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A LINEAR PROGRAMMING MODEL FOR ECONOMIC PLANNING IN NEW ZEALAND

by

T. R. O'Malley

Technical Paper No. 16

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PREFACE

The National Development Conference focused attention on indicative planning of the New Zealand economy. The Department of Agricultural Economics and the Agricultural Economics Research Unit at Lincoln were responsible for formulating an inter-sectoral model of the New Zealand economy which formed the basis of policy projections by the Conference.

Mr O'Malley has carried this work further by fitting the model into a linear programming framework, and this bulletin outlines his results.

J. D. Stewart
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July 1972

A LINEAR PROGRAMMING MODEL FOR ECONOMIC PLANNING IN NEW ZEALAND

INTRODUCTION

A good deal of research into the likely future structure of the New Zealand economy has been carried out in the Agricultural Economics Research Unit. The aim has been to provide realistic, quantitative sectoral targets or guide-lines to centralised policy-making bodies to assist in planning future economic growth in New Zealand. This type of exercise has often been referred to as "indicative planning". Until now, the work has entailed the use of an input-output projection model which has come to be known as "The Lincoln Model". The rationale behind the lincoln model is described in Philpott and Ross [10,11] and Ross and Philpott [13]. Briefly, the procedure is to calculate for some future year an economic structure which satisfies the interindustry relationships and which achieves an exogenously specified increase in the base year consumption level. "Economic structure" in this context means: the level of output of each sector of the model, the level of exports from each sector, the level of investment by each sector, and the level of importing of current and capital goods by each sector. Whenever the Lincoln model has been discussed there has usually been some mention of "the optimum economic structure". It has been said that the structure is optimum when "resources are so allocated between sectors that the highest level of net national product per head is achieved, consistent with the maintenance of overseas balance of payments equilibrium, full employment and a reasonable growth in incomes per head".[1]

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[1] Philpott & Ross [10], p.15.

While many would - justifiably - question this definition, it is probably a reasonable basis on which to begin investigations into the best future shape of the economy and it is certainly where scrutiny of the projected structure should begin.

It has also been suggested that "the most efficient method of investigating the nature of an optimum structure .... is by the use of mathematical programming methods".[1] The purpose of this paper is to demonstrate how the linear programming technique might be used to calculate the optimum economic structure, although it has been found necessary to modify the definition quoted above. Instead of accepting an exogenous target for consumption, programming is used to calculate the maximum level of consumption consistent with the interindustry relationships and resource availabilities. The need to formulate linear functions has prevented optimisation of consumption per head which would be more acceptable theoretically.

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[1] Ibid. p.26.

LINEAR PROGRAMMING AND MACROECONOMICS

There are now many examples of the use of linear programming in models for national economic planning, although the first examples appeared during the mid-1950s. A few such studies are cited in the list of references at the end of this paper.

While the need to preserve linearity is restrictive the technique offers considerable flexibility and the analyst can usually design his model to suit the particular characteristics of the economy he is dealing with. For instance, Chenery and Kretschmer [3] use an objective function which minimises total capital expenditure in their model for Southern Italy because this is the most pressing problem; Manne [5] includes only "key sectors" in his model for Mexico and the objective is the minimisation of overseas borrowing - the outputs of non-key sectors are treated as exogenous; Blyth and Crothall [1] suggest the maximisation of consumption for New Zealand as this is the aggregate most indicative of economic progress for a country at this stage of development. The need for flows of products and resources to balance is common, however, and provided fixed input structures for activities can be assumed to be valid for at least a range of activity levels, linear programming provides a convenient medium to express these relationships.

A general account of the formulation and meaning of linear programming models based on input-output data is given in Chenery and Clark [2] (Chapters 4, 11). The principal difference between Leontief type input-output models (of which the Lincoln projection model is an example) and the programming models now under discussion is that the assumption of each sector producing a distinct and homogeneous "commodity" can be relaxed. Hence it is helpful to refer to "activities" rather than "sectors" or "industries" in programming models. It becomes possible to specify that a commodity may be produced in a number of different ways, i.e. by alternative activities. These may be technological alternatives (e.g. labour intensive versus capital intensive) or alternative sources of supply (e.g. domestically produced goods versus imported goods). In addition, it is possible to approximate increasing marginal costs for production activities and decreasing marginal returns for exporting activities by specifying maximum levels at which relevant activities can be included in the solution. Greater levels of output or exporting can be achieved only by activities with a heavier requirement of resources or lower remuneration. This is evidence that linear programming has something positive to offer to the analysis of real macroeconomic problems.

A summary of linear programming planning models follows:

(1) Variables defined: production activities, investment activities, importing activities, exporting activities, consumption and government expenditure activities.

