The eigenvalue approach to groundwater modelling for resource evaluation at regional scale

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Objectives

- **Identify the dynamic behaviour of an aquifer as storage reservoir**
- **Separate the piezometric effect of land surface recharge from that of river recharge**
- **Provide information about aquifer boundaries**
- **Enable estimation of river recharge relative to land surface recharge**

Assumptions: aquifer dynamics

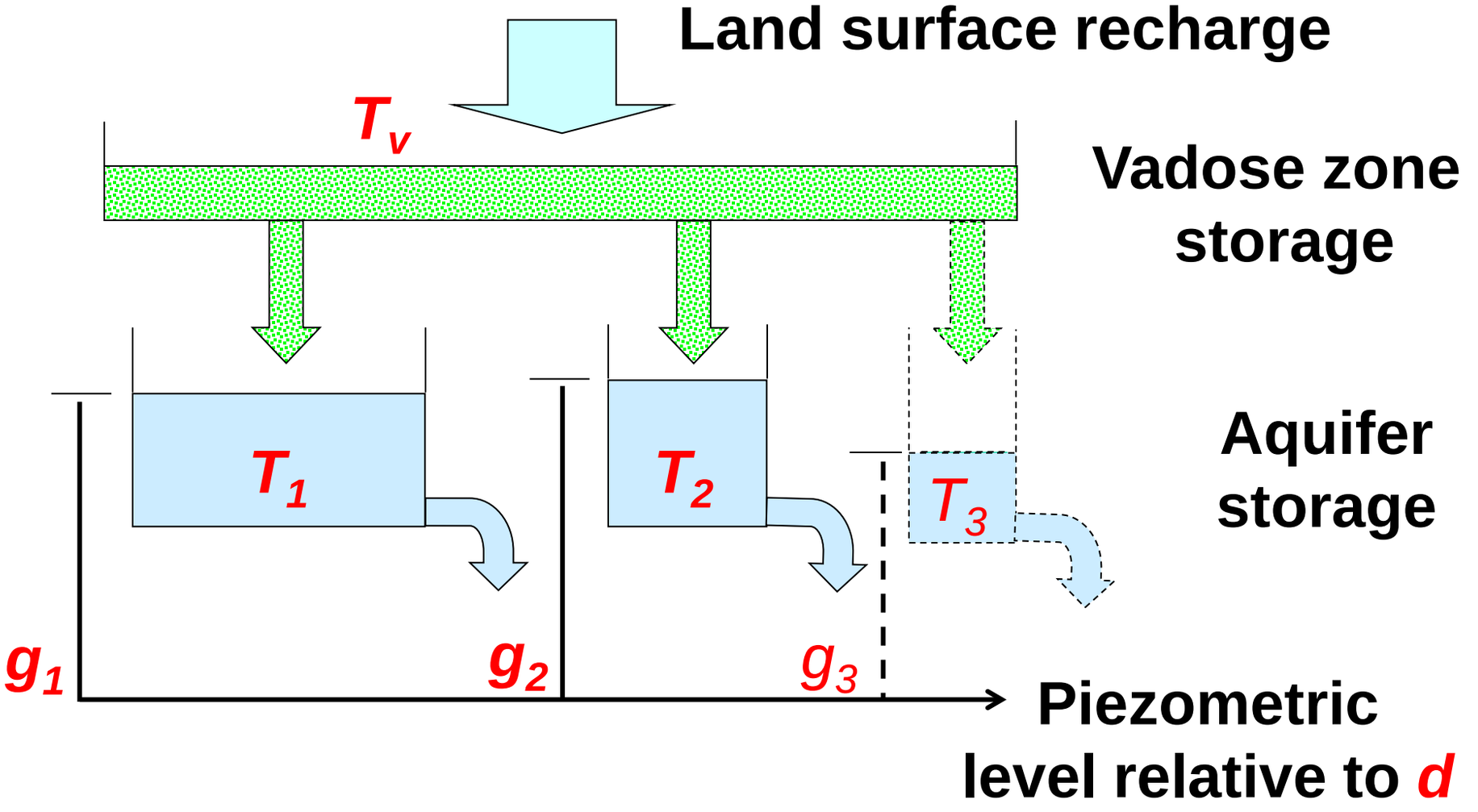
- **Land surface recharge causes most of the temporal variation in piezometric head**
- **River recharge has a steady piezometric effect throughout most of an aquifer**
- **Groundwater resource can be quantified as a dynamic response to land surface recharge**

