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STREAM - AQUIFER SYSTEM ANALYSIS FOR CONJUNCTIVE-USE OPERATIONS

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STREAM-AQUIFER SYSTEM ANALYSIS FOR CONJUNCTIVE-USE OPERATIONS

By Ali Eshett[1] and M. W. Bittinger[2], M. ASCE

SYNOPSIS

Under the Prior Appropriation Doctrine, establishment of rights to use natural stream flows was determined on a basis of time, without regard to relative position on the stream or the existence of a ground water supply stored below the land. Later development of ground water supplies from alluvium below and adjacent to the streams has presented the need for planned operation of the ground water system in conjunction with the surface supplies, such conjunctive operation directed at both an alleviation of water right conflicts and an improvement of over-all efficiency of water use.

The interrelationships of the components of a stream-aquifer system are programmed for computer analysis. A simple hypothetical model is used to present sample results of operational relationships.

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INTRODUCTION

In an era of increasing demands upon scarce water supplies, efficiency of water use must continually be improved. The hydrologist is often faced with the problem of improving efficiency without materially interfering with long-established water rights. The prior-appropriation doctrine of water rights, prevalent in the Western United States, unquestionably served as an impetus to development and beneficial use of water supplies. After completion of development however, it is logical to assume other systems of water allocation may be employed which would result in higher beneficial use of the total water resources. Since water-rights are property rights in the use of water, any proposed change must recognize these rights. Before a plan is initiated the hydrologist or administrator responsible must evaluate the effects upon vested rights and develop proper compensation for those right-holders who will be adversely affected.

THE STREAM-AQUIFER PROBLEM

A case in point is referred to herein as the "Stream-Aquifer Problem." Typically, in the irrigated areas of the Western United States, the natural flows of streams were first diverted in the latter half of the 19th century to irrigate adjacent lands. Establishment of rights to divert the natural flows was determined wholly upon time, without regard to relative position on the stream, and particularly with no appreciation or knowledge of the existence of a potential ground water supply stored below the land and its relationships with surface flows.

In fact, the irrigation activities, due to losses from reservoirs, canals and irrigated fields, tended to increase the amount of ground water in storage in the alluvial aquifers below and adjacent to the streams. As the ground water level rose, drainage back to the streams often converted them from influent to effluent conditions. This "return flow" effect is a recognized phenomenon in irrigated areas. Water users along lower reaches of the stream have become dependent upon and have acquired rights in the return flows.

Thus a stream-aquifer relationship became established in an unplanned way. Water rights were established which are highly dependent upon the ground water situation, but with no official recognition of this relationship. Concern has now developed because of increasing amounts of ground water pumped from wells tapping the same supplies that are providing surface water.

The stream-aquifer problem, as defined herein, is thus not only a problem of improving efficiency by integrated conjunctive use, but also involves many legal, and economic considerations. This paper describes only the physical analysis of a stream-aquifer system including planned operation of a ground water reservoir in conjunction with surface supplies. These techniques will later be applied to field situations with legal and economic aspects taken into consideration.

CONJUNCTIVE-USE PLAN

Certain characteristics of a conjunctive-use management plan designed to increase the efficiency of use of the total water resources within a stream-aquifer system seem logical. These include the following two major operational principles:

Order of surface water diversions determined by location rather than date. In order to obtain maximum benefits from return flows through reuse of the water, it is logical that upstream users should be given first opportunity to divert stream flows and put the water to beneficial use. Capture and reuse of return flow water may occur to the maximum extent under this method of operation.

Manipulation of ground water storage. Obviously, downstream surface-water right-holders would be injured under the above-described scheme of operation. The proposed plan, then, is not complete without adequate compensation to the downstream users by use of ground water storage. Reliance upon ground water supplies would be heaviest during periods of below normal surface flow, and replenishment of ground water storage would be accomplished during periods of above normal runoff.

The question of who should bear the additional cost of providing the compensatory water is pertinent here. Although many ramifications of this question could and should be explored, the basic theory of placing the financial obligation upon those benefiting from the change in the water allocation system should be the guiding factor.

Operation of a ground water reservoir is unavoidably more complex than operation of surface storage facilities. Response to withdrawals or inflow at one point in the reservoir is not immediate throughout the reservoir, making the system extremely time-dependent. The level of water in a ground water reservoir influences rates as well as amounts of natural recharge and discharge - the amounts of which must be predictable at any point in time. Intake at a rate equal to the flow in the stream is not always possible, as with an on-stream surface water reservoir. Similarly, full rate of discharge can not be attained by merely opening valves or gates.

Advantages of operational ground water storage can also be cited. Principal of these (compared to surface storage reservoirs) are protection from evaporation losses with no sacrifice in land area

HYPOTHETICAL STREAM-AQUIFER SYSTEM

To demonstrate the above operational principles, an analysis of a simple stream-aquifer system operation is described below. Figure 1 shows the general plan of the system, consisting of a reach of a river valley in which agricultural land is irrigated by three diversions from the stream. Beneath the irrigated land are alluvial sediments lying in a bedrock trench.

The interactions between surface water and ground water assumed for this model are illustrated schematically in Figure 2 and described below.

