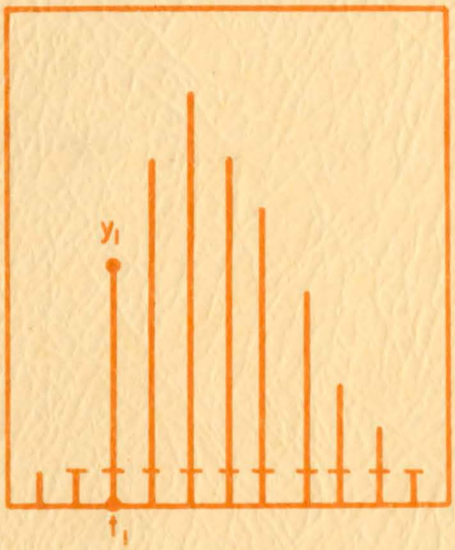
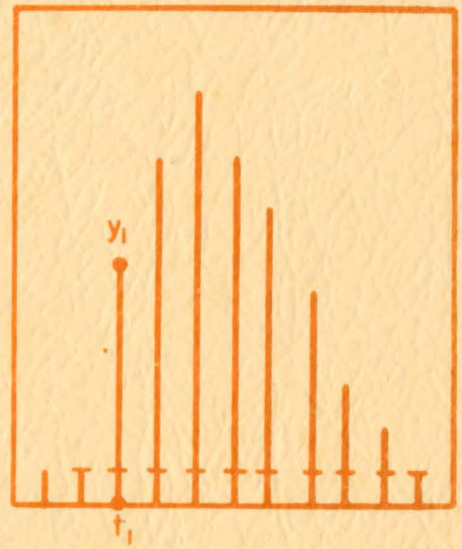
DERIVING A UNIT HYDROGRAPH IN THE ABSENCE OF DETAILED RAINFALL DATA



Runoff A



Runoff B

by

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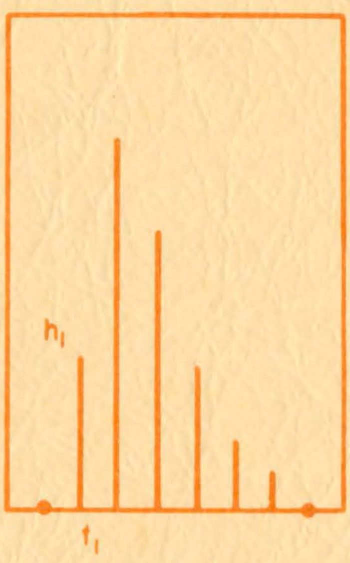

Unit Hydrograph

June 1973

CER72-73KJ-RR-EFS34

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ABSTRACT

A method of deriving a unit hydrograph from two different recorded flood events from the same watershed was tested. The method was originally proposed by De Laine. This technique was tested on floods recorded on a 1.07 square mile watershed in Arizona, on floods simulated on the 1/2 acre Experimental Rainfall-Runoff Facility at Colorado State University and finally on flood simulated on a 40 acre hypothetical watershed. It was found that De Laine's method is practical only on error-free data. If there is any error in the determination of the rainfall excess, or in the measurement of runoff or in the synchronization of rainfall and runoff records, the solution becomes unstable. The method produced unsatisfactory results for the Safford, Arizona and the CSU-ERRF data.

ACKNOWLEDGMENTS

The results on which this report are based were obtained from research sponsored by the Colorado State University Experiment Station under Project 114. Support during a part of this investigation was provided by the U. S. Department of Interior, Office of Water Resources Research as authorized by the Water Resources Research Act of 1964 and pursuant to Grant Agreement No. A-31-0001-3565, OWRR Project No. B-064-COLO, CSU Project 31-1372-1632.

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The concept of the unit hydrograph was first proposed by Sherman (1932). The basic concept was subsequently extended and improved by many including Snyder (1938), Nash (1957), Dooge (1959). The process of deriving a unit hydrograph from concurrent observations of rainfall and runoff is commonplace. Many times difficulty is encountered when the unit hydrographs derived from two different sets of rainfall and runoff data on the same watershed do not agree. The non-agreement of the unit hydrographs may be explained by the nonlinearity of the basic unit hydrograph concept or by errors introduced in the measurement of the rainfall data. Indeed there may be cases where a suitable flood has been recorded on a river but the rainfall information is limited, unreliable or missing entirely. Schulz and Hislope (1972) have shown that many gaged watersheds in the state of Colorado have inadequate rainfall stations so that the precipitation input for any flood hydrograph cannot be known. Under these circumstances it would be highly desirable if a procedure could be developed which would liberate the task of deriving a unit hydrograph from the need for direct measurements of the rainfall.

A method of deriving a unit hydrograph without direct knowledge of the rainfall was proposed in a paper by De Laine (1970). De Laine based his study upon the assumption that the natural watershed system was linear and time-invariant. The runoff hydrograph from the watershed is the result of all periods of rainfall excess (net rainfall in De Laine's paper) being acted upon individually by the characteristic unit hydrograph of the watershed. Each hydrograph resulting from an individual period of rainfall excess is added to all other hydrographs having elements of flow during that time interval. This concept of linear superposition is a consequence of the first assumption of linearity of the watershed system. The assumption of time invariance allows one to predict the runoff hydrograph for a storm occurring at another time given the unit hydrograph and the depth of rainfall excess.

