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Money, Expectations and the Existence of a Temporary Equilibrium

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1. INTRODUCTION

If money and financial assets are to be integrated into general equilibrium theory, it is apparent that the classical Arrow-Debreu framework must be modified. As long as all trading takes place essentially at some initial point in time, with each individual subject only to a present value budget constraint, there is no place in the system for money, either as a medium of exchange or as a store of value, even if uncertainty about future states of the world is introduced as in Debreu [2], for example. It is natural then to consider an economy with sequential trading, and the appropriate Walrasian equilibrium concept becomes the Hicksian temporary equilibrium. This framework underlies the analysis of Patinkin [11] and has been formally developed by several authors, including Grandmont [4], Green [6], Sondermann [12] and Stigum [13, 14].

Clower [1] has pointed out that the classical budget constraint used in neo-Walrasian monetary theory, with money appearing symmetrically with consumption goods, is an inadequate representation of the trading opportunities available to an individual in a monetary economy. Being analytically indistinguishable from other commodities, money plays no essential role in the exchange process. Clower proposed a dichotomized budget constraint to characterize money expenditure and money income; the expenditure constraint requires that the cost of an individual's purchases in a given period does not exceed his initial cash balance. Grandmont and Younès [5] introduced into the Walrasian framework a modification of Clower's expenditure constraint, allowing an individual trader to have access to a fraction of the total proceeds from his sales in the current period. However, their proof of existence of a temporary monetary equilibrium (hereafter TME) does not cover the case where the fraction is zero (Clower's specification). They also employ the very restrictive assumption of " tight expectations " (see below, Section 3), an assumption which has been standard in previous proofs of existence of a TME. (See, for example, Grandmont [4] and Sondermann [12].) It rules out expectations with unitary elasticity and so the case where current prices are expected to hold in the future (as in Patinkin [11]).

In this paper it is shown that, if the actual " Clower " constraint is used to introduce into the Walrasian model the requirement that money be used as the medium of exchange, it is possible to demonstrate the existence of a TME with a much weaker restriction on expectations. More precisely, it is shown that the class of price expectations consistent with the existence of a TME includes expectations with elasticity up to and including unity. If the expenditure constraint is added to the basic Patinkin model we can establish the existence of a short-run equilibrium in that model, subject to a mild condition on initial endowments expressing the desirability of an intertemporal transfer of wealth.

In economic terms, the problem is to establish the existence of an equilibrium in which money has a positive exchange value (Hahn [7]). It has been shown by Grandmont [4] that the short-run price of money will be positive provided that traders expect the price of money to be positive in the future, no matter what prices are currently observed. Here, it will be shown that the short-run equilibrium price of money will be positive provided that traders have a desire to transfer wealth from the present to the future and that money is both a store of value and the institutional medium of exchange. So, in this framework, we can say that, as long as intertemporal trade is advantageous and traders expect the institutional structure (i.e. monetary exchange) to persist in the future, there will be sufficient reason for fiat money to have a positive price even though it has no intrinsic worth.

In Section 2 we describe a model of a monetary economy which is then analysed in Section 3. To simplify the exposition we shall concentrate on the case where individuals have a two-period planning horizon and are subjectively certain about future prices; the sensitivity of expectations can then be expressed in terms of price elasticities. We shall indicate how the analysis can be generalized to the case where individuals have subjective uncertainty about prices, expectations then taking the form of probability distributions. Some concluding remarks are presented in Section 4.

2. A MONETARY ECONOMY

The economy to be studied is essentially the one described by Grandmont and Younès [5] so here we shall summarize it only briefly. We consider a perfectly competitive pure exchange economy operating over a sequence of time periods. In each period there are N perishable consumption goods and money. Money has no direct utility but is the unit of account, a costless means of storing wealth between periods, and the medium of exchange: commodities must be bought or sold for cash. The total money stock M, is constant over time.

