Flexural, Axial Load and Elongation Response of Plastic Hinges in Reinforced Concrete Member

Brian H.H. Peng

University of Canterbury, Christchurch, New Zealand

Rajesh P. Dhakal, Richard C. Fenwick, Athol J. Carr and Des K. Bull

University of Canterbury, Christchurch, New Zealand

ABSTRACT:

Previous research in New Zealand has indicated that elongation of plastic hinges in reinforced concrete (RC) beams can have a significant effect on the seismic performance of RC structures. A number of empirical formulas have been proposed to predict elongation. However, no satisfactory analytical models are currently available which can be used to predict the influence of elongation on the seismic performance. This paper describes a plastic hinge model developed to predict the combined flexural, axial load and elongation response of plastic hinges in RC beam. The model is a filament type element which consists of layers of longitudinal and diagonal axial springs to represent the flexural, shear and elongation response of a plastic hinge. Analytical predictions for beams with different levels of axial load are compared with the experimental results. It is found that the plastic hinge model predicts the response satisfactorily. With this newly formed element, seismic analyses may be performed to assess the significance of beam elongation on seismic performance of RC structures.

1 INTRODUCTION

Recent experimental studies on RC moment resisting frames containing precast prestressed concrete floor units at the University of Canterbury (Lindsay 2004; MacPherson 2005; Matthews 2004) and at the University of Auckland (Lau 2001) in New Zealand have shown that elongation of plastic hinges in RC beams and its interaction with precast-prestressed floor units can lead to undesirable failure mechanisms, such as premature collapse of floor units or the formation of a column side-sway mechanism in a major earthquake. These undesirable failure mechanisms were unable to be modelled using existing structural analysis programmes as there is currently no satisfactory analytical model that can accurately predict the elongation response of RC plastic hinges. Consequently, the effect of elongation on the seismic response is generally overlooked and the design rules often neglect this action.

Extensive experimental studies on the seismic behaviour of RC beams have been carried out at the University of Auckland over the last three decades. Tests have shown that plastic hinges in ductile RC beam designed according to the New Zealand Concrete Structures Standard (NZS3101:1995) typically elongate between 2 and 5 percent of the beam depth before strength degradation occurs. It was found that elongation response differs significantly between two different types of plastic hinges, namely uni-directional and reversing plastic hinges (Fenwick and Megget 1993). Uni-directional plastic hinges may develop in beams where gravity actions dominate over seismic actions, causing the maximum positive and negative moment to occur at different locations. Reversing plastic hinges develop when seismic actions dominate over gravity actions, causing the maximum positive and negative moment to occur at the same location, which is generally next to the column face.

A satisfactory method to predict elongation in uni-directional plastic hinges has been developed (Megget and Fenwick 1989). However, this cannot be applied to reversing plastic hinges where the behaviour is more complex. Empirical equations to predict elongation in reversing plastic hinge were first proposed by Fenwick (Fenwick and Megget 1993). More advanced theory to relate elongation with strain and rotation was then proposed by Lee (Lee and Watanabe 2003). Elongation of a beam

coupled to a floor unit was also introduced (Matthews et al. 2004). While some of these theories predict the elongation behaviour satisfactorily, they cannot be readily incorporated into analysis programmes. An analytical elongation model for RC beams was first proposed by Lau (Lau et al. 2003), but it had limited success in predicting the observed behaviour. Another elongation model for RC beams containing prestressed tendons has also been proposed (Kim et al. 2004). However this could not be applied to predict elongation of plastic hinges in monolithic moment resisting frames where the behaviour is more complex.

With this background, a project has been initiated at the University of Canterbury aiming to develop a suitable plastic hinge model that can predict the combined flexural, axial load and elongation response of reversing plastic hinge. This paper describes the development of this plastic hinge model and it compares the analytical predictions with experimental results obtained from beam tests under different levels of axial load.