(2) Objective function: must be a linear function of the activities defined; e.g. maximisation of national product, consumption, or a linear combination of consumption and investment (indicative of wealth); or minimisation of investment activities (given a constraint for minimum consumption) or foreign exchange deficit.

(3) Linear constraints for each commodity defined: Supply $\geq$ Demand

(4) Linear constraints of resource availability: Resource available + additional supply $\geq$ demand for resources.

(5) Other linear constraints:

Foreign exchange supply (exports) + maximum deficit $\geqslant$ Demand for foreign exchange (imports).

Level of consumption $\geqslant$ specified minimum.

The set of equations defined in (3) will usually be based on interindustry coefficients and will constitute the main part of these models. The set of equations defined in (4) will normally apply to the availability of capital stocks, labour, savings or any primary resource which is limited in supply and has to be allocated between the activities defined.

BLYTH-CROTHALL MODEL, 1965

A brief description of the work of Blyth and Crothall [1] is warranted as their model concerns New Zealand and has considerably influenced the formulation of the present model. Later the two models are compared.

There are four kinds of activity in the Blyth-Crothall model:

(a) Current production activities

(b) Exporting activities

(c) Importing activities

(d) A consumption activity

Investment in the economy is accounted for by providing two types of current production activity - production using existing plant and production using new plant. Production from existing plant is restricted in each sector by a capacity measure representative of the capital stock available for use; the coefficients for these activities are current input requirements per unit of output. The coefficients for production from new plant account for the current input requirements plus the inputs of capital goods required to make a unit of capacity available to that sector. The authors explain that the activities representing production from new capital are composite activities. Consider such an activity, A_{20}, this can be regarded as a combination of two activities A'_{20} and A''_{20} by the relation,

$$A_{20} = A'_{20} + k\, A''_{20}$$

where,

A'_{20} is the level of current production using new plant,

A''_{20} is an investment or capital formation activity which produces the appropriate plant and buildings for A'_{20},

k is the capital-output ratio which states how many units of investment are required to provide capital with the capacity to produce one unit of A'_{20}.

This definition ignores the time lag between capital formation and its use, but this was not considered a serious drawback to the model's ability to give a reasonable indication of the most desirable economic structure.

The exporting, importing and consumption activities follow the pattern of most other studies in this field. However, it is worth noting that to earn a unit of foreign exchange output is required from sectors other than the one producing the commodity being exported. This is to account for ancillary internal costs specifically associated with the activity of exporting, e.g. transport, wharf handling expenses, insurance costs, storage. The model is an annual model so that all variables are defined as levels or amounts for a particular year, and the programme maximises the amount of consumption in that year.

The constraints to the linear programming problem also follow the accepted pattern. There are production reconciliation rows, a foreign exchange row, a labour constraint, a land constraint, and maximum limits on the levels of exporting activities. In addition the output in each sector is restricted by the size of a stock of capital. When this stock is exhausted (it is "used" at the rate determined by a capital-output ratio for each sector) further output of the commodity in question is possible only from activities using new capital.

Total investment (or, in terms of the variables of the model, the activities representing production from new capital) is restricted by the availability of "waiting".[1] This concept is intended to combine

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[1] The concept of "waiting" is rather abstract and is dealt with sparingly in Blyth & Crothall's paper. The present description should be regarded as this author's understanding of it.

the current cost and time (discounting) aspects of investment expenditure. Waiting is measured in years and when multiplied by its price (dollars per year) gives the total interest cost of new investment. The price of waiting is regarded as a constant and is of the nature of an average interest rate, or a rate of preference between a dollar's worth of consumption now and a dollar's worth of consumption a year later. Waiting is therefore representative of the greater opportunity costs (discounted future consumption sacrificed) or "loss" of present value of longer term investments: a large investment for which the payback is quick may have a larger present value than a much smaller investment which does not yield positive cash flows until a considerable time has elapsed. The total amount of waiting available in a given year should be related to the economy's ability to finance new investment and service the interest costs until the capital goods purchased can "pay their own way". The concept is akin to the identity of static economic theory that total savings equals total investment; but it has an additional dynamic attribute which accounts for the fact that investments in different projects have different payback times as well as different initial lump sum purchase requirements.

The data for the model comes mainly from the 1954/55 interindustry study carried out by the New Zealand Department of Statistics [8], which is rather limited in its scope, and the authors give their model the status of "a pilot programming model". Nevertheless they define alternative activities by making arbitrary adjustments to the basic data and thus display clearly the flexibility linear programming gives to the national economic planner. Hence allowance is made for increasing marginal costs in farming, and choice is available between capital intensive and labour intensive technologies for the production activities using new capital.

