

TRANSPPOSITION OF STORMS
FOR ESTIMATING
FLOOD PROBABILITY DISTRIBUTIONS

by

VIJAY KUMAR GUPTA

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ABSTRACT

Contemporary literature in hydrology usually contains the concepts of maximum probable precipitation and maximum probable flood along with methods used to arrive at these limits. These limits signify some physical upper limits for precipitation and flood, however it is difficult to find physical justification for existence of these limits and more so the methods used to compute them. Also, the use of the word 'probable' is incorrect because these 'probable limits' are not assigned any probabilities.

In view of the misconceptions that prevail in such existing concepts, this study attempts to develop a practical methodology with a theoretical framework for estimating the probability of occurrence of floods in a unit time interval, based on the random characteristics of storms. In general, many random characteristics can be defined for a storm, but as a first step only a three-dimensional random vector has been defined for the random characteristics of storms. The random vector is comprised of the coordinates of storm center location and storm orientation. The developed estimation methodology uses all information on historic storms observed in a region that contains the river basin.

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PREFACE

Frequency distributions of various flood descriptors have been studied for decades as random variables. Such variables are the flood volume, peak level or discharge, duration, number of occurrences in a given period, and similar flood characteristics. This long experience shows that there is always a risk involved in flood decisions, which is the probability of exceedance of any value(s) of a random variable(s) describing the flood properties. In classifying all the present or past treatments of flood problems two basic groups of approaches can be distinguished: (1) Each flood characteristic is studied as a random variable, or a set of flood descriptors is studied as a multivariate distribution of these random variables, which represent the probabilistic approach to treatment of flood problems; and (2) Particular flood characteristics are singled out such as the design flood, standard project flood, maximum flood of a transposed storm, probable maximum flood, maximum flood and other characteristics, which represent the deterministic approach to treatment of flood problems.

In the deterministic treatment of floods, one of the particular flood events is the extreme value of any flood descriptor. This value would occur if the most severe storm ever observed in a region were transposed to that river basin position which represents the highest flood producing conditions. This particular flood gives an estimate and/or an outlook of what could have happened in the historic time of storm observations if the largest storm had hit the river basin and in that particular position. Two aspects are of interest in this concept of transposition of a specific storm: (a) The historic largest storm in a region has a very large sampling variation, may be far from the representative largest storm for the size of the historic sample, and much larger storms are expected to occur in the future; and (b) The probability that a storm will be exactly in the position over a river basin, which position produces the maximum flood for that storm location, is a possible event but with the probability of zero, at least when the axioms of probability theory are rigorously applied. In other words, the probability density of a storm being in such a position is finite, but the probability of that exact storm position occurring is zero.

Another concept in obtaining a particular extreme flood event for a river basin, which is a further extension of the deterministic approach to floods, is to maximize the meteorological parameters of the largest historic storm by constructing a still larger storm. The extreme flood produced by this maximized storm for the position with the highest flood producing conditions in a river basin is currently called the probable maximum flood, while the storm precipitation is called the probable maximum precipitation. An earlier concept of locating that maximum storm over a river basin, but with all other river basin factors which influence floods being of the highest flood producing conditions, gives another extreme flood value, namely the maximum flood. This extreme event, as a particular flood characteristic, has been abandoned due to lack of physical evidence for the existence of a definite upper bound for any random variable.

Storms in a region and over a historical sample occur randomly, with the storms described by several characteristics. The probability densities that the center of a storm (the center of the mass of fallen precipitation) is located at any point in the region are finite values. These probability densities can

then be expressed as a function of the point coordinates. Similarly, the storm direction (defined as the axis of the storm isohyetal lines, for which the storm depths have the minimum second mass moment) may have different probability densities for various azimuth angles at any point in the region. Other storm characteristics may be also conceived, when justified by storm properties, as having the probability densities of occurring at a point and/or in a direction over the study area.

Instead of producing only the extreme flood by using the largest storm of the historical sample of storms (with or without a maximization approach for the storm) for only the position in the watershed of highest flood producing conditions, it is feasible to transpose all observed historical storms of a region for various positions and storm directions to that watershed. Instead of determining only a particular, characteristic flood event, it is then possible to determine the entire frequency distribution of each flood descriptor, or the joint frequency distribution of several descriptors using the historical storm data, as estimates of the univariate or joint probability distribution of flood descriptors, as random variables.

The thesis research for the Master of Science degree by Vijay Kumar Gupta, with the original title "Transposition of Storms", is presented as this hydrology paper under the title "Transposition of Storms for Estimating Flood Probability Distributions." The investigations and their results are an attempt to develop a mathematical framework for transposing all observed historical storms in a region to a river basin inside it, and for estimating probability distribution of any flood descriptor as a random variable or estimating the joint distribution of several descriptors. At the same time, an attempt is made to develop a practical, computer-oriented methodology of transposing storms of a region and a historical sample to any river basin inside that region. In other words, a historical sample is used to compute the frequency distribution of any flood descriptor. This, then, represents an extraction of information on a particular flood random variable by using the historical information on storm precipitation. However, a constraint is imposed on the accuracy of the developed results by any existing and approximate method which must be applied to transfer the rainfall data to flood hydrographs.

Instead of using the historical sample of storms and transposing each of these storms to different positions and directions over a river basin, another approach may become feasible in the future. The probabilistic characteristics of occurrence of storms of various properties, both in time and over a region, may be developed in the form of mathematical models, with their parameters estimated from data. When these models are inferred, two approaches may be feasible for deriving the probability distributions of flood descriptors. First, the rainfall-runoff models may be directly applied to the time-area probabilistic model of storms, and second, the Monte Carlo method may be used in generating a large number of storms over a region. Those generated storms which hit a given river basin in various flood producing positions and directions are used to estimate the probability distributions of flood descriptors by applying a rainfall-runoff transfer model.

It becomes easily evident that the storm transposition method, as outlined in this paper, can be only applied if a sufficient number of storms has already been observed in the study region. However, there are few areas with a dense network of precipitation stations and sufficiently long samples of storms. This requirement of a large number of well observed storms, which puts a limitation to extensive application of the method developed, may be circumvented if a condensation of information in the form of time-area stochastic models of storms can be made and the Monte Carlo method in generating new storm samples is applied.

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Chapter 1

INTRODUCTION

1.1 Contemporary Approach Used for Estimating the "Most Critical" Flood Descriptor for a River Basin

Contemporary approaches for estimating the "most critical" flood descriptor of a river basin involve the study of the most severe historic storm occurring in a region that contains the basin. The region is selected on the basis of meteorological homogeneity. Therefore, any storm that occurs somewhere in the region could also occur anywhere else in the region, including the river basin. In order to estimate the most critical flood descriptor, the most severe historic storm is transposed to the river basin. This storm is placed in the most critical position in the basin, after maximizing the precipitation yield in some predetermined manner. "The most critical" position, out of all other positions in the basin, is the one that gives the maximum value of the preselected flood descriptor. The precipitation yield is "maximized" by maximizing the meteorological variables. The maximum precipitation and flood descriptors computed in such a way are called the "most probable precipitation and flood."

Misconceptions in the Contemporary Approach. According to Yevjevich [18]*, "the general philosophy behind these concepts is that there must be an upper limit in the storm intensity or in the total amount of precipitation, that atmospheric circulation could produce at a given place in a given time interval". He [18] also emphasizes that there is no physical justification for upper boundaries in any meteorological factor used in the method of storm maximization. If there is no physical justification for the existence of an upper boundary in precipitation yield from storms, then there is no justification for the resulting floods. Yevjevich further states, "however, if there is a physical boundary to the amount of precipitation, this boundary quite likely is much greater than any maximum precipitation computed by any procedure of maximization used at present". In any event, it is reasonable to assume that at least theoretically the flood phenomenon is an unbounded stochastic process at the upper tail. Also flood descriptors, i.e., flood peak, flood volume, time of occurrence of a flood peak with respect to some origin, etc., associated with the flood phenomenon are continuous or discrete (as the case may be) positive random variables.

1.2 General Description of the Theoretical Base of the Present Study

This study is an attempt to develop a practical methodology, with a theoretical framework, for estimating the probability of occurrence of floods in some unit time interval, say one year. The developed approach uses all information on historic storms observed in a region that contains the selected river basin. The selected storms are based on a predefined criterion, i.e., all storms whose total rainfall at

one or more points over an area exceed some constant. The information is then used in a theoretical framework to estimate the probability of occurrence of floods in the unit time interval that has been selected.

The theoretical formulation defines the sample space to be the collection of all conceivable storms. Each elementary event in the sample space is considered a generic or abstract representation of a storm. An arbitrary subset of the sample space is defined as being comprised of certain types of storms, e.g., all conceivable storms whose total rainfall yield at one or more points over an area that exceeds some constant can form a subset. Associated with a storm are, for instance, k random characteristics, for which a k dimensional random vector can be defined. Each component of the random vector is a random variable, defined as a positive valued function of the storm (the elementary event). The probability distribution of the flood descriptor V , with V also a random variable defined as a function of the storm, can then be theoretically computed from the random variable defined for a storm on the arbitrary subset. The probability of occurrence of floods is computed by using the probability distribution of V , and the probability distribution of the occurrence of storms in the subset, as a function of time. The occurrence of floods is defined as: exceeding V above a certain preselected constant, in some unit time $(0,t)$, from the occurrence of any storm in the arbitrary subset in unit time $(0,t)$.

The computation of the probability distribution function of the flood descriptor, V , requires the identification of the k random variables and the associated joint probability distribution function, defined for storms in the subset. In order to simplify the problem and to demonstrate the concepts underlying the approach, a three dimensional random vector is selected. The random variables as elements of the subset random vector are storm orientation θ , and the two coordinates (X,Y) of the storm center location. The probability distribution of these random variables are studied from observed regional storms. The probability distribution of V is estimated by the inferred variation of the storm center location and the storm orientation of historic storms. The storms are selected on the basis that rainfall yield from each storm at one or more points over an area exceeds a constant. Such a selection is assumed to be a sample of the arbitrary subset. Once criterion for selecting storms is formulated, the subset is automatically defined, and no longer remains an arbitrary subset. Therefore, the selection of an arbitrary subset gives wide flexibility in formulating a criterion for selecting storms. The probability distribution of occurrence of a number of storms in time is not studied here, but is hypothesized based on certain assumptions. Using this hypothesized distribution and the estimated probability distribution of

*[] Designates the reference found in the list of references.

