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THE USE OF LINEAR PROGRAMMING IN LEAST-COST FEED COMPOUNDING

by

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IN LEAST-COST FEED COMPOUNDING

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P R E F A C E

Intensive types of livestock production such as poultry and pigs promise to become of increasing importance in New Zealand agriculture. In such enterprises, feed costs constitute a very high proportion of total variable costs. Any method of computing low cost feeds is obviously of great importance.

Linear programming is one such method which has been widely used overseas. In this bulletin Mr Taylor applies this method to the problem of formulating least-cost commercial feed compounds for broiler chickens.

Further work of a similar nature is proceeding on the formulation of least-cost pig fattening rations.

By publishing Mr Taylor's results in the present form it is hoped that commercial firms, engaged in producing animal feeds, will be encouraged to adopt similar procedures in the development of low cost feeds.

B. P. Philpott

Lincoln College,
October 1965.

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APPENDICES

THE USE OF LINEAR PROGRAMMING IN LEAST-COST FEED COMPOUNDING[1]

1. INTRODUCTION

The development of linear programming and the use of the electronic computer has made a considerable impact on agricultural research in recent years. One important use is in the determination of least-cost feed compounds for livestock. In this paper a broiler starter compound has been used as an example of the effectiveness of this technique in reducing feeding costs in one section of the poultry industry. The impact of even a small reduction in the cost of feed per ton is considerable, since feed costs constitute between 60-80% of the total variable costs in the industry.

Modern poultry compounds are formulated under complex nutrient specifications, which grow even more complex as further advances are made in the poultry nutrition field. Nutrient specifications may include minimum or maximum levels of energy, protein, minerals, vitamins and any number of the wide range of amino acids which are necessary.

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[1] The author wishes to acknowledge the helpful advice and encouragement given by Professor J.D. Stewart, Head of the Department of Farm Management & Rural Valuation, Lincoln College. Also the co-operation of Mr C. Howie of a Christchurch feed compounding firm is acknowledged.

A large number of ingredients may be used in the compound and each of these supplies different amounts of the required nutrients. For example, wheat is relatively high in energy but low in amino acid content while meat meal, relatively low in its energy content, is high in amino acids.

The actual ingredients included in the least-cost ration depend on

(i) their relative prices;
(ii) their composition;
(iii) the nutrient requirements of the ration.

The problem becomes large and complex in nature, when it is required to formulate a least-cost ration from as many as 30 alternative feed ingredients, while complying with up to 40 nutrient restrictions. To add to the complexity of the problem, a new solution must be found each time there are changes in the relative prices of the feed ingredients. As a result the computational burden is considerable and would involve several days or even weeks on a desk calculator, if it were done by arithmetic, using a trial and error procedure. However, by using linear programming the problem can be expressed in such a way that the electronic computer can perform the necessary computations in a very short time.

2. DATA REQUIREMENTS FOR LEAST-COST FEED COMPOUNDING

The data required for least-cost feed compounding are:-

(i) The nutrient requirement specifications for the particular compound. These may include maximum and minimum amounts of protein, fibre vitamins, amino acids, energy, and various minerals.

(ii) The ingredients or sources of nutrients available. For example wheat, barley, oats, lucerne meal, maize, meat meal, pea meal are a few of those available in New Zealand.

(iii) The nutrient content of the available ingredients.

(iv) The prices of these ingredients.

(v) The total weight of the compound required.

It is essential that the input data used in linear programming is as accurate as possible. Poor or inadequate information will yield unreliable results, so that every effort must be made to frame the problem in realistic and meaningful terms. Unfortunately very little research has been done in analysing locally grown ingredients, and in the determination of accurate nutrient requirements for poultry compounds under New Zealand conditions. There is an urgent need for more work in this field in New Zealand.

3. PROGRAMMING A BROILER STARTER COMPOUND

3.1 General

The nutrients required for broiler production are almost wholly supplied by concentrate feedstuffs, purchased by the producer. These feed costs constitute approximately 75% of the total variable costs incurred in broiler production, so that profits to producers depend largely on

(i) the feed conversion ratio;

(ii) the relationship between feed costs per unit and product price.

The feed conversion ratio in New Zealand is approximately 3.3:1 (i.e. 3.3 lb feed consumed for every 1 lb of meat produced), compared with 2.7:1 in the U.K. and 2.5:1 in the U.S.A., so that there is scope for improvement here. Unfortunately improvement in this field can only be made at a relatively slow rate, as a result of research into feeding and management, and improved breeding. The reduction of feeding costs on the other hand has immediate effects on the cost of production in this industry.

Obviously all poultry compound will differ in their nutrient requirements, depending on the type of product being produced. The important nutritional features of a broiler starter compound are the relatively high protein and low energy requirements. This is fed only during the first six weeks and is followed by a high energy-low protein compound for the succeeding six weeks, at the end of which period the birds are killed. At this age the birds weigh approximately 3 lb liveweight.

3.2 The Nutrient Requirements or Restrictions

Correct and meaningful restrictions are absolutely essential if the compound is to be successful. Unfortunately very little nutritional research work has been done on these requirements under New Zealand conditions, so that the author has had to rely on results from overseas research work.

The nutrient requirements for the broiler starter compound are outlined in Table 3.1.

