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Liquidity, Speculation, and the Demand for Money

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1. INTRODUCTION

A basic problem in the general equilibrium theory of money is to justify the existence of a monetary equilibrium, an equilibrium in which traders are collectively prepared to hold the stock of a money with no intrinsic worth (Hahn [13]). Explicit consideration of intertemporal economies with sequential trading (initiated by the studies of Grandmont [6], Green [12] and Stigum [21, 22]) has helped to expose more clearly some of the fundamental requirements. (See Grandmont [7] for a survey.) For example, in [6] and in Grandmont and Younès [10] it is shown that such an equilibrium will exist, for an economy in which money is the unique store of value, provided that people expect money to have positive value in the future. This condition amounts to inelasticity of price expectations, at least as the current relative price of money approaches zero. The analysis is extended in Grandmont and Laroque [8, 9] to an economy in which there is an alternative financial asset, a long-term bond, with the current rate of interest fixed at a positive level by a central bank. Again, inelastic expectations, now extended to the interest rate, permit the compatibility of money and interest-bearing assets.

While the assumption of inelastic expectations is not unreasonable for many circumstances (e.g., when prices have been stable in the recent past), it is sufficiently powerful to dominate some considerations which can be illuminating for the existence problem. The analysis would apply to any asset, paper or otherwise; it does not exploit the distinguishing features of money. From the point of view of monetary theory it seems desirable to develop these properties (Grandmont and Younès [10, 11]). In fact, it can be shown (Hool [17]) that, if money is the sole store of value and if its role as the medium of exchange is explicitly introduced using a dichotomized budget constraint (as suggested by Clower [2] and adopted in modified form in [10]),

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then the equilibrium value of money will be positive if there is a suitable balance between price expectations (which may be unit-elastic), the intertemporal profile of initial endowments and intertemporal preferences. This result prompts the question: Does this continue to be the case if there is an alternative, interest-bearing asset? If the latter asset's yield were uncertain and possibly negative, one might look to speculation to induce the necessary desire to hold money. However, if the asset's yield were surely positive, one would expect people to be prepared to hold money (in the short run) only if they have no choice. Money's value in such a case must derive from its designated role in the exchange process and is thus connected with the illiquidity of physical commodities.

A second, related question is: In an economy in which the rate of interest is determined endogenously by the relative strengths of individuals' desires to borrow or lend, what is the basic force guaranteeing a strictly positive equilibrium rate (or, more generally, an equilibrium rate greater than some given positive level)? We know that inelastic interest-rate expectations—the standard explanation—will do it, and that this is connected with the notion of a "liquidity trap" (Keynes [18], Tobin [23], Grandmont and Laroque [9] and Younès [24]). So the question becomes: Is there anything more fundamental than inelastic interest-rate expectations which will guarantee a rate greater than zero and, if so, how is it related to a liquidity trap?

To analyze these questions a micro-model of the demand for money is developed in the context of an intertemporal economy in which money has its traditional triad of roles and individuals make their intertemporal consumption choice and portfolio choice simultaneously (Section 2). The main implications of the study are to be drawn from the sufficient conditions for existence of a short-run monetary equilibrium (Section 3). This will involve the establishment of properties of the demand for money, individual and aggregate, in various circumstances relating to the behavior of prices and the interest rate. Since the model has some general features in common with Green's [12], several of the assumptions required are similar to his. However, because the forward markets are restricted here to money and bonds, the assumption of "common expectations" ([12]) is not necessary (see Lemma A.1). Also, the conditions of interest here are those which bear specifically on the properties of the demand for money. They will be developed from the necessary conditions for an individual's optimal choice. Concluding remarks are presented in Section 4. All proofs are gathered in the Appendix.

2. A Model of the Demand for Money

We consider a pure exchange economy with I traders ($i = 1, 2, \ldots, I$), operating over a sequence of time periods ($t = 1, 2, \ldots$). In each period there are N perishable consumption goods, fiat money and long-term bonds. Money is the unit of account, the medium of exchange (taken as an institutional fact) and a store of value. The total quantity, M, of money is constant over time. Bonds are perpetuities paying one unit of money in each period after purchase. In each period the only markets operating are spot markets for trading each of the real goods and bonds against money.

The two essential aspects of the demand for money with which the analysis is concerned are (i) the demand arising from the use of money as the means of payment, and (ii) the speculative demand, arising from uncertainty about future bond prices (equivalently, future interest rates) and thus the resultant yield of a bond. Without the possibility of a capital loss greater than the interest payment there can be no speculative demand for money. The distinction of money as the medium of exchange is formalized by assuming that consumption goods are relatively illiquid: there is a delay between the commitment to sell a good and the opportunity to spend the revenue. Specifically, money received from sales conducted in one trading period cannot be spent (on goods or bonds) until markets reopen in the next period. (This corresponds formally to an exchange process in which there is one round of actual trading in goods per period and, in each round, offers to buy and sell are expressed simultaneously on each market, an offer to buy being valid only if backed by cash.) Bonds, like consumption goods, must be purchased with cash. But it will be assumed that bonds, unlike consumption goods, are instantly convertible into cash. (In this sense bonds are as liquid as money, although in another sense (cf. Hicks [16], for example) they have imperfect liquidity because of the uncertainty of their future market values.) This means that an individual can generate immediate purchasing power by borrowing whereas current sales of goods provide income only for future purchases (including debt liquidation). The decision with respect to purchases in the current period implies a *reservation demand* for money; the value of goods supplied constitutes an *income demand* for money (cf. Clower [2]). It is to be stressed that, because of the illiquidity of goods and of the institutional use of money as the means of payment, an income demand for money is the inevitable consequence of the sale of a good. This plays a crucial role in the later analysis. It provides the analytical distinction in the model between money and bonds as means of exchange.