At the beginning of period 1, each trader has an initial endowment of consumption goods, ω_1, and an initial stock of money, m_0, the cash balance held after the close of trading in the previous period; trading takes place in Walrasian spot markets for all consumption goods. Cash payment is a requirement of every transaction and a trader is not permitted to use any part of the proceeds from his current sales to pay for his current purchases. In other words, the total value of his purchases cannot exceed his initial cash balance, and the sum of receipts from current sales forms all or part of his cash balance at the outset of the succeeding period. This will serve to distinguish money from other commodities.

The price of money, the numéraire, is 1. A monetary price system for consumption commodities is denoted by p, a point in R^N. We allow only those price systems which are elements of $P = \{p \in R^N \mid p \gg 0\}$. A TME for the economy in a given period is a list of money prices and final individual demands for all commodities such that all markets are cleared.

3. EXPECTATIONS AND EQUILIBRIUM

A representative trader from the set $\mathscr{I} = \{1, \ldots, I\}$ is assumed to have a two-period planning horizon and to know with certainty his initial endowments in the two periods,

$$\omega = (\omega_1, \omega_2) \in R_+^{2N}.$$

We assume

(a) $$\omega \gg 0.$$

The agent must choose an action $a_1 = (x_1, m_1) \in R_+^{N+1}$, where x_1 is his vector of final demands for current consumption goods and m_1 the amount of cash he desires to hold until period 2. If $p_1 \in P$ is the price system quoted in period 1, a_1 must satisfy the budget equation

$$p_1 \cdot x_1 + m_1 = p_1 \cdot \omega_1 + m_0 \qquad \ldots(3.1)$$

and the expenditure constraint

$$p_1 \cdot [x_1 - \omega_1]^+ \leqq m_0, \qquad \ldots(3.2)$$

where $[x_1 - \omega_1]^+$ is the vector of purchases. The constraint (3.2) reflects the necessity for demand for real commodities to be backed by effective purchasing power, in the form of cash, as suggested by Clower. It requires that the total expenditure on purchases of consumption goods not exceed the amount of cash held before trading and is therefore an immediate consequence of the specification that all purchases be paid for in cash and that cash from sales of consumption goods be unavailable for expenditure until the next period.

(b) The trader's subjectively certain price expectations are represented by a continuous mapping $\psi: P \to P$.

Given p_1 and a_1, the constraints on his planned period 2 consumption $x_2 \in R_+^N$ are then

$$\psi_2 \cdot x_2 = \psi_2 \cdot \omega_2 + m_1 \qquad \ldots(3.3)$$

and

$$\psi_2 \cdot [x_2 - \omega_2]^+ = m_1, \qquad \ldots(3.4)$$

where $\psi_2 = \psi(p_1)$.

In anticipation of the next assumption, (c), the final money balance, m_2, at the end of the planning horizon has been set equal to zero explicitly. From the point of view of planning in the first period, a positive cash balance at the end of period 2 would be worthless since there is no opportunity to spend it. We remark, however, that when period 2 becomes current, the trader will in general choose a positive m_2, his planning horizon having then been extended to period 3. We observe further that (3.3) and (3.4) thus jointly imply that $x_2 \geqq \omega_2$; the trader will add to his initial endowments through the expenditure of m_1.

(c) The trader's preferences for consumption streams (x_1, x_2) are assumed to be represented by a real-valued utility function, u, which is continuously differentiable, bounded, strictly concave and strictly monotone.

Define $A_1(p_1, m_0) = \{a = (x, m) \in R_+^{N+1} \mid (3.1), (3.2)\}$,

$$A_2(\psi_2, m_1) = \{x \in R_+^N \mid (3.3), (3.4)\},$$

and $\gamma(p_1, m_0) = \{(x_1, x_2) \in R_+^{2N} \mid \exists\, m_1 \geqq 0 \text{ such that } (x_1, m_1) \in A_1(p_1, m_0)$ and $x_2 \in A_2(\psi(p_1), m_1)\}$.

