

Using option pricing theory to estimate option value - a preliminary study

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Executive Summary

1. This report examines two approaches to incorporating uncertainty when evaluating natural resource problems. The first approach incorporates option pricing theory which is derived from the financial economics literature. The second approach is based on the concepts of option price and option value which have arisen in the literature of resource economics.
2. Although the discounted cash flow (DCF) project appraisal technique takes account of the timing of cash flows generated by natural resource projects, DCF does not fully capture all of the factors which affect the resource values. The so-called "hard" numbers produced by the DCF approach often do not provide sufficient justification for public policy decisions.
3. Many natural resource valuation problems revolve around the decision over whether to use a resource immediately or reserve it. The decision as to whether the resource should be used (e.g. harvested or mined) immediately or whether its use should be deferred until more information on alternatives become available has a value. The report shows that the ability to defer a decision will have a positive value which DCF analysis overlooks but which option pricing theory values explicitly.
4. One approach to valuing this "right" of deferral is to draw the analogy between the right to invest and a financial call option. Conventional net present value (NPV) models undervalue projects where the benefits of deferring a decision to invest are greater than the income foregone by not investing immediately. This is simply because the value of the right to defer, as well as the value of other operating flexibilities, is ignored in the conventional cost-benefit model. Option value should not be ignored when evaluating investment options in natural resource projects.
5. The general results on option value in the finance and economics literature are not significantly different. For any application the analyst must consider the particular assumptions regarding the stochastic process involved, the availability of data and whether the degree of risk aversion needs to be specified.
6. A valuation model is discussed based on a contingent claims methodology for evaluating a non-renewable resource.
7. Application of the valuation model to the Maui gas field yields a result which suggests that the extraction rate required by the take-or-pay contract between the Crown and the Maui gas partners represents an overproduction of 60-70% above the optimal output rate at the given contract price. Care must be taken when interpreting this result. Assumption with regard to costs and inventory impose some limitations. Better data will allow more robust conclusions to be drawn.
8. These results imply that flexibility in both the management and the legal framework of contract arrangements in natural resource use is essential in ensuring an efficient outcome.
9. The valuation model used provides a sound basis for further development of the methodology with applications in public policy analysis.

CHAPTER 1

Introduction

The purpose of this report is to compare and contrast two approaches to incorporating uncertainty when evaluating natural resource problems. The first approach incorporates option pricing theory which is derived from the financial economics literature. The second approach uses the concept of option value which has arisen in the literature of resource economics. The report introduces each approach separately and then compares and contrasts them. The report is in three parts - Part I deals with the option pricing theory approach to incorporating the value of uncertainty and irreversibility. Part II describes option value as commonly used in the resource economics literature. Part III describes the application of option pricing to a natural resource problem.

Part I

Option pricing theory

CHAPTER 2

Background

Most conventional analyses of natural resource issues use discounted cash flow (DCF) criteria such as net present value¹ (NPV) or internal rate of return (IRR) to assist with the decision making process. In DCF analysis the pecuniary and non-pecuniary costs and benefits of a particular project or asset are calculated for each time period. Future cash flows are then discounted back to the present day at an appropriate discount rate. In practice, accounting for flexibility, or risk, is usually made by adjusting the discount rate or the cash flows. In addition to the *ad hoc* nature of these adjustments there are quite strong theoretical reasons for moving beyond the standard DCF framework. Recognising the impact of decision variables at each stage of a project on the final cash flow profile has led to recent attempts to incorporate a more complete specification of the decision variables, including the information environment (Paddock, *et al.* 1988).

Recent developments in the literature point to DCF being superseded by techniques incorporating option pricing theory as the preferred project and policy appraisal technique. The argument against the DCF approach is well summarised by Pindyck (1991) who makes the point that DCF models of investment behaviour ignore two important characteristics of cash flows associated with natural resource - and other - projects. Firstly, development expenditures are largely irreversible in involving substantial asset-specific investments. Secondly, investment can often be delayed. The ability to delay or defer the project gives the organisation making the investment time to collect more, and better, information.

According to Brennan and Schwartz (1985) the major draw-back of the DCF framework for project appraisal has to do with the analogy between project costs and benefits and the cash flows associated with a portfolio of riskless bonds. Such an analogy is even less appropriate when examining public policy initiatives where the level of investment may be contingent upon climatic or political events. One technique which is increasingly being used incorporates option pricing theory. When a firm evaluates an investment it must decide when to make the investment, if at all. That is, the firm may exercise its option immediately or it may choose to wait for more information thereby delaying investment to an uncertain future date. If the decision to defer adds value for the firm then the "price" of the option is positive. These option-like characteristics are not adequately accounted for in the conventional capital budgeting framework.

Option pricing theory has been applied to two main classes of problem:

- i project evaluation, or capital budgeting problems, and,
- ii contingent liabilities, or claims which involve some form of guarantee.

¹ Net present value calculations use an exogenously given interest rate:

$$NPV = C_0 + \sum_{t=1}^n \left(\frac{C_t}{(1+r)^t} \right)$$

In contrast, an investment's internal rate of return is the rate of interest that solves $NPV = 0$.

The operating decisions faced by firms and government agencies can be viewed as investment and growth options. For example, an oil exploration and refining company may base changes in output on current and expected future oil prices. Each decision is an option from the point of view of the oil company. Likewise, government agencies may expand or mothball a policy initiative depending on how well the policy performed. For example, a health policy may be tested in a small region before being introduced nationwide. Depending on how well the trial goes the policy may either be expanded nationally or scrapped. The flexibility and the right to abandon the policy is worth something and may be evaluated using option pricing theory.

Option pricing theory may also be applied to strategic business and policy problems. Many energy projects entered into in the 1970s and 1980s have negative net present values based on current price projections. However, conventional DCF analysis ignores the right to mothball the plant and also the strategic value of energy self-sufficiency in the event of increased world prices for fossil fuels. Another interesting application of option pricing theory is the valuation of research and development organisations. The various Crown Research Institutes (CRIs) have "portfolios" of research projects which may or may not have commercial value. These projects are similar to call options as buyers, or funders, of those projects have the right, but not the obligation, to implement the outcomes of the research (Shelvin 1991).

The principal advantage of the option pricing framework is that it incorporates the discretion available to policy makers or managers. It does not naively assume that all costs and revenues are known with certainty or that managers and politicians never change their minds based on new information. The value of discretion (an option) depends on:

i How long the project or policy may be deferred

The longer the deferral the more time there is to evaluate the policy, avoid costly errors and make a better decision. By deferring a decision, decision makers have an opportunity to gain superior information, perhaps through investment. Successful adaptations may yield positive NPV outcomes.

ii The risk of the project

Although it seems counter intuitive, options on riskier projects are worth more than options on less risky projects or policies. Only the positive payoffs from the policy are valued because the "option" does not have to be exercised if doing so results in a loss. In contrast, the NPV of risky projects *ceteris paribus* is lower because of the practice of using a higher discount rate for risky projects (Sharp and Cullen, 1991).

iii Interest rates

As mentioned above higher discount rates - often associated with risky projects - mean lower NPVs. However, higher interest rates also reduce the present value of any future cash outlays needed if the policy is to go ahead. Therefore higher interest rates usually raise the value of projects with embedded growth or strategic options.

CHAPTER 3

Option pricing theory and natural resource policy problems

A share option is a right which gives owners the right to buy or sell the option. The *market* price of a share option depends on supply and demand for the rights attached to the option. Black and Scholes (1973) derived a theoretical model which can be used to estimate the *theoretical* option price and to value risky debt, shareholders' equity, and even options on options. Most initial applications of the Black-Scholes model concentrate on pricing share options. However, it was not long before researchers were applying the underlying theory behind option pricing to a number of other valuation problems which had "option-like" characteristics. In general, the new applications were classes of contingent claims. That is, assets whose price is state dependent, for example, on the price of some other asset or occurrence of some event.

Brennan and Schwartz (1985) drew the analogy between natural resource investments and financial options. As summarised by Pindyck (1991) most recently, and many others beforehand, a financial call option has similar features to an investment in a natural resource project. A call option buyer, or holder, has the right (but not the obligation) to buy an asset at a specified price for a specified period of time. Likewise a firm, or organisation, faced with a deferrable project has the right, but not the obligation, to defer its investment for as long as it wants to. Similarly, as well as the right of deferral there are also often the rights to expand, abandon or mothball the project once the firm has undertaken the initial investment. The rights will have a positive value for the firms undertaking the investment. For example, the right to defer a project will allow a firm to accumulate more information on the investment and therefore improve the quality of the decision. The right to expand a project, has value to the firm because the firm will only expand the project if it is profitable. The firm does not have to expand the project if it would result in a decline in the overall value of the firm. These rights have been termed operating or managerial flexibilities by Myers (1987). These flexibilities have option-like characteristics and the next section will deal with how financial options are valued.

CHAPTER 4

Valuing options

4.1 Rights and obligations

An option gives its owner the right to buy or sell some underlying asset, at a given price, for a given period of time. The underlying asset could be anything from a building to a financial contract such as foreign currency, a share or interest rates. The important thing is that options are exercisable at the discretion of the owner (also called the option holder). That is, the holder does not have to buy or sell the underlying asset if he/she is better off not doing so. Options allow holders to do this by splitting the rights and obligations attached to owning any asset.

An option is a contract, between buyer and seller, that gives the buyer the right to buy or sell a specified asset at a predetermined price for a limited time period. An option holder pays for the option, or right, to buy or sell - this is called the premium. The predetermined price is called the strike or exercise price. The limited time is the time to expiry of the option. Options come in two forms. Holders of American options may exercise their options at any time prior to expiry. Holders of European options may only exercise their options at expiry.

The other side of the options contract is the obligation of the option seller. The option seller sells rights to the option buyer. The option seller is obliged to deliver if the option buyer decides to exercise the option. The option seller is also known as the grantor (because the seller is granting rights to someone else) or the writer (as the seller is "writing the business" i.e. selling the contract). In exchange for taking on the obligation the option seller receives a premium. In most markets the right is:

**"the right to buy
or
the right to sell
the underlying asset within a limited time and at a specified price."**

The obligation is:

**"the obligation to sell
or
the obligation to buy
the underlying asset within a limited time and at a specified price."**

The underlying asset may be shares if we are looking at share options or New Zealand dollars if we are considering call options on the New Zealand dollar. A call option is the right to buy the underlying asset at a specified price for a limited time period. The transaction involves the right to buy the underlying asset, not the underlying asset itself. Hence:

call = right to buy

When selling (or writing) a call option the vendor sells the right to buy the underlying asset. The owner of the asset is obliged to sell the asset at the specified price for a limited time period to the

call option buyer if, and when, the option buyer decides to exercise the right to buy. The call option seller receives the premium in exchange for assuming the obligation to sell the underlying asset to the call option buyer. The call option seller is obliged to sell the underlying asset if, and when, the call option buyer decides to exercise the right.

A put option is the right to sell the underlying asset at a specified price for a limited time period. The buyer of a put option obtains the right to sell the underlying asset to the put option seller or writer. The put option buyer pays the premium in exchange for the right to sell the underlying asset. As with the call option, the transaction involves an exchange of rights. In the case of buying a put option the buyer obtains the right to sell the underlying asset. Hence:

put = right to sell

The seller of a put option sells "the right to sell" the underlying asset. The seller is now obliged to buy the asset at a specified price for a limited time period from the put option buyer if and when the buyer wants to sell it. The put option seller receives the premium in exchange for the obligation to buy the underlying asset from the put option buyer. The put option seller is obligated to buy the underlying asset if, and when, the put option buyer decides to exercise the right.

4.2 Examples

Suppose you hold a call option to buy a share for \$1.00. By the time the option expires - say 90 days - the value of the share rises to \$1.50. As owner of the call option you have the right to buy the share for 50 cents less than the market price. Therefore the right to buy the share - the call option - is worth 50 cents. In this case you would exercise the option and buy the share for \$1.00. The grantor of the option has to buy the share in the market for \$1.50 and sell it to you for \$1.00 - therefore incurring a 50 cent loss.

However, what happens if the price of the underlying share falls to 20 cents? You have the right to buy the share for \$1.00, but a rational person would buy the share on the stock exchange for 20 cents. That is you would let your option expire worthless and buy the share yourself on the stock exchange. Therefore the value of the option (C) at its expiry will be either zero or the difference between the market price (S) and the exercise price (X). That is:

$$C = \max[0, S - X]$$

Put options work in the opposite way to call options. Put option holders benefit if the price of the underlying asset falls. For example, assume that you buy the right to sell a building for \$100,000. Six months later, when the option expires, the building is worth \$80,000. Therefore as you still have the right to sell the building for \$100,000 to the person that granted you the put option, the option is worth \$20,000.

Now what would have happened if the value of the building had in fact risen to \$125,000? In this case you could sell the building for \$25,000 more than what you could by exercising the put option. Therefore despite the fact that you have an option you would be better off to let the option expire worthless and sell the building on the open market. The value of the put option (P) at its expiry will be either zero or the difference between the exercise price (X) and the market price (S). That is:

$$P = \max[0, X - S]$$

The payoffs for holding call and put options are shown in Figure 1 and Figure 2. The difference between the exercise price and the market price of the underlying asset is shown on the vertical axis as profit. The market price of the underlying asset is shown on the horizontal axis. As the market price of the underlying asset increases, the value of the call option also increases. At a market price less than \$1.00 the call option is worthless and will not be exercised - which would result in the loss of the 20 cent premium. At a market price above \$1.20 the value of the option covers both the exercise price and the cost of the premium. That is, the option is *in-the-money*. In both cases - the put and the call option - the holders of the options do not have to exercise the option if they are worse off doing so.

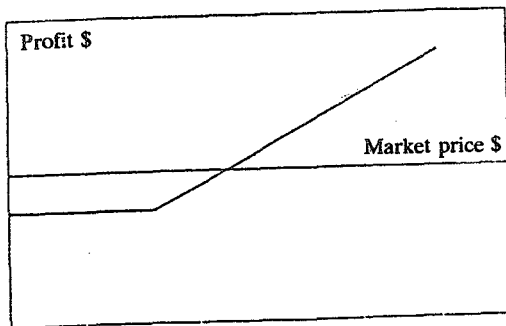


Figure 1 Buy a call option.

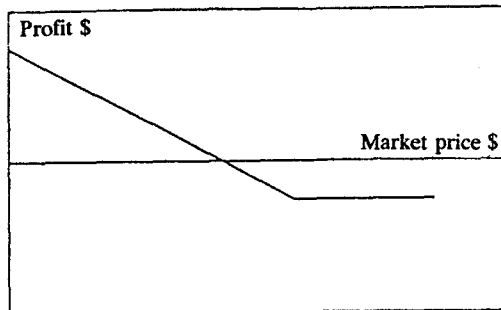


Figure 2 Buy a put option.

Pricing options at expiry is straight forward. The options have a positive value depending on the relation between the exercise and market prices. As long as the market price is greater than the exercise price, call options will be worth something and put options worth nothing. If the exercise price is greater than the market price, put options will be worth something but call options will be worth nothing.

However, what happens when we try to price options which still have some time to run before expiry? Should the option be worth more or less given that we have to wait until expiry to exercise it? What about the effect on the option price of the underlying volatility, or variability, of the asset

price? Fortunately, a robust option pricing formula has been available for pricing options for almost two decades. The original option pricing model was suggested by Black and Scholes (1973) and was based on the notion that the payoffs described above could be replicated by borrowing to buy the underlying share. The pricing model relies on arbitrage - selling in an overpriced market and buying in an underpriced market - to make sure the option price is fair.

Although the Black-Scholes model appears complex it is framed in such a way that there are only four major factors which influence the value of a call option. The value of the call option will rise if:

- i The asset's price becomes more volatile.
- ii The time to expiry increases.
- iii Short term interest rates rise.
- iv The ratio of the market price to the exercise price, i.e. S/X , increases.

Interestingly, neither investor risk aversion nor expected returns on the underlying asset are incorporated in the Black-Scholes model. The only variable which cannot be directly measured is the volatility or variability of the underlying asset's price. Therefore we are left with the following results:

- i Options on assets which are more risky are worth more than options on less risky assets. This is due to there being more chance that the option (either put or call) will end up being worth something.
- ii Options with longer time to expiration are worth more. In this case the option with the longer expiration has a greater chance to end in-the-money.
- iii When short term interest rates rise the present value of paying a call option's exercise price falls. Therefore the value of the option rises.
- iv The ratio of the market price of the asset to the exercise price, (S/X) , becomes very large when a call option is very "in-the-money".

CHAPTER 5

Issues in investment appraisal

5.1 Discounted cash flow analysis

The valuation of investments is often undertaken using the standard technique of DCF analysis. However, this has major shortcomings when used to evaluate natural resource investments. One major deficiency is its total neglect of the stochastic nature of prices and the possibility of managerial adaptation to price variation (Brennan and Schwartz, 1985; Myers, 1987; Shapiro, 1990). At each stage in the investment process the final cash flow is determined by stochastic processes and managerial decisions in response to changing conditions. According to Trigeorgis (1988) conventional time value of money concepts such as NPV fail to recognise either operating flexibility or the strategic value of some policies. Clearly, the most appropriate valuation model is the one with the most structural similarity to what is being valued. According to Brennan and Schwartz (1985) the major failing of DCF analysis is that it compares project costs and benefits to the cash flows from a portfolio of riskless (i.e., default free government) bonds. The comparison is even less appropriate when examining public policy initiatives where the cash flows depend on climatic events, commodity prices, land values or even the rate of economic growth. Even if DCF analysis can adequately account for the time value of money, when calculating the present value of a project or investment, prices and policy are exogenous to the model. Paddock *et al.* (1988) list the major weaknesses of DCF analysis as follows:

- i The proper timing of future cash flows is not clear and the choice of timing used in the DCF calculation is often arbitrary. Therefore different analysts will calculate different present values given the same information.
- ii Analysts will have different forecasts of future prices and costs which are used to construct a profile of future net cash flows. Clearly, this will affect the net present value of the project.
- iii The technique requires selection of the "correct" risk adjusted discount rate. The choice is likely to be subjective and prone to error even if techniques such as the capital asset pricing model are used. The risk adjusted discount rate will also depend on the investment timing rules used by the investor.
- iv DCF techniques sometimes incorporate Monte Carlo simulation to incorporate a stochastic element in the analysis. However, this is often costly and time consuming, even though **expected values** and **variances** can be calculated. Moreover, simulation may not explicitly capture the range of choices available to decision makers, particularly the option of aborting or postponing a project.

5.2 Option pricing

Much of option pricing theory's appeal is that it provides techniques and methods for pricing assets and valuing cash flows which more fully reflect the ability of decision makers to respond to future markets or operating conditions. The operating policy and cash flow profile of many investments will depend on outcomes that are unknown at the start of the project (Shapiro, 1990). Furthermore,

the ability of managers to make decisions contingent on new information is central to most investment decisions. For example, production in the second year of an investment project may be increased, left the same, or cut, depending on the project outcome and the operating environment at the end of the first year. The ability to discontinue a project, or restart a temporarily closed one, has a positive value which DCF analysis cannot incorporate.

