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Testing for Speculative Bubbles in Foreign Exchange Markets

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Honors Program
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Section I. Introduction

1.1 Abstract

Foreign currency speculation has always been a well publicized topic that has captured the attention of people who have not formally studied economics. It is also a topic that has captured the attention of researchers in International Finance because speculative bubbles have often been considered as a possible explanation for the excess volatility of exchange rates. An examination of past studies reveals that different methods have been used by researchers to test for the existence of speculative bubbles in major currencies over the period from 1970-1984. In this paper, I will apply three methods which have been used in the past to reach conclusions about the existence of speculative bubbles in the U.S Dollar/German Mark and the U.S Dollar/Japanese Yen exchange rate over the period from 1982-1992 and the U.S Dollar/British Pound exchange rate from 1987-1992. One objective of this paper is to update previous studies by expanding their scope into the most recent decade. The other objective is to use several testing methods for each currency in order to gain an insight into both the robustness of the conclusions and the dependency of the conclusions on a particular method of testing.

1.2 Definition and Description of a Bubble

1.2.1 Definition

A bubble can be defined as a sustained deviation of the exchange rate from the value determined by the underlying fundamentals. Therefore, acknowledgment of the existence of a fundamentally determined value of an asset underlies any study of asset market bubbles. For the purpose of this study the asset market in concern is the market for foreign exchange. As suggested by their descriptive title, speculative bubbles are caused by speculation by the agents in the foreign exchange market about the future value of a certain currency. Such speculation can often lead to self-fulfilling price expectations and subsequently result in a rapid rise or fall in the value of the currency. In the absence of certain knowledge of the future path of the fundamentals, this process can continue for a period of time before the currency's value returns to its

fundamentally determined value. This process caused by speculation is analogous to a soap bubble which expands rapidly for a finite amount of time before its eventual demise, the 'bursting of the bubble'.

1.2.2 The Creation, Growth and Demise of a Bubble

If an agent who is involved in the market for a particular currency, has reason to believe that the market value of the currency would increase in the future then one would expect that he or she would buy the currency in anticipation of capital gains in the future. If several agents buy the currency even though the expected value of that currency is much higher than indicated by the underlying economic fundamentals, the resultant increase in demand will raise the value of the currency and cause the expected increase in value to actually occur. This process can be described as a self-fulfilling price increase expectation which causes a sustained deviation from the value determined by the fundamentals, or as defined above, a speculative bubble.

Once a bubble begins, the deviations increase with time because the increased possibility of the bubble bursting and the currency returning to the value determined by the fundamentals requires a larger prospective capital gain (a greater price increase) to attract buyers and to encourage the owners of the asset to hang on to their holdings. Eventually the deviation becomes so great that a sharp decrease or increase in value is likely to occur as the bubble bursts.

1.2.3 The Role of Rational Expectations

The process described above has the interesting property that it is not inconsistent with economic theory, though it is not endorsed by most economists. Blanchard and Watson(82) argue that even though economists are inherently prone to believing that the value of an asset must be determined by market fundamentals, any outside event which is perceived by the agents to be of significant concern to the asset market will also have the ability to influence the value. Blanchard and Watson also observe that economists are quick to make ex-ante categorizations of any deviations not caused by movements in the fundamentals as evidence of irrationality on the part of the agents. Speculative bubbles are indeed not inconsistent with rationality on the part of agents in the asset markets. Once a bubble has begun it has to be included in the information

set that is used by the agent in the market to formulate his or her expectations. As described by Copeland(1989), once a bubble occurs, it becomes the reality with which an agent must survive and actions based on the existence of that speculative bubble are consistent with rational behavior. Indeed it would be irrational for the agent to ignore the presence of the bubble and formulate expectations based solely on the fundamentally determined value.

1.2.4 Occurrence of Speculative Bubbles

Speculative bubbles can occur in any market for assets where the value of the underlying fundamentals is hard to discern. Famous historical bubbles like the South Sea Bubble occurred in the share market while the Great Tulip Bubble is the term often used to described the fascinating events that occurred in the flower market in 17th century Holland. In more recent times, the stock market crash of the 1980's as well as excessively high Japanese land values in the 80's, the appreciation of the US Dollar in the 1980's and the collapse of the British Pound and the Italian Lira during the turmoil within the European Monetary System in 1992 are also examples where speculative bubbles have or are suspected to have occurred.

Copeland shows that the overvaluation of the US Dollar during the 1980's to be a good illustration of the above process. Despite almost universal acceptance that the dollar was unnaturally high in value, people clung onto their dollars in the belief that the price would increase and the compensation from holding dollars would be greater than the probability of a loss caused by the bubble bursting. As a result the value of the dollar remained unnaturally high for a long period of time throughout the 1980's.

Just as there are a variety of areas in which bubbles have been suspected to occur, many different types of bubbles have also been defined over the years, rational, irrational, deterministic, probabilistic bubbles, etc. This paper will focus mainly on testing for rational speculative bubbles which are probabilistic in nature. In the methods used in the paper, an attempt is made to derive a structural form to characterize speculative bubbles using rational expectations in an attempt to distinguish them from variations from the fundamentals caused by noise in the error term.

1.3 Reasons for studying this topic and possible insights to be gained from such study.

It is a well known fact in economics that most theoretical models of the exchange rate do not seem to do a very good job of predicting the future spot rate of a currency in the short run. Meese and Rogoff(1983) among others have done extensive work in this area. Over the years several attempts have been made to incorporate various economic variables into exchange rate models in order to improve their forecasting capabilities. Yet exchange rates always seem to move around much more than the underlying fundamentals do. Buiter (87) points out that the presence of bubbles may provide a possible explanation for the failure of these models because they choose a "fundamentals only" solution which characterizes the value of the exchange rate as a function of the current and future expected values of the fundamentals, often ruling out the existence of speculative bubbles.

If a bubble is present and ignored by using such theoretical models, the result will be an underestimation of the changes in exchange rates. If robust conclusions can be reached about the presence of speculative bubbles in major currencies, then that information needs to be taken into account in searching for a good theoretical model of exchange rate determination. Thus, future models of the exchange rate may need to allow for the effects caused by speculative bubbles.

Studying the different methods used to test for bubbles in exchange markets can provide useful knowledge that can be used in the study of another asset market where the inability to determine the appropriate underlying fundamentals may give "crowd psychology" an important role to play in determining the value of the asset.

Section II: Historical Bubbles

3.1 . Famous bubbles of the past:

The South Sea bubble, the Tulip bubble and the Mississippi bubble were studied in depth by Garber(90) who concluded that these bubbles could not be attributed solely to irrational frenzied behavior on the part of market participants. Garber points out that during the great Tulip mania of the 17th

century, which was extremely well documented by Mackay and Kindleberger, the most expensive tulip bulb, Semper Augustus, was selling at the equivalent of $ 16,000, an incredible increase from its base value which was the equivalent of 8 cents. This price increase took place in late 1636 in what was possibly the first instance of large scale speculation that drove the value of an asset to a level far from any reasonable fundamentals driven price. The bursting of the bubble occurred a year later and the price decrease was almost as drastic as the rapid increase which preceded it. Garber claims that the increase and subsequent collapse of tulip prices should not be ascribed solely to market irrationality especially since most of the dramatic price increases occurred in rare varieties of tulips which produced especially beautiful flowers. Such bulbs were in fashion during the day and even when the price was unreasonably high it was acceptable for any rational Dutch trader to buy Semper Augustus bulbs in the belief that these bulbs could be subsequently resold at a much higher price. Garber concludes that irrational bubbles may not characterize the tulip bulb episode, yet one can also conclude that rational speculative bubbles as described above are a plausible explanation for this famous historical event.

The South Sea bubble is another event of historical relevance in a more recent time period than the Tulip "mania". As described in Garber, a company was formed in January 1720, to buy debt issued on behalf of the British Crown. The South Sea Company held the monopoly rights to British trade with the Spanish colonies in South America even though such trade was rendered non existent by the presence of the Spanish armada in the South Atlantic. The company's share offer included a vast number of "gifts" to prominent members of parliament. The resulting public share issues were extremely heavily subscribed, due to the company policy which required less than 20% of the value in cash up front. The value of the shares in this effectively worthless company increased remarkably due to the delayed payments as well as to the 'respectability' that came with the backing of respected members of parliament. The price of the shares reached a height of 775 before collapsing to a level of around 290. Kindleberger describes the mood that underlies the buying craze perfectly with an anecdote about the banker Martin who purchased £500 worth of shares in the South Sea Company with the comment "when the rest of the world are mad, we must imitate them in some measure". This anecdote perfectly captures the reasoning behind a price increase that can happen due to 'crowd psychology'

even when the value of the fundamentals gives absolutely no reason for such an occurrence.

The Great Stock Market crash of 1929 is another event to which the bubble scenario is often applied. Just as important as these historical crashes, manias and panics are modern day effects of bubbles in stock markets and foreign exchange markets. Even though the price increases may not be as dramatic as a $16,000 tulip bulb, long deviations away from the fundamental exchange rate can have significant effects upon a country's economy. This is especially true under a system of managed exchange rates where sustained intervention by central banks fighting a speculative bubble may leave the economy vulnerable to a sudden growth of the bubble. This kind of scenario may be a plausible description of the intense panic that occurred in the summer of 1992 and 1993 within the European Exchange Rate Mechanism.

The above descriptions provide considerable historical justification for devoting time and effort to better understand this phenomenon which is extremely difficult to structurally define and realistically model .

Section III: Theoretical Models of the Exchange Rate

3.1. Brief descriptions of various models of the exchange rate.

In order to study deviations from the fundamentally determined value of the exchange rate, it is important to understand the different models that are used to obtain the fundamentally determined value. Copeland(1992) provides succinct descriptions of four of the most widely used models, the monetary model, the Mundell-Fleming model, the Dornbusch overshooting model and the portfolio balance model.

3.1.1 The Monetary Model

The monetary model is considered as a benchmark because it was the earliest attempt to model the exchange rate and the basic model has undergone several modifications over the years. The derivation of the monetary model begins with a simple specification of a money demand function.

$$m_t^d = \frac{p_t (y_t)^{\mu}}{(r_t)^{\lambda}} \qquad \Rightarrow \text{(MD1)}$$

In the above equation, y is real income, r is the nominal interest rate and p is the price level. μ is the income elasticity of money demand and λ is the interest rate elasticity of money demand. This simple identity says that the demand for money is proportionally related to real income and the price level and inversely related to the nominal interest rate. A similar identity can then be derived for the foreign country where

$$m_t^{*d} = \frac{p_t^*(y_t^*)^\mu}{(r_t^*)^\lambda}. \qquad \Rightarrow \text{(MD2)}$$

Equation (MD1) and equation (MD2) can be re-written equating demand and supply as

$$\frac{m_t^d}{m_t^{*d}} = \frac{\dfrac{p_t(y_t)^\mu}{(r_t)^\lambda}}{\dfrac{p_t^*(y_t^*)^\mu}{(r_t^*)^\lambda}} = \frac{m_t^s}{m_t^{*s}}. \qquad \Rightarrow \text{(MD3)}$$

Taking the logarithm of both sides of equation (MD3) gives the result

$$\log m_t - \log m_t^* = \mu(\log y_t - \log y_t^*) - \lambda(\log r_t - \log r_t^*) + \log p_t - \log p_t^* \Rightarrow \text{(MD4)}$$

In the monetary model, an assumption is made that Purchasing Power Parity(PPP) always holds, therefore similar goods cost the same in both countries, which can be expressed as p=sp*, where s is the spot rate defined as units of domestic currency per unit of foreign currency. Taking Logarithms of the PPP condition yields

log(p) = log(s) + log(p*) or

log(s) = log(p) - log(p*). $\Rightarrow$ (MD5)

From equations (MD4) and (MD5) the following relationship can then be derived

$$\log s_t = \log m_t - \log m_t^* + \mu(\log y_t - \log y_t^*) - \lambda(\log r_t - \log r_t^*)$$

Defining $S_t = \log s_t, M_t = \log m_t, Y_t = \log y_t, R_t = \log r_t$) etc., results in the following relationship

$$S_t = (M_t - M_t^*) - \mu(Y_t - Y_t^*) + \lambda(R_t - R_t^*)$$

In the above identity the asterisks denote the corresponding series for the foreign country. In this model the spot exchange rate between two countries is determined by the relative money supply, by the relative price level and by relative interest rates

Most of the tests used in this paper incorporate the flexible price monetary model as the determinant of the fundamental exchange rate mainly for

the purpose of simplicity. The monetary model is not a very good predictor of short term exchange rate fluctuations because the assumption that Purchasing Power Parity always holds is often not satisfied during the short run. However the monetary model proves to be a fairly accurate predictor of long range exchange rate variations and is considered the easiest model to test empirically.

3.1.2 The Mundell-Fleming Model

The key difference of the Mundell-Fleming model from the monetary model is the absence of an assumption about Purchasing Power Parity. The Mundell-Flerming model assumes that prices are fixed i.e. that the aggregate supply curve is horizontal. The M-F model also assumes that capital mobility is less than perfect and that the current account balance is determined independently of the capital account. According to Copeland this means that it is often used to analyze the appropriate mixture of monetary and fiscal policies to regulate demand and achieve balance in the external sector of an open economy under both fixed and floating exchange rate. As can be expected, the model is of little empirical interest due to the drastic assumption of price inflexibility which is a far more unrealistic assumption than Purchasing Power Parity. The role of the Mundell-Fleming model was further diminished by the development of the Dornbusch model which worked with sticky rather than inflexible prices and consequently is regarded as a vast improvement on the Mundell-Fleming model.

3.1.3 The Dornbusch Model

The Dornbusch model is an improvement on the monetary model in that it allows short run prices to be sticky while allowing prices to adjust in the long run. Dornbusch incorporates the fact that asset markets are often much more flexible than goods markets when it comes to price adjustments. As a result real interest rate differentials have a significant effect on the economy because an increase in the nominal money stock results in an increase in real money with sticky prices and in the short run a fall in interest rates is necessary to clear the excess supply in the money market. This results in a sudden depreciation of the domestic currency caused by a currency outflow from the domestic economy. As time passes the price level adjusts and the price increase brings about a fall in the

real money supply and a gradual appreciation of the currency as the process reverses itself. This sudden change followed by a gradual readjustment is what is called exchange rate overshooting where the currency depreciates past its long run equilibrium value before adjusting as prices move around in the long run.

The Dornbusch model can be considered to be superior to the monetary model in that it explains short run fluctuations better while retaining the long run characteristics of the monetary model and as a result offers intuitively satisfying explanations for exchange rate volatility. Yet, as Copeland points out, the Dornbusch model has to be greatly simplified with many additional assumptions before being tested empirically and as a result yields empirical results that are unsatisfactory.

3.1.4 Portfolio Balance Models

Portfolio Balance models differ from the monetary model in that a lot of weight is given to asset market dynamics with no assumptions about Purchasing Power Parity. The basic premise is that assets in different countries are not perfect substitutes, instead investors will hold assets denoted in different currencies to avoid risks caused by fluctuations of the exchange rate. As described by Copeland, Portfolio Balance models are an integration of the Dornbusch and Mundell-Fleming models incorporating imperfect capital mobility and sticky prices and provides better insights into the working of the economy. Like the Dornbusch model, the Portfolio Balance model proves difficult to test empirically because it contains variables such as the wealth of investors that can not be measured practically.

