Einstein specifically noted that the time of light reflection at position B was established '*by definition* that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A.' Such "imaginary physical experiments," Einstein remarked, provide a way to understand what is meant by synchronous clocks at different places. Einstein also noted that experience indicated that the round-trip speed, namely $c = 2AB/(t_3 - t_1)$, is a universal constant. The operational method clearly associates the synchronisation within the frame with the velocity of light in the frame. Indeed, Einstein later made this explicit in his 1907 review article:

> We now assume *that the clocks can be adjusted in such a way that the propagation velocity of every light ray in vacuum—measured by means of these clocks—becomes everywhere equal to a universal constant* c, provided that the co-ordinate system is not accelerated [23, p. 256].

The empirical content added to these considerations by the "light postulate" was the claim that the velocity of light within a frame was independent of the velocity of the source. In his popular exposition of the special and general theories of relativity [25], Einstein stressed the inherently circular nature of such knowledge: one was prevented from measuring the one-way speed of light in a given direction since that would require the prior synchronisation of clocks and thus a prior knowledge of the speed to be measured. The choice that light travels at equal speeds along the opposite directions of a particular path was "neither a *supposition nor a hypothesis* about the physical nature of light, but a *stipulation*" that can be freely made so as to arrive at a definition of simultaneity.

A number of elements of Einstein's analysis of simultaneity may also be found in essays by Poincaré. In an essay in 1898 Poincaré noted the distinction between deciding on the simultaneity of events that occurred at the same place and those that occurred at distant places [93]. Since there is no access to a universal time to order distant events one must decide on their simultaneity or otherwise on the basis of a convention. For Poincaré, neither light synchronisation nor slow clock transport synchronisation provides an escape from the conventionality of simultaneity. In several essays, both before and after Einstein's 1905 paper (see, for example, references [91, 92]), Poincaré presented a method for synchronising clocks based on the exchange of light signals which was essentially the same as that presented by Einstein. For Poincaré the intent was to explicate Lorentz's notion of "local time" and he noted that when equality of transmission times in the directions A to B and B to A was assumed, then clocks in a frame moving with respect to the aether would be synchronised in a way that would show the local time at that point [91]. While one may surmise that a number of these ideas were important to Einstein's analysis of time and simultaneity, the way Poincaré deals with these issues is essentially different from the approach of Einstein (for a discussion of these differences see the texts by Miller [81], Torretti [111] and Zahar [126]). Nevertheless one sees in the discussions of Poincaré as well as those by Lorentz and Einstein in the decades surrounding the birth of special relativity the intimate manner in which matters to do with time co-ordinates and synchronisation were involved in the forging of the physical and conceptual basis of special relativity.

The special theory of relativity, expressed using the Lorentz transformation assumes a constant, isotropic speed of light in every inertial frame. It also yields the result that time dilation and length contraction effects on a moving body, as seen from any inertial frame, are independent of direction; the only variable being relative velocity. Nevertheless, the experimental measurement of such things as the one-way speed of light (and time dilation and length contraction along an open path) has been a contentious issue. While a large body of literature is in favour of an empirical (and theoretical) determination of the Lorentz transformation, there is an opposing view that some things (such as the one-way speed of light and the parameters in a transformation between two frames of reference) are conventional in nature, and hence cannot be uniquely determined by experiment; the Lorentz transformation itself is regarded as conventional. Claims that the Lorentz transformation may be singled out by experiment for special attention are countered by the point of view that experimental results can only support the existence of many equally valid co-ordinatisations of a special relativity theory, the Lorentz transformation resulting from one particular choice.

The argument against the measurability of the one-way speed of light hinges on the existence of an infinity of possible synchronisation schemes for the setting of the clocks to be used in the measurement. One cannot single out from these some particular choice without assuming something, a priori, about the speed of light. This circularity is most obvious in the use of light signals to synchronise distant clocks with a master clock. This method, where a light signal is sent from a master clock at a time, t, to a distant clock which, on reception of the signal, is set to time $t + \tau$ (where τ is the travel time of the signal), is referred to as "light synchronisation". To know the travel time of the signal, one must make an assumption about the one-way speed of light between the different clocks, thus defeating attempts to measure the one-way speed of light. This situation exists for light travelling along, not only a constant direction, but also any open path. There is no such objection to measuring the return trip (closed path) speed of light since only one clock is used for this case, avoiding synchrony considerations altogether.

Although the difficulties of clock synchronisation have been considered as far back as the ninteenth century, the status of one-way measurements still comes under discussion. In the context of philosophy, the discussion centres on the grounds upon which the natural choice of isotropy may be regarded as obligatory. It is commonly maintained in the community of philosophers of science that theoretical considerations based on the context and symmetries of the causal structure of Minkowski space-time show that the choice of synchronisation under which the one-way speed of light is isotropic is essentially forced upon any reasonable theoretical formulation (for example, Coleman *et al.*[16, 17]. While this has been a recurring theme of many articles over many years, it has become more strident since the work of Malament[73]. Indeed, Friedman[31, page 310] claimed that Malament had shown that Einstein synchronisation is explicitly definable from the conformal structure of the space-time metric, whereas other synchrony conventions are not, and later concluded that dispensing with Einstein synchronisation entailed a denial of the structure of Minkowskian space-time. Havas[40], however, pointed out that all Malament has shown is that "in Minkowski space-time one can always introduce time-orthogonal coordinates". Havas noted that time-orthogonal co-ordinates implied Einstein synchronisation, but correctly pointed out that Malament's result does not imply an inconsistency between Minkowskian space-time and non-standard synchrony conventions.

Parallel to the considerations of space-time structure in relativity, are claims made on empirical grounds. Experiments (such as those on maser stability and two-photon absorption for example) have been declared to give experimental insight into the isotropy of the one-way speed of light. This view dates back to Robertson's pioneering work on testing special relativity[100] which motivated the popular Mansouri-Sexl test-theory[76] which has commonly provided a framework for analysing experiments. It has also gained prominence recently through the work of Krisher *et al.*[55, 54], Will[122, 123] and Haugan *et al.*[39]. The position adopted in this thesis is that the conventionality of the one-way speed of light is unavoidable and forbids such empirical arguments or tests in both areas.

The expansive operational argument underpinning the conventionalist thesis has been discussed at length by Winnie [125], who addressed synchronisation in special relativity, for the two dimensional space-time case. Winnie discussed the consequences of various synchronisation schemes on measurements of relative velocities, showing thay they, along with time dilation and length contraction effects on a one-way trip, are conventional in nature. This is in contrast to the synchrony invariant effects (such as round-trip speeds, and time dilation and length contraction effects on a return trip) which are also considered. Such issues are covered in section 2.3. An important result of Winnie's is his generalisation of the Lorentz transformation to arbitrary synchrony, thus demonstrating explicity that the Lorentz transformation is covariant under synchrony transformations.

Havas[40] pointed out that such arbitrariness in synchrony is implicitly contained in generally covariant formulations of special relativity by appealing to the covariant nature of general relativity and noting that the latter theory contains special relativity in the limit of flat space-time.

The power of covariance in obtaining results such as those of Winnie was demonstrated by Anderson and Stedman[4] who gave a tensor formulation of special relativity with arbitrary synchronisation. They considered an observer who assumes that the velocity of light in the direction $\boldsymbol{n}$ is of the form $c(\boldsymbol{n}) = \boldsymbol{n}/(1+\boldsymbol{\kappa}\cdot\boldsymbol{n})$; the three-vector $\boldsymbol{\kappa}$ is a synchrony vector field which is arbitrary, up to the restriction that it have modulus less than unity if one wishes to keep all speeds positive and finite. It was shown that the kinematics of this observer may be obtained by applying a synchrony transformation tensor (dependent on $\boldsymbol{\kappa}$) to the quantities corresponding to the case $\boldsymbol{\kappa} = \mathbf{0}$; the manner in which various quantities vary with synchrony is then readily available.

Giannoni[33] and Ungar[113] discussed the group properties of the generalised Lorentz transformation (without isotropy assumptions). Giannoni gave a group of transformations which allows differing synchrony conventions in any two frames of reference, but requires no restriction on the magnitude of synchrony vector (so $-\infty < |\boldsymbol{\kappa}| < \infty$) and thus admits infinite speeds, negative speeds (as opposed to velocities), and some conceptually difficult effects such as negative length contractions. Ungar criticised Giannoni's group because it "rules out a causality condition that causes precede effects," and presented a transformation group which has $\kappa < 1$ and thus obeys the causality condition. However, Ungar's group (which is a sub-group of Giannoni's) imposes the same synchrony choice in each reference frame. This is not in keeping with the spirit of the conventionalist thesis, since synchrony choice in one frame should not fix synchrony choice in any other; Giannoni's group is preferable to Ungar's.

In defence against the criticism that Giannoni's group does not obey the causality condition, it should be pointed out that one should distinguish "spatially coincident causality" from "distant causality". The first of these involves a sequence of events occuring at the same spatial point and is unaffected by synchrony. The second brings in the question of distant simultaneity, and there is no contradiction if an occurrence at $\boldsymbol{P}$ at time t causes another occurrence at $\boldsymbol{Q} \neq \boldsymbol{P}$ at time $t' < t$ since the two different times are measured at spatially different locations: Indeed, such apparent inconsistencies are very common on Earth: a plane may fly over the international dateline from the West and arrive at some destination the day before it left—according to the local times at the places of departure and arrival. This is an example of non-obedience of distant causality, corresponding to values of $\boldsymbol{\kappa}$ greater than unity (as allowed by Giannoni's formalism) but there is nothing fundamentally unphysical with this.

The rejection of a temporal ordering in distant causality espouses a point of view in which time at any spatial point flows independently of time at other points, with there being no canonical prescription for the way one links the times at spatially separated points. This viewpoint is in accord with the "fibre-bundle" representation of the conventionality of simultaneity given by Anderson and Stedman[6]. In this representation, the conventionality of simultaneity is identified with the freedom to choose $\boldsymbol{\kappa}$ as the choice of a particular connection in a fibre bundle consisting of a base space of three-space and fibres of the world-lines of particles along which time is represented. Thus a choice of synchronisation is a choice of how the different fibres are to be compared.

Giannoni's group is formulated only for boosts along the x-axis. Section 2.2 contains a generalisation of Giannoni's work (to arbitrary boosts and rotations) using the tensor formulation given by Anderson and Stedman[4].

One area in which clock synchrony has had much consideration is in the experimental testing of special relativity. The currently favoured approach for the testing of relativistic theories is to use a "test-theory" — a theoretical framework which contains a continuum of theories, which are parameterised by a number of functions. For a particular set of parameter values, one has the theory to be tested; all other parameter combinations give rise to alternative (rival) theories. How much one theory differs from another is gauged by the difference in respective parameter values: if the parameters are chosen to correspond to physical effects, then different aspects of a theory can be investigated independently of each other.

Experimental predictions can be made in terms of the test-theory parameters and then experimental data used to put bounds on these parameters, thus eliminating many of the rival theories from contention. This does not single out one theory as correct over all the others: the aim is to use more and more precise experiments to squeeze the parameters' values to a smaller and smaller range around the value set corresponding to the favoured theory. The test-theory approach thus handles all possible theories of a given type simultaneously and verifies a huge number at once, a contrast to the more traditional approach of making predictions with just the theory to be tested and searching for a falsification.

Although the theory of special relativity was formulated before the theory of general relativity, and is indeed assumed, in the latter theory, to be valid in the limit of negligible gravitation, the experimental testing of special relativity with test-theories is not as extensive as in the situation of general relativity, as reviews by Damour[18] and Will[124] indicate. Certainly, more emphasis is placed on dynamics in general relativity than in special relativity: the most popular test-theory of special relativity, the Mansouri-Sexl test-theory[76], does not address dynamical issues to any great extent, concentrating on kinematical considerations and the structure of space-time. However, this bias in favour of the kinematics and space-time structure reflects the importance of both of these properties in the foundational aspects of special relativity.

Synchrony issues are as contentious in test-theories as they are in special relativity, since the way interpretations are made has considerable bearing on the status of the conventionality of measurements. Some authors, such as Mansouri and Sexl[76] (whose test-theory is widely used), have claimed that while the one-way speed of light is not measurable from within a purely special relativistic framework, it becomes so when considered within the more general context of a test-theory. Such claims are discussed in section 3.2 where it is shown that conventionality has not been adequately handled by Mansouri and Sexl[76], and furthermore that the parameters in their theory have, in fact, a degree of conventionality. This is shown by generalising the Mansouri-Sexl test-theory to arbitrary synchrony, and demonstrating covariance for all theories in the Mansouri-Sexl formalism. Section 3.3 briefly reviews some experimental tests and their limitations regarding conventional quantities that they have been claimed to measure. These limitations have been obscured in some analyses of experiments because authors have failed to give proper consideration to synchrony. Thus section 3.4 discusses the analysis and interpretation of experiments in which synchrony-dependent parameters are evaluated. As examples, two experiments (the two-photon absorption[98] and maser phase[54]) are analysed within the generalisation of the Mansouri and Sexl test-theory given in section 3.2.

Chapter 4 looks at synchronisation for non-inertial observers, an aspect which has not been greatly considered in the literature. In section 4.1 the co-ordinate system of an accelerated observer with arbitrary synchrony is developed, not only for the case of special relativity, but also for an arbitrarily curved manifold in a general relativistic theory. Bringing arbitrary synchrony into consideration for a non-inertial observer requires a different prescription from that traditionally used for the observer who sets up a locally Lorentz, locally Einstein synchronised set of co-ordinates. Accordingly, sections (4.2), (4.3) and (4.4) consider, respectively, the assignment of local co-ordinates, tetrad propagation within the observer's frame, and finally the observer's metric. Section (5.1) modifies some of the assumptions in section (4.1) to obtain a test-theory of local Lorentz invariance, taking into account arbitrary synchrony.

Chapter 2

Synchrony in special relativity

2.1 Simultaneity and synchronisation

The conventionality of distant simultaneity, having a long and involved history of debate, is well established as a major point of contention in special relativity. The definition of simultaneity conventions, being intertwined with the concept of speed, became an issue with the advent of attempts to measure the one-way speed of light as a means to verify the existence of the aether—and thus the existence of absolute space. According to Galilean relativity, mechanical motions are insensitive to uniform motion and thus cannot be used to detect a preferred frame. Sklar[105] points out that Maxwell's reduction of light to electromagnetic radiation provided hope for the detection of absolute space: since electromagnetic waves were considered to need a medium—the aether—for propagation, an observer moving with respect to the aether (which was identified with absolute space) would detect a direction-dependent variation in the speed of light.

However, a measurement of light speed would require prior clock synchronisation, for which the only accurate proposed procedure was with the use electromagnetic signals. This method, refered to here as "light synchronisation", encompasses a class of synchrony conventions, of which Einstein synchronisation is a special case, and is usefully explained with the help of a "synchrony-vector", $\boldsymbol{\kappa}$. In light synchronisation an electromagnetic signal is sent at a time t from a master clock at A to another, spatially separated clock located in the direction $\hat{\boldsymbol{p}}$, at B. This second clock is set to the time $t + \Delta t$ where Δt is the time-of-flight of the signal from A to B. For this time of flight to be calculated, the speed of the signal from A to B must be assumed. If the speed of light in the direction $\hat{\boldsymbol{p}}$ is assumed to be $c(\hat{\boldsymbol{p}}) = 1/(1+\boldsymbol{\kappa}\cdot\hat{\boldsymbol{p}})$ then the time at B is set to $t + AB((1+\boldsymbol{\kappa}\cdot\hat{\boldsymbol{p}})$. Einstein synchronisation, imposed by Einstein as the standard for special relativity, arises from the special case $\boldsymbol{\kappa} = \mathbf{0}$. Whichever value of $\boldsymbol{\kappa}$ is chosen, any subsequent measurements made using both those clocks will reflect that choice, and thus defeat attempts to directly determine the one-way speed of light and thus aether-drift effects.

Reichenbach[96], and later Grünbaum[35], promoted this conventionality on the one-way speed of light in the philosophy of science arena, using a different notation from the $\boldsymbol{\kappa}$ above to discuss synchrony conventions. Reichenbach's ϵ characterisation of simultaneity relations, while not as suited to tensorial calculation as the $\boldsymbol{\kappa}$ convention, is a popular choice in the philosophical literature; this characterisation is outlined here. Consider a light beam being sent, at time t_1, from a point A in an inertial frame to a point B in the same frame, arriving at time t_2 and being reflected back to A at time t_3, where all three times are according to a clock at A. The time t_2 must be postulated in terms of t_1 and t_2 and is taken as

$$t_2 = t_1 + \epsilon(t_3 - t_1), \quad 0 < \epsilon < 1. \tag{2.1}$$

When one chooses $\epsilon = 1/2$, one has equality in the to and fro travel times. Taking AB in the positive x direction one has

$$c_+ = \frac{c}{2\epsilon} \quad \text{and} \quad c_- = \frac{c}{2(1-\epsilon)}, \tag{2.2}$$

where $\pm$ denotes the positive and negative x directions respectively and c is the average round-trip speed of light. Thus the values of $c_\pm$ are dependent on the value of ϵ and can be determined only if ϵ can be determined. According to the Reichenbach-Grünbaum thesis of conventionality of simultaneity, any choice for $c_\pm$ which yields a value of c for the (average) round-trip speed of light is as valid as any other.

While ϵ is a natural parameter for comparing to and fro times in a round-trip journey for light, it is not very useful for most dealings with synchrony, especially in covariant formulations of relativistic kinematics and dynamics. A more convenient description is synchrony-vector notation which was adopted by, for example, Anderson and Stedman[4], and Gianonni[33]. The synchrony-vector, $\boldsymbol{\kappa}$, naturally expresses the one-way speed of light in an arbitrary direction and lends itself to the covariant description of synchrony in physics as developed by Anderson and Stedman[4]. This formulation is adapted and used later on in the subsequent chapters.

This circularity inherent in light synchronisation had been realised much earlier by Newcomb—in the later part of the nineteenth century. Newcomb's knowledge on this point was pointed out by Michelson[79], who remarked:

> In the Physikalische Zeitschrift (5 Jahrgang, No. 19, Seite 585-586) a method is proposed by W. Wien for deciding the important question of the trainment of the aether by the earth in its motion through space, by measuring the velocity of light in one direction ... The essentials in the proposed method are two Foucault mirrors, or two Fizeau wheels (one at each station) revolving at the same speed.... The flaw in the proposed method–as was pointed out by Simon Newcomb as long ago as 1880–lies in the fact that the effect which it is proposed to measure is exactly the same as the effect on the light which is to furnish the test of synchronism.

It is widely accepted that the setting of distant clocks using light signals introduces an element of conventionality in the resulting values for the one-way speed of light. Because of the inherent circularity in the method of light synchronisation, other synchronisation methods have been considered, and some authors have made the claims that there are other methods of defining distant simultaneity which can give a clock synchronisation which is not conventional. The most studied alternative to light synchronisation is the method of slow clock transport synchronisation which involves the use of slowly transported clocks, synchronised with a master clock, to synchronise a network of spatially separated clocks. With slow clock transport synchronisation, two separated clocks, at locations A and B are synchronised in the following manner: a third clock, initially set to the time of the reference clock—at A—is moved infinitessimally slowly from A to B. When this moving clock arrives at B, the clock located there is set to the time showing on the moving clock. Such a synchronisation method appears to be independent of light signals and thus has attracted much attention and debate, with no consensus in the literature. Indeed, there is a degree of confusion around as to what it actually achieves: in particular whether it is logically independent of synchronisation using light signals. Of interest also is the significance of the fact that it agrees with Einstein synchronisation within special relativity.

In an essay in 1898, Poincaré[93] mentioned the use of transported clocks to determine the time at different places on the Earth. Such transported clocks provide one of the rules for investigating simultaneity, and by this procedure the problem of simultaneity for Poincaré became one of determining the measure of time that is recorded on the clock. The latter, however, entails the comparison of different time intervals, and given the absence of s direct awareness of the equality or otherwise of two different time intervals, one needs to provide a definition. While pendula and the repetition of certain phenomena, such as the rotation of the earth, provide standards for such comparisons, they do, however, implicitly involve the postulate that "the duration of two identical phenomena is the same." This is a convenient and reasonable way to define equality of intervals but for Poincaré such a definition was not imposed by nature. Thus for Poincaré the conventional element in use of transported clocks reduced to a more basic one of the convention in setting of clock rates.

That the clocks may be affected by movement was not mentioned by Poincaré. Einstein, though, remarked in his 1905 paper[22] that where two separated clocks are synchronised with each other using Einstein synchronisation, if one of them is moved there is a loss of synchronisation. Other than that, however, he did not discuss in that paper, or elsewhere, the procedure of using the transport of clocks for the determination of simultaneity.

Poincare's reasons for the conventionality of clock transport are different from later reasons. For Poincare the conventionality was to do with time intervals: it was a convention to say the times between successive ticks on a pendulum are equal. There is no way of placing them side by side and comparing them. Thus in his 1898 paper[93] he linked the conventionality of simultaneity via clock transport with this sort of conventionality, which is quite different from the discussions of slow clock transport synchronisation.

In a text first published in 1923, Eddington [20] discussed a procedure for synchronisation using the slow transport of clocks. This appears to be the first place where a discussion is made of the use of *slow transport* of clocks. There was no attempt, however, as Ellis and Bowman[28] later tried to do, to try to avoid the circularity present in the determination of simultaneity via light signals.

Eddington proceeded as follows. Consider a clock at rest in a co-ordinate system. Then the interval will be proportional to the time measured by the ticking of a clock,

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2. \tag{2.3}$$

Consider moving the clock from one point $(x_1, 0, 0)$ at the time t_1 to another

$(x_2, 0, 0)$ at time t_2. Whether the clock is at rest or in motion it is considered as recording equal time intervals: thus the time difference between the beginning and end of the journey will be

$$\int_1^2 \mathrm{d}s. \tag{2.4}$$

Setting

$$-\mathrm{d}s^2 = c^2\mathrm{d}t^2 + 2c\kappa\mathrm{d}x\mathrm{d}t - \mathrm{d}x^2, \tag{2.5}$$

means that the difference in the clock readings will be proportional to:

$$\int_{t_1}^{t_2} \mathrm{d}t \left(1 + \frac{2\kappa}{c}\frac{\mathrm{d}x}{\mathrm{d}t} - \frac{1}{c^2}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2\right) \tag{2.6}$$

In the case of a small velocity measured with respect to the co-ordinate system, then,

$$\int_{t_1}^{t_2} \mathrm{d}t \left(1 + \frac{2\kappa}{c}\frac{\mathrm{d}x}{\mathrm{d}t}\right) = (t_2 - t_1) + \frac{\kappa}{c}(x_2 - x_1). \tag{2.7}$$

Eddington[20, p. 15] remarked: "The clock, if moved sufficiently slowly, will record the correct time-difference if, and only if, $\kappa = 0$." (The notation is the same as that used by Eddington except κ has been used instead of α.) He argued that for other directions as well, one "must" have the parameters corresponding to κ in those directions also equal to zero.

Eddington makes no definitive statement as to what he has acheived in this result. His reference to a "correct time" may refer to the assumption that whether clocks are at rest or in motion, then the *intervals* measured by the mechanism of the clock will remain the same: earlier in his text he commented that wherever and whenever, in any frame, clocks with the same mechanism record equal intervals. However, it is clear he sees his result as a "convention" and not as an argument that one must (empirically) have $\kappa = 0$. He noted that both the use of light signals (with equal back and forward light speeds), and the slow clock transport give the same formulae, but stated:

> We can scarcely consider that either of these methods of comparing time at different places is an essential part of our primitive notion of time in the same way that measurement at one place by a cyclic mechanism is; therefore they are best regarded as conventional.

Towards the end of a section on distant simultaneity, Eddington describes the conventions in both of the methods as follows[20, p. 29]:

> (1) A clock moved with infinitesimal velocity from one place to another continues to read the correct time at its new station, *or*
> (2) The forward velocity of light along any line is equal to the backward velocity.

In summary he noted:

> Neither statement is by itself a statement of observable fact, nor does it refer to any intrinsic property of clocks or of light; it is simply an announcement of the rule by which we propose to extend fictitious time-partitions through the world. But the mutual agreement of the two statements is a fact which could be tested by observation, though owing to the obvious practical difficulties it has not been possible to verify it directly.

Thus he claims that although they are conventions, they are empirically related conventions.

In his major text, *The philosophy of space and time* [96], Reichenbach discussed a number of attempts to determine "absolute simultaneity." He presented two criticisms against the use of the transport of clock procedure for attempting to establish absolute simultaneity. One problem is the dependence of clock settings on the path and speed of a transported clock. The other problem is "that even if relativistic physics were wrong, and the transport of clocks could be shown to be independent of path and velocity, this type of time comparison could not change our epistemological results, since the transport of clocks can again offer nothing but a *definition* of simultaneity." Thus even if two originally synchronised clocks are in synchronisation when brought together after motion, there is no knowledge of how their settings compare when they were apart. Ellis and Bowman later maintained that this is a trivial type of conventionality;

> Thus we might allow that infinitely slowly transported clocks, once synchronised, will always be found to be synchronous with each other, but we might deny that they remain synchronous while they are separated. But this only shows that distant simultaneity is conventional in the trivial sense that any quantitative equality between two things at a distance is conventional. If this is all there were to Reichenbach's conventionality thesis, it would be absurd to devote so much time to discussing it. [28]

Bridgman mentions two ways of synchronising clocks other than the standard Einstein method using light signals. One is to use "superlight" velocities. [11, p. 59] The essence of this method is to have a search light sweep out a distant set of clocks all at a given distance from the source and to have the clocks to be set when the sweep reaches them. One does this with different speeds of sweep, and through an extrapolation method extrapolates to the case of infinite velocity for the sweep. Reichenbach has a critique of this method in *The philosophy of space & time* [96]. For Reichenbach such a superlight sweep cannot be a causal sequence. However, for Bridgman this is not decisive for ruling out this method. He notes: "Assumptions, such as Reichenbach's of the necessity for a causal process in setting distant clocks can be peremptorily disproved by actual exhibition, as above, of a method setting distant clocks not involving causal propagation." For Bridgman this method is "definite and unique." More importantly, he notes[11, p. 61]:

> There is no reason in logic why distant simultaneity defined in this way should not be identical with distant simultaneity as defined by Einstein. In fact, the present presumption is that the two are identical. It is ultimately a question for experiment to decide.

Thus Bridgman considered the coincidence of the two methods of determining simultaneity to be an experimental issue; furthermore, his critique of Reichenbach involved a critique of Reichenbach's notion of time and causal order. This is part of Reichenbach's argument that ϵ must be inside the range [0,1].

Brigdman's other method is clock transport. He notes the inadequacy of the comments of Reichenbach and Grümbaum to the effect that because clocks are affected by motion they cannot be used for synchronisation. Instead, he invoked the use of "self-measured" velocities which he claims are "uniquely determinable without further ado". Bridgman used the term "self-measured velocity" following Ives, and likened such a velocity to that determined from the odometer of a car. In essence, a self-measure velocity uses a clock travelling with the object whose velocity is being recorded. Thus time intervals are recorded on one clock only and so the self-measured duration of the trip is not subject to the limitations imposed by the conventionality of simultaneity. This does not, however, provide an escape from the conventionality of simultaneity: the self-measured speed of an observer is in fact conventional because the "self-measured" distance of the trip is conventional. This conventionality arises because the path traversed is not in the rest-frame of the observer, and thus the path-length is subject to length-contraction effects, and these are conventional.

Bridgman's prescription after appealing to self-measured velocities is that one then synchronises clocks using a transported clock, and does so for varying self measured velocities. One then extrapolates these to a zero self-measured velocity. He noted: "By definition, we have now set our clocks for zero velocity of transport. Such a method of setting a distant clock is unique and well defined, involving only actually performable physical operations, and therefore there seems to be no reason whey we should not accept it."

Ives had also considered transported clocks and the concept of self-measured velocities, but had, however, rejected the use of this method because it involved a zero velocity. For Bridgman, Ives had overlooked the possibility of an extrapolating procedure. These two methods do not take away the "conventionality" of simultaneity: a choice of procedure remains. Again (and this is important), Bridgman maintained the correspondence between the two methods is a matter of experiment. In summary he notes:

> Even if the setting of distant clocks defined by the sweeping searchlight or by the clock-transport method agree in giving the value of $\frac{1}{2}$ for ϵ, nevertheless the decision to use one or the other method is a decision in our control, involving a corresponding *definition* of distant simultaneity. The fact that these two methods agree in giving a result obtainable also by another method is in no wise a logical necessary fact, but is something that has to be established by independent experiment.

There are two main issues regarding slow clock transport synchronisation in the recent philosophical literature. The first concerns the logical relationship between the light synchronisation method and the slow clock transport synchronisation method. The second is the significance of the fact that using generalized Lorentz transformations one can show that the slow clock transport synchronisation coincides with Einstein synchronisation. Ellis and Bowman[28], in addressing these issues, made several claims in their 1967 paper which attracted detailed attention from authors such as Winnie[125] and Grünbaum *et al.*[36]. They took the position that slow clock transport synchronisation and light signal procedures are logically independent of each other. They based this claim on a demonstration as to how one can develop a non-standard formulation of STR for arbitrary ϵ that is consistent with the acceptance of slow clock transport (their formalism is quite opaque on this issue). From here they concluded that slow clock transport synchronisation can be used to "test empirically the principle of the constancy of the one-way velocity of light." Ellis and Bowman also regarded Römer's method as a valid method for determining the one-way speed of light. Naturally, then, they took the position that the Reichenbach-Grunbaum thesis of the conventionality of simultaneity is false.

Winnie discusses slow clock transport, especially with reference to Ellis and Bowman[28] who support the notion that this synchronisation method is fundamental. Winnie shows that although slow clock transport synchronisation agrees with Einstein synchronisation within special relativity, it is nevertheless compatible with all synchrony choices, and thus cannot be used to distinguish any particular synchrony choice as correct since a falsification of some synchrony choices results in the falsification of all synchrony choices.Winnie showed that the coincidence of the slow clock transport synchronisation and Einstein synchronisation could be demonstrated using a generalized Lorentz transformation. Salmon[102] made the following comment on the significance of this result:

> ... it follows from the ϵ-Lorentz transformations that standard signal synchrony must coincide with slow clock transport synchrony. From this it follows that Römer's method does not constitute an independent method for ascertaining the one-way speed of light within the special theory. It shows that, whatever value we assign to ϵ, slow clock transport synchrony must agree with standard signal synchrony. Römer's method does not constitute a measurement of the value of ϵ; instead it constitutes a test of the factual content of special relativity. If an experimental determination of the one-way speed of light by Römer's method were to establish it to be other than c, this would not be an experimental proof that $\epsilon \neq \frac{1}{2}$; it would, instead, be an experimental disproof of the special theory of relativity. For Römer's method involves adoption of slow clock transport synchrony, and the ϵ-Lorentz transformations entail that this must coincide with $\epsilon = \frac{1}{2}$.

With this view any experimental divergence between Einstein synchronisation and slow clock transport would mean special relativity theory itself has been experimentally violated.

Ellis [27] made several comments on conventionalism and conventionality of simultaneity. In particular he claimed that the type of conventionalist strategy used by Reichenbach is a *claim of circularity.* To measure the one-way speed of light requires synchronised clocks at different places and to determine such synchrony would require prior knowledge of the one-way speed of light. Ellis commented [27, p. 59]:

> It has now, I think, been conclusively established that Reichenbach was wrong in thinking that one has to make some assumption about the one-way velocity of light to determine the simultaneity of distant events. There are in fact, several procedures which could be used to establish clocks in a relationship of distant synchrony which, logically, do not depend on this assumption (as argued in Ellis and Bowman 1967). Consequently there are several logically independent criteria for distant simultaneity, and the standard signal synchrony criterion is just one of them.

Ellis then noted that conventionalists have a further strategy in that they argue that one still needs to specify a definition while conceding that there are different criteria for defining simultaneity. The conventionality then is on the choice of procedure. It seems that Ellis considered that if one picks the one-way-light principle as conventional, then other criteria (such as slow clock transport synchronisation) become empirical, though he considered the choice of which one method is conventional as arbitrary, remarking:

> Assuming that the special theory of relativity is correct, we know that the 'simultaneity' law cluster exits. In all of the vast literature of distant simultaneity, there is no argument for preferring the standard light signal to the slow-clock- transport definition of distant simultaneity, or conversely; and we may reasonably assume that there is none. Therefore, it is arbitrary which principle we choose to call conventional and which empirical.