(a) Stream Inflow (QI)

The input into the system is denoted by the symbol QI. Monthly streamflow records from a station on the Arkansas River in Colorado were used as

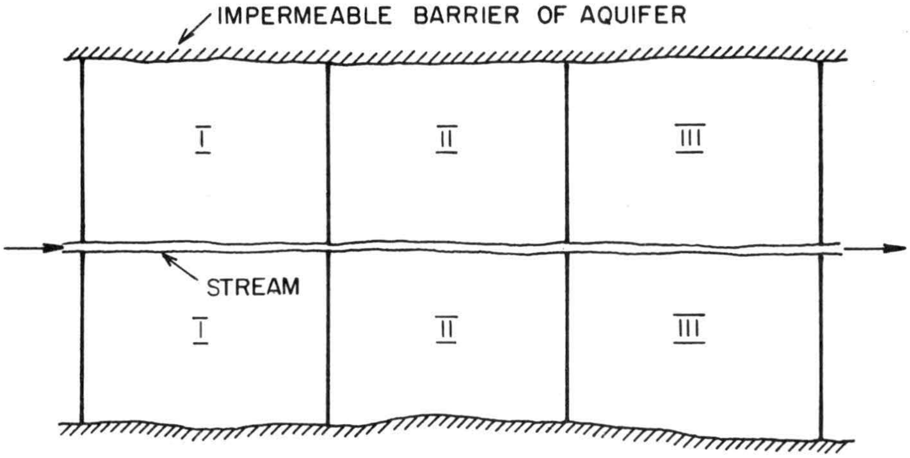

FIG. I SCHEMATIC PLAN OF SYSTEM

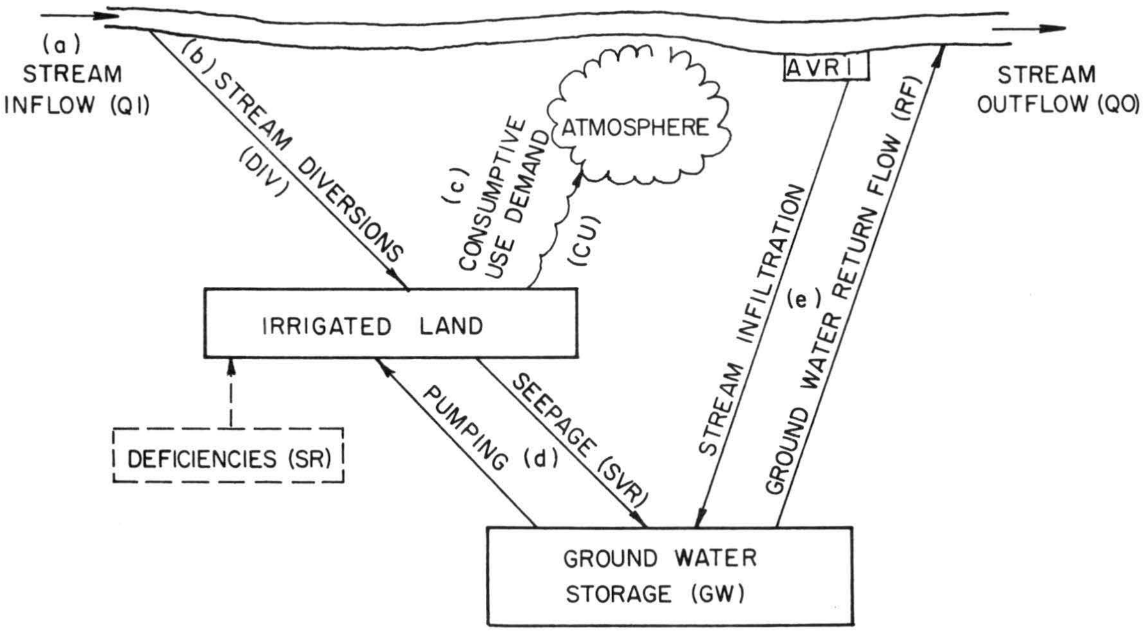

FIG. 2 SCHEMATIC DIAGRAM OF STREAM - AQUIFER SYSTEM COMPONENTS

base data for stream inflow into the hypothetical system. Since only twenty-two years of records were available, additional synthetic data were generated to attain a broad variation in flows and sequences. The synthetic generation of monthly flows utilized the mean, standard deviation and skewness coefficient of the logarithms of each month's flow (January, February, etc.) as well as the first serial correlation of logarithms of successive monthly flows. The technique used is similar to that described by the U.S. Corps of Engineers.[1] Eighteen additional 22-year periods were generated, providing a total of 418 years of data.

(b) Stream Diversions (DIV)

According to the first operational principle stated above, diversions from the stream are based upon position on the stream, rather than order of established priorities. Therefore, Unit I of the hypothetical stream - aquifer model is always allowed to divert an amount equal to its decreed right - or the entire flow of the stream if that flow is less than Unit I's decreed right. Accordingly, Unit II has the opportunity to divert its total decreed right, or the remaining total flow of the river, whichever is the smaller. Unit III, having the lowest point of diversion on the stream, is able to take maximum advantage of return flows from the upstream diversions, but must rely heavily upon ground water storage during months of inadequate surface supplies.

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[1] U.S. Army Engineer District, Corps of Engineers. Technical Bulletin No. 9, "Estimated Long-Term Storage Requirements and Firm Yield of Rivers." October 1963, Sacramento, California.

(c) Consumptive Use Demand (CU)

The consumptive use demand represents the optimum amount of water required to satisfy necessary evaporation and transpiration. For the example herein, the monthly consumptive use demand pattern illustrated in Figure 3 was assumed (precipitation was substracted on an average base). The values shown in the figure represent consumptive use demands relative to the peak month (July).

(d) Ground Water Outflow and Inflow Due to Irrigation

Because supplies are usually large and consumptive use demands low during spring and early summer, an excess of water is often applied to the land. This excess water moves downward (assumed to be uniformly distributed) into the ground water system. During months in which the diversions can not completely satisfy the consumptive use demands, ground water is pumped (also assumed to be uniformly distributed) to supplement the surface water supply. This inflow and outflow to and from the ground water system affects the amount of ground water in storage (the total storage volume available is finite, thus a limit on amount withdrawn may be reached during heavy pumping periods).