3.1.5 Reasons for Choosing the Monetary Model for this Study

Most of the above models have one thing in common, they are all extremely unsatisfactory in explaining variations of exchange rates. In this paper I will use the flexible price monetary model even though it is more suited to modeling long run exchange behavior primarily because of its simplicity but also because both the Dornbusch and the Portfolio Balance models are extremely hard to test empirically. The objective of this paper is to test for the presence of speculative bubbles which is a possible explanation for the lack of success that all

these models of the fundamentals have had. Therefore the simplicity and the empirical testability of the flexible price monetary model provide enough justification for adopting it as the chosen model of the fundamentals, in spite of the weaknesses identified previously.

Section IV: Summary of previous work on bubbles.

4.1. Overview of the different areas in which bubbles have been studied

The earliest instances where speculative bubbles had been studied were the tulip and South Sea incidents as described previously. The phenomenon of hyper inflation offered another area of interest especially the German hyper inflation in the early 20th century. Flood and Garber(88) extensively studied the German hyper-inflation during the period 1920 to 1923 and were unable to conclude that a price-bubble existed during the hyper inflation. Further work on bubbles in German hyper inflation was done by Blackburn(92) and Cassella(89). Hyper-inflations attract researchers looking for price level bubbles because expectations about future rates of inflation take on added significance making it likely that speculative bubbles will occur during such a period. Other important work in the context of rational bubbles that can cause inflation was done by Diba and Grossman(1987 & 1988).

The stock market crashes of the 1920's and the 1980's have also been examined for bubbles as being possible causes for the most spectacular market crises. The literature on stock market bubbles is vast and the literature on speculative bubbles in exchange markets that is relevant to this paper is also fairly extensive.

Other areas where bubbles have been studied include booms in land prices especially in Japan as well as in the markets for securities such as bonds. The section below provides a fairly comprehensive overview of the work that has been done on speculative bubbles in the realm of exchange rates.

4.2 Literature Summary

4.2.1 Important Articles about Rational Bubbles

One of the most influential articles on bubbles was "Bubbles, Rational Expectations and Financial Markets" written by Blanchard and Watson in 1982. Blanchard and Watson point out that certain kinds of bubbles are consistent with rational behavior and as such can be distinguished from instances of irrational speculation. They also offer the opinion that rational bubbles are studied more often simply because modeling bubbles is a hard task even without the additional complexity of modeling irrational behavior.

The classic argument against the existence of deterministic bubbles is that if such a bubble grew forever then the price of the asset would be infinitely high. Since any asset is bound to have a finite value in the long run this means that the deterministic bubble has to end at time T. If all agents are rational then they will drop out of the market at time T-1 knowing that the bubble is going to end at time T. If everybody is going to leave the market at time T-1 then it makes sense for a rational agent to leave the market at time T-2 and by moving backwards one period at a time it can be seen that the deterministic bubble never begins at time 0, i.e. the bubble is "strangled at birth".

Blanchard and Watson answered this criticism by describing an alternative bubble process which has a finite expected lifetime with a probability that the bubble ends at a given time period. Therefore only an extremely small probability is attached to the event that a bubble may grow without bound. At any given time the bubble can continue into the next period with probability π or crash with probability $(1-\pi)$.

This process can be described in the following manner. Let B_t describe the bubble term at time t, i.e. B_t captures the deviation from the fundamentally determined exchange rate at time t. This term B_t has an asymptotic value of zero and the possible outcomes for B_{t+1} the bubble term at time t+1 can be described as follows

$B_{t+1} = (\pi\alpha)^{-1}B_t$ with probability π (this is the event that the bubble continues into the next period) and

$B_{t+1} = 0$ with probability $1-\pi$ (this is the event of the bubble bursting at time t)

where $\alpha = \frac{1}{1+r}$ and r is the return on holding the asset.

From the above description the expected value of B_{t+1} can be calculated as $E_t B_{t+1} = \pi[(\pi\alpha)^{-1} B_t] + (1-\pi)0 = \frac{B_t}{\alpha}$

This was an important counter to the standard argument that bubbles could not exist because their value could not grow forever. The process discussed above has a finite expected lifetime yet there is a remote possibility that it can last forever .

4.2.2 Articles about Testing Methodologies

In this section, I will focus primarily on articles that concern the testing methods and techniques that will be used in my study. Excess variance testing is an important method used to test for speculative bubbles that is based heavily on previous work done by Shiller(85) in looking at volatility in stock market prices. Huang(81) and Wadhwani(87) adapt excess variance tests to test for the possibility of the existence of speculative bubbles in foreign exchange markets. Huang tests the US Dollar/ German Mark, US Dollar/Sterling Pound, Sterling Pound/ German Mark rates over the period from March 1973-1979 and concludes that bubbles may have been present in all three currencies over the specified period.

Meese(1986) adapted the McCallum instrumental variable technique and the Hausman specification test to study the period March 1973-1982 for bubbles in the US Dollar/German Mark, US Dollar/ Sterling Pound, and the US Dollar/Japanese Yen and rejected the null hypothesis of no bubbles for the Pound and the Mark in that period. The Hausman specification test was also used by Kearney and MacDonald who tested for the presence of a bubble in the US Dollar/ Australian Dollar rate for the period January 1984 to December 1986. Kearney and MacDonald were unable to detect the presence of a bubbles in the exchange rate between the United States and Australia. Meese as well as Kearney and MacDonald use econometric techniques and tests that were developed by West(85) and Hausman.

The other method that is prominent in the literature was presented by Evans(86) who uses a Monte Carlo study to look for a non-zero median in excess returns to holding a currency. Evans defines such an occurrence caused by a sustained appreciation or depreciation of a currency as characterizing a speculative bubble. He studied the US Dollar/ Sterling Pound, Sterling Pound/

German Mark for the period December 1981-February 1985 and finds evidence that indicates the presence of bubbles.

4.2.3 Articles That Offer a Unique Perspective

The work described above in section 4.2.2 are the primary sources for the testing methodologies described in detail in Section VI. There have been several other interesting articles on speculative bubbles especially by Buiter(1990) who uses a theoretical portfolio balance model to examine the effect of bubbles within a system of managed exchange rates with target zones for currencies. Buiter analyzes the appropriate policy responses of the central bank to speculative bubbles and introduces the concept of a friendly bubble where a decline in the value of a currency may be halted or reversed by the belief of agents that central bank intervention may occur, thus eliminating the need for intervention in a self-fulfilling manner.

Wing T. Woo uses a different model of the fundamentals, a portfolio balance model to test for the presence of bubbles in major exchange rates. Christopher Towe's study of the Lebanese pound is a marked contrast from the usual studies on major currencies, in that it works with developing country data using a portfolio balance model. Some work done on bubbles in other sectors such as the stock market still remain unapplied to exchange markets, especially the work done for the stock market by Asli Demirguc-Kunt(1988) using a technique developed by Plosser, Schwert and White that modifies the above described methods used by Meese, West and others.

4.2.4 Symposium on Bubbles

The symposium on bubbles is a collection of papers presented in the Journal of Economic Perspectives(1990 -Vol. 4). The collection of papers in this symposium is an excellent introduction to the subject, covering such topics as the history of bubbles, responses to common criticisms that bubbles do not exist, a brief introduction to modeling techniques that are used to study speculative bubbles, and a discussion on the complexities that occur when using some of these techniques.

Since section IV contains mentions many different studies of speculative bubbles it may be useful to summarize previous findings in a table in order to obtain an idea of the scope of past findings. The table given below provides

a summary of relevant studies, the years, currencies, testing methods as well as the results that were obtained from the study.

Table 1: Summary of previous studies on speculative bubbles in foreign exchange markets

AUTHOR(S)	PERIOD	CURRENCY	METHOD	RESULTS
Huang(81)	March 1973-1979	US $/Mark US $/British £ Mark/British £	Excess Variance Tests	Bubbles were Present
Meese(86)	March 1973-1982	US $/British £ US $/Mark US $/ Yen	Hausman Specification Test	Present Weak evidence for Yen/$ rate
Kearney and MacDonald (90)	January 1984 - December 1986	Aus $/ US $	Hausman Specification Test	Unable to detect bubbles
Evans(86)	Dec 1981 - 1985	US $/British £ Mark/British £	Monte Carlo test	Bubbles were present
Woo(84)	June - Oct 78 Dec -March 80	US $/Mark US $/ Franc US $/Yen	Portfolio balance uncertainty model	Present for Mark/$ and franc/$ .Weak for Yen/$
Towe(89)	Dec 1982-1987	US $/ Lebanese £	McCallum instrumental variable technique	Unable to detect bubbles

From Table 1.1 we can see that most of the studies can be updated through to the period leading up to 1992 and that there is scope for different testing methods to be applied to each currency pair to test for the robustness of the conclusions in the table and the conclusions reached from updating the studies. Section V provides a brief overview of the econometric issues that have to be considered in testing for speculative bubbles while Section VI contains the derivation of the flexible price monetary model that is used to produce the fundamentals determined exchange rate. Section VII contains

extremely descriptive derivations of the three testing methods to be used while Section VIII provides a description of the data series and transformations used in the actual testing. Section IX contains the results of carrying out the tests while Section X provides analysis and conclusions.

Section V: Brief overview of relevant areas of time series econometrics

5.1 Stationarity and stochastic processes

Pindyck and Rubinfeld point out that an important assumption that is often used when working with time series models is that the series in concern has been generated by a stochastic process where each observation is randomly drawn from some probability distribution. Perhaps the most commonly used stochastically generated time series is a random walk process where the underlying probability distribution is one with a zero mean, expressed as $W_t = W_{t-1} + \varpi_t$ where the error term ϖ_t has zero mean and constant variance. This means that changes in the series W are independent of past changes in the variable W.

An important factor that has to be considered in dealing with time series models is whether the stochastic process that is assumed to have generated the process is invariant with respect to time. If the underlying properties of the stochastic process happen to change as we move from one time period to the next, then the time series generated by that process is assumed to be non-stationary. Non stationary processes are troublesome because they will cause problems when represented in a simple model with past, present and future values of the variables. This is because the structural relationship between variables as represented in the equations of the model being used may be changing with respect to time. If the structure of the model is changing then standard regression techniques can not be used for the purpose of forecasting. However, techniques do exist for transforming non-stationary processes into stationary ones so that they can be used in regression analysis.

5.2 Some properties of stationary processes

A stationary process is defined to be a process whose conditional and joint probability distributions do not change with time. In a simple algebraic

format it means that $p(W_t,.....W_{t+k}) = p(W_{t+m},.....W_{t+k+m})$ and $p(W_t) = p(W_{t+m})$, where $p(W_t,.....W_{t+k})$ is the joint probability distribution of $W_t,.....W_{t+k}$. This in turn leads into the properties that the mean of the series is also stationary so that if the mean is defined as $\mu_w = E(W_t)$, then it is true that $E(W_t) = E(W_{t+m})$ and also that the variance and covariance must be stationary so that if the variance is defined by $\sigma_W^2 = E[(W_t - \mu_W)^2]$, then it is true that $E[(W_t - \mu_W)^2] = E[(W_{t+m} - \mu_W)^2]$ and $COV(W_t, W_{t+k}) = COV(W_{t+m}, W_{t+k+m})$.

5.3 The Autocorrelation Function

The autocorrelation function provides useful information as to the dependency or correlations between two observations of a time series. The auto correlation function is used to measure the inter dependency and is often subscripted by the time lag between the two observations being considered. The co-efficient is defined as $\rho_k = \frac{COV(W_t, W_{t+k})}{\sigma_{W_t}\sigma_{W_{t+k}}}$. If the process is stationary the co-efficient can be expressed as $\rho_k = \frac{COV(W_t, W_{t+k})}{VAR(W_t)}$

In practice where the true properties of the population are not known, a sample autocorrelation function defined as

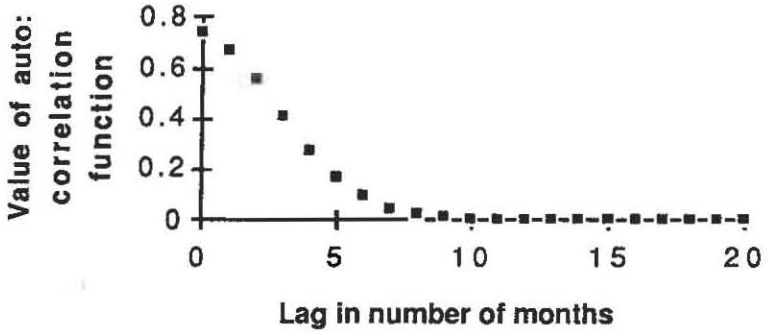
$\hat{\rho}_k = \frac{\sum_{t=1}^{T-k}(W_t - \overline{W})(W_{t+k} - \overline{W})}{\sum_{t=1}^{T}(W_t - \overline{W})^2}$ is often used.

In order to figure out if the true value of the autocorrelation function is zero, a test has to be carried out on the value of the sample autocorrelation function. There are two tests used for this purpose. A test developed by Bartlett is useful in order to determine if a particular value of the autocorrelation function is equal to zero. Bartlett showed that the sample autocorrelation co-efficients are approximately normally distributed with 0 mean and standard deviation of $\frac{1}{\sqrt{T}}$ where T is the number of observations being used. The joint hypothesis that all auto correlation co-efficients upto a lag of K are zero can be carried out by using the Q statistic of Box and Pierce where

$Q = T\sum_{k=1}^{K}\hat{\rho}_k^2$. This Q statistic has approximately a chi square distribution with K degrees of freedom.

The autocorrelation function and the Q statistic are useful in determining the stationarity of a time series. In practice, a stationary time series has the property that values of the sample auto correlation function approaches zero quickly as k increases. This is an informal test that can be used to test for non stationarity. In most instances first differencing a data series can often be adequate to induce stationarity, so that if a given series W has a correlogram (plot of an autocorrelation function) which does not converge to zero then it is very likely that the series ΔW_t has a correlogram that drops off to zero as K increases. A typical correlogram of a stationary series would resemble the figure given below.

Typical correlogram of a stationary series



5.4 Unit Root Tests

Pindyck and Rubinfeld also state that when certain economic variables do not have a long term trend but instead follow random walks, regression of one against another can lead to results that are spurious. This can only be corrected by first differencing which will induce stationarity in the data series in question. The standard method of testing for random walks is by use of unit root tests devised by David Dickey and William Fuller.

The Dickey-Fuller test can be used in the following manner. Let W be the generic data series being considered. Assume that the behaviour of W can

be described by the following equation $W_t = \alpha + \beta T + \varsigma W_{t-1} + \varepsilon_t$, where T is used to capture the time trend and test the null hypothesis that $\beta = 0$ and $\varsigma = 1$..

This is done in a more general format by assuming that W is described by the following regression: $W_t = \alpha + \beta T + \varsigma W_{t-1} + \delta \Delta W_{t-1} + \varepsilon_t$. Use Ordinary Least Squares to run the following unrestricted regression

$$W_t - W_{t-1} = \alpha + \beta T + (\varsigma - 1) W_{t-1} + \delta \Delta W_{t-1}$$

and the restricted regression

$$W_t - W_{t-1} = \alpha + \delta \Delta W_{t-1}$$

The joint restrictions $\beta = 0$ and $\varsigma = 1$ can be tested by calculating the standard F ratio where

$$F = \frac{(N-k)[ESS_R - ESS_{UR}]}{2[ESS_{UR}]}$$

and use a distribution calculated by Dickey and Fuller to test for significance levels. The null hypothesis is of a unit root existing for W (i.e. $\beta = 0$ and $\varsigma = 1$) which is equivalent to saying that W follows a random walk. If the null hypothesis is not rejected then W may follow a random walk and therefore should be first differenced and the differenced series should be tested for stationarity before being used in a regression.

