THESIS

# RESOURCE ALLOCATION FOR WILDLAND FIRE SUPPRESSION PLANNING USING A STOCHASTIC PROGRAM

Submitted by

Alex Taylor Masarie

Department of Forest and Rangeland Stewardship

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Master's committee:

Advisor: Douglas Rideout

Michael Bevers
Michael Kirby

ABSTRACT

RESOURCE ALLOCATION FOR WILDLAND FIRE SUPPRESSION PLANNING USING A STOCHASTIC PROGRAM

Resource allocation for wildland fire suppression problems, referred to here as Fire-S problems, have been studied for over a century. Not only have the many variants of the base Fire-S problem made it such a durable one to study, but advances in suppression technology and our ever-expanding knowledge of and experience with wildland fire behavior have required almost constant reformulations that introduce new techniques. Lately, there has been a strong push towards randomized or stochastic treatments because of their appeal to fire managers as planning tools. A multistage stochastic program with variable recourse is proposed and explored in this paper as an answer to a single-fire planning version of the Fire-S problem. The Fire-S stochastic program is discretized for implementation according to scenario trees, which this paper supports as a highly useful tool in the stochastic context. Our Fire-S model has a high level of complexity and is parameterized with a complicated hierarchical cluster analysis of historical weather data. The cluster analysis has some incredibly interesting features and stands alone as an interesting technique apart from its application as a parameterization tool in this paper. We critique the planning model in terms of its complexity and options for an operational version are discussed. Although we assume no interaction between fire spread and suppression resources, the possibility of incorporating such an interaction to move towards an operational, stochastic model is outlined. A suppression budget analysis is performed and the familiar ``production function'' fire suppression curve is created, which strongly indicates the Fire-S model performs in accordance with fire economic theory as well as its deterministic counterparts. Overall, this exploratory study demonstrates a promising future for the existence of tractable stochastic solutions to all variants of Fire-S problems.

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# 1 Introduction

Much of the fire business on a given district involves initial attack on containable fires. Fire economists and fire managers have long studied the initial attack resource allocation for fire suppression problem. Fire management models that support the Fire Program Analysis (FPA) represent the vast work done on this subject. Models described by Donovan and Rideout in [5] as well as Kirsch and Rideout in [9] are deterministic solutions to the problem. Others have studied initial attack in a probabilistic (or stochastic) framework. A prime example of such stochastic modeling is the California Fire Economics Simulator (CFES). Underlying CFES is the random simulation model presented by Fried, Gilless, and Spero in [8].

Stochastic models can be more complicated than their deterministic counterparts, but also provide significant advantages as realistic planning tools. We present a stochastic programming model that solves a single fire version of the allocation for suppression problem, which is referred to as the Fire-S model. We propose a four-stage stochastic program with variable recourse and explain the underlying mathematics. Section 2 develops the model from a simple example building to the actual model presented in Section 3. Such a stochastic program allows us to (a) capture the dynamic aspect of fire suppression in the four stages and (b) reconcile the reality that decisions made over time demonstrate a hierarchical dependence using recourse. Just like any math programming model, our stochastic program has decision variables and parameters. The decision variables represent resource allocation choices. The parameters represent a fire manager's resource set and the fire behavior he or she encounters. We simulate fire behavior parameters by performing a cluster analysis of historical weather data, which produces representative weather scenarios to use in the Fire Area Simulator (Farsite) software package. We describe the parameterization steps in Section 4, which consists of weather in Section 4.1, fire simulation in Section 4.2, suppression resources in Section 4.3, and escaped fires in Section 4.4. Although the parameterization process is spatially explicit, the stochastic program itself is not. The program does not, for instance, account for the interaction between fire growth and suppression, which we explore in Sections 5.5 and 5.7. As such, our model is most readily interpreted as a fire planning model instead of an operational model. Section 5.2 shows how the Fire-S model supports suppression budget analysis. We use the suppression budget analysis to search for advantages for the model's complexity in Sections 5.3 and 5.4. This exploration concludes with a discussion of promising avenues of further research in Section 5.8.

## 2 Developing the Model

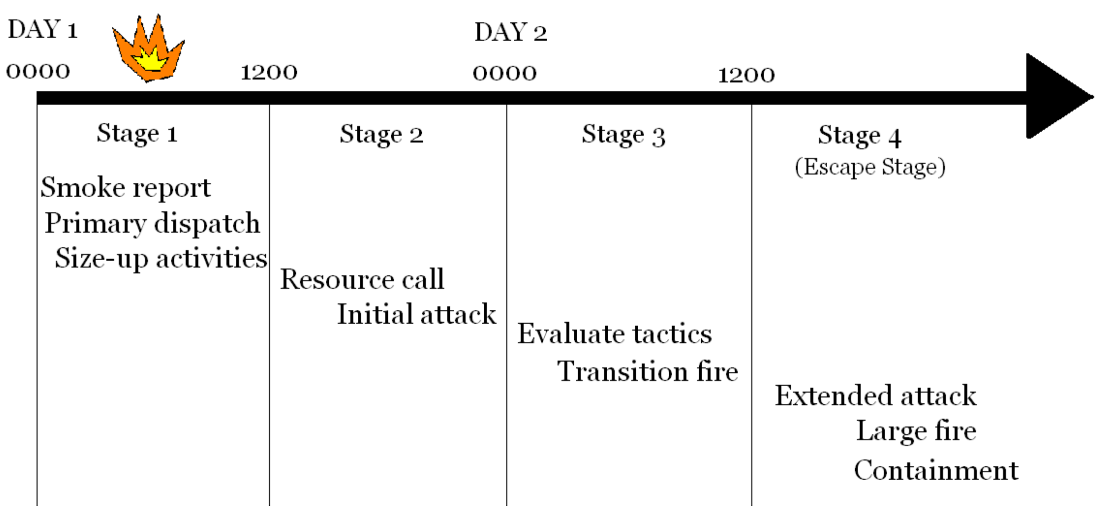



Figure 1: Our single-fire time line.

Figure 1 shows the dynamic aspect of our version of the single-fire allocation for suppression problem. First, notice there are four interdependent stages. For clarity, we assume each stage lasts twelve hours and so the scope of the model is two days, although this assumption is not necessary. An initial smoke report indicates a fire exists on the landscape during Stage 1, which, assuming twelve-hour stage lengths, means the morning of Day 1. Figure 1 also gives some possible terms associated with the fire from initial attack to possible containment. These descriptors are merely a demonstration of one possible scenario; many factors influence fire suppression. Also notice Stage 4 is marked as the ``escape stage." This means the fire has escaped containment in the scope of model. It need not be assumed an extreme fire, just that uncertainty in fire behavior factors such as weather and fuels is quite high for a fire manager applying these techniques at a stage 1 smoke report. We discuss implementing a rolling planning horizon to address this issue later on.

Although the stochastic program follows these stages, the user applies the model just once at the start of Stage 1. All of the user's information about subsequent stages is probabilistic and therefore uncertain. Recourse operates in this program by assuming various possible scenarios occur. Thus, once a scenario is adopted or realized (with its associated probability) the uncertainty is eliminated and a decision can be made. The most basic example of recourse involves containment. Suppose a fire manager has allocated enough resources to contain the fire in Stage 2. Stage 3's recourse decisions must reflect the fact that the fire is contained under the current scenario and perhaps allocate a mop-up crew or do nothing at all.

Scenarios are the key, underlying tool that we use to parameterize the stochastic program and make the model realistic. One of the benefits of a probabilistic approach is that we can work with two basic concepts of fire management:

1. A fire manager *CANNOT* exactly predict fire behavior with complete certainty over time,
2. A fire manager *CAN* incorporate expert knowledge and/or fire behavior software to characterize some likely and unlikely fire behaviors.

Thus, a fire manager is a highly capable predictor of fire behavior and can use his or her numerous, scientific tools to construct a collection of possible fire behavior scenarios, which we call a *scenario tree*.

We will start with a small-scale example of such a scenario tree and proceed to build towards the stochastic program as a whole. Suppose a smoke report indicates an ignition in a given fire management zone. The fire manager uses the ignition's spatial location to obtain detailed fuels and topographical information. Given the ignition's temporal location (time of day, season, etc...) the fire manager also obtains current fire weather information and forecasts. Say, in this simple example, there are two common weather patterns associated with the passage of summertime cold fronts through the area. In reality, the fire manager would have fairly accurate predictions about when the front will pass, but, for the sake of this example, let us assume each of these two weather scenarios (pre-frontal and post-frontal) has a $50\%$ chance of occuring. Furthermore, the fire manager does not know when, during the two day scope of this model, fronts will pass. Using pre-frontal and post-frontal weather data, the fire manager runs Farsite and determines fire behavior for all possible front arrival times during each of the four stages.

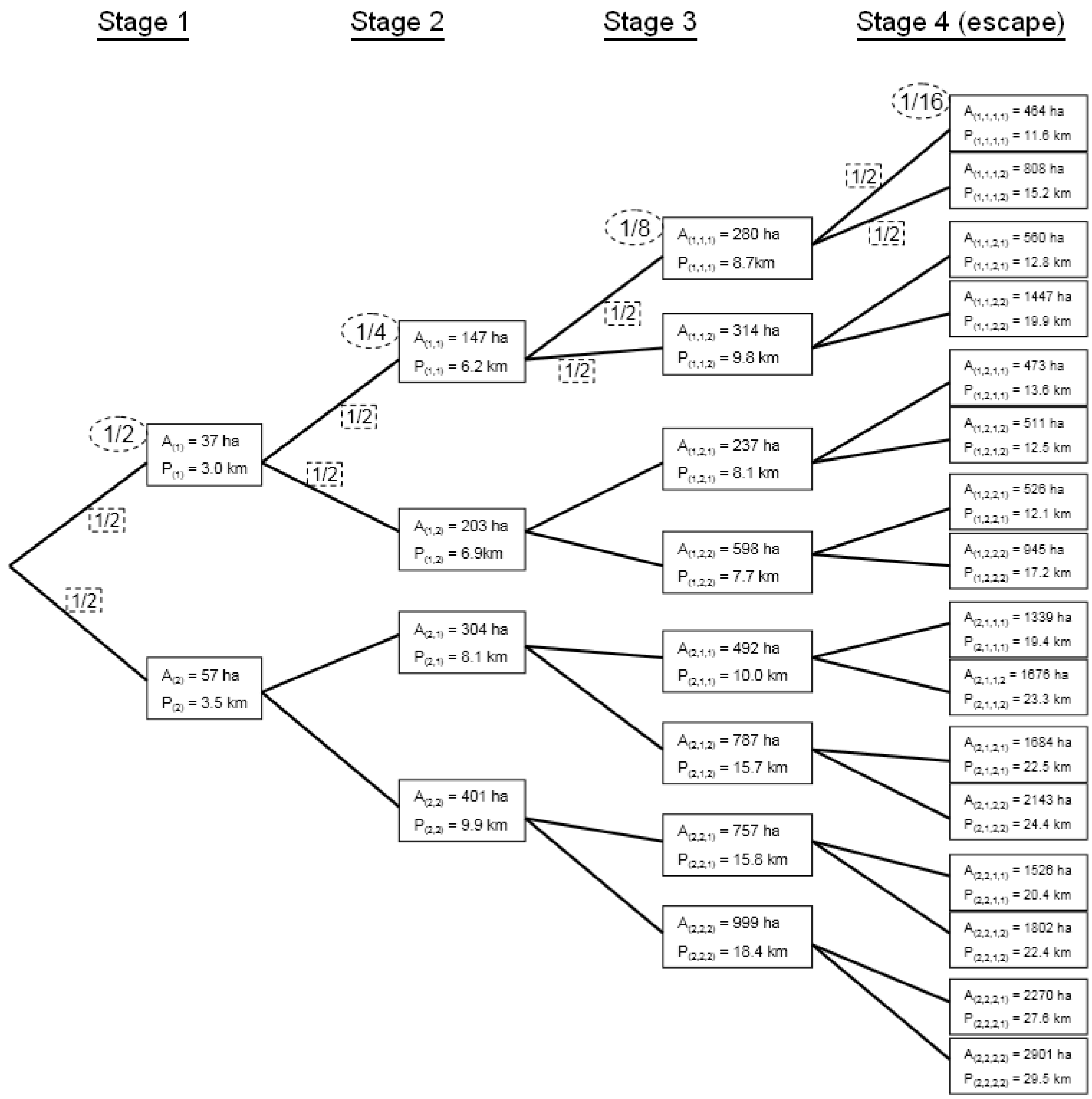



Figure 2: A two-branch scenario tree example.

Figure 2 shows the resulting sixteen fire behavior scenarios. Right now, at the time of the smoke report, the fire manager does not know which branch will best match the actual fire behavior (concept 1), but he or she is confident that Figure 2 represents a clear picture of possible fire behaviors (concept 2) because it is based on the best available weather and a scientifically sound simulation software package.

The scenario tree in Figure 2 is a crucial part of the model so we will explore it thoroughly. The solid lines connecting the diagram together are called *branches*.

Branches initiate and terminate at *nodes*. In terms of the time-line in Figure 1, the branches represent periods of time during a stage and the nodes represent transitions between stages. Decisions are made at nodes and their actions carried out during branches. At any given Stage 1, 2, or 3 node there is one branch that represents moderate fire behavior and another that represents severe fire behavior. These are associated with the fire manager's pre-frontal and post-frontal Farsite simulations. A scenario tree diagram makes the hierarchical nature of the four-stage problem easy to follow. The fire manager can literally trace each of the sixteen fire behavior scenarios by starting at the leftmost node and finishing at each of the rightmost nodes. For example, the Farsite simulation that represents moderate fire behavior for Stages 1, 2, and 3 and more severe fire behavior for Stage 4 is found by tracing the top branch at the first three nodes and then the bottom branch the final, stage 4 node. To represent this scenario we use an ordered pair $(1,1,1,2)$. This way, each scenario has a unique representation $(k_1, k_2, k_3, k_4)$ where $k_1, k_2, k_3,$ and $k_4$ each take the value 1 or $2$. We say scenario $(1,1,1,2)$ has *parent* nodes $(1,1,1)$, $(1,1)$, and $(1)$. In general, we use the notation $\langle k_t \rangle$ to represent an ordered pair at Stage $t$; so $\langle k_1 \rangle = (k_1)$, $\langle k_2 \rangle = (k_1, k_2)$, $\langle k_3 \rangle = (k_1, k_2, k_3)$, and $\langle k_4 \rangle = (k_1, k_2, k_3, k_4)$. Thus, in the equations to follow $(\cdot)$ and $\langle \cdot \rangle$ serve as visual indicators the associated variables come from a scenario tree like Figure 2 and are probabilistic.

The boxes in Figure 2 show cumulative area ($A_{\langle k_t \rangle}$) and perimeter ($P_{\langle k_t \rangle}$) estimates at each node. Under scenario $(1,1,1,2)$ the fire manager will encounter a fire that grows in area as follows:

$$\lfloor A_{(1)} = 37ha \rfloor \rightarrow \lfloor A_{(1,1)} = 147ha \rfloor \rightarrow \lfloor A_{(1,1,1)} = 280ha \rfloor \rightarrow \lfloor A_{(1,1,1,2)} = 808ha \rfloor$$

and perimeter:

$$\lfloor P_{(1)} = 3.0km \rfloor \rightarrow \lfloor P_{(1,1)} = 6.2km \rfloor \rightarrow \lfloor P_{(1,1,1)} = 8.7km \rfloor \rightarrow \lfloor P_{(1,1,1,2)} = 15.2km \rfloor.$$

This scenario may influence the fire manager to call for heavy initial attack because the fire exhibits rapid Stage 4 growth so resources could be dispatched early in order to get the fire contained during Day 1 before fire weather supports more rapid fire growth in the afternoon of Day 2.

Let us now turn the discussion to probabilities. In accordance with the assumption that both types of weather are equally likely, each branch has a dashed box indicating a $\frac{1}{2}$ probability. These are the *unconditional probabilities* ( $\hat{p}_{\langle k_t \rangle}$ ) of each branch. Because each node has a set of parent nodes, nodes are assigned *conditional probabilities* ( $p_{\langle k_t \rangle}$ ), which depend upon all of the parent probabilities. Take scenario $(1,1,1,2)$ as an example again. A Stage 1 node has no parent so

$$p_{(1)} = \hat{p}_{(1)} = \frac{1}{2}.$$

Stage 2's probability is conditional on stage 1 so

$$p_{(1,1)} = \hat{p}_{(1,1)} \cdot p_{(1)} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Similarly for Stage 3

$$p_{(1,1,1)} = \hat{p}_{(1,1,1)} \cdot p_{(1,1)} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

and Stage 4

$$p_{(1,1,1,2)} = \hat{p}_{(1,1,1,2)} \cdot p_{(1,1,1)} = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}.$$

These conditional probabilities are shown in dashed circles on Figure 2. For this simple example, conditional and unconditional probabilities are uniform, but this need not be the case. When a fire manager is using expert knowledge to construct a scenario tree, he or she will come up with a spectrum ranging from highly likely to highly unlikely fire behavior scenarios to account for. Section 4 shows how we create a scenario tree with non-uniform probabilities. This simple example demonstrates two important properties about the conditional probabilities:

$$\forall \langle k_t \rangle \{ 0 < p_{\langle k_t \rangle} \leq 1 \tag{2.1}$$

$$\forall t \left\{ \sum_{\langle k_t \rangle} p_{\langle k_t \rangle} = 1. \right. \tag{2.2}$$

Equation (2.1) is a familiar, general property of fractional probabilities that constrains $p_{\langle k_t \rangle}$ to be from 0 % to 100 %. The property in (2.2) means if we sum across each stage (vertically in Figure 2), we get 100% probability. If we sum scenario probabilities for Stage 3, for instance, Equation (2.2) becomes

$$\sum_{\langle k_3 \rangle} p_{\langle k_3 \rangle} = \sum_{k_1} \sum_{k_2} \sum_{k_3} p_{(k_1, k_2, k_3)} = \sum_{k_1=1}^{2} \sum_{k_2=1}^{2} \sum_{k_3=1}^{2} \frac{1}{8} = 1.$$

Thus, (2.2) ensures that some scenario occurs; in the Fire-S model, some fire behavior occurs with 100% probability.

With this understanding of scenario trees, the next step is to explain how decision-making operates in this framework. Figure 2 is a probabilistic description of fire behavior. A fire manager wants to take this information and make resource allocation decisions that are cost effective and achieve some set of management goals. We follow the classic math programming approach to this problem used in [?, dr]nd [?, kr]nd elect to minimize burned area using fire perimeter as a guide for possible containment through comparison with the amount of fire line constructed. However, due to the stochastic nature of this approach, our objective must be to minimize *expected burned area* because we are working with probabilistic fire behaviors. The conditional probability property in (2.2) is the key to computing expected burned area. Let $E[\cdot]$ denote the *expectation operator*. In general, the expected cumulative burned area at Stage $t$ is

$$E[Area_t] = \sum_{\langle k_t \rangle} \left( p_{\langle k_t \rangle} \cdot A_{\langle k_t \rangle} \right) \tag{2.3}$$

Using Stage 3 of Figure 2 as an example again, (2.3) becomes

$$E[Area_3] = \sum_{\langle k_3 \rangle} \left( p_{\langle k_3 \rangle} \cdot A_{\langle k_3 \rangle} \right) = \sum_{k_1} \sum_{k_2} \sum_{k_3} \left( p_{(k_1,k_2,k_3)} \cdot A_{(k_1,k_2,k_3)} \right)$$

$$= \frac{1}{8} \sum_{k_1} \sum_{k_2} \sum_{k_3} A_{(k_1,k_2,k_3)} = 558 ha.$$

Thus, the best estimate of burned area at Stage 3 is 558 ha. As with any probabilistic estimate, the actual burned area at Stage 3 may not be exactly 558 ha, but the expected value represents our best estimate. As such, it can inform management decisions under our stochastic framework just as exact burned area informs management decisions under a deterministic framework in [5] and [9].

Now, let us suppose the fire manager can allocate enough resources to build $7.0$ km of line during Stage 2. According to Figure 2, containment is now possible for scenarios $(1,1)$ and $(1,2)$ because these scenarios have fire perimeters of $6.2$ km and $6.9$ km respectively. To track containment we introduce the binary decision variable $f_{\langle k_t \rangle}$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Stage 2** | $f_{(1,1)}$ | | $f_{(1,2)}$ | | $f_{(2,1)}$ | | $f_{(2,2)}$ | |
| | *1* | | *1* | | *0* | | *0* | |
| **Stage 3** | $f_{(1,1,1)}$ | $f_{(1,1,2)}$ | $f_{(1,2,1)}$ | $f_{(1,2,2)}$ | $f_{(2,1,1)}$ | $f_{(2,1,2)}$ | $f_{(2,2,1)}$ | $f_{(2,2,2)}$ |
| | *0* | *0* | *0* | *0* | *1* | *1* | *1* | *1* |

Figure 3: One possible containment scenario for Figure 2.

Figure 3 shows the values this decision variable takes when $7.0$ km of line can be built and is called a *containment scenario*. Remember that we are performing this calculation for Stage 3 so $f_{\langle k_2 \rangle}$ and $f_{\langle k_3 \rangle}$ are both relevant. We will formally define $f_{\langle k_t \rangle}$ in Section 3, we emphasize that it indicates the stage during which containment is *declared*. If $f_{\langle k_t \rangle} = 0$, then the fire may be burning uncontained *or* have been previously declared contained. This subtlety is important when we compute expected burned area for Figure 3's containment scenario. Before we do so however, notice this definition of $f_{\langle k_t \rangle}$ supports recourse in the model. If a fire scenario is declared to be contained during Stage 2, then we assume it is contained throughout Stages 3 and 4.