2 EXPERIMENTAL SETUP

Experimental results were extracted from a series of beam tests carried out at the University of Auckland (Fenwick et al. 1981; Issa 1997; Matti 1998). The typical test setup and beam configuration are illustrated in Figure 1. Six tests with axial force of 0kN, 100, 200 and 500kN in compression and 75 and 125kN in tension are considered in this paper. The material properties, yield displacement and theoretical flexural strength obtained from these tests, together with the calculated tensile strength and Young's modulus of concrete, are summarised in Table 1. Here, f_y is the reinforcement yield stress, f'_c and f_t are the compressive and tensile strengths of concrete, M_n is the theoretical nominal flexural strength of the beam and E_c is the Young's modulus of concrete. The value for f_t and E_c are calculated based on the compressive strength of concrete as recommended in NZS3101:1995.
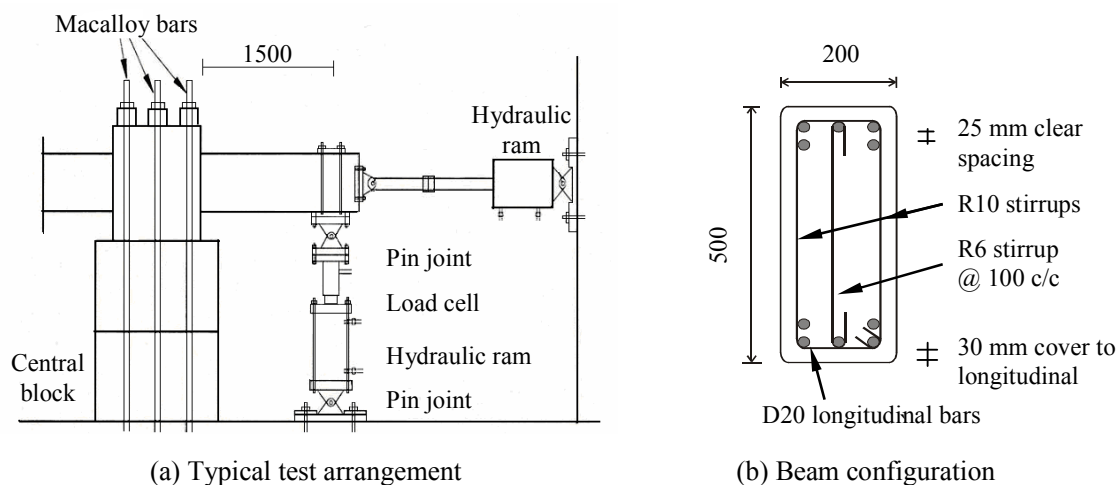

(a) Typical test arrangement   (b) Beam configuration

Figure 1. Experimental setup and beam configuration

Table 1. Summary of measured and calculated properties for selected beam tests

Test	Axial Force (kN)	f_y (MPa)	f'_c (MPa)	Yield Displacement (mm)	M_n (kNm)	Calculated f_t (MPa)	Calculated E_c (GPa)
2A	0	306	37.6	8	202	2.21	27.3
S2A	-100*	332	37.8	9.3	229	2.21	27.3
M1	-200*	318	29.4	9.3	238	1.95	24.9
S1B	-500*	332	37.0	9.1	305	2.19	27.1
M2	75	318	29.4	9.1	193	1.95	24.9
I1B	125	321	40.0	8	181	2.28	27.9

* Negative implies compression force

The loading sequence is illustrated in Figure 2. It started with two elastic cycles where the loading was initially force-controlled. A maximum force corresponding to 75% of the calculated theoretical nominal flexural strength of the beam, M_n, was applied in each direction. From these cycles, the yield displacement corresponding to a displacement ductility of 1, D1, was assessed. The loading history after these elastic cycles became displacement-controlled. In general, two cycles at displacement ductility of two, D2, followed by two cycles at displacement ductility of four, D4, and two cycles at displacement ductility of six, D6, were applied to the beam.
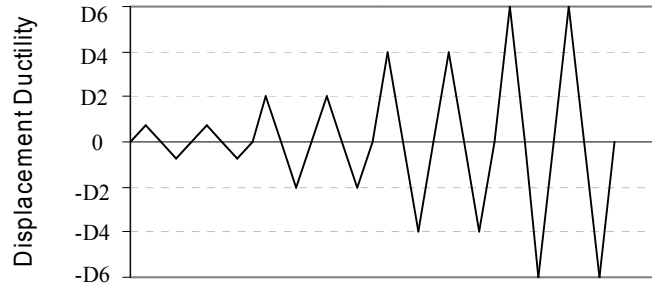
Figure 2. Typical loading history applied in the beam tests

