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# RESOURCE TAXATION WITH HETEROGENEOUS QUALITY AND ENDOGENOUS RESERVES

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This paper analyzes the impact of mining taxation on the intertemporal extraction profile for a heterogeneous ore body. The extraction profile specifies both quantities and qualities extracted in each period. Total extraction is thus endogenously determined. The taxes explicitly considered are severance taxes (ad valorem or per unit; on metal or on ore), property taxes, and profits taxes (with depletion allowance). The intertemporal framework integrates economic and geological factors, generating new results on extraction distortions and providing qualification or reconciliation of previous diverse results in this area.

## 1. Introduction

In this paper we examine some central issues of mineral resource taxation policy. Specifically, we analyze the impact of each of the several major taxes on (i) the timing of extraction with respect to quality, (ii) the rate of extraction, (iii) the total quantity of resource extracted, and (iv) the effectiveness of the tax in collecting resource rents. These are all issues that have been investigated to various degrees and in various contexts, with a diversity of conclusions corresponding to the diversity of premises. As described in more detail below, our model contains some new features, leading to some new results, while having a basic structure that permits a reconciliation or qualification of previous results.

We are concerned with taxes that are applied selectively, i.e. within particular political jurisdictions and, possibly, to particular minerals.[1] Only in exceptional circumstances will a change in a regional tax rate affect

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[1]See Gillis (1977), Stinson (1977) and Conrad (1979b) for a description of various taxes used in the United States, and Gillis et al. (1977) for a description of mining taxes in several other countries.

significantly the market price of the resource.[2] It is therefore appropriate to consider the impact of these taxes on a price-taking firm. The assumption of price exogeneity is further justified by the irregularity of actual price paths for minerals (see footnote 6).

The focus on the firm is also advantageous in allowing for more explicit modelling of geological factors. The mining problem is distinguished above all by these factors. Differences in the composition of the ore bodies cause differences in response to a given economic change. In part because of this, mineral tax policy in some countries has been negotiated on a mine-by-mine basis. Geological features must therefore be an essential part of any model that is to be used for policy or empirical analysis.[3] We emphasize here the grade distribution and the economic, as opposed to physical, quantity of reserves. This quantity, measured relative to a standard quality unit, is *endogenous*, not exogenous (as usually assumed in the literature), except in the very longest of long runs. With regard to any given mine, therefore, taxes will in general affect the size of economically recoverable reserves and the life of the mine, as well as the intertemporal extraction profile.

As an illustration of the diversity of assumptions and conclusions in the literature on mineral economics, consider the investigations of the distortionary effects of a severance tax. In his seminal work, Hotelling (1931) showed that a severance tax (either ad valorem or per unit) causes industry extraction to be reallocated to later periods. This conclusion was also derived by Peterson (1976). Both assume that the tax is on the output of finished metal and that the stock is exogenously determined. Using a model of the firm, with an exogenous reserve, McDonald (1965) argues that the reallocation can be in either direction. Burness (1976) reaches a similar conclusion, again assuming an exogenous reserve, and argues also that the life of the mine will be unaffected. In further contrast, a number of writers have argued that output taxes affect the level of reserves and the life of the mine. This phenomenon of tax-induced 'high-grading' has been analyzed by Lockner (1965), Steele (1967), Laing (1976), Gillis et al. (1977), Gillis (1978) and Conrad (1978a).

What will be shown here is, first of all, how and why both the life of the mine and the intertemporal extraction profile may be affected by such taxes. These results, and the precise nature of the distortions, reflect the empirical fact that ore bodies are not homogeneous with respect to quality. Higher quality ore may be exhausted and accrue rent, while lower quality ore may be left behind. Tax policy will thus affect both the rents on higher quality ores and the quantity of ore extracted. It will also be demonstrated that the

[2]See Gillis (1977) and Shelton and Morgan (1977) for a description of the conditions for shifting taxes forward.

[3]Peterson and Fisher (1977) review the recent literature on the economics of extractive resources and note the importance of increasing the geological content in economic models.

effects of a severance tax will generally differ according to whether it is imposed on ore (at the mouth of the mine) or on concentrate (at the time of sale) and according to whether it is ad valorem or per unit.