5.5 Implications of above for this study

The most important factor that needs to be considered is to test if the data series that will be used in the following sections, namely the fundamental series Z and the spot rate series S are stationary. If they are found to be non stationary then the series should be first differenced and tested for stationarity again before being used. The testing for stationarity can be done by looking at the graphs of the auto correlation functions for the undifferenced series and if the function does not converge to zero as the number of lags increases, the data series should then be first differenced. The first differenced data series' correlograms should then be examined and the process repeated if the sample auto correlation function does not still converge.

Another method of testing to see if first differencing is necessary is to test the spot rate and the fundamentals to see if those series follow random walks. This can be done by the unit root tests described above and if the

tests do not give reason to reject the random walk hypothesis then the data series should be first differenced before being used in regression analysis.

Section VI: The Model of the Fundamentals

The general derivation of the fundamental model of the exchange rate does not differ much between methods, so that derivation can be done first. I will use a flexible price monetary model to model the exchange rate.

In this model we can describe the spot rate of the value of a unit of foreign currency in terms of the domestic currency as

$$S_t = (M_t - M_t^*) - \mu(Y_t - Y_t^*) + \lambda(R_t - R_t^*) \qquad =>(1)$$

where M is the log of money supply, Y is the log of real income and R is the log of the nominal interest rate. The asterisks denote corresponding series for the other country so that M* is the log of foreign money supply, Y* is the log of foreign real income etc. We can define

$\tilde{M}_t = M_t - M_t^*$, $\tilde{R}_t = R_t - R_t^*$ and $\tilde{Y}_t = Y_t - Y_t^*$

From (1) we then have

$$S_t = \tilde{M}_t - \mu\tilde{Y}_t + \lambda\tilde{R}_t \qquad =>(2)$$

Uncovered Interest Rate Parity is said to hold when the expected rate of change in the currency depends on the difference between foreign and domestic interest rates. Assume that Uncovered Interest Rate Parity holds between the countries in the study, therefore

$$R_t - R_t^* = \Delta S_t^e = \tilde{R}_t$$

where ΔS_t^e is the expected change in the spot rate ,

$$(2) \Rightarrow S_t = \tilde{M}_t - \mu\tilde{Y}_t + \lambda\Delta S_t^e$$

which can be re - written as

$$S_t = Z_t + \lambda\Delta S_t^e \qquad =>(3)$$

where $Z_t = \tilde{M}_t - \mu\tilde{Y}_t$ denotes the value of the fundamental variables in the equation.

In order to calculate the series Z_t we need to know the values of μ and in most of the testing methods I will use an interval of parameter estimates taken from existing money demand functions in the literature. This is preferable to estimating a money demand function specifically for this paper because of the accuracy of the parameter estimates taken from money demand functions that have already been estimated specifically for the purpose of looking at the demand for money instead of as a corollary to another study. Using a range of estimates should help overcome some of the problems related to the accuracy of these estimates.

The implication of (3) is that the current spot rate is influenced by the expected future gains of holding the currency. This is because expected future depreciation of the currency can result in people selling their holdings in the currency and thus causing its value to depreciate. From equation (3) we can also derive an equation that shows that the current spot rate is determined by the market's perception of the future value of the underlying fundamentals. If S^e_{t+1} is the agents' expectation (at time t) of the value of the spot rate at time t+1 then the expected depreciation of the currency can be expressed as

$$\Delta S^e_t = S^e_{t+1} - S_t \qquad \Rightarrow (4)$$

From (3) and (4)

$$S_t + \lambda S_t = Z_t + \lambda S^e_{t+1}$$

$$S_t = \frac{1}{1+\lambda} Z_t + \frac{\lambda}{1+\lambda} S^e_{t+1}$$

Define $\beta = \frac{\lambda}{1+\lambda}$ and rewrite the above equation as

$$S_t = (1-\beta) Z_t + \beta S^e_{t+1}$$

If expectations of the agents are assumed to be rational then the agents' expectation of S_t is the mathematical expected value of S_t and therefore it can be derived that

$$S^e_{t+1} = E_t S_{t+1} \text{ and}$$

$$S_t = (1-\beta) Z_t + \beta E_t S_{t+1} \qquad \Rightarrow (A)$$

From (A) it is obvious that

$S_{t+1} = (1-\beta)Z_{t+1} + \beta E_{t+1}S_{t+2}$

So $E_tS_{t+1} = (1-\beta)E_tZ_{t+1} + \beta E_t(E_{t+1}S_{t+2})$

** Since the information at time t+1 is unavailable at time t, the expected value at time t of the forecast at time t+1 of the future spot rate is based upon information available at time t. Therefore $E_t[E_{t+1}S_{t+2}] = E_t[S_{t+2}]$ and we can re-write the above equation as

$E_tS_{t+1} = (1-\beta)E_tZ_{t+1} + \beta E_t(S_{t+2})$ which results in

$(A) \Rightarrow S_t = (1-\beta)Z_t + \beta(1-\beta)E_tZ_{t+1} + \beta^2E_tS_{t+2}$

Since $\beta = \frac{\lambda}{1+\lambda}$ and $\lambda > 0$ it is always true that $0<\beta<1$ and the above equation can be recursively solved to obtain the following

$$S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^iE_tZ_{t+i} + \lim_{i\to\infty}\beta^iE_tS_{t+i} \qquad =>(5)$$

The existence of a speculative bubble is indicated by the second term on the right side of (5). Therefore the bubble term B_t may be defined as $B_t = \lim_{i\to\infty}\beta^iE_tS_{t+i}$. Equation (5) reveals that if the bubble term is non zero in the limit then the exchange rate S, will deviate from the value determined by the fundamentals. Most fundamental models of the exchange rate assume that bubbles do not exist, i.e. that the transversality condition $\lim_{i\to\infty}\beta^iE_tS_{t+i} = 0$ holds, and derive an equation of the form.

$$S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^iE_tZ_{t+i}$$

Define $\tilde{S}_t$ to be the market fundamentals solution to equation (A). That is

$$\tilde{S}_t = (1-\beta)\sum_{i=0}^{\infty}\beta^iE_tZ_{t+i} \qquad =>(6)$$

In the fundamental models it is assumed that bubbles do not exist and therefore that $S_t = \tilde{S}_t$. If bubbles do exist, however, from (5) we can see that in fact $S_t = \tilde{S}_t + B_t$. In order to test for the existence of bubbles we have to test if the transversality condition is satisfied or not, i.e. test to see if $S_t = \tilde{S}_t$ or $S_t = \tilde{S}_t + B_t$. Most of the methods used in this paper are attempts to capture the existence of the bubble term B_t as described above.

Blanchard showed that any process B_t that satisfies the property $E_tB_{t+1} = \frac{1}{\beta}B_t$ makes $\tilde{S}_t + B_t$ a solution to (A). This can be done as follows:

If $S_t = \tilde{S}_t + B_t$ then

$S_{t+1} = \tilde{S}_{t+1} + B_{t+1}$ and $E_tS_{t+1} = E_t\tilde{S}_{t+1} + E_tB_{t+1}$. From the definition for $\tilde{S}_t$ it follows that

$$E_tS_{t+1} = E_t[(1-\beta)\sum_{i=0}^{\infty}\beta^iE_{t+1}Z_{t+i+1}] + \frac{1}{\beta}B_t$$

** As before the condition $E_t[E_{t+1}Z_{t+1+i}] = E_t[Z_{t+i+1}]$ holds and the above equation can be simplified in the following manner

$$\beta E_tS_{t+1} = \beta(1-\beta)\sum_{i=0}^{\infty}\beta^iE_tZ_{t+i+1} + B_t$$

$$\beta E_tS_{t+1} = (1-\beta)\sum_{i=0}^{\infty}\beta^{i+1}E_tZ_{t+i+1} + B_t$$

Setting j=i+1

$$\beta E_tS_{t+1} = (1-\beta)\sum_{j=1}^{\infty}\beta^jE_tZ_{t+j} + B_t$$

which can be re-written as

$\beta E_tS_{t+1} = (1-\beta)\sum_{j=0}^{\infty}\beta^jE_tZ_{t+j} + B_t - (1-\beta)Z_t$. Using the definition that

$\tilde{S}_t = (1-\beta)\sum_{i=0}^{\infty}\beta^iE_tZ_{t+i}$ yields $\beta E_tS_{t+1} = \tilde{S}_t - (1-\beta)Z_t + B_t$ which is equivalent to

$\beta E_tS_{t+1} + (1-\beta)Z_t = \tilde{S}_t + B_t = S_t$

This in turn can be expressed as $S_t = \beta E_tS_{t+1} + (1-\beta)Z_t$ equivalent to (A).

Section VII provides extremely detailed derivations of three of the methods that have been used to test for the existence of the bubble term B_t whose properties were described in the above section.

Section VII: Methods of Detecting Bubbles

7.1 Method One: Excess Variance tests

7.1.1 General overview of method

Define the perfect foresight fundamental exchange rate S_t^* as the fundamental rate that would be predicted if we knew all future values of Z_t excluding the existence of a bubble. Then S_t^* can be written as

$S_t^* = (1-\beta)\sum_{i=0}^{\infty}\beta^{i+1}Z_{t+i+1}$ and from the definition of $\tilde{S}_t$ given in (6) it follows that $E_tS_t^* = \tilde{S}_t$.

Using the assumption that expectations are rational S_t^* will differ from $\tilde{S}_t$ by a random error term v_t that has zero mean and constant variance and is uncorrelated with $\tilde{S}_t$. Therefore $S_t^* = \tilde{S}_t + v_t$. As shown previously, when bubbles do not exist, $S_t = \tilde{S}_t$, therefore

$S_t^* = S_t + v_t$ =>(10)

Equation (10) implies that $VAR(S_t^*) = VAR(S_t) + VAR(v_t)$ which in turn implies that

$VAR(S_t^*) > VAR(S_t)$ =>(12)

If, however, bubbles are present then $S_t = \tilde{S}_t + B_t$ and it follows that

$S_t^* = S_t - B_t + v_t$ =>(11)

In the presence of bubbles equation (11) is satisfied and this implies that

$VAR(S_t^*) = VAR(S_t) + VAR(v_t) + VAR(B_t) - 2COV(S_t, B_t)$

Since S_t and B_t may be positively correlated an inequality of the form of (12) can not be derived from the above equation. Significant violations of inequalities like (12) may indicate the presence of bubbles since the violation may have been caused by the positive correlation between S_t and B_t Failure to violate the inequality does not necessarily mean that bubbles do not exist because the inequality will still hold if S_t and B_t are negatively correlated.

7.1.2 Application of above method to exchange rates

Huang(81) and Wadhwani(87) among others have used a variation of this method to study bubbles in foreign exchange markets. Huang derives a slightly different form of the variance test for exchange rates and uses this test to study the existence of bubbles in the Dollar/Mark, Dollar/Pound and Mark/Pound exchange rates for the period March 1973-March 1979. Huang finds that the spot rate is significantly more volatile than the perfect foresight fundamental exchange rate.

It has been defined that $\tilde{S}_t = (1-\beta)\sum_{i=0}^{\infty}\beta^i E_t(Z_{t+i})$ and therefore that

$$\tilde{S}_t = (1-\beta)\left[\sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) + Z_t\right]$$

$$\tilde{S}_t = (1-\beta)Z_t + (1-\beta)\sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i})$$

The term on the right hand side of the above equation can be re-written in the following manner.

Define $\Delta Z_{t+i} = Z_{t+i} - Z_{t+i-1}$ to be the first difference of the Z series. It follows that

$$\sum_{i=1}^{\infty}\beta^i E_t(\Delta Z_{t+i}) = \sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i} - Z_{t+i-1})$$

$$= \sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) - \sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i-1})$$

$$= \sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) - \beta E_t Z_t - \beta^2 E_t Z_{t+1} - \beta^3 E_t Z_{t+2} - \ldots\ldots$$

$$= \sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) - \beta\sum_{i=0}^{\infty}\beta^i E_t(Z_{t+i})$$

This can be re-written with the summation of the second term expressed from i=1 as

$$\sum_{i=1}^{\infty}\beta^i E_t(\Delta Z_{t+i}) = \sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) - \beta\sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) - \beta Z_t$$

$$\sum_{i=1}^{\infty}\beta^i E_t(\Delta Z_{t+i}) = (1-\beta)\sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i}) - \beta Z_t \qquad \Rightarrow (13)$$

Since $\tilde{S}_t = (1-\beta)Z_t + (1-\beta)\sum_{i=1}^{\infty}\beta^i E_t(Z_{t+i})$, this identity can then be substituted into the definition of $\tilde{S}_t$ to derive the following expression for $\tilde{S}_t$

$\tilde{S}_t = \sum_{i=1}^{\infty}\beta^i E_t(\Delta Z_{t+i}) + Z_t$.

When bubbles are present $S_t = \tilde{S}_t + B_t$ as shown before and S_t can therefore be expressed as $S_t = Z_t + \sum_{i=1}^{\infty}\beta^i E_t(\Delta Z_{t+i}) + B_t$ $\Rightarrow (14)$

Define $A_t = (\sum_{i=1}^{\infty}\beta^i \Delta Z_{t+i})$. It follows then from (14) that

$S_t - Z_t = E_t(A_t) + B_t$ and equivalently that $S_t - Z_t - B_t = E_t(A_t)$. The expression $S_t - Z_t - B_t = E_t(A_t)$ can now be re-written as

$S_t - Z_t + \upsilon_t - B_t = A_t$ $\Rightarrow (15)$

where $\upsilon_t = A_t - E_t(A_t)$ is a random error term with zero mean and constant variance under the assumption of rational expectations. υ_t is also assumed to be independent of B_t and $S_t - Z_t$.

An inequality test as in section 7.1.1 can be derived from (15) as follows.

$$(15) \Rightarrow VAR(S_t - Z_t) + VAR(B_t) + VAR(\upsilon_t) - 2COV((S_t - Z_t), B_t)) = VAR(A_t)$$

When bubbles are not present

$B_t = VAR(B_t) = COV((S_t - Z_t), B_t)) = 0$, and therefore

$$VAR(S_t - Z_t) + VAR(\upsilon_t) = VAR(A_t)$$

Then the relationship $VAR(A_t) > VAR(S_t - Z_t)$ holds. However we cannot test violations of the inequality $VAR(A_t) > VAR(S_t - Z_t)$ because the future values of Z and therefore the value of A_t are unobservable at time t. The value of $VAR(S_t - Z_t)$ has to be compared with an observable value, namely, the value of $VAR(\Delta Z)$.

As A is a moving average which smoothes the ΔZ values the inequality relationship $VAR(\Delta Z) > VAR(A_t) > VAR(S_t - Z_t)$ holds. The exact relationship between $VAR(\Delta Z)$ and $VAR(A_t)$ needs to be determined in order to test for violations of the inequality $VAR(A_t) > VAR(S_t - Z_t)$.

$$\text{As } A_t = \sum_{i=1}^{\infty} \beta^i \Delta Z_{t+i}$$

$$VAR(A_t) = VAR(\sum_{i=1}^{\infty} \beta^i \Delta Z_{t+i})$$

$$= VAR(\beta \Delta Z_{t+1} + \beta^2 \Delta Z_{t+2} + \beta^3 \Delta Z_{t+3} + \ldots\ldots)$$

In order to find the variance described above, it is necessary to find a relationship between the ΔZ's. This is done below.