TABLE 3.1

NUTRIENT REQUIREMENTS PER 1000 lb COMPOUND

Nutrient	Unit	Level	Nutrient	Unit	Level
Minimum requirements:			Maximum restrictions:		
1. Methionine	lb	3.8	23. Sodium	%	.32
2. " + Cystine	lb	7.2	24. Potassium	%	.80
3. Lysine	lb	10.8	25. TFP	lb	60.0
4. Tryptophane	lb	2.1	26. Added Fat	lb	200.0
5. Arginine	lb	10.8	27. Fishmeal + TFP**	lb	120.0
6. Glycine	lb	8.8	28. Lucerne	lb	100.0
7. Protein	%	20.0	29. Ca/P Ratio		2.25
8. Vitamin A	IU	2 mn.	30. Protein	%	24.0
9. Riboflavin	gm	3.5	31. Met. Energy	Cal.	1300.0
10. Pantothenic Acid	gm	7.5	32. Blood Meal	lb	40.0
11. Niacin	gm	24.0	33. Pea Meal	lb	100.0
12. Vitamin B_{12}	mgms	5.0	Equality Requirement:		
13. Choline	gm	700.0	34. Weight	lb	1000.0
14. Fishmeal	lb	25.0			
15. Added Fat	lb	10.0			
16. Biotin	gm	.05			
17. Pyrodoxin	gm	3.0			
18. Folacin	gm	0.3			
19. Met. Energy	Cal.	1000.0			
20. Sodium	%	.24			
21. No. 11*	lb	2.5			
22. Ca/P Ratio		1.75			

* A commercially prepared complex containing distillers' dried solubles, vitamins and minerals.

**A commercially prepared premix containing growth stimulants, antioxidants and vitamins.

3.3 Sources or Ingredients Available

The 25 sources or ingredients from which the least-cost compound must be selected are given in Table 3.2. The costs per lb are those paid by the manufacturer.

TABLE 3.2

INGREDIENTS AVAILABLE FOR COMPOUND FORMULATION

Ingredient	Cost (shgs/lb)	Ingredient	Cost (shgs/lb)
Lucerne	.310	TFP	.800
Barley	.189	Tallow	.350
Maize	.310	Pantothenic acid	40.217
Oats	.271	Folic acid	261.269
Wheat	.241	Sodium	.178
Bran	.178	Calcium	.032
Pollard	.188	Phosphorus	.230
Pea meal	.190	Niacin	15.694
Blood meal	.295	Choline	2.333
Lime	.670	Pyrodoxin	280.093
Meat meal	.210	Riboflavin	93.86
Fish meal	.499	No. 11	8.0
Buttermilk powder	.603		

3.4 Ingredients Selected in the Least-Cost Compound

Given the nutrient composition of the alternative sources or ingredients listed in Table 3.2, a least-cost compound consistent with the restrictions outlined in Table 3.1 was programmed.[1] As with the restrictions used,

[1] This problem was solved on the University of Canterbury I.B.M. 1620 Computer, using the I.B.M. Library Programme 10.1.002.

the nutrient composition of the alternative ingredients available in New Zealand are not accurately known. There is considerable scope for more analysis work on the locally produced ingredients. More accurate information would enable linear programmes to produce even better compounds than is possible at present. As a result of the deficiency in information the author was forced to use overseas analysis figures for the nutrient composition of the alternative sources.

The composition of the final least-cost compound is shown in Table 3.3.

TABLE 3.3

COMPOSITION OF THE LEAST-COST COMPOUND

Variable	Level of Activity lb	Cost of Ingredient shgs/lb	Cost of Feed Ingredient in Compound shgs/1000 lb	Amount of Ingredient in Least-Cost Compound %
Pea meal	100.0000	0.190	19.0000	10.00000
Meat meal	174.4281	0.210	36.6299	17.44281
Fish meal	55.6132	0.499	27.7510	5.56132
Barley	601.7606	0.189	113.7328	60.17606
Lucerne	32.6290	0.310	10.1150	3.26290
Calcium	23.0634	0.030	0.6919	2.30634
Tallow	10.0000	0.350	3.5000	1.0
Folic acid	.000021	261.0	0.0054549	0.0000021
Riboflavin	0.0082	9.400	0.0771	0.00082
Pyrodoxin	0.0044	280.000	1.2320	0.00044
No. 11	2.5000	8.000	20.0000	0.25000
	1000.0069 lb		232.7360/- or £23.27/ton	100.0000%

The major proportion of the compound is composed of barley - 60%, with meat meal 17% and pea meal 10%.

The cost of the compound is £23.27/ton. This is the least-cost compound, there being no other formulation consistent with the given nutrient requirements and lower in cost than that given above.

3.5 Analysis of Nutrient Content of Compound

The nutrient content of the compound is of prime importance. It is often necessary to know the exact levels at which certain nutrients are included in the compound. These levels are given in Table 3.4.

TABLE 3.4

NUTRIENT CONTENT OF LEAST-COST COMPOUND

Nutrient	Level Required in Compound	Level in Least-Cost Compound
Methionine (minimum)	3.8 lb	3.8 lb
" + Cystine "	7.2 lb	7.442 lb
Met. Energy (min-max)	1000-1300 Cal	1123.377 Cal
Lysine (minimum)	10.8 lb	21.932 lb
Tryptophane "	2.1 lb	2.427 lb
Arginine "	10.8 lb	14.432 lb
Glycine "	8.8 lb	19.365 lb
Protein (min-max)	20-24%	22.279%
Vitamin A (minimum)	2 million I.U.	4.414 million I.U.
Riboflavin "	3.5 gm	3.5 gm
Pantothenic acid (minimum)	7.5 gm	7.970 gm
Niacin (minimum)	24 gm	31.2533 gm
Vitamin B_{12} "	5 mgm	45.805 mgm

TABLE 3.4 (Cont'd)

