Analytical Model on Beam Elongation within the Reinforced Concrete Plastic Hinges

2007 NZSEE Conference

B.H.H. Peng, R.P. Dhakal, R.C. Fenwick and A.J. Carr

Department of Civil Engineering, University of Canterbury, Christchurch

D.K. Bull

Department of Civil Engineering, University of Canterbury, Christchurch; Holmes Consulting Group

ABSTRACT: Research in New Zealand over the last two and a half decades has shown that elongation in plastic hinge regions can have a very significant effect on the seismic performance of reinforced concrete buildings. It was found that elongation arises due to (i) plastic strain in reinforcing bars caused by inelastic rotation, and (ii) unrecoverable tensile strains in compression reinforcement. A number of empirical formulas have been proposed to predict elongation. However, there are currently no analytical models that can accurately predict elongation in the plastic hinges. Consequently, the influence of elongation on the seismic performance of reinforced concrete structures cannot be predicted analytically. This paper describes a plastic hinge model for the computer analysis package, 'RUAUMOKO', which can predict both the flexural and elongation response of plastic hinges in reinforced concrete beams. Experimental and analytical results are compared in the study. With the completion of this newly formed element, seismic analyses may be made of structures containing potential plastic hinges to assess the significance of elongation on seismic performance.

1 INTRODUCTION

Recent experimental studies carried out at the University of Auckland (Lau 2001) and at the University of Canterbury (Lindsay 2004; MacPherson 2005; Matthews 2004) have highlighted problems associated with reinforced concrete moment resisting frames containing precast prestressed concrete floor units. It was found that undesirable failure mechanisms, such as premature collapse of floor units or the formation of a column side-sway mechanism, might occur in a severe earthquake. The tests indicated that these undesirable failure mechanisms were primarily due to elongation that occurs in the plastic hinges and the interaction of this elongation with floors containing precast prestressed units. To develop design rules for ductile concrete buildings and to analyse unusual structures it is highly desirable that elongation in beam plastic hinges can be modelled.

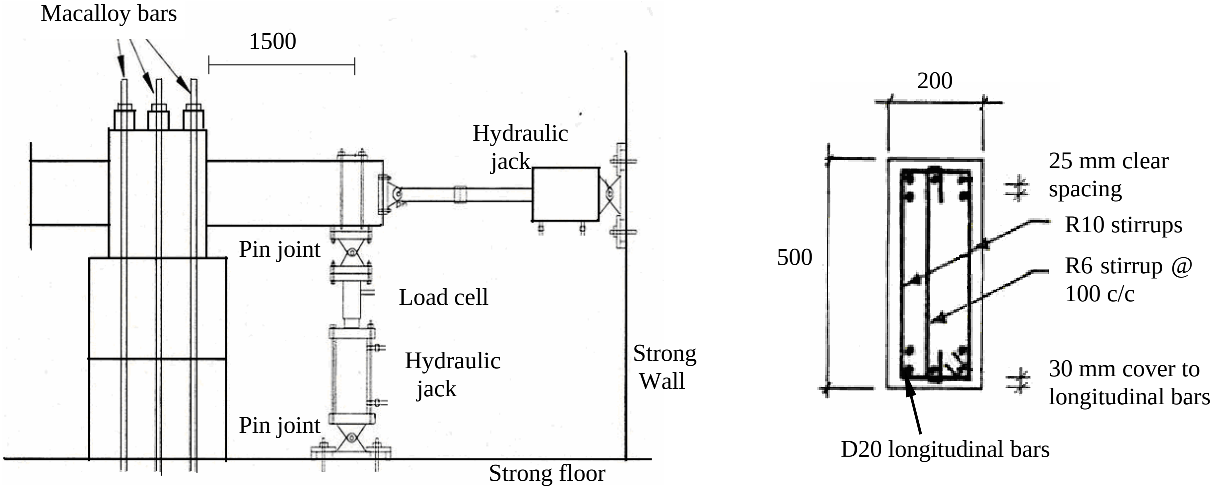
Extensive experimental studies on the seismic behaviour of reinforced concrete beams have been carried out over the last two and half decades. Research has found that elongation occurs in reinforced concrete beams as tensile strains are greater than compression strains. With the formation of plastic hinges the magnitude of elongation greatly increases. There are two different forms of plastic hinge, namely reversing and unidirectional plastic hinges (Fenwick and Megget 1993). In this study the reversing plastic hinge, which is the most common form, is considered. This form of plastic hinge occurs where seismic actions dominate over gravity load actions. A satisfactory method of predicting elongation in unidirectional plastic hinges has previously been developed (Megget and Fenwick 1989) and incorporated in time history analyses of structures (Fenwick and Davidson 1993). However, this does not hold for reversing plastic hinges where the behaviour is more complex.