Let $x_2^*(a_1, \psi(p_1))$ denote the optimal consumption in period 2, given a_1 and p_1, and subject to $x_2 \in A_2(\psi(p_1), m_1)$. Define $v(a_1, p_1) = u[x_1, x_2^*(a_1, \psi(p_1))]$. The function v is continuous in both arguments, strictly concave and monotone in the first (see Grandmont [4]). The trader's demand in period 1, $\xi(p_1)$, is then the solution to the problem: given p_1, choose $a_1 \in A_1(p_1, m_0)$ to maximize $v(a_1, p_1)$. The mapping $\xi: P \to R_+^{N+1}$ is easily shown to be continuous.

In order to prove the existence of a TME we shall want to establish the unboundedness (in norm) of aggregate excess demand sequences corresponding to price sequences which are unbounded or which converge to the boundary of P (i.e., when one or more prices converge to zero). It is the former case which requires a restriction on the sensitivity of expectations to current price changes. It is intuitive that, if current and expected future prices increase without bound, any finite amount of money is ultimately worthless; in order that the money " market " be equilibrated there must be an incentive to carry over, from the present to the future, an increasing amount of money. We introduce here an assumption which characterizes situations in which such an incentive exists. We shall write it first in two parts. The first part is a condition on the rate at which expected future prices increase relative to current prices, at least with respect to some commodity k. The second part can be regarded as a condition on relative endowments in the two periods, namely that, in real terms, taking into account expectations about prices as well as time preference, the trader is sufficiently poorly endowed with commodity k in the future, relative to the present.

(d) for some commodity k,

(1) $\exists B_k < +\infty$ such that for any sequence $\langle p_1^n \rangle$ in P with $\lim_{n\to\infty} \| p_1^n \| = +\infty$,

$$\lim_{n\to\infty} \psi_{2k}^n / p_{1k}^n \leqq B_k;$$

(2) $u'_{N+k}(\omega)/u'_k(\omega) > B_k$, where $u'_k(x)$ denotes the marginal utility of k at x.

Condition (d.1) allows expected prices to increase without bound, admitting any expectations mapping for which some elasticity is no greater than unity. The admissible class therefore includes functions such as (i) that assumed by Patinkin ($\psi(p) = p$), (ii) linear adaptive functions, and (iii) linear homogeneous functions. These functions are ruled out by the " tight expectations " assumption, which in this context takes the form

(*) $\{\pi \in P \mid \pi = \psi(p), \exists p \in P\}$ lies in a compact subset of P.

We can rewrite (d) as

(d′) $\exists k$ such that for any sequence $\langle p_1^n \rangle$ in P with $\lim_{n\to\infty} \| p_1^n \| = +\infty$,

$$u'_{N+k}(\omega)/u'_k(\omega) > \lim_{n\to\infty} \psi_{2k}^n / p_{1k}^n.$$

In this form it is easily recognizable as a limit version of the classical condition for the desirability of substitution in the case of one good and two periods. It is also evident that condition (d) (or (d′)) is implied by (*). Further, it is possible (cf. Grandmont and Younès [5]) to demonstrate the existence of a TME using, as an alternative to (d), a weakened form of (*), namely

(e) $\exists k$ such that for any sequence $\langle p_1^n \rangle$ in P with $\lim_{n\to\infty} \| p_1^n \| = +\infty$, $\lim_{n\to\infty} \psi_{2k}^n < +\infty$.

In either case, we shall want expected prices to be uniformly bounded away from zero, i.e.,

(f) $\exists \sigma \gg 0$ such that $\psi(p_1) \geqq \sigma$ for all $p_1 \in P$.

Finally, we assume

(g) for any sequence $\langle p_1^n \rangle$ in P, if p_{1k}^n is bounded above then ψ_{2k}^n is bounded above.

We shall define a regular trader to be one who satisfies (a), (b), (c), (f) and (g). The following results (Lemmas 3.1 through 3.3) concern the behaviour of a regular trader in the limit as prices of consumption goods go to zero or become arbitrarily high. $\bar{P}$ denotes the closure of P.

