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ANALYTICAL MODEL OF DUCTILE REINFORCED CONCRETE FRAMES ALLOWING FOR ELONGATION OF PLASTIC HINGES

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ABSTRACT : Elongation in reinforced concrete members can have detrimental effect on the seismic performance of reinforced concrete structures. For reliable assessment of seismic performance, analytical models should take into account this elongation phenomenon. In this paper, an analytical model of plastic hinges has been developed and implemented into an analysis package. This model consists of a layer of horizontal concrete and steel springs to simulate the flexural behavior, as well as diagonal springs to represent the diagonal compression struts and to resist the shear force. The model is verified against experimental results of beams and frame subassembly tests found in literature. Comparisons of the analytical predictions and the experimental results show that this model can make significant advancement in predicting elongation in beam plastic hinges. It can be used to assess the effect of beam elongation on the seismic performance of reinforced concrete frame structures.

KEYWORDS: Reinforced concrete frame, Beam elongation/growth, Plastic hinges, Seismic response.

1. INTRODUCTION

Research in New Zealand over the last three decades has shown that plastic hinges in ductile reinforced concrete (RC) members elongate and sustain significant shear deformation when subjected to inelastic cyclic loading. In general, elongations between 2 and 4 percent of the beam depth and shear deformations between 30 and 50 percent of the total displacement had been observed to occur prior to strength degradation in beams with no axial load [1, 2]. Elongation of this order can significantly change the moment and shear force distribution and increase the column deformation demand in the lower storey of a structure. Partial restraint to elongation by floor slabs can also significantly increase the strength of the beams. Consequently, undesirable failure modes such as columns-sway mechanism or unseating of precast floor units from the supporting beams may occur in the event of a major earthquake.

Elongation response differs significantly between two different forms of plastic hinges, namely uni-directional and reversing plastic hinges [1]. The resulting form of plastic hinges depends on the contribution of the gravity and seismic moment and the distribution of the top and bottom reinforcement in the beam. Uni-directional plastic hinges may develop in gravity dominated frame where the maximum positive and negative moments occur at different locations. Whereas reversing plastic hinges may develop in seismic dominated frame where the maximum positive and negative moments occur at the same location, which is generally next to the column face. In this study, the reversing plastic hinge, which is more common in ductile moment frames, is considered.

Previous study has shown that elongation in the reversing plastic hinges arises due to plastic extension of the tension reinforcement from inelastic rotation and unrecoverable extension of the compression

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reinforcement [1]. This is illustrated in Figure 1 where the extension of the top and bottom reinforcement over the plastic hinge region measured in an experiment at first cycle of 2.5% drift is plotted [3]. The unrecoverable extension of the compression reinforcement was found to arise due to two main actions. i) Wedging action of aggregate particles. Aggregate particles become dislodged from crack surfaces around the tension reinforcement which prevent the crack to close fully when the load is reversed. This phenomenon is also known as 'contact stress effect'. ii) Truss like action developed in the plastic hinge where the shear force is carried by diagonal compression struts and shear reinforcement crossing the diagonal cracks. In this case the plane sections do not remain planes and standard flexural theory does not apply. To satisfy force equilibrium at a given section, the flexural tension force in the reinforcement is always greater than the flexural compression force due to the horizontal component of the diagonal compression force. Consequently, inelastic rotation in the plastic hinge occurs predominately by yielding of the tension reinforcement.

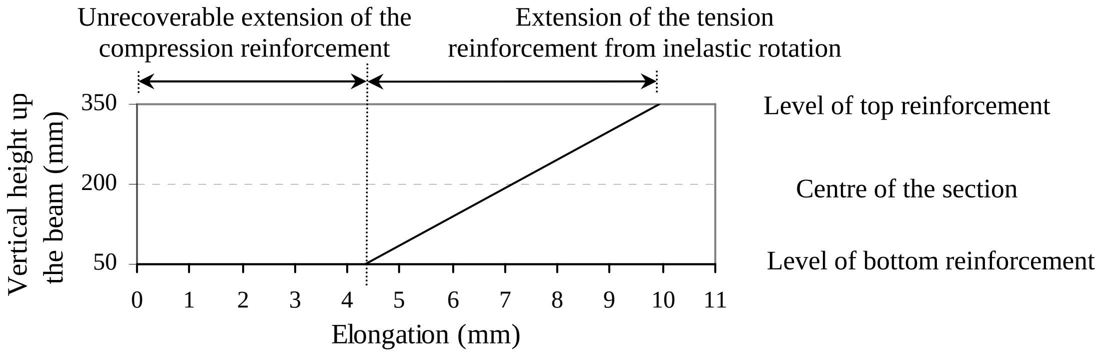

Figure 1. Reinforcement extensions in plastic hinge region at first cycle of 2.5% drift

