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Intertemporal Extraction of Mineral Resources under Variable Rate Taxes

Robert F. Conrad and R. Bryce Hool

1. INTRODUCTION

In this paper we examine the effects of variable-rate taxes on intertemporal extraction patterns and recovery from mineral deposits. Such taxes are being applied with increasing frequency in the United States and other countries. However, as we demonstrate in the following analysis, their effects typically go beyond those that have motivated their use. The formulation of mineral tax policy should take into account the complete list of consequences.

In practice there are currently three basic types of variable-rate taxes: time-dependent output taxes, price-dependent ad valorem taxes and progressive profits taxes. Per-unit output taxes which vary over time are applied in North Dakota and some of the Canadian provinces. Ad valorem taxes which vary with the market price of the mineral are used in Bolivia, Indonesia, and in parts of the United States such as Montana. Progressive income-related taxes have been introduced in Indonesia and Ecuador.[1] These variable taxes have been motivated by several objectives: (1) to acquire for the government a share of the windfall gains that are claimed to accrue to natural resource producers as a result of changes in the economic environment (prices or costs); (2) to capture the "economic" rent that would normally accrue to the owner of the reserves in a market economy;[2] and (3) to preserve the real value of tax revenues. While there is some theoretical justification for expecting the taxes to achieve these ends,[3] there are also incentives created for altering the pattern of exploitation of the taxed resource.

Most minerals are traded on unified world markets, whereas taxes are imposed by "local" governments. So a tax imposed in a particular locality will change the spatial distribution of relative prices of extraction and production, as well as change the economic environment within that locality.[4] Furthermore, characteristics of mineral deposits (such as ore quality, quantity, depth, accessibility) vary within and across deposits.[5] Taxation will therefore affect the magnitude of economically recoverable reserves, as opposed to geological reserves. Accordingly, the model that forms the basis of our analysis is of a price-taking firm engaged in extraction from a geologically heterogeneous deposit.

The model and some of its general properties are described in section 2. In section 3 we demonstrate the consequences of each of the three types of variable-rate tax noted above. Section 4 contains a brief summary and discussion of related issues.

2. BASIC MODEL AND PROPERTIES

In common with most models of the extractive process, the optimal extraction path is taken here to be the solution to the dynamic allocation problem of maximizing the present value of the net cash flow generated by extraction of the mineral resources.[6] The distinctive

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[1]See Gillis (1978), Gillis and Beals (1980), Conrad and Hool (1980) and Stinson (1977) for details of these taxes.

[2]See Hotelling (1931).

[3]See Peterson (1976) and Burness (1976).

[4]See McClure (1978).

[5]See Thomas (1976) for detailed discussion.

[6]See Peterson and Fisher (1977) for a review of this literature.

feature of the model is the treatment of quality variation. With the mineral deposit heterogeneous in quality, the optimal intertemporal extraction profile specifies the quantity of each grade of ore to be extracted in each period. A brief description of the model follows.[7]

Grades of ore are indexed by g ($g = 1, \ldots, G$) in order of decreasing quality. If α_g is the proportion of metal in ore of grade g and X_{tg} is the quantity of ore of grade g extracted in period t, then the total output of metal in period t is

$$\sum_{g=1}^{G} \alpha_g X_{tg} \qquad (t = 1, \ldots, T)$$

where T is the finite but arbitrary planning horizon. The cost of extraction and processing in period t is taken to be a function of the aggregate throughput of ore, independent of quality; i.e.,

$$C_t = C_t(X_t) \qquad t = 1, \ldots, T$$

with $C_t' > 0$ and $C_t'' > 0$, where

$$X_t \equiv \sum_{g=1}^{G} X_{tg}.$$

Profit in period t is then

$$\Pi_t = P_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t\left(\sum_{g=1}^{G} X_{tg} \right) \qquad t = 1, \ldots, T \tag{1}$$

where P_t is the price of output (metal) in period t. Given a discount rate of r, the firm's problem is therefore

$$\underset{\{X_{tg}\}}{\mathrm{Max}}\, \Pi = \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \times \left[P_t \sum_{g=1}^{G} \alpha_g X_{tg} - C_t\left(\sum_{g=1}^{G} X_{tg} \right) \right] \tag{2}$$

