STABILITY SAFETY MARGIN BASED DESIGN OF LOW NOISE MICROWAVE AMPLIFIERS

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Abstract

In low noise microwave amplifier design, particularly when using conditionally stable transistors, it is impossible to simultaneously achieve maximum gain, minimum noise figure and maximum port match. Hence it is necessary to trade-off these performance parameters which is usually accomplished with the aid of stability, gain, noise figure and mismatch (or VSWR) circles described on the reflection coefficient planes. However, use of these graphical aids is often subjective, necessitating final optimisation using circuit design software. In this paper, we propose a different approach whereby the functional behaviour of two objective functions involving gain, noise figure and port mismatch are described as functions of the stability safety margin. This approach leads to an optimum design without requiring sophisticated optimisation algorithms and includes stability safety margin as a design parameter.

I. Introduction

When designing low noise microwave amplifiers, it is the task to select suitable values for the source (input generator) and load reflection coefficients for the transistor. The source and load reflection coefficients are chosen to achieve certain specifications for the amplifier performance parameters, such as gain, noise figure, port match, as well as ensure stability. Normally, it is impossible to simultaneously achieve maximum gain, minimum noise figure and maximum port match; especially when using conditionally stable transistors. Hence a trade-off of these performance specifications is necessary [1].

The low noise amplifier design typically begins with the selection of source reflection coefficient so that a trade-off between noise figure and available gain can be made [1] and conjugate matching can be used at the output [2]. Available gain circles and noise figure circles described on the Γ_s plane are used to aid the selection of the source reflection coefficient [1]. Safety margins between the chosen values of source and load reflection coefficients and their respective stability circles are included to allow for manufacturing variations [3][4]. Although the resulting amplifier is matched at its output port,

the mismatch at the input port is typically severe and necessitates the trade-off of output match for improved input match. Circles of constant mismatch or VSWR [1] can be described on the load reflection coefficient plane to aid this trade-off. The attractiveness of these graphical design methods is that the functional behaviour of gain, noise figure and mismatch with respect to the source and load reflection coefficients is obtained by simply plotting circles whose centres and radii need only be calculated. A draw-back of this approach is that the selection of the source and load reflection coefficients is subjective, particularly when the contours of more than two performance parameters are plotted on the same reflection coefficient plane. "Trial and error" is often required to obtain acceptable performance, and inevitably, circuit optimisers are required.

Other graphical methods have also been proposed recently [5] - [7] to aid the design procedure. Poole & Paul [5], develop expressions for constant noise measure circles which can be used to design low noise amplifiers and considers both noise and gain simultaneously without having to separately plot noise and available gain circles. Albinsson [6] investigated the mapping of noise circles onto the load plane to result in an LNA with matched input. Link & Gudimetla [7] provide explicit design formulae for low noise amplifiers but do not consider stability.

In this paper, we propose a different approach whereby the functional behaviour of noise measure and port mismatch are described as functions of the stability safety margins rather than direct functions of the source and load reflection coefficients. As we shall see, this approach results in improved optimisation without requiring sophisticated circuit optimisers. Further, as the stability safety margins are used as design parameters, stability is directly considered without having to separately plot stability circles. In this work, we will assume that the magnitudes of the transistor S-parameters S_{11} and S_{22} are both less than unity and that the stability circles do not enclose the origin. These assumptions are valid for many microwave transistors used in amplifier circuits.

II. Definitions

In the work that follows, the definitions of the various reflection coefficients refer to the single-stage amplifier depicted in Fig. 1. The transistor is described by its S-parameters and noise parameters. The effective source and load reflection coefficients presented to the input and output ports of the transistor are Γ_S and Γ_L respectively. Γ_S and Γ_L typically result from transforming the external 50 Ω source and load impedances via lossless networks. Γ_1 is the reflection coefficient looking into the input port of the transistor and is a function of Γ_L via the well known bilinear transformation [1]. Similarly Γ_2 is the reflection coefficient looking into the output port of the transistor and is a function of Γ_S via the well known bilinear transformation [1].
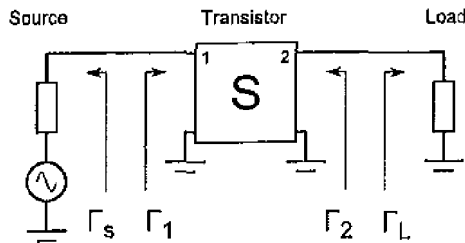

Fig. 1. Microwave amplifier circuit.