Let $p_t \in R_+^N$, $q_t \in R_+$ and 1 be the money prices of goods, bonds and money in period t; $r_t = 1/q_t$ is the rate of interest; $\pi_t = (p_t, q_t, 1)$ is the monetary price system. The set of admissible price systems in period t is $\Pi = \{\pi_t \in R_+^{N+2} \mid \pi_t \gg 0\}$.

Consider a typical individual (whose index i is omitted where inessential). At the start of each period t he receives an initial endowment of consumption goods, $\omega_t \in R_+^N$, and has an initial portfolio of bonds and money, $(b_{t-1}, m_{t-1}) \in R \times R_+$, as a result of his choice in the previous period. To avoid consideration of bankruptcy in the first period ($t = 1$) it is assumed that there is no outstanding debt. It will also be convenient to assume that the commodity endowments in periods 1 and 2 ($\omega = (\omega_1, \omega_2)$) and the initial money balance in period 1 (m_0) are strictly positive. Formally,

(2a) for all i, (i) $(\omega_i, m_{i0}) \gg 0$ and (ii) $b_{i0} = 0$.

The individual's problem in period 1 is to choose an action $a_1 = (x_1, b_1, m_1)$, where $x_1 \in R_+^N$, $b_1 \in R$, $m_1 \in R_+$ are final demands for goods, bonds and money balances, respectively. The expenditure constraint (or liquidity constraint) reflecting money's role as the means of payment, is

$$p_1 \cdot (x_1 - \omega_1)^+ + q_1 b_1 \leqslant m_0 . \tag{2.1}$$

Consequently,

$$m_1 = m_0 - q_1 b_1 - p_1 \cdot (x_1 - \omega_1)^+ + p_1 \cdot (x_1 - \omega_1)^- \tag{2.2}$$

where $(\cdot)^+$, $(\cdot)^-$ denote vectors of purchases and sales, respectively. Note that, in view of (2.1), $m_1 \geqslant p_1 \cdot (x_1 - \omega_1)^- \geqslant 0$. The more familiar form of (2.2) is the budget equation:

$$p_1 \cdot x_1 + q_1 b_1 + m_1 = p_1 \cdot \omega_1 + m_0 . \tag{2.2$'$}$$

The individual evaluates alternative actions according to his intertemporal preferences for consumption. To simplify notation and exposition it will be assumed that, in order to take account of the effects of a current action, the individual considers only his consequent position in the succeeding period and evaluates it as if any credit (or debt) were then realized. In this way his consequent financial position is assessed in real terms; the appropriate intertemporal utility function is therefore one which represents his preferences for combinations of current consumption and "quasi" future consumption. Let this utility function be denoted by u, with the properties

(2b)(i) for all i, u_i: $R_+^{2N} \to R$ is a von Neumann–Morgenstern utility function which is strictly increasing, continuously differentiable, concave and bounded; and

(ii) for all i, if $\langle x_2{}^n \rangle$ is any bounded sequence in $R_+{}^N$ with a strictly positive lower bound, then there exists a k and numbers $\bar{\rho}_k$, $\bar{\bar{\rho}}_k$, with $\bar{\bar{\rho}}_k \geqslant \bar{\rho}_k > 0$, such that $\bar{\rho}_k \leqslant \partial u_i(x_1{}^n, x_2{}^n)/\partial x_{2k} \leqslant \bar{\bar{\rho}}_k$ for all n, for any sequence $\langle x_1{}^n \rangle$ in $R_+{}^N$.

Condition (ii) will be used only in the proof of Proposition 3.4. It implies that the marginal utility of second period consumption is determined primarily by that consumption, and that there is always non-satiation in some good in the second period.

The individual's expectations about prices in period 2 depend on current prices and take the form of a probability distribution, $\psi(\pi_1)$, with the property

(2c) for all i, $\psi_i(\cdot)$ is continuous in the topology of weak convergence.

A borrower in period 1 must generate sufficient money for period 2 to cover his debt, in any circumstance that he considers possible. An action in period 1 is therefore further constrained by

$$(q_2 + 1)\, b_1 + m_1 \geqslant 0, \qquad \text{for all} \quad \pi_2 \in \operatorname{supp} \psi(\pi_1) \tag{2.3}$$

For each $\pi_2 \in \operatorname{supp} \psi(\pi_1)$, the constraints on the individual's choice in period 2 are

$$p_2 \cdot (x_2 - \omega_2)^+ \leqslant (q_2 + 1)\, b_1 + m_1 \tag{2.4}$$

and

$$p_2 \cdot x_2 = p_2 \cdot \omega_2 + (q_2 + 1)\, b_1 + m_1 . \tag{2.5}$$

(Note that because of (2.3) the right-hand sides of (2.4) and (2.5) are non-negative. Further, utility maximization requires (2.4) to hold as an equality.)

So the individual must choose a plan (x_1, b_1, m_1, x_2) which maximizes $\int_\Pi u(x_1, x_2)\, d\psi(\pi_1, \pi_2)$, subject to (2.1) through (2.5), where $\psi(\pi_1, \cdot)$ denotes $\psi(\pi_1)(\cdot)$. Implied by the set of such plans is the set $\xi_1(\pi_1)$ of his optimal actions for period 1.

Of particular relevance to the existence of a temporary equilibrium will be the following relationships implied by the Kuhn–Tucker conditions for the above optimization; k denotes the index of goods in each period, u_k' the marginal utility of good k evaluated at the optimum.

$$\int_\Pi \frac{u_{k_1}'}{p_{1k_1}}\, d\psi(\pi_1, \pi_2) \geqslant \int_\Pi \frac{u_{N+k_2}'}{p_{2k_2}}\, d\psi(\pi_1, \pi_2) \tag{2.6}$$

for all (k_1, k_2) with $x_{1k_1} > 0$;

$$\int_\Pi \frac{u_{k_1}'}{p_{1k_1}/q_1}\, d\psi(\pi_1, \pi_2) = \int_\Pi \frac{u_{N+k_2}'}{p_{2k_2}/(q_2 + 1)}\, d\psi(\pi_1, \pi_2) \tag{2.7}$$

for all (k_1, k_2) with $x_{1k_1} > \omega_{1k_1}$ and $x_{2k_2} > \omega_{2k_2}$;

$$\int_\Pi \frac{q_1 - (q_2 + 1)}{p_{2k_2}}\, u_{N+k_2}'\, d\psi(\pi_1, \pi_2) \leqslant 0 \tag{2.8}$$

for all k_2 with $x_{2k_2} > \omega_{2k_2}$.