Method: eigenvalue approach to groundwater modelling

- A 2-D heterogeneous aquifer can be modelled as a linear system based on the eigenvalues of the solution to the partial differential equations of groundwater flow
- Only a few eigenvalues are significant for modelling piezometric response to land surface recharge that has a fixed spatial pattern

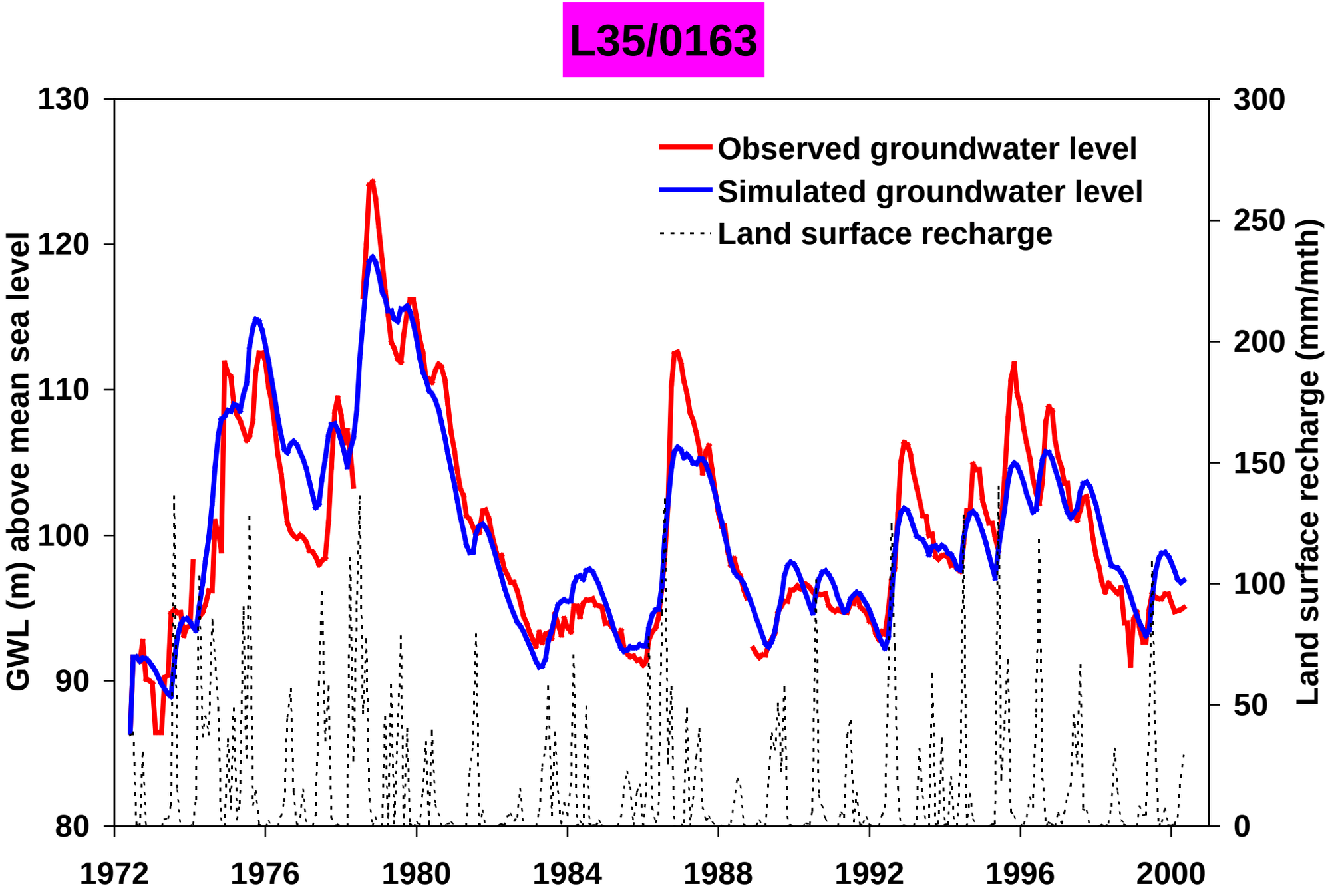
  and time-varying magnitude
- We call this simple version, the “Eigenmodel”

