WIRTSCHAFTS
UNIVERSITÄT
WIEN VIENNA
UNIVERSITY OF
ECONOMICS
AND BUSINESS

EFMD
EQUIS
ACCREDITED

ePub^WU Institutional Repository

Stephan Gasser

Caveat Emptor: Does Bitcoin Improve Portfolio Diversification?

Paper

http://epub.wu.ac.at/

Caveat Emptor:
Does Bitcoin Improve Portfolio Diversification?

Alexander Eisl[*] Stephan M. Gasser[†] Karl Weinmayer[‡]

June 3, 2015

Abstract

Bitcoin is an unregulated digital currency originally introduced in 2008 without legal tender status. Based on a decentralized peer-to-peer network to confirm transactions and generate a limited amount of new bitcoins, it functions without the backing of a central bank or any other monitoring authority. In recent years, Bitcoin has seen increasing media coverage and trading volume, as well as major capital gains and losses in a high volatility environment. Interestingly, an analysis of Bitcoin returns shows remarkably low correlations with traditional investment assets such as other currencies, stocks, bonds or commodities such as gold or oil. In this paper, we shed light on the impact an investment in Bitcoin can have on an already well-diversified investment portfolio. Due to the non-normal nature of Bitcoin returns, we do not propose the classic mean-variance approach, but adopt a Conditional Value-at-Risk framework that does not require asset returns to be normally distributed. Our results indicate that Bitcoin should be included in optimal portfolios. Even though an investment in Bitcoin increases the CVaR of a portfolio, this additional risk is overcompensated by high returns leading to better risk-return ratios.

Keywords: Bitcoin, Portfolio Optimization, Conditional Value at Risk, Virtual Currencies

JEL classification: G11, G15, F31

1 Introduction

Originally introduced in 2008 in a paper titled "Bitcoin: A Peer-to-Peer Electronic Cash System" [1], Bitcoin is currently the most popular unregulated digital currency. While many forms of virtual currencies have been circulating for a number of years, and many more have come and gone more recently in the wake of Bitcoin, only Bitcoin has recently gained a market share that made people outside of a small group of technology enthusiasts aware of its existence.

In recent months, more and more online and offline businesses worldwide started to adopt Bitcoin as an alternative means of payment and even though Bitcoin does not have legal tender status, it has since seen increasing attention by regulators, academics, the media and the general public.[2] This is largely due to the fact that since Bitcoin is based on a decentralized peer-to-peer network to confirm transactions and generate a limited amount of new Bitcoins in due course, it functions without the backing of a central bank or any other monitoring authority. While this of course fuels the ongoing debate whether or not (and if, how) Bitcoin should be regulated, in a recent letter to the US Congress, then Federal Reserve Chairman Ben Bernanke echoed comments from former Federal Reserve Vice Chairman Alan Blinder that Bitcoin and digital currencies in general "may hold long-term promise" through innovations towards a "faster, more secure and efficient payment system." (Bernanke, 2013). He also added that while the Federal Reserve's authority to directly regulate Bitcoin is limited, it monitors the developments closely.

In the academic world, Bitcoin has drawn significant attention from law and computer science scholars. A number of papers has been published focusing for example on descriptive analysis of the Bitcoin network (Ron and Shamir, 2013), the potential risk of double-spending (Karame et al., 2012), as well as the implications of the availability of a public ledger containing all Bitcoin transaction ever made (Meiklejohn et al., 2013). However, few scientific papers have as of yet focused on analyzing Bitcoin from an financial economics point of view, even though

---

[1]The paper was written by an anonymous programmer/a group of programmers only known by the pseudonym of Satoshi Nakamoto in 2008 (Nakamoto, 2008). The accuracy of a recent Newsweek article claiming to have uncovered the true identify of Satoshi Nakamoto can not yet be ascertained.

[2]See for example recent publications in the Wall Street Journal or on bloomberg.com (e.g. Raskin (2013))

certain characteristics certainly provide for Bitcoin to be an interesting research subject (the historical development of the Bitcoin price - especially since the beginning of 2013 - being just one of them). Yermack (2013) for example argues that, even though Bitcoin has several characteristics usually associated with currencies, it does not behave like one and concludes that the high volatility of Bitcoin makes it look more like a "speculative investment similar to the Internet stocks of the late 1990s". Brière et al. (2013) provide a tentative first look at how Bitcoin might be of value in an investment portfolio optimization process.

In this paper, we extend the scarce economically-motivated literature on Bitcoin. An analysis of historic Bitcoin returns shows remarkably low correlation with traditional investment assets such as stocks, bonds, currencies or with commodities such as gold or oil, thus making it an interesting asset for portfolio diversification purposes. We therefore look at the impact an investment in Bitcoin might have on an already well-diversified investment portfolio. Due to the non-normal nature of Bitcoin returns, i.e. with the Bitcoin return distribution showing large excess kurtosis and positive skewness, we do not propose the classic mean-variance approach applied by Brière et al. (2013), but adopt a Conditional Value-at-Risk framework as a risk measure that does have better properties when asset returns are not normally distributed. In addition, rather than picking just one single point in time for our portfolio optimization, we apply a portfolio backtesting technique, tracking and evaluating the out-of-sample monthly and total portfolio performance over an investment horizon of more than 2.5 years, while rebalancing the portfolio weights on a monthly basis.

2 Methodology

In this study, we focus our analysis on the diversification effect of Bitcoin in an otherwise well-diversified portfolio. In other words, we analyze the effect that adding Bitcoin to the set of available assets has on the efficient frontier and the risk-return structure of that portfolio, and we show how the asset allocation changes and the share of Bitcoin develops over time. The

benefits of additional asset classes on portfolio diversification have been analyzed in previous studies. For example, in a related paper, Belousova and Dorfleitner (2012) show that adding different types of commodities to a portfolio can have a beneficial impact.

