

PREDICTION OF SPATIALLY DISTRIBUTED SEISMIC DEMANDS IN SPECIFIC STRUCTURES: GROUND MOTION AND STRUCTURAL RESPONSE

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ABSTRACT

The efficacy of various ground motion intensity measures (IM's) in the prediction of spatially distributed seismic demands (Engineering Demand Parameters, EDP's) within a structure is investigated. This has direct implications to building-specific seismic loss estimation, where the seismic demand on different components is dependent on the location of the component in the structure. Several common intensity measures are investigated in terms of their ability to predict the spatially distributed demands in a 10-storey office building, which is measured in terms of maximum interstorey drift ratios and maximum floor accelerations. It is found that the ability of an IM to efficiently predict a specific EDP depends on the similarity between the frequency range of the ground motion which controls the IM and that of the EDP. An IM's predictability has a direct effect on the median response demands for ground motions scaled to a specified probability of exceedance from a ground motion hazard curve. All of the IM's investigated were found to be insufficient with respect to at least one of magnitude, source-to-site distance, or epsilon when predicting all peak interstorey drifts and peak floor accelerations in a 10-storey RC frame structure. Careful ground motion selection and/or seismic demand modification is therefore required to predict such spatially distributed demands without significant bias.

KEYWORDS

Spatially distributed seismic demand, predictability, efficiency, sufficiency, ground motion selection.

INTRODUCTION

The seismic response of structural systems is complex with ground motion shaking causing a response that varies significantly in space and time. This complex response can result in significantly different acceleration and displacement demands at spatially different locations in a structure, which will depend on its dynamic characteristics, as well as the properties of the ground motion record exciting the structure.

In emerging performance-based frameworks such as the Pacific Earthquake Engineering Research (PEER) Centre performance-based earthquake engineering (PBEE) methodology [1] uncertainties in all aspects (from ground motion to loss estimation) of the seismic analysis of structures can be explicitly incorporated and propagated to obtain performance measures useful for decision making. In such a probabilistic framework, there are transparent advantages in being able to reduce uncertainties in each of the aforementioned aspects, since uncertainties inevitably result in an increase in the risk of structural failure and/or economic losses for infrequent hazards such as those posed by seismic-induced ground shaking.

An area of research in the past decade has been the investigation of ground motion intensity measures (IMs) which provide the link between the seismic hazard curve (which gives the probability/frequency of exceedance of a specific level of IM) and structural response (giving the distribution of the engineering demand parameter, EDP, for a given IM). An ‘optimal’ intensity measure must possess efficiency [2], sufficiency [3], predictability [4] and scaling robustness [5]. The aspects of efficiency and sufficiency have been studied in detail by Cornell and co-workers (e.g. [5-9]) where the seismic response of structures was measured simply via the maximum interstorey drift over all floors (which relates well to joint rotations in structural elements and therefore the potential for structural collapse). Predictability relates to the accuracy in predicting an IM from ground motion prediction equations. With the increased interest in ground motion selection methods (e.g. [10]), scaling robustness seeks to determine if the distribution of EDP using scaled ground motions is biased compared with that obtained using un-scaled ground motions [11]. Optimal intensity measures for total floor accelerations have received less attention than that of peak interstorey drifts, with the exception of Taghavi and Miranda [12] who examined the efficiency of four different IMs at predicting peak floor accelerations using simple elastic structural models.

The significant spatial variation in the response of structural systems with several or more storeys means that separate consideration must be given to each of these demands when rigorously considering the seismic performance of such systems within the PEER framework. As such, loss estimation methods used within the PEER PBEE framework typically employ a vector of EDPs which account for these spatially varying demands (typically maximum interstorey drift ratios and maximum floor accelerations). Furthermore, ground motion IMs which are efficient, sufficient, predictable and robust to scaling for this vector of EDP values are required.

Aslani [13] considered the efficiency and sufficiency of four different IMs for use in predicting spatially distributed demands in structures, and this research is intended to extend the work of Aslani [13] in the following ways: (1) seismic hazard curves for each of the ground motion IMs are developed independently allowing explicit consideration of the predictability of the different IMs; (2) consideration is given to efficiency, sufficiency, predictability, and scaling robustness of the IMs; (3) ground motion selection based on hazard deaggregation is employed; and (4) 50th percentile rotation independent geometric mean (GMRotI50) intensity measures are used in both hazard computations and seismic response analysis.

The purpose of this paper is to use common IMs presented in literature, and for a specific structure investigate their predictability, efficiency, sufficiency, and scaling robustness in predicting peak interstorey drifts and total floor acceleration demands throughout the structure. In a companion paper [14], correction for IM insufficiency, as well as the resulting demand hazard, collapse hazard, and loss estimation results based on the different IMs is considered.

STRUCTURE CONSIDERED

The case study structure used herein is based on the geometry of the ‘Red Book building’ [15], a ten storey reinforced concrete (RC) structure, which acts as a design example of the New Zealand Concrete Code [16]. The primary lateral load carrying system consists of four one-way perimeter moment resisting frames which are 3 bays long. Vertical loads are transferred primarily through interior columns with gravity beams supporting one-way floor units.

A perimeter frame 2D model was developed using the finite element analysis program OpenSees [17]. Due to the symmetry of the structure, it was assumed that the 3D response

could be reasonably approximated by separate 2D analyses in each of the two primary frame directions. The effects of foundation flexibility due to soil-foundation interaction were considered simply by using elastic rotational springs at the base of the columns [18]. The structure was modelled using a lumped mass model and non-linear (beam) elements with the appropriate backbone properties determined using fibre-based section modelling, and stiffness and strength degradation based on calibration with experimental tests [19]. The structural model had a fundamental period of $T_1 = 1.74$ seconds. Based on a pushover analysis it was determined that the ‘yield’ displacement, d_y , of the structure was 10 cm (this is used for the inelastic spectral displacement IM, S_{di}).

The seismic demand due to ground motion excitation was monitored via peak interstorey drift ratios and peak floor accelerations at each floor in the structure (i.e. a total of ten drifts and eleven accelerations).

GROUND MOTION INTENSITY MEASURES AND SEISMIC HAZARD

In order to investigate the prediction of spatially distributed demands in structures a variety of ground motion IMs are selected. As there have been numerous ground motion IMs presented in the literature relating to various different aspects of structural behaviour it is necessary to apply some criteria to determine which IMs to investigate in this research. Firstly, it was desired to consider several IMs which have been used by other researchers when examining structural response from a probabilistic viewpoint. Secondly, and more importantly, all IMs used had to have a ‘robust’ ground motion prediction equation which can be used to develop seismic hazard curves using this IM at a variety of sites. This second point is particularly important as many studies have focused on the consideration of somewhat complex IMs which may be a combination of several ‘standard’ IMs in an effort to achieve better response prediction (i.e. efficiency). However, without a ground motion prediction equation for such an IM, no ground motion hazard curves can be developed, and hence no PBEE assessment can be performed using this IM. The term ‘robust’ has been used in order to differentiate between simple ground motion prediction equations based on limited data, and comprehensive ground motion prediction equations based on large ground motion databases which consider many features known to affect ground motion prediction (e.g. faulting types, hanging wall effects, local soil effects) such as those of the next generation attenuation (NGA) project [20].

Based on the above criteria a total of five different ground motion IMs were selected, namely: peak ground acceleration (PGA); peak ground velocity (PGV), elastic spectral displacement (S_{de}); inelastic spectral displacement (S_{di}); and spectrum intensity (SI). PGA and S_{de} can be predicted from (elastic) spectral acceleration prediction equations, some of which now also include coefficients for computing PGV [20]. Prediction of S_{di} , defined as the peak displacement of a bilinear single degree-of-freedom oscillator [21], is obtained by combining a ground motion prediction equation for S_{de} with a ground motion prediction for the ratio S_{di}/S_{de} [21] (while the empirical equation for S_{di}/S_{de} was determined using one specific ground motion prediction equation Tothong and Cornell [21] argue that it can be used with any prediction equation for S_{de}). Finally, a ground motion prediction equation for SI , defined as the integral of the pseudo-spectral velocity from 0.1-2.5 seconds [22], can be computed directly from ground motion prediction equations for spectral acceleration [23]. The effect of epistemic uncertainties is beyond the scope of this study. However, it should be noted that as all ground motion IM’s have ‘robust’ prediction equations, then the effect of epistemic uncertainty is expected to be of a similar magnitude for all the different IMs.

A simple hypothetical site with a 30-m averaged shear wave velocity of 600 m/s was considered as illustrated in Figure 1a, which is a closest distance of 15 km from a 40 km

strike-slip fault. The fault has a Gutenberg-Richter magnitude distribution with $\alpha = 3.0$ and $\beta = 0.8$; minimum and maximum magnitudes of 5.0 and 7.5, respectively; and events assumed to be Poissonian in time. Based on this hypothetical scenario and using the Boore and Atkinson [24] ground motion prediction equation for PGA , PGV and S_{de} ; the prediction equation of Tothong and Cornell [21] for S_{di} ; and the prediction equation of Bradley *et al.* [23] for SI (with Boore and Atkinson [24] used as the ‘base’ prediction equation for both S_{di} and SI), the ground motion hazard curves shown in Figures 1b-1d were determined using the probability-based formulation for the ground motion hazard [25]. Because of the initial period of the structure it is observed that the elastic and in-elastic spectral displacement hazards are similar in Figure 1c (i.e. the equal displacement rule). Note that the 50th percentile orientation-independent geometric mean of the two horizontal ground motion components, GMRotI50 [26], definition for the ground motion intensity measures has been used in computing the hazard curves for all five IMs.

