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BIBLIOGRAPHY WITH ABSTRACTS OF SUPERCRITICAL FLOW IN OPEN CHANNELS

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INTRODUCTION

During the period when major interest was first shown in the field of rapid flow and air entrainment, it was felt by some that model studies would not materially advance the knowledge of rapid flows. However, as more investigators and researchers became interested in the mechanics of supercritical flow, the model soon became a very useful tool in rapid flow investigations.

Up to the present many studies have been conducted in several countries by various agencies and good use has been made of the hydraulic model in predicting and analyzing some of the phenomena of rapid flows. However, much still remains to be accomplished.

This paper is an attempt to bring together the works of the various researchers in the form of a bibliography with abstracts. Some abstracts are also given for works in related fields, especially use of models and artificial roughnesses. Unfortunately, some of the works and/or translations of these papers were not available to the writer at the time of this compilation. Some of these missing pieces contain contributions from: Vedernikov, V. V. - Comptes rendus (Doklady) de l'Academie des Sciences de l'U.R.S.S.; Halbronn, G. - Houille Blanche; Veparelli, M. - Energia Elett; Arsenishvili, K. I. - Gidrotekh Stroit; Puznov, A. - Rozpravy Ceskoslovenski; Ishihara Tojiro and Iwagaki Yuichi - University of Japan; and others.

STATUS OF THE KNOWLEDGE

Much data has been collected on small rectangular flumes with special regard to air entrainment and some formulations made. Also, some contributions to problems involving waves, effects of roughness, channel junctions, and changes in cross sections have been given. It is felt, however, that the following topics still are in need of investigation or further analysis.

[illegible]

Air Entrainment

It appears that the physical entrapment of air into the water as well as the means by which the air is held in suspension are in need of further investigation and clarification.

How far has it been investigated?

The problem is related to the more fundamental one of the generation of turbulence at the boundary, its diffusion upward through the liquid, and the action of the vertical components of turbulence at the water surface. It should be noted here that perhaps investigations of turbulence should be made without the air before the problem is again complicated by the entrained air.

[illegible] investigation?

Also instrumentation in this area is still in the process of development and needs much further work before we have a means of measuring and evaluating turbulence in water. These instruments should be capable of measuring both the longitudinal and vertical components of turbulent velocities in water. Perhaps hot film anemometers or pressure transducers might be used with some modification. It might also be advisable to investigate photographic techniques as a means of evaluating the problems of turbulence. See "Journal of the Hydraulics Division" Proc. A.S.C.E., May 1961, pages 73-82. The behavior of the turbulence from discrete wall protuberances as the disturbance reaches the surface needs to be studied. Little if [illegible] nothing is known of the effect of joint offsets in channel walls and the effect of piers on overflow spillways with specific regard to air entrainment and generation of turbulence. These problems again necessitate the development and/or modification of the instrumentation.

Some recent studies would lead one to suspect that viscosity and surface tension have an effect on the amount of air entrained. It is suggested that the air content might be increased by about 13% for a temperature change from 40°F to 80°F (Warren DeLapp, University of Colorado). Perhaps these effects could be considered minor, this remains to be determined. It may be that the difference between these properties, of the surrounding air mass and the water itself, are the important criterial rather than just the properties of the water alone. These points are in need of further investigation. See "Journal of the Hydraulics Division" proc. A.S.C.E., Nov. 1961, pages 221-231.

It has been noted that the energy losses for uniform flow with entrained air are higher than without the air for the same water discharge. Is it possible that steep chutes containing flow with high air content could be effectively used as energy dissipators, thus modifying the design of energy dissipators at the foot of such structures? Functional relationships should be formulated such that these losses can be accurately determined as well as the size and type of roughness as it influences the energy losses and amount of air entrained. It appears that the usual friction coefficients such as those used in the Manning or Chezy equations are no longer constant with the type of boundary material, with air entrained flow. The mathematical effect of air content, air distribution and perhaps velocity on these roughness parameters still is in need of investigations. See "Head Loss and Air Entrainment by Flowing Water in Steep Chutes" Proc. Minnesota International Hydraulics Convention, Sept. 1953, pages 467-476 and "Journal of the Hydraulics Division," Proc. A.S.C.E., Nov. 1961, pages 221-231.

Most investigations to date have been made on small rectangular flumes. Studies should be made on larger flumes with variable cross sections. Narrow chutes have been defined as having width less than five times the depth and wide flumes with width greater than five times the depth. Keeping these definitions in mind along with the following assumptions and equations one can estimate the size of structure needed. Proceed by assuming that air entrainment will begin with velocities of from 10-20 ft/sec and will begin development when the turbulent boundary layer from the bottom has reached the water surface. Also it is assumed here that a depth range from 1-2 feet is desirable for accuracy in measurement. This would mean a width range of from 5-10 feet (pick 8 ft for simplicity of construction). For larger values of discharge $Q > 20$ cfs, Ehrenberger observed the water portion of flow to vary from $\approx .4$ to $\approx .6$, the following tabulation is based on a mean value of 0.5. Mannings "n" is computed from $n = .0342\ k^{1/6}$ and the length of flume necessary for air entrainment to begin (x) from $\delta/x = \dfrac{.024}{(x/k)^{.13}}$ where k is the roughness height.

.4 what?

what is δ?

k	Mannings n	V_M Velocity of Air-Water Mixture	Q_M (Mixture)		Q_W (Water)		S_1 slope in degrees		X	
			d=1	d=2	d=1	d=2	d=1	d=2	d=1	d=2
.005	.014	10	80	160	40	80	19	16	160	350
.005	.014	20	160	320	80	160	28	24	160	350
.01	.016	10	80	160	40	80	21	18	150	320
.01	.016	20	160	320	80	160	30	25	150	320
0.1	.023	10	80	160	40	80	25	21	100	220
0.1	.023	20	160	320	80	160	37	31	100	220

Three boundaries, the bottom and two sides, contribute to the generation of turbulence which causes air entrainment in chutes. It is possible that the

d = 1 what?

You are using flumes & chutes interchangeably

effect from one side wall reaches nearly to the opposite wall thus compounding the air-entrainment effect in narrow chutes. Ehrenberger has noted that for wide chutes the cross-sectional water surface is concave upward thus signifying greater air entrainment at the walls than in the center section of flow. The three-dimensional problem could be investigated by first studying the side-wall effect, alone. This could be accomplished by roughening one side and keeping the bottom and other side smooth to isolate the side-wall effect. On the other hand a fairly wide channel with rough bottom and smooth sides would have very little side-wall effect. If an inference can be drawn from the tabulated values above it would appear that a flume or flumes of about 8 ft in width, capable of discharges of from 40-160 cfs, maximum slope of about 40° and length approaching 400 ft could be used to make such studies. The cross-sectional shape of such channels may also influence the air entrainment and studies should be made to isolate these effects. See "Journal of the Hydraulics Division" Proc. A.S.C.E., May 1961, pages 78-82.

The formulation of a general discharge equation for air-entrained, ultra-rapid flow has not been satisfactorily established. It would seem an equation of the form $Q_W = P_W A_M U_M$ would be of considerable value. The subscript (M) denotes air-water mixture and (w) stands for water only. The term "P_W" is the water portion of the total flow ($P_W < 1$) for air-entrained flow. The term "P_W" cannot be satisfactorily evaluated at present, however such evaluation would probably be forthcoming from the satisfactory solution of the problems mentioned earlier. It would certainly be valuable to the designer if the velocity (V_M) could be evaluated from an equation of the Manning or Chezy form. These equations contain parameters of roughness, slope, and hydraulic radius. From an investigation made by Straub (1953) it appears that

What is A & U

velocities computed using the Manning equation give results that are too low when compared to the measured velocity. This could mean a variable roughness parameter or possibly a different treatment given to the hydraulic radius term or both. At any rate it would be well to know the effects of air-entrainment on roughness parameter such as (n) or (c). See "Proc. Minnesota International Hydraulics Convention" Sept. 1953, pages 425-436.