The usual notation of the convolution integral has been followed by De Laine in

$$Y(t) = \int_o^t x(t-\lambda) \quad h(\lambda)d\lambda$$

and in discrete form

$$Y(t) = x(t) * h(t)$$

where $y(t)$ is output, $x(t)$ is input and $h(t)$ is the watershed system unit response. This idea has been presented graphically in Fig. 1 and 2.

The rainfall is averaged over the selected interval of time (δt for which unit graph is desired) and the runoff is sampled as ordinates of the interval. Thus, the unit hydrograph is for the selected interval duration. Following is the description given from the paper.

Let the three discrete representations of $y(t)$, $x(t)$, and $h(t)$ be normalized, such that:

$$\sum_i^m x_i = \sum_i^n h_i = \sum_i^{m+n-1} y_i = 1$$

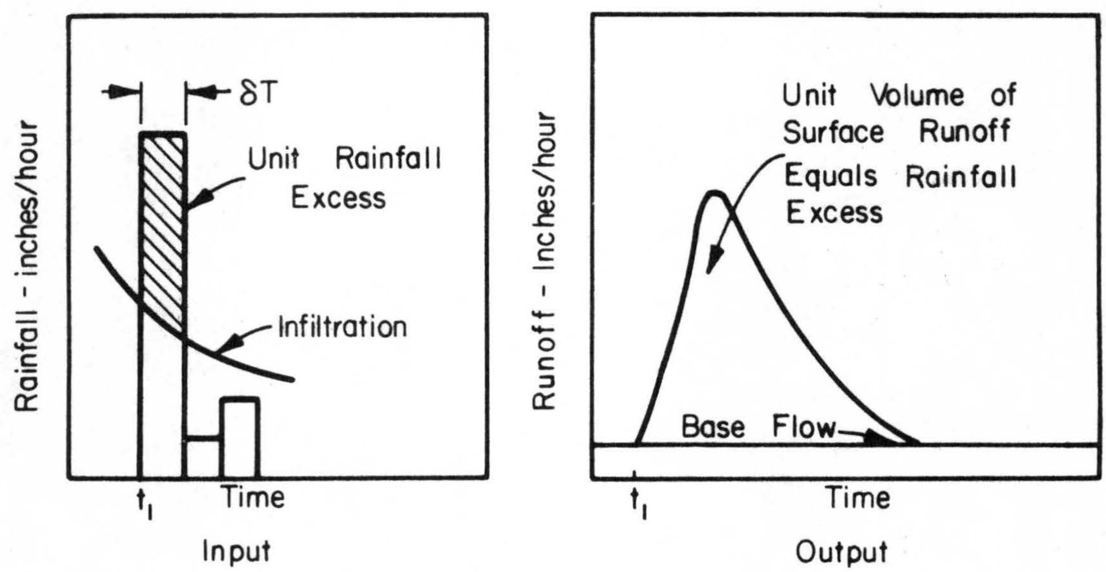

Fig. 1 Unit Hydrograph for δT Duration.