Friedman[31] argued an anticonventionalist position and viewed the slow clock transport synchronisation method as allowing a determination of standard synchrony without a vicious circularity. He regarded slow clock transport synchronisation as exploiting a connection between Einstein synchronisation and proper time, and thus illustrating the manner in which Einstein synchronisation is "deeply embedded in relativity theory." Moreover, he claimed that "one cannot question the objectivity of this relation without also questioning significant parts of the rest of the theory. [31, p. 317]

In the last point Friedman had in mind the result that slow clock transport synchronisation and Einstein synchronisation coincide. One can show this from the standard form for proper time expressed using a co-ordinate system using arbitrary synchrony. For Friedman this is a very important result in that the very formalism of special relativity theory (even when a general-synchrony formalism is used) leads to a result about Einstein synchronisation. These views are woven together in a summary statement [31]:

> *In particular, one cannot maintain that distant simultaneity is conventional without also maintaining that such basic quantities as the proper time metric are conventional as well.*

It appears that Friedman's reading of Winnie's result was quite different from that of Salmon, which is quoted above. All this is part of a general philosophical position argued for by Friedman. He maintained that a "good theoretical" structure is that which is connected to other parts of the theory. The theoretical structure forms an edifice, and if the parts are connected, then testing one means a test for the rest. Thus he argued against geometrical conventionalism in the following manner:

If one allows the possibility of different spatial geometries and takes the position that they are all equally possible, then one has to introduce some universal quantity to account for the form of some of those geometries. But this move is a bad one, the extra quantity has no explanatory significance and it only is used to allow one to entertain different spatial geometries. A similar move in the newtonian context would be to postulate an absolute reference frame, and to introduce a speed parameter, V, which labels the speed of each frame with respect to the absolute frame. But V is arbitrary and disconnected from any other feature of Newtonian physics. It's a "bad" parameter. Similarly ϵ is a "bad" parameter: it is not related to any other part of the theory.

There is a hint of this view in the comment by Torretti [111]: "Reichenbach's rule, as normally understood, does no more than expand it [the simultaneity relation] to a six-parameter family by the cheap expedient of associating *every* inertial frame with the *full* three-parameter family of simultaneity relations adapted to each." To introduce an ϵ is a cheap expediency. Presumably Torretti means that no more physical insight is obtained by its use.

Such a position is inadequate for the following reasons. First, it does not allow that it is a physical feature of the world that allows the introduction of ϵ. Second, many quantities (such as potentials and phases) in physical theories fail to have numerical values determined by the empirical situation but yet they are very significant features of these theories. Third, showing that results are synchrony-independent is by no means a trivial exercise and the history of this issue has shown the important of carefully separating out the dependence of results on the choice of synchronisation.

2.2 Arbitrary synchrony

Havas[40] showed the covariant nature of special relativity by appealing to the same property in general relativity, and noting that special relativity is the flat space-time limit of that gravitational theory. Havas was concerned with the metric properties of space-time under general co-ordinate transformations, and in particular the transformation properties of the metric tensor components as shown by Møller[84], as well as giving a defence for the conventionality of simultaneity. Giannoni[33] had earlier considered aspects of special relativistic dynamics under synchrony transformations, as had Anderson and Stedman[4]

Giannoni considered only boosts along the x-axis: This section extends Giannoni's work to a somewhat explicit discussion of arbitrary boosts and rotations in the context of special relativity with a degree of synchrony freedom. The notation here differs from Giannoni's, and is based on that of Anderson and Stedman. This allows the use of the tensorial approach of the latter paper, which is more powerful than the method used by Giannoni.

It is assumed that in any frame the choice of a constant, spatially invariant, synchrony vector $\boldsymbol{\kappa}$ gives rise to a time, t, related to the Einstein synchronisation time t_0 by $t = t_0 + \boldsymbol{\kappa}\cdot\boldsymbol{x}$; such a synchrony choice gives rise to a velocity of light in the direction $\boldsymbol{n}$ of

$$c(\boldsymbol{n}) = \boldsymbol{n}/(1 + \boldsymbol{\kappa}\cdot\boldsymbol{n}) \tag{2.8}$$

It should be noted that this form admits only some of all possible synchrony schemes; the times corresponding to two different synchrony schemes may differ by an arbitrary function of space, if one wishes (as has been pointed out by Anderson and Stedman). However, the above choice of an invariant $\boldsymbol{\kappa}$ (with unrestricted magnitude) is sufficient for the generalisation to a synchrony modified Lorentz transformation, and for the purposes of demonstrating the conventionality of the one-way speed of light.

A powerful approach to expressing synchrony dependent quantities in terms of their Einstein synchronisation form is the synchrony transformation matrix given by Anderson and Stedman[4]. Here, the synchrony transformation matrix, which yields all necessary synchrony information, is

$$(S_{\boldsymbol{\kappa}})^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \eta^{\mu 0}\kappa_{\nu}, \tag{2.9}$$

where

$$\{\kappa_{\mu}\} = \{0, \boldsymbol{\kappa}\}. \tag{2.10}$$

A change of synchrony from Einstein synchronisation to one corresponding to an arbitrary, constant $\boldsymbol{\kappa}$ corresponds to an operation of $S_{\boldsymbol{\kappa}}$ on the co-ordinates of the frame in question. Since $\boldsymbol{\kappa}$ is a constant, it follows that the transformation from one frame, Σ (with synchrony vector $\boldsymbol{\kappa}$), to another frame, Σ' (with synchrony vector $\boldsymbol{\kappa'}$), is given by

$$\mathcal{L} = S_{\boldsymbol{\kappa}'} \circ L \circ S_{\boldsymbol{\kappa}}^{-1} \tag{2.11}$$

where L is the Einstein synchronisation Lorentz transformation.

Note that for flat space-time and constant $\boldsymbol{\kappa}$,

$$(S_{\boldsymbol{\kappa}})^{-1} = S_{-\boldsymbol{\kappa}}. \tag{2.12}$$

Now, from Møller[84], the Einstein synchronisation Lorentz transformation, for an arbitrary boost has the form

$$\begin{aligned} \mathrm{d}T' &= \Gamma(\mathrm{d}T - \boldsymbol{V}\cdot \mathrm{d}\boldsymbol{X}) \\ \mathrm{d}\boldsymbol{X'} &= \mathrm{d}\boldsymbol{X} + (\Gamma - 1)\frac{\boldsymbol{V}\cdot\mathrm{d}\boldsymbol{X}}{V^2}\boldsymbol{V} - \Gamma \mathrm{d}T\boldsymbol{V} \end{aligned} \tag{2.13}$$

where Γ and $\boldsymbol{V}$ are respectively the Einstein synchronisation contraction factor and three-velocity; capital letters are also used to signify that the co-ordinates correspond to Einstein synchronisation. The transformation matrix for this transformation can be written as

$$L = \begin{pmatrix} \Gamma & -\Gamma \boldsymbol{V}^t \\ -\Gamma \boldsymbol{V} & I + \frac{\boldsymbol{V}\boldsymbol{V}^t}{V^2}(\Gamma - 1) \end{pmatrix} \tag{2.14}$$

where I is the identity matrix.

For an arbitrary boost from frame S to frame S', with respective choices of synchrony vectors $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}'$, the generalised Lorentz transformation (transforming contravariant components) is

$$\begin{aligned} \mathrm{d}t' &= \gamma[1-(\boldsymbol{\kappa}+\boldsymbol{\kappa}')\cdot\boldsymbol{v}][\mathrm{d}t-\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{x}]+\boldsymbol{\kappa}'\cdot\mathrm{d}\boldsymbol{x} \\ &\quad +[\gamma(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})-1]\frac{\boldsymbol{\kappa}'\cdot\boldsymbol{v}}{v^2}\boldsymbol{v}\cdot\mathrm{d}\boldsymbol{x}-\gamma\boldsymbol{v}\cdot\mathrm{d}\boldsymbol{x} \end{aligned} \tag{2.15}$$

$$\begin{aligned} \mathrm{d}\boldsymbol{x}' &= \mathrm{d}\boldsymbol{x}+[\gamma(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})-1]\frac{\boldsymbol{v}\cdot\mathrm{d}\boldsymbol{x}}{v^2}\boldsymbol{v} \\ &\quad -\gamma(\mathrm{d}t-\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{x})\boldsymbol{v} \end{aligned} \tag{2.16}$$

where $\gamma=\gamma_0(1+\boldsymbol{\kappa}\cdot\boldsymbol{v}_0)$ and $\boldsymbol{v}=\boldsymbol{v}_0/(1+\boldsymbol{\kappa}\cdot\boldsymbol{v}_0)$

An (active) rotation of a frame of reference will not change the synchrony vector $\boldsymbol{\kappa}$ as seen from that frame. Hence the corresponding transformation is then

$$\mathrm{d}t' = \mathrm{d}t+(R^t\boldsymbol{\kappa}-\boldsymbol{\kappa})\cdot\mathrm{d}\boldsymbol{x} \tag{2.17}$$

$$\mathrm{d}\boldsymbol{x}' = R\mathrm{d}\boldsymbol{x} \tag{2.18}$$

where R is the necessary rotation matrix.

Since the spinor formalism gives a powerful and elegant way of expressing and combining (standard) Lorentz boosts[82], it is natural to ask (in light of the above generalisations) whether the spinor formalism can be used to express the Lorentz transformations in arbitrary synchrony. This is addressed in section 2.2.1.

The definition of a four-vector is generalised in reference [33] to a quantity which transforms from one frame to another according to the generalised Lorentz transformation. For consistency, all Lorentz covariant quantities must be similarly generalised.

The generalisation of the Einstein synchronisation components of an arbitrary tensor $\check{T}^{\mu\dots}{}_{\nu\dots}$ is related by

$$T^{\mu\dots}{}_{\nu\dots} = (S_{\boldsymbol{\kappa}})^{\mu}{}_{\psi}\cdot s(S_{-\boldsymbol{\kappa}})^{\phi}{}_{\nu}\cdot s\check{T}^{\psi\dots}{}_{\phi\dots}, \tag{2.19}$$

In any frame with synchrony choice $\boldsymbol{\kappa}$, the contravariant components of a four-vector V^{μ}, are given with respect to the Einstein synchronisation components, $\check{V}^{\mu}$, by

$$\begin{aligned} V^0 &= \check{V}^0+\boldsymbol{\kappa}\cdot\check{\boldsymbol{V}} \\ V^i &= \check{V}^i, \end{aligned} \tag{2.20}$$

while the covariant components are expressible as

$$V_0 = \check{V}_0 = -\check{V}^0$$

$$V_i = \check{V}_i - \kappa_i \check{V}_0 = \check{V}_i + \kappa_i \check{V}^0. \tag{2.21}$$

The metric tensor corresponding to arbitrary synchrony may be written

$$g_{\mu\nu} = \eta_{\mu\nu} - (\eta_{0\mu}\kappa_\nu + \eta_{0\nu}\kappa_\mu + \kappa_\mu\kappa_\nu), \tag{2.22}$$

from which one can obtain the interval, as well as other scalar products. It is easily deduced that all Lorentz scalars are also generalised Lorentz scalars (invariant under generalised restricted Lorentz transformations), and are unaffected numerically by synchrony transformations. Similarly, pseudo and proper four-vectors are also generalised pseudo and proper four-vectors respectively. Similarly, as would be expected, any tensor generalises under arbitrary synchrony with behaviour under parity preserved.

Examining the velocity and momentum four-vectors shows that the generalised acceleration and Minkowski-force four-vectors, $a = \mathrm{d}v/\mathrm{d}\tau$ and $f_M = \mathrm{d}p/\mathrm{d}\tau$ respectively (τ being the (synchrony dependent) proper time of the particle in question), are related by $f_M = m_r a$ (m_r being the invariant rest mass) under arbitrary synchrony, the four-vector a having components

$$a = \{\gamma^4[\boldsymbol{a}\cdot\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{a}\cdot\boldsymbol{\kappa}], \gamma^4[\boldsymbol{a}\cdot\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{a}\cdot\boldsymbol{\kappa}]\boldsymbol{v} + \gamma^2\boldsymbol{a}\}, \tag{2.23}$$

where the generalised three-acceleration $\boldsymbol{a} = \mathrm{d}\boldsymbol{v}/\mathrm{d}t$ is related to the Einstein synchronisation three-acceleration by

$$\boldsymbol{a} = [(1 + \boldsymbol{\kappa}\cdot\boldsymbol{v}_0)\boldsymbol{a}_0 - \boldsymbol{\kappa}\cdot\boldsymbol{a}_0\boldsymbol{v}_0]/(1 + \boldsymbol{\kappa}\cdot\boldsymbol{v}_0)^3. \tag{2.24}$$

It is easily seen that the three-vector parts of both a and f are unaffected, numerically, by synchrony, as is the relativistic three-momentum. One also obtains the synchrony generalisation of the relativistic version of Newton's second law as

$$\boldsymbol{f} = m_r\gamma(\boldsymbol{a} + \gamma^2(\boldsymbol{a}\cdot\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{a}\cdot\boldsymbol{\kappa})\boldsymbol{v}) \tag{2.25}$$

where $\boldsymbol{f} = \gamma^- 1\boldsymbol{f}_M$.

Derivative operators, being basis vectors, must transform like four-vector components:

$$\partial_\mu = \check{\partial}_\mu - \kappa_\mu\check{\partial}_0$$

$$\partial^{\mu} = \check{\partial}^{\mu} - \eta^{0\mu}\kappa_i\check{\partial}^i \tag{2.26}$$

Using these generalised derivative operators, together with the generalisation of the electromagnetic four-vector potential formed from the electromagnetic scalar and three-vector potentials ($A = \{\phi, \boldsymbol{A}\}$), one finds that the laws of electromagnetism are covariant under a synchrony transformation. The generalised four-current density is given by $J^{\mu} = \partial_{\nu}(\partial^{\nu}A^{\mu} - \partial^{\mu}A^{\nu})$ and the charge continuity equation $\partial_{\nu}J^{\mu} = 0$ obtains. Gauge variance of the four-potential is preserved, with gauge transformations taking the form $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\alpha(x)$, as in the Einstein synchrony case (see [19]). The generalised electromagnetic field tensor derived from the four-potential according to $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ is seen to agree with the generalisation of the Einstein synchronisation electromagnetic field tensor obtained by Giannoni[33]. The electric and magnetic fields have the following relation to the corresponding Einstein synchronisation quantities:

$$E^i = \check{E}^i + \epsilon^{ijk}\check{B}_j\kappa_k$$

$$E_i = \check{E}_i \tag{2.27}$$

$$B^i = \check{B}^i \tag{2.28}$$

$$B_i = \epsilon_{ijk}(\frac{1}{2}F^{ij} - E^j\kappa^k)$$

Maxwell's equations are given by $\partial_{\nu}F^{\mu\nu} = J^{\mu}$ and $\partial_{\{\mu}F_{\nu\lambda\}} = 0$ (where $\{\}$ denotes cyclic permutation), just as in the Einstein synchronisation case (see [19]). Giannoni[33] derived a solution to the (generalised) Maxwell's equations in free space. The result represents a plane wave travelling at the generalised velocity ascribed to light beforehand, showing that Maxwell's equations are intimately connected to synchrony assumptions.

Giannoni also generalised the Lorentz force relating electromagnetism to mechanics by examining the behaviour of forces under a synchrony transformation. This generalisation is immediately obtainable from the electromagnetic field tensor, since the expression for the electromagnetic four-force on a particle with charge q and velocity $\check{v}$, $\check{f}^{\mu} = q\check{F}^{\mu\nu}\check{v}_{\nu}$, is covariant and so the Lorentz force law keeps its familiar form:

$$f^i = qE^i + \epsilon^{ijk}v_jB_k. \tag{2.29}$$

It can thus be seen that electromagnetism is fully compatible with non-Einstein synchrony in special relativity; the dynamics of light propagation does not prefer any particular synchrony scheme.

There is no reason to impose the condition that the synchrony vector, $\boldsymbol{\kappa}$, be a constant. Under a general time transformation of the form

$$t \longrightarrow t + h, \tag{2.30}$$

where h is an arbitrary, purely spatial, differentiable function, the time differential transforms according to

$$\mathrm{d}t \longrightarrow \mathrm{d}t + \boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{x}, \tag{2.31}$$

where

$$\boldsymbol{\kappa} \equiv \boldsymbol{\nabla} h. \tag{2.32}$$

2.2.1 Spinors

In their development of the spinor formalism for the expression of the Lorentz transformation (in arbitrary synchrony), Penrose *et al.*[88] began with the future and past null cones at the origin of Minkowskian space-time ($T^2 - X^2 - Y^2 - Z^2 = 0$), and then took constant time slices ($T = \pm 1$) to obtain two unit spheres, $S^{\pm}$. They then proceeded to explain how an observer can project what he has seen onto the "past", S^{+}, sphere—a "sky mapping— and how the "future" S^{-} sphere provides a representation of his field of vision—the "anti-sky mapping". The correspondence between the two spheres is given by the antipodal map ($\boldsymbol{X} \leftrightarrow -\boldsymbol{X}$).

Either sphere can be identified with the Riemannn sphere on an Argand plane, and thus provides a representation of the complex numbers. Penrose *et al.*[88] stated that the properties of the Argand plane and the Riemann sphere reflect many of the geometrical properties of Minkowski vector space, and that a restricted Lorentz transformation of Minkowski space is uniquely determinable by its effect on the Riemann sphere (and thus null directions).

The complex co-ordinate, ζ say, on the Riemann sphere, can be expressed as the ratio of a pair of complex numbers (ξ, η):

$$\zeta = \xi/\eta. \tag{2.33}$$

A transformation of these last two co-ordinates on the Riemann sphere is expressible as the action of a two-dimensional "spin-matrix", A, on a "spin-vector" made up of the complex co-ordinates:

$$\begin{pmatrix} \tilde{\xi} \\ \tilde{\eta} \end{pmatrix} = A \begin{pmatrix} \xi \\ \eta \end{pmatrix} \tag{2.34}$$

The co-ordinates in Minkowski space-time are expressible in terms of ξ and η and their complex conjugates as follows:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} T+Z & X+\mathrm{i}Y \\ X-\mathrm{i}Y & T-Z \end{pmatrix} = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \begin{pmatrix} \bar{\xi} & \bar{\eta} \end{pmatrix} \tag{2.35}$$

And thus a Lorentz transformation in Minkowski space-time can be represented in terms of the spinors:

$$\begin{pmatrix} T+Z & X+\mathrm{i}Y \\ X-\mathrm{i}Y & T-Z \end{pmatrix} \mapsto A \begin{pmatrix} T+Z & X+\mathrm{i}Y \\ X-\mathrm{i}Y & T-Z \end{pmatrix} A^* \tag{2.36}$$

where A^* is the conjugate transpose of A.

The transformations of the spin vectors form the two-dimensional complex group SL(2,$\mathbb{C}$), which form a group of conformal transformations, and thus will map a sphere to a sphere. Now, a Lorentz transformation in Minkowski space-time preserves the null cone, and so will induce the mapping of $S^{\pm}$ onto unit spheres. This mapping is conformal, and thus representable by a spin-transformation, which is in accord with the spin-transformations providing a representation of the Lorentz transformations

However, with an arbitrary synchrony choice in each frame, a constant time slice of the null cone no longer gives a sphere, as can be seen by examining equation (2.22). Although the resulting analogues to $S^{\pm}$ can still be given stereographical projections onto the Riemann plane, the transformations between these analogues (which are induced by generalised Lorentz transformations) are no longer conformal, and so cannot be represented by the spin-transformations. Thus it follows that the spinor formalism cannot be used to represent the synchrony-generalised Lorentz transformations. This situation does not discredit the conventionality of simultaneity; it merely reveals the limitations of the spinor formalism in handling arbitrary synchrony.

The result that spinors cannot be used to describe synchrony transformations was arrived at by Zangari[127] using a calculational approach. Zangari went on to claim, incorrectly, that his result disproved the conventionality of distant simultaneity, on the grounds that the existence of spin-half particles provided empirical proof that the synchrony-vector, $\boldsymbol{\kappa}$, could only take the value $\mathbf{0}$.

The argument that Zangari gave to link his two ideas went as follows: Zangari noted that in relativistic quantum mechanics, spin-half particles are described using the Dirac equation which is neccessarily written in terms of spinors and spin matrices. He reasoned that because the SL(2,$\mathbb{C}$) spinor formalism cannot handle arbitrary synchrony, the Dirac equation is not compatible with arbitrary synchrony which would mean that spin-half particles are not compatible with arbitrary synchrony. Then, he argued, that since the existence of spin-half particles has been empirically confirmed, the conventionality of simultaneity was discredited.

The flaw in Zangari's reasoning is that the Pauli matrices in the SL(2,$\mathbb{C}$) spinor representation of the Lorentz transformation, while being used to represent spin in non-relativistic quantum mechanics, do not provide a representation for the spin-matrices in the Dirac equation. This has been pointed out by Gunn and Vetharaniam[37] who, in a refutation of Zangari's thesis, also generalised the Dirac equation to arbitrary synchrony.

The prescription that was used in reference [37] to generalise the Dirac equation is similar to the standard method (see reference [10]), and is briefly outlined below.

The relativistic energy-momentum relation in arbitrary synchrony is

$$g^{\mu\nu}p_\mu p_\nu - m^2 = 0 \tag{2.37}$$

Factorising this expression gives

$$g^{\mu\nu}p_\mu p_\nu = (\gamma^\mu p_\mu + m)(\gamma^\nu p_\nu - m) = 0, \tag{2.38}$$

from which it follows that

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \tag{2.39}$$

This last equation defines an algebra for the γ^μ.

Arbitrarily chosing the second factor in equation (2.38), and applying the quantisation prescription, $p_\mu \to \mathrm{i}\partial_\mu$, and using it to operate on a Dirac spinor, ψ, gives the field equation for a relativistic, spin-half quantum particle:

$$(\mathrm{i}\hbar\gamma^\mu\partial_\mu - m)\psi = 0 \tag{2.40}$$

So far, the value of $g^{\mu\nu}$ in equation (2.39) has been unspecified, and thus corresponds to an arbitrary co-ordinate system. Now under a change of co-ordinates $x^{\tilde{\mu}} = a^{\tilde{\mu}}{}_\alpha x^\alpha$, the metric changes according to $g^{\tilde{\mu}\tilde{\nu}} = a^{\tilde{\mu}}{}_\alpha a^{\tilde{\nu}}{}_\beta g^{\alpha\beta}$. It follows then that if a solution to equation (2.39) exists in one co-ordinate system, solutions exist for all co-ordinate systems:

$$\gamma^{\tilde{\mu}} = a^{\tilde{\mu}}{}_\alpha \gamma^\alpha \tag{2.41}$$

For the case of arbitrary synchrony, the metric components have the values given by equation (2.22),

$$\gamma^0 = \begin{pmatrix} 1 & -\kappa_i\sigma^i \\ \kappa_i\sigma^i & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{2.42}$$

where the σ^i are the 2×2 Pauli matrices which have the algebra $\sigma^i\sigma^j + \sigma^j\sigma^i = 2\delta^{ij}$ For the case of Einstein synchronisation, equation (2.42) reduces to the standard representation of the Dirac matrices, which will be denoted here by $\bar{\gamma}^\mu$. The relationship between the γ^μ for arbitrary $\boldsymbol{\kappa}$ and those for $\boldsymbol{\kappa} = \mathbf{0}$ is given by

$$\gamma^0 = \bar{\gamma}^0 + \kappa_i\gamma^i, \quad \gamma^i = \bar{\gamma}^i \tag{2.43}$$

Examining equation (2.20) shows that the γ^μ transform like the contravariant components of a four-vector, and since the partial derivatives transform like the covariant components of a four-vector, it follows that the operator in equation (2.40) is invariant under a synchrony change, and thus is always Hermitean under any co-ordinate change; this last condition ensures that the associated eigenvalues are real. It also follows from the operator invariance in equation (2.40) that the Dirac spinor, ψ is unaffected by synchrony changes. This can be explained on geometrical grounds for the reason that ψ does not exist in four-dimensional space-time, but rather exists in a "spinor-space" and is thus independent of space-time considerations such as co-ordinatisation issues. Furthermore, given that spin is an internal property of a particle, and that the conventionality of simultaneity is an external, space-time issue, it is to be expected that they have no bearing on each other.

2.3 Conventionality in measurement

The principle of general covariance played a major role in Einstein's development of the general theory of relativity. However, following the results obtained by Kretschmann[52] that general covariance is a mathematical property of any physical theory, it is now regarded that general covariance has no physical content. From this viewpoint, two opposing positions on the conventionality of simultaneity have been taken in the literature, both based on the ability of time transformations to represent clock synchronisation.

On one hand, there is a conventionalist position that the conventionality of simultaneity is mirrored in the time transformations that are a subset of general co-ordinate transformations, and thus there is no physical content in clock synchronisation. On the other hand, the anti-conventionalist view is that beacuse general covariance imposes no physical conditions on a theory, the conventionality of distant simultaneity is a trivial result of the freedom in co-ordinatisation, and that there is a fully determinable temporal structure in space-time.

However, neither of the above position is cogent. The first fails in the context of a theory (such as Newtonian theory) which allows infinite-speed, round-trip propagation, and thus a means of determining simultaneity in spite of general covariance holding. The second position is unwarranted due to the limitations of what can be measured. Although any determination of space-time structure must be empirically based, conclusions can be drawn from measurements which are necessarily invariant of synchrony, and thus able to support any synchrony choice. This is revealed if analysis is performed in the context of a mathematical formalism which is synchrony-covariant.

The philosophy espoused in this work is that the limitation placed on measurements by the conventionality of distant simultaneity is not simply rooted in co-ordinate freedoms; rather, it is the naturally occurring limitation in comparing distant clocks arising from light being a first signal, and the path dependence of time dilation for moving clocks that provides for under-determination of certain quantities. Co-ordinate freedom merely reflects this, as was demonstrated by Winnie [125], who showed quite clearly that convention and measurement interact.

Winnie used Reichenbach's ϵ characterisation of simultaneity relations which was outlined on page 12. As Winnie pointed out, this freedom in the value of ϵ denies the standard relativistic postulate of invariance of the one-way speed of light (although the round-trip speed is invariant), and also makes the relativity of simultaneity conventional, and by suitable choice of ϵ one can obtain agreement on the simultaneity of events in different frames.

The conventionality of ϵ is not only linked to the conventionality of the one-way speed of light, but leads naturally to the conventionality of relative speeds, time dilation and length contraction effects, as discussed at length by Winnie. Generally, relative speeds need a pair of synchronised clocks for their evaluation, and thus the choice of epsilon used in the actual synchronisation is contained in any such measurements. Attempts to "measure" speeds using just one clock, in actual fact require other assumptions (such as relative lengths, for example) which are conventional. The conventionality of length contraction arises from the requirement that a measurement of the length of a moving object needs knowledge of the positions of the extremities at a single instant - a knowledge which entails a concept of distant simultaneity. Time dilation again requires either synchronised clocks for two separated instantaneous comparisons with a moving clock, or a series of signals sent from the moving clock to a stationary clock, and a (conventional) knowledge of signal travel times.

Given the conventionality of time dilation and length contraction, an interesting and important area of focus is that of the effects that time dilation and length contraction have in special relativity. The experimental verification of such effects is not only regarded as confirming the validity of special relativity, but also considered by many to confirm the standard time dilation and length contraction factors corresponding to Einstein synchronisation. However, by using the ϵ-formalism, Winnie[125] showed that such effects, which are widely concluded as indicating the standard time dilation and length contraction factors—and thus $\epsilon = 1/2$—in fact support any value of ϵ. The reason for this is that while a real, physically-measurable quantity may be expressible as a combination of conventional quantities, that combination will be such that the ϵ dependencies cancel, leaving no nett conventional effect. This invariance under synchrony is hidden unless synchronisation issues are taken into account properly. Thus there is no validity to interpretations that $\epsilon = 1/2$ is uniquely verifiable from evaluations of measurables.

In contrast to these synchrony-invariant effects are those that are dependent on convention. Examples of these are the two basic principles of standard special relativity—the invariance of the (isotropic) one-way speed of light and the reciprocity of relative lengths and velocities, which can be used to obtain the Lorentz transformation. These principles do not hold in a formulation where $\epsilon \neq 1/2$; rather, only weaker conditions will hold. The first principle is replaced by the principle of an invariant round-trip speed of light; Winnie replaced the second principle with his principle of "equal passage-time." This last principle states that two rods of equal rest-length in relative motion will take equal times with respect to the other's frame to pass by a point in the other's frame. Winnie used these new principles to obtain his ϵ-Lorentz transformations which describe a boost, in the x-direction, between two frames, each with independently arbitrary synchrony.

It is not always immediately obvious whether some quantity is synchrony-dependent or not; in this respect the status of spatial measures has been subject to much debate. Given the importance of spatial measurement in physical theory, questions of synchrony have implications in a diverse number of topics which are seemingly unrelated to simultaneity issues. An example of this is the Ehrenfest paradox, which involves spatial measurements along the circumference of a rotating disc. The Ehrenfest paradox has been used by Vargas and Torr[116] in an attempt to discredit the conventionality of simultaneity. This topic is considered in section 2.3.1.

Another topic discussed within the context of synchronisation and simultaneity is to do with the relationship between two concepts: proper length and rest-length.

Anderson and Stedman[6] pointed out that a challenge to the conventionalist position has arisen through the assignation of an operational significance to proper lengths. In a differential-geometric approach, one can foliate space-time using surfaces of simultaneity. One may choose to identify distances in physical space with the four-dimensional space-time intervals, or proper lengths, along surfaces of simultaneity, which is clearly dependent on the choice of surfaces of simultaneity, and thus on the choice of synchronisation. Such a prescription makes a numerical distinction between the "proper length" of a rod, and its rest-length.

Anderson and Stedman noted that contrasting positions have been taken on such an ascription. For example, Reichenbach[97], while supporting the conventionality of simultaneity, considered that the proper length of a measuring rod could only be defined within a simultaneity convention, and indeed defined a charcteristic length of a measuring rod as the proper length associated with Einstein synchronisation. Nerlich[87], also considered the proper length of a rod as conventional and made a similar identification of a rod's rest-length and with the proper length arising from Einstein synchronisation, but, in a shift away from Reichenbach's position, regarded such an identification as picking out Einstein synchronisation as preferred. Later, Coleman and Korte[16] claimed that proper length (which they entitled the "spatial metric induced on a hyperplane of simultaneity") is theoretically dependent on simultaneity convention, but in practice empirically measurable, thus providing an empirical determination of simultaneity relations.

Anderson and Stedman[6, 5] commented that such appeals to proper length, as made by Coleman and Korte[16, 17] in order to motivate a unique simultaneity relation, are in fact misplaced since spatial distances can be defined in a synchrony-independent way, in a manner which is in accordance with the standard approach of general relativity, such as given by Møller[84]. That spatial distances can be defined independently of simultaneity choice had already been noted by Havas[40] who not only used Møller's results, but also imposed a Euclidean spatial metric as a neccessary requirement.

Møller[84] had shown that, in the context of general relativistic space-time, the spatial metric tensor components, γ_{ij}, are given in terms of the space-time metric tensor components, $g_{\alpha\beta}$, by

$$\begin{aligned} \gamma_{ij} &= g_{ij} + \gamma_i \gamma_j, \\ \gamma_i &\equiv g_{i0}/\sqrt{-g_{00}} \end{aligned} \tag{2.44}$$

Now, in the standard (Einstein synchronisation) representation of special relativity, the components of the spatial metric tensor have the same value as the spatial components, $\eta_{ij} = \delta_{ij}$ of the space-time metric tensor. However, in a general co-ordinate system with arbitrary synchrony, the metric tensor components are given by equation (2.22).