(e) Interchange of Water Between the Ground Water System and the Stream

Influent or effluent conditions on the stream are dependent upon the ground water elevation in relationship to the stream bed. For influent conditions (water table below the stream bed) the rate of seepage to the ground water system is controlled by the infiltration rate (AVRI). Under effluent conditions (water table above the stream bed) ground water flow rates as influenced by aquifer properties and gradient conditions control the return flow to

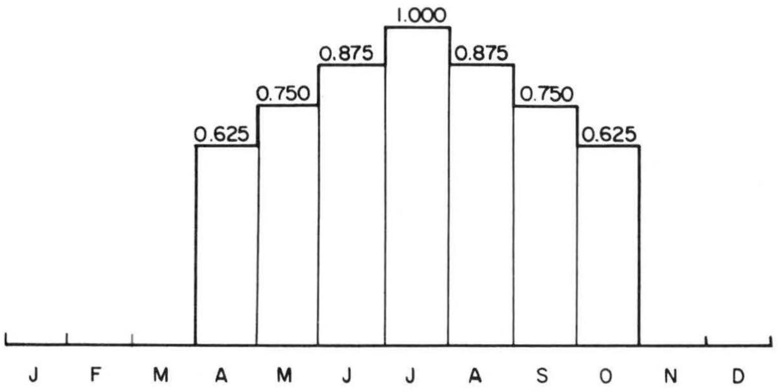

FIG. 3 ASSUMED PATTERN OF CONSUMPTIVE USE DEMAND

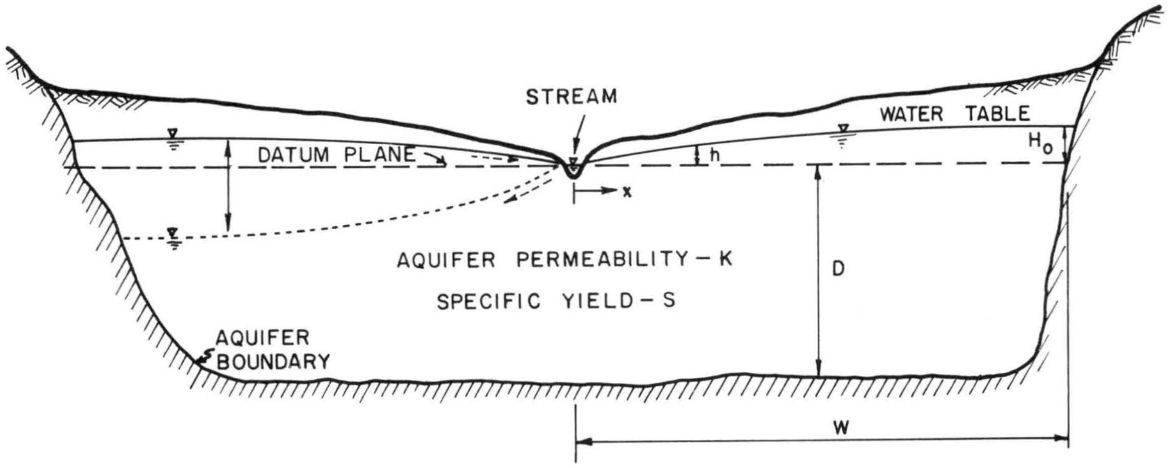

FIGURE 4

SCHEMATIC CROSS-SECTION OF RIVER VALLEY AND GROUND WATER RESERVOIR the stream. Figure 4 illustrates the two conditions, and the pertinent parameters involved. Equations for calculation of stream-aquifer relationships have been developed by Glover,[2] Maasland,[3] and others.[4] A simplification of the Glover equation was chosen which eliminates the need for superposition of incremental additions or withdrawals from the ground water system and allows consideration of the situation in which the water table falls below the stream bed. This simplification is based upon the assumption of a sinusoidal shape of the water table,

$$h = H_o \sin\left(\frac{\pi x}{2w}\right) \exp\left(-\frac{\alpha \pi^2 t}{4w^2}\right) \tag{1}$$

which is a particular solution of the differential equation describing the flow system:

$$\frac{\partial h}{\partial t} = \alpha \frac{\partial^2 h}{\partial x^2} \tag{2}$$

Equation (1) satisfies the following boundary and initial conditions:

$$h = H_o \sin\left(\frac{\pi x}{2w}\right) \quad \text{for} \quad 0 \leq x \leq w \quad \text{at} \quad t = 0 \tag{3}$$

$$h = 0 \quad \text{for} \quad 0 \leq x \leq w \quad \text{at} \quad t \to \infty \tag{4}$$

$$h = 0 \text{ at } x = 0 \text{ and } \frac{\partial h}{\partial x} = 0 \text{ at } x = w \text{ at } t \geq 0 \tag{5}$$

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[2] Glover, R. E., "Ground Water Movement," Engineering Monograph No. 31, Office of the Chief Engineer, U.S. Bureau of Reclamation, February, 1964.

[3] Maasland, Marinus, "Water Table Fluctuations Induced by Intermittent Recharge," Journal of Geophysical Research, Vol. 64, No. 5, May, 1959.

[4] "Proceedings of the Symposium on Transient Ground Water Hydraulics" Civil Engineering Section, Colorado State University, December, 1963.

in which h is the ground water head above the stream bed at time t and distance x from the stream. H_o is the initial height of the water table at the far edge of the aquifer (at distance w), and α is an aquifer constant equal to the product of the permeability and saturated thickness divided by the specific yield.

The part of any ground water accretion introduced at time 0 , remaining at time T , is then;

$$PR = \frac{\left(\int_o^w h\,d\,x\right)_{t\,=\,T}}{\left(\int_o^w h\,d\,x\right)_{t\,=\,0}} = \exp\left(-\frac{\alpha\pi^2 T}{4w^2}\right) \qquad (6)$$

(f) Deficiencies (SR)

One of the results of the stream-aquifer analysis reported herein is the deficiency of water or that amount of water required beyond available surface and ground water supplies to satisfy consumptive-use demands.