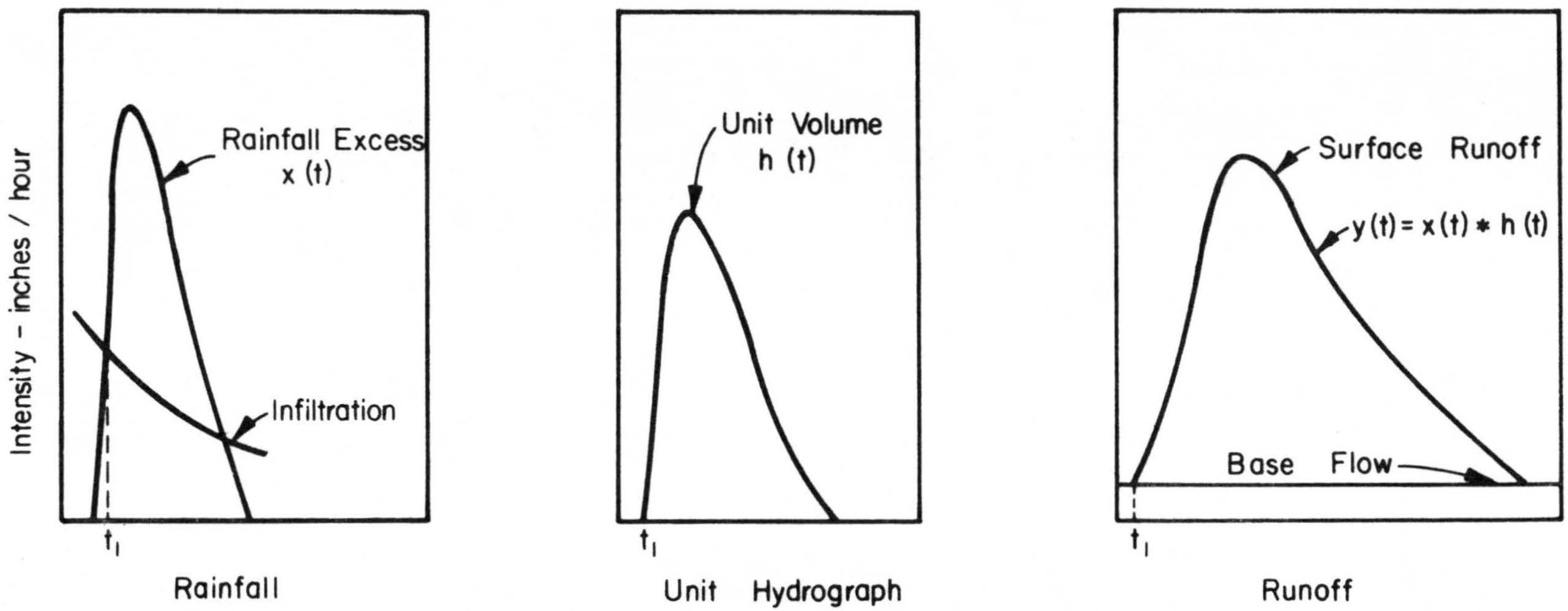

Fig. 2 Continuous Distributions of Rainfall, Unit Hydrograph and Runoff.

$x_1 , x_2 \ldots . x_m$ are successive ordinates of input (rainfall excess)

$h_1 , h_2 \ldots . h_n$ are successive ordinates of unit graph

$y_1 , y_2 \ldots . y_{m+n-1}$ are successive ordinates of output (runoff)

m, n are number of rainfall and unit hydrograph ordinates respectively. For this discrete representation, the convolution becomes a set of simultaneous algebraic equations. If only h's are unknown, the equations are linear. However, if both x's and h's are unknown then the equations are nonlinear. Thus we have

$$y_1 = x_1 h_1$$

$$y_2 = x_1 h_2 + x_2 h_1$$

$$y_3 = x_1 h_3 + x_2 h_2 + x_3 h_1$$

. . . .

$$y_{m+n-3} = \ldots\ldots\ldots\ldots\ldots x_{m-2} h_n + x_{m-1} h_{n-1} + x_m h_{n-2}$$

$$y_{m+n-2} = \ldots\ldots\ldots\ldots\ldots\ldots x_{m-1} h_n + x_m h_{n-1} \qquad (1)$$

$$y_{m+n-1} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x_m h_n$$

$$\sum_{i=1}^{n} = h_1 = 1 \ ; \qquad \sum_{i=1}^{m} x_i = 1 \qquad (2)$$

$$\sum_{i=1}^{m+n-1} y_i = 1 \qquad (3)$$

The Eqs. (1) are the basic equations used in computing the h values by established practices, when the values of x's and y's both are known. De Laine has introduced Eqs. (2) and (3) to help solve the h values when the x values also are not known.

Equations (1), (2) and (3) contain (m+n) independent equations. If the y's are known, we have (m+n) unknowns in the form of x's and h's . If these equations are solved, there will be at least one