V , the probability of the occurrence of floods in a unit time (one year in this study) is computed.

1.3 Storm Transposition (Estimation Methodology)

The methodology used to estimate the probability distribution of the flood descriptor V , developed here, considers only rain storms; snowmelt floods are not taken into account. For snow storms additional stochastic properties and the corresponding random variables associated with snow accumulation and melt must be included, which is beyond the scope of this study. However, concepts underlying the estimation methodology given here are general, so estimation concepts can be extended by incorporating more random variables associated with a storm and by studying their dependence functions (if applicable) and the corresponding probability distributions. The presented methodology can be applied to any small river basin, say up to a thousand square miles or so. For very large basins, the technique may not be useful because the random vector comprised of storm center location and storm orientation may not adequately represent the influence of the random characteristics of storms on the resulting flood. It seems that additional random variables would need to be defined for large basins. Also, the storm characteristics may have large variation, so that the assumptions regarding the probability distribution of random variables associated with a storm may not hold over the entire basin.

Application of the methodology developed in this study is demonstrated by using a simple rainfall to

runoff model. No attempt is made here to study or improve the modeling of rainfall to runoff relationships. The development of the estimation methodology is based on the assumption that a flood descriptor is a function of only selected random variables. Therefore, the randomness in the computed values of the flood descriptor is due to the variation in the values assumed by the selected random variables; the influences of the remaining stochastic characteristics of storms are taken into account in a "lumped" form. The flood descriptor used is the total flood volume V ; the technique is general, however, and any flood descriptor can be selected.

Application of the methodology is demonstrated on the Goose Creek basin in East Central Illinois. The basin is about 50 square miles in size and the Illinois raingage network surrounding the basin is fairly dense.

Summary. The concepts involved in the developed methodology are only first approximations toward estimating the probability of exceeding a flood descriptor magnitude in a unit time interval, by using random characteristics of storms. The estimated probability value is limited by the sampling variations in the data. Predictions of flood frequencies is less reliable for a return period longer than the period of data on historic storms. The reliability of the estimates of probability of "occurrence of floods" would depend on the length of the historic storm records, reliability of the data, and the validity of the assumptions made regarding the probability distributions of random variables defined for a storm.

Chapter II

REVIEW OF PRESENT METHODS OF STORM TRANSPPOSITION AND RELATED SUBJECTS

2.1 Introduction

The need for a reliable estimation of floods plays an important role in planning and designing hydraulic structures and water resources systems. Over the past few decades there has been substantial development of methods that can be used to predict future flows (prediction is generally understood as the use of deterministic methods for flood estimates from storm data) and forecasts of future flows by using stochastic analysis of time series. Much effort also has been made in the area of improving rainfall to runoff models.

Basically two approaches exist in literature that are used to estimate critical flood descriptors. The first is comprised of statistical flood frequency analysis, carried out on the historic flood records. For ungaged sites, the regional flood frequency analysis is in use [1]. The regional analysis consists of evaluating certain regional parameters from past records, which are then used for forecasting future floods. Such an approach is not based on any theory of stochastic process; therefore, the paucity of long records can result in unreliable forecasts or in high uncertainties. The regional flood frequency analysis is not discussed at length, since it is outside the domain of the present investigation.

The second approach is the physical approach to the estimation of future floods. This approach is based on evaluating what is called "the design storm", which is then applied to a river basin, along with the infiltration and unit hydrograph theory, to obtain a design flood [4]. The unit hydrograph theory and other rainfall to runoff models are not reviewed here since they are neither critical nor relevant to the present investigation. On the other hand, the methods in use for computing the design flood producing storms are discussed, since the misconceptions that prevail in such methods show the need for a study such as this.

2.2 Physical Approach to Estimation of Design Flood

The methods of deriving storm rainfall estimates for computing hypothetical flood hydrographs representing major or design flood runoff essentially consider the size, configuration, and runoff characteristics of the basin, as well as the meteorological characteristics of the major storms in the region. Such estimates, when used to compute a flood hydrograph for fixing the design capacity of specific projects, are referred to as "rainfall criteria" or as "design storm rainfall" by the U.S. Corps of Engineers [14]. The steps used to carry out comprehensive design storm investigations are outlined in the U.S. Corps of Engineers manual [14], and summarized as follows:

(1) Analyze the precipitation data and the synoptic situations of major recorded storms in a region surrounding the river basin to determine the

characteristic combinations of meteorological conditions that result in various rainfall patterns and duration-depth-area relations. The duration-depth-area relation gives the maximum average depths of rainfall for given areas and given durations of the storm.

(2) On the basis of an analysis of air-mass properties and the synoptic situations prevailing during the recorded storms, estimate the amount of increase in rainfall quantities that would have resulted if conditions during the actual storm had been as critical as those considered probable to occur in the region.

(3) Estimate the modifications in the meteorological conditions that would have been required for each of the recorded storms to have occurred over the drainage basin under study, considering the topographic features and locations of the respective areas involved.

(4) Take into account the increase in rainfall quantities that might have resulted from more severe meteorological conditions during the recorded storms, with the adjustments necessary to transpose the respective storms to the river basins. Then select the estimates that would represent the design rainfall duration-depth-area relations for the particular drainage area during the various flood seasons of the year. Taking into account an estimate of the maximum quantity and the rate of contribution to flood runoff that might result from snow melt in conjunction with the design storm rainfall, and the minimum infiltration capacities that are likely to prevail during various seasons, the design runoff hydrograph is computed.

The "design rainfall storms" so computed have been called "probable maximum storms" by both hydrologists and meteorologists. The essence of such an approach is to find a physical upper limit of storm rainfall over the basin [4] which perhaps is why the term "most probable" is used; indeed the term "probable" signifies some probability of occurrence of "design storms," which has bypassed the attention of hydrologists. Therefore, in the first case the use of most probable is incorrect, and secondly, as Yevjevich states [18], there is no known physical evidence of existence of an upper limit of rainfall over a basin. Such prevailing misconceptions in existing techniques are briefly discussed subsequently.

2.3 Design Storm and Storm Transposition

The practices of the U.S. Corps of Engineers [14] are most prevailing in developing quantitative estimates of design storms (critical storm adopted for purposes of design). There are three general methods in common use: (I) maximum rainfall depth-duration data and rainfall-excess estimates; (II) transposition of recorded storms and the rainfall-excess estimates,

and (III) the modified storm transpositions and the rainfall-excess estimates.

Briefly, the first method involves the computation of maximum rainfall depth-duration relations for the size of area involved, based on rainfall data of storms that are considered possible to occur in a region. A hyetograph is then computed to represent the critical or design sequence of rainfall quantities corresponding to the adopted depth-duration curve. This method is directly applicable to small basins of less than a few thousand square miles. The other two methods that involve the transposition of storms are discussed in the ensuing text.

Transposition of Record Storm and Rainfall-Excess Estimate. The second method is particularly useful in studies of river basins having areas greater than a few thousand square miles, in which the variations in rainfall intensity and areal distribution during successive time intervals of a storm have a major effect on the infiltration losses and the runoff concentration. A brief review of this method is as follows.

(1) Superimpose an outline of the given drainage basin onto an isohyetal map of the recorded and selected storm in such a way as to place the highest rainfall quantities in a position that would result in maximum runoff. This is shown schematically in Fig. 2.1.

(2) Construct a Thiessen polygon network for raingage stations in and near the basin, also shown in Fig. 2.1.

(3) Prepare the mass rainfall curves for the respective gaging stations; one such curve is shown schematically in Fig. 2.2 for a gage.

The rainfall analysis is carried out by computing areas A_i , enclosed by the basin for the Thiessen polygon of raingage G_i , with $i = 1, 2, \dots$. One such area A_3 is indicated in Fig. 2.1 for the raingage G_3 . The total rainfall observed at the i -th raingage is q_i , with \bar{q}_i the average rainfall depth within the area A_i . The adjustment factor F_i is given by

$$F_i = A_i \left(\frac{\bar{q}_i}{q_i} \right), \quad i = 1, 2, \dots \quad (2.1)$$

The adjustment factor is computed for all i stations in and near the river basin. The volumes of rainfall within area A_i in three hour intervals are computed by multiplying F_i by the total three hour rainfall, obtained from the mass rainfall curve for that interval. This procedure is carried out for all i -stations and all three hour intervals for each station. The observed rainfall values are increased or decreased as considered necessary to assume an estimate of the adequate design storm rainfall for the purposes involved.

The infiltration indices are computed for areas of each polygon by assuming an infiltration rate in inches per three hours for each polygon. These assumed values for polygons are multiplied by the corresponding areas. The values so obtained are then considered as infiltration indices in inches per square mile per three hour interval.

Excess rainfall for any raingage station for each three hour interval is computed by subtracting the infiltration index from the total rainfall volume for that interval. If the infiltration index is greater than the total rainfall volume in an interval, excess rainfall is zero.

The total rainfall and the excess rainfall for each station are then plotted as shown in Fig. 2.3. If tributary flows are to be combined by flood routing methods, the areas selected for each tributary should correspond to the respective tributary basin.

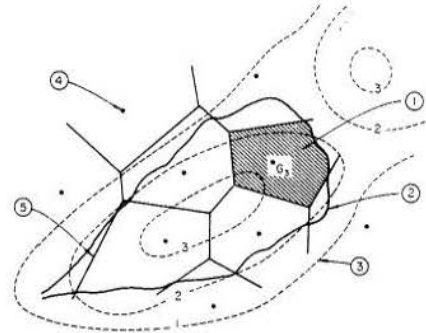


Fig. 2.1 A recorded storm transposed to the "most critical position" on the river basin. (1) Area A_3 ; (2) River basin; (3) Rainfall isohyets in inches; (4) Raingage station; (5) Thiessen polygon.

