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Account of Different Views in Dynamic Choice Processes

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1. Introduction

In the past years, the use of disaggregate choice models has been strongly advocated (see, for example, Golledge and Timmermans 1988, Ben Akiva and Lerman 1985, Bahrenberg, Fischer and Nijkamp 1984, Pitfield 1984, Johnson and Hensher 1982), as such models enable to encapture stochastic and behavioural aspects of spatial decision processes. Starting from the observation that modelling at the level of the individual actor in the spatial system (consumers or suppliers of activities, such as, for example, migrants, travellers, real estate developers or local government decision makers) offers the promise of new insights into decision making and choice behaviour processes, various researchers have devoted considerable efforts to the development of behavioural spatial choice models capable of considering individual choices from a set of discrete alternatives at a point in time. The emphasis of such discrete choice models has been - with very few exceptions - strictly cross-sectional even if the choice processes studied were inherently dynamic in nature.

Quite recently, there has been increasing attention laid on modelling change processes. The reasons for such a focus are well known and relate essentially to a concern with economic, social and environmental change in general and to an interest in identifying the influences on change and understanding the dynamics of choice behaviour in particular. In the last few years several approaches to modelling dynamic choice processes have been developed. These approaches widely differ in scope and in methodology. A major distinction among these approaches can be made with respect to the temporal unit of analysis (continuous versus discrete). Correspondingly discrete-time and continuous-time dynamic approaches may be distinguished. Continuous-time approaches avoid the potentially arbitrary nature of the definition of time of the discrete-time approaches and enable to explicitly incorporate time in specific change points, while discrete-time approaches have to identify 'natural' decision periods which are invariant across the population of sampled individuals. The parameters derived in the latter case are generally not invariant to the positions of and the length between the time separation points. Discrete- and continuous-time approaches may be further disaggregated according to the nature of choice (discrete versus continuous choice). Thus, four broad types of approaches modelling dynamic choice processes may be distinguished (see Figure 1). Only very recently there have been attempts to integrate continuous and discrete choices intertemporally (see, for example, Hensher 1988).

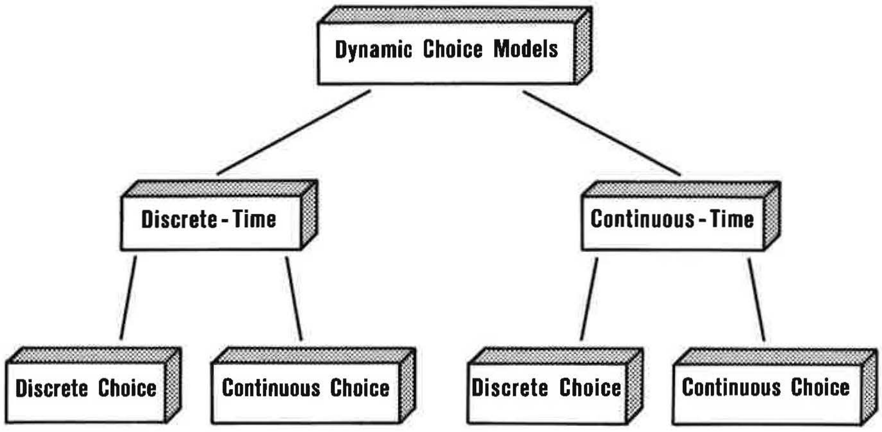

Fig. 1. Different Classes of Dynamic Choice Modelling Approaches

The emphasis in this paper is on discrete-time and continuous-time discrete choice model approaches. First, the panel data-based discrete-time discrete choice model approach will be described. Then, two continuous-time discrete choice modelling approaches will be discussed: the master equation and the ecological deterministic approaches in modelling dynamic choice processes. In section 3 the master equation approach and its relation to the (static) multinomial logit model will be summarized. The ecological deterministic view leading to dynamic extensions and generalizations of the multinomial logit and dogit models will be characterized in section 4. Of course, there are several other important and promising approaches to modelling the dynamics of choice processes which can not be discussed due to space constraints.

2. The Panel Data-Based Discrete Choice Approach to Modelling the Dynamics of Choice Processes

In the recent past social and economic scientists have developed an increasing interest in the potential which the panel data approach offers to measure and model the components of behavioural change at the individual level (see, for example, Coleman 1981, Tuma and Hannan 1984, Hensher and Wrigley 1984, Hensher 1986b, Wrigley 1986). The most proclaimed reason for this approach is the ability to examine the role of temporally-specific phenomena on choice behaviour at different points in time.

The essence of panel data is information on a (more or less) fixed sample of decision-makers across time such that statements can be made about behavioural response at the individual level. Panel data may be obtained by classical panel surveys which involve repeated measurements on the same individuals at different points in time, by rotating panel surveys which are characterized by a process of planned 'retirement' of sample units and systematic 'refreshment' by new representative sample units, or by mixed panel surveys which are hybrids of classical panel surveys on the one hand and rotating panel surveys or repeated cross-sectional surveys on the other hand.

The great potential of panel data for dynamic modelling stems from both the temporal nature of the data and the data linkage for each decision-maker. Panel data enable one to explicitly recognize the intertemporal nature of choice outcomes, especially the role of state dependence and habit persistence (cumulative inertia). Moreover, it is expected that the use of panel data results in greater efficiency, in both statistical and behavioural terms, than the estimation of separate relationships in the case of a repeated cross-sectional sample (see Johnson and Hensher 1982, Coleman 1981). A major shortcoming of repeated cross-sectional surveys refers to the fact that the sample units are not retained from one time period to the next. There is no possibility to decompose observed change in behaviour over time into the two components: changes in population composition and changes in sample behaviour. Thus, there is no doubt that dynamic models of discrete choice have to be based on panel data.

Over the past few years standard random utlity based discrete choice theory has been extended to accomodate a temporal dimension. Panel data-based discrete-time discrete choice models are concerned with a range of intertemporal formulations of the choice processes. The critical issues in an intertemporal specification of a choice model are related to the proper treatment of three types of systematic variation: heterogeneity, non-stationarity and structural state dependency. Heterogeneity refers to the variation among individuals due to both observed and unobserved external influences including variation caused by the censoring of the panel data base . This form of dependency may be treated in a number of different, but not necessarily mutually exclusive ways. For example, the set of decision makers may be disaggregated by exogeneous characteristics or by decision process characteristics in order to account for heterogeneity or taste variation. Alternatively, the presence of heterogeneity may be controlled for through the use of equal likelihood conditioning sequences (see Crouchley, Pickles and Davies 1982). Non-stationarity refers to the variation in individual and aggregate choice probabilities resulting from changes in the behavioural environment affecting the decision maker and/or the choice options. The third type of variation , structural state dependency (also termed feedback effects), refers to the dependency of current individual choice probabilities on preceding individual history. Structural state dependence effects may arise due to several reasons. Choice outcomes may depend on previous choices *(markovian effects*), on the length of time the current state has been occupied (*duration-dependence effects*), on previous interchoice times (*lagged duration-dependence effects*) and on the number of times different states have been occupied (*occurrence-dependence effects*) (Wrigley 1986). For practical reasons it might be useful to assume that one or more of these sources of state dependence are unimportant and, thus, may be neglected for the choice processes under consideration.