This implies that DCF analysis which incorporates "expected value" estimates, according to some statistical distribution, will undervalue projects or cash flows. DCF analysis assumes a given project will go ahead regardless of future events and therefore incorporates expected values of both positive and negative outcomes. Even if some stochastic process is incorporated in deriving an expected value of each project cash flow, the expected value will be the sum of the expected values of positive and negative cash flows for a range of scenarios. However, DCF analysis cannot select one range of outcomes in one period contingent on the outcome in a prior period. Therefore by using DCF techniques analysts will probably undervalue projects and resources. This may increase the chance that feasible projects are incorrectly rejected, or infeasible projects accepted, and assets misallocated as a result.

Myers (1987) summarises this by pointing out that DCF analysis ignores the problem of estimating the links between today's investments and tomorrow's opportunities; that is, the project's impact on the firm's future investments. For example, a firm may undertake a negative NPV project in order to establish a hold in an attractive market. The second stage of the project - e.g., increasing market share - is used to justify the first decision of adopting a project which would be otherwise rejected. What is important to note here is that this type of problem cannot be correctly evaluated by simply taking the NPV's of both projects - the first and second stage - and summing them. The second stage is an option. That is, the firm does not have to expand into the market and the second stage will only go ahead if the first stage works and the opportunity for profit still exists. In effect, if the present value of this option offsets the negative NPV of the first stage, the decision would be justified. The problem is now recast in terms of evaluating an option.

As pointed out by Kensinger (1987) option pricing theory could be used to value the following claims:

- i The right of abandonment. That is, the option to shut down temporarily, or to abandon the entire project if events turn against the developer.
- ii The right of asset redeployment. Should business conditions change, assets may be allocated to other uses. For example, alternative fuels or other inputs may be used in the production process. On the other hand different outputs may be manufactured.
- iii Options on future growth. Current research and development programmes provide future growth opportunities for firms which the firm may, or may not, choose to adopt.
- iv Contingency planning. That is, creating and managing a portfolio of strategic "real options".

5.3 Flexibility and expected value

Suppose the Government is concerned about future energy demand and decides to evaluate a number of energy proposals. Policy advisors might estimate expected cash flows for the projects and calculate a NPV for each policy. If the policy makers are rational and all relevant costs and benefits have been priced, the best unambiguous choice will be the policy with the greatest NPV. **Expected**

cash flows are estimated by weighting possible cash flow outcomes by their probability of occurrence; that is, expected cash flows are mean value cash flows. However, the analyst does not know what those probabilities are. Moreover, if decision makers can discontinue a project if they suspect the project is going to have a negative NPV then it is unrealistic to attach a probability to the negative NPV simply because the outcome will never occur (Shapiro, 1990; Pindyck, 1991). That is, if there is likely to be a negative NPV the project will certainly be mothballed.

The problem can be demonstrated with a simple example. Let's say a project involves investing \$10 million today and generates either a negative cash flow of \$20 million or a positive cash flow of \$30 million, with equal probability, in a year's time. The expected value of the project's cash flow at the end of year one is:

$$(0.5 \times -\$20 \text{ million}) + (0.5 \times \$30 \text{ million}) = \$5 \text{ million}$$

Assuming a 10 per cent discount rate, the project's NPV is:

$$\text{NPV} = -\$10 \text{ million} + \$5 \text{ million}(1 + 0.10)^{-1} = -\$5.45 \text{ million.}$$

Now suppose decision makers have the option to discontinue the project if they know that it will make a loss of \$20 million. In such a case the project can be temporarily closed or mothballed. Incorporating this option not to make a loss increases the expected value of the cash flow at the end of year one to \$15 million. Therefore the net present value of the project increases to:

$$\text{NPV} = -\$10 \text{ million} + \$15 \text{ million}(1 + 0.10)^{-1} = \$3.64 \text{ million.}$$

The project's NPV is now \$3.64 million, \$9.09 million more than the estimate for the same project using the standard DCF approach. The increased value is due to incorporating the option to discontinue the project - i.e., the option not to make a loss. That is, the policy makers can make strategic choices. In such a situation how can the policy analyst build in the value of flexibility? One method which has been demonstrated in the literature, and which is described in this paper, is the method stemming from option pricing theory which values the project as a contingent claim.

5.4 Applying option pricing theory to natural resource investments

Most of the advances in applying option pricing theory to natural resource projects have occurred since the mid-1980s. Brennan and Schwartz (1985) made a major contribution to the literature applying option pricing theory to natural resource problems when they demonstrated how a non-renewable resource, such as a copper mine, could be valued using an option pricing approach. They also examined the optimal operation of a copper mine and, in particular, determined the price at which the mine should be closed or open. Option pricing theory has since been applied to a broad range of valuation problems. For example, Trigeorgis (1990) has demonstrated how an option pricing approach may be practically used to evaluate a natural resource project. Paddock *et al.* (1988) and Ekern (1988) have described techniques for valuing petroleum projects and petroleum leases, respectively. Morck, *et al.* (1989) have used the Brennan-Schwartz model to evaluate forestry investments. Mason and Baldwin (1988) evaluated the value of federal loan guarantees which reduced the cost of capital of energy producers. Bardsley and Cashin (1991) used a model similar to that of Mason and Baldwin to value the Australian Wheat Corporation's minimum price scheme. Seed and Anderson (1991a) have suggested option pricing for evaluating the New Zealand government's primary sector policy.

The following sections give examples of how option pricing theory has been applied to a number of natural resource investment and analysis problems.

5.4.1 Mining projects

One of the first studies to extend the use of option pricing theory to valuing non-financial assets was done by Brennan and Schwartz (1985). Until then, capital budgeting procedures relied almost solely on discounted cash flow techniques. The major problem with DCF analysis is that it neglects the stochastic nature of output prices. Brennan and Schwartz note that price variations in some natural resource industries are commonly between 25% and 40% per year. The practice of replacing distributions of future prices by their expected values is likely to cause errors in the calculation of both expected cash flows, and of appropriate discount rates and thereby lead to suboptimal investment decisions.

Standard DCF analysis also treats a project's expected cash flows as given at the outset by assuming that all operating decisions are set in advance. Many kinds of investment, however, provide management with the opportunity to make decisions that are contingent on information which will become available in the future. As an example, Brennan and Schwartz use an investment in mining. Depending on the future level of the output price, the output rate may be speeded up or slowed down, and the mine may be expanded, temporarily closed, or even abandoned. In fact, the optimal operating policy for many natural resource investments depends on outcomes that are not known at the project's inception.

Such investments bear the characteristics of options on securities and should be valued accordingly. Throughout the life of a planned investment, the owner has the option to change the level of production at any point if it becomes more profitable to do so. By contrast, the traditional approach to investment analysis, which ignores management's ability to respond to future operating conditions may be likened to valuing an option on a share while ignoring the holder's right to avoid exercising the option when it is not profitable. By failing to incorporate the obvious benefits of operating flexibility, the DCF approach to capital budgeting will tend to understate the true value of a possible investment.

The approach taken by Brennan and Schwartz to value the uncertain cash flows from an asset is similar to that of Black and Scholes (1973) in their derivation of a share option valuation model. Rather than using a riskless portfolio to value an option, however, Brennan and Schwartz use a self-financing portfolio whose cash flows replicate those which are to be valued. They show that a long position in the mine and a short position in futures contracts on the commodity to be extracted from the mine, in an appropriate ratio, will have a certain return for any output price. Moreover, to avoid riskless arbitrage opportunities, that return must be equal to the risk free return on the value of the investment. Thus the value of the mine must be set by this equality.

Brennan and Schwartz assume that the mine may have two operating states; it will either be open or closed and each of these operating states can be considered as an option. If the mine is closed, the owner has the option to open it and conversely when it is open the owner has the option to close it. The relationship between the value of these options and the commodity price is presented in Figure 3.

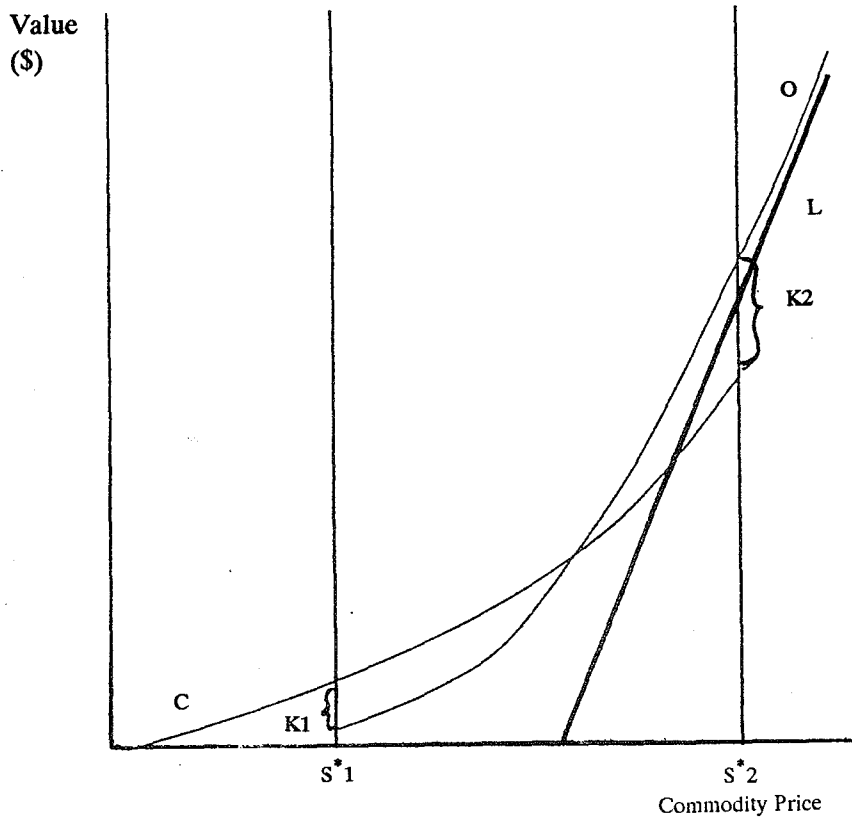


Figure 3 Mine value.

Line C in Figure 3 represents the value of the mine when it is closed and line O represents the value of an open mine. For very low output prices the mine has a higher value when it is closed compared to the open mine value. However, the mine would not be closed until the output price reached S^*1 . At prices lower than this, the difference between the value of the open and closed mine is less than the cost of closing the mine, K1. It would be more profitable to keep the mine operating. Similarly, at the commodity price S^*2 it is worth just enough more to warrant the outlay K2 to open it.

The line L represents the value of the mine if it were open and if the owner did not have the ability to close it. This could be likened to the value of the mine as calculated using the traditional DCF technique. By including the option to either open or close the mine, the implied value is never lower than that calculated from a DCF analysis, and is significantly higher when the commodity price is low.

The option valuation approach also offers a number of other advantages over the DCF technique. The many problems of establishing a relevant risk adjusted discount rate are avoided as risk free bonds are used in the construction of the self - financing portfolio. But most importantly, the valuation model does not rely on the expected rate of change in the output price. This means that future commodity prices do not need to be estimated. As with the Black and Scholes model, all that is required is an assumption regarding the distribution of future commodity prices.

5.4.2 Petroleum projects

The advantages offered by the option valuation approach are considered by Paddock *et al.* (1988) to be especially applicable to the valuation of petroleum projects. They state that the holder of an offshore petroleum lease must pass through three stages before actually recovering hydrocarbons: exploration, development, and extraction. First, if the exploration stage produces favourable results the firm may continue to the development stage. Second, the firm has the choice of incurring development costs, i.e., putting the equipment in place to extract the oil, which converts undeveloped reserves into developed reserves. Finally, the resource is recovered using the installed equipment. Thus both the exploration and development stages represent options of the leaseholder. Depending on information that will become available in the future, the leaseholder has the option to either abandon the project, temporarily suspend the process, or continue towards extraction.

Paddock *et al.* (1988) emphasize the deficiencies that the DCF approach imposes on the valuation of petroleum leases. Because of the nature of a petroleum project, many of the required inputs into a DCF model need to be chosen arbitrarily. For example, the exact timing of exploration and development is very difficult to determine. An appropriate set of discount rates is needed. The complex nature of the project's cash flow and investment timing rules means these are often subject to a great deal of uncertainty and error. Companies often resort to simple rules of thumb, with one constant discount rate applied to the exploration phase and another to the following stages of development and extraction. The use of an option pricing approach to the valuation of these leases can overcome many of these problems.

Before deriving a model which can be used to value a petroleum lease, Paddock *et al.* (1988) provide expressions that give a value for each of the options that make up a three stage process. The exploration stage can be thought of as an option on the undeveloped reserves. In this case the exercise price is the cost of exploration and the expiration date is the time at which the lease must be relinquished. The option will have a value which is calculated as the difference between the expected exploration costs and the expected value of undeveloped reserves:

$$X(V) = \int QX(V, T - t; D(Q))dF(Q)$$

where :

Q	= random quantity of recoverable hydrocarbons in the tract
D(Q)	= per unit development cost
V	= current value of a unit of developed hydrocarbon reserves
F(Q)	= probability distribution over the quantity of hydrocarbons
X(V, T-t, D(Q))	= current per unit value of undeveloped reserves given the current per unit value of a developed reserve and per unit development cost
T, t	= expiration date, current date respectively.

If exploration goes ahead the leaseholder will have some indication of the quantity of reserves available and the costs of developing those reserves. The leaseholder then has the option to pay the development costs and install productive capacity. This option has a value $X(V, T-t; D(Q))$.

If the development option is exercised the leaseholder now owns developed reserves and has the option to extract the hydrocarbons with an exercise price at the cost of extraction. The value of extraction obviously depends on a number of factors such as oil quality, future extraction rates, output prices and taxation treatment. Paddock *et al.* (1988) suggest that as there are active

secondary markets in properties containing developed reserves, the firm will be able to observe values in the market. It should be noted, however, that the value of the extraction option is already incorporated in the current market value of the developed reserve.

Therefore the value of a lease can be calculated as either the value of the exploration option or the development option, depending on the current state of the tract. The development option is valued first as it is used as an input in the valuation of the exploration option. However, unlike the valuation of financial options, this process requires some understanding of equilibrium in the market for petroleum reserves. Paddock *et al.* (1988) construct a model to represent equilibrium in the oil market which determines the manner in which the current value of a unit of developed hydrocarbon reserve should be expected to vary.

The value of the undeveloped reserve $X(V, T-t; D(Q))$ is established using the same arbitrage argument as that used by Brennan and Schwartz (1985). This involves replicating the undeveloped reserve's payoff by holding a portfolio of developed reserves and riskless bonds, which is similar to the approach originally used by Black and Scholes (1973) when valuing shares. As with all options, the calculated value is restricted by a number of boundary conditions. The most important boundary condition to note is that the value of the option to develop the reserve cannot be lower than the difference between the value of developed reserves and the costs of developing. That is, if the costs of developing the reserves are higher than the expected benefit of doing so, development will not take place and the option will be left to expire.

This is one of the main advantages of the option valuation approach to capital budgeting over the traditional DCF approach. The DCF approach, using Monte Carlo simulation, calculates the expected value of developing the oil reserves and adds that value to the expected value of exploring the tract. It is quite possible that the expected value of the undeveloped reserves may be negative, which will obviously reduce the project's overall NPV. However, it is most unlikely that the reserves will be developed at some point in the future if, at that time, the oil price is at a level which makes development unprofitable. Option valuation recognises this flexibility. The lowest possible value of the undeveloped reserve must be zero, as it will only be developed if the prevailing oil price makes development profitable. Thus, the value of the undeveloped reserves calculated using the option valuation model only incorporates the possibility that development will be profitable in the future, and disregards the possibility that development will be uneconomic.

The exploration option is valued in much the same way as the development option. The current value of an unexplored tract is $W(V, T-t; S)$ where S is the combined expected exploration and development cost. Paddock *et al.* (1988) also demonstrate that the comparative statics from the solution to the valuation formula are consistent with those described in Black and Scholes (1973). The value of an unexplored tract of undeveloped reserve will increase given an increase in either the length of the lease, the risk free interest rate, or the variance of the change in the value of a developed reserve.

5.4.3 *Renewable resources*

The application of the option pricing methodology to the valuation of real assets is extended by Morck *et al.* (1989) to incorporate renewable resources such as fisheries and forests. However, the valuation of these types of resources is complicated not only by the uncertainty regarding future output prices but also uncertainty as to the future inventory levels that will be available for harvest. Morck *et al.* (1989) use a lease on a forest to demonstrate their argument.

Classical capital budgeting techniques are inappropriate for the valuation of forestry resources for two main reasons. First, DCF analysis usually assumes that future cash flows follow a rigid pattern and can be accurately predicted far into the future. This is unlikely to be possible due both to the inherent variability in timber prices, and to the budgeting period which typically covers between 20 and 40 years. Second, a DCF approach must incorporate some assumption about the amount of timber that is likely to be harvested. This is obviously reliant on both the natural growth rate of the trees, and the chosen production policy which is in turn a function of the future timber price. Thus, classical budgeting techniques cannot incorporate the option that a forest owner has to halt timber production temporarily if prices are too low. The approach taken by Morck *et al.* (1989) is similar to that of Brennan and Schwartz (1985) and Paddock *et al.* (1988). However, in this case not only is the stochastic nature of output prices allowed for, but the model also incorporates the stochastic nature of the inventory of timber in a forest. The authors derive a formula to value a forestry lease as the value of an option to cut down the trees at the most advantageous time.

Although it is not necessary to formulate predictions of future prices and inventories, some assumption about the future distribution of both variables is needed. Morck *et al.* (1989) derive stochastic differential equations for both variables, and substitute these into a leasehold valuation formula for a given production policy. The lease's value is derived from the valuation formula plus assumptions about the correlations between the price and inventory of timber and aggregate wealth, and an overall assumption that the logging company will choose the optimum production policy.

The derived value of the forest lease must satisfy a number of logical boundary conditions, which may vary from case to case. In the example given, the authors impose conditions which limit the maximum value of an inventory, the minimum value of an inventory due to regulatory constraints, and the terminal date of the lease.

Morck *et al.* (1989) apply their model to a hypothetical lease on a forest and note that the derived value does not depend on any specific predictions about future prices or inventories. The only assumption is that the stochastic processes which govern these variables are stationary. The calculated value takes into account the uncertainty in the underlying timber price and inventories, and the potential strategic reactions of the firm's management to changes in these variables. It is these factors which give the option valuation approach a significant advantage over traditional budgeting techniques.

5.4.4 *Subsidies and guarantees*

A second major type of policy evaluation problem often encountered by policy makers is how to evaluate the costs and benefits of government guarantees or underwriting agreements. Examples of these contingent liabilities are the SOE loans guaranteed by government, proposed export receipt guarantees, the supplementary minimum price (SMP) scheme of the late 1970s and the New Zealand Wool Board minimum price scheme. It is not unusual for governments and other agencies administering guarantee schemes to value them solely on the basis of fiscal cost. The cost of the guarantee, however, is more than simply the fiscal cost of paying exporters for any losses incurred due to importer defaults or wool growers due to lower prices.