Nutrient	Level Required in Compound	Level in Least-Cost Compound
Choline (minimum)	700 gm	1061.678 gm
Fishmeal "	25 lb	55.6132
Added Fat (min-max)	10-200 lb	10.0 lb
Biotin (minimum)	.050 gm	.050 gm
Pyrodoxin "	3.0 gm	3.0 gm
Folacin "	0.32 gm	0.32 gm
Sodium % (min-max)	.24%-.32%	0.32%
$^{Ca}/P$ Ratio "	$1.75/_1$-$2.25/_1$	$2.25/_1$
No. 11 (maximum)	2.5 lb	2.5 lb
Potassium "	0.8%	0.6275%
TFP "	60 lb	0
Fishmeal + TFP (maximum)	120 lb	51.613 lb
Lucerne "	100 lb	32.629 lb
Blood meal "	40 lb	0
Pea meal "	100 lb	100 lb

3.6 Stability of the Solution

The sensitivity of the least-cost solution to changes in prices of the included ingredients, i.e. "stability" of the final solution, is given in Table 3.5. This shows the range over which the cost of each ingredient can alter without causing a change in the composition of the least-cost feed compound.

TABLE 3.5

COST RANGES OF SELECTED INGREDIENTS
IN THE LEAST-COST COMPOUND

Ingredient	Lower Limit	Unit Cost	Upper Limit
Pea meal	.2207	.190	0
Meat meal	.2435	.210	0
Fish meal	.5114	.499	.3664
Barley	.1971	.189	.1495
Lucerne	.3458	.310	.2896
Calcium	.097	.030	0
Tallow	1000.0959	.350	.0968
Folic acid	3053.3	261.0	0
Riboflavin	319.1	9.4	0
Pyrodoxin	11354.5	280.0	0.2
No. 11	1000.2705	8.0	0.2716

For example the unit cost of fish meal is .499 shillings per lb. The cost range calculation indicates that while the cost does not increase to more than .5114 shillings per lb or fall below .3664 shillings per lb, then the 55.613 lb of fish meal per 1000 lb of compound, would remain the optimum level. The unit cost of meat meal is .210 shillings per lb, at which cost 174.4281 lb is included in the compound. This quantity remains optimal for any cost within the range 0 to .2435 shillings per lb. Thus even if meat meal could be obtained free of cost, no more than 174.4281 lb would be included in the compound while the nutritional constraints are rigidly adhered to. It must be emphasised that the range in costs of an ingredient over which a solution is stable only applies if the costs of all other ingredients remain the same. Should two or more ingredient prices change within their cost ranges, the selected combination of ingredients may change, depending on the nutritional relationship between the ingredients.

Important also is the sensitivity of the least-cost compound to changes in cost of <u>excluded</u> ingredients. The reduction in cost of those ingredients which would be necessary before they could enter the correspondingly least-cost solution is shown in column b of Table 3.6. This "shadow price" is the penalty if one unit of the excluded ingredient is forced into the compound. The "shadow price" only holds over a given range of units and these are also shown in Table 3.6.

TABLE 3.6

SHADOW PRICES OF EXCLUDED INGREDIENTS

Ingredient	a. Cost/lb (shillings)	b. Shadow Price	c. Upper Limit	d. Price at which ingredients would enter solution
Buttermilk	.603	.3993	17.726	0.2137
Maize	.310	.1474	9.0533	0.1626
Niacin	15.7	15.5	∞	0.2
Pollard	.188	0.0584	4.0744	0.1296
Choline	2.3	2.19	∞	0.11
Blood meal	0.295	0.0053	10.9799	0.2897
Bran	0.178	0.0441	4.4272	0.1339
Wheat	0.241	0.0805	13.0505	0.1605
Pantothenic acid	0.402	0.401	425.286	0.001
TFP	0.8	0.3474	10.4428	0.4526
Sodium	0.178	1.4696	2.6372	+1.2916
Oats	0.271	0.0253	46.2493	0.2457
Phosphorus	0.230	0.1389	4.6371	0.0911
Liver	0.670	0.3952	6.6781	0.2748

For example Oats has a shadow price of 0.0253/- per lb. This represents the increase in cost of the compound for each pound of Oats forced in, up to a maximum of 46.2493 lbs. Beyond this quantity the shadow price would increase, that is, the marginal cost would rise.

This shadow price also represents the fall in price necessary before a particular ingredient enters the least-cost compound and here the upper limit indicates the amount of that ingredient which will enter the least-cost compound at that new cost. The original cost of Buttermilk powder, for example, was 0.6030 shillings/lb, at which price it was rejected. The shadow price is 0.3993 indicating that if the price was to fall from 0.6030/- lb by 0.3993/- lb, (i.e. to 0.2137/- lb) then buttermilk powder would enter, and at a level of 17.726 lb.

Such a price change would alter the relationship of each ingredient to all the others, so that reprogramming would be required to obtain the composition of the new compound.

The degree of stability of the least-cost formulation in relation to price changes in both the selected and non-selected ingredients is clearly of importance to the manufacturer. An unstable solution of a least-cost formulation would necessitate the reprogramming of the compound after only small fluctuation in ingredient prices. Only by doing this can the manufacturer ensure that he is in fact producing the least-cost compound consistent with the given restrictions. It is much more desirable, from a manufacturing point of view, to have a stable solution. However, once the manufacturer is producing a particular feed formulation, the way in which he will react to a price change in the ingredients will depend on

(a) the proportional increase or decrease in the total cost of the compound resulting from the price changes of the ingredients;

(b) the total volume of the compound being produced;

(c) the level of ingredients being held in stock;

(d) possible digestive repercussions on animals resulting from a change in ingredients of a feel compound, even though this still satisfies the programmed requirements. (This situation arises through imperfect knowledge of the requirements of a particular feed compound and emphasizes the need for testing each formulation.)