Tests have shown that plastic hinges in ductile beams detailed to comply with NZS 3101 typically elongate between 2 and 5 percent of the beam depth before strength degradation occurs. Empirical formulae that could be used to predict elongation were first proposed by (Fenwick and Megget 1993). More advanced micro-mechanics theory was subsequently proposed to predict elongation response of beams (Lee and Watanabe 2003). The rainflow method where elongation of a beam coupled to a floor slab was also introduced (Matthews et al. 2004). While some of these theories predict the elongation behaviour reasonably well, they cannot be readily incorporated into analysis programs.

As there is currently no satisfactory analytical model that can predict the flexural, shear and elongation response of plastic hinges, the effects of elongation on seismic response are generally overlooked. An existing analytical elongation model for reinforced concrete beams has been proposed (Lau et al. 2003), but it had limited success in predicting the observed behaviour. With this background, a research project has been started at the University of Canterbury with the objective of developing a suitable plastic hinge model that can predict the flexural, axial load and elongation response of plastic hinges. This model is being incorporated into the computer analysis package, RUAUMOKO (Carr 2004) and it is described in this paper. Analytical predictions are compared with experimental results obtained from beams subjected to different levels of axial load.

2 EXPERIMENTAL SETUP

Experimental data was extracted from beam tests carried out at the University of Auckland (Fenwick et al. 1981; Issa 1997; Matti 1998). Five tests with axial loading of 0, 100, 200 and 500kN in compression and 75kN in tension are considered. The typical test arrangement, beam configuration and loading sequence are illustrated in Figure 1 and 2.



(a) Test arrangement (b) Cross section of beam

Figure 1. Experimental setup and beam configuration

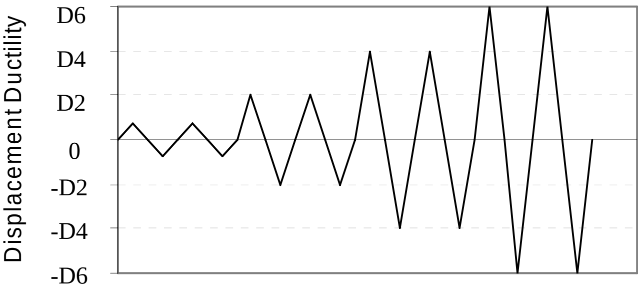


Figure 2. Applied loading history

To determine the displacement, corresponding to a ductility of 1, D1, the initial loading cycles were force controlled in which a maximum force corresponded to 75% of the calculated theoretical flexural strength of the beam was applied in each direction. The loading history following these cycles was displacement-controlled. In general, two cycles at ± displacement ductility of two, D2, followed by two cycles at displacement ductility of four, D4, and two cycles at displacement ductility of six, D6, were applied to the beam. The material properties, theoretical flexural strength and ductility 1 displacement obtained from the selected tests are summarised in Table 1.

Table 1. Summary of the selected beam test properties

Test	Reinforcement Yield Stress (MPa)	Concrete Compressive Stress (MPa)	Axial Force (kN)	Theoretical Flexural Strength (kNm)	Ductility 1 Displacement (mm)
2A	306	37.6	0	202	8
S2A	332	37.8	-100*	229	9.3
M1	318	29.4	-200*	238	9
S1B	332	37	-500*	305	9.1
M2	318	29.4	75	193	9

* Negative axial force represents compression

3 COMPUTATIONAL MODELLING

3.1 Mechanics of Elongation

Elongation within the plastic hinges arises due to plastic rotation in the tension reinforcement and unrecoverable extension in the compression reinforcement (Fenwick et al. 1981). This is illustrated in Figure 3 where the extension of top and bottom reinforcement in the plastic hinge region is plotted for the first cycle at displacement ductility of six for beam 2A.