Mason and Baldwin (1988) suggest that the cost of financial incentives - such as government underwritten loans - are often not well understood by the people who grant them. Their paper demonstrates how incentives which are offered to encourage or discourage resource use may be evaluated using an option pricing approach. The usual approach to valuing guarantee schemes has some problems. The major one is that it implies that if the beneficiaries of the guarantee receive nothing, then the guarantee is worth nothing. In other words, the usual evaluation methods assume

that the reduction in risk is granted free of charge. This is like saying a car insurance policy was worth nothing in January because the owner did not make a claim over the following year. As pointed out in recent studies, the real value of a guarantee should incorporate the value of assistance *potentially* available to the exporters at the time the scheme is announced. Insurance premiums are assessed relative to the risk the policy is covering - i.e. the probability of a claim being made during the life of the policy. Likewise a guarantee must be worth more than just the value of assistance paid out - the value of the guarantee must also reflect the value of the reduction in risk. That is, the total benefit of the guarantee should be assessed as the value of any cash payments *plus* the value of the *right* to sell a commodity at a minimum price or default on a loan - knowing the government will pay the outstanding debt.

5.4.5 *Wildlife habitat applications (endangered species)*

Many environmental and conservation decisions are about whether a non-renewable resource should be exploited immediately or conserved. The decision turns on whether an irreversible decision to exploit should be made immediately, or put off indefinitely, until further information is available. The information may include more accurate data on the size and other characteristics of the resource, research into alternative renewable resources which can be substituted to produce the same outputs, and alternative processes which improve the efficiency of the existing production system and therefore reduce the current rate of exploitation.

As a simple example consider the evaluation of a policy to construct a high dam in an environmentally unique area of the South Island. The project might yield a positive NPV, assuming that a low enough discount rate and high enough energy prices were used. However, the NPV model ignores the value of deferring the decision. If the investment was deferred, other forms of energy efficiency measures could be more adequately evaluated. From a national viewpoint, it may be found that the payoff from energy efficiency measures may exceed that of the dam.

The same approach could be used to evaluate pollution control. A local authority may choose to invest immediately in a sewage outlet. This may have serious implications for rare, or endangered, marine life and the enjoyment of recreational users. The value of deferring the decision arises from being able to come up with pollution control measures which both preserve the ecological balance and achieve the disposal aims.

Although it appears that in most situations the decision to invest should be deferred, Pindyck (1991) points out that in all cases the benefits of the decision to defer must be weighed against the costs measured in terms of forgone revenue or additional environmental damage which occur under the present set of operating conditions. Only when the benefits of deferring the decision exceed the costs should the decision to invest be deferred.

5.4.6 *Summary*

1. Investment models which rely on discounted cash flow and internal rate of return criteria for decision making are shown not to consider two important characteristics - the irreversibility of initial development expenditures and the benefits of delaying investment pending further relevant information.
2. Uncertainty exists in most investment appraisals about the timing of cash flows, future prices and costs, and the discount rate to apply to the cash flows. DCF techniques of investment appraisal do not always adequately account for this uncertainty.

3. Under conditions of uncertainty, managers wish to retain flexibility and use their discretion to alter their investment decisions should conditions within and outside the investment project change. The impact of such decisions on a project can be considerable and therefore has a value which should be incorporated into evaluations of investment proposals.
4. Option pricing theory offers a framework for investment decision making that incorporates the flexibility to delay or defer expenditure until better information is available. Decisions about whether and when to invest in natural resource projects can be treated in the same way as a decision as to whether and when to exercise an option.
5. Black and Scholes (1973) proposed a model for pricing investments which have option-like characteristics. Applications of the model in the financial literature have been extended to natural resource investments including both renewable and non-renewable resources. Structuring the investment problem using the option pricing framework enables the value of managerial flexibility to be calculated.
6. Knowledge of the value of such flexibility may influence decisions including those about strategic management policy, contractual arrangements, and tactical and competitive issues.

Part II

Option value

CHAPTER 6

Background

Measuring consumer benefits associated with a policy or a project in a world where prices and outcomes are known with certainty is, at least in theory, straightforward. Changes in consumer surplus provide an appropriate measure of a project's, or policy's, contribution to consumer welfare. In many natural resource situations, however, prices and outcomes are uncertain making a strong case for concluding that measuring expected consumer surplus alone is inadequate (Bishop, 1982). When there is uncertainty about the demand or supply of an environmental asset, option value is the adjustment, if any, that is made to expected consumer surplus (ECS).

The concept of option value can be traced back to an exchange between Friedman (1962) and Weisbrod (1964). Friedman (1962) identified market imperfections (e.g. monopoly) and neighbourhood effects (e.g. pollution) as two situations that might necessitate government action in a market economy. Sufficient justification for government action was shown to arise when strictly voluntary exchange was technically impossible or exceedingly costly. Friedman contrasted a national park with a central city park to illustrate the need to use the coercive mechanisms of government. Demand-side uncertainty exists in both cases. Individual users of the services in each park would be unable to predict their future demand with certainty. In contrast to users of the national park, the population of central city park users would be more concentrated and individuals' use patterns would tend to be less planned, and more spontaneous and frequent. Supply-side uncertainty can arise from the inherent variability of the natural system (e.g. the lack of snow affecting skiing in the national park) and the decisions of public agencies (e.g. city government reducing the facilities in the central park).

Weisbrod (1964) offered two reasons why private provision might diverge from optimal social provision. First, some commodities are purchased infrequently and demand is uncertain. Second, the cost of increasing production once production has ceased, or curtailed, might be relatively high or technically impossible. Although these two characteristics apply to many natural environments, such as national parks, indigenous forests, wildlife habitats and water resource systems, they may also apply to produced goods such as hospital services, art collections, historic places, and urban transport systems.

With these two characteristics in mind, Weisbrod considered Friedman's example of a national park where it is easy to charge a fee, the park is privately owned, the firm can price discriminate, the good is not storable and there are no external economies. If total cost exceeds total revenue, then it is profitable for the firm to close the park and allocate resources into other uses. Weisbrod notes that the decision to close the park might not be socially optimal because: "... people who anticipate purchasing the commodity at some time in the future, but who, in fact, never will purchase it ... they will be willing to pay something for the option to consume the commodity in the future" (Weisbrod, 1964, p.472).

Weisbrod defined option demand as a person's willingness-to-pay for an option to consume the commodity in the future, even if the individual is not a current user and never actually exercises the option. Economic efficiency requires consideration of both user willingness to pay and option demand. In theory, option demand should influence supply but the lack of a practical non-coercive mechanism by which the firm can collect a fee from non-users (i.e. the option value over and above

the willingness to pay associated with current use), means that these preferences will not be considered by the profit-seeking firm. Therefore, if total revenue is not sufficient to cover total costs, the firm will either seek to adjust its operation to lower costs or it will close down.

The supply of current services associated with the park enters the utility functions of two groups of individuals. Those using the park express their preferences by actually visiting the site and enjoying its benefits. Revenue from charging for access, as is the case in numerous parks throughout the world, provides at least a preliminary estimate of aggregate value. However, continued supply also enters the utility functions of current users and prospective non-users and collecting a fee from them would be expensive.

Friedman's early challenge to the conventional wisdom regarding supply of collective goods and Weisbrod's suggestion that option value should be accounted for in supply decisions, raised some difficult theoretical issues for public sector decision making. Environmental economists were quick to recognise the significance of option value when considering choice where some decisions would have irreversible adverse consequences. They were sceptical of the allocative efficiency of the market mechanism and their conclusions supported the observations made earlier by Weisbrod. Krutilla (1967) illustrated the difficulty that option value poses for project evaluation when irreversibilities exist. Even if perfect price discrimination is feasible, receipts under private ownership are not comparable with estimates of willingness to pay derived from use values. When evaluating alternative actions within an efficiency framework, attention must be given to obtaining estimates of both use-related willingness to pay and use, use-related option value and non-use related option value. For the profit-seeking organisation the practical problem of appropriating non-user values remains. At the time, private non-profit organisations, such as the Nature Conservancy, were active in the preservation market but, as Krutilla suggested, the market is imperfect and does not provide an accurate guide to total value.

Introducing time into the analysis brings into focus the difficulties posed by the dynamics of option value. It is becoming evident, in some forms of recreation at least, that participation today stimulates demand in the future (Davidson *et al.*, 1966). For example, an individual with no history of using a park at time t might be willing to pay nothing for an option for future use. However, it is quite conceivable that at time $t + h$ this individual will be exposed to the enjoyment others derive from use which in turn encourages either use at $t + h$ or results in a positive willingness to pay for an option for future use at $t + k$ where $k > h$. That is, there is an interaction between present demand, future demand, and option demand.

Empirical studies have shown that option value may be as great as half expected consumer surplus (Fisher and Raucher, 1984), indicating that the concept has great significance in determining the optimal allocation of resources. Given the magnitude of option value identified in empirical studies, and its ambiguous sign, serious misallocations are likely to occur if it is not understood and accounted for. In particular, the existence of uncertainty implies that current evaluation procedures, such as cost-benefit analysis, that rely largely on expected values, can be inappropriate and may provide misleading information.

CHAPTER 7

Cost-benefit analysis and uncertainty

Cost-benefit analysis (CBA) is applied welfare economics. In its simplest form CBA attempts to measure and aggregate market preferences so as to identify states of the economy which result in greater levels of aggregate welfare. The recommendation implicit in a project's NPV derive from welfare economics. Theoretical difficulties with CBA are reasonably well-known and are centred on the problems of aggregating welfare across individuals. Practical problems arise when the analyst seeks to value all inputs and outputs associated with a project or policy in terms of the money metric (Kerr and Sharp, 1987).

The task of the cost-benefit analyst is to specify demand functions representing societal willingness to pay, and supply functions representing social marginal cost, in order to identify efficient allocations of resources. To complete this task, to a reasonable degree of accuracy, the analyst requires an understanding of the theoretical constructs that underpin cost-benefit analysis. In particular, the conditions of demand and supply need to be specified, and estimates made of these concepts, in a way that is consistent with the compensation tests used to identify efficient allocations. Concern for uncertainty and time are two characteristics endemic to most natural resource problems.

In a certain environment, each individual knows the quality of the environmental resource (Q) at time t_0 , the future state $Q(t_{0+h})$ and the time path of Q during the intervening period. Under conditions of certainty, each individual also knows how utility, or profit, would change as a result of policy. The compensated variation of the gainers is defined as the largest sum they would be willing to pay (WTP) to enjoy the end-state (e.g. at $Q(t_{0+h})$ = improved water quality) associated with the policy at each point in time. The compensated variation of the losers is the least sum they would be willing to accept (WTA) to live willingly in the end state (e.g. at t_{0+h} profit is lower because of regulated land management practices) implied by the project. A potential Pareto improvement is identified when aggregate benefits (ΣWTP) exceed the costs (ΣWTA). It should be noted that compensation is purely hypothetical and there is no presumption that compensation is actually paid.

Competitive markets in the absence of externalities produce prices that can be used as a measure of the benefit to consumers. Many cost-benefit analyses assume that individuals express their preferences through the pricing mechanism operating in a certain environment. To incorporate uncertainty into cost-benefit analysis, each individual in the economy is assumed to make decisions in the knowledge of the set of possible states (Arrow, 1964). There are assumed to be S possible states of the world, $s = 1, \dots, S$. In the s^{th} state, Q_s is the amount of commodity Q available. Uncertainty is also concerned with the future and, in principle at least, there exists a set of possible states at each point in time. The analyst is now confronted with the task of incorporating both uncertainty and time into the analysis.

Attempts at incorporating the twin problems of uncertainty and time into cost-benefit analysis have centred on adjustments to the discount rate and the basis of the compensation test itself. Let's consider the discount rate first. It is generally accepted that individuals are influenced by uncertainty and will not value outcomes at their expected values. This behaviour is evident in capital markets where individuals do not necessarily maximise the present value of expected returns; rather, they maximise the present value of returns adjusted for risk. Hirshleifer (1965, 1966) argues that investments in perfect capital markets are properly discounted with respect to both time and risk

and these discount rates should be used in public policy analysis. Some economists have argued that uncertainty can be allowed for by adjusting the discount rates attached to the flows of benefits and costs over time (Eckstein, 1965). The analysis of others, such as Arrow and Lind (1970), suggests the use of a riskless discount rate. Both Hirshleifer and Arrow and Lind assume that individual preferences are relevant for public decision making and that these preferences should provide the basis for valuing benefits and costs. The Arrow-Lind result of a riskless discount rate depends on an assumption regarding the ability of governments to spread the risk of public sector decision making among a large number of people.

More recently, the approach of adjusting the discount rate has been challenged by those who consider that the analysis should focus directly on the compensation test itself. Starting with individual valuations of each possible state of the world, Graham (1981) concludes that aggregate willingness to pay in each state is the appropriate measure of benefit. The implication is that a riskless discount rate should be used along with the price (contingent prices) individuals attach to each possible state of the world. In other words, the influence of uncertainty on WTP/WTA is measured directly through the (contingent) pricing mechanism and not through ad hoc adjustments to the discount rate. If the argument for a riskless discount rate is accepted then the appropriate compensation test needs to be carefully considered because uncertainty affects the distribution of costs and benefits and efficiency through its effect on the costs of risk bearing (Ulph, 1982).

Two compensation tests can be applied to uncertain situations where a riskless discount rate is used. One test (*ex ante* compensation) uses an *ex ante* perspective based on what the individual expects to receive/pay before knowing what the state of the world will be. The *ex ante* test examines whether the aggregate WTP (option price) of *ex ante* gainers exceeds the WTA of *ex ante* losers (option price). The policy passes the test if $\Sigma WTP > \Sigma WTA$. A distinguishing feature of the *ex ante* test is that when the policy is undertaken, and a particular outcome occurs, some individuals will be worse off.

The other test (*ex post* compensation) uses an *ex post* perspective based on what the individual expects to receive/pay after knowing what the state of the world will be. Hence, an *ex post* measure of WTP is based on the consumer surplus in the state of the world as if that state occurred. Policy A would pass the compensation test if the gainers could compensate the losers in all states of the world (Bishop 1986).

When a policy is implemented it is possible that the state that actually occurs results in $\Sigma WTA > \Sigma WTP$. That is *ex post* the policy would not be recommended on efficiency grounds. It is at this point in the argument that the Arrow-Lind result is significant because they envisaged government financing the risk of this outcome, via the tax system, over all taxpayers so that the with-tax liability of each individual is negligible. Therefore, *ex post* compensation eliminates the specific risk borne by individuals. Results following from the Arrow-Lind proposition assume that outcomes are expressed purely in terms of financial risk where (risk-averse) individuals can take out insurance against adverse outcomes. In fact, this is the basis of *ex post* compensation where each individual loser is fully reimbursed either through voluntary (market) transactions or through government guarantee.

There is increasing weight of argument now that the *ex ante* compensation test is more appropriate in uncertain situations (Graham, 1981; Ulph, 1982; Bishop, 1987; Smith, 1987a). This is because the *ex ante* test can handle a greater array of economic variables which influence the adjustments that individuals explicitly make to their valuations to allow for uncertainty. The adjustment is option value. Furthermore, if policy makers accept the principle of consumer sovereignty then it behoves analysts to identify and, if possible, incorporate option value into analyses of policy options.

CHAPTER 8

Option value

Option price (OP) is an individual's maximum WTP to maintain an option for future use, or the minimum compensation (WTA) required to give up an option for future use. For uncertain users, OP includes the expected value of future use (ECS) plus option value. Option value (OV) is an adjustment, that is either positive or negative, reflecting uncertainty or risk (Bishop, 1987).

$$OP = OV + ECS$$

There are two broad classes of risk:

- i Demand risk refers to situations where the variables of demand - i.e. the price of other goods, preferences and income - are uncertain. It is possible that a "purchased" option would turn out to be worthless because demand did not eventuate.
- ii Supply risk on the other hand refers to situations where future availability is not guaranteed unless the option is "purchased", but the supply may occur anyway. Of course, a "purchased" option may not be sufficient to guarantee future supply. Natural systems can fail - species become extinct - even though resources have been allocated to their preservation.

To illustrate option value, consider the community drawing water from an aquifer threatened by the prospect of nitrate contamination. Assume that it is possible to avoid contamination by implementing a policy (e.g. land use controls) aimed at curtailing nitrogen use in the region. Because the policy will incur costs (e.g. planning costs, lost production opportunities) the problem is: what is the public's total willingness to pay to prevent uncertain contamination. The situation is characterised by demand and supply uncertainty and option price is the appropriate measure of the policy's economic value.

Option price is the *ex ante* reduction in income necessary to make the individual indifferent about the situation where the policy exists and the alternative state where the policy does not exist for given probabilities of contamination. Option price is an *ex ante* value which represents a state-independent payment that a consumer would be willing to make to obtain the results of the policy. Given the same state probabilities, option value can be positive:

$$OV = ECS - OP > 0$$

or, negative:

$$OV = ECS - OP < 0$$

Many economists agree that measurement of ECS is an inadequate estimate of welfare change and recognise that option value is the appropriate adjustment to ECS under conditions of uncertainty. The possibility of option value having an ambiguous sign has stimulated a great deal of research aimed at resolving the ambiguity.

CHAPTER 9

Sources of uncertainty

If option value is positive then estimating user benefits (i.e. an estimate of expected consumer surplus) would be an underestimate of total benefits. If option value is negative then user benefits would overstate total benefits. Uncertainty can arise either because individual consumers are uncertain about their demand for the environmental asset and/or because of the uncertainty surrounding supply. The adjustment for demand-side uncertainty is demand-side option value (DSOV), and the adjustment for supply-side uncertainty is supply-side option value (SSOV).

9.1 Demand uncertainty

Option price is the maximum amount an individual is WTP now for an option to consume Q in the future. The option, for example, might be a permit allowing a number of visits to a wilderness area or a right to withdraw water from an aquifer. While ECS may vary across states of the world, option price is constant across these states (i.e. option price is a state-independent measure of WTP). Demand is dependent upon variables such as income, preferences and the price of other goods which may themselves be uncertain (Freeman, 1985). To examine demand-side option value (DSOV) assume supply certainty and consider:

$$OP = ECS + DSOV$$

The sign of DSOV is determined by the consumer's attitude to risk (Bishop, 1982). In particular, given relative probabilities, prices and conditional incomes, the sign of DSOV depends on the relationship among the conditional marginal utilities of income. Plummer and Hartman (1986) present a more general model of the problem where the state of the world is described by a state variable which can be used to represent income and tastes. Their results support Bishop's conclusion, showing that the sign of DSOV is the same as the sign of the correlation between ECS and the marginal utilities of income across states of the world. For a risk-averse person facing income uncertainty, the sign is determinate and $DSOV < 0$. Therefore, ECS as a measure of welfare overstates state-independent WTP (option price).