3.7 Shadow Prices of Limiting Requirements

In the formulation of a complex feed compound such as this, several of the restrictions will be limiting. These effective restrictions have a 'cost' in the feed compound.

In the case of a minimum requirement the least-cost feed formulation provides only enough of the specific nutrient to satisfy the requirement. Hence a reduction in the minimum requirement of this nutrient will reduce the cost of the compound. The reductions in cost per 1000 lb of the feed compound for each unit reduction in the minimum requirement and the limit to which these unit reductions can be made are given in Table 3.7 (Columns A and B respectively). Where maximum restrictions are operative, the lifting of the restrictions by one unit will likewise reduce the cost of the feed compound by the amount shown in column A of Table 3.7 and are a range of units indicated by column B.

TABLE 3.7

SHADOW PRICES OF LIMITING REQUIREMENTS AND RANGE OVER WHICH THESE SHADOW PRICES APPLY

	Unit	A Shadow Price (shillings)	B Limit of Reduction (units)
Methionine	lb	20.3428	0.0922
Riboflavin	gms	0.2175	3.5000
Biotin	gms	0.9599	0.2488
Pyrodoxin	gms	0.6170	3.0000
Folacin	gms	0.0070	30.0000
'No. 11'	lb	7.7284	0.4383
Added Fat	lb	0.2532	4.2516
Sodium	%	0.3472	8.0000
$^{Ca}/_{P}$ Ratio	-	0.0713	5.8707
Pea meal	lb	0.0307	6.6090

In the case of Riboflavin, the minimum requirement is 3.5 gms per 1000 lb of compound. This restriction is only just satisfied as Table 3.4 indicates. The shadow price for Riboflavin given in Table 3.7 is 0.2175 shillings, i.e. for each gram the minimum requirement is eased (with a limit of 3.5 gms), the total cost of the compound will be reduced by 0.2175 shillings.

Hence the cost of a restriction or requirement in the compound is indicated by the "shadow price". It may be noted that the shadow price for the methionine minimum restriction could be reduced from 3.8 lb per 1000 lbs to 2.8 lbs, the total cost per 1000 lbs would be reduced by £1 per 1000 lbs.[1] This would be significant if the level of output of the compound was high.

3.8 Summary of Results

The result of this initial study involving the use of linear programming for least-cost compound formulation was encouraging. Not only was the cost of the compound considerably less than a similar feed being produced commercially in New Zealand, but considerable additional information was provided by the programmed solution. This information is of considerable value to feed manufacturers, who can assess the effects (if any) which price changes in alternative ingredients will have on the least-cost compound.

4. CONFIRMATION TRIAL

It is desirable that the least-cost compound selected by linear programming be tested experimentally before it is produced commercially. This is especially important where the least cost compound differs widely from that at present in use as with the present case.

Two trials were conducted to test the programmed compound under commercial conditions. The initial trial involved 200 birds; 2 replicates of 50 birds on the present

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[1] The range over which this shadow price applies however is only 0.0922 units (lbs) as indicated in column B of Table 3.7. The easing of the restriction by more than 0.0922 units (lbs) would change the shadow price.

compound (or control) and 2 replicates of 50 on the least-cost (or experimental) compound.

The birds were weighed weekly over a three week period, and live weight change and weight of feed consumed recorded. These weighings confirmed the hypothesis that the experimental compound would give the same results as the control. Having shown the experimental compound to be the equal of the present commercially available compound nutritionally, any increase in efficiency of production would be shown in a comparison of the cost of feed per lb of liveweight gain. These figures are shown in Table 4.1 and Figure 4.1. Table 4.1 gives the average cost of feed per lb liveweight gain over the three week period, while Figure 4.1 gives the savings in feed costs per 100 birds each week over the same period.

TABLE 4.1

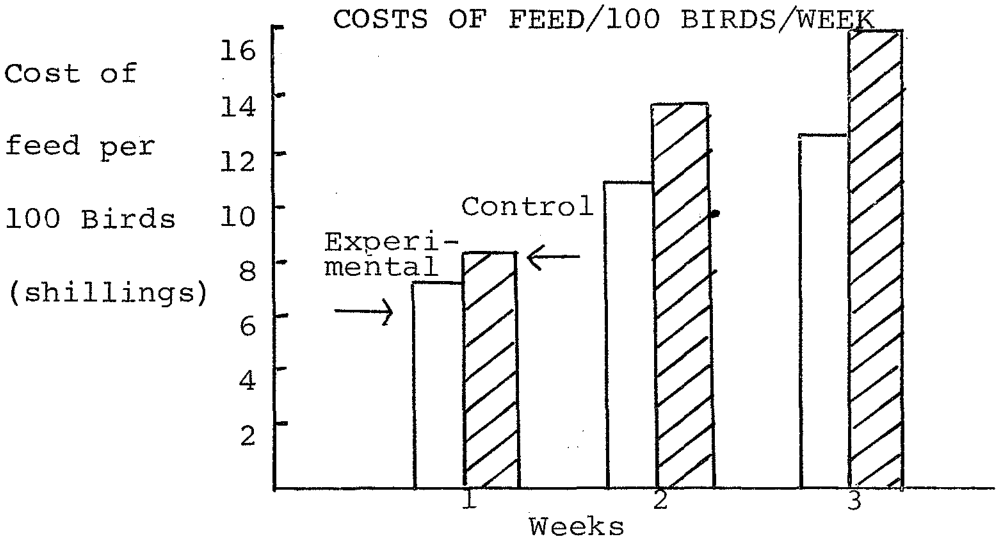
COMPARISON OF FEEDS - COSTS OF FEED/LB LIVEWEIGHT GAIN

	Control	Experimental
Cost of compound/ton	£28	£23.3
Cost of compound/lb liveweight gain	7d	5.5d

This indicates a reduction of 1.5 pence per lb LW gain (or 22.8% reduction) in favour of the experimental compound.