These conditions specify that there should be no remaining advantage to be gained by transfer of wealth from period 1 to period 2 via money, by transfer of wealth between periods 1 and 2 via bonds, or by pure speculation, respectively.

3. Monetary Equilibrium

A temporary monetary equilibrium in period 1 is a price system $\pi_1^* \in \Pi$ and I actions a_{i1}^* $(i = 1, 2,..., I)$ such that

$$a_{i1}^* \in \xi_{i1}(\pi_1^*) \text{ for all } i,$$

and

$$\sum_i a_{i1}^* = \sum_i w_{i1} ;$$

where $w_{i1} = (\omega_{i1}, 0, m_{i0})$ is the initial endowment of individual i, and $\sum_i m_{i0} = M$. Assumptions made below will apply cumulatively for each individual.

For a preliminary result on the properties of individual demand correspondences, we assume

(3a) for every $\pi_1 \in \Pi$, $\exists \bar{\pi}_2 \in \operatorname{supp} \psi(\pi_1)$ such that $\bar{q}_2 + 1 > q_1$, and

(3b) the correspondence $\operatorname{supp} \psi(\cdot)$ is upper-hemicontinuous on Π.

It is then straightforward to establish

Lemma 3.1. (i) $\xi_1(\cdot)$ *is nonempty-, convex-, compact-valued and upper-hemicontinuous on* Π;

(ii) $\pi_1 \cdot \xi_1(\pi_1) = \pi_1 \cdot w_1$, *for all* $\pi_1 \in \Pi$.

Assumption (3a), requiring a positive probability of a positive net yield on bonds, is used to rule out unbounded borrowing (speculative selling of bonds) for $q_1 > 0$ (cf. assumption 2.6(ii) of [12].) Assumption (3b), together with (2c), implies the continuity of $\operatorname{supp} \psi(\cdot)$ (see [12]) required for the continuity of the feasible choice set.

The proof of existence of a monetary equilibrium will require the boundary conditions that (i) aggregate excess demands for goods and bonds become arbitrarily large as their respective prices approach zero, and (ii) the aggregate excess demand for money becomes arbitrarily large when the prices of goods or bonds become arbitrarily high. To ensure the desired behavior as prices go to zero, it is sufficient to assume

(3c) there exists $\sigma \gg 0$ such that $\operatorname{supp} \psi(\cdot)$ is uniformly bounded below by σ, and

(3d) for any sequence $\langle \pi_1{}^n \rangle$ in Π, if for any k the corresponding sequence $\langle p_{1k}^n \rangle$ (respectively, $\langle q_1{}^n \rangle$) is bounded, then $\langle p_{2k}^n \rangle$ (respectively, $\langle q_2{}^n \rangle$) is bounded for any $\langle \pi_2{}^n \rangle$ with $\pi_2{}^n \in \operatorname{supp} \psi(\pi_1{}^n)$ for all n.

Assumption (3d) means that expectations about the future price of any particular commodity (including bonds) are influenced primarily by the current price of the commodity. Together with (3c), (3d) implies that, for any convergent sequence of current price vectors, any sequence of future price vectors considered possible will be contained in a compact subset of Π. (Note: (3d) would be unnecessary if we had assumed some variant of uniform tightness of ψ, as in [6, 8, 24].) Formally, we have

LEMMA 3.2. *Let $\langle \pi_1{}^n \rangle$ be any sequence in Π, and $\langle a_1{}^n \rangle$ any sequence with $a_1{}^n \in \xi_1(\pi_1{}^n)$ for all n. If $\pi_1{}^n \to \pi_1{}^0 \in \bar{\Pi} \backslash \Pi$ then $\| a_1{}^n \| \to +\infty$.*

The boundary condition (i) above then follows from aggregation of individual demand behavior.

It is property (ii) that is most directly relevant to the issues under investigation and it is here that the optimality conditions (2.6)–(2.8) are utilized. If the prices of goods or bonds become arbitrarily high then, ultimately, if these conditions (appropriately specified) are not satisfied an individual will want to take an action that would generate the necessary downward pressure on prices. We establish first that, under certain conditions, when the price level of consumption goods becomes arbitrarily high, (a) the demand for money or bonds becomes arbitrarily large (regardless of the movement of the interest rate); and (b) provided that the bond price is bounded away from zero, the demand for money alone becomes arbitrarily large. The sufficient condition for this result is motivated by (2.6) and (2.7) and imposes a restriction on expectations:

(3e) for any sequence $\langle \pi_1{}^n \rangle$ in Π with $p_{1k}^n \to +\infty$ for all k, there is some commodity k for which *either*

$$\text{(i)} \quad \frac{u_k'(\omega)}{u_{N+k}'(\omega)} < \liminf_{n\to\infty} \int_\Pi \frac{p_{1k}^n}{p_{2k}} \, d\psi(\pi_1{}^n, \pi_2),$$

or

$$\text{(ii)} \quad \frac{u_k'(\omega)}{u_{N+k}'(\omega)} > \limsup_{n\to\infty} \int_\Pi \frac{p_{1k}^n}{p_{2k}} \frac{q_2 + 1}{q_1{}^n} \, d\psi(\pi_1{}^n, \pi_2).$$

PROPOSITION 3.3. *Let $\langle \pi_1{}^n \rangle$ be any sequence in Π and $\langle a_1{}^n \rangle$ any sequence with $a_1{}^n \in \xi_1(\pi_1{}^n)$ for all n.*

(a) *If $\| p_1{}^n \| \to +\infty$ then either $m_1{}^n$ is unbounded above or $b_1{}^n$ is unbounded above.*

(b) *If $\| p_1{}^n \| \to +\infty$ and $q_1{}^n$ is bounded away from zero then $m_1{}^n \to +\infty$.*

The proof of this proposition is essentially an arbitrage argument using (3e), exploiting the fact that goods are illiquid and payments must be made in cash: any sale of a good at an arbitrarily high price implies an arbitrarily large (income) demand for money. The argument does not depend on uncertainty about interest rates and the possibility of a negative bond yield; the result would hold just as well if bonds had a sure positive return.