It may be possible, in the future after the knowledge of air-entrained flows has advanced, to provide a distorted similarity for use in modeling structures where air entrainment is expected. See "Open Channel Flow of Water-Air Mixtures" Trans. A.G.U., 1954, page 235 by Einstein.

Unsteady Flow

Various stability parameters have been formulated to give the point at which one might expect the flow to become unsteady. Probably the most useful of these is:

$$F \begin{bmatrix} \text{unstable} \\ \text{neutral} \\ \text{stable} \end{bmatrix} \begin{matrix} > \\ = \\ < \end{matrix} 1.6 \quad \text{where} \quad F$$

is the Froude number. Questions still arise as to the mechanics of the development of these waves as well as to the parameter that causes the initial instability to exist.

a or e?

The instability that occurs with flow on steep slopes should be effected by air entrainment. Thus far studies have been made with flows of negligible entrained air.

Roll waves are initiated by finite disturbances in the laminar boundary layer, however this process is enhanced by external causes, such as release of air bubbles, roughening of the channel entrance and contact of the water surface with air currents. The mechanics of these variables still remains unsolved.

Confluences

In general it may be said that problems of analysis of flow when two steep chutes are connected, or a steep chute connected to one of mild slope have received little attention. However it should be pointed out here that Oregon State College has received a grant from the Bureau of Public Roads to study supercritical flow in confluences.

Transitions

Problems involving bends, expansions, contractions, and slope changes are in need of investigation when the flow is supercritical and especially if such flows are air entrained.

Non-Uniform Flow

Problems involving the surface profile for supercritical flows that will entrain air, through the range of development of such flows still needs investigation. It is believed that air will begin to be entrained at a point where the turbulent boundary layer develops to the point where it intersects the water surface. See "Some Prototype Observations of Air Entrained Flow" Proc. Minnesota International Hydraulics Convention, Sept 1953, pages 403-414 and "Turbulent Boundary Layer on Steep Slopes" Trans. A.S.C.E., 1954, page 1212 by Bauer and "Air Entrainment on Spillway Faces" Civil Eng. Dec. 1945 by ickox. However studies still need to be conducted on the zones involving "Gradually Varied Aerated Flows" and "Rapidly Varied Aerated Flows." See "Journal of the Hydraulics Division" Nov. 1961, pages 227-229.

Sediment Transport

When confronted with problems of sediment removal, steep chutes may be used where the flow is supercritical. It would be interesting to investigate the behavior of sediment when the flow is rapid to ultra rapid.

General Considerations

In view of all the topics discussed above it can be noted that there is a great variety of experimentation that still needs to be performed in the field of supercritical flow. However, not all of the topics mentioned would require the same instrumentation and laboratory equipment. For example problems involving viscosity and surface tension effects, problems involving behavior and mechanics of turbulence or problems of air distribution would not necessarily require a large, long, wide flume as discussed for studying side-wall effect, joints, and piers. It should also be noted that variety is needed for cross-sectional shapes of flumes as well as work with contractions, expansions and confluences. Thus no one piece of equipment would be satisfactory or even desirable for use in the solutions to the problems yet remaining unsolved. One should also observe at this point that there is a great need of prototype data in the field of supercritical flow and that this would require new or modified instrumentation. If money is to be spent on new instrumentation and laboratory equipment it appears that attention should be focused on individual problems rather than on the solution to all the problems. Perhaps an attack on two or three of the above mentioned problems can be made with essentially the same equipment but certainly others will require special attention.

DEFINITION OF TERMS

This should have been included before previous section.

In order to avoid confusion deviations from the definitions listed here for the same representative symbol are defined in the individual reference.

A = cross-sectional area
C_1, C_2 = constants
F = Froude number $\frac{U}{\sqrt{gh}}$
G = "a function of"
M = $1 - RdP/dA$
P = wetted perimeter
Q = discharge
R = hydraulic radius $\frac{A}{P}$
R_e = Reynolds number
S = energy gradient slope
S' = "distance from leading edge"
U = average velocity
V = average velocity
V' = $\sqrt{gw/b \text{ vs } i}$ (relative velocity of an elementary wave front)

b = width of the free surface
d_1 = depth before jump
d_2 = depth after jump
f = resistance coefficient ($8ghS/V^2$)
h = depth measured normal to the bottom
i = slope of channel bottom
k = height of roughness element
$k/4h$ = relative height of roughness

DEFINITION OF TERMS (Cont'd)

m, n = constants

q = unit discharge

r^2 = constant of proportionality

$-r^2u^2/y$ = form of Chezy resistance

$u-v$ = $-\sqrt{gH/\alpha\beta}$

w = cross-sectional area

x = abscissa measured along the channel bottom

y = depth measured normal to the channel bottom

α = slope angle

β, ρ = exponents in the resistance equation $S = K\,V^{P}/R^{(1+\beta)}$

δ = thickness of turbulent boundary layer

ϵ = sign of U

γ = unit weight of water

λ = $\dfrac{V_1^{2}}{\partial\, d_1}$

ν = kinematic viscosity

ϕ = $2.58 - 0.021\,\lambda$ (1 on 6 slope)

1. Ehrenberger, R. "Flow of Water in Steep Chutes with Special Reference to Self Aeration," Osterreichischen Ingenieur und Architickten vereines Nos. 15/16 and 17/18, 1926. Translation, Proc. ASCE, September 1943, p. 31, by F. Wilsey.

Tests were made on a chute of 0.82 ft in width with slopes from 15.5 to 76.2 percent. The chute decreased in length from 52.5 ft to 18 ft depending on the slope. Discharge varied from 0.353 cfs to 1.57 cfs. Data was also taken from the Rutz wasteway in Austria. The wasteway is constructed of wood on a slope of 76.2 percent and has a trapezoidal cross section with bottom width of 8.2 ft. Suggests that aeration begins at some definite velocity (about 10 ft/sec). An equation of the average water proportion of the flow (C_A) is given:

$$C_A = \frac{2gH}{V^2}$$

where H equals the average velocity head and V the average velocity. Depth measurements made by using a small bar positioned at a point near the "surface," where water droplets would rebound from the bar with a certain force. Velocity determined by using float measurements. Measurements taken on the Rutz Wasteway show a concave transverse water surface profile.

2. Lauffer, Harold, "Druck Energie und Fliesszustand in Gerinnen mit Grossem Gafalle," Wasserkraft und Wasserwirtschaff, Munich, Vol. 30, No. 7, p. 78-82, 1935.

It has been theoretically and experimentally demonstrated that for parallel flow with high gradients the pressure head in the interior of the liquid is no longer equal to the vertical distance from the surface, but is significantly smaller. It follows, therefore, that the (q) line as well as the dynamic capacity are dependent upon the slope. The surface profile corresponding to the minimum dynamic capacity is both the boundary between shooting and streaming flow and the surface profile for maximum discharge. The Froude number for this condition of flow can vary between 0 and I depending upon the slope. Defines (d) as the thickness of the fluid sheet perpendicular to the direction of flow and plots energy line ($H = d \cos \theta + V^2/\partial g$) above the bottom point of each normal section.

3. Lane, E. W., "Recent Studies of Flow Conditions in Steep Chutes," Engineering News Record, January 2, 1936, p. 5-7.

Roughness values for the test section of the UNCOMPAHGRE flume as determined by the Manning and Cutter formulas show variation with discharge. The (n) value increased from .013 to .0177 and then decreased to .0154 as the discharge increased. Observes white water effect as well as cross waves. Points out that due to the decomposition of the boundary of the flume, it is to be expected that the roughness values would vary with discharge (or depth) since different roughnesses would be exposed to the flow at different depths. States that in general roughness values should be about the same for both subcritical and supercritical flow.

