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A Universal Appointment Rule with Patient Classification for Service Times, No-Shows, and Walk-Ins

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This study evaluates patient classification for scheduling and sequencing appointments for patients differentiated by their mean and standard deviation of service times, no-show, and walk-in probabilities. Alternative appointment systems are tested through simulation using a universal Dome rule and some of the best traditional appointment rules in the literature. Our findings show that the universal Dome rule performs better in terms of reducing the total cost of patient's waiting time, doctor's idle time, and overtime, and its performance improves further with the right sequencing of patient groups. Although it is a challenge to find the best sequence, we propose a heuristic rule that successfully identifies the best sequence with an accuracy level of 98% for the universal Dome rule. Sensitivity analyses further confirm that our findings are valid even when assumptions on patient punctuality and service time distributions are relaxed. To facilitate the use of our proposed appointment system, an open source online tool is developed to support practitioners in designing their appointment schedules for real clinics.

Keywords: healthcare; appointment scheduling and sequencing; simulation

1. Introduction

Since Bailey's (1952) pioneering work, several studies have investigated how best to schedule and sequence appointments in healthcare settings. The overall objective is to increase productivity and simultaneously provide high-quality service to patients. On one hand, it is important to minimize idle time and clinic overtime from the doctor's perspective. On the other hand, excessive waiting times are not tolerated by increasingly demanding patients. Operations research techniques offer both practitioners and academicians valuable tools to effectively design appointment systems such that these conflicting goals are balanced.

As long as there are differences in the patient population, such as average service times, the question whether an appointment system (AS) can be improved by recognizing the differences remains. Earlier works usually investigate the added value of patient classification over a single appointment rule using limited criteria based on service time characteristics only. Some commonly used classification schemes include "new/return," "long/short consultation," and "low/high variance" patients (Walter 1973; Cox et al. 1985; Klassen and Rohleder 1996; Vanden Bosch and Dietz 2000; Cayirli et al. 2006, 2008).

The primary goal of this research is to extend these prior works by incorporating four important factors in the literature for patient classification. These include the mean and the variability of service times (denoted by μ and σ, respectively) and the probabilities of no-shows and walk-ins (denoted by PN and PW, respectively). Including these factors to classify patients allows a more comprehensive analysis of the effects of sequencing when conflicting cases exist with one group being "low" in some factors but "high" in others. For example, a patient group may have lower σ and PW but higher μ and PN. To the best of our knowledge, this is the first study to investigate multiple factors simultaneously for patient classification on the total system cost measured as a weighted sum of patients' wait time and the doctor's idle time and overtime.

A major contribution of this research is the development of a "universal" AS, which combines the best appointment rule and sequencing rule for a given clinic once its parameters are provided. This is important as the literature suggests that no AS exists that is flexible enough to offer a good solution for all clinical environments. At best, prior studies have shown that different combinations of appointment and sequencing rules perform well in different environments, with no specific guidelines available on how to choose the "best" combination for a given clinic. The proposed AS builds on the universal Dome rule of Cayirli et al. (2012) by including patient

classification based on the four classification criteria (i.e., μ, σ, *PN*, and *PW*). Its performance is compared against some of the best traditional appointment rules with patient classification, as well as the benchmark universal Dome rule with no patient classification.

Our results show that the universal Dome rule with patient classification performs consistently well under most realistic clinical environments tested over 81 environments and 6 cost ratios that represent different trade-offs between the doctor's time and the patient's time. Thus, there is value in patient classification using the four classification criteria together. However, the choice of best sequence is complex and dependent on the four criteria, as well as their interactions with the cost ratio and group sizes. Since no overall best sequence is identified, we develop a heuristic rule to select the right sequence. Among the three alternatives tested, the rule that sequences the group with the lowest "revised coefficient of variation" earlier in a session emerges as the most promising option. This sequencing rule achieves an accuracy of 98% in finding the best sequence for the universal Dome rule over a wide range of scenarios. Since no existing combination of appointment and sequencing rule is known to perform well in all environments, our proposed AS offers significant advantages. It not only performs well in a wide range of environments but also eliminates the unviable task of having to select the best combination of appointment and sequencing rules for specific environments. The proposed AS performs significantly better than the other combinations of traditional rules with sequencing in most realistic environments. Furthermore, it is easy to use, and its adoption by practitioners is facilitated through an open source tool, the Online Appointment Scheduling Tool (available online at http://www.appointmentschedulingtool.com/, last accessed November 7, 2014).

The rest of this paper is organized as follows: Section 2 gives a literature review on outpatient scheduling, and §3 proposes the AS design approach undertaken in this study. Section 4 describes the simulation methodology, experimental design, and performance measures. Section 5 discusses the performance of the universal Dome rule with patient classification, and §6 analyzes the properties of the best sequence, followed by a new heuristic rule proposed for choosing the best sequence. Section 7 validates the sensitivity of our results to some of the assumptions in the simulation model, and §8 ends with conclusions and suggestions for future research.

2. Literature Review

In this paper, AS design is considered at three decision levels: (i) appointment rules, (ii) patient classification, and (iii) appointment adjustments for no-shows and walk-ins. We discuss some of the more recent prominent papers under each decision category and refer the reader to Cayirli and Veral's (2003) survey on outpatient scheduling for a detailed discussion on earlier papers.

2.1. Appointment Rules

Appointment rules determine the appointment start times for patients, specified in a combination of block size and appointment interval length. Most of the literature concentrates on this problem, starting with the pioneering work of Bailey (1952). A consensus among these studies is that no single rule performs best in all environments. To overcome this limitation, Yang et al. (1998) propose a universal appointment rule that can be parameterized to perform well in a range of clinical environments. The rule is expressed as a mathematical function of four environmental parameters—the probability of no-shows, scheduled number of appointments per session, coefficient of variation of service times, and cost ratio of doctor-to-patient time. Once these parameters are estimated, the universal appointment rule can be used to compute the appointment times. Several studies show that a "dome" pattern, whereby appointment intervals initially increase and then decrease toward the latter part of a session, improves clinic performance (Wang 1993, Denton and Gupta 2003, Robinson and Chen 2003, Kaandorp and Koole 2007, Hassin and Mendel 2008, Klassen and Yoogalingam 2009). More recently, Cayirli et al. (2012) extend the Yang et al. mathematical formulation and suggest a universal Dome rule that combines the advantage of "universality" with dome-shaped appointment intervals.

2.2. Patient Classification

Some studies consider the use of patient classification (PC) with the underlying assumption that the patient population can be classified into distinct groups. This allows sequencing patients at the time of booking, possibly in combination with adjusting the appointment interval lengths based on the characteristics of each group. The practical implementation requires the number of patient groups to be restricted to some reasonable level so that the scheduler can assign patients into the reserved slots for each group. Walter (1973) is one of the earliest studies to suggest separating patients into heterogeneous groups with service times, depending on factors such

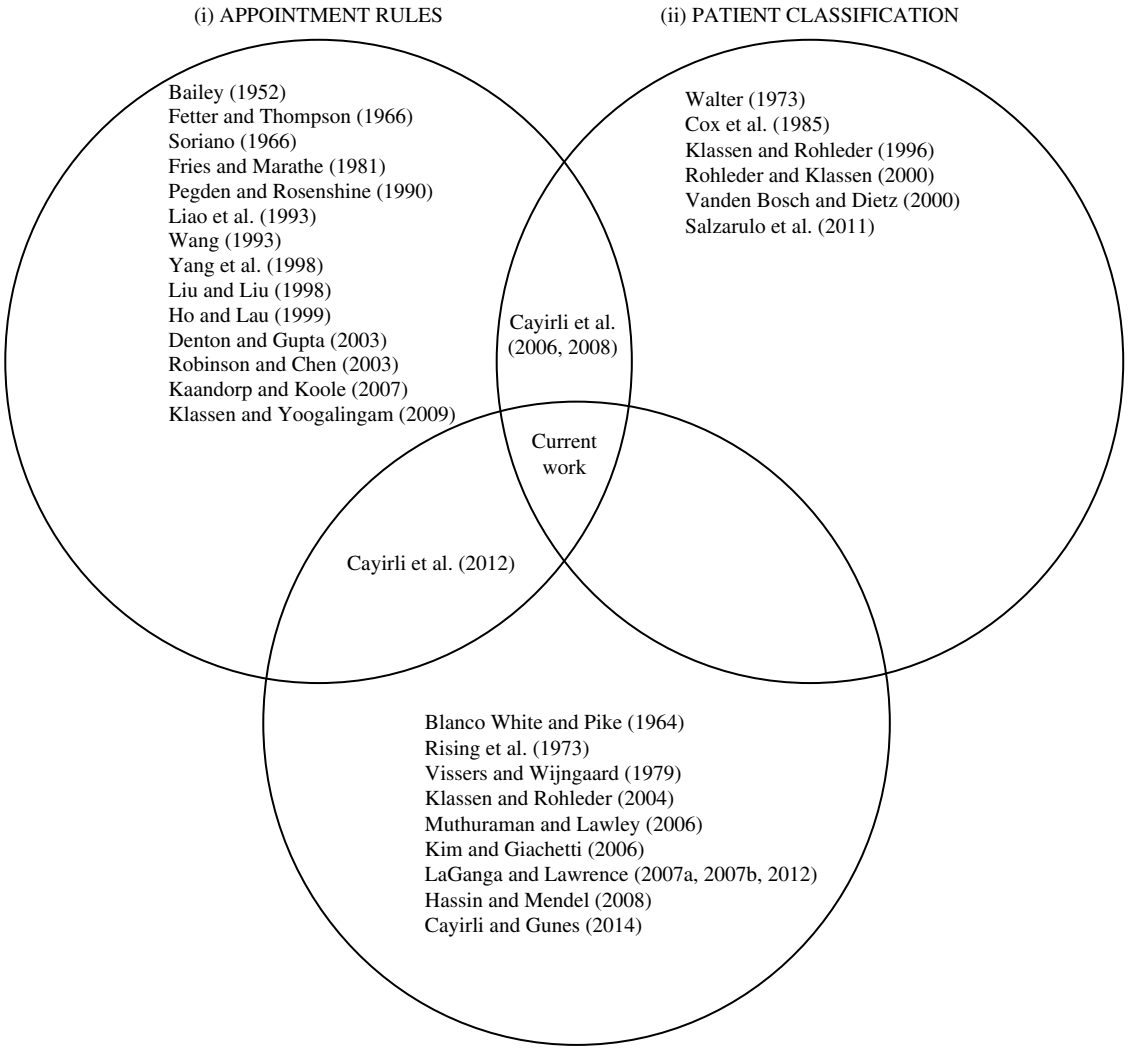
as the patient's age, physical mobility, or type of service. Cox et al. (1985) investigate several approaches on sequencing new and return patients in an ear, nose, and throat clinic, and they report that the proposed AS improves clinic performance in terms of utilization and patient wait times. Klassen and Rohleder (1996) use simulation to test sequencing approaches for patient groups with different service time variances but equal mean service times. Their findings suggest that it is best to sequence the patient group with lower service time variability earlier in a session. Rohleder and Klassen (2000) test the same rules under more realistic assumptions where patients may insist on receiving specific slots. In a case study, Lehaney et al. (1999) evaluate a sequencing approach that orders patients in terms of increasing service times, similar to the shortest processing time rule in job shop scheduling. Vanden Bosch and Dietz (2000) investigate optimal patient mix across several days, as well as the optimal sequencing and scheduling of patients to a discrete set of appointment times. The authors use enumeration to determine the optimal solution when patients differ by type of procedure and conclude that there is no easy rule on sequencing based on the service time mean and variability. Cayirli et al. (2006, 2008) report that the use of information on class-based service times for sequencing and interval adjustment improves clinic performance. Salzarulo et al. (2011) study sequencing based on brief versus comprehensive categorization, depending on whether patients require ancillary services in a multiphase system.