subject to the availability of reserves of each grade,

$$R_g \geqq \sum_{t=1}^{T} X_{tg} \qquad g = 1, \ldots, G \tag{3}$$

and non-negativity,

$$X_{tg} \geqq 0 \qquad t = 1, \ldots, T; \quad g = 1, \ldots, G \tag{4}$$

The Lagrangian function for this problem is

$$L = \Pi + \sum_{g=1}^{G} \lambda_g \left(R_g - \sum_{t=1}^{T} X_{tg} \right)$$

where λ_g ($g = 1, \ldots, G$) is the shadow price of reserves of grade g. The optimal solution is characterized by the following Kuhn-Tucker conditions.

For all (t,g):

$$\frac{\partial L}{\partial X_{tg}} = \frac{1}{(1+r)^{t-1}} \left[P_t \alpha_g - C_t'(X_t) \right] - \lambda_g \leqq 0;$$

$$X_{tg} \geqq 0; \qquad X_{tg} \frac{\partial L}{\partial X_{tg}} = 0 \tag{5}$$

and for all g:

$$\frac{\partial L}{\partial \lambda_g} = R_g - \sum_{t=1}^{T} X_{tg} \geqq 0; \quad \lambda_g \geqq 0; \quad \lambda_g \frac{\partial L}{\partial \lambda_g} = 0 \tag{6}$$

Of particular interest for the later analysis is the following characterization of the intertemporal quality profile, implied by [5]. The timing of extraction of different ore grades corresponds to the time profile of discounted prices: extraction of the highest grade will occur (until reserves are exhausted) in periods when the discounted price is highest; lower grades will be allocated sequentially in a like manner until further extraction is unprofitable. More formally, if some of grade g (respectively, g') is extracted in period t (respectively, t') then

$$(\alpha_g - \alpha_{g'}) \left[\frac{P_t}{(1+r)^{t-1}} - \frac{P_{t'}}{(1+r)^{t'-1}} \right] \geqq 0. \tag{7}$$

so

$$\alpha_g > \alpha_{g'} \quad \text{implies} \quad \frac{P_t}{(1+r)^{t-1}} \geqq \frac{P_{t'}}{(1+r)^{t'-1}}$$

and

$$\frac{P_t}{(1+r)^{t-1}} > \frac{P_{t'}}{(1+r)^{t'-1}} \quad \text{implies} \quad \alpha_g \geqq \alpha_{g'}.$$

The ore quality g^* at which extraction ceases to be profitable is referred to as the cutoff grade. Since the intertemporal ordering of extraction by quality does not necessarily correspond to the natural time sequence, this zero-profit margin (where marginal revenue equals marginal cost of ore extraction) may

[7] See Conrad and Hool (1982) for more detailed development and proofs.

be reached at any time and does not generally imply a closing of the mine at that time. If the cut-off occurs in period k ($1 \leq k \leq T$) after some of grade g^* has been extracted, then g^* is characterized by

$$P_k \alpha_{g*} = C_k'(X_k)$$

i.e.,

$$\alpha_{g^*} = \frac{C_k'(X_k)}{P_k} \qquad [8]$$

and

$$\sum_{t=1}^{T} X_{tg*} < R_{g*}.$$

According to the quality profile determination described above, period k has the lowest discounted price of all periods in which there is some extraction. With the discrete quality variation assumed here, it may happen that extraction becomes unprofitable after reserves of grade g^* have been exhausted but before any of grade g^*+1 has been extracted, in which case

$$P_k \alpha_{g*} > C_k'(X_k) > P_k \alpha_{g*+1}.$$

In the subsequent discussion this boundary case can be ignored without affecting the essential implications of taxation for the cut-off grade and the implied total extraction and output from the mine.