As Plummer and Hartman note, tastes are generally unobservable and there are few grounds, in theory, for restricting taste-related changes in behaviour in other than arbitrary ways. They show that DSOV, for the same individual, can be positive or negative depending on the probability distribution rather than the changes in ECS or the marginal utility of income across states. In general, it is not possible to determine *a priori* the sign of DSOV when uncertain preferences are involved.

9.2 Supply uncertainty

The possibility of supply-side uncertainty, although raised by Cicchetti and Freeman (1971), was first modelled by Bishop (1982). Assuming demand certainty and a two-state world, Bishop showed that supply-side option value (SSOV) is unambiguously positive. Using a more general framework, Freeman (1985) structured his analysis around four possible cases of supply-side uncertainty. Given

a hypothetical project which is assumed to supply a public good, the probability of supply with the project exceeds the probability of supply without the project. On the demand side, individuals are assumed to be risk averse and there is no uncertainty in the determinants of demand. Option price is the payment for the project that equates expected utility with and without the project. Eliminating demand-side uncertainty, the following relationship is of interest:

$$OP = ECS + SSOV$$

Freeman (1985) shows that the sign of SSOV is *a priori* determinate in two cases. In the first case, the probability of supply with the project is one and the probability of supply without the project is zero, and SSOV = 0. There is no uncertainty associated with either state and, therefore, option price equals ECS. In the second case, the probability of supply with the project is one and the probability of supply without the project is between zero and one; without the project the individual holds a lottery ticket (i.e. an uncertain right to consume a public good that might be available). The risk-averse person will pay more than ECS to eliminate the uncertainty of supply, and option value in this case is the risk premium that the person is willing to pay.

9.3 Demand and supply uncertainty

Determining the sign of option value, when both supply and demand are uncertain, is difficult. Freeman (1985) concludes that the sign is ambiguous and depends on:

- i the pattern of supply uncertainty reflected in relative probabilities,
- ii the determinants of demand uncertainty,
- iii the degree of demand uncertainty.

Theoretical research has, to date, been unable to provide unambiguous results. Moreover, it appears that it is unlikely that general statements can be made *a priori* about the sign of option value (Freeman, 1984; Plummer, 1986). This leaves the sign to be determined through empirical analysis based on specific probability distributions and utility functions that represent consumer preferences.

CHAPTER 10

Bounds for option value

Considerable ink has been spilled over the topic of option value as economists have struggled to pin down the nature of the concept and determine its importance. Empirical testing to determine the magnitude and importance of option value has produced widely varying results. An alternative approach is to attempt to provide analytical bounds for option value followed by use of a cardinal utility model and numerical calculations. Freeman (1984) attempts first to determine *a priori* the sign of option value and then to establish whether option value is likely to be large enough to make any difference in a cost-benefit analysis. Smith (1984) attempts to establish an analytical bound for option value and focuses in particular on the uniqueness of the good or service in question.

Freeman employs a standard utility model to explore the relationship between option price and expected value of consumer surplus, and hence the sign of option value in various situations including income uncertainty, price uncertainty, and state-dependent preferences. His principal conclusion from the theoretical analysis is: "... that for risk averse individuals, option value is positive for a plausible model of that case of most interest in the environmental economics literature - where demand uncertainty arises from some exogenous factor (other than price or income) which does not affect the marginal utility of income and attitudes towards risk across states" (Freeman, 1984, p.11).

Theoretical analysis completed, he turns to numerical calculations to determine the size of option value relative to ECS. His numerical simulations examine a considerable number of scenarios including: three income levels, four utility functions reflecting differing levels of risk aversion, four probabilities of the undesirable outcome occurring, and three levels of consumer surplus as a proportion of income. His conclusions are quite clear. For the case of state-dependent preferences, option value exceeds expected value of consumer surplus by 10% only in those cases where the degree of risk aversion is high, the probability of demand for the good or service is low, and consumer surplus as a proportion of income is high (Freeman, 1984, p.9). Where demand uncertainty arises from uncertainty about future income, option value is often negative for risk-averse individuals and use of expected value of consumer surplus in a cost-benefit analysis would give an over-estimate of option price. Freeman notes that, "In many apparently plausible cases, negative option values are quite large relative to expected consumer surplus. In those cases reliance on estimates of expected consumer surplus could lead to substantial overestimates of option prices and benefits" (Freeman, 1984, p.11).

Freeman further notes that his results cast considerable doubt on the validity of the results from empirical estimates of option value reported in earlier studies. Greenley *et al.* (1981), for example, report that option value for recreation in the South Platte River was \$23 per household per year. Freeman employs a cardinal utility function to show that option value could be no more than 7.4c (Freeman, 1984, p.12). In the light of this severe divergence in results he concludes that, "... it appears crucially important that as a first step the nature and source of the demand uncertainty be identified. Then well-specified models of individual choice can be developed and used as the basis of testing with whatever data are being used. Then we will be able to consider whether the data are consistent with our models of option value - or whether the models must be rejected as inconsistent with the empirical evidence" (Freeman, 1984, p.12).

The empirical evidence from a later, carefully conducted empirical study (Edwards, 1988), appears to support Freeman's conclusion that for cases of state-dependent preferences, option value is small relative to expected consumers' surplus. Smith (1984) relies upon a theoretical model to generalise the results of Freeman (1984). By employing an index of uniqueness and a state-dependent utility model, Smith reaches some simple conclusions about the bounds on option value as a fraction of ECS for a two-state world " ... the degree of demand uncertainty and the uniqueness of the good are the key ingredients in determining the magnitude of option value in comparison to the expected user value" (Smith, 1984, p.294).

The ratio of option value to ECS varies inversely with the probability of desiring access to the good or service in question, and varies directly with the degree of uniqueness of the good or service. Smith concludes that, " ... the relationship between option value and expected consumer surplus will vary with the nature of the environmental resource under scrutiny. ... Equally importantly it seems reasonable to expect that there will be differences in the intensity of preferences for these goods and services across individuals. Empirical research that seeks to develop operational rules for estimated ratios of option value to expected user values will need to consider both of these factors if the resulting suggestions for future benefit estimation are to provide plausible approximations for this component of intrinsic values"(Smith, 1984, pp.294-295).

Bishop (1988) reviews the various attempts to determine the sign of option value and reaches a gloomy conclusion about the outcomes and the chances of further progress on this front. "Theoretical analysis on the demand and supply sides have convincingly shown that *a priori* attempts to determine the sign are unsuccessful except in special situations like the Bishop case or where specific information about the utility functions is available" (Bishop, 1988, p.91).

Bishop concludes, however, that the sign of option value is important and option value cannot be ignored in cost-benefit analysis. Given the existence of the brick wall that blocks theoretical determination of the sign of option value, Bishop recommends laboratory and field experiments to investigate further the role of uncertainty in influencing welfare. Controlled experiments using real money, " ... could help establish the validity of contingent option price, option value, and consumer surplus estimates" (Bishop, 1988, p.92).

The messages to be drawn from the work of Freeman and Smith are that if analysts are prepared to make some assumptions about the nature of the utility function and marginal utilities in various states of the world, then some *a priori* insights can be gained as to the likely importance and size of option value relative to ECS. In circumstances where the good or service is not readily replaced (i.e. has few substitutes) and individuals are risk averse, option value can be a significant positive amount. In circumstances where there is considerable demand uncertainty, perhaps because of uncertainty about future incomes and prices, option value can be a significant negative amount.

CHAPTER 11

Quasi-option value

Quasi-option value is a concept associated with maintaining future options. Developed by Arrow and Fisher (1974) and Henry (1974), quasi-option value is the value of emerging information in a situation characterised by irreversibility. Using the example of an aquifer, assume that contamination is irreversible and that:

$$\begin{aligned}t_1, t_2 &= \text{two periods} \\ \alpha &= 0 \Rightarrow \text{aquifer is not contaminated} \\ &= 1 \Rightarrow \text{aquifer is contaminated}\end{aligned}$$

Then, strategy:

A) avoidance $S(\alpha=0, t_1)$ allows either $S(\alpha=0, t_2)$ or $S(\alpha=1, t_2)$

and

B) contamination $S(\alpha=1, t_1)$ allows only $S(\alpha=1, t_2)$

Strategy A, involving a policy aimed at avoiding contamination, keeps future options open whereas Strategy B, perhaps a policy of laissez faire, reduces future options. If contamination is irreversible and new information about the value of avoidance is likely to emerge after Period 1, but before the Period 2 decision must be made, then quasi-option value is positive.

CHAPTER 12

Summary

Uncertainty exists in many situations where economists are asked to estimate the costs and benefits of alternative policies. These uncertainties are particularly evident in the area of environmental policy. A list of applied policy research areas includes: the protection of endangered species, hazardous waste management, air pollution control, groundwater contamination, water resource development and pesticide use policy. The problem of uncertainty is exacerbated by the fact that many of the decisions involved in addressing these problems are irreversible. That is, the decision is often whether to use a natural resource immediately or conserve it until more information is available on the nature of the resource or other alternatives to its use. As we have discussed in this report, techniques which ignore the ability of policy makers to defer such decisions will tend to undervalue projects resulting in a misallocation of resources.

Where uncertainty exists, the policy analyst should use concepts and analytical techniques that survive continued challenge in the literature. Techniques that incorporate uncertainty into cost-benefit analysis, using subjective probability distributions of key parameters, miss the fact that individual welfare can be directly influenced by uncertainty. Discounted cash flow techniques have been strongly criticised in the project appraisal literature (Paddock *et al.*, 1988). Although the techniques of project appraisal take account of the timing of cash flows generated by natural resource projects they do not fully capture the factors which affect the resource values. The point is well made in the literature that the net present value framework draws on the analogy between the project and the riskless cash flows deriving from a default free bond. This is obviously inappropriate where the actual project cash flows depend on present, and future, decisions. Future decisions are contingent upon environmental, political, and social factors which are unknown at the time the project is being evaluated.

Likewise, the use of ECS has found little support in the resource economics literature as a welfare measure. The resource economics literature asserts that the relationship between uncertainty and an individual's willingness to pay, or willingness to accept compensation, needs to be explicitly recognised in the valuation exercise.

Lund (1991) identifies three categories of option value in the literature. First, there is the concept of option value as proposed by Weisbrod (1964) which has been refined and applied to numerous environmental policy problems. The Weisbrod concept of option value focuses attention on the individual valuing alternative states under uncertainty. In resource economics the concept of option value is an adjustment, positive or negative, that the individual makes to ECS when confronted with demand and supply uncertainty. Risk aversion is an important assumption underlying option value.

Option price comprises ECS and option value. Option price therefore includes option value. After lengthy debate in the literature the concept of option price is considered to be consistent with the *ex ante* compensation test which is appropriate when policy analysis is complicated by uncertain outcomes. There is general agreement that option value cannot be ignored in cost-benefit analyses where uncertainty is involved.

Two further categories of option value are found in the finance literature concerned with valuing financial securities such as stocks and commodities. Recent developments in contingent claims

analysis have expanded the range of applications to include assets in general and investment projects. Uncertainty is central to the problem, just as it is in the resource economics literature. Where a firm has the right to defer an investment or make some other form of strategic decision the flexibility has some value. Option pricing theory draws the analogy between the managerial flexibility and a financial option and values the rights accordingly.

The second category of (quasi-) option value is associated with Arrow and Fisher (1974). Quasi-option value is the value of gaining, or taking advantage of, more information before making a decision. This value is always non-negative and can be identified with the value of flexibility. The value of a financial option over and above the value of comparable securities is akin to the Arrow-Fisher quasi-option value concept. That is " ... it derives from the ability to choose in the future whether to exercise the option or not, depending on the stock price which will then be observed." (Lund, 1991, p.145.)

Lund (1991) also shows that the third category of option value (financial option value as in the Black-Scholes model) can be considered as an application of the quasi-option valuation model (e.g. Arrow-Fisher model). The quasi-option model illustrates the value of maintaining flexibility; basing decisions on expected future cash flows may lead to premature investment. Furthermore, option value increases with uncertainty, a result to be found in both financial option value and the Arrow-Fisher model, increasing uncertainty increases option value and postpones investment. Therefore the general results of the Black-Scholes and Arrow-Fisher models are not significantly different. For any application, the researcher must consider the particular assumptions regarding the stochastic processes involved, the availability of data and whether the degree of risk aversion needs to be specified.

In the finance literature, the value of the managerial options, or flexibilities, depends on a number of factors. As a general rule the value of the option depends on:

i How long the project or policy may be deferred

The longer the deferral, the more time there is to evaluate the policy, avoid costly errors and make a better decision.

ii The risk of the project

Riskier projects are worth more than options on less risky projects or policies as only the positive payoffs from the policy are valued. Therefore, the "option" does not have to be exercised if doing so results in a loss.

iii Interest rates

As mentioned above higher discount rates - often associated with risky projects - mean lower NPVs. However, higher interest rates also reduce the present value of any future cash outlays needed if the policy is to go ahead. Therefore, higher interest rates usually raise the value of projects with embedded growth or strategic options.

In summary, the concepts of option value which have arisen in the finance (e.g. Black-Scholes model) and resource economics (e.g. Arrow-Fisher model) literature both attempt to deal with the same problem, i.e. dealing with decision making under uncertainty. One possible integration of the two approaches suggests, as does Lund (1991), that the option value discussed in resource economics is simply the managerial flexibility estimated using option pricing theory from finance. However, relating these two approaches to the Weisbrod concept is not as straightforward. Before this could be done the issue of the sign of option value would have to be resolved because under an option pricing theory approach option value could only ever be positive.

Part III

The valuation model

CHAPTER 13

Rationale for the valuation model

13.1 Introduction

As discussed previously, the option pricing approach is useful when evaluating time distributed uncertain outcomes. Compared to traditional DCF approaches, option pricing provides the opportunity to use more market based data and avoids the need to estimate appropriate risk adjusted discount rates. This section begins by introducing a relatively simple contingent claims valuation model for valuing known, but as yet undeveloped gas reserves. How the technique may be applied is described and a base value for the field as an undeveloped reserve is established. More realistic (and more complex) variations of the model are then introduced to incorporate more realistic assumptions regarding the development time-lags, development costs, production cost functions and management flexibility. As with all models there is a trade-off between realism and simplicity and the option pricing approach to capital budgeting problems is no different.

Starting with a simplified binomial model which operates in discrete time the discussion then moves to a continuous time model which is very similar to the Black and Scholes European call option pricing model. The similarity between these two investments lies in the investment required to develop the lease and the exercise price paid by a call option holder when the call option is exercised. Development expenditures convert undeveloped reserves into developed ones. However, the leases can be relinquished without development. In either case, the decision to develop will depend upon the underlying price of the output compared to the cost of development. The first two models implicitly assume the right to develop, or invest, can only be exercised on the last day of the lease, i.e. at relinquishment. In the third part of the discussion a model is introduced which enables development of the lease at any time prior to relinquishment of the lease. The first three parts of the discussion deal with valuing an undeveloped gas field. In section 13.5 the discussion is extended to valuing developed gas fields where there is the option to shut down or reduce production. This section is based on the work by Brennan and Schwartz (1985).

13.2 A simplified binomial gas price process

To demonstrate a basic application of option pricing to a simple energy example, the option pricing technique is applied to valuing a small undeveloped gas field using a simplified binomial approach. The binomial model is a discrete time representation of the model applied later in the section. This is similar to the approach used by Ekern (1988) with a stylised example to demonstrate how such an option pricing approach may be used to value an undeveloped oil field.

Assume that the price of gas follows a binomial multiplicative random walk. That is, the price of gas, S_t , may go up by u with a probability of q and down by d with a probability of $(1 - q)$. If the initial price of gas, S_0 , is \$300,000 per PetaJoule (PJ)², q is 1/2, u is 3/2 and d is 2/3 then at time $t = 1$ a gas price of \$450,000 per PJ or \$200,000 per PJ will be equally likely. At time $t = 2$, the possible gas prices are \$675,000, \$300,000 or \$133,000 per PJ with respective probabilities 0.25, 0.50 and 0.25.

² One PetaJoule equals 10^9 Joules.

We initially assume the exponential growth rate in the gas price, g , is stochastic and takes on the values $g = \ln u$ and $g = \ln d$ with equal probability q . Therefore as $q = 1/2$ and $d = 1/u$ its expected value, or mean, is $E(g) = 0$ and its standard deviation is $\sigma(g) = \ln u = 0.4055$.

If both the owner of the gas and the owner of the contract for future delivery receive equivalent returns then it is assumed that the gas has no convenience yield. Often the physical price of a commodity, such as copper, gas, oil, or coal, will be significantly higher than its future's price. This is because manufacturers and other producers need to maintain an inventory of commodities to ensure continuity in their production process. Therefore the convenience yield reflects the difference between the cash and futures prices. Note that this assumption will be relaxed later in the discussion. If it is initially assumed that investors do not require a risk premium in order for them to assume risk - i.e. they are risk neutral - then, following Cox and Rubinstein (1985), the probability of up or down movements in price as the hedging probability p is defined as:

$$p = \frac{(r - d)}{(u - d)}$$

Where u and d are as before and r is equal to $(1 + r_F)$ where r_F is the risk free rate.

For simplicity assume that the gas field is modelled by a two period model where development and extraction occur in the second period. That is, the decision to develop is made in period two once the prevailing oil price is known. This is really the crux of the argument for using an option pricing methodology, as opposed to a DCF approach. The decision to develop is *contingent* upon the future gas price. If gas price movements are favourable, the project will proceed. If not, the development will not go ahead. Clearly, this type of process cannot be adequately modeled using a DCF approach which makes the implicit assumption that future cash flows are *not* contingent upon future prices and that the operating environment is static. It is also assumed that the field will either be in full operation or closed. Assume that the costs of development per PJ, D , are as follows:

development cost, $t = 2$	\$140,000
operating cost, $t = 2$	\$240,000
Total unit cost, $t = 2$	$D_2 = \$380,000$

In a conventional discounted cash flow analysis, an analyst would simply calculate the expected value of the gas and compare it to the per unit costs of production. In this case the expected price of gas in time $t = 1$, $E_0(S_1) = \$325,000$ per PJ and at time $t = 2$ $E_0(S_2) = \$352,083$. As the expected gas price in time $t = 2$ is less than the unit costs of development and construction, the analyst would conclude that the field has a negative NPV and therefore should not be developed. This is not necessarily the case as the DCF analysis ignores the option characteristics associated with the field. In the second period the field will be developed, and extraction commenced, only if the price of gas exceeds \$380,000. Recall that the price of gas may take the value \$675,000, \$300,000 and \$133,000 per PJ with probabilities of 0.25, 0.50 and 0.25 respectively. Therefore in the second period, the value of the gas field per PJ of gas will be $\max(S - D_2, 0)$, i.e. the positive difference between the price of gas and its cost of development and extraction or zero. Note this is *not* the same as the weighted average, or expected value of the price of gas less the cost of development and extraction.