There is also related research on patient sequencing and interval adjustment in surgical scheduling. However, some of the basic assumptions are not applicable, such as strict-ordering policies that utilize estimates on *individual* surgery durations for inpatients already admitted to the hospital. As such, surgeries may be ordered in their desired sequence once the full list is available at the start of the day. On the other hand, in outpatient scheduling, without full knowledge of future requests, patients are assigned appointment slots as they call in throughout the day. The cost structure in surgical scheduling is also more complex given that several resources are involved in addition to the surgeon, such as medical equipment and support staff (e.g., nurses, anesthesiologists). Nonetheless, some of the more pertinent studies are discussed briefly. Weiss (1990) is the first to jointly address the problem of optimal scheduling and sequencing of surgical procedures. The sequencing problem is solved for two procedures, and simulation results are provided for larger problems that show that the optimal sequence is in the order of increasing variance. Wang (1999) addresses the problem of sequencing and scheduling for a single-server system using an analytical approach and reports that it is best to order patients in increasing service time means (or variances) when service times are exponential. Denton et al. (2007) use a two-stage stochastic model to solve the sequencing and scheduling of surgical procedures optimally, and they propose a heuristic rule that orders procedures in increasing variance for larger problems. Gul et al. (2011) evaluate several combinations of scheduling and sequencing rules. They report that sequencing surgeries in increasing mean service times yields the best performance, and sequencing in increasing variances of service times performs equally well given the positive correlation between the means and variances in their empirical data. Begen and Queyranne (2011) use multistage stochastic programming to optimally determine schedules for a given surgery sequence. Finally, several studies have addressed heterogeneous patients in the context of diagnostic resources (MRI, CT scan, etc.), where inpatient, outpatient, or emergency patients with different priorities and revenue/cost structures are assigned optimally (Green et al. 2006, Patrick and Puterman 2007, Kolisch and Sickinger 2008, Patrick et al. 2008).

2.3. Adjustments for No-Shows and Walk-Ins

Past literature has investigated two main approaches to accommodate no-shows and walk-ins. One approach is to overbook when no-shows exist and leave some slots open when the possibility of walk-ins exists. In this approach, a subdecision is which slots to overbook or leave open (Blanco White and Pike 1964, Rising et al. 1973, Klassen and Rohleder 2004). A second approach is to adjust the appointment intervals proportionally to the expected probabilities of no-shows or walk-ins (Vissers and Wijngaard 1979). Kim and Giachetti (2006) develop a stochastic model to determine the optimal number to schedule when no-shows exist, but they do not address the scheduling decision. LaGanga and Lawrence (2007b) show that overbooking is more beneficial when the number of patients per session is larger, the no-show rate is higher, and the service time variability is lower. LaGanga and Lawrence (2007a) further evaluate 14 options for adjusting the schedule to accommodate no-shows. These include the general approach of compressing appointment intervals, as well as double-booking, block-booking, and wave scheduling. LaGanga and Lawrence (2012) study the positions of the best slots for overbooking that balance the benefit of seeing additional patients with the costs of patient waiting time and clinic overtime. Muthuraman and Lawley (2006) formulate a scheduling policy for overbooking that uses information on the calling patient's probability of no-show based on historical data. In their optimal analysis of scheduling with no-shows, Hassin and Mendel (2008) find that no-shows greatly affect the schedule and should be considered when designing an AS. All these studies address either no-shows or walk-ins. To overcome this limitation, Cayirli et al. (2012) propose a procedure to compute a revised mean and variance of service times, such that

Figure 1. Taxonomy of Literature Based on Three Decision Levels in Appointment System Design



their universal Dome rule is adjusted for the combined effects of no-shows and walk-ins. In a more recent work, Cayirli and Gunes (2014) investigate the best approaches for adjusting AS when dealing with seasonal walk-ins.

Figure 1 provides a taxonomy of appointment scheduling literature, including the most prominent studies under each decision level in AS design. It also shows that a vast majority of studies have concentrated on finding the best appointment rule, with relatively less emphasis on patient classification and adjustments for no-shows and walk-ins. This study develops a universal AS that can offer a good solution for any given clinical environment once its parameters are provided. In doing so, it fills the void in the literature by addressing all three decision levels simultaneously.

3. Universal Dome Rule with Patient Classification

Our literature review in §2 shows that no single appointment rule performs well in all clinical environments. Yang et al. (1998) introduce the first universal rule in that regard. In an extension, Cayirli et al. (2012) modify it into a universal Dome rule that can be parameterized for different probabilities of no-shows, probabilities of walk-ins, numbers of scheduled patients per session, variations in service times, and cost ratios of the doctor's time to patients' time (see Appendix A for details). Their proposed rule addresses the two decision levels—namely, the *appointment rule* and *adjustments for no-shows and walk-ins*, as discussed in §2 (see Figure 1).

The literature also strongly affirms the benefits of using patient classification for sequencing and interval adjustment in appointment system design. However, there is no research that analyzes the effects of factors beyond the mean and variability of service times, as well as their conflicting effects acting together. Locating our current work at the intersection of the three decision levels in Figure 1, we extend the universal Dome rule to include the *patient classification* rule through the following illustration.

For ease of implementation, patient classification is often restricted to sorting patients into two groups. For example, we may have two patient groups, class A and class B, where class A has a distinct mean and standard deviation of service times μ_A and σ_A and probabilities of no-shows and walk-ins PN_A and PW_A; similarly, class B has μ_B and σ_B and PN_B and PW_B, respectively. In total, there are N_A number of class A patients and N_B number of class B patients, i.e., a total of N patients seeking appointments, where $N = N_A + N_B$. Substituting these parameters into Equations (A2) and (A3) in Appendix A, we can then calculate the revised mean and revised standard deviation of service times of class A and class B patients as μ'_A, σ'_A and μ'_B, σ'_B, respectively. We can also calculate the k-value for each patient class by substituting the respective values of PN, PW, the coefficient of variation of service times (Cv), the number of patients scheduled per session N, and the cost ratio of the doctor's time to patients' time (CR) into Equation (A5) in Appendix A.

Once the clinic has calculated the values of (μ'_A, σ'_A), (μ'_B, σ'_B), and then the values of k_A and k_B as described above, the appointment time for each patient can be determined based on the cumulative mean and standard deviations for the chosen sequencing rule. For example, if the clinic sequences class A patients at the beginning of the session and class B patients at the latter part, the appointment times (A_i) are computed as follows:

$$A_i = \max\{0, k_A(i-1)\mu'_A - \sqrt{i\sigma'^2_A} \cdot \pi\} \quad \text{for (class A) patients } i = 1, \ldots, N_A, \tag{1}$$

$$A_i = k_A(N_A)\mu'_A + k_B(i - N_A - 1)\mu'_B - \sqrt{(N_A+1)\sigma'^2_A + (i - N_A - 1)\sigma'^2_B} \cdot \pi \quad \text{for (class B) patients } i = N_A + 1, \ldots, N. \tag{2}$$

The parameter π serves to create a dome pattern in the appointment intervals, where $\pi = (N + i)/(N - 1)$ and $N = N_A + N_B$. If class A and class B patients are exactly the same (i.e., they have the same service time, no-show, and walk-in characteristics), Equations (1) and (2) will revert to the original formulation of the Dome rule with no patient classification (A4) as in Appendix A. In this example, class A patients are sequenced first such that patient i's appointment time is pulled forward by $\sqrt{i\sigma'^2_A} \cdot \pi$ ahead of $k_A(i-1)\mu'_A$. Similarly, for class B patients, patient i's appointment time is pulled forward by $\sqrt{(N_A+1)\sigma'^2_A + (i - N_A - 1)\sigma'^2_B} \cdot \pi$ ahead of $k_A(N_A)\mu'_A + k_B(i - N_A - 1)\mu'_B$.

As a further illustration, Table 1 presents the appointment times (A_i) for a case where class A patients ($N_A = 4$) have revised mean and standard deviation of service times μ'_A and σ'_A equal to 17.5 and 8.75 minutes, respectively, and class B patients ($N_B = 6$) have μ'_B and σ'_B equal to 5 and 1.5 minutes, respectively, such that the overall revised mean service time of all patients equals 10 minutes. The universal Dome rule assumes $k_A = 1.25$ and $k_B = 1.50$ and sequences class A patients first. Table 1 tabulates the appointment times proposed by the Dome and Individual-Block/Fixed Interval (IBFI) rule (see Equation (3)), with class A-first sequencing.

Table 1. Appointment Times Using the Dome and IBFI Rules, with Class A Patients Sequenced First

	Dome rule		IBFI rule	
Patient no.-class	A_i	Appointment interval	A_i	Appointment interval
1-A	0	—	0	—
2-A	5.376	5.376	17.5	17.5
3-A	21.859	16.483	35.0	17.5
4-A	38.403	16.544	52.5	17.5
5-B	54.891	19.823	70.0	17.5
6-B	60.115	5.435	75.0	5.0
7-B	65.326	5.421	80.0	5.0
8-B	70.525	5.407	85.0	5.0
9-B	75.712	5.394	90.0	5.0
10-B	80.887	5.380	95.0	5.0

4. Methodology

4.1. Simulation Model and Assumptions

Simulation methodology is commonly used in outpatient scheduling literature, as it is generally preferred for the ease of realistically modeling complex clinical environments with variable service times and unpredictable arrivals (Cayirli and Veral 2003). This paper uses a discrete event simulation model, which is written in GPSS/H language. For each environment, 15,000 clinic sessions are simulated (30 replications of 500 clinic sessions). Each clinic session is a terminating batch run with two patient groups scheduled in the initial and latter parts of the clinic session.

The clinic represents a single-server, single-phase queuing system. All walk-ins during the clinic regular hours are accepted, and their interarrival times are exponentially distributed. It is further assumed that a walk-in can be either a class A or a class B patient based on the same overall percentage of each patient class, and this walk-in will be requested to arrive only during the designated times reserved for his or her specific patient class. In other words, if class A patients are scheduled at the first part of a session, class A walk-ins will arrive only during the first part of the session.