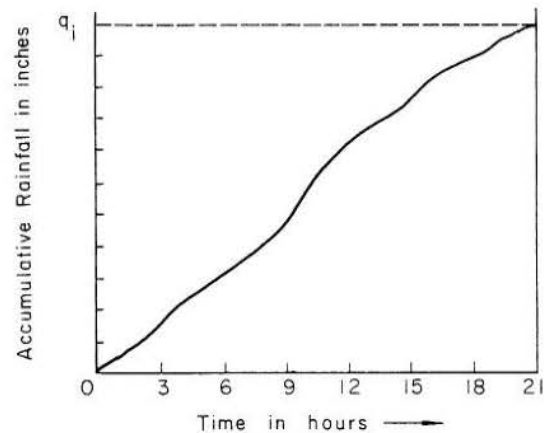


Fig. 2.2 Mass rainfall curve for a raingage G_1 .

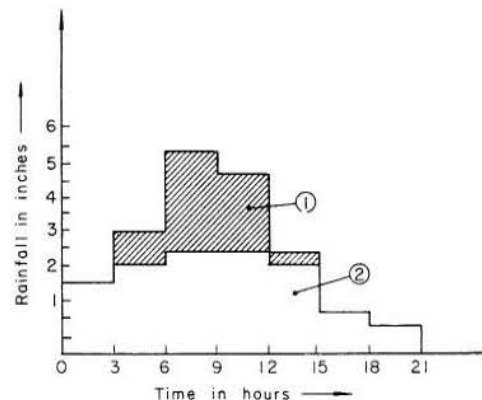


Fig. 2.3 Rainfall excess and rainfall loss per three hour intervals. (1) Rainfall Excess (2) Infiltration.

Modified Storm Transposition and Rainfall Excess Estimates. In the third method, problems involved in determining the critical design storms for a large basin are somewhat different than for small basins. As a rule, the critical design floods in small basins result from extremely intense small-area storms of relatively short duration, whereas the floods of large basins result from a series of less intense large-area storms. Therefore, for large basins, besides computing the critical rainfall volumes in various periods of time, it is also necessary to compute the most critical distribution and location of rainfall that are considered "reasonably probable" during the successive storm periods. The method of modifying transposed storms in light of additional problems encountered for large basins is as follows.

(1) Assemble all precipitation records, isohyetal maps, mass rainfall curves, and duration-depth-area information available for selected storms.

(2) Review the available information regarding the meteorological conditions during the respective storms to determine whether it can be assumed that the movement of the zones of heaviest precipitation during the successive distinct periods of a storm series might have been enough to cause a greater accumulation and a more critical concentration of rainfall over an area comparable to the river basin under study than actually occurred during the recorded storm.

(3) Superimpose the outline of a given drainage basin to the isohyetal patterns that represent the successive rainfall periods of a particular storm, corresponding to the movement of rainfall centers assumed in the above step. The orientations of the basins should be reasonably consistent with the assumptions regarding the meteorological causes of the storm (an arbitrary criterion regarding the assumption is discussed in the next subsection).

(4) Compute the rainfall volumes and the rainfall excess estimates as described in the last section for each transposed storm.

(5) Compare the quantities of rainfall excess for various major storms considered to determine the "critical rainfall series" to be adopted finally for purposes of design.

Although the three methods described above have been widely used, they do have significant limitations.

Limitations of Storm Transpositions. The above descriptions show that the sole purpose of storm transposition techniques presently in use is to increase reliability in computing "the design flood" for a river basin. The word "critical flood" supposedly indicates the estimated upper physical limit of the selected flood descriptor. Bruce [4] mentions two limitations that can be naturally encountered in storm transposition. These limitations are:

(a) proper definition of the region of "hydro-meteorological homogeneity" from which storms can be transposed over the basin, and

(b) the permissible change in the orientation of storm rainfall patterns to obtain the maximum runoff.

Besides these two limitations, the orographic barriers in the region, when present, may also limit storm transposition.

In reference to the "hydrometeorological homogeneous" region, Bruce [4] states that generally the distance limits for a storm transposition are based on atmospheric processes, or synoptic weather patterns and climatological experience within the region. This selection of distances is a subjective decision to some extent. Also, the transposition limits of storms must be based on the hydrometeorological factors for that particular storm, such as whether the same situation could have occurred anywhere else or not.

In regard to storm orientation, Bruce [4] mentions that from studying meteorological factors in a storm, it is sometimes clear that a "slight" change in the orientation of the storm is permissible, without changing the atmospheric conditions that produced the storm. As a rough guide, Paulhaus and Gilman [13] suggest that in attempting to get the maximum runoff effects, the storm orientation should not be adjusted by more than 20°; their criterion seems arbitrary with no physical justification given. The U.S. Corps of Engineers manual [14] states that major changes in the orientation of a storm for some regions can be made without changing the meteorological factors; such cases have been reported for the central United States.

2.4 Misconceptions in Storm Transposition and "Critical Storm" Estimation

The general philosophy behind estimating the "probable maximum precipitation" and in the transposition of these storms is that there must be an upper limit in the precipitation intensity or in the total precipitation amount and in the flood descriptor. The precipitation phenomenon is a stochastic process and so is the flood phenomenon. Probabilistically speaking, any variable defined for these phenomena can be visualized as a random variable, which theoretically has a range from zero to infinity. Even if it is difficult to comprehend an infinite rainfall intensity or infinite flood peak at a point, at least the probability of exceedence of any computed value by methods used at present should be positive, and smaller than 1.00 (a sure event). If there is a physical boundary to the amount of maximum precipitation possible, this boundary quite likely is much greater than any maximum precipitation computed by any procedure of maximization.

The procedure used in storm transposition of placing the storm in the most critical position is also scientifically incorrect. If the location of a storm is defined by some continuous random vector, the probability that this vector would assume any one given position is zero. Similarly, the same can be said for the orientation of storms.

In view of the above misconceptions, it is reasonable to reformulate the problems of computing flood descriptors from precipitation inputs by considering the stochastic nature of the precipitation phenomenon and the entire frequency (probability) distribution curve of the flood descriptor, instead of a value at the distribution tail, with no clear criterion as to how it is selected on this tail. The study presented in the following text is an attempt along these lines.

Chapter III

FORMULATION OF THE PROBLEM

3.1 Theoretical Considerations

Let Ω be the collection of all conceivable storms, such that each element in Ω , denoted by ω , is a generic representation of a storm. A storm can be visualized in general terms as spells of intermittent precipitation. Any uninterrupted intensity in time and over an area of spells may be conceived as a storm. Without explicitly defining a storm, we can envision k random characteristics for each storm, so that a corresponding k -dimensional random vector can be defined. Each element of the random vector is a function of ω and is assumed to be a random variable.

Suppose that a probability space, say (Ω, \mathcal{B}, P) , exists in which Ω is the sample space as defined above, \mathcal{B} is a σ -algebra^(*) of subsets of Ω , and P is a probability measure defined on \mathcal{B} . For any arbitrary event $B \in \mathcal{B}$, $0 \leq P[B] \leq 1$.

Let Ω_0 be some arbitrary subset of Ω , and $\mathcal{B}_0 \in \mathcal{B}$. For instance, Ω_0 might represent the collection of all conceivable storms, whose total rainfall exceeds a certain amount at one or more points over an area. For now Ω_0 is considered arbitrary; it (Ω_0) can also be the space Ω itself.

Let $(\Omega_0, \mathcal{B}_0, P_0)$ denote a probability space, where \mathcal{B}_0 is a σ -algebra of subset of Ω_0 and P_0 is the probability measure defined on \mathcal{B}_0 induced by P .

The k -dimensional random vector is denoted by $X = (X_1, \dots, X_k)$, with each $X_i = X_i(\omega)$ a random variable, $i = 1, 2, \dots, k$. Let Y_1, \dots, Y_d be any subset of X_1, \dots, X_k , such that every $Y_i, i=1, \dots, d$ has as its domain Ω_0 and a real line as its counter-domain. Let $F_{Y_1, \dots, Y_d}(y_1, \dots, y_d)$ be a joint probability distribution function of Y_1, \dots, Y_d . If B is an event and $B \in \mathcal{B}_0$, then the probability of the event B , with respect to Ω_0 , is given by (**)

$$P_0[B] = \int_{R_d} \dots \int P_0[B | Y_1=y_1, \dots, Y_d=y_d] dF_{Y_1, \dots, Y_d}(y_1, \dots, y_d) \quad (3.1)$$

in which R_d is a d -dimensional Euclidean space.

(**) A σ -algebra, denoted by \mathcal{B} , is a collection of subsets of Ω satisfying:

- (i) $\Omega \in \mathcal{B}$, and the null set, $\phi' \in \mathcal{B}$.
- (ii) If event $B \in \mathcal{B}$, then also $\bar{B} \in \mathcal{B}$.
- (iii) If a sequence $\{B_i\} \in \mathcal{B}$, then also $\bigcup_{i=1}^{\infty} B_i \in \mathcal{B}$.

(**) Refers to Parzen, "Modern Probability Theory and Its Applications".

For this study, Ω_0 is considered to be the collection of all conceivable storms whose total rainfall at one or more points over an area exceeds a constant. Consider a three dimensional subset of the random vector X , defined as the two dimensional vector (X, Y) which denotes location of a storm center over a geographic region R , containing the river basin under study, and the random variable ϕ which denotes storm orientation. Let V be a random variable defined on Ω_0 , which denotes a flood descriptor, and let an event B be defined as $[V \geq v_0]$, in which v_0 is a non-negative constant.

Using Eq. 3.1 the probability of the event B is given by

$$P_0[V \geq v_0] =$$

$$\iiint_{R_3} P_0[V \geq v_0 | X=x, Y=y, \phi=\phi] dF_{X,Y,\phi}(x,y,\phi) \quad (3.2)$$

Assuming (X, Y) to be independent of ϕ , and introducing the range of the random vector (X, Y) and the random variable ϕ , Eq. 3.2 can be expressed as

$$P_0[V \geq v_0] =$$

$$\int_0^{\pi} \left\{ \iint_R P_0[V \geq v_0 | X=x, Y=y, \phi=\phi] dF_{X,Y}(x,y) \right\} dF_{\phi}(\phi) \quad (3.3)$$

Equation 3.3 provides a theoretical framework for computing the probability distribution function of the flood descriptor V , which is defined for storms in Ω_0 . The occurrence of storms that constitute the sample space Ω is a function of time, therefore this is also true for the storms in Ω_0 . The time variable is introduced at this stage so that the probability of floods occurring in a time interval $(0, t)$ can be computed.

