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Density Forecasting using Bayesian Global Vector Autoregressions with Common Stochastic Volatility

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Abstract

This paper puts forward a Bayesian Global Vector Autoregressive Model with Common Stochastic Volatility (B-GVAR-CSV). We assume that country specific volatility is driven by a single latent stochastic process, which simplifies the analysis and implies significant computational gains. Apart from computational advantages, this is also justified on the ground that the volatility of most macroeconomic quantities considered in our application tends to follow a similar pattern. Furthermore, Minnesota priors are used to introduce shrinkage to cure the curse of dimensionality. Finally, this model is then used to produce predictive densities for a set of macroeconomic aggregates. The dataset employed consists of quarterly data spanning from 1995:Q1 to 2012:Q4 and includes 45 economies plus the Euro Area. Our results indicate that stochastic volatility specifications influences accuracy along two dimensions: First, it helps to increase the overall predictive fit of our model. This result can be seen for some variables under scrutiny, most notably for real GDP and short-term interest rates. Second, it helps to make the model more resilient with respect to outliers and economic crises. This implies that when evaluated over time, the log predictive scores tend to show significantly less variation as compared to homoscedastic models.

Keywords: Density Forecasting, Stochastic Volatility, Global vector autoregressions.

JEL Codes: C32, F44, E32, E47.

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1 Introduction

Recent episodes of rising volatility of several key macroeconomic quantities revealed that most models employed in policy institutions failed to deliver reliable forecasts under such circumstances. This stems from the fact that practitioners remained largely confined to simple linear models which do not account for structural changes in the behavior of the underlying variables. Two reasons are worth mentioning why the majority of applied researcher still stick to linear models. First, estimation is easy and numerical optimization is often unnecessary. As a consequence, they are easy to implement using standard statistical software packages. Second, linear models are easy to interpret and understand, which makes them valuable for the majority of practitioners. However, the recent global turmoil has proved that more flexible models are needed to fully capture the complex dynamics arising in macroeconomics and finance. Especially for highly volatile financial time series non-linear models are needed to fully capture sudden shifts in volatility commonly observed in financial markets.

Several studies provided evidence for a sudden increase of volatility in industrialized economies after experiencing decades of relatively stable and low volatility of macroeconomic fundamentals. Linear models, like vector autoregressive models (VARs), which have been performing quite well up to the mid 2000s suddenly failed to produce reliable predictions. Ignoring the dynamic behaviour of volatility led to predictive densities which are either too narrow or too wide, resulting in inflated confidence bounds and poorly estimated probabilities for tail events. Thus it might be necessary to account for heteroscedasticy by means of more flexible specifications of the variance covariance matrix. A plethora of studies emphasized the usefulness of such stochastic volatility specifications in terms of point- and density forecasts. Giordani & Villani (2010), Clark (2011) and Carriero *et al.* (2012) all highlight the substantial increases in forecasting accuracy by using SV specifications. Such gains in accuracy typically directly translate into better prediction intervals produced in central banks and other policy institutions, underlining the practical relevance of this approach.

Despite the fact that stochastic volatility VARs introduce additional flexiblity when it comes to macroeconomic modelling, computational needs also increase substantially. Additionally, due to the fact that frequentist estimated VARs typically suffer from parameter proliferation, which translates into the well-known curse of dimensionality, parameters are imprecisely estimated and such models tend to overfit the data dramatically. Thus, Bayesian methods are needed to obtain reliable estimates and impose shrinkage on the parameters. Furthermore, allowing for flexible stochastic volatility specifications in VARs typically leads to non-conjugate situations where forward-filtering-backward-sampling methods (FFBS) are required. This bounds the analysis usually to small- to medium scale models. Especially in forecasting applications, it is of interest to allow for high dimensional models to exploit information originating from other variables or other countries. Several recent contributions aimed for making the estimation of large scale models feasible and still preserve the flexibility of non-linear models. Koop & Korobilis (2013) draw on ideas from the dynamic model averaging literature and utilize forgetting factors to reduce the computational burden. In another contribution, Carriero *et al.* (2012) allow for a simplified version of stochastic volatility, where it is assumed that the volatility of the whole system is driven by a single, latent process. This assumption preserves the conjugacy of the prior and permits a convenient Kronecker structure of the likelihood. This implies significant computational gains using such a simplified prior structure.

In terms of achieving shrinkage in large scale macroeconometric models, the global vector autoregressive model (GVAR) put forward by Pesaran *et al.* (2004) proved to be a convenient

way of reducing the dimensionality of the estimation problem. Several contributions outlined the usefulness of such large scale models to perform forecasting (Pesaran *et al.*, 2009; Greenwood-Nimmo *et al.*, 2012; Crespo Cuaresma *et al.*, 2014) or impulse response analysis (Pesaran *et al.*, 2007; Dees *et al.*, 2007). One disadvantage is, however, that frequentist estimation of GVAR models does not cure the curse of dimensionality at the local level. This implies that even though the dimensionality of the problem is reduced considerably, local models might also suffer from severe overfitting. Crespo Cuaresma *et al.* (2014) proposed a Bayesian variant of the GVAR and evaluated its predictive performance in a forecasting horse race. It is shown that Bayesian shrinkage, in addition to the restrictions imposed by the GVAR, helps to improve point and density forecasts for all variables under consideration.

In the present paper we propose a Bayesian variant of the GVAR which allows for a time varying variance-covariance structure (B-GVAR-CSV) in the spirit of Carriero *et al.* (2012). This implies that the local models, which are stacked in a second stage to yield the global model, are driven by a single latent stochastic process which governs the country specific log-volatities. That means in each country model, that consists of several single equations for the macrovariables at hand, we model one stochastic volatility process as opposed to having stochastic volatility modelled in each equations separately. As a consequence, the global system, which comprises of the $N+1$ local systems, is driven by $N+1$ local latent factors. The contributions of this paper are threefold. First, the possibility to allow for stochastic volatility in the GVAR is introduced. A first attempt to model time varying volatilities has been recently adopted in Cesa-Bianchi *et al.* (2014), where a satellite model for the volatility process is introduced. However, in this paper we take a more coherent approach and model stochastic volatility for each country separately. Second, we propose a simple and efficient algorithm to estimate the local models. In particular, sampling the log volatities is done using the algorithm outlined in Kastner & Frühwirth-Schnatter (2013). As compared to the estimation of standard Bayesian VARs, this method is extremely fast, resulting only in marginally higher computational needs. Finally, we use the B-GVAR-CSV to forecast several key macroeconomic quantities and evaluate their predictive densities. Our results suggest that the introduction of stochastic volatility leads to more precise density forecasts as measured by log predictive scores for various variables under scrutiny at both time horizons, where especially for GDP the GVAR with CSV consistently outperforms its peers.

This paper is structured as follows. Section 2 introduces the econometric framework employed while Section 3 discusses prior setups and the Markov-Chain Monte Carlo (MCMC) algorithm. Section 4 presents the dataset and the results of the density forecasting exercise. Finally, the last section concludes.