Given the current price of gas and the known cost of development and extraction, which occur in period two, what is the current value of the field? The first adjustment to make is to discount the cost of development to the present day. If it is assumed that the price of gas and the costs of

development are such that it is *certain* that the option to develop the gas field will be exercised, then the current value of the gas field might be approximated by:

$$c = S - D_2 r^{-n}$$

Therefore the value of the gas field is simply the current price of gas less the present value of the costs of development and extraction which occur in period 2. When there is uncertainty about the possible paths of future gas prices and costs, it is necessary to introduce weights based on the probabilities of the possible outcomes. Therefore, the value of the gas field can be expressed in the binomial option pricing formula given by Cox and Rubinstein (1985).

$$c = S \cdot B(a; n, p') - D_2 r^{-n} \cdot B(a; n, p)$$

Where: c = the current value of the gas field
 B(.) = complementary binomial distribution
 S = current gas price \$ per PJ
 D₂ = the costs of development and extraction \$ per PJ
 n = the number of periods to relinquishment of the lease
 p = the "hedging" probability
 p' = (u/r)p
 a = the minimum of n and the smallest integer greater than ln(D₂/Sdⁿ)/ln(u/d).

The first term on the right hand side is the current price of gas multiplied by the probability of receiving the current gas price if the field is developed. The second term is the present value of the development cost multiplied by the probability of paying the development cost if the field is developed. For our simplified example, $u = 3/2$, $d = 2/3$, $r = 1.1$, $S = \$300,000$, $D_2 = \$380,000$, $n = 2$, $p = 0.5200$, $p' = 0.7091$ and the critical value of a is $\ln(\$380,000/(\$133,333))/\ln(2.25) = 1.2915$ implying $a = 2$. Therefore:

$$\begin{aligned} c &= \$300,000 \cdot B(2; 2, 0.7091) - \$380,000 \cdot (1.10)^{-2} \cdot B(2; 2, 0.52) \\ &= \$300,000 \cdot 0.5028 - \$380,000 \cdot 0.8264 \cdot 0.2704 \\ &= \$150,840 - \$84,919 = \$65,921 \end{aligned}$$

This implies that, despite the fact that the cost of development exceeds the *expected* gas price, the undeveloped gas field is worth \$65,921 per PJ. This compares with the conventional DCF valuation which would be negative, implying the field was worthless.

13.3 The gas field value with continuous gas prices

So far it has been assumed that gas prices move discretely. The more advanced contingent claim valuation models assume that gas prices change continuously and follow what is often referred to as a Wiener process or geometric Brownian motion. That is, the price of gas evolves through time according to:

$$\frac{dS}{S} = \mu dt + \sigma dz$$

Where S , as before, is the gas price, μ is the trend in the gas price, σ is the instantaneous standard deviation in the gas price and dz is the increment to a standard Wiener process³. Future gas prices are assumed to be lognormally distributed which simply means the lower bound for the gas price is zero while the upper bound can be infinite. It would not make sense to have normally distributed gas prices as this would imply that a *negative* gas price was as likely as a positive price. The change in the price of gas can be expressed as $\mu = \ln(S_t/S_0)$. As μ is the log of a lognormally distributed variable μ is, itself, normally distributed, which is a useful property that simplifies the properties of more advanced models.

The value of the gas field is contingent upon the price of gas, and the value of the field is assumed to behave like any other derivative security, where the price is a function of the stochastic variables underlying the field and time. In the type of model being developed, the value of undeveloped reserves depends on the underlying gas price and time. Using a result known as Ito's lemma, the change in the value of the undeveloped reserves is expressed as a function of the underlying gas price and time. This yields an identical partial differential equation (PDE) to that for the Black and Scholes model which describes the underlying process of a share option.

$$\frac{\partial c}{\partial t} = rc - rS \frac{\partial c}{\partial S} - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2}$$

As with any contingent claim, the solution which satisfies this PDE depends on the boundary conditions. In the case of the undeveloped gas field the investment and development option is similar to a call option. Therefore the appropriate boundary condition is:

$$c = \max(S - D_2, 0) \quad T = t$$

The intuition behind this is that, on the relinquishment date of the lease the value of the field per PetaJoule of gas, c , will be the positive difference between the gas price, S , and the cost of development, D_2 , or zero - whichever is greater. Logically, the field would not be developed if the *known* costs exceeded the *known* gas price. By making similar assumptions to those made by Black and Scholes (1973), when deriving a model for pricing European call options, the following result is obtained which is an exact analytical solution for the valuation of a continuous time process.

$$c = S \cdot N(d_1) - D_2 \cdot N(d_2) e^{-rt}$$

where:

$$d_1 = \frac{\ln(S/D_2) + [r + (\sigma^2/2)]t}{\sigma \sqrt{t}}$$

and

$$d_2 = d_1 - \sigma \sqrt{t}.$$

$N(\cdot)$ is the standardised cumulative normal distribution and the interpretation of the pricing formula is similar to the binomial case. Given the parameter values already used, the Black and Scholes

³ dz is merely a normally distributed random number i.e.

$$dz = \epsilon \sqrt{t} \quad \text{where } \epsilon \sim \Phi(0,1)$$

formula suggests the field is worth \$65,524 per PJ compared with the \$65,921 obtained in the discrete time case. The comparative statics of this model are identical to those of a European option. The value of the field will increase given an increase in the price of gas, the risk free interest rate, the variance of the gas price and the time to relinquishment of the lease. As expected, the value of the field will decline given an increase in the development costs.

So far it is assumed that the development and extraction decisions are simultaneous and identical. By considering them as one single option, exercised on the relinquishment date of the lease, the Black and Scholes model for European call options is appropriate. This assumption is clearly inappropriate where the decision to develop may be implemented at any time before the relinquishment of the lease. This is dealt with in the following section.

13.4 Incorporating the right of development prior to relinquishment

In a similar paper to Ekern (1988) Paddock, Siegel and Smith (1988) also suggest the analogy between the value of undeveloped petroleum reserves and a financial call option. The major difference between the two papers is that while Ekern implicitly assumes the option can only be exercised at the relinquishment date of the lease, Paddock *et al.* propose, more realistically, that the field may be developed at any time prior to the relinquishment date of the lease. In other words that the option to develop may be exercised at any time. Therefore, while Ekern draws the analogy between the option to develop and a *European* option, Paddock *et al.* deal with *American* options. Paddock *et al.* suggest that an offshore petroleum lease has three stages of development: exploration, development, and extraction. They then concentrate on the development stage and suggest that when valuing undeveloped reserves the variables in the Black and Scholes (1973) call option pricing model may be replaced by the following:

Table 1 Comparison of variables for pricing models of call options and undeveloped petroleum reserves.

Financial call option	Undeveloped reserves
Current share price	Values of developed reserve discounted for development lag.
Variance of rate of return on share	Variance of rate of change in value of a developed reserve.
Exercise price	Development cost
Time to expiration	Relinquishment requirement
Riskless rate of interest	Riskless rate of interest.
Dividend	Net production revenue less depreciation

To estimate the value of the field Paddock *et al.* substitute the undeveloped reserve variables for those used in the call option pricing model. As mentioned above, the price paid for a more realistic model is increased complexity and this case is no different. While an exact analytical solution exists for European options, the pricing of American options requires a numerical solution - involving

some form of finite differencing technique - or an analytical approximation which usually involves some form of error correction by scaling.

The option to develop the undeveloped tract is, in effect, an "option on an option". That is, by developing the tract the lease holder has the right, but not the obligation, to extract the condensates and gas. In other words holders of undeveloped reserves hold an option on an option to extract petroleum products. Ekern points out that this could be valued as a form of compound option as suggested by Geske (1979). The major computational complication with this approach is that a cumulative bivariate normal distribution needs to be calculated. This in itself is not that onerous. More importantly, however, the estimates of the compound option model tend to be biased: overpricing out-of-the-money calls and underpricing in-the-money calls (Hull, 1989; p309).

Paddock *et al.* make a major assumption which simplifies their analysis significantly. Once leaseholders have exercised their development option the value of the developed reserve will depend on the assumptions made about oil quality, future extraction rates and costs, taxes and royalties and gas prices. Paddock *et al.* then make the assumption that, if there are secondary markets for developed reserves, then the market values of the developed gas reserves may be readily observed, thereby obviating the need to value the reserves using either Geske's compound option method or that proposed by Brennan and Schwartz. In the United States, the assumption that market prices of developed reserves adequately reflect their value may be a valid one. This assumption presupposes that the markets for developed reserves are informationally efficient, and buyers and sellers are equally well informed and not influenced by motives other than those which influence the decisions of a rational wealth maximiser. Moreover, Sharp and Simon's (1991) account of previous negotiations between the Crown and the Maui gas consortium, suggest that there may well be significant informational asymmetries in the New Zealand market for developed reserves. Therefore this assumption - in New Zealand at least - may be an inappropriate one.

Paddock *et al.* assume that the rate of return from holding the reserve follows the usual diffusion process:

$$\frac{R_t dt}{B_t V_t} = \mu_v^* dt + \sigma_v dz$$

where μ_v^* is the expected rate of return on the gas field investment which adequately compensates the owner for risk equal to $\sigma_v dz$. This is similar to the concept of beta employed in the capital asset pricing model. As μ_v^* is the total expected return, it incorporates a capital gain component μ_v and a payout rate δ . Therefore the return on "moth-balled" or non-producing developed reserves is μ_v ie $(\mu_v^* - \delta)$. B_t is defined as the number of units of petroleum in a developed reserve, and V_t is the value of a unit of developed reserves. R_t is the instantaneous per unit time net payoff from holding the reserve and derives from either a capital gain from holding the remaining inventory and/or the profits from production. In order to deal with the value of the developed reserve V_t , Paddock *et al.* redefine the drift rate in the previous diffusion process in terms of the rate of capital gain μ_v , which yields:

$$\frac{dV}{V} = \mu_v dt + \sigma_v dz$$

The payout rate on the undeveloped reserve can be thought of as being similar to the dividend paid on a share. Thus Ito's lemma is applied to this diffusion process where the change in the value of the undeveloped gas field, c , is a function of the underlying value of developed reserves and time. As before, this yields a PDE similar to that which describes the underlying process of a share option. However, in this case it is for a share assumed to pay a constant dividend yield δ ,

$$\frac{\partial c}{\partial t} = rc - (r - \delta)V \frac{\partial c}{\partial V} - \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 c}{\partial V^2}$$

As with any contingent claim the solution which satisfies this PDE depends on the boundary conditions. The boundary conditions for the solution of this PDE are similar to those for an American call option on a dividend paying share and follow Merton(1973). In particular, Paddock *et al.* impose the condition that says the field should be developed *immediately* if the ratio $C_t = V_t/D$ exceeds some critical value C_t^* . The logic behind this is that the usual NPV rule can be modified in the following way: instead of accepting all projects with positive NPVs, the projects must have NPVs which exceed the *value of keeping the option alive* (Pindyck,1991; p 1112). In other words, gas fields V_t/D ratios must *exceed* the critical value C_t^* in order for the field to be valued immediately. If V_t/D is less than the critical value, the owner of the gas field is better off to defer development. The critical ratio, C_t^* , is relatively constant through time but declines rapidly as the relinquishment date of the lease draws near. Likewise the critical value is higher when higher values of σ_v are used. At the expiry of the lease the critical value *must* equal 1.0, as at that date the cost of development and the value of developed reserves will be known with certainty and the value of the lease will be $\max[V_T - D, 0]$.

As this model has the same underlying assumptions, and return generating process, as the dividend adjusted American call option pricing model, the value of the undeveloped reserve is positively related to the value of developed reserves, the relinquishment requirement, the riskless interest rate and the variance of the rate of change in the value of developed reserves, and is inversely related to the cost of development.

13.5 Summary

Although discounted cash flow approaches are easy to understand and implement they have some conceptual shortcomings when compared to the contingent claims analysis approaches discussed above. The DCF approach draws the analogy between the project's cash flows and the cash flows from a financial instrument such as a bond. DCF cannot adequately incorporate new information on prices, costs or outcomes. Although the cash flow profile of the project may change over time compared to the original projections when managers react to new information, a DCF approach can not take this into account. This concept is central to valuing non-renewable resource projects where part of the value of a reserve may be due to some aspect of managerial flexibility such as, the option to expand production, mothball or close the project.

CHAPTER 14

Applying a contingent claims methodology to the Maui gas field

14.1 Introduction

In the preceding sections Ekern's (1988) binomial model was used to demonstrate that an undeveloped gas field may still have some value even though the conventionally derived NPV was negative. The model assumed that any development took place at the end of the planning period when the lease was relinquished.

More realism (and attendant complexity) was added by assuming that the field could be developed at any time prior to relinquishment of the lease. This model, proposed by Paddock *et al.* (1988), likens investments in oil licenses to American call options which may be exercised at any time prior to expiry. Furthermore, Paddock *et al.* regarded the option to develop the undeveloped tract as, effectively, an "option on an option" which they then simplified by using market equilibrium prices for developed reserves instead of the more complex, but still soluble, compound option approach. While this technique gives values for the undeveloped reserves it tells us nothing about the optimal exploitation policy over time. Therefore, the next logical step is to incorporate some way of valuing the developed reserves and also determining the optimal extraction rate given the market price of gas. Brennan and Schwartz (1985) did this at an analytical level by describing an approach for determining the value of the mine and also the prices at which the mine - or gas reserves - should be open or closed.

Brennan and Schwartz point out that a prerequisite to solving the investment decision problem, such as determining the optimal operating policy for a gas field, is to solve the cash flow problem which estimates the value of the developed resource. Paddock *et al.* avoid the cash flow problem by using the market value of a developed oil reserve, with the proviso that an operating reserve with similar characteristics to the one being planned must be found.

Brennan and Schwartz focus on the problem of simultaneously determining the optimal operating policy and the present value of a developed copper mine. The operating policy includes the possibility of opening, closing or abandoning the mine. They present three versions of the model at different levels of complexity. Their most complex, and most realistic, model consists of two partial differential equations, with seven boundary conditions, which must be solved simultaneously to determine the value of the mine when open or closed and the critical prices at which the mine should be opened, closed or abandoned. They present some numerical results with the assumption that, when open, the mine can operate at a single rate. This obviates the need to account for the effect of extraction rate on operating costs. Brennan and Schwartz also present a model where extraction costs are continuous. In order to obtain a solution they assume an infinite inventory. This approach is used in the case study (discussed in section 14.4).

14.2 Background to the Maui gas project

The Maui gas field was discovered in 1969 and is located 34 km off the Taranaki coast. At the time of discovery the field was estimated to contain 5,446 PJ of recoverable gas reserves. As at December 1991 the Maui field made up about 61.7 per cent of the New Zealand's total known gas reserves of

4,200 PJ, or around 2,594 PJ. Other significant fields are Kupe/Toru and Kapuni which had estimated reserves at the same time of 907 PJ and 435 PJ respectively. Maui is currently serviced by two production platforms: Maui A and Maui B.

The field was discovered by a consortium made up of the Crown and Shell-BP-Todd. After lengthy negotiations the Consortium was granted a 42 year mining license which gave it exclusive rights to the Maui field. According to Sharp and Simon (1992) part of the reason for the protracted length of the negotiations was that the Consortium would not reveal the capital cost of the project and rejected basing the price on a rate of return calculation. Nevertheless, as part of the final agreement the Crown offered to buy a 50 per cent stake in the field for \$30 million, as well as offering 37 cents per MMBtu (or \$350,664 per PJ) and 85 per cent of the take-pay-price for quantities in excess of the agreement. As Sharp and Simon point out, this agreement effectively passed a significant portion of the risk of the project from the Consortium to the Crown. The Consortium (consisting of the Crown and Shell-BP-Todd) were granted a mining licence by the Crown, as owner of the gas. At the same time the Crown was also the Consortium's only customer - not only did the Crown own the reticulation system but it also owned the major gas using industries. From the perspective of this study we are interested in the value of the field to the Consortium.

The value of the field should be influenced by the rate of gas extraction, the price of gas and any royalties paid to the owner of the gas. What makes the Maui project different is the dual involvement of the Crown as owner of the gas and ultimate customer - and the take-or-pay agreement. The royalty rate is nominally set at 5 per cent of the value of production. The effective royalty rate may well be substantially less than 5 per cent as the Consortium are able to deduct operating and other costs. A reliable estimate is difficult to make as information on operating and development costs are unavailable.

14.3 Methodology

To be applicable to the real world a model, aimed at valuing natural resources or determining optimal operating policies, must deal with a large number of variables. The complexity of such a model means exact (analytical) solutions cannot be obtained and numerical approximations must be implemented.

The most common technique used in the numerical solution of the valuation equations is called finite differencing. This method consists of creating a multidimensional mesh, where each dimension represents a different variable, such as time, price of the product or inventory of the resource. The value of the resource is calculated at each point in the mesh. The most serious limitation of finite-difference approximations (FDA) is that they are computationally expensive. To ensure a high degree of accuracy it is necessary to use a very fine grid and enumerate every possible path that could be followed during the remaining planning period (Barone-Adesi and Whaley, 1987). There are many ways to estimate the solution with FDA (Geske and Shastri, 1985) and accuracy of the results can vary dramatically.

Additional complications which arise in the implementation of numerical solutions have to do with consistency, stability, and efficiency. *Consistency* means that the discrete-time process used for computation has the same mean and variance at every time period as the underlying continuous process. Numerical *stability* means that the approximation errors in the computations will be dampened out rather than amplified as the mesh becomes finer. *Efficiency* refers to the amount of computing time needed for accuracy of a given approximation (Trigeorgis, 1991).

These complications mean that there is no 'automated' solution procedure (such as in linear programming). Each particular problem must be programmed and solved individually. A model designed for policy decisions (as opposed to a research model) should be thoroughly tested. The solution procedure should be efficient enough to allow sensitivity analyses to be carried out within reasonable periods of time, and consistency and stability must be ensured within the range of parameter values that will be encountered in the real world.

After some preliminary testing of solution techniques it became obvious that there was a stability problem. Therefore, it was decided to use the version of the Brennan and Schwartz model which assumed an infinite inventory. By solving a simpler model, it was possible to compare solution strategies, perform sensitivity analyses, and gain insight into the behaviour of the system. A more realistic model (suitable for policy analysis) will require additional information concerning the particular characteristics of the resource being modelled. In particular, accurate cost estimates are required, as will become clear in the following sections.

14.4 A simplified model of the Maui gas field

This section describes the simplified Brennan and Schwartz (1985) model as applied to the Maui gas field. The version of the model used assumes an infinite inventory of the resource and therefore does not deal with the possibility of increasing extraction costs as the inventory decreases. Table 2 presents variable definitions as well as base values used for solution of the problem.

Table 2 Model variables and base values.

Exogenous	
r	- Riskless real interest rate: 2%
σ^2	- Variance of output price: 6%
κ	- Convenience yield: 4.1%
tax	- Income tax rate: 33%
q_{MAX}	- Maximum output rate: 170 PJ/Year
s	- Market price of gas (\$/PJ)
S^C	- Fixed contract price: \$350,664/PJ
Operating cost:	
α_0	- Intercept: \$500,000
α_1	- Linear term: \$50,000/PJ
α_2	- Quadratic term: \$1,500/PJ ²
Endogenous	
S^L	- Low critical price (\$/PJ)
S^H	- High critical price (\$/PJ)
q^*	- Optimal output rate (PJ/Year)
v	- Present value of the field (\$)
Notes	The base values used here are intended to reflect either general characteristics of the oil industry (σ^2 and κ after Paddock <i>et al.</i> , 1988), attributes specific to the Maui gas field (q_{MAX} , S^C after Sharp and Simon, 1992) or characteristics of the New Zealand economy (r , tax). Parameters of the cost function were arbitrarily set to provide 'realistic' results.