Lemma 3.1. *Let $\langle p_1^n \rangle$ be any sequence in P such that $p_1^n \to p_1^0 \in \bar{P}\backslash P$, and $\langle a_1^n \rangle$ a sequence with $a_1^n = (x_1^n, m_1^n) = \xi(p_1^n)$ for all n. Consider a regular trader with $m_0 > 0$. Then*

$$\| a_1^n \| \to +\infty.$$

Proof. Assume the proposition false. Then we can find a subsequence

$$\langle a_1^n \rangle = \langle (x_1^n, m_1^n) \rangle$$

converging to $a_1^0 = (x_1^0, m_1^0)$, with $\| x_1^0 \| < +\infty$, $m_1^0 < +\infty$. Let $\langle x_2^n \rangle$ be the corresponding sequence of choices in period 2. Since $m_1^n \to m_1^0 < +\infty$, (f) implies that $\langle x_2^n \rangle$ is bounded. We can therefore assume without loss of generality that $x_2^n \to x_2^0$, $\| x_2^0 \| < +\infty$.

Choose k such that $p_{1k}^0 = 0$. Define $\bar{a}_1 = (x_1^0 + ce_k, m_1^0)$, where $c > 0$ and e_k is the unit vector with 1 in the kth position and 0 elsewhere; $\bar{a}_1 \in A_1(p_1^0, m_0)$. Since A_1 is lower hemicontinuous over $\bar{P}$ for $m_0>0$, there exists $\langle \bar{a}_1^n \rangle = \langle (\bar{x}_1^n, \bar{m}_1^n) \rangle$, with $\bar{a}_1^n \in A_1(p_1^n, m_0$ and $\bar{a}_1^n \to \bar{a}_1$.

Choose $0<\lambda<1$ and define $x_2^\lambda = \lambda x_2^0$. Recall, from the discussion following (3.4), that $x_2^0 \geqq \omega_2$. In view of (g), we can assume that $\psi_2^n \to \psi_2^0$, $\| \psi_2^0 \| < +\infty$. Then

$$\psi_2^n \cdot [x_2^\lambda - \omega_2]^+ \to \psi_2^0 \cdot [x_2^\lambda - \omega_2]^+$$

with $\psi_2^0 \cdot [x_2^\lambda - \omega_2]^+ < m_1^0$, if $x_2^0 > \omega_2$, and $\psi_2^0 \cdot [x_2^\lambda - \omega_2]^+ = 0$, if $x_2^0 = \omega_2$. In either case, $\psi_2^n \cdot [x_2^\lambda - \omega_2]^+ \leqq \bar{m}_1^n$ for n large, and hence $u(\bar{x}_1^n, x_2^\lambda) \leqq u(x_1^n, x_2^n)$. By continuity, when $n \to \infty$ and $\lambda \to 1$, $u(x_1^0 + ce_k, x_2^0) \leqq u(x_1^0, x_2^0)$, which is impossible.

Lemma 3.2. *Let $\langle p_1^n \rangle$ be any sequence in P such that $\| p_1^n \| \to +\infty$, and $\langle a_1^n \rangle$ a sequence with $a_1^n = \xi(p_1^n)$ for all n. Consider a regular trader who satisfies (e). Then $\| a_1^n \| \to +\infty$.*

Proof. Assume false. Then there exist subsequences $\langle x_1^n \rangle$, $\langle m_1^n \rangle$ such that

$$x_1^n \to x_1^0, \ \| x_1^0 \| < +\infty,$$

and $m_1^n \to m_1^0 < +\infty$. Thus, for large n, $m_1^n < m_1^0 + 1$. Note that $m_1^0 < +\infty$, together with (f), implies that $\langle x_2^n \rangle$ is bounded and so we can assume that $x_2^n \to x_2^0$, $\| x_2^0 \| < +\infty$.