FIGURE 4.1

COSTS OF FEED/100 BIRDS/WEEK



Although the comparison was only over three weeks, this is sufficient to indicate the quality of a broiler starter.[1] A second trial involving 2000 birds confirmed the results obtained from the original trial.

An important aspect of compounds used in broiler production is the degree of moisture in the droppings. Where this is high, compaction occurs on the floor of sheds and disease risk becomes high. The experimental or least-cost compound proved to be markedly superior to the control in this respect.

5. CONCLUSIONS

Because the high proportion of total costs in the poultry industry is made up of feed costs (60-80%), any reduction which can be made in this field is of immense value to the producer. As knowledge of the nutritional requirements and limitations of various feeds becomes greater, the problem of determining a least-cost compound, consistent with the given limitations becomes increasingly difficult. Up to 30 alternative sources of nutrients may be available to the manufacturer, who has the complex problem of determining a least-cost compound within limits determined by up to 40 restrictions or requirements - a problem involving days of work on a desk calculator.

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[1] Miller and Edmondson "Development of a Method of Routine Testing of Poultry Feeds", N.Z.J.Ag.Research, Vol. 3, No.4, 1960.

The use of linear programming and the electronic computer has been demonstrated as a quick, powerful and precise technique for the formulation of these least-cost compounds. However, the dependability and precision of the results it yields depends on the accuracy and completeness of the input data used. The imperfect knowledge of nutritional requirements, the wide variation in the composition of feedstuffs, and the differences in the availability of a given nutrient in different feedstuffs, constitutes a definite weakness. There is wide scope for research in this field in New Zealand. As a result of this work, more efficient rations could be produced, using nutrient restrictions and nutrient composition data more specifically related to New Zealand conditions and feed ingredients.

It has been argued that because of this deficiency, the input data which is used is often so incomplete and inaccurate that the use of such a precise technique as linear programming is not justified. This same argument applies however to all other methods of formulating compounds. The major advantage in using linear programming, is in the knowledge that the compound produced is in fact the least-cost compound, it is quick to provide a solution to a complex problem, and it provides a means whereby the effects of price changes can be seen readily.

In the above example, the technique has allowed a rearrangement of ingredients so that a nutritionally similar compound can be produced for £4.15.0/ton less than the comparable commercial mix. The savings in feed costs to a producer using 150 tons per year would be £713. Additional information provided by the computer, e.g. shadow prices, allows the feed manufacturer greater scope in his inventory control, and hence the reduction in his cost structure. Information of this nature was never provided by the more common trial and error methods of feed formulation.

The use of linear programming and the electronic computer for least-cost feed formulation will enable producers to reduce feed costs markedly in the near future. This technique has had wide application in the U.K. and U.S.A., and the poultry industry in New Zealand must recognise and apply this technique if the industry is to increase its efficiency of production. The benefits accruing through this reduction in feed costs are considerable and much more rapid in this effect than, for example, improvements in breeding stock.

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A P P E N D I X

1. THE METHODOLOGY OF LINEAR PROGRAMMING AS APPLIED TO A SIMPLE LEAST-COST COMPOUNDING PROBLEM

1.1 The Least-Cost Linear Programming Model

The objective function here is

(i) To minimise $C = \sum_{j=1}^{n} c_j x_j$

(ii) Subject to

$bi \geqslant \sum r_{ij} x_j$

(iii) and where $x_j \geqslant 0$ for all j

where j = 1, 2, ... n ingredients

i = 1, 2, ... m requirements

C = total cost per unit measurement of the compound

c_j = net cost per unit of the j th ingredient

x_j = level at which the j th ingredient is included in the compound

b_i = level of i th requirement

r_{ij} = level of the i th requirement met by a unit of the j th ingredient.

1.2 The Application of Linear Programming to a Simplified Compounding Problem

A simple compound problem involving two alternative sources or ingredients and two nutrient restrictions is used here to demonstrate the principles behind the use of linear programming for the solving of feed compounding problems.[1]

Table 1.1 indicates the composition of the two alternative sources, wheat and meat meal in terms of the two nutrients, amino acids and vitamin. Restrictions on the amount of amino acids and vitamins entering the feed, or total weight of compound, are indicated. Prices of wheat and meat meal are also given.

It should be noted that in this particular example absolute requirements are placed on the two available nutrients instead of the more usual proportional requirements. This allows a maximum restriction to be placed on the total weight of the compound (in this example, a maximum of 100 lb), whereas using proportional requirements, an equality condition (i.e. 100 lb exactly) would be forced in.

TABLE 1.1

NUTRIENT COMPOSITION OF THE ALTERNATIVE SOURCES

		Amount Required in Compound	Meat Meal	Wheat
(i) Source			Meat Meal	Wheat
(ii) Composition in terms of:-				
- Amino acids	$\geqslant$	1.50 lb	.025	.015
- Vitamins	$\geqslant$	0.75 lb	.018	.005
- Weight	$\leqslant$	100 lb	1.0	1.0
(iii) Cost/lb			12/-	5/-

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[1] A similar explanation is given by Gilson, Yeh, Hodgson, op.cit., and in more detail by Heady & Candler, op.cit.