4. Hedberg, John, "Flow on Steep Slopes," Civil Engineering, September 1937, p. 633.

States that the normal design formulas for flat slopes will not apply to steep slopes. Suggests the following modifications to the theory:

(1) $p = wy' \cos \theta$ where (p) is the pressure, (w) the specific weight, (y') the depth normal to the surface, and (θ) the angle of bed slope.

(2) velocity should be computed from $V = Q/A'$ where (A') is the area normal to the flow.

(3) The velocity head should be corrected by a multiplying factor (k) also notes that air entrainment will influence the design methods but doubts that model studies will ever yield answers to the problems of air entrainment.

5. Keulegan, G. H., "Laws of Turbulent Flow in Open Channels," Research Paper 1151, Journal of Research, National Bureau of Standards, Vol. 21, p. 707-741, December 1938.

The theoretical investigations of Prandtl and Karman and the experimental work of Nikuradse have led to rational formulas for velocity distribution and hydraulic resistance for turbulent flow in circular pipes. While certain assumptions regarding effects of secondary currents and of the free surface and with the adoption of the hydraulic radius as the characteristic length, similar rational formulas are deduced for open channels. The validity and the applications of these formulas are illustrated by a study of Basin's Experiments. In this study equivalent sand roughness of the channels used by Basin are determined. The rational formulas with constants determined from Basin's are expressed in the form of power laws. It is shown that Manning's empirical formula is a good approximation to the rational formula for rough channels when the relative roughness is large.

6. Nelidov, I. M. "Importance of Study of Flow on Steep Slopes," Civil Engineering, February 1938, p. 121.

Concludes that the so-called "laboratory" experiments have a very limited value in establishing the character of the phenomenon, but when it comes to estimating numerical values of more intricate phases of flow, the experiments with flumes of small size usually carried on in a laboratory do not have sufficient value. In order to obtain the coefficients to be applied to full size structures (spillways, siphons, chutes, and so forth) the tests should be made on models as near full-scale as possible. Gives tabulated comparison of percent deviation in depth for a 30° slope using standard methods and those proposed by "Hedberg" (Civil Eng., September 1937) p. 633.

7. Fortson, E. P., "Investigations of Air Entrainment," Civil Engineering, June 1939, p. 371.

States that since the phenomenon of air entrainment depends upon the absolute values of the velocity, depth and so forth, it cannot be studied in the small scale model. Shows photographs of model and prototype to illustrate his point. Photographs presented are of U.S. Waterways Experiment Station overflow structure at the reservoir. The model is 1:20 and shows no air entrainment for a given discharge while the corresponding discharge on the prototype does show considerable entrained air.

8. Lane, E. W., "Entrainment of Air in Swiftly Flowing Water," Civil Engineering, February 1939, p. 89.

States that whether or not the water flowing down the face of a spillway becomes charged with air depends on upon at least four factors, (1) the turbulence set up on the dam crest or upstream from it (2) the roughness of the surface of the dam (3) the thickness of the overflowing sheet and (4) the height of the dam. For chutes the effects of the sides will also play a part in the air-entrainment process. Suggests that air entrainment does not always begin at a velocity of 10 ft/sec - or at any other fixed velocity. Photographs are shown to illustrate the various hypothesis that the author presents. Also refutes the idea that there is a maximum limiting velocity of 80 ft/sec for water flow down steep chutes.

9. Rogers, Thomas, "Friction in Hydraulic Models," Civil Engineer, June 1939, p. 367.

Takes the value of Manning's (n) to vary with slope and hydraulic radius. Tests made in a triangular channel 70 ft long. Both a smooth varnished surface and artificial sand grain roughness was used. One of the conclusions of the investigation is that friction in a hydraulically smooth conduit is governed by the equation

$$1/\sqrt{f} = 2 \log (R\sqrt{f}) - 0.8 \text{ and}$$

suggests an equation for (n) such that

$$n = \frac{0.148\ R^{1/24} v^{1/8}}{g\ V^{1/8}}$$

where (v) is the kinematic viscosity and (V) the mean velocity. For rough surfaces the investigation indicated that even with fully developed turbulence, Manning's (n) varies not only as the one-sixth power of the roughness projections but also in some manner with the geometry of both the roughness and the flow cross section.

10. Durand, W. F., "The Flow of Water in Channels Under Steep Gradients," Transactions of ASME, 1940, p. 9-14. Discussion by Keulegan, G. H., and Eaton, H. N.

This paper gives a mathematical solution to the problems of the development of the velocity along a given reach or after some period of time. Procedure given for both $V_1 = 0$ and $V_1 \neq 0$ and results determined in terms of a coefficient B which relates the resistance to flow at a given point or at a given instant of time to the square of the velocity. The discussion that follows introduces the velocity distribution into the formulated equations. The introduction of the velocity distribution leads to the equation:

$$S = \frac{a_1+a_3}{\beta} \ln \left(\frac{\bar{v}^2-v_o^2}{\bar{v}^2-v^2} \right)$$

where "S" is the distance traveled and a_1 and a_3 are constants which can be determined from the assumed velocity distribution (example cited).

11. Keulegan, G. H., Patterson, G. W., "A Criterion for Instability of Flow in Steep Channels," Transactions of A.G.U., Vol. 21, July 1940, p. 594-596.

Uses Boussinesq's equation for the velocity of propagation of a volume-element of a wave and combines it with both the Manning equation and the Chezy equation. For the Manning criteria $i > 9/8\ \lambda_o$ provides an instability parameter while the Chezy equation yields $i > 2\lambda_o$ where i is the slope of the channel and $\lambda_o = \frac{\partial gHo\ i}{V_o^2}$ and Ho and V_o are the depth and velocity before the wave.

12. Thomas, Harold A., "The Propagation of Waves in Steep Prismatic Conduits," Proceedings of Hydraulics Conference, U. of Iowa studies in Engineering, Bull. 20, 1940, p. 214.

Pulsating flow can only exist in channels having a slope steeper than the "second critical slope" whose value in a wide rectangular channel is $4g/c^2$ or 4 times the ordinary critical slope. Uses the moving belt analogy to develop the wave theory presented. Gives the average discharge (q') which passes down the channel when the train of standing waves is converted to a train of moving waves by superimposing the velocity (U) on the whole system as:

$$q' = \frac{1}{f} \int_0^t (Uy \cos \theta - q)\, dt$$

13. Wilson, Warren E., "Effects of Curvature in Supercritical Flow," Civil Engineering, February 1941, p. 94.

Makes use of the work done by Knapp and Ippen ("Curvilinear Flow of Liquids ---,") and produces the criterion:

$$\frac{dK}{d\theta}\frac{1}{1-K} = 2\sqrt{f}$$

where, if the combinations of factors on the left is greater than $2\sqrt{f}$, the depth gradient will be less than the normal gradient. If, however, this combination of factors is less than $2\sqrt{f}$, the depth gradient is greater than normal. It is apparent that large Froude numbers and short radii, relative to depth of water, contribute to the reduction in wall pressure. Use is made of a total pressure on a vertical plane in the form $P = K\frac{mgd^2}{2}$ where m = unit mass and d the depth. K is used to correct the assumed hydrostatic pressure distribution to the actual.

14. Hall, L. S., "Open Channel Flow at High Velocities," Transactions of ASCE, Vol. 108, 1943, p. 1394.

Measurements were taken on several prototype structures with the intent of formulating a theory of flow for steep chutes. The following assumptions are made in formulating the theory:

1. The value of the roughness coefficient (n) in the Manning or Kutter formula is constant with the particular type of material.
2. The air in and above the water caused no additional loss in energy, the reduction in the specific gravity compensating for the added area.
3. The hydraulic radius is calculated from $R_c = \frac{Q}{VP_c}$ with a smaller value of (n) than normal.

4. The velocity head computed from the mean velocity can be used without substantial error.

Suggests a discharge equation of the type Q = PAV where P = the ratio of water in a mixture of air and water. Results correlated with the parameter $\frac{V^2}{g\ R_c}$. Calculations based on modified values of area, hydraulic radius, velocity, and Manning's (n).

