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Abstract

A stochastic model for generating daily sunshine hours and solar radiation that reaches the earth's surface in dry zone of Sri Lanka has been developed. Historical sunshine data which represent a Weibull probability density function and universal Angström type equations were used in the model giving daily and monthly sunshine and solar radiation. Simulated values clearly follows the astronomical phenomenon and local atmospheric conditions. The lowest solar radiation values (14 to 15 MJ m^{-2} day^{-1}) were obtained during the major rainy season. The highest values (around 20 MJ m^{-2} day^{-1}) were evident around the Vernal Equinox (March and April). Sri Lanka being small in size (65,000 km^2) and small latitudinal extent ($<4^0$), the model may form the basis for more comprehensive solar radiation generating models for other areas in Sri Lanka.

1. Introduction

Measurements of solar radiation from sun and sky (global radiation) are important in developing solar energy devices to supplement existing energy sources. This is of particular importance for country like Sri Lanka where the sunshine is available in abundance. There are also other uses for such information in quantitative ecophysiological studies as the source of energy used in photosynthesis and evapotranspiration. The solar radiation climatology of the world has been extensively studied by numerous workers in the past. The best known and most comprehensive publication is the World Distribution of Solar Radiation (Löf et al., 1965). However, since solar radiation reaching the earth's surface depends on the factors such as cloud cover and turbidity which are not global in nature, on-site radiation data are essential. But, available solar radiation records in Sri Lanka are meagre because of cost and complexity of standard apparatus, cost of maintenance, and the difficulties involved in calibration of the instruments. Although attempts had been made to collect daily solar radiation data in several locations of the country using Actinographs, the records lasted only 3 - 4 years in many cases due to the above mentioned factors. In addition, accuracy of those data are questionable because Actinograph is less accurate than Angström type formula for estimating solar radiation from other weather parameters such as sunshine duration, temperature, and humidity (Stanhill, 1965). It is therefore necessary to supplement solar radiation records by use of related information and approximation formula.

One of the earliest methods for estimating solar radiation is seeking a linear relationship between global radiation levels and the number of bright sunshine hours (Angström, 1924; Fritz and McDonald, 1949; Glover and McCulloch, 1958; Revfeim, 1981) and with cloud cover (Black, 1956). Since cloud cover data are based on the visual estimates, it is probably less useful than bright sunshine duration (Duffie and Beckman, 1980). McQuigg and Decker (1958) has shown that 62% to 91% of the variations in daily global flux of radiation could be explained on the basis of sunshine duration. Fortunately, measurements of daily bright sunshine hours using Campbell-Stokes sunshine recorders are available in useful lengths at several locations of the island and they are widely used because of their simplicity, convenience, and relatively low cost compared to the solar radiation instruments. These records generally cover longer periods in compared to the records of solar radiation.

The objective of this study is to present a model which can generate the daily bright sunshine hours and solar radiation that reaches the earth's surface in a day of dry zone of Sri Lanka. As detailed studies of solar radiation in Sri Lanka are scarce, this model will form the basis for more comprehensive solar radiation generating model in Sri Lanka in future, in particular, it may serve as an indication of what might be expected in other locations as well.

2. Solar Radiation Estimation Equations

The general method by which radiation is estimated from sunshine data involves the determination of regression coefficients of the Angström type equations for a location where both sunshine and solar radiation data are available. However, the use of universal type regression coefficients have been proposed by several workers (Clover and McCulloch, 1958; Black, 1961; Page, 1961) and these universal relationships have been widely used for solar radiation estimates by several workers (Cengiz et al., 1981; Da Mota et al., 1977). The amount of solar radiation coming to the earth's surface depends on several factors such as the solar constant, the latitude of the location, time of the year, influence of the atmosphere, albedo of the earth's surface, and the elevation of the location. As the influences of albedo and elevation are small, compared with the influence of the atmosphere, all three influences are summed up and called "the influence of the atmosphere". Most of the Angström type formula have been formulated by incorporating the "influence of the atmosphere" (De Jong, 1973). The Angstrom type empirical relationship derived by Clover and McCulloch (1958) is valid over the latitudinal range of 0 to 60 degrees and it is given below:

$$R_s = I_o \ (0.29 \cos\phi + 0.52 \ n/N) \qquad (1)$$

where,

R_s = average daily total solar radiation on a horizontal surface of the earth (cal cm^{-2} day^{-1})