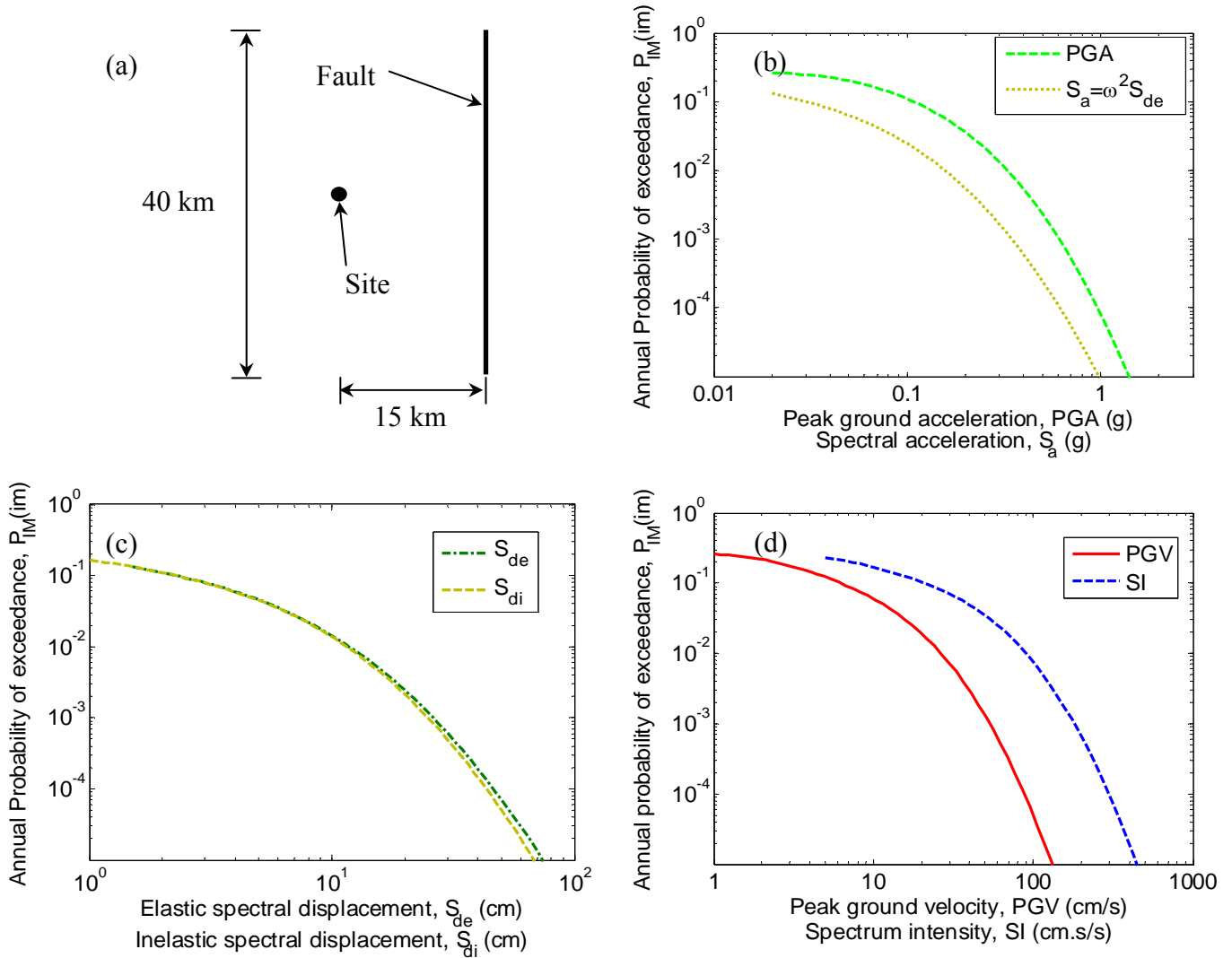


Figure 1: Hypothetical site considered and ground motion hazard curves for the five different ground motion intensity measures (IMs) considered.

GROUND MOTION SELECTION

It is important to carefully select ground motion records for use in dynamic analysis to avoid bias in structural response [27]. Ground motion records should be selected which are representative of those which are most likely to occur at the site of interest in the future.

Hence, the target is to select a suite of ground motions which has the same distribution of ground motion properties as the deaggregation [28, 29] of the seismic hazard at the site. In the remainder of the manuscript particular attention will be given to the spatial distribution of seismic demands for ground motions which have exceedance probabilities of 1/475 and 1/2475 (corresponding to 10% and 2% probabilities of exceedance in 50 years, respectively). Table 1 gives the mean moment magnitude (M_w), (Boore-Joyner) source-to-site distance (R), and epsilon (ϵ , defined as the number of standard deviations a specific ground motion parameter is above the predicted mean) for the five different IMs obtained by deaggregation of the seismic hazard at these exceedance probabilities. It can be seen that for a given IM, the mean M_w , and ϵ are larger and mean R smaller for the 1/2745 exceedance probability compared to the 1/475 exceedance probability. In addition, for a given exceedance probability it is observed that there are quite significant differences between the mean M_w , R and ϵ values for the different IMs, since they are affected by different properties of the ground motion. M_w and R obviously affect the intensity, frequency content and duration of ground motion records, and it has also been shown that when spectral ordinates are used as a ground motion IM (i.e. PGA and S_{de} used in this study) that ϵ has an effect on structural response, as it relates to spectral shape [6]. S_{di} has been shown to be (relatively) insensitive to ϵ (when predicting the peak interstorey drift over all floors), since S_{di} directly accounts for spectral shape in the case of period elongation [5]. The effect of ϵ on PGV and SI has not been researched in detail.

Table 1: Statistics of the ground motion hazard deaggregation and suite of ground motion records used.

	$P_{IM(im)} = 1/475$				$P_{IM(im)} = 1/2475$				Ground motion suite		
	IM	μ_{M_w}	μ_R	μ_ϵ	IM	μ_{M_w}	μ_R	μ_ϵ	μ_{M_w}	μ_R	μ_ϵ
PGA (g)	0.515	6.93	18.8	1.58	0.744	7.01	18.5	2.14	6.49	20.8	1.17
PGV (cm/s)	44.1	7.16	18.8	1.26	66.2	7.24	18.5	1.88	6.49	20.8	1.58
S_{de} (cm)	20.9	7.04	18.9	1.46	33.4	7.10	18.7	2.06	6.49	20.8	1.57
S_{di} (cm)	20.1	7.03	19.0	1.44	31.6	7.10	18.8	2.08	6.49	20.8	1.57
SI (cm.s/s)	148.3	7.11	18.7	1.34	223.3	7.17	18.3	1.89	6.49	20.8	1.76

As the value of ϵ depends on the IM and GMPE used and cannot currently be determined *a priori* (using, for example, the PEER NGA ground motion database [30]) then using the deaggregation results in Table 1, ground motions were initially selected based on wide range of M_w , R , and site 30-m averaged shear-wave velocity (V_{S30}) (specifically $6.0 < M_w < 8.0$; $0 < R < 30$ km; $300 < V_{S30} < 800$ m/s) giving a total of 155 ground motions (each with two orthogonal horizontal components). The ϵ values for the five different ground motion parameters were determined and then the allowable ranges of M_w , R , V_{S30} , and ϵ were further constrained to obtain a set of 25 ground motions (i.e. 50 different horizontal ground motion records for use in dynamic analysis) which were based on $6.2 < M_w < 7.7$; $10 < R < 28$ km; $300 < V_{S30} < 800$ m/s; ϵ (of any IM) > 0.4 . No constraints were placed on the number of recordings from a single event. The adopted ground motion records are presented in Table 2 and their statistics for the different IMs are summarised in Table 1. Note that the process for selecting ground motion records was conducted in an iterative fashion as it was not possible to find a large enough suite of records which match the statistics (mean and standard deviation) of the seismic hazard deaggregation exactly. Therefore a trade-off was required to try and match all M_w , R , and ϵ values relatively well.

