Working the blue gold: a personal journey with mathematical tools for water management

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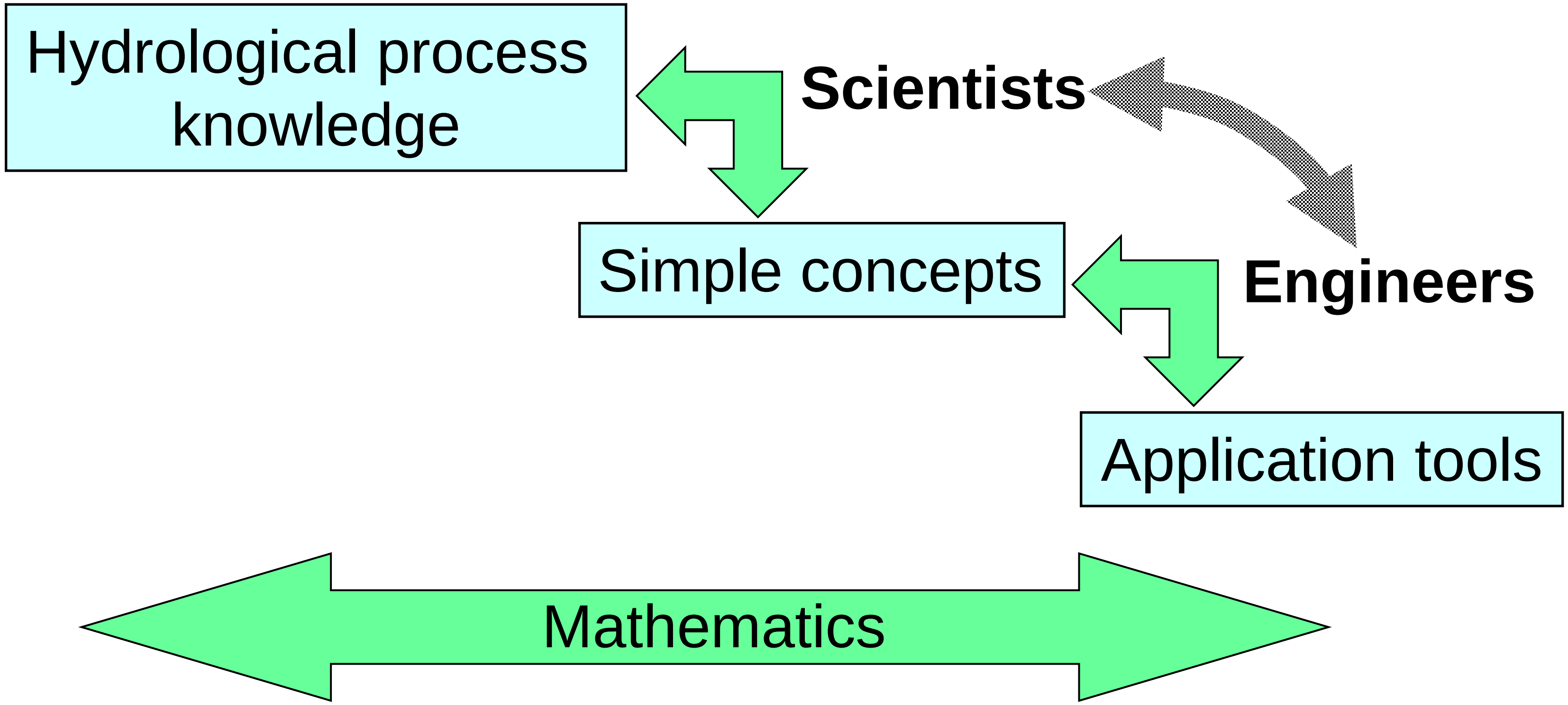
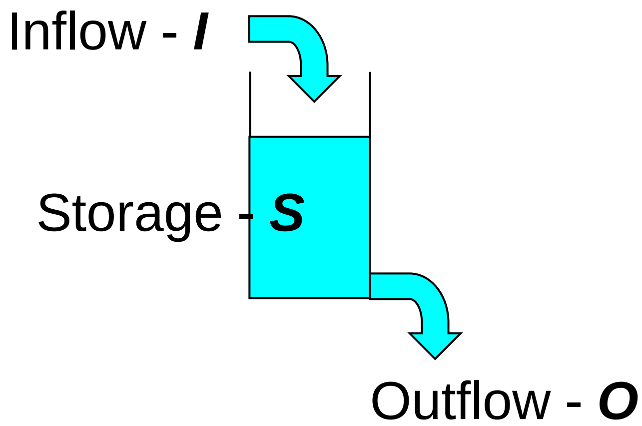
Outline

1. Engineers, tools, and hydrology
2. Water quantity stories
3. We have control
4. Water quality stories
5. We still manage
6. Conclusions

Engineers, tools, and hydrology



Simple concept: water flow and storage



Outflow is proportional to storage:

Storage time $T = \frac{S(t)}{O(t)}$

Coefficients:

a = exp(-1/**T**)

b = 1-exp(-1/**T**)

Mathematical tool for calculating daily flows :

$$O_k = a\, O_{k-1} + b\, I_k$$

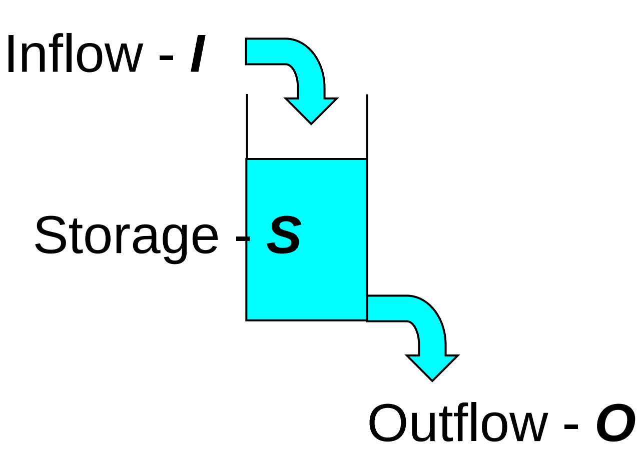
Scientific complexity

- Hydrological science can be complex, in terms of bio-physical processes

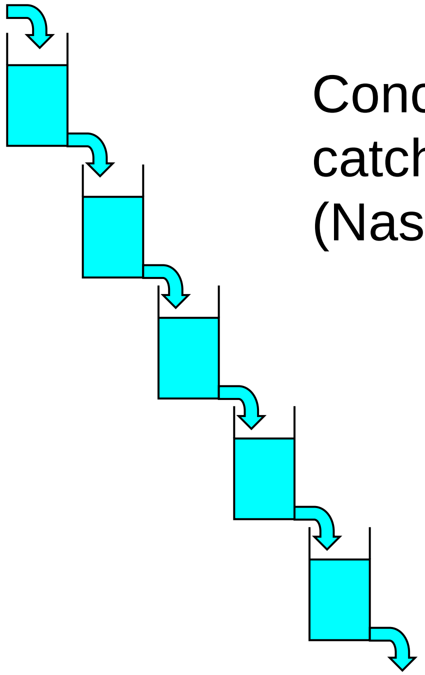
- Some complex processes can be described by complex concepts.

- Some complex concepts can be considered as systems of simple concepts.

A system of conceptual linear water storages



$$\boldsymbol{O}_k = a\,\boldsymbol{O}_{k-1} + b\,\boldsymbol{I}_k$$



Conceptual catchment model (Nash, 1957)

Mathematical tool

$$\boldsymbol{O}_k = a_1\boldsymbol{O}_{k-1} + a_2\boldsymbol{O}_{k-1} + \dots$$
$$+ b_1\boldsymbol{I}_k + b_2\boldsymbol{I}_{k-1} + \dots$$

Stochastic in Colorado (1970–72)

with Vujica Yevjevic ("Dr Y") and his graduate students

- *Stochastic* hydrology refers to random influences
- A principal source is inherent randomness of climate dynamics
- Water resource modelling requires many realisations of likely hydrological time-series

Stochastic in Colorado (1970–72)

with Vujica Yevjevic ("Dr Y") and his graduate students

Stochastic refers to random influences

Input ***I*** can contain random components

Economic & industrial applications

Box, G.E.P. & Jenkins, G.M. (1970). *Time Series Analysis forecasting and control*.

$$\boldsymbol{O}_k = a_1\boldsymbol{O}_{k-1} + a_2\boldsymbol{O}_{k-1} + \ldots$$

$$+ b_1\boldsymbol{I}_k + b_2\boldsymbol{I}_{k-1} + \ldots$$

Auto-**R**egressive, **M**oving **A**verage

ARMA model

can be a "black box"

Flood forecasting in Kuala Lumpur (1972-74)