Methods of predicting elongation in beams has been proposed in a number of studies [1, 4, 5]. However, these methods cannot be readily incorporated into time-history analysis programs to evaluate the effect of elongation on the seismic response of frames. Several analytical models have been proposed [6, 7]. However, these have had limited success in predicting observed behavior. As there is currently no accurate analytical model to predict elongation response of plastic hinges, its effect on seismic performance of RC structures is generally overlooked in the design and analyses. This paper describes a plastic hinge model that can be used to predict elongation of the plastic hinges. The model is verified against experimental beam and frame sub-assembly tests.

2. PROPOSED PLASTIC HINGE MODEL

The development of the plastic hinge model is based on the elongation mechanism described above. The model is incorporated into a non-linear time history analysis program, RUAUMOKO [8]. The plastic hinge is modeled by a layer of longitudinal and diagonal axial springs connected between rigid links at two ends as illustrated in Figure 2. The longitudinal springs are used to represent the flexural behavior and the diagonal springs are used to represent the diagonal compression struts in the web. The concrete spring is adopted from Maekawa hysteretic model [9]. It consists of compression/tension envelopes with unloading and reloading loops. The compression envelope is based Elasto-Plastic Fracture model and the tension envelope is based on a tension stiffening model. The unloading loop from tension envelope includes an allowance for contact stress effect where axial compression stress develops before the tensile strain reverses to zero. The steel spring is based on Dhakal steel hysteretic model [10]. It consists of tension/compression envelopes, based on Mander's strain hardening profile, with unloading/reloading loops based on Giuffre-Menegotto-Pinto model, which includes an allowance for Bauschinger effect.

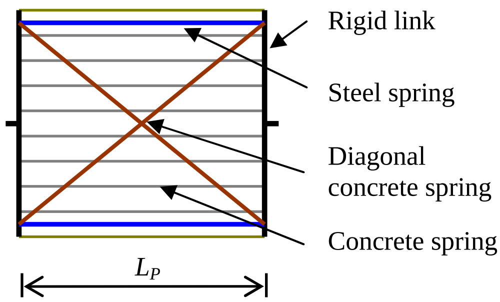

Figure 2. Schematic diagram of plastic hinge model

The length of plastic hinge model, L_p, in Figure 2 was chosen to represent the inclination of the diagonal struts in the plastic hinge as illustrated in Figure 3. This parameter is important as it controls the level of horizontal component of the diagonal force that contributes to the section equilibrium. The expression for L_p is given in Equation 1 where *d-d'* is the distance between the reinforcement centroids, θ is the angle of diagonal strut from the horizontal plane, V_{yc} is the shear force corresponding to the flexural strength of the beam, M_{yc} (given by Equation 2 where L is the shear span), V_c is the shear resistance of concrete, s is the spacing of the stirrups, A_s, A_v, f_y and f_{vy} are the area and yield stress of the longitudinal and shear reinforcement respectively. In this paper, concrete in the plastic hinge region is assumed to have no shear resistance therefore V_c is zero. Note that *(d-d')* is used as the lever arm for calculating the flexural strength because under cyclic loading, concrete is unlikely to resist any compression force (unless the compression bars yield back and the cracks close fully). Thereby rendering the compression reinforcement to carry all compression force required to balance the tensile force.

$$L_P = \frac{d-d'}{\tan\theta} = \frac{(V_{yc}-V_c)s}{A_v f_{vy}} \tag{1}$$

$$M_{yc} = V_{yc}L = A_s f_y (d-d') \tag{2}$$

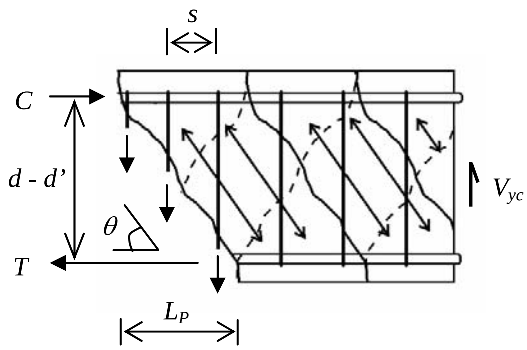

Figure 3. Force equilibrium across a diagonal crack

The actual length over which the reinforcement yield, L_{yield}, is given by Equation 3 where M_{max} is the peak moment that the beam can sustain, L_t is the length of tension shift and L_e is the length of yield penetration into the support. For beams with no axial force, the length of tension shift is approximately equal to *(d-d')/2* [11]. To take into account the difference in lengths, L_{yield} and L_p, the length of steel spring in the plastic hinge model was set as L_{yield} to give the correct stiffness and strain hardening rate.