The expression in (2.3) gives expected burned area for a single stage, but we now have two interdependent stages so the expectation operators must be nested as follows:

$$E[Area] = E[Area_2 + E[Area_3]]. \qquad (2.4)$$

The reader may ask why Equation 2.4 does not take the form $E[Area_2] + E[Area_3]$? Formulating (2.4) as shown, the expected burned area in Stage 2 can depend on the expected burned area in Stage 3, which captures recourse in the fire manager's decision making. A *recourse decision* is a decision made after some uncertainty in the problem has been accounted for. We account for uncertainty each stage by introducing more branches and conditional probabilities into the scenario tree. Stage 2 and Stage 3 are not independent as $E[Area_2]+E[Area_3]$ would suggest. Burned area in Stage 2, may well depend on the expected burned area in Stage 3, so the overall expected burned area must be nested as in (2.4).

These computations can be convoluted so we move through this one in detail. In terms of the scenario tree in Figure 2,

$$E[Area] = \sum_{(k_1,k_2)} [\, p_{(k_1,k_2)} A_{(k_1,k_2)} f_{(k_1,k_2)} + \sum_{(k_1,k_2,k_3)} \left( p_{(k_1,k_2,k_3)} A_{(k_1,k_2,k_3)} f_{(k_1,k_2,k_3)} \right) ]. \quad (2.5)$$

The expression in (2.5) is a two stage version of (2.3). It goes further than (2.3) because it incorporates the nesting of expectation operators in (2.4) and the decision variable $f_{\langle k_t \rangle}$. Area will be added to the total if and only if $f_{\langle k_t \rangle} = 1$. We have dropped the $\langle \cdot \rangle$ notation so as not to double count in the sum. Expanding the summation in (2.5) we see

$$E[Area] = p_{(1,1)}A_{(1,1)} f_{(1,1)} + p_{(1,1,1)}A_{(1,1,1)} f_{(1,1,1)} + p_{(1,1,2)}A_{(1,1,2)} f_{(1,1,2)}$$

$$+ p_{(1,2)}A_{(1,2)} f_{(1,2)} + p_{(1,2,1)}A_{(1,2,1)} f_{(1,2,1)} + p_{(1,2,2)}A_{(1,2,2)} f_{(1,2,2)}$$

$$+ p_{(2,1)}A_{(2,1)} f_{(2,1)} + p_{(2,1,1)}A_{(2,1,1)} f_{(2,1,1)} + p_{(2,1,2)}A_{(2,1,2)} f_{(2,1,2)}$$

$$+ p_{(2,2)}A_{(2,2)} f_{(2,2)} + p_{(2,2,1)}A_{(2,2,1)} f_{(2,2,1)} + p_{(2,2,2)}A_{(2,2,2)} f_{(2,2,2)}.$$

According to Figure 3 half of these terms are zero so

$$E[Area] = p_{(1,1)}A_{(1,1)} + p_{(1,2)}A_{(1,2)} + p_{(2,1,1)}A_{(2,1,1)} + p_{(2,1,2)}A_{(2,1,2)}$$

$$+ p_{(2,2,1)}A_{(2,2,1)} + p_{(2,2,2)}A_{(2,2,2)},$$

which we can calculate with corresponding values from Figure 2 to give

$$E[Area] = \frac{1}{4}(147ha) + \frac{1}{4}(203ha) + \frac{1}{8}(492ha) + \frac{1}{8}(787ha)$$

$$+ \frac{1}{8}(757ha) + \frac{1}{8}(999ha) = 466.875ha.$$

If the fire manager elects to deploy resources and contain the fire under scenarios $(1,1)$ and $(1,2)$, then the expected burned area will be about 467 ha, which is 91 ha less than the expected Stage 3 burn area without any suppression activity (558 ha). The Fire-S model is a four stage model so we will be adding an additional stage to these computations when we discuss the full version of the Fire-S stochastic program in Section 3.

As mentioned, containment involves deployment of resources. Not only do these resources cost money, but they also come from a scarce set and have realistic constraints such as travel time and line production rates. We will continue to build this example by discussing the resource set shown in Table 1.

Table 1

| *r* | Description | FC ($) | VC ($/hr) | Production (chains/hr) |
|---|---|---|---|---|
| 1 | Dozer | 11,600 | 900 | 30 |
| 2 | Type I Hand Crew | 2,050 | 250 | 9 |
| 3 | Type II Hand Crew A | 1,000 | 100 | 6 |
| 4 | Type II Hand Crew B | 1,200 | 100 | 7 |
| 5 | Engine 1 | 8,200 | 500 | 16 |
| 6 | Engine 2 | 7,600 | 550 | 16 |
| 7 | Engine 3 | 4,500 | 300 | 12 |

Table 1: Example resource set.

To build at least 7.0 kilometers of line during Stage 2, the fire manager has various alternatives. Three of these alternatives are shown with the costs they incur in Table 2. Remember that Stage 2 is twelve hours is long so we have incorporated the reasonable assumption of an eight-hour line producing period in these calculations and the full stochastic program. Variable costs will be incurred for each hour the resource is active.

Table 2

| Alternative | Resource Package | Stage 2 Production (km) | Cost ($) |
|---|---|---|---|
| A | 1, 2, and 3 | 7.2 | 29,650 |
| B | 5, 6, and 7 | 7.1 | 36,500 |
| C | 2, 4, 5, and 7 | 7.0 | 29,750 |

Table 2: Alternative resource packages to build at least 7.0 km of line.

The fire manager may deem Alternative A (a dozer and two hand crews) in Table 2 optimal because it is the cheapest way to achieve the line-building requirements. Alternative B deploys three engines, which is costly and probably unnecessary. Alternative C is also attractive because the deployment package calls for two hand crews and two engines, which may be more practical for a wild land urban interface (WUI), at only a slightly higher cost.

While cost minimization is a common objective for fire suppression, we choose to incorporate cost as a constraint, which affords us the natural interpretation of expenditure under a fixed budget. Say the fire manager has a budget goal of \$44,000 for this fire. Let $x_{r,\langle k_t \rangle}$ be a binary decision variable (like $f_{\langle k_t \rangle}$) that tracks resource deployment. If $x_{r,\langle k_t \rangle} = 1$, then resource $r$ is in transit to or active on the fire during Stage $t$ under scenario $\langle k_t \rangle$. If $x_{r,\langle k_t \rangle} = 0$, then it is not. Each resource $r$ has a set of associated parameters: the variable cost of actively building line during Stage $t$ ($VC_{r,t}$), the fixed cost of deployment ($FC_r$), and the line production rate under scenario $\langle k_t \rangle$ ($L_{r,\langle k_t \rangle}$). All three parameters can be read from Table 1 for this example. But, when does a resource actually start to build line? We assume $x_{r,\langle k_t \rangle} = 1$ means the resource is ordered during Stage $t$ and will start producing line at the start of Stage $t+1$. Any preparation and travel time is rolled into the resource ordering stage. For example, if the fire manager wants

Engine 1 ($r = 5$) to contribute fire line during the Stage 2 scenario $\langle k_2 \rangle = (1,1)$, the order will be placed during Stage 1 by specifying $x_{5,(1)} = 1$, perhaps just after the initial smoke report. The fire manager must budget for the fixed cost of Engine 1, $FC_5 = \$8,200$, and the variable cost of operation during the twelve hours of Stage 2,

$$VC_{5,2} = 500 \frac{\$}{hr} \cdot 12 hrs = \$6,000,$$

when he or she orders it. Thus, alternatives A, B, and C must satisfy the following budget constraint

$$\sum_{r=1}^{R} \left( x_{r,(1)} \left[ VC_{r,2} + FC_r \right] \right) \leq \$44,000,$$

where $R$ is the size of the resource set, in our case $R = 7$. According to Table 2, all three alternatives satisfy the budget constraint. Of the three, Alternative A is the most cost effective choice to achieve containment under scenarios $(1,1)$ and $(1,2)$. We can use Alternative A to demonstrate how our containment constraints function in the stochastic program. We follow the classic approach, which means in order to contain a fire, line production must exceed fire perimeter. For scenario $(1,1)$ the containment constraint is

$$\sum_{r=1} \left( x_{r,(1)} L_{r,(1,1)} \right) \geq f_{(1,1)} P_{(1,1)}.$$

We permit containment ($f_{(1,1)} = 1$) if and only if line production exceeds fire perimeter. Choosing Alternative A permits containment for scenarios $(1,1)$ and $(1,2)$ because we see the total Stage 2 line production from Table 2 is $7.2$ km, which exceeds fire perimeter $P_{(1,1)} = 6.2$ km. We do not permit containment for scenario $(2,1)$ nor $(2,2)$ because in each case fire perimeter exceeds line production.

Even though $f_{(2,1)} = f_{(2,2)} = 0$ when Alternative A is deployed, let us illustrate a multistage containment by assuming the fire manager will elect Alternative A for *both* Stage 1 branches: (1) and (2). First, let us examine whether or not Alternative A satisfies the budget. Table 2 shows that Alternative A costs $29,650. With a budget of $44,000 that leaves $14,350 to use for the Stage 3 suppression effort. This may not seem like enough, but remember the fixed costs have already been paid, so only variable costs are incurred for Stage 3. During the twelve-hour Stage 3 the dozer and hand crews incur $15,000 in variable costs. This *does* exceed the budget so Alternative A cannot be used to achieve a multistage containment. Fortunately, we can turn to Alternative C, satisfy the budget, and still achieve the Stage 2 containment we desire. But, does Alternative C permit any Stage 3 containment? Variable costs for Alternative C are low enough that $43,550 covers the Stage 2 and 3 suppression costs. Alternative C is permitted under the budget constraint level of $44,000. Suppose the engines and crews in Alternative C perform strategic attack and construct their 7.0 km of line along the fire's flank during Stage 2 for scenario (2,1). Since $P_{(2,1)} = 8.1$ km, this is not enough line for containment, but if they continue to produce line (at the same rates), they will construct 14.0 km by the end of Stage 3. Since $P_{(2,1,1)} = 10.0$ km, this *is* enough line to contain the fire during Stage 3 under this scenario. Therefore, we say Alternative C can perform a *multistage containment* for scenario (2,1,1).

If we consult Figure 2, we see 14.0 km is not enough fire line to contain the fires in scenarios (2,1,2), (2,2,1), or (2,2,2). This means the new containment scenario differs from that of Figure 3 because

$$f_{(2,1,2)} = f_{(2,2,1)} = f_{(2,2,2)} = 0$$

and the fire will continue to grow into Stage 4 for these scenarios. The fire manager has exhausted the budget so the remaining scenarios represent *escaped fires*. Escaped fires may not necessarily be synonymous with extreme fires, but these fire scenarios extend beyond the time frame that the fire manager has decided is reasonable for fire weather and behavior predictions (Figure 1). How do we account for the six escaped fires in Figure 2? In terms of the Fire-S model, the fire manager was not able to contain the fire under these behavior scenarios because fire spread was too great; so he or she expects each one to transition to a large fire. In order to get an expected burned area like (2.3) we must provide an estimate for a large fire area to account for escape scenarios. Suppose the fire manager consults a Fire Family Plus database and comes up with a large fire area estimate based on historical records of $\hat{A}_{LF} = 7{,}814$ ha. If we track escape scenarios using the binary decision variable $esc_{\langle k_4 \rangle}$ (equals 1 when a fire escapes), then the expected burned area is

$$E[Area] = \sum_{k_1}[\sum_{k_2}(p_{(k_1,k_2)}A_{(k_1,k_2)}f_{(k_1,k_2)} + \sum_{k_3}[p_{(k_1,k_2,k_3)}A_{(k_1,k_2,k_3)}f_{(k_1,k_2,k_3)}$$
$$+ \sum_{k_4}\left(p_{(k_1,k_2,k_3,k_4)}\hat{A}_{LF}esc_{(k_1,k_2,k_3,k_4)}\right)])]. \qquad (2.6)$$

The expression in (2.6) is a four stage version of (2.5) that also accounts for escaped fire scenarios. For the example scenario tree in Figure 2 we denote the six escape scenarios with

$$esc_{(2,1,2,1)} = esc_{(2,1,2,2)} = esc_{(2,2,1,1)} = esc_{(2,2,1,2)} = esc_{(2,2,2,1)} = esc_{(2,2,2,1)} = 1$$

so equation (2.6) gives

$$E[Area] = 919.375ha.$$

In the language of math programming, we have come up with a feasible solution to the problem. Given a budget of $\$44,000$, the fire manager can deploy the resources in Alternative C during Stage 1 and suppress the fire with an expected burned area of about $920$ ha. Do not forget that this entire development was based on a probabilistic scenario tree so in reality, any number of acres could be burned; the value of $920$ ha is our best estimation based on our assumptions about weather and simulated fire behavior.

This example serves as motivation for a stochastic programming approach to the fire suppression resource allocation problem. We have explored a feasible solution, but is it optimal? If we formulate a stochastic program using the basic building blocks presented in this example, then we can search for an optimal solution. The features that make the probabilistic approach attractive are already apparent. The scenario tree in particular lends itself to those two fire management concepts about uncertainty and expert knowledge.

# 3 The Fire-S Stochastic Program

In this section we present the full version of the stochastic program. We encourage the reader to review Section 2 often because a firm understanding of its example will help clarify and motivate the full stochastic program.

We seek to minimize expected burned area and account for fire behavior scenarios that escape containment during the scope of the model using an estimate for large fire area. We adopt classic containment constraints and track overall budget within the constraints as well. Initial dispatch for fixed cost payment and logical dispatch rules are also enforced in the constraints. The problem is discretized using a scenario tree, which is populated with fire spreads and conditional probabilities during the parameterization process.

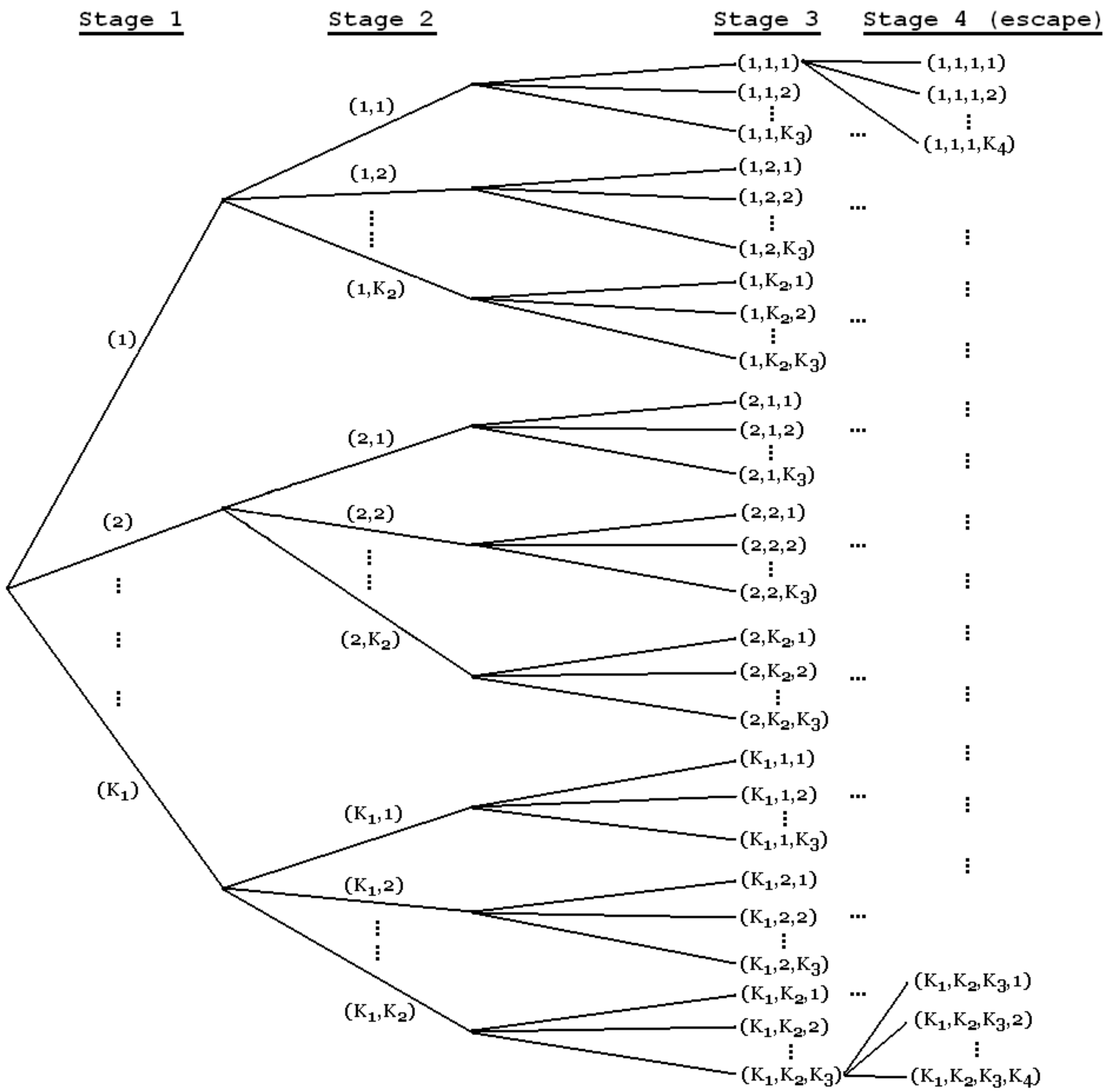



Figure 4: A general, uniform scenario tree.

Figure 4 shows a general, uniform scenario tree. The Fire-S stochastic program has the following mathematical representation:

**Minimize**

$$E[Area] = \sum_{k_1}[\sum_{k_2}(p_{(k_1,k_2)}A_{(k_1,k_2)}f_{(k_1,k_2)} + \sum_{k_3}[p_{(k_1,k_2,k_3)}A_{(k_1,k_2,k_3)}f_{(k_1,k_2,k_3)}$$

$$+\sum_{k_4}\left(p_{(k_1,k_2,k_3,k_4)}\left[A_{(k_1,k_2,k_3,k_4)}f_{(k_1,k_2,k_3,k_4)} + \hat{A}_{LF}esc_{(k_1,k_2,k_3,k_4)}\right]\right)])] \qquad (3.1)$$

**Subject to:**

$$\forall\langle k_t\rangle : \sum_{r=1}^{R}\left(x_{r,\langle k_t\rangle}VC_{r,t+1}\right) + \sum_{r=1}^{R}\left(y^{+}_{r,\langle k_t\rangle}FC_r\right) \le b_{\langle k_t\rangle} \quad t = 1,2,3 \qquad (3.2)$$

$$\forall\langle k_3\rangle : b_{\langle k_1\rangle} + b_{\langle k_2\rangle} + b_{\langle k_3\rangle} \le TC \qquad (3.3)$$

$$\forall\langle k_1\rangle : l_{\langle k_1\rangle} = 0 \qquad (3.4)$$

$$\forall\langle k_t\rangle : \begin{cases} l_{\langle k_{t-1}\rangle} + \sum_{r=1}^{R}\left(x_{r,\langle k_{t-1}\rangle}L_{r,\langle k_t\rangle}\right) = l_{\langle k_t\rangle} \\ l_{\langle k_t\rangle} \ge f_{\langle k_t\rangle}P_{\langle k_t\rangle} \end{cases} \quad t = 2,3,4 \qquad (3.5)$$

$$\forall\langle k_t\rangle : Rf_{\langle k_t\rangle} + \sum_{r=1}^{R}x_{r,\langle k_t\rangle} \le R \quad t = 2,3 \qquad (3.6)$$

$$\forall r,\langle k_t\rangle : x_{r,\langle k_t\rangle} + f_{\langle k_t\rangle} - x_{r,\langle k_{t-1}\rangle} \ge 0 \quad t = 2,3 \qquad (3.7)$$

$$\forall r,\langle k_1\rangle : y^{+}_{r,\langle k_1\rangle} = x_{r,\langle k_1\rangle} \qquad (3.8)$$

$$\forall r,\langle k_t\rangle : \begin{cases} y^{+}_{r,\langle k_t\rangle} - y^{-}_{r,\langle k_t\rangle} = x_{r,\langle k_t\rangle} - y^{+}_{r,\langle k_{t-1}\rangle} \\ y^{+}_{r,\langle k_t\rangle} + y^{-}_{r,\langle k_t\rangle} \le 1 \end{cases} \quad t = 2,3 \qquad (3.9)$$

$$\forall\langle k_4\rangle : f_{\langle k_2\rangle} + f_{\langle k_3\rangle} + f_{\langle k_4\rangle} + esc_{\langle k_4\rangle} = 1 \qquad (3.10)$$

The Fire-S stochastic program has parameters:

- $p_{\langle k_t\rangle}$ ... conditional probability of scenario $\langle k_t\rangle$.

