Structural Health Monitoring using Adaptive LMS Filters

Mostafa Nayyerloo[1], J Geoffrey Chase[1], Gregory A MacRae[2], XiaoQi Chen[1], and Christopher E Hann[1]
[1]*Department of Mechanical Engineering*
[2]*Department of Civil and Natural Resources Engineering*
University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand
mostafa.nayyerloo@pg.canterbury.ac.nz, {geoff.chase, gregory.macrae, xiaoqi.chen, chris.hann}@canterbury.ac.nz

Abstract- **A structure's level of damage is determined using a non-linear model-based method utilizing a Bouc-Wen hysteretic model. It employs adaptive least mean squares (LMS) filtering theory in real time to identify changes in stiffness due to modeling error damage, as well as permanent displacements, which are critical to determining ongoing safety and use. The structural health monitoring (SHM) method is validated on a 4-story shear structure model undergoing seismic excitation with 10% uniform noise added. The method identifies stiffness changes within 0.5-1.0% inside 0.2–1.0 seconds at different sampling frequencies. Permanent deflections are identified to within 10% of the true value in 1.0 second, converging further over the remainder of the record.**

Index Terms- **structural health monitoring; SHM; adaptive filtering; LMS; Bouc-Wen model; damage detection; non-linear structure; computer vision; line scan camera**

I. INTRODUCTION

Structural health monitoring (SHM) is the process of comparing the current state of a structure's condition relative to a baseline state to detect existence, location, and degree of likely damage, particularly after a damaging input [1]. SHM can simplify typical procedures of visual or localized experimental methods, as it does not require visual inspection of the structure. It thus provides valuable data for post-event safety assessments to help optimize recovery planning.

Many current vibration-based SHM methods are based on the idea that changes in modal parameters; frequencies, mode shapes and modal damping, are a result of changes in the physical mass, damping, and stiffness properties of the structure [2]. These modal methods are typically more applicable to steel-frame and bridge structures where vibration response is highly linear [2-3]. Wavelet approaches offer a similar approach as well as determining the time at which damage occurred.

A major drawback of all these approaches is their inability to be implemented in real-time, on a sample-to-sample basis as the event occurs. Further, their reliance on modal properties has potential problems. The modal properties have been shown in some cases to be non-robust in the presence of strong noise and insensitive to small amounts of damage [4].

Adaptive fading Kalman filter [5] and adaptive H_∞ filter techniques [6] to achieve real-time capable, or near real-time capable results, provide identification of modal parameters in real time that comes with significant computational cost and complexity. Moreover, like other linear approaches they are not applicable to the typical non-linearities found in seismic structural responses.

In contrast, direct identification of changes in stiffness and/or permanent deflection would offer the post-earthquake outputs desired by engineers. The goal is to obtain these stiffness changes in real time in a computationally efficient and robust fashion. Model-based methods combined with modern filtering theory offer that opportunity.

Least Mean Squares (LMS) based SHM has been used for a benchmark problem [3], and also for a non-linear rocking structure [7], to directly identify changes in structural stiffness only. They are robust with fast convergence and low computational cost. However, they require full state measurement and do not identify permanent deflections.

Due to a variety of practical constraints, direct high frequency measurement of displacement and velocity is not typically possible. Displacement and velocity are often estimated by integration of measured acceleration and are subject to drift and error. However, this error can be corrected using low frequency displacement data obtained via a variety of sensors, such as ground-based GPS or fibre optics. The work described below is predicated on the idea that emerging high speed line scan cameras can offer a robust and high speed displacement measure required for the modified LMS-based SHM algorithm proposed for non-linear yielding structures undergoing seismic excitation.