2 The B-GVAR with Stochastic Volatility

2.1 From local to global: The GVAR Model

The main building block of the GVAR model put forward by Pesaran *et al.* (2004) are the local macroeconomic models. More specifically, we assume that domestic variables are modeled using a standard VAR with exogenous regressors (VARX*). A typical VARX* model for country

$i = 0, ..., N$ is then given by

$$x_{i,t} = \gamma_{i0} + \gamma_{i1}t + \sum_{s=1}^{S} \psi_{is} x_{i,t-s} + \sum_{k=0}^{K} \Lambda_{ik} x^*_{i,t-k} + \delta_0 d_t + \delta_1 d_{t-1} + \varepsilon_{i,t} \tag{1}$$

where $x_{i,t}$ denotes a $k_i \times 1$ vector of endogenous variables measured in country i at time t. The deterministic part of the model is composed of the coefficient on the constant, γ_{i0} and the coefficient on the time trend, γ_{i1}. Furthermore, ψ_{is} denotes the $k_i \times k_i$ coefficient matrix corresponding to the s'th lag of the endogenous variables. This part of Equation 1 captures domestic dynamics. The $k^*_i \times 1$ vector $x^*_{i,t}$ denotes the so-called *weakly exogenous variables*, which are defined as

$$x^*_{i,t} = \sum_{j \neq i}^{N} \omega_{i,j} x_{j,t} \tag{2}$$

$$\tag{3}$$

where $\omega_{i,j}$ denotes the weight between countries i and j and $\sum_{j \neq i}^{N} \omega_{i,j} = 1$. The $k_i \times k^*_i$ coefficient matrix related to $x^*_{i,t-k}$ is given by Λ_{ik}. The matrix of *strictly exogenous variables* is given by d_t. Note that the discrimination between strictly exogenous and weakly exogenous is crucial because the latter will become effectively endogenous once the model is solved. Finally, $\varepsilon_{i,t} \sim \mathcal{N}(0, \Sigma_{i,t})$ is the usual vector white noise process. The dynamics of the variance covariance matrix, following Carriero *et al.* (2012), are assumed to be driven by a single latent stochastic process $h_{i,t}$. More specifically, we assume that $\Sigma_{i,t}$ evolves according to

$$\Sigma_{i,t} = \exp(h_{i,t}/2) \times \Sigma_i \tag{4}$$

$$h_{i,t} = \eta_i + \xi_i(h_{i,t-1} - \eta_i) + \sigma_i \epsilon_{i,t} \tag{5}$$

$$\epsilon_{i,t} \sim \mathcal{N}(0, 1) \tag{6}$$

where we assume that $\xi_i \in (-1, 1)$. This implies that the stochastic process which governs the log-volatility is mean reverting. It would be possible to assume that the log-volatility follows a random walk process. However, this implies that the log-volatility is unbounded in the limit (for a discussion on whether to model log-volatities as stationary or non-stationary see Eisenstat & Strachan, 2014). Such behavior is ruled out using this more general specification. This completes the discussion of the local models.

To retrieve the global model, we assume the following simplified first-order VARX* model

$$x_{i,t} = \psi_{i1} x_{i,t-1} + \Lambda_{i0} x^*_{i,t} + \Lambda_{i1} x^*_{i,t-1} + \varepsilon_{i,t} \tag{7}$$

In the first step, we define a $(k_i + k^*_i) \times 1$ vector $z_{i,t} := (x_{i,t}\ x^*_{i,t})'$ which permits us to rewrite Equation 7 in terms of $z_{i,t}$

$$A_i z_{i,t} = B_i z_{i,t-1} + \varepsilon_{i,t} \tag{8}$$

with $A_i := (I_{k_i}\ -\Lambda_{i0})$ and $B_i := (\psi_{i1}\ \Lambda_{i,1})$. In the next step we define k-dimensional global vector x_t, where $k = \sum_{i=0}^{N} k_i$. This vector consists of all $N+1$ countries endogenous variables, $x_t = (x_{0,t}, ..., x_{N,t})'$. Finally, we have to define a $(k_i + k^*_i) \times k$ weighting matrix W_i such that $z_{i,t} = W_i x_t$. This allows us to write Equation 8 exclusively in terms of x_t

$$A_i W_i x_t = B_i W_i x_{t-1} + \varepsilon_{i,t} \tag{9}$$

Stacking this equation $N+1$ times yields

$$\Gamma x_t = \Psi x_{t-1} + u_t \tag{10}$$

where $\Gamma := (A_0 W_0, ..., A_N W_N)'$, $\Psi := (B_0 W_0, ..., B_N W_N)'$ and $u_t = (\varepsilon_{0,t}, ..., \varepsilon_{N,t})'$. Note that $u_t \sim \mathcal{N}(0, \Sigma_t)$, where Σ_t is assumed to be a block-diagonal $k \times k$ matrix given by

$$\Sigma_t = \begin{pmatrix} \exp(h_{0,t}/2) \times \Sigma_0 & 0 & \cdots & 0 \\ 0 & \exp(h_{1,t}/2) \times \Sigma_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \exp(h_{N,t}/2) \times \Sigma_N \end{pmatrix} \tag{11}$$

which implies that the log-volatility of the global system is governed by $N+1$ latent stochastic processes. Furthermore, note that this assumption implies that until now, the cross-country covariances are set equal to zero. Solving the model in (Equation 10) for x_t gives

$$x_t = \Upsilon x_{t-1} + e_t \tag{12}$$

with $\Upsilon := \Gamma^{-1}\Psi$ and $e_t := \Gamma^{-1} u_t$. This implies that $e_t \sim \mathcal{N}(0, \Omega_t)$ with $\Omega_t = \Gamma^{-1}\Sigma_t \Gamma^{-1'}$, which is in general not block-diagonal.

The GVAR model in Equation 12 resembles a standard, high-dimensional VAR. Thus we can use (12) to produce forecasts, impulse responses or forecast error variance decompositions. In the following we assume that the GVAR is stable, which would imply that in this case the eigenvalues of Υ lie within the unit circle. Due to the fact that the predictions are produced for one- and four quarters ahead respectively, this assumption is not really crucial.

2.2 General Prior Setup

To conduct Bayesian inference we have to specify prior distributions for all parameters in the model. Following Crespo Cuaresma *et al.* (2014), this is done at the individual country level. For further discussion it proves to be convenient to collect all country specific dynamic coefficients in a matrix $\Psi_i = (\gamma_{i0}\ \gamma_{i1}\ \psi_{i1}\ ...\ \psi_{i,S}\ \Lambda_{i0}\ ...\ \Lambda_{iS}\ \delta_0\ \delta_1)'$.