- $P_{\langle k_t \rangle}$ ... fire perimeter under scenario $\langle k_t \rangle$, cumulative through Stage $t$.
- $A_{\langle k_t \rangle}$ ... area burned under scenario $\langle k_t \rangle$, cumulative through Stage $t$.
- $\hat{A}_{LF}$ ... estimated area of a large fire.
- $R$ ... integer size of resource set.
- $L_{r,\langle k_t \rangle}$ ... line production during Stage $t$ for resource $r$ under scenario $\langle k_t \rangle$.
- $VC_{r,t}$ ... variable cost of using resource $r$ during Stage $t$.
- $FC_r$ ... fixed cost of dispatch for resource $r$.
- $TC$ ... total budget for fire.

and decision variables:

- $x_{r,\langle k_t \rangle}$ ... binary, ``is resource $r$ active on the fire (includes initial dispatch, transit, and fire line construction) during Stage $t$ under scenario $\langle k_t \rangle$?"
- $f_{\langle k_t \rangle}$ ... binary, ``is containment *first* declared under scenario $\langle k_t \rangle$?"
- $y_{r,\langle k_t \rangle}$ ... binary, tracks initial deployment stage for resource $r$ to determine fixed cost payment.
- $esc_{\langle k_4 \rangle}$ ... binary, ``does fire escape under scenario $\langle k_4 \rangle$?" Indicates that the fire was not contained in the scope of this model under scenario $\langle k_4 \rangle$.
- $l_{\langle k_t \rangle}$ ... book-keeping variable that tracks total line production under scenario $\langle k_t \rangle$.
- $b_{\langle k_t \rangle}$ ... ``how much of the budget is spent under scenario $\langle k_t \rangle$?"

This is a mixed integer linear program with a size that depends upon the underlying scenario tree. Endless variations of the general scenario tree in Figure 4 are possible, so the Fire-S program has endless variations as well. The binary containment variable is slightly tricky so we offer the following clarification:

- $f_{\langle k_t \rangle} = 1$ if and only if the fire is declared contained during stage $t$ under scenario $\langle k_t \rangle$ or
- $f_{\langle k_t \rangle} = 0$ if and only if the fire is considered *uncontained* (or has been previously contained) under scenario $\langle k_t \rangle$.

The objective function in Equation (3.1) gives the expected burned area given all the containment decisions. These calculations are discussed extensively in Section 2 and Equations (2.6) and (3.1) are similar. The only difference is that (3.1) introduces the possibility of fourth stage containment with $f_{\langle k_4 \rangle}$ whereas (2.6) assigns the expected large fire area to any active fourth stage fire scenario. Note well that the areas are cumulative, but given the tricky definition of $f_{\langle k_t \rangle}$ this does not lead to double counting. We assume no line is built during Stage 1 so the fire must always grow into Stage 2, which accounts for the abbreviated Stage 1 summation.

The constraints in (3.2) ensure variable and fixed costs for each scenario ($\forall \langle k_t \rangle$) at each dispatch stage ($t = 1, 2,$ and $3$) are within budget. Notice that a resource deployed during Stage $t$ incurs the variable cost associated with Stage $t+1$ because we assume the resource starts building line at the start of the stage immediately following its dispatch. Fixed cost is incurred one time, if the resource is dispatched at all. While (3.2) is a concise formulation, do not forget that it represents a set of many constraints, the number of which depends upon the size of the underlying scenario tree. This is true for each set of constraints (3.2) through (3.10).

Budget decision variables across dispatch stages are constrained to be less than the total budget for each fire scenario in (3.3). The efficacy of our $\langle\cdot\rangle$ notation is apparent in (3.3) when we indicate $\forall\langle k_3\rangle$. Once some $\langle k_3\rangle$ is chosen, $\langle k_1\rangle$ and $\langle k_2\rangle$ are automatically the proper parent scenarios. For example, suppose $\langle k_3\rangle = (3,4,2)$, which indicates $\langle k_1\rangle = (3)$ and $\langle k_2\rangle = (3,4)$. The corresponding constraint in (3.3) is

$$b_{(3)} + b_{(3,4)} + b_{(3,4,2)} \leq TC,$$

which indeed captures the budget decisions across the three dispatch stages for a given scenario branch. It forces them to be less than the total allotment for the fire.

Constraints in (3.4) show the assumption that no line is built during Stage 1. Stage 1 is reserved for size-up and the initial call for resources.

The constraint pairs shown in (3.5) enforce classic containment. For each stage where containment is possible ($t = 2,3,$ and $4$) and each scenario $\langle k_t\rangle$, we permit containment if and only if cumulative fire line production exceeds fire perimeter. Again, observe that an active resource in Stage $t-1$ ($x_{r,\langle k_{t-1}\rangle} = 1$) is assumed to produce line during the following Stage $t$ ($L_{r,\langle k_t\rangle}$). The one stage lag allows for transit and prep time. Notice the book-keeping variable $l_{\langle k_t\rangle}$ facilitates the computation of line accumulation. The cumulative nature of (3.5) allows for multistage containment efforts, as discussed in Section 2.

We refer to the set of constraints expressed in (3.6) and (3.7) as *logical dispatch*. If the fire is declared contained under scenario $\langle k_t\rangle$ ($f_{\langle k_t\rangle} = 1$), then no further resources are dispatched and those resources already there are sent home according to (3.6). If a resource is sent to a fire under scenario $\langle k_t \rangle$ and the fire remains uncontained, the appropriate constraint from (3.7) requires the resource to remain on the fire. These two sets of constraints may be subject to tweaking based on a region's specific dispatch routines. If, for example, the model's scope is much longer than two days, it may be logical to permit resources to leave an uncontained fire and go to another fire, which violates (3.7). An example exception to (3.6) would be if a fire manager wanted to account for mop-up operations in planning. In which case, logical dispatch may involve leaving a crew on a fire after it is declared contained. The Fire-S stochastic program must be calibrated for the problem's scope and the region being modeled, which may include slight changes in the constraints.

The set of constraints in (3.8) and (3.9) govern fixed cost payment. In (3.8), the tracking variable $y^+_{r,\langle k_1 \rangle}$ is initialized. We have utilized a linear programming trick from [3] to detect a change from

$$y^+_{r,\langle k_{t-1} \rangle} = 0 \quad to \quad x_{r,\langle k_t \rangle} = 1,$$

which indicates an initial dispatch of resource $r$ in the variable $y^+_{r,\langle k_1 \rangle}$. This, in turn, triggers fixed cost payment in the associated cost constraint in (3.2).

For each fire scenario, the associated constraint in (3.10) requires either the fire be contained at some node of the scenario tree or escape the scope of the model. Again, we see the convenience of the $\langle \cdot \rangle$ notation in selecting branches that include appropriate parents because these constraints could equivalently be written as

$$\forall (k_1, k_2, k_3, k_4): \quad f_{(k_1,k_2)} + f_{(k_1,k_2,k_3)} + f_{(k_1,k_2,k_3,k_4)} + esc_{(k_1,k_2,k_3,k_4)} = 1.$$

As the Fire-S stochastic program is a potentially large, mixed integer program (MIP), we solve it using ILOG CPLEX, a high powered linear program solver produced by IBM. We detail the solution in Section 5.1. As the extensive parameter list asserts, there is much background work to be done before the stochastic program can be passed to the solver. Section 4 guides the reader through the parameterization process.

# 4 Parameterization

The Fire-S stochastic program in Section 3 reflects the richness and complexity of the mathematics of decision-making, but the parameterization process gives the program context in terms of fire behavior science and suppression resources. We simulate fire behavior using Farsite. Fire simulation is fundamentally based on the Fire Triangle: *fuels*, *topography*, and *weather*. In our study of the single-fire resource allocation for suppression problem, *topography* is fixed because we elect a single ignition location in the Black Hills National Forest (BHNF) in southwestern South Dakota. *Fuels* can be considered mostly fixed because (a) the fire behavior fuel models are drawn from the LANDFIRE database for the study area and (b) before we parameterize the model we draw an ignition date from historical records, which fixes fuel moistures at their historical levels. Weather may cause fuel moistures to vary because Farsite is capable of computing dynamic fuel moistures during a simulation. Therefore, *weather* variables produce *all* the variability in our probabilistic study. The parameterization process involves a cluster analysis of historical weather data to produce representative weather streams with associated conditional probabilities $p_{\langle k_t \rangle}$. Each *representative weather scenario* seeds Farsite to create a *representative fire behavior scenario*, which includes perimeter $P_{\langle k_t \rangle}$ and area $A_{\langle k_t \rangle}$ parameters as output. We explain the cluster analysis in Section 4.1 and tackle fire simulation in Section 4.2. The remaining parameters are associated with the suppression resource set and are discussed in Section 4.3. Finally, we discuss escaped fires in Section 4.4.

## 4.1 Cluster Analysis

Generating feasible weather scenarios from scratch is an enormous, multi-variate correlation problem. We sidestep the problem by studying historical weather records. So when a complicated correlation question arises---such as how the passage of a cold front caused some irregular change in temperature, relative humidity, wind speed, and wind direction---we can default to the fact that the weather pattern actually occurred and, save some sort of data logging error, the weather variables are realistically correlated. While this is a strong advantage, incorporating historical weather introduces some challenges. The Western Regional Climate Center (WRCC) offers hourly data streams from Remote Automated Weather Stations (RAWS), dating back to 1993 for some stations, in and nearby the BHNF. In theory, we could run fire simulations for all possible combinations of this historical weather and parameterize the Fire-S stochastic program with the output using uniform probabilities, but such an approach would be unwieldy and time-consuming. Instead, we use data clustering techniques to pre-process the weather records and create weather classes from which to pull a few, representative weather scenarios. The following discussion is specific to BHNF, but the basic steps can be modified to apply to other locations as well.

Recall from Section 2 that at the outset of the Fire-S model, the fire manager has no deterministic knowledge of weather or fire behavior. Once an ignition is reported, important spatial and temporal information becomes immediately available. The fire manager knows *where*, perhaps very roughly, the smoke is coming from and also knows *when* the fire started. Not only are current weather conditions available, but forecast information is quickly obtainable as well. To parameterize the conditional probabilities $p_{\langle k_t \rangle}$ in the Fire-S stochastic program, we seek to compare historical fire weather to the forecast. We want to take into account types of weather that are historically likely *and* types that are unlikely, but could lead to extreme fire behavior based on our simulations.

The first step we take from the large BHNF weather data set towards a small subset of representative scenarios based on the forecast is to fix our spatial element by electing a specific RAWS to use. For our analysis, Nemo was chosen as the best representative for local fire weather based on proximity and similarity in elevation to our ignition location in the Deerfield management zone [2]. Next, we apply a three month filter to the historical records based on the ignition month. The filter screens out all data except those records that match the ignition month, one month prior, or one month following. For example, this filter avoids the issue of a July ignition somehow pulling a February snow storm from the historical data. This technique is BHNF-specific; the three-month filter works well for the BHNF, but may not work well in a region where adjacent months have very different weather characteristics, if, for example, a monsoon month interrupts the fire season. These two steps: a spatial fix and a month filter, greatly reduce the size of the weather set and we call this starting set of weather records $W$.

In general, multi-variate clustering typically involves some sort of metric that serves as the standard for comparison among vectors. For us, the question is: ``how far is a given weather record from the forecast?" There are many possible answers to this question because there are many weather-related field observables. We elect to work with vectors that consist of four weather variables: temperature ($temp$), relative humidity ($rh$), wind speed ($wspd$), and the cosine of wind direction ($\cos wdir$). This selection is, of course, subjective and based on our experience with BHNF weather data in this specific context; variations are numerous and many may be feasible as well. Let

$$\mathbf{Wx_i} = (temp_i, rh_i, wspd_i, \cos wdir_i) \in W$$

be a weather vector in set $W$ and let

$$\mathbf{Fx} = (temp_0, rh_0, wspd_0, \cos wdir_0)$$

be the forecast vector. To start, the forecast vector also comes from $W$. In Section 5.6 we discuss some forecast considerations for the Fire-S model. Implicit in this discussion is that these vectors come from a specific time of day because we work with hourly weather data. In terms of the scope shown in Figure 1, there are morning and afternoon forecasts, which we assume correspond to weather vectors at 1000 hours and 1400 hours respectively. The fire manager may adjust these forecast points based on the burn period and scope of the model. Our metric to compare $\mathbf{Wx_i}$ and $\mathbf{Fx}$ must account for all four variables, their correlations, and their relative numeric sizes. As such, we compute the $4 \times 4$ covariance matrix $S$ (and its inverse $S^{-1}$) of temperature, relative humidity, wind speed, and wind direction for all the data in set $W$. As a metric, we elect a generalized, Euclidean distance measure

$$d(\mathbf{Wx_i}, \mathbf{Fx}) = \pm\sqrt{(\mathbf{Wx_i} - \mathbf{Fx})^T S^{-1} (\mathbf{Wx_i} - \mathbf{Fx})}. \tag{4.1}$$

We call the scalar distances in Equation (4.1) *forecast errors*. By using the covariance matrix in this way, we resolve correlation issues such as the strong negative correlation between temperature and relative humidity. We also resolve relative numeric size issues such as the differences in weather variable units. Equation (4.1) indicates the multi-valued nature of the square root function in the ``$\pm$." While it is customary to pick the positive square root for a distance measure, if we do this, there will be no distinction between ``better" and ``worse" fire weather. For instance, a dry, windy $\mathbf{Wx_i}$ record could map to the same forecast error numeric value as a wet, calm one. This happens quite readily in fact. Suppose

$$\mathbf{Fx} = (temp_0, rh_0, wspd_0, \cos wdir_0) = (76^\circ F, 25\%, 3mph, \cos 225^\circ), \quad (4.2)$$

$$\mathbf{Wx_{148}} = (59^\circ F, 42\%, 0mph, \cos 225^\circ), \quad (4.3)$$

$$\mathbf{Wx_{643}} = (93^\circ F, 8\%, 6mph, \cos 225^\circ), \quad (4.4)$$

and for the sake of simplicity assume no correlation and uniform covariance so that $S$ and $S^{-1}$ are $4 \times 4$ identity matrices. Then, $d(\mathbf{Wx_{148}}, \mathbf{Fx}) = 24.2281$ and $d(\mathbf{Wx_{643}}, \mathbf{Fx}) = 24.2281$. This is clearly a problem because when we simulate fire behavior colder, wetter $\mathbf{Wx_{148}}$ will likely result in less severe fire behavior than drier, windier $\mathbf{Wx_{643}}$ and we do not want them to fall into the same cluster. To circumvent this issue we introduce a decision rule in order to establish clear separation between $\mathbf{Wx_{148}}$ and $\mathbf{Wx_{643}}$; in general, we assign the negative square root to cooler, calmer weather records and the positive square root to warmer, windier records. The rule manifests as a comparison of the relative error (not to be confused with forecast error) in the relative humidities and wind speeds of $\mathbf{Wx_i}$ and $\mathbf{Fx}$; compute

$$error(RH_i) = \frac{RH_0 - RH_i}{RH_0} \tag{4.5}$$

and

$$error(wspd_i) = \frac{wspd_i - wspd_0}{wspd_0}, \tag{4.6}$$

then make a decision according to the sign of the sum

$$\begin{array}{ll} error(RH_i) + error(wspd_i) \geq 0 & d(\mathbf{Wx_i}, \mathbf{Fx}) \geq 0 \\ error(RH_i) + error(wspd_i) < 0 & d(\mathbf{Wx_i}, \mathbf{Fx}) < 0 \end{array}. \tag{4.7}$$

Let us examine how this decision rule applies to the example vectors from (4.2), (4.3), and (4.4). From (4.5) we have $error(RH_{148}) = -0.68$ and $error(RH_{643}) = 0.68$. From (4.6) we have $error(wspd_{148}) = -1.0$ and $error(wspd_{643}) = 1.0$. According to the rule in (4.7) we assign the negative square root to $\mathbf{Wx_{648}}$ so that $d(\mathbf{Wx_{148}}, \mathbf{Fx}) = -22.2281$ and $d(\mathbf{Wx_{653}}, \mathbf{Fx}) = 22.2281$. This introduces a logical spacing in forecast errors, which helps avoid automatic grouping of weather records that may in fact be dissimilar. One can easily imagine loopholes and canonical cases for the decision rule in 4.7, but it serves as a rough approximation and oftentimes when a questionable decision is made, we are rescued by the next phase of clustering, which we will now describe.

The mathematical machinery of the metric (4.1) and associated decision rule (4.7) combine to create a *logical ordering* of weather data. Given the set of weather vectors $W$ and a forecast vector $\mathbf{Fx}$ we can make the aforementioned assumptions and write a roughly ordered list of forecast errors from least severe to most severe in terms of expected fire behavior. An example of this ordering is shown in Table 3.

Table 3

| $\mathbf{Wx_1}$ | $\mathbf{Wx_2}$ | ... | $\mathbf{Wx_i}$ | ... | $\mathbf{Wx_M}$ |
|---|---|---|---|---|---|
| June 23, 1995 | August 8, 2009 | ... | July 10, 2003 | ... | July 25, 2002 |

Table 3: Example Stage 1 weather record ordering.

Smaller indices in Table 3 indicate cooler, wetter records where we expect more mild fire behavior. Larger indices are indicative of more severe fire behavior because the associated weather records are hotter and drier. Some weather record on the list will be most similar to $\mathbf{Fx}$, i.e. have the forecast error that is closest to 0. While each vector $\mathbf{Wx_i}$ represents specific forecast values, each one corresponds to a date as shown the second row of Table 3. Clearly, to order weather data like this, the dates are taken out of their customary time ordering.

We propose a weather classification scheme to produce a coherent scenario tree like the example from Section 2, which is shown in Figure 2. Our work will produce a scenario tree, which is shown, in general, in Figure 4. Instead of having a rough idea of pre and post-frontal weather patterns, the fire manager now has a vast cache of RAWS data to create more detailed fire growth simulations. Even with the spatial (fixed ignition location) and seasonal (month filter) simplifications used to create $W$, there still may be a large number of weather records on this list. For the BHNF data $M : 1000$. Running Farsite $M$ times is certainly an option for the fire manager, but not a very practical and interpretable one. Instead, we group similar weather records together in a hierarchical clustering and select a *representative weather scenario* from each group. The fire manager sets $K_1$ to be the number of branches he or she wants from the initial node. A hierarchical clustering algorithm starts with each record in its own group ($M$ groups) and begins by pairing the two records that have the most similar forecast errors. On the list, it replaces these two forecast errors with their average and then looks for the next most similar pair of forecast errors. This will continue until there are $K_1$ groups. For a general description of this technique and some very informative diagrams consult [1].

Hierarchical clustering produces $K_1$ subsets of $W$. Let $W_{k_1} \subseteq W$ be the $k_1^{st}$ subset. For each $k_1 = 1,2,\ldots,K_1$ we select a representative scenario from $W_{k_1}$. This could be some sort of average weather record, a modal weather pattern, or some other type of representative. We pick our $k_1^{st}$ representative scenario to be the record with the median forecast error in $W_{k_1}$. But, what probabilities, conditional and unconditional, does this representative scenario carry? We create the Stage 1 probabilities $p_{\langle k_1 \rangle}$ and $\hat{p}_{\langle k_1 \rangle}$ used to parameterize the Fire-S stochastic program from the sizes of each subset. Computing probabilities becomes a record counting endeavor. Let $|\cdot|$ indicate the number of elements in a subset. The Stage 1 node has no parent so the conditional and unconditional probabilities are equal. We define

$$p_{\langle k_1 \rangle} = \hat{p}_{\langle k_1 \rangle} = \frac{|W_{k_1}|}{M}.$$

Notice that this definition is consistent with the two properties for the conditional probabilities we pointed out in Section 2. Property (2.1) from Section 2 holds because

$$\forall \langle k_1 \rangle \quad 0 < p_{\langle k_1 \rangle} = \frac{|W_{k_1}|}{M} \leq 1,$$

since $W_{k_1}$ has at least one (if it was never paired up) and at most $M$ elements (if $K_1 = 1$). Property (2.2) from Section 2 holds because

$$\sum_{k_1=1}^{K_1} p_{\langle k_1 \rangle} = \sum_{k_1=1}^{K_1} \frac{|W_{k_1}|}{M} = \frac{1}{M} \sum_{k_1=1}^{K_1} |W_{k_1}| = \frac{M}{M} = 1,$$

since during the clustering algorithm every record in $W$ is placed in some group. Thus, each representative scenario is assigned a probability based on the size of the weather record cluster that it represents. In terms of the scope of the Fire-S model shown in Figure 1, each Stage 1 representative specifies which morning weather record use in the Farsite simulation.

The next step in our clustering procedure is the key to conditionality in the Fire-S model. With the representative weather record in place for the morning (Stage 1), we must decide which weather record to simulate with in the afternoon (Stage 2). Fix the number of Stage 2 branches from each node $K_2(\langle k_1 \rangle)$ and apply the hierarchical clustering algorithm to each Stage 1 subset. The result will be a collection of new subsets $W_{\langle k_2 \rangle} \subseteq W$ with the added property: $W_{\langle k_2 \rangle} \subseteq W_{\langle k_1 \rangle}$. Just as before, we select the median as a representative scenario and assign it an unconditional probability based on group size:

$$\hat{p}_{\langle k_2 \rangle} = \frac{|W_{\langle k_2 \rangle}|}{|W_{\langle k_1 \rangle}|}.$$

To compute Stage 2 conditional probabilities we see

$$p_{\langle k_2 \rangle} = \hat{p}_{\langle k_2 \rangle} \cdot p_{\langle k_1 \rangle} = \frac{|W_{\langle k_2 \rangle}|}{|W_{\langle k_1 \rangle}|} \cdot \frac{|W_{\langle k_1 \rangle}|}{|W_{\langle k_2 \rangle}|} = \frac{|W_{\langle k_2 \rangle}|}{M}.$$

Conditionality is tracked using the sizes of the subsets. Once a weather record is collected in a Stage 1 cluster, we do not allow it to change clusters. Suppose we derived a representative scenario to be a member of $W$ that was outside the parent subset. This would violate the conditionality we are trying to establish. In general, the hierarchical clustering algorithm is a specific way to create a refinement of the set $W$.