II. DEFINITION OF THE SHM PROBLEM

A seismically excited structure can be modeled using Bouc-Wen hysteretic equations of motion [8-9]:

$$\mathbf{M}\cdot\{\ddot{v}\}+\mathbf{C}\cdot\{\dot{v}\}+\mathbf{K}_e\cdot\{v\}+\mathbf{K}_h\cdot\{z\}=-\underline{\mathbf{M}}\cdot\ddot{x}_g \tag{1}$$

where **M**, **C**, $\mathbf{K_e}$, and $\mathbf{K_h}$ are the mass, damping, pre-yield linear elastic stiffness and post-yield stiffness matrices of the model respectively, $\{v\}$, $\{\dot{v}\}$, $\{\ddot{v}\}$ and $\{z\}$ are the displacement, velocity, acceleration and hysteretic displacement, respectively, and $\ddot{x}_g$ is the ground motion acceleration. $\mathbf{K_e}$ and $\mathbf{K_h}$ can be expressed in terms of the pre-yield stiffness $\mathbf{K_p}$:

$$\mathbf{K}_e = \alpha \mathbf{K}_p \quad (2)$$

$$\mathbf{K}_h = (1-\alpha)\mathbf{K}_p \quad (3)$$

where α is the bi-linear factor which determines the change in slope between elastic and plastic regimes (α=1 represents a purely elastic structure and α=0 represents a fully hysteretic structure).

The vector $\{z\}$ represents hysteretic displacement and is governed by Equation (4) [9]:

$$\dot{z}_i(t) = \dot{r}_i(t)\left\{1 - 0.5\left[1 + sign\left(\dot{r}_i(t) z_i(t)\right)\right]\left|\frac{z_i(t)}{Y_i}\right|^{n_i}\right\},\ i = 1,\ldots,N \quad (4)$$

where $\dot{r}_i(t)$, is the velocity of storey i relative to storey i-1, Y_i is the yield displacement of i^{th} story, and n_i is a shaping parameter determining the curve from elastic to plastic force-deflection behavior of the structure. N is the number of stories which is equal to the number of degrees of freedom for a 1-D shear-type structure.

Permanent displacement in the Bouc-Wen model is defined as:

$$D_i(t) = \frac{r_i(t) - z_i(t)}{1 + \left(\dfrac{\alpha_i}{1-\alpha_i}\right)},\ i = 1,\ldots,N \quad (5)$$

where D_i(t) is the permanent deformation of storey i, and r_i(t) is the displacement of storey i relative to storey i-1.

If damage occurs in the structure from an earthquake, or any other source of damaging excitation, structural properties, such as natural frequency and stiffness may also change, and may be time-varying. For the damaged structure, the equations of motion can be re-defined using Equations (2) and (3):

$$\begin{aligned}&\mathbf{M}\cdot\{\ddot{\overline{v}}\} + \mathbf{C}\cdot\{\dot{\overline{v}}\} + \alpha\left(\mathbf{K}_p + \Delta\mathbf{K}_p\right)\cdot\{\overline{v}\}\\ &+ (1-\alpha)\left(\mathbf{K}_p + \Delta\mathbf{K}_p\right)\cdot\{\overline{z}\} = \mathbf{F}\end{aligned} \quad (6)$$

where $\{\ddot{\overline{v}}\}$, $\{\dot{\overline{v}}\}$, $\{\overline{v}\}$, and $\{\overline{z}\}$ are the responses of the damaged structure, $\mathbf{F}$ is the input load which is equal to $-\underline{\mathbf{M}}\cdot\ddot{x}_g$ in this case, and $\Delta\mathbf{K}_p$ contains changes in the stiffness of the structure and can be a function of time. Identifying the $\Delta\mathbf{K}_p$ term enables the structure's condition to be directly monitored without using modal parameters.

This research examines changes in stiffness properties which are the most likely to change significantly rather than damping or mass for steel-frame structures.

To determine $\Delta\mathbf{K}_p$ using adaptive LMS, following the method proposed in [3], a new form of $\Delta\mathbf{K}_p$ is defined with time-varying scalar parameters $\hat{\alpha}_i$, to be identified using the LMS filter. For instance, $\Delta\mathbf{K}_p$ for a four DOF four-story shear building steel structure is sub-divided into four matrices with entries of 1, −1 and 0 to allow independent identification of changes in stiffness of each story i.e. $(\Delta k_p)_1$, $(\Delta k_p)_2$, $(\Delta k_p)_3$, and $(\Delta k_p)_4$.