Scheduled patients are assumed to arrive punctually, a simplifying assumption that is relaxed in §7. It is assumed that all arriving patients are served, including walk-ins. Service times are modeled using lognormal distribution, a common assumption supported by empirical data in the literature (Cayirli and Veral 2003). The service times are independent and identically distributed for each patient class, whereas in real life, this may not be true, as the doctor may speed up when the queue is long. Finally, it is assumed that the doctor starts the session on time with no time gaps between consultations. In general, these factors are relatively controllable and are less interesting for the scope of this current study.

The simulated clinic uses a queue discipline where scheduled patients are seen in the order of their appointment times, and walk-ins are given a relatively lower priority. Although procedures to accommodate walk-ins and late patients are clinic specific, it is common to assume that walk-ins and late patients will tolerate longer waiting times. On the other hand, when a clinic accepts walk-ins, as a general policy, it is reasonable to assume a limit on their waiting time. To balance these conflicting goals, our model uses a rule of thumb where unless a walk-in arrives into an empty clinic, he or she is made to wait for at most three scheduled patients. If the doctor is free with no scheduled patients waiting, the walk-in patient may be served immediately. Similarly, our simulation model uses a commonsense rule for balancing the need to discourage late arrivals while trying to serve them within a reasonable period, especially when they arrive only a few minutes late. The penalty introduced is directly proportional to a patient's lateness, where patients lose one place for every full μ minutes of lateness.

4.2. Experimental Design

In our simulation experiments, the comparison of our proposed universal Dome rule defined in Appendix A is made against the following traditional rules:

i. The IBFI rule, which schedules patients individually at equal intervals:

$$A_1 = 0; \quad \text{then for } i > 1, \text{set } A_i = A_{i-1} + \mu', \tag{3}$$

where A_i is the appointment time given to patient i, and μ' is the revised mean service time.

ii. 2BEG, commonly referred to as Bailey's rule (Bailey 1952), which schedules two patients at the start of a session and the rest individually at fixed intervals:

$$A_1 = A_2 = 0; \quad \text{then for } i > 2, \text{set } A_i = A_{i-1} + \mu'. \tag{4}$$

In these rules, as well as in the universal Dome rule, the appointment intervals are adjusted for no-shows and walk-ins such that the service time mean μ and standard deviation σ are revised to μ' and σ', respectively, using Equations (A2) and (A3) in Appendix A. Using the revised service time mean and standard deviation is supported by the findings of Cayirli et al. (2012), which affirm the value of interval adjustment for the same appointment rules.

This study further investigates the effects of patient classification on these appointment rules by using two patient classes, referred to as PC1 and PC2. The classification scheme is kept flexible such that each group of patients may take on different values of environmental factors, which include the mean and standard deviation of

Table 2. Appointment Systems

	Description
	Appointment rules
Dome	Universal Dome rule
IBFI	Individual-Block/Fixed-Interval rule
2BEG	Bailey's rule
	Sequencing rules
No PC	No patient classification is used; this benchmark rule is tested on Dome only
PC1-first	Patient class 1 is sequenced first with appointment intervals adjusted accordingly
PC2-first	Patient class 2 is sequenced first with appointment intervals adjusted accordingly

service times (μ and σ), no-show probability (PN), and walk-in probability (PW). Prior studies on PC consider only service time differences based on either μ or σ alone (Klassen and Rohleder 1996; Cayirli et al. 2006, 2008). Including four factors to classify patients allows a more comprehensive analysis of conflicting cases where one group may be low in some factors but high in others.

We restrict our analysis to sequencing one patient group at the beginning and the other at the latter part of a session, in line with findings from earlier studies that show that other more complicated alternating patterns are inferior (Cayirli et al. 2008). The number of patients in each group is fixed at 5 and 10, such that the groups represent one-third and two-thirds of the total clinic size $N = 15$. This means that a fixed number of appointments are available for each group per clinic session when PC is used. Each appointment slot is reserved for a specific patient group with the appointment intervals adjusted according to the unique group characteristics in terms of μ, σ, PN, and PW.

In contrast, when no PC is used for scheduling, requests for appointments can be scheduled into any slot in a session. The appointment intervals are computed based on the overall mean characteristics of all patients, such that μ_o, PN_o, and PW_o are the weighted overall averages. In other words, $\mu_o = (N_1 \cdot \mu_1 + N_2 \cdot \mu_2)/(N_1 + N_2)$. Similarly, the overall σ_o is calculated as $\sqrt{((N_1 \cdot o_1^2) + (N_2 \cdot o_2^2))/(N_1 + N_2)}$. Although the average number of patients scheduled for each group will remain the same over time, the actual number of patients scheduled for each patient group may vary across days. This benchmark Dome rule with "No PC" is tested and referred to in short as "Dome–No PC." Table 2 summarizes the appointment systems, i.e., the different combinations of appointment and sequencing rules, tested in our simulation experiments.

In summary, our study investigates the effects of four classification criteria used for PC—namely, the ratios of the mean service time, standard deviation, no-show, and walk-in probabilities of patient classes 1 and 2 (i.e., μ_1:μ_2, σ_1:σ_2, PN_1:PN_2, and PW_1:PW_2). The cost ratio CR is also included because prior research shows that this factor affects the performance and choice of AS. The range of cost ratio evaluated, $1 \le CR \le 50$, provides a reasonable coverage of real clinics. Table 3 summarizes the environmental factors and their levels tested in our simulation experiments, resulting in a total of 486 cases (81 environments $\times$ 6 cost ratios).

4.3. Performance Measures

A primary performance measure used in this study is the patients' mean waiting time ($WAIT$). When scheduled patients are assumed to arrive punctually, the waiting time of a patient is calculated as the difference between the time when service starts and the appointment time (or, equivalently, the patient's arrival time). For walk-ins, the waiting time is the time interval between the time when service starts and the time of arrival, since walk-ins arrive without appointments. When unpunctual patients are assumed (in §7), the waiting time of a late patient is calculated from the time of arrival, whereas the waiting time of an early patient is calculated from the time of appointment. In the latter case, negative waiting times are possible, and these are truncated to zero to calculate

Table 3. Environmental Factors

Factor	Explanation	Levels
μ_1:μ_2	Ratio of the mean service time of PC1 to the mean service time of PC2	1:1, 1:2, 2:1
σ_1:σ_2	Ratio of the standard deviation of service time of PC1 to PC2	0.4:0.4, 0.4:0.8, 0.8:0.4
PN_1:PN_2	Ratio of the no-show probability of PC1 to PC2 (in %)	15:15, 15:30, 30:15
PW_1:PW_2	Ratio of the walk-in probability of PC1 to PC2 (in %)	15:15, 15:30, 30:15
CR	Cost ratio of the value of the doctor's time relative to patients' time	1, 2, 5, 10, 25, 50

the overall mean waiting time for patients. This is based on the presumption that a clinic is not responsible for early arrivals, and excessive earliness should also be discouraged because it leads to congestion in the waiting area, as well as higher perceived waiting times for patients.

Two other primary performance measures are collected in this study. They are the doctor's mean idle time and doctor's mean overtime. Idle time is the proportion of time that the server is not busy serving patients during the target session, and overtime is the extra time required to serve all patients beyond the target session end. These two server-related performance measures should be balanced against the patients' waiting time on a per-patient basis. *IDLE* is thus calculated by dividing the total idle time in a session by the number of patients served. Similarly, *OVER* is calculated by dividing the total overtime (actual session end time minus the target session length) by the number of patients served. As a result, the expected total cost of the system per patient (*TC*) is represented as

$$TC = (WAIT)Cp + (IDLE)Cd + (OVER)Co. \tag{5}$$

The relative cost of patients' wait times (*Cp*), doctor's idle time (*Cd*), and doctor's overtime (*Co*) need to be estimated by the decision maker. The cost ratio *CR* represents the cost of the doctor's time to patients' time (i.e., Cd/Cp), and in parallel with the common practice, this study fixes the overtime cost of the doctor at a 50% premium (i.e., $Co = 1.5 \times Cd$ in (5)). The cost ratios are analyzed in a range of 1–50, which provides reasonable coverage of the general trade-offs between the doctor's and patient's time.

5. Performance of the Proposed Appointment System

This section analyzes the performance of the proposed AS, which combines the universal Dome rule with PC for both sequencing and interval adjustment. Given two classes of patients, sequencing is defined in terms of which group is placed earlier in the session, referred to as "PC1-first" or "PC2-first" rules. As discussed in §4, the set of criteria used for classification include μ, σ, *PN*, and *PW*. First, the Dome rule with PC is compared with the benchmark case of no PC (i.e., Dome–No PC). This allows an assessment of the value of patient classification. Next, the Dome rule with PC is compared with the best traditional rules with PC, including the IBFI and 2BEG rules. This allows an assessment of the improvements due to the universal Dome rule itself.

Table 4 illustrates how the output data were analyzed using 2 representative environments out of the 81 simulated. The Dome rule with the best sequencing, either Dome–PC1 or Dome–PC2, is listed (first row) and compared with the Dome–No PC rule (second row), as well as the best traditional rule with PC (third row). The corresponding total costs *TC* of each AS are noted in parentheses, and the relative improvements in total cost performance are represented by Imp(No PC) and Imp(Trad), respectively. Statistically insignificant differences are identified using the Bonferroni test at the 95% confidence level and are indicated by a superscript dagger.

In the first environment, where $\mu_1{:}\mu_2 = 2{:}1$, $\sigma_1{:}\sigma_2 = 0.8{:}0.4$, $PN_1{:}PN_2 = 15{:}15$, and $PW_1{:}PW_2 = 15{:}15$, the first group of patients with size $N_1 = 5$ has a higher mean and standard deviation in service time compared with the second group with size $N_2 = 10$, with equal no-show and walk-in probabilities. In this environment, the best AS is Dome with PC2-first sequencing. Given the characteristics of the two patient groups, the best sequencing corresponds to placing the group with the lower service time mean and variability earlier in the clinic session, all else being equal. This is true regardless of the cost ratios evaluated from 1 through 50. When $CR = 1$, representing equal costs for the doctor's and patient's time, the improvement as a result of this AS is 18.44% compared with Dome–No PC and 11.87% compared with the best traditional rule, IBFI–PC2. However, when $CR = 10$, the Dome–PC2 rule performs nearly as well as the best traditional rule 2BEG–PC2, and the difference (0.31%) is not statistically significant at $\alpha = 5\%$.

The second environment in Table 4 is similar to the first with the factor levels reversed such that the second group, which is larger in size ($N_2 = 10$), has a higher mean and standard deviation than the first group, all else being equal. For $CR \leq 10$, the best sequencing is PC1 that assigns the group with lower mean and variability in service times first for low/moderate cost ratios. However, the preference shifts to PC2-first sequencing for $CR \geq 25$. This example shows that the group size and cost ratio can interact and affect the choice of sequencing.[1]

The analysis described above for Table 4 is extended to the complete set of environments in Appendix B that lists results for three *CR* values of 1, 10, and 50. Since the Dome rule with PC consistently outperforms the best traditional rules with PC in almost all environments, details identifying the best combination of traditional

[1] Although $N_1{:}N_2$ is fixed at 5:10 in our simulated environments, as illustrated in this example, we can identify and analyze the effect of group size on sequencing using the same output.