The conclusions of Proposition 3.3 will hold, *a fortiori*, when the bond price also becomes arbitrarily high. But it is not yet established that either $\| a_1 \|$ or, in particular, m_1 becomes arbitrarily large when $q_1 \to +\infty$ but p_1 is bounded. For this, the similarity with Proposition 3.3 suggests an assumption based on the optimality condition (2.8). But for bounded sequences $\langle p_1{}^n \rangle$ (and hence all associated $\langle p_2{}^n \rangle$), there is no interpretation independent of the particular sequence. This condition must be strengthened in several ways in order to guarantee that m_1 is unbounded above. Define, for any sequence $\langle \pi_1{}^n \rangle$ in Π and any $\tau > 0$,

$$B^n = \{\pi_2 \in \Pi \mid q_2 + 1 \leqslant q_1{}^n\};$$

$$B_\tau{}^n = \{\pi_2 \in \Pi \mid q_2 + 1 \leqslant (1 - \tau)\, q_1{}^n\};$$

and

$$D^n = \Pi \backslash B^n = \{\pi_2 \in \Pi \mid q_2 + 1 > q_1{}^n\}.$$

It will be assumed first that, as the bond price gets arbitrarily high (i.e., as the interest rate approaches zero), the individual becomes subjectively certain that there will be a fall in the capital value of a bond at least equal to the nominal interest payment. It will be further assumed that, when $q_1{}^n$ is arbitrarily large, there is a positive subjective probability that there will be a negative rate of return whose magnitude, $1 - (q_2 + 1)/q_1{}^n$, is greater than some arbitrarily small positive number. Formally,

(3f)(i) for any sequence $\langle \pi_1{}^n \rangle$ in Π with $q_1{}^n \to +\infty$

(a) $\lim\limits_{n \to \infty} \psi(\pi_1{}^n, B^n) = 1;$

(b) $\exists \tau, \alpha > 0$ such that $\psi(\pi_1{}^n, B_\tau{}^n) \geqslant \alpha$ for n sufficiently large.

It will also be required that

(3f)(ii) for any sequence $\langle \pi_1{}^n \rangle$ in Π and any sequence $\langle \pi_2{}^n \rangle$ such that $\pi_2{}^n \in \operatorname{supp} \psi(\pi_1{}^n)$ for all n,

$$\frac{q_2{}^n}{q_1{}^n} \text{ is uniformly bounded above.}$$

The conditions (3f) rule out inflationary bond price expectations. On the other hand, they are implied by, but do not imply, inelastic expectations.

PROPOSITION 3.4. *Let $\langle \pi_1{}^n \rangle$ be any sequence in Π with $q_1{}^n \to +\infty$, and $\langle a_1{}^n \rangle$ any sequence with $a_1{}^n \in \xi_1(\pi_1{}^n)$ for all n. Then $m_1{}^n \to +\infty$.*

Thus, an individual with expectations consistent with (3f) becomes a speculative seller of bonds as the interest rate approaches zero. In this situation the demand for money, by the individual and thus in the aggregate, is unbounded, i.e., there is a "liquidity trap" at a zero rate of interest. Assumption (3f) could have been strengthened to apply to sequences $\langle \pi_1{}^n \rangle$ with $q_1{}^n \to \bar{q}_1 < +\infty$ (i.e., $r_1{}^n \to \bar{r}_1 > 0$) and Proposition 3.4 restated to apply to such sequences. There would then be a liquidity trap at some strictly positive rate of interest, $\bar{r}_1$ (cf. Younès [24]). The preceding results concerning individual behavior can be aggregated without further assumptions (see Properties of the aggregate excess demand correspondence and Lemma A.1 in the Appendix). In particular, it is not necessary to assume the existence of a region of common interest rate expectations [12, pp. 1114–1117].

We can now assert the existence of an equilibrium in which money has a positive exchange value and the interest rate is positive.

THEOREM 3.6. *There exists a temporary monetary equilibrium in period* 1.

In an economy in which the interest rate is a target of central bank policy with money supply variable, rather than being endogenously determined with a fixed money supply, the above analysis could be applied to show that achieving a rate arbitrarily close to zero would require an arbitrarily large money stock. This is the implication found by Grandmont and Laroque [9]. The result here suggests that the class of economies for which this applies is broader than that described in [9]. With the stronger form of (3f) remarked on above, leading to unbounded speculation in the limit as the interest rate falls to some $\bar{r}_1 > 0$, a temporary equilibrium would exist with $r_1 > \bar{r}_1$. Correspondingly, a monetary authority could not depress the interest rate to $\bar{r}_1$ or below with a finite stock of money.

4. CONCLUSION

The model analyzed above presented a theory of the demand for money within a general framework of intertemporal choice. It was shown that money's institutional role as the medium of exchange is sufficient to assure its positive value in terms of consumption goods, provided that an intertemporal transfer of wealth is desirable. This is in spite of the existence of an alternative financial asset, even if that asset bears a surely positive rate of return. (A speculative demand will further reinforce money's positive value.) This conclusion stems from the way in which illiquidity of consumer goods leads inevitably to an income demand for cash. The latter is a rather technical demand for money but might be expected to be present even with more complete characterizations of the trading process. A consideration of the nature of the central argument (in Proposition 3.3) indicates that the *ad hoc* expenditure constraint (2.1) could be replaced by a more general technological requirement of transactions without altering the conclusion. However, while it has been taken as a fact that money is *the* medium of exchange, this situation is unlikely to persist over time unless there are transaction costs or there is a possibility of negative yields for other paper assets.