I_o = solar radiation on a horizontal plane at the top of the atmosphere (cal cm^{-2} day^{-1})

ϕ = the latitude of the location in degrees

n = actual sunshine hours

N = maximum possible sunshine hours

Number of day light hours or maximum possible sunshine hours (N) is dependent upon the latitude (ϕ) of the location and sun's declination angle (δ).

$$N = (2/15)\cos^{-1}(-\tan\phi \tan\delta) \quad (2)$$

where,

δ = sun's declination angle in degrees

The sun's declination angle (δ) is dependent on the position of the earth, therefore on the day of the year, and can be defined as follows (Duffie and Beckman, 1980):

$$\delta = 23.45\sin\left(360\frac{284+n}{365}\right) \quad (3)$$

To calculate the solar radiation that reaches the earth surface in a day, it is necessary to have the total daily extraterrestrial radiation on a horizontal surface, I_0. Duffie and Beckman (1980) developed following relationship to compute the solar radiation at the top of the atmosphere over the period of sunrise to sunset.

$$I_0 = \frac{23.9\times 24\times 3600}{\pi} \; S\left[1+0.033\cos\left(\frac{360n}{365}\right)\right]\left[\cos\phi\cos\delta\sin\omega_s + \frac{2\pi\omega_s}{360}\sin\phi\sin\delta\right] \quad (4)$$

where,

I_0 = Solar radiation at top of the atmosphere (cal cm^{-2} day^{-1})
S = Solar constant (1353 MJ m^{-2})
n = Julian day of the year
ϕ = Latitude in degrees
δ = Declination angle in degrees
ω_s = Sunset hour angle in degrees

For sunset, the sun's hour angle (ω_s) is

$$\cos\omega_s = -\tan\phi\tan\delta \quad (5)$$

3. Concepts and Development

Estimation of solar radiation using average daily values of sunshine hours do not yield promising results because, sunshine duration is a random variable as all other meteorological variables. It is necessary to represent this randomness by a probability distribution function rather than by its mean. Use of generated data from the probability distributions which are developed from observed data bases will be more advantageous as they provide all possible outcomes. This approach ultimately leads to a clear understanding on the temporal variability of the system being studied.

It is possible to fit different theoretical probability distributions to the observed data. Conventionally, sunshine data analysis have been based on the fitting a normal probability density function which is bell-shaped and thus symmetric to the observed data. Popular use of normal probability density function in sunshine data analysis is probably due to the fact that it enables the use of certain statistical techniques such as Analysis of Variance (ANOVA), regression analysis and certain types of hypotheses. But in practical situations sunshine duration does not fit very well into such a symmetric distribution and most of the time it represents a skewed distribution.

4. Methodology

Sunshine duration data for each day of 17 years were fitted into six different continuous probability distributions namely, Extreme Value Type-A, Log-Logistic, Normal, Gamma, Exponential and Weibull distributions. Each distribution was assigned a relative evaluation score from 0 to 100 (best) based on the heuristic algorithms available in UNIFIT II, a statistical software to determine an appropriate probability distribution for observed data (Law and Vincent, 1993). The higher the score of a distribution, the better it is relative to the other fitted distributions. As the determination of the most appropriate probability distribution for observed data beyond the scope of this study, a common distribution with a moderately high score that represents the each day of the year was selected. Out of six probability distributions studied, Weibull distribution was the most appropriate distribution based on the heuristic and Anderson-Darling test for goodness of fit. Thus, on each day the number of sunshine hours was assumed to have a Weibull distribution with density function given in equation (6).

$$f(x) = \begin{cases} \alpha\, \beta^{-\alpha}\, x^{\alpha-1}\, e^{-(x/\beta)^{\alpha}} & \textit{if } x>0 \\ 0 & \textit{otherwise} \end{cases} \tag{6}$$

In addition, with the location parameter (γ) preset to zero, there are other two parameters, the shape parameter (α), and scale parameter (β) which need to be determined *for each day* of the year for 17 years from January 1976 to December 1992. These two parameters were then determined using Guided model selection mode of UNIFIT II (Law and Vincent, 1993). Using the estimated shape (α) and scale (β) parameters of the Weibull distribution, the amount of sunshine hours for each day were generated as random variates of the Weibull distribution (see Appendix 1). Solar radiation that reaches the earth's surface for each day was then computed using simulated values of sunshine duration along with the variables obtained from eqns (1,2,3,4 and 5). Solar radiation coming to the earth was corrected for cloud transmittance with a coefficient of 0.8 (Schulze, 1976).