The occurrence of at least one flood is defined here as the exceedence of V above v_0 , i.e.,

$V \geq v_0$, in the time interval $(0,t)$, for some storms occurring in $(0,t)$. Denote this event by F_0 , and denote the complement of this event by \bar{F}_0 .

Let $Z(t)$ be the random variable denoting the number of storms occurring in the time interval $(0,t)$ whose total rainfall at one or more points over an area exceeds a fixed value. Then the probability of the event F_0 can be expressed as

$$P_0[F_0] = 1 - P_0[\bar{F}_0] =$$

$$1 - \sum_{v=0}^{\infty} P_0[\bar{F}_0 | Z(t) = v] P_0[Z(t) = v]. \quad (3.4)$$

Assuming the number of storms in the time interval $(0,t)$ to be independent of the random variable V denoting a flood descriptor, and the random variables $V_1, \dots, V_v^{(*)}$, $v = 1, 2, \dots$, being mutually independent and identically distributed, the following identity holds,

$$P_0[\bar{F}_0 | Z(t) = v] =$$

$$\{P_0[V < v_0]\}^v, \quad v = 0, 1, \dots \quad (3.5)$$

Substituting Eq. 3.5 into Eq. 3.4 then

$$P_0[F_0] = 1 - \sum_{v=0}^{\infty} \{P_0[V < v_0]\}^v P_0[Z(t) = v]. \quad (3.6)$$

Substituting Eq. 3.3 into Eq. 3.6, the probability of floods occurring in an interval $(0,t)$ can be computed.

3.2 Treatment of Regional Storms

Regional storms can be treated by hypothesizing the joint probability distributions of all random variables associated with a storm. "Regional" is used here to imply the study of such storms, not only for a given river basin but also over a large region surrounding it. Such a region may be selected after considering statistical homogeneity of different random variables associated with a storm. The main advantage of selecting a region containing a given river basin is that more information is available about historic storms in the region. This information can be used statistically to infer the marginal or joint probability distributions of various random variables associated with each storm.

The inferred joint probability distribution of these random variables can then be used in Eq. 3.1. As outlined in Section 3.1, for the present study,

only three random variables are selected: the coordinates of the storm center location, (X,Y) , and the storm orientation ϕ . The ensuing text refers to the study of properties of these random variables and the appropriate statistical tests that can be used in making inferences about their probability distribution functions.

Regional Distribution of Storm Centers. A storm center has been denoted by a two-dimensional random vector (X,Y) . The storm center is a hypothetical point with various possible definitions. In this study, a storm center is defined as follows.

Consider a system of m masses distributed over an area, as shown in Fig. 3.1. In this case the total rainfall at each point (raingage) denoted by $q_i, i=1, 2, \dots, m$, would constitute the system of m masses. Let the location of each point with respect to some origin be denoted by (x_i, y_i) for $i=1, 2, \dots, m$. Then the location of the storm center is defined as the center of gravity for this system, denoted by (x_g, y_g) and given by

$$\left. \begin{aligned} x_g &= \frac{\sum_{i=1}^m x_i q_i}{\sum_{i=1}^m q_i} \\ y_g &= \frac{\sum_{i=1}^m y_i q_i}{\sum_{i=1}^m q_i} \end{aligned} \right\} \quad (3.7)$$

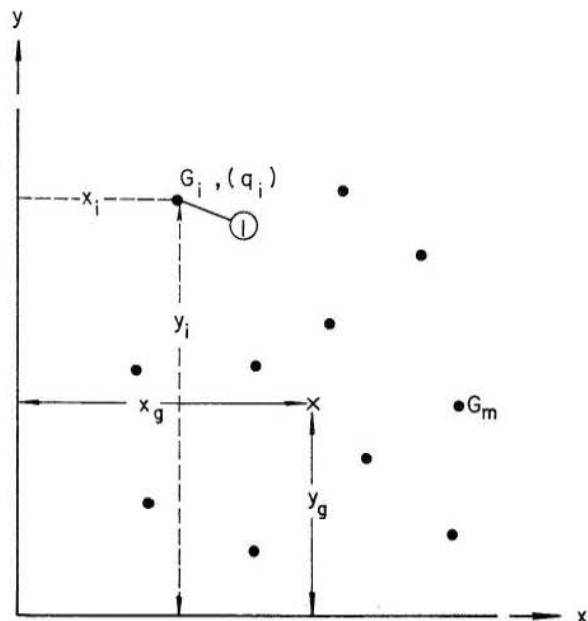


Fig. 3.1 m rainfall measurements over an area: (1) G_i raingage, $i=1, \dots, m$; X Storm center location; q_i Total rainfall at raingage G_i .

(*) The sequence of random variables V_1, \dots, V_v , is formed by defining one variable for each storm, but all denote the same flood descriptor.

Because the location of a storm over a river basin greatly influences the resulting runoff, one component of the subset random vector, as outlined in Section 3.1, is considered to be the random vector (X,Y) denoting the storm center's location.

In general, if major orographic barriers, climatological variations, etc., are absent from the selected region it is quite likely that storms occur over the region at random. Such a region is called homogeneous with respect to the occurrence of storms. However a selected region may not always be homogeneous. Therefore, the study of the probability distribution of the random vector (X,Y) can be based either on phenomenological considerations, which is not attempted here, or fitted empirically to the historic data. In either case, a probability distribution function must be hypothesized. The hypothesized distribution should be tested by appropriate statistical tests. One such statistical test that can be used to test the hypothesized probability distribution of the random vector (X,Y) is presented.

Consider a region shown in Fig. 3.2, that is divided into "a" segments along the x-axis and "b" segments along the y-axis. Let n_{ij} , $i=1,\dots,b$, $j=1,\dots,a$ be the number of storm centers observed over the ij -th segment, then

$$n = \sum_{i=1}^b \sum_{j=1}^a n_{ij} \quad (3.8)$$

Let p_{ij} be the probability of occurrence of a storm center in the ij -th segment, such that

$$\sum_{i=1}^b \sum_{j=1}^a p_{ij} = 1 \quad (3.9)$$

The random quantity Q given by

$$Q = \sum_{i=1}^b \sum_{j=1}^a \frac{(n_{ij} - n \cdot p_{ij})^2}{n \cdot p_{ij}} \quad (3.10)$$

has a limiting chi-square distribution with $(ab-2)$ degrees of freedom.

To test the hypothesized probability distribution, namely $p_{ij} = p_{ij}^0 = a$ constant, $i=1,\dots,b$, $j=1,\dots,a$, Eq. 3.10 would be used. Such a hypothesis is not tested for the region used in this study. However, it will be assumed that the storm center occurrence over the region is homogeneous and the probability density function for the random vector (X,Y) is uniform, given by

$$f(x,y) = \frac{1}{A_R} I_R[(x,y)] \quad (3.11)$$

in which A_R is the area of the region denoted by R , and I_R is an indicator function defined over the region, given by

$$I_R[(x,y)] = \begin{cases} 1 & \text{if } (x,y) \in R \\ 0 & \text{otherwise} \end{cases} \quad (3.12)$$

Regional Distribution of Storm Orientation. The orientation of a storm, denoted by the random variable ϕ , is obtained from rainfall isohyets over the region with respect to some fixed orientation. The N-S line is taken to be the fixed orientation in this study. The influence of storm orientation on the resulting runoff depends to a large extent on the shape and size of the river basin and the shape of the storm. This concept is schematically shown in Fig. 3.3; in case (a) the basin covered by the large rainfall magnitude isohyets is greater than for case (b). As a result, it is reasonable to assume greater runoff in case (a) than in (b), assuming all other variables influencing runoff remain the same in both cases.

Usually, the rainfall isohyets for a storm form complicated patterns and rarely have regular geometrical shapes. Huff [7] shows four major types of isohyetal patterns observed over East Central Illinois. These patterns are reproduced in Fig. 3.4. According to Huff, "There exists a trend for the storm pattern to become more complex with increasing rainfall duration and rainfall volume". In view of this complexity, it is essential to design an objective criterion for defining storm orientation. For the present study the following criterion is designed.

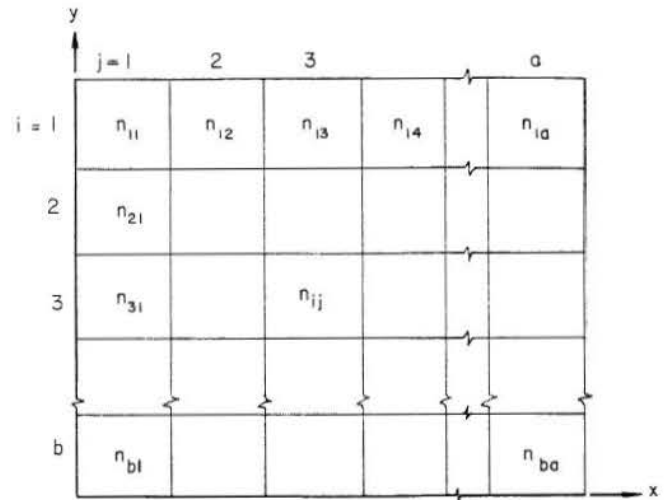


Fig. 3.2 Schematic representation of a region divided into segments to test the hypothesized probability distribution of locations of storm centers. n_{ij} = number of storm occurrences in the ij -th segment.