As already mentioned, Bitcoin has recently experienced an increasing amount of attention, also by the investment community, at least partly due to the fact that Bitcoin has shown a significant price increase with an exceptional returns pattern, especially since mid-2013. From a portfolio management perspective, however, we are not solely interested in the impact Bitcoin might have on an already well-diversified investment portfolio, but mostly in its effect on that portfolio's risk-return ratio, i.e. its Sharpe Ratio, as it is commonly being referred to. Therefore, our research questions can be summarized as follows:

Q1: How does the inclusion of Bitcoin affect the asset allocation of an already well-diversified portfolio?

Q2: Is the weight of Bitcoin in an already well-diversified portfolio robust with regard to the optimization procedure used?

Q3: Can Bitcoin improve the risk-return profile of an already well-diversified portfolio?

The details of our approach are explained in the following subsections. As Bitcoin's return distribution exhibits large deviations from a normal distribution, we choose a more robust optimization framework based on Conditional Value-at-Risk.

2.1   Mean-CVaR Approach

In the context of the standard CAPM, a mean-variance approach is used to calculate Sharpe Ratios and to determine the optimal portfolio given a number of risky assets. However, the mean-variance analysis requires that returns follow a normal distribution in order to allow for the use of variance as a risk measure. Otherwise, the variance is likely to underestimate the potential loss resulting from additional tail-risk and, hence, can lead to sub-optimal portfolio decisions (see, for example, Jorion (2001) and McNeil et al. (2005)).

In such a case, a risk measure better reflecting the downside risk is more favorable. One possible measure proposed in the literature is Value-at-Risk (VaR), which is the loss that will not be exceeded over a given time horizon at a given confidence level. This measure has received a lot of attention due to its inclusion in financial regulation (see, for instance, Jorion (1996) and Campbell et al. (2001)). Despite its popularity, VaR suffers from various shortcomings, like instability and difficult numerical estimation, when losses are not normally distributed. Additionally, VaR does not further quantify the amount by which this quantile can be exceeded (Rockafellar and Uryasev, 2002). Moreover, Artzner et al. (1999) show that VaR is not a coherent risk measure.

Therefore, we follow Rockafellar and Uryasev (2000) and adopt a different approach for our analysis, using Conditional Value at Risk (CVaR) as risk measure. We try to adhere to their notation to make the definitions comparable.

Define the cumulative distribution function $\Psi(\omega, \zeta)$ of a loss $z = f(\omega, y)$ as

$$\Psi(\omega, \zeta) = P\{y | f(\omega, y) \leq \zeta\}, \tag{1}$$

where ω = decision vector (i.e., portfolio weights),

ζ = a specific loss,

y = uncertainties (e.g. market variables) that affect the loss.

Then, the Value at Risk for a given confidence level α (ζ_α) is defined as

$$\zeta_\alpha(\omega) = \min\{\zeta | \Psi(\omega, \zeta) \geq \alpha\}. \tag{2}$$

The Conditional Value at Risk (CVaR$_\alpha$) is now the expected value of the loss, given that the loss is weakly exceeding the Value at Risk $\zeta_\alpha(\omega)$[3]:

---

[3]Rockafellar and Uryasev (2002) also refer to this as CVaR$^-$ or Tail VaR. In case of a continuous distribution, this coincides with the Expected Shortfall, a widely used risk measure.

$$CVaR_\alpha(\omega) = \frac{1}{1-\alpha} \int_{f(\omega,y) \geq \zeta_\alpha(\omega)} f(\omega,y)p(y)dy, \tag{3}$$

We then form optimal portfolios by minimizing CVaR$_\alpha$:

$$\min_\omega \text{CVaR}_\alpha(\omega) \tag{4}$$

$$\text{s.t.}$$

$$\omega^T \hat{\mu} = \overline{r}$$

$$\omega^T \mathbf{1} = 1$$

where $\quad \hat{\mu} =$ vector of excepted asset returns,

$\overline{r} =$ expected total return of the portfolio.

On the basis of the CVaR$_\alpha$ as defined in Equation 3, we calculate a risk-return ratio S similar to Campbell et al. (2001), which can be used as a performance indicator to evaluate the risk-return efficiency of portfolios in the same way as the Sharpe Ratio. For our analysis we choose a confidence level α of 95%. The optimal combination of assets is then found, when the risk-return ratio S is maximized, in other words, the higher the risk-return ratio S of a portfolio of risky assets, the better.

2.2 Portfolio Strategy

In order to evaluate the diversification effect of including Bitcoin into portfolios based on the described mean-CVaR approach, we adopt the view of a US Investor and construct well-diversified portfolios including various broad indices for equity, fixed-income, money market, commodity, real estate and alternative investment opportunities. We then apply a backtesting technique to calculate monthly out-of-sample portfolio returns based on the optimal weights w_i of each asset

i given the maximized risk-return ratio. We use the first 12 months of data from our sample period for the initial weights estimation, apply a 12-month rolling horizon to estimate portfolio weights throughout our investment period, and calculate the out-of-sample expected monthly returns as well as CVaRs. Afterwards we explore the effect of adding Bitcoin by comparing the risk-return ratios of the optimal portfolios. The weight optimization process for each optimal portfolio is thereby subject to various parameters defined in four different portfolio optimization frameworks described below. Additionally, in three out of four strategies, 3-month weights smoothing is applied based on an exponentially-weighted moving average (EWMA). Hence we end up with eight portfolios in total, four of which include Bitcoin.