$$\begin{aligned}\Delta\mathbf{K}_p = \hat{\alpha}_1\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} + \hat{\alpha}_2\begin{bmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\\ + \hat{\alpha}_3\begin{bmatrix}0&0&0&0\\0&1&-1&0\\0&-1&1&0\\0&0&0&0\end{bmatrix} + \hat{\alpha}_4\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&-1\\0&0&-1&1\end{bmatrix}\end{aligned} \quad (7)$$

$$\Delta\mathbf{K}_p = \begin{bmatrix}\hat{\alpha}_1+\hat{\alpha}_2 & -\hat{\alpha}_2 & 0 & 0\\ -\hat{\alpha}_2 & \hat{\alpha}_2+\hat{\alpha}_3 & -\hat{\alpha}_3 & 0\\ 0 & -\hat{\alpha}_3 & \hat{\alpha}_3+\hat{\alpha}_4 & -\hat{\alpha}_4\\ 0 & 0 & -\hat{\alpha}_4 & \hat{\alpha}_4\end{bmatrix} \quad (8)$$

or

$$\Delta\mathbf{K}_p = \begin{bmatrix}(\Delta k_p)_1+(\Delta k_p)_2 & -(\Delta k_p)_2 & 0 & 0\\ -(\Delta k_p)_2 & (\Delta k_p)_2+(\Delta k_p)_3 & -(\Delta k_p)_3 & 0\\ 0 & -(\Delta k_p)_3 & (\Delta k_p)_3+(\Delta k_p)_4 & -(\Delta k_p)_4\\ 0 & 0 & -(\Delta k_p)_4 & (\Delta k_p)_4\end{bmatrix} \quad (9)$$

where

$$\hat{\alpha}_1 = (\Delta k_p)_1, \hat{\alpha}_2 = (\Delta k_p)_2, \hat{\alpha}_3 = (\Delta k_p)_3, \hat{\alpha}_4 = (\Delta k_p)_4 \quad (10)$$

hence equations (7)-(9) can be expressed as:

$$\Delta\mathbf{K}_p = \sum_{i=1}^{n}\hat{\alpha}_i\mathbf{K}_i \quad (11)$$

where n is the number of degrees of freedom (DOF) of the model, and $\mathbf{K}_i$ is the corresponding matrix to i^{th} DOF with entries of 1, -1, and 0 in Equation (7). Rewriting (6) using (7)-(11) yields:

$$\begin{aligned}&\sum_{i=1}^{n}\hat{\alpha}_i\mathbf{K}_i\cdot\{\alpha\{\overline{v}\} + (1-\alpha)\{\overline{z}\}\}\\ &= \mathbf{F} - \mathbf{M}\cdot\{\ddot{\overline{v}}\} - \mathbf{C}\cdot\{\dot{\overline{v}}\} - \alpha\mathbf{K}_p\cdot\{\overline{v}\} - (1-\alpha)\mathbf{K}_p\cdot\{\overline{z}\}\end{aligned} \quad (12)$$

In Equation (12), responses of the damaged structure $\{\ddot{\bar v}\}$, $\{\dot{\bar v}\}$, and $\{\bar v\}$ are measured. The vector $\{\bar z\}$ at each time step is calculated as following using Equation (4) and assuming constant $\{\dot{\bar z}\}$ at each time step:

$$\dot{\bar z}_i(t) = \frac{\bar z_i(t+\Delta t) - \bar z_i(t)}{\Delta t} \tag{13}$$

where Δt is the time step. Substituting (13) in (4) yields:

$$\bar z_i(t+\Delta t) = \dot{\bar r}_i(t)\left\{1 - 0.5\left[1 + sign\left(\dot{\bar r}_i(t)\bar z_i(t)\right)\right]\left|\frac{\bar z_i(t)}{Y_i}\right|^{n_i}\right\}\Delta t + z_i(t) \tag{14}$$

$$i = 1, \ldots, N$$

The damaged structure stiffness, or the effective stiffness changes due to non-linear behavior such as yielding or hysteresis, can then be determined by identifying the $\hat\alpha_i$ at every discrete time step using Equation (15).