Consider an isohyetal pattern for a storm as shown in Fig. 3.5. The storm center for this storm is denoted by C_g , and the precipitation gages are denoted by G_i , $i=1,2,\dots,m$, where m is the total number of precipitation gages. Let the range of storm orientation, from 0 to π , be divided into s intervals. Select one orientation from each interval, denoted by D_j , $j=1,\dots,s$. Denote the total rainfall amounts at each raingage by q_i , $i=1,\dots,m$ and the

$$S_j = \sum_{i=1}^m q_i [L_i^j]^2 \quad (3.13)$$

The statistic is computed for all the s directions. The storm orientation, denoted by D_s is then given by

$$D_s = \text{Min}[S_1, \dots, S_s] \quad (3.14)$$

Huff [7] in his study on the East Central Illinois network indicates that the movement of storms in general was observed to be closely associated with the orientation of storm isohyetal patterns. The storm movements were roughly along the major axis in case of elliptical isohyetal patterns of storm rainfall. In view of this observed phenomenon regarding the direction of storm movement, consider a specific case of an isohyetal pattern, given as concentric ellipses, and shown in Fig. 3.6. If the statistic s is computed about different directions passing through the center of the concentric ellipses, including the major axis of the ellipses, the statistic s computed about the major axis is always the minimum value. Therefore the major axis of the concentric ellipses is the orientation in this specific case.

Major storms in general cover large areas at least up to a 1,000 sq. mi. or more. Therefore the rainfall isohyets observed on small areas, say less than 500 sq. mi. are not well defined and do not give a fair idea regarding the direction of the storm movement. In the absence of a well defined storm isohyetal pattern, Eq. 3.13 provides an objective basis for computing orientations.

Having defined the storm orientation, the probability distribution of the random variable ϕ , denoting a storm orientation, can be studied either (i) by

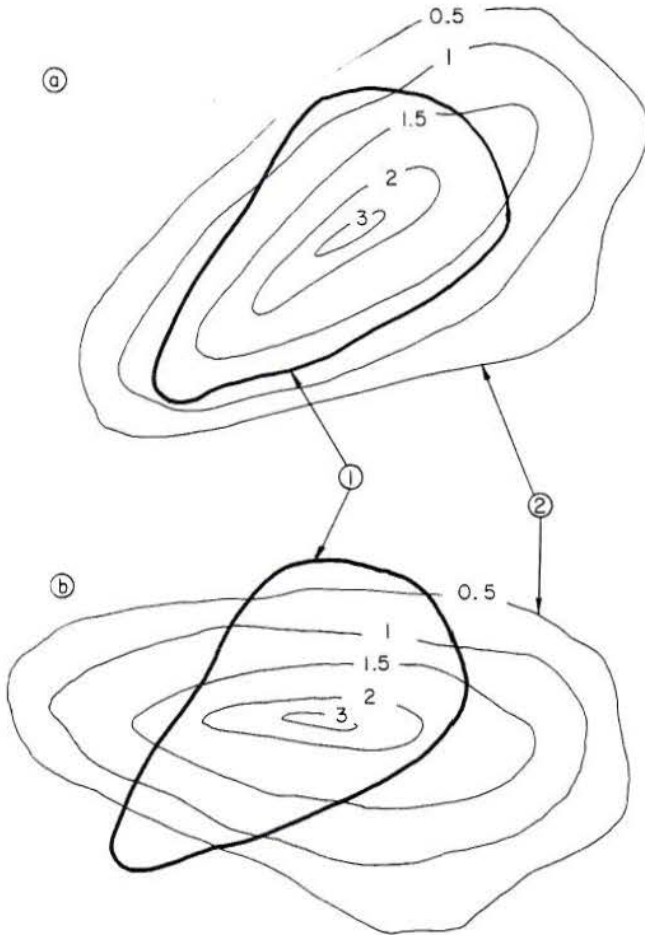


Fig. 3.3 Schematic effect of storm orientation on resulting runoff. (1) River basin; (2) Rainfall isohyets in inches.

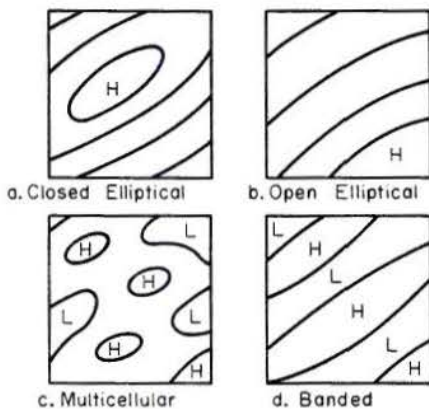


Fig. 3.4 Major storm patterns observed over East Central Illinois, according to Huff [7].

normal distances of the raingages for the j -th orientation by L_i^j , $i=1, \dots, m$, $j=1, \dots, s$. Consider a statistic S_j for the j -th direction, given by

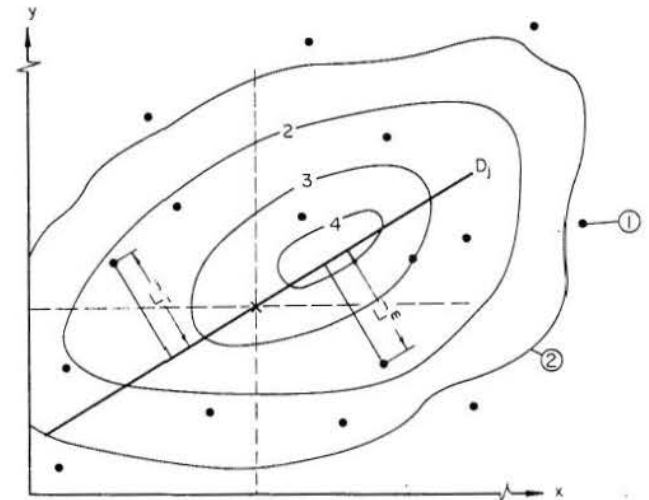


Fig. 3.5 Conceptual representation for computing storm orientation. (1) G_i raingage, $i=1, \dots, m$; (2) Rainfall isohyets in inches; D_j Storm orientation in j -th interval; L_i^j Normal distance of raingage G_i from the orientation line D_j , $i=1, \dots, m$; C_g Location of storm center.

hypothesizing a probability distribution function and making appropriate statistical tests for testing the hypothesized distribution, or (ii) by fitting mathematical functions to the observed frequencies of the storm orientation and converting it to a probability density function by dividing it by the total area of the fitted curve. One such curve is schematically shown in Fig. 3.7.

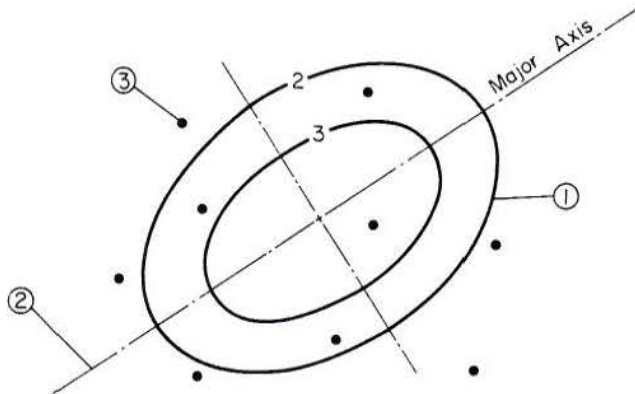


Fig. 3.6 Storm orientation for an elliptical isohyetal map. (1) Rainfall isohyets in inches; (2) Major axis is the storm orientation; (3) Raingage.

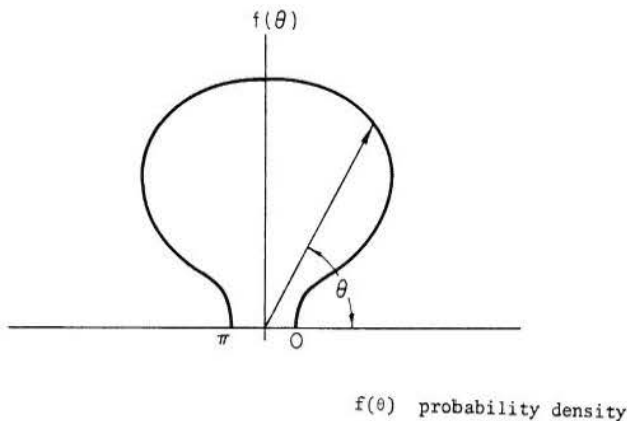


Fig. 3.7 Probability density function for storm orientation.

In general, it may be difficult to fit a mathematical function to the observed frequencies of the orientation variable ϕ . The hypothesis regarding the probability density function of the random variable ϕ can be either governed by the physics of the phenomenon of storm movements in the region, or can be empirically based on personal judgment and experience on the observed storm orientations. In the following pages, a hypothesized probability density function for storm orientations in the East Central Illinois region is included. An appropriate statistical test is also included to test the hypothesized probability distribution.

For this study, the data on observed frequencies of historic storm orientations are taken from the study by Huff [7] on the East Central Illinois raingage network. Table 3.1 gives the observed frequencies of 100 storm orientations.

The probability density function for the random variable ϕ selected in this example is the Beta-density functions. Because the range of values assumed by the random variable ϕ is from 0 to π , and in

general the Beta-density function is defined for a random variable with a range of values from 0 to 1, a new random variable θ is defined as

$$\theta = \frac{\phi}{\pi}, 0 \leq \theta \leq 1 \quad (3.15)$$

The random variable θ is equivalent to the random variable ϕ/π with probability 1, since π is a constant. The Beta-probability density function for θ with the parameters a_1 and b_1 is given by

$$f(\theta, a_1, b_1) = \frac{1}{B(a_1, b_1)} \theta^{a_1-1} (1-\theta)^{b_1-1} I_{(0,1)}(\theta) \quad (3.16)$$

in which $I_{(0,1)}(\theta)$ is an indicator function. The range of 0 to 1 is divided into nine intervals. Each storm orientation of Table 3.1 is representative of the corresponding interval. In Table 3.1, the diametrically opposite directions of storms have not been identified separately, but in this study the range of the values of θ is from 0 to 1, therefore the north and south directions must be separately identified. In view of this, the total number of storms observed in the south direction has been equally divided between north and south. The 17 storms in Table 3.1, identified as complex with respect to their orientations have been also distributed equally in each direction interval, except for the direction WSW, to which three storms are assigned arbitrarily. All this information is summarized in Table 3.2 including the relative frequencies for storm orientations.