Framework 1: Unconstrained Portfolio ($w_i \in \mathbb{R}$)

This portfolio framework does not apply any weight-related constraints to the optimization process and should therefore yield optimal portfolios with the highest risk-return ratios of all portfolio frameworks. The results from this strategy show the unbiased effect of adding a new asset to the portfolio mix. However, the risk-return maximization process might lead to both fairly large asset weights as well as extremely high weight rebalancing over time and render an implementation of such a strategy unfeasible. Due to the first 12 months of the total sample period being used to calculated the initial weights estimations, the investment period is 12 months shorter than the total sample period.

Framework 2: -100%/+100% Portfolio ($w_i \in \mathbb{R} : -1 \leq w_i \leq 1$)

This framework allows for asset weights to shift between +100% and -100%. Compared to the unconstrained portfolio optimization framework, we expect to see smoother weights rebalancing, which makes this strategy more interesting in terms of feasibility. Here, the first 12 months of data are again used to compute the initial portfolio weights, reducing the investment period to a time frame that is 12 months shorter than the total sample period.

Framework 3: Long-Only Portfolio ($w_i \in \mathbb{R}_0^+$)

In this framework, a short-selling constraint is imposed in order to reflect possible restrictions involved with short-selling certain assets that are included in each portfolio, thus effectively limiting both the sum of all asset weights as well as the weight of each asset in a portfolio to 100%. As of now, it is also not clear, whether or not a short position in Bitcoin is actually feasible. Again, the first 12 months of data are not part of the investment period, since they are used for the initial estimation of portfolio weights.

Framework 4: Equally-weighted Portfolio ($w_i = 1/N \quad \forall \quad i$)

The first three frameworks rely on the maximization of the risk-return ratio to find the optimal portfolio weights. In the equally weighted $1/N$ framework (with N being the number of assets available for the portfolio optimization process), the portfolios consist of equally-weighted assets, with the weights being constant over time. Therefore no optimization process is applied. We include this strategy based on the findings of DeMiguel et al. (2009), who show that an equally-weighted portfolio leads to comparable or even higher Sharpe Ratios in comparison to various portfolio optimization techniques, due to the poor predictive capacity of many commonly used risk and return measures. By including this approach, we acknowledge the prediction risks involved with the risk and return measures we apply in this paper (and consequently any resulting bias in our findings) and provide a possible solution with this framework not depending on estimating risk and return measures. As the portfolio weights are held constant over time, also no smoothing is here applied. Furthermore, the investment period equals the total sample period, since of course no initial portfolio weights estimation is necessary here.

3 Data

For Bitcoin price data we use the CoinDesk Bitcoin USD Price Index, a simple average of global Bitcoin/USD exchange prices. It is expressed as the midpoint of the bid/ask spread

across a number of global exchanges meeting certain minimum criteria with regard to minimum trade size, trading volume and others[4]. Since historical price data on Bitcoin becomes available starting on July 18, 2010 on CoinDesk.com, the sample period covers just under 57 months until April 30, 2015. Figure 1 depicts the historical development of the Bitcoin price quoted in USD starting on July 18, 2010.

Bitcoin Price Index
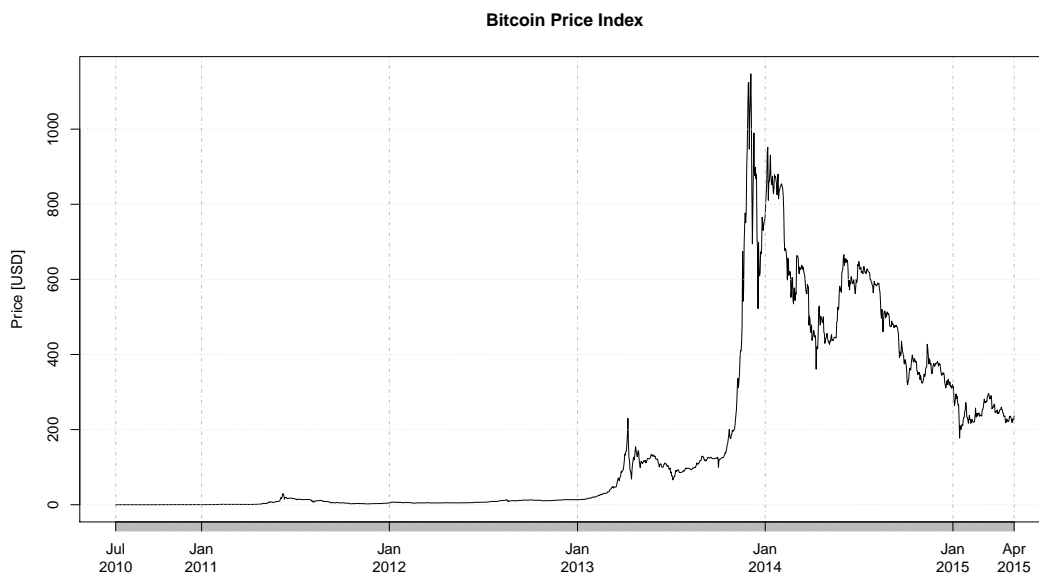
Figure 1: This figure shows the historical development of the Bitcoin price in USD from July 18, 2010 until April 30, 2015. We use the Bitcoin USD Price Index created by CoinDesk as a market wide representation of the Bitcoin price. The index represents the average of the Bitcoin price quoted in USD across a number of exchanges that fulfill basic data requirements.

For the portfolio optimization process we assume the position of a US investor. In order to allow for a well-diversified and international portfolio, we include a broad range of asset classes in our sample. Our asset class lineup therefore covers equity, fixed-income, money market, commodity, real estate and alternative investment opportunities, each represented by at least one or a number of broad and liquid financial indices. All assets are required to be quoted in USD and data is gathered using Thomson Reuters Datastream and Bloomberg. See Table 1 for a detailed overview of all sample assets. [5]

---

[4]See http://www.coindesk.com/price/bitcoin-price-index/ for more detailed information.