The prior setup for all coefficients in country i is then given by

$$\text{vec}(\Psi_i)|\Sigma_i \sim \mathcal{N}(\text{vec}(\underline{\mu}_\Psi), \Sigma_i \otimes \underline{V}_\Psi), \tag{13}$$

$$\Sigma_i^{-1} \sim \mathcal{W}(\underline{v}, \underline{S}^{-1}) \tag{14}$$

$$\eta_i \sim \mathcal{N}(\underline{\mu}_\eta, \underline{V}_\eta) \tag{15}$$

$$\frac{\xi_i + 1}{2} \sim \mathcal{B}(a_0, b_0) \tag{16}$$

$$\sigma_i \sim \mathcal{G}(1/2, 1/2B_\sigma) \tag{17}$$

Note that we assume prior dependence between Ψ_i and Σ_i, which implies that we can exploit a Kronecker structure for the likelihood. The Kronecker structure, as mentioned in Carriero *et al.* (2012), leads to large increases in computational efficiency, especially when the number of endogenous variables is increased at the local level. Several choices for $\underline{\mu}_\Psi$ and $\underline{V}_\Psi$ are possible. However, we will restrict our analysis to the well-known Minnesota prior, which shrinks the system towards a naïve random walk process. Exact details for the implementation can be

found below. The prior on the time-invariant part of the precision Σ_i^{-1} is of standard Wishart form with prior degrees of freedom $\underline{v}$ and scale matrix $\underline{S}^{-1}$. Furthermore, for the level η in the log-volatility equation we impose a normal prior with mean $\underline{\mu}_\eta$ and variance $\underline{V}_\eta$. Following Kastner & Frühwirth-Schnatter (2013) we impose a beta prior on the persistence parameter ξ_i. Formally, the prior density is

$$p(\xi_i) = \frac{1}{2B(a_0, b_0)} \frac{(1+\xi_i)}{2}^{(a_0-1)} \frac{(1-\xi_i)}{2}^{(b_0-1)} \tag{18}$$

where $B(a_0, b_0)$ denotes the beta function. The support of this distribution is the unit ball, which implies stationarity of the log-volatility process. The prior mean and variance are equal to

$$E(\xi_i) = \frac{2a_0 - 1}{a_0 + b_0} - 1$$
$$\text{Var}(\xi_i) = \frac{4a_0 b_0}{(a_0 + b_0)^2 (a_0 + b_0 + 1)}$$

Note that if $\frac{2a_0-1}{a_0+b_0} < 1$, the prior mean is negative. Obviously, this case would coincide with setting $b_0 > a_0$. A positive prior mean would correspond to the case when $a_0 > b_0$. For typical datasets arising in macroeconomics the exact choice of the hyperparameters a_0 and b_0 is quite influental, due to the short time series available. Finally, we conclude the prior section with a non-conjugate gamma prior for σ_i. This choice has the advantage that it does not bound σ_i away from zero and increases sampling efficiency considerably. Further details can be found in Frühwirth-Schnatter & Wagner (2010) and Kastner & Frühwirth-Schnatter (2013). We will discuss the exact choice of the hyperparameters at length in Section 3.

2.3 Posterior Distributions

For the present application the conditional posteriors for Ψ_i and Σ_i are of a well-known form. Namely a multivariate normal distribution for Ψ_i and a inverse-Wishart distribution for Σ_i. This implies that those parts can be sampled using a Gibbs sampling scheme. Drawing the parameters of the stochastic volatility equation is then done following Kastner & Frühwirth-Schnatter (2013) using a ancillarity-sufficiency interweaving strategy (ASIS).

Let us define some additional notation used to describe the posterior moments of Ψ_i and Σ_i. Assume that the data for each country i is stored in $Z_{i,t} = (1, t, x_{i,t-1}, ..., x_{i,t-S}, x^*_{i,t}, ..., x^*_{i,t-K}, d_t, d_{t-1})$. Stochastic volatility is introduced by dividing $Z_{i,t}$ and $x_{i,t}$ by $\exp(h_{i,t}/2)$:

$$\tilde{x}_{i,t} = \exp(-h_{i,t}/2)\ x_{i,t}$$
$$\tilde{Z}_{i,t} = \exp(-h_{i,t}/2)\ Z_{i,t}$$

In the following, we denote the full-data matrices of $\tilde{x}_{i,t}$ and $\tilde{Z}_{i,t}$ as $\tilde{X}_i = (\tilde{X}_{i,1}, ..., \tilde{X}_{i,T})'$ and $\tilde{Z}_i = (\tilde{Z}_{i,1}, ..., \tilde{Z}_{i,T})'$. Given $\tilde{x}_{i,t}$ and $\tilde{D}_{i,t}$ it is straightforward to describe the conditional posterior distributions for Ψ_i and Σ_i:

$$\text{vec}(\Psi_i)|\Sigma_i, h_{i,t}, \eta_i, \xi_i, x_i \sim \mathcal{N}(\text{vec}(\overline{\mu}_\Psi), \Sigma_i \otimes \overline{V}_\Psi) \tag{19}$$
$$\Sigma^{-1}|\Psi_i, \eta_i, \xi_i, x_i \sim \mathcal{W}(\overline{v}, \overline{S}) \tag{20}$$

where x_i is a $k_i T \times 1$ vector containing the data for country i. Using standard results for the natural conjugate prior (Zellner, 1976), the posterior mean and variance on the dynamic coefficients are given by

$$\overline{\mu}_{\Psi_i} = \overline{V}_{\Psi_i} \left(\underline{V}_{\Psi}^{-1} \underline{\mu}_{\Psi} + \tilde{Z}_i' \tilde{Z}_i \hat{\Psi}_i \right) \tag{21}$$

$$\overline{V}_{\Psi_i} = \left(\underline{V}_{\Psi}^{-1} + \tilde{Z}_i' \tilde{Z}_i \right)^{-1} \tag{22}$$

where $\hat{\Psi}_i = (\tilde{Z}_i' \tilde{Z}_i)^{-1} \tilde{Z}_i' \tilde{X}_i$ denotes the GLS estimate of Ψ_i. For the variance-covariance matrix the posterior degrees of freedom and scale matrix are given by

$$\overline{v}_i = \underline{v} + T \tag{23}$$

$$\overline{S}_i = \underline{S} + S + \hat{\Psi}_i' \tilde{Z}_i' \tilde{Z}_i \hat{\Psi} + \underline{\mu}_{\Psi}' \underline{V}_{\Psi}^{-1} \underline{\mu}_{\Psi} - \overline{\mu}_{\Psi_i}' (\underline{V}_{\Psi}^{-1} + \tilde{Z}_i' \tilde{Z}) \overline{\mu}_{\Psi_i} \tag{24}$$

where $S = (\tilde{X}_i - \tilde{Z}_i \hat{\Psi}_i)'(\tilde{X}_i - \tilde{Z}_i \hat{\Psi}_i)$. Finally, the components of the log-volatility equations are of no-well known form, which precludes simple Gibbs sampling schemes.