Eigenmodel concept



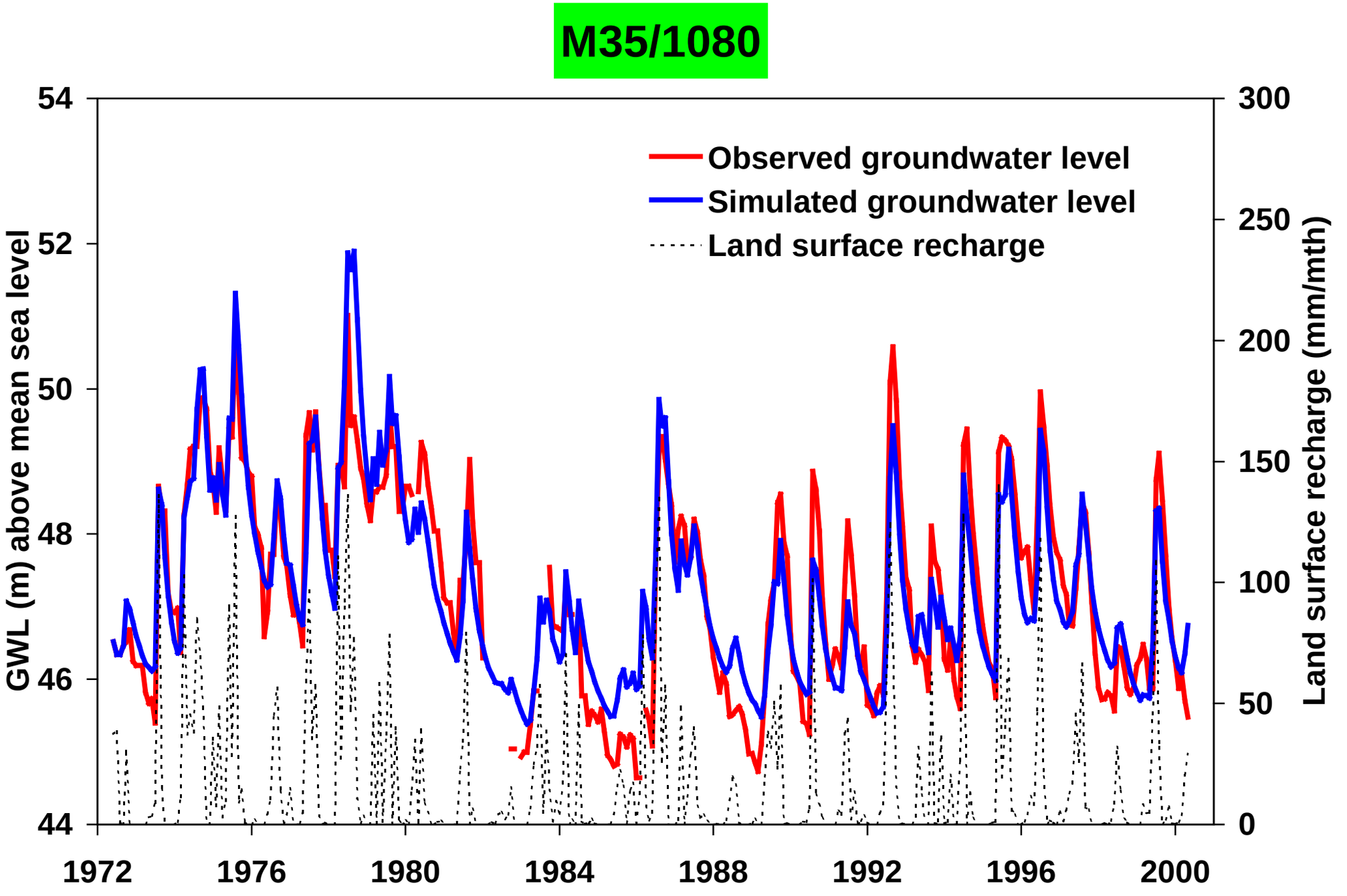
Eigenmodel parameters

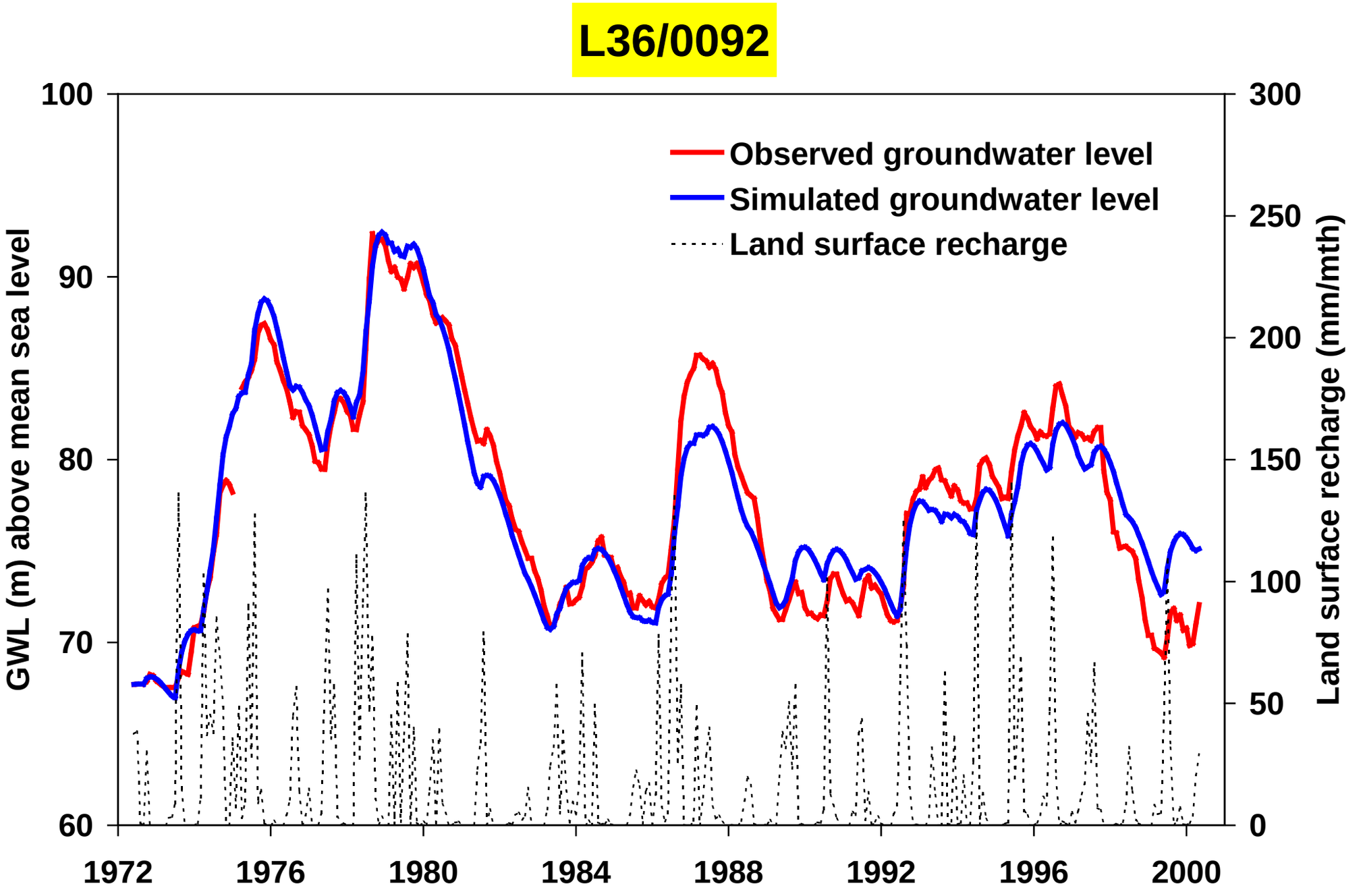
- **Eigenvalues are displayed here as the inverse T_i, called the storage residence time**
- **Eigenvalues are, theoretically, the same everywhere in the aquifer**
- **Gain coefficients g_i are specific to a location, and determine the proportions of linear element response**
- **Piezometric response is relative to a datum parameter *d* at each location**
- **Vadose zone storage parameter T_v varies with location**