$$\begin{aligned}&\sum_{i=1}^{n} \hat\alpha_i \mathbf{K}_i \cdot \{\alpha \bar v_k + (1-\alpha)\bar z_k\} = \mathbf{F}_k - \mathbf{M}\cdot\ddot{\bar v}_k - \mathbf{C}\cdot\dot{\bar v}_k \\ &-\alpha\mathbf{K}_p \cdot \bar v_k - (1-\alpha)\mathbf{K}_p \cdot \bar z_k = y_k\end{aligned} \tag{15}$$

where $\mathbf{F}_k$ is the input load at time k, and $\bar v_k$, $\dot{\bar v}_k$ and $\ddot{\bar v}_k$ are the measured displacement, velocity and acceleration at time k, respectively. $\bar z_k$ is calculated sample-to-sample using Equation (14). The elements of the vector signal y_k can be readily modeled in real-time using an adaptive LMS filter so that the coefficients $\hat\alpha_i$, and then using Equation (11), $\mathbf{\Delta K_p}$, changes in stiffness of the damaged structure, can be readily determined.

III. ADAPTIVE LMS FILTERING

Adaptive filters are digital filters with coefficients that can change over time. The general idea is to update filter coefficients and assess how well the existing coefficients are performing in modeling a noisy signal, and then adapt the coefficient values to improve performance. The least mean squares (LMS) algorithm is one of the most widely used of all the adaptive filtering algorithms and is relatively simple to implement. It is an approximation of the Steepest Descent Method using an estimator of the gradient instead of its actual value, considerably simplifying the calculations and to be readily performed in real-time applications. The goal in this case is to model the individual, scalar elements of the signal y_k of (15) using the adaptive LMS filter.

In adaptive LMS filtering, the coefficients are adjusted from sample-to-sample to minimize the mean square error (MSE), between a measured noisy scalar signal and its modeled value from the filter.

$$\hat e_k = \hat y_k - W_k^T X_k = \hat y_k - \sum_{i=o}^{m-1} w_k(i) x_{k-i} \tag{16}$$

where W_k is the adjustable filter coefficient vector or weight vector at time k, $\hat y_k$ is the measured noisy scalar signal at time k, to be modeled or approximated, X_k is the input vector to the filter, model of current and previous filter outputs, x_{k-i}, so $W_k^T X_k$ is the vector dot product output from the filter at time k to model a scalar signal $\hat y_k$, and m is the number of prior time steps or taps considered. The Widrow–Hopf LMS algorithm for updating the weights to minimize the error, e_k, is defined as [10]:

$$W_{k+1} = W_k + 2\mu \cdot e_k \cdot X_k \tag{17}$$

where μ is a positive scalar that controls the stability and rate of convergence.

To identify $\mathbf{\Delta K_p}$ at time k, using LMS adaptive filters we will follow the One-Step method [3] and re-write Equation (16) in matrix form by substituting $W_k^T X_k$ with its equivalent from Equation (15):

$$e_k = y_k - \sum_{j=0}^{m-1}\sum_{i=1}^{n} \hat\alpha_{ij} \mathrm{K}_i (\alpha \bar v_k + (1-\alpha)\bar z_k) \tag{18}$$

Minimizing the mean square error (MSE) with respect to $\hat\alpha_{ij}$ using Equation (17) yields the following weight update formula for the SHM problem:

$$w_{k+1} = w_k + 2\mu\left[e_k^T \mathrm{K}_i(\alpha \bar v_{k-j} + (1-\alpha)\bar z_{k-j})\right] \tag{19}$$

In this fashion, summing $\hat\alpha_{ij}$ over j, will be $\hat\alpha_i$ in Equation (15) which are changes in stiffness of each story.

IV. INPUTS TO THE SHM PROBLEM

Inputs to this SHM problem are acceleration, velocity, and displacement of the structure. Acceleration can be easily measured with low cost accelerometers at high sampling rates, but due to practical constraints direct high speed measurement of displacement and velocity is not typically possible. A high speed displacement sensor would provide displacement, and could be used to derive velocity at low added computational cost. Estimating the velocity using both acceleration and displacement data would provide a more precise estimation of the velocity. To measure displacement of a real structure at high rates, this paper proposes but does not explore the method proposed in [11]. Using only one high speed line scan camera and a special pattern explained in [11], multiple displacements and motions can be determined in real-time at rates of up to 15kHz. This is more than sufficient for the structural seismic SHM problem.