3 MECHANICS OF ELONGATION WITHIN REVERSING PLASTIC HINGES

Elongation within the reversing plastic hinges arises due to two main factors: i) unrecoverable extension of the compression reinforcement and ii) plastic extension of the tension reinforcement due to inelastic rotation (Fenwick and Megget 1993). This is illustrated in Figure 3 where the extension of the top and bottom reinforcement measured over the plastic hinge region is plotted for the first half cycle at displacement ductility of six for beam 2A.
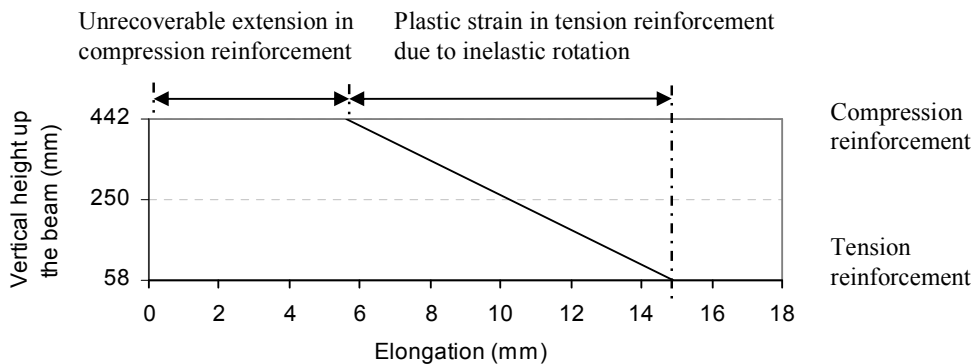
Figure 3. Reinforcement extension in plastic hinge region at first D6 cycle in specimen 2A

The unrecoverable extension of the compression reinforcement observed in Figure 3 was found to arise due to two main actions. Firstly, intersecting diagonal cracks in the plastic hinge region destroy the shear resistance of concrete (i.e. $v_c = 0$). Consequently, truss like actions, illustrated in Figure 4a, developed in the plastic hinge where the shear force is solely resisted by the shear reinforcement and the diagonal compression struts in the web. In this figure, T and C are the flexural tension and compression forces in the reinforcement, θ is the angle of the diagonal struts measured from the longitudinal reinforcement and V is the shear force acting in the beam. It can be seen from Figure 4a that at a given section, the flexural tension force in the reinforcement is always greater than the flexural compression force due to the horizontal component of the diagonal compression forces in the web. Consequently, inelastic rotation in the plastic hinges is resisted predominately by yielding of the tension reinforcement rather than by yielding of the compression reinforcement. Secondly, aggregate particles around the tension reinforcement become dislodged at the crack surface. These aggregate

particles restrain the closure of the cracks when subjected to compression as shown in Figure 4b.
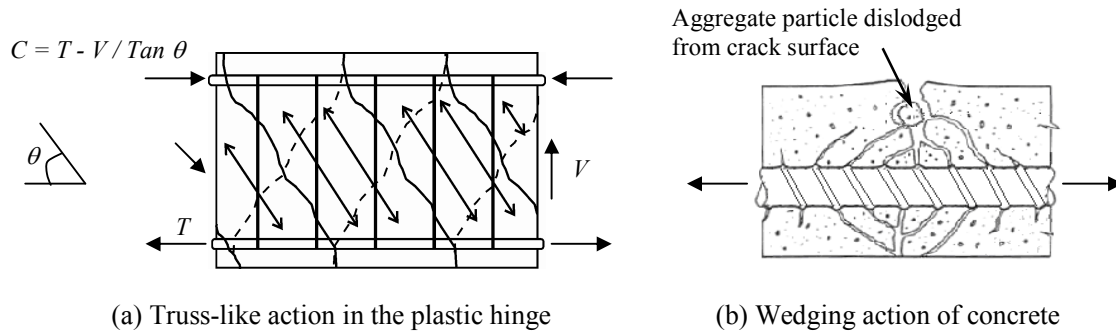

$C = T - V / Tan\ \theta$

θ

T

V

Aggregate particle dislodged from crack surface

(a) Truss-like action in the plastic hinge   (b) Wedging action of concrete

Figure 4. Mechanisms associated with unrecoverable extension in compression reinforcement

4 PLASTIC HINGE MODEL

The analytical model is incorporated into a non-linear time history analysis program, RUAUMOKO, (Carr 2007) and is illustrated in Figure 5a where Δ and P are the applied displacement and applied axial force respectively, L is the length of the beam and L_P is the length of the plastic hinge model. The beam is divided into two parts, namely an elastic region and a plastic hinge region. The elastic region is modelled using the existing Giberson beam element in RUAUMOKO. The plastic hinge region is modelled by a series of axial spring elements connected between rigid links at two ends as illustrated in Figure 5b. It can be seen that the plastic hinge model consists of two longitudinal concrete spring elements located at the centroid of cover concrete to represent un-confined concrete, eight longitudinal concrete spring elements distributed evenly between the reinforcement to represent confined concrete and two steel springs located at the centroid of the top and bottom reinforcement to represent reinforcing bars. These longitudinal springs are used to represent the flexural behaviour of the plastic hinge. In addition, two diagonal concrete spring elements are inserted to represent the diagonal compression forces in the web, which resist the shear force.