3. EFFECTS OF VARIABLE-RATE TAXATION

The introduction of any sort of tax, other than a pure proportional profits tax, will in general alter the optimal extraction profile. The profile change will reflect three basic and interrelated effects. First, it is possible that the profile of effective (i.e., net-of-tax) output prices will be different, in which case the analogue under taxes of condition [7] will imply a different sequence of quality selection. Second, the reduction in effective prices may cause the zero-profit margin to be reached after a smaller quantity of total extraction. The cut-off grade, characterized by the analogue of [8], may be higher as a result. Third, even if the quality ordering and cut-off grade are unchanged, the change in the relationship between prices and costs will affect the optimal quantity of extraction in each period and thus the entire intertemporal profile.

The precise effects of a discrete or marginal change in a tax rate follow from the tax analogues of conditions [5] through [8]. For the marginal effects we can simplify the analysis, while bringing out clearly the allocative incentives, by determining the quantity responses explicitly using a two-period, two-grade version of the model. In particular, we shall suppose that the higher grade of ore (grade 1) is extracted in both periods until reserves are exhausted and that grade 2 is extracted in the second period but not exhausted. (In the analysis below, this will be referred to as profile P.) It is implicit then, from the optimal grade selection characterization given above, that the effective output price in period 1 is at least as high as the discounted effective price in period 2. The results for the reverse situation can be easily inferred. We shall also note the differences that would appear if reserves of grade 2 also are exhausted in period 2 (profile P′). Together these cases generate all the nontrivial reallocation incentives.

3.1 *Variable Per-Unit Severance Taxes*

This per-unit tax is set in nominal terms as a dollar amount per unit of output sold. The rate, τ_t in period t, may vary from period to period.[8] With a per-unit tax, the intertemporal extraction problem, [2]–[4], is modified by the substitution of $P_t - \tau_t$ for P_t in [2] and correspondingly in [5], [7], and [8].

The tax results in a general reduction in profitability of extraction. In particular, the cut-off grade, g_τ^*, will tend to be higher than g^*, according to the counterpart of [8],

$$\alpha_{g_\tau^*} = \frac{C_k'}{P_k - \tau_k}$$

The timing of grade selection may also be affected by the introduction of (or discrete change in) this tax. Referring to the ordering condition [7], it will be the case that

[8]For example, in North Dakota the Coal Severance Tax is tied to the Wholesale Price Index. See Link (1978).

$$\frac{P_t}{(1+r)^{t-1}} - \frac{P_t'}{(1+r)^{t'-1}} \gtreqqless 0$$

implies

$$\frac{P_t - \tau_t}{(1+r)^{t-1}} - \frac{P_t - \tau_{t'}}{(1+r)^{t'-1}} \gtreqqless 0$$

provided that

$$\tau_{t'}/\tau_t \gtreqqless (1+r)^{t'-t} \qquad [9]$$

If γ is the (geometric) average growth rate of τ from t to t', [9] can be rewritten as

$$(1+\gamma)^{t'-t} \gtreqqless (1+r)^{t'-t}$$

so that [9] requires

$$\gamma \gtreqqless r \quad \text{if} \quad t' > t \qquad [9']$$

and $\gamma \lesseqqgtr r$ if $t' < t$.

Condition [9′] will be met if, in particular, the tax increases at the rate of interest.[9] But otherwise [9′] will not generally be satisfied and it is consequently plausible that the profile of discounted net-of-tax prices, and hence the grade profile, will differ from the no-tax situation.

In general, the intertemporal extraction profile will be affected by a marginal change in the sequence of tax rates. In a two-period context, the change in taxes can be thought of as being either a change in the base tax (τ_1) with the growth rate (γ) unchanged, or vice versa. In the former case, the relationship between the tax growth rate and the interest rate will appear again as a central factor in the determination of the quantity reallocation. In discounted terms, a given increase in the initial tax rate will lead to a larger or smaller increase in the future tax rate according to whether or not the growth rate exceeds the interest rate. In addition to the general decline in profitability which results from a tax increase, there will be a tendency for intertemporal reallocation to the period with the lower discounted value of the increase. At the margin the tax increase will tend to produce an absolute reduction in quantity extracted. These two effects may or may not be reinforcing.