DEFINITION OF VARIABLES

The following definitions and symbols apply to the variables of the present model:

N - the number of activities

R - the number of sectors (commodities)[1]

P_j - the value in constant prices of the output of activity j in the target year;

$j = 1, 2, \ldots, N$

I_j - the value in constant prices of net investment purchases by activity j in the target year;

$j = 1, 2, \ldots, N$

C - the total value in constant prices of consumption plus government expenditure in the target year;

E_j - the level of exporting, valued in constant domestic prices, of the output of sector j in the target year;

$j = 1, 2, \ldots, R$

M - the number (thousands) of immigrants during the planning period.

a_{ij} - the output of sector i required per unit of output of activity j,

$i = 1, 2, \ldots, R$
$j = 1, 2, \ldots, N$

a_{mj} - the level of importing, valued in constant domestic prices, required per unit of output of activity j,

$j = 1, 2, \ldots, N$

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[1] In this paper one commodity is associated with one sector as in a Leontief model. However, a sector may be composed of several activities each producing the same commodity. Thus $N \geqslant R$.

b_{ij} - the output of sector i required per unit of net investment by activity j,

$i = 1, 2, \ldots, R$
$j = 1, 2, \ldots, N$

b_{mj} - the level of importing, valued in constant domestic prices, required per unit of net investment by activity j,

$j = 1, 2, \ldots, N$

c_i - the output of sector i required per unit of consumption plus government expenditure,

$i = 1, 2, \ldots, R$

c_m - the level of importing, valued in constant domestic prices, required per unit of consumption plus government expenditure.

e_{ij} - the output of sector i required per unit of exporting of the output of sector j,

$i = 1, 2, \ldots, R$
$j = 1, 2, \ldots, R$

k_j - capital output ratio appropriate to activity j,

$j = 1, 2, \ldots, N$

g_i - capital formation required by activity i per unit of immigration,

$i = 1, 2, \ldots, N$

d_j - depreciation is a proportion of total output for activity j, $j = 1, 2, \ldots, N$

s - amount of savings achieved per unit of consumption plus government expenditure

l_j - labour-output ratio appropriate to activity j in the target year (inverse of labour productivity ratio),

$= 1, 2, \ldots, N$

K_i - the capital stock available for production by activity i at the beginning of the planning horizon,

$i = 1, 2, \ldots, N$

Q_i - maximum level of exporting of the output of sector i in the target year,

$i = 1, 2, \ldots, R$

D - maximum deficit in current account of overseas transactions in the target year.

L - the projected labour force available in the target year assuming zero net annual immigration.

ALGEBRAIC STATEMENT OF MODEL

Objective function: maximise $Z = C$

Restrictions:

(1) Reconciliation of current production,

$$0 > \sum_{j}^{N} (a_{ij} - S_{ij})P_j + \sum_{j}^{N} b_{ij} I_j + \sum_{j}^{R} e_{ij} E_j + c_i C,$$

$i = 1, 2, \ldots, R$

S_{ij} is the Kronecker delta.

(2) Reconciliation of capital stocks,

$$K_i > k_i P_i - 6.6667\, I_i + g_i M,$$

$i = 1, 2, \ldots, N$

(3) Reconciliation of overseas exchange transactions,

$$D > \sum_{j}^{N} a_{mj} P_j + \sum_{j}^{N} b_{mj} I_j + C_m C - \sum_{j}^{R} E_j,$$

(4) Savings reconciliation,

$$0 > \sum_{j}^{N} d_j P_j + \sum_{j}^{N} I_j - s C,$$

(5) Labour force reconciliation,

$$L > \sum_{j}^{N} l_j P_j - .5 M,$$

(6) Maximum exporting restrictions,

$$Q_i > E_i,$$

$i = 1, 2, \ldots, R$

(7) Non-negativity requirements,

$$P_j, I_j, E_i, M, C > 0$$

$j = 1, 2, \ldots, N$

$i = 1, 2, \ldots, R.$

EXPLANATION OF THE MODEL

As will have been observed from the above definitions there are five groups of variables in the model:

(a) Current production activities.

(b) Net investment activities.

(c) A consumption activity.

(d) Exporting activities.

(e) An immigration activity.

The coefficients for the current production activities are derived from input-output data and are adjusted so that they include expenditure on the replacement of worn or obsolete capital equipment.[1] Hence the investment activities refer to net rather than gross investment and the whole of the investment purchases can be considered to be added to the capital stock. The consumption activity assumes fixed consumption proportions and the exporting activities are similar to those used by Blyth and Crothall. An immigration activity is included as a means of augmenting the labour supply; the activity has capital coefficients to account for the housing and public facilities (hospitals, schools etc.,) required by the additional population. There is no need to have coefficients for other capital requirements caused by immigrants such as those calculated by the Monetary and Economic Council [6], as the interdependent properties of the model ensure that these are accounted for.