Table 4. Performance Analysis of the Best Appointment Systems ($N_1{:}N_2 = 5{:}10$)

$CR = 1$	$CR = 2$	$CR = 5$	$CR = 10$	$CR = 25$	$CR = 50$
Environment 1: $\mu_1{:}\mu_2 = 2{:}1$, $\sigma_1{:}\sigma_2 = 0.8{:}0.4$, $PN_1{:}PN_2 = 15{:}15$, $PW_1{:}PW_2 = 15{:}15$					
Dome–PC2 (1.38)	Dome–PC2 (1.88)	Dome–PC2 (3.00)	Dome–PC2 (4.56)	Dome–PC2 (8.78)	Dome–PC2 (15.38)
Dome–No PC (1.69)	Dome–No PC (2.26)	Dome–No PC (3.46)	Dome–No PC (5.13)	Dome–No PC (9.67)	Dome–No PC (16.81)
IBFI–PC2 (1.56)	IBFI–PC2 (1.95)	IBFI–PC2 (3.10)	2BEG–PC2 (4.58)	2BEG–PC2 (8.94)	2BEG–PC2 (16.21)
Imp(No PC): 18.44%	Imp(No PC): 16.88%	Imp(No PC): 13.14%	Imp(No PC): 11.10%	Imp(No PC): 9.20%	Imp(No PC): 8.51%
Imp(Trad): 11.87%	Imp(Trad): 3.30%	Imp(Trad): 3.05%	Imp(Trad): 0.31%†	Imp(Trad): 1.73%	Imp(Trad): 5.11%
Environment 2 : $\mu_1{:}\mu_2 = 1{:}2$, $\sigma_1{:}\sigma_2 = 0.4{:}0.8$, $PN_1{:}PN_2 = 15{:}15$, $PW_1{:}PW_2 = 15{:}15$					
Dome–PC1 (1.77)	Dome–PC1 (2.40)	Dome–PC1 (3.76)	Dome–PC1 (5.68)	Dome–PC2 (10.81)	Dome–PC2 (18.78)
Dome–No PC (2.02)	Dome–No PC (2.72)	Dome–No PC (4.16)	Dome–No PC (6.17)	Dome–No PC (11.63)	Dome–No PC (20.19)
IBFI–PC1 (1.96)	IBFI–PC1 (2.44)	2BEG–PC2 (3.85)	2BEG–PC2 (5.62)	2BEG–PC2 (10.91)	2BEG–PC2 (19.73)
Imp(No PC): 12.04%	Imp(No PC): 11.67%	Imp(No PC): 9.67%	Imp(No PC): 8.06%	Imp(No PC): 7.09%	Imp(No PC): 6.99%
Imp(Trad): 9.45%	Imp(Trad): 1.49%†	Imp(Trad): 2.47%	Imp(Trad): -1.06%†	Imp(Trad): 0.95%†	Imp(Trad): 4.81%

Notes. The first row lists the best Dome rule with patient classification, the second row lists the best Dome rule with no PC, and the third row lists the best traditional rule with PC. Total costs are indicated in parentheses. Imp(No PC), percent improvements in total cost of Dome with PC relative to Dome with no PC; Imp(Trad), percent improvements in total cost of Dome with PC relative to the best traditional rule with PC.

†Indicates statistically insignificant difference at $\alpha = 5\%$ using the Bonferroni test.

rule and sequencing in different environments are excluded to simplify the presentation. To further simplify the identification of the best sequencing rule for the Dome rule under each environment and cost ratio, the Bonferroni test is used to identify results where PC1- or PC2-first sequencing is not significantly different at $\alpha = 5\%$. These results are indicated with a superscript asterisk in Appendix B. For example, in the first environment in Table 4 (which corresponds to Environment 1 in Appendix B), the PC2-first rule is significantly better than the PC1-first rule for all cost ratios and therefore remains as a strictly "dominant" choice. On the other hand, for the second environment in Table 4 (which corresponds to Environment 6 in Appendix B), the sequencing choices are not significantly different across the cost ratios. This means that the decision maker can be indifferent to choosing either sequencing option across all cost ratios without affecting the total cost significantly.

Based on this parsing of information, we can determine the best dominating sequence for a given environment across all cost ratios, as listed under the last column in Appendix B. As a result, we are able to reduce the best choice for sequencing to a single option in all but one environment (Environment 20). Our comparison of the AS in the 81 simulated environments leads to the following findings, discussed in two parts: (i) improvements as a result of patient classification and (ii) improvements as a result of the universal Dome rule.

5.1. Improvements as a Result of Patient Classification

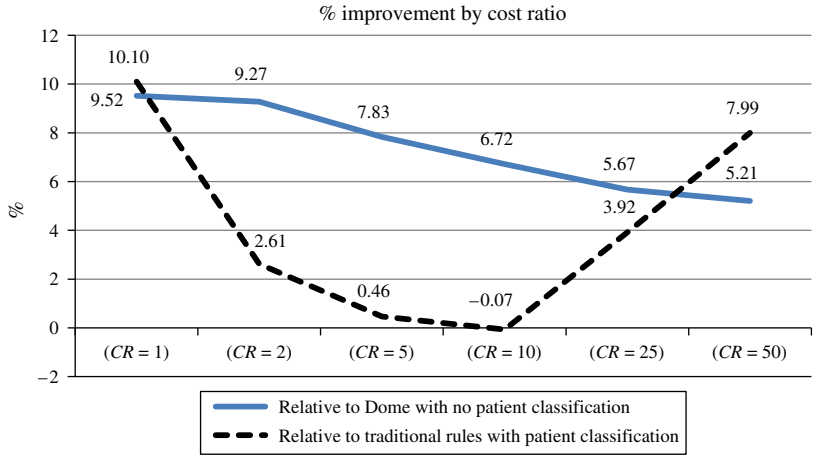
The universal Dome rule was originally proposed for clinics with no PC. It is thus interesting to examine the improvement stemming from the extension of its use with PC. The universal Dome rule with PC significantly improves performance relative to the benchmark Dome rule with no PC. The average total cost improvement as a result of using patient class information is 7.37%, and the maximum is approximately 22% (see Appendix B). It is observed that the improvements are lower when both groups have equal mean and standard deviation in service times ($\mu_1{:}\mu_2 = \sigma_1{:}\sigma_2 = 1$), and in a few of these environments, "No PC" is not significantly different (such as in Environment 17). This suggests that there is less benefit in sequencing when the service time characteristics of the two groups are similar and the only differentiating factors are no-show and/or walk-in probabilities. Apart from a few cases where sequencing hurts the total cost performance (e.g., by a maximum 4.9% in Environment 73 when $CR = 50$), the universal Dome rule with PC remains beneficial or at least equally best for 98.8% of the cases (81 environments $\times$ 3 cost ratios). Overall, μ has a higher impact on the improvements stemming from PC compared with σ, all other factors being equal. The impact of *PW* and *PN* occur indirectly through their interactions with μ and/or σ.

Figure 2 graphically represents the magnitude of improvements by cost ratios averaged across all environments. The average improvements resulting from using PC (comparison relative to the Dome with no PC) decrease from 9.52% to 5.21% as *CR* increases from 1 to 50. It is also observed that in 23 of 81 environments, the Dome rule with PC performs equally well regardless of the sequencing used. These are cases where the Dome–PC1 rule is not significantly different from Dome–PC2 (i.e., both PC1 and PC2 are listed as dominant sequences in the last column in Appendix B). In essence, although it is important that sequencing is used, choosing the right sequence is less critical in some circumstances. This observation is especially valid at lower cost ratios (i.e., approximately three-fourths of all cases when $CR = 1$ versus one-half of cases when $CR = 50$). We hence conclude that although there is more value in practicing PC in patient-centered clinics that assume a lower *CR* value, choosing the right sequence is less important in such clinics. At the other extreme, sequencing may be less beneficial in clinics that place a higher *CR* value, such as surgical clinics, yet choosing the right sequence is more important under those circumstances.

5.2. Improvements as a Result of the Universal Dome Rule

In a majority of the environments, the universal Dome rule performs better than the traditional rules IBFI and 2BEG when all rules are combined with PC. The average and maximum improvements in total cost are 4.17% and 16.99%, respectively. From Figure 2, these improvements are highly dependent on the cost ratio, showing a concave pattern. The average improvements peak at $CR = 1$ and 50 (i.e., 10.1% and 7.99%, respectively), and they reach almost zero at $CR = 5$ and 10 (i.e., 0.46% and -0.07%, respectively). Although the traditional rules may occasionally perform better than Dome under moderate cost ratios (such as by approximately 3% in Environment 31 when $CR = 10$), the decision maker still faces the challenge of choosing the best traditional rule and the right sequencing to complement the selected traditional rule for those clinical environments and cost ratios. Consequently, the real advantage of the Dome rule is its universality. When it is combined with the best sequencing as suggested in §6, it offers a good solution for any given clinical environment.

Figure 2. Total Cost Improvements for Dome with Patient Classification



6. Choosing the Best Sequence with Multiple Criteria

6.1. Best Sequencing When One or Two Factors Differ

In this part of the analysis, we derive properties for best sequencing for the simpler cases where groups differ in one or two of the multiple criteria used for patient classification, including μ, σ, *PN*, and *PW*. This analysis is conducted only for the Dome rule, since it consistently performs better than or nearly as well as the traditional rules 2BEG and IBFI with PC, as discussed in §5.

The cost ratio and the relative group size are factors that are also included for their impact on choosing the best sequencing. Obviously, if we divide patients with the same characteristics into two "artificial" groups, sequencing either group first with Dome will result in the same appointment schedule as that of Dome with no PC. Consequently, group size affects the total cost only indirectly when the two groups have different characteristics as specified by the other factors.