Choose k such that $p_{1k}^n \to +\infty$, and k' such that $\psi_{2k'}^n \to \psi_{2k'}^0 < +\infty$. Note that

$$p_1 \cdot [x_1 - \omega_1]^+ \leqq m_0,$$

together with $m_1 < +\infty$, implies that $x_{1k}^0 = \omega_{1k} > 0$. Define $\bar{x}_1^n = x_1^n - e_k/p_{1k}^n$, $\bar{m}_1^n = m_1^n + 1$, and $\bar{x}_2^n = x_2^n + e_{k'}/\psi_{2k'}^n$. We have $\bar{x}_1^n \to x_1^0$, $\bar{m}_1^n \to m_1^0 + 1$, $\bar{x}_2^n \to \bar{x}_2^0 = x_2^0 + e_{k'}/\psi_{2k'}^0$, and

$$(\bar{x}_1^n, \bar{x}_2^n) \in \gamma(p_1^n, m_0)$$

for large n. So $u(\bar{x}_1^n, \bar{x}_2^n) \leqq u(x_1^n, x_2^n)$, and by continuity, when $n \to +\infty$,

$$u(x_1^0, \bar{x}_2^0) \leqq u(x_1^0, x_2^0),$$

which is impossible.

Lemma 3.3. *Let $\langle p_1^n \rangle$ be any sequence in P such that $\| p_1^n \| \to +\infty$, and $\langle a_1^n \rangle$ a sequence with $a_1^n = \xi(p_1^n)$ for all n. Consider a regular trader who satisfies (d′). Then $\| a_1^n \| \to +\infty$.*

Proof. In view of (g) and Lemma 3.2, we need only consider the case where, for all k, $p_{1k}^n \to +\infty$ and $\psi_{2k}^n \to +\infty$.

Assume the proposition false. Then there exist subsequences $\langle x_1^n \rangle$, $\langle m_1^n \rangle$ such that $x_1^n \to x_1^0$, $\| x_1^0 \| < +\infty$, and $m_1^n \to m_1^0 < +\infty$. Thus for large n, $m_1^n < m_1^0 + 1$.

Choose k satisfying (d′). Note that $p_{1k}^n \to +\infty$ for all k implies $x_1^0 \leqq \omega_1$, which, together with $m_1^0 < +\infty$, implies $x_1^0 = \omega_1$. Also then $x_2^n \to x_2^0 = \omega_2$. Define

$$\bar{x}_1^n = x_1^n - e_k/p_{1k}^n, \ \bar{m}_1^n = m_1^n + 1, \ \bar{x}_2^n = x_2^n + e_k/\psi_{2k}^n.$$

Then $\bar{x}_1^n \to \omega_1$, $\bar{m}_1^n \to m_1^0 + 1$, $\bar{x}_2^n \to \omega_2$. For large n, $(\bar{x}_1^n, \bar{x}_2^n) \in \gamma(p_1^n, m_0)$ and

$$u(\bar{x}_1^n, \bar{x}_2^n) - u(x_1^n, x_2^n) \simeq u'_{N+k}(\omega)/\psi_{2k}^n - u'_k(\omega)/p_{1k}^n > 0,$$

by (d′), contradicting $a_1^n = \xi(p_1^n)$.

We remark that this result could have been achieved with an assumption weaker than (d′), namely

(d″) $\quad \exists (k_1, k_2)$ such that for any sequence $\langle p_1^n \rangle$ in P with $\lim_{n \to \infty} \| p_1^n \| = +\infty$,

$$u'_{N+k_2}(\omega)/u'_{k_1}(\omega) > \lim_{n \to \infty} \psi_{2k_2}^n / p_{1k_1}^n.$$

We are concerned with the existence of a TME in the economy composed of the set $\mathscr{I}$ of regular traders and their initial resources. We shall call this a regular economy.

Characteristics of the ith trader will now be distinguished by the subscript i, and aggregations will be over the index set $\mathscr{I}$. We define $\tilde{P} = \{\tilde{p} = (p, 1) \in R^{N+1} \mid p \in P\}$.

The aggregate excess demand function $\zeta: \tilde{P} \to R^{N+1}$ is defined by

$$\zeta(\tilde{p}_1) = \Sigma\xi_i(p_1) - (\Sigma\omega_{i1}, M).$$

Lemma 3.4.