The methodological problem posed to the analyst by the presence of all three types of systematic variation in the data is very considerable. It is already not an easy task to disentangle the influences of intertemporal state dependence and heterogeneity, especially when some choice-relevant influences are unobserved (i.e. neglected or unmeasurable) and if they are temporally invariant and , thus, correlated with any time invariant observable variable. Moreover, omitted variables may and most likely do introduce a spurious time-dependence effect and bias into the parameter estimates of the observed exogeneous variables. The so-called 'cumulative inertia' effect identified in residential mobility studies is a typical example of spurious time-dependence effects resulting from omitted variables (see Wrigley 1986, Wrigley, Longley and Dunn 1988). It is clear that the identification of the three types of systematic variation and in particular of state dependence effects is of vital importance for satisfactory modelling the dynamics in choice processes in the framework of a panel data-based discrete-time discrete choice context.

In the rapidly growing field of panel data-based discrete-time choice models four major categories of intertemporal formulations of the choice process may be distinguished: first, Bernouilli models; second, markov models and their generalizations in form of Polya process models; third, models with habit persistence, and finally renewal models of structural state dependence (see Heckman 1981, Hensher and Wrigley 1984).

Bernouilli model approaches including the independent trials, the random effects and the fixed effects Bernouilli models are the simplest and most familiar models of dynamic stochastic behaviour (especially in the context of stochastic buying behaviour). The independent trials model is based on the assumption that the probability of choosing an alternative $a \varepsilon A = \{1,...,A'\}$ is constant over time. This model version does neither account for heterogeneity nor for structural state dependency and non-stationarity. The rigid homogeneity assumption has been ralaxed by the more sophisticated random and fixed effects model versions which account for the presence of unobserved temporally correlated error components (heterogeneity). But they do not generate structural relationships between choice outcomes in different time periods. The random effects model assigns to each individual an 'incidental ' or individual-specific parameter drawn from a population density whereas the fixed effects model permits the analyst to estimate rather than to impose the population density for the incidental parameters.

Structural dependence among time-ordered discrete choice outcomes can be analysed by all the other model categories which account for structural state dependence effects. *Markov models* (including time-homogeneous and time-inhomogeneous model versions) have been used quite frequently to study the dynamics of choice behaviour. This is especially true for the first order models which assume that choices made in the last time period are the only prior choices relevant to current choices. Of course, they are accounting for what has been termed markovian effects. The conventional model versions assume homogeneity and stationarity, i.e. that the transition probabilities apply to all individuals in the population and that the transition probability matrix is independent of time. It is worth noting that most of the non-inventory-based variety-seeking models are based on the concept of first-order markov chains in attempting to predict switching probabilities from concepts of variety-seeking (see Timmermansand Borgers 1985). Where an entire event history of the choice outcomes is relevant to current decision making, as it is suggested in several human capital models in labour economics, then a Polya process is assumed. *Models for Polya type processes* might be considered to be generalizations of markov models (see Heckman 1981 for more details on this issue). In the more sophisticated versions heterogeneity in unmeasured variables is introduced.

Models with habit persistence (including Coleman's 'latent markov' model) form the third category. They assume that prior propensities to choose a state rather than prior occupancies per se influence the current probability that a state is occupied or changed. They ignore markov effects but account for lagged effects and allow relative evaluations in other periods to determine current choice outcomes. The models capture the notion of 'naive' habit persistence contrasting with the first order markov model or the Polya process models which capture only the chosen-state dependencies (Hensher and Wrigley 1984). The model version outlined in Heckman (1981) might be considered as a discrete data analogue to the distributed lag models.

The final model category, the *renewal process models* of structural state dependence, assumes that the only effect of previous state occupancy on current choices is from the most recent current spell in the state or in other words that the current continuous duration in a state is influencing the decision to continue in or to leave the state. When the decision maker leaves the state the experience is lost and, thus, irrelevant to future decisions. Heterogeneity not accounted for in the conventional model version can easily be introduced (see Heckman 1981).

Consequently, *a general intertemporal representation of individual choice behaviour* ideally requires to include terms to represent all the dimensions of intertemporal causality captured by the four model categories and importantly to enable to separate these intertemporal relationships from persistent individual-specific effects (heterogeneity) (see Hensher and Wrigley 1984), i.e.

$$\text{Current Choice} =$$

$$f\left\{ f_1\left(\begin{matrix}\text{current, past}\\ \text{and/or future}\\ \text{levels of}\\ \text{exogenous variables}\end{matrix}\right), f_2\left(\begin{matrix}\text{effects of the}\\ \text{relevant entire}\\ \text{(or part) past}\\ \text{history}\end{matrix}\right), f_3\left(\text{habit persistence}\right), f_4\left(\begin{matrix}\text{cumulative of the}\\ \text{most recent}\\ \text{continuous}\\ \text{experience}\\ \text{in a state}\end{matrix}\right), f_5\left(\begin{matrix}\text{accounting for}\\ \text{heterogeneity}\end{matrix}\right) \right\}$$

Markov/Polya Process Model — Model with Habit Persistence — Renewal Process Model

A model which fulfills this requirement has been developed by Heckman (1981). His general model of discrete-time individual choice behaviour is sufficiently flexible to take into account time-dependent explanatory variables and to account for complex structural state dependence inter-relationships and for a general characterization of heterogeneity.

Heckman's model is based upon the following ideas. It is assumed that from a random sample $I=\{1,...,I'\}$ of choice makers or individuals information on the presence or absence of an event (i.e. choice outcome) in each of T equi-spaced time periods is assembled. The key assumption of the model is that discrete outcomes are generated by continuous variables with cross-thresholds ;or more precisely that an event for decision maker $i \varepsilon I$ in time period t occurs, if and only if a continuous latent random variable y_{it} crosses a threshold. In applications, such continuous variables may be related to well defined economic concepts. For example, in Domencich and McFadden (1975) the continuous variables producing discrete choices are differences in utilities of possible choice.

Only for convenience this threshold may be assumed to be zero. The random variable y_{it} is supposed to consist of two components: a deterministic component v_{it} which is a function of exogeneous, predetermined and measured endogeneous variables affecting current choices; and a purely random disturbance component ε_{it} , i.e.

$$y_{it} = v_{it} + \varepsilon_{it} \qquad (1)$$

with

$$y_{it} \geq 0 \quad \text{if and only if} \quad d_{it} = 1 \qquad (2)$$

and

$$y_{it} < 0 \quad \text{if and only if} \quad d_{it} = 0 \qquad (3)$$

where d_{it} is a dummy variable denoting the occurrence of the event under consideration. The distribution of the d_{it}'s is generated by the distributions of the ε_{it}'s and v_{it}'s where adopting a multinomial probit formulation it is assumed that $\varepsilon_{i.}$ is normally distributed with mean zero and a (T,T)-positive definite covariance matrix. This normality assumption generates a model which admits a rather general characterization of heterogeneity. It is worthwile mentioning that alternative assumptions of v_{it} and ε_{it} give rise to a variety of other interesting models useful for analysing discrete panel data.