Notice the number of Stage 2 branches is a function of the node, that is $K_2(\langle k_1 \rangle)$. In our example scenario tree shown in Figure 2 we have $\forall \langle k_1 \rangle : K_2(\langle k_1 \rangle) = 2$, but this need not be the case. In fact, the nature of our clustering procedure essentially guarantees that these branches will not be uniform due to its sensitivity to outliers. Our scenario trees will even differ from the general scenario tree in Figure 4 because Figure 4 shows a uniform tree.

Forecast error indicates how ``far'' a weather record $\mathbf{Wx_i}$ is from the forecast $\mathbf{Fx}$. Common weather patterns will create large groups of weather records with similar forecast errors. These will tend to cluster together. Extreme weather (on both ends of the spectrum) will stand out with large forecast errors and tend to cluster separately. Both types of weather are very important to the fire manager. Fire weather will most likely match one of the typical groups, but the fire manager needs to consider extreme fire weather, however unlikely, because it may cause safety concerns for personnel involved in suppression. Extreme weather scenarios will have lower associated conditional probabilities because their underlying groups will be smaller. Dividing a small subset could result in *singleton clusters*, which are clusters with a single element. In which case, we may not be able to create multiple subsets because the branch has become data poor. We explore these singleton cases in Section 5.8 because if they occur during a dispatch stage, they eliminate the possibility for recourse decisions, which is a crucial feature of our model.

Clustering continues in this way to fill out Stages 3 and 4. Although large, non-uniform scenario trees are complex and difficult to represent graphically, we attempt to do so in the exploded tree diagrams of Section 5.1. Each Stage $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ path $(k_1, k_2, k_3, k_4)$ specifies a weather stream. We form the stream by splicing four representative weather records together. The dates will quite possibly be discontinuous. Table 4 shows two examples of this splicing.

Table 4

| Path | Stage 1 | Stage 2 | Stage 3 | Stage 4 |
|---|---|---|---|---|
| (3,5,1,6) | August 12, 2010 | July 31, 1998 | July 3, 2001 | August 8, 2004 |
| $(1,4,1,1)^*$ | June 9, 2005 | June 10, 1994 | June 11, 1994 | June 11, 1994 |

Table 4: Possible representative weather scenarios.

The path marked with a * indicates a singleton case. Starting at Stage 2 the same record is being drawn as a representative because $W_{(1,4)}$ has a single member. This annuls the multistage set-up of the Fire-S stochastic program because the Stage 3 and 4 weather is known starting at Stage 2. Again, we refer the reader to Section 5.8 for a better discussion.

Before we move onto simulation, let us study some features of the cluster analysis procedure as applied to the BHNF data.

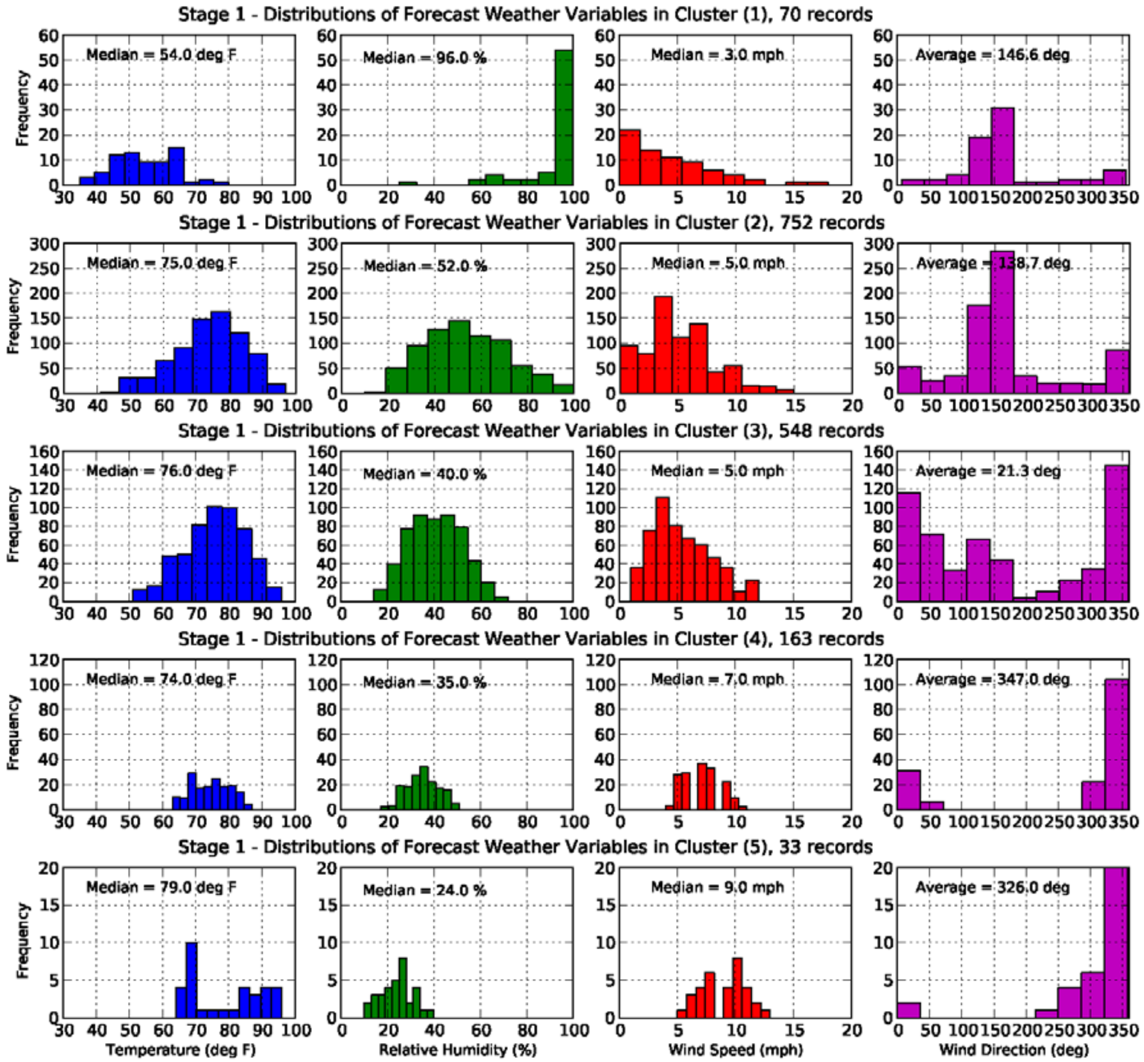



Figure 5: Stage 1 clusters for a July ignition using the Nemo RAWS. $|W| = 1639$ and $K_1 = 5$.

Figure 5 displays a wealth of information in histograms of the stage 1 clusters for a July ignition. After applying the month filter to the Nemo RAWS data we found $M = 1639$ weather records. Hierarchical clustering produced $K_1 = 5$ groups of varying sizes. The four classifying variables in $\mathbf{Wx_i}$ (temperature, relative humidity, wind speed, and the cosine of wind direction) make up the columns of Figure 5. The vertical axis in each plot is frequency, which indicates that each histogram bar gives a record count for the corresponding bin on the horizontal axis. For example, cluster $(k_1) = (1)$ has a lot of rainy days because of the 70 records in cluster $(1)$ approximately 55 show relative humidity near $100\%$. Figure 5 allows us to visually inspect the degree to which the clustering technique accomplished our goals. Recall that our initial ordering established a rough ranking system for fire weather. Greater indices were indicative of more severe fire weather in terms of simulated fire behavior. As we collapsed the records into the groupings shown in Figure 5, that relationship was maintained. If our classification scheme worked, then we expect to find mildly severe fire weather in cluster $(k_1) = (1)$ and increasingly worse fire weather until cluster $(k_1) = (5)$, which should represent extreme fire weather and behavior. Study the medians and distribution shapes for the temperature, relative humidity and wind speed columns in Figure 5 and we see this is indeed what occured. For example, consider wind speed. The medians increase from $3.0$ mph to $9.0$ mph monotonically from cluster $(k_1) = (1)$ to $(k_1) = (5)$. Furthermore, the distributions appear to trend towards higher wind speeds as well. Considering our desire to separate cool, wet, and calm days from hot, dry, and windy using the metric in (4.1) and decision rule in (4.7), Figure (5) is strong evidence in support of the cluster analysis approach.

Thus far, we have not considered the wind direction column of Figure 5, but it creates a somewhat different lens through which to view these fire weather clusters. Recall that the analysis was performed on the cosine of wind direction, so these plots represent the compass rose. For instance, at first glance the histogram of wind direction in cluster $(k_1) = (3)$ looks odd and strongly bimodal, but it actually reflects strong central tendency about north or $0^\circ$. Two dominant wind directions emerge when you take this linearization of the compass rose into account: north $(:0^{\circ})$ and south-southeast $(:150^{\circ})$. Based on general wind patterns in this region of the country we expect that a north wind represents the passage of a cold front near the RAWS station. The prevailing winds are likely represented by the south-southeast spike. With this in mind, look again at Figure 5 and some general categorizations of fire weather become apparent. We offer an explanation for these categories in Table 5.

Table 5

| | |
|---|---|
| Cluster $(k_1) = (1)$ | Low prevailing winds; precipitation. |
| Cluster $(k_1) = (2)$ | Stronger prevailing winds; higher temperatures. |
| Cluster $(k_1) = (3)$ | Moderate frontal winds; similar temperatures to cluster $(k_1) = (2)$ |
| Cluster $(k_1) = (4)$ | Dry cold front; strong winds. |
| Cluster $(k_1) = (5)$ | Very dry cold front; very high temperatures; strong winds. |

Table 5: Interpretation of fire weather categories in Figure 5.

These categories should be viewed more as descriptors than rules. In terms fire suppression however, such categories are highly meaningful because they follow the type of discourse heard on a radio in the field. For instance, say fire weather predictions indicate a dry cold front is to move through the area during the burn period. The fire manager could decide to run detailed analysis based on historical weather patterns in cluster $(k_1) = (2)$ and cluster $(k_1) = (4)$ to best approximate fire behavior during frontal conditions. Notice that this model run consists of all available weather data. Even though we use a specific forecast in the forecast error computation, this type of model run reflects fire behavior prediction in absence of a forecast. We will further discuss forecasting and the contrast between operational and planning models in Section 5.6. A fire weather forecast would indicate which weather category from Table 5 to expect. The fire manager would then run a restricted model in which he or she used just the historical data from this category. By restricting the number of data records to allow, the scenario tree would quickly become data poor. Since one of our goals is to explore recourse and the probabilistic nature of this model, we elect to use all the data, which assumes a forecast is unavailable.

With a description like Figure 5 in hand, we can critique our use of the median as a representative for each cluster. Mean forecast error would not be a good candidate to dictate the choice of representatives because these distributions are not normal. Most are asymmetric to some degree and skewness is quite common. Selecting the median assumes central tendency in these distributions, which is observable to be roughly the case in Figure 5, without assuming normality. The median forecast error may not always reflect the median of all four weather variables. For example, we may notice that our representative for cluster $(k_1) = (1)$ has a wind speed of $3.0$ mph, but happens to have a relative humidity of $31\%$, which is far from the median. Our hope is that selecting a single representative using the median captures the basic category of weather, while maintaining the natural variations associated with complicated weather interactions.

With a proxy for each cluster at each stage in place as well as the associated probability parameters $p_{\langle k_t \rangle}$ for the Fire-S stochastic program, we are ready to simulate fire behavior. We use Farsite to create the area $A_{\langle k_t \rangle}$ and perimeter $P_{\langle k_t \rangle}$ parameters in Section 4.2.

## 4.2 Fire Simulation

Section 4.1 explains the procedure we use to create a scenario tree diagram. Each $\langle k_4 \rangle = (k_1, k_2, k_3, k_4)$ path through the scenario tree represents a possible path through reality. This path has a probability of $p_{\langle k_4 \rangle}$ of actually occurring. Each node is a decision point and everything that occurs along the branches from one node to the next is dictated by the historic weather record that was chosen as the representative. Refer to Table 4 for two examples of these spliced weather streams. The reader may be slightly troubled by issues of continuity that this splicing process creates. Butting weather records up against each other like this violates the notion that hourly weather data should change gradually in a smooth manner. This objection is valid, but becomes less relevant considering Farsite's simulation environment.

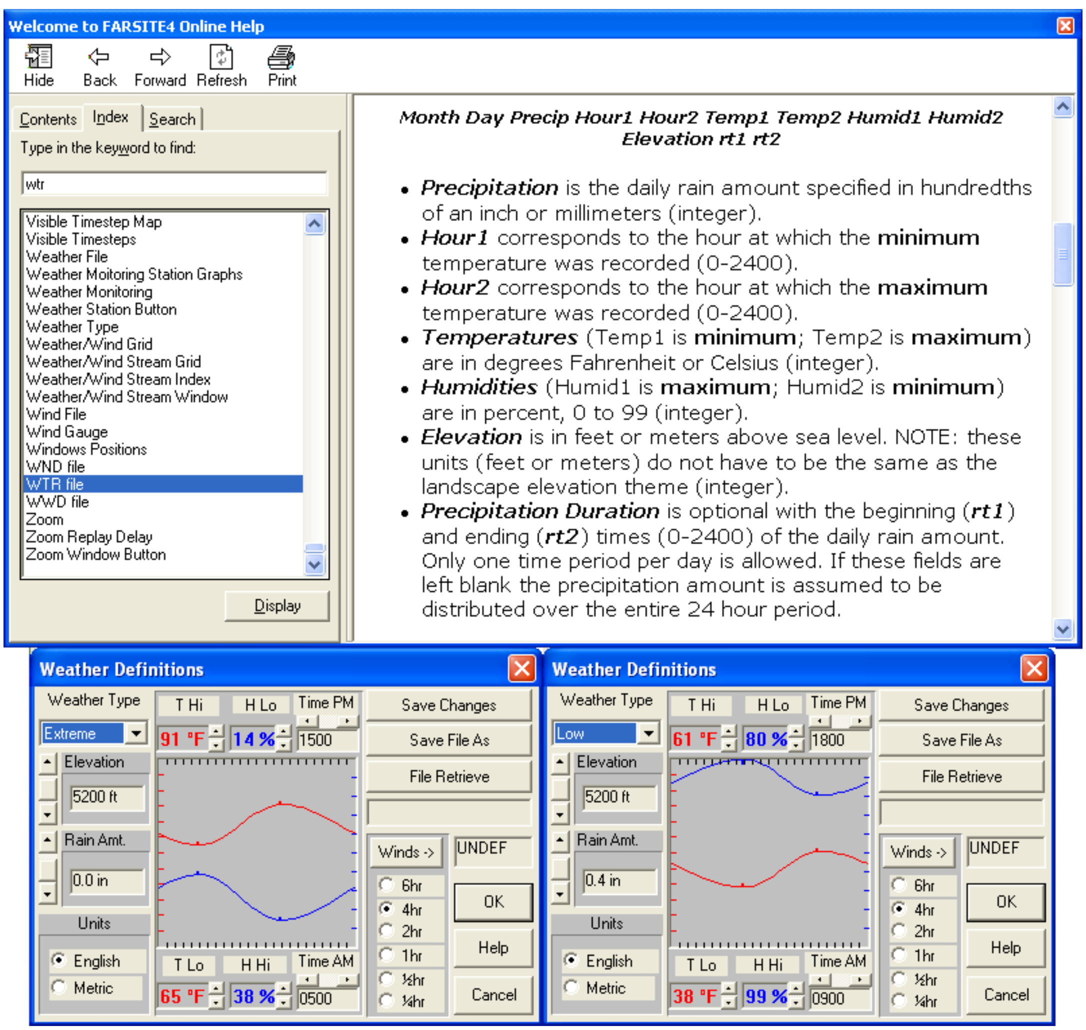



Figure 6: Screen shots of Farsite's .wtr weather file format.

Figure 6 shows Farsite's protocol for generating continuous weather streams for simulation from a small set of inputs. Daily minima and maxima are used to create sinusoidally varying weather on a daily basis. This technique helps smooth out discontinuities in temperature and relative humidity. Hourly wind observations are submitted and used directly in a Farsite simulation without this smoothing. Discontinuities at nodes in wind behavior are less of a concern because winds tends to change abruptly, at least more abruptly than temperature or relative humidity.

Farsite requires initial fuel moistures for 1-hour, 10-hour, 1000-hour, live herbaceous, and live woody fuels. Once a smoke report is received, the fire manager will be able to obtain or calculate these values appropriate to the ignition location. Farsite incorporates a dynamic fuel moisture calculator that runs before the fire simulation. We rely on Farsite to derive probabilistic fuel moisture scenarios from our probabilistic weather scenarios.

Spatial data is not randomized. As noted in Section 4, once a smoke report is made, the spatial aspects of the problem are fixed. We construct a landscape (.lcp) file from LANDFIRE raster grid data for BHNF [10]. There are eight data layer requisites for a landscape file:

1. Elevation
2. Slope
3. Aspect
4. Fuel Model (Scott and Burgan 40 from [11])
5. Canopy Cover
6. Stand Height

7. Crown Base Height
8. Crown Bulk Density

Each data input grid has 30-meter resolution. The Northern Great Plains Interagency Dispatch Center divides the BHNF into Initial Attack Response Zones. Section O ``GPC Pre-planned Dispatch Cards'' of the 2006 Black Hills Fire Management Plan [2] contains run cards that assign RAWS representatives to each zone. We simulate a pixelated line of ignition in the Deerfield response zone, which relies on the Nemo RAWS for initial weather data. Once a fire manager is managing a fire, he or she will likely obtain more spatially specific weather data for fire behavior prediction, but at the time of the smoke report, the RAWS data serve as the best available proxy for fire weather.

We have two versions of the Farsite software with which to simulate fire behavior. Farsite 4 is a free software package available from firemodels.org [6]. It has a high-level, graphical user interface. It is enormously useful for single simulations, which are important in the Fire-S calibration process. For example, to optimize computation speed for large scenario trees, it is important to restrict the extent of the landscape file to match the extent of the largest fires. Farsite's graphical interface is ideal for ironing out these sorts of issues.

However, if the landscape is too small, the fire can move out of the grid and render the perimeter and area parameters meaningless for larger fires. The second version we have access to is a DLL that runs through an interface with the C programming language. This version is enormously useful for the batch runs that are required to realize a large-scale scenario tree, but less detail about each run is available. To achieve a batch run we create weather scenario (.input) files for each path on the scenario tree; each scenario file contains all the weather information required for the corresponding Farsite simulation. The Farsite DLL creates grid files of simulation results. We derive $A_{\langle k_t \rangle}$ and $P_{\langle k_t \rangle}$ from these grids directly.

Figures 7, 8, 9, and 10 show simulation results for an ignition south of Deerfield Lake in the BHNF. One can readily observe the weather patterns from Figure 5 (as discussed in Table 5) support different types of fire growth. These pictures give an operational feel to the model because they are spatially explicit, but remember the stochastic program itself only uses scalar values $P_{\langle k_t \rangle}$ to determine containment. This lends a level of detail to the planning model that is very useful, but creating an operational model would require a different stochastic program, which we will look at in Section 5.7. For example, a fire manager may look at the footprints in Figure 8 and anticipate line-building tactics that would avoid the fast-moving flaming front on the lower left extent of the fire and pinch the spread until containment was achieved. Our model cannot account for such pinching; in fact, the Fire-S stochastic program ignores the spatial interaction between fire line and the fire itself, which is why it is not an operational level model. See Section 5.5 for further discussion.

Figures 7, 8, 9, and 10 represent only a small sampling of the Farsite simulations that were run. To visualize them all on a landscape file individually or simultaneously would be impractical.