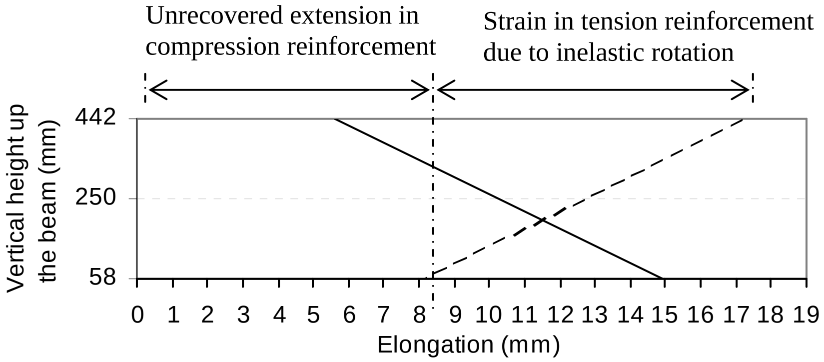
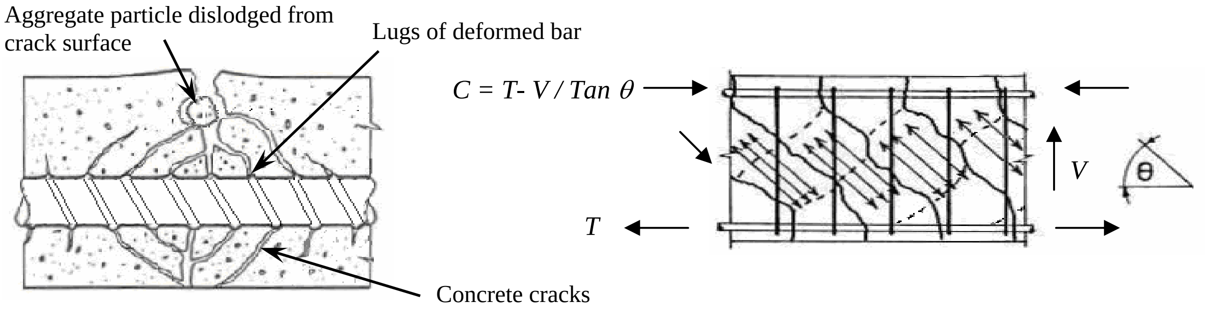


Figure 3. Elongation of top and bottom reinforcement in plastic hinge region at first D6 cycle in beam 2A

The unrecoverable extension of the reinforcement in the compression region was found to arise due to two actions (Fenwick and Megget 1993). First of these was due to the wedging action of aggregate particles that become dislodged from crack surfaces when tension was applied. These particles restrain the closure of the cracks when subjected to compression, see Figure 4a, and it is referred to as the contact stress effect. Secondly, intersecting diagonal cracks in the plastic hinge region destroy the shear resistance of concrete (i.e. $v_C = 0$). Consequently, a truss like actions develops in the plastic hinge to resist the shear as shown in Figure 4b. In this figure, C and T are the flexural compression and tension forces, θ is the angle of the diagonal struts and V is the shear force acting on the plastic hinge. In this region the shear force is solely resisted by the stirrups and diagonal compression forces in the web. To satisfy force equilibrium for a given section, the flexural tension force is always greater than the flexural compression force due to the longitudinal component of the diagonal compression forces in the web. Consequently, inelastic rotation in the plastic hinge is resisted predominately by yielding of the tension reinforcement rather than by yielding of reinforcement in the compression zone.