$$L_{yield} = L\frac{M_{max}-M_{yc}}{M_{max}} + L_t + L_e \tag{3}$$

Parametric study was carried out to determine the sensitivity of some of the parameters used in the model. It was found that the result is very sensitive to the step size but not to the mesh discretization. Parameters such as L_{yield} and L_p also have a major influence on the overall response. The area or the stiffness of the diagonal spring has little influence to the elongation response. This is because the deformation of the struts only contributes to a small portion of the total displacement.

3. VERIFICATION OF THE MODEL

3.1. Cantilever Beam Tests

Experimental results were extracted from a series of beam tests carried out at the University of Canterbury and the University of Auckland [2, 3]. Typical test setup is shown in Figure 4. The beams have equal top and bottom longitudinal reinforcement. To prevent yield penetration of the reinforcement into the support, additional bars were welded to the longitudinal reinforcement. Cyclic displacement was applied at the end of the beam. In general, two elastic cycles were applied initially to determine the yield displacement, D1. Then, two cycles at displacement ductility of two, D2, followed by two cycles at displacement ductility of four, D4,

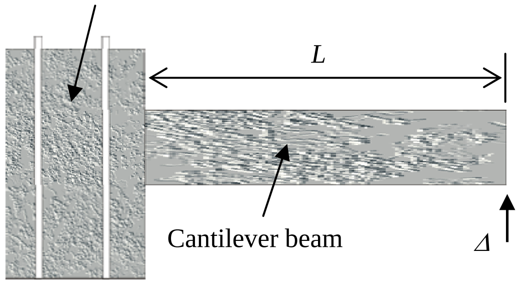

Figure 4. Test set up

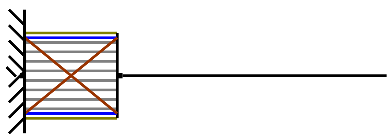

Figure 5. Analytical beam model

and four cycles at displacement ductility of six were applied to the beams. The measured material properties and beam configuration is summarized in Table 1 where *b* and *h* are the width and depth of the section, *d'* is the cover distance to the centroid of the longitudinal bars, *L* is the span length and f_u is the ultimate stress of the longitudinal bars. Additional material properties and calculated plastic hinge properties are summarized in Table 2 where ε_y, ε_{sh} and ε_u are the strains at yield, strain hardening and ultimate respectively and f'_c is the concrete compressive strength.

Table 1. Beam configuration and material properties

Test	*b* (mm)	*h* (mm)	*d'* (mm)	*L* (mm)	*Flexural steel*	Stirrups arrangement	f_{vy} (MPa)	f_y (MPa)	f_u (MPa)
2A	200	500	58	1500	5D16	2R10 + R6 @ 100c/c	298(1) 357(2)	306	459
S1A	200	500	58	1500	5D16	2R10 + R6 @ 100c/c	344(1) 391(2)	331.6	476
1A	200	500	58	1500	5D16	2R10 + R6 @ 100c/c	298(1) 357(2)	311	460
1B	200	500	58	1500	5D16	2R10 + R6 @ 100c/c	298(1) 357(2)	311	460
AA1	250	400	50	1420	3D25	HR10 @ 175c/c	445	350	525
AA2	250	400	50	1420	3D25	HR10 @ 100c/c	445	350	525

(1) Yield stress for R10 stirrup (2) Yield stress for R6 stirrup

The analytical beam model is illustrated in Figure 5. It can be seen that the beam is divided into a plastic hinge region and an elastic region. The plastic hinge region is modeled using the plastic hinge element developed in this research and the elastic region is modeled using an elastic Giberson element.