Although Angström type formula have been based on the clear day sky conditions, presence of clouds in the sky is very common in Sri Lanka throughout the year. Being closer to the equator where the trade winds tend to converge causes development of low pressure cells and consequently the formation of clouds. When clouds are present, water droplets effectively absorb the energy from downward radiation. Thus, inclusion of a correction factor in to the model for cloud transmittance is necessary in order to model the system accurately. In this study, the other major assumptions made were sunshine duration to be dependent upon the

Julian day of the year and the year was assumed to have exactly 365 days, 29 February being ignored (Excell, 1981) in order to make the model simple.

Data procurement

The daily data for the computation of parameters of the appropriate probability distribution were observed during the 17 years period, January 1976 to December 1992 at a agro-meteorological station located at the Dry Zone Agricultural Research Institute, Maha-Illuppallama ($8^0 07'$N, $80^0 28'$E), Sri Lanka. The station is situated on a flat to gently undulating land and relatively free from obstructions to record the sunshine duration. The site has remained unchanged during the years under investigation.

In order to understand the temporal variability of sunshine duration, a climatological background of the region is necessary. Sri Lanka is divided into Dry Zone in northeastern part and a Wet Zone in southwestern part. Wet zone experiences rains all over the year while in the dry zone, it mainly rains during the period of October to mid January. The weather pattern at Maha-Illuppallama, the observation site, which is located at 137 m above sea level with an average annual rainfall of 1500 mm and annual mean temperature of 27 ^{0}C, is typical for dry zone. Three basic types of rainfall mechanisms are identifiable in the region: monsoonal, convectional, and cyclonic. The northeast monsoon rains occur from mid November to January and it brings substantial amount of rains to the area. Meanwhile rainfall during October and December period is generally augmented by the frequent formation of cyclonic depressions in the Bay of Bengal. Since the central highlands act as an orographic barrier, the southwest monsoon which occurs during May to September period blows over this region as a dry desiccating wind. First intermonsoonal convectional rains which occur due to the northward migration of Intertropical Convergence Zone bring considerable amount of rains during March to April. October to November period experiences substantial amount of rains as second intermonsoonal convectional rains due to the southward migration of the Intertropical Convergence Zone. These two intermonsoonal convectional rains occur as late afternoon, short duration, high intensity thunder storms, sometimes attain intensities even up to 100 mm per hour. During the dry periods (February and May-September) the climate is hot and sunny with high humidity.

5. Model Validation

The model was run for 300 times in order to obtain an unbalanced point estimate of simulated mean of daily bright sunshine hours. The 95% confidence interval band for monthly mean of daily bright sunshine hours was also calculated. These simulated values were compared with the observed monthly mean values of bright sunshine hours recorded during 1993-1994. Reliable on-site solar radiation data were not available for model validation. However, fairly precise monthly mean values of solar radiation during 1976 measured by an Actinograph were available. These observed values were then compared with simulated solar radiation values. Nevertheless, Accuracy of values recorded by Actinographs are also questionable because it always carries a 10% error (Stringer, 1972) and may go even up to 20 percent (Mani et al., 1967). Therefore, these observed values may not represent true mean values of the location.

6. Results and Discussion

Sunshine Duration

Simulated mean monthly sunshine hours computed from daily simulated values agree well with the observed values except during the period of October to November (Figure. 1). In all other months, observed values were very closer to either side of the simulated values. Generally, the band width of 95% confidence interval is very narrow throughout the year suggesting a uniform variability of simulated values. However, a little increase in variability among simulated values was evident during the period of October to December. Observed values always lies in between or very closer to the limits of the 95% confidence interval band of simulated values (Figure. 2). The highest observed and simulated sunshine hours were evident during February to March and then there is a decreasing trend up to December. The lowest observed and simulated values were recorded during the period of November to December.