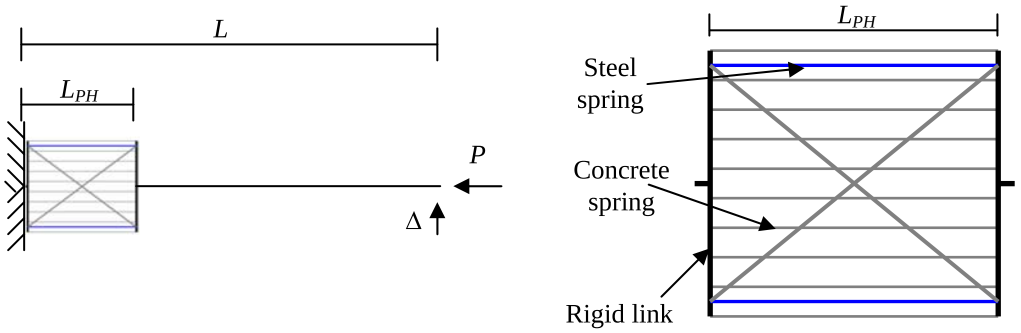


(a) Wedging action of concrete (b) Truss-like action in the web

Figure 4: Inelastic action in the plastic hinge region

3.2 Plastic Hinge Model

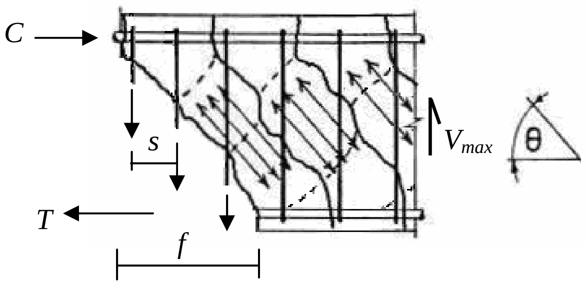
The displacement history and the axial force used in the selected tests were applied to the analytical models. The analytical beam model is illustrated in Figure 5a, where Δ is the applied displacement and P is the applied axial force. The beam with length, L, is divided into two parts, namely a plastic hinge region and an elastic region. The elastic region is modelled using a Giberson beam element. The plastic region is modelled by mounting axial spring elements between rigid arms located at the ends of the plastic hinge, as illustrated in Figure 5b. Ten longitudinal concrete spring elements are evenly distributed over the depth of the section and two steel spring elements are located at the centroids of the top and bottom reinforcement. In addition two diagonal concrete spring elements are inserted to represent the diagonal compression forces in the web which resist the shear force.



(a) Analytical beam layout (b) Plastic hinge model

Figure 5. Analytical model of reinforce concrete beam

The length of plastic hinge model, L_{PH}, in Figure 5 was chosen to obtain the correct inclination of the diagonal compression members in the high moment region of the plastic hinge. This value is taken equal to the longitudinal length of the diagonal crack, f, as shown in Figure 6. It is calculated on the basis that the shear force corresponding to the theoretical flexural strength is resisted by the theoretical yield force in the stirrups. An expression for f is given on Figure 6 where A_v and f_{vy} are the area and yield stress of the stirrups.