[5]For the risk free rate we assume a value of 0%, since interest rates on US one-month Treasury bills have been floating in the range 0% - 0.1% in recent years.

9

Name	Mnemonic	Asset Class
CoinDesk Bitcoin Price Index (BTC)	coindeskBPI	-
MSCI World Index	msciworld	Equity
MSCI Emerging Markets Index	msciem	Equity
MSCI Frontier Markets Index	mscifm	Equity
Bloomberg Glob. Dev. Sovereign Bond Index	bgsv	Fixed-Income
Bloomberg Emerg. Mark. Sovereign Index	blcsv	Fixed-Income
Bloomberg Glob. Investm. Grade Corp. Bond Index	bcor	Fixed-Income
Bloomberg Glob. High Yield Corp. Bond Index	bhyc	Fixed-Income
iShares TIPS Bond ETF (US only)	tipus	Fixed Income
MSCI EAFE Currency Index (USD)	mxea	Money Market
S&P World Commodity Index	spwici	Commodities
FTSE EPRA/NAREIT Developed Real Estate Index	engl	Real Estate
Global Hedge Fund Index	hfrxgl	Alternative

Table 1: This table gives details on the investment assets included in the sample. The mnemonic column gives the abbreviations used in tables and figures later on, while the asset class column indicates each asset's respective asset class. Data on these assets is gathered using Thompson Reuters Datastream and Bloomberg. The sample period over which data is gathered starts on July 18, 2010 and ends on April 30, 2015.

4   Results

Tables 2 and 3 provide descriptives on all assets used in our portfolio optimization frameworks. As already mentioned, the Bitcoin return distribution exhibits large excess kurtosis (14.94) and is positively skewed (3.19). With a view to whether or not Bitcoin returns are correlated with the returns of other assets included in this paper, Table 3 indicates that the Bitcoin correlation coefficients are exceptionally small and only insignificantly different from zero. Our findings in this regard are thus largely in line with the results of Brière et al. (2013).

	coindeskBPI	bcor	bgsv	bhyc	blcsv	engl	hfrxgl	msciem	mscifm	msciworld	mxea	spwici	tipus
Mean	30.18	0.32	0.09	0.64	0.32	0.99	0.14	0.23	0.33	0.88	0.56	-0.08	0.32
Median	3.71	0.51	0.13	0.76	0.45	1.4	0.39	0.06	0.48	1.2	0.76	-0.26	0.44
Maximum	465.21	3.5	3.08	5.91	4.08	12.38	2.36	13.01	7.37	10.26	9.59	14.11	3.9
Minimum	-39.26	-3.55	-4.64	-4.3	-4.93	-6.79	-3.47	-14.78	-6.83	-8.99	-12.09	-13.3	-4.19
Volatility	84.19	1.52	1.46	2.07	1.88	3.72	1.14	5.21	3.43	3.82	4.53	5.98	1.54
Skewness	3.19	-0.5	-0.65	-0.32	-0.42	0.14	-0.74	-0.17	-0.16	-0.22	-0.34	-0.12	-0.48
Kurtosis	14.94	3.18	4.01	3.05	2.88	3.69	4.03	3.84	2.41	3.56	3.26	3.05	3.88
# Observations	57	57	57	57	57	57	57	57	57	57	57	57	57

Table 2: This table shows descriptive statistics for the monthly returns of the assets included in our portfolios.

| Correlation Matrix - Coefficients | | | | | | | | | | | |
	coindeskBPI	bcor	bgsv	bhyc	blcsv	engl	hfrxgl	msciem	mscifm	msciworld	mxea	spwici
bcor	0.13											
bgsv	0.09	0.83										
bhyc	0.15	0.80	0.43									
blcsv	0.00	0.79	0.59	0.70								
engl	-0.03	0.40	0.22	0.35	0.46							
hfrxgl	0.15	0.47	0.15	0.74	0.50	0.32						
msciem	0.03	0.68	0.36	0.83	0.82	0.39	0.69					
mscifm	0.12	0.39	0.15	0.60	0.43	-0.02	0.65	0.60				
msciworld	0.16	0.62	0.25	0.87	0.67	0.32	0.83	0.84	0.65			
mxea	0.14	0.68	0.33	0.88	0.72	0.33	0.80	0.85	0.65	0.97		
spwici	0.09	0.41	0.25	0.60	0.40	-0.12	0.55	0.58	0.50	0.61	0.60	
tipus	0.06	0.63	0.70	0.30	0.47	0.46	0.01	0.24	-0.01	0.04	0.12	0.02

| Correlation Matrix - P-Values | | | | | | | | | | | |
	coindeskBPI	bcor	bgsv	bhyc	blcsv	engl	hfrxgl	msciem	mscifm	msciworld	mxea	spwici
bcor	0.33											
bgsv	0.52	0.00										
bhyc	0.26	0.00	0.00									
blcsv	0.98	0.00	0.00	0.00								
engl	0.84	0.00	0.10	0.01	0.00							
hfrxgl	0.25	0.00	0.26	0.00	0.00	0.02						
msciem	0.83	0.00	0.01	0.00	0.00	0.00	0.00					
mscifm	0.37	0.00	0.26	0.00	0.00	0.87	0.00	0.00				
msciworld	0.25	0.00	0.06	0.00	0.00	0.01	0.00	0.00	0.00			
mxea	0.29	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00		
spwici	0.51	0.00	0.07	0.00	0.00	0.39	0.00	0.00	0.00	0.00	0.00	
tipus	0.66	0.00	0.00	0.02	0.00	0.00	0.92	0.07	0.95	0.76	0.39	0.89

Table 3: This table shows the pairwise correlation coefficients of the assets included in our portfolio. In addition, we test whether these correlations are significantly different from zero and report the p-values in the lower part of the table.