PREDICTION OF STRUCTURAL RESPONSE: DETERMINISTIC HAZARD SCENARIO

Before investigating the efficacy of the five different ground motion IMs in predicting the spatially varying structural response for the 1/475 and 1/2475 probability of exceedance hazard levels, it is first necessary to test for any bias when comparing the different ground motion IMs. Potential sources of bias could be whether the selected ground motion record suite is equally representative for all five different ground motion IMs or whether the different ground motion prediction equations used to determine the scale factors (used to scale the amplitude of motion only) for each ground motion are consistent. To investigate the above points a deterministic earthquake scenario is considered with a moment magnitude of $M_w = 7.0$ and a source-to-site distance of $R = 18$ km. Note that this deterministic scenario is intentionally similar to the mean magnitude and distance obtained from deaggregation of the seismic hazard in Table 1. Table 3 gives the median and dispersion (lognormal standard deviation) for the ground motion IMs obtained from the ground motion prediction equations for the deterministic scenario. Figures 2a and 2b illustrate the median (specifically, the mean of the logarithms which is the median assuming a lognormal distribution) and dispersion for the GMRotI50 acceleration response spectra obtained by scaling the ground motion records to the median IM for the deterministic scenario. Since the median response spectra of the 25 ground motion records scaled based on the five different IMs are very similar, it indicates that the selected ground motion suite and the different ground motion prediction equations do not introduce any significant bias when comparing the results of the structural analyses to follow. Note that this result was to be expected since all of the IMs use (or are derived from) the Boore and Atkinson [24] ground motion prediction equation and ground motions were selected to match hazard deaggregation. Figure 2b provides insight into the effect of IM scaling on the dispersion in response spectra amplitudes as a function of vibration period. Obviously, *PGA* scales all the ground motions to have the same spectral acceleration at $T = 0$ so the dispersion is zero at $T = 0$, and similarly for S_{de} at $T = T_1$. Also since the inelastic spectral displacement for this scenario, $S_{di} = 6.09$ cm, is less than the yield displacement of the inelastic single-degree-of-freedom (SDOF), $d_y = 10$ cm, then the dispersion is also zero at $T = T_1$ for ground motions scaled to S_{di} (i.e. the S_{de} and S_{di} lines in Figure 2b are coincident). An increase in response spectra dispersion with period is observed for ground motions scaled based on *PGA*, while in a similar fashion the dispersion increases as T moves away from T_1 for S_{de} - and S_{di} -based scaling. While scaling based on *PGV* or *SI* does not ‘fix’ the dispersion to zero at any point of the response spectra it is observed that apart from the small period window around $T = T_1$ (approximately $1.0 < T < 2.5$) the dispersion for both *PGV* and *SI* is lower than when ground motions are scaled to S_{de} .

Figures 3a and 3b illustrate the median response (in terms of maximum interstorey drift ratios and maximum floor accelerations) of the case study structure based on ground motion scaling using the five different IMs. As expected, the median response, both for interstorey drifts and floor accelerations are approximately the same for all five IMs. The minor exception being that ground motions scaled using *PGA* give slightly larger interstorey drifts and floor accelerations over the lower portion of the structure. This is consistent with the slightly larger median response spectra over $1.0 < T < 3.0$ s using *PGA*-scaling in Figure 2a. Figure 3c illustrates the dispersion in the maximum interstorey drifts for the deterministic scenario. It can be seen that over the lower half of the structure where the peak responses are primarily due to the first mode of vibration, that S_{de} (and S_{di}) are the most efficient in predicting the interstorey drifts, while *PGA* is the worst, and *SI* is marginally better than *PGV* as it contains spectral velocity information at periods around that of the first mode. In the upper-half of the structure, where the effects of higher vibration modes are more significant it is clearly seen that the efficiency of the spectral displacement IMs (S_{de} and S_{di}) reduces and the efficiency of

PGA increases. Figure 3d illustrates the dispersion in the peak floor accelerations for the deterministic scenario. It is apparent that *PGA* has the best efficiency for all floors, although the difference is less pronounced in the upper floors (where the structures dynamic characteristics have significantly modified the ground motion input at the base). On the other hand, spectral displacement IMs (S_{de} and S_{di}) are the worst at predicting the maximum floor accelerations over all floors. Note that the dispersion in the ground floor peak acceleration when using *PGA* is not zero as the ground motion IMs are based on GMRotI50 [26] (i.e. the geometric mean of the two components), while only a single component is applied in each of the structural analyses. It is necessary to use the GMRotI50 definition for the IMs to be consistent with the ground motion hazard curves [31].

Table 2: Properties of the ground motions adopted

ID*	Earthquake	Year	Recording station	M _w	R (km)	V _{S30} (m/s)
125	Friuli, Italy	1976	Tolmezzo	6.50	14.97	425
265	Victoria, Mexico	1980	Cerro Prieto	6.33	13.80	660
339	Coalinga	1983	Parkfield - Fault Zone 15	6.36	28.00	376
359	Coalinga	1983	Parkfield - Vineyard Cany 1E	6.36	24.83	339
369	Coalinga	1983	Slack Canyon	6.36	25.98	685
587	New Zealand	1987	Matahina Dam	6.60	16.09	425
755	Loma Prieta	1989	Coyote Lake Dam (SW Abut)	6.93	19.97	598
776	Loma Prieta	1989	Hollister - South & Pine	6.93	27.67	371
952	Northridge	1994	Beverly Hills - 12520 Mulhol	6.69	12.39	546
963	Northridge	1994	Castaic - Old Ridge Route	6.69	20.10	450
995	Northridge	1994	LA - Hollywood Stor FF	6.69	19.73	316
1003	Northridge	1994	LA - Saturn St	6.69	21.17	309
1010	Northridge	1994	LA - Wadsworth VA Hospital South	6.69	14.55	414
1077	Northridge	1994	Santa Monica City Hall	6.69	17.28	336
1485	Chi-Chi, Taiwan	1999	TCU045	7.62	26.00	705
2461	Chi-Chi, Taiwan	1999	CHY028	6.20	23.44	543
2495	Chi-Chi, Taiwan	1999	CHY080	6.20	21.34	553
2618	Chi-Chi, Taiwan	1999	TCU065	6.20	25.17	306
2619	Chi-Chi, Taiwan	1999	TCU067	6.20	27.66	434
2626	Chi-Chi, Taiwan	1999	TCU075	6.20	18.47	573
2627	Chi-Chi, Taiwan	1999	TCU076	6.20	13.04	615
2655	Chi-Chi, Taiwan	1999	TCU122	6.20	18.10	475
2661	Chi-Chi, Taiwan	1999	TCU138	6.20	21.11	653
3300	Chi-Chi, Taiwan	1999	CHY074	6.30	27.57	553
3507	Chi-Chi, Taiwan	1999	TCU129	6.30	22.69	664

*ID as given on the NGA database. <http://peer.berkeley.edu/nga/earthquakes.html>

Table 3: Median and dispersion in the ground motion IMs for the deterministic $M_w = 7.0$, $R = 18$ km scenario.

IM	Median, $exp(\mu_{\ln IM})$	Dispersion, $\sigma_{\ln IM}$
PGA (g)	0.187	0.564
PGV (cm/s)	15.78	0.560
S_{de} (cm)	6.19	0.690
S_{di} (cm)	6.09	0.672
SI (cm.s/s)	52.4	0.592

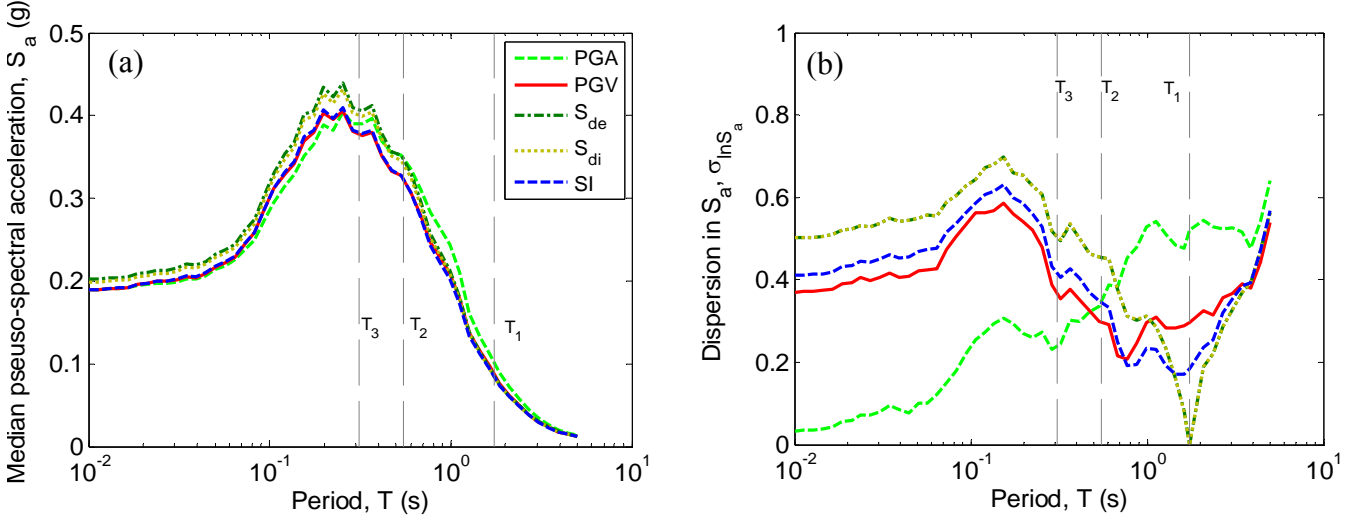


Figure 2: Response spectra of the ground motion suites scaled based on the different IMs for a $M_w = 7$, $R = 18$ km scenario

The results of Figures 3c and 3d are consistent with the results obtained by Aslani [13] and Taghavi and Miranda [12], and clearly indicate that the ability of various ground motion IMs to predict structural response EDPs depends on the similarity of the frequency range of the motion which dominates the EDP and that of the IM. For example, peak accelerations are dominated by high frequency content so PGA is the most efficient IM, while displacements are dominated by lower frequency content so IMs in the lower frequency region (i.e. S_{de} and S_{di} in this case) are more efficient in displacement prediction. The velocity IMs (PGV and SI) being in the ‘medium’ frequency range (i.e. between accelerations and displacements) provide ‘moderate’ efficiency in predicting both maximum interstorey drifts and maximum floor accelerations.