V. Simulated Structure

The algorithm was tested using simulated input data in order to provide proof of concept and quantify the effects of noise in measured data on the accuracy of the identified parameters $\mathbf{\Delta K_p}$ and permanent displacement. MATLAB® was used to simulate the responses of the structure shown in Figure 1 using Newmark-β integration method. Each storey in this building has a pre-yield stiffness of 1610N/m and mass of 1kg, resulting in an undamped fundamental natural period of 0.45s for the structure. This period was chosen to closely match the natural period of the laboratory structure, which has been calculated at 0.47s [12]. A diagonal mass matrix was used in simulation.

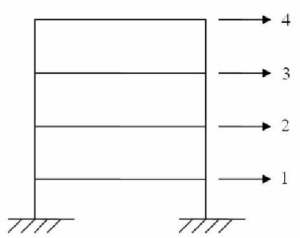


Figure 1. Simulated four-story 4-DOF shear building

The laboratory structure has been calculated to have between 0.9% and 2.9% damping, depending on the magnitude of response [12]. Thus, the following damping matrix was constructed for the simulated structure using the equivalent viscous damping approach assuming 2% damping in each mode:

$$C=\begin{bmatrix} 2.18 & -0.63 & -0.11 & -0.05 \\ -0.63 & 2.07 & -0.68 & -0.17 \\ -0.11 & -0.68 & 2.01 & -0.79 \\ -0.05 & -0.17 & -0.79 & 1.39 \end{bmatrix} \text{[N.s/m]} \qquad (20)$$

Each storey was given a yield displacement, Y, of 0.04m, shaping parameter, n, of 2, and a bilinear factor, α=0.9. These parameters were chosen to provide realistic non-linear structural behavior.

The simulated structure was subjected to the El Centro earthquake record, with a 10% reduction in pre-yield stiffness applied to the bottom story at a time of 10 second. Data was recorded at 500Hz. Noise was applied after simulation using the following equation for each story response:

$$\bar{v}_{noisy}(t)=\bar{v}_{true}(t)+pf_d\sigma,\ 0\le t\le T \qquad (21)$$

where $\bar{v}_{noisy}(t)$ is the noisy displacement, $\bar{v}_{true}(t)$ is the 'true' displacement from simulation, T is the total time span of the earthquake, p is the percentage of noise to be applied, f_d the mean absolute value of $\bar{v}_{true}(t)$ over the time span $0 \le t \le T$, and σ is a uniformly distributed random variable lying between -1 and 1. Noisy velocity and acceleration were also obtained using the same procedure for velocity with the mean absolute value of $\dot{\bar{v}}_{true}(t)$, f_v, and for acceleration with the mean absolute value of $\ddot{\bar{v}}_{true}(t)$, f_a.

VI. Results

Typical responses of the bottom story of the simulated four-story shear building under the El Centro earthquake are shown in Figure 2. Simulated responses of the structure in damaging event, with 10% uniformly distributed noise added, have been used to identify changes in structural stiffness using the adaptive LMS method. As shown in Figures 3 and 4, in both sudden and gradual failure cases, $\mathbf{\Delta K_p}$ converges to the actual value within less than a second using 10 taps at 500 Hz sampling rate.

Figures 5 and 6 show that filter approaches faster to the final values of the pre-yield stiffness changes after damage when higher sampling rate or a greater tap number is used to identify the stiffness changes.

The proposed adaptive LMS SHM is robust in the presence of noise as Figures 7(a) and (b) represent. Even with 10 percent uniformly distributed noise on all the responses of the structure, which is conservative, outputs of the filter are still almost identical with true values for changes in stiffness of the structure.