Table 4 displays an overview of the main results of our empirical analysis. For all four portfolio optimization frameworks introduced above (see Section 2) and over the total 45-month investment period (exception: under the equally-weighted framework the investment period equals the total sample period of 57 months), the table shows the mean portfolio weights of Bitcoin, the mean monthly portfolio returns, the mean monthly portfolio CVaRs at the 95% level and the corresponding mean monthly risk-return ratios of the portfolios. As already explained in more detail in Section 2, all of the result presented in the following are out-of-sample results computed using our portfolio backtesting technique.

Portfolio Optimization Framework	Mean Monthly BTC Weight	Mean Monthly Return	Mean Monthly CVaR	Mean Monthly Risk-Return Ratio
Equally-Weighted BTC	7.69%	1.93%	1.01%	3.75%
Equally-Weighted No BTC	-	0.38%	0.64%	2.32%
Long-Only BTC	2.09%	0.43%	0.60%	1.04%
Long-Only No BTC	-	0.28%	0.51%	0.60%
Unconstrained BTC	6.65%	5.21%	1.94%	5.88%
Unconstrained No BTC	-	0.43%	0.34%	1.81%
-100%/+100% BTC	1.65%	1.03%	0.53%	2.77%
-100%/+100% No BTC	-	0.43%	0.34%	1.81%

Table 4: This table details the main results for each portfolio and every portfolio optimization framework. All data presented are monthly means for the investment period starting on July 1, 2011 and ending on April 30, 2015.

Overall, we find that the results for all the portfolio optimization frameworks are rather similar. The mean monthly weight of Bitcoin in the portfolios is relatively low across the board, with values between 1.65% in the -100%/+100% framework and 7.69% in the equally-weighted portfolios. It is interesting to not here that the equally-weighted framework yields the highest mean BTC weight of all optimization frameworks. Concerning the mean monthly portfolio returns, an increase is clearly established across all optimizations when comparing the portfolios excluding Bitcoin to the portfolios including Bitcoin. With the average increase in return being 1.77 percentage points, the effect is most prominent in the completely unconstrained optimization framework, where the mean monthly return increases by 4.78 percentage points from 0.43% to 5.21%. In line with the results reported earlier, our results confirm that the mean portfolio risk also increases when Bitcoin is added to the asset mix. With the mean CVaR in BTC-portfolios being 0.56 percentage points higher on average across all frameworks, this effect is again most prominently illustrated by the completely unconstrained portfolios, with the CVaR increasing from 0.34% to 1.94%, i.e. an increase of 1.6 percentage points. Most importantly, as can finally be seen from the risk-return ratios, the higher returns of portfolios including Bitcoin seem not to be completely offset the evident increases in risk. In fact, the risk-return ratios improve consistently over all optimization frameworks when adding Bitcoin to the asset mix (average increase: 1.7 percentage points). Again, the effect is most prominent

for the completely unconstrained portfolio. However, even in the naive equally-weighted $1/N$ framework, the risk-return ratio increases by 1.43 percentage points from 2.32% to 3.75% and in the -100%/+100% portfolio we find an increase of 0.96 percentage points, up from 1.81% in the portfolio excluding BTC to 2.77% in the portfolio including BTC.
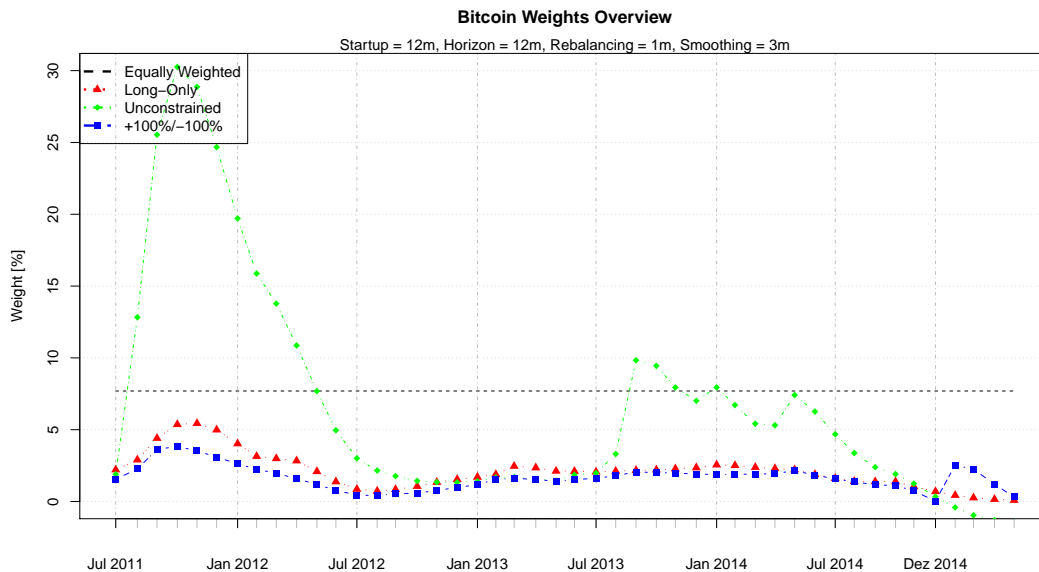

Figure 2: This figure shows the optimal portfolio weights of Bitcoin under the four different portfolio optimization frameworks (equally-weighted, long-only, unconstrained and -100%/+100%) over the investment period. Of the total sample period of 57 months, the first 12 months of data are used for the initial estimation of optimal portfolio weights, thus yielding a 45-month investment period starting on July 1, 2011 and ending on April 30, 2015. Furthermore a rolling 12-month window is used for the optimization throughout the investment period, with monthly portfolio rebalancing and 3-month EWMA smoothing on asset weights. This is true for all optimization frameworks except for the equally-weighted framework, where no estimation of optimal portfolio weights takes place and no smoothing is applied.