For a precise illustration of these general effects, consider the initial extraction profile described above. The corresponding necessary conditions yield the quantity responses

$$\frac{\partial X_{11}}{\partial \tau_1} = -\frac{\partial X_{21}}{\partial \tau_1} = \frac{1}{C_1''}\left[\left(\frac{1+\gamma}{1+r} - 1\right)\alpha_1 - \left(\frac{1+\gamma}{1+r}\right)\alpha_2\right] \qquad [10.1]$$

and

$$\frac{\partial X_{22}}{\partial \tau_1} = -\frac{\partial X_{21}}{\partial \tau_1} - \frac{1}{C_2''}(1+\gamma)\alpha_2 \qquad [10.2]$$

Equation [10.2] represents the adjustment at the zero-profit margin; total extraction in period 2 ($X_{21} + X_{22}$) must fall sufficiently that marginal cost is reduced by the amount of the effective tax increase on *ore* of grade 2. Equation [10.1] reveals the intertemporal adjustment. The first term in the square brackets represents the pure reallocative effect (the sole effect in the absence of the second ore quality or in a situation like profile P′) and shows the dependence on the relative magnitudes of Y and r discussed above. The second term is a consequence of the decline in total extraction in period 2 which, ceteris paribus, makes extraction of grade 1 more profitable in period 2 relative to period 1 at the margin, because of the reduction in marginal cost in period 2.

It is apparent from [10.1] and [10.2] that the net effect on the intertemporal profile depends on the grade distribution as well as the tax growth and interest rates. In particular, for profile P,

$$\frac{\partial X_{11}}{\partial \tau_1} \gtreqless 0 \quad \text{as} \quad 1+\gamma \gtreqless \frac{1}{\alpha_1 - \alpha_2}(1+r)$$

The presence of an economic margin with respect to reserves may therefore reverse the qualitative response. Note also that if the tax rate is not variable ($\gamma = 0$) then, from [10.1] and [10.2], some of grade 1 is reallocated to the second period. In this case it is apparent from [10.2] that extraction of grade 2, and hence total recovery in the two periods, will decline. In general, however, when there is a

[9]Severance taxes with this property have been discussed by Peterson (1976) and Burness (1976), both of whom also assume that all ore is extracted.

reallocation of grade 1 from the second period to the first it is quite possible for extraction of grade 2, and therefore total recovery, to increase.

The effects of an increase in the tax growth rate, holding constant the initial tax rate, are more straightforward to determine since the onus of the tax is then unambiguously increased over time. Accordingly, there is a reallocation from future to present. Again there is also a reduction in total second-period extraction to preserve the zero-profit margin. For profile P the quantity responses are

$$\frac{\partial X_{11}}{\partial \gamma} = -\frac{\partial X_{21}}{\partial \gamma} = \frac{1}{C_1''} \cdot \frac{\tau_1}{1+r}(\alpha_1 - \alpha_2) \quad [11.1]$$

and

$$\frac{\partial X_{22}}{\partial \gamma} = -\frac{\partial X_{21}}{\partial \gamma} - \frac{\tau_1 \alpha_2}{C_2''} \quad [11.2]$$

So extraction unambiguously rises in the first period and falls in the second, with the change in total recovery again ambiguous.

It is worth emphasizing that, when either γ or τ_1 increases, extraction of grade 2 does not necessarily decline. The intertemporal reallocation changes the marginal cost structure and, furthermore, a given output tax implies a tax on ore that declines proportionately with the ore quality. The ultimate impact on total recovery therefore depends on both the grade distribution and cost structure. It should be emphasized that this conclusion is strictly applicable only in the short run, since the intertemporal pattern of capital investment and, *a fortiori*, the choice of technology are both taken as given.

3.2 *Variable Ad Valorem Severance Taxes*

An ad valorem severance tax is specified as a percentage of the market value of the output sold. For any given tax rate, the nominal value of the tax on a unit of output thus fluctuates with the market price. But also the rate itself, β_t in period t, may vary with the price.[10] The intertemporal extraction problem and optimality conditions are modified by the replacement of P_t by $(1-\beta_t)P_t$ in [1], [2], [5], [7], and [8].