In section 2 we describe the basic model without taxes and obtain a characterization of the extraction profile. This will be the reference point for distortions caused by the introduction of taxes. In section 3 we analyze the impacts of several taxes frequently imposed on mining operations. Implications for policy are summarized in section 4.

## 2. The model

The firm's objective is taken to be the maximization of present value of profits from extraction over an arbitrary planning horizon, subject to constraints imposed by the technology and resource availability. The firm is assumed to know the quality distribution of the ore body, i.e. the tonnage of each grade of ore. It must choose the quality and quantity of ore to be extracted in each period of time.

Extraction costs in each period will depend on the total throughput of ore in that period, but not on the quality of ore extracted. The latter will determine the quantity of (final) output obtained from a given quantity of ore, output being defined as metal or 'concentrate' of a given purity. So, if $X_{tg}$ is the quantity of ore of grade $g$ ($g=1,\ldots,G$, in order of decreasing quality) extracted in period $t$ ($t=1,\ldots,T$), the extraction cost in period $t$ is $C_t(X_t)$, where $X_t \equiv \sum_{g=1}^{G} X_{tg}$ and with $C_t'>0$, $C_t'' \geqq 0$. The output in period $t$ is $\sum_{g=1}^{G} \alpha_g X_{tg}$, where $\alpha_g$ is the proportion of metal in ore of grade $g$.[4]

The firm's decision problem is

$$\max_{(X_{tg})} \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \left[ p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) \right], \tag{1}$$

subject to

$$R_g \geqq \sum_{t=1}^{T} X_{tg}, \qquad g=1,\ldots,G,$$

$$X_{tg} \geqq 0, \qquad t=1,\ldots,T, \quad g=1,\ldots,G,$$

where $p_t$ is the market price of output in period $t$ (usually an expectation if

[4]For example, if ore is 50% metal and concentrate is 80%, then $\alpha_g=0.625$. For examples of grade variation models with endogenous reserves see Thomas (1976), Walduck (1976) and Conrad (1978a). Concentrate and final output will be used synonymously in the text. For a description of the two stages involved in the mining process see Gillis et al. (1978) and Conrad (1979a).

$t>1$), $R_g$ is the total tonnage of ore of grade $g$ physically available, and $r$ is the interest rate used by the firm (which may incorporate a risk factor).[5]

The optimal extraction profile is determined by the Kuhn–Tucker conditions for (1). Letting $\mathscr{L}$ denote the Lagrangian of (1), with $\lambda_g$ the shadow price of reserves of grade $g$, these conditions are

$$\mathscr{L}'_{X_{tg}} = \frac{p_t\alpha_g - C'_t(X_t)}{(1+r)^{t-1}} - \lambda_g \leqq 0, \quad X_{tg} \geqq 0, \quad X_{tg}\mathscr{L}'_{X_{tg}} = 0,$$

$$t=1,\ldots,T, \quad g=1,\ldots,G, \qquad (2)$$

and

$$\mathscr{L}'_{\lambda_g} = R_g - \sum_{t=1}^{T} X_{tg} \geqq 0, \quad \lambda_g \geqq 0, \quad \lambda_g \mathscr{L}'_{\lambda_g} = 0, \qquad g=1,\ldots,G. \qquad (3)$$

The following grade selection rule is obtained from these conditions. Suppose ore of grade $g_1$ (respectively, $g_2$) is extracted in period $t_1$ (respectively, $t_2$). Then

$$(\alpha_{g_1} - \alpha_{g_2})\left(\frac{p_{t_1}}{(1+r)^{t_1}} - \frac{p_{t_2}}{(1+r)^{t_2}}\right) \geqq 0. \qquad (4)$$

So $\alpha_{g_1} > \alpha_{g_2}$ implies $p_{t_1}/(1+r)^{t_1} \geqq p_{t_2}/(1+r)^{t_2}$ and, conversely, $p_{t_1}/(1+r)^{t_1} > p_{t_2}/(1+r)^{t_2}$ implies $\alpha_{g_1} \geqq \alpha_{g_2}$; the grade selection profile corresponds to the profile of discounted prices. The firm wants to take the highest grade of ore when the discounted prices are greatest; lower grades will be sequentially allocated in like manner until further extraction is no longer profitable.[6] The reason that costs do not appear in (4) is that they are a function of the quantity of ore processed in the given period, independent of the grade. Costs do, however, affect the quantity of ore processed and the lowest grade taken in any period.