Assuming that the latent variable y_{it} is a linear function of observed choice-relevant attributes (including past exogeneous variables, current exogeneous variables and expectations of future exogeneous variables), represented in the vector x_{it}, of lagged values y_{it} and of past outcomes $d_{it'}$ with $t' \leq t$, Heckman's general model may be written as

$$v_{it} = x_{it}\beta + \sum_{j=1,\dots,\infty} \gamma_{t-jt}\, d_{it-j} + \sum_{j=1,\dots,\infty} \lambda_{jt-j} \prod_{l=1,\dots,j} d_{it-l} + G(L)\, y_{it} \qquad i \varepsilon I; t=1,\dots,T \qquad (4)$$

where β is a vector of parameters of x_{it}; $G(0)=0$ and $G(L)=g_1 L+g_2 L^2+\dots+g_k L^k$ is a general lag operator, $L^k y_{it}=y_{it-k}$. The initial conditions $d_{it'}$ and $y_{it'}$ for $t'=o,-1,-2,\dots$ (in other words, the relevant presample history of the process) are assumed to be predetermined or exogeneous. This assumption, however, is only valid if the unobserved choice-relevant charateristics generating the process are serially independent.

The first term at the right-hand side of (4) may incorporate past and current information and future expectations on exogeneous choice-relevant attributes affecting current choices, as already mentioned above. The second term represents the effects of the entire past history on choice behaviour at time t and, thus, structural state dependence effects. This term is assumed to be finite. The coefficients for past events (i.e. γ_{t-jt}) are considered to be functions of the current time period t and the time period t-j in which the event occurred. The third term denotes the cumulative effect on current choices of the most recent experience in a state. It is assumed to be finite. The λ's denote parameters. Finally, the last term in (4) representing the effect of previous relative evaluations of the two states on current choices captures the action of habit persistence.

Heckman (1981) has shown that the above mentioned models, namely the Bernouilli models, the markov and Polya process models, the models with habit persistence and the renewal process models of structural state dependence, emerge as restricted versions of this general panel data-based discrete-time choice model by imposing certain restrictions on the parameters.

Even if a probit formulation requiring a fairly general error covariance matrix is theoretically rather attractive to handle state dependence and heterogeneity, it is in practice only of limited use for more than three choice options per time period (see Hensher 1988) . Thus, current efforts in computationally tractable discrete-time discrete choice models for multiple (unordered) choices in the presence of state dependence and serial correlation are directed to take the cross-sectional multinomial logit rather than the multinomial probit model as a starting point. An important example is the generalized beta-logistic model for longitudinal data which permits heterogeneity to be controlled in the estimation of the structural parameters of the determinants of choice behaviour and incorporates time-varying exogeneous variables as well as feedback effects. An application to residential mobility within the County Borough of Leeds is described in Davies (1984).

In these efforts three broad modelling approaches may be distinguished according to Hensher and Wrigley (1984) (see also Hensher 1986a). The first approach involves estimating wave-specific (i.e. time period specific) models separately for each wave and taking the estimated choice probabilities in order to identify a choice sequence probability. Although exogeneous variables can be included to represent previous period choice outcomes or propensities to occupy states in previous periods, strong assumptions such as zero serial correlation are invoked (see Hensher and Le Plastrier 1985). In the second approach the data is pooled, with inter-period linkages represented by a lagged index, one for each exogeneous variable. The use of such a lagged index is a way to deal with state dependence without serial correlation attributable to lagged endogeneous variables (for more details see also Davies 1984). The third approach considers the data as an explicit sequence wherein the likelihood function associated with the choice sequence probability has two major components. One component accounts for the time-invariant influences (including the initial conditions) and the other one incorporates the time-varying influences. The separation of these components provides the formal mechanism for explicitly handling heterogeneity (for more details see Smith, Hensher and Wrigley 1985). Of course, the modelling style increases in complexity as one moves from the first to the third approach.

Much progress has been made in panel data-based discrete-time discrete choice modelling in the last few years. But unquestionably, there are several problems which are not yet satisfactorily solved up to now, such as, for example, the problem of attrition bias, the problem of initial conditions, the problem to account for heterogeneity due to variation outside the sample period, the problem of non-stationarity etc.

3. The Master Equation View in Dynamic Choice Processes

An interesting alternative to the panel data-based discrete-time approach for analysing dynamic choice processes can be found in the so-called master equation approach. This approach which has already a long tradition in physics (especially in the context of laser theory and spin relaxation) has been brought to the attention of the regional science community in the early 1980s by Smith (1981), Kanaroglou, Liaw and Papageorgiou (1986a,b) and especially by Haag and Weidlich (1983, 1984, 1986). In the last few years much research has been undertaken by Haag and Weidlich and their associates (see inter alia Weidlich 1987, Munz and Reiner 1987, Haag 1988, Weidlich and Haag 1988) to open a large field of applications in the social and economic sciences in general and regional science in particular where special emphasis has been laid on the dynamics of migration processes.

A *master equation* describes the evolution of the probability function, representing the transition probabilities for well defined states of a dynamic micro-based system of actors. By using, for example, a mean value approach an elegant link can be established between micro -

levels and macro--levels of a system, so that structural changes in dynamic systems can be analysed in a statistically satisfactory way.

There are several cogent reasons for using the master equation approach in analysing dynamic choice processes. A first reason is its flexibility and generality. The ranges of possible behaviours embodied in master equations is almost unlimited (Smith 1981). In the second place, this approach allows to take account of synergetic effects in the behaviour of different individuals (such as adaptation processes and learning effects). The socio-configuration includes then the individual transition probabilities based on joint interaction effects. A third major advantage of the master equation approach is that it links the micro-level decisions of individuals with the macro-level behaviour of collective variables (seeFigure 2). Feedback elements, heterogeneity (variation between individuals) and non-stationarity (variation over time) can be taken care of.