In general, Γ_S and Γ_1 are mismatched with mismatch factor M_1:

$$M_1 = \frac{\left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_1|^2\right)}{|1 - \Gamma_S\Gamma_1|^2} \qquad \ldots 1$$

and Γ_L and Γ_2 are mismatched with mismatch factor M_2:

$$M_2 = \frac{\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_2|^2\right)}{|1 - \Gamma_L\Gamma_2|^2} \qquad \ldots 2$$

The available gain G_A and noise figure F are functions of Γ_S via the well known formulae [1]. The centres and radii of the stability circles, and circles of constant available gain, noise figure, and mismatch are also given by well known formulae [1]. These circles are plotted so that the functional behaviour of the circuit with respect to Γ_S and Γ_L may be ascertained. It is of use to consider the noise measure which incorporates both G_A and F:

$$NM = \frac{F - 1}{1 - \dfrac{1}{G_A}} \qquad \ldots 3$$

The centres and radii of constant noise measure are available [5] but are generally unwieldy.

In the presence of statistical variation of both transistor and circuit parameters, it is important when selecting Γ_S and Γ_L that an adequate safety margin with respect to their respective stability circles be maintained. Eccleston [3][4] has previously developed equations for maximum gain design using conditionally stable transistors and defined a new performance parameter, the stability safety margin. Let C_S and r_S be the centre and radius of the source stability circle, and C_L and r_L be the centre and radius of the load stability circle. The source safety margin (SSM) is the distance between the nominal (and allowable) value of Γ_S and the source stability circle. Likewise, the load safety margin (LSM) is the distance between the nominal (and allowable) value of Γ_L and the load stability circle. A contour of constant SSM is a circle with centre C_S and radius $r_S + SSM$ when the origin is stable and outside the source stability circle [4]. Similarly for contours of constant LSM.

III. Development

In the approach that we propose the functional behaviour of noise measure and port mismatch are described as functions of the stability safety margins rather than direct functions of Γ_S and Γ_L. Since we assume that the transistor S-parameters S_{11} and S_{22} are both less than unity and that the stability circles do not enclose the origin, we can write Γ_S and Γ_L in terms of stability circle centres and radii, SSM and LSM :

$$\Gamma_S = C_S + (r_S + SSM)\, exp(j\theta) \qquad \ldots 4$$

$$\Gamma_L = C_L + (r_L + LSM)\, exp(j\phi) \qquad \ldots 5$$

where θ and ϕ are additional design parameters. With Γ_S and Γ_L described in this manner, G_A and F can be described as functions of SSM and θ. For a given value Γ_S, M_1 and M_2 can be described as functions of LSM and ϕ. It is evident that SSM and LSM are design parameters as well as performance parameters.

To further simplify matters, two objective functions can be described. One of these being the noise measure and is described as a function of SSM and θ :

$$F_1(SSM,\theta) = \frac{F(SSM,\theta) - 1}{1 - \dfrac{1}{G_A(SSM,\theta)}} \qquad \ldots 6$$

The other being the product of M_1 and M_2 and is a function LSM and ϕ, for a given value of Γ_S:

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$$F_2(LSM, \phi) \quad = \quad M_1\left[\Gamma_S, LSM, \phi\right] M_2\left[\Gamma_2(\Gamma_S), LSM, \phi\right]$$

$$...7$$

where Γ_S is given by (4) after SSM and θ have been determined using (6). The expressions for G_A, F, M_1 and M_2 are well known and can be found in various texts [1]. Clearly we need to minimise F_1 and maximise F_2 for the constraint that SSM and LSM are both greater than zero (or required safety margin), and the magnitudes of the resulting Γ_S and Γ_L are both less than unity.

IV. Numerical Example

To illustrate the application of (4) to (7), we can consider an example of an LNA design that employs a transistor whose S-parameters and noise parameters are:

$$S_{11} = 0.809 \ \underline{/-148^\circ} \qquad S_{12} = 0.111 \ \underline{/-4^\circ}$$
$$S_{21} = 1.940 \ \underline{/\ 57^\circ} \qquad S_{22} = 0.634 \ \underline{/-48^\circ}$$
$$\Gamma_{Sopt} = 0.50 \ \underline{/\ 130^\circ} \qquad F_{min} = 0.97 \ \text{dB}$$
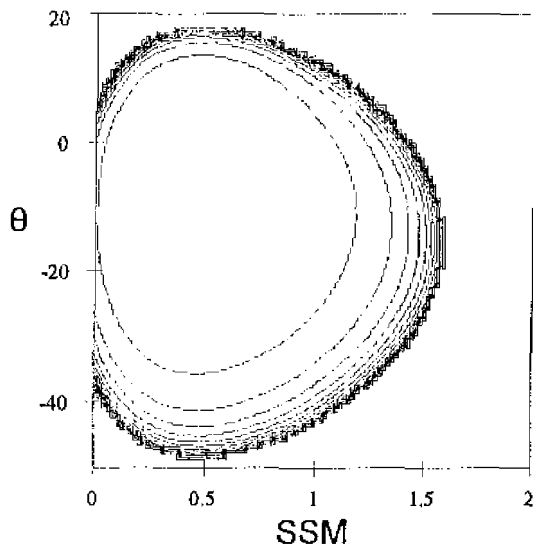$$R_N = 2 \ \Omega$$

Fig 2. F_1 as a function of SSM and θ (deg).