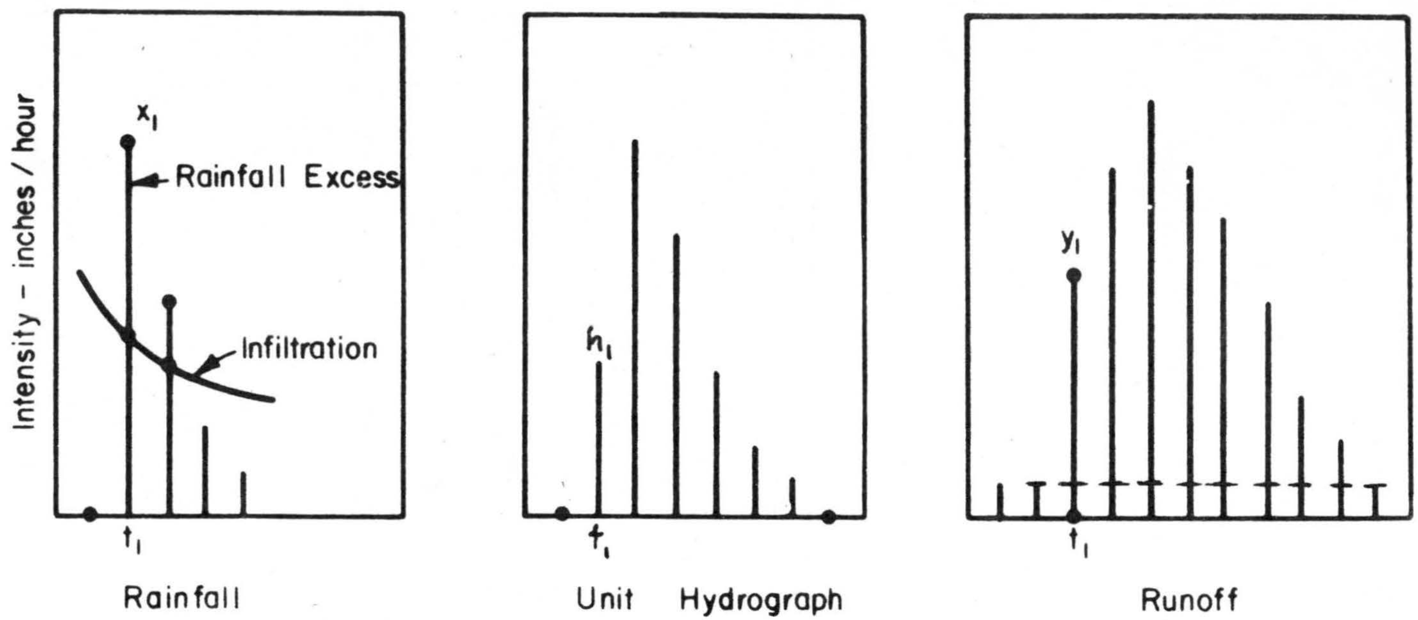

Fig. 3 Discrete Representation of Rainfall, Unit Hydrograph and Runoff.

set of real values that satisfy these equations but practically it is not possible to find such a set. To overcome this difficulty it has been suggested to consider another output from the same system and solve Eqs. (1), (2), and (3). These two groups of sets of equations will have common values of h because the outcomes are from the same system (which was assumed to be time invariant).

The procedure adopted for solution is:

put $$x(k) = x_1 + x_2k + x_3k + \dots\dots\dots\dots x_mk^{m-1} \tag{4}$$

$$h(k) = h_1 + h_2k + h_3k^2 + \dots\dots\dots\dots k_nk^{n-1} \tag{5}$$

from which $x(k) \cdot h(k) = x_1h_1 + (x_1h_2 + h_1x_2)k + .(x_mh_n)k^{n+m-2}$

$$= y_1 + y_2k \dots\dots\dots\dots\dots y_{m+n}-1k^{m+n-2} \tag{6}$$

This means that polynomial (6) has ordinates of observed output and is equal to product of two polynomials (4) and (5) that have coefficients which are the successive ordinates of input and the system response. Thus the factors of polynomial (6) will also be the factors of polynomial (4) and (5). By equating (6) equal to zero, the complex roots of the polynomial can be solved. Now consider another polynomial of type (6) i.e., on the output, and solve for complex roots. Then the roots common to these equations will lead to the factors of polynomial (5). The remaining roots of each of the polynomials will lead to the factors of equation (4).

The steps for solution can be summarized as: -

1) select two sets of observed data, say

y_1 , y_2 . . . . y_n

x_1 , x_2 . . . . x_n

2) let

$$0 = y_1 + y_2 k + \dots\dots\dots\dots . y_n k^{n-1}$$

$$0 = x_1 + x_2 k + \dots\dots\dots\dots . x_n k^{n-1}$$

and find the complex roots of these equations.

3) Select the roots common to both equations.

4) Suppose the roots are:

$$(K + a + i\,b)\ (K + a - i\,b)\ (K - c + i\,d)\ (K - c - i\,d)\ \dots$$

(the complex roots will be in pairs)

5) Multiply the above roots as:

$$a_o + a_1 k + a_2 k_2 \dots\dots\dots\dots . a_n k^{n+1} .$$

6) Normalize the coefficients of the above equation and these normalized coefficients are the ordinates of the unit graph.

7) For finding rainfall ordinates, take the remaining roots for each set of data and obtain the equation of the form

$$a_o' + a_,' k \dots\dots\dots\dots . a_n' k^{n+1}$$

8) Multiply the coefficients of the above equation by a factor such that the coefficients add up to total rainfall excess (volume of observed flows). The coefficients thus computed will be the ordinates of rainfall excess.

Mathematically, the procedure is correct. A system behaving under the assumptions of linearity and time invariance should give rise to correct ordinates of the unit graph by this method. This procedure would have practical application on many observed floods in Colorado where good rainfall data are lacking. In practice, however, the conditions are different. Initially, we do not have a perfectly linear system and secondly, there are data errors which may inhibit a correct solution. De Laine has given an example of a nonlinear system and inexact data but his data are not so inexact that the validity of the method for practical problems is demonstrated. The ordinate of the unit graphs obtained by him by considering three events are quite close to one another. De Laine has also tried to show effects of random errors in the input data.

The procedure has been tested on watersheds in Arizona and also for events recorded in the CSU Experimental Rainfall-Runoff Facility. The steps followed in applying the method are:

(1) A program for finding the complex roots of the polynomial was obtained from the CSU Computer Library and tested with De Laine's original data. The CSU computer program gave exactly the same roots as shown in De Laine's paper.