Table 2. Calculated plastic hinge parameters

Test	ε_{sh} / ε_y	ε_u / ε_y	f'_c (MPa)	M_{yc} (kNm)	L_p (mm)	M_{max} (kNm)	L_{yield} (mm)
2A	13	130	37.6	185	220	217	463
S1A	14	62	37	200	210	233	452
1A	14	130	33.2	188	220	233	532
1B	14	130	42.1	188	220	227	498
AA1	10	80	41.5	155	280	170	328
AA2	10	80	42.2	155	160	179	390

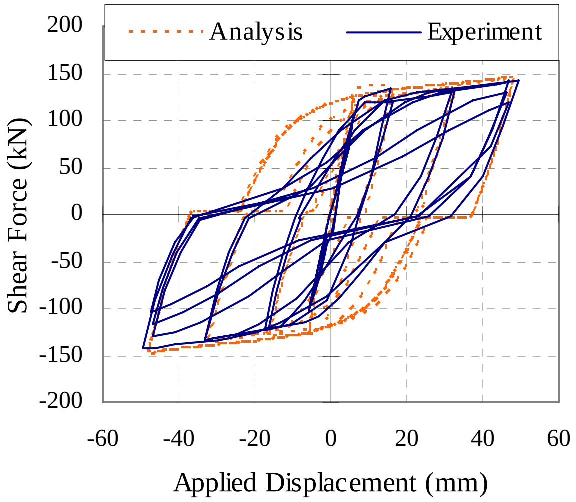

Figure 6. Force-displacement relationship

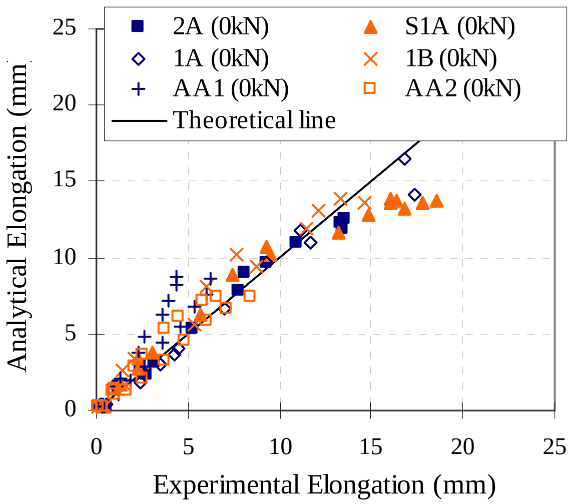

Figure 7. Elongation histories

The comparison of force-displacement relationship for Beam 2A is plotted in Figure 6. It can be seen that the model predicts the initial elastic stiffness and the peak force well. It does not consider either strength degradation due to buckling of reinforcing bars or shear deformation due to yielding of stirrups. Consequently, the strength degradation as well as some pinching behavior due to shear deformation is not captured. It should be noted that shear deformation arising from elongation is included in the model. It was found that this contributes to about half of the total shear deformation before strength degradation occurs. The analytical and experimental elongation histories are compared in Figure 7. It can be seen that the analytical elongation generally matches well with the experiment.

3.2. 2D Frame Test

The test was conducted by Lau [7] at the University of Auckland. The test set up is shown in Figure 8. The columns were designed to remain elastic throughout the test. Loading was displacement controlled; displacements were applied at the top and bottom of each column through hydraulic actuators. Two cycles at ±0.2%, ±0.35%, ±0.5%, ±1%, ±1.5%, ±2%, ±2.5% and ±3% drifts were applied. The applied displacement was corrected for beam elongation so the columns would remain parallel and the axial force in the beams would be minimized. The measured material properties, beam dimension and calculated plastic hinge properties are summarized in Table 3 and Table 4.