February is the driest month for this region as it is not under the influence of any of the rainfall mechanisms discussed in a preceding section. The monthly mean rainfall in February seldom exceeds 20 mm in this region. Therefore, the highest number of sunshine hours in February, both observed and simulated, is due to the clear sky condition that exists during this month. Both March and April also maintain a relatively high amount of sunshine hours. Generally, these two months have been categorised as minor rainy months because of the convectional rains. Since first intermonsoonal convectional rains is not a strong event for this region and cloud formation occurs only at late afternoon due to the convectional currents, cloudiness do not hinder the bright sunshine hours significantly during March and April. However, there is a noticeable over-estimation of bright sunshine hours during October and November. These two months come within the major rainy season for the region due to strong convectional activity and northeast monsoonal blow. Therefore, it can be reasoned that there may be a strong correlation between sunshine hours and a day being wet or dry. Hence, during this period, the major assumption on which model was developed that is the number of sunshine hours depend only on Julian day of the year may not be valid. Thus, further development of this model has to be done by investigating the correlation structure of the system in wet and dry days separately.

In general, the discrepancy between simulated and observed values throughout the year may be due to the improper representation of sunshine hours by a Weibull distribution. In this model development, third parameter (location parameter) of the Weibull distribution, γ , was preset to zero as the lower boundary. Although this assumption could be applied to the weather variables such as rainfall, it may not be a good assumption for sunshine hours which seldom exhibits zero values. Similar kind of observation has made by Larsen and Pense (1982) with Gamma distribution for solar radiation.

Solar Radiation

Both simulated and observed values of solar radiation follow the same trend (Figure. 3) which is very similar to the behaviour of bright sunshine hours. However, it is interesting to note that the model has under estimated solar radiation values during first half of the year while there is an over estimation at second half of the year. Thus, the correction factor introduced in to the model for cloud transmittance is not strong enough to generate the precise data during the period from August to November. During September to November, due to the convectional activity, northeast monsoonal blow and formation of depressions in Bay of Bengal, thick and dense clouds (ie, Cumulus and Cumulonimbus) are very common in this region. Therefore, absorbence and transmittance by clouds during this period of the year may be much more higher than the what is expected. Although first half of the year consists of a minor rainy season during March and April, development thick and dense clouds is rare because of the weak convectional nature of first inter-monsoonal rains. Thus, influence of clouds for downward solar radiation is minimal during the first half of the year.

The highest solar radiation values (around 20 MJ m^{-2} day^{-1}) have been obtained throughout the period of February to May, followed by a peculiar depression in July. Despite being a rainy month, January value also higher than that of the values in November and December. The high insolation during February to May is invariably due to the increased number of bright sunshine hours per day during this period. Naturally, solar radiation in March and April must have lower values compared to February value because these two months have been categorised as a convectional rainy season. But it is known that March-April convectional currents in this region are not strong as in the case of October-November (Suppaiah, 1989). Higher insolation during March and April could also be due to the fact that relative position of the earth with respect to the sun. As the sun is directly overhead of the equator on March 21 (Vernal Equinox), during the period of March-April sun rays are nearly perpendicular to the earth surface of the equatorial regions. Since vertical incidence of sunrays always brings more insolation intensity than inclined incidence, solar radiation that reaches the earth's surface during these two months are comparatively higher than February.

Peculiar depression of solar radiation values during the June-July months can be explained astronomically. Since the orbit of the earth is elliptical, the sun-earth distance varies throughout the year and causes a variation of the amount of solar energy reaching the earth surface. The sun-earth distance reaches its maximum on July 3 (Aphelion), its minimum on January 3 (Perihelion). Although the eccentricity of the orbit is small (only 0.01673), there is about 7% difference in the solar energy flux at the top of the atmosphere between Perihelion and Aphelion (Coulson, 1975). Therefore the flux is highest in early January and lowest in early July. Hence, reduced solar radiation interception during the period of June-July could be due to the earth's position with respect to the sun. Despite being a rainy month, higher solar radiation during the January could be due to relatively high extraterrestrial flux compared to other months.

The discrepancies between the observed and simulated values could also be due to the improper representation of global radiation by Angström models. Although Angström type models have been based on the clear sky conditions, there is ample evidence that sufficient number of dust, haze, and other type of non-Rayleigh particles exist in even clearest cases of the natural atmosphere to produce significant scattering and absorption of incoming solar radiation (Coulson, 1975). It has also been shown that tropical atmosphere is rich in turbid particles especially during hot and dry periods (Mani et al., 1973). Therefore, the presence of relatively high amount of turbid particles in the tropical atmosphere reduces solar radiation coming to the earth's surface which can not be explained by Angström type models. De Jong

(1973) has shown that use of Angstrom type models in estimating solar radiation causes 10-15% error. Observed values are also not accurate enough for a thorough validation of the model as they always carry a 10%-20% error in measurements (Stringer, 1972 and Mani et al., 1967).