The objective function is the maximisation of consumption plus government expenditure. This is definitely not a satisfactory criterion for optimisation. Consumption <u>per head</u> would, perhaps,

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[1] Details of the adjustments are given in Appendix I.

be acceptable, but due to the immigration activity population is variable and it is not possible to express consumption per head as a linear function of the other variables. In its present form the model can only approximate the optimum economic structure. A possible solution to this difficulty would be to specify a minimum level of consumption for each unit of immigration. The objective would then be to maximise consumption for the indigenous population. Again, this would not be a true optimum but it could represent a step in the right direction. No claim is made that the model does anything to distribute income among the population, and it is very likely that in a real situation immigrant workers would receive a share of the maximand.

There are seven groups of equations which restrict the model:

1. The production reconciliation rows ensure that sufficient is produced in the target year to match the total amount of each commodity used. The rationale of these constraints has already been explained. There is one equation of this type for each commodity; thus in the formalised model there are R such equations.

2. The capital stock reconciliation rows ensure that the level of output in each sector does not exceed its base year capacity unless the capital stock has been augmented by investment during the planning period. When investment does occur in a particular sector, the corresponding investment activity makes certain that the current output for each unit of investment is accounted for in the production reconciliation rows. Since this is an annual model investment is measured as an annual flow for the final year of the planning horizon; but the capital stocks available for production in that year will include capital formed due to

investment flows in all the years of the period. Consequently, for programming purposes, each unit of investment is regarded as contributing 6.6667 units rather than 1.0000 units to the stock of capital for that sector.[1] As each activity in the model is considered to have its own specialised capital stock there is an equation of this type for each activity; thus there are N equations.

3. Reconciliation of current overseas exchange transactions prevents growth in the economy while massive deficits occur in the current overseas account, a situation which cannot be allowed to continue indefinitely and certainly should be guarded against in a quantitative planning exercise. For the sake of development, however, some maximum deficit on current account will normally be tolerated. There is one equation of this type in the model; import requirements of the production, investment and consumption activities tend to exhaust or "use up" the supply of foreign exchange while the exporting activities augment it.

4. There is one equation in the model labelled the savings reconciliation row; its role is to prevent the ratio of consumption to investment from becoming unrealistic. Investment is often thought of as a sacrifice of consumption "now" in order that consumption might occur at some future time. This is a basic

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[1] The value of 6.667 is not a generalised value but is the particular value chosen for the eight year planning horizon for which this model has been used. 6.6667 is the inverse of .15 which was calculated (according to the method described by Manne [5], p. 384) as a linear approximation to the proportion of total investment that is likely to occur in the target year.

motive behind all saving and it is a well known identity of static economics that savings equal total gross investment. It is assumed here that, in aggregate, consumers tend to save a constant proportion of their income so that the ratio between savings (available for investment) and consumption is a constant. Each dollar of net investment requires a dollar of savings as well as a set of current inputs (equal to one dollar in value) from the producing sectors; each dollar of depreciation expenditure also requires a dollar of savings. In the programming model savings are "allowed to be available" in proportion to the level of consumption; they are "used up" by the production activities in amounts required for depreciation, and by investment activities to account for net investment expenditure. This row restricts total gross capital formation in the model.

5. There is one labour constraint which reconciles the total demand for labour with the estimated labour supply. The labour requirements of producing activities are expressed as labour-output ratios (inverse of labour productivity coefficients). The model has the capacity to generate more labour by means of the immigration activity.

6. Each of the exporting activities has an upper limit in recognition of the fact that opportunities to sell exports are not unlimited. The upper limits set should be based on projected trends and on knowledge gained from studies into the markets for individual products.

5. Non-negativity requirements for the activities of the model are necessary in order that the solution makes economic sense. The linear programming optimising routine fore-ordains that these constraints are satisfied.

COMPARISON WITH BLYTH-CROTHALL MODEL

The Blyth-Crothall model has been one of the impelling forces behind the present one, and the two have many attributes in common. The production reconciliation rows, described in the last section, the labour constraint, the foreign exchange constraint, and the constraints on the levels of the exporting activities are all analagous to the corresponding rows of the Blyth-Crothall model.