(2) Two events on Safford W-II Watershed in Arizona were selected (01 and 04). The unit graph using this procedure was attempted. The results are shown in Table 1. It is obvious from the computed roots that it is very difficult to find the common roots from two events because all of them differ to the same degree and one does not know how many ordinates the unit graph is going to have. For checking the applicability of the procedure and also to get a trial estimate of how much difference could be tolerated for the selection of roots, the rainfall records of these events were consulted. It was found that event 01 and 04 had 8 and 4 rainfall ordinates, respectively. Hence the number of unit graph ordinates were estimated from the equation/

observed output ordinates = m + n - 1. The equations obtained by selecting 21 common roots in this example are shown in Table 1. The final equation gave a few negative coefficients and therefore it could not be normalized. The negative coefficients are due to data errors. Another set of two events (05, 06) was tried but this pair also gave doubtful results.

(3) Errors could be introduced by the base flow separation. Data were taken from the Experimental Rainfall-Runoff Facility. The events are shown in Table 2. It is again apparent from the three sets of roots that it is quite difficult to select the common roots. Nevertheless, an attempt has been made to find common roots for Experimental Run Nos. 88-A and 98. The polynomial obtained is:

$$6.680 + 17.450k + 57.470\,k^2 + 91.989k^3 + 80.019k^4 - 27.227\,k^5 - 18.128\,k^6 - 76.211k^7 - 121.675k^8 - 138.507k^9 - 124.470k^{10} - 94.340k^{11} - 55.835k^{12} - 25.435k^{13} - 5.715k^{14} + 1.880k^{15} + 3.300k^{16} + 2.094k^{17} + 1.622k^{18} + 0.911k^{19} + k^{20}$$

The above polynomial has negative coefficients and it is difficult to find logical ordinates of the unit graph.

(4) At this stage, it was realized that the decimal places to which output data is computed has an effect on the values of the roots. Therefore, the next attempt was made to use Experiment Run Nos. 97 and 98 with the same rainfall intensity and observed hydrograph ordinates normalized to four decimal places. The idea of selecting the same intensity was to have less differences in common roots. The results of these runs are shown in Table 2. The scrutiny of the roots show that it is quite difficult to select the common roots.

(5) Next attempt was made with a hypothetical case. A unit hydrograph was assumed for an area of 40 acres. Two rainfall events

(0.3, 0.4, 0.5 and 0.2 inches) and (0.7 and 0.5 inches) with rainfall excess given at 5 minute intervals were taken and the observed hydrographs were obtained using the assumed unit graph. Table 3 shows these synthetic hydrographs and the roots of simultaneous equations obtained from these equations. It is very easy to delineate the common roots.

The computed unit graph ordinates are very close to the originally assumed unit graph ordinates.

For event 1, the remaining roots are:

$$(K + 1.95189)\ (K + .32055 + .83874i)\ (K + 32.55 - .83874i)$$

$$= 1.5735 + 2.0577K + 2.5930K^2 + K^3$$

Rainfall excess for event 1 $= \frac{\text{Volume} \times \Delta t}{\text{area}} = \frac{352.24 \times 10 \times 60}{40 \times 4840 \times 9} \times 12 = 1.42''$

Normalizing and multiplying the above polynomial by 1.42" we have the rainfall ordinates as:

0.311 ; 0.399 ; 0.503 ; 0.194

which are very close to given rainfall ordinates.

For event 2, the remaining factors are:

$$(K + 1.4215)$$

Normalizing $\qquad 0.587 + .413K$

The rainfall excess for event 2 $= \frac{301.92 \times 10 \times 60}{40 \times 4840 \times 9} \times 12 = 1.2''$

$\therefore$ ordinate of rainfall are: 0.704, 0.496 which are very close to 0.7 and 0.5.

CONCLUSIONS

1. The method is mathematically correct if the assumptions of linearity and superposition hold for the catchment and its application is demonstrated by taking a hypothetical example (see Table 3). It is

essentially the same concept as used in any matrix inversion procedure for finding the unit graph. In a matrix inversion we solve the simultaneous equation by matrix operation whereas in this method complex root techniques are employed.

2. The errors in the data are magnified in the solutions of the simultaneous equations so that it is not practical to use the method for any observed events. This is shown in Tables 1 and 2.

3. De Laine has shown its use for the actually observed events but he might have fortuitously used data which yielded comparable complex roots.

4. In the examples of flood events in Arizona, it is clear that the difficulty arises in selecting the proper number of common roots. Even if the number of input ordinates is known a priori, it becomes quite difficult to decide which of the roots is to be retained or to be rejected.

5. The roots are quite sensitive to the number of places to which the normalized ordinates are computed. This was found for run 98 and 94 made on the rainfall-runoff facility. The roots obtained by taking the normalized ordinates up to three decimal places were quite different from those taken up to four decimal places.

6. Data errors, selection of wrong common roots and sensitivity of the number of places to which the ordinates are normalized, lead to the negative coefficients of the polynomials, from which it is then not feasible to compute a unit hydrograph.

7. In view of these difficulties, it appears that this method does not offer much hope for practical application for the derivation

of unit hydrographs from observed flood data lacking concurrent rainfall recorder data.