Running the simulation with estimated noisy values for changes in pre-yield stiffness of the structure to obtain identified responses of the damaged structure using Newmark-β integration method and Equation (14), and then using Equation (5) to get the permanent deflections of the structure, yields Figure 8. In this figure, the error between the true simulated permanent deflection of the bottom story of the structure simulated with actual changes in stiffness, and identified permanent deflection simulated with noise contaminated estimated changes in stiffness for bottom story has also been shown. The figure clearly shows that as the filter approaches its final value for changes in stiffness, the permanent deflection approaches its actual value and the error becomes smaller. Even immediately after the damage, the error is within 20% of the true simulated values and falls within 10% after one second.

VII. Conclusion

LMS Adaptive filters and non-linear Bouc-Wen structural model enable identification of changes in stiffness within 0.5-1.0% of actual values inside 0.2-1.0 seconds plus determination of permanent deflection of the damaged structure to within 10% of true values in 1.0 second in simulated case studies with realistic sensor noise on responses of the structure. The proposed filter based identification approach to SHM problems in comparison with existing adaptive methods makes permanent deflection identification possible, which is critical for determining ongoing safety of the structure.

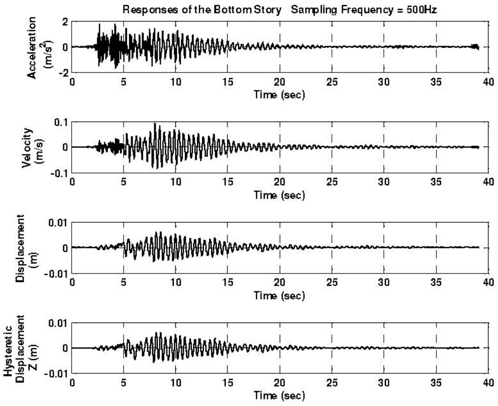


Figure 2. Responses of the bottom story of the simulated structure subject to the El Centro earthquake and 10% sudden failure in the bottom story

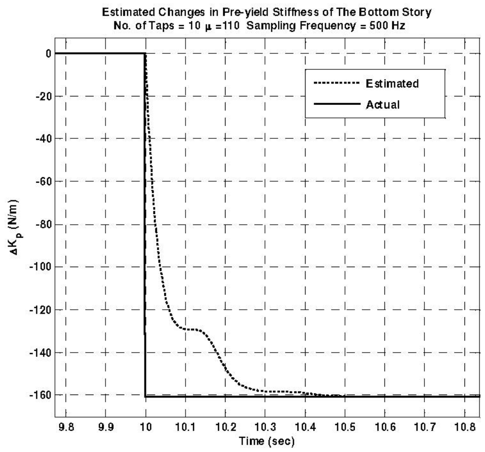


Figure 3. Identified changes in pre-yield stiffness of the bottom story with 10% sudden failure using adaptive LMS algorithm

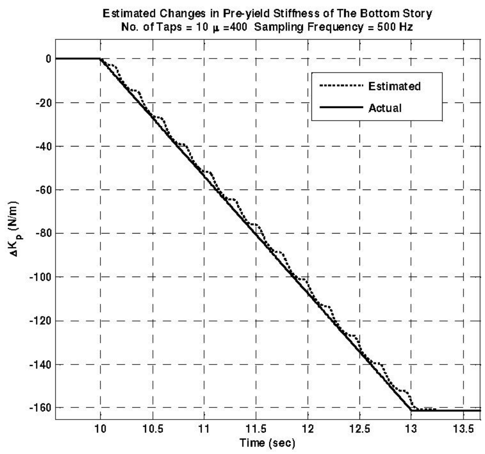


Figure 4. Identified changes in pre-yield stiffness of the bottom story with 10% gradual failure using adaptive LMS algorithm

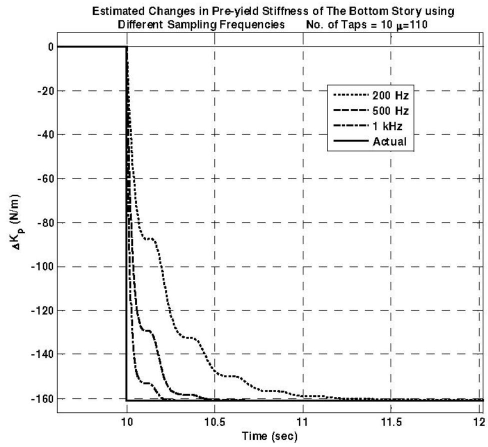


Figure 5. Identified changes in pre-yield stiffness of the bottom story with 10% sudden failure using adaptive LMS algorithm at different sampling rates