Assume that the fundamentals' behaviour is described by the AR(1) process $\Delta Z_t = \varphi \Delta Z_{t-1} + \delta_t$, where δ is a well behaved error term with zero mean and constant variance. Then it follows that $\Delta Z_{t+1} = \varphi \Delta Z_t + \delta_{t+1}$ and

$$\Delta Z_{t+2} = \varphi \Delta Z_{t+1} + \delta_{t+2}$$

$$= \varphi(\varphi \Delta Z_t + \delta_{t+1})$$

$$= \varphi^2 \Delta Z_t + \varphi \delta_{t+1} + \delta_{t+2}$$

This process can be expressed in a more general format as

$$\Delta Z_{t+i} = \varphi^i \Delta Z_t + \sum_{k=1}^{i} \varphi^{i-k} \delta_{t+k}$$

The value of $VAR(A_t)$ can therefore be calculated as follows

$$VAR(A_t) = VAR(\beta\Delta Z_{t+1} + \beta^2\Delta Z_{t+2} + \beta^3\Delta Z_{t+3} + ...)$$
$$= VAR(\beta[\varphi\Delta Z_t + \delta_{t+1}] + \beta^2[\varphi^2\Delta Z_t + \varphi\delta_{t+1} + \delta_{t+2}] + \beta^3[\varphi^3\Delta Z_t + \varphi^2\delta_{t+1} + \varphi\delta_{t+2} + \delta_{t+3}] ....)$$
$$= VAR(\Delta Z_t[\varphi\beta + \varphi^2\beta^2 + \varphi^3\beta^3 + ..] + \delta_{t+1}[\beta + \varphi\beta^2 + \varphi^2\beta^3 + ..] + \delta_{t+2}[\beta^2 + \varphi\beta^3 + \varphi^2\beta^4 + ..] + ..)$$
$$= VAR(\Delta Z_t[\frac{\varphi\beta}{1-\varphi\beta}] + \delta_{t+1}[\frac{\beta}{1-\varphi\beta}] + \delta_{t+2}[\frac{\beta^2}{1-\varphi\beta}] + \delta_{t+3}[\frac{\beta^3}{1-\varphi\beta}] + ...) \qquad => (15.1)$$

Since the δ_t's are white noise VAR δ_{t+i} =VARδ_{t+j} and COV(δ_{t+i},δ_{t+j})=0 for $i \neq j$. In addition since $\Delta Z_t = \varphi\Delta Z_{t-1} + \delta_t$, COV(Z_t,δ_{t+i})=0 for i>1. All this information can be combined to simplify (15.1) and obtain the result

$$VAR(A_t) = VAR(\beta\Delta Z_{t+1} + \beta^2\Delta Z_{t+2} + \beta^3\Delta Z_{t+3} + ...)$$
$$= (\frac{\varphi\beta}{1-\varphi\beta})^2 VAR(\Delta Z_t) + \frac{VAR(\delta_t)}{(1-\varphi\beta)^2}[\beta^2 + \beta^4 + \beta^6 + ...]$$
$$VAR(A_t) = (\frac{\beta}{1-\varphi\beta})^2[\varphi^2 VAR(\Delta Z_t) + \frac{1}{1-\beta^2} VAR(\delta_t)]$$

From this it can be seen that it is necessary to test for significant violations of the inequality

$$(\frac{\beta}{1-\varphi\beta})^2[\varphi^2 VAR(\Delta Z_t) + \frac{1}{1-\beta^2} VAR(\delta_t)] > VAR(S_t - Z_t) \qquad => (16)$$

From previous studies of money demand functions Huang uses a range of point estimates μ = 0.5, 1.0 and 1.5 to calculate three different series of Z. These series of Z are used to obtain different estimates of φ from the regression $\Delta Z_t = \varphi\Delta Z_{t-1} + \delta_t$. β = 0.75 is calculated from a value of λ=3.0 taken from the studies and a total of 3 different inequality tests were carried out by Huang for each pair of currencies.

7.2 Method Two: The Hausman Specification Test.

This method is used by Meese to study the Dollar/Mark, Dollar/Yen and Dollar/Pound exchange rates and by Kearney and MacDonald to study the Australian and US Dollar rates, using monthly data for the period 1973-1982. The method involves obtaining two different estimates of the co-efficient β, one of which is consistent irrespective of bubbles being present and

the other is consistent only in the absence of bubbles i.e. when the null hypothesis of no bubbles is true. Then the Hausman statistic is used to test for a significant difference in the two estimates of the co-efficients.

As before, the spot rate can be expressed as $S_t = Z_t + \lambda \Delta S_t^e + u_t$ where u_t is a random error term with zero mean and constant variance. The expected change in the spot rate can be expressed as $\Delta S_t^e = E_t S_{t+1} - S_t$. Accordingly,

$S_t = Z_t + \lambda E_t S_{t+1} - \lambda S_t + u_t$ and

$$S_t = \frac{1}{1+\lambda} Z_t + \frac{\lambda}{1+\lambda} E_t S_{t+1} + \frac{1}{1+\lambda} u_t.$$

Define $\beta = \frac{\lambda}{1+\lambda}$, which means that

$S_t = (1-\beta) Z_t + \beta S_{t+1}^e + (1-\beta) u_t$. The spot rate at time t-1 can then be expressed as

$S_{t-1} = (1-\beta) Z_{t-1} + \beta S_t^e + (1-\beta) u_{t-1}$.

Let $\Delta S_t = S_t - S_{t-1}$, $\Delta Z_t = Z_t - Z_{t-1}$ etc.. From the two equations given above, given that expectations are rational, then it is true that

$\Delta S_t = (1-\beta) \Delta Z_t + \beta [E_t S_{t+1} - E_{t-1} S_t] + (1-\beta)[u_t - u_{t-1}]$

$\Delta S_t = (1-\beta) \Delta Z_t + \beta [E_t S_{t+1} - E_{t-1} S_t] + (1-\beta) \varepsilon_t$

$\Delta S_t = (1-\beta) \Delta Z_t + \beta [E_t S_{t+1} - E_{t-1} S_t] + \varsigma_t$ => (17)

where $\varsigma_t = (1-\beta) \varepsilon_t$

Once again, a simple process is used to describe the behavior of the fundamentals. Assume that the fundamentals follow a time path given by

$\Delta Z_t = \varphi \Delta Z_{t-1} + \delta_t$ => (18)

where δ is an error term with zero mean and constant variance.

Since the expected values of the spot rate in equation (17) can't be directly estimated, it is necessary to find a recursive solution to equation (17) in order to obtain an estimate of β. This is done as follows:

$(17) \Rightarrow \Delta S_t = (1-\beta) \Delta Z_t + \beta [E_t S_{t+1} - E_{t-1} S_t] + (1-\beta) \varepsilon_t$

which can be re-expressed as $S_t = S_{t-1} + (1-\beta) \Delta Z_t + \beta [E_t S_{t+1} - E_{t-1} S_t] + (1-\beta) \varepsilon_t$

The spot rate at time t+1 can then be expressed as

$S_{t+1} = S_t + (1-\beta) \Delta Z_{t+1} + \beta [E_{t+1} S_{t+2} - E_t S_{t+1}] + (1-\beta) \varepsilon_{t+1}$.

Taking expectations of this equation yields that

$E_t S_{t+1} = S_t + (1-\beta) E_t Z_{t+1} - (1-\beta) Z_t + \beta [E_t S_{t+2} - E_t S_{t+1}]$, and therefore

$E_{t-1}S_t = S_{t-1} + (1-\beta)E_{t-1}Z_t - (1-\beta)Z_{t-1} + \beta[E_{t-1}S_{t+1} - E_{t-1}S_t]$. The value of $E_tS_{t+1} - E_{t-1}S_t$ can then be derived as

$$E_tS_{t+1} - E_{t-1}S_t = \Delta S_t + (1-\beta)[E_tZ_{t+1} - E_{t-1}Z_t] - (1-\beta)\Delta Z_t + \beta[E_t\Delta S_{t+2} - E_{t-1}\Delta S_{t+1})$$

Then a substitution for $E_tS_{t+1} - E_{t-1}S_t$ can be made from the above equation into equation (17), which was $\Delta S_t = (1-\beta)\Delta Z_t + \beta[E_tS_{t+1} - E_{t-1}S_t] + \varsigma_t$ in the following manner.

$$\Delta S_t = (1-\beta)\Delta Z_t + \beta[\Delta S_t + (1-\beta)[E_tZ_{t+1} - E_{t-1}Z_t] - (1-\beta)\Delta Z_t + \beta[E_t\Delta S_{t+2} - E_{t-1}\Delta S_{t+1})] + \varsigma_t \quad \Rightarrow (18.1)$$

This equation can be simplified with a bit of difficulty as follows. Since $(1-\beta)\Delta Z_t = (1-\beta)\beta^0[E_tZ_{t+0} - E_{t-1}Z_{t-1+0}]$, equation (18.1) can be re-written as

$$\Delta S_t = (1-\beta)\sum_{i=0}^{1}\beta^i[E_tZ_{t+i} - E_{t-1}Z_{t+i-1}] + \beta\Delta S_t + \beta^2(E_t\Delta S_{t+2} - E_{t-1}\Delta S_{t+1}) - \beta(1-\beta)\Delta Z_t + \varsigma_t$$

$$\Delta S_t = (1-\beta)\sum_{i=0}^{1}\beta^i[E_tZ_{t+i} - E_{t-1}Z_{t+i-1}] + \beta\Delta S_t + \beta^2E_tS_{t+2} - \beta^2E_tS_{t+1} - \beta^2E_{t-1}S_{t+1} + \beta^2E_{t-1}S_t - \beta(1-\beta)\Delta Z_t + \varsigma_t$$

$$\Delta S_t = (1-\beta)\sum_{i=0}^{1}\beta^i[E_tZ_{t+i} - E_{t-1}Z_{t+i-1}] + \beta[\Delta S_t - (1-\beta)\Delta Z_t - \beta E_tS_{t+1} + \beta E_{t-1}S_t] + \varsigma_t + \beta^2[E_tS_{t+2} - E_{t-1}S_{t+1}] \quad \Rightarrow (18.2)$$

Once again equation (17) is useful in simplifying this messy expression. Since (17) can be re-written as

$\varsigma_t = \Delta S_t - (1-\beta)\Delta Z_t - \beta[E_tS_{t+1} - E_{t-1}S_t]$, this can be substituted into (18.2) to yield

$$\Delta S_t = (1-\beta)\sum_{i=0}^{1}\beta^i[E_tZ_{t+i} - E_{t-1}Z_{t+i-1}] + \beta\varsigma_t + \varsigma_t + \beta^2[E_tS_{t+2} - E_{t-1}S_{t+1}]$$

Similar simplifications can be used to solve the above equation forward to obtain

$$\Delta S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^i(E_tZ_{t+i} - E_{t-1}Z_{t-1+i}) + B_t + \varsigma_t\sum_{i=0}^{\infty}\beta^i \quad \Rightarrow (18.3)$$

where the bubble term B_t can now be described as follows:

$B_t = \lim_{i\to\infty}(E_tS_{t+i} - E_{t-1}S_{t+i-1})$. Another little simplification can be made since

$\varsigma_t\sum_{i=0}^{\infty}\beta^i = \frac{\varsigma_t}{1-\beta} = \frac{(1-\beta)\varepsilon_t}{1-\beta} = \varepsilon_t$, equation (18.3) can be written as

$$\Delta S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^i(E_tZ_{t+i} - E_{t-1}Z_{t-1+i}) + B_t + \varepsilon_t \qquad \Rightarrow (19)$$

This still presents a problem since the expectations of the Z series are also unobservable. However equation (18) specifies the driving process for the Z series and can be used to derive an expression for the expected value of Z_{t+i} that can be used to finally find a recursive solution to (17).

(18) $\Rightarrow Z_t = Z_{t-1} + \varphi\Delta Z_{t-1} + \delta_t$. It follows then that $Z_{t+1} = Z_t + \varphi\Delta Z_t + \delta_{t+1}$ and therefore

$$\begin{aligned} Z_{t+2} &= Z_{t+1} + \varphi\Delta Z_{t+1} + \delta_{t+2} \\ &= Z_{t+1} + \varphi(\varphi\Delta Z_t + \delta_{t+1}) + \delta_{t+2} \\ &= Z_{t+1} + \varphi^2\Delta Z_t + \varphi\delta_{t+1} + \delta_{t+2} \end{aligned}$$

Substituting in the term for Z_{t+1} results in

$Z_{t+2} = Z_t + \varphi\Delta Z_t + \varphi^2\Delta Z_t + \varphi\delta_{t+1} + \delta_{t+2}$ and that $E_tZ_{t+2} = Z_t + \varphi\Delta Z_t + \varphi^2\Delta Z_t$. In general the identity $E_tZ_{t+i} = Z_t + \sum_{j=1}^{i}\varphi^j\Delta Z_t$ holds and (19) can be simplified as

(19) $\Rightarrow \Delta S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^i(E_tZ_{t+i} - E_{t-1}Z_{t-1+i}) + B_t + \varepsilon_t$. Using the identity derived above $\Delta S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^i[(Z_t + \sum_{j=1}^{i}\varphi^j\Delta Z_t) - (Z_{t-1} + \sum_{j=1}^{i}\varphi^j\Delta Z_{t-1})] + B_t + \varepsilon_t$, and

$\Delta S_t = (1-\beta)\sum_{i=0}^{\infty}\beta^i\Delta Z_t + (1-\beta)\sum_{i=0}^{\infty}\sum_{j=1}^{i}\beta^i\varphi^j(\Delta Z_t - \Delta Z_{t-1}) + B_t + \varepsilon_t$. Moving the terms that do not involve i and j from the summation signs yields

$$\Delta S_t = \Delta Z_t(1-\beta)\sum_{i=0}^{\infty}\beta^i + (\Delta Z_t - \Delta Z_{t-1})(1-\beta)\sum_{i=0}^{\infty}\sum_{j=1}^{i}\beta^i\varphi^j + B_t + \varepsilon_t \qquad \Rightarrow (20)$$

Assume now that bubbles do not exist i.e. that $B_t=0$ and further simplify the above equation by examining the term $\sum_{i=0}^{\infty}\sum_{j=1}^{i}\beta^i\varphi^j$ which can be shown to be equal to $(\frac{1}{1-\beta})[\frac{\varphi\beta}{1-\varphi\beta}]$ in the following way.

$$\begin{aligned} &\sum_{i=0}^{\infty}\sum_{j=1}^{i}\beta^i\varphi^j = \varphi\beta + \varphi\beta^2 + \varphi^2\beta^2 + \varphi\beta^3 + \varphi^2\beta^3 + \varphi^3\beta^3 + \ldots \\ &= \varphi(\beta + \beta^2 + \beta^3 + \beta^4 + \ldots) + \varphi^2(\beta^2 + \beta^3 + \beta^4 + \ldots) + \varphi^3(\beta^3 + \beta^4 + \beta^5 + \ldots) \\ &= \varphi\beta(\frac{1}{1-\beta}) + \varphi^2\beta^2(\frac{1}{1-\beta}) + \varphi^3\beta^3(\frac{1}{1-\beta}) + \ldots \end{aligned}$$

$$\sum_{i=0}^{\infty}\sum_{j=1}^{i}\beta^i\varphi^j=(\frac{1}{1-\beta})[\frac{\varphi\beta}{1-\varphi\beta}] \qquad \Rightarrow (21)$$

Equation (21) can then be used to rewrite equation (20) in simplified form as

$\Delta S_t=\frac{(1-\beta)}{(1-\beta)}\Delta Z_t+(\Delta Z_t-\Delta Z_{t-1})[\frac{\varphi\beta}{1-\varphi\beta}]+\varepsilon_t$ or equivalently as

$$\Delta S_t=\Delta Z_t+(\Delta Z_t-\Delta Z_{t-1})[\frac{\varphi\beta}{1-\varphi\beta}]+\varepsilon_t \qquad \Rightarrow (B1).$$

As mentioned previously the behaviour of the fundamentals is assumed to be described by the following process

$$\Delta Z_t=\varphi\Delta Z_{t-1}+\delta_t \qquad \Rightarrow (B2)$$

Equations (B1) and (B2) represent the system of equations that can be used to obtain the first estimate of β which can be labeled $\hat{\beta}$. This can be done using indirect least squares by first running the regression $\Delta S_t=\Delta Z_t+k(\Delta Z_t-\Delta Z_{t-1})$ and then regressing $\Delta Z_t=\varphi\Delta Z_{t-1}$. Since $\hat{k}=\frac{\varphi\beta}{1-\varphi\beta}$ it follows that $\hat{k}=\varphi\beta+\varphi\beta\hat{k}$ and $\hat{\beta}=\frac{\hat{k}}{\hat{\varphi}(1+\hat{k})}$.