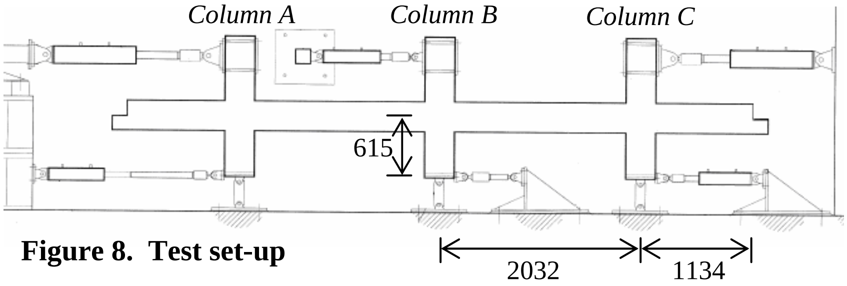

Figure 8. Test set-up

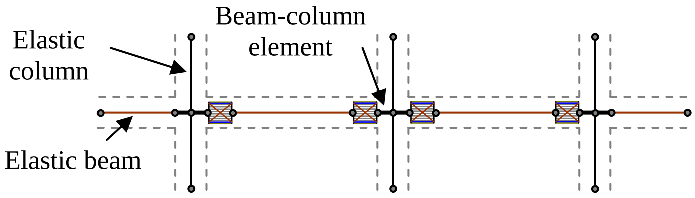

Figure 9. Analytical 2D frame model

Table 3. Beam configuration and material properties

Member	*B* (mm)	*h* (mm)	*d'* (mm)	*L* (mm)	*Flexural steel*	Stirrups arrangement	f_{vy} (MPa)	f_y (MPa)	f_u (MPa)
Beam	130	300	27.5	866	3D12	3R6 @ 65c/c	364	315	442

Table 4. Calculated plastic hinge parameters

Test	ε_{sh} / ε_y	ε_u / ε_y	f'_c (MPa)	M_{yc} (kNm)	L_p (mm)	M_{max} (kNm)	L_{yield} (mm)
Beam	18	159	26.1	26.2	64	32.9	569

The analytical model is illustrated in Figure 9. The columns, elastic beams and the beam-column joints are modeled using elastic Giberson element. Shear deformation in all of the elastic members is suppressed. The effective moment of inertia is taken as the cracked section moment of inertia given by the NZ Concrete

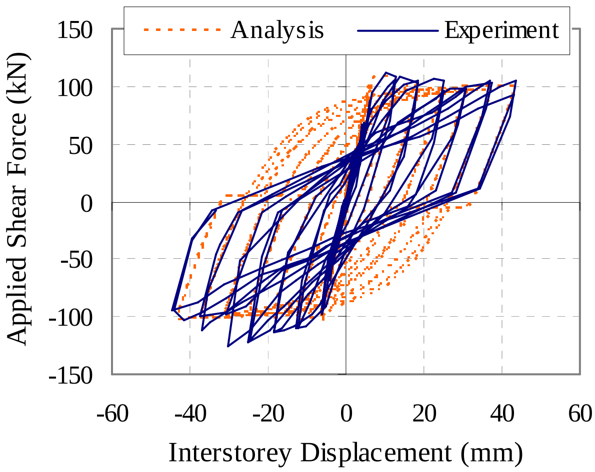

Figure 10. Force-displacement of the overall frame

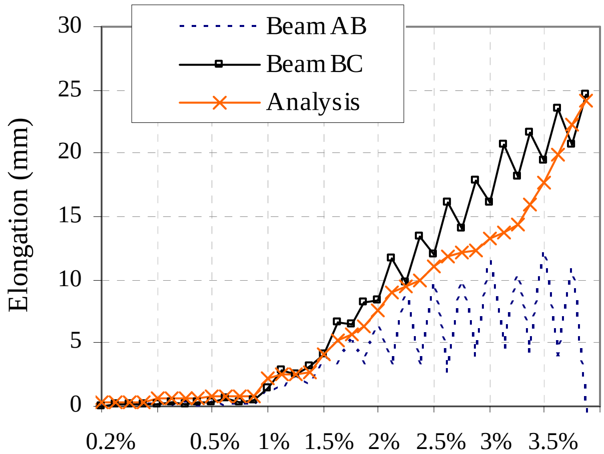

Figure 11. Elongation comparison

Structures Standard [12]. The effective moment of inertia of the beam-column joint is assumed to be twice that of the elastic beam. Rotation is applied to the centre of each beam-column joint as the input displacement.

Analytical and experimental force-displacement and elongation comparisons are shown in Figure 10 and Figure 11. It can be seen that the model predicts the initial stiffness and the peak force well; however the pinching is underestimated as shear deformation due to yielding of stirrups in the plastic hinges as well as bond slip in the beam-column joints are not considered in the model. Elongation matches well with the experiment till the end of 1.5% drift. The analytical elongation is larger than the experimental elongation of *Beam AB* but slightly smaller than that of *Beam BC*. This is because in the experiment, *Beam AB* was under large axial compression force which would have restricted the growth whereas *Beam BC* was under minor axial tension force which would have enhanced elongation slightly.

4. CONCLUSIONS

A plastic hinge model has been developed and implemented into a time-history analysis program to predict elongation in the plastic hinges. The comparisons of the analyses with experimental beams and 2D frame tests have shown that the model predicts elongation response satisfactorily. This model can be used to assess the significance of elongation on the seismic performance of RC frame buildings.

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