First, we discuss the simplest case where patient groups differ only in one factor and the rest of the factors are equal or at least similar. The important findings derived from the results in Appendix B are summarized as follows:

i. *Service time variability* (σ): When PC is based on σ with the other factors being equal, it is best to sequence "low-σ first" for all group sizes and cost ratios (Environments 7 and 9). Intuitively, sequencing the low-σ group first and the high-σ group later will help reduce congestion by minimizing patients' wait time—but it could be at the cost of doctor's idle time and overtime later in the session. When the high-σ group is larger in size (Environment 9), we observe a shift toward "high-σ first" being as good a sequence as "low-σ first" at $CR = 50$. This is because with an increased size for the group with higher variability, the high cost of overtime at a high *CR* requires shifting the group with high variability to the earlier part of a session to reduce the risk of overtime. We also note that at this extreme cost ratio, it may not be worth doing any sequencing because the total cost improvement over "No PC" is insignificant.

ii. *Service time mean* (μ): When PC is based on μ with all else being equal, the best sequence depends on the relative size of each group. If the group with shorter service time is larger in size (Environment 2), the "low-μ first" rule is the dominant sequence, although it is not statistically better than "high-μ first" across all *CR* values (see Appendix B). On the other hand, if the group with longer service time is larger in size (Environment 5), it is best to sequence "high-μ first" in the session, especially if the doctor's time is deemed highly valuable ($CR = 50$). Intuitively, such an ordering will reduce overloading the latter part of the session and minimize the risk of overtime. As a rule of thumb for μ-based sequencing, sequencing "low-μ first" or "high-μ first" will help achieve equally significant benefits over no PC unless the cost ratio is high. If the cost ratio is high and the high-μ group is larger in size, it is best to place the high-μ group earlier in the session so as to minimize the risk of overtime. It is also intuitive that the impact of "high μ" on the risk of overtime is stronger than it is for "high σ," as the latter affects the service time in both directions, resulting in shorter as well as longer service times than the mean. This explains why the shift of the best sequence from "low-first" to "high-first" occurs relatively faster for μ compared with σ, when the *CR* and size of the "high" group increase.

iii. *No-show probability* (*PN*): When PC is based on *PN*, all else being equal, "low-*PN* first" performs consistently better, especially when the low group is smaller in size (Environments 35 and 62). As a general pattern, the impact of this factor can be explained through the congestion it causes in different parts of a session.

Figure 3. Best Sequence by Cost Ratio When One Factor Differs (All Else Being Equal)

Cost ratio	Factor	Best sequencing rule
Low (CR = 1)	Service time variability (σ)	Low-σ first
	Service time mean (μ)	Indifferent between low and high-μ first
	No-show probability (PN)	Indifferent between low and high-PN first or No PC
	Walk-in probability (PW)	Indifferent between low and high-PW first or No PC
Moderate (CR = 10)	Service time variability (σ)	Low-σ first
	Service time mean (μ)	Indifferent between low and high-μ first
	No-show probability (PN)	Indifferent between low and high-PN first
	Walk-in probability (PW)	Indifferent between low and high-PW first or No PC
High (CR = 50)	Service time variability (σ)	Low-σ first or No PC
	Service time mean (μ)	High-μ first
	No-show probability (PN)	Low-PN first or No PC
	Walk-in probability (PW)	High-PW first or No PC

The choice shifts from "high-PN first" (i.e., low congestion) to "low-PN first" (i.e., high congestion) as the size of the latter group and the cost ratio increase. However, it may not matter which group is sequenced first because "No PC" remains statistically an equally best alternative under most cases when PN is the only difference between the two groups.

iv. *Walk-in probability* (PW): When walk-in probability is the only difference between the two patient groups, the resulting total cost improvements over "No PC" are statistically insignificant for all the factor levels evaluated (Environments 17 and 26). In terms of sequencing preference, if the low-PW group is larger in size (i.e., low congestion group), the "low-PW first" rule is the best for all CR values, although it is not statistically better than the "high-PW first" rule (Environment 26). However, if the high-PW group dominates in size, it is best to assign this group early in the session to reduce the risk of overtime, especially if the CR is high (Environment 17 for $CR = 50$).

In summary, when patient groups differ only by a single factor, μ-based sequencing provides the highest benefit over the benchmark case of "No PC." The improvement measured in terms of total cost averaged across all scenarios is 11.31%, which is statistically significant for all cost ratios. This is followed by σ, PN, and PW in decreasing order, with total cost improvements of 3.15%, 1.7%, and 0.5%, respectively. The improvements as a result of σ-based sequencing are statistically significant across most cost ratios with the exception of $CR = 50$. "No PC" is usually a viable and simpler option when patient groups differ by PN or PW only.

Given that the best sequence often depends on the cost ratio used by the clinic, we summarize the above results based on cost ratio using a decision tree format (see Figure 3). If a clinic determines a single dominating criterion for classifying patients and estimates a cost ratio for the trade-off between the patient's and the doctor's time, the decision makers may simply use the decision tree in Figure 3 for choosing the best sequencing rule.

We extend this analysis to cases where two factors differ in opposite directions and the remaining two are equal. For example, a group may have higher mean service times yet lower service time variability and similar no-show and walk-in probabilities compared with the other group. For brevity purposes, the results are presented in a decision tree format in Figure 4. Our findings suggest that the choice of sequence matters more for the high cost ratio, $CR = 50$. On the other hand, for the low to moderate cost ratios, $CR = 1$ to 10, it is often statistically insignificant which group is sequenced first. The reader is referred to Figure 4 for complete details on the best choice of sequencing when two conflicting factors exist, all else being equal.

6.2. Heuristic Rule for Sequencing with Multiple Criteria

As discussed in §6.1, it is not an easy task to choose the right sequence even when patient groups differ by one or two factors. This is further complicated by the fact that cost ratio and group size may affect the final choice of the best sequence. In an effort to simplify the identification of the best sequence, we suggest three heuristic rules. Given the general dominance of the "low-σ first" sequence in our results, as well as the popularity of this well-studied rule in the literature, we test the following three variants:

i. the "low-σ' first" rule, which sequences the group with the lower revised standard deviation (σ') first;
ii. the "low-$\sigma'\mu'$ first" rule, which sequences the group with the lower product of σ' and μ' first; and
iii. the "low-Cv' first" rule, which sequences the group with the lower revised coefficient of variation (i.e., σ'/μ') first.

Figure 4. Best Sequence by Cost Ratio When Two Factors Differ in Opposite Directions (All Else Being Equal)

Cost ratio	Factors	Best sequencing rule
Low (CR = 1)	σ & μ	Indifferent between low σ & low μ
	σ & *PN*	Indifferent between low σ & low *PN*
	σ & *PW*	Low-σ first
	PN & μ	Indifferent between low *PN* & low μ
	PW & μ	Indifferent between low *PW* & low μ
	PN & *PW*	Indifferent between low *PN* & low *PW*, or No PC
Moderate (CR = 10)	σ & μ	Low-σ first
	σ & *PN*	Indifferent between low σ & low *PN*
	σ & *PW*	Low-σ first
	PN & μ	Indifferent between low *PN* & low μ
	PW & μ	Indifferent between low *PW* & low μ
	PN & *PW*	Indifferent between low *PN* & low *PW*, or No PC
High (CR = 50)	σ & μ	Low-σ first
	σ & *PN*	Indifferent between low σ & low *PN*, or No PC
	σ & *PW*	Low-σ first
	PN & μ	Low-*PN* first
	PW & μ	Low-*PW* first
	PN & *PW*	Low-*PN* first

Notably, these rules incorporate the effects of all factors, i.e., μ, σ, *PW*, and *PN*, simultaneously within their formulations based on Equations (A2) and (A3) in Appendix A. These three rules are compared against the base-case sequencing rule "low-σ first" across 486 cases (81 environments $\times$ 6 cost ratios). Although the "low-σ first" rule is able to find the best sequence in 324 cases with an accuracy of 97.8%, it cannot be used in the remaining 162 cases where both patient groups have the same service time variability but differ in other dimensions. The overall usefulness of the "low-σ first" rule in choosing the right sequence thus reduces to 65.2% when all cases are included.

Our results shows that the "low-σ' first" and "low-$\sigma'\mu'$ first" rules identify the best sequence in 417 and 409 of the 486 cases, respectively (i.e., with respective accuracies of 85.8% and 84.2%). The "low-Cv' first" rule is most successful in finding the best sequence in 477 cases (i.e., with an accuracy of 98.1%). Most exceptions occur when $CR = 50$; when these exceptions are excluded, the accuracy level of the "low-Cv' first" rule increases to 99.0%. We thus conclude that the universal Dome rule combined with the "low-Cv' first" sequencing rule is a good universal AS for most outpatient environments where cost ratios are likely to be less than 50.

As a further illustration, let us assume a clinic with two patient groups with $N_1{:}N_2 = 5{:}10$, $\mu_1{:}\mu_2 = 2{:}1$, $\sigma_1{:}\sigma_2 = 0.8{:}0.4$, $PN_1{:}PN_2 = 15{:}30$, and $PW_1{:}PW_2 = 30{:}15$ (Environment 46 in Appendix B). The revised coefficients are computed as $Cv'_1 = \sigma'_1/\mu'_1 = 1.564/2.30 = 0.680$ and $Cv'_2 = \sigma'_2/\mu'_2 = 0.7507/0.85 = 0.883$ for patient groups 1 and 2, respectively, using Equations (A2) and (A3) in Appendix A. The "low-Cv' first" rule suggests that the first patient group should be sequenced earlier in the session independent of the choice of the cost ratio. This is consistent with simulation results listed in Appendix B, which shows "PC1-first" as the dominant best sequence for all CR values from 1 through 50. This corresponds to sequencing the group with high μ, high σ, high *PW*," and low *PN* earlier in the session. This result also shows that some of the best rules identified in prior research, such as the "low-σ first" or "low-μ first" rules, are not necessarily the best when evaluated with multiple factors. The "low-Cv' first" rule, which considers the combined effects of μ, σ, *PW*, and *PN*, is a better alternative.

Another advantage of the proposed sequencing rule is that it can be easily incorporated into a decision support tool. An open source online tool developed for the universal Dome rule (i.e., the Online Appointment Scheduling Tool) is extended with a new module on patient classification. Practitioners can use such tools to design good appointment schedules for their clinics.

7. Sensitivity Analyses

In this section, we relax some of the assumptions in the simulation model to assess the sensitivity of the results discussed in §6. First, we test two additional service time distributions in comparison with the lognormal

distribution used in our main experiments. Next, we introduce unpunctual patients, given that patients may arrive early or late for their appointments. Last, we relax the assumption that walk-ins of the same patient class can arrive *only* during the periods preassigned to their class. We call this scenario "walk-ins with mixed arrivals" to represent cases where clinics may not completely enforce walk-ins of the same patient class to arrive in the designated periods of a session.

In our sensitivity analysis, we examine a clinic with two patient classes, PC1 and PC2, such that the size of each group is $N_1{:}N_2 = 10{:}5$, the mean and standard deviation of service times are $\mu_1{:}\mu_2 = 10{:}20$ minutes and $\sigma_1{:}\sigma_2 = 3{:}12$ minutes (i.e., $Cv_1{:}Cv_2 = 0.3{:}0.6$), and the no-show and walk-in probabilities are $PN_1{:}PN_2 = 15{:}15$ and $PW_1{:}PW_2 = 15{:}15$, respectively. Hence, the target length of the clinic is 200 minutes, which represents a typical half-day session. The mean number of walk-ins per session is 2.25 walk-ins, with 1.5 PC1 and 0.75 PC2 walk-ins. When a walk-in occurs with mixed arrivals, the probability is 2/3 for a PC1 and 1/3 for a PC2 walk-in.