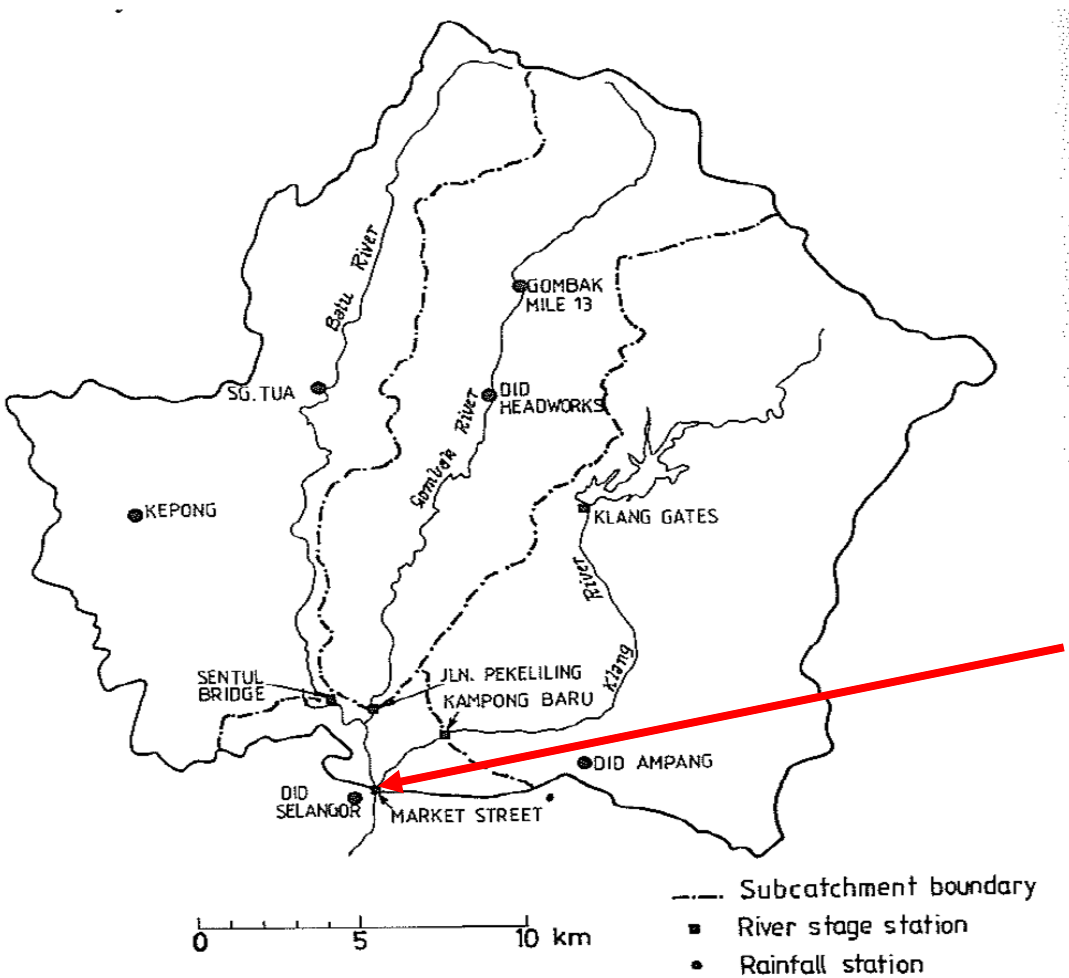
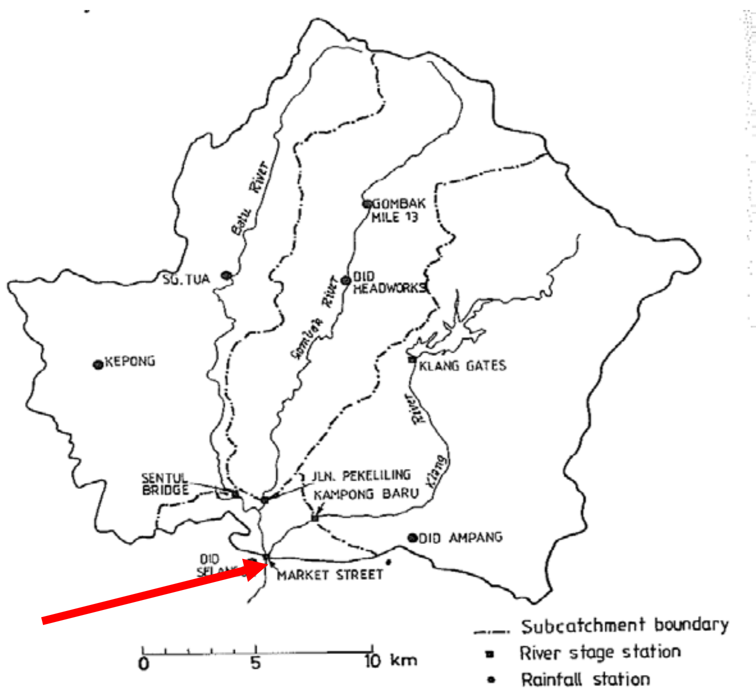


FIG. — 1 The catchment of the Klang River at Kuala Lumpur

*NZ Journal of Hydrology 18(1):*29-35.

Flood forecasting in Kuala Lumpur (1972-74)



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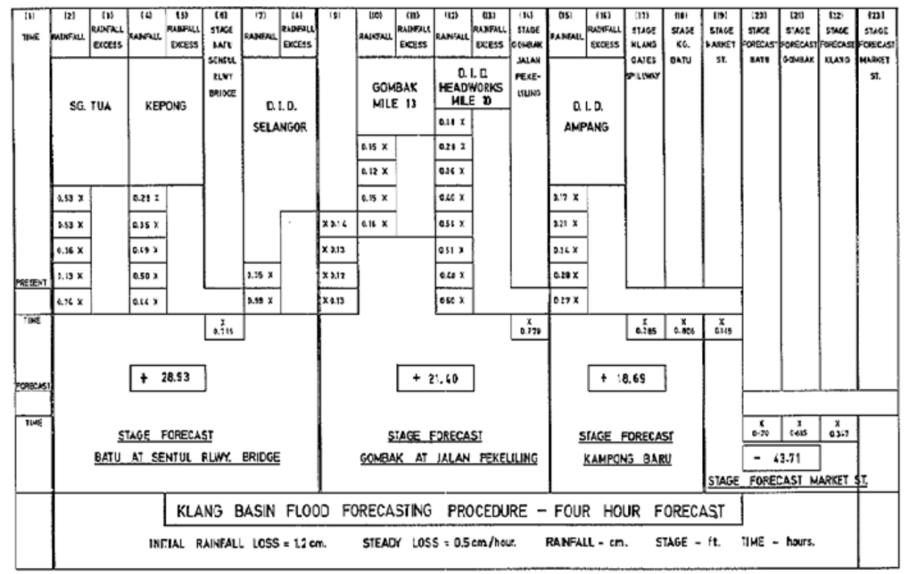


FIG. – 2 The template used for flood forecasting

Cardboard template placed over real-time record of rainfall & river stage observations, sent by telephone.

ARMA model calculations by electronic calculator

*NZ Journal of Hydrology 18(1):*29-35.