It is also clear from (2) that it is possible (and indeed empirically likely) that some lower grades of ore will not be extracted; that is, there will be a $g^*$ with $X_{tg^*} > 0$ in some period $t^* \leqq T$, and $\lambda_{g^*} = 0$, so that

$$\alpha_{g^*} = \frac{C'_{t^*}(X_{t^*})}{p_{t^*}}. \qquad (5)$$

[5]For examples of how risks are evaluated see Bennett et al. (1970).

[6]Schulze (1974), Herfindahl (1967) and Hartwick (1978) discuss rules where high grades are always taken first. In their models prices are assumed to be rising at a rate not greater than the rate of interest. However, in fact, prices vary substantially from period to period. For instance, the price of cobalt rose 253%, lead 87%, plutonium 159%, and copper 47% between June 1978 and June 1979 (see *Business Week*, July 2, 1979, p. 50).

This $g^*$ is the 'cut-off' grade in the current model and is analogous to that found frequently in the mining engineering literature.[7] The firm reaches the profitability margin while extracting ore of grade $g^*$ and will not extract ore of lower grades. This cut-off condition determines the aggregate quantity of reserves extracted and, together with the period rates of extraction, the life of the mine.

Note that it is not generally the case that the mine is closed at the time the cut-off grade is extracted. Condition (5) determines total extraction and thus capacity utilization in period $t^*$. But if $t^* < T$ then, by the very nature of the determination of the grade selection profile, the discounted price in period $t^*+1$ is higher and at that time a grade of ore higher than $g^*$ will be extracted.

## 3. The effects of taxation

The introduction of taxes will modify the objective function in (1) in ways that vary according to the type of tax. Discrete changes in the tax parameters (in particular, a jump from zero to some positive number corresponding to the introduction of a tax) may bring about a change in the grade selection profile, affecting also the cut-off grade. The intertemporal allocation of grades in the presence of taxes is determined by a relationship analogous to (4), with the relevant 'price' being the effective net-of-tax price of output.

Marginal changes in the tax parameters will not, except in boundary cases, alter the quality profile. But they can affect the quantity of ore extracted in any given period, this quantity response depending not only on prices and costs, but also on the nature of the intertemporal allocation prevailing prior to the tax change. In view of the numerous possible initial situations, the quantity responses will be illustrated in a two-period, two-grade context.[8] For the same reason, it will be further assumed for this part of the analysis that the discounted 'effective' price is higher in the first period, so that the higher grade of ore is taken first. The results for the cases of constant or rising discounted effective price can be easily inferred. Indeed, the qualitative character of the quantity distortions will not depend on the path of prices

[7]For a discussion of cut-off grades from the engineering perspective see Thomas (1973) and Lane (1964). Lane notes that the cut-off grade is a function of several factors and is more complicated than (5). However, (5) yields a convenient summary measure used in the industry that we can adopt for our purposes. It should also be noted that the planning horizon may be reduced because of risk and the use of such factors as pay-back periods. The lowest grade of ore planned to be extracted may therefore exceed the cut-off grade given by (5). See Bennett et al. (1970) for discussion.

[8]The use of two-period models to demonstrate the effects of dynamic processes is well established in the economics literature. It does not impose any significant restriction in the present context.

except for two of the tax variants we consider. These exceptions will be emphasized as they occur.

Even with the above simplifications and ignoring boundary cases, there are eight possible extraction profiles, according to whether one or both grades are extracted in each period and to whether or not one or both of the resource constraints are binding (i.e. the corresponding grades scarce). We shall note the general principles involved in the reallocation, and confine specific illustration to the following two cases.

*Case 1.* Grade 1 (high grade) ore is extracted in both periods, being exhausted in the second. Grade 2 (low grade) ore is extracted only in the second period, and exhausted. Both grades are therefore scarce. I.e., $X_{11}>0$, $X_{12}=0$, $X_{21}>0$, $X_{22}>0$; $X_{11}+X_{21}=R_1(\lambda_1>0)$; $X_{12}+X_{22}=R_2(\lambda_2>0)$.