The comparisons of the results are based on cost ratios $CR = 1, 15,$ and 50 to respectively represent low, medium, and high valuations of a doctor's time relative to a patient's time. Under this scenario, our proposed universal Dome rule coupled with the low revised coefficient of variation (Cv') sequencing rule, referred to in short as the "Dome," recommends scheduling the PC1 patients first, independent of the cost ratio. This corresponds to sequencing the group with low μ and low σ first, all else (i.e., *PN* and *PW*) being equal. We compare this appointment system with the traditional rules (i.e., 2BEG and IBFI) combined with both sequencing options tested, i.e., PC1 or PC2 patients first. Our results confirm that the "PC1-first" sequencing rule combined with our universal Dome rule is the best appointment system in the sensitivity analysis. Similarly, 2BEG and IBFI rules also perform better under the same "PC1-first" sequencing rule. Therefore, for purposes of brevity, we tabulate results for "PC1-first" sequencing only. We summarize our important findings below.

7.1. Service Time Distribution

Although some studies have reported that the lognormal distribution is the best-fitting distribution for service times collected empirically (O'Keefe 1985, Klassen and Rohleder 1996, Dexter 1999, Cayirli et al. 2006), it is important to validate the efficacy of our proposed Dome rule under different service time distributions. We test two other service time distributions that are commonly cited in the literature: the Gamma (Walter 1973, Yang et al. 1998, Vanden Bosch and Dietz 2000) and Beta (Williams et al. 2014) distributions. Table 5 shows the total cost performance for cost ratios $CR = 1, 15,$ and 50 for each AS under the three service time distributions. Superscripts indicate homogeneous groups that are not statistically different using the Bonferroni test ($\alpha = 5\%$). For example, the total cost performance of the Dome rule with PC1 sequencing is statistically indifferent when either the lognormal or Gamma distribution is used to model the service times, yet its performance deteriorates significantly when Beta distribution is used. The second column tabulates the percentage differences in total cost in comparison with the lognormal distribution used in the main experiments, with the statistically insignificant differences indicated by a superscript dagger in Table 5.

The results indicate that the total cost performance is rather insensitive to the choice of service time distribution. Most importantly, the relative performance of the appointment systems remains the same, and the Dome rule continues to outperform under different service time distributions. The largest percentage increase in total cost is 3.67%, with higher differences observed for the Beta distribution (2.11%–3.67%) and lower differences for the Gamma distribution (0.47%–2.45%). The Dome rule is more robust than the 2BEG and IBFI rules to different service time distributions. Overall, these results affirm the findings by Ho and Lau (1999) that system performance is affected primarily by the coefficient of variation of the service time distribution but not by its skewness, kurtosis, or other shape parameters.

7.2. Unpunctual Patients

To assess the sensitivity of the results to unpunctual patients, two unpunctuality scenarios are examined; we call these Unp1 and Unp2. In the Unp1 scenario, most patients arrive early, with a mean earliness of 13 minutes and a standard deviation of 20 minutes. In the Unp2 scenario, patients are on average punctual, with a mean earliness of 0 minutes and a standard deviation of 20 minutes. The latter scenario represents a more adverse situation where 50% of patients are late. These parameters are chosen within the ranges reported by empirical studies, which confirm that patients are usually on time or early for their appointments (Blanco White and Pike 1964, Fetter and Thompson 1966, Cox et al. 1985, Harper and Gamlin 2003, Williams et al. 2014).

Although several other distributions have been cited in the literature for modeling patient unpunctuality (Tai and Williams 2009, Eftakhari et al. 2012, Cheong et al. 2013), the normal distribution is the most commonly

Table 5. Total Cost Performance Under Different Service Time Distributions

AS service time distribution	$TC(CR=1)$	% difference	$TC(CR=15)$	% difference	$TC(CR=50)$	% difference
Dome–PC1_Lognormal	13.842[a]	—	63.888[e]	—	166.873[i]	
Dome–PC1_Gamma	13.987[a]	1.05%[†]	64.888[ef]	1.56%[†]	169.511[ij]	1.58%[†]
Dome–PC1_Beta	14.216	2.71%	65.671[f]	2.79%	171.526[j]	2.79%
2BEG–PC1_Lognormal	19.382[b]	—	63.715	—	174.546	—
2BEG–PC1_Gamma	19.473[bc]	0.47%[†]	64.840[g]	1.77%	178.258[k]	2.13%
2BEG–PC1_Beta	19.790[c]	2.11%	65.669[g]	3.07%	180.367[k]	3.33%
IBFI–PC1_Lognormal	15.345[d]	—	72.992	—	217.110	—
IBFI–PC1_Gamma	15.507[d]	1.05%[†]	74.626[h]	2.24%	222.424[l]	2.45%
IBFI–PC1_Beta	15.805	3.00%	75.598[h]	3.57%	225.082[l]	3.67%

Note. Superscripts indicate homogeneous groups that are not statistically different at $\alpha = 5\%$; "% difference" calculated for Gamma and Beta in comparison to the lognormal distribution.

[†]Indicates statistically insignificant difference at $\alpha = 5\%$ using the Bonferroni test.

used and supported empirically (Swartzman 1970, Dexter 1999, Cayirli et al. 2006, Klassen and Yoogalingam 2014) and therefore chosen for our sensitivity analysis. Whereas Alexopoulos et al. (2008) report that the Johnson distribution is a better fit for their empirical data on patient unpunctuality, they also note that the penalty may not be high as long as the first two moments, i.e., the mean and variance, are estimated correctly.

The results in Table 6 show that the Dome rule still performs the best when patients are unpunctual. In general, the total costs improve when patients arrive early on average under scenario Unp1, yet they deteriorate when more patients arrive late in scenario Unp2. The IBFI rule is the least robust, whereas the 2BEG rule also deteriorates fairly rapidly in scenario Unp2. The Dome rule handles the late arrivals fairly well, with total costs increasing by 7.66% to 13.27% in scenario Unp2, and it even increases its dominance over the traditional rules at the higher unpunctuality level.

7.3. Walk-Ins with Mixed Arrivals

Our baseline simulation model restricts the arrivals of walk-ins such that walk-ins of each patient class are allowed to arrive *only* during the period assigned to that class. The underlying assumption is that clinics that implement patient classification will inform their patients to come only during the periods assigned to them. For example, if a clinic uses an appointment system that schedules PC1 patients first, walk-ins who belong to patient class 1 are advised to arrive only during the first part of the clinic session.

When we relax this assumption, our results indicate that the changes in total cost are not statistically significant at $\alpha = 5\%$ except for when there is a low cost ratio (see Table 7). The Dome rule is the most sensitive to this assumption, with an increase of 6.13% at $CR = 1$. This is expected, since the Dome rule fine-tunes the appointment intervals for each patient class arriving in different parts of a session under the original assumption. This is followed by IBFI, whereas 2BEG is the most robust when mixed arrivals of walk-ins are assumed. For 2BEG, the total cost differences are statistically insignificant at $\alpha = 5\%$ under all cost ratios. This is likely due

Table 6. Total Cost Performance Under Different Scenarios for Patient Unpunctuality

AS unpunctuality	$TC(CR=1)$	% difference	$TC(CR=15)$	% difference	$TC(CR=50)$	% difference
Dome–PC1_Punctual	13.842[a]	—	63.888	—	166.873[e]	
Dome–PC1_Unp1	13.646[a]	−1.42%[†]	62.829	−1.66%[†]	165.140[e]	−1.04%[†]
Dome–PC1_Unp2	15.679	13.27%	70.719	10.69%	179.655	7.66%
2BEG–PC1_Punctual	19.382[b]	—	63.715[d]	—	174.546[f]	—
2BEG–PC1_Unp1	19.370[b]	−0.06%[†]	63.185[d]	−0.83%[†]	172.721[f]	−1.05%[†]
2BEG–PC1_Unp2	19.213	−0.87%	72.766	14.21%	206.650	18.39%
IBFI–PC1_Punctual	15.345[c]	—	72.992	—	217.110	—
IBFI–PC1_Unp1	15.420[c]	0.49%[†]	70.280	−3.72%	207.431	−4.46%
IBFI–PC1_Unp2	17.189	12.02%	88.693	21.51%	267.454	23.19%

Note. Superscripts indicate homogeneous groups that are not statistically different at $\alpha = 5\%$; "% difference" calculated for unpunctuality scenarios 1 and 2 in comparison to the punctual case.

[†]Indicates statistically insignificant difference at $\alpha = 5\%$ using the Bonferroni test.

Table 7. Total Cost Performance When Walk-In Arrivals Are Mixed

AS service time distribution	$TC(CR = 1)$	% difference	$TC(CR = 15)$	% difference	$TC(CR = 50)$	% difference
Dome–PC1_Orig	13.842	—	63.888[b]	—	166.873[e]	—
Dome–PC1_Mixed	14.690	6.13%	63.387[b]	−0.78%[†]	164.774[e]	−1.26%[†]
2BEG–PC1_Orig	19.382[a]	—	63.715[c]	—	174.546[f]	
2BEG–PC1_Mixed	19.160[a]	−1.15%[†]	63.560[c]	−0.24%[†]	174.560[f]	0.01%[†]
IBFI–PC1_Orig	15.345	—	72.992[d]	—	217.110[g]	—
IBFI–PC1_Mixed	15.872	3.44%	73.937[d]	1.29%[†]	219.098[g]	0.92%[†]

Note. Superscripts indicate homogeneous groups that are not statistically different at $\alpha = 5\%$; "% difference" calculated for mixed walk-in arrivals in comparison to the original case.

[†]Indicates statistically insignificant difference at $\alpha = 5\%$ using the Bonferroni test.

to 2BEG preloading the session with more patients such that it is least affected by increased randomness in the latter parts of the session. Last, but not the least, the Dome rule with PC1 sequencing remains the best choice under all cost ratios regardless of the assumed pattern of walk-in arrivals.

8. Conclusions and Future Directions

This research proposes an appointment system that combines the universal Dome rule with patient classification for further improvements through sequencing and interval adjustment. To the best of our knowledge, this is the first study that applies multiple classification criteria simultaneously to differentiate patient groups, including the mean and the standard deviation of service times (μ and σ), the probability of no-shows (PN), and the probability of walk-ins (PW). Using the scenario of two patient groups, the rule sequences either the first or the second group earlier in a session while adjusting appointment interval lengths for the unique characteristics of each group.

First, we compare our proposed AS, the Dome rule with PC, against the Dome rule with no PC. Our results show that the inclusion of patient classification is better or at least equally good for 98.8% of the cases examined. The average improvement in total cost as a weighted sum of patients' wait time and the doctor's idle time and overtime is 7.37%, and it can be as high as 22% for a wide range of environments simulated in our study. It can be observed that the improvement increases as the cost ratio decreases, which suggests greater value in PC in patient-centered clinics. Whereas the value of PC decreases by almost half (i.e., from 9.5% to 5.2%) in our simulated clinics as the cost ratio increases from 1 to 50, choosing the *right* sequence becomes increasingly more important. In other words, in environments with high cost ratios, such as surgical clinics, the value of patient classification may decrease, but if it is used, it is important to choose the right sequence. Otherwise, using a wrong sequence is worse than having no patient classification.