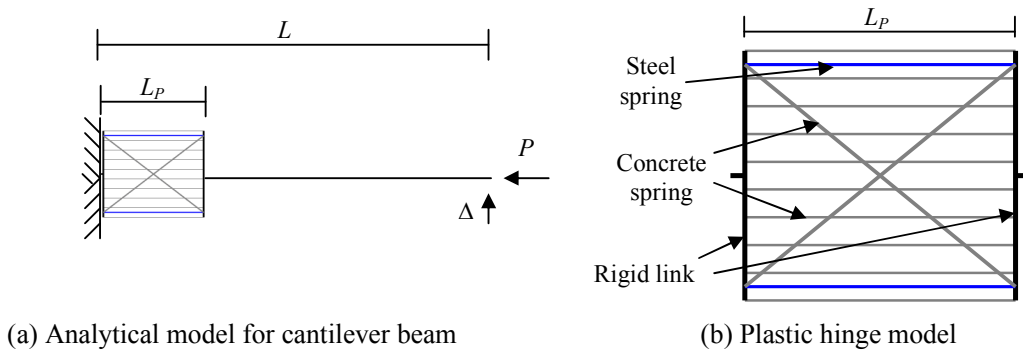(a) Analytical model for cantilever beam   (b) Plastic hinge model

Figure 5. Analytical model for RC beam

The concrete spring elements employed in the plastic hinge is based on Maekawa concrete hysteric model developed in University of Tokyo (Maekawa et al. 2003). This model uses path-dependent averaged stress-strain relationship, which consists of compression/tension envelopes with unloading and reloading loops. The compression envelope is based on Elasto-Plastic Fracture model and the tension envelope is based on a tension stiffening model. The unloading loop from tension envelope takes into account the contact stress effect where axial compression stress develops before the tensile strain reverses to zero. This arises due to wedging action of dislocated aggregate particles in the cracks as described earlier. The steel spring elements are based on steel hysteric model proposed by Dhakal (Dhakal and Maekawa 2002a,b). It uses path-dependent averaged stress-strain relationship, which consists of tension/ compression envelopes with unloading and reloading loops. The tension envelope has the option of using Mander or Rodriguez strain hardening profile. The unloading and reloading loops are based on Giuffre-Menegotto-Pinto Model which takes into account the Bauschinger effect.

The length of plastic hinge model, L_P, in Figure 5 was calculated using Equation 1, where d-d' is the distance between the centroid of reinforcing bars, θ is the angle of the diagonal struts, V_{yc} is the shear force corresponding to the theoretical flexural strength of the beam, M_{yc}, where the compression steel has been previously yielded in tension, V_c is the shear resistance of concrete and is assumed to be zero in this paper, s is the spacing of the shear reinforcement, A_v is the area of the shear reinforcement and f_{vy} is the yield stress of the shear reinforcement. This length is calculated to represent the inclination of the diagonal compression struts in the plastic hinge region as illustrated in Figure 6.

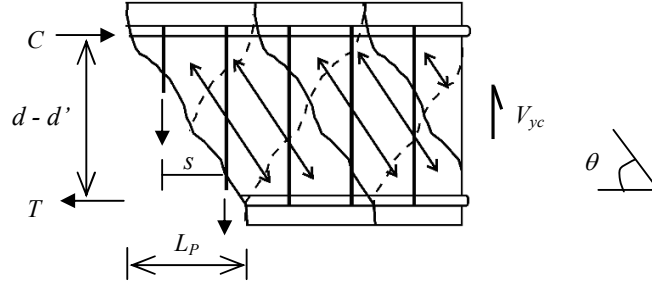$$L_P = \frac{d-d'}{\tan\theta} = \frac{(V_{yc}-V_c)s}{A_v f_{vy}} \qquad (1)$$

Figure 6. Analytical model for RC beam

The actual length over which the reinforcement yields, L_{yield}, is given by Equation 2, where L is the length of shear span, $M_{\max}$ is the maximum moment sustained in the beam, L_t is the length for tension shift effect and L_e is the length for yield penetration of reinforcement into the supporting member. For a beam with no axial force, L_t is approximately equal to $(d$-$d')/2$ (Fenwick and Dhakal). This is based on the assumption that the diagonal crack extends over a distance d-d' along the member at the low moment end of the plastic hinge. For beam with axial compression force, the diagonal crack angle would decrease and the length of tension shift would increase. Unfortunately, the relationship between the crack angle and the applied axial force in the high moment end of plastic hinges is not available in the literature. To estimate the length of tension shift under different levels of axial force, the diagonal crack angle is calculated using principle stress method assuming un-cracked concrete section in the web. In the experiment, two additional deformed bars were welded onto the longitudinal bars in the supporting column to prevent yield penetration. Therefore, the yield penetration length in this paper was estimated as two times the diameter of the longitudinal bar. To take into account the difference in lengths, L_P and L_{yield}, the length of steel spring in the plastic hinge model was set as L_{yield} to give the correct stiffness and strain hardening rate for the reinforcement. Values for L_P and L_{yield} are summarised in Table 2.