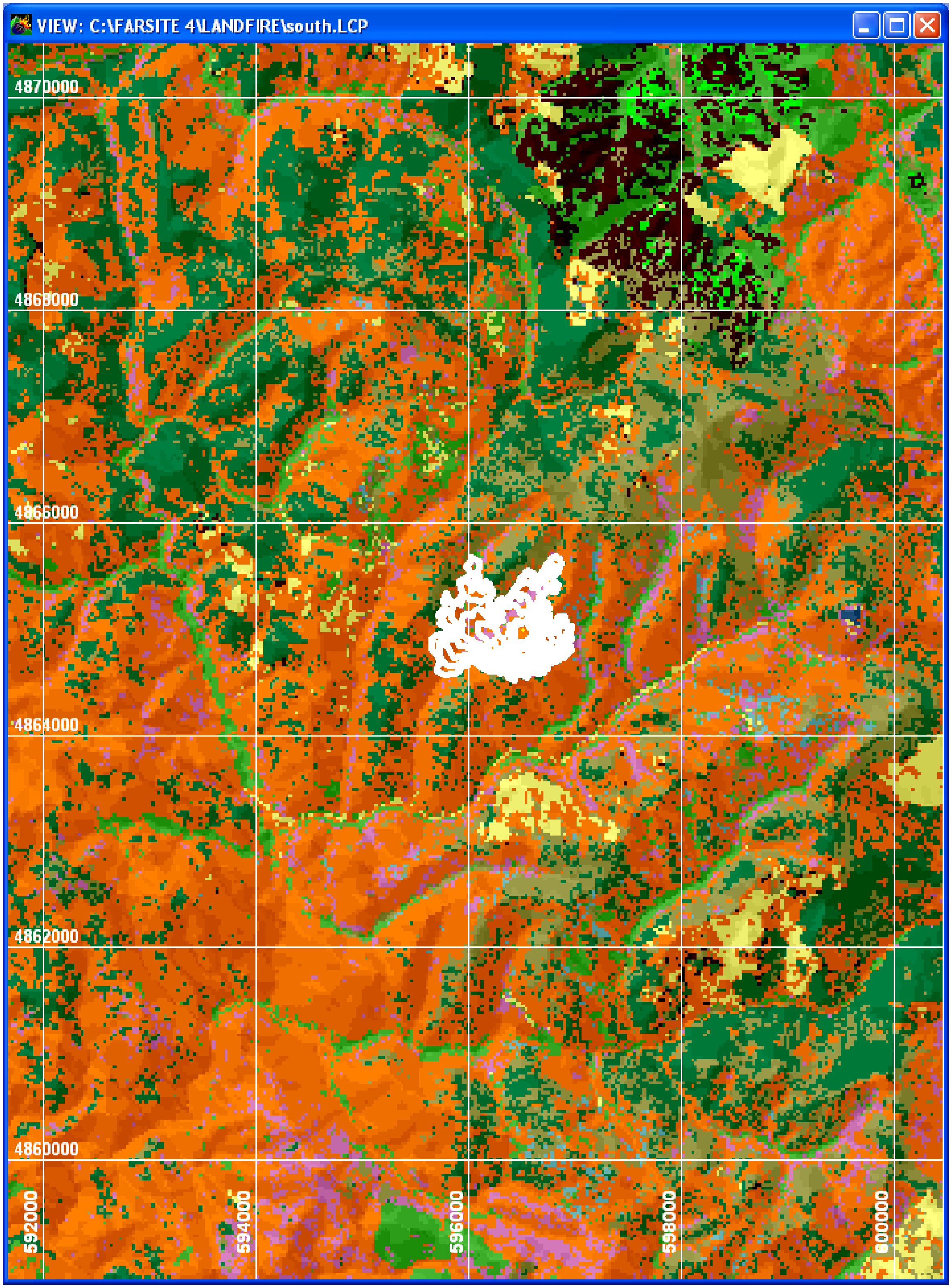



Figure 7: Farsite simulation for $\langle k_4 \rangle = (1,2,2,3)$. Wet cold front conditions.

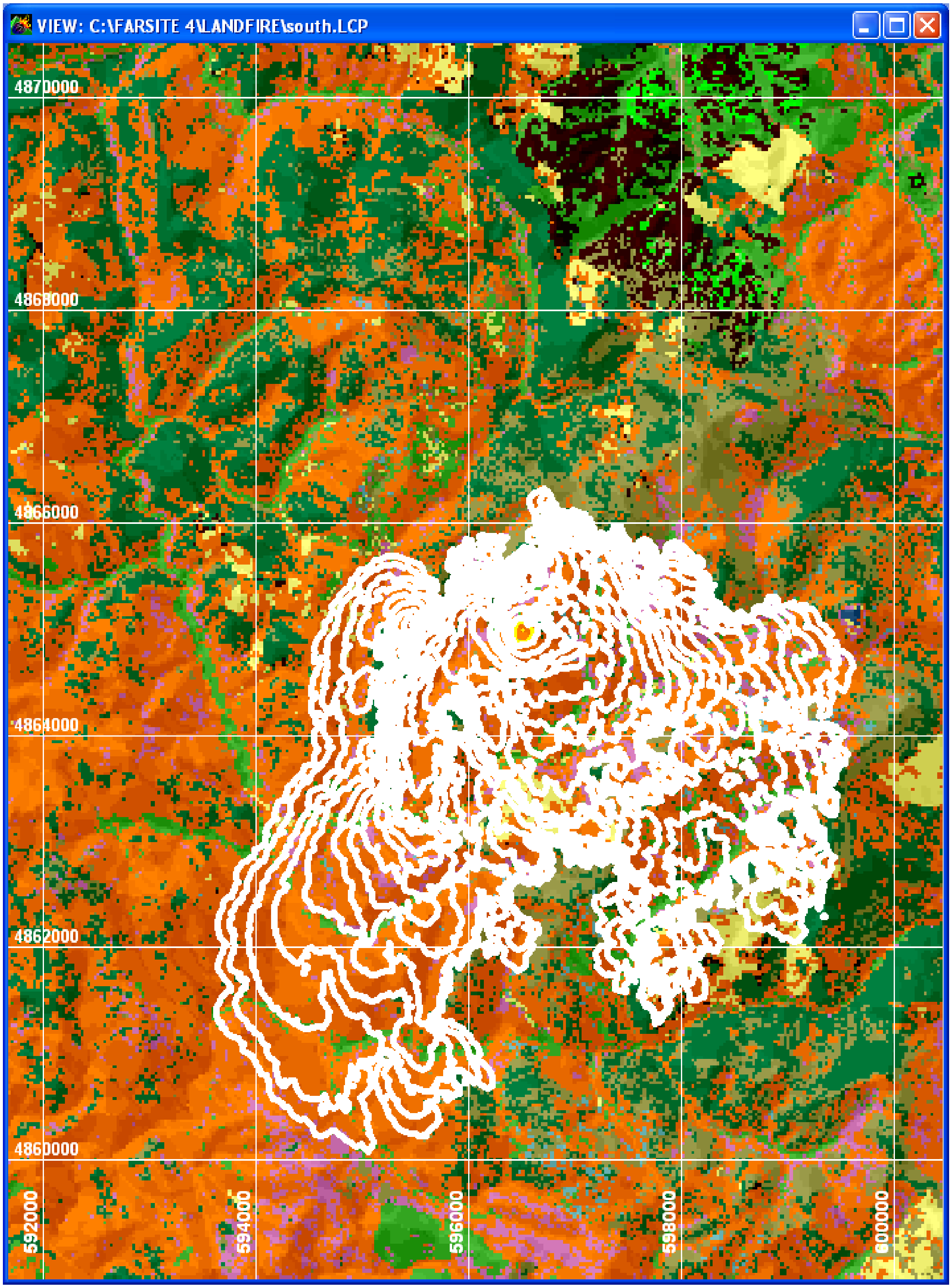



Figure 8: Farsite simulation for $\langle k_4 \rangle = (5,3,2,1)$. Dry cold front conditions; strong, north winds.

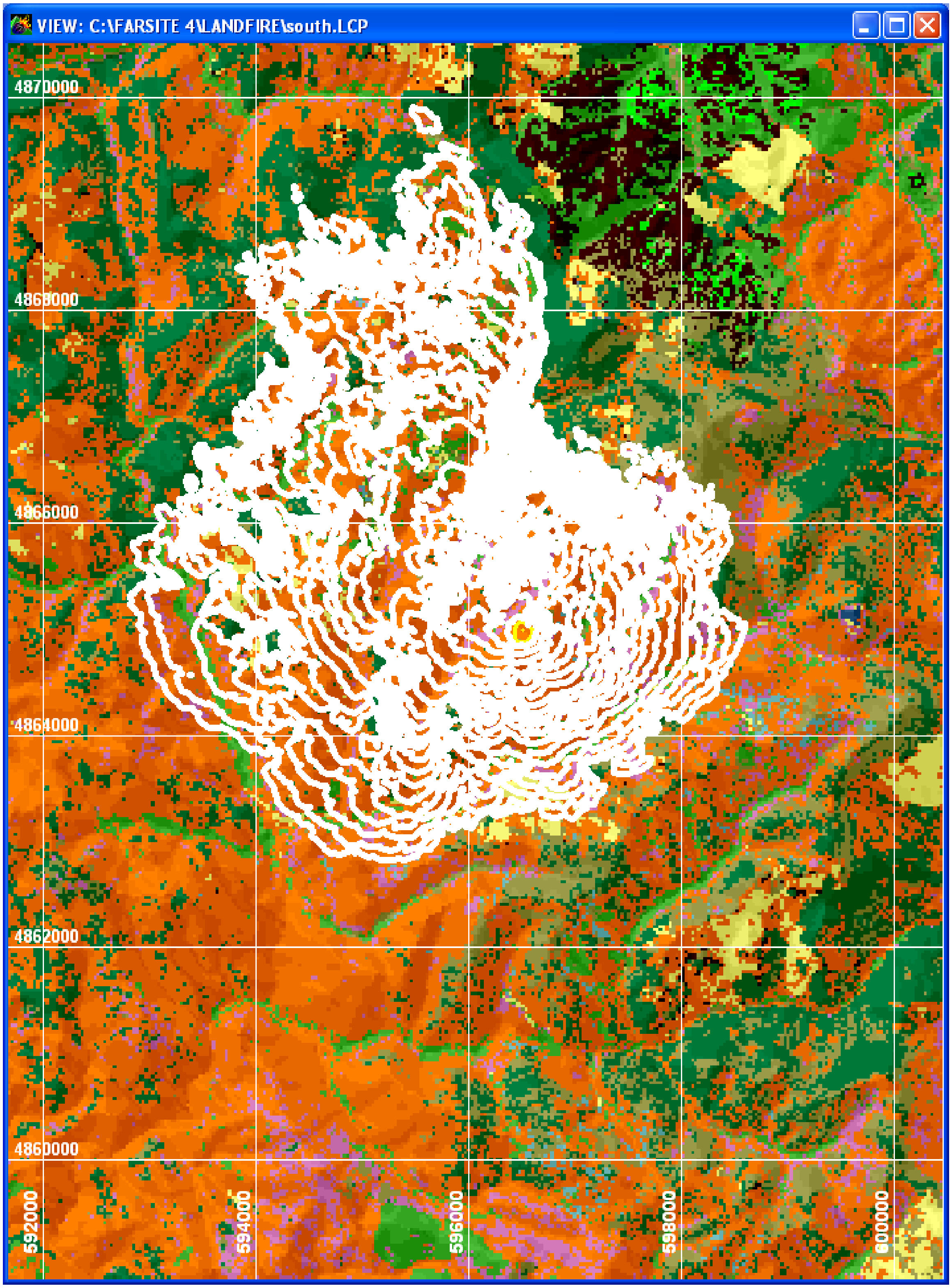



Figure 9: Farsite simulation for $\langle k_4 \rangle = (6,3,3,2)$. Dry, prevailing conditions with high winds.

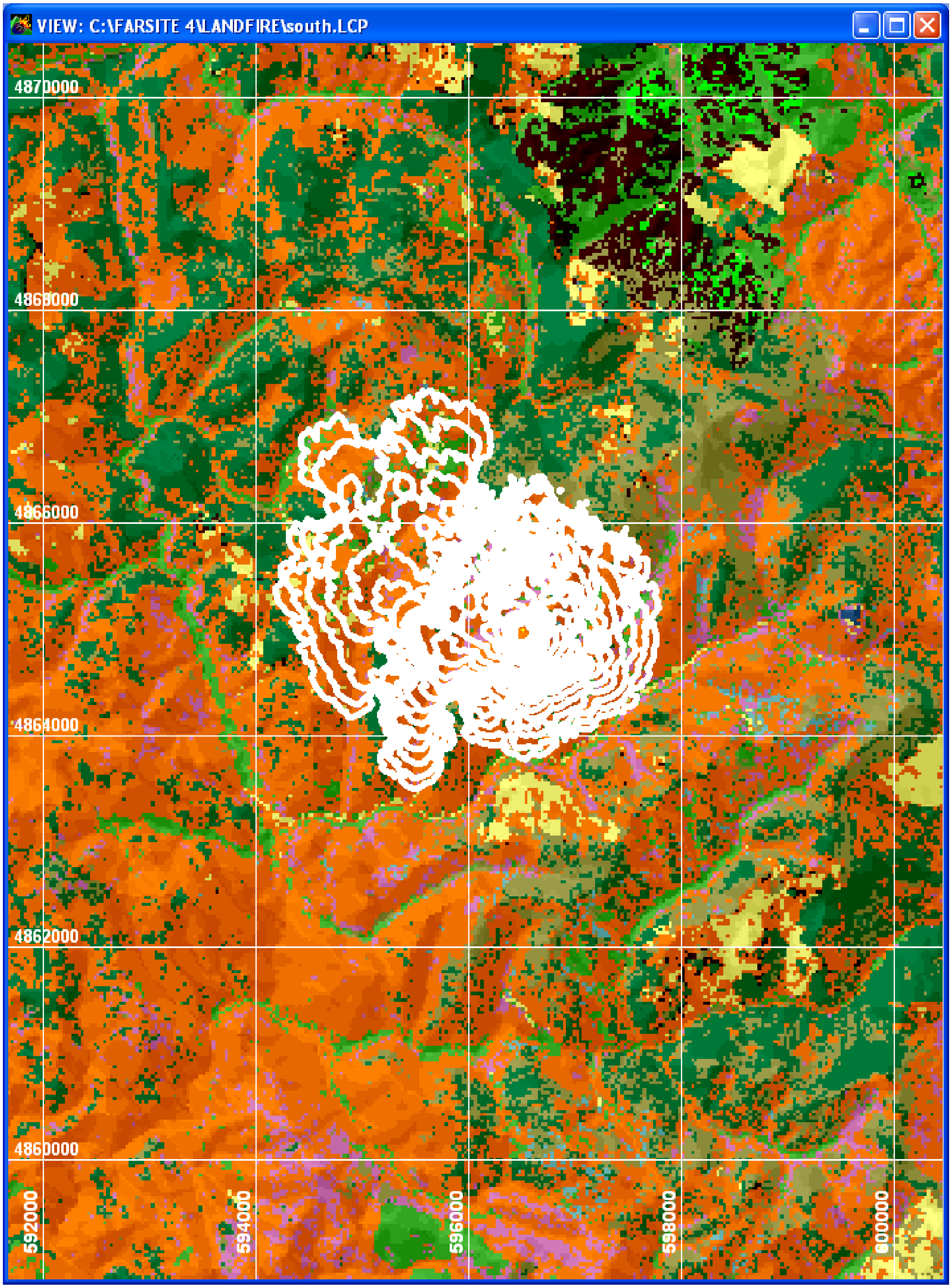



Figure 10: Farsite simulation for $\langle k_4 \rangle = (2,2,2,4)$. Wet conditions turning dry and windy.

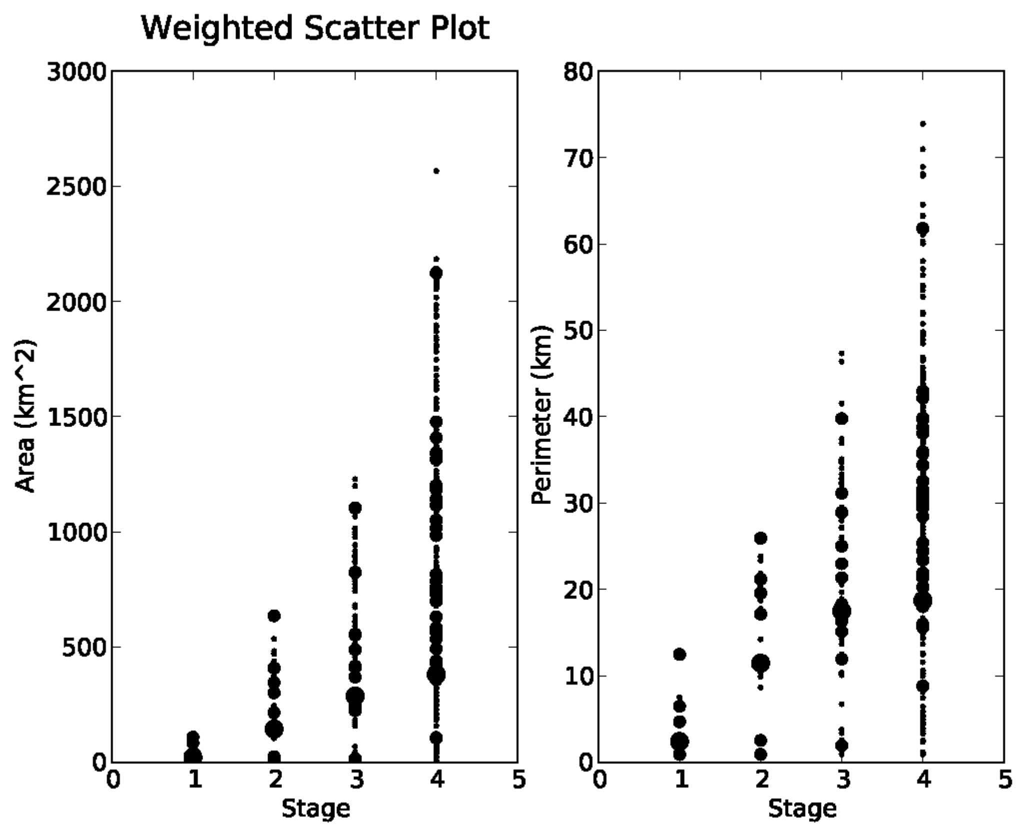



Figure 11: A probabilistic view of the fire behavior simulations in Farsite.

Instead, consider Figure 11. It shows a weighted scatter plot of the fire growth parameters $A_{\langle k_t \rangle}$ and $P_{\langle k_t \rangle}$. Probabilities are captured by the weight of the dots. We can track typical fire growth by connecting large dots across the stages. We can track fringe fire growth by connecting smaller dots. These are the scalar values that the Fire-S stochastic program takes into account, which means it will be sensitive to all types of simulated fire behavior.

Farsite also creates raster grids describing projected fire behavior such as flame lengths, intensity, and rates of spread.

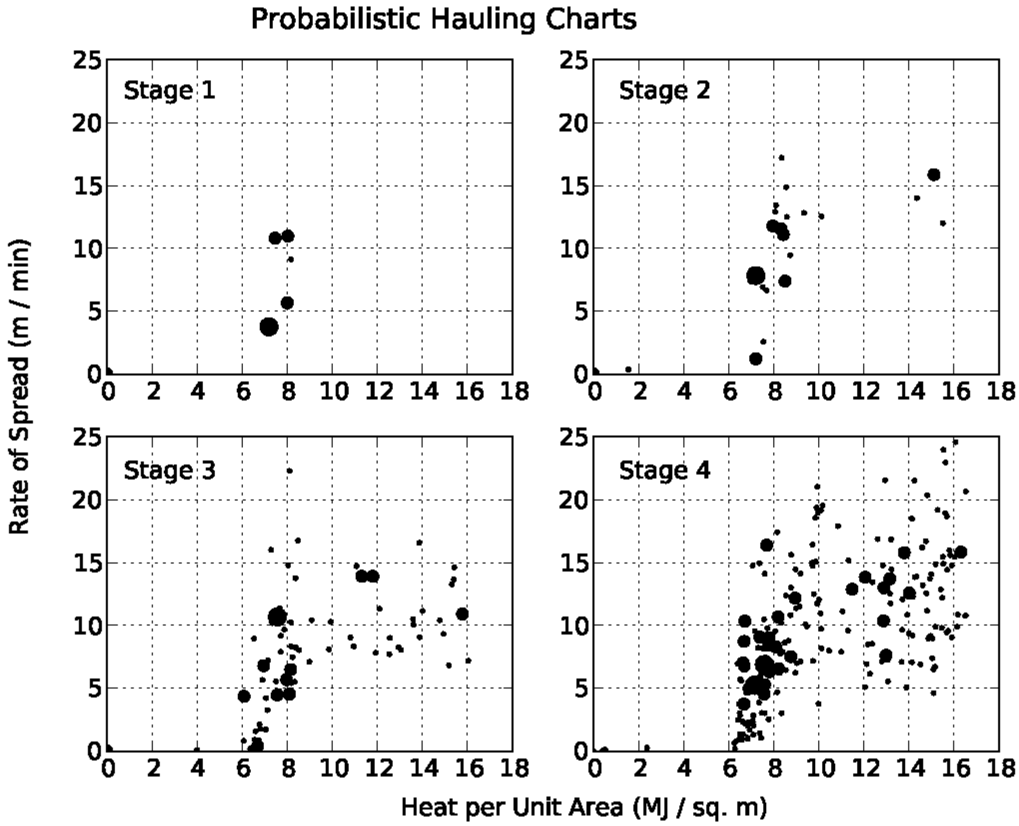



Figure 12: Probabilistic hauling charts.

Figure 12 displays some of this information in a probabilistic hauling chart format. The fire manager can use such diagrams in combination with Appendix B of the Fireline Handbook [7] or a software package like BehavePlus to assess fire severity and address safety considerations under each fire behavior scenario. Consider Stage 4. Based on the density of the points, it seems most likely that the fire will move from $5$ to $10$ meters per minute and create between $6$ and $9$ mega joules of heat per square meter. However, there are fire behavior scenarios where the simulations show much more extreme rates of spread and heats.

These simulations parameterize $P_{\langle k_t \rangle}$ and $A_{\langle k_t \rangle}$ in the Fire-S stochastic program. The remaining parameters involve resources, financing, and escaped fire scenarios.

## 4.3 Suppression Resources

The Fire-S stochastic program exhibits enormous flexibility in terms of the underlying resource set. Parameterizing the resources in the program is equivalent to populating a table like Table 1 of Section 2. A fire manager must specify which resources he or she has available and characterize their costs and line production rates.