As with the per-unit severance tax, the cutoff grade, $g\beta$, with an ad valorem tax will tend to be higher than g^*, since

$$\alpha_{g\beta} = \frac{C_k'}{(1-\beta_k)P_k}$$

The order of grade selection may also change. Referring to [7] again,

$$\frac{P_t}{(1+r)^{t-1}} - \frac{P_{t'}}{(1+r)^{t'-1}} \gtreqless 0$$

implies

$$\frac{(1-\beta_t)P_t}{(1+r)^{t-1}} - \frac{(1-\beta_{t'})P_{t'}}{(1+r)^{t'-1}} \gtreqless 0$$

if

$$\frac{\beta_{t'}P_{t'}}{\beta_t P_t} \gtreqless (1+r)^{t'-t} \quad [12]$$

In terms of average rates of growth, if υ and ρ are the respective growth rates for β and P, [12] becomes

$$[(1+\nu)(1+\rho)]^{t'-t} \gtreqless (1+r)^{t'-t}$$

which requires

$$(1+\nu)(1+\rho) \geqq 1+r \quad \text{if} \quad t' > t \quad [12']$$

and

$$(1+\nu)(1+\rho) \leqq 1+r \quad \text{if} \quad t' < t.$$

To a first approximation then, the order will be unaffected if

$$\nu + \rho \geqq r \quad \text{if} \quad t' > t \quad [12'']$$

and

$$\nu + \rho \leqq r \quad \text{if} \quad t' < t$$

Unless the growth rate of the nominal value of the tax per unit of output $(\upsilon + \rho)$ is equal to the rate of interest, condition [12″] will not generally be satisfied and, accordingly, the quality profile may be affected by the intro-

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[10] The New Mexico severance tax on uranium is an example. See Conrad and Hool (1980) for details.

duction of the tax. Stated in this form, the condition is seen to be equivalent to that in the preceding case of a per-unit tax, as would be expected.

This correspondence between the per-unit and ad valorem taxes continues in the incentives for quantity reallocation. With the appropriate compounding of price and tax growth rates, and the inclusion of the price factor, the quantity responses are completely analogous. For profile P, the responses to a change in the initial tax rate, β_1, are

$$\frac{\partial X_{11}}{\partial \beta_1} = -\frac{\partial X_{21}}{\partial \beta_1} = \frac{1}{C_2''} P_1 \left[\left(\frac{(1+\nu)(1+\rho)}{1+r} - 1 \right) \alpha_1 - \frac{(1+\nu)(1+\rho)}{1+r} \alpha_2 \right] \quad [13.1]$$

and

$$\frac{\partial X_{22}}{\partial \beta_1} = -\frac{\partial X_{21}}{\partial \beta_1} - \frac{1}{C_2''} P_1 (1+\nu)(1+\rho)\alpha_2 \quad [13.2]$$

Those for a change in the growth rate, υ, of the tax rate are

$$\frac{\partial X_{11}}{\partial \nu} = -\frac{\partial X_{21}}{\partial \nu} = \frac{1}{C_1''} P_1 \left(\frac{1+\rho}{1+r} \right) \beta_1 (\alpha_1 - \alpha_2)$$

and

$$\frac{\partial X_{22}}{\partial \nu} = -\frac{\partial X_{21}}{\partial \nu} - \frac{1}{C_2''} P_1 (1+\rho) \beta_1 \alpha_2$$

The general conclusions regarding reallocation are thus the same in this case as for the per-unit tax. The adjustments depend on the grade distribution, the cost structure, and the real growth rate of the unit tax. Note that in the particular case where the tax rate is not variable ($\upsilon = 0$), profile P requires $\rho \leqslant r$. It therefore follows from [13.1] and [13.2] that, as with the per-unit tax, grade 1 is extracted more slowly and total extraction declines.

The implicit assumption that discounted prices are falling over time (profile P) is potentially important here. The fundamental distinction between the implications of ad valorem and per unit severance taxes is that a higher percentage tax, as with the ad valorem, translates into a larger absolute decrease in effective price in the period that initially has the higher effective price; whereas a higher nominal tax, as with the per unit, results in an effective decline that diminishes with time.