The parameters of the Beta-density function are estimated by forming two simultaneous equations in terms of the parameters

$$E[\theta] = \frac{a_1}{a_1+b_1} \quad , \text{ and} \quad (3.17)$$

$$\text{Var}[\theta] = \frac{a_1 b_1}{(a_1+b_1)^2 (a_1+b_1+1)} \quad (3.18)$$

TABLE 3.1
Percentage Frequency of Storm Pattern Orientations,
According to Huff [7]

Direction	Percent	Direction	Percent
S	1	WNW	5
SSW	11	NW	6
SW	22	NNW	3
WSW	23	COMPLEX	17
W	12		

TABLE 3.2
Relative and Cumulative Frequencies
of Observed Storm Orientations

Direction	Intervals of θ	Percent of Storms	Relative Frequency	Cumulative Frequency
S	0-1/16	1.5	.015	.015
SSW	1/16-3/16	13	.130	.145
SW	3/16-5/16	24	.240	.385
WSW	5/16-7/16	26	.260	.645
W	7/16-9/16	14	.140	.785
NWN	9/16-11/16	7	.070	.855
NW	11/16-13/16	8	.080	.935
NNW	13/16-15/16	5	.050	.985
N	15/16-1	1.5	.015	1.000

The expectation and variance of θ , denoted by $E[\theta]$ and $\text{Var}[\theta]$, are computed from the observed frequencies and then Eqs. 3.17 and 3.18 are solved simultaneously for a_1 and b_1 , giving $a_1 = 1.576$ and $b_1 = 2.306$. The observed frequency distribution and the fitted Beta-distribution function are shown in Fig. 3.8.

The hypothesized or the empirically fitted probability distribution for the random variable θ can be tested by performing the following statistical test.

Divide the range of values assumed by θ into s intervals. Let $n_j, j=1, \dots, s$ be the number of storms observed in each interval and let n be the total number of storms, such that

$$\sum_{j=1}^s n_j = n \quad (3.19)$$

Let $p_j, j=1, \dots, s$ be the probability of occurrence of a storm orientation in the j -th interval, such that

$$\sum_{j=1}^s p_j = 1 \quad (3.20)$$

The random quantity Q' , given by

$$Q' = \sum_{j=1}^s \frac{(n_j - np_j)^2}{np_j} \quad (3.21)$$

has a limiting chi-square distribution with $(s-1)$ degrees of freedom.

To test the empirically fitted Beta-density function, namely $p_j = p_j$, $j=1, \dots, s$, Eq. 3.21 is used. The hypothesized probabilities $p_j, j=1, \dots, s$ and the computation of the quantity Q' are given in Table 3.3.

In computing the statistic Q' only eight intervals are considered, the first and the last intervals are grouped together.

Assuming the level of significance is equal to .05, $\chi^2(.05) = 11.1$ for five degrees of freedom. Two additional degrees of freedom are lost from $(s-1)$, since the parameters a_1 and b_1 are estimated from the observed data.

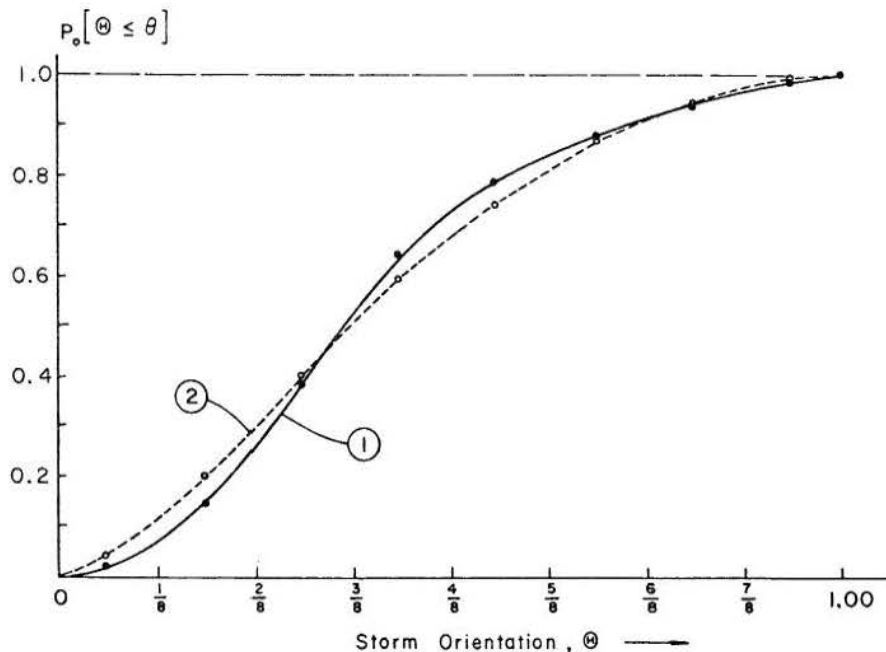


Fig. 3.8 Observed and fitted probability distributions for storm orientations. (1) Observed frequency distribution; (2) Fitted probability distribution.

Since $11.1 > 7.06$, the null hypothesis is accepted and the fitted Beta-distribution is assumed to be adequate to estimate the distribution of θ for the East Central Illinois region.

3.3 Estimation of Probability of Floods

Equation 3.3 gives the theoretical probability distribution of the flood descriptor, denoted by V , and defined on the arbitrary subset Ω_0 . The random variable θ , for storm orientation, is identically equal to the variable ϕ/π with probability 1, as outlined in Section 3.2. Therefore, in the subsequent development of the estimation model, the random variable ϕ will be replaced by θ , without any loss of generality.

Let n be the total number of storms observed in a region R in some time interval $(0,t)$, with the total rainfall from each storm exceeding a constant value at one or more points over an area. The constant can be the value v_0 . All those storms, for which the total rainfall yield at one or more points over an area does not exceed the constant v_0 , would not produce a flood descriptor value greater than or equal to v_0 in general. This selection specifies the composition of Ω_0 . The total number of observed storms simply denotes a random sample of Ω_0 . The practical advantage of this selection in the estimation procedure is that all small bursts and storms do not need to be considered in estimating the probability distribution of a flood descriptor at the upper tail. To simplify computations, the flood descriptor selected is the total flood volume, denoted by V . A difficulty arises in the definition of the beginning and the end of a storm. For example, very low and prolonged storm intensities before or after a major storm contribute to the total runoff volume. Therefore, for a more precise definition of total runoff volume V , the base flow should be included and then the total flow above the base flow will constitute the total flood volume. The instantaneous flood peak is a simple flood descriptor to define, because for any flood it is the maximum discharge of the entire hydrograph. However, the flood volume is used in this text only as an example. Any flood descriptor can be used instead of flood volume, without any loss of generality.

The function $P_0[V \geq v_0 | X=x, Y=y, \theta=\theta]$, which will be denoted by $t'(x,y,\theta)$, can be estimated by using the transposition of the historic storms over the river basin of interest, as follows.

Assume that the flood descriptor V is a deterministic function of all variables used in the rainfall-to-runoff model in computing it. For any storm the randomness in the computed values of the flood descriptor is only due to the variation in the values assumed by the random vector (X,Y) and the random variable θ , which denote the storm center location and the storm orientation, respectively. Therefore, any one computed value $v = v(x,y,\theta)$. Let each one of the n storms be placed in the basin at some $X=x, Y=y$ and $\theta=\theta$ and let a sequence of flood descriptors values be computed for each storm in that location and that orientation, denoted by $v_j, j=1,2,\dots,n$. Recall that $v_j = v_j(x,y,\theta)$ for $j=1,2,\dots,n$. An indicator function for the $v_j^{(*)}(x,y,\theta)$ is defined by

$$I_{(v_0, \infty)}(v_j) = \begin{cases} 1 & \text{if } v_j \geq v_0 \\ 0 & \text{otherwise} \end{cases} \quad (3.22)$$

(*) $v_j(x,y,\theta)$ will be generally written as v_j , unless otherwise necessary for better clarification.

The function $t'(x,y,\theta)$ can now be estimated as

$$\hat{t}'(x,y,\theta) = \frac{1}{n} \sum_{j=1}^n I_{(v_0, \infty)}[v_j(x,y,\theta)] \quad (3.23)$$

The estimate given by Eq. 3.23 is an unbiased estimate, provided the sample of size n is a representative sample of Ω_0 , since for any arbitrary event, say B , $E[I_B] = P_0[B]$.

Substituting Eq. 3.23 into Eq. 3.3, an estimate of the probability distribution function of the flood descriptor, denoted by $\hat{P}_0[V \geq v_0]$, is obtained as

$$\hat{P}_0[V \geq v_0] = \int_0^1 \left[\iint_R \left\{ \frac{1}{n} \sum_{j=1}^n I_{(v_0, \infty)}(v_j) \right\} dF_{X,Y}(x,y) \right] dF_{\theta}(\theta) \quad (3.24)$$

$$= \frac{1}{n} \sum_{j=1}^n \int_0^1 \left[\iint_R I_{(v_0, \infty)}(v_j) dF_{X,Y}(x,y) \right] dF_{\theta}(\theta) \quad (3.25)$$

The integration scheme of Eq. 3.25 is developed as follows.