To introduce fire suppression resource sets let us consider how the Fire-S stochastic program formulation in (3.1) through (3.10) responds to small and large values of $R$, or equivalently, small and large resource sets. Start with the extreme small case: $R = 0$. If there are no resources available to suppress a fire, the fire will grow in every scenario. We will have $f_{\langle k_t \rangle} = 0$ for every scenario $\langle k_t \rangle$ at every stage $t$. As a result, the set of constraints in (3.10) will indicate $esc_{\langle k_4 \rangle} = 1$ for every $\langle k_4 \rangle$, which means every fire behavior scenario escapes the scope of the model. The largest possible expected burned area will be computed in the objective function (3.1). So in a sense, this parameterization results in the worst possible optimal solution to our minimization problem. As we increase $R$ by adding resources to the available set, we expect to start catching more and more fires and thus, lower the optimal expected burned area.

Next, let us explore the opposite extreme. Suppose $R$ is huge. Say we parameterize the Fire-S stochastic program with a national resource list that includes every possible firefighting resource the fire manager could possibly obtain. This parameterization will not result in zero expected burned area because our travel and prep time assumption (3.4), that says $\forall r, \langle k_1 \rangle : l_{r,\langle k_1 \rangle} = 0$, will allow the fires to burn into stage 2 regardless of how many resources are deployed in Stage 1. Given the classic containment constraints in (3.5) we might imagine such a large resource set to carry out the most effective suppression possible under the travel time assumption and produce a containment scenario where $f_{\langle k_2 \rangle} = 1$ for every $\langle k_2 \rangle$. This type of *total Stage 2 containment* for all scenarios is certainly possible, but we need to take budget considerations into account. Regardless of what is available in the resource set, the program can only operate with resources it can afford based on the budget constraints in (3.2) and (3.3). If $R$ and $TC$ are both huge, then total Stage 2 containment could occur.

The preceding discussion helps us to narrow down the size of the resource set to use. In reality, the fire manager's budget will not be huge, but he or she will have affordable suppression resources to work with. To reflect this reality we choose to make $R$ large to represent availability of resources, then restrict their use naturally with a reasonable budget. A typical resource set is essentially an augmented version of Table 1 based on the approximate numbers of resources that the area has available. For the BHNF we consult the Section M of the Fire Management Plan [2], which contains Most Efficient Level forms.

Table 6

| $r$ | Description | Quantity | FC ($) | VC ($/hr) | Rate (ch/hr) |
|---|---|---|---|---|---|
| 1 | Dozer | 1 | 18,000 | 900 | 30 |
| $2-13$ | T6 Engine | 12 | 8,000 | 400 | 16 |
| $14-17$ | TI Hand Crew | 4 | 2,050 | 250 | 9 |
| $18-21$ | TII Hand Crew | 4 | 1,000 | 100 | 3 |

Table 6: Resource set derived from MEL forms in BHNF Fire Management Plan [2], $R = 21$.

Table 6 shows the working resource set that was derived from these forms. Notice there are twelve identical Type 6 engines, four identical Type I hand crews, and four identical Type II hand crews. Within these groups the linear solver will select an arbitrary resource to use, but it may matter realistically which specific element is chosen. For instance, if the model were updated to include prepositioning of resources or resource travel time, beyond the current assumption of a single stage travel time, there could be incentive to choose Engine B versus Engine K, if B was closer to the ignition than K, for example. Also note that variable costs and line production rate estimates are given per hour. The parameters $VC_{r,t}$ and $L_{r,\langle k_t \rangle}$ should be scaled to be per stage, which will depend on the scope of the model. We refer the reader to Section 2 for an example of scaling from hourly to per-stage variable cost, but we delve deeper into the line production parameter here.

Line production rates are approximated from guidelines in Appendix A of the Fireline Handbook [7]. We assume line production is performed at the given rate during the stage regardless of terrain, current fire behavior, and fatigue level of resources. Given all that happens on a fire line, this is a broad assumption. Even the simplest factors, such as slope or rocky soil, can greatly affect line production rates. These broad assumptions are presumed to be acceptable at the planning level, but unacceptable at the operational level. In planning, the interactions can be estimated, and we discuss options for doing so in Section 5.5, to further the realism of the model. In operation, this model would need to address such interactions in a more rigorous way because sometimes they are critical to firefighter safety. We operate the model by assuming $L_{r,\langle k_t \rangle}$ could be expressed as $L_{r,t}$, but leave the scenario dependence notation to demonstrate the possibility of scenario-by-scenario adjustment (see Section 5.5). Remember the Fire-S model has twelve-hour stages, but we assume an eight-hour line building period within each stage. This assumption accounts for resource positioning, fatigue, breaks, and other logistics.

## 4.4 Escaped Fires

All of the Fire-S stochastic program's parameters, as listed in Section 3, have been filled save one. Section 2 gives a discussion of escaped fires defining them as fire behavior scenarios that escape the predictive scope of the model. Recall that these are not necessarily large fires, just longer-lasting than the two-day, four-stage scope of the Fire-S model. Such fires must be accounted for in the objective function because it seeks to minimize burned area and an escaped fire will likely have a significant contribution to this value. As described in Section 2, the user supplies an estimate for an escaped fire $\hat{A}_{LF}$. We set $\hat{A}_{LF}$ to be an area estimate of a large fire in the BHNF because if the program does not contain the fire, then it has continued to grow. Our best estimate for the fire's area after it escapes the model's scope must assume the growth continues. One possible way to derive this estimate is to use a Fire Family Plus database to calculate the average fire areas from its top four size classes (D, E, F, and G). This method applied to the BHNF database yields $\hat{A}_{LF} = 7{,}814$ ha, which, for reference, is about $20{,}000$ acres. Equation (3.1) shows how $\hat{A}_{LF}$ is associated with the decision variables $esc_{\langle k_4 \rangle}$ to add this escaped fire estimate to the expected area burned in the objective function.

# 5 Discussion

We will conclude this paper with a detailed discussion of the Fire-S model outputs (Section 5.1). We examine outputs in general and then track a specific scenario through the model. The Fire-S model is applied to a suppression budget analysis in Section 5.2. We use two different model versions and the suppression budget analysis to explore the advantages of complexity of the model in terms of better optimal solutions in

Sections 5.3 and 5.4. The remaining sections suggest avenues of continued exploration. In reading through the results, we urge the reader to keep in mind that this is a ``proof-of-method" type paper as opposed to one designed to answer a specific research question so that he or she can critique the model itself instead of focusing on the results.

## 5.1 Stochastic Program Outputs

As we pointed out in Section 3, the Fire-S stochastic program is a large size, mixed integer program (MIP). To give the reader an idea of how large it can get, consider a uniform scenario tree (like Figure 4) with $K_1 = \cdots = K_4 = 4$ that utilizes the resource set in Table 6 where $R = 21$. A program of this size would have $5{,}884$ decision variables and $13{,}216$ constraints in the formulation expressed in Equations (3.1) through (3.10) from Section 3. This is a useful size considering the program that results from the cluster analysis parameterization with Stage 1 shown in Figure 5 ran with $6{,}838$ decision variables and $7{,}308$ constraints.

Despite their large sizes, these programs can be solved extremely quickly using the high-powered linear solver CPLEX. CPLEX is distributed by IBM and incorporates state-of-the-art branch-and-cut methods in its MIP solver. The user is further empowered in terms of computing time with the ability to set relative error tolerances for solutions. As such, these programs can be solved and studied relatively quickly. Each solver routine is seeded with a ``.lp" file, which contains the entire program with numerical values for all its parameters. CPLEX outputs a wealth of information about the optimal solution of the stochastic program. Solution diagnostics and the objective function value can be printed. With our C-interface version of CPLEX a ``.sol" solution file is written for each routine; it contains the value of each decision variable in the program. While there is not as much post-processing work to be done as parameterization work, care must be taken to interpret the output information in a useful context.

First, we examine the optimal values of the decision variables in the objective function: $f_{\langle k_t \rangle}$ and $esc_{\langle k_t \rangle}$. Solving the program creates a set of recourse allocation decisions ($x_{r,\langle k_t \rangle}$) that achieve an *optimal containment scenario*. One way to explore an optimal containment scenario is by using an exploded scenario tree diagram as shown in Figure 13.

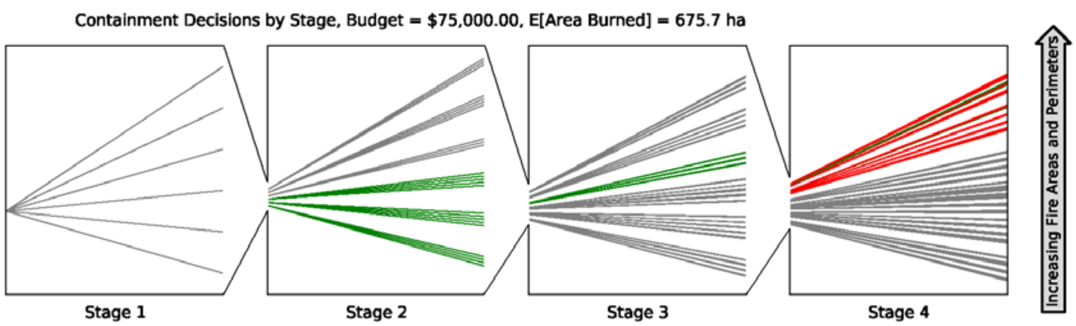



Figure 13: Exploded tree diagram, $TC = \$75,000$.

This figure seeks to emulate the small scale scenario tree diagram shown in Figure 2 of Section 2, but there are so many branches that the visualization is more difficult to achieve in a graphics program. Figure 13 shows, in general, how containment was achieved or escape occurred in the Fire-S stochastic program. A gray branch indicates $f_{\langle k_t \rangle} = 0$, a green branch indicates $f_{\langle k_t \rangle} = 1$, and a red branch indicates $esc_{\langle k_4 \rangle} = 1$. Moving upward through the branches in Figure 13 progresses roughly from smaller to larger fires. Stage 4 in Figure 13 has many escape scenarios for the largest fires. Some of the smaller fire growth scenarios are contained during Stages 2 and 3.

There are also fourth stage containments, but very few of them. Notice the model has performed in accordance with our assumption in (3.4) that no Stage 1 containment is possible. We will return to the exploded tree diagram as a tool to study the budget constraint in Section 5.2.

The objective function value and budget constraint level are also printed in Figure 13. These indicate that given a budget of $TC = \$75{,}000$ to spend on fire suppression resources, the fire manager can expect a minimum burned area of 675.7 ha. The next logical question to ask is: under each scenario, which resources from Table 6 were dispatched to suppress the fire and when were they ordered?

Table 7

| *r* | Description | Stage 1 | Stage 2 | Stage 3 |
|---|---|---|---|---|
| 1 | Dozer | 16.7 % | 0.0 % | 0.0 % |
| 2 | T6 Engine A | 16.7 % | 12.5 % | 13.5 % |
| 3 | T6 Engine B | 16.7 % | 4.2 % | 4.1 % |
| 4 | T6 Engine C | 16.7 % | 4.2 % | 5.4 % |
| 5 | T6 Engine D | 0.0 % | 0.0 % | 0.0 % |
| 6 | T6 Engine E | 33.3 % | 0.0 % | 0.0 % |
| 7 | T6 Engine F | 0.0 % | 4.2 % | 4.1 % |
| 8 | T6 Engine G | 0.0 % | 0.0 % | 0.0 % |
| 9 | T6 Engine H | 0.0 % | 0.0 % | 0.0 % |
| 10 | T6 Engine I | 16.7 % | 4.2 % | 4.1 % |
| 11 | T6 Engine J | 50.0 % | 16.7 % | 4.1 % |
| 12 | T6 Engine K | 66.7 % | 16.7 % | 4.1 % |
| 13 | T6 Engine L | 50.0 % | 0.0 % | 0.0 % |
| 14 | T1 Hand Crew A | 33.3 % | 29.2 % | 17.6 % |
| 15 | T1 Hand Crew B | 16.7 % | 29.2 % | 20.3 % |
| 16 | T1 Hand Crew C | 16.7 % | 29.2 % | 17.6 % |
| 17 | T1 Hand Crew D | 16.7 % | 29.2 % | 17.6 % |
| 18 | T2 Hand Crew A | 0.0 % | 4.2 % | 4.1 % |
| 19 | T2 Hand Crew B | 0.0 % | 4.2 % | 5.4 % |
| 20 | T2 Hand Crew C | 0.0 % | 8.3 % | 8.1 % |
| 21 | T2 Hand Crew D | 0.0 % | 4.2 % | 5.4 % |

Table 7: Resource dispatch and use rates. $TC = \$75{,}000$, minimum expected burned area = 675.7 ha, 22.2% of fire scenarios escape.

The most straightforward answer to this question is to tabulate how often each resource is used during each stage across all scenarios. Table 7 shows just that. The tabulated values indicate the percentage of $\langle k_t \rangle$ for which $x_{r,\langle k_t \rangle} = 1$. For example, we see Type 6 Engine K ( $r = 12$ ) was used in 66.7% of scenarios in Stage 1, then used in 16.7% during Stage 2, and then used in 4.1% during Stage 3. Recall, that this indicates the engine was on the fire and constructing line during Stages 2, 3 and 4 for those percentages of scenarios. There are two factors that can cause the percentages to decrease for this engine. One, the number of scenarios increases as the scenario tree branches out. Two, some of the stages acheived containment so the engine was sent home. Table 7 is ambiguous as to which was the true case. Overall, Table 7 should be viewed as a summary of the overall tendencies of the dispatch decisions. It can guide further investigation into the actual values of decision variables specific to a resource of interest. For instance, suppose we are interested the resource prescription for scenario $\langle k_t \rangle = (6,2,4,1)$. For the ignition in the BHNF's Deerfield zone, the stochastic program found fourth stage containment to be optimal, that is $f_{(6,2,4,1)} = 1$. Let us examine this scenario closely and dissect the model's performance in this specific case.

Scenario $(6,2,4,1)$ can be deemed fringe fire behavior because it has a very low conditional probability $p_{(6,2,4,1)} = 6.1 \times 10^{-4}$. Nonetheless, it represents a possible weather stream under the cluster analysis of the Nemo RAWS historical data so a simulation was performed.

Table 8

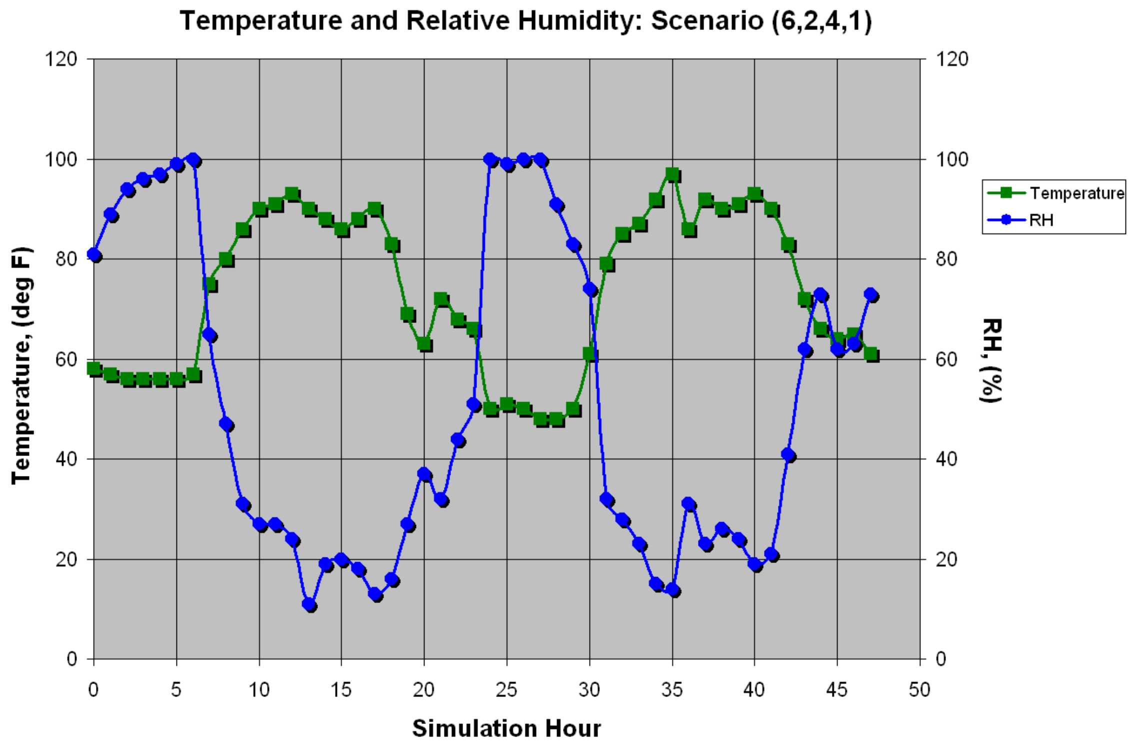
| Stage | Weather Record |
|---|---|
| 1 | August 12, 2003 |
| 2 | August 31, 2002 |
| 3 | June 23, 2001 |
| 4 | June 23, 2001 |

Table 8: Date splice description for scenario $(6,2,4,1)$.

Table 8 shows the date records that were spliced together as a representative for the weather patterns found with the cluster analysis. We notice right away that this is likely a singleton cluster in stage 3 because stage 3 and 4 share a date record. Next, let us examine the RAWS data directly to describe the weather during the simulation.

Figure 14 plots hourly measurements of temperature, relative humidity, and winds for scenario $(6,2,4,1)$. There was no precipitation recorded. The data show the typical negative correlation of temperature and relative humidity. This weather stream shows hot, dry, prevailing winds dominate the burn periods of both days. The winds are stronger the first day than the second. A fire manager may be troubled when he or she sees the wind plot of Figure 14 because it shows some significant changes in wind direction throughout the day. This could be a common, up-slope/down-slope diurnal pattern, but needs to be noted for firefighter safety. The Farsite simulation associated with this weather scenario is shown in Figure 15.

The fire footprint does not give any strong clues about the driving weather, but the fire front seems to be pushed by northerly winds and moves quite quickly during the hot dry portions of each day. Farsite output also indicates torching, but the majority of the fire is confined to surface spread.



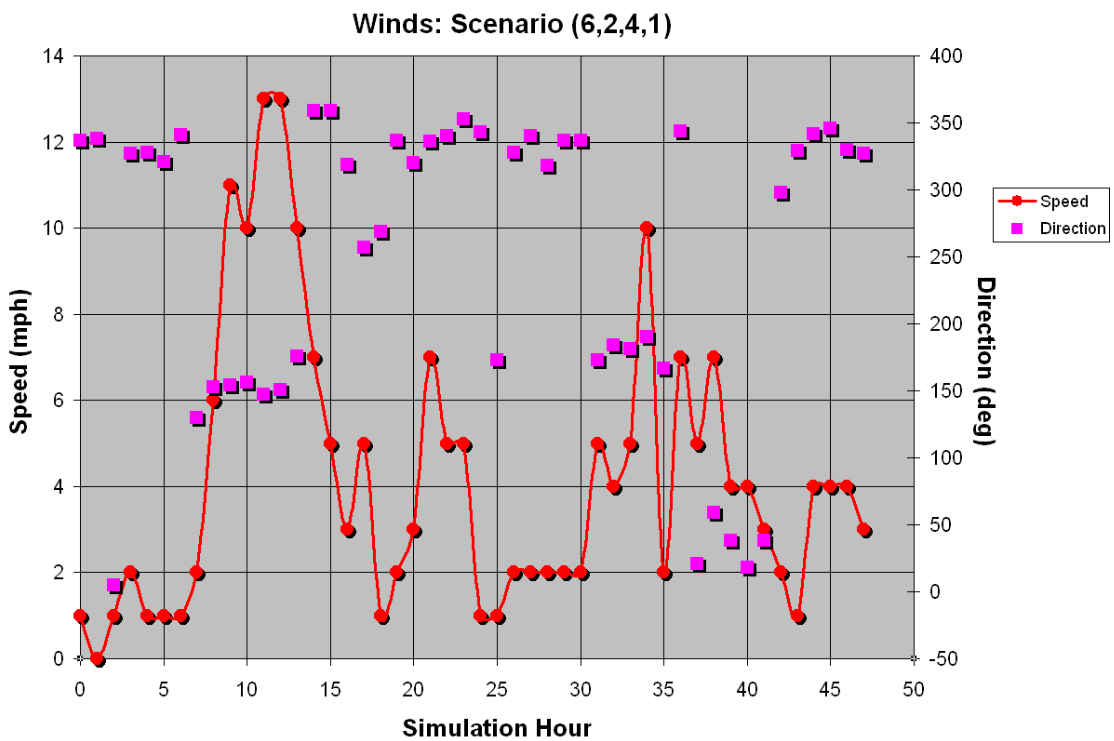




Figure 14: RAWS data for $\langle k_t \rangle = (6,2,4,1)$.