3 Implementation & Estimation

3.1 Prior Implementation

Until now we have remained silent on the exact prior settings. For the B-GVAR-CSV we utilize a standard implementation of the well-known Minnesota prior to achieve shrinkage at the local level. Following Karlsson (2012) this implies setting the prior moments according to

$$[\underline{\mu}_{\Psi}]_{i,j} = \begin{cases} 1 & \text{for the first, own lag of a variable} \\ 0 & \text{in all other cases} \end{cases} \tag{25}$$

$$\underline{V}_{i,j} = \begin{cases} \frac{\underline{\alpha}_1}{(r^{\underline{\alpha}_2} \varsigma_j)^2} \text{ for coefficients on lag } s = 1, ..., S \text{ of variable } j \neq i \\ \frac{\underline{\alpha}_1}{(\varsigma_1 (1+k))^2} \text{ for coefficients on lag } k = 0, ..., K \text{ of the weakly exogenous variables} \\ \underline{\alpha}_3 \text{ for the deterministic part of the model} \end{cases} \tag{26}$$

where the hyperparameters are set such that $\underline{\alpha}_1 = 0.2^2$,$\underline{\alpha}_2 = 1$ and $\underline{\alpha}_3 = 50^2$. This prior setup has several implications. First, the endogenous part of the model is shrunk towards a random walk model. Second, the weakly exogenous variables are assumed to be non-influential a priori, however, given the scale of the data the prior setup chosen implies a fairly diffuse prior on the contemporaneous part, whereas higher lag orders are shrunk aggressively towards zero. Note that the ς_{ij} refer to the standard deviations obtained by running univariate autoregressions in a given country. Variants of this prior setup has been used extensively in the literature with great success in forecasting applications (see, for example, Kadiyala & Karlsson, 1997; Bańbura *et al.*, 2010; Koop, 2013, among others).

For the time-invariant part of the variance-covariance part, we stay relatively uninformative, assuming that $\underline{S} = cI_{k_i}$, where $c = 1/1000$ is set to a small constant. The prior degree of freedom parameter is set equal to $\underline{v} = k_i$. These hyperparameter render the prior effectively non-influential in our analysis.

The hyperparameters for the log-volatility equation are set as follows. For the level η_i we set the mean $\underline{\mu}_\eta = 0$ and the variance $\underline{V}_\eta = 10^2$. This implies a non-informative prior distribution on η_i. For the persistence parameter ξ_i we set $a_0 = 5$ and $b_0 = 1.5$, resulting in a prior mean of around 0.54 and standard deviation 0.31. Finally, for σ_i we set the $B_\sigma = 1$. For our application, the exact choice of B_σ is not critical, as long as it is set sufficiently large.

3.2 Posterior Simulation: The MCMC algorithm

The MCMC algorithm for country i works as follows.

1. Draw $\Psi_i|\Sigma, h_{i,t}, \eta_i, \xi_i, D$ from $\mathcal{N}(\overline{\mu}_{\Psi_i}, \overline{V}_{\Psi_i})$
2. Draw $\Sigma^{-1}|\Psi_i, \eta_i, \xi_i, D$ from $\mathcal{W}(\overline{v}_i, \overline{S}_i)$
3. Draw the parameters of the log-volatility equations and $h_i = (h_{i,0}, ..., h_{i,T})'$ using the AWOL-sampler described in Kastner & Frühwirth-Schnatter (2013)

Steps (1) - (2) can be implemented in a straightforward fashion by sampling from Normal and Wishart distributions, respectively. The parameters of the stochastic volatility equation are updated using the ASIS algorithm. The main idea behind this interweaving strategy is that depending on which parametrization we use for the stochastic volatility process (i.e. whether we use the centered parameterization shown above or a non-centered parameterization), sampling efficiency is increased by combining "the best of two worlds".

Sampling $h_{i,t}$ and the parameters of the log-volatility equation is done in three steps. In the first step, we draw the latent volatilities all without a loop. Conditional on the other parameters, h_i is multivariate normal, where the first-order autoregressive nature makes it possible to write this density in terms of a tridiagonal-precision matrix Ω_h, which makes it possible to sample h_i all without a loop. Note that this is true for the centered and non-centered parameterizations.

In the second step, we sample the parameters of the stochastic volatility equation using Metropolis Hastings steps. Centered and non-centered parameterizations require a 2- and 3-block sampler respectively.

Finally, note stochastic volatility models are usually non-Gaussian. This implies that the innovations are $\log \chi(1)^2$ distributed. Standard practice in the literature is to circumvent this problem by using a mixture of Gaussian distributions to approximate the $\log \chi(1)^2$-distribution. In this case, we have to sample indicators which govern the normal distribution to use through inverse transform sampling. For further details we refer the reader to Kastner & Frühwirth-Schnatter (2013). These steps are implemented using the `stochvol` package, which is available on CRAN.

Treating the local models as separated estimation problems facilitates the usage of parallel computing. That is, we can distribute the estimation of individual country model across different CPU cores. This implies that in our case, if we would have $N + 1$ available CPU cores on a cluster it would take approximately the same time as the estimation of a single VARX* model.[1]

This delivers draws from the country specific joint posterior. However, interest centers on the posterior for the global model, which can be readily obtained using draws from $p(\Psi_i|\bullet)$, $p(\Sigma_i|\bullet)$

[1] One would also have to take into account overhead times from distributing the data to the nodes, but compared to the whole estimation time this part is negligible.

and $p(h_i|\bullet)$ for all $i = 0, ..., N$, where the dot indicates conditioning on all other coefficients and the data. Applying the algebra outlined in Section 2 to the individual country posterior draws yields valid draws from the global posterior of Ω_t and Υ.

4 Empirical Forecasting Application

4.1 Data Overview and Forecasting design

We use the same dataset as Crespo Cuaresma *et al.* (2014), which consists of 45 economies plus the Euro Area (EA). The dataset spans the time period from 1995Q1 to 2012Q4, which are 72 quarterly observations. Table 3 provides an overview of the countries included in the sample. These 46 cross sectional units account for more than 90 % of global output. However, this strong country coverage comes at a cost. Namely due to the fact that reliable data for most economies in the Emerging European country group is only available from 1995Q1, the available time series are rather short as compared to the dataset used in Dees *et al.* (2007).

Following the literature on GVARs (Pesaran *et al.*, 2004; Dees *et al.*, 2007; Pesaran *et al.*, 2009), we use a fairly standard set of macro aggregates in our individual country models. Table 4 provides a brief description of the variables employed. Note that most variables are available for nearly all countries in our dataset, with the exception of long-term bond yields. As a global control variable we include the price of oil. For a more complete description of the data see Feldkircher (2013) and Crespo Cuaresma *et al.* (2014).

The weakly exogenous part in the local model is symmetric, implying that all weakly exogenous variables show up in every country model. This also includes the exchange rate. The reason for this choice is that, as pointed out by Carriero *et al.* (2009), exchange rate forecasts tend to profit from cross-sectional information, i.e. the co-movement with other exchange rates. Thus we include the trade weighted real effective exchange rate in all country models to exploit information on the co-movement of exchange rates efficiently.