Differences occur in that the present model has no land constraint and that savings are endogenously generated in proportion to consumption, rather than a fixed amount being available in the form of "waiting". The land constraint has been omitted because, for the present there is only one farming activity and land is not considered to be one of the important factors limiting the level of activity of farming as a whole. Blyth and Crothall included the constraint as a means of attributing higher marginal costs to more intensive farming. Should the present model be expanded to include several types of farming activity or to feature increasing marginal costs for farming the desirability of a land constraint should be investigated. The inclusion of endogenously generated savings is thought to be an improvement, although there is some argument as to the correct ratio of savings to consumption. However, the "waiting" concept of the earlier model has a dynamic attribute which is not present when a single savings constraint replaces it.

Another noticeable difference between the models is the manner of treating investment. Blyth and Crothall do not provide distinct investment activities but incorporate investment expenditure into production activities which exceed the existing capital capacities. The reasoning supporting the investment activities of the present model is similar but the new capital formation is associated with the same input-output coefficients for current production as the original capital stocks. The current component of the coefficients for production-from-new-capital activities of the Blyth-Crothall model reflects new technologies and are therefore different from the coefficients for production from existing capital.

The immigration activity is a feature not included in the Blyth-Crothall model, although the authors suggest at the end of the article that the shadow price of labour could be restricted by adding a constraint to the dual problem. This would be equivalent to adding an immigration activity and a leisure (slack labour) activity to the primal. Immigration would have a cost which would be accounted for as a negative component of the objective function rather than as a specific requirement of capital and consumption.

There is considerably more choice of activities in the Blyth-Crothall model: alternative technologies, competing imports and diminishing returns. However, the coefficients for these activities were largely arbitrary and were included mainly for exposition of the technique. These refinements can easily and quickly be included in the present model, but it was thought unnecessary to do so unless realistic data were available.

LIMITATIONS OF THE MODEL

Given interindustry data for a sufficiently large number of sectors, this model provides a potentially powerful tool for economic policy makers and their advisers. However, there are a number of theoretical and practical drawbacks which should always be in the mind of users of the method.

1. Capital and Replacement of Capital

Organisation of the purchase and use of capital equipment is critical to economic development, but unfortunately effective measurement of capital is difficult, if not impossible. Thus the capital stock estimates, the capital input-output coefficients and the capital output ratios can only be thought of as the "best estimates available" - there is absolutely no way of estimating how close to the truth these estimates are.

A feature of capital which is frequently overlooked is that it is not generally substitutable between different processes. This has been acknowledged in the model to the extent that there is a separate, non-transferable capital stock for each sector, but the specialised nature of plant within sectors is probably no less important. Also the embodiment of technological change in capital has been virtually ignored. Arbitrary reduction of capital-output ratios is a possible means of isolating those industries in which technological change (in the form of increased capital productivity) would be most beneficial, and hence pinpointing those industries in which improved machinery would be of greatest value. However, technological change is a phenomenon which is inadequately understood. It may be that technical advances will be labour saving rather than capital saving, or that they will be neutral.

The treatment of depreciation is another difficulty closely allied to the above discussion. Depreciation coefficients related to the level of output and based on accounting depreciation rather than statistics of actual capital replacement have been suggested - mainly because no others are readily available. These may be totally unrelated to patterns of capital replacement - which is what really affects resource requirements, the major concern of the model. Physical replacement of capital is certainly lumpy at the firm level, and it is hoped that in the aggregated sectors these "lumps" tend to level out due to different firms replacing capital at different times. Differences in the durability of capital is another problem in this area. As different capital items have different economic life spans, the structure of a unit of replacement capital in any one year is likely to be vastly different from the structure of a unit of net investment. The former will have a lower average proportion of the more durable goods.

2. Depth of Analysis

The model oversimplifies the economic process and patterns of economic behaviour. Before economic projects can be initiated the financing arrangements have to be feasible; there are no constraints in the model to ensure this. It is left to the policy makers using the results of the model to consider the financial problems, but it would be more satisfactory if equations representing attitudes to borrowing and lending were part of the main analysis.

Secondly, the model in no way accounts for the existence of lags, imperfect knowledge and other ways in which economic reality differs from perfect competition. It is difficult to specify how the target values given in the solution are likely to be affected by these inevitable imperfections.

Thirdly, there is no satisfactory statistical test that can be applied to the optimum solution as the stochastic characteristics of the base year data are unknown.

3. Comparative Statics

The model gives no indication of the dynamics of the optimum solution - that is, the time paths of the variables involved. This information is as important as knowledge of the target year optimum values, as the degree of choice over time is much greater than at a point of time. Consequently, a polyperiod linear programming model would be much more suitable, but the data and computer capacity requirements are markedly greater.

4. Prices

The whole analysis is done in constant base year prices. It is certain that the price level will not remain constant, and there is no guarantee that the same input-output coefficients will be appropriate at the new price level. The existence of shadow prices in the optimum solution means that there is pressure on relative prices as well as on price levels. This may also invalidate the original coefficients. Allowing prices to vary would introduce non-linearities into the equations so that more sophisticated programming techniques would be required.