Justifying a lower bound to the rate of interest was seen to be an essentially separate issue (Proposition 3.4) because of an asymptotic Keynesian dichotomy between those forces determining the desirability of an intertemporal transfer of purchasing power (the consumption/saving decision) and those which determine the form in which this purchasing power is held (the portfolio decision) (cf. [24]). A motive for speculation was shown to be a more primitive explanation for a positive rate of interest than inelasticity of interest-rate expectations or any direct postulate of a liquidity trap.

Finally, it has been observed that the existence of a monetary equilibrium in Patinkin's [19] simpler model (without bonds) can be guaranteed only by giving up the unit-elasticity assumption on expectations which is vital for Patinkin's comparative static analysis (see [6, 7]), or by adding more structure (as in [17]). The first alternative clearly still applies to Patinkin's model with bonds. The result of the present paper shows that the second alternative is also available: the unit-elasticity assumption can be retained if formal account is taken of money's role in the exchange process.

Appendix

Properties of the Individual's Choice Correspondence

The feasible choice set for an individual in period 1, corresponding to any given $\pi_1 \in \Pi$, is

$$A_1(\pi_1) = \{a_1 = (x_1, b_1, m_1) \in R^{N+2} \mid (2.1), (2.2') \text{ and } (2.3)\}.$$

It is straightforward to show that, for all $\pi_1 \in \Pi$,

(i) $A_1(\pi_1)$ is nonempty and convex; and

(ii) $A_1(\pi_1)$ is compact if and only if (3a).

Further, given the continuity of the correspondence supp $\psi(\cdot)$ (assumption (3b)),

(iii) the correspondence $A_1(\cdot)$ is continuous on Π.

Consider any sequence $\langle \pi_1{}^n \rangle$ in Π and any sequence $\langle a_1{}^n \rangle$ with

$a_1{}^n \in A_1(\pi_1{}^n)$. Together, (2.3) and (3a) imply that $q_1{}^n b_1{}^n + m_1{}^n \geqslant 0$ for all n. Consequently,

(iv) $q_1{}^n b_1{}^n \to -\infty$ implies $m_1{}^n \to +\infty$.

It also follows from (2.3) that $b_1{}^n$ is bounded below if $m_1{}^n$ is bounded above; so

(v) $b_1{}^n \to -\infty$ implies $m_1{}^n \to +\infty$.

Properties of the Individual's Demand Correspondence

For any given $a_1 \in A_1(\pi_1)$ and $\pi_2 \in \Pi$, let $x_2(a_1, \pi_2)$ denote any $x_2 \in R_+^N$ which maximizes $u(x_1, x_2)$ subject to (2.4) and (2.5). The expected utility index, $v(a_1, \pi_1) := \int_\Pi u(x_1, x_2(a_1, \pi_2))\, d\psi(\pi_1, \pi_2)$, is continuous in both arguments, concave and monotone in the first (see [6]). It then follows, using the above properties of $A_1(\cdot)$, that the individual's demand correspondence $\xi_1(\cdot)$, defined by

$$\xi_1(\pi_1) := \{a_1 \in A_1(\pi_1) |\ v(a_1, \pi_1) \geqslant v(a_1', \pi_1) \text{ for all } a_1' \in A_1(\pi_1)\},$$

is nonempty-, convex-, compact-valued and upperhemicontinuous on Π. From the budget equation (2.2′), $\xi_1(\cdot)$ must satisfy $\pi_1 \cdot \xi_1(\pi_1) = \pi_1 \cdot w_1$, for all $\pi_1 \in \Pi$. Lemma 3.1 is thus established.

Proof of Lemma 3.2 (cf. [6]). In view of (3c) and (3d) we can assume that there exists some compact subset of Π which contains supp $\psi(\pi_1{}^n)$ for all n, and therefore that $\psi(\pi_1{}^n)$ converges weakly to some ψ^0. Then we can define $v^0(a_1) = \int_\Pi u(x_1, x_2(a_1, \pi_2))\, d\psi^0(\pi_2)$; $\lim_{n\to\infty} v(a_1, \pi_1{}^n) = v^0(a_1)$ and $v^0(\cdot)$ is continuous and strictly increasing. We consider two cases. (i) If $p_{k1}^0 = 0$ for some k we can apply the reasoning of [6, Proposition 4.2 of Section IV]. (ii) Suppose $q_1{}^0 = 0$. Assume that the sequence $\langle a_1{}^n \rangle$ converges to $a_1{}^0 = (x_1{}^0, b_1{}^0, m_1{}^0)$ with $b_1{}^0 < +\infty$; $a_1{}^0$ is feasible for $\pi_1{}^0$ and $v^0(a_1{}^0) \geqslant v^0(a_1)$ for all a_1 feasible for $\pi_1{}^0$. But $\bar{a}_1 = (x_1{}^0, \bar{b}_1, m_1{}^0)$, with $\bar{b}_1$ arbitrarily large, is also feasible for $\pi_1{}^0$. Since $v^0(\cdot)$ is strictly increasing, $v^0(\bar{a}_1) > v^0(a_1{}^0)$, a contradiction. Q.E.D.