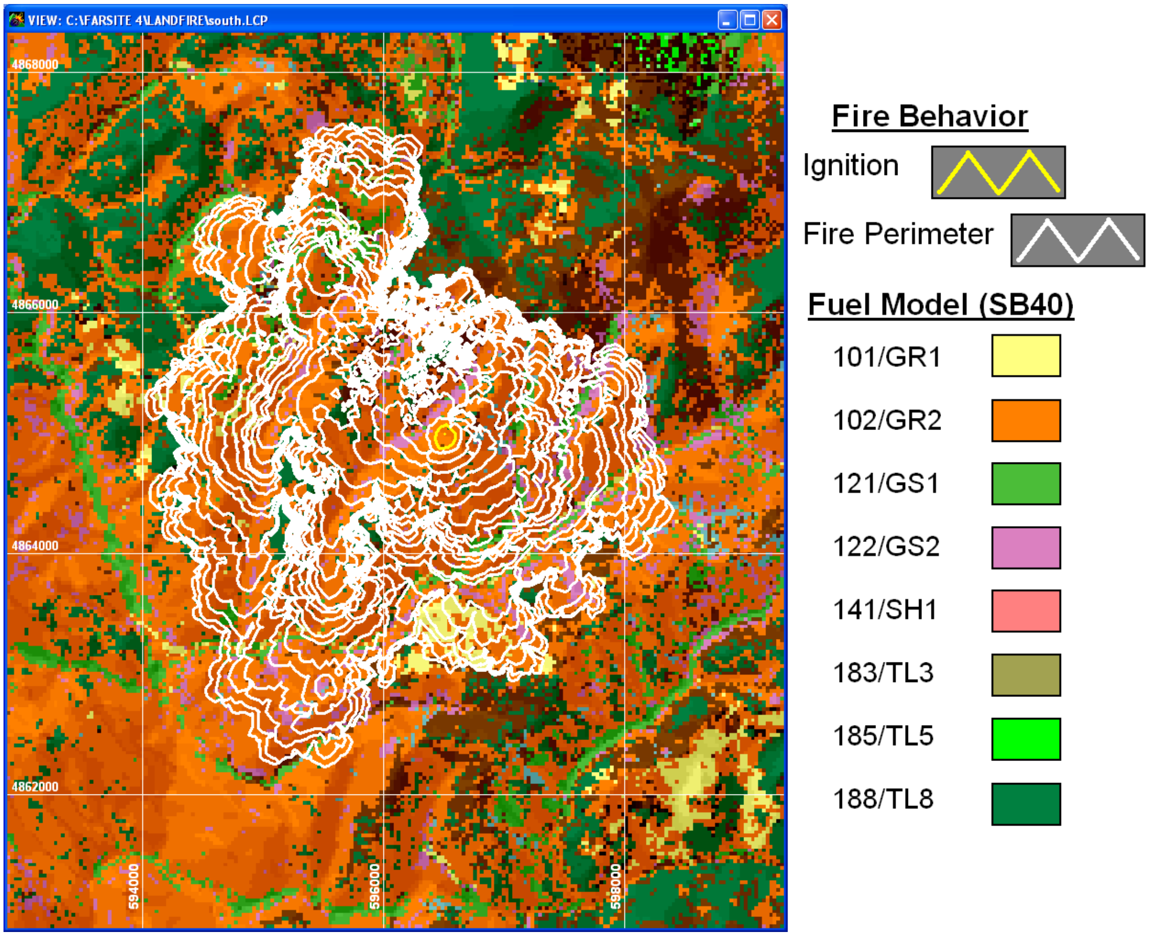



Figure 15: Farsite fire perimeters for $\langle k_t \rangle = (6,2,4,1)$.

Figure 16 quantifies the fire spread and gives a description of fire behavior throughout the scope of the model.

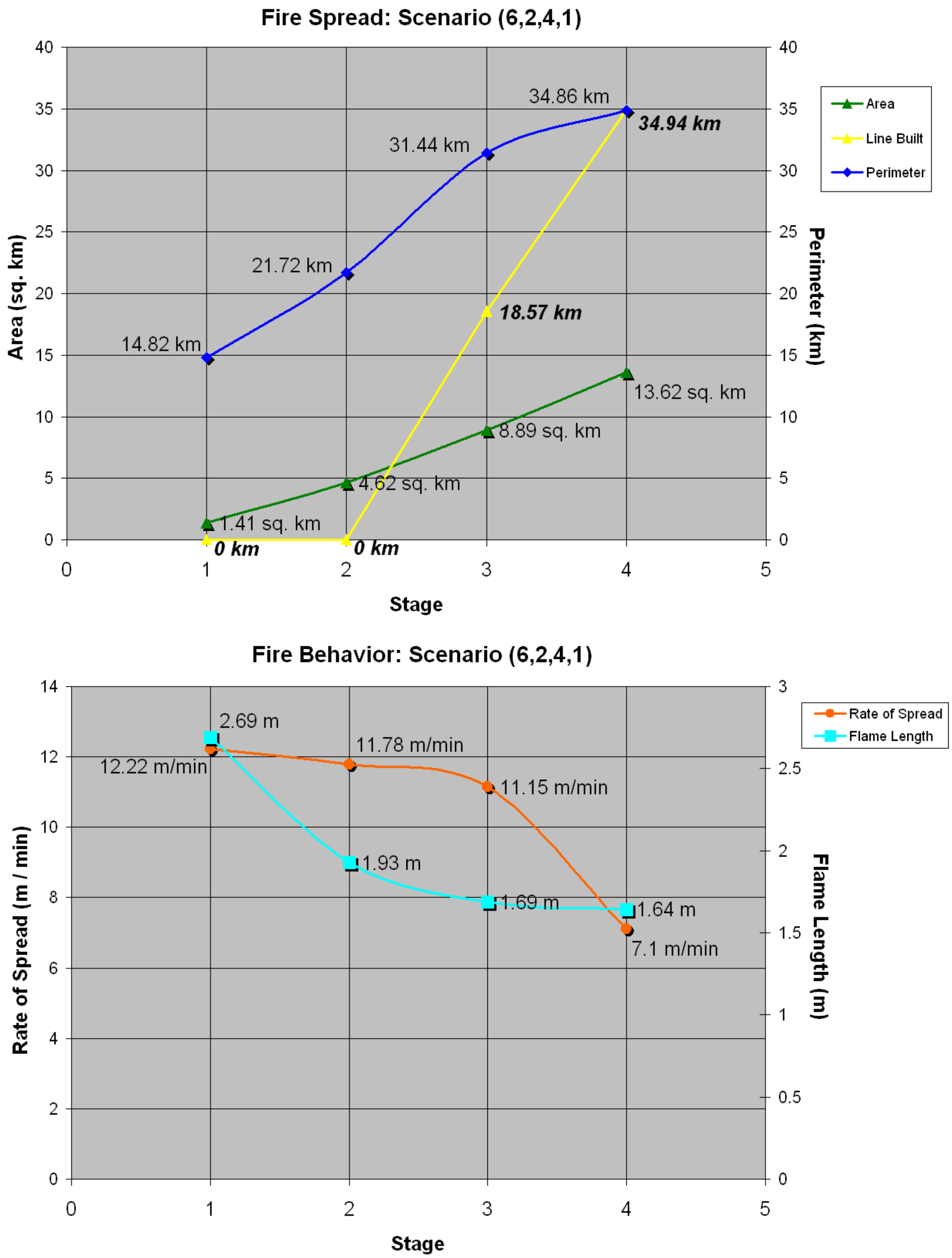



Figure 16: Simulation results for $\langle k_t \rangle = (6,2,4,1)$.

Now that we have a rigorous description of the weather stream and resulting fire behavior in scenario $(6,2,4,1)$, let us examine what the stochastic program outputs show as the optimal resources to achieve $f_{(6,2,4,1)} = 1$. Figure 16 also shows the progress of the resources as they build line. We see that fourth stage containment is achieved by a very small margin; the resources achieve 34.94 km of line at the end of Stage 4, which just barely exceeds the Stage 4 fire perimeter of 34.86 km.

Table 9

| Stage | Package | Description | Cost |
|---|---|---|---|
| 1 | $r = \{\varnothing\}$ | No resources | \$ 0 |
| 2 | $r = \{2,4,14,15,16,17,20\}$ | Two engines, five hand crews | \$40,400 |
| 3 | $r = \{2,4,14,15,16,17,18,20\}$ | Two engines, six hand crews | \$17,000 |

Table 9: Dispatch packages for scenario $(6,2,4,1)$.

Table 9 shows the resources that were used for containment. With the budget of $TC = \$75,000$ these dispatch packages were affordable because they cost $\$57,400$. At this point, we can critique the Fire-S program's choices. Sending so many hand crews may be slightly illogical unless the fire is in rough terrain, which the program would have no way of discerning. Likewise, we can make some comments about the practicality of the line-building tasks. Building 35 km of lines would take considerable time, but to be commensurate with fire sizes that is the required amount. We can imagine ``line building" to be a loose term and assume it includes natural barriers, but again, the program itself has no way of knowing the spatial aspects of the problem. Issues such as these indicate the importance of tight, overall calibration so that the outputs are as realistic as possible.

A fire manager can use these outputs in many ways, but the context of a fire *planning* model should always be considered. The level of detail in terms of stage-by-stage decision-making and specific resource packages is a benefit of the Fire-S stochastic program's formulation. But, as we have already mentioned and indeed will explore again in Section 5.7, to make this model function on an operational level, we would need to account for interactions between the fire and suppression resources. Section 5.2 proposes a study that fits well in the planning framework.

## 5.2 Suppression Budget Analysis

As a fire planning model, this approach lends itself to suppression budget planning. This type of analysis is routinely carried out using deterministic models, but the appeal of using such a detailed, stochastic model is great. We run the Fire-S stochastic program with various values of the $TC$ parameter to show fire suppression performance at different levels of the budget constraint in (3.3).

Figure 17 demonstrates the efficacy of the exploded tree diagram that we mentioned in Section 5.1. The results in Figure 17 are created by finding four separate solutions to the Fire-S stochastic program with budget constraint levels of \$20,000, \$60,000, \$75,000, and \$150,000. First, notice the escaped fire branches decrease as the budget constraint level is relaxed. More money means more containment options are available because more resources can be dispatched. We also see a general trend towards early containment, for scenarios that can be contained. More money allows more resources to be dispatched right away to a fire and thus, lower the expected burned area. The spectrum ranges from $TC = \$20{,}000$, which displays limited third stage containment, to $TC = \$150{,}000$, which displays total Stage 2 containment.

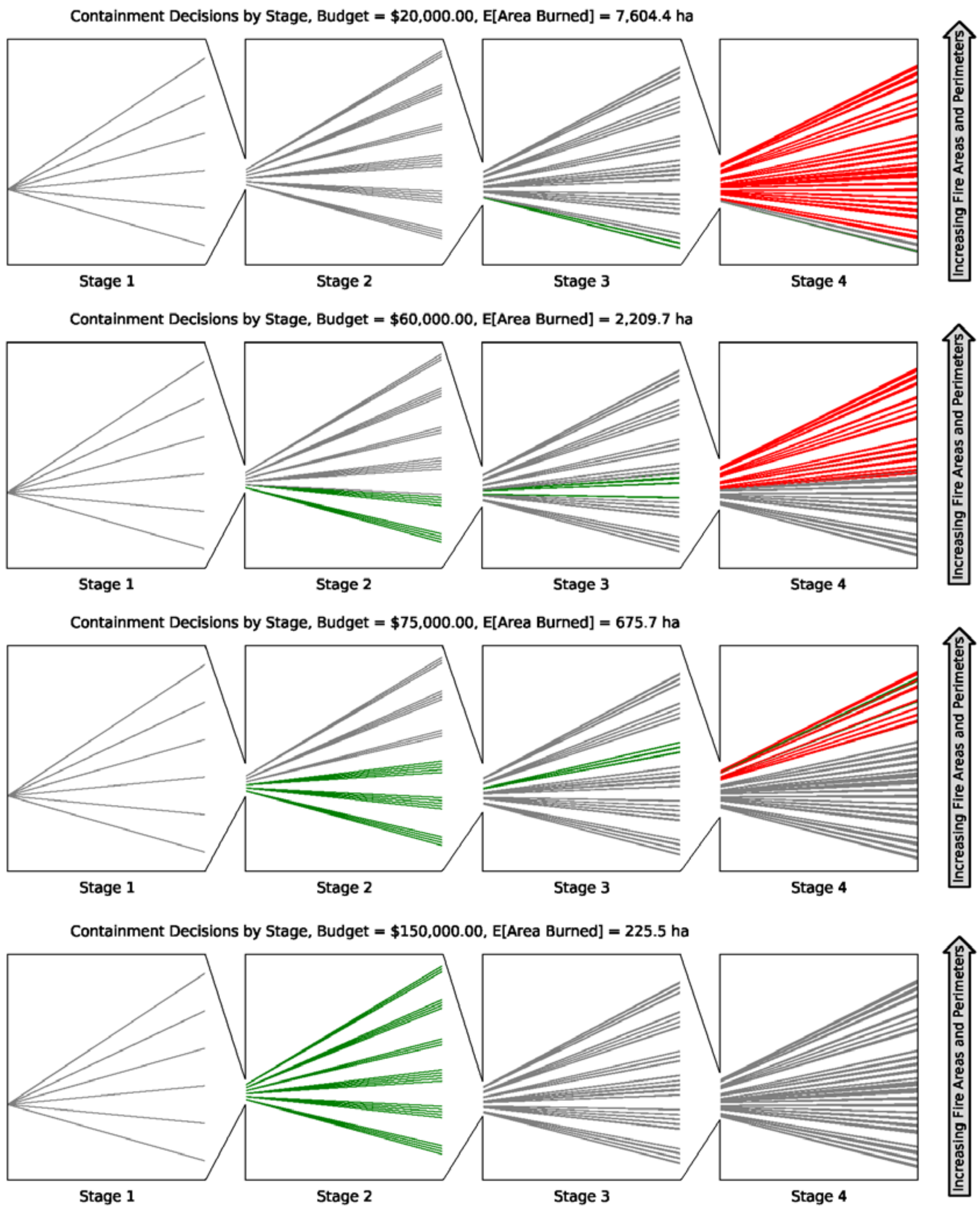



Figure 17: Exploded tree diagrams with four levels of $TC$ parameter.

We can further elucidate the relationship between suppression performance, as captured in the objective function value, and the level of the budget constraint by solving the Fire-S stochastic program for many values of $TC$.

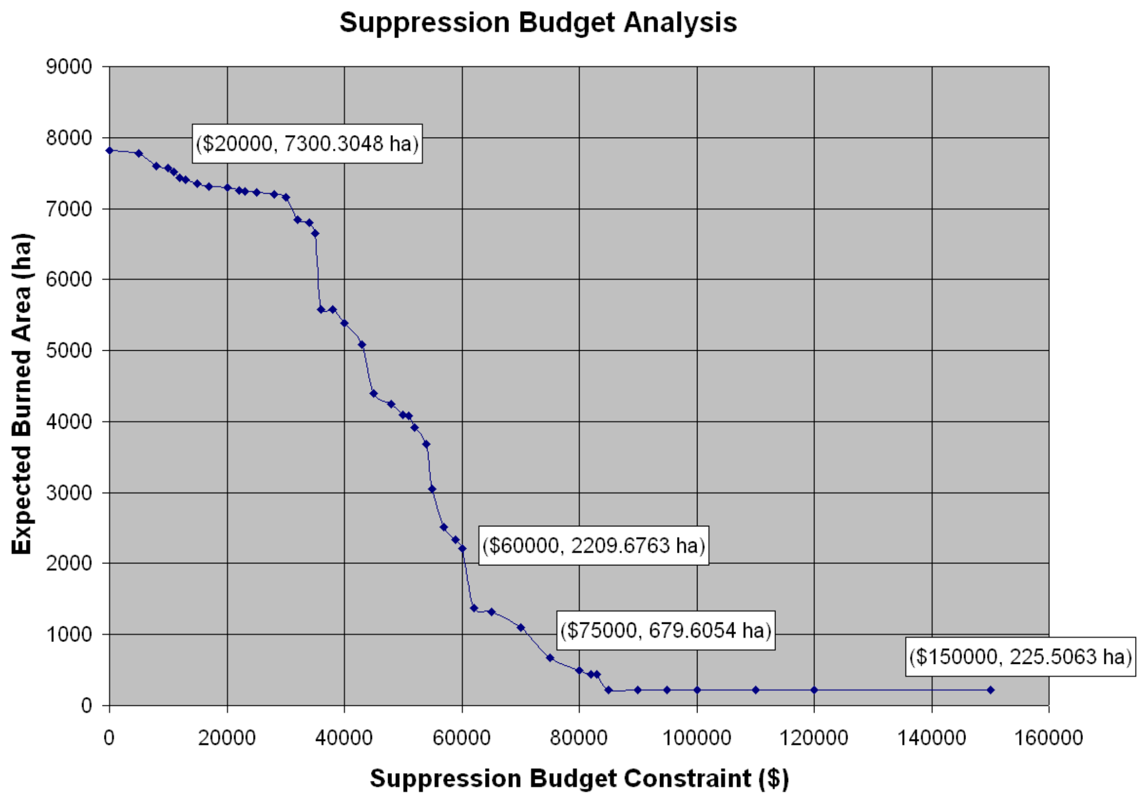



Figure 18: Expected burned area for various suppression budget levels.

Figure 18 shows the results of the suppression budget analysis. The shape of the curve is familiar to fire economists as a basic form of a production function. A suppression curve is upside down and backwards from a typical economic production function due to our minimization context in which ``production" depends on dollars input. For suppression budget constraints below about $35,000, the marginal decrease in expected burned area is quite small. From $35,000 to roughly $60,000, the marginal decrease in expected burned area is much larger. In this region, the discrete nature of resource dispatch is readily apparent because some of the drops are large compared to others. For example, when $TC = \$60{,}000$, expected burned area is over 2,000 ha, but adding $2,000 dollars to the budget constraint allows the program to afford some resource package that reduces the expected burned area to well below 1,500 ha. Above

$60,000 marginal decrease in expected burned area becomes nominal because we are moving towards total Stage 2 containment and the model cannot perform any better. A suppression curve like Figure 18 is a fundamental fire planning tool and can help demonstrate budget requirements to funding agencies. For an ignition in the Deerfield zone, a fire manager may want to realize an area-based suppression goal of 3,000 to 4,000 ha and can use this simulation, in combination with others if necessary, to justify a budget request of $55,000 for the fire. Since a planning process makes more sense on a seasonal level, we will discuss options for a multiple fire version of the Fire-S model in Section 5.8.

## 5.3 Version Without Recourse

Not only is the statement in Section 3 of the Fire-S stochastic program complicated, but the entire parameterization process discussed at length in Section 4 is complex. All the complications arise from the multistage stochastic program formulation with recourse, but what benefit, if any, does the complexity afford? We study the results of a program without recourse and then one without multiple stages to study this question in the context of our specific BHNF ignition. The suppression budget analysis of Section 5.2 provides a wonderful venue to compare these two variations with the full model.

To make each of these variations we start with the stochastic program in (3.1) through (3.10) and add an extra set of constraints in each case. A common theorem from [3] says that adding constraints to a math program cannot result in a better optimal solution. So neither of these versions will lower expected burned area, but the exercises are informative nonetheless because we can study the extent of the worsening in the optimal solutions.

To start, let us eliminate the possibility for recourse decisions. By definition, a recourse decision gets made after some random event is realized. In our case, the random events are fire perimeters and in each Stage $t$ we realize many perimeters $P_{\langle k_t \rangle}$ with different conditional probabilities $p_{\langle k_t \rangle}$. In order to disallow recourse, we must enforce an extra set of constraints that require all dispatch decisions to be the same for each stage. In other words, the fire manager is allowed to know the probabilities $p_{\langle k_t \rangle}$, but must make only one set of dispatch decisions at each stage. Given $t$, we force $x_{r,\langle k_t \rangle}$ to be the same for each $\langle k_t \rangle$. For example, in Stage 1

$$\forall r: \quad x_{r,(1)} = x_{r,(2)} = \cdots = x_{r,(K_1)}.$$

In Stage 2, the constraints become more complicated so we will state them as well:

$$\forall r: \quad \begin{cases} & x_{r,(1,1)} & = & x_{r,(1,2)} & = & \cdots & = & x_{r,(1,K_2)} \\ = & x_{r,(2,1)} & = & x_{r,(2,2)} & = & \cdots & = & x_{r,(2,K_2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ = & x_{r,(K_1,1)} & = & x_{r,(K_1,2)} & = & \cdots & = & x_{r,(K_1,K_2)} \end{cases}$$

An analogous set of constraints is added to make Stage 3 dispatch decisions uniform too.

The parameterization is exactly the same as the full model. We solve the Fire-S stochastic program for various levels of the $TC$ just as in Section 5.2. Results are shown in Figure 19. We will discuss them after presenting the single stage version in Section 5.4.

## 5.4 Single Stage Version

Next, let us implement an extra set of constraints that eliminate the opportunity to make distinct, stage-specific dispatch decisions. Equations (3.1) through (3.10) allow the

the fire manager to make distinct decisions at each node along a branch. We can turn the program into a single stage version by forcing all dispatch decisions along each branch to match. In this version the fire manager can still see the entire tree, but the decisions will be made for the duration of the model right after Stage 1, following the smoke report. Single-stage recourse still applies because decisions will be made based on the simulation outcomes and their associated probabilities $p_{\langle k_t \rangle}$. The extra set of constraints force $x_{r,\langle k_t \rangle}$ to be the same for each $t$. These constraints can be written

$$\forall r, \langle k_3 \rangle : \quad x_{r,\langle k_1 \rangle} = x_{r,\langle k_2 \rangle} = x_{r,\langle k_3 \rangle}.$$

Using a branch from Figure 2 as an example the constraint is

$$\forall r : \quad x_{r,(2)} = x_{r,(2,1)} = x_{r,(2,1,2)}.$$

We implement this set of constraints and perform the budget analysis described in Section 5.2.