If one assumes that, as in the Einstein synchronisation case, the two sets of metric components coincide one introduces an apparent synchrony dependence into spatial measures. The resulting expression for spatial distances in this situation is

$$\mathrm{d}\sigma^2_{|\mathrm{d}t=0} = \mathrm{d}\boldsymbol{x}^2 - (\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{x})^2 \tag{2.45}$$

This quantity is the space-time interval between two events on a plane of constant time, t, and as such should be independent of $\boldsymbol{\kappa}$ for two fixed events. However, the two events in equation (2.45) in fact vary with $\boldsymbol{\kappa}$, thus bringing in an apparent synchrony-dependence of space-time intervals.

On the other hand, the use of Møller's prescription and the substitution of equation (2.22) into equation (2.45) gives the following form for the spatial metric components and metric[40, 6]:

$$\gamma_{ij} = \delta_{ij}, \quad \mathrm{d}s^2 = \mathrm{d}\boldsymbol{x}^2 \tag{2.46}$$

Thus, the traditional and intuitive Euclidean metric obtains for any spatial distances in any synchrony choice, and accords with the operational approach where, in light synchronisation, the spatial separation of two clocks is specified before the clocks are synchronised.

The choice of a synchrony-dependent spatial measure leads to the situation that the numerical value of the distance between two synchronised clocks would change if they were reset using a different synchrony scheme. Such a choice would be untenable if one wished to use devices such as rods for taking spatial measurements, and thus warrants rejection, leaving the synchrony-independent metric, equation (2.46) as the natural choice.

2.3.1 Ehrenfest paradox

Ehrenfest's paradox[21] concerns length contraction effects on a rigid disc which is set in rotation. As seen from an inertial observer at the centre of rotation, the disc's circumference will appear Lorentz contracted since it is moving transversely to the observer. However, the radius of the disc, since it is not in transverse motion with respect to the observer will not appear contracted. Thus the ratio the of disc's circumference to its radius will no longer be 2π, apparently violating Euclidean geometry. Einstein[24] had argued that the paradox was resolvable on the basis that since a rotating disc was non-inertial, its geometry was necessarily non-Euclidean. However, the paradox still remains in the case of an inertial observer viewing the disc, and has been considered by a number of authors offering a variety of solutions, which range from the purely kinematical to the dynamical. (For example see references [12, 13, 34, 101].)

Vargas and Torr[116] made an attempt to discredit the conventionality of simultaneity by claiming that Grøn[34] had resolved the Ehrenfest paradox for the case of special relativity with Einstein synchronisation, and that this resolution is incompatible with other synchrony schemes. Grøn's strategy for resolving the paradox was to show that the motion that would realise contraction of the periphery of the disc is inconsistent with special relativistic kinematics. However, there is a flaw in Grøn's proof, and thus Vargas' and Torr's argument is not cogent.

Now, Grøn[34] stated that a body which is put into motion undergoes a contraction equal to the Lorentz contraction corresponding to its instantaneous velocity only when its acceleration programme keeps the rest length of every part of the body constant. He then considered a dust of n particles (representing the periphery of a disc) moving in a circular path, and found that the acceleration programme that would give circular motion to the dust and at the same time "keep the rest length between neighbouring particles constant, represents kinematically self-contradicting boundary conditions." Thus Grøn found that in the special theory of relativity the motion corresponding to a contraction of the periphery of the disc cannot be realised, apparently resolving the paradox.

However, in his reasoning to obtain a contradiction in the above mentioned acceleration programme, Grøn considered an arbitrary pair of neighbouring dust particles and then made a transformation from "their instantaneous inertial rest frame" to the inertial rest frame of the centre of the disc using the Lorentz transformation. But as Peres[90] had pointed out in a paper concerning clock synchronisation on a rotating disc, no two points on the periphery of a disc would share an instantaneous rest frame since each would have a different velocity. Therefore, Grøn's method of argument is, in itself, inconsistent and so fails to provide a resolution. It then follows that the appeal by Vargas and Torr to Grøn's work as a means of denying the conventionalist stance is without substance.

Peres' note may in itself provide some insight towards a resolution: the usual procedure for predicting the contraction of the circumference of a rotating disc is to consider the length contraction suffered by an infinitessimal section of the circumference and then to integrate through 360 degrees. To evaluate the length contraction of an infinitessimal section of arc in this manner requires the assumption that the entire section is at rest in the same inertial frame, which, as stated above, is false.

On a more speculative note, one might make the observation that since length contraction effects are conventional, any prediction of the contraction of the circumference of a rotating disc would also be conventional, and so dependent on the prescription one used in the analysis.

Chapter 3

Synchrony and experimental tests

3.1 Test-theories of special relativity

Historically, it was the fact that the Lorentz group of transformations were a symmetry group of electrodynamics that led to the acceptance of the validity of the Lorentz transformation for the description of non-gravitational physics in inertial frames. Lorentz showed, in 1904, that the Lorentz group was a symmetry group for Maxwell's equations in vacuum, though he did not consider that group as fundamental to nature[3]. This role for the Lorentz transformation was realised in the following year by Poincaré who showed that electrodynamics was covariant with respect to the Lorentz group[3]. Einstein demanded the fundamentality of the Lorentz group by postulating, first, the principle of relativity: "... that in all coordinate systems in which the mechanical laws are valid, also the same electrodynamical and optical laws are valid ..."[22] and, second, the constancy of the one-way speed of light in vacuum (independent of the motion of the source), in all inertial frames.

The neccessity of Einstein's second postulate has been refuted as far back as 1911, according to Berzi *et al.*[8]. Certainly the relativity principle plus the assumptions of the isotropy and homogeneity of space-time, together with the requirement that the transformations have various group properties, yield the Lorentz and Gallilean transformations as the only possible candidates for relating one inertial frame to another; for example, see references [8, 61, 62, 45, 19]). With this approach, the difference in physics between these two theories manifests itself in the presence of a parameter identified as the limiting speed for matter. The Gallilean transformation results when this limiting speed is taken to be infinite, else the Lorentz transformation obtains. As Lee *et al.*[61] point out, this distinguishes two concepts: the speed of light and the limiting speed of matter.

Using an even more different approach, Lalan[58, 59] derived the Lorentz transformation without using the relativity postulate or reciprocity. Requiring the usual conditions of space-time homogeneity and isotropy of space, Lalan demanded that the transformations formed a parity-invariant group which preserved causality and arrived at the Galilean and Lorentz groups of transformations[59]. In this context, the two kinematics are distinguished, according to Lalan, by the fact that the Galilean transformation forms a three-parameter group while the Lorentz transformation only forms a collection of transformations of one-parameter groups.

Although the assumptions and postulates used in the theoretical derivation of the Lorentz transformation are based on experimental evidence, there has been great interest in using experiments to directly test the Lorentz transformation, invariably by trying to measure conventional quantities. The two separate considerations of synchrony and the validity of special relativity, which are involved in this area, are often regarded in the literature as inseparable. Some authors go so far as to claim that two transformations which share the same kinematics but differ in synchronisation convention represent different theories. The issues involved are discussed in this section.

Formulations of special relativity usually begin with the Lorentz transformation being used to relate any two frames in relative motion. It then follows that the Minkowski metric is an invariant in every frame (if orthonormal bases are used in each frame) and is thus taken as the space-time metric. Therefore, the most natural way to test special relativity is to postulate a parameterised deviation from it, most obviously by relaxing the constraint that the Lorentz transformation links any two frames of reference (and thus denying the invariance of the Minkowski metric under a boost). However, in order to achieve a meaningful test-theory, this arbitrariness must be constrained in some way by imposing enough structure on the theory to allow different frames of reference to be compared, and so letting useful experimental predictions be made. This structure, which is inputed at the beginning, defines a general class of theory, and is taken as axiomatic and used with experiment to cull members of this class. The assumptions made at the outset do have a bearing on the end results, and so any conclusions must be made only within the context of those assumptions.

An attempt to infer the Lorentz transformation from experimental observation was made by Robertson[100] who presented a test-theory of special relativity which motivated the important Mansouri-Sexl test-theory[76, 77, 78]. Robertson started with a postulated rest-system, with a Lorentzian metric, in which light propagated rectilinearly and isotropically. This rest-system was supposed to have "preferred" (convenient) physical properties and all analysis was carried out with reference to this frame when physical predictions were made. Assuming isotropy and homogeneity of space-time, Robertson considered a linear transformation (with unknown parameters) linking the rest-system to an arbitrary moving (laboratory) frame. The number of parameters was reduced by making various physical and operational demands. Values were then found for the remaining parameters (as functions of the relative speed of the two frames) by appealing to the results of the Michelson-Morley, Kennedy-Thorndike, and Ives-Stillwell experiments (all of which are tests of second order in relative velocity).

Robertson's approach suggests the view that the empirical observations from these experiments could be translated into axiomatic statements about kinematical behaviour which would then give exact functional values for the parameters in the arbitrary transformation. Experimental uncertainty and the continuous nature of the parameters were not taken into account to give ranges for the parameter values. In effect, in Robertson's test-theory, observations from a small number of experiments played the same role as high-level, group-property demands in theoretical derivations of the Lorentz transformation. This ideal is generally not espoused in test-theories: the more usual approach is to use different experiments to constrain the range of values that test-theory parameters can take. It should be noted, however, that this latter path was not available to Robertson: the laboratory frame was naturally identified with the Earth, but Robertson did not have a viable candidate for the hypothesised rest-frame. Thus there was no empirical value for the relative velocity of the two frames, and since the other parameters in the theory are functions of this velocity, they are not, in principle, determinable. In this situation, where experimental outcomes could only be predicted to the extent of whether or not they had some velocity dependence (which would show up as sidereal variations because of the Earth's motion), Robertson's idealisation appears to have been the only viable option. (This dilemma was rectified with the discovery of the cosmic microwave background[89] enabling the development of a more sophisticated test-theory, such as that of Mansouri and Sexl[76] discussed later in this section.

In order to reduce the degrees of freedom in the test-theory, Robertson also made the assumption that the one-way speed of light is isotropic in all inertial frames. This assumption is therefore built into the observational derivation of the Lorentz transformation which in turn induces isotropic one-way light propagation in all inertial frames. This circularity has been noted by, amongst others, Vargas[115], and Maciel and Tiomno[70]. Vargas[115] examines inadequacies in Robertson's paper, and revises the Robertson test theory. Vargas claims to obtain the Lorentz transformation using the same three second-order experiments considered by Robertson, but without making any assumptions about convention. However, there is a decidedly conventional assumption in Robertson's work, which is overlooked by Vargas, and in fact included in Vargas' version of Robertson's test theory: it is supposed that light propagates isotropically in the "rest" frame used as a reference for physical analysis of the "moving" frame. The conventionality of simultaneity applies to all frames, even a preferred one, and thus the choice of Einstein synchronisation in the rest frame is conventional. It should be noted that the parameters in the linear transformation linking two frames are dependent on synchrony choices in both the frames, since, for example, contraction factors for one frame, as seen from another, are dependent on the synchronisation conventions of the latter frame. The Lorentz transformation corresponds to Einstein synchronisation in both frames and thus can only be obtained by appeal to experiment if Einstein synchronisation is chosen in both frames. Vargas' equations (4) and (5) of reference [115] are derived in [114] for Einstein synchronisation in the rest frame, the contraction factor, γ, taking on the standard value [114]. Thus at the initial stages, Vargas has introduced an element of conventionality and so claims that his work is free from conventional assumptions are not founded, irrespective of possible synchrony assumptions in the moving frame.

Mansouri and Sexl[76, 77, 78] developed a test-theory which refined and extended Robertson's test-theory, allowing synchrony to be varied in the laboratory frame. They analysed first-order experiments as well as those of second order, discussed the manner in which a variety of experiments constrained the parameters in the theory, and also motivated the experimental comparison of slow clock transport and Einstein synchronisation as a test of special relativity. Indeed, their test-theory has been a popular choice as a foil for special relativity, for example in the analysis of experimental tests as those of Riis *et al.*[98] and of Krisher *et al.*[54].

The Mansouri-Sexl test-theory, like Robertson's, has a rest-system (exhibiting preferred behaviour) which is referred to for the purposes of making prediction, a laboratory frame in which the analysed experiment is carried out and a relative-velocity-dependent transformation between the two systems. However, Mansouri and Sexl went a step further by postulating an aether (or preferred) frame, Σ to serve as the rest-system. Robertson had no motivation for making such an identification; however the cosmic microwave background, discovered in 1965[89], performed this function for Mansouri and Sexl, who substituted a measured value for the velocity of the microwave background into their test-theory, thus eliminating a degree of freedom and enabling them to bound the parameters in their theory empirically.

It was assumed that the equality, in one inertial frame, of measuring devices of differing composition implied their equality in all inertial frames; that there is no preferred direction in Σ; and that the velocity of light is independent of the motion of the source. Homogeneity of space-time implies the linearity of the transformation from Σ to the laboratory frame S. This transformation can be writen in the form

$$\mathrm{d}t = A\mathrm{d}\mathcal{T} + \boldsymbol{E}\cdot\mathrm{d}\boldsymbol{X} \tag{3.1}$$

$$\mathrm{d}\boldsymbol{x} = B(\boldsymbol{\Xi} - \boldsymbol{V}\mathrm{d}\mathcal{T}) \tag{3.2}$$

where the Greek variables represent co-ordinates in Σ and the Roman variables, co-ordinates in S; $\boldsymbol{V}$ is the velocity of S with respect to Σ; A is a time dilation parameter, and B is a length contraction matrix. For the only case Mansouri and Sexl considered (where the x and Ξ axes are co-linear, the y-z and H-Z planes parallel, and the spatial origin of S is moving along the Ξ axis), B is a diagonal matrix: $B = \mathrm{diag}(\beta, \delta, \delta)$. (Note that, for purposes of consistency with other chapters, the notion given here is an adaptation of Robertson's and differs from that used by Mansouri and Sexl who used capital Roman letters for the Σ co-ordinates).

It is assumed that light propagation is isotropic in the aether frame but, unlike Robertson, Mansouri and Sexl make no assumption about the speed of light in the second frame: The vector $\boldsymbol{E}$, which varies with synchrony choice, reflects this. Mansouri and Sexl investigated the functional values $\boldsymbol{E}$ would take

($\boldsymbol{E}_E$ and $\boldsymbol{E}_T$, respectively) under the two most important synchrony schemes, Einstein synchronisation and slow clock transport synchronisation, showing that, in general, the two schemes are not equivalent synchrony methods:

$$\boldsymbol{E}_E = (\frac{-AV}{\beta(1-V^2)}, 0, 0)$$
$$\boldsymbol{E}_T = (\frac{1}{\beta}\frac{\mathrm{d}A}{\mathrm{d}V}, 0, 0) \tag{3.3}$$

The Mansouri-Sexl result for the agreement of slow clock transport synchronisation and Einstein synchronisation that the time dilation parameter must take on its Lorentz transformation value ($A = \sqrt{1-V^2}$) is too restrictive, as they do not consider non-Einstein synchronisation in the aether frame; agreement holds for the time dilation value $\sqrt{1-V^2}/(1-\boldsymbol{\kappa}\cdot\boldsymbol{V})$ where $\boldsymbol{\kappa}$ is a synchrony vector, and $\boldsymbol{V}$ is the Einstein synchronisation velocity of the moving frame, in the aether frame [117]. However, the result that experimental equivalence of these two methods would obtain only if time dilation was special relativistic in its behaviour is a very powerful result in itself. Mansouri and Sexl claim that the various parameters in their transformation can be determined by experiment, and thus that the speed of light is measurable. This is not an accurate claim, for the same reason that Vargas'[115] similar claim is false; there is a degree of conventionality in the isotropy of light propagation in the aether frame[117]. This conventionality is discussed in section 3.2 which gives a reworking of the Mansouri-Sexl test-theory[76] with no synchrony assumptions in either frame explicitly showing the synchrony-independence of any actual physical measurement. For each of the different theories (corresponding to different sets of values for the parameters in the transformation) in the Mansouri-Sexl test class, there exists an equivalence class of theories related by kinematic agreement. Thus the assertion made by Mansouri and Sexl that slow clock transport synchronisation has a preferred status amongst all synchrony schemes is seen to be false, since a measurable result (such as agreement or disagreement between Einstein synchronisation and slow clock transport synchronisation) cannot be used to distinguish members of an equivalence class of theories related by agreement in measurable quantities.

A few comments about the application of tests by Mansouri and Sexl[76] are in order here. In the second paper of their series (on first-order tests), Mansouri and Sexl consider two experiments: the Römer experiment (supposedly a measurement of the one-way speed of light using observations of the moons of Jupiter), and the transverse Doppler effect. In their discussion of the former of these experiments, Mansouri and Sexl reject the work of Karlov[46] (who maintains that the Römer method does not allow the determination of the one-way speed of light) Mansouri and Sexl[77], by explicitly disagreeing with Karlov, regarded their work as re-establishing the observability of the one way-speed of light. In effect, Mansouri and Sexl[77] chose slow clock transport synchrony to be fundamental, and produced an expression for one-way light speed to be tested by Römer type experiments. It might be argued that since slow clock transport synchrony has the potential to disagree with Einstein synchrony in the Mansouri-Sexl framework, its claim to fundamental status has been lessened rather than strengthened as a result of their work. As it is, their choice has been widely accepted and its conventional content suppressed. For example, Krisher *et al.*[98] title their paper "Test of the isotropy of the one-way speed of light ..." and Gabriel and Haugan[32] and Will[122] retain similar terminology.

However, in developing special relativity, Einstein originally set the one-way speed of light to be isotropic by convention, that is, by an appropriate choice of synchronisation scheme. Because of its dependence on this convention the one way speed of light, as opposed to the round trip speed of light, is not observable: Karlov[46, 47], for example, showed that Römer's measurement for the one-way speed related to a round trip speed. This conventionality is equally inescapable in any test theory.

It should be noted here that Karlov on the one hand, and Mansouri and Sexl on the other, look at two different experimental set-ups, as discussed below. In Karlov's version (which is formulated in special relativity), Jupiter and the Sun are at rest in an inertial system. An observatory on Earth receives periodic signals from Jupiter (periodic eclipses of one of Jupiter's moons) as the Earth travels around the sun, and measures the delay between successive signals. The result $\tau_r - \tau_r' \approx d/c$ is deduced, where the left hand side is the cumulative delay of signals (as measured by one who expects a periodicity in their arrival), d is the diameter of the circle described by the Earth, and c is the speed of light. Karlov holds that c is a measure of only the return trip speed of light. Mansouri and Sexl disagree. It is helpful to regard the observer's time measurements as equivalent to those from a network of slow clock transport synchronised clocks, because the Earth is moving slowly with respect to the sun's inertial system (indeed Karlov works only to first order). Then Karlov's version can be seen to predict an apparent one-way speed of light which is isotropic because in special relativity slow clock transport synchronisation gives results equivalent to those of Einstein synchronisation, though this is not a measurement of the one-way speed of light since the choice of slow clock transport synchronisation, like the experimental set-up which induces it, is conventional. In the version considered by Mansouri and Sexl, the Earth is at rest in an inertial frame, moving at a velocity $\boldsymbol{v}$ with respect to the aether. Jupiter and its moons are considered to be a clock, moving in circles around the Earth, sending signals to Earth. Mansouri and Sexl, in their analysis of this situation (reference [77]), use their result for the one-way speed of light resulting from the use of slow clock transport synchronisation (equation 6.16 of [76, part I]): $c(\theta) = 1 - v(1 + 2\alpha)\cos\theta$, where θ is the angle of light direction to the velocity of the frame through the aether, and α is the second-order coefficient in the speed expansion of the time dilation factor. However, in this description all measurements are made using one clock which is stationary in the Earth's frame. Since no separated and synchronised clocks are involved in the procedure, there is no natural synchrony choice, and so Mansouri and Sexl are not justifed in using their equation 6.16 which is obtained explicitly for use with slow clock transport synchronisation. Instead, it would have been more appropriate to use the more general equation 6.15 of [76], [1] which has in it the unknown synchrony parameter ϵ (whose functional dependence on the parameters a, b, and d is governed by synchrony choice). Then their analysis of Römer's experiment would not yield their stated result, unless a synchrony assumption is made. A point to remember is that the even powers of speed expansion performed by Mansouri and Sexl is not valid for non-Einstein synchronisation in the rest frame, since only Einstein synchronisation predicts direction independent effects on one-way trips.

In the second of the first order tests Mansouri and Sexl consider a rotor experiment in which a rotating source and absorber are equidistant from, and on opposite sides of the point of rotation, with which they are co-linear. Mansouri and Sexl modify an equation derived by Møller[83] for the Doppler effect in classical aether theory, adapting it to their theory by replacing the Galilean expression for the speed of light ($c(\theta) = 1 - \boldsymbol{n}\cdot\boldsymbol{v}$) by the expression derived in [76] for the slow clock transport synchronisation speed of light, $c(\theta) = 1 - (1 + \alpha)\boldsymbol{n}\cdot\boldsymbol{v}$ where $\boldsymbol{n}$ is the direction of light propagation from source to absorber, and $\boldsymbol{v}$ the velocity, with respect to the aether, of the centre of rotation. The Mansouri-Sexl

[1] note that in this equation, as well as in equation 6.17 of the same paper, d^2 should be read as d^{-2}

formula $\nu/\nu_0 = 1+2(1+2\alpha)\boldsymbol{u}\cdot\boldsymbol{v}$ (where $\boldsymbol{u}$ is the instantaneous velocity of the absorber) for the ratio of the frequency detected to the frequency emitted predicts a non-null result for transverse Doppler effects unless the time dilation parameter approaches the special relativity value (the special theory of relativity predicting no frequency shift [82], p63). This is of course the right expression to use for the rotor situation since the ends of the rotor, at which measurements are being made, are moving slowly with respect to the inertial frame of the rotor centre. The rotor experiment gives an indication of the validity of special relativity, but cannot determine either the time dilation factor (since this is dependent on the conventional assumption of Einstein synchronisation in the aether frame) or the one-way speed of light in the moving frame, since this is again conventional, the choice of slow clock transport synchronisation being conferred by the experimental set-up. Vargas and Torr[116] question the presentation by Mansouri and Sexl of the rotor experiment and reject it as not being a meaningful first order test, regarding it instead as one of second order. What Vargas and Torr mean by this is unclear. Certainly, the formula corresponding to the test contains a term with two velocity factors, $\boldsymbol{u}\cdot\boldsymbol{v}$, but one of these velocities, $\boldsymbol{u}$, is known and so that term is linear in the unknown aethereal velocity, $\boldsymbol{v}$; experiments would be affected to first order. However, since the term also contains the unknown parameter α, which is the quantity which is really being measured, little information is given about $\boldsymbol{v}$.

In the third paper of the series, Mansouri and Sexl[78] analyse the Kennedy-Thorndike and Michelson-Morley experiments. These second order experiments involve to and fro light trips, and hence synchrony does not enter into the analysis of these experiments. Yet, as in the their analysis of Römer's experiment, Mansouri and Sexl impose a synchrony choice without justification. In this case they choose Einstein synchronisation by using equation 6.17 of [76], thus incorrectly making it appear that measurements of the isotropy of the return trip speed of light gives an indication of the isotropy of the one-way speed of light.

It is interesting to note that the philosophy espoused by Mansouri and Sexl[76] curiously evolved from the starting position of acknowledging the conventionality of simultaneity to the opposing position that the each theory has associated with it a uniquely determinable synchrony convention. The Mansouri and Sexl test-theory cannot be used to give empirical determination of simultaneity relations between spatial points. However, it provides a useful framework for comparing different theories and thus for verifying special relativity. The vector $\boldsymbol{\mathcal{E}}$ compares the different synchronies in the Σ and S frames, and this conventionality excludes its measurement, although it's functional form *within* a synchrony choice can be evaluated. Similarly the parameters A and B, are dependent on the conventionality in Σ and so are determinable only within a synchrony convention. Rather than values for test-theory parameters defining a unique synchrony, the values for the parameters are determined only after a synchrony is defined. This is covered in section 3.2 where the Vetharaniam-Stedman[117] generalisation of the Mansouri-Sexl test-theory to arbitrary synchrony in the aether frame is discussed.

Maciel and Tiomno[70] give a review of some absolute frame theories (such as that of Mansouri and Sexl) developed for testing special relativity. They find many of these theories are, in fact, special relativity "in different coordinate systems" because they agree kinematically with special relativity. Maciel and Tiomno find that this is the case for the Mansouri and Sexl test theory[76], pointing out that Mansouri's and Sexl's "Relativity without Relativity"[76, §4] is the special theory of relativity with a re-synchronisation of clocks. Mansouri and Sexl produce the transformation [76, equation 4.1],

$$\mathrm{d}t = (1 - v^2)^{1/2}\mathrm{d}T \tag{3.4}$$

$$\mathrm{d}x = (1 - v^2)^{-1/2}(\mathrm{d}X - v\mathrm{d}T) \tag{3.5}$$

as corresponding to an aether theory, kinematically equivalent to special relativity. It can easily be seen that this transformation can be reached from the Lorentz transformation

$$\mathrm{d}t = (1 - v^2)^{1/2}\mathrm{d}T - v\mathrm{d}x \tag{3.6}$$

$$\mathrm{d}x = (1 - v^2)^{-1/2}(\mathrm{d}X - v\mathrm{d}T) \tag{3.7}$$

by the time transformation

$$t \rightarrow t + vx \tag{3.8}$$

where v is the Einstein relative speed of the frames. Certainly, to equate the former of the two transformations with an absolute frame theory, and the latter with special relativity is to misunderstand the role of conventionality. Because of the impossibility of determining the one-way speed of light, distant simultaneity is conventional. Absolute simultaneity between two frames in uniform translational motion is neither mandatory for, nor exclusive to, a preferred frame theory with finite light speeds, and is compatible with special relativity. However, Maciel and Tiomno seem to take exception to the use by Mansouri and Sexl of special relativistic values of time dilation and length contraction factors in an aether theory. (See also Maciel and Tiomno[71, 72].) This criticism of Mansouri and Sexl is not justified; for example, the Lorentz aether theory discussed by Erlichson[29] (who distinguishes two types of Lorentz aether theory) uses the Lorentz transformation, although it subscribes to a preferred frame. This work seems to have been ignored by many, such as Spavieri[106], who continued to regard simultaneity conventions as distinguishing theories. The conventionality of the one-way speed of light prevents the distinguishing between special relativity and its equivalence of all inertial frames, and a preferred frame theory kinematically equivalent to special relativity. Mansouri and Sexl also reach the conclusion of the indistinguishability of the two above theories, although for reasons other than the immeasurability of the one-way speed of light. In this light, it appears that at the kinematic level at least, any distinction between special relativity and a kinematically equivalent aether theory is only philosophical. Then the function of the Mansouri-Sexl test-theory is not so much a test for a preferred frame as a test of Lorentz invariance.

Both the Robertson[100] and Mansouri-Sexl[76] test-theories are restricted to comparing frames in uniform motion with respect to a preferred frame. Although this is a satisfactory arrangement for many situations, certain experiments (such as the two-photon absorption experiment, page 69) require a more general framework in order that they be more accurately modelled than the Mansouri-Sexl test-theory would allow. This limitation was recognised by Abolghasem *et al.*[2] who extended the Mansouri-Sexl formalism, deriving a transformation from an inertial, aether frame to a constantly rotating frame. These authors also investigated Einstein synchronisation and slow clock transport synchronisation in rotating frames, arriving at the result that, as is the case in inertial frames, those two synchronisation schemes are equivalent only if special relativity holds true. Such an equivalence does not imply a preferred synchrony scheme for the same reasons that a similar equivalence in inertial frames would fail to establish a unique simultaneity relation: the equivalence of the two holds within any synchronisation scheme, and thus cannot be used to falsify any particular convention.

While the Abolghasem *et al.* theory extends the work of Mansouri and Sexl[76] to rotating frames, it neglects the conventionality of simultaneity in the aether frame, thus again suppressing the conventionality of the test-theory parameters. The test-theory in sectiontestnoninert, developed from differential geometric methods by Vetharaniam and Stedman[119], not only incorporates arbitrary, space-varying synchronisation in all frames, but also allows the to laboratory frame to exhibit arbitrary non-inertial motions. The synchrony extension in this last test-theory sheds light on the operational significance of the various parameters in the Mansouri-Sexl test-theory; the generality of the motions allowed enables more accurate modelling of experiments.

3.2 Generalising the Mansouri-Sexl test-theory

The observables in any test-theory, like those of special relativity, must be independent of clock settings, just as the experimental predictions of any theory with gauge freedom must be gauge-independent.

This disqualifies both the one-way speed of light and time dilation factors involved in a one-way trip as observables in any theory with freedom in simultaneity conventions, since separated clocks are ultimately needed in the measurements of both quantities. Time dilation, for example, requires comparison of a slowly transported clock with another clock whose setting is conventional. In special relativity, slow clock transport and Einstein synchronisation coincide in any synchronisation choice. All experiments based on such a comparison will, according to special relativity confirm the apparent isotropy of the one-way speed of light. As an experimental test of special relativity, this is a highly significant result, particularly in view of the Mansouri-Sexl counter-theory. As a test of isotropy of the speed of light, however, it is an illusion.

According to special relativity, if the one-way speed of light is chosen to be anisotropic, a slowly transported clock suffers a nett time dilation that conspires to hide the anisotropy in the formalism, and thus to deny slow clock transport a fundamental status by negating its apparent ability to determine the "true" (conventional) one-way light speed[125, 121].

The freedom of synchronisation convention is an example of a gravitational (metric) gauge transformation. Einstein synchronisation in special relativity corresponds to the choice of a time-orthogonal metric, the gravitational vector potential g_{i0} being set to zero[84, 4].

Mansouri and Sexl[76] and Mansouri[75] acknowledged the conventionality of synchronisation in a laboratory frame S through the introduction of their parameter ϵ. The logically distinct conventionality of synchronisation in the preferred frame Σ is of equal significance. However, Mansouri and Sexl[76] and subsequent authors simply chose Einstein synchronisation in Σ. While such gauge fixing is perfectly acceptable in analysing experiment, it obscures the conventional content of the formalism, in particular that of the claim to test the isotropy of the one-way speed of light. In fact, within the context of their test-theory, the Mansouri-Sexl ϵ is not purely dependent on synchrony choice in S, but also on that of Σ, as is shown later in this chapter. Thus the Mansouri-Sexl ϵ performs a different function from Reichenbach's ϵ, which is a one-frame synchrony parameter.

The conventionality of the one-way speed of light is shown below by recasting the Mansouri-Sexl test-theory for general synchrony choice in Σ as well as S. This is developed to verify that the results of experiments (for example those [98, 54] which involve a local comparison of synchronisation convention[32]) are not affected by gauge fixing. Hence, just as in special relativity, it is impossible to measure the one-way speed of light appropriate to an arbitrary gauge even when the fundamental status accorded to slow clock transport in the Mansouri-Sexl tradition is accepted; a similar conspiracy operates, as a consequence of the gauge dependence of time dilation, for a slowly transported clock. The observables of a Mansouri-Sexl type test theory are gauge-independent, and include measurement of the round-trip speed of light [76], and tests of reciprocity for the relative speeds of two frames.

Following Mansouri and Sexl[76], consider, within the context of a homogeneous space-time, an aether (or preferred) frame Σ which has co-ordinates $\{\xi^\mu\}=\{\tau,\xi^1,\xi^2,\xi^3\}$, in which there is no experimentally determinable preferred direction, and a frame S ($\{x^\mu\} = \{t,x,y,z\}$) which is in uniform motion with respect to Σ. The assumption of space-time homogeneity restricts any mapping from Σ to S to a transformation linear in spatial and temporal co-ordinates[8]. Further physical conditions imposed by Mansouri and Sexl (in order to restrict the class of theories) are adopted here: It is assumed that the velocity of light is independent of the motion of the source and also that if temporal and spatial measuring devices of differing composition agree in one inertial frame, they will agree in all inertial frames.

The absence of a preferred direction indicates that such a transformation can be expressed in terms of functions of the relative velocity between the two frames:

$$\mathrm{d}t = a\mathrm{d}\tau + \boldsymbol{\epsilon}\cdot\mathrm{d}\boldsymbol{x} \tag{3.9}$$

$$\mathrm{d}\boldsymbol{x} = b(\mathrm{d}\boldsymbol{\xi} - \boldsymbol{v}\mathrm{d}\tau), \tag{3.10}$$

where $\boldsymbol{v}$ is the velocity of S as measured in Σ. The parameter $a = a(\boldsymbol{v}, \boldsymbol{\kappa})$ is a differentiable function which tends to unity when v is zero; $b = b(\boldsymbol{v},\boldsymbol{\kappa})$ is an invertible matrix which equals the identity for zero v; and $\boldsymbol{\epsilon}=\boldsymbol{\epsilon}(\boldsymbol{v},\boldsymbol{\kappa})$ reflects the synchrony choice in S with respect to the synchrony choice in Σ.