Fig. 2 shows a plot of F_1 as a function of SSM and θ (for $|\Gamma_S| < 1$). The minimum of F_1 was located by a direct search of the SSM, θ plane with a grid size of 0.02 in the SSM direction and 0.7° in the θ direction. The minimum of F_1 was found to

be located at SSM equal to 0.46 and θ equal to -4.5° and results in Γ_S being equal to 0.52 $\underline{/136^\circ}$, G_A equal to 10.3 dB and F equal to 0.97 dB.

With Γ_S equal to 0.52 $\underline{/136^\circ}$, conjugate matching can be used at the output and results in an LSM of 0.33; however the input mismatch is 0.571. ie only 57.1% of the available power from the input generator is delivered to the input of the transistor. We can however, plot F_2 as a function of LSM and ϕ with Γ_S equal to 0.52 $\underline{/136^\circ}$ as depicted in Fig. 3. The maximum of F_2 was located by a direct search of the LSM, ϕ plane with a grid size of 0.02 in the LSM direction and 0.05° in the ϕ direction. The maximum of F_2 was found to be located at LSM equal to 0.54 and ϕ equal to -84.6° and results in Γ_L being equal to 0.47 $\underline{/35.9^\circ}$. Under this condition, M_1 and M_2 are 0.786 and 0.903 respectively and has resulted in a 1.39 dB improvement in the input match at the expense of only 0.44 dB deterioration in the output match.
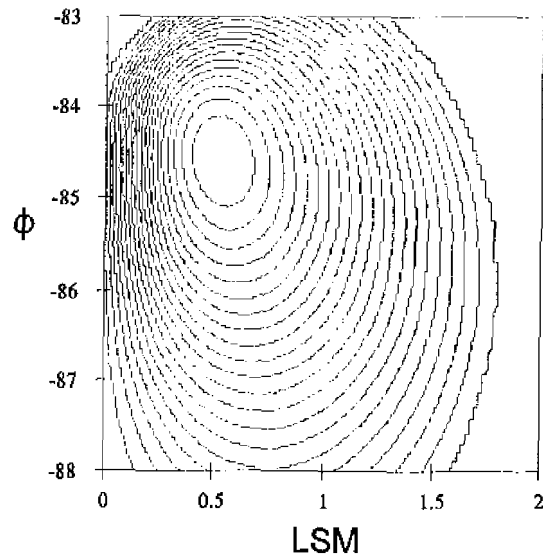

Fig. 3. F_2 as a function of LSM and ϕ (deg).

Figs. 2 and 3 clearly show that the behaviour of F_1 and F_2 allows their minimisation and maximisation respectively to be achieved without resorting to sophisticated optimisers. Indeed, location of the minimum and maximum of F_1 and F_2 respectively only took a few seconds on a Pentium II. Since SSM and LSM are design parameters, stability margin is automatically included in the design process without plotting stability circles. So long as SSM and LSM are greater than

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zero, then Γ_S and Γ_L respectively are within the stable their regions.

We could have subjected the optimisation to constraints on *SSM* and *LSM*, of which Figs. 2 and 3 indicate the ease with which this could have been done. In this case, one would only need to perform a one-dimensional search for θ and ϕ.

One will observe looking at Figs. 2 and 3 that the ranges of θ and ϕ are both narrow. The reason stems from the fact that these angles are with respect to the stability circle centres. Only small ranges of θ and ϕ are necessary to *sweep* the entire unit circle. This range decreases with increasing magnitudes of stability circle centres.

V. Conclusion

In this paper we have shown a method for selecting the values of the source and load reflection coefficients to achieve minimum noise measure, and optimum input and output match. This method is based on objective functions that are described in terms of stability safety margins, thereby incorporating stability into the design parameters rather than consider them separately. The objective functions are straightforward to optimise without sophisticated optimisers.

Work is continuing to extend this design approach to more generality as well as efficient means to locate the optimum values *SSM, LSM,* θ and ϕ.

References:

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