$$L_{yield} = L\frac{M_{\max} - M_{yc}}{M_{\max}} + L_t + L_e \qquad (2)$$

Table 2. Summary of plastic hinge and reinforcement yield length for the selected beam tests

Test	Shear Reinforcement Arrangement	f_{vy} (MPa)	M_{yc} (kNm)	V_{yc} (kN)	L_P (mm)	$M_{\max}$ (kNm)	L_t (mm)	L_{yield} (mm)
2A	2R10 + R6 @100c/c	298[1] 357[2]	182	121	213	217	192	516
S2A	2R10 + R6 @100c/c	344[1] 391[2]	219	146	224	265	254	596
M1	3R6 @55c/c	377[2]	230	153	263	287	317	697
S1B	2R10 + R6 @100c/c	344[1] 391[2]	296	197	303	323	519	726
M2	3R6 @55c/c	377[2]	177	118	203	228	151	569
I1B	3R6 @55c/c	331[2]	170	113	221	233	130	618

[1] Yield stress for R10 stirrup   [2] Yield stress for R6 stirrup

It should be noted that the current model is set up to predict flexural, axial load, elongation response and shear deformation associated with elongation. However, it does not predict shear deformation due to extension of the stirrups. For beam with no axial force, the shear deformation from extension of stirrups measured at the end of D6 cycles is 12mm. This shear deformation will increase with axial tension force and decrease with axial compression force.

5   ANALYTICAL PREDICTIONS AND COMPARISON WITH EXPERIMENTAL RESULTS

5.1   Beam with No Axial Force

Comparison of the analytical and experimental moment-rotation, force-displacement and elongation behaviour for beam 2A is shown in Figure 7. It can be seen that the analytical predictions match well with the experimental results. The model over-estimates the moment in moment-rotation diagram and under-estimates the pinching behaviour in force-displacement diagram. These discrepancies can be attributed to the exclusion of shear deformation from stirrup extension in the current model.
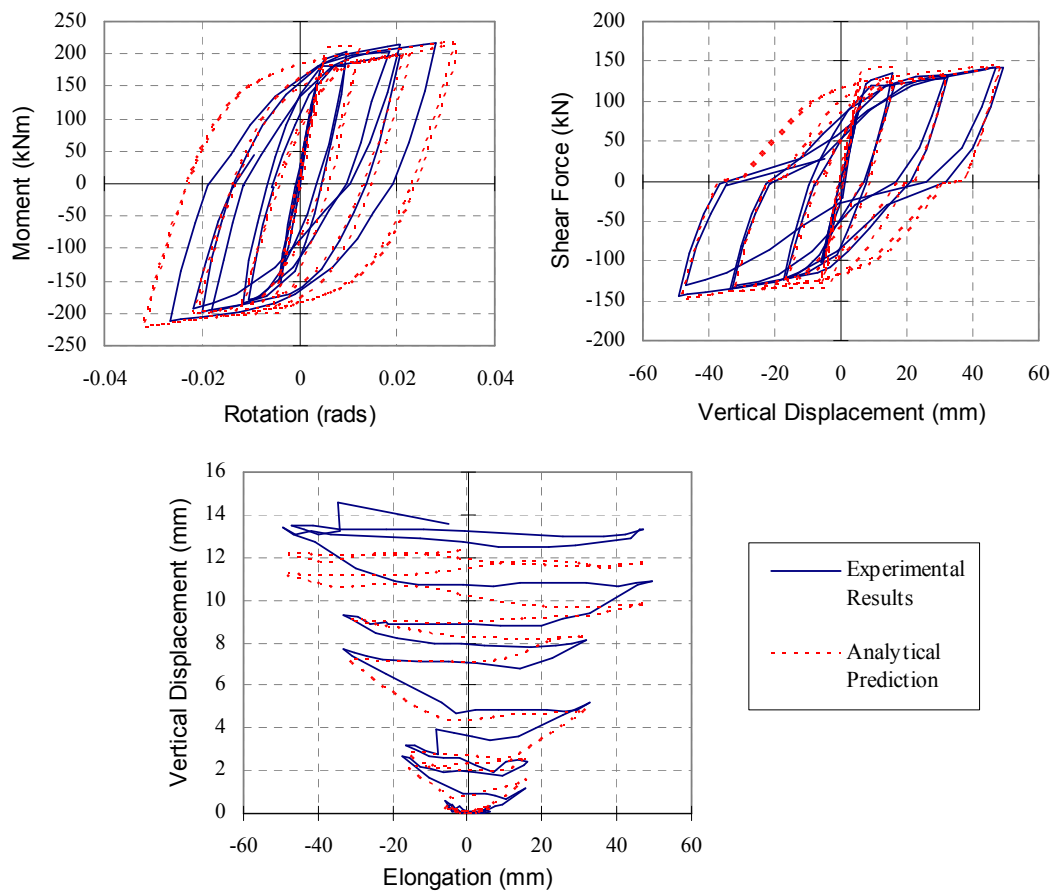