7. Conclusion

The model predicts the mean monthly global solar radiation of dry zone of Sri Lanka within a range of 15-20 MJ m^{-2} day^{-1}. This is in agreements with the values suggested by Löf et al., (1965) for the south Indian region and Samuel (1991) for Sri Lanka. Although model is capable of providing even daily solar radiation values, use of monthly averages is highly advisable as the sky is not always clear in this region. Further development of model should consider the correlation structure of the day being wet or dry in order to account the influence of rainy days on global solar radiation. Also, weight of the correction factor for cloud transmittance should be varied with the season approaching as the year advances. As the long series of bright sunshine duration measurements from different regions of dry zone in Sri Lanka are available, this model could be used for generation of solar radiation data in the dry zone which is not available or reliable.

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References

Angström, A. (1924). Solar and terrestrial radiation. Quart. J.R. Met. Soc. **50,** 121-126.

Black, J.N. (1961). Some aspects of climatology of solar radiation. Paper presented at UN conference on New Sources of Energy, Rome

Black, J.N. (1956). The distribution of solar radiation over the earth's surface. Arch. Meteorol. Geophys. Bioklim. **B 7,** 165-189.

Cengiz, H.S.; Gregory J.M. and Sebaugh, J.L. (1981). Solar radiation prediction from other climatic variables. Transactions of the ASAE. **24(2)**,1269-1272.

Coulson, K.L. (1975). Solar and terrestrial radiation. Methods and measurements. pp 242. Academic Press. New York, London.

Da Mota, F.S.; Beirsdorf, M.I.C. and Acosta, M.J.C. (1977). Estimates of solar radiation in Brazil. Agricultural Meteorology. **18,** 241-254.

De Jong, B. (1973). Net radiation received by a horizontal surface at the earth. Monograph, Delft University Press.

Duffie, J.A.; and Beckman W.A. (1980). Solar engineering of thermal processes. pp 1-75. John Wiley and Sons, New York.

Excell, R.H.B. (1981). A mathematical model for solar radiation in South-East Asia (Thailand) Solar Energy **26,** 161-168.

Fritz, S.; and McDonald T.H. (1949). Average solar radiation in the United States. Heating and Ventilation. **46,** 61-64.

Glover, J.; and McCulloch, J.S.G. (1958). The empirical relationship between solar radiation and hours of sunshine. Quart. J. R. Met. Soc. **84,** 172-175.

Larsen, G.A.; and Pense, R.B. (1982). Stochastic simulation of daily climatic data for agronomic models. Agronomy Journal. **74,** 510-514.

Law, A.M.; and Vincent, S.G. (1993). Unifit II user's guide. Averill M. Law Associates, Tuscon.

Löf, G.O.G.; Duffie, J. and Smith, C.O. (1965). World distribution of solar radiation, Engineering station Report No. 21, University of Wisconsin, Madison.

Mani, A.; Chacko, O. and Iyer, N.V. (1973). Atmospheric turbidity over India from solar radiation measurements. Solar Energy **14,** 185-195.

Mani, A.; Chacko, O.; Krishnamurthy, K. and Desikan, V. (1967). Distribution of global and net radiation over the Indian ocean. Arch. Met. Geoph. Bioklm. **B 15,** 83-98.

McQuigg, J.D.; and Decker, W.L. (1958). Solar energy. A summary of records at Colombia, Missouri. Mon. Agr. Exp. Sta. Bull. 671.

Page, J.K. (1961). The estimation of monthly mean values of daily total short wave radiation on vertical and inclined surfaces from sunshine records for latitudes 40^0 N to 40^0S. Paper presented at the UN conference of New Sources of Energy, Rome.

Ravfeim, K.J.A. (1981). Estimating solar radiation income from bright sunshine records. Quart. J. R. Met. Soc. **107,** 427-435.

Samuel, T.D.M.A. (1981). Estimation of global radiation for Sri Lanka. Solar Energy. **47,** 333-337.

Schulze, R.E. (1976). A physically based method of estimating solar radiation from suncards. Ibid. **16,** 85-101.