Next, the Dome rule with PC is compared against the traditional rules IBFI and 2BEG with PC. The results affirm that the Dome rule with PC performs best, with average (maximum) total cost improvements of 4.17% (17%). These improvements show a concave pattern with the largest benefits at the extreme low and high cost ratios tested at 1 and 50. On the other hand, at moderate cost ratios tested at 5 and 10, the traditional rules with PC perform as well as the proposed Dome rule with PC—even slightly better in a few exceptions. However, this does not diminish the real advantage of our proposed AS, which is its "universality" that allows the decision maker to select the right combination of appointment rule and sequence, once the clinic parameters for different patient groups are estimated in terms of their coefficient of variation of service times (Cv), probability of no-shows (PN), probability of walk-ins (PW), and cost ratio of doctor-to-patient time (CR).

The choice of the best patient sequence for the Dome rule depends on the specific environment, as well as the cost ratio and relative group size through their interactions with the other factors. When two groups differ by only one factor, sequencing the low group first is usually the best, with some exceptions that occur at high cost ratios. For example, when the high-σ group is larger in size at a high cost ratio (such as $CR = 50$), the "high-σ first" sequence is best. Similarly, when the high-μ group is larger in size at a high cost ratio, the "high-μ first" sequence is best. These results are interesting because they dispute previous findings in the literature that "low-σ first" and/or "low-μ first" rules are the overall best within the limited set of factors and cost ratios examined in their studies.

When two groups differ by more than one factor, the decision of choosing the right sequence can be very complex. We thus propose and compare three heuristic rules that are variants of the "low-σ first" rule for deciding which patient group to sequence first. Our results show that the "low-Cv' first" rule, which sequences the group with the lower revised coefficient of variation (σ'/μ') earlier in a clinic session, is the best. This rule indirectly considers the combined effects of *all* four factors, i.e., μ, σ, *PW*, and *PN*, through the σ' and μ', revised based on the no-show and walk-in probabilities. This heuristic successfully chooses the best sequence with an accuracy of 98% for the cases investigated in our simulation experiments. In summary, the universal Dome rule combined with the new heuristic for sequencing offers a simple and highly accurate approach to build a good AS for a wide range of clinical environments. An online decision support tool called the Online Appointment Scheduling Tool has been developed to help disseminate its adoption among practitioners.

Our sensitivity analyses conducted to test the robustness of our results indicate that general findings are valid when assumptions on punctual patients and service time distributions are relaxed. The proposed universal Dome rule with sequencing continues to outperform its traditional counterparts when unpunctual patients are allowed, or even if the service times are better represented by distributions other than the lognormal, such as Gamma or Beta distributions. We also investigate the effects of mixed arrivals of walk-ins of different patient classes throughout a clinic session. Although the total cost performance of the Dome rule may deteriorate slightly, its overall superiority over the traditional appointment rules remains.

Future research may investigate the use of other criteria for patient classification and test their efficacy in AS design. Patient classification with more than two groups may be evaluated for added value of more refined groupings within the patient population. Finally, real applications of our proposed AS will offer further insights to validate its robustness in generating good appointment schedules for real clinics.

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Appendix A. The Universal Dome Rule

The universal Dome rule introduced by Cayirli et al. (2012) explicitly takes into account the presence of no-shows and walk-ins to mitigate their disruptive effects by adjusting the appointment rule. The following formula is used to calculate the number of appointments scheduled, N, in order to serve a target number of T patients in a clinic session, given the probabilities of no-shows *PN* and walk-ins *PW*:

$$N = T/(1 - PN + PW). \tag{A1}$$

The mean and variability of service times are revised to adjust the lengths of appointment intervals accordingly. Assuming a no-show to be a patient with zero service time and a walk-in as two patients arriving to the same appointment slot, the revised mean (μ') and variance (σ'^2) are computed based on the original mean and standard deviation of service times (μ and σ) as follows:

$$\mu' = (1 - PN + PW)\mu, \tag{A2}$$

$$\sigma'^2 = (1 - PN - PW)(\sigma^2 + (PN - PW)^2\mu^2) + PN(1 - PN + PW)^2\mu^2 + PW(2\sigma^2 + (1 + PN - PW)^2\mu^2). \tag{A3}$$

The universal Dome rule calculates the appointment time given to patient i, A_i, using the following mathematical formulae:

$$A_i = \max\{0, k(i-1)\mu' - \sigma'\sqrt{i}\cdot\pi\}, \quad \text{with } \pi = (N+i)/(N-1), \tag{A4}$$

$$k = \{0.9973 - 0.103[0.005765 \cdot CR(1-PN) + (CR(1-PN))^{-0.3481}] - 0.10699[Cv^{1.257}] - 0.627[(N(1-PN))^{-0.8579}] - 0.007574[|CR(1-PW) - 2.143|^{0.9682} - 0.622 \cdot CR(1-PW)] + 0.004855(CR^{0.8913})\}^{-1.898}, \tag{A5}$$

where k is the planning constant, which is a function of a set of clinical parameters, including the number of patients scheduled per session N, the probability of no-shows *PN*, the probability of walk-ins *PW*, the coefficient of variation of service times *Cv*, and the cost ratio of server's time relative to patient's time *CR*. The original formulation (A4) is based on the logic that with a higher standard deviation in service times, patients are called to arrive earlier given the increased variability (Yang et al. 1998). Cayirli et al. (2012) introduce the parameter π in Equation (A4) that serves to create a dome pattern in the appointment intervals. In application, once these parameters (i.e., N, *PN*, *PW*, *Cv*, and *CR*) are estimated for a given clinic, the k-value is calculated using Equation (A5) and then used in Equation (A4) to compute the appointment times, with the revised mean and standard deviation of service times (μ' and σ', respectively) from (A2) and (A3). It should be noted that *Cv* (i.e., σ/μ) in Equation (A5) is computed using the original values of the mean and standard deviation of service times.

A methodology consisting of simulation and nonlinear regression is used by Cayirli et al. (2012) to derive the k-equation (A5). Since one should not extrapolate in regression analysis, it is important to note that the universal rule is valid only for clinical environments represented within the range of parameters used to derive the k-equation (A5). Specifically, it is applicable for clinics where N is in a range of 5 to 25, *Cv* is 0.2 to 1, and both *PN* and *PW* are within 0%–40%.

Appendix B. Dome with Best Sequencing Identified and Compared with Benchmark and Traditional Rules (N_1:N_2 = 5:10)

Environment					$CR = 1$			$CR = 10$			$CR = 50$			Dominant
No.	μ_1:μ_2	σ_1:σ_2	PN_1:PN_2 (%)	PW_1:PW_2 (%)	Seq	No PC (%)	Trad (%)	Seq	No PC (%)	Trad (%)	Seq	No PC (%)	Trad (%)	best sequence[a]
1	2:1	0.8:0.4	15:15	15:15	PC2	18.44	11.87	PC2	11.10	0.31†	PC2	8.51	5.11	PC2
2	2:1	0.4:0.4	15:15	15:15	PC2*	16.46	13.35	PC2*	11.44	−0.42†	PC2*	9.96	4.82	PC2/PC1
3	2:1	0.4:0.8	15:15	15:15	PC1*	10.05	10.16	PC1	10.37	0.92†	PC1*	8.28	8.67	PC1
4	1:2	0.8:0.4	15:15	15:15	PC2*	9.02	9.36	PC2*	10.69	−0.79†	PC2	8.36	4.36	PC2
5	1:2	0.4:0.4	15:15	15:15	PC1*	11.07	10.69	PC2*	10.09	−2.23	PC2	8.81	2.05	PC2
6	1:2	0.4:0.8	15:15	15:15	PC1*	12.04	9.45	PC1*	8.06	−1.06†	PC2*	6.99	4.81	PC1/PC2
7	1:1	0.8:0.4	15:15	15:15	PC2	7.23	10.87	PC2	2.88	−0.39†	PC2	0.78†	6.79	PC2
8	1:1	0.4:0.4	15:15	15:15	N/A	N/A	13.35	N/A	N/A	−0.86†	N/A	N/A	5.31	N/A
9	1:1	0.4:0.8	15:15	15:15	PC1	5.43	9.14	PC1	1.98	0.29†	PC1*	0.58†	9.45	PC1
10	2:1	0.8:0.4	15:15	15:30	PC2	21.73	15.28	PC2*	14.90	−0.03†	PC2*	14.30	5.90	PC2
11	2:1	0.4:0.4	15:15	15:30	PC2*	20.28	16.99	PC1*	16.87	−0.13†	PC1*	15.77	5.89	PC1/PC2
12	2:1	0.4:0.8	15:15	15:30	PC1*	14.86	11.06	PC1	15.15	1.34	PC1	13.00	9.46	PC1
13	1:2	0.8:0.4	15:15	15:30	PC2*	0.80†	10.63	PC2*	3.75	−0.69†	PC2	3.70	7.31	PC2
14	1:2	0.4:0.4	15:15	15:30	PC1*	3.89	8.29	PC1*	2.32	−1.85	PC2	2.92	5.57	PC2
15	1:2	0.4:0.8	15:15	15:30	PC1*	5.79	8.26	PC1*	2.28	−0.66†	PC2	1.65	7.30	PC2
16	1:1	0.8:0.4	15:15	15:30	PC2*	5.85	12.69	PC2	2.85	−0.57†	PC2	2.24	8.18	PC2
17	1:1	0.4:0.4	15:15	15:30	PC1*	0.23†	12.07	PC1*	−0.25†	−0.85†	PC2	0.48†	6.91	PC2/No PC
18	1:1	0.4:0.8	15:15	15:30	PC1*	4.42	9.26	PC1	1.14†	0.27†	PC2*	10.06	−0.73†	PC1
19	2:1	0.8:0.4	15:15	30:15	PC2	12.81	9.91	PC2	4.38	0.15†	PC2*	0.34†	4.83	PC2
20	2:1	0.4:0.4	15:15	30:15	PC2	10.15	10.66	PC2*	4.23	0.59†	PC1	2.26	5.60	None[1]
21	2:1	0.4:0.8	15:15	30:15	PC1*	2.28†	11.87	PC1*	4.28	0.95†	PC1	3.79	9.96	PC1
22	1:2	0.8:0.4	15:15	30:15	PC2*	13.62	8.80	PC2	15.76	−0.16†	PC2	12.45	4.90	PC2
23	1:2	0.4:0.4	15:15	30:15	PC1*	16.45	13.77	PC2*	16.17	−1.20	PC2	13.89	2.99	PC2
24	1:2	0.4:0.8	15:15	30:15	PC1*	17.03	12.00	PC2*	12.80	−1.27	PC2*	11.84	5.49	PC1/PC2
25	1:1	0.8:0.4	15:15	30:15	PC2	6.79	9.98	PC2	2.57	−0.12†	PC2*	−0.85†	6.92	PC2
26	1:1	0.4:0.4	15:15	30:15	PC2*	1.19†	11.84	PC2*	1.34†	−0.45†	PC2*	−0.02†	5.91	PC2/PC1/No PC
27	1:1	0.4:0.8	15:15	30:15	PC1	5.66	10.88	PC1	2.73	0.22†	PC1*	1.81	9.70	PC1
28	2:1	0.8:0.4	15:30	15:15	PC2*	17.09	10.84	PC2*	11.51	1.03†	PC2*	10.67	8.31	PC2/PC1
29	2:1	0.4:0.4	15:30	15:15	PC1*	15.86	11.08	PC1*	13.86	0.37†	PC1*	12.87	8.52	PC1/PC2
30	2:1	0.4:0.8	15:30	15:15	PC1*	13.42	6.48	PC1	13.64	2.66	PC1	11.39	11.26	PC1
31	1:2	0.8:0.4	15:30	15:15	PC2*	6.72	9.27	PC2*	4.22	−2.69	PC2*	3.18	7.66	PC2/PC1
32	1:2	0.4:0.4	15:30	15:15	PC1*	11.95	7.79	PC1*	5.66	−1.84	PC1*	3.15	6.16	PC1/PC2
33	1:2	0.4:0.8	15:30	15:15	PC1*	12.25	6.07	PC1	5.76	−0.52†	PC1*	3.05	8.68	PC1
34	1:1	0.8:0.4	15:30	15:15	PC2*	4.73	9.97	PC2*	1.78	0.22†	PC2*	0.92†	9.62	PC2/PC1
35	1:1	0.4:0.4	15:30	15:15	PC1*	2.59†	9.01	PC1*	1.34†	0.08†	PC1*	0.88†	9.45	PC1/PC2/No PC
36	1:1	0.4:0.8	15:30	15:15	PC1	6.60	5.64	PC1	3.00	1.83	PC1*	1.04†	12.04	PC1