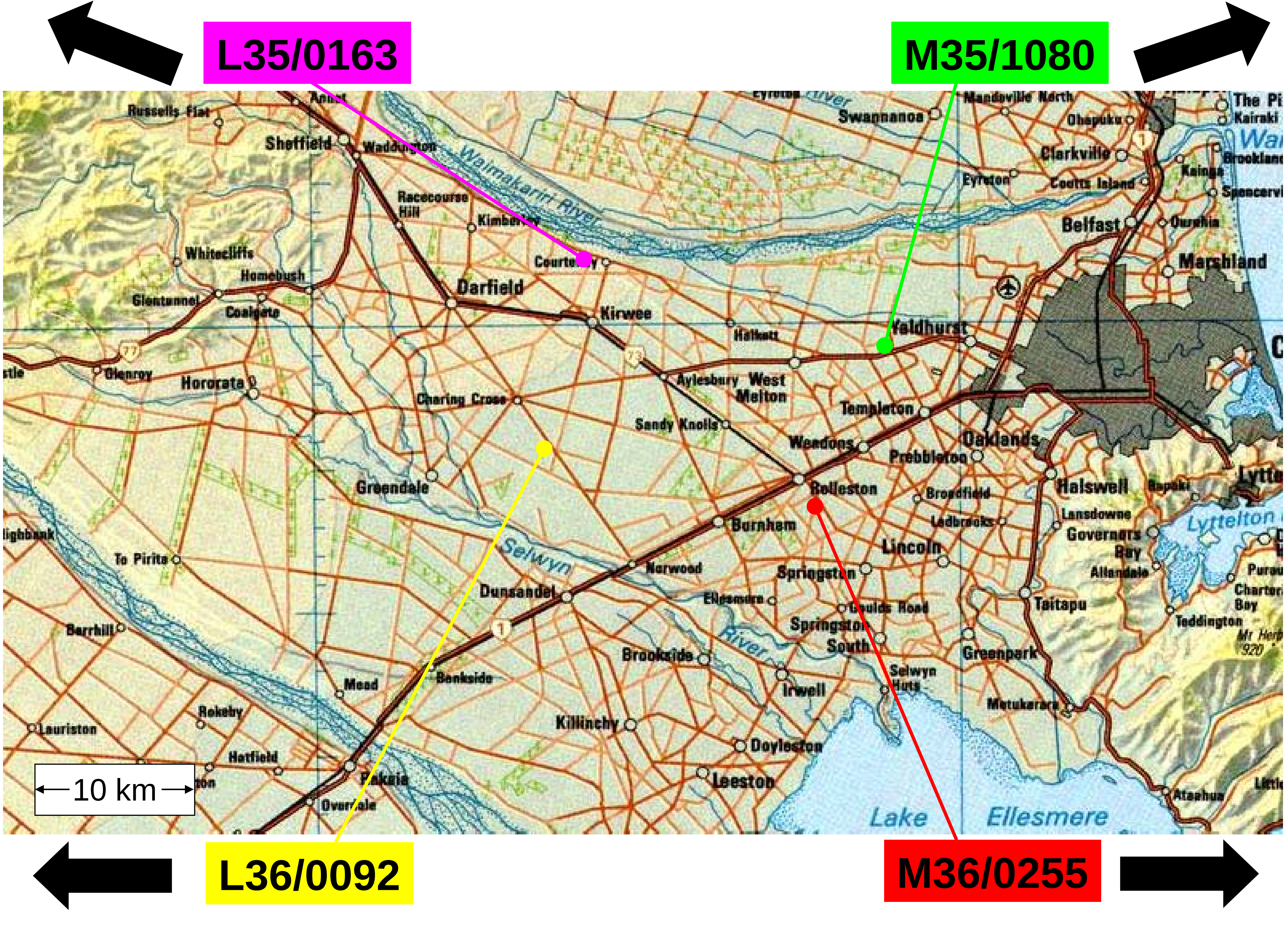

L35/0163
Observed groundwater level
Simulated groundwater level
Land surface recharge
GWL (m) above mean sea level
Land surface recharge (mm/mth)
130
120