15. DeLapp, Warren, Discussion of "Open Channel Flow at High Velocities," by Hall, Transactions of ASCE, 1943, p. 1448.

Suggests there is questionable significance in relating the percentage of entrained air to the parameter $V^2/g\ Re$. Also does not agree with the use of (Re) in the computations. Points out that the fact that (n) is relatively constant for a given channel is not significant since the values of (Ro) are so close to unity.

16. Douma, J. H., Discussion of "Open Channel Flow at High Velocities," by Hall, Transactions of ASCE, Vol. 108, 1943, p. 1462.

In general the results of studies on open channels are conclusive in showing that depths and velocities in steep chutes and spillway channels cannot be calculated by the usual application of Manning's formula without consideration of entrained air. Hydraulic design of channel walls, horizontal curves, vertical curves, super elevated inverts, and stilling basins should be based on new design assumptions based on entrained air. Suggests use of Manning's formula in the formula $V = \frac{N}{n} R^{2/3} S^{1/2}$ where R and N are functions of velocity and air content. Expresses $N = 1.486 - 0.000248\ u^2$. Treatment also given to hydraulic jump with entrained air.

17. Knapp, Robert T., Discussion of "Open Channel Flow at High Velocities," by Hall, Transactions of ASCE, 1943, p. 1455.

Treats the problem of flow around horizontal curves as well as problems of air entrainment. Suggests that the use of his treatment of flow around horizontal bends will eliminate the need for model tests as proposed by Mr. Hall. States that the problem of air entrainment is essentially a wave phenomenon. Also, states that the entrainment and transportation of air and sediment have many points in common and that parallel treatments should yield many useful results.

18. McConaughy, D. C., Discussion of "Open Channel Flow at High Velocities," by Hall, Transactions of ASCE, Vol. 108, 1943, p. 1484.

Concludes that better and more accurate means of taking measurements would yield results that would be more reliable in forming conclusions. Suggests that the formula for aeration should allow for channel roughness. States that the velocity of an air-water mixture is less than for water alone which is opposite the opinion of Mr. Hall. Suggests that there could be problems of viscosity and density currents involved and that there should be some definition given to the term (depth) for air-entrained flows.

19. Stevens, J. C., Discussion of "Open Channel Flow at High Velocities," by Hall, Transactions of ASCE, Vol. 108, 1943, p. 1474.

Gives an alternative analysis of the observed data presented by Mr. Hall. Uses the subscript (w) to denote water without air and (m) the mixture of water and air. Then defines the percentage of water

$$(P) = \frac{Q}{A_m V_m} \quad \text{and} \quad A_w = p\, A_m \, .$$

Revised calculations are tabulated and an example design calculation is given. Suggests that the water in the bottom of the channel is moving faster than the mixture. States that if the friction slope is greater than the bed slope, pulsation of flow is likely.

20. Johnson, J. W., "Rectangular Artificial Roughness in Open Channels," Transactions of A.G.U., 1944, p. 906-912.

    Tests made in a redwood flume 9-1/2 inches by 9-1/2 inches in cross section and 70 feet long. The average bottom slope was 0.00245. Roughness was obtained by nailing redwood strips to the bottom of the flume. States that with a given ratio b/a there is a value c/a where the roughness coefficient is a maximum. (b) is the width of the strip of artificial roughness, (a) is the height and (c) is the spacing. Results correlated with earlier investigations. Discussion by Powell follows the paper.

21. Kindsvater, C. E., "The Hydraulic Jump in Sloping Channels," Transactions of ASCE, 1944, p. 1107.

    Tests made in a glass walled flume, 30 in. wide, 3 ft deep, and 30 ft long. Slope was 1 on 6. Studies made of three general cases of hydraulic jump in sloping channels. Paper presents a generalized analysis of the jump leading to a practical method of computing the essential dimensions of the jump. Complete equation of the hydraulic jump in sloping rectangular channels is given by

$$\frac{\gamma(d_2)^2}{2} - \frac{r(d_1)^2}{2\cos^2\alpha} - \Phi\left[r(d_2)^2 - \frac{r(d_1)^2}{\cos^2\alpha}\right] \tan\alpha$$

$$= \frac{r(U_1)^2\, d_1 \cos\alpha}{g\, d_2} \left[d_2 - \frac{d_1}{\cos\alpha}\right]$$

Discussions by Messrs. Joe W. Johnson, Karl R. Kennison, J. C. Stevens, C. J. Posey, Jerome Fee, Frank S. Bailey, G. H. Hickox, and Carl E. Kindsvater follow the paper.

22\. Hickox, G. H., "Air Entrainment on Spillway Faces," Civil Engineering, December 1945, p. 562.

Makes comparison of the point at which air entrainment begins on the Norris Dam Spillway and on the 1:72 scale model of the dam. Suggests that the air entrainment begins at the point where turbulence, generated at the water concrete interface, finally reaches the surface. Notes that the ratio of L/D, length along spillway face to depth of water, is nearly constant for all discharges observed, indicating that the rate of expansion of turbulence is in the order of about 1 to 100.

23\. Powell, R. W., "Flow in a Channel of Definite Roughness," Transactions of ASCE, Vol. III, 1946, p. 531.

Tests made in a flume 50 ft long, 8 in. wide and 7 in. deep. Artificial roughness formed by square steel strips, which extended down the sides and across the bottom. Various arrangements of the strips are shown. Slopes ranged from 0.0312 to .0005. Runs were made on both smooth and rough channels and tabulation of data is given. Equations are presented for tranquil flow in smooth and rough channels. Discussion by Messrs. Joe W. Johnson and E. A. LeRoux, Garbis H. Keulegan, C. J. Posey, and Ralph W. Powell.

24\. Powell, R. W., "Vedernikov's Criterion for Ultra-Rapid Flow," Transactions of A.G.U., Vol. 29, December 1948, p. 882.

Powell gives the expression $\underline{V} = (1 + B)\, MV/p\, (u-v)$ which he calls the Vedernikov number. This expression comes from papers written in Russian by

Vedernikov (1945 and 1946). When this criterion is less than one, waves tend to be dampened out but when it is equal to or exceeds one the flow is ultra rapid, roll waves form, and the flow cannot be steady. An empirical formula for Chezy's (c) in ultra-rapid flow is derived from data already published by Powell. A tabulation of the data used and computations made is given. States that a resistance law for ultra-rapid flow should depend on the Vedernikov number. The formulation given for Chezy's (C) is:

$$C = 41.2 \log_{10} (R/C) + 42.3 \underline{V} (F_c - 0.515) - 113.7$$

Discussions by Vedernikov, V. V., Powell, R. W., Owen, W. M., Thijsse, J. and Halsey, J. F. in Transactions of A.G.U., 1951, p. 603.

25. Gumensky, D. B., "Air Entrained in Fast Water Affects Design of Training Walls and Stilling Basins," Civil Engineering, December 1949, p. 35.

Suggests using Manning's equation for the design with a value of (n) equal to .008 for computation of velocities and net depths. When air entrainment is to be expected the following equation is offered $m = \frac{V^2}{200\ gD}$ where (m) is the ratio of volume of entrained air to water. Suggests also that a freeboard allowance of at least 100 percent of the computed entrained air plus 5 ft. Also investigates the hydraulic jump with entrained air and states for practical design it is permissible to use the actual velocity and net depth of water without air entrainment in hydraulic jump computations. Net depth is the depth with no air entrainment.

26. Bowers, C. E., "Studies of Open Channel Junctions," Hydraulic Model Studies for Whiting Field Naval Air Station, Milton, Florida, St. Anthony Falls Hydraulic Laboratory - University of Minnesota, Part V - Project Report No. 24 - January 1950.

Studies made in trapezoidal channels with maximum discharge ranging from 380 to 960 cfs in the main channel and from 27 to 70 cfs in the terrace channels. Some flows were supercritical with maximum velocities reaching 30 fps. Several designs were studied using the momentum analysis. Suggests more experimentation in this area may lead to some general design criteria. The pressure-momentum relationships seemed to give only partial solutions to the problem.