Allowing synchrony freedom for the observer in Σ, it is assumed that the one-way speed of light in the direction $\boldsymbol{n}$ in Σ is $\mathcal{C}(\boldsymbol{n}) = 1/(1 + \boldsymbol{\kappa}\cdot\boldsymbol{n})$, as in equation (2.8), where $\boldsymbol{\kappa}$ is a spatially-constant vector of arbitrary magnitude. This definition of the speed of light is reflected in the way clocks are synchronised in Σ, in order to measure the time τ. Now, under a change of synchrony given by $\tilde{\tau} = \tau + \Delta\boldsymbol{\kappa}\cdot\boldsymbol{\xi}$, there will be a corresponding change in synchrony from $\boldsymbol{\kappa}$ to $\tilde{\boldsymbol{\kappa}}$. Thus some of the quantities evaluated in Σ will differ. For example, the new velocity is given by

$$\tilde{\boldsymbol{v}} = \frac{\boldsymbol{v}}{1 + \Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}, \tag{3.11}$$

with the resulting identity

$$1 + \Delta\boldsymbol{\kappa}\cdot\boldsymbol{v} = (1 - \Delta\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}})^{-1} \tag{3.12}$$

By considering equation (3.11) in the limit of light propagation, one sees that

$$\tilde{\mathcal{C}}(\boldsymbol{n}) = \frac{1}{1 + (\boldsymbol{\kappa} + \Delta\boldsymbol{\kappa})\cdot\boldsymbol{n}}, \tag{3.13}$$

and hence

$$\tilde{\boldsymbol{\kappa}} = \boldsymbol{\kappa} + \Delta\boldsymbol{\kappa} \tag{3.14}$$

The way the test-theory parameters change can be seen by considering its transformation equations: since dt and $\mathrm{d}\boldsymbol{x}$ will not change with a change in $\boldsymbol{\kappa}$, from equations (3.9) and (3.10),

$$\begin{aligned} a\mathrm{d}\tau + \boldsymbol{\epsilon}\cdot\mathrm{d}\boldsymbol{x} &\equiv \tilde{a}\mathrm{d}\tilde{\tau} + \tilde{\boldsymbol{\epsilon}}\cdot\mathrm{d}\boldsymbol{x} \\ &= \tilde{a}(\mathrm{d}\tau + \Delta\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{\xi}) + \tilde{\boldsymbol{\epsilon}}\cdot\mathrm{d}\boldsymbol{x} \\ &= \tilde{a}(1 + \Delta\boldsymbol{\kappa}\cdot\boldsymbol{v})\mathrm{d}\tau + (\tilde{a}b^{-1}\mathrm{d}\boldsymbol{x}\cdot\Delta\boldsymbol{\kappa} + \tilde{\boldsymbol{\epsilon}}\cdot\mathrm{d}\boldsymbol{x}). \end{aligned} \tag{3.15}$$

Equating the coefficents of $\mathrm{d}\tau$ and $\mathrm{d}\boldsymbol{\xi}$ on the two different sides gives

$$\tilde{a} = \frac{a}{1 + \Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}} \tag{3.16}$$

and, for any vector $\boldsymbol{p}$,

$$\tilde{\boldsymbol{\epsilon}}\cdot\boldsymbol{p} = \boldsymbol{\epsilon}\cdot\boldsymbol{p} - \frac{ab^{-1}\boldsymbol{p}\cdot\Delta\boldsymbol{\kappa}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}} \tag{3.17}$$

Similarly, using equations (3.9), (3.10) and (3.11),

$$\begin{aligned} b(\mathrm{d}\boldsymbol{\xi} - \boldsymbol{v}\mathrm{d}\tau) &\equiv \tilde{b}(\mathrm{d}\tilde{\boldsymbol{\xi}} - \tilde{\boldsymbol{v}}\mathrm{d}\tilde{\tau}) \\ &= \tilde{b}\left(\mathrm{d}\boldsymbol{\xi} - \frac{\boldsymbol{v}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}(\mathrm{d}\tau + \Delta\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{\xi})\right) \end{aligned} \tag{3.18}$$

Comparing coefficients of the differentials in this equation gives

$$\begin{aligned} b\boldsymbol{v} &= \frac{\tilde{b}\boldsymbol{v}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}, \\ b\mathrm{d}\boldsymbol{\xi} &= \tilde{b}\mathrm{d}\boldsymbol{\xi} - \frac{\tilde{b}\boldsymbol{v}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}\Delta\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{\xi} \end{aligned} \tag{3.19}$$

from which it follows that, for any vector $\boldsymbol{p}$,

$$\tilde{b}\boldsymbol{p} = b\boldsymbol{p} + \Delta\boldsymbol{\kappa}\cdot\boldsymbol{p}\, b\boldsymbol{v} \tag{3.20}$$

$$\tilde{b}^{-1}\boldsymbol{p} = b^{-1}\boldsymbol{p} - \frac{\Delta\boldsymbol{\kappa}\cdot b^{-1}\boldsymbol{p}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}\boldsymbol{v} \tag{3.21}$$

These results are also easily obtainable by expressing $\mathrm{d}\tau$ and $\mathrm{d}\boldsymbol{\xi}$ in terms of $\mathrm{d}t$ and $\mathrm{d}\boldsymbol{x}$, using the test-theory equations—(3.9) and (3.10)—and operating on the corresponding transformation matrix,

$$\frac{1}{a}\begin{pmatrix} 1 & -\boldsymbol{\epsilon}^{\mathrm{t}} \\ \boldsymbol{v} & \begin{matrix} ab^{-1} \\ -\boldsymbol{v}\boldsymbol{\epsilon}^{\mathrm{t}} \end{matrix} \end{pmatrix}, \tag{3.22}$$

with the synchrony transformation matrix in equation (2.9) discussed in section 2.2.

It is useful to relate back these results to the Einstein synchronisation quantities used by Mansouri and Sexl. At a position $\boldsymbol{\xi}$ in Σ, the time τ is related to the Einstein synchronisation time, $\mathcal{T}$, (corresponding to an isotropic light speed) by

$$\tau = \mathcal{T} + \boldsymbol{\kappa}\cdot\boldsymbol{\xi} \tag{3.23}$$

and a velocity $\boldsymbol{v}$, measured using τ is related to the velocity $\boldsymbol{V}$, corresponding to T, by

$$\boldsymbol{v} = \frac{\boldsymbol{V}}{1+\boldsymbol{V}\cdot\boldsymbol{\kappa}}. \tag{3.24}$$

In this notation, lower-case letters correspond to quantities dependent on general $\boldsymbol{\kappa}$ and upper-case to the conventional $\boldsymbol{\kappa}=\boldsymbol{0}$ case.

From equations (3.16), (3.20) and (3.17) the parameters in equations (3.9) and (3.10) can be written in terms of the terms of the $\boldsymbol{\kappa}=\mathbf{0}$ values used by Mansouri and Sexl:

$$
\begin{aligned}
a &= \frac{A}{1+\boldsymbol{\kappa}\cdot\boldsymbol{V}}, && (3.25)\\
b &= B\iota, && (3.26)\\
\iota\boldsymbol{p} &= \boldsymbol{p}+\boldsymbol{\kappa}\cdot\boldsymbol{p}\,\boldsymbol{V}, &&\\
\iota^{-1}\boldsymbol{p} &= \boldsymbol{p}-\frac{\boldsymbol{\kappa}\cdot\boldsymbol{p}}{1+\boldsymbol{\kappa}\cdot\boldsymbol{v}}\boldsymbol{V}, &&\\
\boldsymbol{\epsilon}\cdot\boldsymbol{p} &= \boldsymbol{E}\cdot\boldsymbol{p}-\frac{AB^{-1}\boldsymbol{p}\cdot\boldsymbol{\kappa}}{1+\boldsymbol{\kappa}\cdot\boldsymbol{V}}, && (3.27)
\end{aligned}
$$

Note that there are misprints in equations (6) and (7) of Vetharaniam*et al.*[117] where these results were presented; also note the difference in convention for the sign of $\boldsymbol{\kappa}$ in that paper.

One of the main considerations that Mansouri and Sexl[76] made was how slow clock transport synchronisation and Einstein synchronisation compared within the test-theory. Within special relativity, these two methods result in the same simultaneity relations; in general, however they give rise to distinct synchronisation schemes. Mansouri and Sexl showed this by deriving the corresponding values for the synchrony parameter in S. Their derivation is adapted below to the more general case being discussed here, where the synchrony in Σ is not assumed.

In the case of slow clock transport synchronisation, two separated clocks are synchronised using a third clock which is moved from one to the other at an infinitesimally small velocity. Consider a clock at an arbitrary point, P, with position vector $\boldsymbol{P} = P\hat{\boldsymbol{p}}$ in S. Suppose that this clock is synchronised with a master clock at the origin in S, using a slowly moving clock which has constant velocity and which first passes through the origin and which is set to the master clock at that event. Let the "moving" clock be at the origin, O', of an inertial frame S', and assume that at $t' = t = \tau = 0$, the spatial origins of S', S and Σ coincide.

Now, even though slow clock transport is being used for the purposes of defining a synchronisation in S, some synchrony choice must be initially made in S, in order that the velocity of the clock can be measured — to ensure that it is moving "slowly," (which is gauged within the initial synchronisation). Choose $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_{\mathrm{i}}$, and denote the velocity of S' in S by $\boldsymbol{u}_{\boldsymbol{\epsilon}_{\mathrm{i}}} = u_{\boldsymbol{\epsilon}_{\mathrm{i}}}\hat{\boldsymbol{p}}$. Let S' move at velocity

$\boldsymbol{w}$ in Σ, so from equation (3.9),

$$\mathrm{d}t'|_{\boldsymbol{x}'=\boldsymbol{0}} = a(\boldsymbol{w})\mathrm{d}\tau \tag{3.28}$$

Now, in S, S' has the equation of motion

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{u}_{\epsilon_\mathrm{i}}\mathrm{d}\boldsymbol{t}_{\epsilon_\mathrm{i}}, \tag{3.29}$$

so comparing the motions of S' in S and Σ using equations (3.9) and (3.10),

$$\begin{aligned}
\mathrm{d}t_{\epsilon_\mathrm{i}} &= \frac{a(\boldsymbol{v})\mathrm{d}\tau}{1-\boldsymbol{\epsilon}_\mathrm{i}\boldsymbol{\cdot}\boldsymbol{u}_{\epsilon_\mathrm{i}}} \\
\Longrightarrow \quad (b(\boldsymbol{v}))\,\mathrm{d}\boldsymbol{\xi} &= (b(\boldsymbol{v}))\,\boldsymbol{v}\mathrm{d}\tau + \frac{\boldsymbol{u}_{\epsilon_\mathrm{i}}a(v)\mathrm{d}\tau}{1-\boldsymbol{\epsilon}_\mathrm{i}\boldsymbol{\cdot}\boldsymbol{u}_{\epsilon_\mathrm{i}}} \\
&\approx (b(\boldsymbol{v}))\,\boldsymbol{v}\mathrm{d}\tau + u_{\epsilon_\mathrm{i}}a(v)\mathrm{d}\tau
\end{aligned} \tag{3.30}$$

where the last step holds for small u_{ϵ_i}. Then it follows that for $\boldsymbol{w} \equiv \mathrm{d}\boldsymbol{\xi}/\mathrm{d}\tau$

$$\boldsymbol{w} \approx \boldsymbol{v} + a(\boldsymbol{v})\left(b^{-1}(\boldsymbol{v})\right)\boldsymbol{u}_{\epsilon_\mathrm{i}}, \tag{3.31}$$

$$w \approx v + a(\boldsymbol{v})\left(b^{-1}(\boldsymbol{v})\right)\boldsymbol{u}_{\epsilon_\mathrm{i}}\boldsymbol{\cdot}\,\hat{\boldsymbol{v}}, \tag{3.32}$$

using the binomial theorem to make a further approximation in the last step.

The clock at P is reset such that $t(P) = t'(O')$ when O' coincides with P in S. The new time, $t(P)$, corresponds to slow clock transport synchronisation, denoted here by ϵ_T; integrating equations (3.9) and (3.29), using $\boldsymbol{u}_{\epsilon_\mathrm{i}}\mathrm{d}\boldsymbol{t} = \mathrm{d}\boldsymbol{x}$, and noting that the space-time origins of all three frames coincide, one obtains for the point, P

$$\begin{aligned}
t = \frac{a(\boldsymbol{v})}{1-\boldsymbol{\epsilon}_\mathrm{T}\boldsymbol{\cdot}\boldsymbol{u}_{\epsilon_\mathrm{i}}}\tau &= a(\boldsymbol{w})\tau = t' \\
\Longrightarrow \quad \boldsymbol{\epsilon}_\mathrm{T}\boldsymbol{\cdot}\hat{\boldsymbol{p}} &= \frac{a(\boldsymbol{w})-a(\boldsymbol{v})}{u_{\epsilon_\mathrm{i}}a(\boldsymbol{w})}
\end{aligned} \tag{3.33}$$

The time dilation parameter,$a(\boldsymbol{v}) \equiv a(\boldsymbol{v},\boldsymbol{\kappa})$, is dependent on both the magnitude and direction of $\boldsymbol{v}$ with respect to $\boldsymbol{\kappa}$, where $\boldsymbol{\kappa}$ is considered as arbitrary but fixed. Since $\boldsymbol{v}$ already contains $\boldsymbol{\kappa}$-dependence, $a(\boldsymbol{v})$ can be considered as a function of v and $\boldsymbol{\kappa}\boldsymbol{\cdot}\hat{\boldsymbol{v}}$. Similarly, $a(\boldsymbol{w})$ may be considered as function of $\boldsymbol{w}$ and $\boldsymbol{\kappa}\boldsymbol{\cdot}\hat{\boldsymbol{w}}$.

Now, from equations (3.31) and 3.32, the following approximation can be made when $\boldsymbol{u}_{\epsilon_\mathrm{i}}$ is small:

$$\begin{aligned}
\boldsymbol{\kappa}\boldsymbol{\cdot}\hat{\boldsymbol{w}} &\approx \frac{\boldsymbol{v}\boldsymbol{\cdot}\boldsymbol{\kappa} + a(\boldsymbol{v})(b^{-1}(\boldsymbol{v}))\boldsymbol{u}_{\epsilon_\mathrm{i}}\boldsymbol{\cdot}\boldsymbol{\kappa}}{v + a(\boldsymbol{v})(b^{-1}(\boldsymbol{v}))\boldsymbol{u}_{\epsilon_\mathrm{i}}\boldsymbol{\cdot}\hat{\boldsymbol{v}}} \\
&\approx \boldsymbol{\kappa}\boldsymbol{\cdot}\hat{\boldsymbol{v}} + a(\boldsymbol{v})\frac{u_{\epsilon_\mathrm{i}}}{v}\left(b^{-1}(\boldsymbol{v})\right)\hat{\boldsymbol{p}}\boldsymbol{\cdot}\left(\boldsymbol{\kappa} - \boldsymbol{\kappa}\boldsymbol{\cdot}\hat{\boldsymbol{v}}\hat{\boldsymbol{v}}\right).
\end{aligned} \tag{3.34}$$

Using this result and equation (3.32) to make a first-order Taylor's expansion of $a(w, \boldsymbol{\kappa}\cdot\hat{\boldsymbol{w}})$ about $(v, \boldsymbol{\kappa}\cdot\hat{\boldsymbol{v}})$, and writing $q \equiv \boldsymbol{\kappa}\cdot\hat{\boldsymbol{v}}$, one obtains

$$\begin{aligned} a(w, \boldsymbol{\kappa}\cdot\hat{\boldsymbol{w}}) \approx\ & a(v, \boldsymbol{\kappa}\cdot\hat{\boldsymbol{v}}) + a(\boldsymbol{v}) \left(b^{-1}(\boldsymbol{v})\right) \boldsymbol{u}_{\epsilon_{\mathrm{i}}}\cdot \hat{\boldsymbol{v}} \frac{\partial a(\boldsymbol{v})}{\partial v} \\ & + a(\boldsymbol{v}) \frac{u_{\epsilon_{\mathrm{i}}}}{v} \left(b^{-1}(\boldsymbol{v})\right) \hat{\boldsymbol{p}}\cdot (\boldsymbol{\kappa} - \boldsymbol{\kappa}\cdot\hat{\boldsymbol{v}}\hat{\boldsymbol{v}})) \frac{\partial a(\boldsymbol{v})}{\partial q}. \end{aligned} \tag{3.35}$$

Substituting this into equation (3.33) gives

$$\begin{aligned} \epsilon_{\mathrm{T}}\cdot\hat{\boldsymbol{p}} \approx\ & \left[\left(b^{-1}(\boldsymbol{v})\right) \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{v}} \frac{\partial a(\boldsymbol{v})}{\partial v} + \frac{1}{v} \left(b^{-1}(\boldsymbol{v})\right) \hat{\boldsymbol{p}}\cdot (\boldsymbol{\kappa} - q\hat{\boldsymbol{v}})) \frac{\partial a(\boldsymbol{v})}{\partial q}\right] \\ & \times \left[1 + \left(b^{-1}(\boldsymbol{v})\right) \boldsymbol{u}_{\epsilon_{\mathrm{i}}}\cdot \hat{\boldsymbol{v}} \frac{\partial a(\boldsymbol{v})}{\partial v} + \frac{u_{\epsilon_{\mathrm{i}}}}{v} \left(b^{-1}(\boldsymbol{v})\right) \hat{\boldsymbol{p}}\cdot (\boldsymbol{\kappa} - q\hat{\boldsymbol{v}})) \frac{\partial a(\boldsymbol{v})}{\partial q}\right]^{-1}. \end{aligned}$$

For slow clock transport synchronisation, $\boldsymbol{u}_{\epsilon_{\mathrm{i}}}$ has infinitesimally small magnitude, and so can be ignored in the above equation, which then yields the following definition for ϵ_{T}:

$$\epsilon_{\mathrm{T}}\cdot\boldsymbol{p} \approx b^{-1}\boldsymbol{p}\cdot\hat{\boldsymbol{v}} \frac{\partial a}{\partial v} + b^{-1}\boldsymbol{p}\cdot \frac{\boldsymbol{\kappa} - q\hat{\boldsymbol{v}}}{v} \frac{\partial a}{\partial q}, \tag{3.36}$$

where $\boldsymbol{p}$ is an arbitrary vector. Note that this result is independent of the initial synchronisation corresponding to ϵ_{i} in S, since a, b, $\boldsymbol{\kappa}$ and $\boldsymbol{v}$ are measured in Σ. Despite the $\boldsymbol{\kappa}$ dependence of ϵ_{T}, the actual synchrony convention resulting in S is independent of $\boldsymbol{\kappa}$ as will be shown later on page 57. There is no inconsistency in this situation since ϵ is not a synchrony vector playing the same role as $\boldsymbol{\kappa}$, but rather compares the synchronies in the two frames. This is discussed on page 99.

The most commonly used synchronisation is Einstein synchronisation, where the assumption of an isotropic one-way speed of light is made in the synchronisation of clocks using light signals. In the frame S, consider a clock at a point P which has a position vector $\boldsymbol{P} = P\hat{\boldsymbol{p}}$. If this clock is Einstein synchronised with a clock at the origin O, of S. Suppose the light signal is sent from O at time t_1, and that it is received at P at t_2, whence it is immediately reflected back to O, being recieved back at O at time t_3.

Now, by integrating equations (3.9) and (3.10), one can relate the co-ordinates in S of these three events to their respective co-ordinates in Σ:

$$\begin{aligned} \{t_1, \boldsymbol{0}\} &= \{a\tau_1, b(\boldsymbol{\xi_1} - \boldsymbol{v}\tau_1)\}, \\ \{t_2, \boldsymbol{P}\} &= \{a\tau_2 + \epsilon_{\mathrm{E}}\cdot\boldsymbol{P}, b(\boldsymbol{\xi_2} - \boldsymbol{v}\tau_2)\}, \\ \{t_3, \boldsymbol{0}\} &= \{a\tau_3, b(\boldsymbol{\xi_3} - \boldsymbol{v}\tau_3)\} \end{aligned} \tag{3.37}$$

where ϵ_{E} denotes the choice, in S, of Einstein synchronisation, which results from setting

$$\begin{aligned} t_2 - t_1 &= 1/2(t_3 - t_1) \\ \Longrightarrow \quad P\epsilon_{\mathrm{E}}\cdot\hat{\boldsymbol{p}} &= \frac{1}{2}a\left((\tau_3 - \tau_2) - (\tau_2 - \tau_1)\right) \end{aligned} \tag{3.38}$$

Denote $\boldsymbol{\mathcal{C}}_+$ as the velocity of light in Σ in the journey from $\boldsymbol{\xi_1}$ to $\boldsymbol{\xi_2}$, and $\boldsymbol{\mathcal{C}}_-$ as the velocity from $\boldsymbol{\xi_2}$ to $\boldsymbol{\xi_3}$ (with τ_+ and τ_- being the corresponding travel times). Then, using equations (3.37)

$$\mathcal{C}_\pm\tau_\pm = \boldsymbol{v}\tau_\pm \pm \boldsymbol{P} \tag{3.39}$$

from which, by substituting the definition (2.8) for $\mathcal{C}$,

$$1 + \boldsymbol{\kappa}\cdot\frac{\boldsymbol{v}\tau_\pm \pm b^{-1}\boldsymbol{P}}{|\boldsymbol{v}\tau_\pm \pm b^{-1}\boldsymbol{P}|} = \frac{\tau_\pm}{|\boldsymbol{v}\tau_\pm \pm b^{-1}\boldsymbol{P}|} \tag{3.40}$$

From here, solving for $\tau_\pm$ gives two quadratics, which when compared give the following relation containing the difference between τ_- and τ_+:

$$\begin{aligned} &(\tau_-^2 - \tau_+^2)\left((1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2\right) \\ &+2(\tau_- - \tau_+)\left((1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa} + \boldsymbol{v}\right)\cdot b^{-1}\boldsymbol{P} = 0 \end{aligned} \tag{3.41}$$

Thus, expressing $\tau_\pm$ in terms of time differences, and substituting the preeceeding result into equation (3.38), and noting that P can be varied, one gets a result for ϵ_{E}:

$$\epsilon_{\mathrm{E}}\cdot\boldsymbol{p} = -ab^{-1}\boldsymbol{p}\cdot\frac{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa} + \boldsymbol{v}}{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2} \tag{3.42}$$

where $\boldsymbol{p}$ is any arbitrary vector.

Comparing equations (3.36) and (3.42) shows that in general, slow clock transport synchronisation and Einstein synchronisation produce totally distinct simultaneity conventions. Equating the two corresponding values of ϵ allows one to investigate of the conditions under which these two synchronising procedures produce equivalent results. Substituting (3.36) and (3.42) into $\boldsymbol{\epsilon_T} = \epsilon_{\mathrm{E}}$ gives

$$b^{-1}\boldsymbol{p}\cdot\left(\hat{\boldsymbol{v}}\frac{\partial a}{\partial v} + \frac{\boldsymbol{\kappa} - q\hat{\boldsymbol{v}}}{v}\frac{\partial a}{\partial q} + a\frac{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa} + \boldsymbol{v}}{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2}\right) = 0 \tag{3.43}$$

Since $b^{-1}\boldsymbol{p}$ is common to each term in the above equation, and since it is arbitrary and in general not the zero-vector (since $\boldsymbol{p}$ is arbitrary), the equality in the above equation holds only if the vector sum inside the brackets is zero. Then, because the directions of $\boldsymbol{\kappa}$ and $\boldsymbol{v}$ are independent, the coefficients of those two vectors separately add to zero. Writing $\boldsymbol{\kappa}\cdot\hat{\boldsymbol{v}} \equiv q$,

$$\begin{aligned} \frac{\partial a}{a} &= -\frac{q(1-qv)+v}{(1-qv)^2-v^2}\partial v, \\ \frac{\partial a}{a} &= -\frac{(1-qv)v}{(1-qv)^2-v^2}\partial q \end{aligned} \tag{3.44}$$

By taking partial sums of fractions in these differential equations one finds that the most general solution to a for the equivalence of slow clock transport synchronisation and Einstein synchronisation is

$$a_{\mathrm{TE}} = \sqrt{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2} \tag{3.45}$$

This value for a is the special relativistic form for time dilation, showing precisely the $\boldsymbol{\kappa}$ dependence obtained when special relativity is considered in a general gauge as considered in chapter 2.2.

Many authors, following Mansouri and Sexl, conclude that if, as all experimental evidence indicates, special relativity holds and slow clock transport and Einstein synchronisation give equivalent results, then the time dilation parameter has been determined to have a unique value based on the agreement of these different synchronisation schemes. However, because of the synchrony dependence of such a conclusion, this can hold only after the gauge in Σ is fixed (usually one takes $\boldsymbol{\kappa}=\mathbf{0}$, with the result that $a = \sqrt{1-v^2}$).

Thus, the equivalence of Einstein synchronisation and slow clock transport synchronisation cannot uniquely define that synchrony with an isotropic one-way speed of light as correct to the falsification of all other conventions. However the experimental comparison of these two schemes can serve to falsify theories: as equation (3.45) indicates, agreement of those schemes discards all theories whose time dilation behaviour is not in accord with special relativity, which is a potentially powerful result.

A general expression for the speed of light, within the formalism of this test-theory, shows that there are both conventional and theory-dependent, non-conventional components in light propagation; the latter being revealed only in round-trip propagation. Furthermore, the operational significance of the vector, $\boldsymbol{\epsilon}$, is revealed by such an expression which can be obtained as follows.

Consider a light ray propagating from the origin of S to a point P with position vector $\boldsymbol{P} = P\hat{\boldsymbol{p}}$. Using equations (3.37) to transform the propagation time, $t_2 - t_1$, in S to the corresponding interval $\tau_2 - \tau_1 \equiv \tau_+$ in Σ, one sees that the velocity, in S, of this ray is given by

$$1/c(\hat{\boldsymbol{p}}) = a\tau_+/P + \boldsymbol{\epsilon}\cdot\hat{\boldsymbol{p}} \tag{3.46}$$

where $\boldsymbol{\epsilon}$ is arbitrary. Solving for τ_+ in equation (3.40), one has

$$\begin{aligned}\left((1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2\right)\tau_+ \;&=\; b^{-1}\boldsymbol{P}\cdot((1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}+\boldsymbol{v}) \\ &+ \left(\begin{array}{c}[(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2]\,[(b^{-1}\boldsymbol{P})^2 - (b^{-1}\boldsymbol{P}\cdot\boldsymbol{\kappa})^2] \\ +\,[b^{-1}\boldsymbol{P}\cdot((1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}+v)]^2\end{array}\right)^{\frac{1}{2}}\end{aligned} \tag{3.47}$$

Substituting this back into equation (3.46) gives the one-way speed of light along an arbitrary direction, $\hat{\boldsymbol{p}}$, in S as

$$1/c(\hat{\boldsymbol{p}}) \;=\; \boldsymbol{\epsilon}\cdot\hat{\boldsymbol{p}} + a[d\gamma^2 + k], \tag{3.48}$$

$$\begin{aligned} d(\boldsymbol{p}) \;&\equiv\; b^{-1}\hat{\boldsymbol{p}}\cdot(\boldsymbol{v}+(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}), \\ 1/\gamma^2 \;&\equiv\; (1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2, \\ k^2(\boldsymbol{p}) \;&\equiv\; d^2\gamma^4 - \gamma^2[(b^{-1}\hat{\boldsymbol{p}}\cdot\boldsymbol{\kappa})^2 - (b^{-1}\hat{\boldsymbol{p}})^2]. \end{aligned} \tag{3.49}$$

Now, the appearance of $\boldsymbol{\kappa}$ in this expression may be mistakenly interpreted as showing that the one-way speed of light in S is necessarily $\boldsymbol{\kappa}$-dependent. Such a dependence would in principle, within the framework of their test-theory, enable a particular value of $\boldsymbol{\kappa}$, in Σ, to be singled out by experiment in S. Such an event, on the one hand, might suggest that the conventionality of simultaneity in Σ could be falsified (since experiment in S need not refer to Σ). On the other hand, since synchrony choices in Σ and S should not influence each other,this $\boldsymbol{\kappa}$-dependence of c might be used to discredit all values of a and b except for those whose functional form causes a cancelling of this $\boldsymbol{\kappa}$-dependence. However, equation (3.48) is intriniscally independent of $\boldsymbol{\kappa}$, as it must be since it is measured in S. That this holds mathematically can be seen by substituting equations (3.25), (3.26) and (3.27) into (3.48) and performing a tedious calculation to give an expression in which all quantities have their values at $\boldsymbol{\kappa} = \mathbf{0}$.

$$1/c(\hat{\boldsymbol{p}}) \;=\; \boldsymbol{E}\cdot\hat{\boldsymbol{p}} + A[D\Gamma^2 + K], \tag{3.50}$$

$$\begin{aligned} D \;&\equiv\; B^{-1}\hat{\boldsymbol{p}}\cdot\boldsymbol{V} \\ 1/\Gamma \;&\equiv\; 1 - V^2 \\ K^2 \;&\equiv\; D^2\Gamma^4 + \Gamma^2(B^{-1}\hat{\boldsymbol{p}})^2. \end{aligned} \tag{3.51}$$

Hence the speed of light in S, for arbitrary synchrony in Σ, is expressible in a form dependent on only the parameters corresponding to Einstein synchrony in Σ. The fact that this expression is equal to the expression for the one-way speed of light in S written in terms of arbitrary $\boldsymbol{\kappa}$, does not give Einstein synchronisation a preferred status; rather it shows that that expression takes the same value for all synchronies. Thus numerical determination of the one-way speed of light in S (say by reference to the choice of slow clock transport synchrony in S) cannot then favour a particular choice of synchrony in Σ: the numerical value of $c(\boldsymbol{p})$ is independent of the choice of synchrony in Σ, except for when an "external synchronisation" scheme is adopted in S where clocks in S are synchronised by comparing them to clocks in Σ.

External synchronisation was discussed by Mansouri and Sexl[76, p. 502 and §4] who concentrated, in particular, on absolute simultaneity resulting from the choice $\boldsymbol{\epsilon} = \mathbf{0}$. With this latter scheme, observers in both frames would agree on whether two events were simultaneous or not; however they would not generally agree on the equivalence of time intervals. Mansouri and Sexl considered the case of special relativistic kinematics combined with absolute simultaneity to be a distinct theory from the same kinematics combined with Einstein synchronisation, rather than the same theory in two different gauges. The various positions that they and others took on this issue were discussed on pages 46–47.

The particular results for the one-way speed of light in S arising from differing synchrony conventions are easily obtained by substituting the corresponding values of $\boldsymbol{\epsilon}$ into equation (3.48). In particular, when Einstein synchronisation is chosen, $\boldsymbol{\epsilon}_{\mathrm{E}}$ takes on the value $-a\boldsymbol{d}\gamma^2$, cancelling out any linear dependence on direction. Thus the sense in which a light ray moves along a line does not affect its velocity. However, there is still a degree of direction dependence, since the remaining term, ak, contains squares of direction-dependent terms, as can be seen from equation (3.49). As Mansouri and Sexl[76] showed, with Einstein synchronisation, the degree of this direction-dependence is governed by the length contraction factors and is independent of a.

However, the opposite is true if clock transport is used: Mansouri and Sexl give, to first order in velocity, an equation for the one-way speed of light in S which results from the use of slow clock transport synchronisation. This expression, [76, equation (6.16)] takes the form

$$c(\theta) = 1 - V(1 + 2A_2)\cos\theta \tag{3.52}$$

where A_2 is coefficient of V^2 in a velocity expansion of A, and θ is the angular deviation from the X-axis. This form shows a potential direction-dependence of the speed of light, and Mansouri and Sexl conlude from here that the "one-way velocity of light is a measurable quantity in this case". However, any measurement in this situation is only within the prior assumption of slow clock transport synchronisation, and can test only for the degree of isotropy conferred by slow clock transport.