For forecasting purposes we have to simulate the predictive density of the global model. The h-step ahead predictive density is given by

$$p(x_{t+h}|x_{1:t}) = \int_{\Gamma} \int_{\Omega_{t+h}} p(x_{t+h}|\Gamma, \Omega_{t+h}, x_{1:t}) p(\Gamma, \Omega_{t+h}|x_{1:t}) \; d\Gamma d\Omega_{t+h} \tag{27}$$

Due to the fact that the posterior of Γ and Ω_t is available we can obtain (27) using Monte Carlo integration. To evaluate the predictive capabilities in terms of density forecasts we utilize the well known log predictive score (LPS), given by

$$LPS = \sum_{t=t_0}^{T-h} \log p(x_{t+h} = x^o_{t+h}|x_{1:t}) \tag{28}$$

where x^o_{t+h} denotes the observed outcome of x at time $t + h$. The LPS provides guidance on the overall predictive fit of the model. However, we are more interested in differences in terms of predictive accuracy across the different variables. That is, we investigate the marginal predictive distribution across variables, which can be obtained by using the predictive density of some variable under scrutiny. Judgement of the models is done exclusively based on log predictive scores. The reasons for this are twofold. First, model evaluation based on point

forecasts typically disregards the uncertainty surrounding the predictions. Second, the point of stochastic volatility specification is to increase the robustness of the model with respect to changing magnitudes of economic shocks. Thus, judging the models by their point forecasts would result in a situation where the variability is averaged out, indicating that the value added of a stochastic volatility specification delivers is effectively purged out.[2]

The forecasting design employed is the following: We start with an initial estimation period $t_0 = 1995Q1$ to $t_1 = 2009Q1$ and simulate the h-step ahead predictive density for $p(x_{t_1+h}|x_{1:t})$. After retrieving density estimates we proceed by setting $t_1 = t_{1+h}$ and again producing the h-step ahead predictive density. This procedure is repeated until we reach $t_1 = T - h$. Let us denote the LPS specific to variable k in country i by $LPS_{i,k}$. To judge the overall predictive power of the model in terms of variable k we use the cross-country average

$$LPS_k = \frac{1}{N+1} \sum_{i=0}^{N} LPS_{i,k} \tag{29}$$

As competitors we include mainly linear Bayesian GVARs. Our goal is to show that, especially when benchmarked against a standard Minnesota-GVAR, that the Common Stochastic Volatility specification increases the predictive fit in terms of log predictive scores considerably. In addition, we also include the SSVS prior specification excelled in forecasting in the linear GVAR framework provied in Crespo Cuaresma *et al.* (2014).

4.2 Results

Table 1 and Table 2 present the results of the forecasting exercise. Diffuse refers to a GVAR estimated using maximum likelihood, Minnesota is a standard, homoscedastic GVAR with Minnesota prior, NC denotes a GVAR with natural conjugate prior and SSVS denotes a GVAR with stochastic search variable selection prior. Finally, CSV denotes the GVAR with common stochastic volatility specification.

Table 1: Forecasting Performance, One-Quarter-Ahead: Log Predictive Score

	Diffuse	M	NC	SUR	SSVS	CSV
y	1.5736	2.4200	0.8594	1.5171	2.4612	**2.8846**
π	2.0111	**3.1508**	0.8950	1.8303	2.6527	3.0617
e	-0.3236	**1.3526**	0.6996	-0.3206	1.1459	1.2782
ρ_S	1.4452	2.8469	0.8121	1.2966	2.2382	**3.0351**
ρ_L	2.3839	**3.5306**	0.8768	2.3818	2.4940	3.2080

Notes: The figures refer to the average log predictive score specific to variable k. Results based on rolling forecasts over the time period 2010Q1-2012Q4. NC stands for the normal conjugate prior, M stands for the Minnesota prior, SUR stands for the single unit root prior, SSVS stands for the SSVS prior, Diffuse stands for the model estimated using maximum likelihood, CSV stands for the B-GVAR specification with Common Stochastic Volatility. Bold figures refer to the highest value across models for a given endogenous variable.

For real GDP (y), the outperformance in terms of log predictive score is large at the one quarter ahead time horizon. Comparing columns 5 and 6 of the GDP row in Table 1 reveals that the log predictive score of the CSV specification is around 20 % higher. Furthermore, comparing

[2]We have computed point forecasts and the results corroborate this statement: The differences in the accuracy of the point forecasts between the homoscedastic Minnesota prior GVAR and the CSV variant are quite small.

columns 2 and 6 indicates that the CSV specification also improves upon the homoscedastic Minnesota-GVAR to a large extent. In terms of one-year ahead forecasts for real GDP, the CSV is again ahead, outperforming the SSVS specification by around 14 %.

For CPI inflation (π), the prediction gains are less pronounced for the one-quarter ahead time horizon. Especially when comparing the homoscedastic variant with the CSV specification reveals a small advantage of the linear Minnesota-GVAR, but this difference in terms of log scores is rather negligible. However, comparison with most other specifications reveals that the CSV and Minnesota specifications tend to do well when it comes to forecasting inflation. For the one-year ahead inflation forecasts the Minnesota specification outperforms the CSV specification by around 25%, providing further evidence of strong gains in predictive accuracy when forecasting GDP.

Table 2: Forecasting Performance, One-Year-Ahead: Log Predictive Score

	Diffuse	Minnesota	NC	SUR	SSVS	CSV
y	0.4155	1.1113	0.5063	-0.3851	1.9932	**2.2441**
π	1.0099	**3.0115**	0.6650	0.0971	2.4771	2.3619
e	-1.6818	0.3429	0.2506	-2.3025	0.6150	**0.8576**
ρ_S	0.3262	**2.4380**	0.4925	-0.5348	2.0571	2.3316
ρ_L	1.4567	**2.7463**	0.6616	0.4998	2.5323	2.4789

Notes: The figures refer to the average log predictive score specific to variable k. Results based on rolling forecasts over the time period 2010Q1-2012Q4. NC stands for the normal conjugate prior, M stands for the Minnesota prior, SUR stands for the single unit root prior, SSVS stands for the SSVS prior, Diffuse stands for the model estimated using maximum likelihood, CSV stands for the B-GVAR specification with Common Stochastic Volatility. Bold figures refer to the highest value across models for a given endogenous variable.

One-quarter ahead exchange rate forecasts are surprisingly more accurately forecasted using the standard Minnesota implementation. As can be seen in the row corresponding to e in Table 1, Minnesota outperforms its peers by large margins, with the CSV specification showing the second-best performance. This outperformance especially vis-a-vis the CSV specification can be explained by noting that most variables share the same pattern of estimated volatilities, with the exception of the exchange rate. This is a disadvantage of the CSV specification as compared to specifications where we have k_i distinct latent processes in country i. The weak performance thus is mainly due to a lack of flexibility when it comes to modelling the volatility of real effective exchange rates. However, this does not carry over to the 4-steps ahead exchange rate forecasts. On that time horizon, CSV performs best, closely followed by the SSVS prior specification.