Proof of Proposition 3.3. There are three possible situations to consider when $\| p_1{}^n \| \to +\infty$: (i) $p_{1k}^n \to +\infty$ for some k and $p_{1k'}^n$ bounded above for some k'; in which case, because of (3d), $p_{2k'}^n$ will be bounded for any $\langle \pi_2{}^n \rangle$ with $\pi_2{}^n \in$ supp $\psi(\pi_1{}^n)$ for all n; (ii) $p_{1k}^n \to +\infty$ for all k and $p_{2k}^n \to +\infty$ for all k, for all $\langle \pi_2{}^n \rangle$ with $\pi_2{}^n \in$ supp $\psi(\pi_1{}^n)$; (iii) $p_{1k}^n \to +\infty$ for all k but there are sequences $\langle \pi_2{}^n \rangle$, with $\pi_2{}^n \in$ supp $\psi(\pi_1{}^n)$, such that p_{2k}^n is bounded for some k. In case (i), a diminishing, ultimately negligible, amount of k in period 1 can be traded (via m_1) for a given finite amount of k' in period 2. The arbitrage argument of [17, Lemma 3.2] can be applied to establish that $m_1{}^n \to +\infty$. Case (ii) is the critical one and uses the condition (3e). If (a) of the proposition is false, then $(b_1{}^n, m_1{}^n)$ is bounded above. From property (v) of $A_1(\cdot)$, $b_1{}^n$ must therefore be bounded below. Since $m_1{}^n$ is also bounded below, one can assume $(b_1{}^n, m_1{}^n) \to (b_1{}^0, m_1{}^0)$. Since $p_{1k}^n \to +\infty$ for all k, it then follows, using (2.1), that $x_1{}^n \to \omega_1$. Likewise, $x_2{}^n \to \omega_2$ for any $x_2{}^n = x_2(a_1{}^n, \pi_2)$ with $\pi_2 \in \operatorname{supp} \psi(\pi_1{}^n)$ for all n. Suppose (i) of (3e) holds. Define $\bar{x}_1{}^n = x_1{}^n - (1/p_{1k}^n)\, e_k$ (where e_k is the k-th unit vector), $\bar{b}_1{}^n = b_1{}^n$, $\bar{m}_1{}^n = m_1{}^n + 1$, and $\bar{x}_2{}^n = x_2{}^n + (1/p_{2k})\, e_k$ for all $(x_2{}^n, \pi_2)$ such that $\pi_2 \in \operatorname{supp} \psi(\pi_1{}^n)$ and $x_2{}^n = x_2(a_1{}^n, \pi_2)$. For large n, $(\bar{x}_1{}^n, \bar{x}_2{}^n)$ is feasible, and

$$\begin{aligned} &v(\bar{a}_1{}^n, \pi_1{}^n) - v(a_1{}^n, \pi_1{}^n) \\ &\quad = \int_\Pi \left[u\left(x_1{}^n - \frac{1}{p_{1k}^n}\, e_k\,,\, x_2{}^n + \frac{1}{p_{2k}}\, e_k\right) - u(x_1{}^n, x_2{}^n)\right] d\psi \\ &\quad \cong \int_\Pi \left[-\frac{1}{p_{1k}^n}\, u_k'(\omega) + \frac{1}{p_{2k}}\, u_{N+k}'(\omega)\right] d\psi > 0, \end{aligned}$$

by (i), contradicting $a_1{}^n \in \xi_1(\pi_1{}^n)$. Similarly, suppose (ii) of (3e) holds and define

$$\bar{x}_1{}^n = x_1{}^n + \frac{1}{p_{1k}^n}\, e_k\,,\ \bar{m}_1{}^n = m_1{}^n,\ \bar{b}_1{}^n = b_1{}^n - \frac{1}{q_1{}^n}\,,\ \bar{x}_2{}^n = x_2{}^n - \frac{1}{q_1{}^n}\,\frac{q_2 + 1}{p_{2k}}\, e_k\,.$$

For large n,

$$v(\bar{a}_1{}^n, \pi_1{}^n) - v(a_1{}^n, \pi_1{}^n) \cong \int_\Pi \left[\frac{1}{p_{1k}^n}\, u_k'(\omega) - \frac{q_2 + 1}{q_1{}^n p_{2k}}\, u_{N+k}'(\omega)\right] d\psi > 0,$$

contradicting $a_1{}^n \in \xi_1(\pi_1{}^n)$.

Part (b) of the proposition follows immediately from (a), with the observation that (2.1) implies $q_1{}^n b_1{}^n \leqslant m_0$.

The result for case (iii) follows as a combination of (i) and (ii). Q.E.D.

Proof of Proposition 3.4. Given (b) of Proposition 3.3 it is necessary to consider only those $\langle \pi_1{}^n \rangle$ with $\langle p_1{}^n \rangle$ bounded. Suppose the proposition false, i.e., $m_1{}^n \nrightarrow +\infty$. Then there is a subsequence $\langle a_1{}^n \rangle = \langle x_1{}^n, b_1{}^n, m_1{}^n \rangle$ with $\langle m_1{}^n \rangle$ convergent and hence $\langle b_1{}^n \rangle$ convergent to 0.

For any $\langle \pi_2{}^n \rangle$ with $\pi_2{}^n \in \operatorname{supp} \psi(\pi_1{}^n)$, $(q_2{}^n + 1)\, b_1{}^n$ is bounded above, since $q_1{}^n b_1{}^n$ is bounded above (by m_0) and $q_2{}^n / q_1{}^n$ is bounded above (3f(ii)). Also $p_2{}^n$ is bounded below by $\sigma \gg 0$ ((3c)); thus any feasible $\langle x_2{}^n \rangle$ is bounded above. Assumption (2b)(ii) then implies that there is a k such that $u_{N+k}'(x_1{}^n, x_2{}^n)$ is bounded above 0 for any feasible $\langle x_2{}^n \rangle$, any $\langle x_1{}^n \rangle$.