The present value of the field and the optimal operating policy for a given market price are determined by solving the partial differential equation:

$$\frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2} + (r-\kappa)s \frac{\partial v}{\partial s} - rv + (1-tax) \max[qs - \alpha_0 - \alpha_1 q - \alpha_2 q^2, 0] = 0$$

The first three terms in this equation result from the underlying stochastic process, they differ from the Black and Scholes equation in the presence of a convenience yield term, κ , and the fact that the time derivative has been eliminated by assuming a constant inflation rate and using discounted variables. The fourth term in the equation represents after-tax profit. It is assumed that no costs are incurred when there is no gas extraction. As no cost data for the Maui field are available, the true cost function for the field cannot be reliably estimated. Following a common assumption, a quadratic form of the cost function is used (after Brennan and Schwartz). The quadratic nature of

the cost function implies linearly increasing marginal costs. That is, extraction costs rise quadratically as output is increased.

The optimal operating policy (q^*) is given by:

$$q^*(s) = \begin{cases} q_{MAX}, & s > S^H \\ \frac{s - \alpha_1}{2\alpha_2}, & \text{if } S^L < s < S^H \\ 0, & s < S^L \end{cases}$$

This simply means that the optimal operating policy is bound by two critical prices: a low price (S^L) and a high price (S^H). At prices above the high price, production proceeds at maximum capacity while at prices below the low price, production is halted. The critical prices are determined by:

$$S^L = \alpha_1 + 2\sqrt{\alpha_0\alpha_2} ; \quad S^H = \alpha_1 + 2\alpha_2 q_{MAX}$$

The after-tax cash flow under the optimal operating policy is:

$$P(s) = \begin{cases} (1-tax)[q_{MAX}s - \alpha_0 - \alpha_1 q_{MAX} - \alpha_2 q_{MAX}^2], & s \geq S^H \\ (1-tax)\frac{(s - \alpha_1)^2}{4\alpha_2 - \alpha_0}, & \text{if } S^L < s < S^H \\ 0, & s \leq S^L \end{cases}$$

The solution to this system of equations is presented by Brennan and Schwartz (1985) and is not replicated here. The numerical solution was implemented using the base values presented in Table 2. Sensitivity analysis was performed by varying each exogenous variable in turn and estimating the value of the field at the critical prices $v(S^L)$ and $v(S^H)$ and at the fixed contract price $v(S^C)$. The optimal output rate at the contract price $q^*(S^C)$ was also estimated. Elasticities with respect to the exogenous variables were estimated as the percent change in each dependent variable (v , S^L , S^H or q^*) caused by a one percent change in the value of each exogenous variable in turn, with the remaining variables kept at base values.

14.5 Results

Figure 4 presents the base model results. The low and high critical prices occur at \$104,770 per PJ and \$560,000 per PJ, respectively; at these prices, the value of the field increases from \$35.87 million to \$780.33 million.

At the fixed contract price of \$350,664 per PJ the optimal extraction rate is 100.22 PJ per annum and the value of the field is \$358.42 million. This point is indicated by the arrows in Figure 4.

The contract between the Crown and the Consortium specifies take-or-pay quantities increasing from 42 PJ in 1978-79 to 170 PJ in 1995-96, but gradually decreasing thereafter to 68 PJ in 2007-08 (Sharp and Simon: 1992).

Compared to these results, the extraction rates required by the contract in the period 1988-98, represent an over-production of between 60% and 70% above the optimal output rate at the given contract price. Furthermore, the simplified model we have utilised assumes that the gas inventory

is infinite. The implication of this is that the gas's scarcity value and the costs of extraction remain constant. Therefore the optimal extraction rate of 100.22 PJ per annum could be an overestimate of the optimal rate given an actual inventory of around 2,500 PJ in 1991.

It should be emphasised at this point that these estimates are based only partially on the true Maui gas field data and rely to a large extent on data supplied in papers by Brennan and Schwartz and Paddock *et al.* Obviously, these results also depend on the assumed base values, so it is necessary to determine how the solution is affected as those values change. The elasticities presented in Table 3 indicate the sensitivity of the solution to changes in the base values. The value in each cell represents the percent change in the variable on the top row caused by a one percent change in the exogenous variable in the left column.

Table 3 Elasticity of dependent variables with respect to exogenous variables at base values.

	S^L	$V(S^L)$	S^H	$V(S^H)$	$q^*(S^C)$	$V(S^C)$
r	0.000	0.555	0.000	0.208	0.000	0.280
σ^2	0.000	1.310	0.000	0.296	0.000	0.485
κ	0.000	-2.761	0.000	-1.495	0.000	-1.744
tax	0.000	-0.493	0.000	-0.493	0.000	-0.493
q_{max}	0.000	0.403	0.911	1.990	0.000	0.467
S^C	0.000	0.000	0.000	0.000	1.167	1.735
α_0	0.261	0.465	0.000	-0.011	0.000	-0.020
α_1	0.475	0.663	0.089	0.027	-0.167	-0.164
α_2	0.261	-0.133	0.910	0.983	-1.000	-0.553

The value of the low critical price (S^L) is affected only by the parameters of the cost function, with α_1 having the greater effect. The value of S^H is affected mostly by the quadratic cost term (α_2) and the maximum output rate q_{MAX} ; the magnitude of the effect is the same with both variables (0.91). The value of the field is affected by all exogenous variables, with the magnitude of each effect being different as the gas price changes. At S^L , the most important effect on the field value is price variability (σ^2), with an elasticity of 1.31. In contrast, at S^H , q_{MAX} has the largest effect (1.99).

At the contract price (S^C), the optimal extraction rate is affected only by the cost parameters, with α_2 being dominant. The value of the field at S^C is negatively affected by cost, convenience yield, κ , and income tax (with values of -0.493, -1.744 and -0.553 for tax, κ and α_2 respectively). The fact that a 1% increase in the quadratic cost term causes optimal output to decrease by 1% underscores the need to obtain realistic estimates of the cost function.

14.6 Sensitivity analysis

The effects of exogenous variables on the value of the field can be analysed in more detail by referring to Figures 5 to 10. Figure 5 shows the sensitivity of the present value of the gas field to changes in the exogenous parameters. The value of the field is positively related to both the real interest rate and price variability (Rho_2), with the slope decreasing at higher values of the exogenous variables. This is similar to what happens to the value of a call option given an increase in the risk free interest rate or the variance. Increases in the interest rate merely discount any future costs thereby increasing the net benefit. On the other hand, increases in variance have a positive effect. The discussion in Part I noted that in contingent claims analysis interest lies in the positive payoffs given the assumption that decision-makers take action to avoid or mitigate negative outcomes. For example, if the price of gas were to drop below \$104,770 per PJ, it is implicitly assumed the field will be closed. This flexibility is worth something and the greater the variability in the gas price, the greater the value. As expected, the income tax rate has a linear negative effect on the value of the field, with a steeper slope at higher prices. The convenience yield has an exponentially negative effect on the field value. This is similar to the effect convenience yield has on call option values. Convenience yield may be likened to a continuous dividend payment. In general, the higher this dividend payment, the lower the value of the asset. The value of the field is positively related to the maximum output rate (q_{MAX}), with the effect being more pronounced at the high critical price SH . At the lower critical price, SL , the value of the field remains relatively flat over a range of output levels. At the contract price, SC , there is a gradual increase in value as output rises. The present value profile for the higher critical price, however, shows an almost exponential increase.

Figure 6 shows the parameter elasticities for the present value of the field. These were included to give more insight into the relative sensitivity of the value of the field to changes in the parameter variables given the differing prices. The elasticity estimates are independent of the units of measurement and deal with percentage changes. For example, although in price terms the present value of the field is more sensitive at the higher critical price, the field value has a higher elasticity with regard to real interest rates and variance when evaluated at the lower critical price. By evaluating the elasticity measures over a range of parameter values, it is possible to identify whether the elasticity is constant (as in the case of maximum output at the higher critical price), decreasing, (as in the income tax example), or increasing (as in the case of real interest rates). If an elasticity is increasing it implies the sensitivity of the present value of the field to a change in the parameter is increasing and vice versa. Constant elasticity implies the field value is uniformly sensitive to the parameter of interest.

Figure 7 shows the relationship between the present value of the gas field and the cost parameters a_0 , a_1 and a_2 . Recall that as there are no reliable cost data, a quadratic cost function has been specified and the parameters set in such a way as to give "reasonable" answers (i.e. in line with those produced by Brennan and Schwartz). A sensitivity analysis of the parameters with respect to the present value of the field was undertaken to test the robustness of the model. These results are shown in the left hand panels of Figure 7. It was found that a_1 and a_2 had differing impacts on the value of the field depending on whether the high or low critical prices, or the contract price prevailed. (See the left hand panels of Figure 7). The top panel suggests that fixed costs, a_0 , have little or no effect on the value of the field. The linear cost term, a_1 , suggests that the value of the field increases slightly as costs increase when the high and low critical prices prevail but decreases at the fixed contract price. The most interesting example is the quadratic cost term, a_2 . While the value of the field at SH increases as costs increase, the value of the field at the contract price (SC) decreases. This is because while SH and SL adjust in response to cost changes, the value of SC remains fixed. This result explains differences in the sign of the elasticities of $v(SL)$, $v(SH)$ and $v(SC)$ with respect to the cost parameters in Table 3.

Figure 8 shows the effect on the optimal output rate of maximum output, the contract price, and the linear and quadratic cost terms. None of the other variables had any impact on the optimal operating policy. As would be expected optimal output is constant at 100.22 PJ per annum once the maximum output constraint rises above this level and is therefore non-binding. Optimal output also increases linearly with contract price. As the linear cost term (a_1) increases the optimal output rate at SC decreases linearly. Also, the effect of the quadratic cost term on the optimal output rate is curvilinear. At values below approximately \$1,000 per PJ² it is optimal to operate at the maximum extraction rate and as a_2 increases beyond \$1,000, $q^*(SC)$ decreases. An optimal output rate of around 50 PJ per annum occurs at around \$3,000 per PJ². These results suggest that, when the effect of inventory on extraction costs is incorporated into the model, the optimal output rate would decrease over time. The extent of this decrease will depend on the specific shape of the cost function and the extent to which it is affected by decreasing inventory. Usually as inventory, or reserves fall, it becomes more difficult to extract the gas from the field. Therefore in a finite inventory example extraction costs will be negatively related to the level of reserves.

The impact that maximum output and the three parameters in the cost function have on high and low critical prices is shown in Figure 9. The high critical price, SH, increases linearly with the maximum level of output, at a relatively constant rate, while the lower critical price, SL, is relatively stable over the range. SH and SL are relatively unresponsive to changes in the fixed cost parameter but are, predictably, more responsive to the linear and quadratic parameters.

As discussed in section 14.2, the rate of royalty payment made to the Crown by the Consortium is nominally 5 per cent of the field production. However, legislation allows for deductions to be made from this figure and an undisclosed net percentage is arrived at. As the actual royalty rate is unavailable we have assumed a figure of 0 per cent in the previous analysis and examined the sensitivity of the value of the field, and the optimal operating policy, to changes in the royalty rate. This is shown in Figure 10. Over the feasible range of 0 to 5 per cent the value of the field falls from \$358 million to around \$325 million. Over the same range the optimal output falls from around 100 PJ per annum to around 94 PJ per annum. Therefore, for feasible values of the royalty rate, changes in the value of the field and the optimal operating policy are relatively small.

CHAPTER 15

Discussion and conclusions

The purpose of the case study was to examine the applicability and usefulness of a contingent claims approach when applied to a problem such as evaluating the optimal extraction rate and the value of a non-renewable resource. The objective of the analysis was to examine solution methodologies and expose any practical or methodological weaknesses. While the study is only preliminary, the results suggest that a number of policy implications may follow.

15.1 Managerial flexibility

With respect to the objective, it was found that the methodology required a relatively small number of inputs when compared to a discounted cash flow (DCF) methodology which would have required detailed cash flow estimates for the life of the project as well as an appropriate discount rate. Moreover, the DCF approach would have involved the implicit assumption that there was no managerial flexibility and that all cash flows were known with certainty at the time the study commenced. Specifically, the DCF approach ignores any options the Crown, as owner of the gas field, might have to optimally alter gas output given changing market conditions, demand or environmental policy directives. For example, extraction could have been expanded, reduced, or abandoned depending on a range of outside influences which influence resource use policy. This point is central to estimating the "cost" of the take-or-pay agreement in terms of foregone flexibility. Although the Consortium is given some certainty in its cash flows, the take-or-pay agreement locks it into a fixed regime of gas price and rate of supply. The Crown, as owner of the field is also locked into a fixed regime of gas field depletion. Therefore the agreement effectively cancels out any flexibility. In such a situation the DCF approach is probably quite a reasonable technique for evaluating the present value of the field - given the existence of the take-or-pay agreement. To estimate the value which has been foregone by the Crown, a contingent claims approach is required. Contingent claims theory implies that managerial flexibility in a project has a positive value over and above the DCF derived net present value. From a theoretical perspective, the approach looks promising simply because it explicitly attempts to model the way decision makers are likely to behave. Furthermore, the more advanced applications such as that by Brennan and Schwartz suggest some optimal extraction path or profile.

15.2 Implementation of the model

With regard to the implementation of the method, contingent claims analysis employs an approach based on Ito stochastic control. This specifies models in the form of partial differential equations (PDE's) which describe the trajectory of the variable of interest through time given some initial conditions and feasible ranges. The solution of the PDE's is by no means straight forward and usually involves some form of numerical search technique. As discussed in the previous section this will not necessarily provide a robust stable solution. In fact, in this study, problems of instability require that the analysis be limited to the Brennan and Schwartz technique employing the assumption of infinite inventory. This is obviously unrealistic given that the estimated reserves of the Maui field are about 2,500 PJ. It was a necessary assumption to make the problem tractable within the time available.

A second limitation was that detailed information on the cost structure of the field is unavailable. This is hardly surprising given the reluctance of the commercial partners in the Consortium to divulge such details. Furthermore, reliable estimates of the royalty payments from the Consortium to the Crown as owner of the field are not available. Nominally these are 5 per cent but unquantified costs are deducted from this figure and the final payment is not made public. Therefore, it was assumed that the cost function for the field is quadratic and parameter values were used which gave "sensible" answers in the sense that they were consistent with the results obtained by Brennan and Schwartz.

Given these limitations, what does the model tell us? Realistically, in its present form and with the existing data, the model can only indicate the relative magnitudes of the estimated values and the broad dynamics of the system as the variables change. With these provisos and the data from the existing agreements, it does suggest that the level of output under the existing take-or-pay agreement is significantly higher than it might optimally be. In fact, the optimal extraction rate is probably overstated because of the infinite inventory assumption implying that the optimal extraction rate should be less than 100.22 PJ per annum. Furthermore, the value of the field reacts to changes in the exogenous variables such as the variability of the gas price and the real interest rate in a similar way that an option would. That is, higher interest rates and higher variances lead to higher values for the field.

It has been necessary to make some major simplifying assumptions to obtain a useful model. Therefore any further research should concentrate on two major areas. Firstly, the nature of the cost structure of the Maui field should be investigated and the more reliable parameter estimates obtained. Secondly, the solution technique should be refined to investigate the implications of having a finite gas reserve.

15.3 Energy demand

A sustained supply of energy is of vital social, economic and commercial importance to New Zealand. The energy demand forecasts published by the Ministry of Commerce (1992) indicate that total primary energy demand will grow by nearly 50 percent from 537 PJ in 1990 to 789 PJ in 2020. Forecasts indicate that this demand will be met from four main sources - liquid fuels, gas, coal, and electricity. The proportion of total demand that each of these will contribute is expected to change over the 30 year period. Liquid fuels will rise from 28% to 38%, gas will decline from 24% to 7%, coal will rise from 8% to 10%, and electricity will rise from 40% to 44%.

Demand for natural gas is expected to decline mainly because of the closure of the synthetic gasoline, methanol, and ammonia/urea plants. These closures mean that demand for gas will fall from 127 PJ in 1990 to 55 PJ in 2020 as gas prices rise sufficiently to encourage many bulk industrial consumers of low grade energy to switch from gas to coal.

Published estimates put the remaining total recoverable gas reserves at about 4200 PJ as at December 1991. The Maui field, with about 60 percent of remaining recoverable reserves, is by far the largest resource from which most future supplies will come. Exploratory effort to locate new reserves has been sparse due largely to the plentiful supplies of gas from both the Maui and Kapuni fields. Further exploration activity may occur in response to increased commercial potential, but this will be strongly influenced by prospecting potential and by expected profitability in relation to overseas opportunities.

Table 4 sets out the depletion dates under varying initial gas reserve estimates:

Table 4 Impact of varying gas reserve estimates on depletion date.

Reserve estimate (PJ)	Percentage of base case reserve	Year gas reserves are depleted
3500	87.5	2014
3700	92.5	2015
4000 (base)	100.0	2016
4500	112.5	2018

Source: Ministry of Commerce, 1992.

The principal impact of varying the estimate of gas reserves is on the reserves' depletion date. Lower reserves will bring forward the depletion date and gas and electricity prices will rise sooner, fuel share shifts will occur earlier, the synthetic gasoline and petrochemical plants will cease operation sooner, LNG imports will begin earlier, and electricity generation will use less gas and more coal.

Table 5 Major users of natural gas (1990).

	Petrochem	Petralgas	Syngas	Electricity	Reticulation	Total
Quantity (PJ)	4.7	19.5	57.8	61.8	41.3	185.1
Percent	2.5	10.5	31.0	34.0	22.0	100.0

Source: Ministry of Commerce, 1992.

Electricity will continue to be the largest contributor to total energy demand. On the basis of currently known reserves, supplies of natural gas will be fully depleted by 2016. It is assumed that major gas users will trade their rights to marginal quantities of gas as reserves approach depletion (assuming there are no significant new discoveries). As reserves are finally depleted the gas price will reach a sufficiently high level for a substitute fuel to displace gas fully. Gas prices are a major determinant of both the mix of electricity generation fuels chosen and the consequent profile of electricity price rises. Fuel shares in the other demand sectors are similarly influenced by the relative price of gas, as is the closure of the synthetic gasoline and petrochemical plants.

While the largest proportion of electricity generation will continue to be hydro based, gas fired power stations generate almost 20 percent of electricity consumed. Gas fired stations produce the marginal unit of electricity about 90 percent of the time (MoC, 1992) so the supply and use of gas have a significant effect on the short run marginal cost of electricity supply.