*Case 2.* Grade 1 ore is extracted in both periods, exhausted in the second. Grade 2 ore is extracted only in the second period, and not exhausted. Only grade 1 is scarce. I.e., $X_{11}>0$, $X_{12}=0$, $X_{21}>0$, $X_{22}>0$; $X_{11}+X_{21}=R_1$ $(\lambda_1>0)$; $X_{12}+X_{22}<R_2(\lambda_2=0)$.

Consideration of these cases will be sufficient to demonstrate both the intertemporal distortions and the tax-induced high-grading effects.[9]

We now analyze in turn the allocative effects of severance taxes (including royalties), property taxes and proportional profits taxes with depletion allowances. In each case it will be assumed that the tax is to be applied at the same rate (or nominal value, as appropriate) in all periods.

### 3.1. *Severance taxes*

Severance taxes imposed on mining operations are generally of three types: (a) a fixed payment per ton of metal, (b) a fixed payment per ton of ore extracted, and (c) a proportion of the metal price, in which case the tax is often termed a royalty. Severance taxes have developed to replace property taxes on account of the relative ease of their administration [see Lockner (1965)].

It was noted in the introduction that previous analysis has produced an assortment of results on the allocative effects of these taxes. Further, Boyle (1977) has argued that such taxes can be used to collect rent accruing to extraction of the scarce resource. Below, it will be shown how the impacts of a severance tax will depend on the variant used and on the prevailing context of economic and geological conditions.

[9]Results for the other cases as well as derivations are contained in a longer paper, available from the authors.

*3.1.1. Per unit tax on output*

This form of severance tax is a fixed nominal charge ($\tau > 0$) per ton of output. With it, the firm's profit in period $t$ is

$$\pi_t = (p_t - \tau) \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t). \tag{6}$$

Note that the only effect on problem (1) is the replacement of $p_t$ by $p_t - \tau$. The grade selection (condition (4)) is now characterized in terms of this *effective* price, $p_t - \tau$:

$$(\alpha_{g_1} - \alpha_{g_2}) \left( \frac{p_{t_1} - \tau}{(1+r)^{t_1}} - \frac{p_{t_2} - \tau}{(1+r)^{t_2}} \right) \gtreqless 0.$$

Consequently, there will be no grade selection distortion[10] provided that $(p_{t_1} - \tau)/(1+r)^{t_1} > (p_{t_2} - \tau)/(1+r)^{t_2}$ whenever $p_{t_1}/(1+r)^{t_1} > p_{t_2}/(1+r)^{t_2}$. This will always be the case if $t_1 > t_2$ (i.e. if prices are rising at a rate greater than the rate of interest) since then $\tau/(1+r)^{t_1} > \tau/(1+r)^{t_2}$. However, if $t_1 < t_2$ (i.e. if discounted prices are falling) it is possible that the lower discounted tax in a later period may now more than compensate for the lower discounted price, inducing the firm to postpone extraction of the higher grade ore. The tax rate sufficient to bring about this reversal is lower the smaller is the amount by which the firm's interest rate exceeds the expected rate of inflation of metal prices in the short run. In the particular case where prices are rising at the rate of interest, so that in the absence of the tax the firm would be indifferent about the timing of extraction, introduction of the tax will definitely cause postponement of high-grade extraction. (If the interest rate were less than the anticipated rate of price inflation, the firm would already have been postponing high-grade extraction.) On the other hand, the cut-off grade is now given by

$$\alpha_{g*} = \frac{C_t'}{p_t - \tau},$$

and so may be greater than in (5). Empirical confirmation of such increases in the cut-off grade (tax-induced 'high-grading') has been made, for example,

[10]We shall say that there is no grade selection distortion if the ordering of time periods according to (4) is the same before and after the imposition of the tax. Of course, any quantity changes induced by the tax will in general change the time periods in which any particular grade is planned to be extracted, i.e. even if the ordering of periods according to (4) is unchanged, the association of time periods with qualities of ore extracted will change.

in Bolivia [see Gillis et al. (1977)], where mines have been closed prematurely.[11]

The general features of the response to an increase in this tax are as follows.

(i) If a grade of ore would have been exhausted had there not been a tax increase, this will continue to be so, i.e. the total quantity extracted of that grade will not be affected by a marginal change.