PREDICTION OF STRUCTURAL RESPONSE: PROBABILISTIC HAZARD

The nature of the deterministic scenario presented in the previous section allowed Figures 3c and 3d to be viewed solely to investigate the efficiency of the five different IMs (in this case for a relatively small level of ground motion shaking). As mentioned previously however, the determination of an optimal IM contains several other criteria, one of which is the predictability of the IM. Predictability relates to the magnitude of the aleatory uncertainty in the ground motion prediction equation used to compute the ground motion hazard for a specific site. Predictability is an important property of a ground motion IM since it will affect the probability of a specific level of ground motion occurring. Bommer and Abrahamson [32,

Figure 3] illustrate that the effect of a large uncertainty in a ground motion prediction equation (i.e. poor predictability) is to increase the likelihood of a specific level of ground motion intensity occurring, with the increase in likelihood becoming more significant at long return periods. As a direct indication of the predictability of the five different IMs considered here, Table 3 indicates the dispersion in the ground motion prediction equations for the deterministic scenario considered in the previous section. As is typical, the standard deviation of a response spectral ground motion prediction equation increases with response spectral period, as long period motion is more deterministically related to the earthquake source [33, p101] than short period motion (i.e. the predictability of S_{de} and S_{di} is worse than PGA), while the predictability of PGA , PGV and SI are similar.

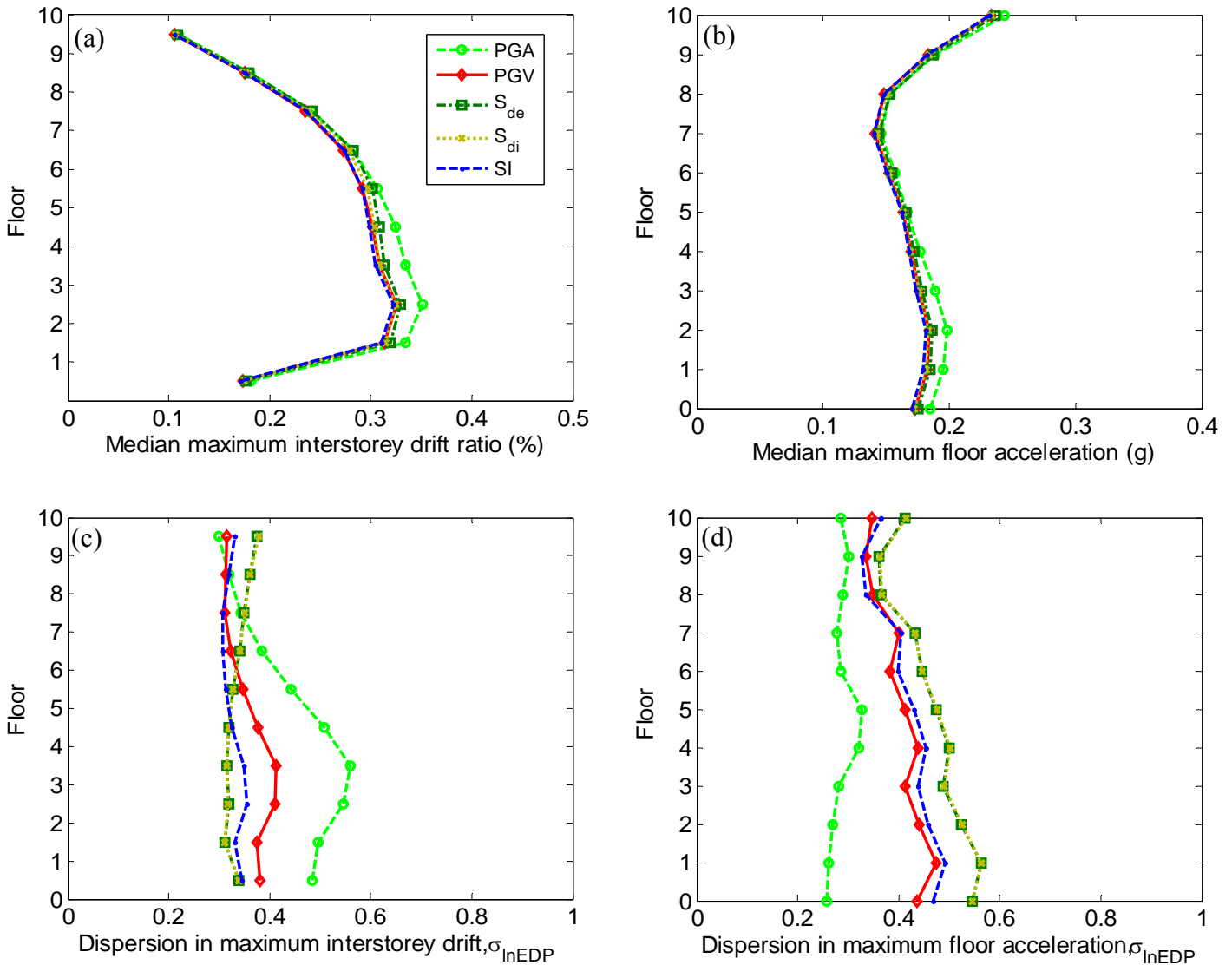


Figure 3: Median and dispersion in maximum interstorey drift and maximum floor acceleration demands for the various IMs for the deterministic scenario

Interstorey drift response

To illustrate the effects of predictability on the results of structural response analyses the ground motion records given in Table 2 were scaled to ground motion intensities which had 1/475 and 1/2475 exceedance probabilities (using the hazard curves in Figure 1), the values

for which are given in Table 1. Figure 4 illustrates the median and dispersion in the maximum interstorey drifts predicted using the various IMs for the two different exceedance probabilities. Unlike the deterministic scenario where all of the five IMs produced similar median demands, it is clear from Figures 4a and 4b that there is a significant difference when the ground motions are scaled to the same exceedance probabilities. For example, scaling ground motions to SI gives median values for the maximum interstorey drift between the 2nd and 3rd floors of 0.9% and 1.25% at the two different exceedance probabilities compared to 1.1% and 1.6% using ground motions scaled to S_{de} (i.e. 22% and 28% differences, respectively). It should be clear that the relative magnitude of the median values of the maximum interstorey drifts between the different IMs is closely related to the predictability of the different IMs.