Recall that the above estimator of β was obtained under the assumption that bubbles do not exist. The next step is to use McCallum's instrumental variable technique on equation (17) to obtain another estimator of β named β_{IV} which is consistent even when bubbles are present. The process of obtaining β_{IV} is described below.

(17)=> $\Delta S_t=(1-\beta)\Delta Z_t+\beta[E_tS_{t+1}-E_{t-1}S_t]+\varsigma_t$. Assume that the actual spot rate at time t+1 differs from the expected value by a random value η_{t+1} based on the assumption of rational expectations of the agents. Then

$S_{t+1}-E_tS_{t+1}=\eta_{t+1}$ and $S_t-E_{t-1}S_t=\eta_t$ and from equation (17), it follows that

$\Delta S_t=(1-\beta)\Delta Z_t+\beta[E_tS_{t+1}-E_{t-1}S_t]+\varsigma_t$

$\Delta S_t-\Delta Z_t=-\beta\Delta Z_t+\beta[S_{t+1}-\eta_{t+1}-S_t+\eta_t]+\varsigma_t$

$$\Delta S_t-\Delta Z_t=\beta[\Delta S_{t+1}-\Delta Z_t]+[\varsigma_t-\beta(\eta_{t+1}-\eta_t)] \qquad \Rightarrow (21.1)$$

Define the composite error term θ_t to be $\theta_t=[\varsigma_t-\beta(\eta_{t+1}-\eta_t)]$. Equation (21.1) can be re-written as

$$\Delta S_t - \Delta Z_t = \beta[\Delta S_{t+1} - \Delta Z_t] + \theta_t \qquad \Rightarrow (C)$$

Since the composite error term θ_t is not independent of ΔZ_t, an instrument has to be used for $[\Delta S_{t+1} - \Delta Z_t]$. In order to pick this instrumental variable the composite error term θ_t needs to be examined because it can be shown that θ_t depends on ΔZ_t as follows.

$$\eta_{t+1} - \eta_t = \Delta S_{t+1} - (E_t S_{t+1} - E_{t-1} S_t) \qquad \Rightarrow (D)$$

Equation (17) can be re-written as $[E_t S_{t+1} - E_{t-1} S_t] = \frac{1}{\beta}\Delta S_t - \frac{(1-\beta)}{\beta}\Delta Z_t - \frac{1}{\beta}\varsigma_t$

Using (D) and (17)

$$\eta_{t+1} - \eta_t = \Delta S_{t+1} - \frac{\Delta S_t}{\beta} + \frac{(1-\beta)\Delta Z_t}{\beta} + \frac{1}{\beta}\varsigma_t \qquad \Rightarrow (D1)$$

Recall that equation (B1) was $\Delta S_t = \Delta Z_t + (\Delta Z_t - \Delta Z_{t-1})[\frac{\varphi\beta}{1-\varphi\beta}] + \varepsilon_t$ which can be simplified to yield $\Delta S_t = \frac{1}{1-\varphi\beta}\Delta Z_t - (\frac{\varphi\beta}{1-\varphi\beta})\Delta Z_{t-1}) + \varepsilon_t \qquad \Rightarrow (D2)$

and therefore that $\Delta S_{t+1} = \frac{1}{1-\varphi\beta}\Delta Z_{t+1} - (\frac{\varphi\beta}{1-\varphi\beta})\Delta Z_t) + \varepsilon_{t+1}. \qquad \Rightarrow (D3)$

Equations (D2) and (D3) can be used to simplify equation (D1) as follows:

$$\eta_{t+1} - \eta_t = \frac{1}{1-\varphi\beta}\Delta Z_{t+1} - \frac{\varphi\beta}{1-\varphi\beta}\Delta Z_t + \varepsilon_{t+1} - \frac{1}{\beta(1-\varphi\beta)}\Delta Z_t + \frac{\varphi\beta}{\beta(1-\varphi\beta)}\Delta Z_{t-1} - \frac{\varepsilon_t}{\beta} + \frac{(1-\beta)}{\beta}\Delta Z_t + \frac{1}{\beta}\varsigma_t$$

Since $\varsigma_t = (1-\beta)\varepsilon_t$ by definition, $\frac{\varsigma_t}{\beta} - \frac{\varepsilon_t}{\beta} = \frac{1}{\beta}[(1-\beta)\varepsilon_t - \varepsilon_t] = -\varepsilon_t$ and the above can be simplified as

$$\eta_{t+1} - \eta_t = \frac{1}{1-\beta\varphi}\Delta Z_{t+1} - \frac{1}{\beta(1-\beta\varphi)}\Delta Z_t[\beta^2\varphi + 1 - (1-\beta)(1-\beta\varphi)] + \frac{\varphi}{(1-\beta\varphi)}\Delta Z_{t-1} + \varepsilon_{t+1} - \varepsilon_t$$

which is equivalent to

$$\eta_{t+1} - \eta_t = \frac{1}{1-\beta\varphi}\Delta Z_{t+1} - \frac{(1-\varphi)}{(1-\beta\varphi)}\Delta Z_t + \frac{\varphi}{(1-\beta\varphi)}\Delta Z_{t-1} + \varepsilon_{t+1} - \varepsilon_t \text{ and also to}$$

$$\eta_{t+1} - \eta_t = \frac{1}{1-\beta\varphi}[\Delta Z_{t+1} - \varphi\Delta Z_t] - [\frac{1}{(1-\beta\varphi)}](\Delta Z_t - \varphi\Delta Z_{t-1}) + \varepsilon_{t+1} - \varepsilon_t$$

Recall now that (B2) was $\Delta Z_t = \varphi\Delta Z_{t-1} + \delta_t$

$$\eta_{t+1} - \eta_t = \frac{1}{1-\beta\varphi}\delta_{t+1} - \frac{1}{1-\beta\varphi}\delta_t + \varepsilon_{t+1} - \varepsilon_t, \text{ equivalently}$$

$$\eta_{t+1} - \eta_t = \frac{1}{1-\beta\varphi}[\delta_{t+1} - \delta_t] + \varepsilon_{t+1} - \varepsilon_t. \qquad \Rightarrow (D4)$$

Since by definition the composite error term θ_t is defined as

$$\theta_t = [\varsigma_t - \beta(\eta_{t+1} - \eta_t)].$$

$$(D4) \Rightarrow \theta_t = \varsigma_t - \beta[\frac{1}{1-\beta\varphi}[\delta_{t+1} - \delta_t] + \varepsilon_{t+1} - \varepsilon_t] \qquad \Rightarrow (D5)$$

As the composite error term depends on δ_t, as seen in equation (D5), it follows that it is not independent of ΔZ_t but it is independent of ΔZ_s for $s \leq t-1$ and therefore ΔZ_{t-1} can be used as instruments for $\Delta S_{t+1} - \Delta Z_t$. in equation (C) $\Delta S_t - \Delta Z_t = \beta[\Delta S_{t+1} - \Delta Z_t] + \theta_t$. The following regression is then run and another estimate of β is obtained.

$$\Delta S_t - \Delta Z_t = \beta_{IV}\Delta Z_{t-1} + \theta_t \rightarrow (E)$$

The Hausman statistic can be used to test for a significant difference in the two estimates of β, namely $\hat{\beta}$ and $\hat{\beta}_{IV}$. $\hat{\beta}_{IV}$ is a consistent estimator of β even when bubbles are present while $\hat{\beta}$ is consistent only if the assumption that bubbles do not exist is satisfied. Testing for a significant difference in the two statistics is done using the Hausman statistic where the Hausman statistic has a χ_1^2 distribution and is defined as

$$\text{Hausman} = \frac{N[\hat{\beta}_{IV} - \hat{\beta}]^2}{\frac{\hat{\beta}_{IV}{}^2(1+\hat{\varphi})^2}{\hat{\varphi}^2} + \frac{\sigma_\varepsilon^2(1+\hat{\varphi})^3(1-\hat{\beta}_{IV}\hat{\varphi})^2[(1-\hat{\beta}_{IV}\hat{\varphi})^2 + 2\hat{\beta}_{IV}{}^2(1-\hat{\varphi})]}{2\sigma_\delta^2(1-\hat{\varphi}^2)\hat{\varphi}^2}}$$

The Hausman statistic is calculated by using parameters from the money demand equations as specified previously and by the residuals of (B1), (B2) and equation (E). Meese derives the denominator based upon the variance of $\hat{\beta}$ and $\hat{\beta}_{IV}$ provided by Hausman.

7.3: Method Three: Evans' method of testing for non zero medians in excess returns

Evans takes a different approach from the methods described in sections 7.1 and 7.2 to test for the presence of speculative bubbles. Evans defines a period during which there is a consistent run of negative or positive returns to holding a particular currency as a period during which a speculative bubble may exist. Evans' claims that such a period is characterized by a speculative bubble because the extent of the appreciation or depreciation of the currency is often greater than can be explained by differentials in interest rates or price level differentials between the respective countries.

This method is different from the previous two studies in that it is not dependent on parameter estimates of a money demand function or on a specific model of the fundamentally determined exchange rate. The advantage of this method is that it avoids dealing with a fundamentals only value of the exchange rate that was generated by a model which has not proven to be an accurate short run predictor of the exchange rate. However, as can be seen later, this model has a considerable weakness in that it relies on an assumption about the existence of efficient markets which has often been contradicted empirically. The other main flaw in the method of Evans is the assumption that a non zero median in excess returns necessarily implies the existence of a speculative bubble in the economy. There are a number of alternative explanations for a the existence of a non zero median including non-efficient markets and non rational expectations. The method is described below.

Assume that agents are risk neutral with rational expectations and that the market for foreign exchange is efficient. Even though the risk neutrality assumption can be relaxed, the assumption of efficient markets is used throughout and can reduce the power of the test. An excess return is defined as the difference between the actual spot rate and the one period ahead forward rate at which transactions were conducted during the previous time period. Let excess returns at time t+1 be denoted as X_{t+1}. Then by definition $X_{t+1} = S_{t+1} - F^{t}_{t+1}$ where F^{t}_{t+1} is the one period ahead forward rate at time t.

Under the efficient markets hypothesis $E_t S_{t+1} = F^{t}_{t+1}$ and therefore $E_t X_{t+1} = 0$.

Evans looks for a non zero median in the distribution of X by using a procedure which attempts to make allowances for the possibility that a data series which follows a random walk can show a sustained period of negative or positive deviations purely by chance with a small probability. The test that Evans develops is basically a sign test, i.e. a test that looks at the difference between the number of positive and negative values of the variable in question, which can be applied to a specific sub period as well as to the whole sample. The null and alternate hypothesis are given by

H_0: $m_t = 0$ for $t = 0,\ldots,T$
Ha: $m_t \neq 0$ for T1...T2, where $t = 0 \leq T1 \leq T2 \leq T$
where m_t is the median value of excess returns $X_{t+1} = S_{t+1} - F_t^{t+1}$.

The excess return on holding a currency X_t can be adjusted to allow for risk premia and a test can also be carried out for a non zero median in risk adjusted excess returns X_t'. The risk adjusted excess return is defined as follows: Assume that the markets follow Covered Interest Rate Parity, which imply that the difference between the forward rate and the corresponding future spot rate depends upon relative interest rates between the domestic country and the foreign country. This implies that $F_t = S_t + R_t - R_t^*$ where R_t is the short term nominal interest rate in the domestic country and R_t^* is the short term nominal interest rate in the foreign country. Then $X'_{t+1} = S_{t+1} - S_t + R_t^* - R_t$ and the null hypothesis of a zero median for the risk adjusted exchange rate can also be tested.

If these hypothesis tests were to be carried out using a standard t-test the results would depend on the validity of the assumptions of constant variance and simple kurtosis of a small sample. Evans uses a Monte Carlo study to directly estimate the significance level of the observed excess returns and claims that his method is superior since it does not depend on assumptions about various properties of a small sample. The Monte-Carlo study that is used to obtain the significance level is described below.

7.3.1 Using the Monte Carlo study to estimate the significance level

Ten thousand samples of random numbers are generated for the number of months involved in the study. Since Evans uses a 12 year and 11 month period for his study, he generates 10,000 samples of 155 positive and negative numbers from a distribution that is uniform in the interval [-1..1]. This means that each number is positive or negative with probability p=1/2. For each k year sub period k=1,2.....12 Evans calculates the value of N_k where N_k=Number of positive observations - Expected number of positives

In general we would find 12k months in a k year sub sample and expect that 1/2*12k=6k of those months will have positive excess returns. Therefore N_k=Number of positive observations-6k.

Define $L_k = MAX(N_k)$ to be the largest number of positive deviations for a k year sub-sample. Therefore L_k is the most extreme deviation from the expected value in a sample over periods of length k. We use the 10,000 samples to generate 10,000 values of L_k and obtain a_k a cumulative density function (CDF) for L_k Evans provides a table which contains the values of $a_k(L_k)$ in his article, and that table is given below. Each entry of the table provides the value of $a_k(L_k)$, the number of times that value was exceeded in 10,000 simulations of 155 months each.

Table 2: Cumulative Distribution of L_k

Length of sub-period: k years

L_k	1	2	3	4	5	6	7	8	9	10	11	12
0	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
1	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
2	10000	10000	10000	10000	10000	10000	9999	10000	9998	9987	9936	9663
3	9978	9996	9987	9992	9984	9980	9964	9927	9842	9667	9333	8561
4	8348	9694	9770	9744	9735	9656	9585	9376	9152	8729	8179	7273
5	3033	7789	8682	8818	8854	8731	8539	8314	7941	7418	6838	6043
6	353	4460	6384	7050	7257	7270	7203	6975	6570	6134	5581	4913
7		1808	3883	4957	5409	5568	5616	5420	5170	4875	4466	3957
8		566	1950	3037	3620	3967	4110	4063	3985	3802	3538	3100
9		119	801	1685	2290	2662	2863	2947	2916	2863	2673	2348

L_k	1	2	3	4	5	6	7	8	9	10	11	12
10		13	278	807	1307	1657	1902	2046	2081	2073	1977	1756
11		2	97	363	687	1000	1210	1368	1450	1429	1386	1268
12			28	132	334	584	748	868	961	973	971	891
13			11	42	139	292	423	520	604	648	653	588
14			3	20	58	145	218	294	348	392	429	400
15				4	23	60	111	158	199	243	268	260
16				1	7	22	51	80	112	144	158	163
17					2	7	23	37	62	84	93	98
18						3	8	19	27	48	58	50
19							4	13	15	25	37	30
20							3	4	6	12	15	21
21							1	3	3	6	10	8
22								1	3	3	4	6
23									1	1	2	4
24									1	1	1	3
25											1	1
26												1

From the exchange rate data, one can obtain the number of positive excess returns in excess of the expected number. The above table can then be used to calculate the test statistic Y which is the number of times such a value was observed in the Monte Carlo random number sample. An example given by Evans will help to clarify this idea better.