The CSV specification again possesses advantages when used to conduct one-quarter ahead forecasts of the short-term interest rates (ρ_S). As compared to its linear counterpart, CSV outperforms all competitors by large margins. The outperformance against the Minnesota specification is around 19%. For the one-year ahead forecasts the log predictive scores between Minnesota and CSV are comparable, with minor advantages for the homoscedastic variant of the model. Finally, for long-term interest rates (ρ_L) the linear Minnesota specification is slightly ahead for both time horizons considered. This advantage in terms of log scores, however, is negligible. Note that at the one-year ahead horizon the second strongest specification is the SSVS-GVAR, closely followed by the CSV, which ranked third. It is evident that the CSV and Minnesota specifications perform extraordinary well when used to forecast short- and long-term interest rates at both time horizons. This is due to the fact that in financial economics it is typically assumed that those quantities tend to follow random walk processes, which indicates

that a prior that centers the system on a random walk is the best choice to forecast interest rates (see Fama, 1990, for a prominent contribution outlining the difficulty to forecast interest rates at short time horizons).

Comparing the differences between the one- and four-steps time horizon reveals that when used to conduct short-term forecasts, the CSV specification provides increases in predictive accuracy which are significant, especially for GDP and (short-term) interest rate forecasts. However, and this corroborates the findings in Carriero *et al.* (2012), for one-year ahead forecasts the CSV specification has a slight disadvantage when used to forecast inflation and interest rates. This is due to the fact that for longer forecasting horizons, the predicted volatilities approach their long-run mean, implying that the differences in conditional volatilities between the homoscedastic Minnesota-GVAR and the CSV specification disappears.

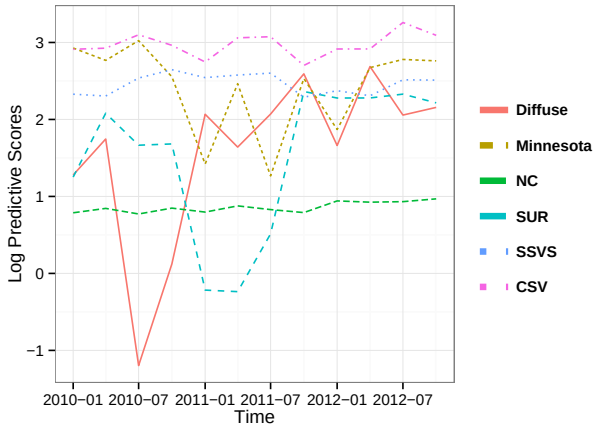
In our forecasting exercise we have made some efforts to robustify our results with respect to different estimation periods. Some interest results are worth discussing. First, reducing the length of the initial estimation sample (thus effectively including the crisis of 2008/2009 in the hold-out period) leads to qualitatively similar results, although the log predictive scores show significantly more variation over this time period. Furthermore, the inclusion of stochastic volatility helps to robustify the results with respect to the crisis. This implies that over the period 2008 to 2009 the LPS associated to the CSV specification drops significantly less as compared to other specifications.

Finally, the inclusion of stochastic volatility should make the forecasts more robust. This implies that in times of economic crisis, stochastic volatility would lead to wider predictive densities, whereas in "normal" times, the predictive density would be less dispersed. Figure 1 and Figure 2 present the evolution of the log predictive score across different prior structures for the GVAR model. Note that the dashed pink line corresponds to the CSV-specification. Taking a look at the evolution of the LPS for real GDP on both time horizons indicates that the GVAR with stochastic volatility is quite robust, in the sense that it forecasts similarly well during tranquil and crisis times. Comparison with all other specifications, which tend to show a much higher variability of LPS over time, reveals that the CSV specification helps to robustify the analysis with respect to different time periods. This result holds true for all variables under scrutiny, with the exception of the real exchange rate at the one-step ahead time horizon. It can be seen that in that case, the dashed pink line is also quite volatile, dipping below zero twice over the time period 2010Q1 to 2012Q4. Again, the reason here is that due to the single latent process, which also incorporates some information of the variability of the exchange rate equation, is mainly driven by GDP, inflation and both interest rates, which are more homogeneous.

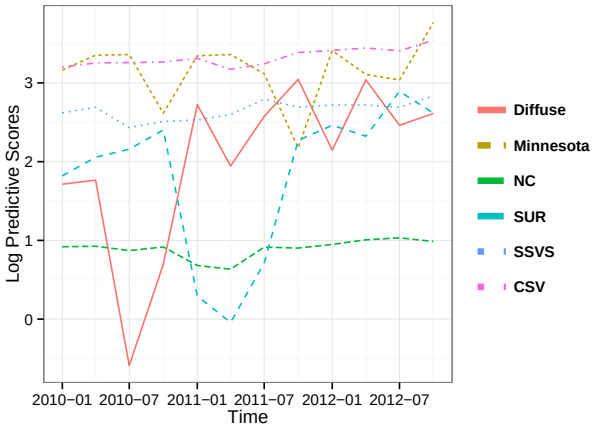
For the one-year ahead time horizon, it can be seen that the pattern is again similar to the one-step ahead case. However, even though the CSV specification is marginally weaker for interest rates and inflation, the variability over time is lower as compared to most other specifications. Another interesting regularity is that across different variables, the shape of the LPS-curves of different specifications is quite similar, indicating that if a model works well to forecast some variable at a given time, it also works well for other variables at that time. This can be seen by comparing the evolution of log predictive scores for the Diffuse prior specification. For both time horizons, this specification tends to produce weak forecasts for the beginning of the sample (2010Q1 to 2010Q4), but then recovers from 2011Q1 onwards.

Figure 1: Evolution of Log Predictive Scores over time - 1-step ahead predictive density.