5. Objectives of Policy

The inadequacy of maximising consumption as an economic goal is well known. However, the difficulty of finding realistic alternatives is equally well known and the author feels that this is an area in which allot of research energy would be well spent. Some socially oriented objectives fit readily into a linear programming framework however: for example, minimum balance of payments deficits, minimum levels of output in problem industries, restrictions on labour transfer between sectors, specification of consumption and capital requirements for immigrants.

FIVE SECTOR PILOT MODEL

The model was designed with the 16 sector interindustry data derived by Ross and Philpott [12] in mind. It has now been assembled in this form and solved to contribute, it is hoped, something positive to problems of economic policy. In this paper we are concerned mainly with technique and these results will be reported in another publication. However, in the early stages the model was solved as a five sector pilot example and this should supplement the above discussion.[1]

The linear programming tableau is given in Table I. The columns of the tableau represent activities and the rows are the restrictions.

Activities

(a) Activities A_1 to A_5 represent current output from the five sectors and the block of coefficients bounded by A_1 to A_5 and R_1 to R_5 is the matrix of current input-output coefficients (including the depreciation requirements of each sector from each other).

(b) Activities A_6 to A_{10} represent net investment by the five sectors. The block of coefficients bounded by A_6 to A_{10} and R_1 to R_5 is the relevant matrix of capital input-output coefficients. Thus A_6 shows the requirements of current output from each sector to product $1 net investment in primary industry.

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[1] This example was the basis of a paper presented to the New Zealand Association of Economists Conference, August 1970, by Philpott and O'Malley [9].

TABLE I LINEAR PROGRAMMING TABLEAU FOR FIVE SECTOR MODEL

Activities / Restraints				Current Output					Capital Provision					Consumption		Exports			Immigration
				Farm	Forest	Mfg	Bldg	Services	Farm	Forest	Mfg	Bldg	Services			Farm	Forest	Mfg	
				A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}
Maximise C+G														1.0000	1.0000				
R1)		0	≥	-.5915	.0031	.0348	.0258	.0136	.0457	-	-	-	.0003	.0878	.0878	.9324	-	-	
R2)	Use	0	≥	.0142	-.7043	.0272	.0948	.0120	.0281	-	-	.0031	.0056	.0041	.0041	-	.9223	-	
R3)	of	0	≥	.0844	.0403	-.8504	.1581	.0754	.3162	.0303	.1873	.3223	.1162	.2531	.2569	-	-	.9200	
R4)	Output	0	≥	.0080	.0174	.0067	-.8781	.0362	.2712	.5481	.4039	.1379	.6738	.0162	.0162	.0002	.0002	.0002	
R5)		0	≥	.1003	.1638	.1521	.1512	-.8120	.1656	.1823	. 393	.2304	.0660	.5395	.5395	.0674	.0775	.0798	
R6)		2100	≥	1.1500					-6.6667										
R7)	Initial	200	≥		.5900					-6.6667									
R8)		600	≥			.3500					-6.6667								
R9)	Capital	1100	≥				1.5200					-6.6667							
R10)		15200	≥					5.4800					-6.6667						2.9930
R11	B/P	60	≥	.0391	.0786	.2043	.0534	.0425	.1732	.2393	.2695	.3063	.1382	.0758	.0720	-.8000	-1.0000	-1.0000	
R12	Savings	0	≥	.0371	.0487	.0269	.0218	.0498	1.0000	1.0000	1.0000	1.0000	1.0000	-.3900	-.3900				
R13	Labour	1131	≥	.0726	.0843	.1122	.1068	.1536											-.5000
R14)	Export	1019	≥													1.0000			
R15)	Restraints	49	≥														1.0000		
R16)		223	≥															1.0000	

All values in million dollars,
except Labour which is in thousands.

(c) Activities A_{11}, A_{12} represent the fixed 1972/73 bill of consumption goods to be maximised. In A_{12} five percent import substitution of manufactured goods has been allowed for.

(d) Activities A_{12}, A_{14}, A_{15} represent exports of the primary, forestry and manufacturing sectors respectively. The contributions of tertiary industries (building, services) to export earnings are margins associated with the handling of exports from other industries and are accounted for as fixed proportions of exports.

(e) Activity A_{16} is immigration which requires capital for houses, social services etc., and provides additions to the labour force as in Row 13.

Restraints

(a) Restraints R_1 - R_5 ensure that total annual requirements of production is not greater than what is produced.

(b) Restraints R_6 - R_{10} reconcile capital stocks, investment and capital use.