Assume that the sample length is four years and that the number of observed excess returns that were positive was 39. Since the expected value was 6k=24, this provides a value of L_4=15 for the excess returns series. From the table above, the value of Y can be calculated as 4, i.e. the entry in column k=4 and L_k=15. So the value of the test statistic is 4.

Evans provides another table for the true significance level of the test statistic Y and that table is given below. The significance level that is given in each entry of the table is the estimated probability of obtaining a value of Y less than or equal to the table entry given that the null hypothesis of a zero median in excess returns from holding a currency is satisfied.

Table 3: True Significance Levels of Test Statistic Y

Y	Significance Level	Y	Significance Level
0	0.0002	111	0.0269
1	0.0005	112	0.0283
2	0.0009	119	0.0335
3	0.0011	132	0.0369
4	0.0015	139	0.0396
6	0.0022	144	0.0412
7	0.0027	145	0.0427
8	0.0029	158	0.0452
10	0.0030	163	0.0480
11	0.0037	199	0.0513
12	0.0041	218	0.0534
13	0.0057	243	0.0553
15	0.0061	260	0.0594
19	0.0063	268	0.0620
20	0.0067	278	0.0707
21	0.0073	292	0.0743
22	0.0078	294	0.0776
23	0.0090	334	0.0814
25	0.0094	348	0.0854
27	0.0099	353	0.1070
28	0.0108	363	0.1106
30	0.0110	392	0.1156
37	0.0119	400	0.1213
42	0.0135	423	0.1248
48	0.0143	429	0.1267
50	0.0149	520	0.1312
51	0.0159	566	0.1486
58	0.0182	584	0.1538
60	0.0187	588	0.1602
62	0.0195	604	0.1643
80	0.0205	648	0.1675
84	0.0213	653	0.1708

Y	Significance Level	Y	Significance Level
93	0.0224	687	0.1773
97	0.0247	748	0.1832
98	0.0259	801	0.2012

From this table, the true significance can be obtained. In the previous example, the value of Y was 4. From the above table, it can be seen that the probability of obtaining a value of Y=4 if the null hypothesis of a zero median in excess returns were true is about 0.0015 which makes it likely that the null hypothesis can be rejected and the conclusion reached that a non-zero median in excess returns was observed and that such an observation corresponds to the existence of a speculative bubble.

Section VIII: Description of Data Series

8.1 Data Sources

The data series used in the first two testing methods are the logarithm of the end of the month spot rate expressed as units of domestic currency per unit of foreign currency. The national output series is an industrial production index with a common base of January 1972 used for both countries. The money supply series is seasonally adjusted nominal money supply data (M1). Both the nominal money supply and the income data are expressed in terms of logarithms. The majority of the data was obtained from the International Financial Statistics published by the International Monetary Fund. An attempt was made to use data series that were consistent across all countries in the sample. The industrial production data are from line 6..c and the seasonally adjusted money supply figures (M1) are from line 34..b. The exchange rate data used were end of the period spot rates and not period averages. The end of period spot-forward rates were obtained from a database at the Federal Reserve Bank of Cleveland, thanks to the generosity of Mr. Owen Humpage of the Cleveland Federal Reserve Bank.

The main problem that exists in the data set is the unavailability of money supply figures for the U.K for the period Jan 1980 - December 1986. This is due to an accounting change in 1986 which saw the U.K change its methods of

calculating narrow money (M1). Post 1987 figures are not comparable with the pre 1987 figures and often money supply data prior to 1987 is omitted for the U.K in various databases including the IFS data set. Even the database at the Cleveland Federal Reserve Bank did not have the appropriate series and the research assistant at the Fed was only able to come up with only post 1987 data. This necessitates the restriction of the U.K study to the period January 1987-October 1992, a sample that is almost 60 fewer observations than the German or Japanese data.

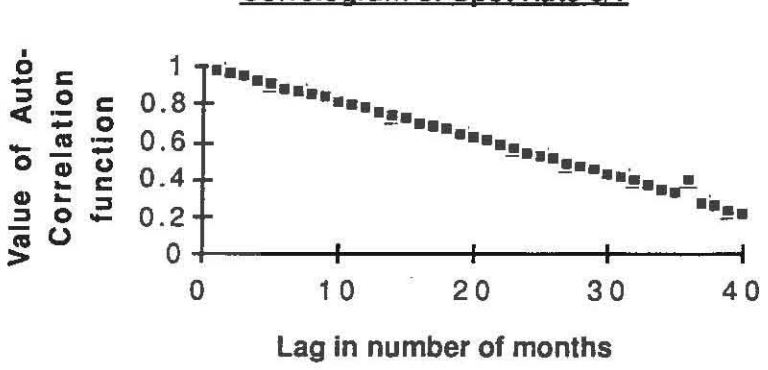
Recall that the fundamentals denoted as the Z series was defined as $Z_t = (M_t - M_t^*) - \mu(Y_t - Y_t^*)$. Ten different Z series were constructed for different values of the income elasticity of money demand obtained from previous studies. i.e. values of μ=0.6,0.7,.....,1.4,1.5, and these series were used in sections 9.2 and 9.3. Graphs of the data series are given in Appendix I in case the reader wishes to obtain an idea about the behavior of the data series used in the model over the sample period.

Section IX: Results

9.1 : Results of Stationarity Tests

Section V described the importance of using stationary time series for the models. This section contains correlograms of the spot rate and the extreme values of the ten values of the fundamentals, i.e. the Z series constructed by using values of μ = 0.6 and μ = 1.5. The need for first differencing the data can be determined by examining the correlograms and by use of the Dickey-Fuller tests for unit roots in the data.

9.1.1 Correlograms

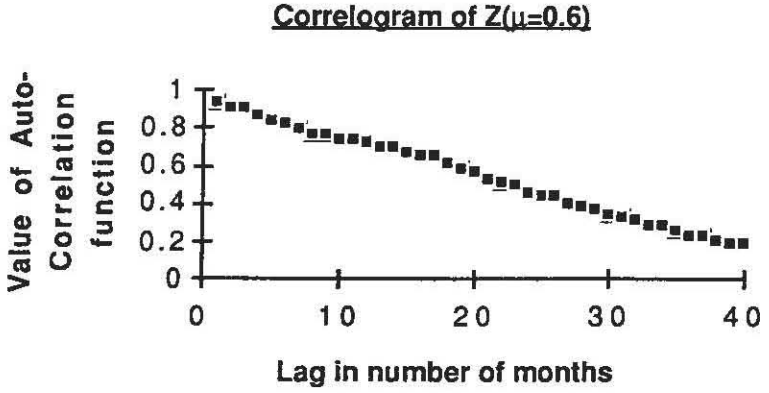


The chart given above shows that the correlogram of the Japanese spot rate seems to be converging to zero as the number of lags increases. The shape of the graph, however, seems to indicate a linear convergence rather than the exponential convergence of the typical shape of a correlogram of a stationary series as described in section 5 and the number of lags needed to reach a value of zero is fairly high. When the data is first differenced, however, the correlogram seems to fluctuate around zero and does not show the typical shape of a stationary series. This raises an interesting quandary in that the data series does not seem to be stationary but first differencing does not induce stationarity instead causing the data series to be over differenced. This observation is pretty consistent for the spot rate and the fundamentals for all three currencies, with an exception being made for the $/£ exchange rate.

Since the degree of differencing that is required to induce stationarity seems not to be an integer, it is not immediately obvious that first differenced data should be used in a regression equation. However, the unit root tests provide another method to figure out the degree of differencing that should be applied to the data.

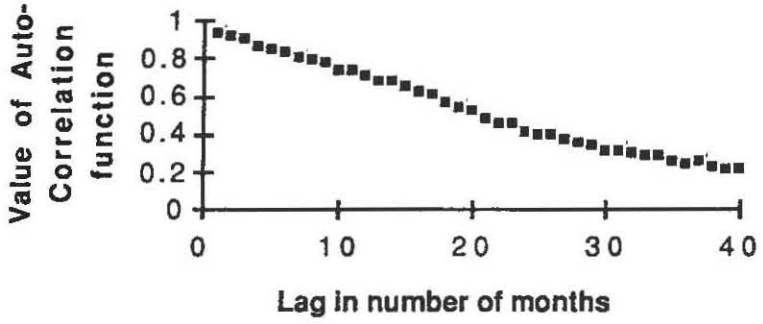
The charts below give the correlograms for the Z series obtained for the extreme parameter values μ=0.6 and μ=1.5

Fundamental Series with μ=0.6 for Japan



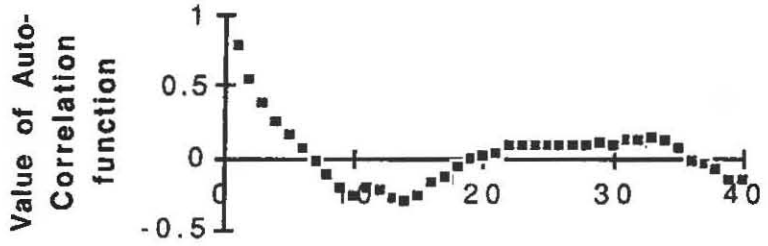
Fundamental Series with μ=1.5 for Japan

Correlogram of Z(μ=1.5)



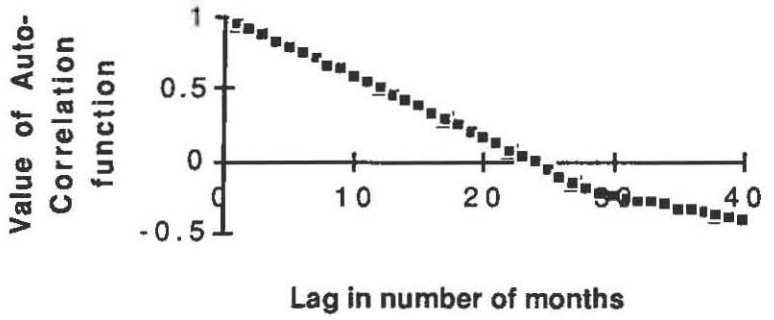
The correlograms for the English spot rate show the signs of a stationary data series while the correlograms for the fundamentals series are very similar to the Japanese data even though the shorter sample period (by almost 60 months) shows a faster convergence.

Correlogram of Spot Rate ($/£)



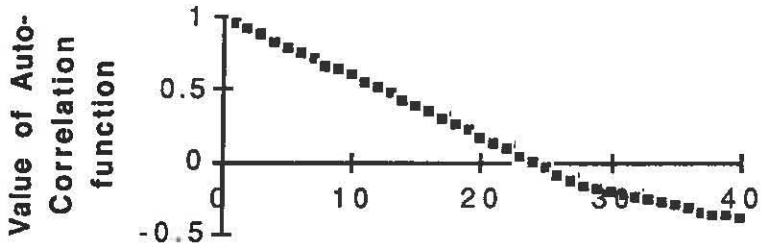
Fundamental Series with μ=0.6 for the U.K

Correlogram of Z(μ=0.6)



Fundamental Series with μ=1.5 for the U.K

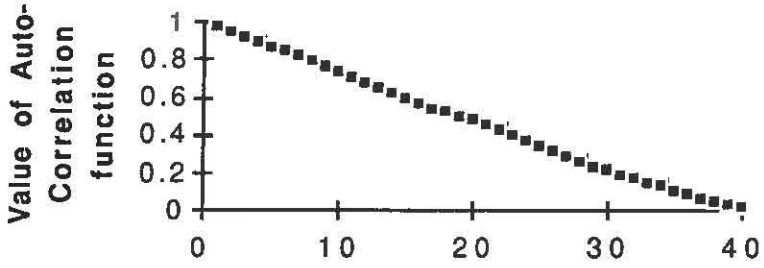
Correlogram of Z(μ=1.5)



Lag in number of months

The final set of correlograms is for the German data series which is also fairly similar to the data on Japan in that the convergence to zero seems to occur in a slow linear pattern that closely resembles a non stationary data series.

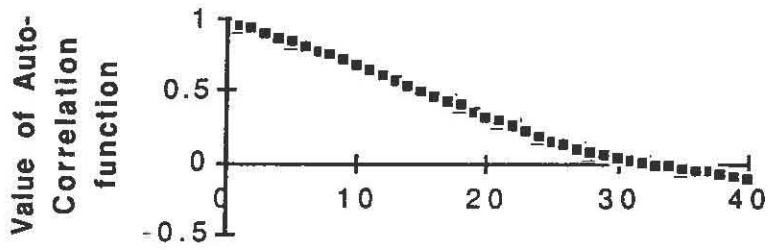
Correlogram of Spot Rate($/DM)



Lag in number of months

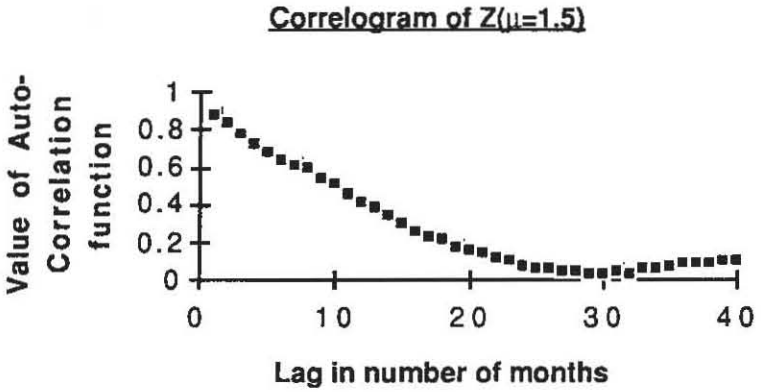
Fundamental Series with μ=0.6 for Germany

Correlogram of Z(μ=0.6)



Lag in number of months

Fundamental Series with μ=1.5 for Germany



9.1.3 Results of Dickey-Fuller Tests

Recall that the Dickey Fuller test is a test of the null hypothesis

H_0: The data series in question follows a random walk with no time trend

against the alternate hypothesis described by

H_a: The data series in question does not follow a random walk or has a time trend

The results of the Dickey-Fuller tests seem to indicate that the null hypothesis which states that the variables in question follow a random walk can not be rejected at a 95% level of significance. This would imply that the variables in question have to be first differenced before being used in a regression equation since regressing one random walk against another in a regression can render it spurious. Even though there are Dickey-Fuller test, the results from this, coupled with the uncertainty that arose when looking at the correlograms about the degree of differencing necessary to induce stationarity imply that first differenced data may yield less spurious results than level data when used in regression equations.