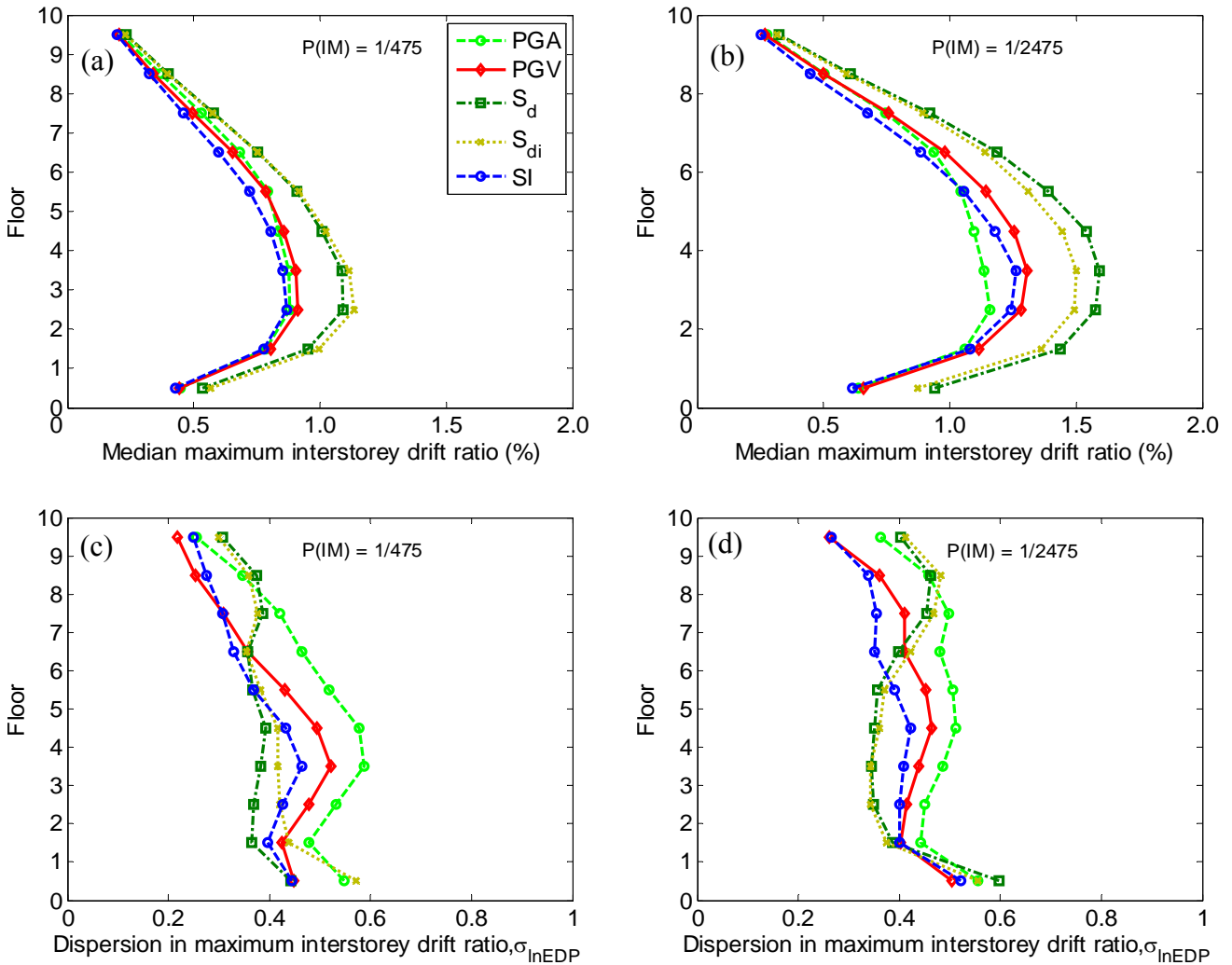


Figure4: Median and dispersion in the maximum interstorey drifts for ground motions scaled to the 1/475 and 1/2475 exceedance probabilities using the various IMs.

Figures 4c and 4d illustrate the dispersion in the maximum interstorey drifts obtained using the five different IMs for the 1/475 and 1/2475 exceedance probabilities. The trends regarding efficiency in predicting maximum interstorey drifts are similar to those for the deterministic scenario with the key difference being that as the intensity of the ground motion increases inelastic response causes changes in the vibration characteristics of the structure. These changing vibration characteristics subsequently affect the efficiency in predicting the

EDPs at different locations in the structure. For example, comparing the dispersion in the interstorey drifts at the 1/475 and 1/2475 exceedance probabilities illustrates that increasing inelastic behaviour reduces the dispersion in the prediction of the interstorey drifts in the lower half of the structure and increases the dispersion in the upper floors.

Floor acceleration response

Figures 5a and 5b illustrate the median values of the maximum floor accelerations for the various IMs. Similar to the median values of the interstorey drifts, it is observed that the relative magnitude of the maximum floor accelerations is directly related to the predictability of the IMs. A comparison of Figures 3b, 5a and 5b illustrates the change in the spatial distribution of the peak floor accelerations as the ground motion intensity increases. For the aforementioned deterministic scenario the largest floor accelerations occur at the top of the structure due to the presence of significant higher mode effects. However as the ground motion intensity increases, inelastic behaviour in the lower floors of the structure (which causes an elongation in the effective period of the structure) effectively acts as a filter on the high frequency components of the ground motion. The same logic also explains why the maximum interstorey drift demands in the upper floors of the structure reduce (relative to the maximum interstorey drifts in the lower floors) as the ground motion intensity increases. Figures 5c and 5d illustrate the dispersion in the prediction of the maximum floor accelerations at the 1/475 and 1/2475 exceedance probabilities. Similar trends are observed compared to the deterministic scenario with *PGA* being the most efficient IM and spectral displacements the worst. However, due to significant inelastic behaviour at the 1/2475 exceedance probability, it is seen that the efficiency of *PGA* in predicting peak accelerations on some floors is reduced.

It is interesting to observe in Figures 4 and 5 that there is little difference between the predictive capacity of S_{di} and S_{de} for the structure considered. Comparison with the results of Tothong and Luco [5, Table 2] however illustrates that S_{di} provides little improvement over using S_{de} for structures with fundamental period above 1.5 seconds. Also as careful ground motion selection has been used with respect to ε , then the results presented here for S_{de} are equivalent to somewhere between S_{de} and $S_{de\&\varepsilon}$ presented in Tothong and Luco [5, Table 2]. In such cases an intensity measure, $IM_{1I\&2E}$, which combines information of the inelastic first mode response and elastic higher modes can give better approximations of drift demands [5]. However, as $IM_{1I\&2E}$ combines different mode contributions in terms of spectral displacements, then those spectral displacements due to higher modes are relatively small and it is not likely that $IM_{1I\&2E}$ will be efficient in predicting floor acceleration demands. $IM_{1I\&2E}$ was not considered here as an IM since current ground motion prediction equations for $IM_{1I\&2E}$ require the use of a first-order Taylor-series expansion (which is known to be inaccurate for the large uncertainty in ground motion prediction equations), and the ‘equal displacement’ assumption [34].

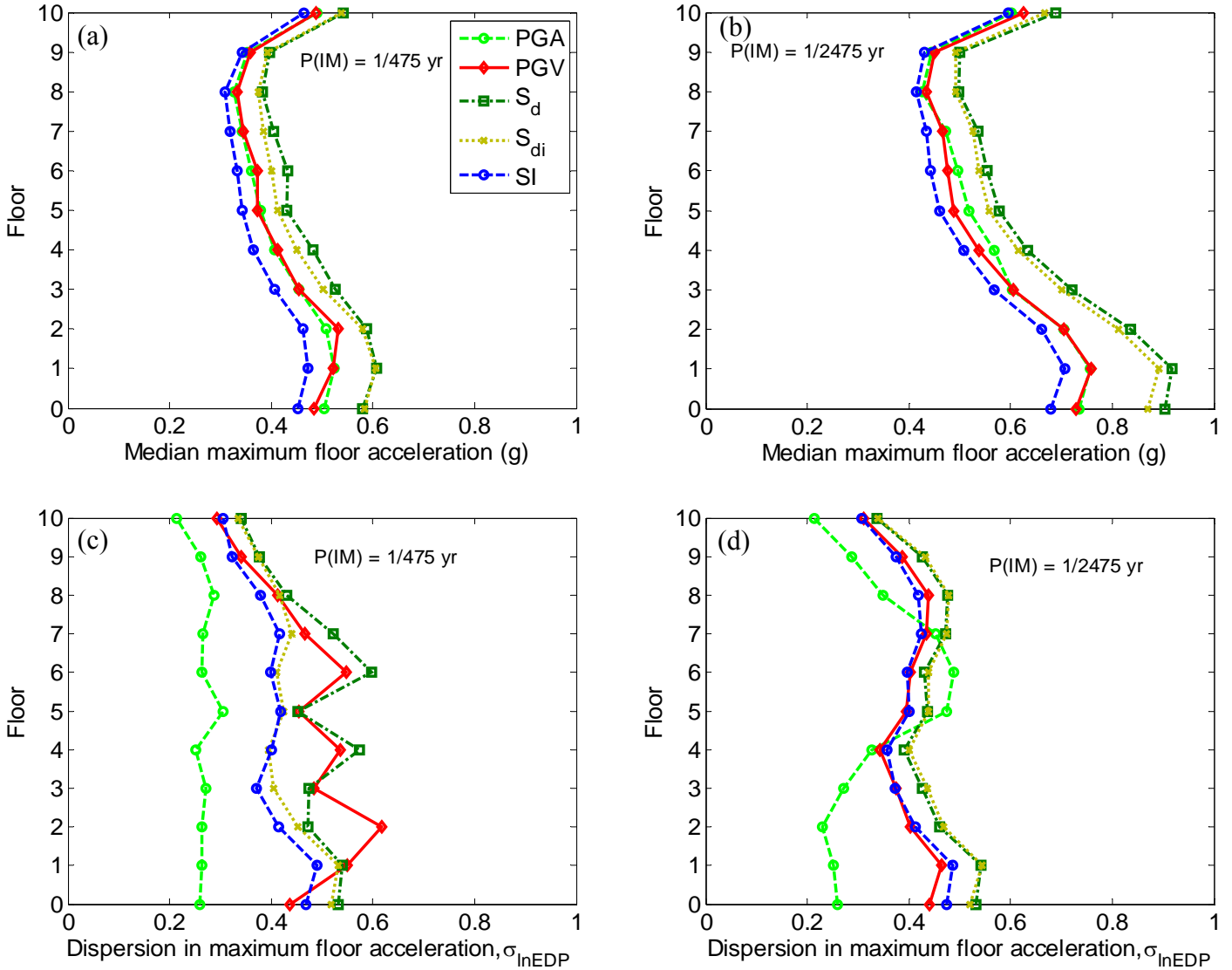


Figure 5: Median and dispersion in the maximum floor accelerations for ground motions scaled to the 1/475 and 1/2475 exceedance probabilities using the various IMs.