27. Powell, R. W., "Resistance to Flow in Rough Channels," Transactions of A.G.U., Vol. 31, 1950, p. 575.

Presents formulas for turbulent flow which apply to both tranquil and rapid flow. States that possibly the resistance to Ultra-rapid flow may follow a somewhat different law than the one presented. The resistance coefficient is presented in the form $C = 42 \log_{10} (c/R + E/R)$, however, Powell suggests that the list of proper values of E to use in this formula needs further investigation. Graphs are given for the solution of this equation with three different groups of given quantities. Discussion by Owen, W. M., Thijsse, J., Halsey, J. F., and Powell, R. W., in Transactions of A.G.U., Vol. 32, August 1951, p. 607.

28. Ippen, A. T., "Mechanics of Supercritical Flow," Transactions of ASCE, Vol. 116, 1951, p. 268.

Surface disturbances are treated by two methods of approach:

1. Gradual surface changes may be analyzed on the basis of constant specific head; and
2. Standing wave fronts of appreciable height can be computed, considering the energy dissipation involved.

Graphical aids for the solution of both types of problems are given in detail. The primary features of supercritical flow and general characteristics of standing wave patterns are treated. Shock wave interesections and reflections are explained. Mechanics of wave propagation is given a thorough analysis. Discussions by Messrs. Paul Baumann, N. N. Bhandari, T. Blench, Clarence A. Hart, Fred W. Blaisdell, J. H. Douma, Arthur T. Ippen, Robert J. Knapp, Hunter Rouse, B. V. Bhoota and EN-YUN HSU.

29. Ippen, A. T., and Dawson, J. H., "Design of Channel Contractions," Transactions of ASCE, Vol. 116, 1951, p. 362.

Treatment given to contractions formed by circular arcs as well as straight wall contractions. For supercritical flow the accent of design is on the reduction of the standing wave patterns. Suggests that the selection of both deflection angles and length for given reductions in width will avoid excessive standing wave heights in the contraction. States that additional experimental work considering warping of the bottom and large longitudinal changes in slope should be conducted. Discussions by Messrs. Paul Baumann, N. N. Bhandari, T. Blench, Clarence A. Hart, Fred W. Ippen, Robert T. Knapp, Hunter Rouse, B. V. Bhoota and En-Yun Hsu.

30. Knapp, R. T., "Design of Channel Curves for Supercritical Flow," Transactions of ASCE, Vol. 116, 1951, p. 296.

Supercritical flow around curved sections of channel produce cross-wave disturbance patterns which also persist for long distances in the downstream tangent. Outlines two basic methods of eliminating the disturbance patterns. One method consists of applying a lateral force simultaneously on all filaments. This can be accomplished by bottom banking or the use of vertical curved vanes. The other method makes use of compound curves, spiral transitions, and sills which set up interference patterns. Points out that results may be erratic with Froude numbers between 1 and 1.5 because of the instability of the flow in this region. Discussions by Messrs. Paul Baumann, N. N. Bhandari, T. Blench, Clarence A. Hart, Fred W. Blaisdell, J. H. Douma, Arthur T. Ippen, Robert T. Knapp, Hunter Rouse, B. V. Bhoota and En-Yun Hsu.

31. Rouse, Hunter, Bhoota, V. V., and En-Yun Hsu, "Design of Channel Expansions," Transactions of ASCE, Vol. 116, 1951, p. 347.

Discussion given to characteristics of flow at abrupt expansions as well as the efficient curvature of expanding boundaries. Concludes that application of the elementary wave theory to the analysis of high-velocity flow in open-channel expansions may be expected to yield results in essential agreement with experiment as long as the assumptions involved in the theory are approximately satisfied. Makes use of the initial Froude number and relative coordinate location for design purposes. Treatment also given to the elimination of disturbances at the end of an expansion. Jump stablization is accomplished by a drop in the floor level. Discussions by Messrs. Paul Baumann, N. N. Bhandari, T. Blench, Clarence A. Hart, Fred W. Blaisdell, J. H. Douma, Arthur J. Ippen, Robert T. Knapp, Hunter Rouse, B. V. Bhoota and En-Yun Hsu.

32. Craya, A., "The Criterion for the Possibility of Roll-Wave Formation," National Bureau of Standards Circular 521, 1952, p. 141-151.

Treats Quasi-steady regimes, stability of elementary waves, and criteria for the possibility of roll waves, by making use of the dynamic equation and the equation of motion in the forms:

$$\frac{\partial}{\partial x}\left(h \cos i + \frac{U^2}{2g}\right) + \frac{1}{g}\frac{\partial U}{\partial t} = \sin i - \epsilon \lambda \frac{U^2}{gR}$$

$$\frac{\partial Q}{\partial x} + \frac{\partial w}{\partial t} = 0$$

Formulates a general criteria for the possibility of roll waves of the form

$$\frac{V'}{U} < w \frac{d}{dw} \log U$$

If $\lambda = K(UR)^{-\alpha}$ the criterion becomes

$$\frac{V'}{U} < \left(1 - R\frac{dx}{dw}\right)\frac{1+\alpha}{2-\alpha}$$

or if $\lambda = KR^{-\beta}$ the criterion becomes

$$\frac{V'}{U} < \left(1 - R\frac{dx}{dw}\right)\frac{1+\beta}{2}$$

33\. Dressler, R. F., "Stability of Uniform Flow and Roll-Wave Formation," National Bureau of Standards Circular 521, 1952, p. 237-241

Using nonlinear shallow water equations of the form:

$$u_t + uu_x + g \cos\theta\, y_x = g \sin\theta - \frac{r^2u^2}{y^m}$$

$$yu_x + uy_x + y_t = 0$$

A stability criterion is proposed in the form:

$$m^{2n}\, g^{2-n}\, Y^{2m-n} \sin^2\theta \lesseqgtr n^{2n}\, r^4 \cos^n\theta$$

where $\leqq$ implies stability and $>$ implies instability. By treating the above stability criterion from two standpoints, (a) $m \longrightarrow 0$ $n > 0$ and (b) $n \longrightarrow 0$ $m > 0$, the following conclusions are made:

1. When turbulent resistance effects behave directly with any power of the velocity and inversely with any power of the depth, there

will always exist an angle of declination beyond which the uniform flow becomes unstable.

2. This critical angle where instability occurs is the same angle which is obtained as a condition for the existence of roll waves by satisfying the shock energy inequality.
3. Instability cannot occur if resistance depends only upon velocity variation, or only upon depth variation; the simultaneous action of both effects is required. This can be concluded either from the stability analysis, or the shock energy approach.

The Jeffrey's criterion for instability is given as $\tan \theta > 4 r^2$ which is also formulated from the above conditions. Thus the resulting roll waves are subcritical at the peaks and supercritical in the valleys.

34. Lamb, Owen P., "Experimental Channel for Study of Air Entrainment in High Velocity Flow," Project Report No. 34, November 1952, St. Anthony Falls Hydraulic Laboratory, University of Minnesota.

A large open channel designed for the study of self aeration of high velocity flows has been built and installed at the laboratory. This 50 ft. channel has a cross section 12 in. deep and 18 in. wide and can be set at any slope from the horizontal to the vertical. The slope is controlled by means of a hydraulic system and is indicated with a servo system with accuracy of 1/4 degree. The initial flow depth in the test flume is controlled by an electrically driven sluice gate with a rounded entrance located at the head of the flume. The depth can be controlled and indicated within .0018 through its 1/4 in. to 6 in. range. The water discharge is related by two hydraulically operated gate valves in the supply line and is measured with accuracy of about

1-1/2 percent. The inlet region is designed to produce an initially uniform jet at terminal velocity and the flume is long enough to permit the aeration process to reach equilibrium for a range of discharges at all slopes. The selection of these flume dimensions and performance limits of the installation are explained from present knowledge of air-entrained flows and from aerated flow measurement requirements. Points out that for the flume breadth requirements that strict two dimensionality would of course occur only at very large ratios of channel width to depth where the presence of the side walls could not be felt in any manner over the large central region of the flow. However, for his purpose a region of the flow may be considered two dimensional if there is no appreciable change in the profiles of air concentration or velocity at successive intervals across the region which implies that the turbulent boundary layers from the side walls have not yet infected the region. Suggests that further studies be made in a channel other than rectangular.