Furthermore, in contrast to the $\boldsymbol{\kappa}$-independence of light velocity in S, both the time dilation factor, a, and the length contraction factors in b, all of which apply to S, depend explicitly on $\boldsymbol{\kappa}$, and thus cannot be determined independently of the synchrony choice of $\boldsymbol{\kappa}$ in Σ. Thus no measurement can be used to restrict the time dilation parameter further than an infinite class of special relativistic values: experimental constraints can only limit synchrony-dependent parameters to an equivalence class of functions. Further selection is possible only if a particular convention is assumed. There is also an important sense in which (3.50) is inapplicable to an observer who chooses a non-standard synchronisation in Σ, for the quantities a and b, rather than A and B, are those obtained from his measurements.

Since experiments in the laboratory frame, S, often involve other moving frames, it is useful to consider how quantities transform when a third frame is added to the test-theory. Consider a frame S' which is moving with velocity $\boldsymbol{u}$ with respect to with respect to S, and moving with velocity $\boldsymbol{w}$ in Σ. The form of the transformation from Σ to S' takes the same form as equations (3.9) and (3.10), with t, $\boldsymbol{x}$ and $\boldsymbol{v}$ being replaced by t', $\boldsymbol{x}'$ and $\boldsymbol{w}$ respectively. Since $\boldsymbol{u} \equiv \mathrm{d}\boldsymbol{x}/dt$ and $\boldsymbol{w} \equiv \mathrm{d}\boldsymbol{\xi}/\mathrm{d}\tau$,

$$\boldsymbol{u} = b(\boldsymbol{v})\frac{\boldsymbol{w} - \boldsymbol{v}}{a(\boldsymbol{v})}(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}), \tag{3.53}$$

from which it follows that

$$\boldsymbol{w} = \frac{a(\boldsymbol{v})}{1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}}(b^{-1}(\boldsymbol{v}))\boldsymbol{u} + \boldsymbol{v}. \tag{3.54}$$

Note that $\boldsymbol{w}$ is invariant with respect to the choice of $\boldsymbol{\epsilon}$ and thus is independent of the synchrony in S, since $\boldsymbol{w}$ is measured in Σ. Mathematically, $\boldsymbol{u}$ has the appropriate dependence on $\boldsymbol{\epsilon}$ to bring about this independence: let $\tilde{t}$ correspond to $\tilde{\boldsymbol{\epsilon}}$, then $\mathrm{d}\tilde{t} = \mathrm{d}t + (\tilde{\boldsymbol{\epsilon}} - \boldsymbol{\epsilon})\cdot\mathrm{d}\boldsymbol{x}$, and so

$$\frac{u}{1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}} = \frac{(\mathrm{d}\boldsymbol{x}/\mathrm{d}\tilde{t})\,(\mathrm{d}\tilde{t}/\mathrm{d}t)}{1 - \boldsymbol{\epsilon}\cdot(\mathrm{d}\boldsymbol{x}/\mathrm{d}\tilde{t})\,(\mathrm{d}\tilde{t}/\mathrm{d}t)}$$

$$= \frac{\tilde{\boldsymbol{u}}/(1-(\tilde{\epsilon}-\epsilon\cdot\tilde{\boldsymbol{u}})}{1-\epsilon\cdot\tilde{\boldsymbol{u}}/(1-(\tilde{\epsilon}-\epsilon\cdot\tilde{\boldsymbol{u}})}$$
$$= \frac{\tilde{\boldsymbol{u}}}{1-\tilde{\epsilon}\cdot\tilde{\boldsymbol{u}}} \tag{3.55}$$

It also follows from equation (3.9) and the definition of $\boldsymbol{u}$ that the time interval for a fixed spatial point in S' is related to the corresponding time interval in S by

$$\mathrm{d}t' = \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})}(1-\epsilon\cdot\boldsymbol{u})\mathrm{d}t. \tag{3.56}$$

This time interval, being measured in S', should be independent of the synchrony choices in either frame, and thus invariant under a change of $\boldsymbol{\kappa}$ or ϵ. Although this does not appear to be the case, since both ϵ-dependent and $\boldsymbol{\kappa}$-dependent quantities appear explicitly in equation (3.56), that expression is mathematically invariant under a change in either vector, as can be seen by considering changes in synchronies.

Following the approach in equation (3.55), consider a change of ϵ to $\tilde{\epsilon}$ with the concomitant change in differentials of $\mathrm{d}\tilde{t} = \mathrm{d}t + (\tilde{\epsilon}-\epsilon)\cdot\mathrm{d}\boldsymbol{x}$. Then

$$(1-\bar{\epsilon}\cdot\bar{u})\mathrm{d}\bar{t} = \left(1-\bar{\epsilon}\cdot(1+(\bar{\epsilon}-\epsilon)\cdot u)^{-1}\right)\mathrm{d}t(1+(\bar{\epsilon}-\epsilon)\cdot u)$$
$$= (1-\epsilon\cdot\boldsymbol{u})\mathrm{d}t, \tag{3.57}$$

as would be expected.

Similarly, by using equations (3.12) and (3.16), then substituting in equation (3.54), and finally using expressions (3.17) and (3.21), it can be shown that $\mathrm{d}t'$ does not change value under a change of $\boldsymbol{\kappa}$ by an amount $\Delta\boldsymbol{\kappa}$:

$$\frac{a(\tilde{\boldsymbol{w}})}{a(\tilde{\boldsymbol{v}})}(1-\tilde{\epsilon}\cdot\tilde{\boldsymbol{u}}) = \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})}\frac{1-\Delta\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{w}}}{1-\Delta\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}}(1-\tilde{\epsilon}\cdot\boldsymbol{u})$$
$$= \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})}\left(1-\tilde{\epsilon}\cdot\boldsymbol{u}-\frac{a(\tilde{\boldsymbol{v}})(b^{-1}(\tilde{\boldsymbol{v}}))\boldsymbol{u}\cdot\Delta\boldsymbol{\kappa}}{1-\Delta\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}}\right)$$
$$= \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})}\left(1-\tilde{\epsilon}\cdot\boldsymbol{u}-\frac{a(\boldsymbol{v})(b^{-1}(\boldsymbol{v}))\boldsymbol{u}\cdot\Delta\boldsymbol{\kappa}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}\right)$$
$$= \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})}(1-\epsilon\cdot\boldsymbol{u}) \tag{3.58}$$

Thus, as is physically reasonable, and indeed expected, quantities measured in one frame are, in general, independent of the synchrony choices in other frames (unless an external synchronisation process has been chosen, which specifically refers to another frame's clocks). Hence, synchrony cannot be determined experimentally, and claims to be able to do so are misguided. Such claims can arise from situtations where an approximation has been made to a measurable, but the approximation has not been made in a synchrony-invariant manner, thus leaving a residue which is, mathematically, synchrony-dependent. The natural, but false, conclusion after such an approximation would be the claim that a measurable quantity can empirically define a "correct" synchrony, and therefore could lead to spurious results.

The making of approximations in analyses is dealt with in section 3.4.

3.3 Experimental tests of special relativity

The test theories produced by Robertson[100] and Mansouri and Sexl have motivated many experimental tests of various special relativistic predictions in the sense that these test theories (or modifications of them) are used as a framework for analysis. Most physical interpretations of the results of such experiments erroneously attribute a measurable status to the conventional quantities in the test theory being used. Some confusion also exists in cases where authors acknowledge the role of conventionality, and then proceed to deny it (a precedent set by Mansouri and Sexl). For example, Will[123] states that a direct measurement of the absolute value of the speed of light in S between two points will depend on the synchronisation of the clocks, but that "a test of the *isotropy* of the speed between the same two clocks as the orientation of the propagation path varies relative to Σ should not depend on how they were synchronized, as long as they were synchronized by some procedure initially." Will also states that experimental results should not depend on synchronisation procedures, so one understands that the measurables in the test referred to above are of a synchrony-invariant nature. (Of course, if the test were such that this was not the case, it would be immediately discredited.) Now, since the same (synchrony-invariant) experimental results are a consequence of all possible synchronisation procedures (describing an infinity of anisotropies in light propagation), no synchrony-invariant test can be used as a test of the isotropy of light propagation.

Some more recent examples of these are discussed below.

MacArthur *et al.*[69] analyse (within the Robertson formalism) an interesting experiment in which a beam of hydrogen atoms in their ground state is intersected at a variable angle θ by an ultaviolet laser beam whose ionisation effects on the hydrogen atoms are measured. The ratio of the energy of the laser beam as seen by the atoms to its rest frame energy is obtained by the authors to be $(\gamma/g_0)(1+\beta cos\theta)$. By varying the angle of intersection of the two beams, one can test the sinusoidal variation predicted by the authors although this is not a test of special relativity since this variation is universally predicted. The differentiation of theories is contained in the time dilation parameter g_0, which is velocity dependent. Unfortunately in this experiment only one velocity is considered. The authors acknowledge the Mansouri and Sexl test theory which is more comprehensive than Robertson's, but chose to work in the latter. Presumably MacArthur *et al* follow MacArthur[68] who erroneously finds the test theories of Robertson[100] and Mansouri and Sexl[76] equivalent on the grounds that Einstein synchronisation in both frames of the Mansouri and Sexl test theory produces a resulting test theory which can be identified with Robertson's. However, while Robertson assumes isotropy of light in the moving frame Mansouri and Sexl make no such assumption and hence have a more general framework. This has been pointed out by Maciel and Tiomno[70] who also dismiss MacArthur's handling of absolute time for the Doppler and lifetime experiments: whereas the test theories being discussed provide an "aether" transformation from a generally non-accessible preferred frame to an arbitrary one (requiring one, when considering two different moving frames, to transform from one to the other via the preferred frame), MacArthur (and also MacArthur *et al.*) use the aether transformation to link directly two accesible frames (for example laboratory rest frame and rest frame of atomic beam).

The following authors all use the Mansouri and Sexl formalism for the theoretical framework in which their experiments are analysed. Hils and Hall[43] describe an improved Kennedy-Thorndike experiment (using an interferometer with unequal arm lengths to search for sidereal variations between the frequencies of two lasers locked to different references); Einstein synchronisation is assumed through the choice of their expression for the one-way speed of light [76, equation 6.17]. The authors state that this experiment allows purely experimental determination of the Lorentz transformation, when in fact the dilation and contraction parameters in the Lorentz transformation are dependent on the synchrony choice in the aether frame, and ϵ (the moving frame's synchrony vector in the Mansouri and Sexl formalism) is also a conventional quantity.

Kaivola *et al.*[44] claim to have measured the relativistic Doppler shift for neon. Their experiment compares the frequency difference between two lasers, one locked to a two-photon absorption transition in a fast beam of neon, the other to the same transition in thermal neon. A similar experiment is performed by Riis *et al.*[98] who look for sidereal variation in the frequency difference of a rotating and a stationary laser locked, respectively, to the resonant frequencies of the two photon absorption in neon and of an Iodine cell(?). It is maintained that the measured frequency variation gives a restriction on the anisotropy of light propagation. The claims made by Riis *et al.*[98] have been disputed (*cf* Bay and White[7], and Riis *et al.*[99]).It is an important fact that all experimental measurements are compatibile with all synchrony schemes and hence cannot differentiate between different synchrony conventions. It is instructive to analyse this experiment (taking into account synchrony considerations in all frames) in order to see the conventional nature of the various parameters involved, and the compatibility of all synchrony schemes with experimental results; this is done later in this section.

Another experiment, which here is analysed in the above manner, is that discussed by Krisher *et al.*[54],[56] who perform an experiment where two distant masers situated at either end of a fibre optic cable simultaneously send signals to each other. An analyser is situated at each end of the fibre-optic cable. Each analyser is used to compare the phase of the incoming signal with that of the outgoing signal. The observable in this experiment is the variation in phase (or difference in these comparisons). The authors claimed that experimental constraints applied to the predicted relative phase variation to constrain the (one-way) time dilation parameter to close agreement with the standard special relativistic value and also give a measurement of the difference between length contraction factors for directions parallel and perpendicular to motion with respect to the preferred frame. The authors also claimed that this gives an indication of the isotropy of the one-way speed of light. However, as well as these claims being questionable, so is the analysis of this experiment given by Krisher *et al.* (and expanded on by Will[123].) In their treatment, they considered one maser to be at rest in the non-rotating frame (S say) comoving with the centre of rotation of the Earth, while the other maser is moving with respect to S (in frame S', say), the latter's motion being due to the Earth's rotation. This assumption cannot be justified. Krisher *et al.*[54] and later Will[123] acknowledged that neither maser is at rest in frame S, but claimed that assuming one of them is at rest in S makes no difference to the theoretical prediction. However, their simplified model predicts (within the Mansouri and Sexl formalism) greater experimental constraints on parameters in the test-theory than is actually the case. This is discussed in section (jpl) in a re-analysis of this experiment which Will referred to as the "JPL" experiment.

It should be stressed that the above experiments' inability to measure conventional quantities (such as the one-way speed of light, and one-way time dilation effects) does not detract fromn their importance in the verification of theories. No experiment can measure synchrony-dependent quantities, and thus the emphasis that authors put on such measurements is misplaced. All the above experiments have the potential to distinguish between special relativity and some preferred frame theories, and once conventionality is taken into account, real physical effects can be exposed to testing.

Since the analyses of the results of the experiments mentioned above do not take into account synchrony considerations in the hypothesised preferred frame, it is not explicit that the dilation and contraction factors (the parameters a and b in the Mansouri and Sexl test theory) are dependent on the synchrony choice in the aether frame[117] and thus definitely not measurable - in contrast to the claims of MacArthur *et al.*[69], Hils and Hall[43], Kaivola *et al.*[44] and Krisher *et al.*[54]. As a consequence of this, the Lorentz transformation is not inferable by experiment. And although synchrony choice in the preferred frame does not affect the results of experiments in the moving frame, one should be aware that these results themselves may be dependent on the conventionality in the moving frame in a way which is not immediately transparent. For example, the choice of experimental set-up can induce a synchrony convention which is reflectsed in the result. This is the case in Römer's experiment, where in effect, slow clock transport synchronisation is used, as is discussed on pages 42–44.

Thus the parameters in the frame transformation are evaluable only within the Einstein synchronisation "gauge" in the preferred frame; the one-way speed of light is isotropic within the Einstein synchronisation gauge in the moving frame (and within the slow clock transport synchronisation gauge in special relativity). Attempts to experimentally disprove non-Einstein synchronisation are futile since, as Weingard[120] points out, non-Einstein synchronisation simply corresponds to a change of coordinates and, by covariance of physical laws, must be compatible with any experiment compatible with standard synchrony.

3.4 Analysing and interpreting experiments

The analysis of results is, of course, a vital part of the experimental testing of any theory, and inappropriate reasoning can lead to unfounded claims. In the area of experimental testing of special relativity, synchrony issues often have caused confusion, and have been improperly handled, leading to misunderstandings of what the experiments in question actually test.

For example, a recent paper by Will[123] contains several such misunderstandings, with unjustified conclusions. This paper analyses several of the experiments mentioned in section 3.1: the two-photon absorption, maser phase, Mössbauer-rotor and rocket-redshift experiments. Will pointed out that these experiments have the potential to set bounds on Lorentz-violating, preferred-frame, alternative theories to special relativity. However Will has made several incorrect claims regarding these important experiments as well as synchrony in general. These claims have been addressed by Vetharaniam and Stedman[118] and the analyses in sections 3.4.1 and 3.4.2 draw from that paper.

Will, like Mansouri[75], noted that observables cannot be affected by synchrony choice within the laboratory frame. He accepted that a measurement of the one-way speed of light in the laboratory frame using "a time-of-flight technique" between two clocks is synchrony dependent, but stating that " ... a test of the *isotropy* of the speed between the same two clocks as the orientation of the propagation path varies relative to Σ should not depend on how they were synchronized, ..." he maintained that this allows a determination of the isotropy of one-way light speeds. However, this argument neglects the effects of a synchrony choice, within the Mansouri-Sexl test-theories or even within special relativity, on the cumulative time dilation experienced by a slowly transported clock[117, 125]. The nett change in synchronisation induced under slow clock transport is itself synchrony dependent in such a way as not to affect experiment. It is precisely synchrony invariance which prevents an experimental determination of conventional quantities.

Will tried to distinguish a measurement of the value of the one-way speed of light from a test of the isotropy of the one-way speed of light, saying that the former is conventional, but that the latter is not, and is measurable. But since the return-trip speed of light is measurable there can be no such distinction, because knowledge of the isotropy of one-way light propagation would allow determination of the numerical value of the one-way speed of light from the return trip value.

The misleading proposition of "testing the one-way speed of light" should be avoided. No experiment tests the one-way speed of light. Indeed, no experiment is a "one-way" experiment. It is just as incorrect for authors like Will to refer to the two-photon absorption experiment as a one-way experiment, as it is for Mansouri and Sexl[77] to so refer to Römer's experiment. Karlov[46] showed explicitly that not even this experiment could be considered as measuring the one-way speed of light. In rejecting Karlov's (correct) resolution Mansouri and Sexl set a precedent for all the subsequent misinterpretations which have dogged the use of their test-theory.

Concomitantly, the suppression of conventionality in the aether frame, within the Mansouri-Sexl formalism, has served only to disguise the conventional nature of the parameters in that test-theory. For this reason the generalisation of the Mansouri-Sexl test-theory given in section 3.2 would be useful as a standard, to reveal this conventionality, just as Mansouri[75] expounded the conventionality of synchrony in the laboratory frame.

One synchrony-related aspect in experimental analysis which has the potential to mislead is the making of approximations, where consistency with regard to synchrony has generally been overlooked. The analysis of experiments invariably requires approximations necessary for the simplification of algebra to the stage where predictions can be made. It is important in this to take care that all approximations are independent of synchrony choice if one is dealing with arbitrary synchrony, otherwise misleading results may be obtained. A synchrony-dependent approximation may disguise the synchrony-invariance of measurable quantities. This is the case, with Mansouri and Sexl[76], and Will[123], who both make approximations to first or second order in a speed measured using a synchrony scheme involving an arbitrary ϵ, referring to the speed as small. The Mansouri-Sexl formulation contains an expansion in powers of a velocity which is synchrony dependent (on the choice of gauge in S). Isolating individual terms in such an expansion immediately introduces a lack of synchrony invariance into analyses, and can be justified only if the gauge is fixed in the *relevant* frame (with the result that conventionality is obscured). Using the Mansouri-Sexl expansion, Will[123, equation (3.5)] gave an expression of the following form as an approximation of $A'(\boldsymbol{u})$, the time dilation factor for a frame S' moving at velocities $\boldsymbol{u}$ and $\boldsymbol{W}$ in S and Σ respectively:

$$A' \approx A(v)[1 + (2\tilde{\alpha} - \Gamma^2)(A/\beta)\boldsymbol{V}\cdot\boldsymbol{u} + O(u^2)] \tag{3.59}$$

$$\tilde{\alpha} v = \frac{1}{2}(A^{-1}\partial A/\partial \boldsymbol{V} + \Gamma^2 \boldsymbol{V})$$

Note that the symbols in this equation have been translated from Will's notation into the notation of this chapter for the purpose of continuity, and in order to avoid confusion: Will's $\boldsymbol{w}$ and $\boldsymbol{v}$ in his paper correspond respectively to the $\boldsymbol{V}$ and $\boldsymbol{u}$ conventions used by Mansouri and Sexl and adopted here). The quantity β is the length contraction in the direction of $\boldsymbol{V}$. Will derived his expression by approximating to first order in u, which is dependent on the synchrony ϵ in S. It is easily seen that the left hand side of equation (3.59) is independent of ϵ since it contains just the time dilation factor for S' as measured in Σ. However, the right hand side of that equation contains the velocity $\boldsymbol{u}$, which is intriniscally dependent on ϵ since $\boldsymbol{u}$ is measured in S. Now, none of the other quantities in the equation are dependent on ϵ, and so the right hand side is ϵ-dependent. Will's equation seems to suggest that an observer in Σ can, by making measurements, determine a "true" synchrony for S. This is of course unphysical, and the disparity in the expression is the result of failing to maintain synchrony-invariance throughout a calculation. It is also the case that a speed which is small in one synchrony scheme may be very large in another, and thus if one is dealing with arbitrary synchrony, it may be inappropriate to make an approximation to first-order in velocity.

Now using equation (3.54), the velocity, $\boldsymbol{w}$, in Σ of a frame S' moving with velocity $\boldsymbol{u}$ in S can be written in the form

$$\boldsymbol{w} = \boldsymbol{j} + \boldsymbol{v} \tag{3.60}$$

where the vector

$$\boldsymbol{j} = a\, b^{-1}\boldsymbol{u}/(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}) \tag{3.61}$$

is unchanged by a change of ϵ, as is shown by equation (3.55). If an approximation must be made to an ϵ -dependent quantity which is a function of $\boldsymbol{w}$, then approximations may be made to some order in $\boldsymbol{j}$ on the basis that $\boldsymbol{u}/(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u})$ is small, since the latter is independent of choice of ϵ. Making an approximation in such a manner would not introduce spurious synchrony effects into one side of an equation, thus avoiding misinterpretations of experimental results.

For example, using equation (3.60), the Taylor's series expansion of $a(\boldsymbol{w})$ about $\boldsymbol{v}$ is

$$a(\boldsymbol{w}) = a(\boldsymbol{v}) + \sum_{n=1}^{\infty} \frac{1}{n!}(\boldsymbol{j}\cdot\frac{\partial}{\partial \boldsymbol{v}})^n a(\boldsymbol{v}) \tag{3.62}$$

Each term on the right hand side is independent of the choice of ϵ, and so the series may be truncated to any order and remain consistent as far as synchrony-dependence in S is concerned.

Note however, from equations (3.16), (3.17) and (3.20), the manner in which $\boldsymbol{j}$ transforms under a change of $\boldsymbol{\kappa}$:

$$\begin{aligned}\tilde{\boldsymbol{j}} &\equiv \tilde{a}\tilde{b}^{-1}\boldsymbol{u}/(1-\tilde{\boldsymbol{\epsilon}}\boldsymbol{\cdot}\boldsymbol{u}) \\ &= \frac{a}{1+\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{v}}\left(b^{-1}\boldsymbol{u}-\frac{\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}b^{-1}\boldsymbol{u}}{1+\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{v}}\boldsymbol{v}\right)\Bigg/\left(1-\boldsymbol{\epsilon}\boldsymbol{\cdot}\boldsymbol{u}-\frac{ab^{-1}\boldsymbol{u}\boldsymbol{\cdot}\Delta\boldsymbol{\kappa}}{1+\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{v}}\right).\end{aligned} \tag{3.63}$$

Now, from equations (3.11) and (3.12),

$$\frac{\partial}{\partial\tilde{\boldsymbol{v}}} = (1+\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{v})\left(\frac{\partial}{\partial\boldsymbol{v}}+\Delta\boldsymbol{\kappa}\,\boldsymbol{v}\boldsymbol{\cdot}\frac{\partial}{\partial\boldsymbol{v}}\right). \tag{3.64}$$

Therefore, using the last two results and equation (3.61) it follows that

$$\begin{aligned}\tilde{\boldsymbol{j}}\boldsymbol{\cdot}\frac{\partial}{\partial\tilde{\boldsymbol{v}}} &= \left(1-\frac{ab^{-1}\boldsymbol{u}\boldsymbol{\cdot}\Delta\boldsymbol{\kappa}}{(1-\boldsymbol{\epsilon}\boldsymbol{\cdot}\boldsymbol{u})(1+\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{v})}\right)^{-1}\frac{ab^{-1}\boldsymbol{u}}{1-\boldsymbol{\epsilon}\boldsymbol{\cdot}\boldsymbol{u}}\boldsymbol{\cdot}\frac{\partial}{\partial\boldsymbol{v}} \\ &= \left(1-\frac{\boldsymbol{j}\boldsymbol{\cdot}\Delta\boldsymbol{\kappa}}{1+\Delta\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{v}}\right)^{-1}\boldsymbol{j}\boldsymbol{\cdot}\frac{\partial}{\partial\boldsymbol{v}}\end{aligned} \tag{3.65}$$

From here it can be seen that the terms in equation (3.62) will vary differently from each other under a synchrony transformation in Σ. So an approximation to $a(\boldsymbol{w})$ may not exhibit the same synchrony-covariance as $a(\boldsymbol{w})$. This discrepancy is unavoidable in such an approximation. However it is minor in its effect when compared with the inappropriate introduction or deletion of synchrony-dependence on only one side of an equation: the $\boldsymbol{\kappa}$ -dependence of a—and hence its conventionality–has been preserved while at the same time no spurious synchrony-dependence has been inserted.

Using the results of this, and the previous section, the two-photon absorption experiment[98] and the maser phase experiment [54] discussed by Will[123] are re-analysed in the rest of this section, in order that several misconceptions be cleared. In addition, an unnecessary and somewhat curious geometrical assumption has been introduced into Will's analysis of the maser phase experiment, in which the relative timekeeping of two clocks connected by a precision link is also monitored over the sidereal day: one station is deemed not to move. The consequences of this assumption are shown in the discussion of the maser phase experiment.

As well as the above two, Will[123] analysed two more experiments—the rocket red-shift and the Mössbauer rotor experiments, which are, as are all experiments, compatible with arbitrary synchrony, having measurables which are synchrony invariant. While these are not re-analysed here, it is in order to make a synchrony-related comment on Will's model for the Mössbauer rotor experiment. In this experiment, an absorber is positioned at the centre of a rotating disc, and measurements are made of the change in transmission of gamma rays through the absorber as a function of the propagation direction of these rays from an emitter placed on the rim of the disc. Will assumed that the disc rotates rigidly in the laboratory frame, but this is an unjustified assumption because Will was supposed to be using an arbitrary synchrony convention: a body which rotates rigidly according to some synchrony convention does not do so according to all others. As stated above, all assumptions one makes must be synchrony invariant when one is dealing with arbitrary synchrony. The model Will uses for the maser phase experiment is appropriate for the Mössbauer experiment, since that model does not assume rigid rotations of a disc, and its assumptions of relative motions are manifest in the Mössbauer experiment.

3.4.1 The two-photon absorption

The two-photon absorption experiment performed by Riis *et al.* [98] involved a beam of fast atoms travelling collinearly in a laboratory frame with two counter-propagating laser beams, both produced by one laser. Both beams have the same frequency in the laboratory frame in which the laser was at rest. The frequency of the laser was continually varied (if necessary) to maintain resonance in a two-photon transition between two energy levels of the atoms via an intermediate level, the velocity of the atomic beam being adjusted for resonance in the intermediate state. The variation in laser frequency required to maintain resonance in the two-photon transition was recorded. The constraints on the parameters in the transformations (3.9) and (3.10), given by a null variation in ν are examined below.

In an ideal model, the laboratory frame, S, should display variable, non-inertial motion owing to the rotation of the Earth, while the atomic beam should be taken as stationary in S. However, in the Mansouri-Sexl formalism, S is assumed to be in uniform motion with respect to Σ and this has a bearing on the form of the theory: for example, the matrix $b(\boldsymbol{v})$ is dependent in part on the relative orientation of the axes in Σ and S which varies in this case, and in doing so makes analysis intractable. For this reason, the Mansouri-Sexl test-theory can not properly handle experiments such as the two-photon absorption experiment, where the laboratory frame rotates with the Earth. The need for a test-theory which allows S to exhibit non-inertial behaviour motivated Abolghasem *et al.*[1, 2]) who gave a limited extension to the Mansouri-Sexl framework in order to account for a rotating Earth. A more general test-theory given by Vetharaniam and Stedman[119] is discussed in chapter 5.

For the purpose of correcting Will's analysis[123], the synchrony extension to the Mansouri-Sexl formalism given in section 3.2 is used here instead of the Mansouri-Sexl test-theory. Since, this extension assumes S is inertial, it is convenient to make the assumption that the laboratory frame, S, is moving at constant velocity with respect to Σ, and that the frame with the atomic beams, S', has a varying velocity in S. While this does not accurately model the actual situation, the essential element—the rotation of the atomic beam relative to the fixed stars—is preserved.

Consider an atom (in frame S') moving at a velocity $\boldsymbol{u}$ with respect to the laboratory frame, S. It receives crests from two collinear, anti-propagating laser beams which have the same frequency in S. In S, the laser beams and velocity $\boldsymbol{u}$ are all collinear. Let the atom have an energy state, E_A, which it currently occupies, and also a higher state, E_C. Further suppose that the atom posseses a virtual energy level, E_B, which is intermediate between the other two:$E_A < E_B < E_C$. The atom can move from level E_A to level E_C in two transitions via the virtual state, E_B if it receives quanta of the corresponding energy differences. The two laser beams (which, in the atom's frame, experience different Doppler shifts) provide these required transition energies, subject to the atomic beam velocity, and the frequency output of the laser, thus enabling two-photon resonance. Such a resonance would be sensitive to any change in the Doppler frequencies that the atom may experience as the beam orientation changes. Thus the dye-laser frequency was continually varied to maintain resonance, and this frequency examined for sidereal dependence.

Denote ν_1' and ν_2' as the frequencies associated with the respective transitions from the lower to the virtual states and the virtual to the higher states:

$$\nu_1{\prime} = \frac{E_B - E_A}{\hbar}, \text{ and } \nu_2{\prime} = \frac{E_C - E_B}{\hbar} \tag{3.66}$$

For the atom to resonate, the two frequencies supplied by the laser beams, ν_+' and ν_-' (where $\nu_\pm'$ is the frequency of the beam in the direction $\mp\hat{\boldsymbol{u}}$), must be the two frequencies ν_1' and ν_2'.

Let $\nu_\pm$ be the frequencies at which S sees the atom receive successive crests from the $\pm$ beams. These frequencies are distinct from the frequencies of the beams in S. From equation (3.56), $\nu_\pm$ are related to the corresponding frequencies in the atomic rest frame by

$$\nu_\pm = \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})}(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u})\nu'_\pm. \tag{3.67}$$

But, in S, both $\pm$ beams have the same frequency, ν, so

$$\lambda_\pm \equiv \frac{c(\mp\hat{\boldsymbol{u}})}{\nu} = \frac{c(\mp\hat{\boldsymbol{u}}) \pm u}{\nu_\pm} \tag{3.68}$$

and one thus derives the relation

$$\nu = \nu_\pm \Big/ \left(1 \pm \frac{u}{c(\mp\hat{\boldsymbol{u}})}\right). \tag{3.69}$$

Now note, from equation (3.49), that the quantities d and k in the expression for c given by equation (3.48) are, respectively, odd and even functions of their vector argument. Substituting equation (3.48) into the expression for ν gives

$$\nu = \nu_\pm \Big/ \Big(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u} - ua\gamma^2 d(\hat{\boldsymbol{u}}) \pm uk(\hat{\boldsymbol{u}})\Big), \tag{3.70}$$

so that

$$\nu_+ + \nu_- = 2\nu\left(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u} - \gamma^2 ab^{-1}\boldsymbol{u}\cdot(\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa})\right). \tag{3.71}$$

From here, rearranging the expression and substituting in equation (3.54) gives

$$\begin{aligned}
\nu &= \frac{\nu_+ + \nu_-}{2(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u})} \Bigg/ \left(1 - \gamma^2 \frac{ab^{-1}\boldsymbol{u}\cdot(\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa})}{(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u})}\right) \\
&= \frac{\nu_+ + \nu_-}{2(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u})} \Big/ \Big(1 - \gamma^2(\boldsymbol{w} - \boldsymbol{v})\cdot(\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa})\Big) \\
&= \frac{1}{2\gamma^2} \frac{(\nu_+ + \nu_-)/(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u})}{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})(1 - \boldsymbol{\kappa}\cdot\boldsymbol{w}) - \boldsymbol{w}\cdot\boldsymbol{v}}.
\end{aligned}$$

Making use of equation (3.67) in the above expression then gives

$$\nu = \frac{1}{2\gamma^2}\frac{a(\boldsymbol{w})}{a(\boldsymbol{v})} \frac{(\nu'_+ + \nu'_-)}{(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})(1 - \boldsymbol{\kappa}\cdot\boldsymbol{w}) - \boldsymbol{w}\cdot\boldsymbol{v}} \tag{3.72}$$

Note that this result is invariant of the choice of $\boldsymbol{\kappa}$. This can be seen by using equations (3.49) together with equations (3.11), (3.14) and (3.16, and then considering a change, $\Delta\boldsymbol{\kappa}$, in $\boldsymbol{\kappa}$:

$$\frac{a(\tilde{\boldsymbol{w}})}{a(\tilde{\boldsymbol{v}})} \frac{1/\tilde{\gamma}^2}{(1 - \tilde{\boldsymbol{\kappa}}\cdot\tilde{\boldsymbol{v}})(1 - \tilde{\boldsymbol{\kappa}}\cdot\tilde{\boldsymbol{w}}) - \tilde{\boldsymbol{w}}\cdot\tilde{\boldsymbol{v}}}$$

$$
\begin{aligned}
&= \frac{a(\tilde{\boldsymbol{w}})}{a(\tilde{\boldsymbol{v}})} \frac{(1-\tilde{\boldsymbol{\kappa}}\cdot\tilde{\boldsymbol{v}})^2 - \tilde{v}^2}{(1-\tilde{\boldsymbol{\kappa}}\cdot\boldsymbol{v})(1-\tilde{\boldsymbol{\kappa}}\cdot\boldsymbol{w}) - \tilde{\boldsymbol{w}}\cdot\tilde{\boldsymbol{v}}} \\
&= \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})} \frac{(1+(\Delta\boldsymbol{\kappa}-\tilde{\boldsymbol{\kappa}})\cdot\boldsymbol{v})^2 - (v^2)}{(1+(\Delta\boldsymbol{\kappa}-\tilde{\boldsymbol{\kappa}})\cdot\boldsymbol{v})\,(1+(\Delta\boldsymbol{\kappa}-\tilde{\boldsymbol{\kappa}})\cdot\boldsymbol{w}) - \boldsymbol{w}\cdot\boldsymbol{v}} \\
&= \frac{a(\boldsymbol{w})}{a(\boldsymbol{v})} \frac{1/\gamma^2}{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})(1-\boldsymbol{\kappa}\cdot\boldsymbol{w}) - \boldsymbol{w}\cdot\boldsymbol{v}}. \qquad (3.73)
\end{aligned}
$$

It follows from here that ν is invariant under synchrony choices in all frames, since $\nu'_\pm$ is trivially synchrony-independent. Thus any experimental testing for variation of ν cannot measure the time dilation parameter uniquely, since the latter is synchrony dependent. Experiment can restrict time dilation only to a class of functions which are related by equation (3.16).