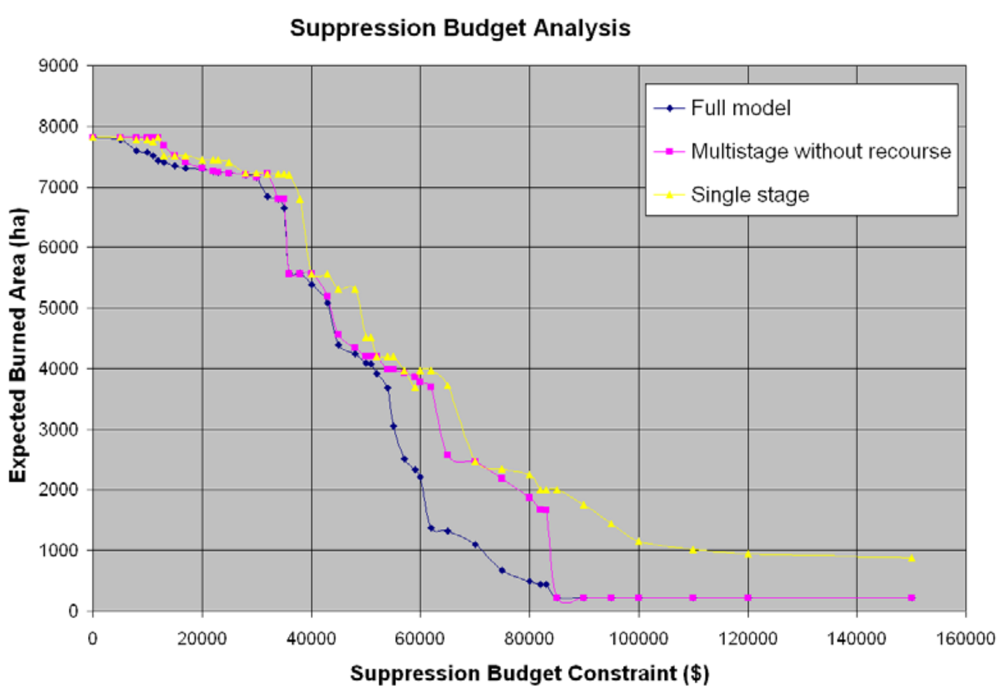



Figure 19: Suppression budget analysis for restricted models.

Figure 19 shows the results of the comparison between the full model and both restricted versions. Let us first consider the extremities of these curves. All models tend to perform about the same for the region below $35,000. For small budgets, dispatch is choked by lack of funds, so the complications of the full model are not a significant advantage. Above about $85,000 recourse is not a factor because we are observing total Stage 2 containment so recourse lends no significant advantage to the program. We see the single stage version of the model does not reach the same total Stage 2 containment floor after $85,000 as the other two versions. This is somewhat of a fabrication due to the set of constraints in (3.7). If we force dispatch decisions to be the same along each branch, then the constraints in (3.7) forbid Stage 2 and Stage 3 containment scenarios. On one hand, this is exactly what we want to assume for a single stage version because we cannot have staged decisions, which includes declaring containment before the end of model. On the other hand, this is not very realistic. To study a single stage model, we would most likely work with one of shorter duration, given the amount of detail we incorporate in the program. Regardless, the tail of the single stage version's suppression curve reflects *total Stage 4 containment*, as opposed to total Stage 2 containment.

Next, consider the region from $35,000 to $85,000 where the curves diverge. The full model shows no clear advantage over the one without recourse until about $55,000. At this point the optimal solutions without recourse are consistently higher expected burned areas than the full model until total Stage 2 containment can be achieved at $85,000. This is strong support, in this specific case and context, for recourse. Recourse gives the program an ability to navigate alternatives when there are many to choose from. In this region, there is sufficient budget money available to realize a wide variety of containment scenarios without total Stage 2 containment. Letting the model make recourse decisions according to typical and fringe fire behaviors allows for a more accurate and possibly realistic reflection of the spectrum of suppression tasks. Whether or not this is an advantage for the fire manager during the planning process becomes a question of specificity of planning instead of modeling limitations.

The single stage version begins to lag near $35,000, catches up at $40,000, lags again, catches up at $50,000 or so, and then lags for all higher values of $TC$. We can interpret this as an indication of the advantage of multiple stages in this type of model. When a single dispatch is made, there will be losses when the fire grows rapidly in the late stages of the model. According to Figure 19 these losses outweigh the gains of simultaneously preparing for all typical and fringe fire behaviors in a single dispatch. The flexibility of a multistage decision process is apparent.

Figure 19 and the associated discussion represent a single case study, which is insufficient to make conclusions about the methods described in this model in general. In this case, the added complexity of multiple stages and recourse do change outputs. If we assume multiple stages and recourse increase the realism of the model, then this allows us to conclude, in this case, that the added complexity exhibits strong gains.

## 5.5 Interactions

Thus far we have alluded to interactions as an important component of the resource allocation for fire suppression problem by noting this formulation lacks an interaction term. We will explore why an interaction term introduces a greater level of complexity in the model and suggest two ways to capture interactions within the current framework.

Recall the classic containment constraints in (3.4) and (3.5) of Section 3. We initialize line-building with the assumption

$$\forall \langle k_1 \rangle : \quad l_{\langle k_1 \rangle} = 0,$$

track line production for $t = 2,3$ and $4$ with the book-keeping variable $l_{\langle k_t \rangle}$ using

$$\forall \langle k_t \rangle : \quad l_{\langle k_{t-1} \rangle} + \sum_{r=1}^{R} \left( x_{r,\langle k_{t-1} \rangle} L_{r,\langle k_t \rangle} \right) = l_{\langle k_t \rangle},$$

and then decide containment for $t = 2,3$ and $4$ based on the following set of constraints

$$\forall \langle k_t \rangle : \quad l_{\langle k_t \rangle} \geq f_{\langle k_t \rangle} P_{\langle k_t \rangle}. \qquad (5.1)$$

Say we want to include an interaction between line production and fire spread. We seek some function $g$ that gives the interaction between $l_{\langle k_t \rangle}$ and $P_{\langle k_t \rangle}$ so that we have an adjusted estimate for perimeter $\hat{P}_{\langle k_t \rangle}$ based on what the suppression resources have previously done on the fire. In other words,

$$\hat{P}_{\langle k_t \rangle} = g\left( P_{\langle k_t \rangle}, \hat{P}_{\langle k_{t-1} \rangle}, l_{\langle k_{t-1} \rangle} \right).$$

There are countless ways to create the function $g$, all with varying levels of complexity. We will explore a relatively simple choice. Suppose we approximate the interaction by assuming line production decreases fire perimeter according to some scalar attack parameter $\alpha(t)$ that describes the effectiveness of line building at each Stage $t$. This allows us to create a family of functions $g_{\alpha(t)}$ to describe the interaction:

$$\hat{P}_{\langle k_t \rangle} = g_{\alpha(t)}\left( P_{\langle k_t \rangle}, \hat{P}_{\langle k_{t-1} \rangle}, l_{\langle k_{t-1} \rangle} \right)$$

$$= \begin{cases} P_{\langle k_t \rangle} - \alpha(t) \cdot l_{\langle k_{t-1} \rangle} & if \quad P_{\langle k_t \rangle} - \alpha(t) \cdot l_{\langle k_{t-1} \rangle} > \hat{P}_{\langle k_{t-1} \rangle} \\ \hat{P}_{\langle k_{t-1} \rangle} & if \quad P_{\langle k_t \rangle} - \alpha(t) \cdot l_{\langle k_{t-1} \rangle} \leq \hat{P}_{\langle k_{t-1} \rangle} \end{cases}. \qquad (5.2)$$

The function $g_{\alpha(t)}$ is piecewise so that we do not somehow decrease fire perimeter from one stage to the next. For this discussion we will assume $\alpha(t)$ is small enough that the top option in (5.2) holds. For example, take $\alpha(t) = 0.2$ for each Stage $t$ and say the fire simulation shows a spread from $\hat{P}_{\langle k_{t-1} \rangle} = 6.6$ km to $P_{\langle k_t \rangle} = 10.3$ km. If we allocate enough resources to build $l_{\langle k_{t-1} \rangle} = 5.2$ km of line, then the adjusted perimeter would be

$$\hat{P}_{\langle k_t \rangle} = g_{0.2}(10.3, 6.6, 5.2) = 10.3 - 0.2 \cdot 5.2 = 9.26 \quad km.$$

In this way, we could continuously adjust the fire perimeters as the model progresses.

These ideas are sound, but implementing the model with $\hat{P}_{\langle k_t \rangle}$ in place of $P_{\langle k_t \rangle}$ has two critical pitfalls. First, in Farsite we simulate fire spread without line interaction. Farsite does contain the features to implement barriers that act as fire line, but this would complicate simulation immensely. We saw in Section 4 that the parameterization process is spatially explicit, so line building parameters would also have to be spatially explicit, which means careful consideration of terrain, fuel model, and strategy. We could choose to be ignorant of this pitfall and work with the attack parameter $\alpha(t)$ to avoid complicating the fire simulation process, but the second pitfall remains.

The second pitfall is that even this simple treatment of interaction introduces a non-linearity in the set of constraints from (3.5). Suppose we substitute $\hat{P}_{\langle k_t \rangle}$ for $P_{\langle k_t \rangle}$ in (5.1). Then we have

$$\begin{aligned} \forall \langle k_t \rangle : \quad & l_{\langle k_t \rangle} \geq f_{\langle k_t \rangle} \hat{P}_{\langle k_t \rangle} \\ &= f_{\langle k_t \rangle} \left( P_{\langle k_t \rangle} - \alpha(t) \cdot l_{\langle k_{t-1} \rangle} \right) \\ &= f_{\langle k_t \rangle} P_{\langle k_t \rangle} - \alpha(t) f_{\langle k_t \rangle} l_{\langle k_{t-1} \rangle}. \end{aligned} \tag{5.3}$$

The first term in expression (5.3) is familiar, but the second term has the product of two decision variables, which is non-linear. Of course, non-linear programs can be solved, but additional techniques would be required.

An alternative way to incorporate an interaction term and sidestep non-linearity would be to use the line production rate parameter $L_{r,\langle k_t \rangle}$. Recall from Section 4.3 that the values of $L_{r,\langle k_t \rangle}$ are assumed constant across all stages. This need not be the case and varying $L_{r,\langle k_t \rangle}$ for each scenario will not increase the problem size whatsoever. The crux of this idea is to account for resource safety in the line production rate parameters. How do we propose to do this? As we saw in Figure 12 in Section 4.2, each Farsite simulation generates a wealth of information about fire behavior under each scenario. Any of these outputs could be used in the parameterization process. Suppose we add a flame length parameter for each scenario $FL_{\langle k_t \rangle}$ to the list in Section 3. Now we can create resource-specific line production rates as a function of flame length

$$L_{r,\langle k_t \rangle}\left(FL_{\langle k_t \rangle}\right)$$

as a proxy for a true interaction term. We propose some possible line production functions based on the values from Table 6 in Section 4.3 and the general safety guidelines in Appendix B of the Fire Line Handbook [7].

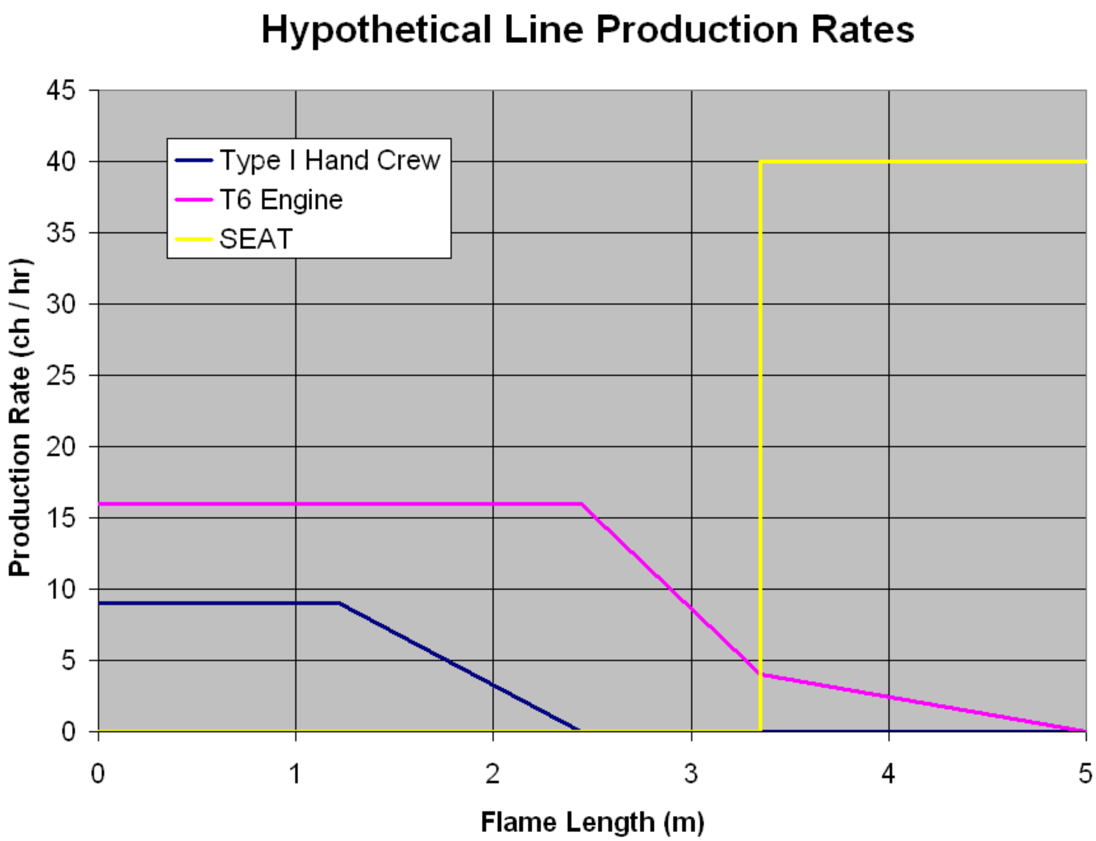



Figure 20: Hypothetical line rates as a function of flame length.

Figure 20 shows a hypothetical example for a Type I hand crew, a Type 6 engine, and a Single Engine Air Tanker (SEAT). Each resource has a maximum production rate at which it builds line under practical and safe conditions. A hand crew might be able to build line for low flame lengths, but then their abilities taper as the fire becomes more intense until some threshold, shown in Figure 20 to be about 2.5 meters or about 8 feet, where they must leave the fire line for safety. An engine can produce longer, but at lower and lower rates as flame length increases. Although we have not included aircraft in our exploration, a SEAT is shown in Figure 20 as well. It may not be practical to use such an aircraft on small fires, but it is immediately and highly effective at some threshold flame length and can be used until fire behavior becomes extreme.

Using flame length is a simple demonstration, but any combination of fuels, weather, and topographic information could be combined to scale $L_{r,\langle k_t \rangle}$. This would be an incredibly interesting avenue of study.

## 5.6 Forecast Availability

In Section 4.1 we discussed the generalized, Euclidean distance measure in Equation (4.1) and how it serves as a metric to compare forecast $\mathbf{Fx}$ and weather vectors $\mathbf{Wx_i}$. But, where do each of these vectors come from? Each $\mathbf{Wx_i}$ comes from the hourly RAWS data at the given forecast time (recall that we used 1000 and 1400 hours). In our treatment of the problem $\mathbf{Fx}$ also comes from the RAWS hourly data set. Since we study a fixed ignition, it makes sense to develop the model using some historical ignition date and treat the associated weather stream as the fire weather forecast. For example, we can calculate an energy release component (ERC) of 57, a spread component (SC) of 14, and an afternoon 1-hour fuel moisture of 4 % for July 10, 2003. The dryness and winds on this day, as indicated by these ERC, SC, and 1-hour fuel moisture values, indicates an ignition on this day, whatever the cause, would likely begin to spread in the dry fuel bed. So we use the actual hourly observations from July 10, 2003 and July 11, 2003 as the forecast stream and create $\mathbf{Fx}$ vectors for differencing directly from the historical data.

Even though Section 4.1 gives a rigorous treatment of the mathematics behind the forecast errors, our model runs are essentially ignorant of a true fire weather forecast. For instance, the clusters we use from Figure 5 show some days with precipitation and some without. A true fire weather forecast will state, with relatively low uncertainty, whether there will be rain or not. By including all the weather days, with precipitation and without, we ignore this forecast. In terms of a planning level model, this is exactly what we want to do because we cannot predict very well what the forecasts will be. In terms of an operational level model, we should whittle down the initial set $W$ so that it contains records from the category that best matches the qualitative forecast.

By using historical weather as the forecast weather we are also assuming the forecast is perfect. $\mathbf{Fx}$ will actually match the observed weather. While fire weather forecasting is an amazingly accurate process, it has some associated uncertainty. This is reflected in the way fire weather forecasts are relayed through dispatch. Rarely does a forecast read, ``Temperature at 1000 hours will be $68^\circ$F with a humidity of $41\%$, wind speed of $4.5$ mph out of $198^\circ$." A fire weather forecast is more likely to say, ``Morning temperatures in the high $60$s to low $70$s with winds of about $4$ mph out of the south. A cold front will move moisture into the area by $1500$ hours." This categorical statement of a forecast actually fits very nicely into the cluster framework we have already established. In Figure 5 we observed underlying weather categories and listed them in Table 5. This suggests an algorithm that would account for the forecast and move the model towards the operational realm. Suppose there is a smoke report today, then

1. Find an historical weather record with a similar ERC and SC.
2. Use this record as $\mathbf{Fx}$ and create the Stage 1 clusters from $W$.
3. Obtain a real fire weather forecast.
4. Match qualities of the real forecast to one of the weather categories suggested by the cluster analysis.
5. Adjust $W$ to only include members of this cluster.
6. Parameterize and run the Fire-S stochastic program.

Running the model in this way will reduce the amount of data in the scenario tree, but will reflect a spectrum of possible fire suppression tasks that agrees with the forecast on an operational level.

Although we did not track them down for this project, we expect some sort of forecast archive database exists within the National Weather Service. Historical forecast data could be combined with historical RAWS to create a more operational historical analysis of this problem.

## 5.7 Operational Limitations

As a whole, this work seems attractive as an operational fire suppression model. Section 2 with its discussion of multistage containments and specific resource packages sounds especially like the functionality features of an operational model. As is, the model is structured for planning purposes, but we will briefly lay out some suggestions for the interested reader to move towards an operational version.

An operational version would be most successful with

- A rigorous treatment of suppression resource and fire spread interactions (see Section 5.5).
- A way to incorporate fire weather forecasting (see Section 5.6).
- Selection of a realistic resource set for the region of interest (like Table 6).
- Careful calibration of line production rates and fire spread.

## 5.8 Moving Forward

As an exploratory model, we have opened several intriguing avenues of study. Interactions and forecasting have already been suggested, but we would not need to create an operational version for any of the ideas proposed in this section.

A *multiple fire version* would be an immediately useful planning application. To move towards a seasonal budget analysis, expanding the single fire analysis in Section 5.2, we would need to model multiple fires with the possibility for simultaneous ignitions. The deterministic equivalent of the multi-fire problem has been formulated in [5] and [9]. A solid treatment of simultaneous ignitions would introduce the possibility of planning for a *``lightning bust'' event*. A lightning bust occurs when a dry lightning storm creates multiple ignitions on a landscape. A fire manager needs to plan for such an event because it typically requires more resources than normal fire business. With this model, the fire manager could use a large resource set (perhaps a regional or national set) and examine the outputs to decide dispatch levels during a lightning bust.

Throughout our entire exploration, we have used the same scope from Figure 1. The twelve hour stage length and four stage assumption are not requisite for the Fire-S model. This model exhibits a *strong flexibility in the temporal nature of the scope*. One could elect any scope of interest and study a single fire in more detail or do a longer duration analysis of many test cases.

Another option to adjust the scope and duration of the model would be to implement the model on a *rolling planning horizon*. Recall that a fire manager runs this model at the time of the smoke report with best available knowledge about likely and unlikely fire weather. As the fire grows, the weather changes, and suppression resources build fire line, the manager could update the parameterization and forecasts creating a new model to run given that some random events (such as weather and fire behavior) since the first run had been realized.

Section O of the BHNF Fire Management Plan [2] contains run cards for each fire response zone that the Interagency Dispatch Center in Rapid City, South Dakota is responsible for. These *pre-defined dispatch decisions* are based on many factors such as proximity to Wildland Urban Interface (WUI), road access, previous experience with fire on the landscape, and fuel characteristics. Resource packages depend on the ERC or burn index (BI) for time of the smoke report becoming more significant as the danger of a severe fire increases. This pre-planning is important and should be incorporated into the first stage dispatch. If some district were interested in the planning capabilities of the Fire-S model, then part of the customization would involve a careful account of any pre-defined dispatch decisions.

Lastly, the issue of *singleton clusters* during the hierarchical clustering process warrants attention. A singleton cluster is one with a single weather record in it. Should a singleton cluster occur before the fourth stage of the model, then the subsequent clusters do not branch any further. Sometimes singleton clusters reflect a data poor scenario tree, but not always. Singleton clusters may indicate an extreme weather pattern that could be relevant to planning and safety and so they should not be discarded as outliers until it can be determined that the weather represents a data-logging error or can be accounted for in some other way. When the model encounters a singleton with only a single branch, it is no longer stochastic because the unconditional probability associated with the branch is 100%. Perhaps this is acceptable because if such extreme or bizarre weather is occuring, then our best guess is to follow the historical weather stream to the end of the model's duration even if recourse and other probabilistic features are dropped.

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