The total cost of the least-cost compound will be

$$C = 12\ x_1 \times 5\ x_2$$

where x_1 = level of meat meal in the compound

x_2 = level of wheat in the compound

The information in Table 1.1 may be expressed in algebraic terms:

1) $1.5 \leqslant .025\ x_1 + .015\ x_2$

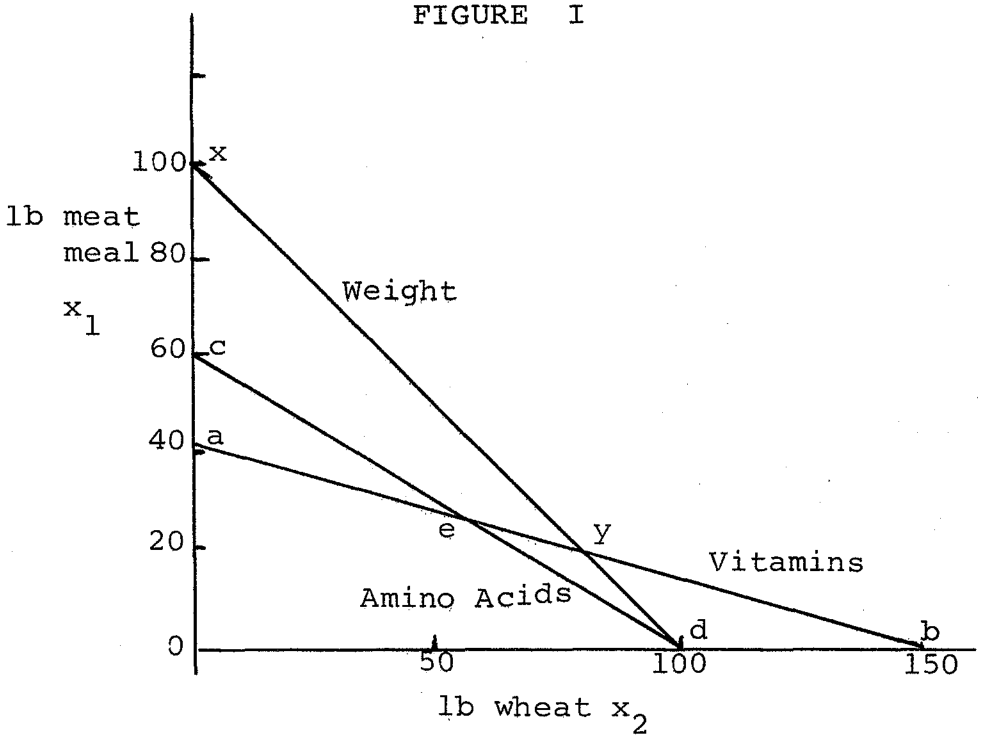
2) $0.85 \leqslant .018\ x_1 + .005\ x_2$

3) $100 \geqslant 1\ x_1 + 1\ x_2$

i.e. equation 1) states that the amount of amino acid provided by meat meal (x_1) and wheat (x_2) must be greater than, or equal to, 1.50 lbs.

When the above equations are graphed as in Figure I, the various combinations of x_1 and x_2 which satisfy the three restrictions or requirements, can be seen.

FIGURE I



The line cd indicates the amount of wheat and meat meal which provide exactly 1.5 lb of amino acid. Any point on or above line cd will provide sufficient amino acid to satisfy the requirement of this in the ration. (Points such as these are termed "technically feasible points".) Line ab has the same meaning with respect to quantities of vitamins required. Line xd specifies that the weight of compound cannot exceed 100 lb in total, so that any point on or below xd is a technically feasible point.

Considering the three equations, the technically feasible points lie in the area c e y x. Any point in the area a c e will satisfy the vitamin requirement, but will not satisfy the amino acid level. Similarly any combination of wheat and meat meal falling in area e y d will satisfy the amino acid requirement, but will not meet the vitamin requirement. Points in area d y b will satisfy the amino acid requirement but will violate the weight and vitamin restriction. Any point below line a e d will of course satisfy neither the vitamin or amino acid requirement.

Obviously there are many combinations which are technically feasible (i.e. in area c x y e). However the problem is to determine one compound - the least-cost compound in this feasible area.

2. THE ESTABLISHMENT OF AN INITIAL FEASIBLE SOLUTION

2.1 Derivation of Equalities

Now in order to use linear programming as a means of solving the problem as outlined in the previous section, the inequalities

1 $\quad 1.5 \leqslant .025\ x_1 + .015\ x_2$

2 $\quad 0.75 \leqslant .018\ x_1 + .005\ x_2$

3 $\quad 100 \geqslant 1\ x_1 + 1\ x_2$

must be converted to equations by the addition of disposal activities. In equation 1′ the disposal act. x_3 allows for an excess of amino acid over and above the 1.5 lb minimal requirements of equation 1 should the combination produce the least-cost compound. In equation 2′, x_4 represents any excess of vitamins which may be provided in the least-cost compound and in equation 3′, x_5 represents the amount by which the total weight of meat meal and protein is less than the 100 lb maximum weight specified.

The equations now appear in the following form:

$$
\begin{aligned}
1'\quad 1.5 &= .025\,x_1 + .015\,x_2 \overbrace{-\,1\,x_3 + 0\,x_4 + 0\,x_5}^{\text{Disposal Activities}} \\
2'\quad 0.75 &= .018\,x_1 + .005\,x_2 + 0\,x_3 - 1\,x_4 + 0\,x_5 \\
3'\quad 100 &= 1\,x_1 + 1\,x_2 + 0\,x_3 + 0\,x_4 + 1\,x_5
\end{aligned}
$$

2.2 Establishment of a Basis

Unfortunately the negative disposal activities do not allow easy solution of the above equations. The difficulty is in arriving at an "initial feasible solution". One way of achieving this is by the use of artificial activities. Very high costs (m) are assigned to these activities so that they are forced out of the final solution. These artificial activities are added to equations which have no disposal (i.e. an equality condition) or equations having a disposal activity with a negative coefficient (i.e. minimum restrictions).