Cardboard mathematical tool - ARMA model

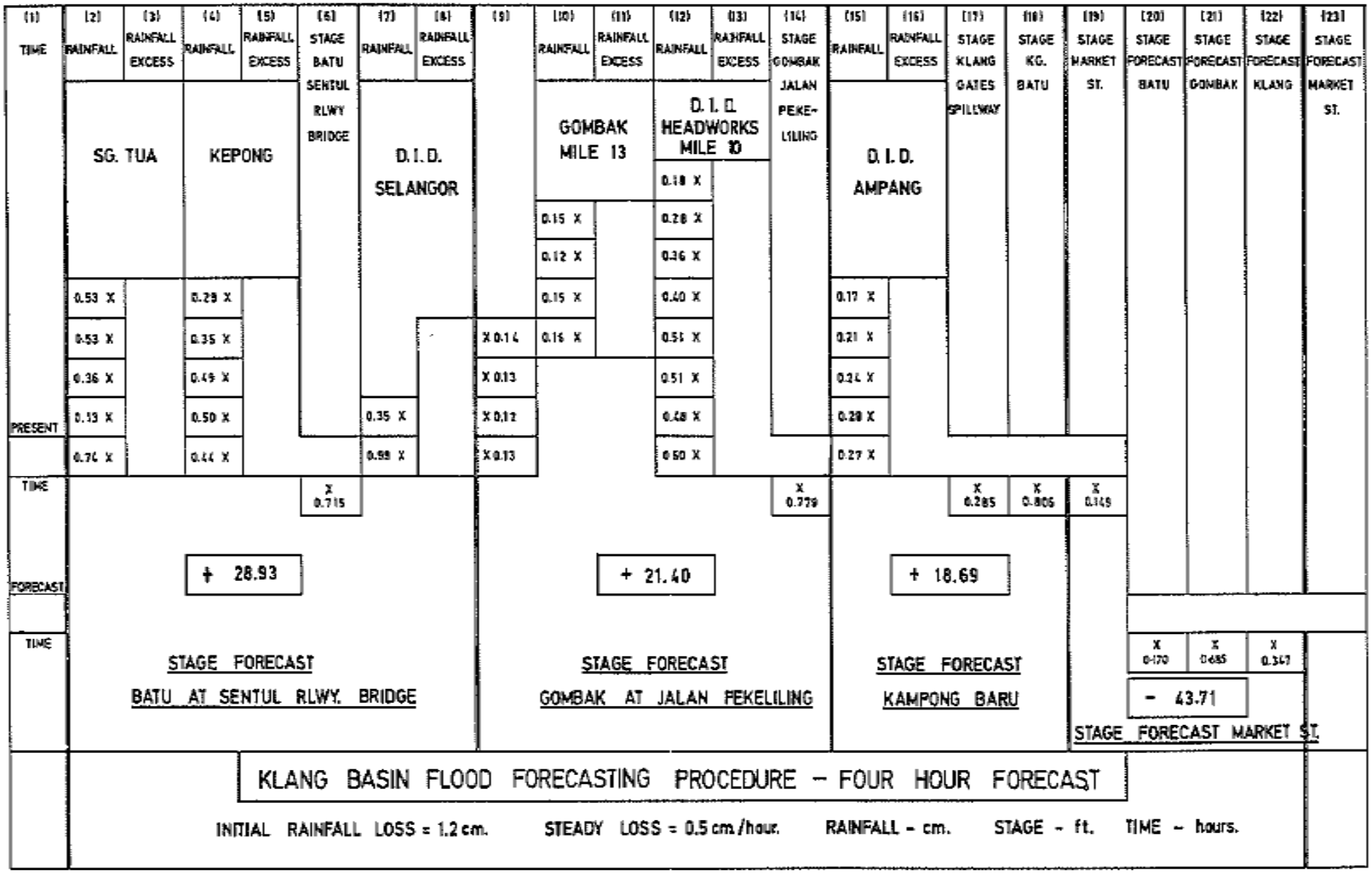


FIG. – 2 The template used for flood forecasting

Guided model for flood forecasting

with George Griffiths, 1994

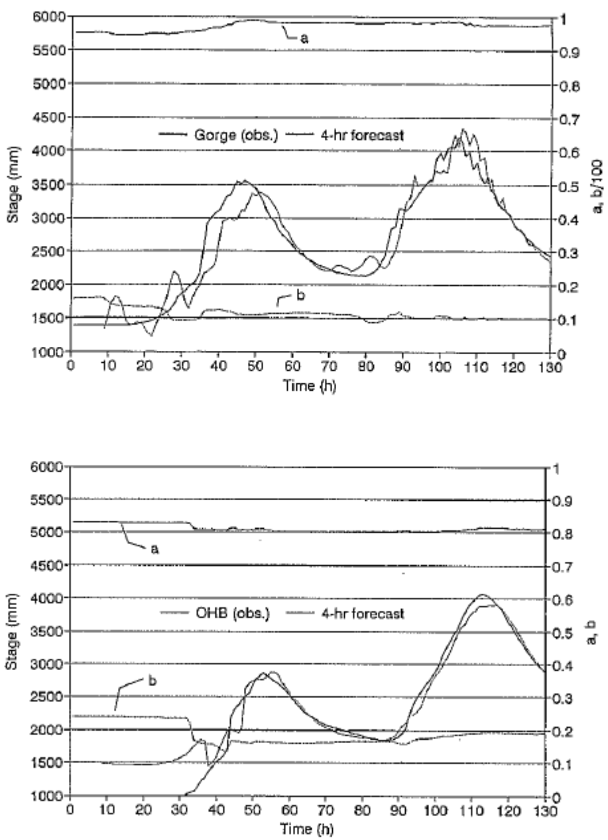


FIG. 1—Comparison of 4-hour forecast and observed stage, and time variation of model parameters, at Waimakariri Gorge and Old Highway Bridge, for a flood on 31 August 1970.

Real-time flood stage forecasts for Waimakariri River, by simple model:

$$\boldsymbol{O_k} = a_t \boldsymbol{O_{k-1}} + b_t \boldsymbol{I_k}$$

with time-varying parameters.

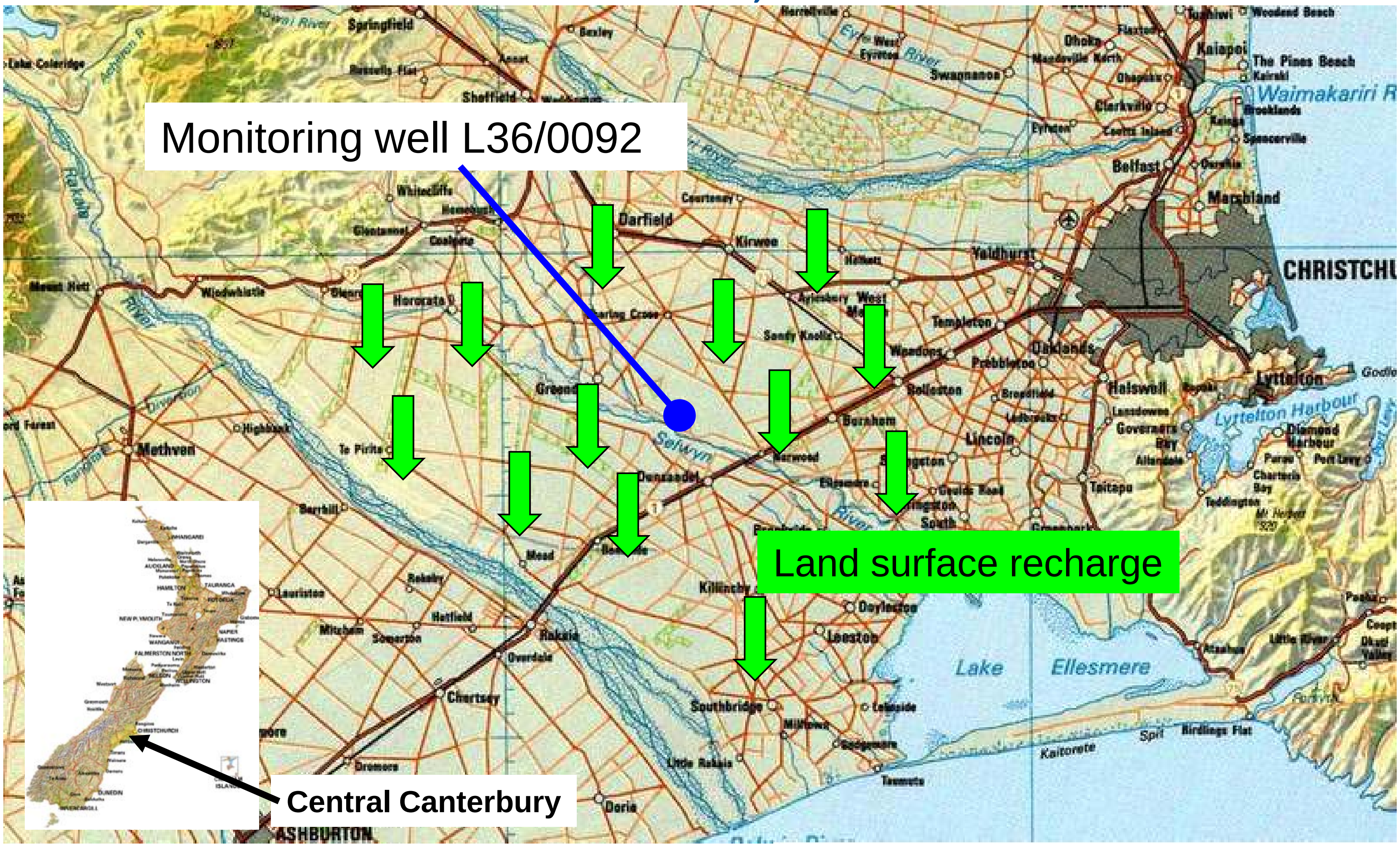
Real-time, adaptive forecasting by use of the ***Kalman Filter*** algorithm

Kalman Filter algorithm developed by engineers. Initial applications in aerospace and guided missile control.

*NZ Journal of Hydrology 32(2):*1-15

Discovery at Well 92

with Peter Callander and Catherine Moore, 1991



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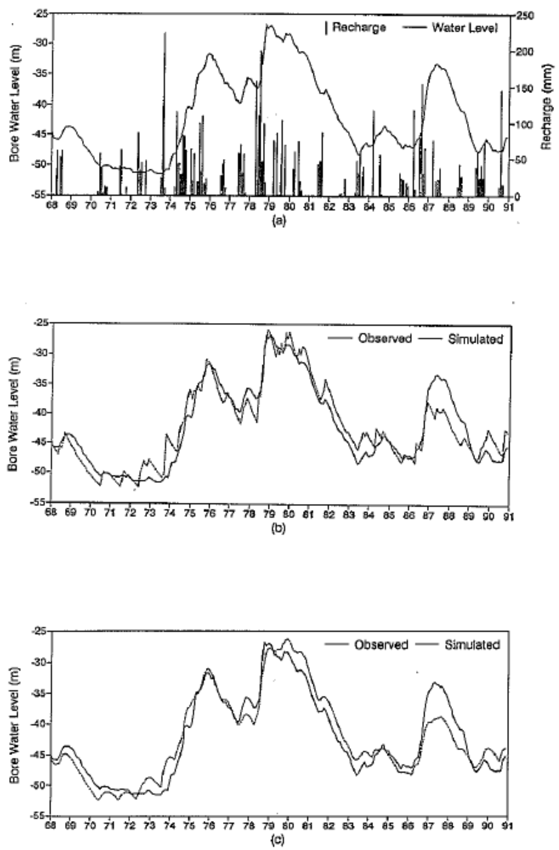

FIG. 4—The (a) water level and recharge series, (b) (1, 1, 2) simulation, and (c) (2, 1, 1) simulation, for monthly data January 1968–December 1990.