(ii) However, if a grade is being extracted in more than one period, whether ultimately exhausted or not, there will be a quantity reallocation from present to future. An increase in the tax reduces the marginal revenue (effective price) in each period, but this reduction is greater in the first period than in the second because the same nominal change is being discounted in the second.

(iii) If some grade is not being exhausted there will be a reduction in the extraction of that grade, this being governed by the condition that marginal cost be equal to the net-of-tax price. This will be accompanied by the reallocation from present to future, described in (ii), of any grade being extracted in both periods. Total extraction, and hence output, is therefore reduced whenever some grade is not viewed by the firm as scarce in an economic sense.

(iv) The shadow prices of scarce grades, measuring the present value of profit from an additional unit, are reduced, by more for higher grades than for lower. The reason for the relative difference is that a given tax on *metal* means a higher tax per ton of higher grade *ore*.

These principles are not affected by the assumption that the discounted effective price is falling over time. The details of the adjustments in the representative situations will now be described. The corresponding derivatives are presented in table 1.

In case 1 the reserves of both grades are binding constraints. Total extraction will therefore not change. However, extraction of the high grade in period 1 will fall, i.e. the rate of extraction will be reduced, with a corresponding reallocation to period 2. In case 2, where the low grade is not fully extracted, extraction becomes less profitable at the margin and is therefore cut back. With high-grade extraction being merely reallocated from present to future, total recovery will fall by the reduction in low-grade extraction.

[11]Lockner (1965) notes that, as long as reserves are exogenous, a severance tax may lengthen the life of the mine by decreasing extraction each period and thus reducing costs. Such a phenomenon is not precluded a priori in the present model. But for it to occur the fall in marginal costs would have to exactly equal the increase in the tax, implying no change in the scarcity price of reserves. However, we show that the scarcity price is reduced, if anything, increasing the cut-off grade.

Table 1

Output effects of a marginal change in the severance tax on output.

| | Case 1 | Case 2 |
|---|---|---|
| $\frac{dX_{11}}{d\tau}$ | $\frac{-r\alpha_1}{(1+r)C_1''+C_2''}<0$ | $-\frac{r\alpha_1+\alpha_2}{(1+r)C_1''}<0$ |
| $\frac{dX_{21}}{d\tau}$ | $-\frac{dX_{11}}{d\tau}>0$ | $-\frac{dX_{11}}{d\tau}>0$ |
| $\frac{dX_{22}}{d\tau}$ | 0 | $-\frac{(1+r)\alpha_2 C_1''+(r\alpha_1+\alpha_2)C_2''}{(1+r)C_1''C_2''}<0$ |
| $\frac{d\lambda_1}{d\tau}$ | $-\frac{\alpha_1(C_1''+C_2'')}{(1+r)C_1''+C_2''}<0$ | $-\frac{\alpha_1-\alpha_2}{1+r}<0$ |
| $\frac{d\lambda_2}{d\tau}$ | $\frac{-(1+r)\alpha_2 C_1''-(r\alpha_1+\alpha_2)C_2''}{(1+r)((1+r)C_1''+C_2'')}<0$ | n.a. |

The value of grade 1 reserves will fall by more than that of grade 2 (which will be zero in case 2), although the former value must always exceed the latter. A tax increase of \$1 per ton on metal of 80% purity, for example, translates to a \$0.75 tax increase per ton of 60% ore ($\alpha=0.6/0.8=0.75$) but only a \$0.50 tax increase per ton of 40% ore ($\alpha=0.4/0.8=0.50$).

It is also worth noting that, aside from the possible quality profile effects noted before, continued increases in the tax would eventually transform a case 1 situation into case 2, and so on, until ultimately, if the tax were sufficiently high, the mine would be closed. Thus, instead of serving only to collect resource rents, severance taxes may reduce the size of economically recoverable reserves. Only in a situation where each grade of ore is extracted and exhausted within a given period will a marginal tax change have no distortionary effects, i.e. have only a pure (lump sum) rental effect.

### *3.1.2. Per unit tax on ore*

This severance tax is on ore as it is extracted from the mine, before processing, independent of the grade.[12] Because of ore quality differences, its impacts are qualitatively and quantitatively different from those of a tax on output.