Figures 4a, 4b, 5a and 5b have illustrated that for a given exceedance probability, a significant reduction in the median response (specifically for this structure), both interstorey drifts and floor accelerations, can be obtained by using a predictable intensity measure such as *PGA*, *PGV*, and *SI*. This has a direct implication for current code-based applications for time-history analysis which require that the average response be used for design if seven or more ground motion records are used [34]. Note that design codes state that the ‘average’ of the structural responses from the different ground motions should be used, where it is assumed that ‘average’ refers to the arithmetic mean of the responses (and not the mean of the logarithms of the responses used here to get the ‘median’). Since for a lognormal distribution the ratio of the mean to the median is $\exp(\sigma_{\ln X}^2 / 2)$, where $\sigma_{\ln X}$ is the dispersion, then it was found that *PGA*, *PGV* and *SI* give lower ‘mean’ responses than spectral displacements, S_{de} and S_{di} .

Conditional distribution of seismic demand given intensity measure

In order to proceed from seismic response analysis results and compute demand hazard

and loss estimation it is necessary to know the conditional distribution of the demand (EDP) given ground motion intensity (IM), i.e. $f(\text{EDP}|\text{IM})$. Due to the numerous permutations of the 21 different EDPs, five IMs and two different hazard levels considered, no attempt is made here to rigorously illustrate the observed distribution although it is pertinent to discuss such a topic. Reference is given to the numerous studies which have found that interstorey drifts [e.g., 2, 35] and floor accelerations [e.g., 35] are lognormally distributed. Note that Taghavi and Miranda [12] argue that peak floor accelerations are normally distributed, however their comparisons between empirical and analytic distributions were for ground motion IMs with dispersions which were less than 0.3 (in which case the normal and lognormal distributions are similar). Although not explicitly illustrated here, it was found that the conditional demand distribution, $f(\text{EDP}|\text{IM})$ was satisfactorily estimated using the lognormal distribution based on the Kolmogorov-Smirnov goodness-of-fit test [36] at a 5% significance level (i.e. $\alpha=0.05$) for all of the EDP|IM permutations (which as shown in Figures 3-5 included dispersions from 0.25-0.75).

SUFFICIENCY OF INTENSITY MEASURES

The previous two sections have focused on the efficiency and predictability of an IM for estimating the spatially distributed demands in structures. The remaining property of an optimal IM is sufficiency; scaling robustness [5] can be thought of as sufficiency with respect to scale factor. Sufficiency (in this context) relates to the conditional independence of the distribution of EDP given IM on other parameters such as earthquake magnitude (M_w), source-to-site distance (R), and epsilon (ϵ). IM sufficiency is important since the distribution, $f(\text{EDP}|\text{IM})$ is obtained from the results of a finite number of seismic response analyses. Thus if the distribution $f(\text{EDP}|\text{IM})$ is dependent on the M_w , R and ϵ values of the ground motions used, then the distribution will be biased if the distribution of M_w , R and ϵ of the ground motion records used in the seismic response analysis is not the same as that of the ground motions which will occur at the site in the future. Thus, mathematically speaking, sufficiency requires that $f(\text{EDP}|\text{IM}) \cong f(\text{EDP}|\text{IM}, M_w, R, \epsilon)$, where the ‘approximately equals’ sign is intended to mean ‘practically equal to’. In order to test for sufficiency linear regression is typically performed between some property of the ground motion records (i.e. M_w , R , ϵ) and the observed EDPs from the seismic response analysis [3]. Thus the coefficient b from the linear regression line $E[\ln \text{EDP}] = a + bx$ (where x is one of M_w , R , ϵ) indicates the dependence of the observed EDP values on the parameter x . Since the linear regression is based on a finite number of observations it is necessary to use statistical tests to determine the significance of the coefficient b . As an underlying assumption of linear regression is that the observations, $\ln \text{EDP}$, are normally distributed, and since there are a finite number of observations, it follows that the coefficient, b has a student-t distribution and the F-test can be used to determine the statistical significance of b [36]. The F-test gives a ‘p-value’ which is the probability of the coefficient b having a value at least as large as that observed, given that its underlying true value is zero. Typically p-values less than 0.05 are used to indicate a statistically significant value. For example, Figure 6a illustrates the sufficiency of PGV with respect to source-to-site distance, R , for the peak interstorey drift between the 2nd and 3rd floors, while Figure 6b illustrates the sufficiency of S_{de} with respect to epsilon for the peak 2nd floor acceleration (both for the 1/475 exceedance probability). Figure 6a indicates that based on the finite observations the positive correlation between $\ln \text{EDP}$ and R is statistically insignificant (p-value = 0.12 > 0.05), while the negative correlation between $\ln \text{EDP}$ and ϵ in Figure 6b is statistically significant (p-value = $2.4 \times 10^{-6} \ll 0.05$). Because of the numerous permutations for the five different IMs over the 21 different EDPs monitored in the case study

structure, discussion in the remainder of the section is given with respect to p-values directly.

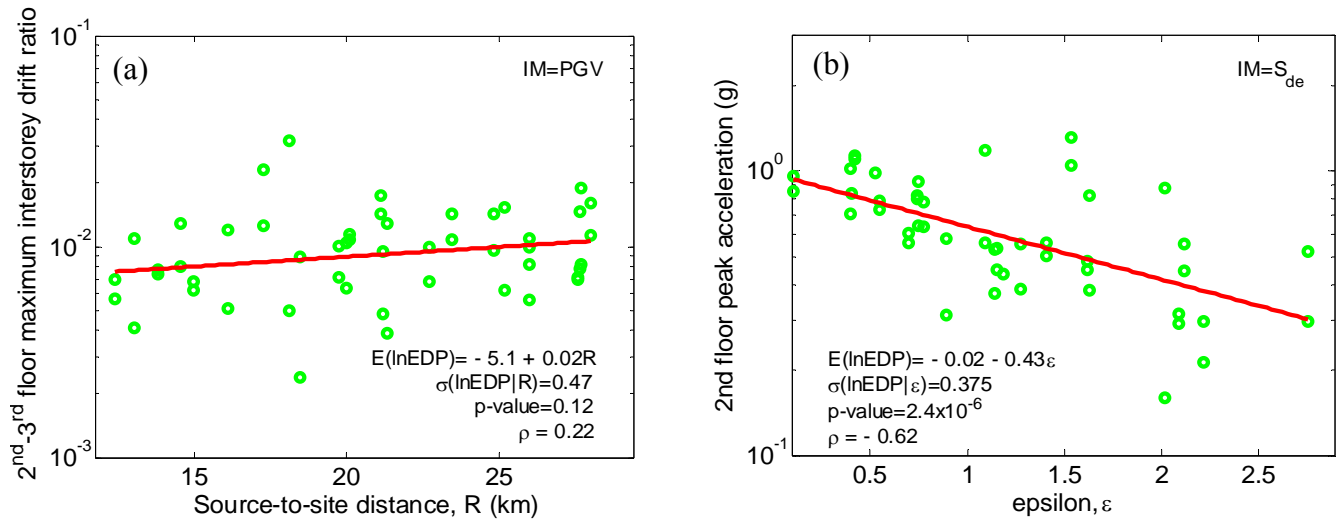


Figure 6: Sufficiency of: (a) peak ground velocity (*PGV*) with respect to source-to-site distance in predicting the peak interstorey drift ratio between the 2nd and 3rd floors; and (b) spectral displacement (*S_{de}*) with respect to epsilon in predicting the 2nd floor peak acceleration.

Sufficiency with respect to magnitude and source-to-site distance

Several of the ground motion IMs used here have been investigated (regarding M_w and R sufficiency) previously for the peak interstorey drift and floor accelerations over all floors and are briefly reviewed here. Aslani [13] showed that for a seven-storey non-ductile frame structure S_{de} is sufficient with respect to M_w and R when used for predicting the maximum interstorey drift on the ground floor and the maximum floor acceleration at the roof level. Aslani [13] however notes that the sufficiency of S_{de} with respect to M_w will decrease as the influence of higher modes increases. This was illustrated by Luco and Cornell [7] who found S_{de} to be sufficient with respect to M_w for a 9-storey structure, but insufficient for a 20-storey structure. Aslani [13] illustrated that an IM equivalent to PGA was insufficient with respect to M_w for predicting maximum interstorey drifts, but is sufficient with respect to R . PGA was also found to be sufficient with respect to M_w and R for predicting maximum floor accelerations. Luco and Cornell [7] illustrate that S_{di} is sufficient with respect to both M_w and R when used for predicting peak interstorey drift ratio over all floors.

The majority of the above trends found by previous research were also observed for the seismic response analysis of the case study structure presented in this manuscript. Figure 7 illustrates the sufficiency of the five different IMs with respect to M_w and R when predicting peak interstorey drifts and peak floor accelerations. Similar to the variation in efficiency of the different IMs over the height of the structure, there is also variation in sufficiency of the IMs for different EDPs. For example, S_{di} is found to be less sufficient with respect to M_w and R in the upper floors of the structure, where higher mode effects are more prominent (as S_{di} cNum>159</RecNum><recor period-lengthening effects). Both S_{de} and S_{di} were found to be sufficient for predicting peak floor accelerations in the structure (having p-values less than 0.05 with respect to R in Figure 7d), which is controlled by higher mode vibrations. PGA is sufficient for both M_w , and R at predicting peak floor accelerations but less sufficient at predicting peak interstorey drifts in the central half (i.e. storeys 2-7) of the structure which is dominated by the first-mode response. The velocity-based IMs (PGV and SI) are generally found to be sufficient with respect to M_w and R for predicting peak interstorey drifts, and relatively sufficient at predicting peak floor accelerations (sufficient with respect to M_w , but

marginally insufficient with respect to R). Thus it appears to be a clear trend that the better the efficiency of an IM at predicting a specific EDP, the better its sufficiency with respect to M_w and R .