Figure 2 depicts the optimal portfolio weights of Bitcoin under all of the four portfolio optimization frameworks we applied in this paper in more detailed over the investment period. With the mean Bitcoin weights lying somewhere between 1.65% and 7.69% as already mentioned above, it is interesting to note, that Bitcoin has a positive weight under all frameworks over the whole investment period. Furthermore, while it is obvious that Bitcoin is included under the equally-weighted framework at a stable weight of 7.69% (100%/13 assets = 7.69% per asset), Bitcoin weights are are also more or less stable in the three other frameworks, hovering in the area of 1% to 5% across all other optimization frameworks and throughout the entire investment

13

period. The only exception to both of these results is the completely unconstrained framework, where besides two phases of Bitcoin weights above the equally-weighted share of 7.69% (reaching a high of 30.26% in November 2011) the Bitcoin weight also becomes negative during the last four months of the investment period. In general, the relatively low and stable BTC weights shown here might be beneficial from a liquidity perspective: low and stable Bitcoin portfolio shares (i.e. thus requiring only infrequent rebalancing) might thus make Bitcoin investments more feasible for both institutional and private investors.

Following this, we take a more detailed look at the results of one specific optimization framework, the -100%/+100% framework. We present these results as representative examples of all four optimizations, since the results of the other optimization frameworks (i.e. equally-weighted, long-only, unconstrained) are qualitatively comparable in almost all areas.