For two-photon resonance to be maintained over a period of time, ν'_+ and ν'_- must be invariant. Assume that the change in $\boldsymbol{v}$ is negligible over the period of one Earth rotation when compared with the change in $\boldsymbol{w}$ (where $\boldsymbol{\kappa}$ is considered arbitrary but fixed. Then a null variation in ν together with resonance being maintained will establish that the right hand side of equation (3.72) is invariant under a change in $\boldsymbol{w}$. This resonance condition can be expressed as:

$$\frac{a(\boldsymbol{w})}{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})(1-\boldsymbol{\kappa}\cdot\boldsymbol{w}) - \boldsymbol{w}\cdot\boldsymbol{v}} = g(\boldsymbol{v}) \qquad (3.74)$$

where

$$g(\boldsymbol{v}) \equiv \frac{2\gamma^2 a(\boldsymbol{v})\nu}{\nu'_+ + \nu'_-} \qquad (3.75)$$

Since the relationship between ν and $\nu'_\pm$ is not yet known, g has a degree of freedom and is treated as an unknown to be solved. Note, with the use of equation (3.49), that when $\boldsymbol{w} = \boldsymbol{v}$

$$a(\boldsymbol{v}) = g(\boldsymbol{v})/\gamma^2 \qquad (3.76)$$

Using equation (3.60) in the resonance condition (3.74) and using the identity (3.76) gives

$$a(\boldsymbol{w}) = a(\boldsymbol{v}) - \boldsymbol{j}\cdot[\,(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa} - \boldsymbol{v}]\,g(\boldsymbol{v}). \qquad (3.77)$$

By substituting equation (3.62) into equation (3.77) one gets

$$-\boldsymbol{j}\cdot[\,(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa} - \boldsymbol{v}]\,g(\boldsymbol{v}) = \sum_{n=1}^{\infty} \frac{1}{n!}(\boldsymbol{j}\cdot\frac{\partial}{\partial \boldsymbol{v}})^n a(\boldsymbol{v}). \qquad (3.78)$$

Comparison of the coefficients of $\boldsymbol{j}$ in the above expression, together with the use of equation (3.76) gives

$$\begin{aligned}\frac{\partial}{\partial \boldsymbol{v}}\left(\frac{g(\boldsymbol{v})}{\gamma^2}\right) &= -\left[(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}-\boldsymbol{v}\right]g(\boldsymbol{v}) \\ &= \frac{1}{2}\frac{\partial}{\partial \boldsymbol{v}}\left(\frac{1}{\gamma^2}\right)g(\boldsymbol{v}) \qquad (3.79)\end{aligned}$$

Now substituting equation (3.76) into this differential equation, and solving with the requirement that $a(\mathbf{0})=1$, gives the form of the time dilation parameter as

$$a(\boldsymbol{v}) = 1/\gamma \equiv \sqrt{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2} \qquad (3.80)$$

which is the expression given in section 2.2 for the special relativistic time dilation in arbitrary synchrony.

If $\boldsymbol{u}/(1-\boldsymbol{\epsilon}\cdot\boldsymbol{u})$ (which is invariant of $\boldsymbol{\epsilon}$) is small then all orders of $\boldsymbol{j}$ greater than one may be neglected and so no more information is gained by a null variation in ν; otherwise, the accuracy of the model permitting, one may place restrictions on b also, by considering higher orders in $\boldsymbol{j}$.

3.4.2 The maser phase

The maser phase experiment involved two identical masers (which both output the same rest frequency of 100MHz). The masers were located on the surface of the Earth and were linked by a single, underground, fibre-optic cable. The output from each maser was transmitted to the position of the other maser; at both ends, detectors measure the phase differences between the incoming and outgoing signals. The masers were fixed to the Earth's surface and thus their relative orientation with respect to the fixed stars changed as a consequence of the Earth's rotation. The variation in phase difference at each end of the cable was monitored for sidereal variations. The consequences of a null variation at both detectors are discussed below.

A convenient way to model this experiment is to identify Earth's centre of rotation with the laboratory frame, S, moving at a constant velocity, $\boldsymbol{v}$, with respect to the aether (over the period of a day, the change in the Earth's velocity with respect to the aether is negligible). The two masers, then, would be tracing a common circular path in S; however their velocities with respect to S can be approximated as being constant over the time periods between the emission and reception of two consecutive signals (wave crests).

Consider an emitter, e, and an absorber, a, at rest in frames S'_e and S'_a respectively, which are moving at the respective velocities $\boldsymbol{u}_e$ and $\boldsymbol{u}_a$ in the laboratory frame, S. Suppose an observer in S sees two consecutive signals being emitted by e at time t_1 and t_3 from the respective positions $\boldsymbol{x}_1$ and $\boldsymbol{x}_3$. In S these signals will be recieved by a at two distinct times, say t_2 and t_4, with respective positions of reception, $\boldsymbol{x}_2$ and $\boldsymbol{x}_4$. Let the rest-frequency of the emitter (measured in S'_e) be $\nu'_e \equiv 1/(t'_3 - t'_1)$ and let the frequency of the signals received by the absorber, as measured in S'_a be $\nu'_a \equiv 1/(t'_4 - t'_2)$. From equation (3.56), these two frequencies are related to the corresponding time measurements in S by

$$\begin{aligned} \frac{1}{\nu'_e} &= \int_{t_1}^{t_3} \frac{a(\boldsymbol{w}_e)}{a(\boldsymbol{v})}(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}_e)\mathrm{d}t, \\ \frac{1}{\nu'_a} &= \int_{t_1}^{t_3} \frac{a(\boldsymbol{w}_a)}{a(\boldsymbol{v})}(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}_a)\mathrm{d}t. \end{aligned} \tag{3.81}$$

Now consider the phase comparison made between the signals a receives from e and the signals that a, itself, emits. This comparison is made by a, for whom the phases of the incoming signals are $\phi - 2\pi\nu'_a t'_a$ where ϕ is arbitrary. Since a's own signal has a rest frequency of ν'_e, its phase in S'_a is $\theta - 2\pi\nu'_e t'_a$. Thus the variation in phase difference between the incoming and outgoing signals, over a period of $1/\nu'_e$ is

$$\Delta = 2\pi(1 - \nu'_a/\nu'_e). \tag{3.82}$$

Both ν'_e and ν'_a are invariant of synchrony choice in any frame, and thus the quantity Δ (which is the measurable in the maser phase experiment) is unaffected by choice of $\boldsymbol{\kappa}$ and $\boldsymbol{\epsilon}$. Hence experimental measurements of Δ cannot distinguish either a preferred value of $\boldsymbol{\kappa}$ or a preferred value of $\boldsymbol{\epsilon}$. It then follows that the maser phase experiment cannot measure the $\boldsymbol{\kappa}$-dependent time dilation factor beyond a class of synchrony-dependent functions; nor can it give a measure of the one-way speed of light since this speed is also synchrony-dependent.

Following the formula of equation (3.62), one may expand $a(\boldsymbol{w_a})$ and $a(\boldsymbol{w_e})$ in equations (3.81) using Taylor's series expansions about $\boldsymbol{v}$. If the quantities $\boldsymbol{u_a}/(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u_a})$ and $\boldsymbol{u_e}/(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u_e})$ are both small (as is assumed to be the case here), then it is justifiable and convenient to make first-order approximations to these expansions of $a(\boldsymbol{w_a})$ and $a(\boldsymbol{w_e})$. Such a course of action then gives

$$\begin{aligned} \frac{1}{\nu'_a} &\approx \int_{t_2}^{t_4} (1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}_a + b^{-1}\boldsymbol{u}_a\cdot\frac{\partial a}{\partial \boldsymbol{v}})\mathrm{d}t, \\ \frac{1}{\nu'_e} &\approx \int_{t_2}^{t_4} (1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}_e + b^{-1}\boldsymbol{u}_e\cdot\frac{\partial a}{\partial \boldsymbol{v}})\mathrm{d}t. \end{aligned} \tag{3.83}$$

Note that the right hand sides of both these approximations are independent of the choice of ϵ since each term of the series expansion in equation (3.62) is independent of the choice of ϵ. Similarly, the approximations are unaffected by choice of $\boldsymbol{\kappa}$ since if $\boldsymbol{\kappa}$ were changed by an amount $\Delta\boldsymbol{\kappa}$, the integrands in equations (3.83) would remain unchanged. This follows from the use of equations (3.11), (3.12), (3.16), (3.17) and (3.21) in the approximations:

$$\begin{aligned} 1 - \tilde{\boldsymbol{\epsilon}}\cdot\boldsymbol{u} + \tilde{b}^{-1}\boldsymbol{u}\cdot\frac{\partial\tilde{a}}{\partial\tilde{\boldsymbol{v}}} &= 1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u} + \frac{a\,b^{-1}\boldsymbol{u}\cdot\Delta\boldsymbol{\kappa}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}} \\ &\quad + \left(b^{-1}\boldsymbol{u} - \frac{b^{-1}\boldsymbol{u}\cdot\Delta\boldsymbol{\kappa}}{1+\Delta\boldsymbol{\kappa}\cdot\boldsymbol{v}}\boldsymbol{v}\right)\cdot\left(\frac{\partial a}{\partial\boldsymbol{v}} + \left(\frac{\partial a}{\partial\boldsymbol{v}}\cdot\boldsymbol{v} - a\right)\boldsymbol{\kappa}\right) \\ &= 1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u} + b^{-1}\boldsymbol{u}\cdot\frac{\partial a}{\partial\boldsymbol{v}} \end{aligned} \tag{3.84}$$

Let the final displacements, in S, of the two emitted signals be $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. Then

$$\boldsymbol{r}_1 \equiv \boldsymbol{x}_2 - \boldsymbol{x}_1; \qquad \boldsymbol{r}_2 \equiv \boldsymbol{x}_4 - \boldsymbol{x}_3. \tag{3.85}$$

Also, define the difference in displacements of the two consecutive signals to be

$$\begin{aligned} \boldsymbol{\mu} &\equiv \boldsymbol{r}_2 - \boldsymbol{r}_1 \\ &= \int_{t_2}^{t_4} \boldsymbol{u}_a \mathrm{d}t - \int_{t_1}^{t_3} \boldsymbol{u}_e \mathrm{d}t. \end{aligned} \tag{3.86}$$

This vector, $\boldsymbol{\mu}$, is unaffected by synchrony changes, as can be easily seen by performing a change of variable from t to $\bar{t}$, where $\bar{t}$ corresponds to a different synchrony. It is reasonable to assume that the magnitude of $\boldsymbol{\mu}$ is small because of the high frequency of the masers involved (since $\boldsymbol{\mu}$ is synchrony-invariant, this assumption does not depend on clock synchronisation). Hence approximations can be made to first order in $\boldsymbol{\mu}$.

Now the respective frequencies of a and e, as measured in S, are given by:

$$\frac{1}{\nu_a} = t_4 - t_2; \qquad \frac{1}{\nu_e} = t_3 - t_1. \tag{3.87}$$

The propagation times of the signals are given by

$$t_2 - t_1 = \frac{r_1}{c(\hat{\boldsymbol{r}}_1)}; \qquad t_4 - t_3 = \frac{r_2}{c(\hat{\boldsymbol{r}}_2)}. \tag{3.88}$$

Then, with the use of equations (3.48), (3.49) and (3.86), it follows that

$$\begin{aligned} \frac{1}{\nu_a} - \frac{1}{\nu_e} &= \frac{r_2}{c(\hat{\boldsymbol{r}}_2)} - \frac{r_1}{c(\hat{\boldsymbol{r}}_1)} \\ &= \boldsymbol{\epsilon}\cdot\boldsymbol{\mu} + a(v)\gamma^2 d(\boldsymbol{\mu}) + a(v)\left[k(\boldsymbol{r}_2) - k(\boldsymbol{r}_1)\right] \end{aligned} \tag{3.89}$$

Returning to equations (3.83), using equations (3.87) and (3.86), and then using equation (3.89) gives

$$\begin{aligned}
\frac{1}{\nu_a'} - \frac{1}{\nu_e'} &\approx \int_{t_2}^{t_4} (1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}_a)\,\mathrm{d}t - \int_{t_1}^{t_3} (1 - \boldsymbol{\epsilon}\cdot\boldsymbol{u}_e)\,\mathrm{d}t \\
&\quad + b^{-1}\left(\int_{t_2}^{t_4} \boldsymbol{u}_a \mathrm{d}t - \int_{t_1}^{t_3} \boldsymbol{u}_e \mathrm{d}t\right)\cdot\frac{\partial a}{\partial \boldsymbol{v}} \\
&= \frac{1}{\nu_a} - \frac{1}{\nu_e} - \boldsymbol{\epsilon}\cdot\boldsymbol{\mu} + b^{-1}\boldsymbol{\mu}\cdot\frac{\partial a}{\partial \boldsymbol{v}} \\
&= a(\boldsymbol{v})\gamma^2 d(\boldsymbol{\mu}) + a(\boldsymbol{v})\left[k(\boldsymbol{r}_2) - k(\boldsymbol{r}_1)\right] + b^{-1}\boldsymbol{\mu}\cdot\frac{\partial a}{\partial \boldsymbol{v}}. \qquad (3.90)
\end{aligned}$$

Making the definition

$$\boldsymbol{r} \equiv \frac{\boldsymbol{r}_2 + \boldsymbol{r}_1}{2}, \qquad (3.91)$$

and using equation (3.86), one can write

$$k(\boldsymbol{r}_2) - k(\boldsymbol{r}_1) = k(\boldsymbol{r} + \boldsymbol{\mu}/2) - k(\boldsymbol{r} - \boldsymbol{\mu}/2) \qquad (3.92)$$

By substituting in the expression for k given by equations (3.49) and then approximating to first order in $\boldsymbol{\mu}$ (which was assumed to be small), one deduces that

$$\begin{aligned}
k^2(\boldsymbol{r} \pm \boldsymbol{\mu}/2) &= \gamma^4 \left(b^{-1}(\boldsymbol{r} \pm \boldsymbol{\mu}/2)\cdot\left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right]\right)^2 \\
&\quad + \gamma^2 \left(\left[b^{-1}(\boldsymbol{r} \pm \boldsymbol{\mu}/2)\right]^2 - \left[b^{-1}(\boldsymbol{r} \pm \boldsymbol{\mu}/2)\cdot\boldsymbol{\kappa}\right]^2\right) \\
&\approx \gamma^4 \left(b^{-1}\boldsymbol{r}\cdot\left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right]\right)^2 \\
&\quad \pm \gamma^2 b^{-1}\boldsymbol{r}\cdot\left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right] b^{-1}\boldsymbol{\mu}\cdot\left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right] \\
&\quad + \gamma^2 \left[\left(b^{-1}\boldsymbol{r}\right)^2 \pm b^{-1}\boldsymbol{r}\cdot b^{-1}\boldsymbol{\mu} - \left(b^{-1}\boldsymbol{r}\cdot\boldsymbol{\kappa}\right)^2 \mp b^{-1}\boldsymbol{r}\cdot\boldsymbol{\kappa}\, b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{\kappa}\right] \\
&= k^2\left(\boldsymbol{r} \pm b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{s}\right), \qquad (3.93)
\end{aligned}$$

where $\boldsymbol{s}$ is defined as

$$\begin{aligned}
\boldsymbol{s} \equiv\ & b^{-1}\boldsymbol{r}\cdot\left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right]\gamma^4\left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right] \\
& + \gamma^2 b^{-1}\boldsymbol{r} - \gamma^2\boldsymbol{\kappa}\cdot b^{-1}\boldsymbol{r}\ \boldsymbol{\kappa}. \qquad (3.94)
\end{aligned}$$

Hence, application of the binomial theorem gives $k(\boldsymbol{r} \pm \boldsymbol{\mu}/2)$ to first order in $\boldsymbol{\mu}$:

$$k(\boldsymbol{r} \pm \boldsymbol{\mu}/2) \approx k(\boldsymbol{r}) \pm \frac{1}{2k(\boldsymbol{r})} b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{s} \qquad (3.95)$$

Using this approximation in conjunction with equations (3.90) and (3.92) gives the following expression for the difference between a's and e's measurements for the temporal separation of the two signals:

$$\frac{1}{\nu'_a} - \frac{1}{\nu'_e} \approx a(\boldsymbol{v})\gamma^2 d(\boldsymbol{\mu}) + \frac{a(v)b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{s}}{k(\boldsymbol{r})} + b^{-1}\boldsymbol{\mu}\cdot\frac{\partial a}{\partial \boldsymbol{v}} \tag{3.96}$$

Define the following quantity, α, in terms Δ, given by equation (3.82) as the variation in the signals' phase difference at a:

$$\begin{aligned} \alpha &= \frac{\Delta}{2\pi - \Delta} \\ &= \frac{\nu'_e}{\nu'_a} - 1; \end{aligned} \tag{3.97}$$

(α is well defined if $\Delta \neq 2\pi$; that is, if $\nu'_a \neq 0$). From this definition, $\alpha = 0$ if and only if $\Delta = 0$, which equates to no variation in the phase difference between the signals.

For the maser at a, from equations (3.96) and (3.97),

$$\alpha_a \approx \nu'_e \left(a(\boldsymbol{v})\gamma^2 d(\boldsymbol{\mu}) + \frac{a(v)b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{s}}{k(\boldsymbol{r})} + b^{-1}\boldsymbol{\mu}\cdot\frac{\partial a}{\partial \boldsymbol{v}} \right) \tag{3.98}$$

Since the experiment involves a degree of symmetry, in that comparison of the phases of incoming and outgoing signals occurs at both masers, an analagous quantity, α_e exists for the measurements made by the detector at e:

$$\alpha_e \approx \nu'_e \left(-a(\boldsymbol{v})\gamma^2 d(\boldsymbol{\mu}) + \frac{a(v)b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{s}}{k(\boldsymbol{r})} - b^{-1}\boldsymbol{\mu}\cdot\frac{\partial a}{\partial \boldsymbol{v}} \right). \tag{3.99}$$

This last expression was obtained from equation (3.98) by making the substitutions $\boldsymbol{\mu} \to -\boldsymbol{\mu}$ and $\boldsymbol{r} \to -\boldsymbol{r}$, and noting that d, b^{-1} and $\boldsymbol{s}$ are odd functions of their vector arguments, while k is an even function.

If α_a and α_e are both zero, then from equations (3.49), (3.98) and (3.99),

$$a\gamma^2 b^{-1}\boldsymbol{\mu}\cdot(\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}) + b^{-1}\boldsymbol{\mu}\cdot\frac{\partial a}{\partial \boldsymbol{v}} = 0 \tag{3.100}$$

and

$$b^{-1}\boldsymbol{\mu}\cdot\boldsymbol{s} = 0 \tag{3.101}$$

Equation (3.100) gives

$$a\gamma^2 \left[\boldsymbol{v} + (1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}\right] + \frac{\partial a}{\partial \boldsymbol{v}} = 0 \tag{3.102}$$

which is the same equation as (3.79), as can be seen from equation (3.76). Thus the solution for a is given by equation (3.80), the special relativistic form for arbitrary synchronisation. Thus this experiment has the same limitations as the two-photon absorption experiment, and cannot pick out the functional value corresponding to Einstein synchronisation.

Similarly, equation (3.101) can only yield a restriction on b within an arbitrary synchrony choice. For the sake of simplicity, place $\boldsymbol{\kappa} = \mathbf{0}$, $\boldsymbol{V} = \{V, 0, 0\}$, and x^i to be parallel to ξ^i. Then the length contraction matrix, B has only diagonal components: β, δ and δ respectively. Equation (3.101) becomes

$$\frac{B^{-1}\boldsymbol{\mu}\cdot\boldsymbol{V}}{1 - V^2} B^{-1}\boldsymbol{r}\cdot\boldsymbol{V} + B^{-1}\boldsymbol{\mu}\cdot B^{-1}\boldsymbol{r} = 0 \qquad (3.103)$$

From here

$$\mu_x r_x \left(\frac{\delta^2}{\beta^2(1 - V^2)} - 1\right) + \boldsymbol{\mu}\cdot\boldsymbol{r} = 0. \qquad (3.104)$$

Now, the definitions of $\boldsymbol{\mu}$ and $\boldsymbol{r}$ show that

$$\boldsymbol{\mu}\cdot\boldsymbol{r} = ({\boldsymbol{r}_2}^2 - {\boldsymbol{r}_1}^2)/2. \qquad (3.105)$$

Assuming, as does Will, that one maser is at rest in S with the other maser circling it, automatically imposes $r_1 = r_2$, and one is led to the unjustified conclusion that equation (3.101) gives a measure of a direct relation, $\beta^2 = \delta^2/(1 - V^2)$ between β and δ. However, in principle, one does not have such a strong constraint because, in general, $r_1 \neq r_2$.

As an aside, note that there has been some confusion as to the reason for the synchrony-independence of the phase differences of the signals measured at one location. For example, Will[123], in his analysis of the maser phase experiment, made the following statement concerning the phase difference measured at one location:

> Notice that the result is *independent* of the synchronization procedure embodied in the vector $\boldsymbol{\epsilon}$. This is because the *initial* relative phase of the two oscillators must be chosen arbitrarily; this is tantamount to choosing a convention for synchronization.

Will's reasoning is incorrect: the initial relative phase of the masers (oscillators) has nothing to do with the synchrony convention in an inertial frame with which they are not co-moving. The reason for the synchrony invariance of the measured phase difference is that only one clock, and not a system of spatially separated clocks is needed for such a measurement, and thus synchrony considerations do not influence the result.

Chapter 4

Synchrony and non-inertial observers

4.1 Relativistic non-inertial observers

So far, in the preceding chapter, arbitrary synchrony has been considered only for inertial frames in flat space-time (the freedom of synchrony in curved space-time being widely accepted). However, this conventionality of synchronisation also extends to non-inertial frames, and is relevant to experimentation on the rotating Earth, and other accelerated frames of reference[1, 2]. This chapter discusses the case of arbitrary synchrony for an accelerated observer in a space-time of unspecified curvature.

When investigating the co-ordinates of an accelerated observer, one examines two separate problems. First, in this situation, the observer's co-ordinate system is derived from using a tetrad (a set of basis vectors that the observer is postulated to choose and transport along his worldline in space-time). The nature of this tetrad is important in that the choice of tetrad defines, amongst other things, the one-way speed of light at the observer's location, and the propagation of the tetrad along the observer's worldline must take arbitrary synchrony into account. A modification of the Frenet frame method (see Kreyszig[53] for a discussion of the standard techniques) is used for these considerations. The second problem is the assignment of the co-ordinates themselves, in a way that is natural and unrestrictive. In the following sections, the Riemann normal co-ordinates type approach used by Misner *et al.*[82] for the co-ordinates of an accelerated observer in general relativity is adapted to produce a prescription that allows one to take arbitrary synchrony into account.

Misner *et al.*[82] require that their accelerated observer propagates a tetrad, and state the requirements of that tetrad, but do not give a formal analysis of how that tetrad is realised. However, an example of the use of the Frenet frame method is given by Scorgie[103] who, working within special relativity, uses the standard Frenet frame formulae to obtain the tetrad for an accelerated observer. Scorgie then follows Misner *et al.*[82] in their flat space analysis of an accelerated observer, obtaining the observer's metric by appealing to the invariance of the interval in going from an arbitrary inertial frame (in which the observer is analysed) to the accelerated frame.

Central to both the analyses given by Scorgie and by Misner *et al.* is the consideration of a (constant-time) three-dimensional slice of space-time as the observer's physical space. This precludes the use of their approaches in considering arbitrary synchrony because constant time slices are synchrony dependent and thus the above methods do not lend themselves to arbitrary synchrony. Similarly, one cannot use the curved space-time prescription given by Misner *et al.* who, for any point on the observer's worldline, take as constant time curves those geodesics whose tangent vectors at that point have zero temporal component (purely "spatial" tangent vectors).

This chapter first presents a desired set of local co-ordinates which will handle space-varying synchrony (section 4.2). Section 4.3 contains a generalisation of the Frenet frame, and section 4.4 then combines the previous results to obtain the co-ordinates of an accelerated observer.

The following convention is used for basis vectors. Both for the case of local co-ordinates and the accelerated observer, basis vectors are denoted by $\{g_\mu\}$ with the basis vectors at the spatial origin denoted by $\{e_\mu\}$. The standard orthonormal basis vectors are written as $\{\eta_\mu\}$. The corresponding metrics are then written respectively as $g_{\mu\nu}$, $e_{\mu\nu}$, $\eta_{\mu\nu}$.

4.2 Local co-ordinates

The choice of co-ordinate system has no physical significance and is a matter of convenience for the description of events. However, co-ordinates do reflect to some extent the assumptions made in this description, and so some freedom in prescriptions for co-ordinatising a set of events in a general way can be helpful.

In this section, the assignment of co-ordinates to the neighbourhood of an event is considered by generalising the Riemann normal co-ordinates method in order to facilitate synchrony considerations. Initially this generalisation is discussed on an arbitrarily curved manifold. However, a manifold with zero curvature is used to obtain an actual set of co-ordinates, in order to produce a co-ordinate system for an accelerated observer with arbitrary synchrony, for application in section 4.4 and 5.1.

Riemann normal co-ordinates form a system of co-ordinates local to a point on a manifold (assigned in a neighbourhood of that point) and are defined in terms of a vector basis defined at that point. There is no requirement that the manifold be Riemannian or semi-Riemannian for the construction of such a co-ordinate set to work (although this point has no bearing in the present context of the special and general theories—which both assume a Lorentzian metric—it allows the results of this chapter to be generalised for the purposes of producing a test-theory of local Lorentz invariance in section 5.1 where a wide range of theories is considered).

The motivation for Riemann normal co-ordinates comes from the exponential map (see Kobayashi and Nomizu[49, Section 8]). This is a mapping from the tangent space, T_P, of a point, P, on a manifold to a neighbourhood of that point, and is defined by

$$\exp \lambda V = \gamma(\lambda) \tag{4.1}$$

where $\gamma(\lambda)$ is a geodesic starting at P with tangent vector V at P. In particular, the exponential map, for all the geodesics through P, maps each geodesic's tangent vector at P to the point a unit parameter distance along that geodesic[110]. If a vector basis is defined at P, then normal components along such a geodesic can be defined to be proportional to the components of its tangent vector at P in the following manner:

Consider a point P on a manifold with a torsion-free connection. There exists a neighbourhood, N, of P such that for all Q in N (where Q is distinct from P) there exists a unique geodesic, γ say, connecting Q and P (this follows from the definition, equation (4.1), and the property that the exponential map maps a neighbourhood of the zero vector in the tangent space at a point *onto* a neighbourhood of that point[49, Proposition 8.2]).

Let λ be an affine parameter along $\gamma(\lambda)$ with $\gamma(0) = P$ and $\gamma(\lambda) = Q$ and let

$\frac{\mathrm{d}}{\mathrm{d}\lambda}_{|\lambda=0} = V = N^\alpha \eta_\alpha$, where the basis vectors η_α are the conventional, orthonormal choice, corresponding to Einstein synchronisation at the observer's spatial origin: $\langle \eta_\alpha | \eta_\beta \rangle = \eta_{\alpha\beta}$. The Riemann normal co-ordinates X^α centred at P are taken as proportional to the parameter distance λ from P to Q and also proportional to the components of V. That is, $X^\alpha(Q) = \lambda N^\alpha$ (see Misner *et al.*[82], for example, or Laugwitz[60] who gives a different derivation of Riemann normal co-ordinates using a Taylor's expansion of a prior co-ordinate system).

The discussion in this chapter will consider only manifolds with torsion-free connections. The existence of torsion does not prevent one from finding a sytem of normal co-ordinates centred on a point on a manifold; however, if the connection is torsion-free (and thus symmetric), then there exist normal co-ordinates such that the connection coefficients vanish at that point[49]

A set of co-ordinates different from Riemann normal co-ordinates can be obtained by using a different choice of synchrony at P. This corresponds to the use of an alternative tetrad, e_α, where

$$\langle e_0 | e_0 \rangle = -1, \quad \langle e_0 | e_m \rangle = \phi_m, \quad \langle e_m | e_n \rangle = \delta_{mn} - \phi_m \phi_n, \tag{4.2}$$

for three arbitrary numbers ϕ_n.

The basis vectors, e_α, can be related to a set of orthonormal basis vectors, η_α, by

$$e_0 = \eta_0, \quad e_n = \eta_n - \phi_n \eta_0 \tag{4.3}$$

with the result $V \equiv N^\alpha \eta_\alpha = V^\alpha e_\alpha$ where

$$V^0 = N^0 + \phi_n N^n, \quad V^n = N^n. \tag{4.4}$$

Co-ordinates $\{x^\alpha\} = \{t, x^i\}$ can then be assigned to $Q = \gamma(\lambda)$ according to the formula $x^\alpha(Q) = \lambda V^\alpha$. These are related to the "Einstein synchronisation"[1] Riemann normal co-ordinates, $\{X^\alpha\} = \{T, \boldsymbol{X}\}$ by $t = T + \phi_i X^i$ and $x^i = X^i$. The choice of basis vectors in these cases defines the surfaces of simultaneity in the whole neighbourhood N of P and thus determines the synchrony choice for all points covered by the co-ordinate system. A more general (space-varying) choice of synchrony requires a description of the propagation of the spatial basis vectors (which are tangent to the surfaces of simultaneity at each point on the manifold). Now these propagation laws are defined by and require the knowledge of the connection coefficients at all points, and so cannot be handled by the method

[1] On a curved manifold, Einstein synchronisation holds only at P

mentioned above: initially, it specifies values for $\Gamma^{\alpha}{}_{\beta\gamma}$ at only one point, P, with geodesic deviation corrections applied later[82].