(g) Stream Outflow (QØ)

Stream outflow is the surface water flowing out of the hypothetical system. Operation of the system should strive to minimize this factor. Because of limits of stream bed infiltration, all surface flows are not used, even though the water table may be drawn below the stream bed. Greater efficiency (less stream outflow) could be accomplished through surface storage and/or planned artificial recharge. These two items are not considered in the example reported herein.

The block diagram for the computer program is shown in Figure 5. Although not reported in this paper many intermediate time and location

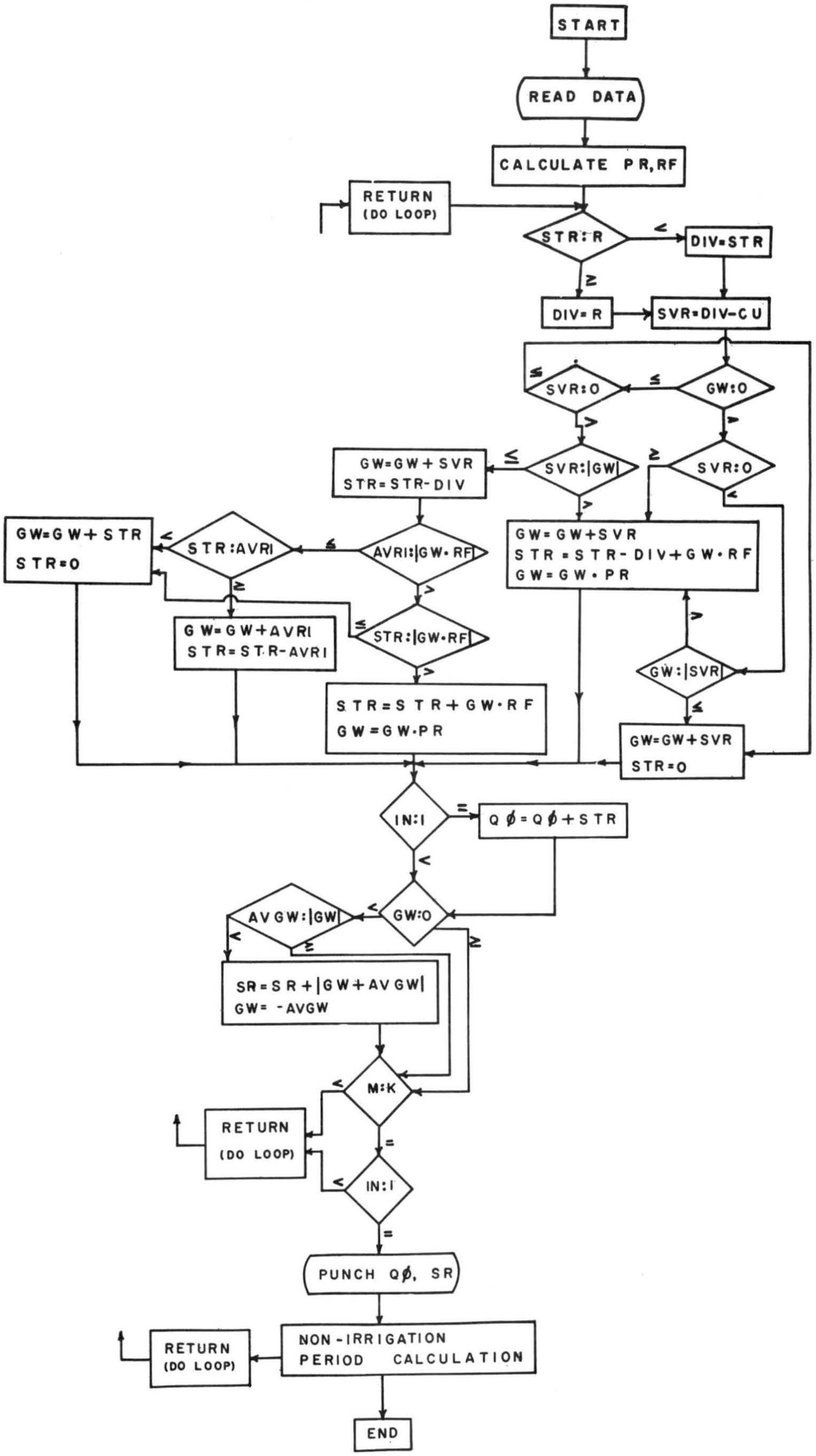

FIG. 5 BLOCK DIAGRAM OF COMPUTER PROGRAM

outcomes such as volume of diversions, amount of ground water pumped, ground water deficits and stream flows can be obtained. Also, the program may be varied as to the number of diversions, amount of decreed rights of each, pattern and amount of consumptive-use demand, ground-water reservoir storage limits, the starting volume of ground water storage, aquifer characteristics and limits of stream bed infiltration rates.

Some additions that may be desired include provision for ground water inflow and outflow from the system, precipitation, surface storage, and artificial ground water recharge.

HYPOTHETICAL MODEL SYSTEM ANALYSIS

Because word descriptions of the variables used and the results obtained become quite lengthy, the following abbreviations are defined:

F0	the percentage of the 418 events (years) in which a deficiency occurred.
S0	the percentage of the 418 events in which no deficiency occurred (F0 + S0 = 100 percent)
AI	the probability that deficiencies will be equal to or less than a specified amount, SRM.
DS	degree of satisfaction, defined as S0 + AI (100 - S0) or S0 + AI · F0 .

PCU	the annual consumptive-use requirement or demand stated as the percentage of the average annual stream inflow.
PGW	the usable volume of ground water storage capacity stated as a fraction of the average monthly stream inflow.
SR	annual deficiency, difference between total consumptive use requirement and water available from surface and ground water supplies.
SRM	the maximum amount of supplemental water required to obtain a specified degree of satisfaction
PSRM	the SRM as a fraction of the average monthly stream inflow.