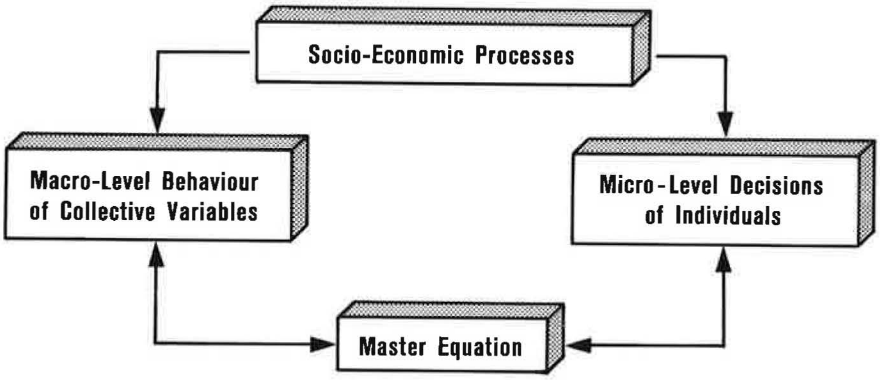

Fig. 2. The Master Equation Point of View: The Relationship between the Micro- and Macro-Level in Decision Processes

The purpose of this section is to briefly outline this approach in general terms and to illustrate its relationship between the master equation approach and the static multinomial logit model. Let us start with some preliminary notational remarks. As usually $I_s = \{1,...,I'_s\}$ may denote a set of decision makers or individuals belonging to population segment s=1,...,S'. Without loss of generality the subindex s is dropped in the sequel in order to facilitate notation. Each individual has to select one alternative a out of a set A={1,...,A'} of choice options. The macro state of the decision system at any time t may be described by the so-called *decision configuration* (i.e. the distribution of choice frequencies):

$$\mathbf{n} = \{n_1, n_2, \ldots, n_{A'}\} \qquad (5)$$

with

$$I' = \sum_{a \varepsilon A} n_a \qquad (6)$$

consisting of A' integer variables n_a where n_a denotes the number of individuals who have chosen option a.

In the course of time transitions can take place from any initial decision configuration **n** into one of the neighbouring configurations $\mathbf{n} + \mathbf{k} = \{ n_1+k_1, n_2 + k_2, ..., n_{A'} + k_{A'}\}$ where k_a (a=1,...,A') is positive or negative integer with $k_1+k_2+ ... +k_A = 0$. These possible transitions arise because any one of the I' decision makers who originally preferred alternative b now makes a transition to alternative a. The individual transition rates between all alternatives give rise to a total transition rate $\omega(\mathbf{n} + \mathbf{k}, \mathbf{n})$, per unit time, for the transition from decision configuration **n** to decision configuration **n+k**.

Since these transition processes are probabilistic rather than deterministic in nature, the decision configuration evolves with time stochastically. For this type of motion there is the well-established general theory for stochastic markov systems in terms of an equation of motion, the so-called master equation, for the probability distribution over the configurations of such systems.

Let us introduce the probability distribution function as

$$P(\mathbf{n},t) = P(n_1, n_2, ..., n_{A'}; t) \qquad (7)$$

which is, by definition, the probability that the decision configuration **n** is realized at time t. Of course, P(**n**, t) must satisfy at all times the following probability normalization condition

$$\sum_{\mathbf{n}} P(\mathbf{n},t) = 1 . \qquad (8)$$

If the configurational transition rates $\omega(\mathbf{n+k},\mathbf{n})$ from any **n** to all neighbouring **n+k** are given, then an equation of motion for P(**n**, t) can be derived. This equation which describes the dynamics for P(**n**,t) at the probabilistic level follows by specification of the general master equation for markov systems to the decision system at hand (see for more details Haag and Weidlich 1984, Haag 1988). It reads

$$dP(\mathbf{n},t)/dt = \sum_{\mathbf{k}} [\omega(\mathbf{n}, \mathbf{n+k}) \; P(\mathbf{n+k},t) - \omega(\mathbf{n+k}, \mathbf{n}) \; P(\mathbf{n},t)] \qquad (9)$$

where the sum on the right-hand side of (9) extends over all **k** with non-vanishing configurational transition rates $\omega(\mathbf{n},\mathbf{n}+\mathbf{k})$ and $\omega(\mathbf{n}+\mathbf{k},\mathbf{n})$, respectively. The change with time of the probability of decision configuration **n** is caused by two effects of opposite direction, first by the probability flux from all neighbouring configurations **n**+**k** into **n** (first term of the right hand side of the equation) and second by the probability flux from **n** to all **n**+**k** (second term of the right hand side). The solution of this equation (9), namely the time-dependent distribution P(**n**, t), contains all information about the choice process at the most detailed level. In particular not only the mean value of **n** (t), but also their mean square deviations due to fluctuations in the decision process can be calculated.

In order to make the model explicit the transition rates $\omega(\mathbf{n}+\mathbf{k},\mathbf{n})$ governing the dynamics of the system by (9) have to be constructed. This can be done as follows: First, so-called *dynamic advantage* or *utility functions* (describing the advantage for an individual to adopt choice option a) have to be introduced; second, the *individual transition rates* (describing changes of probability per unit time with the dimension 1/(time), namely that an individual will choose alternative b at $t+\tau$ given that alternative a has been chosen at t) have to be defined; and finally the *total transition rates between decision configurations* (describing changes of probability per unit time from one decision configuration to a neighbouring decision configuration) have to be specified.

The desirability of an alternative a for a choice maker may be described by a so-called dynamic advantage or utility function u_a. Of course, the utility of an alternative a for an individual depends on the socio-economic situation of the system at hand , to be expressed by the configuration **n** (t) and by certain trend parameters which in turn depend on various push/pull terms. It is important to note that the concept of dynamic advantage utilities is not an ordinal, but a cardinal one. Moreover, it should be emphasized that interaction effects among choice makers may be taken into account via their dependence on **n** (t).

The dynamic advantage functions remain purely theoretical quantities unless their influence on the dynamics of the decision process is specified. The dynamics are governed by the individual transition rates (per unit of time), the p_{ba}'s , of any individual who originally preferred alternative a and now makes a transition to alternative b. In order to make the relationship between the master equation approach and the static multinomial logit model clear, it is assumed that the p_{ba} 's are functions of the above mentioned utilities, namely that

$$p_{ba}(\mathbf{n}) = \nu \exp\left[u_b(\mathbf{n}) - u_a(\mathbf{n})\right] \qquad a, b \varepsilon A \qquad (10)$$

where ν is the overall flexibility parameter of subpopulation I_s with respect to changes in attitude and essentially accounts for global effects facilitating or impeding a transition from alternative a to alternative b. The construction of the individual transition rates has the purpose to attribute the information contained in the choice behaviour of certain individuals to a few parameters embedded in the corresponding utility functions.

The crucial element of a dynamic decision process is the configurational conditional probability $P(\mathbf{n}+\mathbf{k}, t+\tau \mid \mathbf{n}, t)$, i.e. the probability to find a certain decision configuration $\mathbf{n}+\mathbf{k}$ at time $t+\tau$, given that the decision configuration $\mathbf{n}$ was realized at time t, because it describes how the probability spreads out in the time interval τ. The conditional probability may also depend upon the previous history of the system under consideration. But in this general case the probability evolution process becomes very complex. Thus, in general the markov assumption is made, which implies that only the very recent past is considered to be relevant, and not the whole past history.

Under the assumption that the individuals make their choices statistically independent of each other, the configurational transition rate is given by the product of the individual conditional transition rates. This analytically convenient assumption appears to be rather rigid in many decision contexts where individuals interact in their decisions. This is especially the case in a migration context.