However, in flat space-time, one can first define simultaneity relations within this context by specifying how spatial basis vectors propagate along the geodesics, since all curvature effects of the manifold are already known. Now, by definition of the connection coefficients,

$$\nabla_{\nu} g_{\mu} \equiv \nabla_{g_{\nu}} g_{\mu} \equiv \Gamma^{\alpha}{}_{\mu\nu} g_{\alpha}. \tag{4.5}$$

The temporal basis vector, g_0, plays no role in determining simultaneity relations, and should be unchanged by synchronisation transformations, as suggested by equation (4.3). Thus the choice can be made that g_0 is parallelly propagated along the geodesics emanating from P: $\nabla_{\nu} g_0 = 0$, which by symmetry of the connection coefficients gives the result that synchrony choice will be independent of time. As is also indicated by equation (4.3), a change in synchronisation alters the spatial basis vectors only in the g_0 direction: $\nabla_n g_m \propto g_0$. These considerations leave unrestricted only the propagation of spatial basis vectors in the temporal direction, and together with equation (4.5) suggest the following general form for the connection coefficients:

$$\Gamma^{\alpha}{}_{\mu 0} = 0, \quad \Gamma^{l}{}_{mn} = 0, \quad \Gamma^{0}{}_{mn} = F_{mn} \tag{4.6}$$

where the F_{mn} are some differentiable functions of position.

The basis vectors along the geodesics can obtained from the definitions of the connection coefficients in the following manner: from equations (4.5) and (4.6), it follows that

$$\begin{aligned} \frac{\mathrm{D}}{\mathrm{d}\lambda} g_0 &= 0 \\ \frac{\mathrm{D}}{\mathrm{d}\lambda} g_n &= \frac{\mathrm{d}x^m}{\mathrm{d}\lambda} F_{mn} g_0. \end{aligned} \tag{4.7}$$

These differential equations then give

$$\begin{aligned} g_0 &= e_0 \\ g_n &= e_n + \int_0^{\lambda} \frac{\mathrm{d}x^m}{\mathrm{d}\lambda} F_{mn} g_0 \mathrm{d}\lambda. \end{aligned} \tag{4.8}$$

The symmetry of the connection coefficients, and the requirement that the last equation be integrable, suggest a constraint of the form $F_{mn} = F_{,mn}$ for some differentiable function F.

From here, an appropriate neighbourhood of a point P can be co-ordinatised by using the above values for the connection coefficients and solving the geodesic equation,

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} = -\Gamma^\mu{}_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda} \frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda}, \tag{4.9}$$

to obtain a co-ordinate set. For a point $\gamma(\lambda)$ such that $\gamma(0) = P$ and $\frac{\mathrm{d}}{\mathrm{d}\lambda}(0) = V$, Equations (4.6) and (4.9) give

$$\frac{\mathrm{d}^2 x^n}{\mathrm{d}\lambda^2} = 0, \quad \frac{\mathrm{d}^2 x^0}{\mathrm{d}\lambda^2} = -\frac{\mathrm{d}^2 F(x^p)}{\mathrm{d}\lambda^2} = 0 \tag{4.10}$$

which, together with the conditions that $x^n = 0$ when $\lambda = 0$ and that $V = V^\alpha e_\alpha$, gives

$$x^0 = \lambda(V^0 + F_{,n}(0)V^n) + F(0) - F(\,x^n(\lambda)\,), \quad x^n = \lambda V^n. \tag{4.11}$$

It is usual to require that the connection coefficients vanish at the point on which the co-ordinate system is centred, which imposes the condition that the F_{mn} vanish at that point (the origin). Furthermore, taking the inner product of g_0 and g_n gives $g_{0n} = \phi_n + F_{,n}(0) - F_{,n}$. In principle, so that the synchrony at parameter value λ along the geodesic is independent of the synchrony definitions at the origin, the inner product g_{0n} should be independent of ϕ which is satisfied by the placing $F_{,m}(0) = -\phi_m$. Equations (4.11) show that F can be replaced by $f = F(0) - F$, which gives the final form of the connection coefficients in equation (4.6) as

$$\Gamma^\alpha{}_{\mu 0} = 0, \quad \Gamma^l{}_{mn} = 0, \quad \Gamma^0{}_{mn} = -f_{,mn} \tag{4.12}$$

where

$$f(0) = 0, \quad f_{,m}(0) = \phi_m, \quad f_{,mn}(0) = 0 \tag{4.13}$$

for some differential function $f = f(x^n)$.

The co-ordinates in equations (4.11) then simplify to

$$x^0 = \lambda(V^0 + \phi_n V^n) + f(\,x^n(\lambda)\,), \quad x^n = \lambda V^n. \tag{4.14}$$

Similarly, from equations (4.9), the basis vectors along the geodesic are seen to be

$$g_0 = e_0, \quad g_n = e_n + (\phi_n - f_{,n}(x^q(\lambda))\,)e_0, \tag{4.15}$$

and the metric tensor components (obtained by taking inner products of the basis vectors) are

$$g_{00} = -1, \quad g_{0m} = f_{,m}(x^n(\lambda)), \quad g_{mn} = \delta_{mn} - f_{,m}f_{,n}, \tag{4.16}$$

giving the local one-way speed of light in the direction $\boldsymbol{p}$ as

$$c(\boldsymbol{p}) = \frac{1}{1 + f_{,m}p^m}. \tag{4.17}$$

While the co-ordinates equations (4.14) have been derived in a flat space-time, they provide a valid co-ordinate choice for an accelerated observer in a space-time of arbitrary curvature, and indeed they are used in the section 4.4 as a set of "flat" space-time co-ordinates for an observer whose accelerations and synchrony are both arbitrary.

4.3 Tetrad propagation

Let P be the observer's worldline. The parameter, t, along P is the observer's proper time. When constructing a tetrad to be propagated with the observer, it is natural to take the temporal basis vector to be the tangent to his worldline, for in the observer's rest frame his space-time motion is purely in a temporal direction:

$$e_0 \equiv \frac{\mathrm{d}}{\mathrm{d}t} \tag{4.18}$$

Given the definition of e_0, the other basis vectors are chosen subject to whatever conditions are required of the co-ordinate frame.

One way of obtaining spatial basis vectors along this worldline is the Frenet frame method where a set of orthonormal basis vectors is developed sequentially by requiring that the k^{th} derivative of the worldline lies in the span of the first k basis vectors[48]. (See Kreyszig[53] for a discussion of the usual case where each derivative of the worldline is, by definition, proportional to a basis vector.) An orthonormal basis corresponds to a local imposition of Einstein synchronisation; using a modification of the Frenet frame method, one can construct a tetrad which allows more synchrony freedom: instead of the usual orthonormality requirement for the basis vectors of a Frenet frame, the inner product relations given in equation (4.2) are used.

From the definition, (4.5), of the connection coefficients, propagation of the tetrad along the observer's worldline is given by[82]

$$\dot{e}_\mu \equiv \frac{\mathrm{D}e_\mu}{\mathrm{d}t} = \nabla_{e_0} e_\mu \equiv \Gamma^\alpha{}_{\mu 0} e_\alpha. \tag{4.19}$$

Now, define, in Frenet fashion[48],

$$\begin{aligned} \dot{e}_\mu &= A_\mu{}^\nu e_\nu, \\ A_\mu{}^\nu &= 0 \mathrm{if} \nu \geq \min(\mu+1, 3), \\ A_0{}^1 = \chi_1, \quad A_1{}^2 &= \chi_2, \quad A_2{}^3 = \chi_3. \end{aligned} \tag{4.20}$$

Covariant differentiation of the relations in equation (4.2) gives

$$\langle \dot{e}_\mu | e_\nu \rangle + \langle e_\mu | \dot{e}_\nu \rangle = 0. \tag{4.21}$$

By substituting equation (4.21) into this last equation and solving for $A_\mu{}^\nu$ successively for the cases $\mu = 0$ to $\mu = 3$ one obtains

$$\begin{aligned} \dot{e}_0 &= \chi_1 \phi_1 e_0 + \chi_1 e_1 \\ \dot{e}_1 &= (\chi_1(1 - {\phi_1}^2) + \chi_2 \phi_2) e_0 - \phi_1 \chi_1 e_1 + \chi_2 e_2 \\ \dot{e}_2 &= -(\chi_1 \phi_1 \phi_2 + \chi_2 \phi_1 - \chi_3 \phi_3) e_0 - (\chi_1 \phi_1 + \chi_2) e_1 + \chi_3 e_3 \\ \dot{e}_3 &= -(\chi_1 \phi_1 \phi_3 + \chi_3 \phi_2) e_0 - \chi_1 \phi_3 e_1 - \chi_3 e_2 \end{aligned} \tag{4.22}$$

(A Frenet frame results when $\phi_i = 0$ for all i.) Comparing equation (4.22) with equation (4.19) gives values for $\Gamma^\alpha{}_{\mu 0}$ at the origin. These may be expressed as[82]

$$\Gamma_{\mu\nu 0} = (a \wedge u)_{\mu\nu} + \epsilon_{\alpha\beta\mu\nu} u^\alpha \omega^\beta \tag{4.23}$$

where u, a and ω are respectively the four-velocity, four-acceleration, and angular velocity four-vector of the observer, and $\wedge$ denotes the wedge product. (No restrictions are placed on the other components of the affine connection by this method.) The observer's self-measured three-acceleration and spatial angular velocity are $\{\chi_1, 0, 0\}$ and $\{\chi_3, 0, \chi_2\}$, respectively. In the case of zero angular velocity the observer is in Fermi-Walker transport[82], and then with the choice of an orthonormal basis ($\phi_i = 0$) one obtains the standard description of a Fermi-Walker tetrad.

4.4 Accelerated observer

Here the development of a co-ordinate system for an accelerated observer, with arbitrary acceleration and rotation is considered within the context of special relativity or general relativity. Metric tensor components are obtained to first order. The co-ordinate set given in equations (4.14) is used to label events, and the main task here is to find a prescription for assigning these co-ordinates in a consistent manner. The observer's worldline, P, parameterised by proper time, t, is modelled by a curve on a semi-Riemannian manifold with signature (1,3). It is natural to take P as the time axis for the observer, who is assumed to propagate a tetrad along P according to the modified Frenet frame prescription given in section 4.3. This tetrad reflects the choice for the one-way speed of light at his spatial origin, which is represented as moving along P on the manifold.

Misner *et al.*[82], who consider only an observer with an orthonormal frame, assign co-ordinates in the following manner. They consider all geodesics, at each point $P(t)$ on the worldline, whose tangent vectors at $P(t)$ have no temporal component according to the observer's tetrad at that point. These geodesics are considered curves of constant time, t, and have spatial co-ordinates assigned along them which are proportional to the geodesic parameter and the tangent vector components at $P(t)$, in a similar fashion to the Riemann normal co-ordinates case (see section 4.2).

This prescription has two properties which preclude its use for formulating co-ordinates with arbitrary synchrony. First, if an arbitrary tetrad is used, the geodesics picked out as curves of constant time (on the basis of having a tangent vector at $P(t)$ with zero temporal component) will vary with choice of ϕ_i and thus an undesirable transformation in spatial co-ordinates would be concomitant with a temporal transformation corresponding to a redefinition of the one-way speed of light along P (in principle, the spatial co-ordinates should be independent of a change in clock setting). This problem arises because that prescription is not geometric in nature. Second, with that prescription, since the tetrad choice determines spatial surfaces (which are surfaces of simultaneity) this choice determines how clocks are synchronised all along the geodesics, removing any freedom in varying clock settings from point to point.

Since it is convenient to assign co-ordinates along geodesics emanating from the observer, a geometric property is used to distinguish a set of geodesics along which spatial co-ordinates are assigned independent of synchrony choice. Once a set of geodesics is chosen, the co-ordinates assigned along members of this set are those of the corresponding "local co-ordinates" centred at $P(t)$ (equation (4.14)) with the observer's time, t, being added to the x^0 co-ordinate.

There are two sets of geodesics that might be expected to be useful for this purpose: those orthogonal to the observer's worldline (which do not necessarily coincide with those having tangent vectors with no time component) and null geodesics.

Of the two sets of geodesics, the null geodesics might initially seem the preferable choice to work with because their use is natural in that it corresponds to the operational approach of obtaining distant information from electromagnetic radiation, where the observer assigns co-ordinates to only those events which he sees. An advantage of the use of the null geodesics over the orthogonal geodesics would be that while orthogonal geodesics intersecting an accelerated observer's worldline at different events eventually interesect, even in flat space-time, and thus limit the validity of the co-ordinate system to a region around the worldline[82], null geodesics will not cause this limitation. However, the use of null geodesics in this approach results in an inconsistency: the connection coefficients are singular along the world line, as is shown below.

Take one of the "past" null geodesics (with parameter λ) intersecting $P(t)$ and consider trying to co-ordinatise it using the prescription in section 4.2. If the tangent vector at $P(t)$ is $V = V^\alpha e_\alpha$, then since it is a null vector, $V^\alpha V^\beta e_{\alpha\beta} = 0$ where the values of $e_{\alpha\beta}$ are given in equations (4.2). This null condition together with the fact that V is pointing in towards the "past" places a constraint on the components of V:

$$V^0 - V^m \phi_m = -\sqrt{V^m V^n \delta_{mn}} \tag{4.24}$$

From equation (4.14) the required co-ordinates along this geodesic would be

$$x^n = \lambda V^n, \quad x^0 = t - \lambda\sqrt{V^m V^n \delta_{mn}} + f(\,x^n(\lambda)\,) \tag{4.25}$$

Now, the connection coefficients $\Gamma^\alpha{}_{\mu 0}$, which are involved with the obeserver's tetrad propagation along his world line are known at $P(t)$ from equations (4.22). The rest of the coefficients can be found using the geodesic equation (4.9) and equation (4.25). In particular, evaluation of $\Gamma^r{}_{mn}$ along the geodesic gives

$$\Gamma^r{}_{mn} = (\hat{V}^p \delta_{pn} + f_{,n})(\Gamma^r{}_{m0} + \Gamma^r{}_{00}(\hat{V}^p \delta_{pm} + f_{,m})) + (\hat{V}^p \delta_{pm} + f_{,m})\Gamma^r{}_{n0}, \tag{4.26}$$

where $\hat{V}^p$ signifies a unit three-vector. Clearly these connection coefficients are dependent on the tangent vector along the geodesic. This, in itself is not unexpected, but in the limit as $\lambda \to 0$, this tangent vector dependence does not vanish at $P(t)$ and thus the $\Gamma^r{}_{mn}$ are not uniquely determinable and so not defined along the worldline. This problem appears to rule out the use of null geodesics in assigning co-ordinates for an observer in arbitrary motion.

However, for an inertial observer, co-ordinates may be assigned along null geodesics since then all the connection coefficients are zero along P. The time co-ordinate depends on the one-way speed of light along the geodesics and a locally flat metric is obtained.

This leaves the choice of geodesics orthogonal to the worldline as the only one to give tractable results. Again assume the results of sections 4.3 and 4.2.

Orthonormality of a geodesic with the worldline at $P(t)$ requires that for the geodesic's tangent vector V, $V \cdot e_0 = 0$. So

$$V^0 = \phi_n V^n, \tag{4.27}$$

and so the co-ordinates along the geodesics are, using equations (4.14),

$$x^n = \lambda V^n \quad \text{and} \quad x^0 = t + f(x^q). \tag{4.28}$$

Using the chain rule, equations (4.28) and the geodesic equation (4.9), allows the connection coefficients $\Gamma^\mu{}_{nm}$ to be expressed in terms of the other connection coefficients $\Gamma^\mu{}_{\nu 0}$ which themselves have already been determined along P by the application of equations (4.19) and (4.22) to this situation:

$$\begin{aligned} \Gamma^m{}_{pn} &= -(\Gamma^m{}_{p0} f_{,n} + \Gamma^m{}_{n0} f_{,p} + \Gamma^m{}_{00} f_{,p} f_{,n}) \\ \Gamma^0{}_{pn} &= -(\Gamma^0{}_{p0} f_{,n} + \Gamma^0{}_{n0} f_{,p} + \Gamma^0{}_{00} f_{,p} f_{,n} + f_{,pn}). \end{aligned} \tag{4.29}$$

From equations (4.22) and (4.30) the connection coefficients along the worldline are then

$$(\Gamma^\mu{}_{\nu 0})^t(P) = \begin{pmatrix} \chi_1\phi_1 & \chi_1 & 0 & 0 \\ \chi_1(1-(\phi_1)^2) + \chi_2\phi_2 & -\chi_1\phi_1 & \chi_2 & 0 \\ -\chi_1\phi_1\phi_2 - \chi_2\phi_1 + \chi_3\phi_3 & -\chi_1\phi_2 - \chi_2 & 0 & \chi_3 \\ -\chi_1\phi_1\phi_3 - \chi_3\phi_2 & -\chi_1\phi_3 & -\chi_3 & 0 \end{pmatrix}. \tag{4.30}$$

Although the coefficients $\Gamma^\mu{}_{\nu 0}$ are known along P by reason of tetrad propagation, they are not known along the orthogonal geodesics at this stage; their values might be obtained by integrating the expression for the Riemann tensor in terms of the connection coefficients and their derivatives[74]. From the definition of the connection coefficients, one could then obtain the basis vectors exactly along all the geodesics. Unfortunately the solution for the connection coefficients from the Riemann tensor expression is generally intractable (but not in the case of an inertial observer[74]).

The method used by Misner *et al.*[82] at this stage consists of solving the differential equations which result from expressing the connection coefficients in their Christoffel symbols form and does not readily lend itself to space-varying synchrony since the connection coefficients are evaluated at only one spatial point.

Therefore an approximation must be used to obtain values for the basis vectors away from the worldline. Instead of making an approximation to the $\Gamma^{\mu}{}_{\nu 0}$ and thus the other connection coefficients, one can make an appropriate approximation to the covariant derivative which would require only the connection coefficients' values along the worldline. Metric tensor components can then be found by taking inner products of the resultant basis vectors.

Now, the basis vectors, $\{g_\mu\}$, along the orthogonal geodesics are obtained by parallel transport of the set $\{e_\mu\}$ along these geodesics. From equations (4.5) and (4.28)

$$\frac{\mathrm{D}}{\mathrm{d}\lambda}g_0 = \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda}\Gamma^0{}_{00} + V^n\Gamma^0{}_{n0}\right) g_0 + \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda}\Gamma^m{}_{00} + V^n\Gamma^m{}_{n0}\right) g_m \tag{4.31}$$

and

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{d}\lambda}g_n &= \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda}f_{,n}\Gamma^0{}_{00} - V^p\Gamma^0{}_{p0}f_{,n} - \frac{\mathrm{d}f_{,n}}{\mathrm{d}\lambda}\right) g_0 \\ &\quad - \left(f_{,n}\frac{\mathrm{d}f}{\mathrm{d}\lambda}\Gamma^m{}_{00} + f_{,n}V^p\Gamma^m{}_{p0}\right) g_m \end{aligned}\tag{4.32}$$

Comparing these last two equations gives

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{d}\lambda}g_n &= \frac{\mathrm{D}}{\mathrm{d}\lambda}(f_{,n}g_0) \\ \Rightarrow g_n &= f_{,n}g_0 + e_n + \phi_n e_0. \end{aligned}\tag{4.33}$$

Substituting (4.33) into (4.31) yields

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{d}\lambda}g_0 &= \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda}\Gamma^m{}_{00} + V^n\Gamma^m{}_{n0}\right)(f_{,m}g_0 + e_m + \phi_m e_0) \\ &\quad + \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda}\Gamma^0{}_{00} + V^n\Gamma^0{}_{n0}\right) g_0 \end{aligned}\tag{4.34}$$

In principle, equation (4.34) can be used to give an exact expression for g_0, but because the form for the coefficients $\Gamma^{\mu}{}_{\nu 0}$ is not in general the correct expression, even in flat space-time, any result for g_0 will be an approximation, valid only near the observer's worldline.

Now, by definition of the covariant derivative in terms of limits[82, page 208],

$$\frac{\mathrm{D}}{\mathrm{d}\lambda}g_0|_P = \lim_{\lambda\to 0}\left(\frac{g_0(\lambda)_{\|} - g_0(0)}{\lambda}\right), \tag{4.35}$$

where $\|$ signifies that $g_0(\lambda)$ has been parallelly-propagated to $P(t)$ for the purpose of comparing it with $g_0(0)$.

Making an approximation to this covariant derivative of g_0 for small λ, equating it to equation (4.34),

$$\frac{g_0(\lambda) - e_0}{\lambda} \approx \frac{\mathrm{D}}{\mathrm{d}\lambda} g_0{}_{|P} \tag{4.36}$$

$$\begin{aligned} &= \left[\left(\frac{\mathrm{d}f}{\mathrm{d}\lambda} \Gamma^m{}_{00} + V^n \Gamma^m{}_{n0} \right) (f_{,m} g_0 + e_m + \phi_m e_0) \right. \\ &\quad \left. + \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda} \Gamma^0{}_{00} + V^n \Gamma^0{}_{n0} \right) g_0 \right]_{|P} \\ &= \left[V^1 \chi_1 g_0 + (V^1 \chi_2 - V^3 \chi_3)(e_2 + \phi_2 e_0) \right. \\ &\quad \left. + V^2 \chi_3 (e_3 + \phi_3 e_0) - V^2 \chi_2 (e_1 + \phi_1 e_0) \right]_{|P} \end{aligned} \tag{4.37}$$

From the above expression, and equations (4.33) and (4.28), the basis vectors can be written in the following form which is valid only near the worldline:

$$\begin{aligned} g_0 &= (1 + x^1 \chi_1) e_0 - R^n (e_n + \phi_n e_0), \\ g_n &= f_{,n} g_0 + e_n + \phi_n e_0, \end{aligned} \tag{4.38}$$

where the following definitions have been used.

$$\begin{aligned} R^1 = x^2 \chi_2, \quad R^2 = -x^1 \chi_2 + x^3 \chi_3, \quad R^3 = -x^2 \chi_3, \\ R^n \equiv \epsilon^{njk} x_j \omega_k, \end{aligned} \tag{4.39}$$

and where ω is the observer's angular four-velocity, as mentioned in section 4.3. (In this last equation, the observer's self-measured spatial angular velocity has components $\{\omega^k\} = \{\chi_3, 0, \chi_2\}$.)

The metric tensor components are obtained by taking inner products of the basis vectors (4.38) and using the relations in equation (4.2):

$$\begin{aligned} g_{00} &= -(1 + x^1 \chi_1)^2 + R^n R^m \delta_{nm}, \\ g_{0m} &= -f_{,m} g_{00} - R^p \delta_{pm}, \\ g_{mn} &= f_{,m} f_{,n} g_{00} + f_{,m} R^p \delta_{pn} + f_{,n} R^p \delta_{pm} + \delta_{mn}. \end{aligned} \tag{4.40}$$

These metric components ignore curvature effects on the manifold since they do not take geodesic deviation into account. For a manifold with non-zero curvature, curvature effects come into play at second order[82], and so the expressions (4.41) are, in general, valid only to first order. However, in a flat space-time theory such as special relativity they are in fact exact, agreeing with the metric derived by Nelson[86], Hehl *et al.*[41] and Scorgie[103] who all assume an orthonormal basis (in which case $f = 0$).

Chapter 5

Tests of local Lorentz invariance

5.1 A test-theory

The Mansouri-Sexl test-theory was formulated for inertial frames only, and attempts to generalise it for the analysis of non-inertial observers[1, 2] have been of the form of a co-ordinate transformation, and only for constant rotational motion. The approach presented here has a geometrical foundation from which a co-ordinate transformation is derived, as opposed to simply postulating a co-ordinate transformation. The advantage of this is that all physically possible motions of an observer can be accommodated, giving a more general theory which contains the Mansouri-Sexl transformations[76] as a subclass; the geometric perspective proves to be complementary to the traditional approach in the Mansouri-Sexl formalism.

When special relativity is formulated on a four-dimensional manifold, its kinematics results from the existence of two geometric structures on the manifold: an affine connection corresponding to a flat space-time (having zero curvature) and a semi-Riemannian metric, of signature (1,3), which is compatible with the connection. In a standard construction, using orthonormal bases for all observers, inertial co-ordinates can be found in which the connection coefficients are zero and the metric tensor components have the familiar form $\{\eta_{\mu\nu}\} = \mathrm{diag}\{-1, 1, 1, 1\}$. The invariance group of transformations of the metric (which preserves the form of the metric) contains the Lorentz transformations, and these form the group of transformations from one set of inertial co-ordinates to another (see Friedman[31] for a discussion). If the restriction to orthonormal bases is relaxed, for example by introducing a synchrony change, the connection coefficients are not in general zero, and the metric components no longer keep their diagonal form (since they are the inner products of the basis vectors).

Now, any metric theory possesses a symmetric, locally Lorentzian metric, with test particles following geodesics of that metric. On a manifold this corresponds to the existence of a torsion-free affine connection, compatible with a semi-Riemannian metric of signature (1,3). (Other structure may be needed to impart the full character of the corresponding space-time: for example general relativity also needs the addition of a stress-energy tensor[31]). Thus the existence of local Lorentz invariance in a theory depends only on the affine connection and the metric. If an affine connection is torsion-free, there always exist co-ordinates centred on any event such that at that event the connection coefficients are all nought[49] (but not necessarily their derivatives) and then a (1,3) metric is Lorentzian at that event. For the purpose of testing local Lorentz invariance, one must operate in a framework of theories that in general does not possess local Lorentz invariance. Such a test-theory can be obtained by removing structure which corresponds to local Lorentz invariance, thus admitting a wider class of theories. Since the analysis of kinematics requires comparison of vectors propagated along curves, an affine structure is needed, and so it is natural to remove the metric as geometric structure in order to produce a test-theory.

The omission of a metric as a geometric structure on the manifold is not in conflict with relativistic kinematics, nor is it a denial of the existence of a space-time metric. Rather, it allows the test-theory to examine both theories which have a metric and those which do not, with the aim that experimental tests be used to restrict these theories and determine the validity of local Lorentz invariance at a given level of precison or confidence.

The metric and affine structures on a general Riemannian manifold select classes of geodesic which are, in general, distinct. The metric singles out those curves of extremal distance, while the connection selects those curves which parallel transport their tangent vectors. These two classes coincide only if the connection is torsion-free and metric compatible[42]. Thus a theory with just affine structure does possess geodesics, which naturally correspond to unaccelerated motion, and whose existence does not automatically demand any metric structure. However, a formalism of only affine structure does not suffice for a test-theory; it is too general to have the necessary predictive power and falsifiability. Thus, in accord with the Mansouri-Sexl test-theory discussed in chapter 3, an aether is postulated and extra structure corresponding to a preferred frame is required. A natural candidate for the aether is the cosmic background radiation, as discussed in section 3.1, and the philosophy of this approach is not so much a test for a preferred frame as a test of local Lorentz invariance (special relativistic kinematics is compatible with an aether).

From a geometric perspective, the most natural way to impose this structure is to postulate the existence of a preferred vector field, X, whose integral curves are geodesic on the entire manifold. Here X is interpreted as the four-velocity of the aether at each event, and the integral curves of X model the worldlines of the spatial points of the aether.

Thus a torsion-free affine connection and the preferred vector field, X are imposed as the structure for the test-theory. By setting up a co-ordinate system, an observer has a set of basis vectors at each point of that system by simply taking partial derivatives with respect to the co-ordinates. Then that observer may define an inner product rule for these basis vectors, and thus define a metric. This has no physical meaning and does not suggest a metric for the manifold, but is rather a matter of description, which may or may not be invariant under a co-ordinate transformation. For mathematical convenience, it is assumed that each observer who propagates a tetrad defines a special relativistic type inner product relation similar to equation (4.2) between the basis vectors in that tetrad, and thus the formalism of sections 4.2 to 4.4 can be used here. Note, however, that while such an inner product definition does define a sense of orthogonality, it makes no sense to talk about a metric in an observer's space.

While no space-time metric is discussed, this is independent of the existence of a metric in physical three-space; it is assumed that the physical three-space of each inertial observer in the theory is Euclidean. It is assumed that light travels along geodesics of the connection and furthermore, that in the aether frame, Σ, the return-trip speed of light is isotropic (having value unity) to first order.

Consider an observer, S, who is in a laboratory frame in arbitrary motion, and let S' worldline be P, parameterised by his proper time t. Suppose that S defines a set of basis vectors, e_μ say, along P and let S define the inner product relation equations (4.2) along P. Such a definition in this context is purely mathematical and not neccessarily related to any intrinsic property on the manifold. S can then assign the co-ordinates $\{x^\alpha\} = \{t, x, y, z\}$ given in equations (4.28), and his basis vectors, g_μ near P are then given by equations (4.38). Thus S formally has the same set of basis vectors and co-ordinates as the noninertial observer discussed in section 4.4.

Writing

$$L = 1 + x^1\chi_1 - R^n\phi_n, \tag{5.1}$$

one can express S' basis vectors (4.38) as

$$g_0 = L - R^n e_n, \quad g_n = (\phi_n - f_{,n}L)e_0 + ({\delta_n}^m + f_{,n}R^m)e_m \tag{5.2}$$

Now, consider the aether, which is represented by a vector field X. Any geodesic which is an integral curve of X can be used to represent the worldline of spatial point fixed with respect to the aether. Let Σ be an observer at rest in the aether (and thus in an inertial frame). Choose a geodesic integral curve , Π say, of X which intersects P at $P(0)$ and set the parameter, τ, of Π to be nought at $P(0)$. This corresponds to the choosing the spatial origins of both frames to coincide when $t = \tau = 0$.

Σ can define a co-ordinate system $\{\xi^\alpha\} = \{\tau, \xi, \eta, \zeta\}$, in a manner similar to S, although because Σ is inertial the system will be simpler. Denote the basis vectors of Σ near Π by G_μ, with those along Π labelled $\mathcal{E}_\mu$, and let Σ define the following inner products and co-ordinates along a geodesic with tangent vector $\frac{\mathrm{d}}{\mathrm{d}\lambda} = V^\alpha\mathcal{E}_\alpha$, perpendicular to Pi:

$$\langle\mathcal{E}_0|\mathcal{E}_0\rangle = -1, \quad \langle\mathcal{E}_0|\mathcal{E}_m\rangle = \kappa_m, \quad \langle\mathcal{E}_m|\mathcal{E}_n\rangle = \delta_{mn} - \kappa_m\kappa_n,$$

$$G_0 = \mathcal{E}_0, \quad G_n = h_{,n}\mathcal{E}_0 + \mathcal{E}_n + \kappa_n\mathcal{E}_0, \tag{5.3}$$

$$\xi^n = \lambda V^n \quad \text{and} \quad \xi^0 = t + h(x^q), \tag{5.4}$$

where h and $\kappa_n \equiv h_{,n}(0)$ are the counterparts in Σ of f and ϕ_n in S.

None of these steps requires a global metric; in particular, orthogonality along worldlines is arbitrarily defined.

Now, let E_μ be Σ's basis vectors along P, the worldlline of S. At an event Q, lying a distance λ along a geodesic through $P(t)$, Σ's basis vectors G_μ are given in terms of E_μ by

$$\begin{aligned} G_0 &= E_0, \\ G_n &= E_n - (\, h_{,n}(Q) - h_{,n}(\,P(t)\,)\,)E_0 \\ &\equiv E_n - \Delta h_n E_0 \end{aligned} \tag{5.5}$$

Now, the basis vectors E_μ and e_μ at $P(t)$ are related by some transformation:

$$E_\mu = T_\mu{}^\nu e_\nu. \tag{5.6}$$

Considering S' four-velocity in terms of Σ's co-ordinates, using equation (5.6), and comparing coefficients of e_μ gives

$$\begin{aligned}
u \equiv e_0 = Y^\alpha E_\alpha &= Y^\alpha T_\alpha{}^\nu e_n u \\
\implies \quad T_0{}^0 = \frac{1 - Y^n T_n{}^0}{Y^0}, &\quad T_0{}^m = -\frac{Y^n T_n{}^m}{Y^0} \\
\implies \quad T_0{}^0 = 1/Y^0 - v^n T_n{}^0, &\quad T_0{}^n = -v^m T_m{}^n, \qquad (5.7)\\
v^n &\equiv Y^n/Y^0 \qquad (5.8)
\end{aligned}$$

where $\{v^m\}$ are the components of the three-velocity of S with respect to Σ.