Adding the artificial activities the above equations appear as follows:

$$
\begin{aligned}
1''\quad 1.5 &= .025\,x_1 + .015\,x_2 \overbrace{-\,1\,x_3 + 0\,x_4 + 0\,x_5}^{\text{Disposal Activities}} + \overbrace{1\,x_6 + 0\,x_7}^{\text{Artificial Activities}} \\
2''\quad 0.75 &= .018\,x_1 + .005\,x_2 + 0\,x_3 - 1\,x_4 + 0\,x_5 + 0\,x_6 + 1\,x_7 \\
3''\quad 100 &= 1\,x_1 + 1\,x_2 + 0\,x_3 + 0\,x_4 + 1\,x_5 + 0\,x_6 + 0\,x_7
\end{aligned}
$$

These artificial activities x_6 and x_7 are added only to give an initial solution from which cheaper and eventually the cheapest or optimum solution can be determined.

Disposal activities have no prices; however artificial activities have high costs to ensure their elimination from the final solution.

The cost function to be minimised then is:

$$C = 12\ x_1 + 5\ x_2 + 0\ x_3 + 0\ x_4 + 0\ x_5 + m\ x_6 + m\ x_7$$

Having a means of achieving an initial feasible solution, the optimum or least-cost solution can readily be determined using the simplex routine of linear programming.

2.3 An Alternative Method of Establishing a Basis

An alternative and more efficient method for establishing an initial feasible solution is available. The inequalities are converted to equations by the addition of disposal activities as outlined - section 2.1 above (equations $1', 2', 3'$).

In this form however, the negative disposal activities of equations $1'$ and $2'$ make it difficult to obtain an initial basic feasible solution. This problem can be overcome by multiplying equations $1'$ and $2'$ by minus one, giving:

$$1''' \quad -1.5 = -\ .025\ x_2 - .015\ x_2 + 1\ x_3 - 0\ x_4 - 0\ x_5$$

$$2''' \quad -0.75 = -\ .018\ x_1 - .005\ x_2 - 0\ x_3 + 1\ x_4 - 0\ x_5$$

$$3''' \quad 100 = 1\ x_1 + 1\ x_2 + 0\ x_3 + 0\ x_4 + 1\ x_5$$

The disposal activities, x_3 in equation $1'''$, x_4 in equation $2'''$, and x_5 in equation $3'''$, are used as the basic variables while the b or requirement column is allowed to become negative in respect of equation $1'''$ and $2'''$.

With negatives b's in the initial basic solution the Dual algorithm is used until a feasible basic solution is obtained, at which point the Primal algorithm takes over and obtains the optimum basic feasible solution by the normal simplex procedure.

In terms of storage space used and computing time involved, this second method is more efficient than that involving the artificial activities. With large problems this can be vitally important and the second method explained above is to be preferred.

There is one weakness however with this method. This arises when it is necessary to include equality conditions in the basis. Where the method employing artificial activities is used, equality requirements can be treated as for minimum requirements, i.e. artificial activities are added to the equation, thus allowing an initial feasible solution to be established. These are then forced out by the high cost (m) placed on them.

When the second method is used however, only those requirements specifying minimum conditions are multiplied by minus one and it becomes necessary to use two rows to force in the equality condition,

i.e. $b_i \geqslant \sum r_{ij} x_j$

and $b_i \leqslant \sum r_{ij} x_j$

Normally there are only a very small number of these equality conditions in a feed compounding problem so that this method involves only small increases in the size of the problem.

For a given problem, the differences in storage space required when the two methods described above are used to establish an initial feasible solution, can be described in the two generalised equations below.

Method 1.

Where artificial activities are added to all those requirements specifying minimum or equality conditions.

Storage space required = $(m + n + 1)m$

Where m = no. of requirements in problem
n = no. of real activities in problem
1 = no. of artificial activities in problem.

Method 2.

Where those requirements specifying minimum conditions are multiplied by minus one.

Storage space requirement = (m + n)m

Where m = no. of requirements in the problem
n = no. of real activities in the problem.

3. THE INITIAL TABLEAU AS USED FOR DETERMINING THE LEAST-COST BROILER STARTER [1]

3.1 General Description

The nutrient requirements for a Broiler Starter compound, the alternative feed ingredients available to the manufacturer, and the nutrient composition of these ingredients are indicated in Table 3.1. This is the initial tableau from which the least-cost compound is eventually determined.

The nutrient composition of the alternative feed ingredients available are listed in columns P_1 to P_{25}. For example, one pound of lucerne meal contains .0035 lb of methionine, .0079 lb of methionine and cystine, .01 lb of lysine and so on.

Disposal activities are set out in columns P_{26} to P_{58}. Those with negative coefficients in the columns indicate a minimum requirement for the particular nutrient in that row, while those with positive coefficients indicate maximum restrictions for the nutrient in the corresponding row. Obviously where equality conditions are required, disposal activities do not appear.

In the disposal activity column, P_{26} for example, the -1 indicates that the least-cost solution must contain at least 3.8 lb of methionine, i.e. a minimum requirement. The +1 in disposal activity column P_{48} indicates that the sodium percentage must be no greater than 0.32%, i.e. a maximum restriction.

[1] This tableau is included at the end of the Appendix.