The three plots shown in Figure 3 outline the main results of the -100%/+100% portfolio optimization framework. The first plot of Figure 3 depicts the cumulative returns of both the portfolios including Bitcoin as well as the portfolios excluding Bitcoin on a monthly basis. As can be expected by now, it can clearly be seen that the portfolios including Bitcoin exhibit a higher total cumulative return at the end of the investment period (47.44%) than their counterparts excluding Bitcoin (20%). The second plot of Figure 3 details the development of monthly CVaRs of both the Bitcoin- and the No-Bitcoin portfolios, with the Bitcoin portfolios always having a higher risk exposure (except for 10 months out of the total investment period of 45 months). The mean CVaRs of 0.53& (incl. BTC) and 0.34% (excl. BTC) of course confirm this as well.
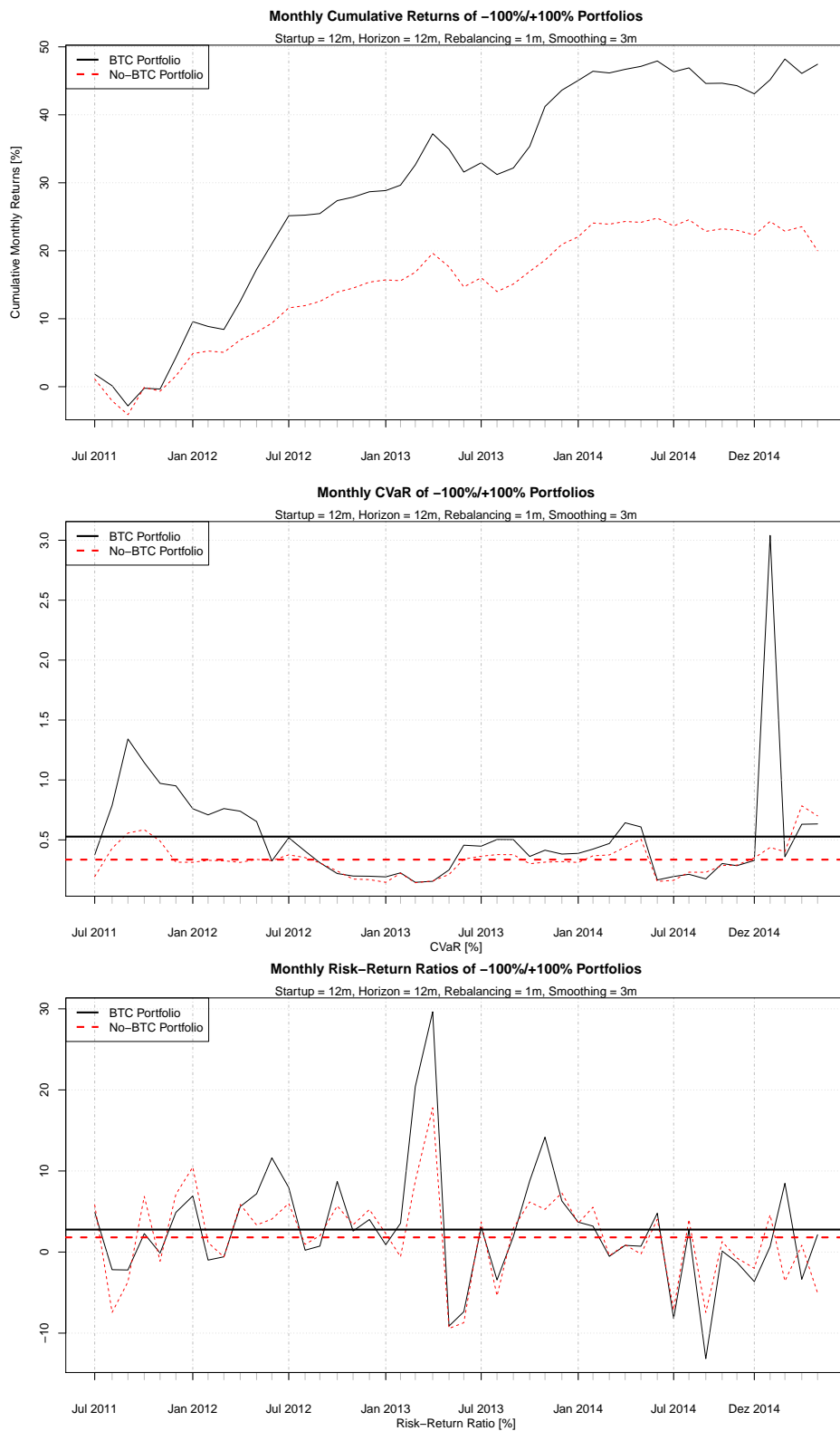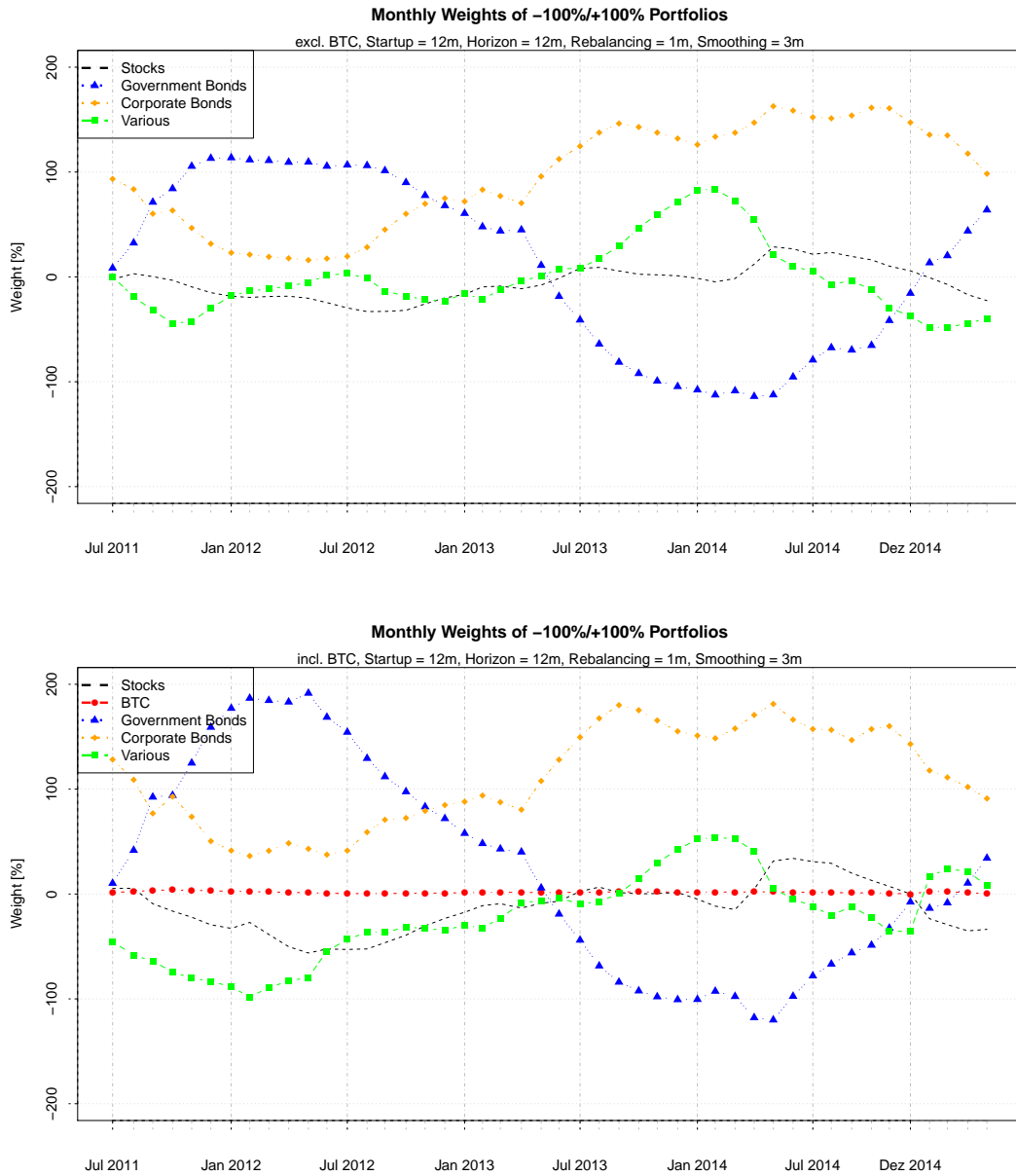
Figure 3: This figure outlines the backtesting results of the portfolios including Bitcoin and the portfolios excluding Bitcoin under the -100%/+100% portfolio optimization framework. Firstly we plot cumulative monthly returns of the two portfolios, secondly we plot the time-series of CVaRs of the two portfolios, and thirdly we plot the time series of risk-return ratios of the two portfolios. Of the total sample period of 57 months, the first 12 months of data are used for the initial estimation of optimal portfolio weights, thus yielding a 45-month investment period starting on July 1, 2011 and ending on April 30, 2015. Furthermore a rolling 12-month window is used for the optimization throughout the investment period, with monthly portfolio rebalancing, and 3-month EWMA smoothing on asset weights.

15

The third plot of Figure 3 graphs the development of the monthly risk-return ratios over the investment period, with the portfolios including Bitcoin constantly having higher risk-return ratios than the portfolios excluding Bitcoin. Again, this is also evidenced by the respective mean risk-return ratios with 2.77% (incl. BTC) and 1.81% (excl. BTC; see Table 4).