Gas used in electricity generation is forecast to rise from 62 PJ in 1990 to 73 PJ in 2000 and then to fall to 18 PJ in 2020 as the gas price rises high enough to justify the substitution of coal for gas as the least expensive generation fuel. Demand for coal as a source of primary energy is forecast to rise from 44 PJ in 1990 to 81 PJ in 2020. The primary energy sources used to generate electricity are forecast to rise from 213 PJ in 1990 to 351 PJ in 2020 (MoC, 1992).

A major concern of the Ministry for the Environment that the rise in demand for electricity will largely be met from increased use of coal fired generation and that this will be accompanied by increased emissions of carbon dioxide, thus adding to the concentration of greenhouse gases. The energy sector in New Zealand contributes over 80 percent of carbon dioxide emissions from human activities. The present Government announced a planning target of reducing total carbon dioxide emissions to 80 percent of the 1990 level by the year 2000.

Concern about what part New Zealand should play in reducing global greenhouse gases while maintaining competitiveness in world markets has led to debate about what can be done at this point to address the issue i.e. should there be a change in energy policy, and if so, what should it be?

Government's energy policy in recent years has been consistent with wider moves to deregulate areas of the economy where intervention and controls had previously characterised the operation of markets. A key element of the Government's energy policy is to ensure the continuing availability of energy to commercial, industrial, and domestic consumers at the lowest cost to the economy as a whole, consistent with sustainable development.

15.4 Output rate from the Maui field

While New Zealand is currently well endowed with gas resources (courtesy of the Maui field), by the middle of the next decade indigenous gas will be in critically short supply unless substantial new sources are proved and developed. The market instruments used to bring gas projects into existence include preferential, and commonly exclusive, contractual arrangements that can act as a deterrent to exploration. Gas reserves cannot be discovered and proved on demand so there is no basis for expecting the market to deliver them given the inability of new suppliers to compete until such time as there is an actual shortage.

The contractual arrangements that were established initially between the Crown and gas wholesalers, and subsequently between Maui Developments Ltd and its customers (e.g. National Gas Corporation, ECNZ) preclude a competitive pricing mechanism guiding decisions about current uses of gas or conservation for future use. The question arises as to whether the contract prices have provided efficient signals for the use of gas. Sharp and Simon (1992) show that inflexibility in the legal framework of the contract prevented adjustments being made for changes in market conditions including prices of alternative fuels, costs of production for alternative sources of supply, and demand conditions facing major gas users. The lack of a transmission mechanism for these signals and the consequent lack of output adjustments suggests that a less-than-efficient outcome is likely.

The contractual arrangements have also established a depletion profile for gas reserves without provision for adjustment in response to changes in reserve estimates. Estimates of recoverable reserves in the Maui field have been revised several times from an initial level of 5446 PJ in 1970 to 2170 PJ in 1993 (pers. com. MoC). Depletion rates based on the take-or-pay quantities in the Maui gas purchase contract indicate that reserves should have declined from the initial level of 5446 PJ to 3572 PJ by 1992-93. In the absence of adjustments to the depletion rate, the implication of

the lower reserve estimate is that the date of depletion will occur earlier than expected, thus forcing earlier consideration of alternative energy supplies by gas users.

The indications from the model that the output rate from Maui is higher than it might optimally be, and the consequences for energy users of an earlier-than-planned depletion date point to there being substantial opportunity costs associated with the existing agreement.

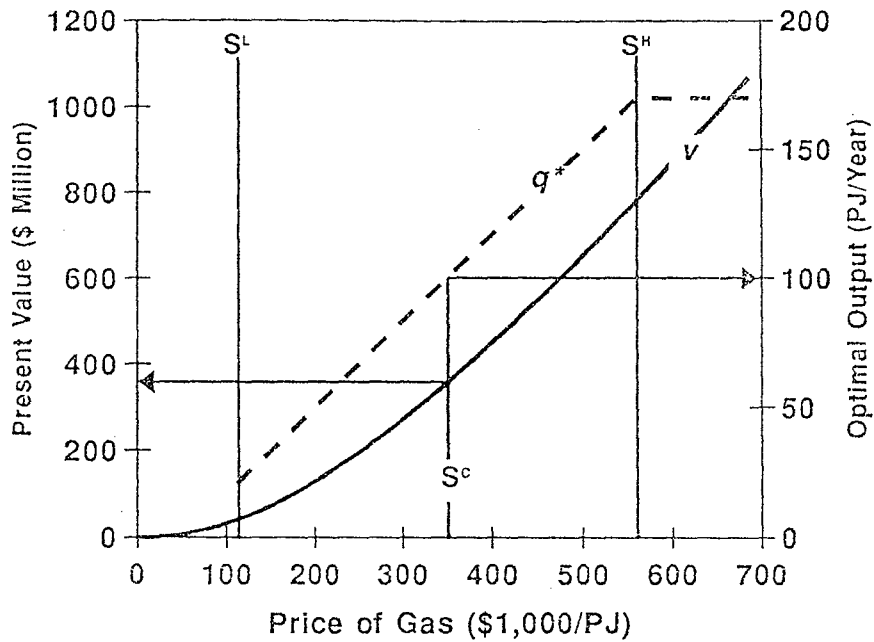


Figure 4 Value of the gas field (v) and optimal operating policy (q^*) as affected by market price.

Present Value of Gas Field (\$ Million)

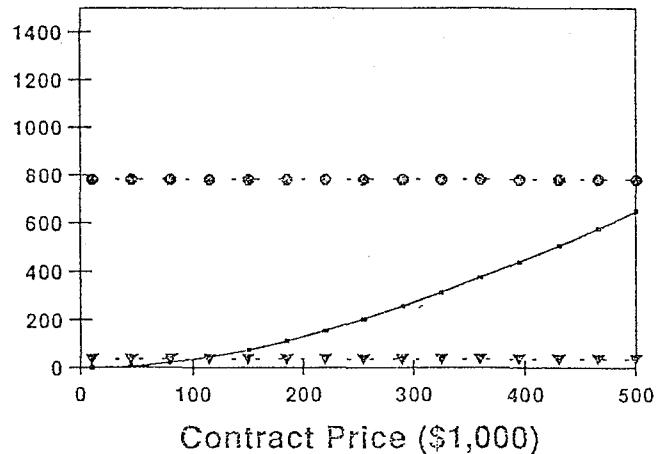
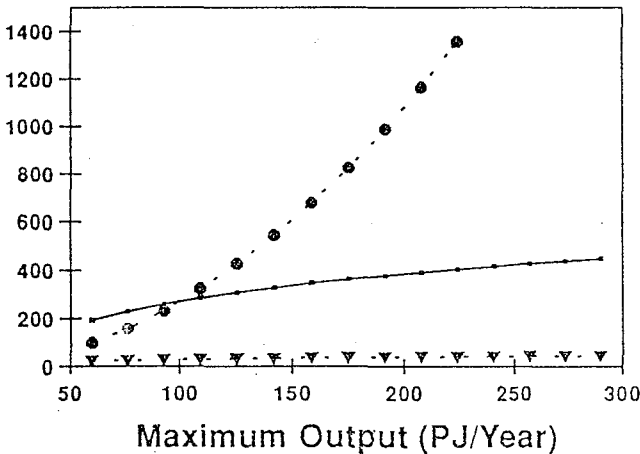
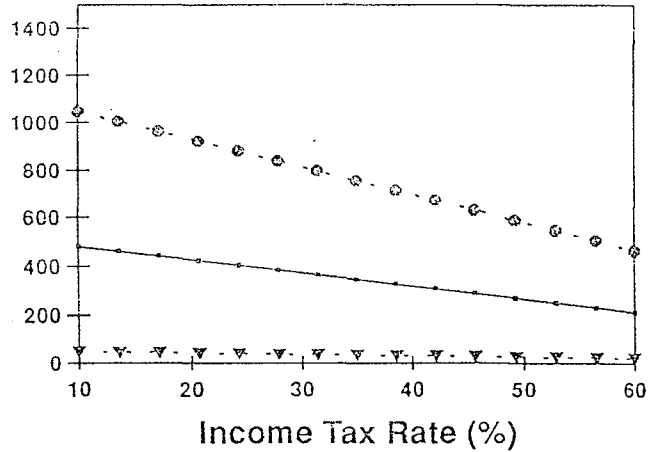
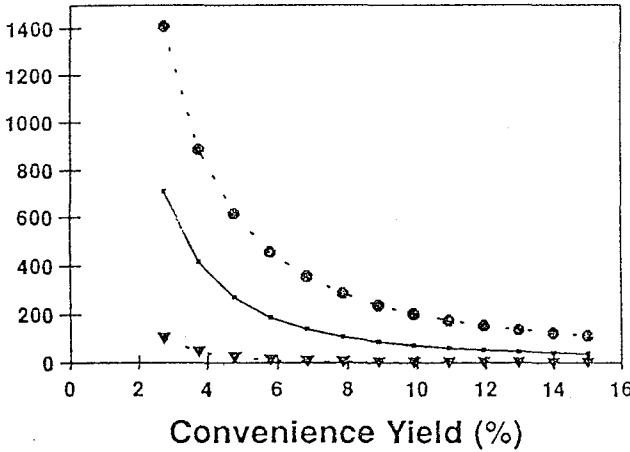
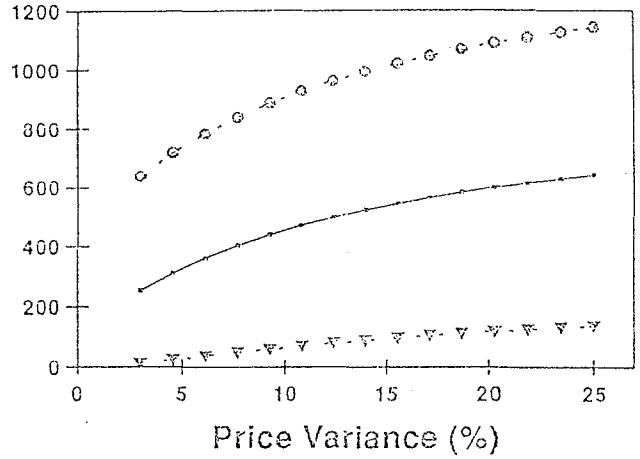
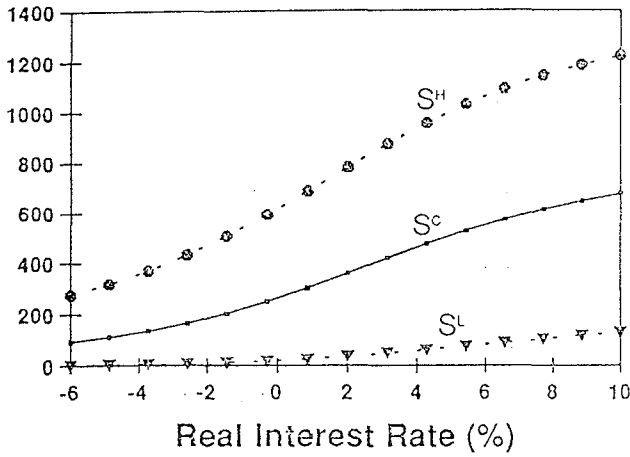


Figure 5 Effect of exogenous variables on the present value of the gas field. At the fixed contract price (S^C) and critical prices (S^L , S^H).

Present Value Elasticities

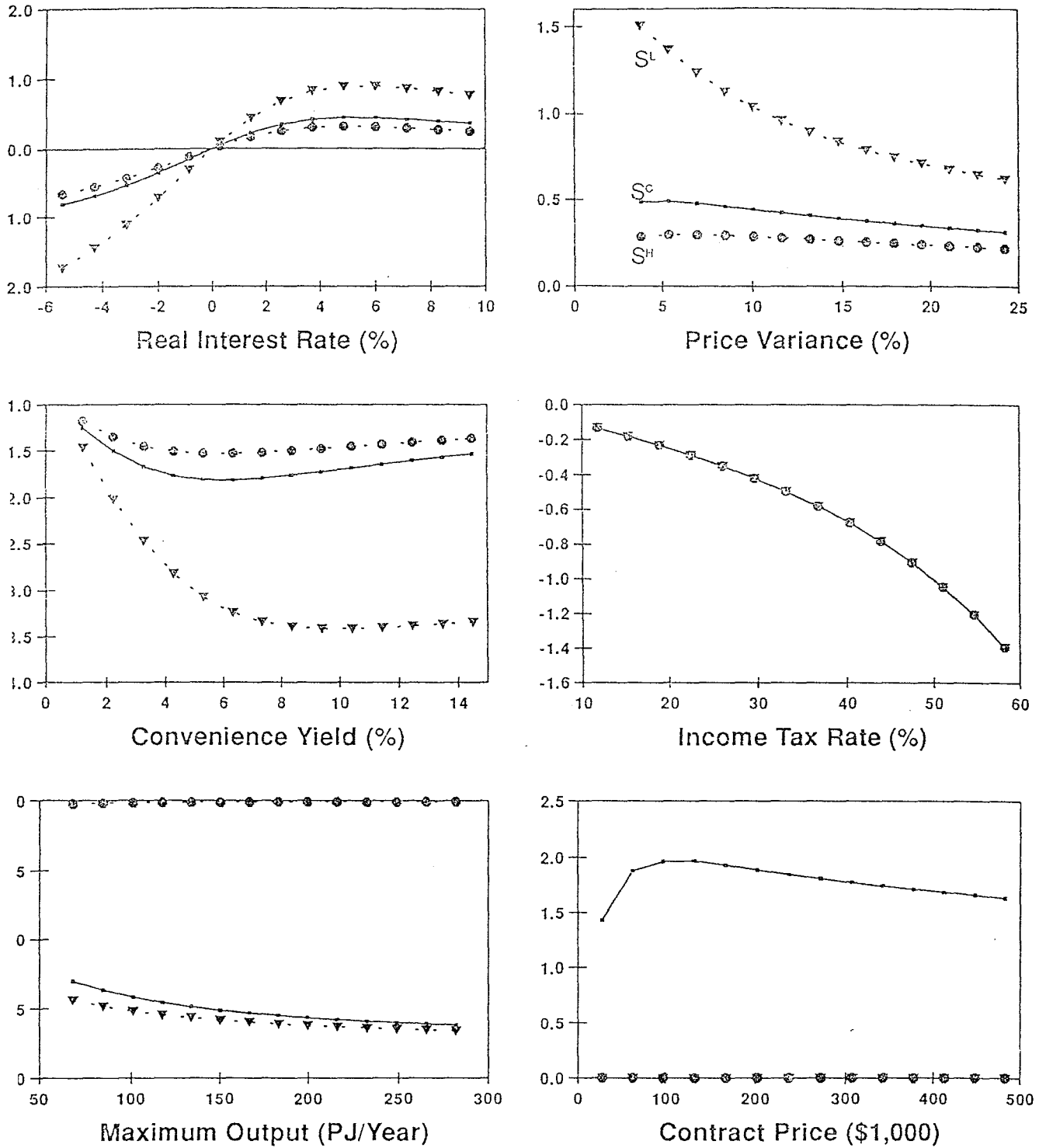
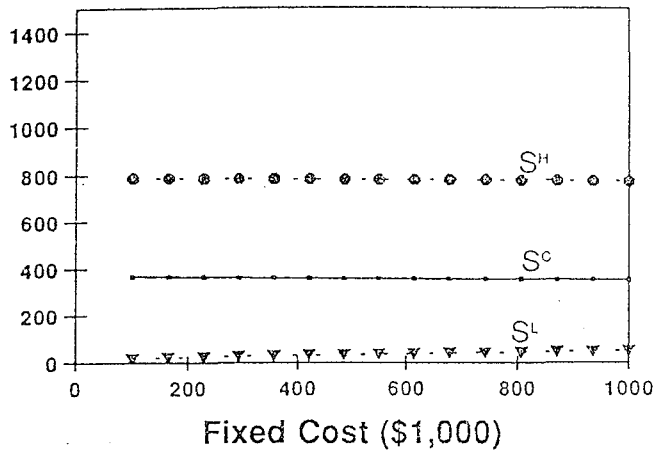


Figure 6 Elasticities of the present value of the gas field with respect to exogenous variables. At the fixed contract price (S^C) and the critical prices (S^L , S^H).

Present Value of Gas Field
(\$ Million)