Figure 7. Analytical and experimental comparison for beam 2A (with no axial force)

5.2   Beams with Axial Tension/ Compression Force

It can be seen from Figures 8 and 9 that the analytical predictions for beam I1B and S1B, with 125kN axial tension force and 500kN axial compression force respectively, provide a reasonable match with the experimental results. Again, the model over-estimates the moment in moment-rotation diagram and under-estimates the pinching behaviour in force-displacement diagram due to extension of the stirrups not being captured in the current model. This also resulted in a larger predicted elongation at the last few cycles.
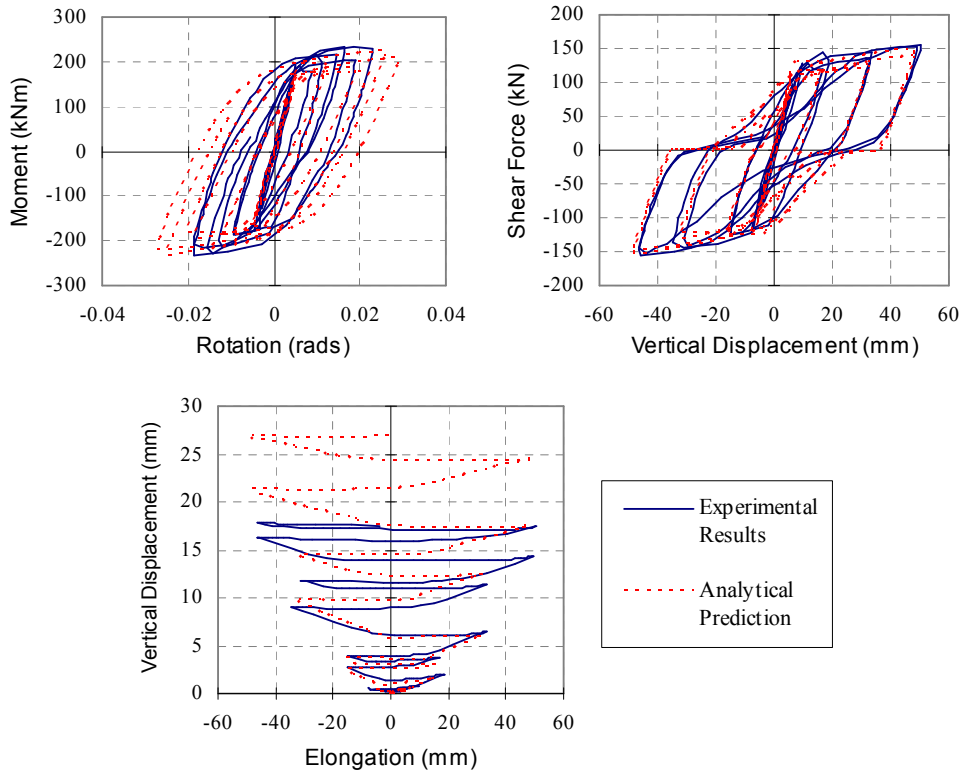
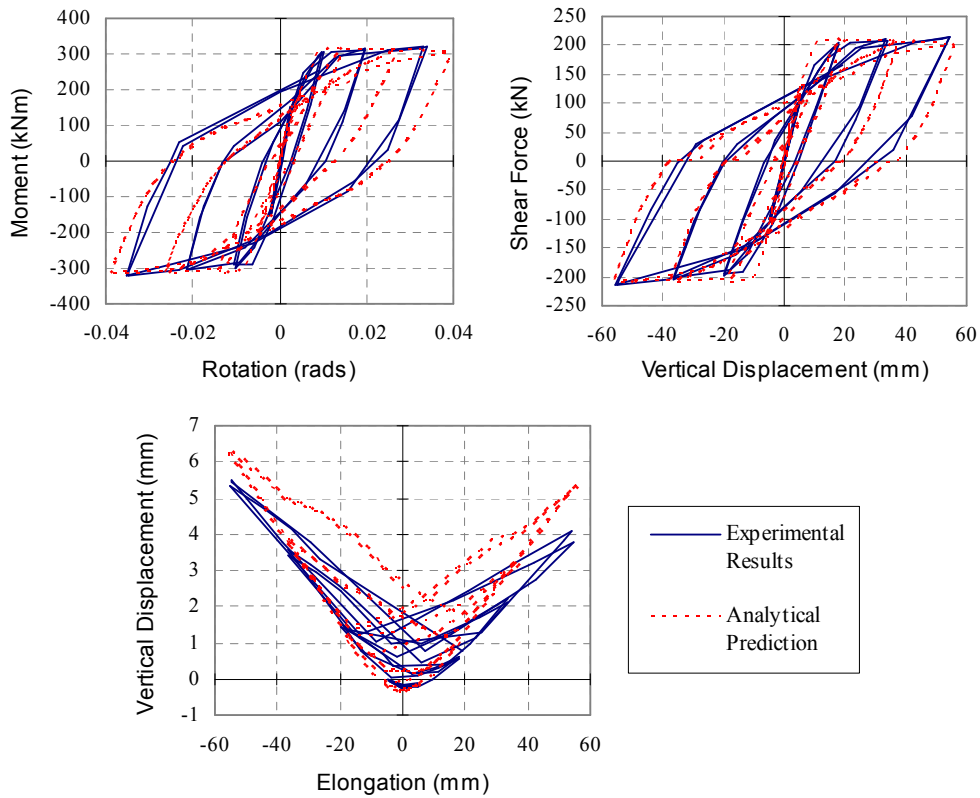Figure 8. Analytical and experimental comparison for beam I1B (with 125kN axial tension force)