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Simple ARMA relationships between monthly rainfall recharge and groundwater level at L36/0092

-initially a "black box" approach!

-concept building came later.

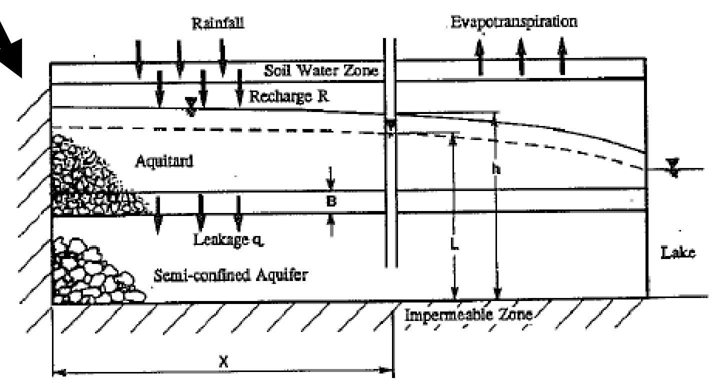
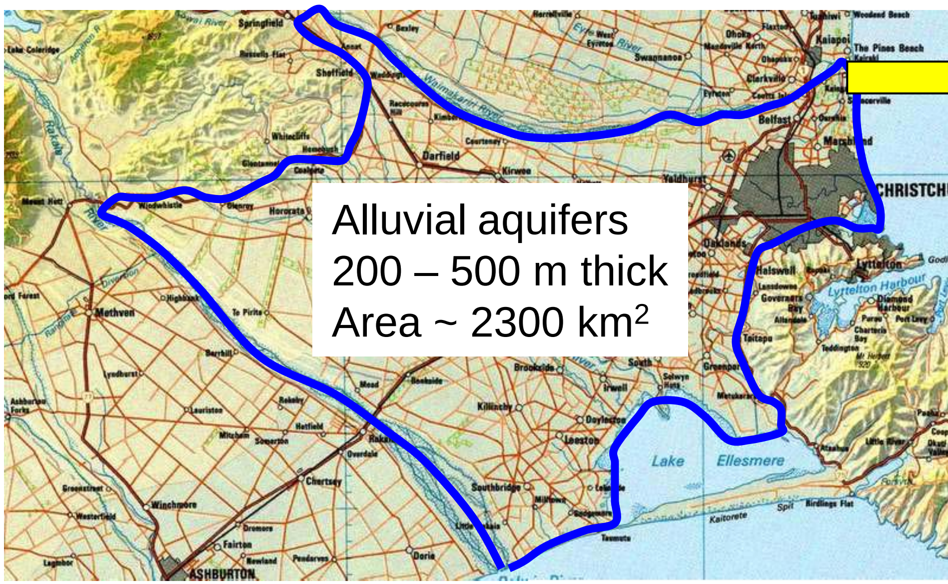


FIG. 2—A simple physical model of the Greendale aquifer.

NZ Journal of Hydrology 30(1): 16-36

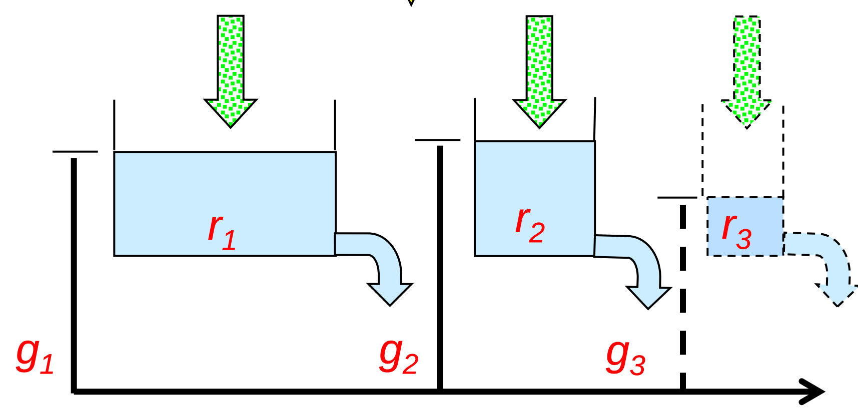
Buckets of groundwater

with Matthew Morgan, for the Canterbury Strategic Water Study, 2002



$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) + R = S\frac{\partial h}{\partial t}$$

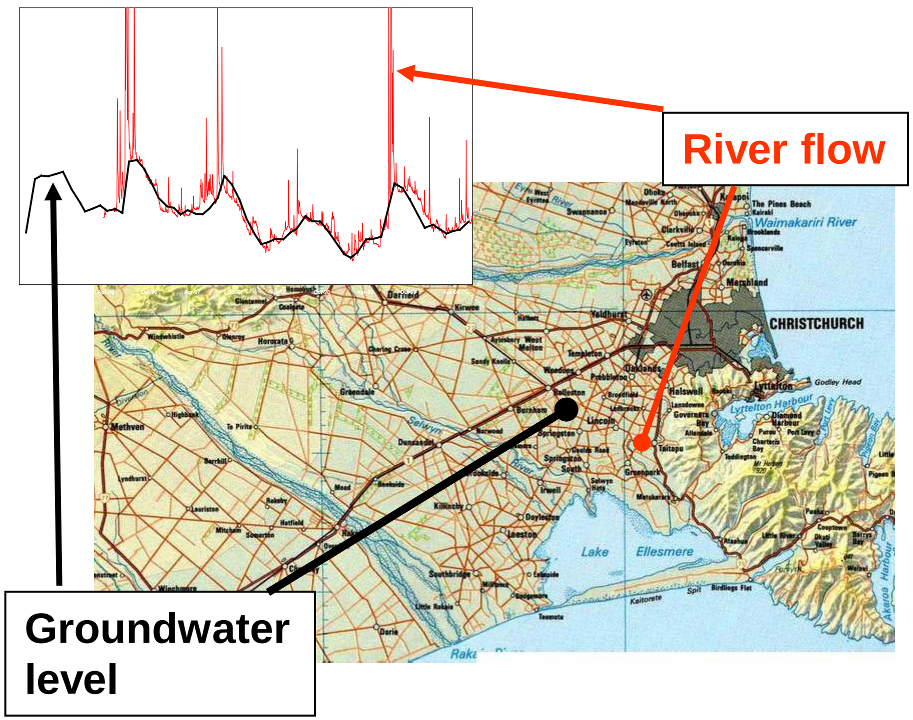
Mathematical theory
(Sahuquillo, 1983)

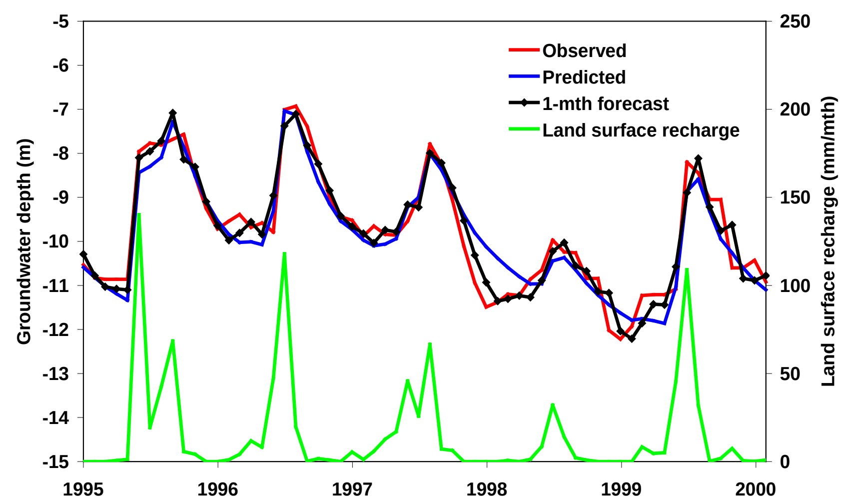


The resulting **Eigenmodel** is a system of conceptual linear water storages.

Forecasting groundwater level

with ECan groundwater staff





We have control
– mathematics for water management

- Many water resource quantity issues can be described by systems of conceptual linear water storages
- The mathematical basis for analysis, prediction, and control of linear systems is a well developed engineering science
- This discipline includes algorithms, such as Kalman filter, which take account of knowledge uncertainty as well as imperfect monitoring

Waves in the vadose zone

with Hugh Thorpe, 1998-2001

Non-linear kinematic wave in macropores with sorption to micropores

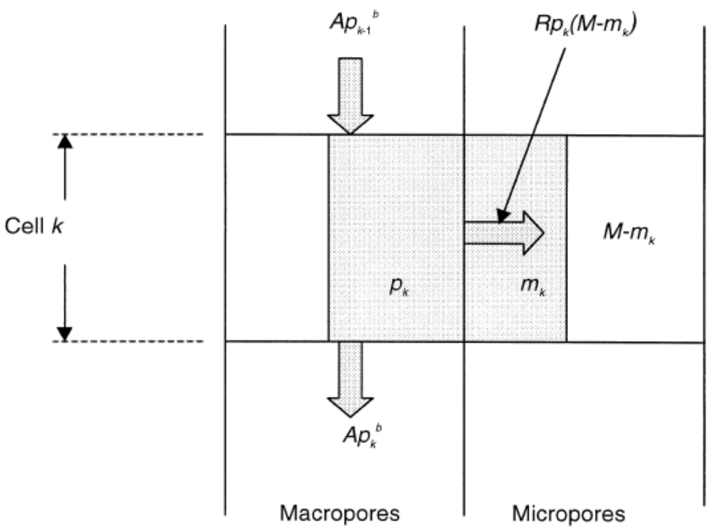


Fig. 3. Water fluxes and volumes for the macropore and micropore regions of a typical cell in the model.