With the help of the individual transition rates p_{ba}, defined in (10), it is easy to construct the transition rates between decision configurations. Each of the n_a individuals making a transition from alternative a to alternative b with a transition rate $p_{ba}(n_a, n_b)$ induces a configuration transition of the following type

$$(n_1, n_2, \ldots, n_a, \ldots, n_b, \ldots, n_{A'}) \rightarrow (n_1, \ldots, (n_a-1), \ldots, (n_b+1), \ldots, n_{A'}) . \tag{11}$$

Consequently, the n_a members contribute the following term

$$\omega_{ba}(\mathbf{n}+\mathbf{k}, \mathbf{n}) = \begin{cases} n_a\, p_{ba}(\mathbf{n}) = \omega_{ba}(\mathbf{n}) & \text{for } \mathbf{k} = (0,\ldots,-1_a,\ldots,+1_b,\ldots,0) \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

to the corresponding configurational transition rate (per unit time) $\omega(\mathbf{n}+\mathbf{k},\mathbf{n})$. Since the transitions between all alternatives take place simultaneously and independently, the total transition rate $\omega(\mathbf{n}+\mathbf{k}, \mathbf{n})$ is the sum of all contributions (12) so that

$$\omega(\mathbf{n}+\mathbf{k},\mathbf{n}) = \sum_{a,b\in A} \omega_{ba}(\mathbf{n}+\mathbf{k}, \mathbf{n}) . \tag{13}$$

It is worthwile to note that for very short time intervals the configurational conditional probability can be traced back to configurational transition rates and individual transition probabilities to individual transition rates (see Haag and Weidlich 1984).

The explicit form of the master equation corresponding to the static multinomial logit model is immediately obtained if the total transition rates (13) with (12) are included in equation (9). Since only transitions between decision configurations $\mathbf{n} = (n_1, \ldots, n_a, \ldots, n_b, \ldots, n_{A'})$ and adjacent configurations $\mathbf{n}^{(ba)} = (n_1, \ldots, n_a-1, \ldots, n_b+1, \ldots, n_{A'})$ for all pairs $(b,a) \varepsilon (A \times A)$ are involved, the master equation describing the full dynamics in probabilistic terms may be formulated in the following more convenient form

$$d\, P(\mathbf{n}, t)/dt = \sum_{a,b \varepsilon A} \omega_{ba}(\mathbf{n}^{(ba)})\; P(\mathbf{n}^{(ba)}, t) - \sum_{a,b \varepsilon A} \omega_{ab}(\mathbf{n})\; P(\mathbf{n}, t) \,. \qquad (14)$$

where ω_{ba} and ω_{ab} are configurational transition rates,$P(\mathbf{n},t)$ is the probability that the decision configuration $\mathbf{n}$ is realized at time t and $P(\mathbf{n}^{(ba)},t)$ is defined analogously. The master equation (14) can be interpreted as a probability rate equation. It evidently provides the link between the micro-level of individual decisions and the macro-level of the system under consideration (see also figure 2.2) and, thus, gives insight into how decisions on the micro-level of choice makers induce probabilistic fluctuations on the macro-level of mean values of the decision configurations.

The master equation (14) establishes a set of $\binom{I'}{A'}$ coupled linear differential equations for the probabilities $P(\mathbf{n}, t)$ of the $\binom{I'}{A'}$ possible configurations $\mathbf{n}$. The exact stationary solution $P^{st}(\mathbf{n})$ of (14) is reached for $t \rightarrow \infty$ (for more details on this issue see, for example, Haag 1988).

Usually, the full information contained in the probability distribution (configurational probability) $P(\mathbf{n},t)$ is not exploited in an empirical analysis due to lack of sufficiently comprehensive empirical data.Thus, generally a transition to a less exhaustive description in terms of equations of motion for the mean values of the decision configurations is made and consequently corresponding equations for the quasi-deterministic level of mean values $\bar{n}_b(t)$ with $b \varepsilon A$ rather than the master equation for the probabilistic level are solved. These equations of motion can be derived from the master equation (14) in a straightforward manner as

$$d\,\bar{n}_b(t)/dt = \nu \sum_{a \varepsilon A} \bar{n}_b(t) \exp\left(u_b(\mathbf{n}) - u_a(\mathbf{n})\right) - \nu \sum_{a \varepsilon A} \bar{n}_a(t) \exp\left(u_a(\mathbf{n}) - u_b(\mathbf{n})\right) \qquad b \varepsilon A. \quad (15)$$

The *mean value equations* (15) belonging to (14) may have one or several stationary states. It can be shown (see Haag 1988) that they coincide with the maximum (the maxima) of the stationary distribution $P^{st}(\mathbf{n})$ in the considered case (not in general). All time-dependent solutions approach for $t \rightarrow \infty$ one of these stationary states. But it is depending on the initial conditions which of the equilbrium states of (15) is approached.

The conditions for the stationary solution $\bar{\mathbf{n}}^{st} = (\bar{n}_1^{st}, \bar{n}_2^{st}, \ldots, \bar{n}_{A'}^{st})$, for which the right hand side of (15) has to be equal to zero, can be read off immediately

$$\bar{n}_b^{st} = C \exp\left[2\, u_b(\bar{\mathbf{n}}^{st})\right] \qquad b \varepsilon A \qquad (16)$$

with the normalization factor

$$C = I' / \sum_{a \varepsilon A} \exp [2 \, u_a(\bar{n}^{st})] \tag{17}$$

where n_b is the most probable number of individuals who have decided for alternative b. Thus, the quantity

$$p_b = \bar{n}_b^{st} / I' = \exp [2 \, u_b(\bar{n}^{st})] \, / \sum_{a \varepsilon A} \exp [2 \, u_a(\bar{n}^{st})] \qquad b \varepsilon A \tag{18}$$

is equivalent to the probability that any one individual selects alternative b.

Comparing the stationary solution (18) of this dynamic theory of choice processes with the outcome of the static multinomial logit model approach[1]

$$p_b = \exp (\mu \, v_b) / \sum_{a \varepsilon A} \exp (\mu \, v_a) \qquad b \varepsilon A \tag{19}$$

with v_b denoting the systematic component of utility attached to alternative b and μ a positive scale parameter of the Gumbel distribution, then one has to identify

$$u_b(\bar{n}^{st}) = (\mu \, v_b) / 2 \qquad b \varepsilon A \, . \tag{20}$$

Thus, both 'utilities' coincide up to an ordinary rescaling, if the same utility function can be assigned to all individuals of the decision configuration. The coincidence of the stationary formula (18) with the multinomial logit model (19) under appropriate rescaling (20) of the utility concepts has the meaning that the static multinomial logit model describes the limiting case for $t \to \infty$ in the special case of non-interacting individuals where u_b does not depend on n.