35. Robinson, A. R., and Albertson, M. L., "Artificial Roughness Standard for Open Channels," Transactions of A.G.U., Vol. 33, December 1952, p. 881.

Demonstrates that a roughness standard such as exists for pipes may be set up for open channels with rough boundaries. States that the resistance coefficient is not a constant for a given channel but varies with velocity and depth. Also, points out that viscosity influence is not taken into account with formulas in present use. Study was made with a boundary so rough that the viscous effects were negligible. The equation $C = 26.65 \log_{10} (1.891\ d/a)$ was established.

36. Frankovic, Ante, "Head Loss and Air Entrainment by Flowing Water in Steep Chutes," Proceedings Minnesota International Hydraulics Convention, September 1-4, 1953, University of Minnesota, p. 467.

The maximum flow velocities for the measurements taken appears to occur at a little above half the depth. This fact caused taking into account the loss of head due to the friction of the free water surface with air. The higher the water velocity in a chute or the more it is mixed with air, the greater the loss of energy. At a double depth of mixture, the energy loss amounts to 75 percent and at higher velocities it is in proportion higher and equals: $Eg = \frac{MV^2}{2}(1-u)$ where $u = \frac{t}{t+z}$ (u) equals the volume ration of the water and the air-water mixture and (M) equals the mass of the air-water mixture. Concludes that the main energy loss in the stilling basin does not arise by the formation of the hydraulic jump, but in the steep chute owing to the mixing of air with flowing water. Also points out thattemperature and viscosity play a part in the amount of air entrained. Example computations are cited using the equations presented.

37. Halbronn, D. R. and Cohen de Lara, G., "Air Entrainment in Steeply Sloping Fumes," Proceedings Minnesota International Hydraulics Convention, September 1-4, 1953, University of Minnesota, p. 455.

Studies made on a tilting platform 3 ft wide and 53 ft long with slope adjustments from the horizontal to the vertical. Channels of various cross sections could be placed on this platform with one of 1-2/3 ft in width selected for the particular study described. Gives detailed description of measuring devices and instrumentations used for the study. Data presented only for a 14° slope. States that for the 14° slope, the passage from the emulsion zone to the droplet zone always corresponds to an air concentration of about 60 percent. The emulsion zone is described as (air bubbles in water) while the droplet zone is described as (drops of water in air).

38. Yevdjevich, V. and Levin, L., "Entrainment of Air in Flowing Water and Technical Problems Connected with it." Proceedings Minnesota International Hydraulics Convention, September 1-4, 1953, p. 439-454.

States that the presence of air in water decreases frictional resistance among the strata and on the boundary. Also, suggests that model tests provide little help in studies of air entrainment due to our ignorance of the laws of hydrodynamic similarity of aerated flow. Gives relationship of wall roughness, total roughness of aerated flow, and roughness of non-aerated flow in terms of Manning's (n_o) such that $n_w > n > n_o$. Also, suggests that the magnitudes involved in the preceding relationships for (n) is a function of the Froude number. For specific measurements taken at Imotsko Polje $n \approx n_o$ for super rapid flow. Treatment also given to stilling basin behind steep chute and water inlet into shafts.

39. Michels, V. and Lovely, M., "Some Prototype Observations of Air Entrained Flow," Proceedings Minnesota International Hydraulics Convention, September 1-4, 1953, p. 404-414.

Suggests seven (7) classifications of air entrainment in terms of the observed characteristic kinds of turbulent flow associated with each. The seven classifications are:

1. Rippled flow
2. Choppy flow
3. Scarified flow
4. Emulsified flow
5. Ebullient flow
6. Spraying flow
7. Separation flow

States that for considerable air entrainment to occur the following conditions should be fulfilled: (1) the flow velocity should exceed a certain minimum stated to range from 10 fps to 20 fps; (2) turbulence caused by boundary layer development should extend throughout the whole depth of flow; (3) for uniform flow, the channel bed slope should exceed a certain minimum value. For air entrainment to begin on a spillway face the equation $\delta = \frac{S' \cdot q^{1/2}}{157.7}$ is given. Excellent photographs of prototype observations are shown.

40. Straub, L. G., Killen, J. M., and Lamb, O. P., "Velocity Measurement of Air-Water Mixtures," Proceedings of ASCE, Vol. 79, May 1953, No. 193.

Explains in detail the development of the St. Anthony Falls (SAF) velocity meter. This meter makes use of the salt velocity principle as well as electronic methods for measuring very short time intervals that were developed extensively during World War II for radar, sonar and other similar uses. The SAF velocity meter used with the cathode-ray tube is quite satisfactory for laboratory observations and delivers an individual reading accurately on the order of 2 percent of the actual velocity.

41. Straub, L. G. and Lamb, O. P., "Experimental Studies of Air Entrainment in Open Channel Flow," Proceedings Minnesota International Hydraulics Convention, September 1-4, 1953, p. 425-439.

For purpose of measurement defines the surface of the air-water mixture to be at the point where the concentration of air is 0.95. Tests made in a 1.5 ft wide by 50 ft long flume at varying discharges and slopes. Suggests that the use of the Manning equation for air-water mixtures will yield values of velocity that are too low. Comparison of data seem to show that the air concentration is not particularly sensitive to slope changes. Tables and graphs of the data are presented. Also presented in Trans. ASCE, Vol. 121, 1956, p. 30. Computation presented seems to show that velocity calculated from the Manning equation is usually too low when compared to measured values.

42\. Viparelli, Michele, "The Flow in a Flume with 1:1 Slope," Proceedings Minnesota International Hydraulics Convention, September 1-4, 1953, p. 415-423.

Makes use of two values for the roughness coefficient in the Manning equation (n, n') where $n' > n$ and where average velocity is computed using (n) and depth with (n'). Breaks the cross section of flow into: (1) a layer of water near the bottom in which only little quantities of bubbles are carried, (2) an intermediate layer in which water and air are in almost equal quantities and (3) an upper layer of air in which large water drops are in movement. Describes in detail the experimental apparatus used in the investigation. In the upper layer concludes that the distribution of drops depend on the velocity and slope of the current.

43\. Bauer, William J., "Turbulent Boundary Layer on Steep Slopes," Transactions of ASCE., 1954, p. 1212. Discussions by Messr. G. Halbronn, and William J. Bauer.

The slope of the channel and the magnitude of the discharge have little effect in determining the boundary layer thickness as a function of the Reynolds number or as a function of X/K . The significant parameter in this regard appears to be the boundary roughness. Suggests velocity might better be expressed as $u = Jy^{i}$ than the form $u = b \log \cdot y$. Application is made to the design problem and an example is given. Points out that boundary layer development is related to the process of air entrainment: $x = \frac{U^2}{2gS}$ K = roughness height value. An application to a design problem is given.

44\. Einstein, H. A., and Sibul, O., "Open Channel Flow of Water-Air Mixtures." Transactions of A.G.U., 1954, p. 235.

Gives in detail the system used for measuring the air concentration in an air-water mixture. The problem is analysed in analogy with that of sediment transport. Found that the vertical distribution of air in a turbulent water mass follows the laws of suspension. Suggests that for similarity conditions the Froude law breaks down and that perhaps a distorted similarity exists that would suffice for model-prototype relationships. This distorted similarity might then apply to air entrainment problems.