The influences of two factors (PCU and PGW) on the system are reported herein. PCU was varied in amount, but the pattern of relative monthly consumptive use demand was held as shown in Figure 3. For each set of conditions, 418 values of SR were obtained. Frequency distributions of SR's were found to follow the gamma probability law represented by the density function:

$$f(u) = \frac{e^{-v} v^{p}}{\Gamma(p+1)} \tag{7}$$

in which p and a are parameters of the distribution with mean = $a(p+1)/p$ and standard deviation = $a(p+1)^{1/2}/p$, $v = u(p+1)^{1/2}$, u is the quotient of the variable and the standard deviation of the distribution.

When p is small, but larger than zero, the gamma density function is a positively skewed function represented by the general curve of Figure 6. The gamma distribution parameters, p and a, were computed from the

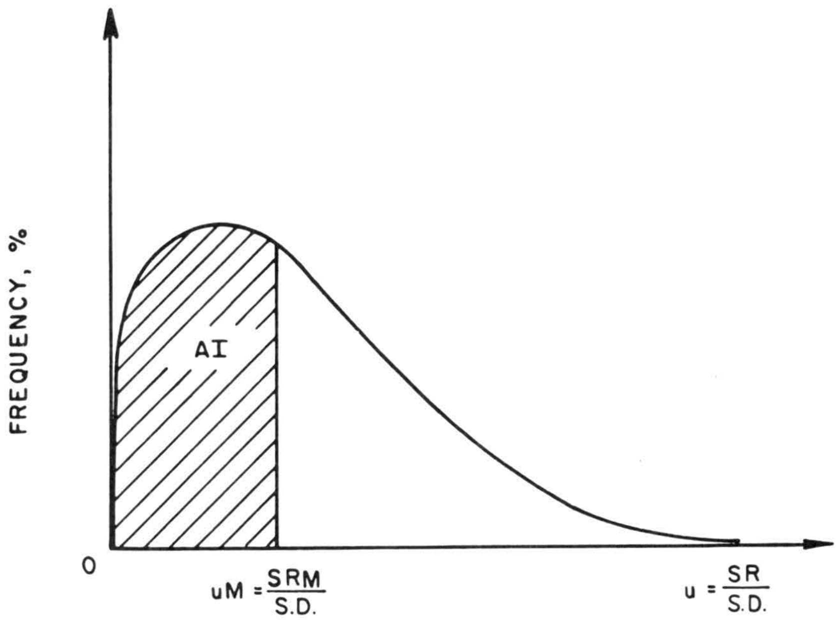

FIGURE 6. TYPICAL RELATIVE DISTRIBUTION OF DEFICIENCIES

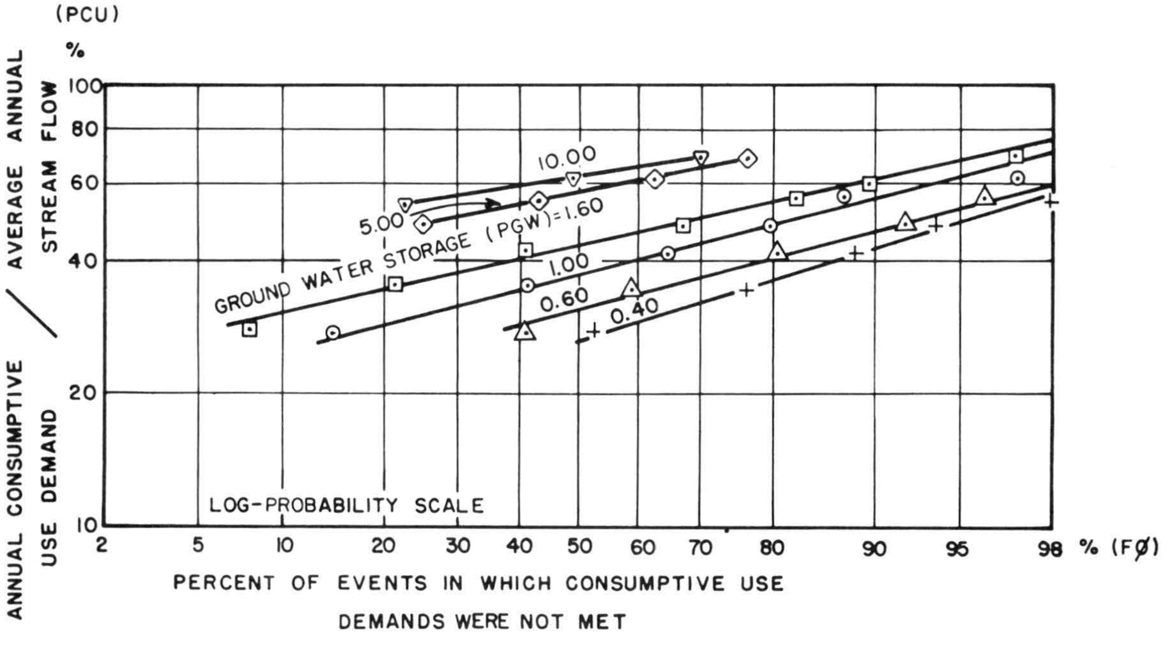

FIGURE 8. RELATIONSHIP OF PCU AND PGW TO PERCENT OF FAILURES

mean and standard deviation of each of the SR distributions. Using these parameters, the value of SRM for particular values of AI (or DS) were determined from tables of the incomplete gamma function,[5] giving relationships of DS and SRM for a range of PCU and PGW values.

SUMMARY OF RESULTS

Before presenting the relationships obtained between PCU, PGW, DS and SRM, it is of interest to observe the variation in results as a function of the number of 22-year periods (historical plus generated data). Figure 7 shows graphically the tendency for percentage of failures, average and standard deviation of deficiencies and SRM's for various DS to approach constant values as the number of 22-year periods increased. All of the curves, with the exception SRM's for DS = 70 and 90 percent, appear to asymptotically approach a horizontal line when all 19 periods are considered. The percentage of failures and standard deviation of deficiencies could have been as accurately determined with about 15 periods. An accurate PSRM value for a degree of satisfaction 90 percent would require several more periods than were used in this analysis. All of the curves in Figure 7 are for a PCU of 41.3 percent and PGW of 0.60.