In Haag and Weidlich (1986) it is described how the master equation approach can be used for analysing the dynamics of inter-regional migration systems using data for the Canadian system. Similar in spirit is the analysis of migration systems undertaken by Kanaroglou, Liaw and Papageorgiou (1986a,b). These authors adopt the master equation approach for dealing with the evolution of the migratory system and provide an operational framework in which a somewhat more explicit link between the macro-properties of the population system and human behaviour is given. Quite recently, Haag and Weidlich initiated an international project in which the master equation approach has been applied to compare and evaluate the dynamics of migration processes in six countries (Canada, France, the FRG, Israel, Italy and Sweden) (see Weidlich and Haag 1988).

4. From *Homo Economicus* to *Homo Socialis* : The Ecological Approach to Dynamic Choice Processes

The ecological (deterministic) view in dynamic choice processes suggested and put forward in a series of papers by Sonis (1983, 1984, 1986, 1987) is based on the consideration of the individual choice behaviour as a choice behaviour of *homo socialis* instead of behaviour of *homo economicus.*

Homo economicus is a totally egoistic rational omniscient creature who is supposed to accomplish a rational free choice between different competitive alternatives on the basis of the individual's utility maximization principle. *Homo socialis* is an individual whose (collective) behaviour is based on the interaction among choice-makers and on the imitation and learning within an active uncertain environment. The choice behaviour of *homo socialis* is directed by the subjective mental evaluation of the marginal temporal utilities (individual's expectations of gains in the future). This mental evaluation is heavily influenced by the enormous information flows through mass media presenting 'ready' opinions and solutions and making difficult the rational evaluation of alternatives and their utilities for an individual.

The choice behaviour of *homo socialis* in space-time continuum generates the spatio-temporal spread of alternatives (alternative innovations). Therefore, a 'duality' exists between the individual choice behaviour and the behaviour of the system generating, supporting and introducing the alternative choice options. This duality leads to the interpretation of the relative distribution of choice-makers between alternatives as individuals' choice frequencies of alternatives. Moreover, the choice and spread of alternatives occur within an *active social and physical environment* which changes the behaviour of systems supporting and individuals adopting an alternative by filtering the information flows about alternatives and by social, physical, cultural, administrative, economic, political etc. restrictions and stimulations. Thus, three major actors are participating in the dynamic choice process: *alternatives, choice-makers* and *active environment* (see Figure 3).

The behaviour of choice alternatives includes the behaviour of the systems generating, supporting and introducing alternatives and organizing their spread. The spread of alternatives incorporates features of the ecological competition between alternatives in the form of antagonistic or cooperative zero-sum games between different subsets of alternatives. The result of the ecological competition is the competitive exclusion of non-efficient alternatives.

The understanding of the choice-makers' behaviour is based on the consideration of an individual as *homo socialis* and the rejection of the concept of *homo economicus.* The external intervention of an active environment restricts the choice behaviour of individuals and changes the competitive abilities of supporting systems by generating the redistribution of choice-makers between alternatives. An active environment is weakening essentially the action of the individual's utility considerations, smoothing out the extreme action of competitive exclusion of alternatives and generating socio-economic niches preserving and supporting the existing tendencies of choice.

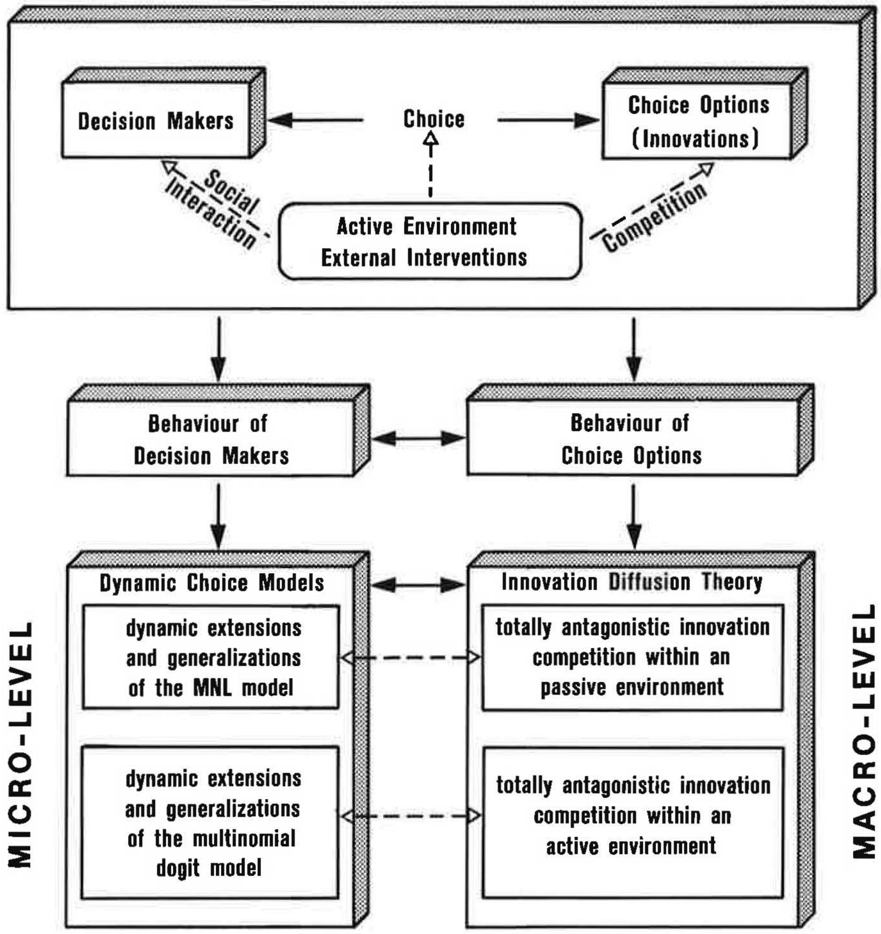

Fig. 3. The Relationship between Innovation Diffusion and Individual Choice:
The Duality Point of View

Pursuing the ecological deterministic view in dynamic choice processes it can be shown that the system of partial differential equations of the Volterra-Lotka type arising from the relative dynamics of portions of adopters of competitive (i.e. mutually exchangeable and mutually exclusive) alternatives can be reformulated in an analytical form resembling the static multinomial logit and dogit choice models. Conceptually, however, the derived dynamic extensions are different from their static counterparts. The static model versions are based on the micro-level principle of individual utility maximization, while the dynamic versions are based on the macro-level variational principle determining the balance between the cumulative social spatio-temporal interactions among choice-makers and the cumulative equalization of the choice alternatives. This balance condition is governing the dynamic choice process and constitutes the dynamic macro-level counterpart of the individual utility maximization principle. On the micro-level a somewhat different behavioural principle can be derived, namely the principle that an individual chooses an alternative not on the basis of a comparison of utilities, but on the basis of a comparison of the temporal marginal utilities (interpreted as the expectations of a gain in the future) which may be influenced by social interaction, imitation and learning processes between choice makers.