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Table 1

Data From Safford W-II Watershed, Arizona

	EVENT NUMBERS	1-03-06-002-01		1-03-06-002-04		1-03-06-002-06		1-03-06-002-05		1-03-06-002-01		1-03-06-002-04	
	Time in mins.	Runoff in/hr.	Normal-ized	Runoff in/hr.	Normal-ized	Runoff in/hr.	Normal-ized	Runoff in/hr.	Normal-ized	Roots Real	Imaginary	Roots Real	Imaginary
1	5	.190	.046	.120	.029	.280	.026	.24	.157	-.04199	-1.18110	.08434	-.67837*
2	10	.507	.124	.347	.084	.605	.055	.41	.268	-1.03865	.54513	-1.04189	.68086
3	15	.798	.195	.480	.116	.900	.082	.32	.209	1.19851	-.21980	1.28484	.28620
4	20	.886	.216	.964	.233	1.204	.110	.18	.118	-1.16570	-.26400	-1.19479	-.32295
5	25	.503	.123	.810	.196	1.060	.097	.095	.062	.25128	1.20760	.13796	1.22250
6	30	.338	.083	.570	.138	.490	.045	.062	.041	.25128	-1.20760	.13796	-1.22250
7	35	.217	.053	.290	.070	.330	.030	.048	.031	.75312	.99788	.89311	.96673
8	40	.152	.037	.147	.035	.230	.021	.037	.024	1.12054	-.53529	1.14887	-.65787
9	45	.106	.026	.103	.026	.145	.013	.028	.018	-.64898	.98085	-.78710	.94609
10	50	.075	.018	.070	.017	.093	.008	.022	.014	-.85002	-.80153	.08434	.67837*
11	55	.053	.013	.052	.013	.058	.005	.018	.012	.75312	-.99788	.89311	-.96673
12	60	.040	.010	.040	.010	.035	.003	.016	.010	-.34297	1.08870	-.49749	1.10449
13	65	.032	.008	.030	.007	.025	.002	.013	.008	-1.23856	-.00000	-1.28158	.00000
14	70	.025	.006	.025	.006	.014	.001	.011	.007	1.12054	.53529	1.14887	.65787
15	75	.021	.005	.020	.005	.006	.001	.008	.005	-.64898	-.98085	-.78710	-.94609
16	80	.019	.005	.015	.004	0		.006	.004	.51517	1.14386	.55825	1.15164
17	85	.018	.004	.010	.002			.005	.003	.51517	-1.14386	-.55825	-1.15164
18	90	.017	.004	.009	.002			.004	.003	-.85002	.80153		*
19	95	.016	.004	.008	.002			.003	.002	-1.03865	-.54513	-1.04189	-.68086
20	100	.014	.003	.007	.002			.002	.001	1.19851	.21980	1.28484	-.28620
21	105	.013	.003	.006	.001			.001	.001	.96443	-.78524		*
22	110	.012	.003	.005	.001			0	0	-.04199	1.18110		*
23	115	.010	.002	.004	.001					-.34297	-1.08870	-.49749	-1.10449
24	120	.009	.002	.003	.001					.96443	.78524		*
25	125	.008	.002	.002	.001					-1.16570	.26400	-1.19479	.32295
26	130	.007	.002	.002						-.24205	-.72693	-.22429	-1.24193
27	135	.006	.001	.001						-.24205	.72693	-.22429	1.24193
28	140	.004	.001	.001						-.70677	.00000	-.44205	-.00000*
29	145	.003	.001	0									
30	150	.002	0										
31	155	.001											
32	160	0											

Equation from 1-03-06-002-01 $= [(K + 1.039 - .5450)(K + 1.039 + .545i)]\,[(K - 1.199 - .220i)(K - 1.199 + .220i)]\,[(K + 1.166 - .264i)$
$(K + 1.166 + 2.64i)]\,[(K - .251 - 1.208i)(K - .251 + 1.208i)]\,[(K - .753 - .998i)(K - .753 + .998i)]\,[(K - 1.121 - .535i)(K - 1.121 + .535i)]$
$[(K + .649 - .981i)(K + .649 - .981i)]\,[(K + .343 - 1.089i)(K + .343 + 1.089i)]\,[K + 1.239]\,[(K - .515 - 1.144i)(K - .515 \_ 1.144i)]$
$[(K + .242 - .727i)(K + .242 + .727i)]$

$= K^{22} + 1.046K^{21} - .382K^{20} - .103K^{19} - 1.057K^{18} - 2.535K^{17} - 1.754K^{16} + 1.587K^{15} + 7.124K^{14} + 21.015K^{13} + 6.933K^{12} + 7.405K^{11} - 3.119K^{10} - 25.022K^{9}$
$- 20.582k^{8} - 17.194K^{7} - 2.514K^{6} - 2.988K^{5} + 11.088K^{4} + 51.026K^{3} + 55.766K^{2} + 41.347K + 15.690$