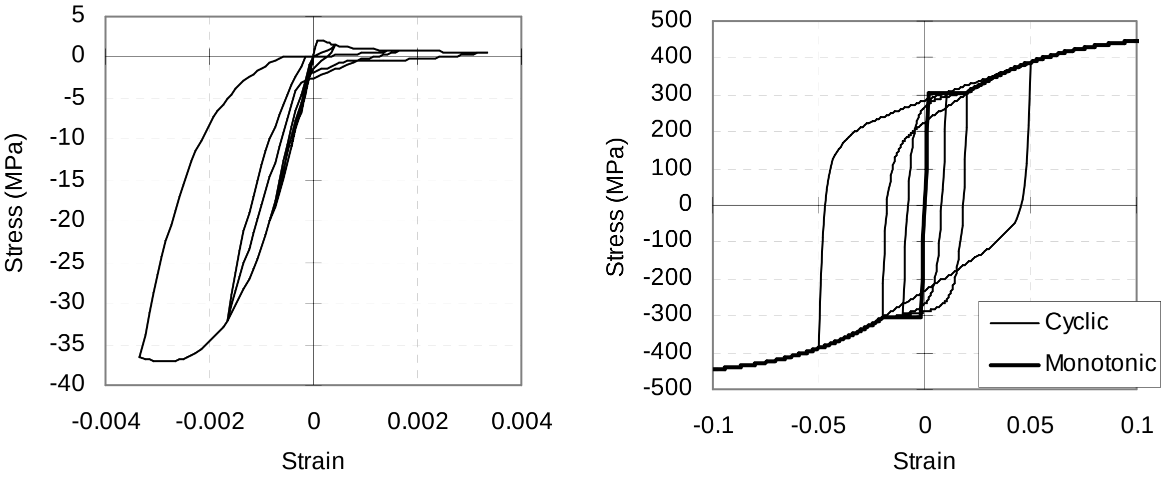
$$f = V_{max}\, s / A_v\, f_{vy}$$

Figure 6. Force equilibrium across a diagonal crack

The actual length over which reinforcement yields, L_{yield}, is given by Equation 1 where L is the length of beam, M_{max} is the maximum moment sustained up to the current stage and M_y is the first yield moment. Here, the term *d/2* is included to represent the "tension shift" effect, which is based on the assumption that the diagonal crack extends over a distance *d* along the member at the low moment end of the plastic hinges. The value *e* allows for the yield penetration of the reinforcement into the supporting member. As strain levels in the reinforcement decrease approximately linearly from the maximum value at the face of the support to the point where the strain is equal to the yield strain, the effective plastic hinge length, L_{eff}, is equal to approximately half L_{yield}. To take into account the difference in the lengths, L_{PH} and L_{eff}, the stiffness of the steel springs was modified to give the correct hinge stiffness and strain hardening rate for the reinforcement.

$$L_{yield} = L\frac{M_{\max} - M_y}{M_{\max}} + \frac{d}{2} + e \qquad (1)$$

New concrete and steel hysteretic models, which are illustrated in Figure 7, have been incorporated into the RUAUMOKO hysteretic library. The concrete hysteretic model is based on Maekawa model (Maekawa et al. 2003). It uses path-dependent average stress-average strain relationships and it contains compression and tension envelopes, with unloading and reloading loops that include contact stiffness effects. The compression envelope is based on Elasto-Plastic Fracture model, EPF, and the tension envelope is based on a tension stiffening model. The reloading loop includes an allowance for contact stress effect, which arises due the wedging action of dislocated aggregate particles in the cracks. The steel hysteresis, developed by Dhakal (Dhakal 2000), also uses path dependent average stress-average strain relationship. It consists of tension envelope, which is based on Mander strain hardening model, compression envelope and an unloading/reloading loop that allows for the Bauschinger effect using the Giuffre-Menegotto-Pinto Model.



(a) Concrete hysteresis (b) Steel hysteresis

Figure 7. Material hysteretic model for concrete and steel

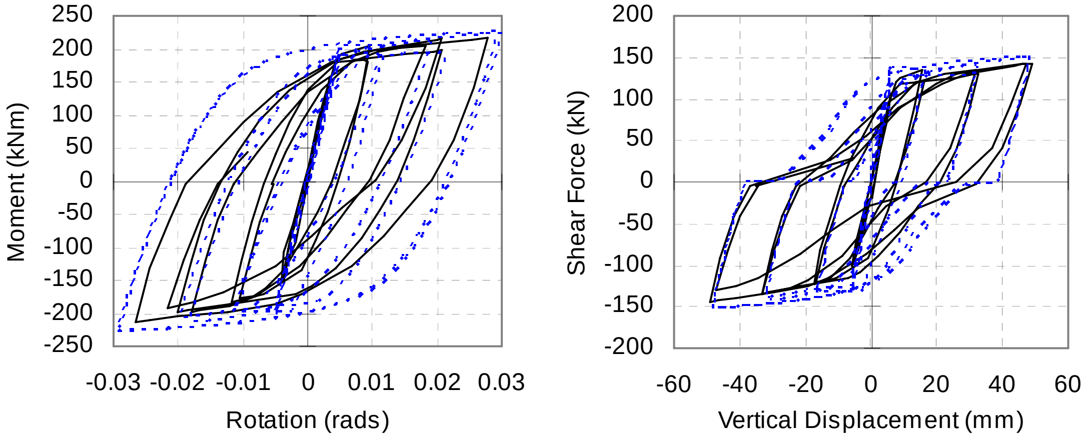
A parametric study was carried out to determine the effect of incremental loading step size, and cross sectional area of diagonal compression strut on the overall behaviour. It was found that very small incremental displacement (0.0005mm) was required in order to satisfy force equilibrium. The area of the diagonal compression strut was found to have little influence on the overall behaviour and it was chosen to be 50mm deep.