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[5] "Tables of the Incomplete Gamma Function" edited by Karl Pearson, Cambridge University Press, 1957.

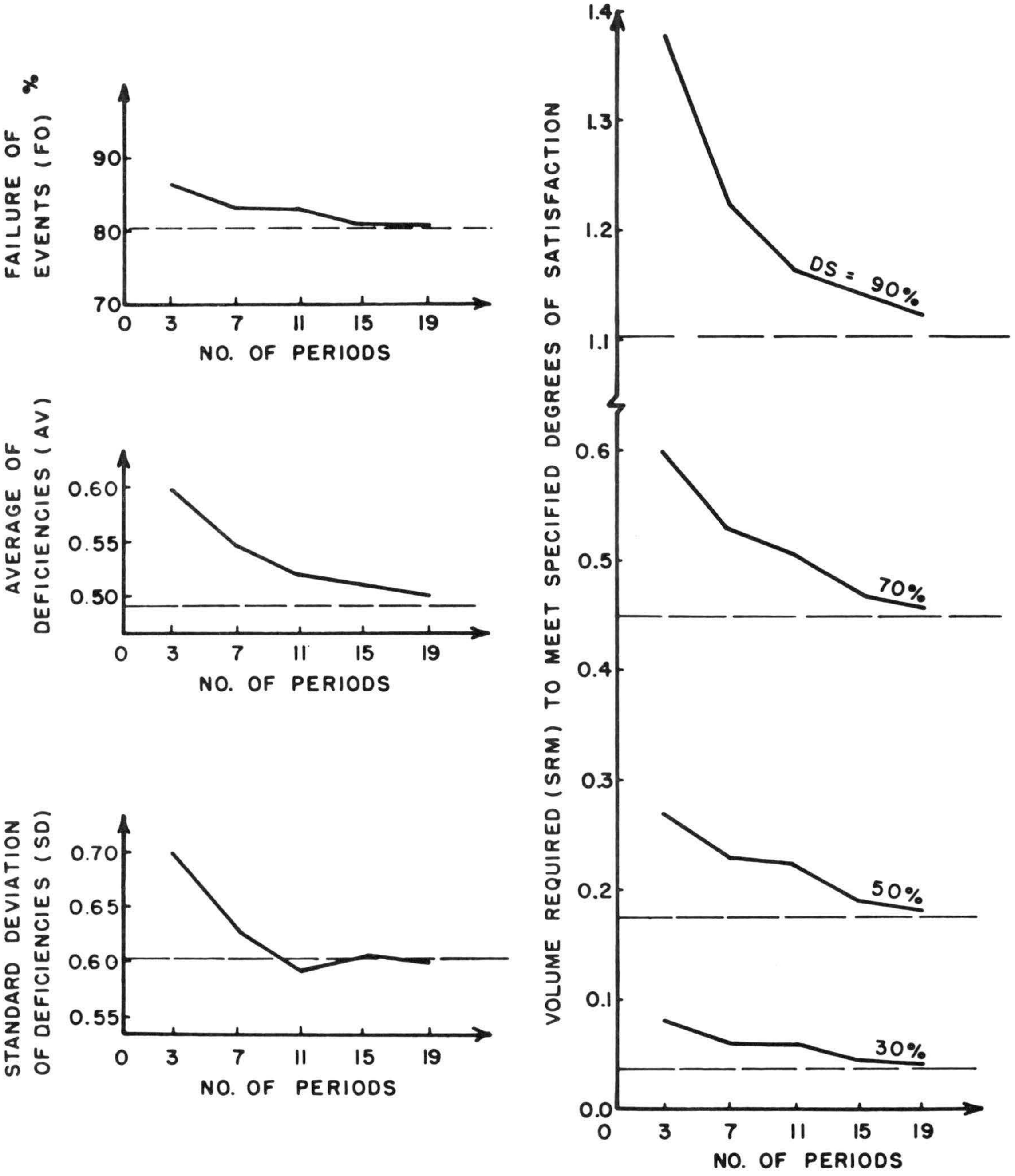

FIGURE 7

VARIATION OF THE OUTCOME AS A FUNCTION OF THE NUMBER OF PERIODS FOR PARTICULAR CONSUMPTIVE USE DEMAND AND GROUND WATER STORAGE

The influence of the two variables (consumptive use demand, PCU, and ground water storage, PGW) upon the percentage of events or years in which the consumptive use demands were not met (F0) is shown in Figure 8. The plot illustrates the importance of ground water storage volume. As can be seen, however, a storage volume of over 5 times the average monthly stream inflow has little influence on the percentage of failures. This is because of the infiltration restriction imposed upon the stream bed in this example.

Figures 9a and 9b illustrate relationships between PSRM, PGW, DS and PCU. Figure 9a is plotted for a particular degree of satisfaction (DS = 70 percent) showing the effects of PCU and PGW upon PSRM.

As an example, assume the annual consumptive use demand is 60 percent of the average annual volume of stream inflow (PCU) and the usable ground water storage volume is equal to the average monthly stream inflow (PGW = 1.0). From Figure 9a the maximum volume of supplemental water required in any year to insure a 70 percent degree of satisfaction is about 1-3/4 of the average monthly stream inflow, i.e., PSRM = 1.75.

For the same assumptions, if PGW is 5.0 then PSRM becomes only 0.25. Or using the graph a different way, assume a PGW of 1.60, a PSRM that can be supplied at 1.50 and a desired degree of satisfaction of 70 percent, then PCU can be as high as 61.6 percent.