Similarly equation for 103-06-001-04 $= K^{22} + 1.168K^{21} + 0.729K^{20} + 0.579K^{19} + 0.750K^{18} + 1.843K^{17} + 1.109K^{16} + 0.373K^{15} - 4.928K^{14} + 3.486K^{13} +$
$3.857K^{11} - 15.451K^{10} - 7.793K^{9} - 7.446K^{8} - 30.297K^{7} + 1.940K^{6} + 46.091K^{5} + 61.019K^{4} + 59.668K^{3} + 170.301K^{2} + 124.195K + 23.350$

*Roots assumed uncommon to both sets

Table 2

Data From CSU Experimental Rainfall - Runoff Facility

	EXPERIMENTAL RUN NUMBERS									
	Run No. 85 Runoff		Run No. 88-A Runoff		Run No. 98 Runoff		Run No. 94 Runoff		Run No. 98 Runoff	
Time seconds	in/hr.	Normal-ized	in/hr.	Normal-ized	in/hr.	Normal-ized	in/hr.	Normal-ized	in/hr.	Normal-ized
30	.002	.001	.006	.001	.035	.002	.035	.0035	.035	
60	.003	.001	.030	.007	.140	.008	.140	.0138	.140	.0076
90	.020	.006	.096	.021	.550	.029	.480	.0476	.550	.0294
120	.070	.021	.275	.061	1.475	.079	1.113	.1100	1.475	.0789
150	.155	.047	.460	.102	3.075	.164	1.822	.1802	3.075	.1646
180	.277	.084	.645	.142	3.420	.183	2.011	.1992	3.420	.1829
210	.397	.120	.720	.159	2.840	.152	1.570	.1556	2.840	.1518
240	.450	.136	.615	.139	2.270	.121	1.001	.0992	2.270	.1216
270	.425	.128	.446	.098	1.575	.084	.707	.0701	1.575	.0842
300	.342	.103	.325	.072	1.015	.054	.434	.0403	1.015	.0528
330	.260	.079	.244	.054	0.625	.033	.239	.0238	.625	.0334
360	.206	.062	.178	.039	0.420	.022	.166	.0166	.420	.0225
390	.164	.050	.138	.030	0.300	.016	.115	.0114	.300	.0160
420	.128	.039	.100	.022	0.220	.012	.076	.0075	.220	.0118
450	.098	.030	.074	.016	0.170	.009	.055	.0055	.170	.0092
480	.074	.022	.054	.012	0.135	.007	.042	.0042	.135	.0072
510	.059	.018	.040	.008	0.110	.006	.030	.0029	.110	.0059
540	.047	.014	.028	.006	0.080	.004	.020	.0020	.080	.0044
570	.036	.011	.020	.004	0.070	.004	.012	.0012	.070	.0037
600	.028	.008	.014	.003	0.050	.003	.009	.0009	.050	.0029
630	.022	.007	.010	.002	0.045	.002	.005	.0005	.045	.0024
660	.018	.005	.007	.002	0.035	.002	.002	.0002	.035	.0019
690	.014	.004	.004	.001	0.020	.001	.001	.0001	.020	.0012
720	.010	.003	.002	.001	0.015	.001	.001	.0001	.015	.0008
750	.003	.001	.001	0	0.010	.001	.001	.0001	.010	.0005
780	0	--	0		0	0	.001			

	Roots of simultaneous equations						Roots of simultaneous equations			
	Real	Imaginary	Real	Imaginary	Real	Imaginary	Real	Imaginary	Real	Imaginary
1	1.20082	.43322	1.28921	.45248	1.28944	.35503	-.87858	-1.10116	-.65125	-1.07775
2	1.20082	-.43322	1.28921	-.45248	1.28944	-.35503	-.87858	1.10116	-.65125	1.07775
3	-.18798	-1.03419	-.20449	-1.23969	-.26608	-1.20512	1.42083	-.46431	1.28906	-.36826
4	-.18798	1.03419	-.20449	1.23969	-.26608	1.20512	1.42083	.46431	1.28906	.36826
5	.61641	1.05442	.68797	1.09988	.80206	1.03777	.53798	-1.29575	.43886	-1.23579
6	.61641	-1.05442	.68797	-1.09988	.80206	-1.03777	.53798	1.29575	.43886	1.23579
7	.95935	-.79728	1.06083	-.81229	1.11146	-.77279	-1.28867	-.15067	-1.16371	.17245
8	.95935	.79728	1.06083	.81229	1.11146	.77279	-1.28867	-.15067	-1.16371	-.17245
9	-.88409	.22851	-1.04909	+.16879	-1.13205	.14404	1.33123	.92241	1.09301	.8044
10	-.88409	-.22851	-1.04909	.16879	-1.13205	-.14404	1.33123	-.92241	1.09301	-.80444
11	-.76843	.75266	-.83607	.72815	-.87411	.73703	-.41887	-1.33602	-.28675	-1.25555
12	-.76843	-.75266	-.83607	-.72815	-.87411	-.73073	-.41887	1.33602	-.28675	1.25555
13	-.88352	.43515	-.98401	.53284	-1.13003	.41425	.94594	-1.13854	.76268	-1.09095
14	-.88352	-.43515	-.98401	-.53284	-1.13003	-.41425	.94594	1.13854	.76268	1.09095
15	-.45263	.89312	-.58014	1.09124	-.57848	1.02808	-1.08765	.64690	-.94626	.78282
16	-.45263	-.89312	-.58014	-1.09124	-.57848	-1.02808	-1.08765	-.64690	-.94626	-.78282
17	.13162	1.15861	.25153	+1.23511	.11448	1.23363	.10814	1.34886	.11371	1.28842
18	.13162	-1.15861	.25153	-1.23511	.11448	-1.23363	.10814	-1.34886	.11371	-1.28842
19	.07234	-.78321	.01755	-.80049*	.46372	-1.15793*	-.76901	-.43660	-1.16172	-.46696
20	.07234	.78321	.01755	.80049*	.46372	1.15793*	-.76901	.43660	-1.16172	.46696
21	.17553	-.30870	-.04219	-.43155	.05376	-.36108	-.00430	-.42296	.04300	.35424
22	.17553	.30870	-.04219	-.43155	.05376	-.36108	-.00430	.42296	.04300	.35424
23	-1.47940	-.46440	-.22219	.00000*	-.35418	-.29625*	-.39705	-.59808	-.33064	.31011
24	-1.47940	.46440			-.35418	.29625*	-.39705	.59808	-.33064	-.31011