4 ANALYTICAL AND EXPERIMENTAL COMPARISON

4.1 Beam Test with No Axial Force

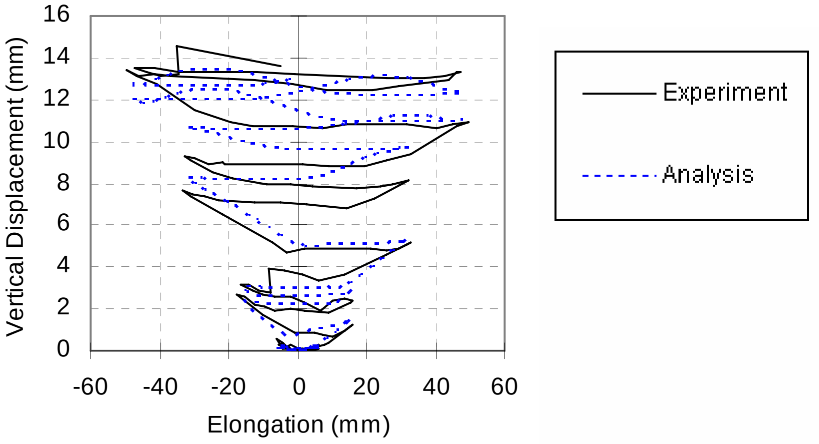
It can be seen from Figure 8 that the analytical moment-rotation, force-displacement and elongation behaviour are in reasonable agreement with the experimental results. The significance of the model not allowing for shear deformation due to yielding of stirrups can be seen from the results shown in

Figure 8. This results in under-estimates of pinching in the force displacement diagram (see Figure 8b) and an over-estimates of the rotation in the moment rotation diagram (see Figure 8a).