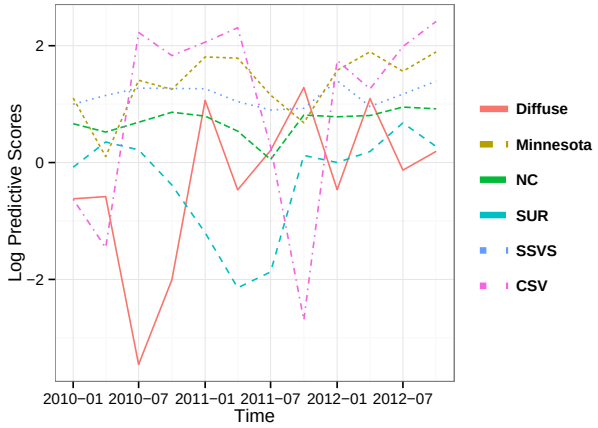
(a) Real GDP



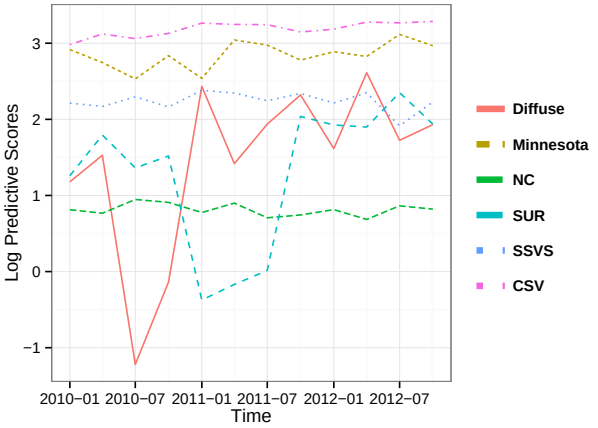
(b) Inflation



(c) Real Exchange Rate



(d) Short-term interest rates



(e) Long-term interest rates

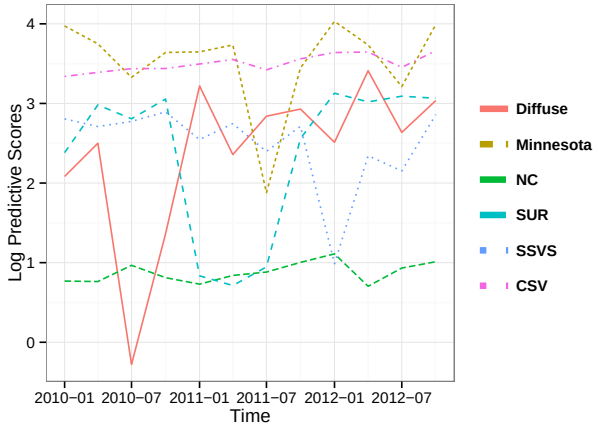
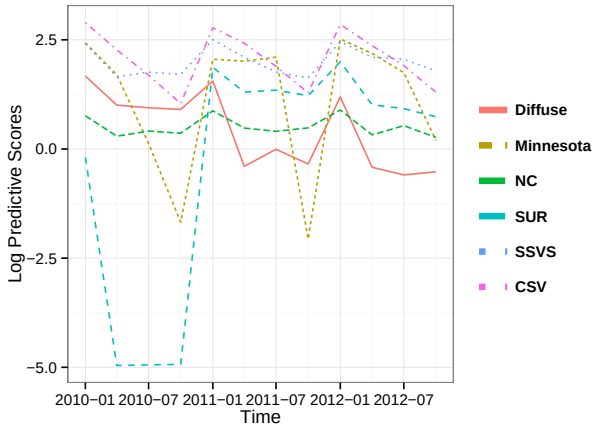
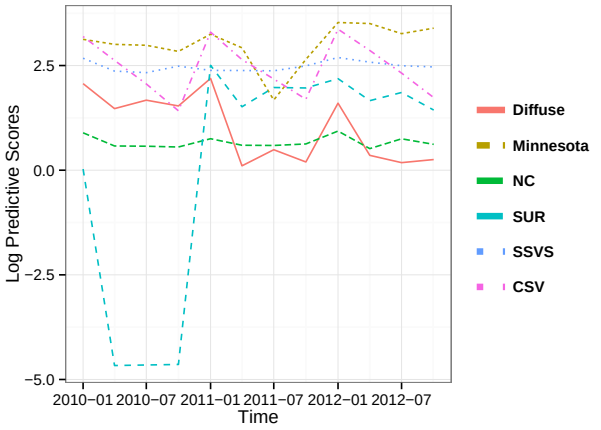


Figure 2: Evolution of Log Predictive Scores over time - 4-steps ahead predictive density.