*Roots uncommon to both sets

Table 3

Synthetic Data for 40 acre Watershed

Area = 40 acres.

	Time in minutes	Unit graph ordinates assumed cfs	Normalized U.G. ordinates	Event No. 1 ordinate in cfs	Event No. 1 Normalized	Event No. 2 cfs	Event No. 2 Normalized
1	0	13.6	.0541	4.08	.0116	9.52	.0316
2	20	40.0	.1590	17.44	.0495	34.8	.1152
3	30	44.0	.1749	36.00	.1022	50.8	.1682
4	40	38.5	.1530	51.87	.1472	48.95	.1621
5	50	33.0	.1312	55.30	.1570	42.35	.1403
6	60	27.5	.1093	49.50	.1405	35.75	.1184
7	70	22.0	.0874	41.80	.1187	29.15	.0965
8	80	16.5	.0656	34.10	.0968	22.55	.0747
9	90	11.0	.0437	26.40	.0749	15.95	.0528
10	100	5.5	.0215	18.70	.0531	9.35	.0310
11	110	0	0	11.00	.0312	2.75	.0091
12	120			4.95	.0141	0	
13	130			1.10	.0031		
14				0			

Roots

Real	Imag.	Real	Imag.
.86284	-.86861	.86279	-.86857
.86284	.86861	.86279	.86857
-.63632	-1.01002	-.63969	-1.01185
-.63632	1.01002	-.63969	1.01185
.12827	-1.23611	.12674	-1.23668
.12827	1.23611	.12674	1.23668
-1.03140	-.36479	-1.04123	.37105
-1.03140	.36479	-1.04123	-.37105
-.60223	-.00000	-.60232	-.00000
-1.91184	.00000	-1.42150	.00000*
-.32055	-.83874		*
-.32055	-.83874		*

Simultaneous equations -

Event 1

$$0 = .0116 + .0495K + .1022K^2 + .1472K^3 + .1570K^4 + .1405K^5 + .1187K^6 + .0968K^7 + .0749K^8 + .0531K^9 + .0312K^{10} + .0141K^{11} + .0031K^{12}$$

Event 2

$$0 = .0316 + .1152K + .1682K^2 + .1621K^3 + .1403K^4 + .1184K^5 + .0965K^6 + .0747K^7 + .0749K^8 + .0528K^8 + .0310K^9 + .0091K^{10}$$

The roots of these equations are shown as real and imaginary parts. The roots uncommon are crossed.

Taking the common roots for event 2:

$$(K - .8628 + .8686i)\ (K - .8628 - .8686i)$$
$$(K + .6363 + .1010i)\ (K + .6363 - 1.010i)$$
$$(K - .1283 + 1.236i)\ (K - .1283 - 1.236i)$$
$$(K + 1.0314 + .3648i)\ (K + 1.0314 - .3608i)$$
$$(K + .6022)$$

$$= 2.3533 + 7.0363K + 7.8263K^2 + 6.8279K^3 + 5.8811K^4 + 4.8813K^5 + 3.9188K^6 + 2.9354K^7 + 1.9574K^8 + K^9$$

$$\text{OR } 2.3533 + 7.0363K + 7.8263K^2 + 6.8279K^3 + 3.9188K^6 + 2.9354K^7 + 1.9574K^8 + K^9$$

Sum of the coeffs = 44.6701

$\therefore$ Normalizing

$$.0526 + .1575K + .1752K^2 + .1529K^3 + .1317K^4 + .10393K^5 + .0877K^6 + .0657K^7 + .0438K^8 + .0224K^9$$

thus unit graph coordinates are:

.0526, .1575, .1752, .1529; .1317, .1093, .0877, .0657; .0438, .0224

which compares well with the original ordinates.