(a) Moment rotation

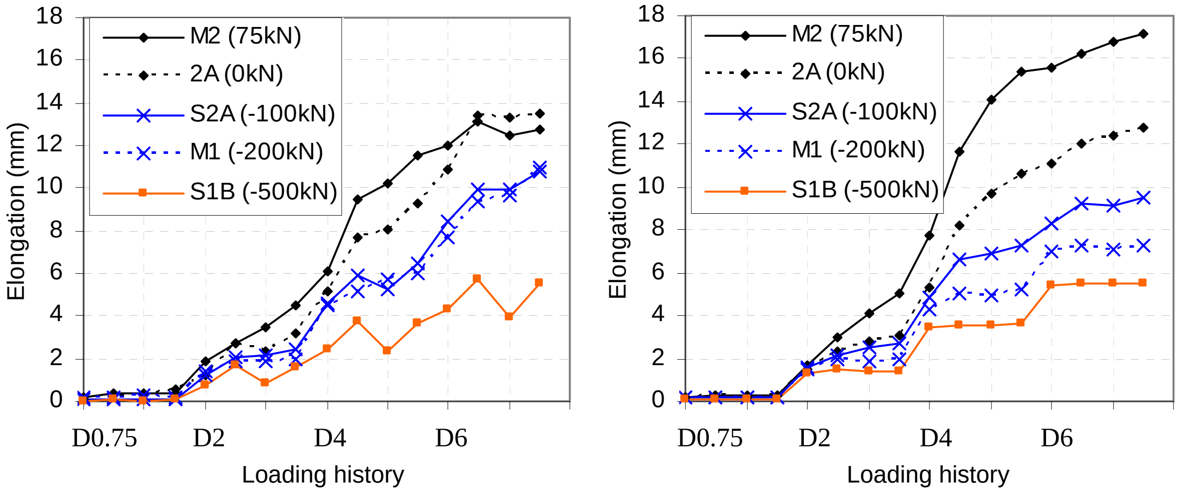
(b) Force Displacement



(c) Displacement Elongation

Figure 8. Analytical and experimental comparison for beam 2A

4.2 Axial Force Effect



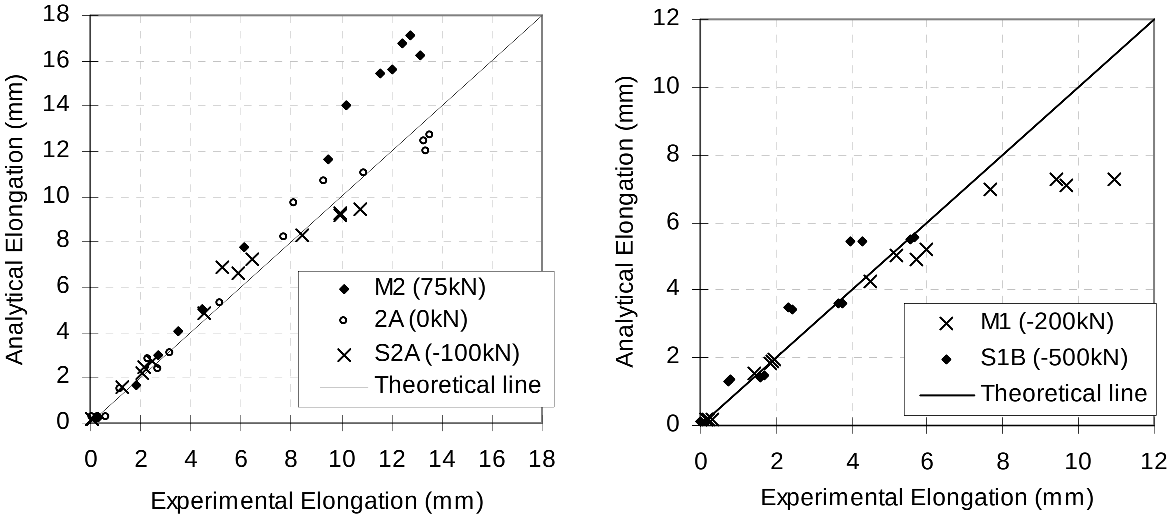
(a) Experimental elongation

(b) Analytical elongation

Figure 9. Axial force effect on experimental and analytical elongation response at peak displacements

Figure 9 summarises the experimental and analytical elongation values at peak displacements for beams with different axial forces (negative values represent compression). As expected the elongation reduces as axial compression force increases. However, this trend is less obvious in the experiments with small variation of axial force. Beams with 100kN and 200kN compression force elongate by a similar amount, and likewise beams with 0kN and 75kN tension force have similar elongation behaviour. In comparing the measured and predicted responses it should be noted that there is appreciable variability in elongation measured from similar tests.

It can be seen from Figure 10 that the analytical predictions are in reasonable agreement with the experimental results. For beam M2, where 75kN axial tension force is applied, the elongation is over predicted in the high ductility displacement cycles. This is due to the model not allowing for shear deformation from yielding of stirrups to occur. In beams M1, where axial load was 200kN in compression, the elongation is under predicted in the last few cycles. The reason for this discrepancy is not known at this stage.



(a) Elongation comparison for beam M2, 2A and S2A (b) Elongation comparison for beam M1 and S1B

Figure 10. Analytical and experimental elongation comparison at peak displacements for different axial force

4.3 **Discussions**

While the model shows promising results it does have some limitations;

- Shear deformation due to elastic and inelastic strains in the stirrups is not included. In the test where there is axial tension or no axial load, appreciable shear deformation occurs in the high ductility load cycles due to yielding of the stirrups. Consequently, the rotation applied to the plastic hinges in the analysis in these load cycles is higher than the experimental values. This results in a higher elongation prediction as observed in the comparisons above.
- Reinforcement length, based on the L_{yield} value, increases with ductility cycles. This effect is ignored in the current analysis and a constant average value was used.