Consider a vector $\frac{\mathrm{d}}{\mathrm{d}l}$ at Q:

$$\frac{\mathrm{d}Q}{\mathrm{d}l} = \frac{\mathrm{d}x^\alpha}{\mathrm{d}l} g_\alpha = \frac{\mathrm{d}\xi^\alpha}{\mathrm{d}l} G_\alpha \tag{5.9}$$

From here, using equations (5.2), (5.5) and (5.6),

$$\begin{aligned}
\mathrm{d}x^0(Le_0 - R^n e_n) + \mathrm{d}x^n((\phi_n - f_{,n}L)e_0 + (\delta_n{}^m + f_{,n}R^m)e_m) \\
= \mathrm{d}\xi^0 E_0 + \mathrm{d}\xi^n(\Delta h_n E_0 + E_n) \\
= (\mathrm{d}\xi^0 + \mathrm{d}\xi^n \Delta h_n) T_0{}^\nu e_\nu + \mathrm{d}\xi^n T_n{}^\nu e_\nu \qquad (5.10)
\end{aligned}$$

Substituting equation (5.7) into the above equation gives

$$\begin{aligned}
[\mathrm{d}x^0 L + \mathrm{d}x^n(\phi_n - f_{,n}L)]e_0 - [\mathrm{d}x^0 R^m - \mathrm{d}x^n(\delta_n{}^m + f_{,n}R^m)]e_0 = \\
[(\mathrm{d}\xi^0 + \mathrm{d}\xi^n \Delta h_n)(1/Y^0 - v^n T_n{}^0) + \mathrm{d}\xi^n T_n{}^0]e_0 \\
- [(\mathrm{d}\xi^0 + \mathrm{d}\xi^n \Delta h_n)v^p + \mathrm{d}\xi^p] T_p{}^m e_m \qquad (5.11)
\end{aligned}$$

By comparing coefficients of e_0 and e_m, one gets

$$\begin{aligned}
L\mathrm{d}t + (\phi_m - f_{,m}L)\mathrm{d}x^m = \\
(\mathrm{d}\tau - \mathrm{d}\xi^n \Delta h_n)(1/Y^0 - v^n T_n{}^0) + \mathrm{d}\xi^n T_n{}^0 \qquad (5.12)
\end{aligned}$$

and

$$\begin{aligned}
-R^m \mathrm{d}t + \mathrm{d}x^n(\delta_n{}^m + f_{,n}R^m) &= b^m{}_p(\mathrm{d}\xi^p - (\mathrm{d}\tau - \mathrm{d}\xi^n \Delta h_n)v^p), \qquad (5.13)\\
b^m{}_p &\equiv T_p{}^m \qquad (5.14)
\end{aligned}$$

where $\{b^m{}_p\}$ is assumed to be an invertible matrix

Note that when S is inertial (when $\chi_1 = \chi_2 = \chi_3 = 0$), and when $h = \boldsymbol{\kappa}\cdot\boldsymbol{\xi}$ and $f = \boldsymbol{\phi}\cdot\boldsymbol{x}$, one has $R^n = 0$, $\Delta h_n = 0$, and $f_{,n} = \phi_n$. Then equations (5.12) and (5.13) give

$$\begin{aligned} \mathrm{d}t &= 1/Y^0\ \mathrm{d}\tau + T_p{}^0(b^{-1})^p{}_m \mathrm{d}x^m \\ \mathrm{d}x^m &= b^m{}_p(\mathrm{d}\xi^p - v^p \mathrm{d}\tau) \end{aligned} \tag{5.15}$$

which are the synchrony-generalised transformations of the Mansouri-Sexl test-theory given in equations (3.1) and (3.2), as may be verified upon making the identifications

$$a = 1/Y^0, \quad \epsilon_m = T_p{}^0(b^{-1})^p{}_m, \tag{5.16}$$

where a is the time dilation parameter, ϵ depends on the synchrony choice in both frames, and $b^m{}_p \equiv T_p{}^m$ are length contraction parameters.

It is interesting to note the significance of ϵ. This can be seen from the definition (5.16) above by noting that, from equation (5.6), if $T_p{}^0 = 0$ then, at any point along P, S' spatial basis vectors span the same surface as do Σ's spatial basis vectors. In the context of the Mansouri-Sexl test-theory[76] (flat space-time) and since ϕ and κ are constant, Σ and S share the same foliation of space-time along P and thus they agree on whether two events are simultaneous or not. This perspective explicitly shows the conventional nature of simultaneity, and is in accord with Mansouri-Sexl test-theory where ϵ was introduced as a measure of the difference in time intervals between Σ and S. (For zero ϵ there is agreement on simultaneity, but not necessarily on time intervals.)

An expression for ϵ may be obtained in terms of the other parameters as indicated below. Such an expression shows that ϵ adds no degress of freedom to the theories in the test-theory, and thus has operational significance only.

First, expressing S' four-velocity as $e_0 = Y^\alpha E_\alpha$ and using equation (5.6) gives

$$(T^{-1})_0{}^\alpha = Y^\alpha \tag{5.17}$$

From the invertibility of $T_\alpha{}^\mu$, and using equation (5.14), one has

$$\begin{aligned} & T_n{}^0(T^{-1})_0{}^m + b^q{}_n(T^{-1})_q{}^m = \delta_n{}^m \\ \Longrightarrow\quad & (T^{-1})_q{}^m = (b^{-1})^m{}_q - \epsilon_q Y^m; \end{aligned} \tag{5.18}$$

and similarly

$$\begin{aligned} & (b^{-1})^n{}_m T_n{}^0(T^{-1})_0{}^0 = (T^{-1})_m{}^0 \\ \Longrightarrow\quad & (T^{-1})_m{}^0 = Y^0\epsilon_m \end{aligned} \tag{5.19}$$

For simplicity, place $h = \boldsymbol{\kappa}\cdot\boldsymbol{x}$ so that $\Delta h = 0$. Equation (5.13) can be re-expressed as

$$\begin{aligned} & T_p{}^0(b^{-1})^p{}_m(\mathrm{d}x^n(\delta_n{}^m + f_{,n}R^m) - \mathrm{d}x^0 R^m) = (\mathrm{d}\xi^p - \mathrm{d}\xi^0 v^p)T_p{}^0 \\ \Longrightarrow \quad & \mathrm{d}x^0 L + \mathrm{d}x^n(\phi_n - f_{,n}L) - a\mathrm{d}\xi^0 = \epsilon_m(\mathrm{d}x^n(\delta_n{}^m + f_{,n}R^m) - \mathrm{d}x^0 R^m) \\ \Longrightarrow \quad & \mathrm{d}x^0 = \frac{a\mathrm{d}\xi^0 + \mathrm{d}\boldsymbol{x}\cdot(\boldsymbol{\epsilon} - \boldsymbol{\phi})}{L + \boldsymbol{\epsilon}\cdot\boldsymbol{R}} - \mathrm{d}f \end{aligned} \tag{5.20}$$

where equations (5.16) and (5.12) have been used in the second step.

Both $a = 1/Y^0$ and $\xi^0 \equiv \tau$ are measured in Σ, and since from equation (4.39), $\boldsymbol{R}$ is dependent only on $\boldsymbol{x}$, χ_2 and χ_3. Thus all three quantities a, $\mathrm{d}\xi^0$ and $\boldsymbol{R}$ are independent of the synchrony function, f, in S. Hence, upon substitution of equation (5.1) into (5.20), and by comparing equation (5.20) with its expression when f is varied by an amount δf, one sees that

$$\frac{a\mathrm{d}\xi^0 + \mathrm{d}\boldsymbol{x}\cdot(\boldsymbol{\epsilon} - \boldsymbol{\phi})}{1 + \chi_1 x^1 + (\boldsymbol{\epsilon} - \boldsymbol{\phi})\cdot\boldsymbol{R}} = \frac{a\mathrm{d}\xi^0 + \mathrm{d}\boldsymbol{x}\cdot(\bar{\boldsymbol{\epsilon}} - \boldsymbol{\phi} - \boldsymbol{\delta\phi})}{1 + \chi_1 x^1 + (\bar{\boldsymbol{\epsilon}} - \boldsymbol{\phi} - \boldsymbol{\delta\phi})\cdot\boldsymbol{R}} \tag{5.21}$$

where the barred quantities correspond to the varied f. Since $\mathrm{d}\xi^0$ varies independently of $\mathrm{d}\boldsymbol{x}$, and $a\mathrm{d}\xi^0$ is independent of f, it follows that

$$\boldsymbol{\epsilon}_{\boldsymbol{\phi}} + \boldsymbol{\delta\phi} \equiv \bar{\boldsymbol{\epsilon}} = \boldsymbol{\epsilon}_{\boldsymbol{\phi}} + \boldsymbol{\delta\phi} \tag{5.22}$$

Consider a vector $p^n e_n$, lying on P, which is purely spatial in S. Postulate that the inner product of this vector and e_0, when evaluated in S, is proportional to their inner product in Σ for arbitrary p^n, and all along P. That is,

$$\langle e_0 | p^n e_n \rangle_{|\Sigma} = G \langle e_0 | p^n e_n \rangle_{|S} = G p^n \phi_n, \tag{5.23}$$

where G is some function and equation (4.2) has been used in the last step.

Now, first using equations (5.6) and (5.17), and then using equations (5.18)) and (5.19) to get to the third step,

$$\begin{aligned} \langle e_0 | p^n e_n \rangle_{|\Sigma} &= P^n \langle Y^\alpha E_\alpha | (T^{-1})_n{}^\beta E_\beta \rangle \\ &= Y^\alpha p^n (T^{-1})_n{}^\beta E_{\alpha\beta} \\ &= -p^n Y^\alpha Y^0 \epsilon_n E_{\alpha 0} + p^n Y^\alpha ((b^{-1})^m{}_n - \epsilon_n Y^m) E_{\alpha m} \\ &= (Y^0)^2 \boldsymbol{\epsilon}\cdot\boldsymbol{p}(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v}) + p^n (b^-1)^m{}_n Y^0 (\kappa_m + v^q \delta_{qm} - \boldsymbol{\kappa}\cdot\boldsymbol{v}\kappa_m) \\ &\quad -(Y^o)^2 \boldsymbol{\epsilon}\cdot\boldsymbol{p} v^m (\kappa_m + v^q \delta_{qm} - v^q \kappa_q \kappa_m) \\ &= \frac{1}{a^2} \boldsymbol{\epsilon}\cdot\boldsymbol{p}((1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2) + \frac{1}{a} b^{-1} \boldsymbol{p}\cdot((1 - \boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa} + \boldsymbol{v}) \end{aligned} \tag{5.24}$$

where equations (5.5)) and (5.4) have been used in the elimination of $E_{\alpha\beta}$, and the identities (5.8) and (5.16) applied.

Substituting the expression derived above into the postulated relation (5.23) between inner products in Σ and S gives

$$\boldsymbol{\epsilon}\cdot\boldsymbol{p} = \frac{Ga^2\boldsymbol{\phi}\cdot\boldsymbol{p}}{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2} - \frac{ab^{-1}\boldsymbol{p}\cdot((1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\boldsymbol{\kappa}+\boldsymbol{v})}{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2} \tag{5.25}$$

Now, a, b, $\boldsymbol{\kappa}$ and $\boldsymbol{v}$ are independent of a change in $\boldsymbol{\phi}$, and so, from equation (5.22),

$$\delta\boldsymbol{\phi}\cdot\boldsymbol{p} = \frac{Ga^2\delta\boldsymbol{\phi}\cdot\boldsymbol{p}}{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2} \quad\Longrightarrow\quad G = \frac{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - v^2}{a^2}. \tag{5.26}$$

The last two results give the following expression for $\boldsymbol{\epsilon}$:

$$\epsilon_m = \phi_m - a(b^{-1})^n{}_m \frac{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})\kappa_n + v^l\delta_{ln}}{(1-\boldsymbol{\kappa}\cdot\boldsymbol{v})^2 - \boldsymbol{v}^2}, \tag{5.27}$$

which is in agreement with the expressions for $\boldsymbol{\epsilon}$ obtained using an operational approach in section 3.2. Note that when Einstein synchronisation is imposed ($\boldsymbol{\phi} = \mathbf{0}$), the above equation coincides with equation (3.42).

Thus $\boldsymbol{\epsilon}$, while depending on a and b, is not a discriminator of theories, but rather a reflector of relative synchrony conventions.

As is shown by section 3.2, synchrony choice does not affect experimental predictions for measurables within the Mansouri-Sexl test-theory(and thus the parameters a and b are only measurable within equivalence classes). Hence, for the purpose of making predictions, it is convenient to put $\boldsymbol{\kappa} = \mathbf{0}$ and $f = 0$ to simplify analysis. This gives $\boldsymbol{\epsilon}$ the simple form in equation (3.42). Such a choice is not in conflict with the conventionality of distant simultaneity as long as there is then no attempt to claim experimental distinction of this synchrony choice.

Equation (5.20) becomes (using 3.42 and 5.1):

$$\begin{aligned} \mathrm{d}x^0 &= a\frac{(1-v^2)\mathrm{d}\xi^0 - b^{-1}\mathrm{d}\boldsymbol{x}\cdot\boldsymbol{v}}{(1+x^1\chi_1)(1-v^2) - ab^{-1}\boldsymbol{R}\cdot\boldsymbol{v}} \\ &= a\frac{\mathrm{d}\xi^0 - \mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v}}{(1+x^1\chi_1)(1-v^2)} \end{aligned} \tag{5.28}$$

Now, the velocity of a point at $\boldsymbol{x}$ in S with respect to the instantaneously comoving, non-rotating frame, S_{S}, is $\boldsymbol{\omega}\times\boldsymbol{x} = -\boldsymbol{R}$. So the spatial co-ordinates assigned to an event will be

$$\boldsymbol{x} = \boldsymbol{x}_{\mathrm{S}} + \int_0^t \boldsymbol{R}\mathrm{d}t \quad\Longrightarrow\quad \mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t = \mathrm{d}\boldsymbol{x}_{\mathrm{S}} \tag{5.29}$$

Writing Equation (5.13) in the form (according synchrony choices)

$$\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t = b(\mathrm{d}\boldsymbol{\xi} - \boldsymbol{v}\mathrm{d}\tau), \tag{5.30}$$

shows that although S is rotating with respect to Σ, b is not affected by the changing orientation of the two axis sets (since, from (5.29), $\mathrm{d}\boldsymbol{x}-\boldsymbol{R}\mathrm{d}t$ has constant orientation with respect to a nonrotating frame and thus Σ). Hence, requiring that the space axes of S and Σ are parallel when $t = 0$ ensures that the "length contraction" matrix b in equation (5.13) has no rotational components.

It is natural to separate the action of b into two parts by assuming that b acts on the direction $\boldsymbol{v}$ independently of its action in the plane perpendicular to $\boldsymbol{v}$. Furthermore, assume that all directions perpendicular to $\boldsymbol{v}$ are acted on in the same way and that the action of b is purely to scale by factors β and δ in directions parallel and perpendicular to $\boldsymbol{v}$:

$$b\boldsymbol{w} = \begin{cases} \delta\boldsymbol{w} & \text{if } \boldsymbol{w} \perp \boldsymbol{v} \\ \beta\boldsymbol{w} & \text{else.} \end{cases} \tag{5.31}$$

Now, from this equation and the identity

$$(\boldsymbol{v} \times \boldsymbol{p}) \times \boldsymbol{v} = v^2\boldsymbol{p} - \boldsymbol{v}\cdot\boldsymbol{p}\ \boldsymbol{v}, \tag{5.32}$$

one arrives at an expression for the action of b on a arbitrary (spatial) vector:

$$b\boldsymbol{p} = \beta\frac{\boldsymbol{v}\cdot\boldsymbol{p}}{v^2}\ \boldsymbol{v} + \delta\frac{\boldsymbol{v} \times \boldsymbol{p}}{v^2} \times v \tag{5.33}$$

Since the effect of b can be broken down into the sum of independent scaling effects, it follows that the inverse action of b is given by

$$b^{-1}\boldsymbol{p} = \frac{1}{\beta}\frac{\boldsymbol{v}\cdot\boldsymbol{p}}{v^2}\ \boldsymbol{v} + \frac{1}{\delta}\frac{\boldsymbol{v} \times \boldsymbol{p}}{v^2} \times v \tag{5.34}$$

Restating equation (5.28), and using expression 5.33 in equation (5.30), gives

$$\mathrm{d}t = a\frac{\mathrm{d}\tau - \mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v}}{(1 + x\chi)(1 - \boldsymbol{v}^2)} \tag{5.35}$$

$$\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t = (\ \beta\frac{\mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v}}{\boldsymbol{v}^2} - \mathrm{d}\tau)\boldsymbol{v} + \delta\frac{\boldsymbol{v}\times\mathrm{d}\boldsymbol{\xi}}{\boldsymbol{v}^2} \times \boldsymbol{v} \tag{5.36}$$

where β, δ and a are functions of $\boldsymbol{v}$, and χ_1 has been written as χ. These last two equations are given as the final form of the transformations of the test-theory, for the case where all observers propagate orthonormal tetrads.

When the special relativistic values of the three parameters ($a = 1/\gamma$, $\beta = \gamma$ and $\delta = 1$ — where $\gamma = 1/\sqrt{1-V^2}$) are substituted into equations (5.35) and (5.36) the resulting transformation is in agreement with the co-ordinate transformation from an arbitrarily accelerating frame to an inertial frame within special relativity given by Nelson[86]. This can be seen by taking infinitesimals of equations (19) of that paper.

An expression for the one-way speed of light for S, corresponding to the choice $f = 0$, is obtainable by using the value chosen in Σ's frame and transforming from that frame to the S-frame. Equations (5.30) and 5.34 give

$$\mathrm{d}\boldsymbol{\xi} - \boldsymbol{v}\mathrm{d}\tau = \frac{\boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)}{\beta v^2}\boldsymbol{v} + \frac{\boldsymbol{v}\times(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)}{\delta v^2}\times\boldsymbol{v} \tag{5.37}$$

$$\Longrightarrow \quad \mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v} - v^2\mathrm{d}\tau = \boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)/\beta. \tag{5.38}$$

From here, using equation (5.35),

$$\mathrm{d}\tau = \frac{(1+x\chi)\mathrm{d}t}{a} - \frac{\boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)}{\beta(1-v^2)}. \tag{5.39}$$

Using this and equation (5.37),

$$\mathrm{d}\boldsymbol{\xi} = \left(\frac{\boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)}{\beta v^2(1-v^2)} + \frac{1+x\chi}{a}\mathrm{d}t\right)\boldsymbol{v} + \frac{\boldsymbol{v}\times(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)}{\delta v^2}\times\boldsymbol{v}. \tag{5.40}$$

These two results for $\mathrm{d}\tau$ and $\mathrm{d}\boldsymbol{\xi}$ then give

$$\begin{aligned}\mathrm{d}\boldsymbol{\xi}^2 - \mathrm{d}\tau^2 = {} & \frac{(\mathrm{d}\boldsymbol{x} - \boldsymbol{R}\mathrm{d}t)^2}{\delta^2} - \frac{(1+x\chi)^2\mathrm{d}t^2}{a^2\gamma^2} \\ & + \frac{[(\mathrm{d}\boldsymbol{x} - \boldsymbol{r}\mathrm{d}t)\cdot\boldsymbol{v}]^2}{v^2}\left(\frac{\gamma^2}{\beta^2} - \frac{1}{\delta^2}\right), \qquad (5.41) \\ \gamma^2 \equiv {} & (1-v^2)^{-1} \qquad (5.42)\end{aligned}$$

Since the one-way speed of light in Σ has been chosen as unity, in S its value in a direction $\hat{\boldsymbol{p}}$ is given by putting the left hand side equal to zero in 5.42 and solving for $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = c\hat{\boldsymbol{p}}$:

$$\begin{aligned} & c^2\left(\frac{1}{\delta^2} + \frac{(\boldsymbol{p}\cdot\boldsymbol{v})^2}{\boldsymbol{v}^2}(\frac{\gamma^2}{\beta^2} - \frac{1}{\delta^2})\right) - 2\left(\frac{\boldsymbol{p}\cdot\boldsymbol{R}}{\delta^2} + \frac{\boldsymbol{p}\cdot\boldsymbol{v}\,\boldsymbol{R}\cdot\boldsymbol{v}}{\boldsymbol{v}^2}(\frac{\gamma^2}{\beta^2} - \frac{1}{\delta^2})\right) \\ & = \frac{(1+x\chi)^2}{a^2\gamma^2} - \frac{\boldsymbol{R}^2}{\delta^2} - \frac{(\boldsymbol{R}\cdot\boldsymbol{v})^2}{\boldsymbol{v}^2}(\frac{\gamma^2}{\beta^2} - \frac{1}{\delta^2}). \qquad (5.43)\end{aligned}$$

Note that when β and δ take on their special relativistic values all velocity dependence in the above equation vanishes.

While the present formalism would suffice for any kinematical analysis, dynamical considerations (such as electromagnetic or gravitational effects) require the postulation of dynamical behaviour. The most natural way to input this is to require a particular dynamical behaviour in the aether frame and to transform to the test frame via equations (5.35) and (5.36).

5.2 Sagnac effect

A novel feature of recent experimental laser research is the improvement in the accuracy with which noninertial effects are measured. The Sagnac effect has now been seen in a wide variety of interferometers including, in particular, SQUIDs or superconducting Cooper pair interferometers[128], optical interferometers[94], neutron interferometers[15] and most recently atomic interferometers[38]. Large ring laser experiments are presently earth-bound and so inevitably are rotating with respect to the local Lorentz frame. The detection of the Sagnac effect arising from the rate Ω_E of rotation of the earth was initially performed optically[80], and the detection by small ring lasers predated the vivid and better-documented demonstration by neutron interferometry[107]. By now, several ring laser systems are reported to have detected the associated Sagnac effect (for example, see [9, 67, 66]). In the Canterbury ring laser, the earth-induced Sagnac effect is detectable at the level of $10^{-6}\Omega_E$[108], and it now seems to be feasible to detect the secular variations in the rate of the rotation of the earth (at the level of $10^{-8}\Omega_E$) in a somewhat larger device. Optical interferometry still leads the field for relative accuracy in such a measurement. In addition, several studies of ring lasers under significant acceleration have been reported[57, 14, 112, 30] and some elegant experiments by Kowalski *et al.*[50, 51] explore at novel precision the effect of acceleration or of gravity, applied to some or all of the optical components of the ring laser system, on the beat frequency of ring lasers containing dielectrics.

The nature of ring interferometric effects within a preferred frame theory and the potential of ring lasers in bounding deviation from local Lorentz invariance can be seen from the analysis given in section 5.3 for a vacuum ring.

5.3 Ring laser tests

In the context of the test-theory of section 5.1, with the imposition $h = 0$ for Σ (corresponding to the choice of an isotropic one-way speed of light for Σ), the parameters a, β, and δ are expandable in terms of $\boldsymbol{v}^2$ (see Mansouri and Sexl I.[76]). Thus it is seen from equation (5.43) that the expression for the one-way speed of light in S has only even powers of $\boldsymbol{v}$ in a velocity expansion. This suggests that any closed-loop optical test covered by the test-theory in section (5.1) can, at best, be of second order in the aethereal velocity. While some choices of synchrony in S would introduce first-order velocity dependence in the one-way speed of light in S, this dependence would cancel out over a closed path (because of the covariance of the formalism). Similarly, a choice other than Einstein synchronisation in Σ (while resulting in odd powers of $\boldsymbol{v}$ in the expressions for the test-theory parameters) will have the same effect in S.

Within this test-theory, an analysis of the Sagnac effect in a ring laser predicts sidereal, $\boldsymbol{v}$-dependent variations in the measured beat frequency: following Scorgie[104], consider an arbitrary, smooth closed path $\mathcal{C}$ along which laser beams travel in both senses. Denote T_+ and T_- as the times taken for light to traverse $\mathcal{C}$ in anti-clockwise and clockwise senses respectively and let C_+ and C_- be the (position-dependent) speed of light in those respective senses. Taking $\mathrm{d}l$ to be an element of arc along $\mathcal{C}$ in an anti-clockwise sense (so that $\boldsymbol{c}_+ \equiv c_+\boldsymbol{p} = \mathrm{d}\boldsymbol{l}/\mathrm{d}t$), one has, from equation (5.43), the following expression for the difference in transit time for the two directions:

$$\begin{aligned} T_+ - T_- &= \oint_{\mathcal{C}} \left(\frac{1}{c_+} - \frac{1}{c_-}\right) \mathrm{d}l \\ &= 2\oint_{\mathcal{C}} \frac{\boldsymbol{p}\cdot\boldsymbol{R}/\delta^2 + (\gamma^2/\beta^2 - 1/\delta^2)\,\boldsymbol{p}\cdot\boldsymbol{v}\boldsymbol{R}\cdot\boldsymbol{v}/v^2}{R^2/\delta^2\,(\gamma^2/\beta^2 - 1/\delta^2)\,(\boldsymbol{R}\cdot\boldsymbol{v})^2/v^2) - (1+x\chi)^2/(a\gamma)^2}\mathrm{d}l \\ &= 2\oint_{\mathcal{C}} \frac{(\boldsymbol{R} + (Q\boldsymbol{R}\cdot\boldsymbol{v}/\boldsymbol{v}^2)\,\boldsymbol{v})\cdot\mathrm{d}\boldsymbol{l}}{R^2 + Q(\boldsymbol{R}\cdot\boldsymbol{v})^2/\boldsymbol{v}^2 - \delta^2(1+x\chi)^2/a^2\gamma^2}, \end{aligned} \tag{5.44}$$

where

$$Q \equiv \delta^2\gamma^2/\beta^2 - 1 \tag{5.45}$$

For an Earth-bound ring laser having a rotation of the order of the Earth's, the linear acceleration is due to gravity and is small, $\chi \approx 10^{-16}\mathrm{m}^{-1}$, and contributes negligibly in equation (5.44). The Earth's rotation has magnitude, $\omega \approx 2 \times 10^{-13}m^{-1}$, and so $\boldsymbol{R} = \boldsymbol{x} \times \boldsymbol{\omega}$ can be ignored at second order.

Now, noting Stokes' theorem for the closed-loop integral of a vector field, $\boldsymbol{F}$,

$$\oint_{\mathcal{C}} \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{l} = \oint_{A} \boldsymbol{\nabla} \times \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{A} \tag{5.46}$$

where $\mathrm{d}\boldsymbol{A}$ is an element of the vector area, $\boldsymbol{A}$ enclosed by $\mathcal{C}$, using the vector identities

$$\nabla \times (\boldsymbol{x} \times \boldsymbol{\omega}) = -2\boldsymbol{\omega}, \qquad (5.47)$$
$$\nabla \times (\boldsymbol{x} \times \boldsymbol{\omega}\cdot\boldsymbol{v}\ \boldsymbol{v}) = \boldsymbol{\omega}\cdot\boldsymbol{v}\ \boldsymbol{v} - \boldsymbol{v}^2\boldsymbol{\omega},$$

using equation (4.39) and assuming that the change in $\boldsymbol{v}$ is negligible over the time it takes a light signal to traverse $\mathcal{C}$ (which allows some of the v-dependent parameters to be treated as constants of integration), one makes the following approximation to equation (5.44):

$$\begin{aligned} T_+ - T_- &\approx -\frac{2a^2\gamma^2}{\delta^2} \oint_{\mathcal{C}} \left(\boldsymbol{R} + Q\frac{\boldsymbol{R}\cdot\boldsymbol{v}}{v^2}\boldsymbol{v}\right)\cdot \mathrm{d}\boldsymbol{l} \\ &= -\frac{2a^2\gamma^2}{\delta^2} \oint_{A} \left(-2\boldsymbol{\omega} + \frac{Q}{v^2}(\boldsymbol{\omega}\cdot\boldsymbol{v}\ \boldsymbol{v} - v^2\boldsymbol{\omega}\right)\cdot \mathrm{d}\boldsymbol{A} \\ &= \frac{a^2\gamma^2}{\delta^2}(4+2Q)\boldsymbol{\omega}\cdot\boldsymbol{A} - \frac{2a^2\gamma^2}{v^2\delta^2} Q\boldsymbol{\omega}\cdot\boldsymbol{v}\ \boldsymbol{v}\cdot\boldsymbol{A} \end{aligned} \qquad (5.48)$$

When a, β and δ take on the special relativistic values ($1/\gamma$, γ and 1, respectively), with $Q = 0$ as a result, (5.48) becomes

$$T_+ - T_- \approx 4\boldsymbol{\omega}\cdot\boldsymbol{A} \qquad (5.49)$$

which is the standard result for special relativity.

Following the arguments of Mansouri and Sexl[76, I], a, β and δ are decomposable in terms of v^2 only. Working to first order in v^2, $a \approx 1 + a_2\boldsymbol{v}^2$, $\beta \approx 1 + \beta_2\boldsymbol{v}^2$, $\delta \approx 1 + \delta_2\boldsymbol{v}^2$, and so

$$\frac{a^2\gamma^2}{\delta^2} \approx 1 + (1 + 2(a_2 - \delta_2))\, v^2, \quad Q \approx (1 + 2(\delta_2 - \beta_2))\, v^2, \qquad (5.50)$$

and so

$$\begin{aligned} T_+ - T_- &\approx 4\boldsymbol{\omega}\cdot\boldsymbol{A} + (6 + 8a_2 - 4(\delta_2 + \beta_2))\, \boldsymbol{v}^2\boldsymbol{\omega}\cdot\boldsymbol{A} \\ &\quad -2(1 + 2\delta_2 - 2\beta_2)\boldsymbol{\omega}\cdot\boldsymbol{v}\ \boldsymbol{v}\cdot\boldsymbol{A} \\ &= 4\boldsymbol{\omega}\cdot\boldsymbol{A} + 4(1 + 2(a_2 - \delta_2)\,)\boldsymbol{v}^2\ \boldsymbol{\omega}\cdot\boldsymbol{A} \\ &\quad +2(1 + 2(\delta_2 - \beta_2))(\boldsymbol{v} \times \boldsymbol{\omega}) \times \boldsymbol{v}\cdot\boldsymbol{A} \end{aligned} \qquad (5.51)$$

where equations (5.48) has been used.

The first term in equation (5.51) is the diurnal constant predicted by special relativity. The second term varies with time as the ring laser's velocity with respect to the aether changes. The third term exhibits sinusoidal-like variation as $\boldsymbol{A}$ varies in direction with respect to $\boldsymbol{v}$. Taking the Earth's velocity with respect to the aether to be 2.4×10^{-3} at a declination of $-26°$[85], consider sidereal variation in equation (5.51). Since $\boldsymbol{\omega}\cdot\boldsymbol{A}$ is constant, only $\boldsymbol{v}^2$ contributes to variation in the second term. Now, the change in $\boldsymbol{v}$ is due to the rotational velocity of the ring, and is of the order 10^{-6}, and thus $\boldsymbol{v}^2$ may be approximated as constant. However, the orientation of $\boldsymbol{A}$ with respect to $\boldsymbol{v}$ would change notably over the period of a sidereal day. Thus in equation (5.51), variation in the second term can be ignored, as any variation would be dominated by the third term. Assuming a small difference between the two light travel times around the ring (needed for a beat frequency in the laser), and restricting attention to values of a, β and δ not greatly differing from the special relativistic values (which are the ranges one is interested in), one obtains the expression $|T_+ - T_-| = f\lambda P$, where f is the beat frequency measured for a ring laser having perimeter P and using a beam of wavelength λ. Then, from equation (5.51)

$$\Delta f \lambda P \geq 2(1 + 2(\delta_2 - \beta_2))\boldsymbol{v}\cdot\boldsymbol{\omega}\Delta(\boldsymbol{v}\cdot\boldsymbol{A}). \tag{5.52}$$

Using the above assumption that $\boldsymbol{v}$ has magnitude 2×10^{-3} and a declination of $-26°$ gives, and taking into account the geometry of the Canterbury ring laser,

$$1 + 2(\delta_2 - \beta_2) \leq 9.1 \times 10^{17}\mathrm{m} \times \frac{\lambda P}{A}\Delta f \tag{5.53}$$

$$= 3.0 \times 10^{9}\mathrm{s} \times \frac{\lambda P}{A}\Delta f, \tag{5.54}$$

where equation (5.53) corresponds to natural units, and equation (5.54) to S. I. units. Substituting the values for the Canterbury ring laser [108] into this expression, it follows that measures of the variations in beat frequency can bound $1+2(\delta-\beta)$ to an accuracy of 10^{-3}, given the presently achieved 140 nHz resolution in the beat frequency[109].

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