[12]Most ores, even coal, are processed at the mine site before being transported. Thus the tax on ore is actually a tax on an input into the processing operation. See Gillis et al. (1978) for a description.

The firm's after-tax profit in period $t$ will be

$$\pi_t = p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) - \tau X_t, \tag{7}$$

where $\tau$ now denotes the tax per unit of ore. Unlike the tax on output, which effectively reduces the output price, this tax amounts to a uniform upward shift in the firm's marginal cost curve.

Since the effective price of output is not changed, the relationship (4) is unaffected and so there is no distortion of the grade selection profile. However, the magnitudes of the quantity distortions are increased when the tax is on ore rather than metal and is thus partially a tax on wasted material.

The cut-off grade, $g^*$, is now given by

$$\alpha_{g^*} = \frac{C_t' + \tau}{p_t},$$

whereby the tax is seen to induce a tendency toward high-grading. In terms of (7), the effect of the difference in the location of the tax can be regarded alternatively as a decline in the value of ore of grade $g$ (before extraction costs) in period $t$, from $p_t\alpha_g - \tau\alpha_g$ to $p_t\alpha_g - \tau$. In the expressions for rates of change with respect to the tax, the derivatives being all of the same form as in table 1, the $\alpha_g$'s are correspondingly replaced by 1's. A unit tax on ore is, for any grade less than 100%, greater than a unit tax on metal and is proportionately greater for lower grades of ore.

### *3.1.3. Ad valorem severance tax*

This severance tax is a proportion $\beta$ (assumed fixed, less than 1) of the output price. After-tax profit in period $t$ is then

$$\pi_t = (1-\beta) p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t). \tag{8}$$

The effect of the tax is to reduce the effective output price in period $t$ from $p_t$ to $(1-\beta)p_t$. It is clear from (4) that the imposition of the tax will therefore not affect the order in which grades are extracted. However, the cut-off grade is now given by

$$\alpha_{g^*} = \frac{C_t'}{(1-\beta)p_t},$$

so there may be a reduction in economic reserves.

The quantity distortions induced by an ad valorem tax differ from those of the per unit output tax. Because the size of the ad valorem tax varies with the output price, its effects will depend on the path of discounted prices. In particular, the intertemporal reallocation of a scarce grade, always from present to future with the per unit tax, will now be in the direction of the period with the lower discounted market price. The reduction in effective output price due to an increase in the royalty rate is less when the market price is less.

These comparative static effects are given in table 2 for the two illustrative cases. Note that in case 1 there is a pure reallocation of grade 1 in favor of period 2, which has the lower discounted price. There would have been no impact if prices had been rising at the rate of interest, an impact in the reverse direction (i.e. toward period 1) if prices had been rising faster than the rate of interest. In view of the varied patterns of actual mineral prices it is clear that the response to an increase in the royalty rate will not be uniform, either over time or across minerals.

A comparison of the three types of severance taxes shows that only the ad valorem tax may produce a distortion in either direction. The result of McDonald (1965) and Burness (1976), that severance taxes have this ambiguous effect, does not apply to the per unit taxes.

### 3.1.4. *A note on variable severance taxes*

It has been shown by Peterson (1976) and Burness (1976) that, with an exogenously determined reserve base, an output tax that rises at the rate of interest will be non-distortionary. Such a tax creates the obvious practical difficulty of determining the proper discount rate. But it may also have distortionary effects, not examined by the preceding authors, when economic reserves are endogenous.

Consider, for instance, the effect on the cut-off grade. With a per unit severance tax, the cut-off grade is given by

$$\alpha_{g*} = \frac{C_t'}{p_t - \tau(1+r)^t},$$

while for an ad valorem tax it is given by

$$\alpha_{g*} = \frac{C_t'}{p_t[1 - \beta(1+r)^t]}.$$

Table 2

Output effects of a marginal change in the ad valorem tax rate.