Figure 9. Analytical and experimental comparison for beam S1B (with 500kN axial compression force)

5.3 Elongation Summary

Analytical and experimental elongations at peak displacement cycles for the beams are plotted in Figure 10. It can be seen that the analytical elongations are in reasonable agreement with the experimental results. When axial tension force is applied, the elongation is over predicted in the high displacement cycles. This is in part due to the model not allowing for shear deformation from yielding of stirrups. For beams M1 and S2A, the elongation is under predicted in the high displacement ductility cycles. The reason for this discrepancy is not known at this stage.
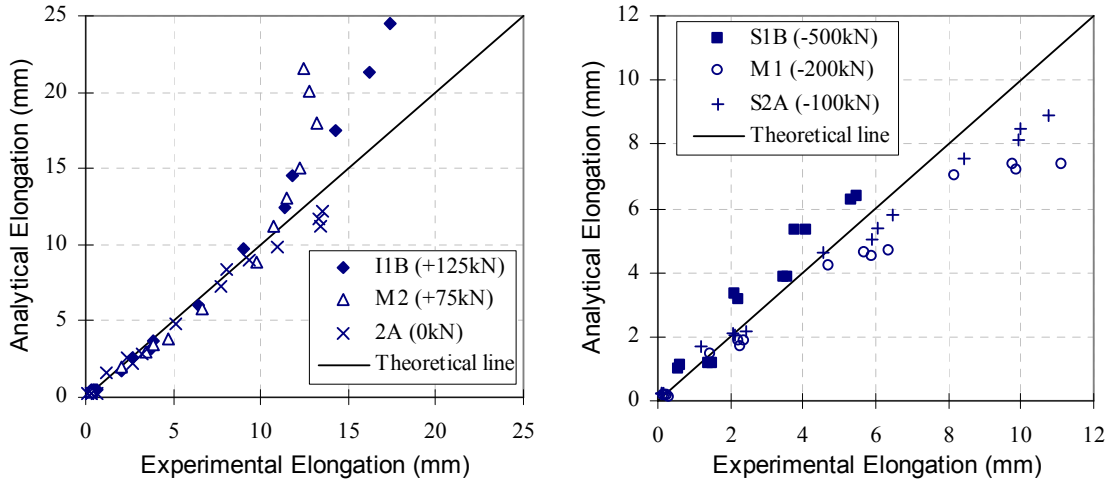
Figure 10. Analytical and experimental elongation comparison at peak displacements

5.4 Discussions

While the current model has shown some promising results, it does have some limitations that required further development:
- Shear deformation associated with extension of the shear reinforcement is not included. In the test where there is axial tension or no axial load, appreciable shear deformation occurs in the high ductility cycles due to inelastic extension of the stirrups. Consequently, a larger rotation is being applied in the analysis which leads to over prediction of elongation.
- For beam with axial compression force, the assumption of $v_c = 0$ for calculating L_P is not accurate as concrete would resist a portion of the shear force. This is currently ignored in the analysis. Consequently, a smaller elongation prediction would be expected if the effect is included.
- Reinforcement yield length increases with increasing ductility cycles. This is not modelled in this analysis and a constant maximum value is used.