Stanhil, G. (1965). A comparison of four methods of estimating solar radiation. In Methodology of plant eco-physiology., pp 55-61. Proc. Montpellier Symposium, UNESCO Paris.

Stringer, E.T., (1972). Techniques in Climatology. pp 200. W.H. Freeman and Company, San Fransisco. USA.

Suppaiah, R. (1989). Relationships between the southern oscillation and the rainfall of Sri Lanka. International Journal of Climatology. **9,** 601-618

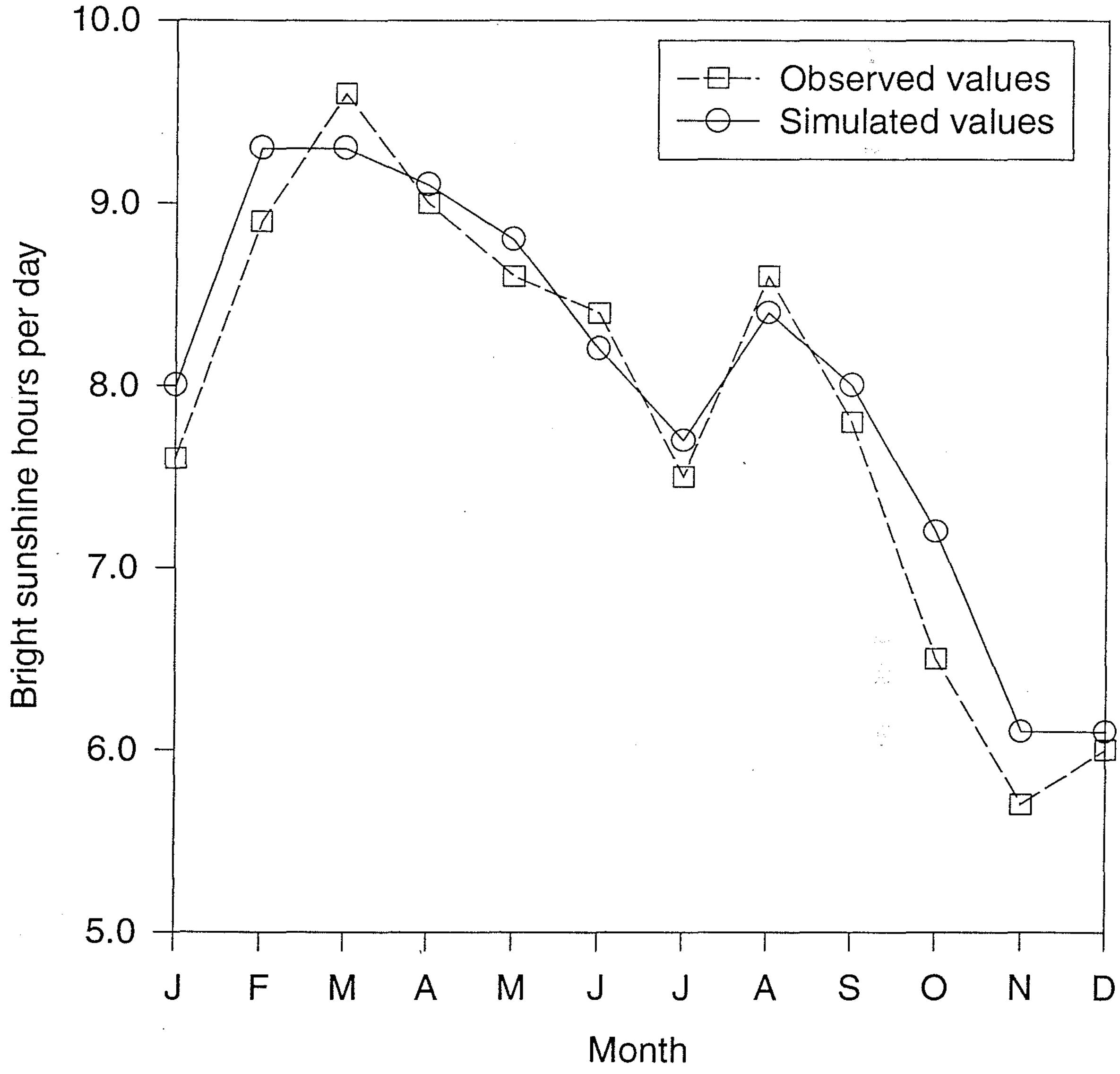

Figure. 1
Simulated and observed values of bright sunshine hours

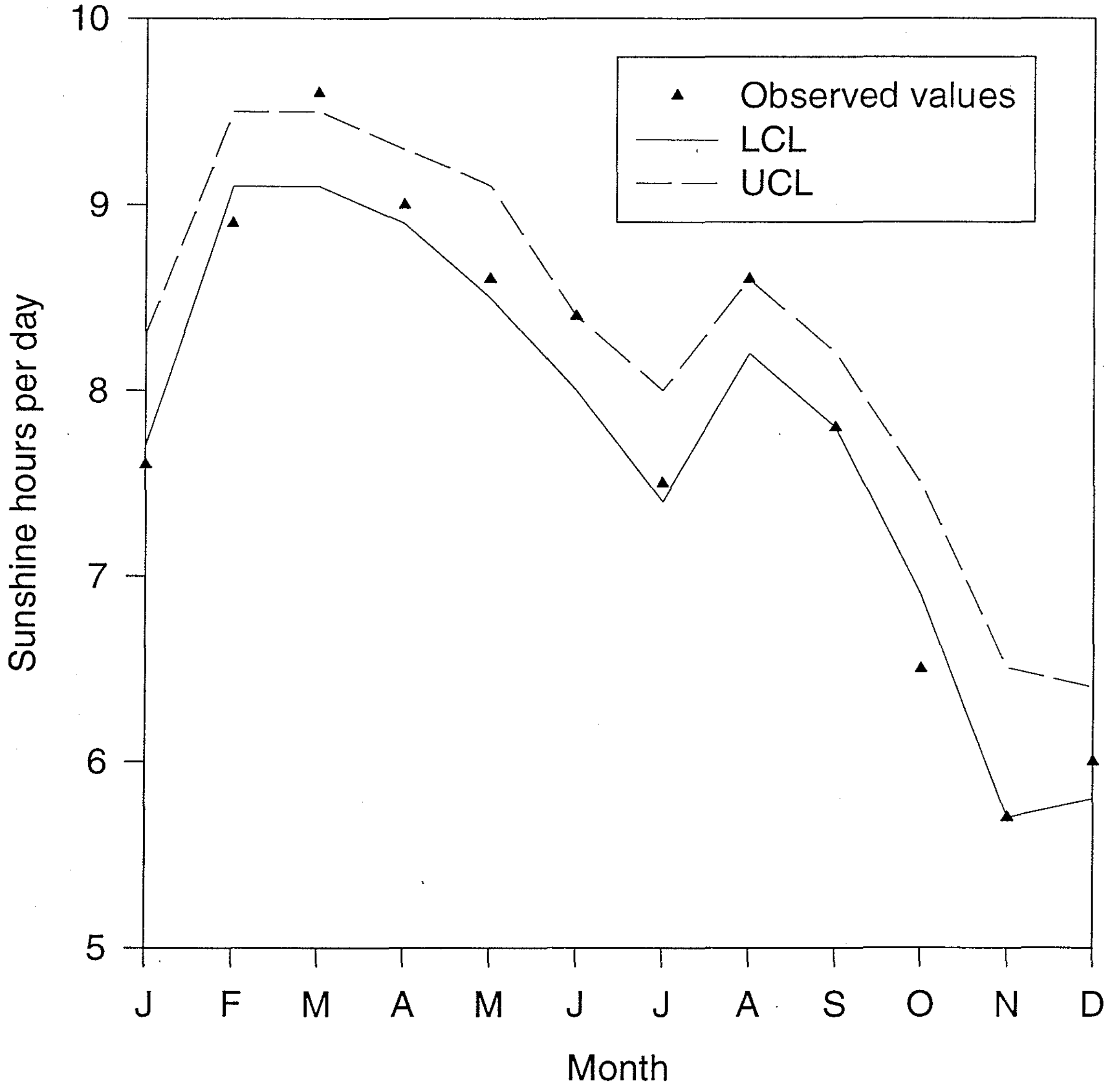

Figure 2.
95% confidence interval band for simulated values and observed values of sunshine hours