| | Case 1 | Case 2 |
|---|---|---|
| $\frac{\mathrm{d}X_{11}}{\mathrm{d}\beta}$ | $-\frac{\alpha_1((1+r)p_1-p_2)}{(1+r)C_1''+C_2''}<0$ | $-\frac{\alpha_1((1+r)p_1-p_2)+\alpha_2 p_2}{(1+r)C_1''}<0$ |
| $\frac{\mathrm{d}X_{21}}{\mathrm{d}\beta}$ | $-\frac{\mathrm{d}X_{11}}{\mathrm{d}\beta}>0$ | $-\frac{\mathrm{d}X_{11}}{\mathrm{d}\beta}>0$ |
| $\frac{\mathrm{d}X_{22}}{\mathrm{d}\beta}$ | 0 | $-\frac{(1+r)p_2\alpha_2 C_1''+[((1+r)p_1-p_2)\alpha_1+p_2\alpha_2]C_2''}{(1+r)C_1''C_2''}<0$ |
| $\frac{\mathrm{d}\lambda_1}{\mathrm{d}\beta}$ | $-\frac{\alpha_1(p_2 C_1''+p_1 C_2'')}{(1+r)C_1''+C_2''}<0$ | $-\frac{p_2(\alpha_1-\alpha_2)}{1+r}<0$ |
| $\frac{\mathrm{d}\lambda_2}{\mathrm{d}\beta}$ | $-\frac{(1+r)p_2\alpha_2 C''+(\alpha_1((1+r)p_1-p_2)+\alpha_2 p_2)C_2''}{(1+r)((1+r)C_1''+C_2'')}<0$ | n.a. |

The cut-off grade is increased in either case. Also, for reasons discussed above, the grade selection profile may now be distorted by an ad valorem royalty, as well as by a per unit output tax.

Similar distortions will be induced by other forms of variable severance tax that have been adopted, such as those which vary with the consumer or wholesale price index, or with the market price of the mineral.[13]

### *3.2. Property taxes*

Property taxes cause a tendency to shift extraction from future to present and are thus, ceteris paribus, detrimental to conservation.[14] This result has been established in the literature and follows also from the present model. Once again, however, property taxes have some associated distortions that have not been noted and that may be significant in off-setting the effects of the forward-shifting tendency.

Suppose that the tax rate and the estimated value per ton of metal are constant, and that the tax in any period is imposed on the reserves remaining at the end of the period. The tax paid at the end of period $t$ is therefore

$$\theta \sum_{g=1}^{G} \alpha_g \left[ R_g - \sum_{j=1}^{t} X_{jg} \right],$$

where $\theta > 0$ is the tax rate (incorporating the constant estimate of metal value). After-tax profit in period $t$ is then, apart from a constant term,

$$\pi_t = \left[ p_t + \theta \sum_{j=t}^{T} \frac{1}{(1+r)^{j-t}} \right] \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t). \tag{9}$$

It is clear from (9) that for each ton of ore extracted in any period, the firm saves the present value of future tax payments. This results in an increase in the net-of-tax price in the period of extraction, i.e. it is in effect a negative per-unit severance tax whose value changes from period to period. The direction of the tax-induced distortions are accordingly the opposite of those reported in 3.1.1; the tax will encourage extraction of higher grades in the early periods. But it is also the case that the cut-off grade is *lowered.* This is an important consideration since it is known that in many instances the

[13]For instance, New Mexico's coal severance tax is a function of the consumer price index, while North Dakota's oil severance tax is a function of the wholesale price index. See Conrad (1979b) for details.

[14]See Stinson (1977) and Peterson (1976) for discussion.

quantity of ore increases as the grade decreases.[15] The increase in economic reserves of the mine may be sufficient to offset the reallocation of extraction to the present, and thus increase the life of the mine. The net effect will depend upon the grade-tonnage distribution of the particular deposit, but it is an effect that certainly should not be ignored in planning tax policy.

It is worth emphasizing that, since the taxable body of ore is not fixed but rather depends on the extraction decision, a property tax is not a lump sum tax on the mine. However, a lump sum tax could be achieved with a combination of suitably chosen property and severance taxes, for example, in view of their qualitatively opposite distortionary effects.

### *3.3. Proportional profits taxes with depletion allowances*[16]

It is well established that pure profits taxes without depletion allowances are non-distortionary. That the introduction of depletion allowances tends to increase the rate of extraction in the early periods is equally well documented.[17] However, as in the case of property taxes, there are additional effects that have been ignored in the literature.