To explain the relationship between efficiency and sufficiency described above assume that EDP is a deterministic function of multiple explanatory variables; $EDP = f(\boldsymbol{\theta})$. Uncertainty in the EDP for a given IM occurs because the IM does not account for all of the explanatory variables which influence EDP. Now if a particular IM correlates strongly with a large majority of the explanatory variables, $\boldsymbol{\theta}$, then given IM the uncertainty in EDP will be relatively small (i.e. IM will be efficient at predicting EDP). Because of the strong correlation of IM and many of the explanatory variables, then the response conditioned on IM will already implicitly account for many of the other explanatory variables and hence EDP conditioned on IM will be (practically speaking) independent (i.e. sufficient) with respect to these explanatory variables.

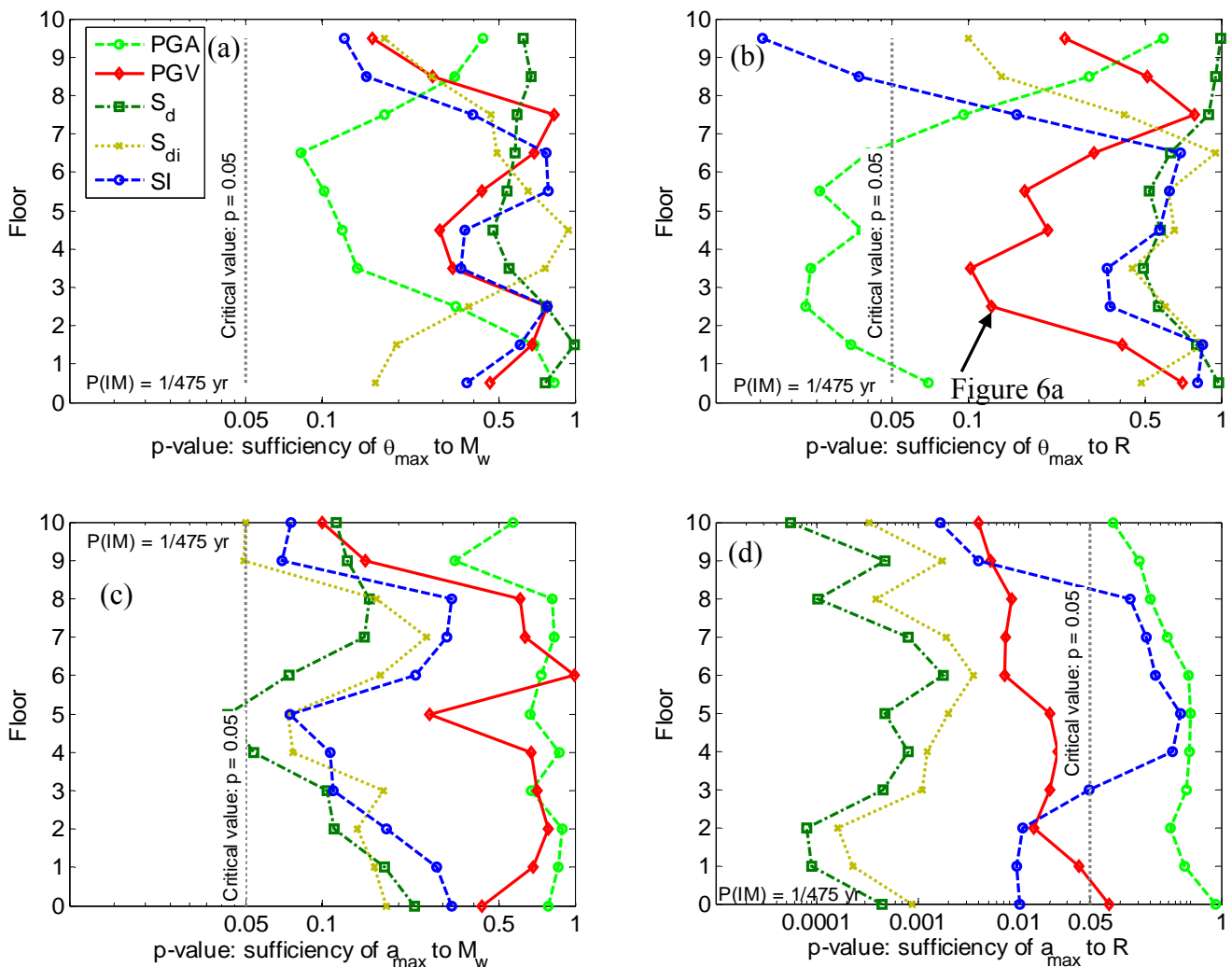


Figure 7: Sufficiency of the various IMs with respect to magnitude and distance when predicting peak interstorey drifts and peak floor accelerations

Sufficiency with respect to ‘epsilon’

As previously noted, the parameter epsilon (ϵ) is defined as the number of standard deviations a specific ground motion parameter is above the mean predicted by a ground

motion prediction equation. Epsilon is one of three parameters which can be obtained from seismic hazard deaggregation (the other two being M_w and R). In particular, because of the partial-correlated nature of spectral acceleration ordinates at different periods, Baker and Cornell [6] have shown that when ground motion records are scaled to a common $S_a(T)$ value (this includes S_{de} and PGA of the IMs examined here), ϵ can be used as a proxy for spectral shape, and therefore the extent to which higher-modes and period elongation will affect the seismic response for a given ground motion. Tothong and Luco [5] illustrate why S_{di} accounts for spectral shape at periods longer than that which the ground motions are scaled to (but not for spectral shape at higher mode periods). While Baker and Cornell [6] and Tothong and Luco [5] focus on the effect of epsilon when investigating peak interstorey drift ratios, the same logic can be directly applied to its effect on peak floor accelerations.

Figure 8a illustrates the sufficiency of the five IMs with respect to ϵ , when predicting peak interstorey drifts. Since when ground motion records are scaled to PGA , ϵ (being an indicator of spectral shape) will directly relate to the magnitude of the response spectra at longer periods, then PGA is insufficient with respect to ϵ for predicting interstorey drifts (which are mostly dominated by first mode vibration). Similarly, S_{de} and S_{di} are insufficient with respect to ϵ in predicting peak floor accelerations and peak interstorey drifts in upper floors which are dominated by higher mode vibration. PGV and SI are found to be sufficient to ϵ for predicting peak interstorey drifts, but insufficient for peak floor accelerations, as illustrated in Figure 8b.

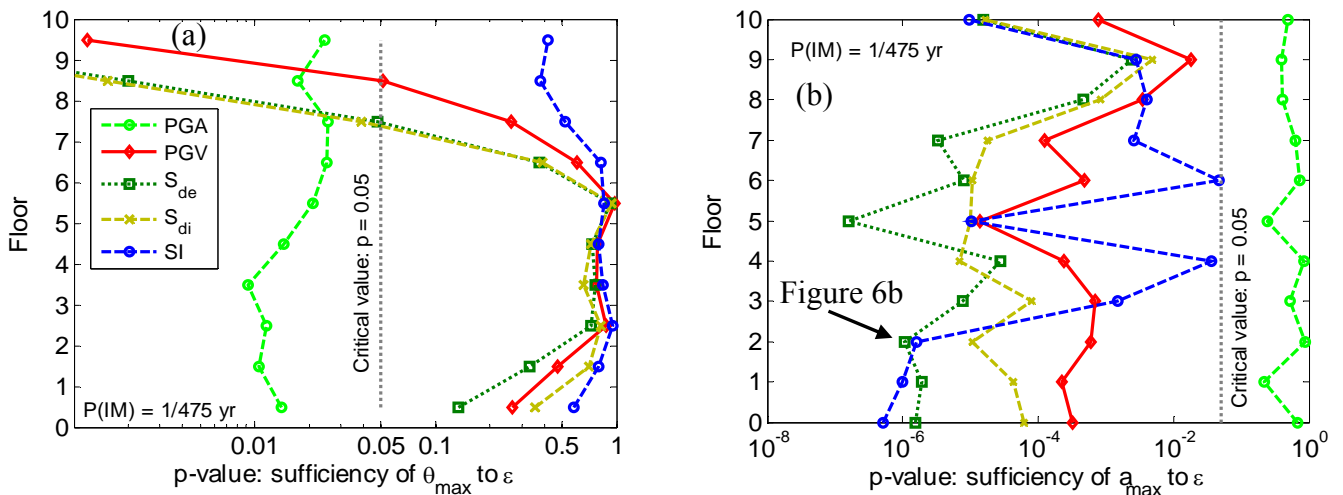


Figure 8: Sufficiency of the various IMs with respect to epsilon, ϵ

Sufficiency with respect to scale factor (SF)

The use of linear scaling (i.e. scaling amplitude without modifying frequency content or duration) ground motion records to a specific IM level has been scrutinized for introducing bias compared with some ‘true’ response that would be obtained using un-scaled ground motions. Luco and Bazzurro [11] have illustrated that when using S_{de} as an IM, scale factors that are significantly different from 1.0 can introduce significant bias in seismic response analysis. Baker [37] has shown however that careful ground motion selection (similar to what is used here) can significantly reduce such bias. Figure 9 illustrates the sufficiency of the five IMs with respect to SF for predicting peak interstorey drifts and peak floor accelerations. It can be seen that despite PGA for peak interstorey drifts, and PGV for peak floor accelerations, there is little dependence of the IMs on SF. As mentioned above this is likely the result of ground motion selection based on deaggregation of the seismic hazard. Also, because of the

ground motion selection employed here, the scale factors required to scale the ground motions are not significantly large, with means of 2.1 and 3.2 (averaged over all five IMs) and ranges of 0.6-5.7 and 0.81-9.1 for the 1/475 and 1/2475 exceedance probabilities, respectively.