The C_j now indicates the cost per unit of the real activity, e.g. lucerne, .31 shillings per lb. Artificial activities (P_{59} - P_{80}) have been ascribed a high cost of 1000.0 shillings, while disposal activities have zero cost.

Where the range of coefficients in a matrix is extremely wide, it is desirable to scale those rows and columns containing coefficients at the extremities of the range. By reducing this range, the computing time involved in solving the problem is markedly reduced and the likelihood of rounding errors is minimised.

In the above matrix several rows and columns have been scaled. The minimum biotin (row 16 in the matrix) required per 1000 lb of compound is .05 gms, while 1 lb of lucerne supplies .00015 gms of biotin, 1 lb of barley .00006 gms of biotin, etc. All coefficients in this row were multiplied up by 1000 (10^3), as indicated in the matrix above. The calcium column (P_{22}) has been scaled down by a factor 10, i.e. all coefficients in the column including the C_j (or net revenue), pertain to .1 lb calcium.

3.2 Minimum Requirement

(i) The Generalised Algebraic form:

$$b_i = \sum r_{ij} x_j - x_{j+i} + x_k$$

where

b_i = level of the i th requirement

i = 1, 2, ... m requirements

j = 1, 2, ... n ingredients

r_{ij} = level of the i th requirement met by a unit of the j th ingredient

x_j = level at which the j th ingredient is included in the compound

x_{j+i} = disposal activity for the i th requirement (or the amount by which the i th requirement in the compound, exceeds the minimum amount required).

x_k = artificial activity for the i th requirement where $j + i + 1 \leqslant k \leqslant j + i + i$

(ii) A particular case from the Broiler Starter problem, e.g. minimum requirement of biotin of 50 gms (row 16).

The relationship can be expressed by the following equation:

$$50 = \underbrace{.15\ x_1 + .06\ x_2 + .03\ x_5 + .13\ x_8 + .04\ x_9 + .09\ x_{13} + .2182\ x_{14}}_{\text{real activities}}$$

$$\underbrace{-\ 1\ x_{41}}_{\substack{\text{disposal}\\ \text{activity}}} \quad \underbrace{+\ 1\ x_{24}}_{\substack{\text{artificial}\\ \text{activity}}}$$

where x_{41} measures the quantity by which the biotin in the compound exceeds the minimum of 50 gms and x_{74} represents the positive artificial activity, which enables an initial feasible solution to be established.

3.3 Maximum Restrictions

(i) The Generalised Algebraic form:

$$b_i = \sum r_{ij} x_j + x_{j+i}$$

where

b_i = level of the i th requirement

i = 1, 2, ... m requirement

j = 1, 2, ... n ingredient

r_{ij} = level of the i th requirement met by a unit of the j th ingredient

x_j = level at which the j th ingredient is included in the compound

x_{j+i} = disposal activity for the i th requirement (or the amount by which the level of the i th requirement in the compound is less than the maximum allowed).

(ii) A particular case from the Broiler Starter problem, e.g. maximum protein allowed, 24% (row 30).

The relationship can be expressed by the following equation:

$$24 = \underbrace{.02\,x_1 + .01\,x_2 + .08\,x_3 + .032\,x_4 + .0096\,x_5 + .065\,x_6 + .055\,x_7 + .011\,x_8 + .0105\,x_9 + .015\,x_{10} + .016\,x_{11} + .065\,x_{12} + .024\,x_{13} + .028\,x_{14}}_{\text{real activities}} + \underbrace{1\,x_{55}}_{\substack{\text{disposal}\\ \text{activity}}}$$

where x_{55} measures the amount by which the protein percentage in the compound is less than the maximum protein percentage allowed.

3.4 <u>The Calcium/Phosphorus Ratio</u>

To enable the $^{Ca}/_{P}$ ratio to be handled by the normal simplex procedure the following method was used.

$$\text{The total Calcium provided in the compound} = \sum_{j=1}^{25} r\,Ca_j x_j$$

$$\text{The total Phosphorus provided in the compound} = \sum_{j=1}^{25} r\,P_j x_j$$

where $r\,Ca_j$ = level of Calcium provided by a unit of the j th activity.

$r\,P_j$ = level of Phosphorus provided by a unit of the j th activity

x_j = level of j th activity in the compound.

1. Maximum ratio $^{Ca}/_{P} = {}^{2.25}/_{1}$

$$\sum_{j=1}^{25} r\,Ca_j\,x_j \Big/ \sum_{j=1}^{25} r\,P_j x_j \leqslant 2.25/1$$

$$2.25 \sum_{j=1}^{25} r\,P_j x_j \geqslant \sum_{j=1}^{25} r\,Ca_j x_j$$

$$\therefore\quad 0 \geqslant \sum_{j=1}^{25} r\,Ca_j x_j - 2.25 \sum_{j=1}^{25} r\,P_j x_j$$

$$\text{i.e. } 0 \geqslant \sum_{j=1}^{25} (r\,Ca_j - 2.25\,P_j)\,x_j$$

2. Minimum ratio $^{Ca}/_{P} = {}^{1.75}/_{1}$

$$\sum_{j=1}^{25} r\,Ca_j x_j \Big/ \sum_{j=1}^{25} r\,P_j x_j \geqslant 1.75/1$$

$$1.75 \sum_{j=1}^{25} r\,P_j x_j \leqslant \sum_{j=1}^{25} r\,Ca_j x_j$$

$$\text{i.e. } 0 \leqslant \sum_{j=1}^{25} (r\,Ca_j - 1.75r\,P_j)\,x_j$$

In the above form the ratios can be handled by two rows, one for the minimum restriction and one for the maximum.

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