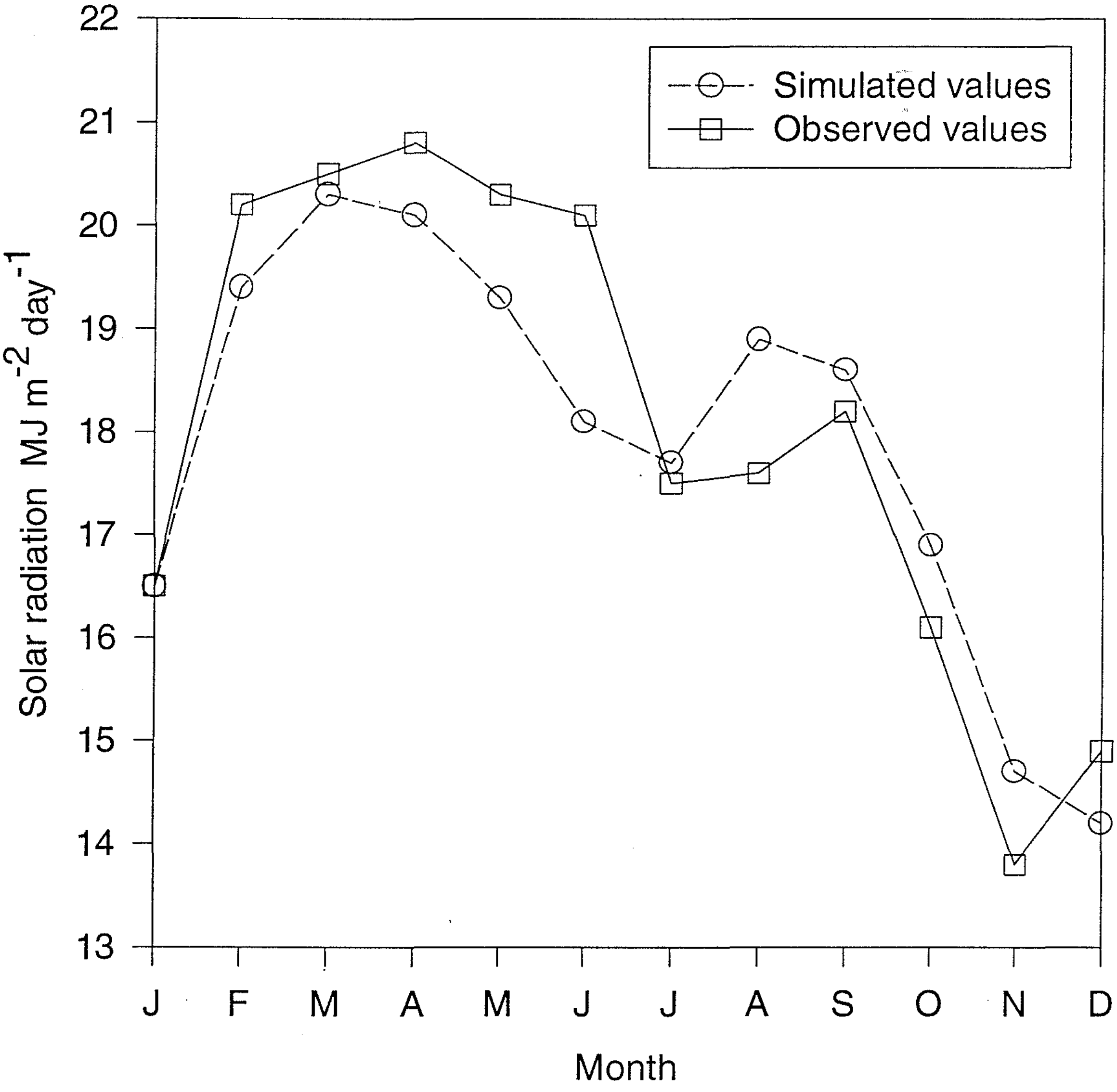

Figure 3.
Simulated and observed values of solar radiation

Appendix 1

Maximum likelihood estimates of parameters of Weibull distribution for representative dates (Duffie and Beckman, 1980) of each month.

Month	Shape parameter(α)	Scale parameter (β)
January	2.8	8.2
February	3.2	9.7
March	13.8	10.4
April	5.0	9.6
May	4.3	9.3
June	4.8	9.1
July	5.1	8.5
August	4.9	9.5
September	2.7	8.1
October	2.5	7.6
November	1.7	7.5
December	1.3	6.1