110
100
90
80
300
250
200
150
100
50
0
1972
1976
1980
1984
1988
1992
1996
2000

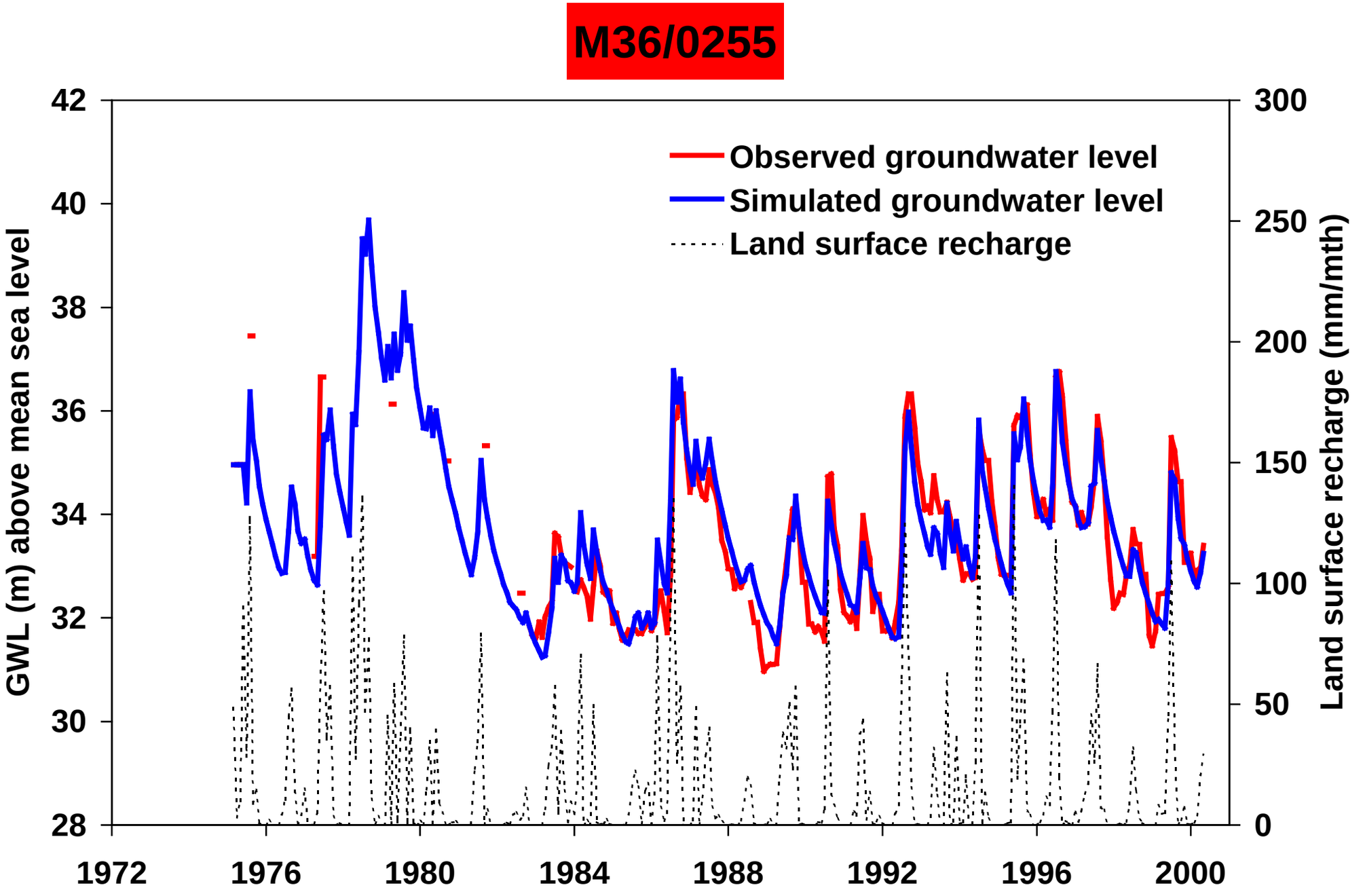
Results: wells in a 2000 km^2 aquifer in Central Canterbury, New Zealand