(c) Restraint R_{11} gives the imports required by each activity and the exports provided by the exporting activities. The difference between these two amounts must not exceed the permissible balance of payments deficit of $60 m in the target year.

(d) Restraint R_{12} ensures that savings available, generated as a function of consumption, are not exceeded by investment requirements which are reflected by depreciation coefficients for activities A_1 - A_5 and capital formation due to activities A_6 - A_{10}.

(e) Restraint R_{13} restricts labour use to the 1,131,000 labour force estimated for 1972/73 unless the labour force is added to by immigration. The coefficient for immigration is -.5 on the assumption that half the immigrant population becomes part of the labour force.

(f) Restraints R_{14}, R_{15}, R_{16} impose upper limits on each of the exporting activities.

SOLUTION OF FIVE SECTOR MODEL

The solution to the linear programming problem is given in column (i) of Table II and the shadow prices of the restrictions are given in column (i) of Table III.

The value of this kind of analysis however is not so much in the absolute levels of the variables in the solution as in the relative levels of solutions in which some of the assumptions are altered.

Interindustry data is difficult to compile and there are many likely sources of error. Similarly, there can be much argument as to the suitability of such data for investigation into the economic structure. Many of the parameters in the model involve a high degree of uncertainty. Terms of trade for exported goods, the size of overseas markets, the rate of change of technology, the rate of growth of investment, are a few examples. For illustration only, a solution was obtained in which all the maximum exporting limits were doubled. This would be most unlikely, of course, but the behaviour of the model for changes of this type might reveal some interesting information about the forces of cause and effect within the economy. The export limits for the original solution were based on National Development Conference Targets [14], so the second solution (column (ii) of Table II) reflects the most desirable economic structure for 1972/73 if New Zealand is able to rapidly expand export outlets for all products; the shadow prices of two solutions are compared in Table III. The structure projected by Ross and Philpott [13] for 1972/73 is given in column (iii) of Table II for comparison with the linear programming solutions.

It is not intended to consider these results as realistic guide-lines for economic policy makers. However, there are a number of observations to be made which should demonstrate how a larger model of this type could be useful.

TABLE II Activity Levels of Linear Programming Solutions

Activity		(i) N. D. C. Export Limits	(ii) Expanded Export Limits	(iii) Projected Structure N. D. C.
P_1	Farming Output	2,402	2,236	2,452
P_2	Forestry Output	396	506	415
P_3	Manufacturing Output	2,197	2,571	2,184
P_4	Building Output	797	1,015	936
P_5	Services Output	3,549	3,789	3,652
P_6	Net Farming Investment	99	71	106
P_7	Net Forestry Investment	5	15	7
P_8	Net Manufacturing "	25	45	26
P_9	Net Building Investment	16	66	68
P_{10}	Net Services Investment	637	882	652
P_{11}	Cons. (no import subst.)	-	-	-
P_{12}	Cons. (5% subst.)	3,641	3,760	3,415
P_{13}	Farming Expenditure	1,019	879	1,019
P_{14}	Forest Expenditure	49	98	49
P_{15}	Manufacturing Exp.	223	446	223
P_{16}	Immigration	-	106	40

Column (iii) is an aggregation of structure given in Ross & Philpott [13].

All values in $mn except immigration which is in thousands of people.

TABLE III Shadow Prices of Restrictions for Linear Programming Solutions

		(i) N.D.C. Export Limits	(ii) Expanded Export Limits
R_1	Farming	.713	.774
R_2	Forestry	.642	.688
R_3	Manufacturing	.718	.693
R_4	Building	.804	.850
R_5	Services	1.150	1.252
R_6	Farm Capital	.145	.137
R_7	Forest Capital	.159	.148
R_8	Manufacturing Capital	.159	.142
R_9	Building Capital	.166	.145
R_{10}	Services Capital	.139	.136
R_{11}	Foreign Exchange	1.626	1.008
R_{12}	Savings	-	.028
R_{13}	Labour	-	.815
R_{14}	Max. Farming Exp.	.558	.276
R_{15}	Max. Forestry Exp.	.954	.276
R_{16}	Max. Manufacturing Exp.	.874	.270

(1) The structure of column (i) is very similar to the structure of column (iii); the only noticeable difference is that building output and investment are considerably reduced in the linear programming solution. This could mean that resources should be more efficiently used in the building industry or that businesses have tended to over-capitalise in building. However, the main observation should be that projection models such as the Lincoln model do not seriously deviate from providing optimum structures.

(2) Farming output and exports are heavily reduced in the expanded exporting situation. This is evidence that, provided markets are expanding, manufacturing and forestry are the key sectors in New Zealand's economic development.