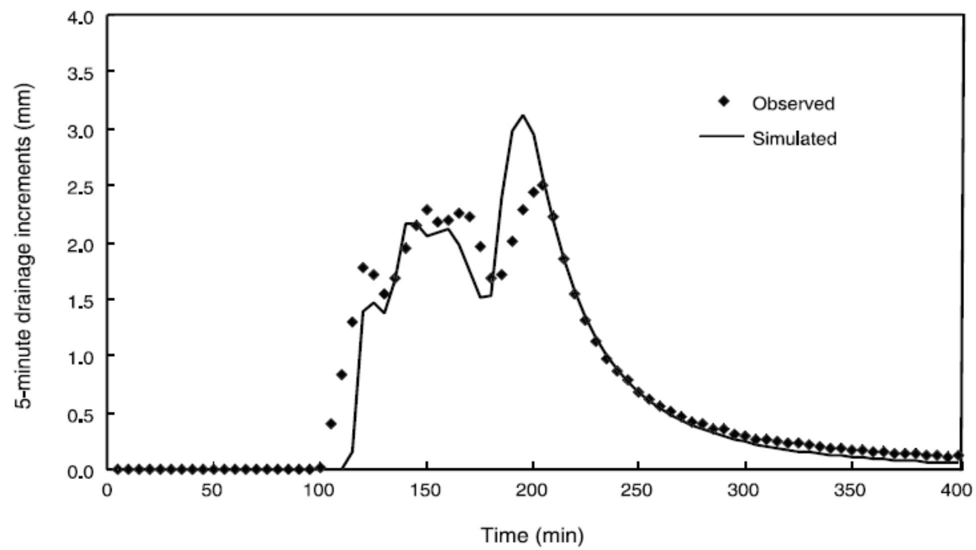
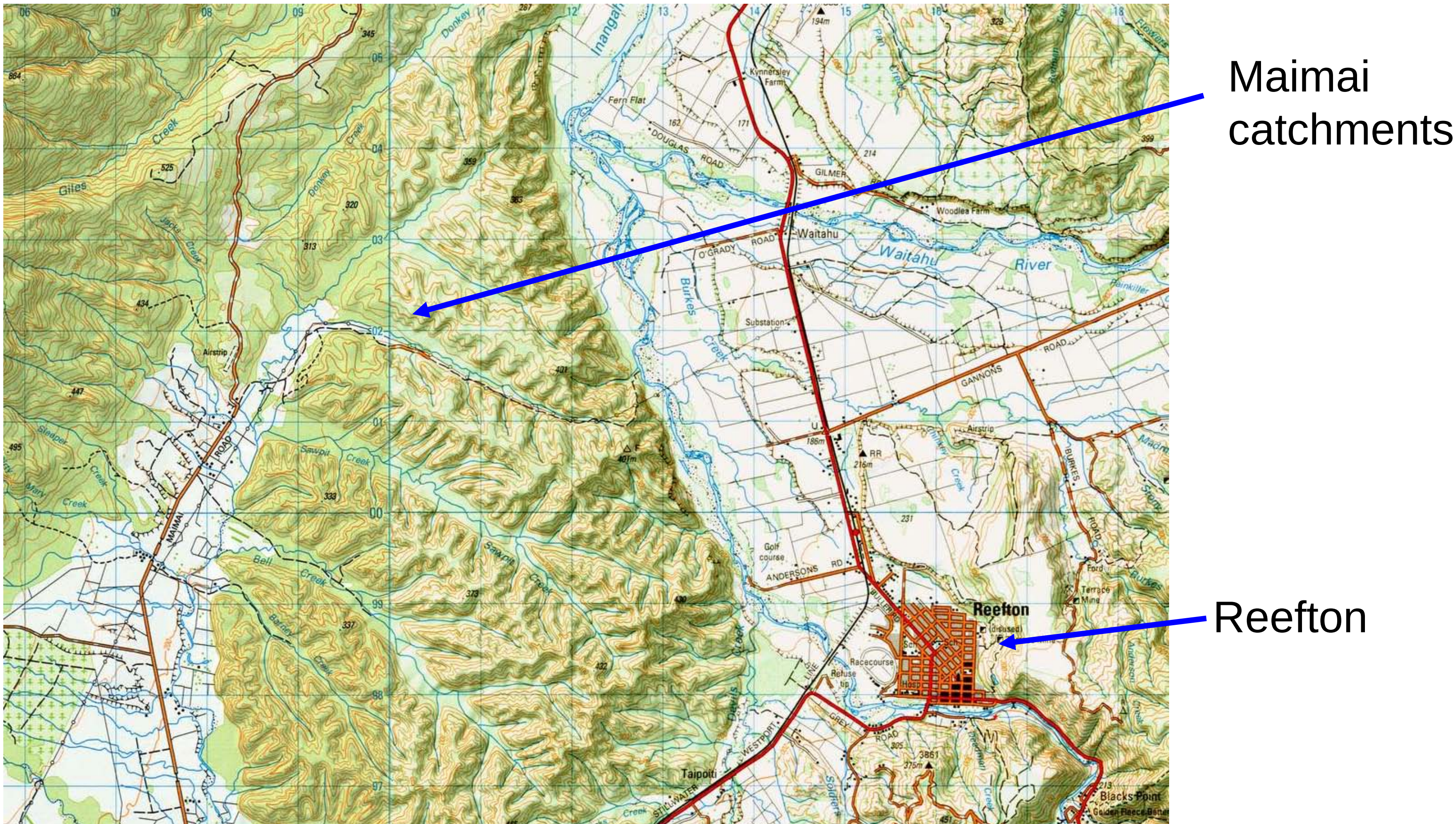


Fig. 5. Result of calibration of the kinematic cell model to the data from Lysimeter 4 for the irrigation of 9 May 1996.

Australian Journal of Soil Research, *39:* 837-849.

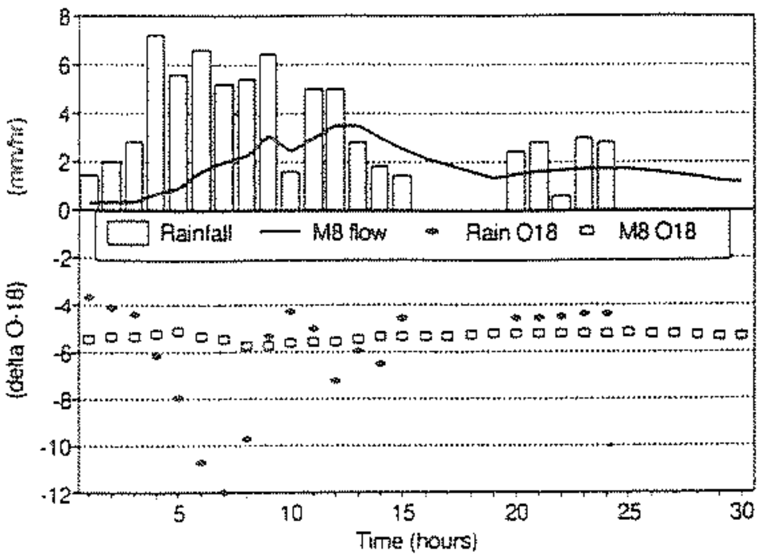
Old water at Maimai
with Mike Stewart, 1997



Old water at Maimai

with Mike Stewart, 1997

Oxygen-18 transport through
a steep hillslope during rainstorms



Hydrometric

O-18 tracer

Figure 1: The hydrometric and oxygen-18 tracer data for the stream M8 during the storm of 18-19 January 1994.