Well No.	L35/0163	L36/0092	M36/0255	M35/1080
T_v (mth)	2.7	4.8	0.6	1.6
T_1 (mth)	18.9	20.4	19.2	20.4
g_1	1.22	1.12	0.28	0.15
T_2 (mth)	-	-	0.61	0.62
g_2	-	-	0.02	0.04
d (m)	80.2	59.6	28.9	44.0


M35/1080
Observed groundwater level
Simulated groundwater level
Land surface recharge
GWL (m) above mean sea level
Land surface recharge (mm/mth)
54
52
50
48
46
44
300
250
200
150
100
50
0
1972
1976
1980
1984
1988
1992
1996
2000


L36/0092
Observed groundwater level
Simulated groundwater level
Land surface recharge
GWL (m) above mean sea level
Land surface recharge (mm/mth)
100
90
80
70
60
300
250
200
150
100
50
0
1972
1976
1980
1984
1988
1992
1996
2000


L35/0163
M35/1080
L36/0092
M36/0255
10 km
Russells Flat
Sheffield
Waddington
Waimakariri River
Racecourse Hill
Whitecliffs
Homebush
Glentannel
Coalgate
Darfield
Kirwee
Halkett
Glenroy
Hororata
Charing Cross
Aylesbury
West Melton
Sandy Knolls
Greendale
To Pirita
Selwyn
Dunsandel
Norwood
Burnham
Rolleston
Weedons
Templeton
Prebbleton
Yaldhurst
Swannanoa
Mandeville North
Ohapuku
Clarkville
Eyreton
Coutts Island
Belfast
Ouruhia
Marshland
Kaiapoi
Oaklands
Halswell
Broadfield
Ladbrooks
Lincoln
Springston
Ellesmere
Gaulds Road
Springston South
Brookside
River
Irwell
Selwyn Huts
Greenpark
Taitapu
Lansdowne
Governors Bay
Allandale
Teddington
Metukarara
Barrhill
Mead
Bankside
Rokeby
Lauriston
Hatfield
Killinchy
Doyleston
Leeston
Rakaia
Overdale
Lake
Ellesmere
Atahua


M36/0255
Observed groundwater level
Simulated groundwater level
Land surface recharge
GWL (m) above mean sea level
Land surface recharge (mm/mth)
42
40
38
36
34
32
30
28
300
250
200
150
100
50
0
1972
1976
1980
1984
1988
1992
1996
2000

Discussion

- **This aquifer has a hydraulic residence time of about 20 months (T_1)**
- **High vadose zone storage (T_v) indicates presence of an aquitard at L35/0163 and L36/0092**
- **Higher gain (g_1) at L35/0163 and L36/0092 means no interaction with rivers (perched above aquifer)**
- **Datum values (d) are height above sea level, at each well, of piezometric effect of river recharge**

Model calibration and implementation

- **Eigenmodels are transformed to difference equations for implementation in Microsoft Excel**
- **Calibration utilises the “solver” tool in Excel**
- **Simulation error is modelled as the noise term of a stochastic difference equation (SDE)**
- **SDE structure enables model updating for forecasting groundwater levels**
- **Calibration can be assisted by eigenvalues being the same at all observation sites**

Summary

- **The eigenvalue approach can quantify the storage characteristics of an aquifer from as few as one observation well record**
- **The piezometric record at each well can be separated into the effects of river recharge and land surface recharge**
- **The dynamic models are simple, and suitable for updating in forecast versions**
- **Models are calibrated and implemented in spreadsheet form**