3.3 *Progressive Profits Taxes*

As noted above, a constant-rate profits tax will not create any incentive to distort the intertemporal extraction profile that is optimal in the absence of taxes. This is no longer the case when the tax rate itself is a function of the level of profits, as with a progressive tax.[11] Further, compared to the taxes already discussed, the effects of the variable profits tax are more difficult to characterize because the tax on a unit of output or ore is not parametric to the firm. The rate is endogenous to the extent that the firm determines its gross profit in any period and, in turn, the intertemporal path of profits does not stand in any simple relationship to the path of output price or any other exogenous variable.

The implications of these differences will be developed explicitly using a quadratic approximation to the general tax function, namely

$$T_t = \eta \Pi_t + \tfrac{1}{2} \mu \Pi_t^2$$

where T_t denotes total taxes paid in period t and Π_t is gross profit in period t, as defined in [1]. The Lagrangian function for the optimization problem is then

$$L = \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \left(1 - \eta - \frac{1}{2} \mu \Pi_t \right) \Pi_t + \sum_{g=1}^{G} \lambda_g \left(R_g - \sum_{t=1}^{T} X_{tg} \right)$$

and accordingly, in the first-order conditions for an optimum,

[11]Examples of progressive income-related taxes are the "windfall" profits tax on mineral extraction in Indonesia (see Gillis and Beals [1980]), the Net Proceeds Tax in Wisconsin (see Strasma [1975]), and the rent tax proposed by Garnaut and Ross (1975).

$$\frac{\partial L}{\partial X_{tg}} = \frac{1}{(1+r)^{t-1}}$$
$$\times\left[(1-\eta-\mu\Pi_t)(P_t\alpha_g - C_t'(X_t))\right] - \lambda_g$$

The zero marginal profit condition determining the cut-off grade is unaffected by a profits tax. However, the same is not true of the condition characterizing the quality profile. With the profits tax, the counterpart of condition [7] is

$$(\alpha_g - \alpha_{g'})\left[\frac{(1-\eta-\mu\Pi_t)P_t}{(1+r)^{t-1}} - \frac{(1-\eta-\mu\Pi_{t'})P_{t'}}{(1+r)^{t'-1}}\right] \geqq 0 \qquad [14]$$

So in this case the rule for allocating grades intertemporally involves profits as well as discounted prices, since the effective price reflects the tax on profit. The rule is therefore no longer a function solely of exogenous variables. How the time path of endogenous profits relates to the time path of prices depends, among other things, on the path of extraction costs. In particular, however, the gross profit may be higher when the discounted price is higher, in which case it is evident from [14] that the introduction of a profits tax may alter the grade selection profile.

The optimal extraction profile will respond to a change in the progressivity of the profits tax, i.e., in the parameter μ. Moreover, there will also be a response to a change in the base tax rate, η, which would not have been the case in the absence of progressivity (i.e., when μ is zero). The difference is due to the fact that, with a nonlinear tax, the firm does not allocate each grade so that the discounted value of its marginal profitability is equated across the time periods in which that grade is extracted. In broad terms, the optimal response to an increase in either of the tax parameters will involve a reallocation away from periods with higher gross profits. However, as shown in more detail below, the reallocation of a higher grade will require some compensating adjustment in extraction of a lower grade. Whether total extraction will increase or decrease will be seen to depend on the path of profits prior to the tax change.

For the representative profile P, the quantity responses to a change in η are given by

$$\frac{\partial X_{11}}{\partial \eta} = -\frac{\partial X_{21}}{\partial \eta} = -\frac{1}{\Delta}(1-\eta-\mu\Pi_2)$$
$$\times C_2''\left[M\Pi_{11} - (1+r)^{-1}M\Pi_{21}\right] \qquad [15]$$

and

$$\frac{\partial X_{22}}{\partial \eta} = -\frac{\partial X_{21}}{\partial \eta}$$

where

$$\Delta \equiv \left[(1-\eta-\mu\Pi_1)C_1'' + \mu(P_1\alpha_1 - C_1')^2\right]$$
$$\times\left[(1-\eta-\mu\Pi_2)C_2'' + \mu(P_2\alpha_2 - C_2')^2\right]$$

is a positive determinant, and $M\Pi_{tg} \equiv P_t\alpha_g - C_t'$ denotes the marginal profitability in period t of ore of grade g.