It should be noted that the opening and closing of the diagonal struts associated with elongation results in some shear deformation. It is planned to extend this model to include shear deformation due to strains of stirrups at a later stage of the project.

5 **CONCLUSIONS**

A plastic hinge model has been developed and incorporated into a structural analysis package. Comparisons of analytical predictions of flexural, and elongation response with experimentally measured values indicates the model generally behaves satisfactorily. At a later stage, it is intended to extend the model to included shear deformation associated with elastic and inelastic deformation of shear reinforcement. This model should enable analyses to be made to assess the significance of elongation on the behaviour of structural assemblages consisting of ductile fames and floor slabs.

ACKNOWLEDGEMENT:

The authors would like to acknowledge the financial support provided by the Tertiary Education commission, TEC, and the University of Canterbury

REFERENCES:

Carr, A. J. 2004. *RUAUMOKO2D - Inelastic Dynamic Analysis.* Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

Dhakal, R. P. 2000. Enhanced Fiber Model in Highly Inelastic Range and Seismic Performance Assessment of Reinforced Concrete. *PhD. Thesis*, University of Tokyo, Tokyo, Japan.

Fenwick, R.C. and Davidson, B.J. 1993, Elongation in ductile seismic resistance reinforced concrete frames, *Paulay Symposium,* ACI Special publication SP-157, pp. 141-168.

Fenwick, R. C., and Megget, L. M. 1993. Elongation and load deflection characteristics of reinforced concrete members containing plastic hinges,. *Bulletin of the New Zealand National Society for Earthquake Engineering*, 26(1), 28-41.

Fenwick, R. C., Tankut, A. T., and Thom, C. W. 1981. The deformation of reinforced concrete beams subjected to inelastic cyclic loading: experimental results. *Report No. 268*, Department of Civil Engineering, University of Auckland, Auckland, New Zealand

Issa, M. S. 1997. The deformations of reinforced concrete beams containing plastic hinges in the presence of axial compression or tension load, *Master of Engineering Report*, Department of Civil and Resource Engineering, University of Auckland, Auckland, New Zealand

Lau, D. B. N. 2001. The Influence of Precast-prestressed Flooring on the Seismic Performance of Reinforced Concrete Perimeter Frame Buildings. *Report No. 604,* Department of Civil and Resource Engineering, University of Auckland, Auckland, New Zealand.

Lau, D. B. N., Davidson, B. J., and Fenwick, R. C. 2003. Seismic Performance of RC Perimeter Frames with Slabs Containing Prestressed Units. *Pacific Conference on Earthquake Engineering*, Christchurch.

Lee, J.Y., and Watanabe, F. 2003. Predicting the longitudinal axial strain in the plastic hinge regions of reinforced concrete beams subjected to reversed cyclic loading. *Engineering Structures*, Vol 25(7), 927-939.

Lindsay, R. 2004. Experiments on the Seismic Performance of Hollow-core Floor Systems in Precast Concrete Buildings. *Master of Engineering Thesis,* Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

MacPherson, C. 2005. Seismic Performance and Forensic Analysis of a Precast Concrete Hollow-core Floor Super-assemblage, *Master of Engineering Thesis*, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

Maekawa, K., Pimanmas, A., and Okamura, H. 2003. *Nonlinear Mechanics of Reinforced Concrete*, Spon Press, London.

Matthews, J. 2004. Hollow-core Floor Slab Performance following a Severe Earthquake, *PhD. Thesis*, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.

Matthews, J. G., Mander, J. B., and Bull, D. K. 2004. Prediction of beam elongation in structural concrete members using a rainflow method. *New Zealand Society of Earthquake Engineering Conference*, Rotorua.

Matti, N. A. 1998. Effect of axial loads on the behaviour of reversing plastic hinges in reinforced concrete beams, *Master of Engineering Report*, Department of Civil and Resource Engineering, University of Auckland, Auckland, New Zealand.

Megget, L.M. and Fenwick, R.C. 1989, "Seismic behaviour of a Reinforced Concrete Portal Frame Sustaining Gravity Loads", *Bulletin of NZ National Society for Earthquake Engineering,* Vol. 22, No. 1, Mar., pp. 39-49.