45. Haberman, W. L., and Morton, R. R., "An Experimental Study of Bubbles Moving in Liquids." Proceedings ASCE, Vol. 80, No. 387, 1954.

Gives three size categories for the bubbles: spherical, ellipsoidal, and spherical cap. As the size of the bubbles increased the shapes given above would take form. The most important parameters effecting the rate of rise were: viscosity for spherical, surface tension for ellipsoidal, and the spherical cap bubbles rise independently of the fluid properties. Various liquids were used and results put in graphical form in the appendix. Photographs of bubble shapes in various liquids are also given.

46. Ippen, Arthur T., and Harleman, R. F., "Verification of Theory for Oblique Standing Waves." Proceedings of ASCE., Vol. 80, 1954, No. 526.

Studies made of oblique hydraulic jumps and expansion waves with Froude numbers ranging from two to seven. Analytical and experimental observations show the transition from undular jump to the roller type jump takes place at a depth ratio of two. Summary of experimental results on oblique standing waves is given in tabular form. Photographs and charts shown in the appendix.

47. Morris, H. N., "A New Concept of Flow in Rough Conduits." Proceedings of ASCE., Vol. 80, 1954, No. 390.

Suggests the existance of three basic types of flow over rough surfaces. The three types are: (1) Isolated-Roughness Flow, (2) Wake-Interference Flow, and (3) Quasi-Smooth (skimming) Flow. Mathematical treatment given to these three types and equations for friction factor given for each type. States that isolated roughness flow would occur over most commercial conduit surfaces, wake-interference flow over corrugated surfaces and sand or gravel coated surfaces, and Quasi-smooth flow over surfaces that are nearly smooth but have depressions at the joints. Subject also presented in Trans. ASCE, Vol. 120, 1955, p. 373, with discussions by Messrs. V. L. Streeter, Walter Rand, Harry H. Ambrose, and Henry M. Morris, p. 399.

48. Priest, Melville S., and Baligh, Aly., "Free-Surface Instability of Liquids in Steep Channels." Transactions of A.G.U., 1954, p. 133.

Uses the function - Φ $(UD/v\ ,\ U^2/gd\ ,\ \theta) = 0$ to express the variables in question. "D" is the measured depth of flow, "U" is discharge divided by the product of depth and width, "v" is the kinematic viscosity, and "θ" the angle of inclination of the channel bottom. The three parameters given above are plotted on a single graph for the data taken. Suggests further study be made before any commentary is justified. Relatively short channel was used which placed some limitations on the observations made.

49. Robertson, J. M., "More Research on Aerated Flow Needed." Civil Engineering, Dec. 1954, p. 55.

Further studies needed of the mechanics of the entrainment process as well as the processes by which the air is kept in the water. Many specific problems involving open and closed conduit structures are in need of further research. Thus, additional information on the rate of rise of air bubbles, especially in turbulent water, and the movement of air slugs along pipes is needed to help in locating air vents in pipes. In spillways and chutes, the effect of side walls in causing air entrainment is not adequately known. Suggests turbulence is extremely significant in keeping the air in the flow and in many cases in entraining it--a point in need of further research.

50. Wilsey, E. F., "Flow in Open Channels." Proceedings of ASCE, Vol. 80, 1954, No. 466.

Use is made of "Ehrenberger's" data as well as data from the Kittitas wasteway (slopes of 11°-12' and 33°-10') Suggests that B (the resistance coefficient) is independent of the Reynolds number for channels set at steep slopes. For wooden chutes at angles from 8 to 42 degrees the water portion of the flow (P_w) can be expressed as $P_w = .06\ B \cos\theta$ (function of roughness and slope) and velocity (V) as $V = 16.7\ P_w \sec\theta - \sqrt{2g\ R \sin\theta}$. For concrete chutes, $P_w = .0466\ B \cos\theta$ and $V = 21.4\ P_w \sec\theta - \sqrt{2g\ R \sin\theta}$. Also, the resistance coefficient largely depends on the amount of air entrained in the flow and that as the water portion of the flow decreases, the air portion $(1 - P_w)$ increases. Treatment also given to flow in channels of low slope. The ratios of numerical constants in the equation for velocity for wooden chutes to that for concrete chutes is $\frac{21.4}{16.7} = 1.28$. A plot is made of P_w vs angle of inclination in degrees for both chutes where the ratio of the slope is 1.32. There might be some relation that could be developed here.

51. Anderson, A. G., "The Distribution of Air in Self-Aerated Flow in a Smooth Open Channel." St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Project Report No. 48, 1955.

Suggests that in the lower portion of the flow the entrainment of air is in the form of bubbles distributed through the flow. Above the region water particular projected outward from the flow by the intense normal velocity fluctuations gives rise to the highly agitated appearance of the flow. Properties and parameters of the distribution curve are correlated with the bulk flow characteristics. The mean depth for the smooth channel based upon the air distribution appears to be a function of $q/s^{1/2}$. It was also found imperically that the mean air concentration is related to $S/q^{2/3}$ where q is the unit discharge and S is the sine of angle of bed slope. The results indicate that the resistance to aerated flow is less than non-aerated flow. Based upon the mean depth these results indicate that the resistance to aerated flow is about 70 percent of that for non-aerated flow. Tests were made for discharges ranging from 3.2 to 9.6 cfs on slopes ranging from 7.5° to 45° in a painted steel channel 1.5 ft wide and 50 ft long.

52. Amein, M. A. and Prist, M. S., "Free Surface Disturbances Along A Channel Wall," Journal of ASCE, 1956, Proc. Paper 1005.

Experiment conducted in an 18 in. wide by 12 ft long flume. Slopes from 10 to 45°; discharge from 0.0177 to 0.0730 cfs; and water temperature from 38 to 44°F. Experimental data given in the appendix. Free-surface disturbances initiated by fixing a thin wire to an otherwise smooth channel wall. Equations presented for initiation of the free-surface disturbance at the channel wall as well as for the growth of such a disturbance. Data also used in graphical form for both conditions listed above.

53. Grace, John L. and Prist, Melville S., "Division of Flow in Open Channel Junctions," Engineering Experiment Station, Alabama Polytechnic Institute, Auburn, Alabama, Bull. No. 31, June 1958.

Studies made with rectangular channels having plane horizontal bottoms. Runs were made with low Froude Number and with junctions of 30, 60, 90 and

120 degrees. Results presented in graphical form at the end of the paper. Scatter of data attributed propagation of surface disturbances from the junction and the method of depth measurement.

54. Koloseus, H. J., "The Effect of Free-Surface Instability on Channel Resistance," Ph.D. Dissertation, Iowa State University, 1958.

Using purely analytical considerations and the equation of motions as given by "Keulegan" (Effect of Turbulence and Channel Slope on Translation Waves-National Bureau of Standards paper 1544, 1943) and a continuity equation of the form $\frac{\partial h''}{\partial +} + \frac{\partial (U''h'')}{\partial x} = 0$, developes a stability criterion of the form

$$F \begin{bmatrix} \text{Unstable} \\ \text{Neutral} \\ \text{Stable} \end{bmatrix} \begin{matrix} > \\ = \\ < \end{matrix} \left\{ \left[(.434\ C_1)^2 f \left(1 - \frac{2}{C_2 e^{\frac{4h}{k}}} \right) -0.5 - (.434\ C_1) f^{1/2} \right]^2 - \left[1 + (.434\ C_1)^2 f \left(1 - \frac{2}{C_2 e^{\frac{4h}{k}}} \right) \right] \left[(.434\ C_1)^2 f \left(1 - \frac{2}{C_2 e^{\frac{4h}{k}}} \right) \right] \right\}^{-1/2}$$

Experimental work is then conducted in a glass walled tiltable flume, 2 ft wide and 30 ft long. 3/16 in. brass cubes were used to form the roughness. Depth measurements were made with a specially constructed vibrating needle point gage.