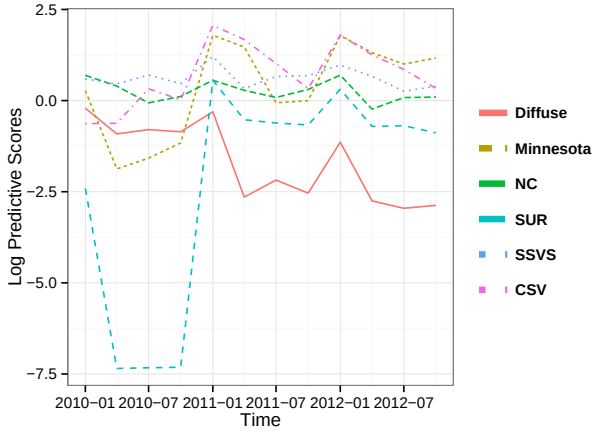
(a) Real GDP



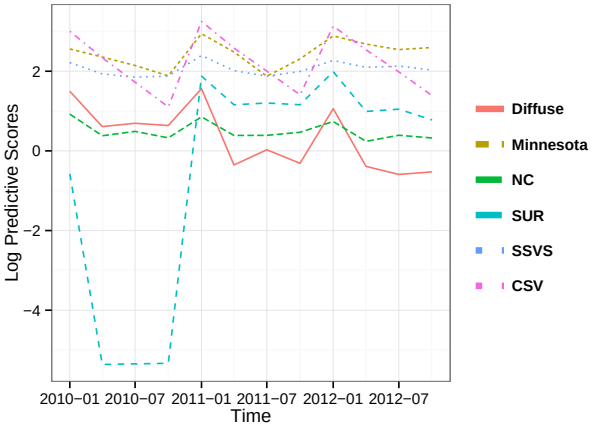
(b) Inflation



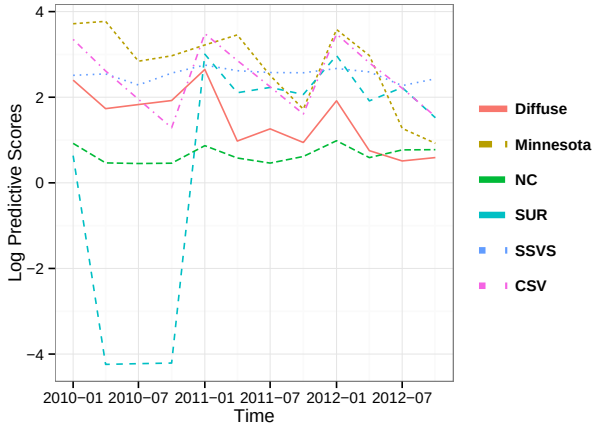
(c) Real Exchange Rate



(d) Short-term interest rates



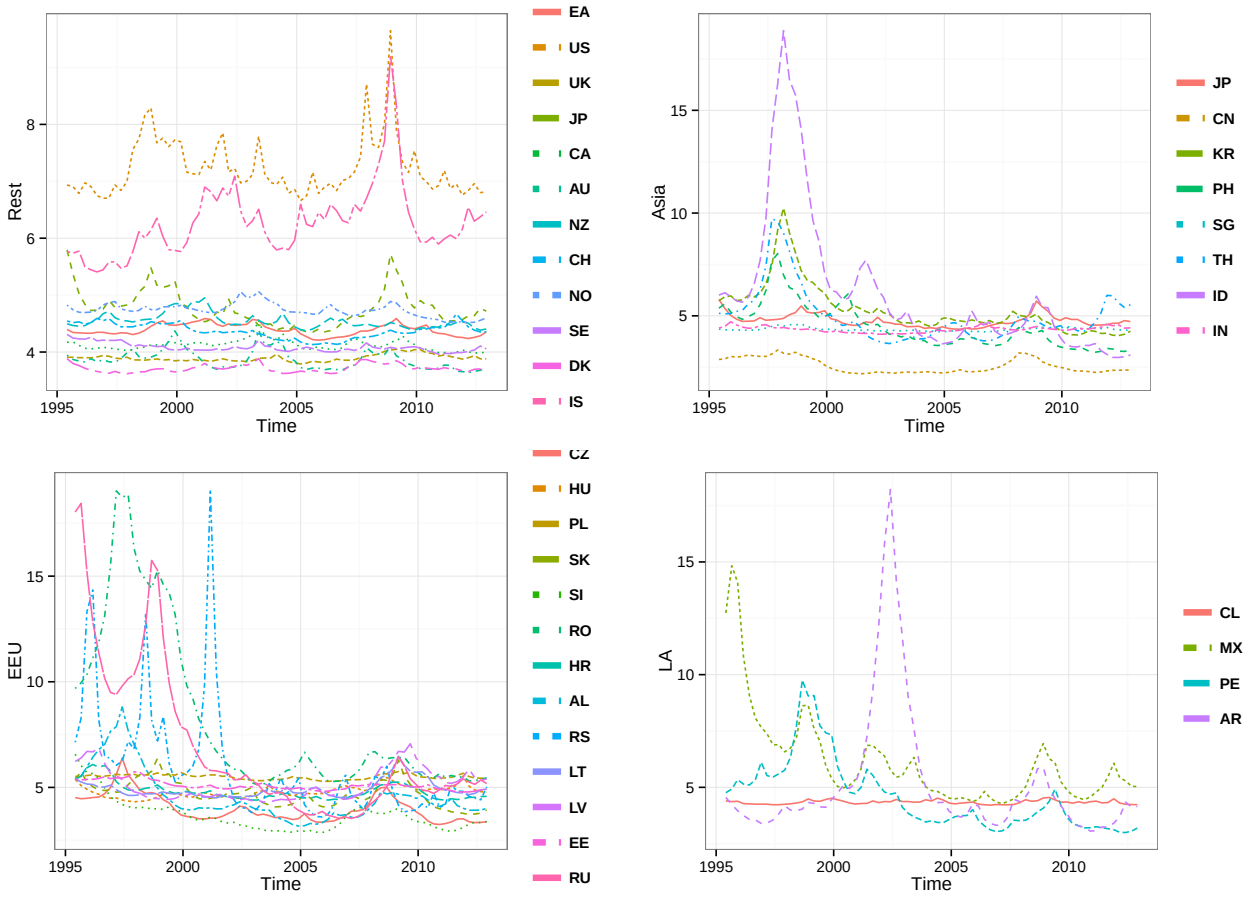
(e) Long-term interest rates



4.3 Taking a look at the second moment

The previous subsection highlighted the strong, positive effect of stochastic volatility on density forecasting accuracy. The reason why increases in log scores are common for most variables is mainly due to the fact that stochastic volatility allows for another dimension of flexibility, namely accounting for structural changes in the volatility of the underlying time series at the local level. However, because the GVAR framework also allows us to exploit the cross section, one further explanation of this accuracy-premium could be due to information originating from other countries. For example, our framework implicitly allows for contagion effects in terms of rising macroeconomic volatility, which implies that cross-sectional information is exploited efficiently. This is achieved by noting that other countries influence domestic volatility through the inclusion of weakly exogenous factors. As a consequence, our model succeeds in exploiting

Figure 3: Posterior Mean of country specific standard deviations over time



this information and translates this into stronger forecasting performance. It is evident that some countries tend to react faster to economic shocks then other countries. In terms of forecasting this would imply that if country j experiences a sudden rise in volatility at time $t+h$, country i would also be influenced at time $t+h$ through the weakly exogenous factors. This would contemporaneously affect the predictive densities in country i, leading to wider prediction intervals.

Figure 3 presents the posterior mean of country specific volatilities across country groups for selected economies. Note that our model succeeded in finding most significant economic events

in all countries under scrutiny. This includes the economic crises in Argentina and Russia, the Asian crisis and for the developed economies the recent downturn of 2008/2009. Furthermore, within-group-volatility appears to be quite homogeneous. This implies that most economies within a group tend to follow the same pattern in terms of volatility dynamics.

5 Conclusion and Further Remarks

This paper has shown that adding stochastic volatility to the GVAR modelling framework tends to improve the accuracy of density forecasts by large margins. Furthermore, as expected, stochastic volatility specifications tend to produce more robust predictions with respect to the underlying forecasting period. Our GVAR with common stochastic volatility improves upon a battery of linear Bayesian GVARs for several variables under scrutiny at the one-quarter ahead time horizon, with the main exception being the real effective exchange rates. This result is mainly due to the heterogeneous nature of exchange rate volatility as compared to GDP, inflation and interest rate volatility. Thus, to further increase the predictive capabilities when it comes to forecasting real exchange rates it might be needed to introduce more flexibility and allow for different stochastic processes across the variables within the local level models. The results found for the one-quarter ahead horizon carry over to the one-year ahead horizon. For predicting GDP and interest rates, the CSV specification is still the best model, whereas inflation and real exchange rates it falls behind its homoscedastic counterpart, the linear Minnesota specification. To account for the prevailing heterogeneity observed in the world economy it would also be possible to replace the Minnesota prior used for the CSV specification with a hierarchical SSVS prior specification. This could combine the virtues of a specification which is robust towards heteroscedasticity and a specification which accounts for model uncertainty at the individual country level. As a possible avenue of further research the inclusion of time varying dynamic coefficients could also prove to have a positive influence on the accuracy of point and density forecasts.

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A Data Appendix

Advanced Economies (11):	US, EA, UK, CA, AU, NZ, CH, NO, SE, DK, IS
Emerging Europe (14):	CZ, HU, PL, SK, SI, BG, RO, HR, AL, RS, TR, LT, LV, EE
CIS and Mongolia (6):	RU, UA, BY, KG, MN, GE
Asia (9):	CN, KR, JP, PH, SG, TH, ID, IN, MY
Latin America (5):	AR, BR, CL, MX, PE
Middle East and Africa (1):	EG

Abbreviations refer to the two-digit ISO country code. Source: Feldkircher (2013)

Table 3: Country coverage

Table 4: Data description

Variable	Description	Min.	Mean	Max.	Coverage
y	Real GDP, average of 2005=100. Seasonally adjusted, in logarithms.	3.465	4.516	5.194	100%
π	Consumer price inflation. CPI seasonally adjusted, in logarithms.	-0.258	0.020	1.194	100%
e	Nominal exchange rate vis-à-vis the US dollar, deflated by national price levels (CPI).	5.699	-2.220	5.459	97.8%
ρ_S	Typically 3-months-market rates, rates per annum.	0.000	0.100	4.332	93.5%
ρ_L	Typically government bond yields, rates per annum.	0.006	0.060	0.777	39.1%
poil	Price of oil, seasonally adjusted, in logarithms.	-	-	-	-
Trade flows	Bilateral data on exports and imports of goods and services, annual data.	-	-	-	-

Summary statistics pooled over countries and time.

The coverage refers to the cross-country availability per country, in %.

Source: Feldkircher (2013).