(i) *ζ is continuous on $\tilde{P}$;*

(ii) *ζ is bounded below;*

(iii) $\tilde{p}_1 \cdot \zeta(\tilde{p}_1) = 0$, *for all $\tilde{p}_1 \in \tilde{P}$;*

(iv) *Assume that at least one trader satisfies (d′) or (e). If $\langle p_1^n \rangle$ is a sequence in P such that either $p_1^n \to p_1^0 \in \bar{P} \backslash P$ or $\| p_1^n \| \to +\infty$, then $\| \zeta(\tilde{p}_1^n) \| \to +\infty$.*

The proof of Lemma 3.4 is straightforward and will be omitted. We just observe that ξ_i is bounded below by $(-\omega_{i1}, 0)$, and $m_{i0} > 0$ for at least one trader.

A TME in period 1 is an $(I+1)$-tuple $(\tilde{p}_1^*, a_{11}^*, \ldots, a_{I1}^*)$ of points in R^{N+1} such that $\tilde{p}_1^* = (p_1^*, 1)$ with p_1^* in P, $a_{i1}^* = \xi_i(p_1^*)$ for all i in $\mathscr{I}$, and $\Sigma a_{i1}^* = (\Sigma\omega_{i1}, M)$.

Theorem 3.5. *There exists a* TME *for a regular economy if at least one trader satisfies either* (*d′*) *or* (*e*).

With the properties of ζ established in Lemma 3.4, the proof of the theorem is a direct application of Grandmont [4], Theorem 1 of the Appendix.

The arguments applied above for the case of subjective certainty can be generalized to the situation where traders are subjectively uncertain about future prices. Expectations can then be described by a (weakly continuous) mapping $\psi: P \to \mathscr{M}(P)$, where $\mathscr{M}(P)$ is the set of all probability measures defined on the measurable space $(P, \mathscr{B}(P))$, $\mathscr{B}(P)$ denoting the Borel σ-field of P. For B in $\mathscr{B}(P)$, $\psi(B;\ p_1)$ denotes the probability of B given that p_1 is the price system in period 1. If u has the von Neumann-Morgenstern property, the trader's decision problem can be summarized as: given p_1, choose a_1 to maximize the expected utility $v(a_1, p_1) := \int_P u[x_1, x_2^*(a_1, p_2)]d\psi(p_2;\ p_1)$, subject to constraints (3.1) through (3.4), where (3.3) and (3.4) apply for each p_2 in supp $\psi(p_1)$.

The assumptions (d) through (g) are generalized in the obvious way. In particular, we have

(d‴) $\exists k$ such that for any sequence $\langle p_1^n \rangle$ in P with $\lim_{n\to\infty} \| p_1^n \| = +\infty$,

$$\lim_{n\to\infty} \int_P \frac{p_{1k}^n}{p_{2k}} d\psi(p_2;\ p_1^n) > \frac{u_k'(\omega)}{u_{N+k}'(\omega)}.$$

Condition (d‴) requires that, in the limit, the ratio of the average (expected) price of some commodity in period 2 to its price in period 1 be greater than the corresponding ratio of marginal utilities evaluated at the initial endowment point $\omega = (\omega_1, \omega_2)$. As was the case with (d′), (d‴) can be weakened by allowing the commodity in period 2 to differ from the one in period 1. So we now admit expectations whose elasticity, on the average, does not exceed unity; the introduction of uncertainty strengthens the result.

4. CONCLUDING REMARKS

The preceding analysis shows that, by introducing a constraint on money expenditure, it is possible to incorporate the role of money as the medium of exchange in a Walrasian equilibrium system. The existence of an equilibrium in Patinkin's basic model can then be established subject to a mild restriction on initial endowments. Expectations about the future are clearly of great importance in determining the current state of the economy. The results obtained above indicate that the case of unit-elastic expectations marks the boundary between those expectations which are consistent with the existence of a temporary monetary equilibrium and those which are not. This agrees with our intuition since inelastic expectations would tend to moderate inflationary or deflationary trends while elastic expectations would tend to aggravate them.

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