Similarly, Figure 9b uses the same variables, showing a family of PSRM curves in relation to PCU and PGW. Of particular interest in Figure 9b

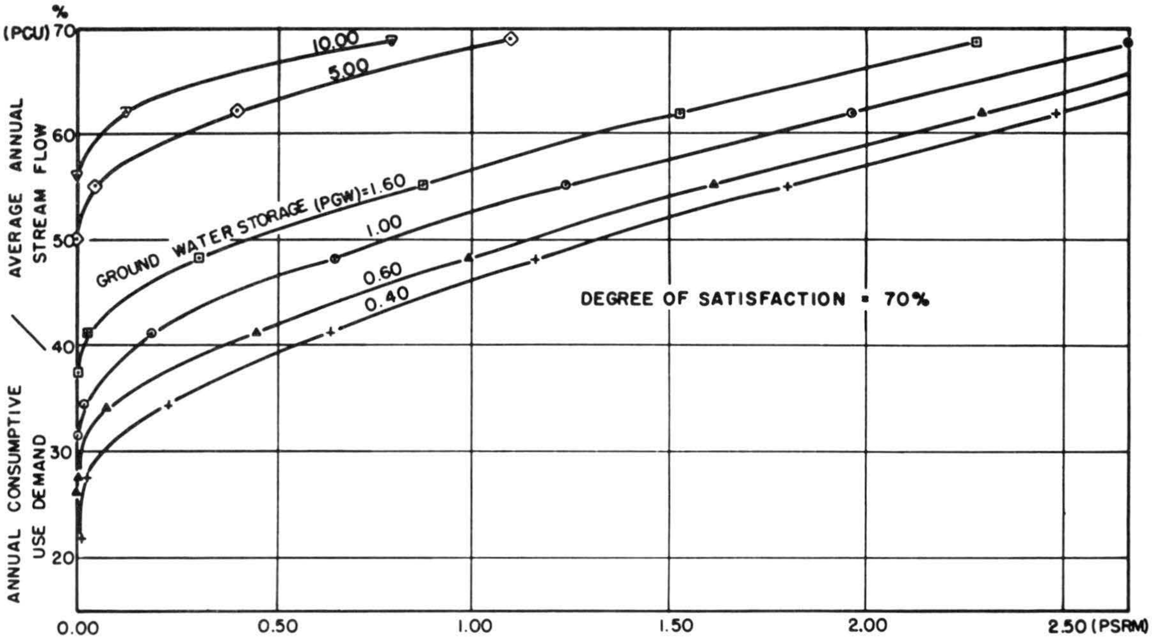

FIG. 9a MAXIMUM ANNUAL VOLUME REQUIRED / AVERAGE MONTHLY STREAM INFLOW TO OBTAIN 70% DEGREE OF SATISFACTION

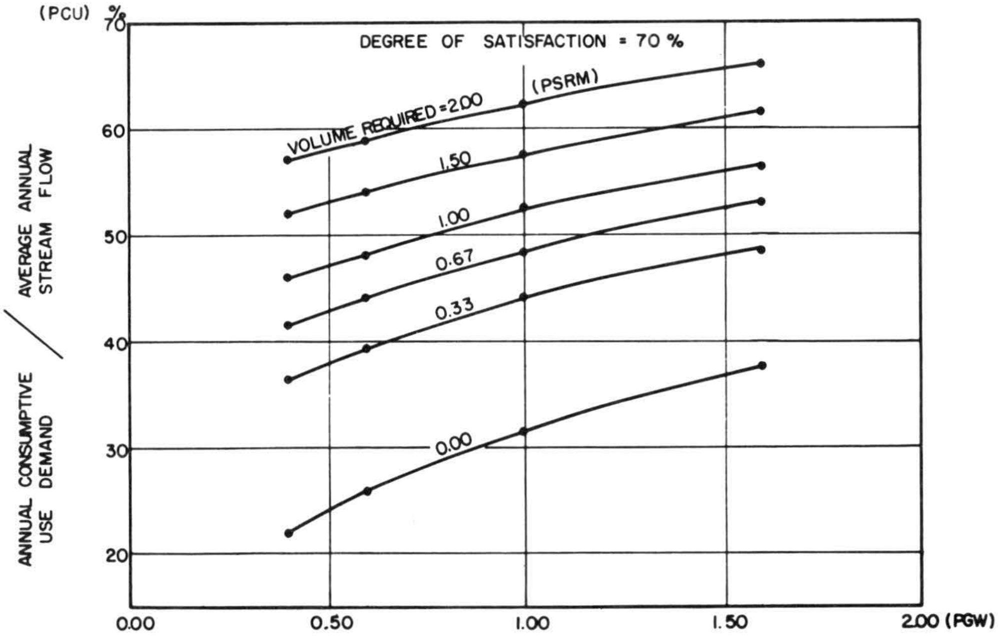

FIG. 9b GROUND WATER STORAGE, Ratio of usable storage volume to average monthly stream flow

is the lower curve representing PSRM = 0.0. This curve shows the maximum relationship between PCU and PGW in which no supplemental water is required in order to obtain a 70 percent degree of satisfaction. Notice that the values of PCU are rather low (less than 38 percent) for the range of ground water storage volume shown.

Figures 10a and 10b show the influence of PCU and PGW upon the degree of satisfaction (DS). Figure 10a is drawn for PSRM = 0, a condition in which no additional water beyond the stream flows and ground water pumping should be needed to meet a specified degree of satisfaction. If a point of intersection between PCU and PGW is under a desired DS curve, (which represents the expectation of a full supply from the system itself) there is no need for supplemental amounts of water. Figure 10a shows that to reach a reasonable degree of satisfaction when PCU is larger than 40 percent, the ground water storage must be larger than 1.0. Figure 10b presents a summation of the extreme values used illustrating the dependence of PSRM upon PCU, PGW and DS.