(3) Large scale immigration does not appear to be necessary to promote growth. Foreign exchange is the most urgent limiting factor; it is only after export markets have expanded that immigration is brought into the solution and that the labour restriction has a shadow price (equilibrium wage rate) greater than zero.

To show that all flows computed by the model balance, the interindustry table for the solution given in column (i) of Table II is presented in Table IV.

TABLE IV LINEAR PROGRAMMING SOLUTION No. 1 IN NATIONAL INCOME ACCOUNTS FORM

($ mn 1964/65 Prices including Exports)

Sales to: / Purchases From:	Farming	Forestry	Mfg.	Building	Services	Final Demand: Consumption	Final Demand: Exports	Final Demand: Gross Investment	TOTAL OUTPUT
Farming	976	1	77	21	48	320	950	10	2403
Forestry	31	117	60	76	41	15	45	12	396
Manufacturing	168	15	313	120	226	935	205	216	2197
Building	12	2	6	97	68	59	1	553	797
Services	223	59	322	116	644	1964	90	131	3549
Imports	75	23	426	37	102	262		224	1147
Depreciation	89	19	59	17	177	-			362
Domestic Value Added	830	160	935	315	2244	86			4569
TOTAL INPUT	2403	396	2197	797	3549	3641	1291	1146	15423
Annual gross Investment Required	189	24	85	34	814				

Totals may not add exactly due to rounding errors.

NOTE This table is expressed in 1964/65 prices including exports and in these terms the balance of payments show a surplus of $144 m. When however allowance is made for the 20% decline in export prices adopted in the model, the fall in export income and farm income converts this into a $60 m deficit as programmed with appropriate fall in g. n. p. of $204 m.

CONCLUSION

A framework has been suggested for an optimistic model of the New Zealand economy; a demonstration of its use has been made with a five sector example. The policy statements in the last section should not be taken seriously as, although the coefficients are based on real data, the model is not sufficiently disaggregated to isolate key relationships in the economy. The model has now been formulated for 16 sectors and these results will soon be published.

Exposure of the underlying economic relationships rather than the actual numerical solution is the main purpose of this type of analysis. Blyth and Crothall stressed the importance of sensitivity analysis and parametric programming in the interpretation of solutions to national economic linear programmes. Other writers in the field have done likewise. For instance, Moustacchi [7] devotes a large part of his article to the interpretation and significance of the dual solution to the model he describes. Due to the restricted scope for choice between activities in the five sector model, there would be little point in parametric methods in this paper - the optimum solution is stable with respect to the variables in the basis. Nevertheless, the relative levels of the activities under different conditions are of interest, so the procedure suggested is to compare solutions for discrete changes in key parameters. For illustration, a solution has been presented in which the assumptions concerning export markets are changes, and the resultant changes in activity levels and shadow prices considered. It is of little consequence that actual doubling of export targets for 1972/73 is improbable. The nature of the changes in the optimum solution reflect cause, effect and interaction within the economy if the model has been adequately specified.

APPENDIX

Data of the Five Sector Model

The five sectors are aggregations of the 16 sectors used by Ross and Philpott 13 :

1. Farming:	Farming, Primary Produce Processing, Hunting and Fishing, Mining.
2. Forestry:	Forestry, Forestry Processing.
3. Other Manufacturing:	Other Manufacturing.
4. Building:	Building and Construction
5. Services:	Public Utilities, Transport and Communication, Distribution, Banking and Insurance, Services, Services to Households, Services to Government, Ownerships of Propert y.

The current input-output coefficients are derived from Ross and Philpott [12] , Table VI, as are the coefficients for the consumption activities, and import coefficients. Capital input-output coefficients and capital-output ratios are derived from Francis [4]. The capital coefficients for immigration are derived from Monetary and Economic Council [6] .

Estimates of capital stocks of sectors were not readily available, so artificial "capital stocks" were obtained by dividing capital-output ratios into the 1964/65 levels of output. This means that it is assumed that all sectors were operating at full capacity in that year.

Labour coefficients are also the same as those used in the Lincoln projection work.

The savings coefficient is set at .39 times consumption. This reflects the average ratio of savings to consumption assumed by the National Development Conference.

Agricultural export prices for 1972/73 are assumed to be 80 per cent of 1964/65 prices.

All values in the model are expressed in 1964/65 prices.

The nature of the adjustments to the current input-output coefficients for depreciation needs to be more clearly explained. The adjustment takes the form of an addition to each interindustry and import coefficient. The sum of the additions to the coefficients of any sector equals the depreciation coefficient for that sector. This value is distributed among the inputs in the same proportions as would be the inputs of net investment goods for the sector in question. These proportions are determined by the capital input-output matrix.

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