Divide the region R into r arbitrary subregions, denoted by $R_i, i=1,2,\dots,r$; such that the function $t'(x,y,\theta)$ is assumed to be approximately constant with respect to (x,y) within the subregion. Equation 3.25 can now be expressed as

$$\hat{P}_0[V \geq v_0] = \frac{1}{n} \sum_{j=1}^n \int_0^1 \left[\sum_{i=1}^r \iint_{R_i} I_{(v_0, \infty)}[v_j(x,y,\theta)] dF_{X,Y}(x,y) \right] dF_{\theta}(\theta) \quad (3.26)$$

Denoting the values (x,y) by (x_i, y_i) within the subregion R_i , Eq. 3.26 is then

$$\hat{P}[V \geq v_0] = \frac{1}{n} \sum_{j=1}^n \int_0^1 \left[\sum_{i=1}^r I_{(v_0, \infty)}[v_j(x_i, y_i, \theta)] \iint_{R_i} dF_{X,Y}(x,y) \right] dF_{\theta}(\theta) \quad (3.27)$$

In Eq. 3.27, the expression $\iint_{R_i} dF_{X,Y}(x,y)$ denotes the area of the subregion divided by the total area of the region, since the vector (X,Y) has been assumed to have a uniform probability density function as outlined in Section 3.2. Denoting A_i as the area of the subregion R_i , for $i=1,\dots,r$,

$$\iint_{R_i} dF_{X,Y}(x,y) = \iint_{R_i} \frac{dx dy}{A_R} = \frac{A_i}{A_R} \quad (3.28)$$

Substituting Eq. 3.28 into Eq. 3.27

$$\hat{P}_0[V \geq v_0] \doteq \frac{1}{n} \sum_{j=1}^n \int_0^1 \left[\sum_{i=1}^r I_{(v_0, \infty)}(v_j) \frac{A_i}{A_R} \right] dF_\theta(\theta) \quad (3.29)$$

Now let $\sum_{i=1}^r I_{(v_0, \infty)}(v_j) A_i$ be denoted by $A_0^j(\theta)$. Then $A_0^j(\theta)$ is the area, in the region R , where the flood descriptor v_j exceeds the value v_0 , for the j -th storm with an orientation $\theta = \theta$. Equation 3.29 is now written as

$$\hat{P}_0[V \geq v_0] \doteq \frac{1}{n} \sum_{j=1}^n \int_0^1 \frac{A_0^j(\theta)}{A_R} dF_\theta(\theta) \quad (3.30)$$

Let the range of values assumed by the random variable θ be divided into s mutually exclusive intervals. Denote these intervals by J_i , $i=1, \dots, s$, and the probability of θ falling in the i -th interval J_i , by $F[J_i]$. Also assume that the area $A_0^j(\theta)$ is approximately a constant for any orientation $\theta = \theta_i$ in the i -th intervals, and denote it by $A_0^j(\theta_i)$. Equation 3.30 is now written as

$$\hat{P}_0[V \geq v_0] \doteq \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^s \frac{A_0^j(\theta_i)}{A_R} \int_{J_i} dF_\theta(\theta) \quad (3.31)$$

$$\hat{P}_0[V \geq v_0] \doteq \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^s \frac{A_0^j(\theta_i)}{A_R} F[J_i] \quad (3.32)$$

The area $A_0^j(\theta_i)$ can be estimated by transposing the j -th storm over and around the basin in a certain orientation θ_i , selected from the orientation interval J_i , by computing the flood descriptor value at each transposed position, and finally by interpolating the isoline for the flood descriptor value v_0 , such that the flood descriptor values $v_j \geq v_0$ within the area enclosed by the isoline of v_0 . This concept is represented schematically in Fig. 3.9. The same procedure is repeated for the j -th storm for each of the s orientation intervals and is similarly repeated for all the n storms. The $n \cdot s$ areas computed in this way can then be used in Eq. 3.32 to estimate $P_0[V \geq v_0]$, denoted by $\hat{P}_0[V \geq v_0]$.

The following assumptions are made in order to hypothesize a probability distribution for the number of storm occurrences, $Z(t)$, in an interval $(0, t)$.

(1) The probability that exactly one storm will occur in a time interval of length Δt is approximately $\lambda \Delta t$, or the probability that one or more storms will occur in the interval Δt is $\lambda \Delta t + o(\Delta t)$, where $o(\Delta t)$ is some function of a smaller order than Δt , such that $\lim_{\Delta t \rightarrow 0} \frac{o(\Delta t)}{\Delta t} = 0$, λ is the mean rate of occurrence of storms.

(2) The number of storm occurrences in non-overlapping time intervals are assumed to be mutually independent.

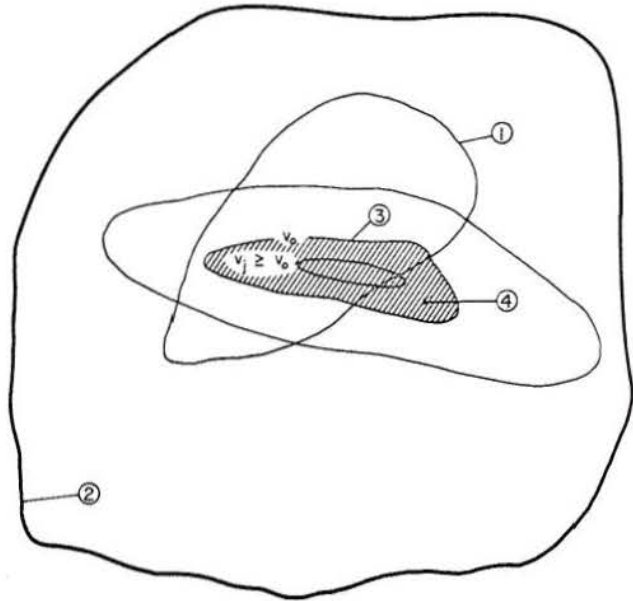


Fig. 3.9 Isolines of a flood descriptor generated by transposing the j -th storm in the θ_i orientation. (1) River basin; (2) Region boundary; (3) Isoline for flood descriptor magnitude equal to v_0 ; (4) Area $A_0^j(\theta_i)$.

(3) No two storms can occur in a small time interval Δt , i.e., the probability that two or more storms will occur in Δt is $o(\Delta t)$.

Based on the above assumptions, $Z(t)$ follows a Poisson distribution, or

$$P_0[Z(t) = v] = \frac{e^{-\lambda t} (\lambda t)^v}{v!} \quad (3.33)$$

The parameter λ can be estimated by dividing the total number of observed storms n by the total length of the time of the observed storms, denoted by T , i.e., $\lambda = \frac{n}{T}$.

Substituting $\hat{P}_0[V < v_0]$ for $1 - P_0[V_0 \geq v_0]$ and Eq. 3.32 into Eq. 3.6,

$$\hat{P}_0[F_0] = 1 - \sum_{v=0}^{\infty} \left\{ \hat{P}_0[V < v_0] \right\}^v \frac{e^{-\lambda t} (\lambda t)^v}{v!} \quad (3.34)$$

$$= 1 - e^{-\lambda t} \sum_{v=0}^{\infty} \frac{\{\lambda t \hat{P}_0[V < v_0]\}^v}{v!} \quad (3.35)$$

$$= 1 - e^{-\lambda t} e^{\lambda t \hat{P}_0[V < v_0]} \quad (3.36)$$

or

$$\hat{P}_0[F_0] = 1 - e^{-\lambda t} \hat{P}_0[V \geq v_0] \quad (3.37)$$

For the time interval t to be one year, Eq. 3.37 reduces to

$$\hat{p}_0[F_0] = 1 - e^{-\lambda P_0[V \geq v_0]} \quad (3.38)$$

The probability of a flood occurring in one year can be estimated using Eq. 3.38.

3.4 Criteria for Selecting a Region

The term "region" as used in contemporary approaches to the transposition of storms is defined to include, "the area surrounding the given river basin in which storm producing factors are substantially comparable; i.e., the general area within which meteorological influences and topography are sufficiently alike to permit adjustment of storm data to a common basis of comparison with a practical degree of reliability."(*) Such a "region" may include a very large geographic area in the eastern half of the United States where relief is generally moderate and it may include relatively small areas in the western United States where extreme topography is encountered.

Using a geographic region that contains the river basin, as outlined in Sections 3.2 and 3.3, has somewhat the same purpose as mentioned above. Strictly speaking for the presented model, however, the precise allocation of a region surrounding a river basin is arbitrary to some extent. The selection of a region will depend to a great extent on the availability of rainfall data from historic storms; the primary objective in selecting a region is to gather more information about these historic storms. Therefore, the region and the river basin should have similarities in meteorological influences and topography, as well as sufficient data. Such a region is defined to be meteorologically homogeneous, i.e., the occurrence of storms over the region is random, or the probability of occurrence of a storm is the same over the entire region.

It may not always be possible to ascertain the similarities in meteorological influences associated with various storms. In some cases the topography may have significant variations in the areas surrounding a river basin. If this is the case, one criterion for selecting a region is to select one that contains all the major storms that have occurred in the past in the vicinity of the river basin. The region so selected may not be meteorologically homogeneous, but, in this case, appropriate probability distribution functions can be hypothesized for the random variables defined

for a storm, thus taking into account heterogeneity in the meteorology and topography within the region. For such cases the estimation model as outlined in Section 3.3 will need to be modified; this modification must be based on appropriate probability distribution functions.

Once the extent of the region is selected, the criterion needed for selecting the precise boundary of this region is that there is enough information on historic storm rainfall all around the region. This criterion primarily governs the limits of transposition of a storm over a basin. For example, in transposing a storm over the basin for which there is no rainfall data outside the region, the storm cannot cover the entire basin at some positions. This concept is schematically shown in Fig. 3.10. Therefore, the area surrounding the region on which the rainfall information is also needed should be at least large enough to cover the size of the basin.

In summary, the selection of a region in the case of small basins, say less than 1,000 square miles, is more important in comparison with large basins of a few thousand square miles. Large basins can have pronounced differences in meteorological and topographic influences, that result in different types of storms for which it may be difficult to even hypothesize regional probability distribution functions for random variables defined on storms. Therefore, the applicability of the estimation model developed here is primarily confined to small river basins.

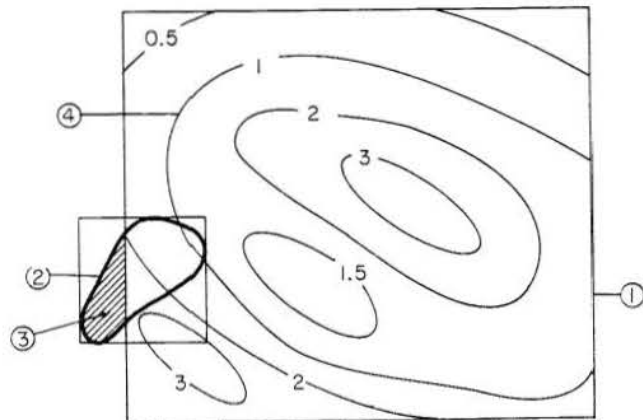


Fig. 3.10 Conceptual representation of needed rainfall data outside the selected region. (1) Region boundary; (2) River basin; (3) Area of the basin without available rainfall data in the above transposed position and orientation of a storm; (4) Rainfall isohyets in inches.