Moreover, an active environment may alter the choice behaviour of individuals, *implicitly* by filtering and/or intensifying the information flows between individuals (social interaction) and between individuals and choice options (such as, for example, informational constraints), and *explicitly* by different forms of physical, socio-economic, cultural and legal restrictions or by different forms of stimulation and support.

The dynamic continuous-time choice models which are dynamic deterministic counterparts and generalizations of the well-known static multinomial logit and dogit models will be derived in the sequel. Their discrete-time equivalents as well as issues of statistical estimation and testing may be found in Sonis (1987). It is important to mention that the discrete-time choice processes present analytically the particular cases of the universal discrete-time multistock/multilocation relative socio-temporal dynamics (see Dendrinos and Sonis 1989).

Let us formalize now the macro-level choice hypothesis, postulated by this ecological deterministic view. For this purpose consider an exhaustive set $A=\{1,\ldots,A'\}$ of A' different mutually exchangeable and mutually exclusive choice alternatives and, moreover, a multidimensional space R of space-time parameters and all decision-relevant attributes characterizing both the choice maker and the choice options. The frequency vector $\mathbf{p}(r)$, $r\varepsilon R$, may represent the relative distribution of choice frequencies in each point $r\varepsilon R$:

$$\mathbf{p}(r) = [p_1(r), p_2(r), \ldots, p_{A'}(r)] \qquad r\varepsilon R \qquad (21)$$

with

$$0 \leq p_a(r) \leq 1 \qquad a\varepsilon A,\ r\varepsilon R \qquad (22)$$

and

$$\sum_{k\varepsilon A} p_k(r) = 1 \qquad r\varepsilon R. \qquad (23)$$

The relative change in frequency $p_a(r)$ in some direction s is

$$\frac{\partial p_a(r)}{\partial s} \Big/ p_a(r) = \frac{\partial \ln p_a(r)}{\partial s} \qquad a\varepsilon A;\ r\varepsilon R \qquad (24)$$

where $\partial/\partial s$ is the directional derivative in the arbitrary direction s in the space of all explanatory choice-relevant variables and space-time parameters.

The main hypothesis which constitutes the conceptual framework of the ecological approach is as follows: The choice behaviour of the homo socialis is the collective macro-level choice behaviour such that the relative changes in choice frequencies depend on the distribution of alternatives between choice-makers, i.e, depend on all components of frequency distribution vectors. This hypothesis means analytically that the dynamic continuous-time choice model can be presented in the form of the following system of partial differential equations for each direction s:

$$\partial \ln p_a(r) / \partial s = f_{sa}(r; p_1(r), \dots, p_{A'}(r)) \qquad a\varepsilon A;\ r,s\varepsilon R \qquad (25)$$

where f_{sa} is a non-linear function in r and p (r), depending on the direction s.

The integrability conditions for (25) are the usual ones for each pair of directions q,s:

$$\partial^2 \ln p_a(r) / \partial q\, \partial s = \partial^2 \ln p_a(r) / \partial s\, \partial q \qquad a\varepsilon A;\ r\varepsilon R \qquad (26)$$

or equivalently

$$\partial f_{qa}(\mathbf{p}) / \partial s = \partial f_{sa}(\mathbf{p}) / \partial q \qquad a\varepsilon A;\ r\varepsilon R . \qquad (27)$$

These conditions mean that for each $a\varepsilon A$ there is a function $V_a(r)$ - the so-called scalar interaction potential - such that (see Sonis 1986 for more details)

$$\partial V_a(r)/\partial s = f_{sa}(\mathbf{p}) \qquad a\varepsilon A,\ r\varepsilon R \qquad (28)$$

and

$$\partial p_a(r)/\partial s = \sum_{b\varepsilon A} g_{sab}(r)\ p_a(r)\ p_b(r) \qquad a\varepsilon A,\ r\varepsilon R \qquad (29)$$

with

$$g_{sab}(r) := \partial (V_a(r) - V_b(r)) / \partial s \qquad a,b\varepsilon A;\ r\varepsilon R \qquad (30)$$

where g_{sab} represents the marginal influence (in the direction s) of the a-th choice alternative on the adoption of the b-th alternative and thus measures the actual portion of contacts stimulating the transition from a to b. The marginal interaction coefficient g_{sab} between the alternatives a and b depends on changeable (in space and time) attributes of the alternatives a and b and socio-economic characteristics of the choice-makers. Due to

(23) the interaction matrix $G_s := (g_{sab})$ has to be antisymmetric (i.e. $g_{sab}+g_{sba}=0$ for $a,b\varepsilon A$). This antisymmetry may be interpreted in such a way that each pair of choice alternatives a and b participates in an antagonistic zero-sum game with the interaction coefficient g_{sab} being the payoff (expectation of gain) for the a-th choice alternative. Moreover, the antisymmetry of G_s implies the existence of the competitive exclusion equilibrium states, i.e. the transfer of all individuals to one alternative (see Sonis 1984).

A solution of the system of differential equations (29)-(30) together with (23) is given by

$$p_a(r) = C_a \exp V_a(r) / \sum_{b\varepsilon A} C_b \exp V_b(r) \qquad a\varepsilon A,\ r\varepsilon R \qquad (31)$$

with

$$C_a = p_a(0) \exp(-V_a(0)) \qquad a\varepsilon A. \qquad (32)$$

Evidently (31) resembles analytically the static multinomial logit model. Consequently, one may interpret $p_a(r)$ as choice frequencies of a dynamic extension of the logit model and V_a as the systematic component of an individual's utility. From this point of view the interpersonal interactions $V_{ab} := V_a - V_b$ are the utilities of transition from alternatives a to b and $\partial V_a(r)/\partial s$ the dynamic (dynamic-space) marginal utilities which represent the expectation of future gains. It is important to stress that the dynamic extension (31)-(32) of the logit model corresponds to the specific state of totally antagonistic competition between alternative choice options within an indifferent (i.e. passive) environment, the simplest case of competition which generates the equilibrium states according to the principle of competitive exclusion. This implies that each subset of the choice set participates in an antagonistic non-cooperative zero-sum game, and an individual cannot gain anything by exchanging alternatives and returning to the initial one.

Expression (29) means that the frequency p_a increases, i.e. $\partial p_a(r)/\partial s > 0$, if

$$\sum_{b\varepsilon A} g_{sab}(r)\ p_b(r) > 0 \qquad a\varepsilon A;\ r\varepsilon R. \qquad (33)$$

The behavioural interpretation of this fact is as follows. The choice-maker compares alternative a with all other choice options b ($b \neq a$) not by comparing the utilities V_a and V_b only, but also by comparing the dynamic marginal utilities $\partial V_a(r)/\partial s$ and $\partial V_b(r)/\partial s$. Moreover, the consideration of only expected transitional utilites is not sufficient. The individual observes the choice of other individuals and takes into account how many individuals are choosing the other alternatives. Thus, the term $p_b(r)\,[(\partial V_a(r)/\partial s) - (\partial V_b(r)/\partial s)]$ represents a measure of the transitional expected growth in utility and the degree of imitation or influence of adopters of alternative b on the decision to change from alternative a to b.