6 CONCLUSIONS

A plastic hinge model has been developed and incorporated into a structural analysis package. Comparisons of analytical predictions of flexural, axial force and elongation response with experimental results indicate the model generally behaves well. It is intended to refine the model to allow for shear deformation associated with extension of shear reinforcement and the effect of axial force on the length of plastic hinge at a later stage. This model may be used to assess the significance of elongation on seismic performance of RC buildings.

8

REFERENCES:

Carr, A. J. 2007. *RUAUMOKO2D - Inelastic Dynamic Analysis.* Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

Dhakal, R. P. and Maekawa, K. 2002a. Path-dependent cyclic stress-strain relationship of reinforcing bar including buckling. *Engineering Structures*, Vol 24, Page 1383-1396.

Dhakal, R. P., and Maekawa, K. 2002b. Modeling for postyield buckling of reinforcement. *Journal of Structural Engineering*, Vol 128, No. 9, Septemeber.

Fenwick, R. C., and Dhakal, R. P. 2007. Material strains limits for seismic design of concrete structures. *Journal of the Structural Engineering Society New Zealand*, Vol. 20, No.1, Page 14-28.

Fenwick, R. C., and Megget, L. M. 1993. Elongation and load deflection characteristics of reinforced concrete members containing plastic hinges. *Bulletin of the New Zealand National Society for Earthquake Engineering*, 26(1), 28-41.

Fenwick, R. C., Tankut, A. T., and Thom, C. W. 1981. The deformation of reinforced concrete beams subjected to inelastic cyclic loading: experimental results. *Report No. 268*, Department of Civil Engineering, University of Auckland, Auckland, New Zealand

Issa, M. S. 1997. The deformations of reinforced concrete beams containing plastic hinges in the presence of axial compression or tension load. *Master of Engineering Report*, Department of Civil and Resource Engineering, University of Auckland, Auckland, New Zealand

Kim, J., Stanton J., and MacRae, G. 2004. Effect of beam growth on reinforced concrete frames. *Journal of Structural Engineering ASCE*. Vol. 130, No.9, September.

Lau, D. B. N. 2001. The Influence of Precast-prestressed Flooring on the Seismic Performance of Reinforced Concrete Perimeter Frame Buildings. *Report No. 604*, Department of Civil and Resource Engineering, University of Auckland, Auckland, New Zealand.

Lau, D. B. N., Davidson, B. J., and Fenwick, R. C. 2003. Seismic Performance of RC Perimeter Frames with Slabs Containing Prestressed Units. *Pacific Conference on Earthquake Engineering*, Christchurch.

Lee, J.Y., and Watanabe, F. 2003. Predicting the longitudinal axial strain in the plastic hinge regions of reinforced concrete beams subjected to reversed cyclic loading. *Engineering Structures*, Vol 25(7), 927-939.

Lindsay, R. 2004. Experiments on the Seismic Performance of Hollow-core Floor Systems in Precast Concrete Buildings. *Master of Engineering Thesis,* Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

MacPherson, C. 2005. Seismic Performance and Forensic Analysis of a Precast Concrete Hollow-core Floor Super-assemblage, *Master of Engineering Thesis*, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

Maekawa, K., Pimanmas, A., and Okamura, H. 2003. *Nonlinear Mechanics of Reinforced Concrete*, Spon Press, London.

Matthews, J. 2004. Hollow-core Floor Slab Performance following a Severe Earthquake, *PhD. Thesis*, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

Matthews, J. G., Mander, J. B., and Bull, D. K. 2004. Prediction of beam elongation in structural concrete members using a rainflow method. *New Zealand Society of Earthquake Engineering Conference*, Rotorua.

Matti, N. A. 1998. Effect of axial loads on the behaviour of reversing plastic hinges in reinforced concrete beams, *Master of Engineering Report*, Department of Civil and Resource Engineering, University of Auckland, Auckland, New Zealand.

Megget, L.M. and Fenwick, R.C. 1989, Seismic behaviour of a Reinforced Concrete Portal Frame Sustaining Gravity Loads, *Bulletin of NZ National Society for Earthquake Engineering,* Vol. 22, No. 1, Mar., pp. 39-49.

Standards New Zealand. 1995. *Concrete structures standard NZS 3101*, Standards New Zealand, Wellington, New Zealand.