Mixing cell
for tracer
dynamics
(250 mm)

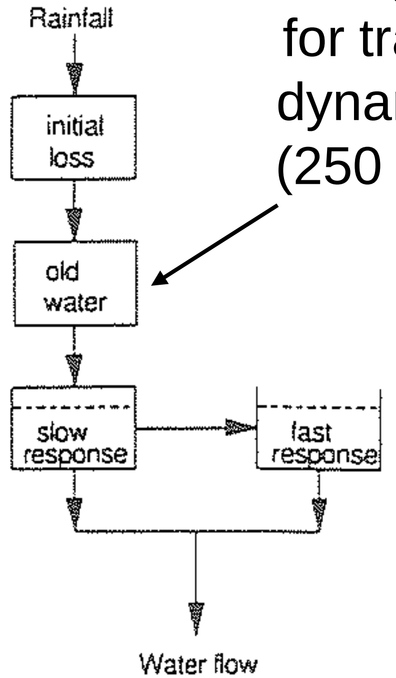
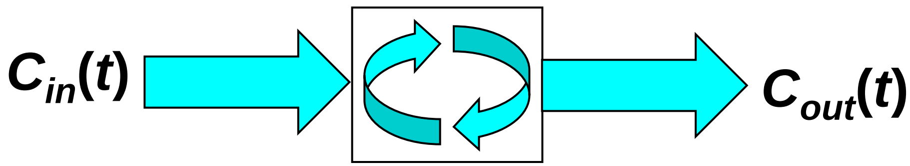


Figure 4: The model structure for testing the "old water" hypothesis

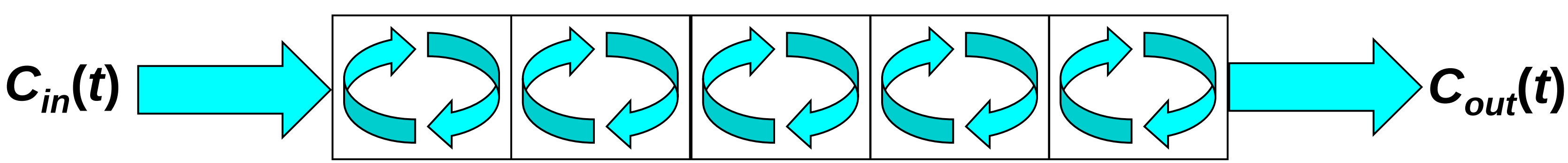
MODSIM 97, 42-47

Mixing cell is a simple concept for water quality



Concentration dynamics
at intervals of time or flow

$$C_{out}(k) = a\, C_{out}(k-1) + b\, C_{in}(k)$$



System of mixing cells simulates advective-dispersive solute transport

Peclet number = 2 x (number of mixing cells)

Mixing cells in the soil

with soil scientists at Lincoln University

Conceptual model of advective-dispersive solute transport and transformations in soil, designed for prediction and management with use of Kalman filter.

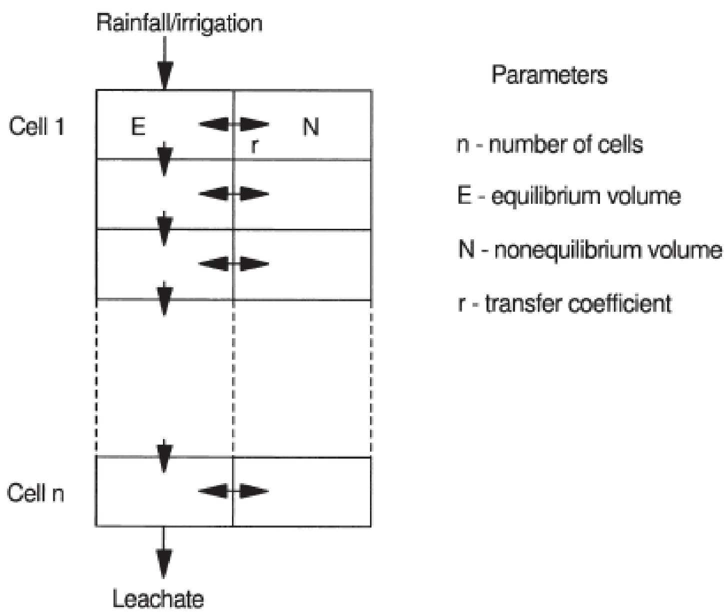
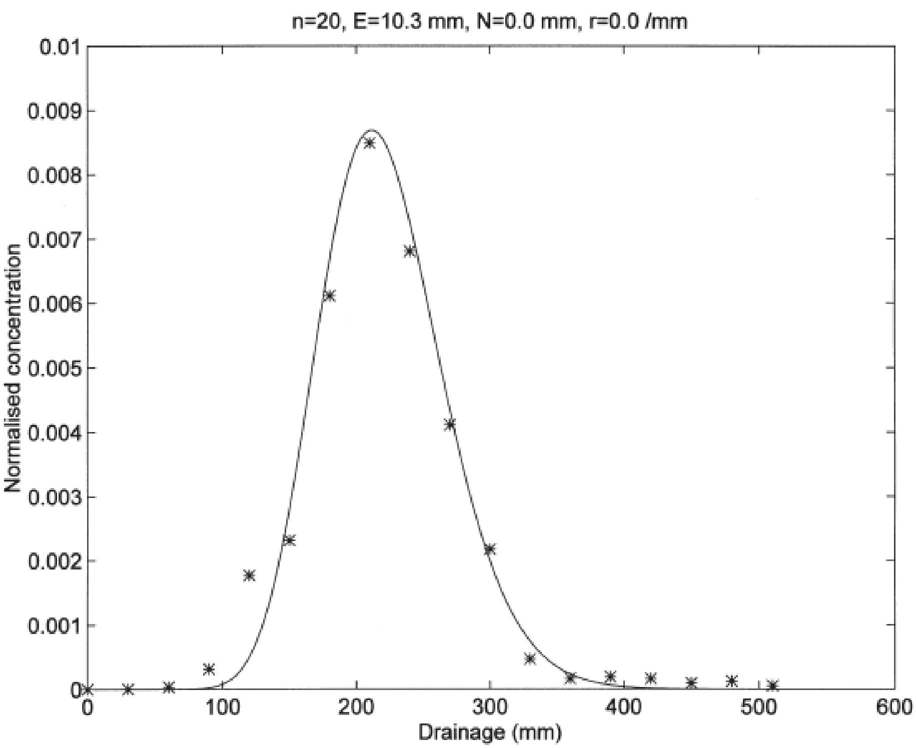


Fig. 1. Conceptual structure of the state–space mixing cell (SSMC) model.

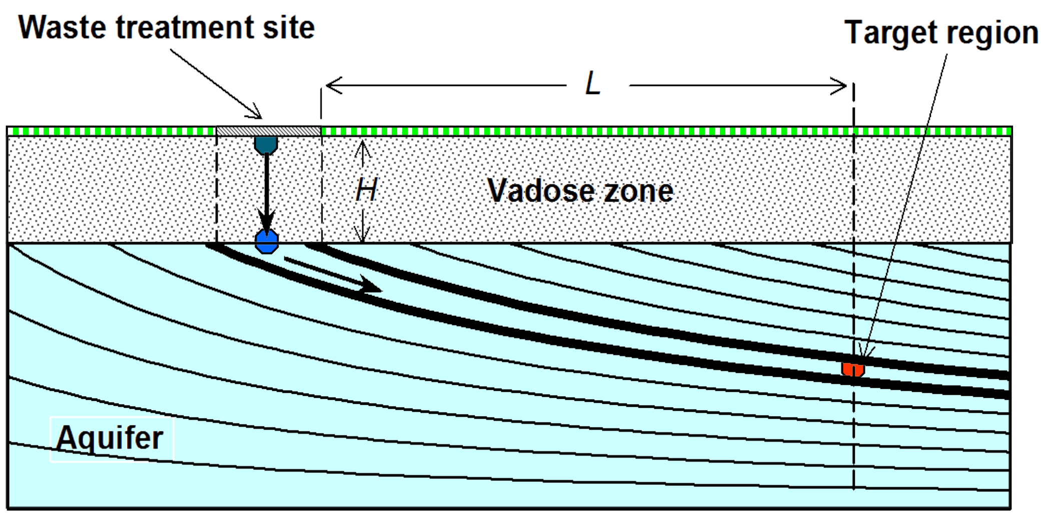


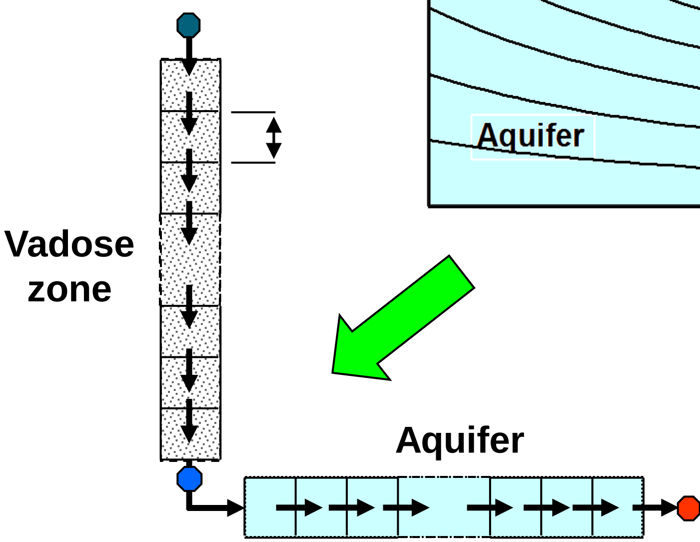
Environmental Modelling & Software 14 (1999)161-169