From the data collected it is concluded that the stability criterion is a function of the resistance coefficient and the Froude number. For practical considerations the stability criteria reduces to

$$F \begin{bmatrix} \text{Unstable} \\ \text{Neutral} \\ \text{Stable} \end{bmatrix} \begin{matrix} > \\ = \\ < \end{matrix} 1.6$$

It was concluded from this investigation that:

1. For two dimensional flow over a hydraulically rough boundary, the stability criterion is a function of both the Froude number and the resistance coefficient. However for all practical purposes, the flow could be regarded as being stable when the Froude number was less than 1.6 and unstable when the Froude number was greater than 1.6
2. The resistance coefficient is independent of the Froude number when the flow is classified as being stable.
3. The resistance coefficient is a function of the Froude number when the flow is said to be unstable.

55. Straub, L. G. and Anderson, A. G., "Experiments on Self-Aerated Flow in Open Channels," Journal of ASCE, 1958, HY7, No. 1890.

Data is given for tests made in a 1.5 ft wide, 1.0 ft deep, and 50 ft long channel. An artificial roughness was used which was constructed with a commercial non-slip fabric coated with granular particles. Slopes ranged from 7.5 to 75 degrees and discharge from 2.2 to 9.6 cfs. with up to 15 cfs on some of the slopes. The upper limit of depth defined as the point where the air concentration equals 0.95. States that the mean concentration of air is a function of $S/q^{1/5}$ where (S) is the sine function of the slope angle and (q) is the unit discharge. Concludes that the mean velocity of an air-entrained flow is greater than that of a corresponding non-aerated flow by an amount that increases with the air concentration and corresponds to a decrease in the mean depth. Also that self-aerated, high velocity, open channel flow appears to consist of two distinct phases, an upper and lower region in the vertical provile, where the water-air agglomerate of the upper may be described by the Gaussian cumulative probability and that of the lower with an equation for turbulent mixing. Discussion by Viparelli, M., ASCE, HY6, No. 2076, p. 75 and by Straub, L. G., and Anderson, A. G., ASCE, HY11, No. 2269, p. 119.

56. Irmay, S., "Accelerations and Mean Trajectories in Turbulent Channel Flow," ASME, Series D. Vol. 81-82, 1960, p. 961.

Shows that even in steady uniform turbulent channel flow there exist on the average mean accelerations and forces acting on fluid particles. Flow is divided into essentially three different flows (a) Zone I - laminar sublayer--where there are practically no external forces acting on the particle (b) Zone II - negative accelerations--where each particle is subjected to a retarding force, lifting force and to a couple tending to rotate it forward (c) Zone III - positive accelerations--where each particle is subjected to a constant force in the direction of flow. It is also acted upon by a lift force in the lower half of the depth and a force directed downward in the upper half. Reference is made to sediment transport application. Discussion by Messrs. Delleur, Kline, and Townsend follow the paper.

57. Peterson, D. F., Mohanty, P. K., "Flume Studies of Flow in Steep Rough Channels," Proceedings of ASCE, November 1960, p. 55-76.

Studies made in a flume 2 ft wide, 16 in. deep, and 64 ft long. Maximum discharge of 4 cfs, maximum slope of 8.5 percent. Artificial roughness consisted of bars and cubes and several different spacings were tested. Suggests the flow be classified into three major regimes; (1) tranquil flow, (2) tumbling flow and (3) rapid flow. Roll waves occurred during the transition stages of the flow regimes. Suggests the tranquil and rapid regimes may be described by an equation of the type $F_1 = C' S^n$ in which C' and n depend on the roughness intensity and configuration. Proposes that the use of large roughness elements could result in improved hydraulic design. Discussion by Engel, F. V. A., Rajaratnam, N., and Koloseus, H. J., Proceedings of ASCE, July 1961, p. 245-250.

58. Koloseus, H. J. and Davidian Jacob, "Flow in an Artificially Roughened Channel," Geological Survey Professional Paper No. 424-B, 1961.

Roughness elements consisting of 3/16 in. cubes, arranged in diamond patterns and placed normal to the mean direction of flow were used in the tests. Tests were conducted in two rectangular tiltable flumes, one 2 ft wide and 30 ft long, and the other 2.5 ft wide and 85 ft long.

States that for supercritical flow the resistance coefficient for a rough channel is independent of gravitational effects ehen the Froude number is less than 1.6 and is independent of viscous effects when $\sqrt{f}\ R\ \frac{k}{4h}$ exceeds 600. With these limitations assumes $f = G(\frac{k}{4h}, h)$. For a 4-fold variation in $\frac{k}{4h}$ and a 64-fold variation in n the equation:

$$\frac{1}{\sqrt{f}} = 2 \log \left[0.14\ \lambda^{-09} \left(\frac{4h}{k}\right)\right]$$

gives the best fit for the data taken.

59. J. Ernest Flack, Jan Inge Kveisengen, and John H. Nath, "Air Entrainment in Turbulent Liquids," Science, Aug. 11, 1961, p. 392-393.

A 12" diameter container 24" deep with an oscillating grid suspended in the liquid was used to study the effects of viscosity, surface tension, and density on air entrainment. An equation $e = .013\ (R^{0.21}\ w^{0.58}\ F^{o})$ was formulated to fit the data taken. "C" is the mean air content by percent of volume in the container, "R" the Reynolds number, "w" the Weber number and "F" the Froude number. The "O" exponent on "F^O" indicates that gravity had a negligible effect on the air entrainment for the tests made. Decreasing the surface tension resulted in greater air entrainment. Increasing the turbulence and decreasing the viscosity had the same effect.

60. Sayre, W. W., and Albertson, M. L., "Roughness Spacing in Rigid Open Channels," Proceedings ASCE, 1961, Journal of the Hydraulics Division, p. 121.

Experiments conducted in an 8 ft wide, 2 ft deep rectangular flume, 72 ft long. Sheet metal baffles, 6 in. wide and 1-1/2 in. high were used as roughness elements. Baffles were placed so as to form symmetrical repeating patterns. Discharges ranged from 2 to 6 cfs and slopes from 0.001 to 0.003. The equations $\frac{c}{\sqrt{g}} = 6.06 \log \frac{Y_n}{a} + c_2$ and $\frac{c}{\sqrt{g}} = 6.06 \log \frac{Y_n}{X}$ were found to apply over the range of roughness conditions tested. Suggests application of the Manning equation to the range of roughness tested would lead to serious error. Graphical and tabular forms of the data are presented. Discussions of the paper appear in Journal of the Hydraulics Division, ASCE, November 1961.

61. Task Committee on Air Entrainment in Open Channels, Committee on Hydromechanics, "Aerated Flow in Open Channels," Proceedings of ASCE, Journal of the Hydraulics Division, Vol. 87, May 1961, p. 73.

Divides air entrainment problems into three general classes. Problems with narrow chutes (width less than five times the depth), problems with wide chutes (width greater than five times the depth) and overflow spillways. Suggests more studies directed toward the development and effects of turbulence, also studies in wide channels where side wall effects can be isolated, as well as effects of wall protuberances, joints, and piers. Bibliography on aerated flow given in appendix. Discussion in Journal of the Hydraulics Division, ASCE, November 1961, p. 221.

62. Discussion of "Aerated Flow in Open Channels" (Proc. Paper 2814, May 1961) by Jan Inge Kveisengen; N. Rajaratnam; and Mikio Hino. Proceedings ASCE, Journal of the Hydraulics Division, November 1961, p. 221.

Suggests that little or no light has yet been shed on the basic mechanics of the generation of turbulence and of the actual phenomenon of the air entrainment. Mentions that water temperature seems to play a roll in the amount of air entrained in a steep channel. "Rajaratnam" states that entrainment of air in chutes and spillways may also modify the design of energy dissipators at the foot of such structures. Suggests also that the laws of flow resistance of aerated flows are likely to be different from those of non-aerated flows. Postulates a critical value for the Froude number for air inception that may vary with the nature of the boundary. Using the two region concept developed by Straub and Anderson, energy equations are developed for both regions.