COUNTRY	DATA SERIES	DF STATISTIC	95% CRIT. VAL
Germany	Spot Rate	4.3088	6.49
	Z (μ=0.6)	2.2583	6.49
	Z (μ=1.5)	4.4857	6.49
United Kingdom	Spot Rate	0.14874	5.61
	Z (μ=0.6)	5.4199	5.61
	Z (μ=1.5)	3.8192	5.61

I A SERIES	DF STATISTIC	95% CRIT. VAL
Spot Rate	1.6056	6.49
Z (μ=0.6)	5.1292	6.49
Z (μ=1.5)	8.2938	6.49

1.4	1.5
-0.539	-0.539
8.34E-3	8.90E-3

Variance Tests.

previous studies of money demand functions I have obtained a
nates μ = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3 ,1 .4 and 1.5 to calculate ten
Z. The version of the excess variance test that I will carry out is the
Huang in his study and was described in detail in section 7.1.2.
en values of Z are used to obtain different estimates of φ by
ing regression $\Delta Z_t = \varphi \Delta Z_{t-1} + \delta_t$. Three different values of the
f money demand λ =2.0 , 2.5 and 3.0 are obtained from past studies
se values a corresponding value of $\beta = \frac{\lambda}{1+\lambda}$ is calculated from three
the inequality being tested is

$$Z \quad \frac{1}{1-\beta^2} VAR(\delta_t)] > VAR(S_t - Z_t). \qquad \Rightarrow (F)$$

test is carried out for each of the 10 Z series as well as the three
of β , which results in a total of thirty different inequality tests. The
as the variance of the u_t series is given below for all three pairs of

results of the

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LE OF VALUES FOR φ AND FOR VAR(δ_t)

	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
.253	-0.276	-0.294	-0.31	-0.323	-0.333	-0.342	-0.349	-0.355
6E-3	1.46E-3	1.46E-3	1.47E-3	1.50E-3	1.54E-3	1.59E-3	1.67E-3	1.76E-3
9	-0.095	-0.099	-0.104	-0.109	-0.113	-0.118	-0.121	-0.125
2E-3	3.40E-3	3.57E-3	3.76E-3	3.96E-3	4.16E-3	4.37E-3	4.57E-3	4.78E-3

Case 3: λ=3.0

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.01252438	0.00142849	Violated
0.7	0.01246119	0.001392553	Violated
0.8	0.01240906	0.001372618	Violated
0.9	0.01236799	0.001369591	Violated
1	0.01233798	0.001385528	Violated
1.1	0.01231904	0.001420959	Violated
1.2	0.01231115	0.001476661	Violated
1.3	0.01231433	0.001553906	Violated
1.4	0.01232857	0.001653014	Violated
1.5	0.01235387	0.001774853	Violated

U.S. Dollar - Japanese Yen Rate

Thirty different tests of inequality (F) were carried out, and the results were more varied. The inequality violations were not as significant as for the Mark/$ rate and for extreme values of μ and λ, the inequality is narrowly violated. A value of μ=1.6 will cause the inequality to be satisfied, therefore the results in this instance are somewhat dependent upon the parameter values.

Case 1: λ=2.0

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.007534045	0.002084769	Violated
0.7	0.0073536	0.002238972	Violated
0.8	0.00718469	0.002404731	Violated
0.9	0.007027316	0.002581957	Violated
1	0.006881477	0.002770536	Violated
1.1	0.006747173	0.002969337	Violated
1.2	0.006624404	0.003180123	Violated
1.3	0.006513171	0.00340178	Violated
1.4	0.006413473	0.003635315	Violated
1.5	0.00632531	0.003879442	Violated

Case 2: λ=2.5

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.007534045	0.002616679	Violated
0.7	0.0073536	0.002809161	Violated
0.8	0.00718469	0.003016161	Violated
0.9	0.007027316	0.003237582	Violated

1	0.006881477	0.003473307	Violated
1.1	0.006747173	0.003722139	Violated
1.2	0.006624404	0.003985944	Violated
1.3	0.006513171	0.004263541	Violated
1.4	0.006413473	0.004555997	Violated
1.5	0.00632531	0.004861953	Violated

Case 3: λ=3.0

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.007534045	0.003143211	Violated
0.7	0.0073536	0.003373491	Violated
0.8	0.00718469	0.003621222	Violated
0.9	0.007027316	0.003886304	Violated
1	0.006881477	0.004168615	Violated
1.1	0.006747173	0.004466914	Violated
1.2	0.006624404	0.004783134	Violated
1.3	0.006513171	0.005116054	Violated
1.4	0.006413473	0.005466779	Violated
1.5	0.00632531	0.005833899	Violated

U.S. Dollar - Sterling Pound Rate

The results were similar to the Dollar/Mark exchange rate with thirty violations of inequality (F) being observed. However the magnitude of the violations was considerably less than for the Dollar/Mark exchange rate but are more significant than for the Dollar/Yen rate.

Case 1 λ=2.0

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.004248521	0.002188895	Violated
0.7	0.004395297	0.002295274	Violated
0.8	0.00454546	0.002405052	Violated
0.9	0.004699011	0.002516722	Violated
1	0.00485595	0.002633292	Violated
1.1	0.005016276	0.002753312	Violated
1.2	0.00517999	0.002875103	Violated
1.3	0.005347092	0.003002013	Violated
1.4	0.005517581	0.003128795	Violated
1.5	0.005691457	0.003260723	Violated

Case 2: λ =2.5

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.004248521	0.002828384	Violated
0.7	0.004395297	0.002964583	Violated
0.8	0.00454546	0.003105061	Violated
0.9	0.004699011	0.003248143	Violated
1	0.00485595	0.003397173	Violated
1.1	0.005016276	0.003550535	Violated
1.2	0.00517999	0.003706367	Violated
1.3	0.005347092	0.003868381	Violated
1.4	0.005517581	0.004030763	Violated
1.5	0.005691457	0.004199355	Violated

Case 3: λ =3.0

μ	VAR(S-Z)	L.H.S	Inequality
0.6	0.004248521	0.00347111	Violated
0.7	0.004395297	0.003637107	Violated
0.8	0.00454546	0.003808256	Violated
0.9	0.004699011	0.003982745	Violated
1	0.00485595	0.004164186	Violated
1.1	0.005016276	0.00435083	Violated
1.2	0.00517999	0.004540671	Violated
1.3	0.005347092	0.004737708	Violated
1.4	0.005517581	0.00493568	Violated
1.5	0.005691457	0.005140878	Violated

Overall, the results from the excess variance tests seem to strongly indicate the presence of bubbles for the Dollar/Mark exchange rate and somewhat less strongly indicate the presence of bubbles for the Dollar/Pound rate. The results for the Dollar/Yen exchange rate are susceptible to variations in the income and interest elasticity of money demand parameters with extreme values of the parameters failing to provide convincing proof about the existence of bubbles.

Section 9.2 Hausman Specification Test

Recall that the Hausman specification test involved obtaining two different estimates of the co-efficient β , one of which is consistent only if the null hypothesis of no bubbles is true. This estimator which was named $\hat{\beta}$ was obtained using Indirect Least Squares from using OLS on the following system of equations:

$$\Delta S_t - \Delta Z_t = [\frac{\varphi\beta}{1-\varphi\beta}](\Delta Z_t - \Delta Z_{t-1}) + \varepsilon_t$$

$$\Delta Z_t = \varphi \Delta Z_{t-1} + \delta_t$$

The second estimator of β, which was named β_{IV} was obtained by using Instrumental Variables on the following regression equation:

$$\Delta S_t - \Delta Z_t = \beta_{IV}(\Delta Z_{t-1}) + \theta_t$$

The Hausman statistic is then used to test for a significant difference in the two estimators of β. Recall that the Hausman statistic had a Chi-Square distribution with one degree of freedom and was explicitly derived by Meese as

$$\text{HAUSMAN} = \frac{N[\hat{\beta}_{IV} - \hat{\beta}]^2}{\frac{\hat{\beta}_{IV}^2(1+\hat{\varphi})^2}{\hat{\varphi}^2} + \frac{\sigma_\varepsilon^2(1+\hat{\varphi})^3(1-\hat{\beta}_{IV}\hat{\varphi})^2[(1-\hat{\beta}_{IV}\hat{\varphi})^2 + 2\hat{\beta}_{IV}^2(1-\hat{\varphi})]}{2\sigma_\delta^2(1-\hat{\varphi}^2)\hat{\varphi}^2}}$$

As in the excess variance testing, ten series of Z were constructed for values of $\mu = 0.6, 0.7, 0.8 \ldots 1.5$ from the previous studies. The numerator of the Hausman statistic is calculated with the number of observations N as well as the two estimators of β , namely $\hat{\beta}$ and β_{IV}.

The denominator was calculated by means of estimates of β_{IV} from equation(E), estimates of φ from the system of equations (B1) and (B2), estimates of σ_δ^2 obtained from the residuals of equation (B2) and estimates of σ_ε^2 obtained from the residuals of equation (B1).

Case 1 The Dollar-Mark Rate:

Ten values of the Hausman statistic are calculated below for a range of values of μ .from 0.6,0.7, ...1.5 over the period from January 1980 to October 1992.

μ	HAUSMAN statistic	Result
0.6	2.04702435	NON SIGNIFICANT DIFFERENCE
0.7	2.06089263	NON SIGNIFICANT DIFFERENCE
0.8	2.23862519	NON SIGNIFICANT DIFFERENCE
0.9	2.54712263	NON SIGNIFICANT DIFFERENCE
1.0	2.92397959	NON SIGNIFICANT DIFFERENCE
1.1	3.40102293	NON SIGNIFICANT DIFFERENCE

1.2	3.95423234	SIGNIFICANT DIFFERENCE
1.3	4.58589493	SIGNIFICANT DIFFERENCE
1.4	5.28069874	SIGNIFICANT DIFFERENCE
1.5	6.05080542	SIGNIFICANT DIFFERENCE

The results for the Dollar/Mark exchange rate are more varied with the Hausman statistics being significant at a 5% level of confidence only for some values of μ and all ten statistics are insignificant at a 99% level of confidence. This is rather different from the strength of the results attained by the excess variance testing which provided a strong argument for the presence of bubbles.

Case 2 The Dollar-Pound Rate:

This time the sample period is from January 1987 to October 1992 and the values of the calculated Hausman statistic are given below. This time there is a difference in that the results are **not** significant at a 95% level of confidence for all ten instances.

The results are given below and once again provide an interesting contradiction with the results of the excess variance tests where the results seemed to strongly indicate the presence of bubbles. This is a result that mirrors that for the \$/DM rate where a similar contradiction was observed.

Value of μ	Hausman Statistic	Result(95%)
0.6	0.009601695	NOT SIGNIFICANT
0.7	0.008983297	NOT SIGNIFICANT
0.8	0.006721713	NOT SIGNIFICANT
0.9	0.002567286	NOT SIGNIFICANT
1.0	0.000304008	NOT SIGNIFICANT
1.1	0.026208052	NOT SIGNIFICANT
1.2	0.21977313	NOT SIGNIFICANT
1.3	1.101202947	NOT SIGNIFICANT
1.4	2.653064802	NOT SIGNIFICANT
1.5	3.235103081	NOT SIGNIFICANT

Case 3: The Dollar-Yen Rate:

The complete results of the Hausman test are given below. This time the results are strongly significant in all ten instances at a 95% level of significance with little variation caused by different values of μ. This outcome is again an interesting contradiction of the results obtained by the excess variance method where the results for the Dollar/Yen rate were far less robust against variations in μ than seems to be the case here.

Value of μ	Hausman Statistic	Result
0.6	24.6077423	SIGNIFICANT DIFFERENCE
0.7	24.7057568	SIGNIFICANT DIFFERENCE
0.8	25.25123	SIGNIFICANT DIFFERENCE
0.9	25.8996324	SIGNIFICANT DIFFERENCE
1.0	26.6131402	SIGNIFICANT DIFFERENCE
1.1	27.4152747	SIGNIFICANT DIFFERENCE
1.2	28.1791187	SIGNIFICANT DIFFERENCE
1.3	29.0986894	SIGNIFICANT DIFFERENCE
1.4	30.0399618	SIGNIFICANT DIFFERENCE
1.5	30.9416195	SIGNIFICANT DIFFERENCE

Overall the results for this section are interesting with strong evidence for the presence of bubbles being indicated for the Yen/Dollar exchange rate, conflicting evidence emerging for the Dollar/Mark rate where the results are dependent upon the different values of μ at a 95% level of confidence with insignificant differences for all the tests emerging at a 99% level of confidence and strong evidence against bubbles shown for the $/¥ rate. The interesting fact that emerges is that all three results contradict the results obtained from excess variance testing.

9.3 Results of the Evans Test for Speculative Bubbles

Recall that Evans characterizes a period during which there exists a non-zero median in excess returns as an indication of the presence of a speculative bubble. For this test I used spot/forward rate data for the period Feb 1981-Oct1992 for the German Mark/U.S Dollar and Japanese Yen/U.S Dollar and data for the period Jan 1987 - Oct 1992 for the British Pound/U.S Dollar. The period for the Pound/Dollar was

chosen to co-incide with the two previous tests although the inability to obtain money supply data prior to 1987 was not an issue in carrying out this test. I was unable to obtain data for the year 1980 for the mark and the yen and therefore these results are for a sample period that is roughly a year less than the previous two methods.

The sample length for Germany and Japan is 12 years and for the United Kingdom it is 5 years. Therefore the expected number of positive excess returns (= 6k) for the respective countries are 72 for Germany and Japan and 30 for the U.K. The actual number of observed positive excess returns X_t are given below:

Germany:	76
Japan:	76
U.K:	43

This results in a value of $L_{12} = 4$ for Germany and Japan and $L_5 = 13$ for the United Kingdom. Recall that L_k is the number of observed positive excess returns that exceeded the expected number of positive excess returns. Since my sample length does not exceed the 155 month period for which Evans performed the Monte-Carlo study the tables provided by Evans can be used to estimate the significance level of the observations. Recall that the test statistic Y provides the number of times that our observed value of L was attained or exceeded in the Monte Carlo study. From Table 2, provided in section 7.3.1, three values of the test statistic Y can be obtained for the Dollar/Mark, Dollar/Yen and Dollar/Pound Exchange Rates.

Therefore Y_{Ger} can be calculated by looking at the table entry given in column k=12 and row L=4 which means that Y_{Ger} =7273. Similarly Y_{Jap} is calculated to be equal to 7273 and Y_{Eng} = 139. Once the values of the test statistic have been obtained then the true significance of the test statistic can be obtained by looking at the table of true significance values in section 7.3.1.

From this we can see that the test statistics for Germany and Japan are extremely high meaning that there is a high probability of the observed variations in excess returns occurring given that the null hypothesis is satisfied. So even at a 99% level of confidence, the null hypothesis of a zero median in excess returns occurring for Japan and Germany can not be rejected. The test statistic for the U.K is 0.0396 which implies that there is a 4% chance of the observed variations in excess returns occurring if the null hypothesis is true which in turn implies that the null hypothesis of a zero median in excess returns can be rejected at a 95% level of confidence.

These results however do not allow for the risk aversion of agents in the exchange market and therefore it is important to test the null hypothesis of a non zero median in excess returns after allowing for risk adjustment.

The number of risk adjusted excess returns $X_t^{\cdot}$ can also be calculated to be

Germany:	114
Japan:	114
U.K:	70

This results in a value of $L_{12} = 42$ for Germany and Japan and $L_5 = 40$ for the United Kingdom. The test statistic Y which provides the number of times that our observed value of L was attained or exceeded in the Monte Carlo study can be obtained from Table 2, provided in section 7.3.1.

Therefore Y_{Ger} ,Y_{Jap} and Y_{Eng} have values less than 1. Therefore the true significance values of the test statistic have been obtained then the true significance of the test statistic can be obtained by looking at the table of true significance values in section 7.3.1. From this we can see that the significance levels for Germany, the United Kingdom and Japan are extremely low (i.e. less than 0.02) which implies that there is only a very small probability of the observed variations in excess returns occurring given that the null hypothesis is satisfied. Therefore the null hypothesis of a zero median in excess returns can be rejected at a 95% level of confidence for all three pairs of exchange rates

If one were to accept Evans' claim that the occurrence of a non zero median proven by using this test necessarily implies the existence of a speculative bubble, and if no adjustment was made for the risk averseness of agents in the economy then the final conclusions reached by using this method is that a speculative bubble exists for the Dollar/Pound Exchange Rate between 1987 and 1992, while no such bubble seems to exist for the Mark/Dollar and Yen/Dollar rates. Furthermore the results are so insignificant for the Dollar/Mark and the Dollar/Yen rate that it does not seem likely that the sample length being short by one year affects the ability to compare the results with the results of the two methods described previously. However the adjustment for risk aversion produces results which seem to strongly indicate the presence of bubbles for all three currency pairs, thus presenting an interesting contradiction between various applications of the Evans' method as well as between the results of the Evans' test and the other two methods studied previously. An attempt is made to make sense out of these contradictory results in the next section.