Appendix B. (Continued)

Environment					$CR = 1$			$CR = 10$			$CR = 50$			Dominant
No.	μ_1:μ_2	σ_1:σ_2	PN_1:PN_2 (%)	PW_1:PW_2 (%)	Seq	No PC (%)	Trad (%)	Seq	No PC (%)	Trad (%)	Seq	No PC (%)	Trad (%)	best sequence[a]
37	2:1	0.8:0.4	15:30	15:30	PC1*	21.01	14.19	PC1*	16.45	1.14	PC1	16.04	9.75	PC1
38	2:1	0.4:0.4	15:30	15:30	PC1*	20.06	10.88	PC1	19.61	1.60	PC1	17.99	9.93	PC1
39	2:1	0.4:0.8	15:30	15:30	PC1*	17.48	7.67	PC1	17.74	2.92	PC1	15.09	11.87	PC1
40	1:2	0.8:0.4	15:30	15:30	PC2*	0.67†	7.48	PC2*	0.47†	−1.02†	PC2	0.73†	10.30	PC2
41	1:2	0.4:0.4	15:30	15:30	PC2*	4.95	5.96	PC2*	0.45†	−1.17†	PC2	−0.08†	9.21	PC2
42	1:2	0.4:0.8	15:30	15:30	PC2*	6.00	5.71	PC2*	0.45†	0.04†	PC2	−0.43†	11.25	PC2
43	1:1	0.8:0.4	15:30	15:30	PC2*	4.71	11.66	PC2*	2.03	0.29†	PC2	2.41	10.91	PC2
44	1:1	0.4:0.4	15:30	15:30	PC1*	2.93†	8.43	PC1*	1.54†	0.30†	PC1*	0.82†	10.43	PC1/PC2/No PC
45	1:1	0.4:0.8	15:30	15:30	PC1*	5.77	6.27	PC1	2.12	1.68	PC1*	0.12†	12.74	PC1
46	2:1	0.8:0.4	15:30	30:15	PC1*	11.05	9.10	PC1*	5.26	0.98†	PC1	4.69	9.21	PC1
47	2:1	0.4:0.4	15–30	30:15	PC1*	9.06	9.81	PC1*	6.34	0.53†	PC1	6.75	10.02	PC1
48	2:1	0.4:0.8	15:30	30:15	PC1*	5.58	8.52	PC1	8.16	2.85	PC1	7.30	12.51	PC1
49	1:2	0.8:0.4	15:30	30:15	PC2*	11.55	11.21	PC2*	10.06	−1.59	PC2*	7.46	7.69	PC2/PC1
50	1:2	0.4:0.4	15:30	30:15	PC1*	17.08	9.91	PC1*	10.24	−2.50	PC1*	8.81	6.90	PC1/PC2
51	1:2	0.4:0.8	15:30	30:15	PC1	17.19	8.01	PC1*	10.37	−0.59†	PC1*	8.51	8.91	PC1
52	1:1	0.8:0.4	15:30	30:15	PC2*	4.30	9.34	PC2*	1.86	0.73†	PC2*	−0.42†	9.50	PC2/PC1
53	1:1	0.4:0.4	15:30	30:15	PC1*	1.87†	10.73	PC1*	0.99†	−0.07†	PC1*	1.31	9.96	PC1/PC2
54	1:1	0.4:0.8	15:30	30:15	PC1*	6.71	7.45	PC1*	3.87	1.72	PC1*	2.51	12.38	PC1/PC2
55	2:1	0.8:0.4	30:15	15:15	PC2	18.52	9.91	PC2	8.01	−0.47†	PC2	4.01	7.03	PC2
56	2:1	0.4:0.4	30:15	15:15	PC2	16.91	11.11	PC2	7.91	−0.77†	PC2	4.62	6.59	PC2
57	2:1	0.4:0.8	30:15	15:15	PC2*	7.72	10.89	PC2*	4.21	−0.62†	PC2*	3.17	9.49	PC2/PC1
58	1:2	0.8:0.4	30:15	15:15	PC2*	11.09	7.68	PC2	12.71	0.25†	PC2	10.21	5.93	PC2
59	1:2	0.4:0.4	30:15	15:15	PC2*	10.76	9.80	PC2*	12.42	−0.96†	PC2	10.71	4.23	PC2
60	1:2	0.4:0.8	30:15	15:15	PC1*	11.19	10.06	PC1*	8.96	−1.05†	PC1*	8.57	6.56	PC1/PC2
61	1:1	0.8:0.4	30:15	15:15	PC2	8.30	9.20	PC2	3.75	0.19†	PC2	1.24	8.24	PC2
62	1:1	0.4:0.4	30:15	15:15	PC2*	2.53†	10.93	PC2*	1.89	−0.46†	PC2	0.95†	7.34	PC2
63	1:1	0.4:0.8	30:15	15:15	PC1*	3.55	9.97	PC1*	1.38†	0.81†	PC1*	0.28†	9.94	PC1/PC2
64	2:1	0.8:0.4	30:15	15:30	PC2	22.31	12.97	PC2	12.55	−0.82†	PC2	10.42	7.39	PC2
65	2:1	0.4:0.4	30:15	15:30	PC2	21.03	14.19	PC2*	12.23	−1.11†	PC2*	10.83	7.06	PC2
66	2:1	0.4:0.8	30:15	15:30	PC1*	12.29	14.27	PC1*	9.91	0.01†	PC1*	8.34	9.59	PC1/PC2
67	1:2	0.8:0.4	30:15	15:30	PC2*	2.54†	9.29	PC2*	6.00	0.22†	PC2	5.40	8.30	PC2
68	1:2	0.4:0.4	30:15	15:30	PC2*	3.36	9.14	PC2*	4.81	−0.80†	PC2	4.81	6.96	PC2
69	1:2	0.4:0.8	30:15	15:30	PC2*	8.92,	4.96	PC1*	3.12	−0.47†	PC2	3.35	8.47	PC2
70	1:1	0.8:0.4	30:15	15:30	PC2*	6.73	10.86	PC2*	3.40	−0.03†	PC2	2.09	9.41	PC2
71	1:1	0.4:0.4	30:15	15:30	PC2*	1.30†	12.57	PC2*	0.60†	−0.72†	PC2	0.77†	8.58	PC2/No PC
72	1:1	0.4:0.8	30:15	15:30	PC1*	2.33†	9.89	PC1*	0.81†	1.05†	PC1*	−0.54†	10.80	PC1/PC2

Appendix B. (Continued)

Environment					$CR = 1$			$CR = 10$			$CR = 50$			Dominant
No.	μ_1:μ_2	σ_1:σ_2	PN_1:PN_2 (%)	PW_1:PW_2 (%)	Seq	No PC (%)	Trad (%)	Seq	No PC (%)	Trad (%)	Seq	No PC (%)	Trad (%)	best sequence[a]
73	2:1	0.8:0.4	30:15	30:15	PC2	13.03	8.43	PC2	1.42	−0.31†	PC2*	−4.90	6.36	PC2
74	2:1	0.4:0.4	30:15	30:15	PC2	11.17	9.07	PC2	1.27	−0.65†	PC2*	−3.93	6.05	PC2
75	2:1	0.4:0.8	30:15	30:15	PC1*	2.46†	9.42	PC1*	−1.11†	−0.92†	PC1	−1.19	9.79	PC1
76	1:2	0.8:0.4	30:15	30:15	PC2	15.40	7.05	PC2	17.64	0.71†	PC2	13.97	6.03	PC2
77	1:2	0.4:0.4	30:15	30:15	PC2*	15.97	9.48	PC2	18.36	−0.10†	PC2	15.56	4.75	PC2
78	1:2	0.4:0.8	30:15	30:15	PC2*	16.04	12.26	PC2*	14.66	−0.29†	PC2	13.21	6.85	PC2
79	1:1	0.8:0.4	30:15	30:15	PC2	8.46	8.59	PC2	3.42	0.51†	PC2*	−0.65†	8.24	PC2
80	1:1	0.4:0.4	30:15	30:15	PC2*	3.54	9.85	PC2	2.54	−0.20†	PC2*	0.56†	7.61	PC2
81	1:1	0.4:0.8	30:15	30:15	PC1*	11.62	4.15	PC1*	2.07	0.84†	PC1*	1.64	10.52	PC1/PC2

Note. Seq, best sequencing is listed as "PC1 or PC2-first"; No PC, total cost improvement relative to Dome with "No PC"; Trad, total cost improvement relative to the best traditional rule with PC.

[a]"Dominant best sequence" refers to the sequencing rule that dominates as the best choice across *all CR* values; "None" means that there is no dominant sequence where different sequencing rules are desirable for different *CR* values, and "N/A" means that not applicable (in Environment 8, where both groups are identical).

*Indicates PC1 and PC2 are statistically indifferent at $\alpha = 5\%$; †indicates statistically insignificant difference at $\alpha = 5\%$ using the Bonferroni test.

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