Present Value Elasticity

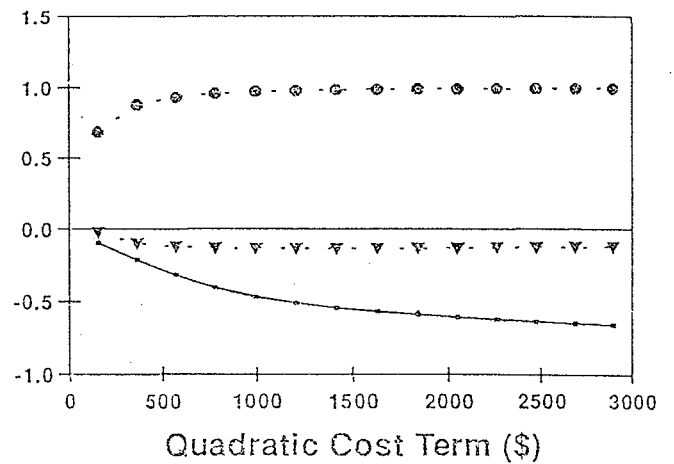
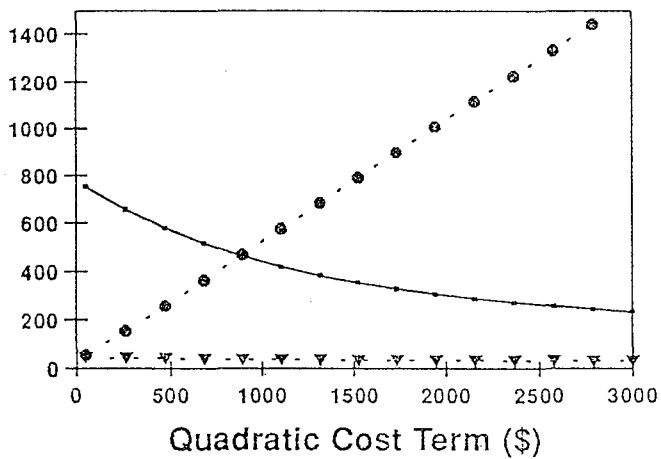
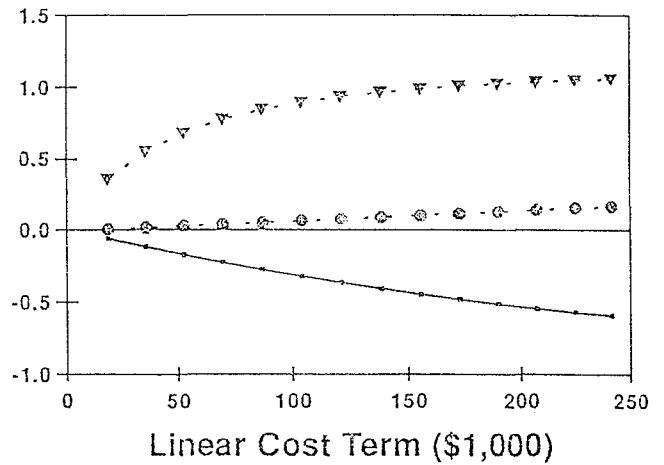
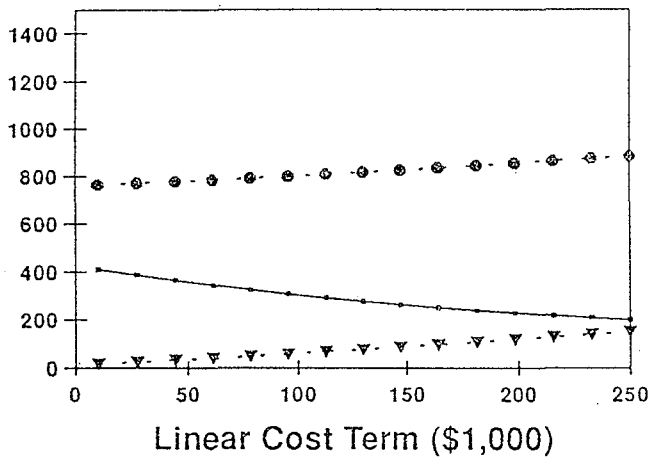
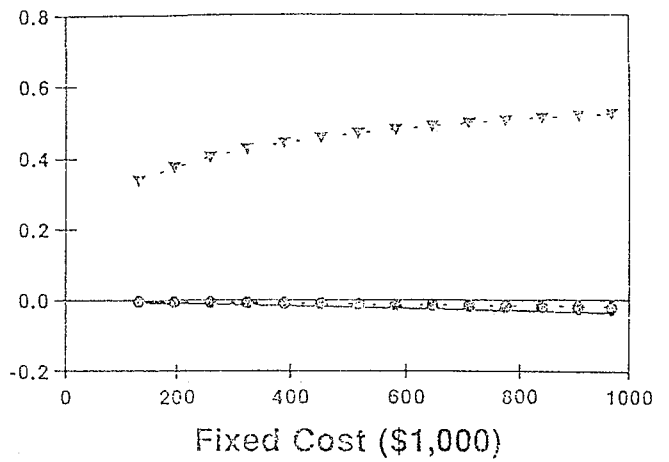
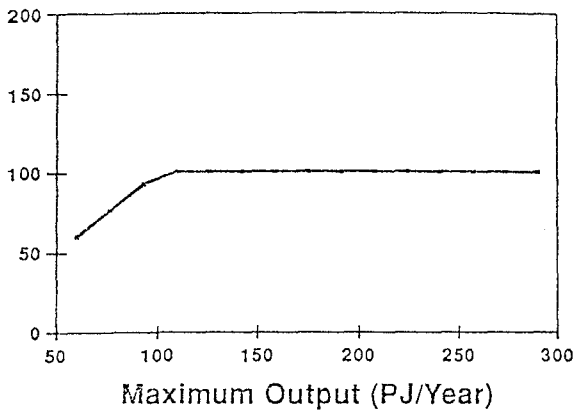


Figure 7 Effect of cost parameters on the present value of the gas field. At the fixed contract price (S^C) and critical prices (S^L , S^H).

Optimal Output [$q^*(S^c)$, PJ/Year]



Optimal Output Elasticity

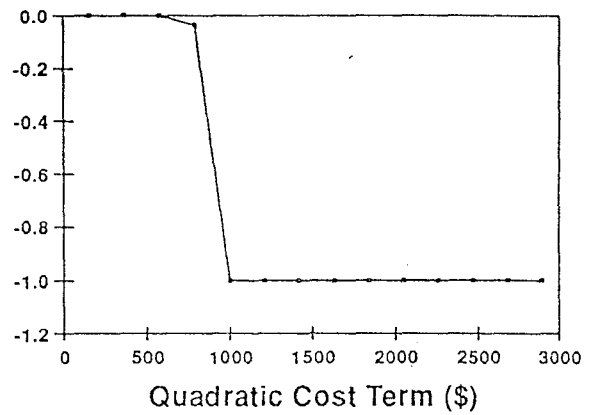
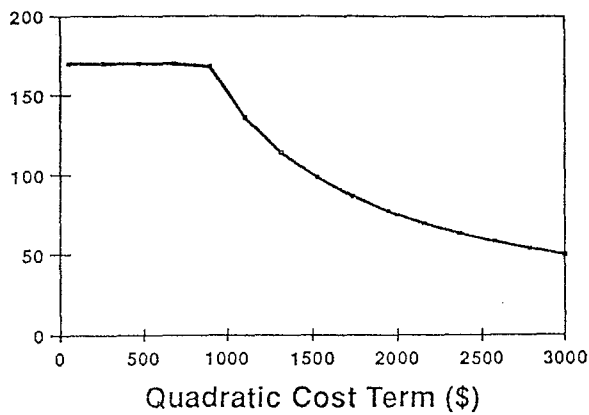
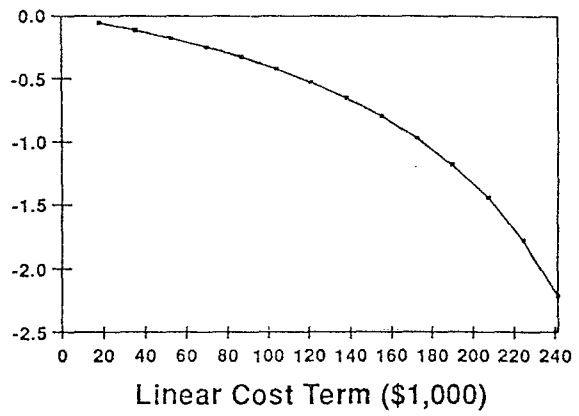
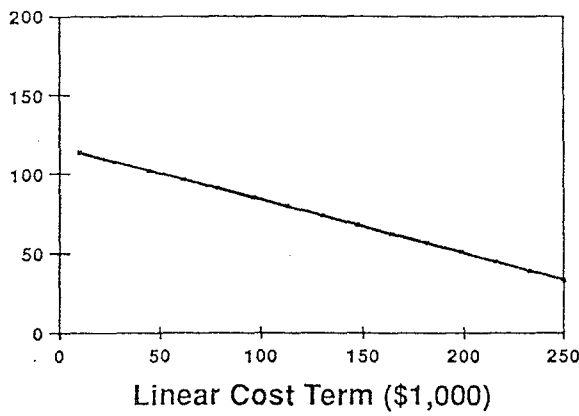
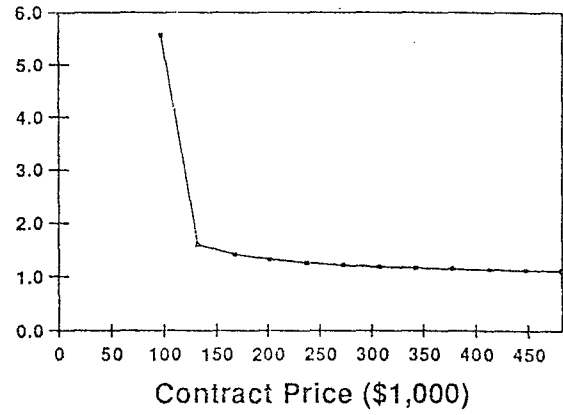
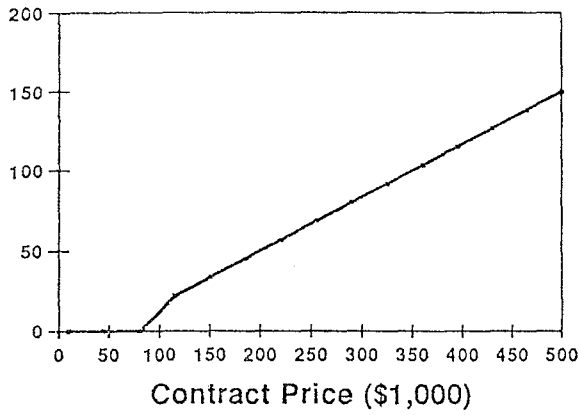
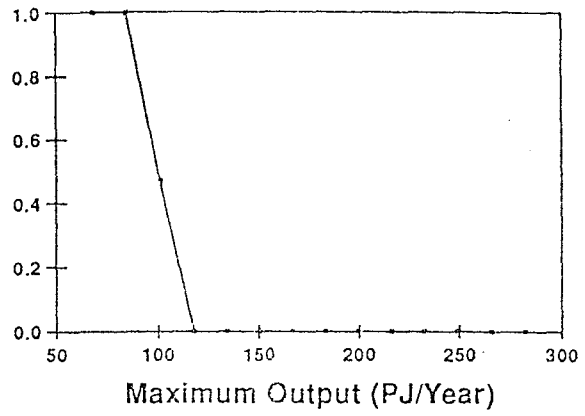
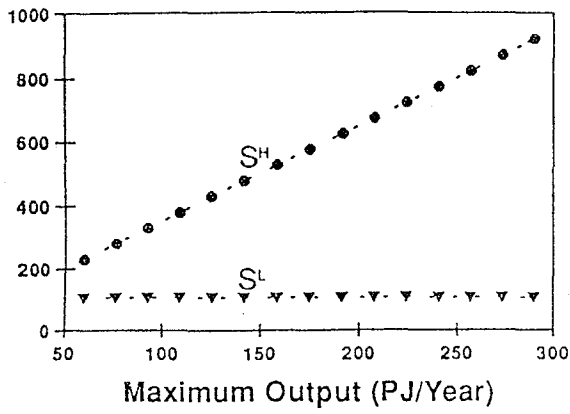


Figure 8 Effect of parameters on the optimal output rate at the fixed contract price (S^c).

Critical Prices (\$1,000)



Critical Price Elasticity

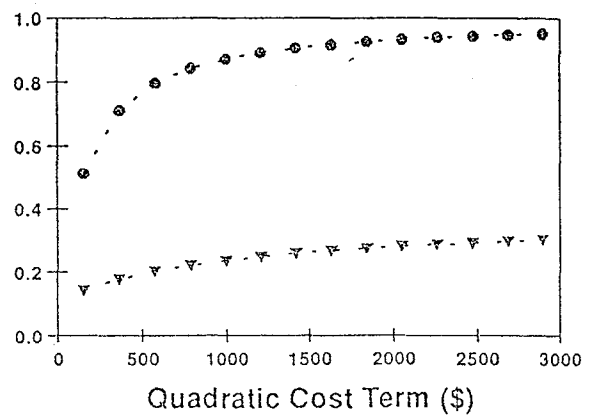
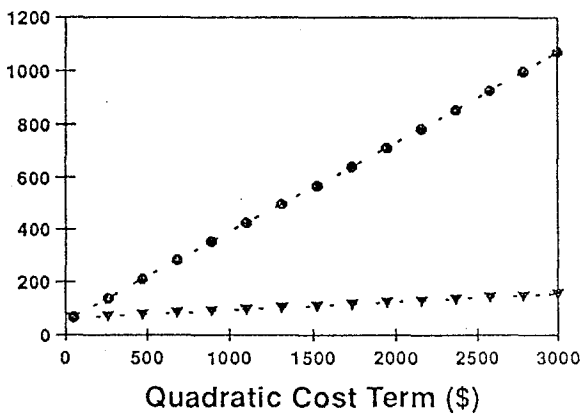
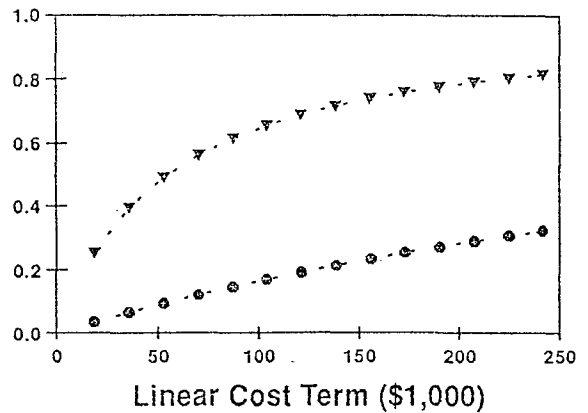
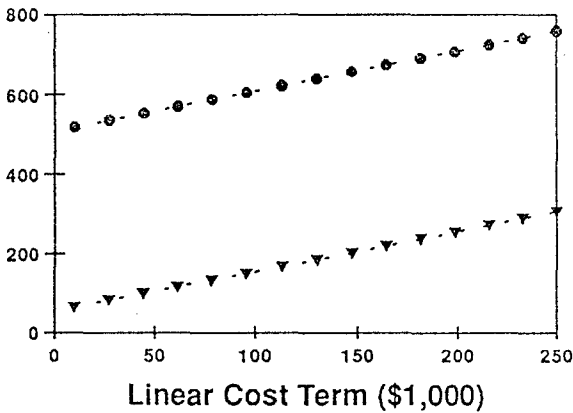
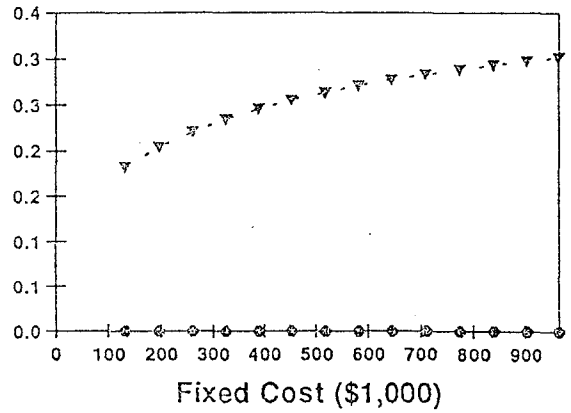
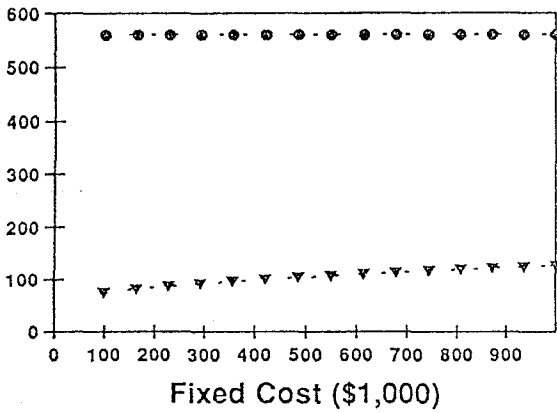
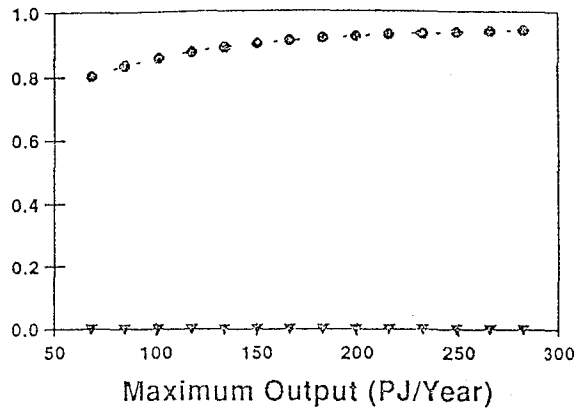


Figure 9 Effect of parameters on the critical prices (S^L and S^H).

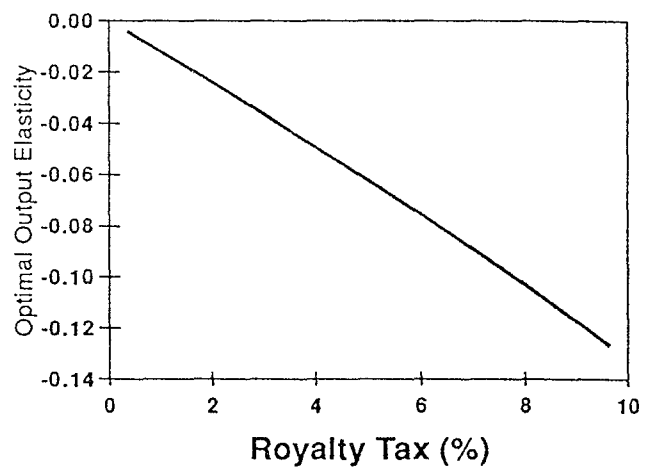
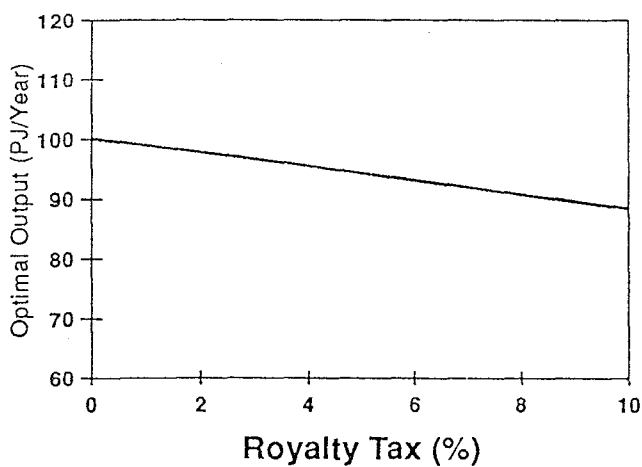
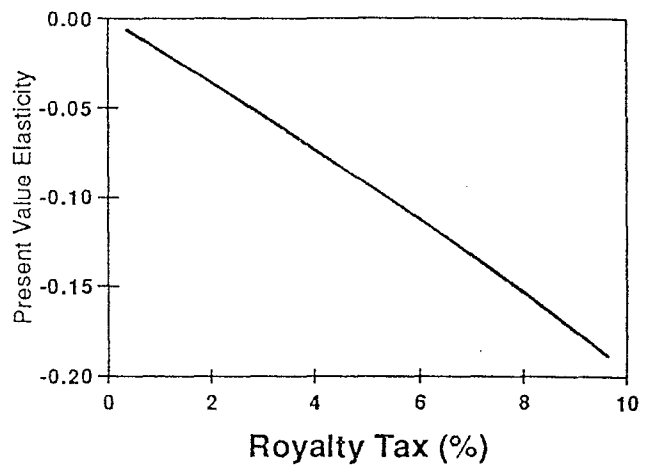
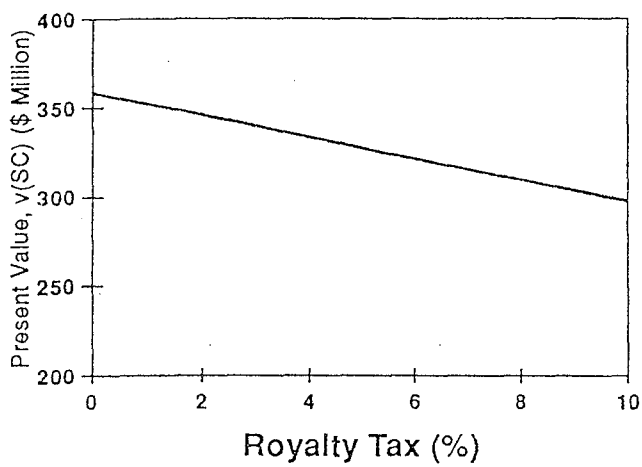


Figure 10 Effect of royalty tax on the optimal operating policy and present value of the gas field.

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