Section X Analysis and Conclusions:

Section 10.1 Some weaknesses of the testing methods

In order to analyze the results of the various tests and understand their importance, it is important to discuss the weaknesses of each of the three testing methods used in this paper. The primary criticisms are likely to be about the assumptions that underlie the monetary model which is used as the model of the fundamentally determined exchange rate. As mentioned before the monetary model has not proven to be a very good predictor of short term fluctuations in the exchange rate mainly because the assumption that Purchasing Power Parity holds in the short run has been shown to be untrue. This raises an interesting question about the value of a study that attempts to study speculative bubbles, defined as systematic deviations from the fundamentally determined exchange rate, with a model of the fundamentally determined exchange rate whose validity has often been questioned. This places the value of "fundamentally determined exchange rate" derived from this model and the measurements of deviations from this value in some doubt.

Another issue that arises with respect to the value of the fundamentally determined exchange rate is that the Z series contains only values of output and money supply even though those two variables may only be a small subset of the true economic fundamentals that affect and drive the value of a currency. Even interest rates are not factored in to the Z series because Uncovered Interest Parity is assumed in order to introduce the spot rate into the derivation of the monetary model from a standard money demand function. This is another assumption that has been questioned by some researchers but not on the scale that PPP has been doubted.

Also causing some consternation is the assumption that agents in the market for foreign exchange are rational because some of the methods are used to test the joint null hypothesis that speculative bubbles do not exist and expectations are rational. Therefore a rejection of the null hypothesis can happen either because a speculative bubble exists or because expectations are not rational and incorrect conclusions reached about the existence of speculation in the foreign exchange market.

These general criticisms apply primarily to the excess variance test and the Hausman specification test which use the monetary model of the exchange rate and assume that expectations of the agents in the market for foreign exchange are rational. Some of these general criticisms can be answered somewhat satisfactorily. The use of the monetary model is often justified, as described before, by the assertion that almost

all models of the exchange rate are imperfect and some models are empirically hard to test, therefore the simplicity of the monetary model makes it attractive. The monetary model is useful to the task at hand because of both its empirical testability and the ability to use it to provide a structured form for the bubble term.

Following the advice of Blanchard and Watson as described in Section 4.2.1, the assumption of rational expectations can also be justified to a certain extent because it is often easier to model rational bubbles rather than model irrational speculation. In the context of other areas of economics where rational expectations are assumed in a far more carefree manner, assumptions of rational expectations in modeling the exchange market can also be justified by the fact that there are comparatively few agents in the market for foreign exchange, with most of the more significant players being large banks and wealthy investors who often have access to enough information upon which to base expectations about the future value of the currency that are fairly accurate to the actual mathematical expected value. Another criticism that can be raised with respect to the first two methods is the assumption that first differences in the Z series can be characterized by an AR(1) process. This assumption can also be justified for the sake of simplicity and for its usefulness in providing the ability to structure and characterize the bubble process in an elegant manner.

The Evans test has some advantage over the other two methods in that it is not dependent upon a particular model of the fundamentals and as such avoids much of the criticism given above. However, the underlying assumption about the existence of efficient markets and the claim that any deviation of a zero median characterizes a speculative bubble makes the results of the Evans' test vulnerable to criticism, especially since his method does not provide an explicit characterization of the bubble term unlike the other two methods.

The implication created by all these criticisms is that there is no true test for bubbles and that it is difficult to accept the results of one testing method at the expense of another. This paper which uses three of the best known tests for bubbles has found that the overall results vary widely depending on the method used. A summary of the results of the three different tests will be very useful at this point, both to understand the different results that came about from using a particular test across currencies but also to look at cross test summary of results for a particular currency. The tables below provide a summary of the results of this study.

Section 10.2: Cross Currency Results for Each Testing Method

Comparison using Excess Variance Tests

Exchange Rate	Period	Result
Dollar-Mark	1980-1992	Strong evidence for the presence of bubbles
Dollar-Pound	1987-1992	Evidence for the presence of bubbles, results are not as strong as for the $/Mark rate
Dollar Yen	1980-1992	Evidence for the presence of bubbles, yet results are dependent on values of the parameters with higher values providing evidence against bubbles

Comparison using Hausman Tests

Exchange Rate	Period	Result
Dollar-Mark	1980-1992	Mixed evidence for bubbles with results that are susceptible to change
Dollar Pound	1987-1992	Fairly strong evidence against the presence of bubbles
Dollar-Yen	1980-1992	Strong evidence for the presence of bubbles with little susceptibility to changes in the parameters

Comparison using the Evans test

Exchange Rate	Period	Result with no risk premia	Result allowing for risk premia
Dollar-Mark	1980-1992	Strong evidence against bubbles	Strong evidence against bubbles
Dollar Pound	1987-1992	Evidence for bubbles (95% significance) Evidence against bubbles (99% significance)	Strong evidence against bubbles
Dollar-Yen	1980-1992	Strong evidence against bubbles	Strong evidence against bubbles

Cross Method Comparison

Exchange Rate	Excess Variance Test	Hausman Test	Evans' Test
Dollar-Mark	Strong evidence for bubbles	Results are indeterminate	Evidence against bubbles
Dollar Pound	Some evidence for bubbles	Strong evidence against bubbles	Some evidence for bubbles before allowing for risk premia
Dollar-Yen	Somewhat weak evidence for bubbles	Evidence for bubbles.	Evidence against bubbles

Looking at the tables above, the most striking fact is that the results from tests for bubbles are not robust across currencies and across testing methods. Since it is difficult to say that a particular testing method is superior to all other methods the primary finding of this study has to be that any conclusions that have been reached about the existence of speculative bubbles should be questioned in light of the lack of robustness of the results and their dependency on the testing method that was used to

achieve that result. However, this finding should not be misinterpreted to mean that all claims for the existence of speculative bubbles should be summarily dismissed. Even if the methods show conflicting results, the next logical step should be an attempt to improve current methods or develop new methods that can be used to test for and identify speculative bubbles. After all just as there is no conclusive evidence that indicate the presence of bubbles, there is no conclusive evidence against the presence of bubbles. It is important to understand that speculative bubbles could well explain the excess volatility of exchange rates.

Researchers who make a priori assumptions that bubbles do not exist, and proceed to derive models that fail to predict short run variations in the exchange rate would be better served by acknowledging that there is imperfect evidence for the presence of bubbles based on the results obtained from using current tests for bubbles. Future research effort to both develop better tests for bubbles and to incorporate the possible existence of speculative bubbles into the building of a more accurate fundamental model of short term exchange rate fluctuation seem entirely justified on the basis of the results of this paper.

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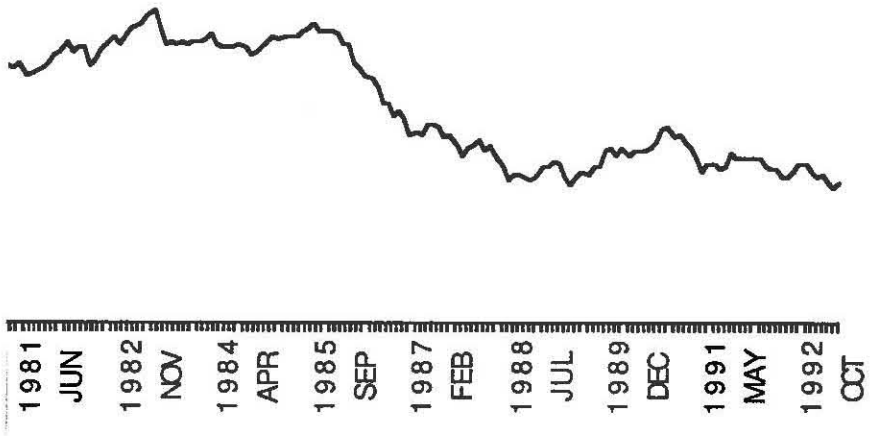
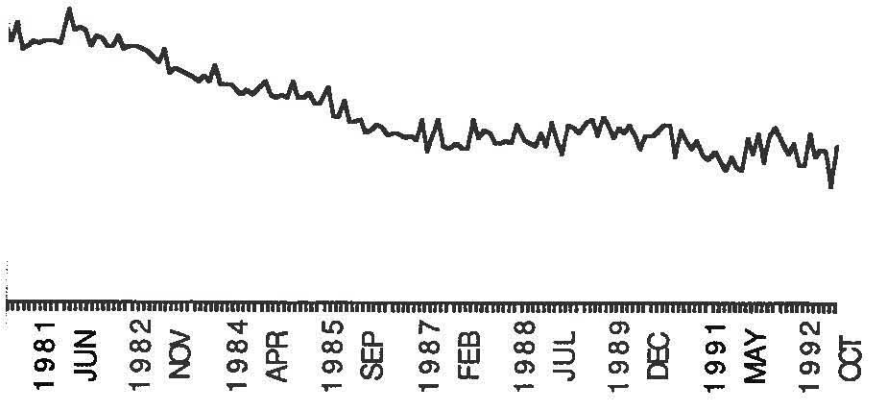
Spot Rate $/¥



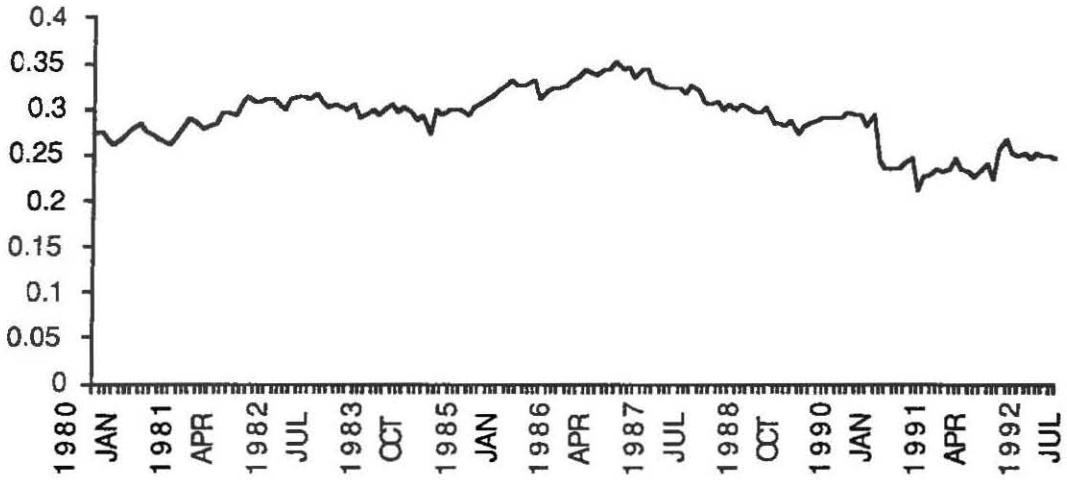
Japan Z (μ=0.6)



Section 3 :German Spot Rates and Fundamentals.

Spot Rate $/DM

German Z (μ=0.6)



SECTION XI: Bibliography

Asako, Kazumi. "The Land Price Bubble in Japan"
Ricerche Economiche 45(2-3) April-Sept. 1991 pages 167-84.

Blackburn, Keith.*Market Fundamentals Versus Speculative Bubbles, a New Test Applied to the German Hyperinflation*
University of Southampton, Dept of Economics, 1992

Blanchard, Olivier J and Watson, Mark W. *Bubbles, Rational Expectations and Financial markets: in Crises in the Economic and Financial Structure*
Lexington MA, Lexington Books: 1982

Booth, Elizabeth B; Booth, G. Geoffrey "Exchange Rate Bubbles: Some New Evidence"
Rivista-Internazionale-di-Scienze-Economiche-e-Commerciali 34(11-12), Nov-Dec 1987, pages 1053-68

Buiter, William *Rational Speculative Bubbles in an Exchange Rate Target Zone*
National Bureau of Economic Research 1990, Cambridge, Massachusetts

Cassella, Alessandra. "Testing for Rational Bubbles with Exogenous or Endogenous Fundamentals: The German Hyper-inflation Once More"
Journal of Monetary Economics 24(1) July 1989 pp109-122

Copeland, Laurence S. *Exchange Rates and International Finance*
Addison Wesley Publishing Company, England.

Demirguc-Kunt, Asli *Testing for Speculative Bubbles in Stock Prices*
Research Dept, Federal Reserve Bank of Cleveland 1988

Dezhbakhsh, Hashem ; Demirguc-Kunt, Asli. "On the Presence of Speculative Bubbles in Stock Prices"
Journal of Financial and Quantitative Analysis 25(1), March 1990

Diba, Behzad T; Grossman, Herschel I. "On the Inception of Rational Bubbles"
Quarterly Journal of Economics 102(3), August 1987

Evans, George W. "A Test for Speculative Bubbles in the Sterling-Dollar Exchange Rate: 1981-84"
American Economic Review 76(4), September 1986, pages 621-36

Flood, Robert P; Hodrick, Robert J. "On Testing for Speculative Bubbles"
Journal of Economic Perspectives 4(2), Spring 1990, pages 85-101

Flood, Robert P; Garber, Peter-M. "Market Fundamentals versus Price-Level Bubbles: The First Tests"
Journal of Political Economy 88(4), August 1980, pages 745-70

Froot, Kenneth A., Obstfeld, Maurice "Intrinsic Bubbles: The Case of Stock Prices"
American Economic Review 81(5); December 1991 pages 1189-214

Garber, Peter M. "Famous First Bubbles"
Journal of Economic Perspectives 4(2), Spring 1990, pages 35-54

Garber, Peter M. "Tulipmania"
Journal of Political Economy 97(3), June 1989, pages 535-60

Hausman, Jerry A. "Specification tests in Econometrics"
Econometrica, Vol. 46(November 1978) PP 1251-71

Huang, Roger D. "The Monetary Approach to Exchange Rates in an Efficient Foreign Exchange Market:Tests based on Volatility"
The Journal of Finance, Vol. 36 (March 1981)

Kearney, Colm; MacDonald, Ronald. "Rational Expectations, Bubbles and Monetary Models of the Exchange Rate: The Australian/US Dollar Rate during the Recent Float"
Australian Economic Papers 29(54), June 1990, pages 1-20

Kindleberger, Charles. *Manias, Panics and Crashes: a History of Financial Crises*
N.Y Basic Books 1978

Mackay, Charles F. *Extraordinary Popular Delusions and the Madness of Crowds*
L.C Page & Co. Boston 1932.

Meese, Richard A. "Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates?"
Journal of Political Economy 94(2), April 1986, pages 345-73

Pindyck, Robert and Rubinfeld, Daniel. *Econometric Methods and Economic Forecasts*
McGraw-Hill Inc, 1991

Shiller, Robert J. *Market volatility*
Cambridge, Mass and London: MIT Press, 1989, pages xii, 464

Stiglitz, Joseph E. "Symposium on Bubbles"
Journal of Economic Perspectives 4(2), Spring 1990, pages 13-18

Towe, Christopher M. "Exchange Rate "Fundamentals" versus Speculation in a Developing Economy: An Illustrative Example Using Lebanese Data"
International Monetary Fund Staff Papers 36(3), September 1989

Wadhwani, Sushil B. "Are Exchange Rates 'Excessively' Volatile?"
Journal of International Economics 22(3/4), May 1987, pages 339-48

West, Kenneth D. "A Specification Test for Speculative Bubbles"
Quarterly Journal of Economics 102(3) August 1987 pp553-80

Woo, Wing T. *Speculative Bubbles in the Foreign Exchange Markets*
The Brookings Institution 1984