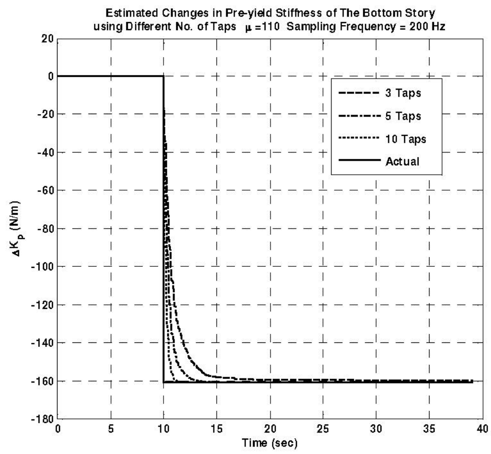
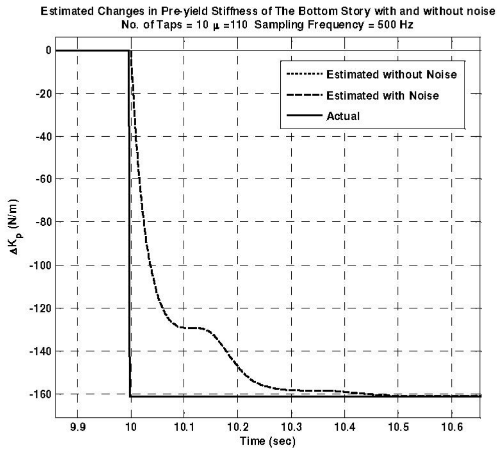
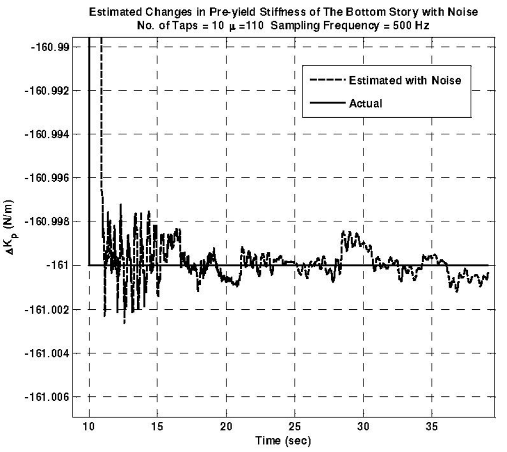


Figure 6. Identified changes in pre-yield stiffness of the bottom story with 10% sudden failure using adaptive LMS algorithm with different tap numbers



(a)



(b)

Figure 7. Identified changes in pre-yield stiffness of the bottom story with 10% sudden failure using adaptive LMS algorithm with and without noise on the structural responses: (a) total convergence period (b) close to the final value for stiffness changes

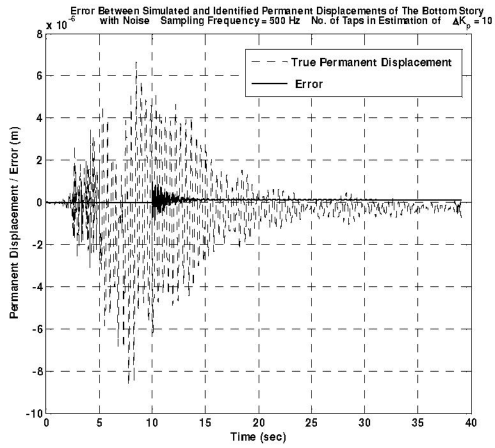


Figure 8. Identified permanent displacement of the bottom story with 10% sudden failure using estimated changes in pre-yield stiffness of the structure with realistic noise on the structural responses using adaptive LMS algorithm, and error between identified and true simulated permanent displacements for the bottom story

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