Define an alternative subsequence $\langle \bar{a}_1{}^n \rangle$ by $\bar{x}_1{}^n = x_1{}^n$, $\bar{b}_1{}^n = b_1{}^n - \delta / q_1{}^n$, $\bar{m}_1{}^n = m_1{}^n + \delta$, where $\delta > 0$ is arbitrarily small. For each $x_2{}^n = x_2(a_1{}^n, \pi_2)$, define

$$\bar{x}_2{}^n = x_2{}^n + \frac{q_1{}^n - (q_2 + 1)}{q_1{}^n}\, \delta e_k\,.$$

Since $x_2{}^n \geqslant \omega_2 \gg 0$ for all n and $q_2/q_1{}^n$ is bounded above, $\bar{x}_2{}^n$ is feasible for δ sufficiently small. Then

$$v(\bar{a}_1{}^n, \pi_1{}^n) - v(a_1{}^n, \pi_1{}^n) = \int_{\Pi} [(u(x_1{}^n, \bar{x}_2{}^n) - u(x_1{}^n, x_2{}^n)]\, d\psi(\pi_1{}^n, \pi_2)$$

and, with δ sufficiently small,

$$u(x_1{}^n, \bar{x}_2{}^n) - u(x_1{}^n, x_2{}^n) \cong \frac{q_1{}^n - (q_2 + 1)}{q_1{}^n p_{2k}}\, \delta u'_{N+k}(x_1{}^n, x_2{}^n).$$

$$\therefore v(\bar{a}_1{}^n, \pi_1{}^n) - v(a_1{}^n, \pi_1{}^n) \cong \int_{B^n} \frac{q_1{}^n - (q_2 + 1)}{q_1{}^n p_{2k}}\, \delta u'_{N+k}(x_1{}^n, x_2{}^n)\, d\psi(\pi_1{}^n, \pi_2)$$

$$- \int_{D^n} \frac{(q_2 + 1) - q_1{}^n}{q_1{}^n p_{2k}}\, \delta u'_{N+k}(x_1{}^n, x_2{}^n)\, d\psi(\pi_1{}^n, \pi_2).$$

Denote these two integrals by A^n and E^n, respectively. So $v(\bar{a}_1{}^n, \pi_1{}^n) - v(a_1{}^n, \pi_1{}^n) \cong A^n - E^n$.

Take $\tau > 0$ satisfying (3f)(i). Then

$$A^n \geqslant \int_{B_\tau{}^n} \frac{q_1{}^n - (q_2 + 1)}{q_1{}^n p_{2k}}\, \delta u'_{N+k}(x_1{}^n, x_2{}^n)\, d\psi(\pi_1{}^n, \pi_2).$$

From (3d), $\exists \bar{p}_{2k} > 0$ such that $p_{2k} \leqslant \bar{p}_{2k}$, for all $\pi_2 \in \operatorname{supp} \psi(\pi_1{}^n)$, all n. From (2b)(ii), $\exists \bar{\rho}_k > 0$ such that $u'_{N+k}(x_1{}^n, x_2{}^n) \geqslant \bar{\rho}_k$.

So $A^n \geqslant (\tau\delta\bar{\rho}_k/\bar{p}_{2k})\, \psi(\pi_1{}^n, B_\tau{}^n)$ and therefore, from (3f)(i)(b), $\exists \gamma > 0$ such that $A^n \geqslant \gamma$ for n sufficiently large. Consider E^n. From (3f)(ii), $\exists \beta$ such that $(q_2 + 1) - q_1{}^n \leqslant \beta q_1{}^n$. From (2b)(ii), $\exists \bar{\bar{\rho}}_k$ such that $u'_{N+k}(x_1{}^n, x_2{}^n) \leqslant \bar{\bar{\rho}}_k$. From (3c), $p_{2k} \geqslant \sigma_k > 0$, for all $\pi_2 \in \operatorname{supp} \psi(\pi_1{}^n)$, all n.

$$\therefore\ E^n \leqslant \frac{\beta\delta\bar{\bar{\rho}}_k}{\sigma_k}\, \psi(\pi_1{}^n, D^n) \to 0 \text{ as } n \to \infty,$$

using (3f)(i)(a). Thus $A^n > E^n$ for n sufficiently large, contradicting the optimality of $a_1{}^n$. Q.E.D.

Properties of the Aggregate Excess Demand Correspondence

The economy's aggregate excess demand correspondence, $\zeta_1 \colon \Pi \to R^{N+2}$, is defined by $\zeta_1(\pi_1) = \sum_i \xi_{i1}(\pi_1) - \sum_i w_{i1}$. An element of $\zeta_1(\pi_1)$ is $z_1 = (x_1 - \omega_1, b_1, m_1 - M)$, where $x_1 = \sum_i x_{i1}$, $\omega_1 = \sum_i \omega_{i1}$, $b_1 = \sum_i b_{i1}$ and $m_1 = \sum_i m_{i1}$. It follows from Lemma 3.1 that (i) $\zeta_1(\cdot)$ is nonempty-, convex-, compact-valued and upper hemicontinuous on Π; and (ii) $\pi_1 \cdot \zeta_1(\pi_1) = 0$, for $\pi_1 \in \Pi$. From Lemma 3.2 and Propositions 3.3 and 3.4 we have further: for any sequence $\langle \pi_1{}^n \rangle$ in Π, (iii) if either $\pi_1{}^n \to \pi_1{}^0 \in \bar{\Pi} \backslash \Pi$ or $\| \pi_1{}^n \| \to +\infty$, then $\| \zeta(\pi_1{}^n) \| \to +\infty$. [If $\| x_{i1}^n \| \to +\infty$ or $m_{i1}^n \to +\infty$ for some i then, correspondingly, $\| x_1{}^n \| \to +\infty$ or $m_1{}^n \to +\infty$. If $b_{i1}^n \to +\infty$ for some i and there is no j for which $b_{j1}^n \to -\infty$, then $b_1{}^n \to +\infty$. If $b_{j1}^n \to -\infty$ for some j then $m_{j1}^n \to +\infty$ (Property (v) of $A_1(\cdot)$) and hence $m_1{}^n \to +\infty$.]