Managing land treatment of wastewater

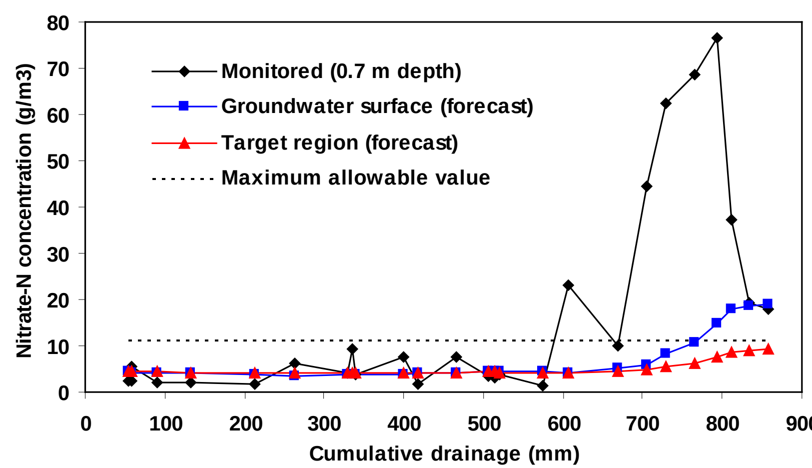
with meat processing industry

Physical processes

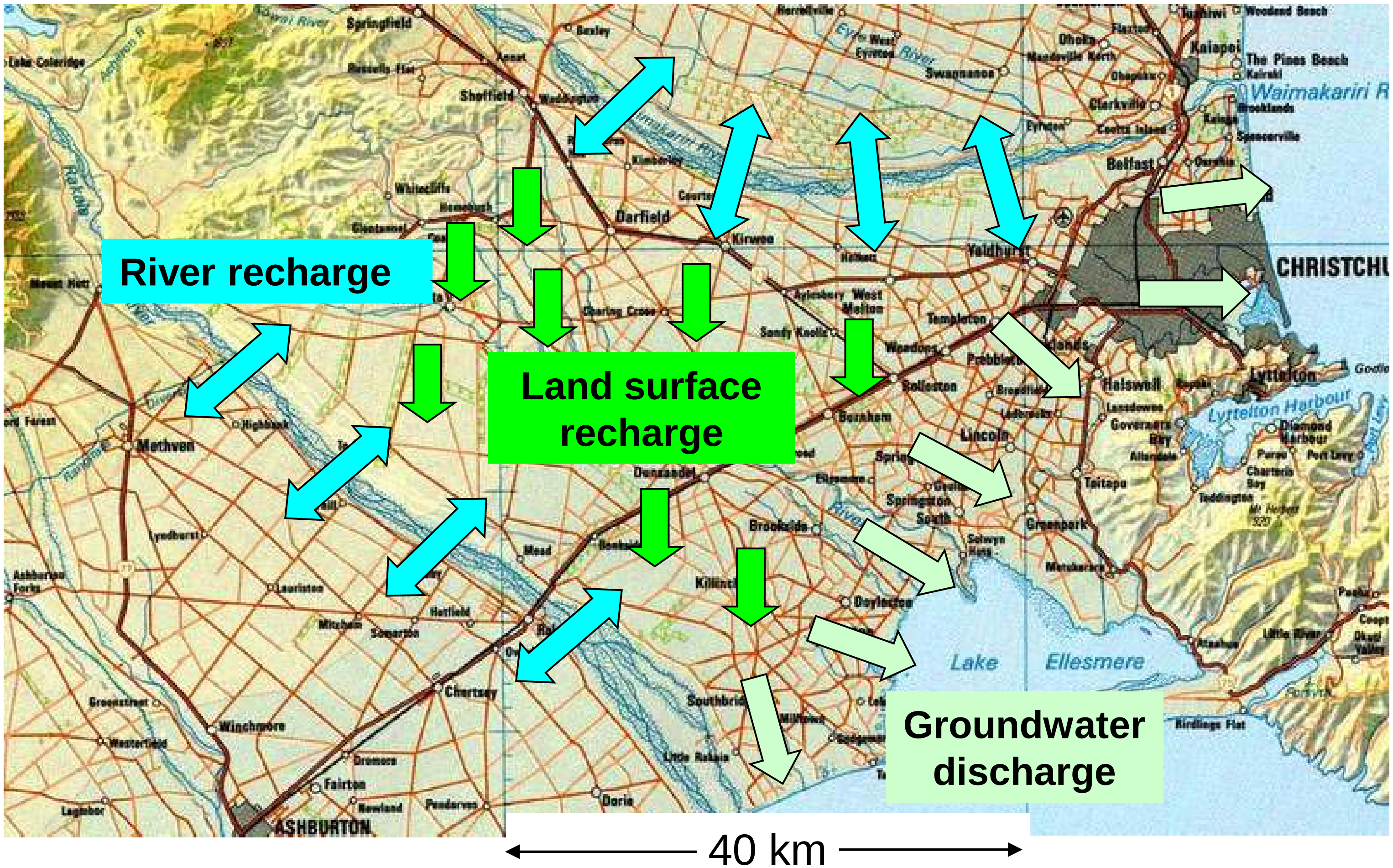




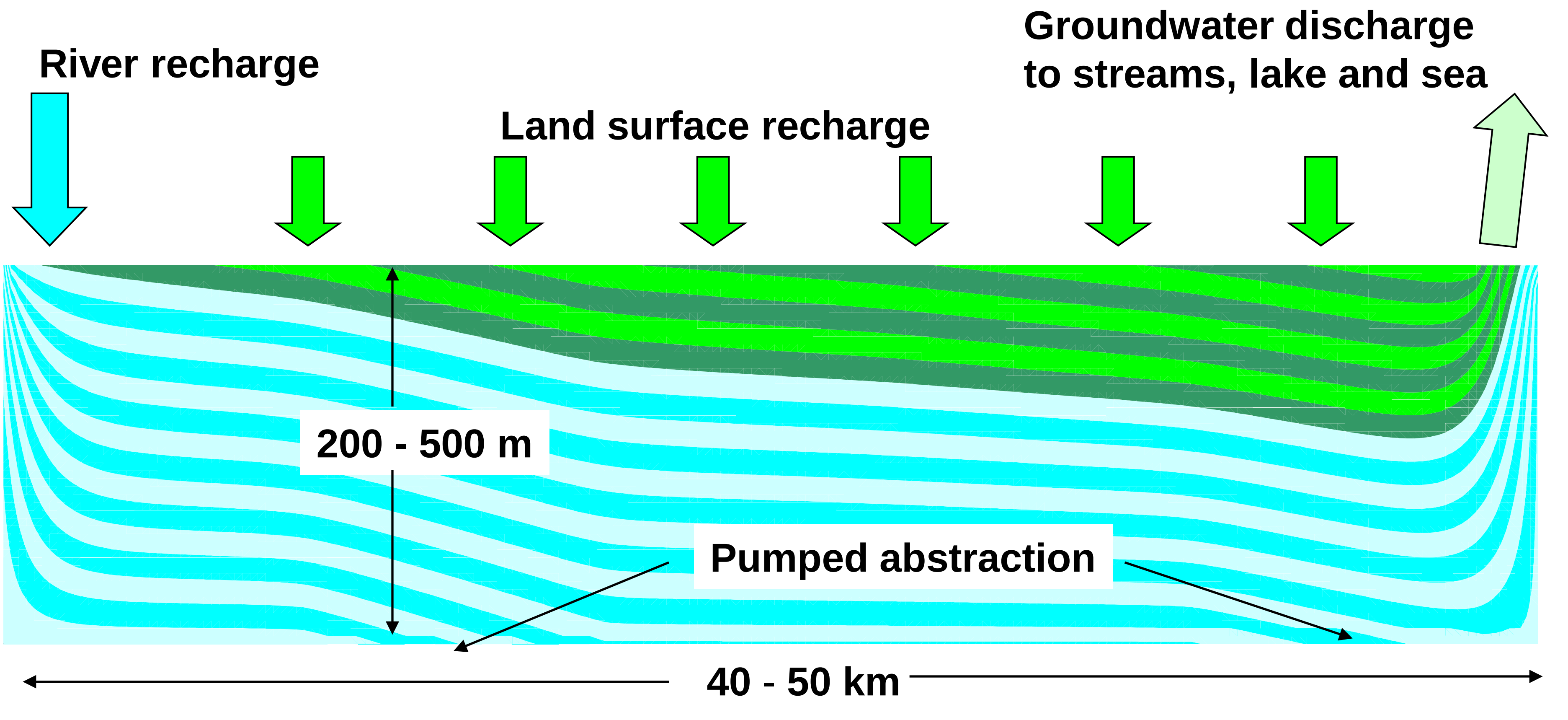
Mixing-cell concept



Managing effects of land use on water quality

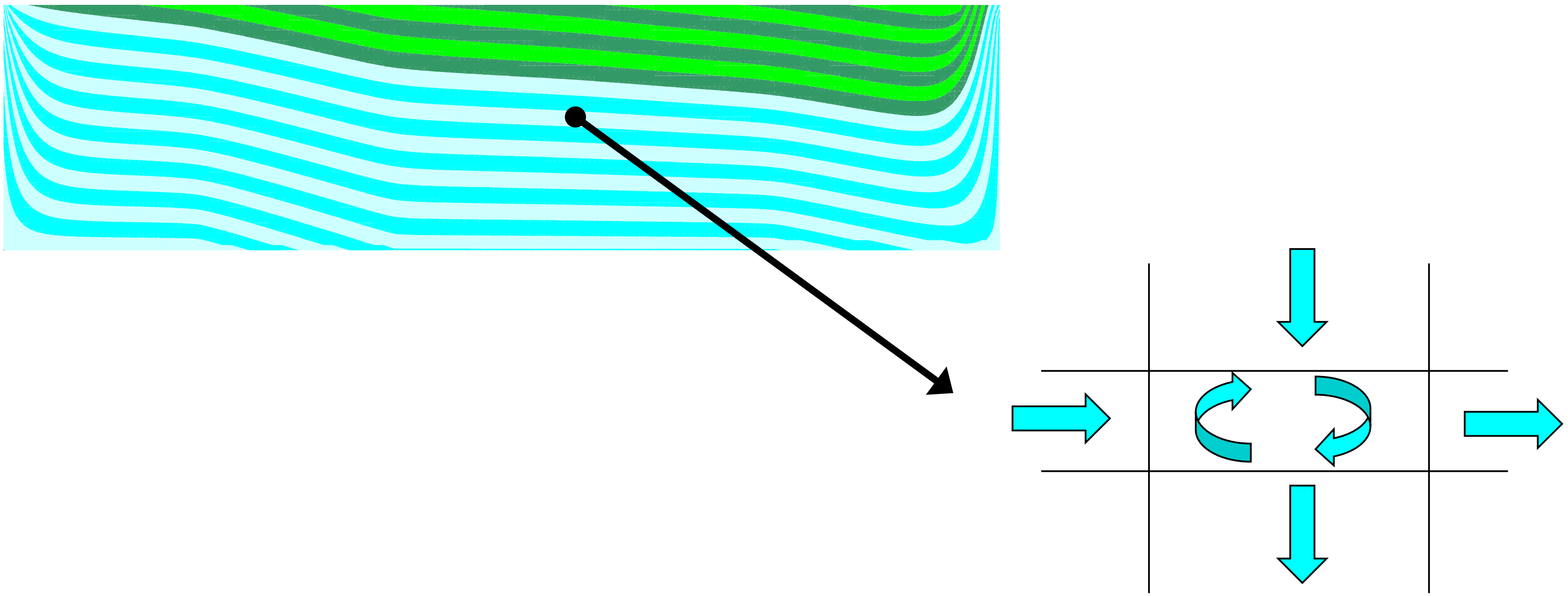


Streamlined groundwater flow

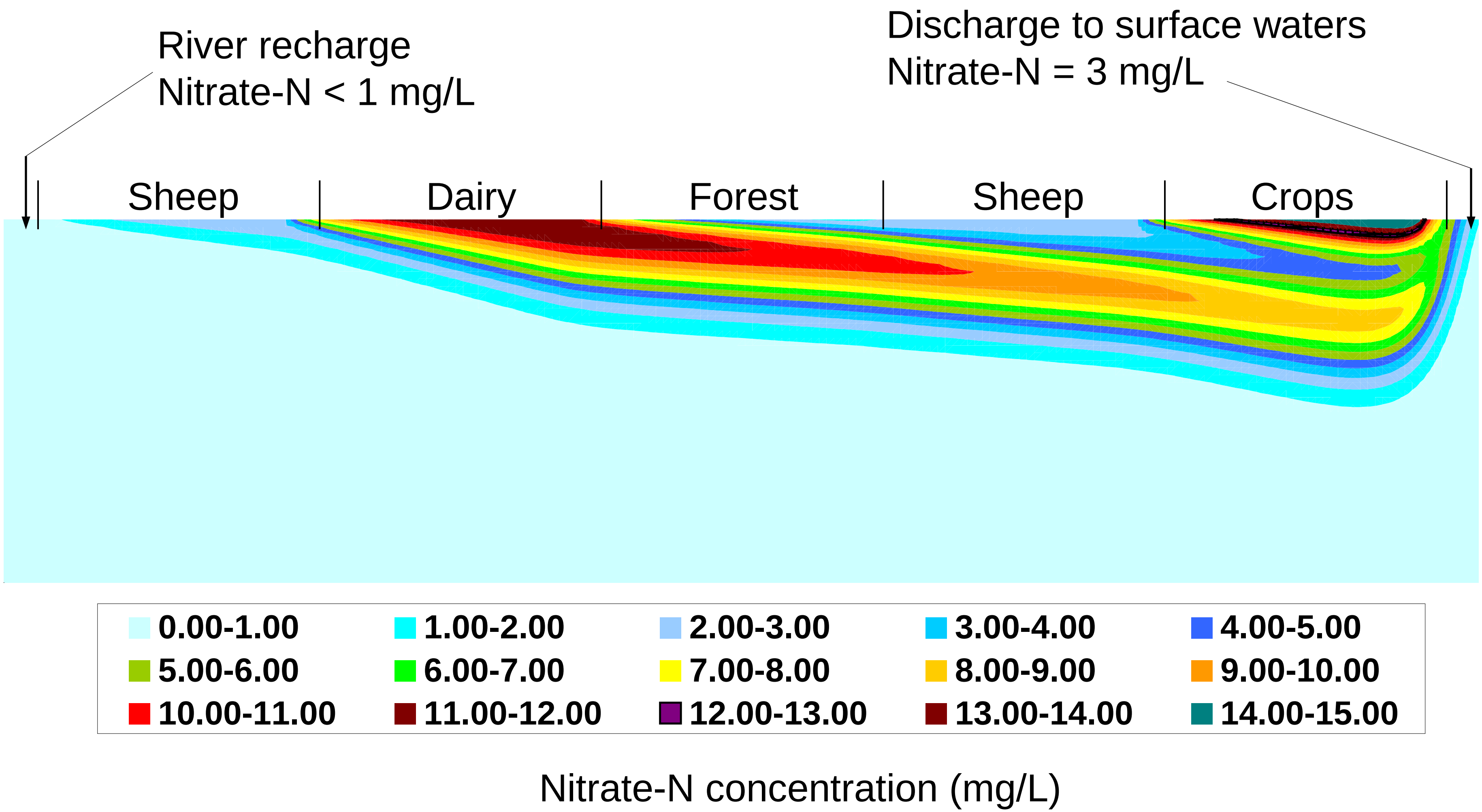


Mixing cells in the aquifer

Simulate horizontal & vertical, advective-dispersive transport, and transformations

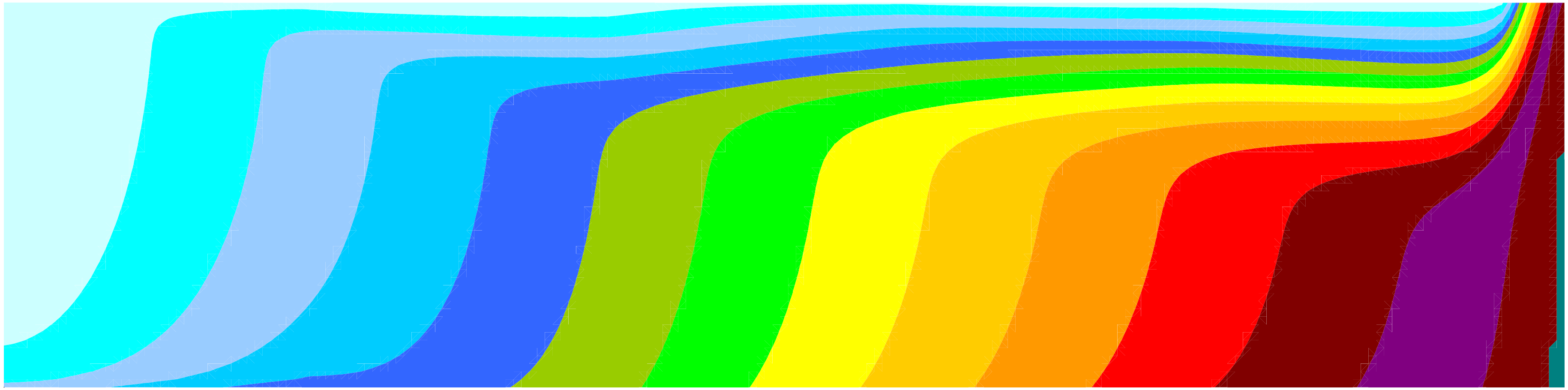


Streamlines + mixing cells = nutrient transport



Groundwater age is a contaminant that grows in the mixing cells

Aquifer is 300 m thick
Porosity is 0.15



0-10	10-20	20-30	30-40	40-50	50-60	60-70
70-80	80-90	90-100	100-110	110-120	120-130	130-140

Groundwater age (years)

We still manage - water quality tools

- Mixing cells don’t exist in the bio-physical world
- Stream functions are a mathematical construct that can’t be directly observed
- Mathematics links these concepts to processes
- Mathematics describes systems of concepts and enables management

Conclusions from the journey

- “Scientists” pursue knowledge
- “Engineers” apply knowledge to decisions
- Concepts are the meeting points of these minds
- Mathematics links knowledge to solution of real problems by means of abstract concepts
- Mathematical tools are the expressions of these linkages

Mathematics enables human benefit from hydrological science