Figure 4 finally compares the optimal asset allocation of the portfolios excluding Bitcoin and the portfolios including Bitcoin under the -100%/+100% portfolio optimization framework over the investment period. To allow for easier illustration, we combine the weights of the original investment assets into a number of merged asset classes: equity (stocks), fixed-income (government bonds and corporate bond), and various (money market, commodities, real estate and alternative asset). It can clearly be seen that the introduction of Bitcoin into the portfolio optimization process has some impact on the weights of all other included asset classes. Throughout the investment period, the optimal portfolios contain between 0.3% and 3.8% Bitcoin, with only one month showing a Bitcoin weight of 0%. At the same time, the equity portfolio weights do not change very much as a result of introducing Bitcoin to the asset mix. For most of the investment period, the same can be said about the weights of the government bonds, the exception here being the period between November 2011 and July 2012, where the investment in government bonds nearly doubles in the portfolios including Bitcoin in comparison to the portfolios excluding Bitcoin. The weights of corporate bonds are roughly 15 percentage points higher on average in the portfolios including BTC, while the weights of the various securities are 21 percentage points lower on average.

Figure 4: This figure compares the optimal asset allocation of the portfolios including Bitcoin and the portfolios excluding Bitcoin under the -100%/+100% portfolio optimization framework. Using our backtesting approach, we show the optimal weights of individual asset classes over time, combining the Money Market, Commodities, Real Estate and Alternative asset classes into one group named "Various", to allow for easier illustration. Of the total sample period of 57 months, the first 12 months of data are used for the initial estimation of optimal portfolio weights, thus yielding a 45-month investment period starting on July 1, 2011 and ending on April 30, 2015. Furthermore a rolling 12-month window is used for the optimization throughout the investment period, with monthly portfolio rebalancing, and 3-month EWMA smoothing on asset weights. Due to the combining of specific assets into asset classes, the weights of specific asset classes can reach values below -100% and above 100% notwithstanding the applied portfolio constraints.

5   Conclusion

Bitcoin is currently the most popular unregulated digital currency. Given Bitcoin's interest-
ing characteristics (e.g. the historical development of the Bitcoin price or its surprisingly
low correlation with other, more well-known and widely-used investment assets), the scarce
economically-motivated literature on the subject and the high level of media coverage, we aim
to answer three specific research questions on Bitcoin: how does the inclusion of Bitcoin affect
the asset allocation of already well-diversified portfolios (Q1), is the weight of Bitcoin in an
already well-diversified portfolio robust with regard to the optimization procedure used (Q2),
and is Bitcoin able to improve the risk-return profile of an already well-diversified portfolio?

To account for Bitcoin's highly non-normal return distribution, we adopt a more robust
portfolio optimization approach built on the Conditional Value-at-Risk (CVaR) approach. We
apply a portfolio backtesting technique to calculate monthly out-of-sample returns and risk-
return ratios based on the CVaR. As a robustness check, we calculate our results under a number
of different portfolio optimization frameworks, including completely unconstrained portfolios,
portfolios featuring short selling constraints, and equally-weighted portfolios as a naive bench-
mark.

In answer to our research questions and on the basis of a data set covering all available
data on Bitcoin (July 18, 2010 until April 30, 2015) we find that even in already well-diversified
portfolios our optimizations lead to Bitcoin being included in efficient portfolios with mean
weights ranging from 1.65% to 7.69%. The relatively low and stable Bitcoin weights are also
beneficial from a liquidity perspective. Especially the recent market turmoils have cast doubt
on the liquidity of Bitcoin markets. Low Bitcoin portfolio shares with infrequent rebalancing
requirements might thus make Bitcoin investments more feasible for both institutional and
private investors. Our results furthermore indicate that Bitcoin can contribute to the risk-
return ratios of optimal portfolios. Including Bitcoin increases both the expected return as well
as the risk of the portfolios. However, as documented by the increase of the risk-return ratios,

the return contribution seems to outweigh the additional risks faced by the investor. Our results are thereby largely robust with regard to the optimization framework applied.

References

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9(3):203–228.

Belousova, J. and Dorfleitner, G. (2012). On the diversification benefits of commodities from the perspective of euro investors. *Journal of Banking & Finance*, 36(9):2455 – 2472.

Bernanke, B. (2013). Letter to US Congress on virtual currencies. https://www.documentcloud.org/documents/835843-virtual-currency-hearings.html.

Brière, M., Oosterlinck, K., and Szafarz, A. (2013). Virtual currency, tangible return: Portfolio diversification with bitcoins. CEB Working Paper.

Campbell, R., Huisman, R., and Koedijk, K. (2001). Optimal portfolio selection in a value-at-risk framework. *Journal of Banking & Finance*, 25(9):1789–1804.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *The Review of Financial Studies*, 22(5):1915–1953.

Jorion, P. (1996). Risk2: Measuring the risk in value at risk. *Financial Analysts Journal*, 52(6):47–56.

Jorion, P. (2001). *Value at Risk Second Edition*. McGraw Hill.

Karame, G., Androulaki, E., and Capkun, S. (2012). Two bitcoins at the price of one? Double-spending attacks on fast payments in bitcoin.

McNeil, A., Frey, R., and Embrechts, P. (2005). *Quantitative Risk Management*. Princeton University Press.

Meiklejohn, S., Pomarole, M., Jordan, G., Levchenko, K., McCoy, D., Voelker, G., and Savage, S. (2013). A fistful of bitcoins: Characterizing payments among men with no names. Proceedings of the 2013 conference on Internet measurement conference.

Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system.

Raskin, M. (2013). U.S. agencies to say bitcoins offer legitimate benefits. http://www.bloomberg.com/news/2013-11-18/u-s-agencies-to-say-bitcoins-offer-legitimate-benefits.html.

Rockafellar, R. T. and Uryasev, S. (2000). Optimization of conditional value at risk. *Journal of Risk*, 2.

Rockafellar, R. T. and Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, 26(7):1443–1472.

Ron, D. and Shamir, A. (2013). Quantitative analysis of the full bitcoin transaction graph. Proceedings of Financial Cryptography 2013.

Yermack, D. (2013). Is bitcoin a real currency? Working Paper 19747, National Bureau of Economic Research.