LEMMA A.1. *Let $\langle \Pi^n \rangle$ be a non-decreasing sequence of compact, convex subsets of Π such that $\Pi \subset \bigcup_{n=1}^{\infty} \Pi^n$ and each Π^n has a nonempty interior. Let $\langle \pi_1{}^n \rangle$ and $\langle z_1{}^n \rangle$ be sequences such that, for each n, $\pi_1{}^n \in \Pi^n$, $z_1{}^n \in \zeta_1(\pi_1{}^n)$, $\pi_1{}^n \cdot z_1{}^n = 0$, and $\pi_1 \cdot z_1{}^n \leqslant 0$ for all $\pi_1 \in \Pi^n$. Then $\langle z_1{}^n \rangle$ is bounded.*

Proof. (a) $\langle b_1{}^n \rangle$ is bounded. With (3a), $b_1{}^n$ is bounded on any compact subset of Π. For any sequence $\langle \pi_1{}^n \rangle$, (3a) (for each trader) implies the existence of a sequence $\langle \bar{q}_2{}^n \rangle$ such that, for all n, $\bar{q}_2{}^n + 1 > q_1{}^n$ and $(b_1{}^n, m_1{}^n)$ must satisfy (because of (2.3) for each trader) $(\bar{q}_2{}^n + 1)\, b_1{}^n + m_1{}^n \geqslant 0$ (1). Take $\bar{\pi}_1 = (\bar{p}_1, \bar{q}_1, 1)$ such that $\bar{\pi}_1 \in \Pi^n$ for n sufficiently large, in which case $\bar{\pi}_1 \cdot z_1{}^n \leqslant 0$ and thus $\bar{q}_1 b_1{}^n + m_1{}^n \leqslant \bar{p}_1 \cdot \omega_1 + M$ (2). Since $m_1{}^n \geqslant 0$ for all n, (2) implies $b_1{}^n \leqslant (1/\bar{q}_1)(\bar{p}_1 \cdot \omega_1 + M)$ for all n sufficiently large $\therefore$ $\langle b_1{}^n \rangle$ is bounded above for any $\langle \pi_1{}^n \rangle$. (1)–(2) implies $[(\bar{q}_2{}^n + 1) - \bar{q}_1]\, b_1{}^n \geqslant -(\bar{p}_1 \cdot \omega_1 + M)$. Choose $\bar{\pi}_1$ such that $\bar{q}_1 < 1$ and hence $(\bar{q}_2{}^n + 1) - \bar{q}_1 > 1 - \bar{q}_1 > 0$. Then

$$b_1{}^n \geqslant -\frac{\bar{p}_1 \cdot \omega_1 + M}{1 - \bar{q}_1}$$

for n sufficiently large. $\therefore$ $\langle b_1{}^n \rangle$ is bounded below for any $\langle \pi_1{}^n \rangle$. So $\langle b_1{}^n \rangle$ is bounded.

(b) $\langle m_1{}^n \rangle$ is bounded. $\langle m_1{}^n \rangle$ is bounded below by 0. Take $\bar{\bar{\pi}}_1 = (\bar{\bar{p}}_1, \bar{\bar{q}}_1, 1) \in \Pi^1$; so (3) $\bar{\bar{p}}_1 \cdot x_1{}^n + \bar{\bar{q}}_1 b_1{}^n + m_1{}^n \leqslant \bar{\bar{p}}_1 \cdot \omega_1 + M$ for all n and $\therefore$ $m_1{}^n \leqslant \bar{\bar{p}}_1 \cdot \omega_1 + M - \bar{\bar{q}}_1 b_1{}^n$ for all n. Since $b_1{}^n$ is bounded below, $m_1{}^n$ is bounded above. So $\langle m_1{}^n \rangle$ is bounded.

(c) $\langle x_1{}^n \rangle$ is bounded below by 0 and bounded above, from (3), since $\bar{\bar{p}}_1 \gg 0$ and $b_1{}^n$ and $m_1{}^n$ are both bounded. Q.E.D.

Proof of Theorem 3.6. Let $\langle \Pi^n \rangle$ be a sequence of subsets of Π as in the statement of Lemma A.1. For each n, take Z^n to be a compact, convex set containing the range of $\zeta_1{}^n$. Then, using the result of [3], there exists for each n a pair $(\pi_1{}^n, z_1{}^n)$ such that $z_1{}^n \in \zeta_1{}^n(\pi_1{}^n)$, $\pi_1{}^n \cdot z_1{}^n = 0$ and $\pi_1 \cdot z_1{}^n \leqslant 0$ for all $\pi_1 \in \pi^n$. By Lemma A.1, the sequence $\langle \pi_1{}^n \rangle$ is bounded. The sequence $\langle \pi_1{}^n \rangle$ must therefore be bounded, otherwise Property (iii) of ζ_1 would be contradicted. So there exists a subsequence of $\langle \pi_1{}^n \rangle$ converging to π_1^*. Since the corresponding subsequence of $\langle z_1{}^n \rangle$ is bounded, Property (iii) of ζ_1 implies $\pi_1^* \notin \bar{\Pi} \backslash \Pi$. Therefore $\pi_1^* \in \Pi$. Since ζ_1 has closed graph and $\langle z_1{}^n \rangle$ is bounded, there is a subsequence of $\langle z_1{}^n \rangle$ converging to $z_1^* \in \zeta_1(\pi_1^*)$; $\pi_1^* \cdot z_1^* = 0$ since $\pi_1{}^n \cdot z_1{}^n = 0$ for all n; $\pi_1^* \cdot z_1^* = 0$ prohibits both $z_1^* > 0$ and $z_1^* < 0$. So if $z_1^* \neq 0$ and $\pi_1^* \cdot z_1^* = 0$, there exists $\pi_1 \in \Pi$ such that

$\pi_1 \cdot z_1^* > 0$ and hence, for n sufficiently large, $\pi_1 \cdot z_1{}^n > 0$. But this contradicts $\pi_1 \cdot z_1{}^n \leqslant 0$ for all $\pi_1 \in \Pi^n$. Therefore $z_1^* = 0$. Q.E.D.

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