For present purposes cost depletion is defined as a fixed dollar allowance $d$ per ton of final output produced. If the profits tax rate is $k$, the after-tax profit in period $t$ is then

$$p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) - k\left[ p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) - d \sum_{g=1}^{G} \alpha_g X_{tg} \right]$$

$$= (1-k)\left[ \left( p_t + \frac{kd}{1-k} \right) \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) \right].$$

The depletion allowance is seen to effectively raise the output price by a constant amount, $kd/1-k$, when profits are positive. It therefore operates as a negative per-unit severance tax, or 'severance subsidy'. The combined effects of tax and allowance are consequently the opposite of those reported in 3.1.1. As with a property tax, the cut-off grade is increased and so there may be an increase in the life of the mine, depending on the grade-tonnage distribution.

[15]See Brooks (1976) and the references in that paper for a discussion of the grade-tonnage relationship.

[16]States have begun imposing progressive profits taxes as well. For an analysis of this tax see Conrad and Hool (1979b).

[17]Depletion allowances are the most studied incentive in the mining industry. Peterson (1976) and Sweeney (1977) show that depletion allowances tend to reallocate extraction to the present. Others, including Harberger (1955), Steiner (1959, 1968) and McDonald (1961, 1967, 1976) concentrate on the allocative distortions of percentage depletion.

Percentage depletion, on the other hand, is a fixed proportion $h$ of the current value of output. In this case, after-tax profit in period $t$ is

$$p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) - k\left[p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t) - hp_t \sum_{g=1}^{G} \alpha_g X_{tg}\right]$$

$$= (1-k)\left[\left(1+\frac{kh}{1-k}\right)p_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t(X_t)\right].$$

When profits are positive the allowance effectively increases the price per ton by a factor $kh/1-k$ and therefore operates as a negative ad valorem severance tax.[18] The effects of the allowance are then the opposite of those reported in 3.1.3. In addition to decreasing the cut-off grade and increasing reserves, the allowance may or may not defer extraction, depending on the path of discounted prices. Thus, in periods of rapidly increasing prices, such an allowance may be an additional incentive to conserve the resource which has not been previously noted.

## 4. Summary

The results of this investigation are summarized in table 3. They offer a synthesis of previous work as well as new insights into the effects of tax

Table 3

Summary of effects of taxes on mining decisions.

| Tax | Grade selection profile | Reserves (high-grading effects) | Extraction profile |
|---|---|---|---|
| Per unit severance on output | Present to future | Decreased | Present to future |
| Per unit severance on ore | None | Decreased | Present to future |
| Ad valorem severance | None | Decreased | Depends on path of discounted prices |
| Profits tax with cost depletion | Future to present | Increased | Future to present |
| Profits tax with percentage depletion | None | Increased | Depends on path of discounted prices |
| Property tax | Future to present | Increased | Future to present |

[18]The fact that percentage depletion is a negative severance tax has been noted by Davidson (1963, 1970) and Hause (1963).

policy on mining firms. Each tax (subsidy) has been shown to have the potential for inducing distortions in the intertemporal allocation of reserves and the economically recoverable reserve base. The results are based on the fact that reserves are not homogeneous, and that any one mine is planned to have a finite life. Ore which is extracted and exhausted does accrue rents, and thus a tax may distort the extraction profile of these ores. On the other hand, the size of the deposit is endogenous at this level and a tax may well affect the life of the mine and the total reserves extracted.

The effect that a particular tax will have in an actual situation will depend on the price–cost profile and the geological composition of the ore body. For instance, a severance tax imposed on a porphyry copper deposit, where the ore is evenly distributed and marginal costs are constant to capacity, would only affect the later years of production. The same tax imposed on a small silver deposit, where the variance in grades is high and marginal costs are increasing, may force the mine to close prematurely.

A number of empirical questions are suggested. First, how much 'high-grading' is induced by these taxes for different types of deposits? Second, do property taxes and depletion allowances significantly lengthen or shorten the life of the mine? Third, can states or countries with marginal deposits use tax policy to compete effectively for mineral investments? Fourth, to what degree can states or countries with large reserves collect substantial taxes without significantly affecting investment? Finally, given that reserves of any mine are endogenous, is there a combination of taxes which will cause a minimum of distortions yet require acceptable administration procedures and costs? These are all questions that warrant further research.

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