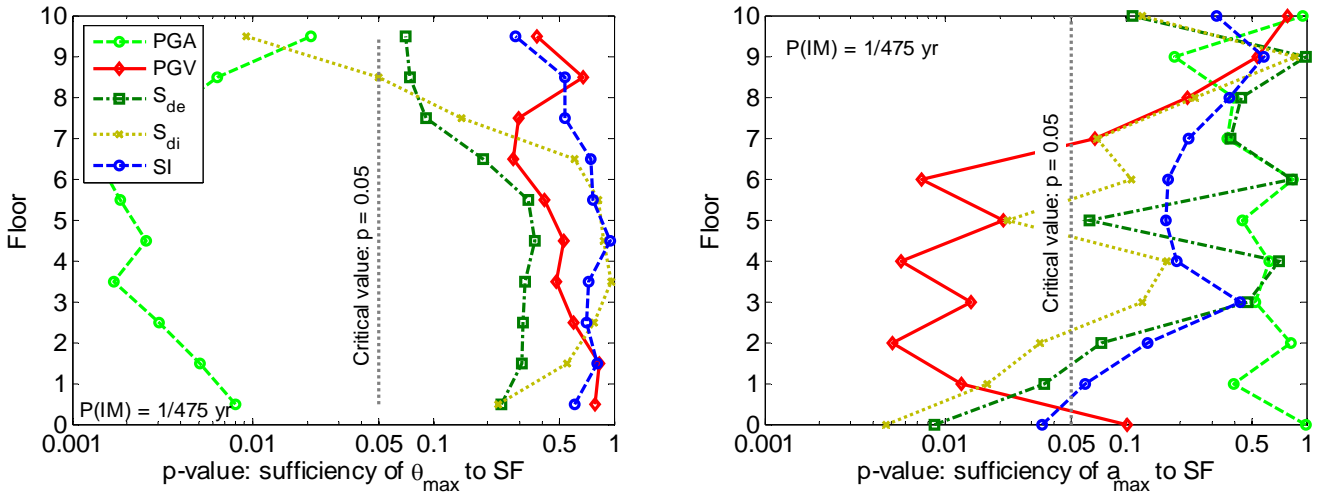


Figure 9: Sufficiency of the various IMs with respect to scale factor, SF

Sufficiency with respect to site shear wave velocity, V_{S30} .

With contemporary ground motion prediction equations [e.g., 20] giving soil site classification in terms of a 30-m averaged shear wave velocity, V_{S30} , as opposed to a qualitative alphabet-based classification, it is possible to investigate the sufficiency of the various IMs with respect to V_{S30} . Figure 10 illustrates the sufficiency of the IMs investigated with respect to V_{S30} for both peak interstorey drifts and peak floor accelerations. In general all IMs are sufficient with respect to V_{S30} . It is also worth noting that the considered range $V_{S30} = 300 - 800$ m/s represents relatively stiff soils. Further studies are needed to investigate the sufficiency with respect to V_{S30} for soft soil deposits ($V_{S30} < 300$ m/s) and significant nonlinearity in the soil response

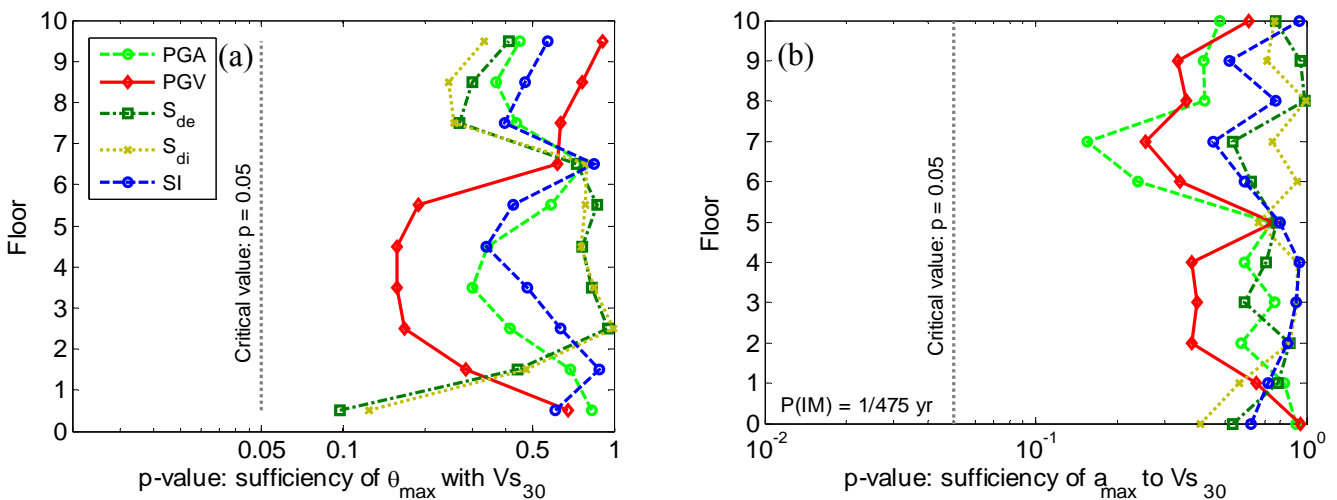


Figure 10: Sufficiency of the various IMs with respect to 30m-averaged site shear wave velocity, V_{S30} .

‘Correcting’ seismic demand distributions

The aforementioned results have clearly illustrated that no single IM is sufficient with respect to M_w , R , ε , for all drift and floor acceleration demands in the case study structure. Therefore bias will be introduced in the computed response when the (M_w, R, ε) distribution of the ground motion suite is different than that of the hazard deaggregation when predicting EDPs using an insufficient IM. It was also noted that it may be difficult to obtain a desired number of ground motion records which match the hazard deaggregation. In such cases, Bradley *et al.* [23] show that it is possible to regress on the results of the seismic response analyses to ‘correct’ the resulting distribution of EDP to reflect the M_w, R, ε distribution from hazard deaggregation.

An alternative option is to avoid the “IM approach” and simply develop a prediction equation for the vector of EDPs directly as a function of ground motion parameters (e.g. magnitude, distance) (e.g. [38]). While this approach will not suffer from some of the problems of the IM approach discussed in this manuscript it should be made clear that the development of such a prediction equation for a vector of EDPs is complex, requiring significantly more seismic response analyses, complex regression analyses, expertise in strong ground motion modelling and access to ground motion metadata and earthquake fault databases. The IM approach bypasses these difficulties by separating seismic hazard analysis and seismic response analysis. Both approaches have their respective pros and cons and will thus be useful in different situations.

CONCLUSIONS

Prediction of the seismic response of multi-degree-of-freedom structures is a complex task due to the spatially distributed seismic demands which are sensitive to different frequency contents of the imposed ground motion. This manuscript investigated the efficacy of five different ground motion intensity measures (IMs): peak ground acceleration, PGA ; peak ground velocity, PGV ; elastic and inelastic spectral displacement, S_{de} , S_{di} ; and spectrum intensity, SI , for which robust ground motion prediction equations are available. The concepts of predictability, efficiency, and sufficiency were investigated for each of the IMs when applied to the seismic response analysis of a 10-storey RC frame structure. It was illustrated that the efficiency (uncertainty in seismic response prediction) of an IM can be qualitatively determined based on the frequency range of the ground motion which controls the IM to that which controls the EDP being monitored. The predictability (uncertainty in ground motion prediction) of an IM was clearly shown to be an important factor in reducing the median response demand for ground motions scaled to an IM with a given probability of exceedance. Ground motion IMs which are predictable (namely PGA , PGV and SI), result in lower median seismic demands when ground motions are scaled to a specific probability of exceedance using a seismic hazard curve. The sufficiency of the IM (the dependence of the seismic response on parameters such as M_w , R , and ε) was found to be closely related to the efficiency of the IM (the more efficient the IM, the higher the sufficiency), and it was observed that all IMs were insufficient with respect to at least one of M_w , R , or ε when predicting both peak interstorey drifts and peak floor accelerations. As a result of the insufficiency of an IM, careful ground motion selection (compatible with the hazard deaggregation) and/or appropriate response modification is therefore needed to reduce bias and dispersion.

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