Note first that total extraction in period 2 ($X_{21} + X_{22}$) will remain the same, a consequence of the zero marginal profit condition for extraction of grade 2, which requires that the marginal cost of extraction in period 2 be unchanged. So whether or not the quantity of grade 2 extracted (and hence, total quantity extracted over the two periods) will be higher or lower as a result, depends on the direction of reallocation of grade 1. From [15] it is seen that this depends on the difference between the discounted marginal profitability of grade 1 in the two periods. From the first-order conditions for grade 1

$$\frac{\partial L}{\partial X_{t1}} = 0$$

for $t = 1,2$) it follows that

$$M\Pi_{11} - (1+r)^{-1}M\Pi_{21}$$
$$= (1+r)^{-1}M\Pi_{21}\left(\frac{1-\eta-\mu\Pi_2}{1-\eta-\mu\Pi_1} - 1\right)$$
$$\gtreqless 0 \quad \text{as} \quad \Pi_1 \gtreqless \Pi_2$$

so

$$\frac{\partial X_{11}}{\partial \eta} \gtreqless 0 \quad \text{as} \quad \Pi_1 \lesseqgtr \Pi_2$$

i.e., extraction of grade 1 is reallocated toward the period with lower gross profit and total extraction rises or falls according to whether the cut-off grade (grade 2 here) is being extracted in the period of higher or lower profit.

The effects of an increase in the progressivity of the tax are similar. For profile P the reactions are given by

$$\frac{\partial X_{11}}{\partial \mu} = - \frac{\partial X_{21}}{\partial \mu}$$

$$= - \frac{1}{\Delta}(1 - \eta - \mu \Pi_2)$$

$$\times C_2''[\Pi_1 \cdot M\Pi_{11} - \Pi_2(1 + r)^{-1} M\Pi_{21}$$

and

$$\frac{\partial X_{22}}{\partial \mu} = - \frac{\partial X_{21}}{\partial \mu}$$

Reasoning analogous to that above shows

$$\Pi_1 \cdot M\Pi_{11} - \Pi_2(1 + r)^{-1} M\Pi_{21} \gtreqless 0$$

$$\text{as} \quad \Pi_1 \gtreqless \Pi_2$$

so that again the reallocation of the higher grade is to the period with lower gross profit and extraction of the lower grade adjusts so that total quantity extracted in the cut-off period is constant. As was the case with an increase in the base rate of the profits tax, this may mean either an increase or decrease in total recovery from the deposit.

4. SUMMARY

It is evident from the preceding analysis that variability in tax rates, whether for output, value, or profits taxes, may create allocation incentives that are qualitatively different from those under fixed-rate taxation. The differences appear in the order in which the various grades are extracted, the rate at which they are extracted, and the total quantity extracted.

In the case of per-unit severance taxes, a constant-rate tax induces a reallocation from present to future (when the discounted unit tax will be smaller) and a decline in total extraction. These effects are reversed if the tax rate is variable and has a sufficiently high growth rate (higher than the discount rate by an amount that depends on the grade distribution). The degree of intertemporal progressivity of the tax may be critical also for the grade selection profile, but the outcome in any particular case depends on the path of output prices. A constant ad valorem severance tax has no effect on grade selection but reallocates extraction in the direction of periods with lower discounted prices and decreases total recovery. In contrast, a variable-rate ad valorem tax may change the grade selection, alter the intertemporal profile in either direction, and increase total recovery. Finally, progressivity destroys the neutrality of a profits tax with respect to grade selection, extraction rate, and recovery, with distortions that depend qualitatively on the pre-existing path of profits.

It must be emphasized that these conclusions are based on an analysis that is short run in the sense that there is no scope for an investment response to diminished profitability. This applies particularly to the conclusion that total extraction may be higher under more progressive taxes. In the longer run, smaller profits will discourage investment in the taxing jurisdiction and result in lower recovery from the deposits. Quite aside from this, however, it is clear that the implications of variable-rate taxation are sensitive to both geological and economic conditions. Unless these conditions are accounted for in the formulation of tax policy, none of the usual taxes can be relied upon to produce the desired results in terms of tax revenue or conservation of the resource.

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