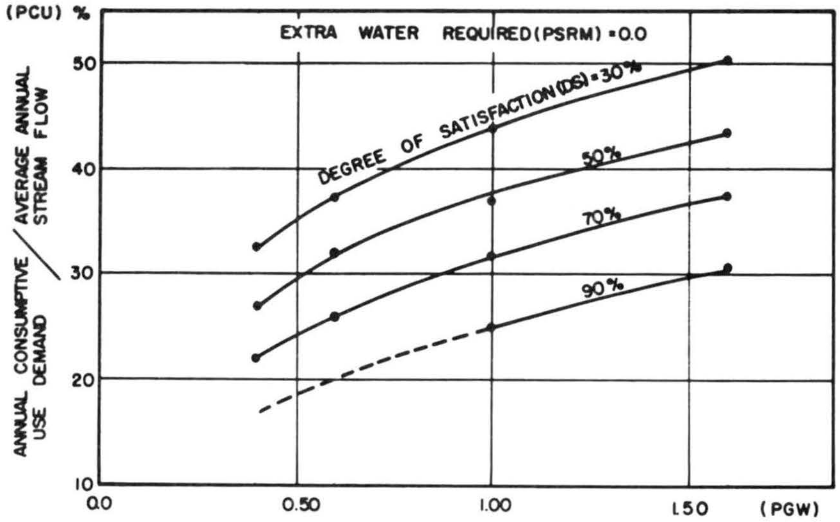

FIG. 10a GROUND WATER STORAGE, Ratio of usable storage volume to average monthly stream flow

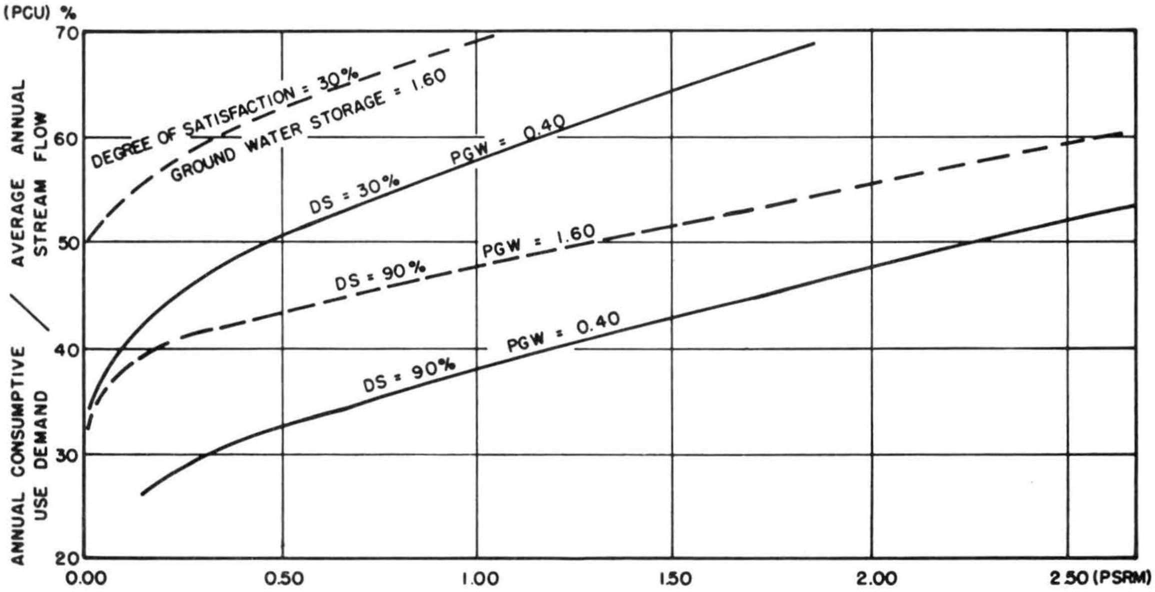

FIG. 10b MAXIMUM ANNUAL VOLUME REQUIRED / AVERAGE MONTHLY STREAM FLOW

CONCLUSIONS

Useful relationships between the components of a stream-aquifer system can be developed for analysis and design purposes. For practical application operational limits must be known, such as infiltration restrictions, usable ground water storage and the degree of satisfaction desired. In addition, legal and economic constraints need to be considered before application to specific areas.

ACKNOWLEDGEMENTS

Work described herein is part of a research study sponsored by the Colorado Water Conservation Board.

APPENDIX - Notation

The following symbols have been adopted for use in this paper:

$\Gamma(p+1)$	Gamma function of $(p+1)\int_0^\infty e^{-v} v^p \, dv$
v	$\frac{pz}{a}$
z	variable
p&a	parameters of Gamma distribution
u	variable/standard deviation of distribution
h	water table height above stream bed elevation
H_o	initial water table height at edge of aquifer
x	horizontal distance from stream
t, T	time
w	distance from stream to edge of aquifer
α	$\frac{kD}{s}$
k	aquifer permeability
D	saturated thickness of aquifer
s	specific yield of aquifer

SUMMARY FOR CIVIL ENGR.

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by

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The operation of a ground water reservoir in conjunction with surface-water supplies, where the two are in hydraulic connection is discussed. A computer program which describes the interrelationships of effects and responses within a stream-aquifer system is presented. Results from an analysis of the operation of a simple system are reported.

Key Words:
Ground Water, Management, Stream-Aquifer System, Computers, Conjunctive Use, Reservoir Operation.

Abstract:

The operation of a ground water reservoir in conjunction with surface water supplies, where the two are in hydraulic connection, is discussed. A computer program which describes the interrelationships of effects and responses within a stream-aquifer system is presented. Results from an analysis of the operation of a simple system are reported.

References:

Eshett, Ali, and Bittinger, M. W., "Stream-Aquifer System Analysis for Conjunctive-Use Operations."