The transition from a passive to an active environment in which the decision process takes place generates inter alia the dynamic extension of the multinomial dogit model. This transition may be accomplished technically with the help of a (invertible) stochastic redistributional matrix $S=(s_{ab}(r))$ where the coefficient $s_{ab}(r)$ may be interpreted as the frequency of individuals rejecting alternative a and, instead, shifting to alternative b under the influence of external influences.

Introducing external forces into the dynamic individual choice models (31)-(32) yields the following generalizations:

$$\tilde{p}_a(r) = [\sum_{b\varepsilon A} s_{ba}(r)\ C_b\ \exp V_b(r) / \sum_{b\varepsilon A} C_b\ \exp V_b(r)] \qquad a\varepsilon A,\ r\varepsilon R \qquad (34)$$

with C_b defined by (32) and where

$$\tilde{p}_a(r) := \sum_{b\varepsilon A} s_{ba}(r)\ p_b(r) \qquad a\varepsilon A,\ r\varepsilon R \qquad (35)$$

denotes the relative distribution of choice frequencies in $r\varepsilon R$ transformed by the markov matrix $S'=(s_{ba}(r))$.

Different specifications of the stochastic matrix S result into different rather general dynamic model specifications, i.e. different generalizations of the above mentioned dynamic multinomial logit model. If the elements of S are chosen in the following form of

$$s_{ab} = \begin{cases} s_a / s & \text{for } a \neq b \\ (1 + s_a) / s & \text{for } a = b \end{cases} \qquad (36)$$

where $s := s_1 + s_2 + \ldots + s_{A'}$, then the dynamic version of the logit model (31)- (32) will be transfomed into a dynamic version of the random utility based dogit model. The above mentioned specification of S may be interpreted as to stimulate the conservative choice behaviour in the form of the 'captivity' of the alternative.

Itisimportant to stress that the transition from one original choice model to another with the help of (non-stable) stochastic matrices is very helpful operationally because it opens up the possibility to generate a wide range of rather general dynamic choice models which enable to take into account various external interventions of the active environment.

This section will be concluded with the presentation of the variational principle which is the dynamic counterpart of the utility maximization principle for static models of utility choice. Consider the simplest case of the system of differential equations (29) where the space R includes only the time dimension t, and the influence functions g_{tab} are constant in t:

$$d \ln p_a(t) / dt = \sum_{b \varepsilon A} g_{ab}\, p_a(t) \qquad a \varepsilon A,\ 0 \leq t \leq T \qquad (37)$$

with

$$\sum_{a \varepsilon A} p_a(t) = 1 \qquad 0 \leq t \leq T. \qquad (38)$$

The derivation of this system of log-linear differential equations can be done with the help of the following Hamilton type variational principle (Dendrinos and Sonis 1986). Consider the cumulative portions of relative populations of choice-makers prefering alternative $a \varepsilon A$:

$$P_a(t) = \int_0^t p_a(t)\, dt \qquad (39)$$

and the integral

$$\int_0^T \Big(-2 \sum_{a \varepsilon A} p_a(t) \ln p_a(t) + \sum_{a,b \varepsilon A} g_{ab}\, p_a(t)\, P_b(t)\Big)\, dt \qquad (40)$$

This integral plays the role of a *welfare function* arising from the cumulative social interaction between choice-makers

$$\int_0^T \Big(\sum_{a,b \varepsilon A} g_{ab}\, p_a(t)\, P_b(t)\Big)\, dt \qquad (41)$$

and from the process of the equalization of alternatives measured by the *cumulative temporal entropy*

$$\int_0^T \Big(-\sum_{a,b \varepsilon A} p_a(t) \ln p_a(t)\Big) dt \qquad (42)$$

If the first variation of the integral (40) vanishes then the system of Euler differential equations coincides with the system (37) -(38) representing the dynamic choice process.

The most important fact is that the stationary value of the integral (40) turns out to be the cumulative entropy (42). This fact implies that in the actual dynamic choice process the cumulative social interaction and the cumulative entropy balance each other:

$$\int_0^T \left(-\sum_{a \varepsilon A} p_a(t) \ln p_a(t)\right) dt = \int_0^T \left(\sum_{a,b \varepsilon A} g_{ab} p_a(t)\, p_b(t)\right) dt. \qquad (43)$$

It is worthwile to elaborate the probabilistic version to the above mentioned ecological deterministic view.

5. Concluding Remarks

It is evident that the modelling of dynamics in choice processes is getting increasingly more attention in geography and regional science. The primary objective of this paper has been to discuss the three modelling approaches which appear to predominate the discussion in geography and regional science in the recent past.

We think that each of the approaches described above has appealing features in studying the dynamics in choice processes, but suffers also from some shortcomings and limitations. There is no doubt that from an analytical point of view the two continuous-time approaches are much more general and flexible than the discrete-time approach. This attractive feature partly comes from the fact that time is dealt with in a continuous way. Consequently, the potentially arbitrary nature of the definition of discrete time is avoided and a more accurate representation of the duration of events is guaranteed. The parameters derived are invariant to the time unit selected. The differential equations do not only describe the development towards stationary states, but also a variety of phase transitions of transient states and provide a deeper understanding of the dynamics in choice processes and of the relationship between the micro- and macro-behaviour of spatial systems.

Both continuous-time approaches contain the two major ingredients of a truly integrated dynamic discrete choice model, the accounting framework in form of differential equations and behavioural assumptions. Moreover, the master equation approach takes into account the interaction between individual choice behaviour and collective state variables, while the ecological deterministic one the interaction between the environment and the decision maker. Although these approaches have considerable appeal due to their generality and flexibility the price paid for this attractiveness seems to be a the rather high degree of abstractness implying a lack in operational terms. The approaches are fundamentally analytic and do not yet explicitly provide a fully developed operational framework. This is especially true for the ecological deterministic approach. But there is hope that serious applications in the near future might pave the way in translating the approaches into satisfactory operational frameworks. In this respect the panel data-based discrete time approach appears to be superior. Evidently it is computationally more tractable (at least the logit-type formulations) and especially operationally more flexible and provides a richness of information on the dynamics of choice processes.

Finally, some remarks concerning the underlying behavioural assumptions should be made. The panel data-based discrete-time approach is explicitly based upon the random utility maximization principle. Although there is no explicit choice-behavioural assumption inherent in the master equation approach, the general form suggested for the transition rates of the dynamic equations is consistent with utility maximization. In contrast to these two approaches the ecological deterministic one is based on a different macro-level behavioural principle of balance between the cumulative social interaction and the cumulative entropy of choice makers' distributions, which is a measure of the equalization of competing choice alternatives. On the micro-level this principle means that an individual chooses an alternative not on the basis of a comparison of utilities, but on the basis of a comparison of the temporal marginal utilities. Unfortunately, there is no empirical evidence available up to now either for or against the validity of these behavioural principles.

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