TABLE 3.3

Chi-Square Test for Hypothesized Beta-Probability Distribution Function

Interval j	n_j	p_j^0	np_j^0	$\frac{(n_j - np_j^0)^2}{np_j^0}$	$\sum_{j=1}^8 \frac{(n_j - np_j^0)^2}{np_j^0}$
1	3	.055	5.5	1.136	7.06
2	13	.16	16	0.562	
3	24	.2	20	0.800	
4	26	.195	19.5	2.166	
5	14	.145	14.5	0.017	
6	7	.12	12	2.100	
7	8	.085	8.5	0.029	
8	5	.04	4.0	0.250	

(*) Refer to Handbook of Applied Hydrology by V. T. Chow, Chapter 9, page 65.

DEVELOPMENT OF A STORM TRANSPOSITION TECHNIQUE4.1 Brief Description of a Rainfall-Runoff Model

Various rainfall-to-runoff models described in literature are deterministic in nature. Such models generate the runoff hydrograph from the rainfall hyetographs. In reality this relationship is not deterministic. The response of a river basin to the rainfall input has stochastic components resulting from various sources of randomness. For example, the soil moisture level of a basin prior to the initiation of rainfall is a random variable, since it is a function of the previous rainfall. A probabilistic interpretation can be given to many variables involved in the rainfall-to-runoff relationship. Such random variables are evaporation, evapotranspiration, infiltration, the vegetal cover, the surface roughness, etc. Since the major stochastic variations in rainfall-to-runoff relationships are due to the random characteristics associated with a storm, it is reasonable to simplify the basin response by defining an average response in a deterministic way.

No attempt is made in this study to study the problem of rainfall-to-runoff modeling. The estimation model given in Section 3.3 is also based on the assumption that the randomness in the values of the flood descriptor for a given storm is only due to the variation in the values assumed by the random vector (X,Y) and the random variable Θ , i.e., for a given set of values of (X,Y) and Θ , the value of the flood descriptor V is computed by a deterministic function of the basin response.

To demonstrate the application of the estimation model developed in this study, a simple rainfall-to-runoff model is selected. Any deterministic model can be used to generate the corresponding runoff without a loss of generality in the estimation concepts.

The flood descriptor used is the total surface-runoff volume, denoted by V. Included here is the model given by Betson, Tucker, and Haller [2] which is a mathematical version of the graphical rainfall-to-runoff model developed by the U. S. Weather Bureau. The model computes the surface-runoff volume by relating it to the rainfall input, season index, and the antecedent precipitation index. The complete model is comprised of two equations having these three variables and five parameters. The equations are:

$$RI = c' + (a' + f' \cdot SI) \cdot e^{-b' \cdot API} \quad (4.1)$$

$$SRO = (RF^{n'} + RI^{n'})^{1/n'} - RI \quad (4.2)$$

With the five parameters a' , b' , c' , f' , and n' , API is the antecedent precipitation index, SI is

defined as the season index, which is a function of the sequential week number, RI is the rainfall index, SRO is the surface runoff volume, and RF is the average rainfall on the basin^(*).

The API value is computed according to Linsley, Kohler, and Paulhaus [10] by

$$I_u = I_0 K_1^u \quad (4.3)$$

in which I_0 is the initial value of the antecedent precipitation index, I_u is the value u days later than the day of initial value, and K_1 is a recession factor ranging usually between 0.85 and 0.98.

The five parameters given in Eqs. 4.1 and 4.2 can be computed by using the historic rainfall data over the river basin and the runoff records at a corresponding river gaging station. Once the parameters have been determined from the historic data, usually as average values, the surface runoff volume can be computed for any storm simply by using the input variables, RF, API, and SI.

4.2 Basic Concepts of the Storm Transposition Technique

The technique of transposing storms over a basin is computer oriented and is developed as a compromise between simplified computer programming and accuracy of results.

Consider a region that contains a river basin under study, as schematically shown in Fig. 4.1. The symbols G_i , $i=1,2,\dots,m$, denote the rain gages over the region, with the observed rainfall amounts. The region is divided into rectangular grids which are not necessarily of equal size, equal sized grids, however, do facilitate computer programming. A grid can be visualized as a subregion, over which the function $P[V \geq v_0 | X=x, Y=y, \Theta=\theta]$ is approximated to be a constant with respect to the values assumed by the random vector (X,Y) in the subregion. Therefore, "small" sized grids would provide a better approximation of the function. However, very small sized grids may not necessarily increase the accuracy, while they would increase the necessary computer time. A rigid criterion cannot be laid down for selecting grid sizes. As an example of selecting a grid size, the East Central Illinois region, selected to demonstrate the presented technique, is about 400 square miles in size and is close to being a square. The region is divided into 28 x 28 equal size grids.

The total observed rainfall at a raingage G_i , $i=1,2,\dots,m$, is assigned to every grid point in the computer program that falls within the Theissen

(*) The two and three letter symbols used in Eqs. 4.1 and 4.2 for the five variables are not in conformity with the symbol terminology used in the rest of the text. These symbols are used here simply because the authors of the model used them; the same symbols have been also used in the computer program included in Appendix I.

polygon corresponding to that raingage. For example, the total rainfall observed at raingage G_2 is assigned to all those grid points that fall within the Thiessen polygon corresponding to the raingage G_2 . All such grid points to which the observed rainfall at G_2 is assigned have been numerically identified by the number 2 in Fig. 4.2. The rainfall-to-runoff model governs the type of rainfall input required in the computer program. For example, the rainfall input can be the total rainfall observed at a raingage or the rainfall amounts in time intervals of specified length. The rainfall-to-runoff model used in this study requires the total rainfall in inches at each raingage. The rainfall input at a grid point is denoted by $GP(IK,JK)$ in the computer program developed for this example; IK,JK are the subscripts that numerically identify that grid point within the program.

The location of raingages in the East Central Illinois raingage network are in a square pattern and are fairly uniform. As a result the number of grid points within each Thiessen polygon is approximately the same for the entire raingage network. Consequently, the rainfall input is specified at each raingage in the computer program. The rainfall input at each raingage is denoted by $P(I,J)$ in the program, in which I,J are the subscripts identifying each raingage. The rainfall amount at each raingage is transferred to the grid points (as outlined above) in the computer program. In general, the raingages may not be located in a uniform pattern in the raingage network. In such cases, it is easier (from the point of view of computer programming) to directly specify the rainfall for each grid point outside the program as compared to first specifying the rainfall for each raingage and then transferring it to the respective grid points in the computer program.

The river basin is also divided into grids which are of the same size as those over the region. The shape of the basin is numerically specified by the numbers 0 and 1 in the computer program, i.e., if a grid point falls within or on the boundary of the basin, it is assigned a value 1, otherwise 0 as demonstrated in Fig. 4.3. The configuration of the basin can be better represented by a small sized grid; therefore, the grid size selected for the region should also consider the extent of approximation required in representation of the basin configuration. The variable $A(I,J)$ represents the numerical value 0 or 1 assigned to grid points of the basin in the computer program.

Because most storms cover an area much larger than that of a small river basin, it is more convenient to transpose the basin over the isohyetal map of a storm, rather than transposing the storm over the river basin. However, for the river basins that are much larger than the areas covered by storms it would be easier to transpose storms over the basin. Basically, the two approaches are identical. Because the Goose Creek basin selected for this study is about 50 square miles in area, the computer program is written so as to transpose this basin over an area covered by a storm. In general, a storm covers areas much larger than 50 square miles, usually a thousand square miles or more.

The transposition of a storm is required in each direction selected from intervals J_1, \dots, J_s as given by Eq. 3.24. Therefore, the total number of directions required for transposition of any storm is a selective parameter s . Before transposing a storm in any one of the s directions, the storm is rotated such that the storm orientation coincides with that

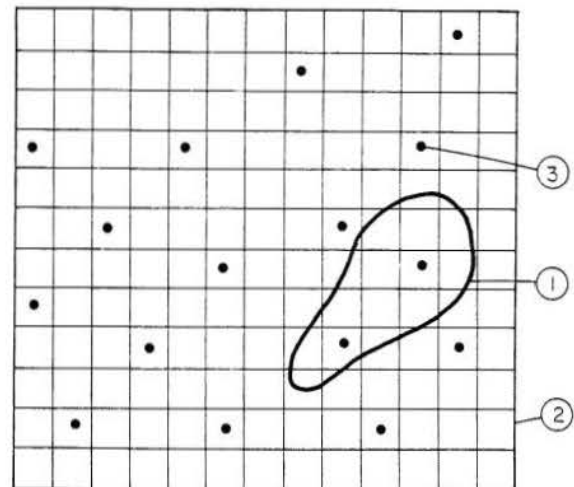


Fig. 4.1 A region divided into grids containing the river basin. (1) River basin; (2) Region boundary; (3) G_1 Raingage

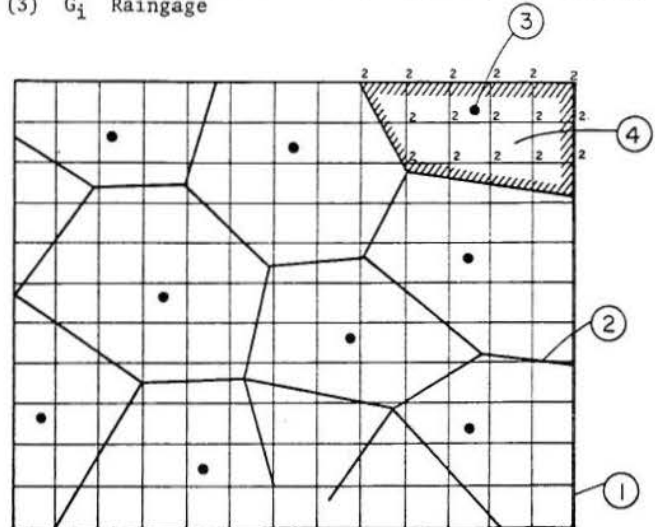


Fig. 4.2 Thiessen polygons for a region divided into grids. (1) Region boundary; (2) Thiessen polygon; (3) G_2 Raingage; (4) The grid points within this Thiessen polygon are assigned rainfall values corresponding to the one observed at G_2 .

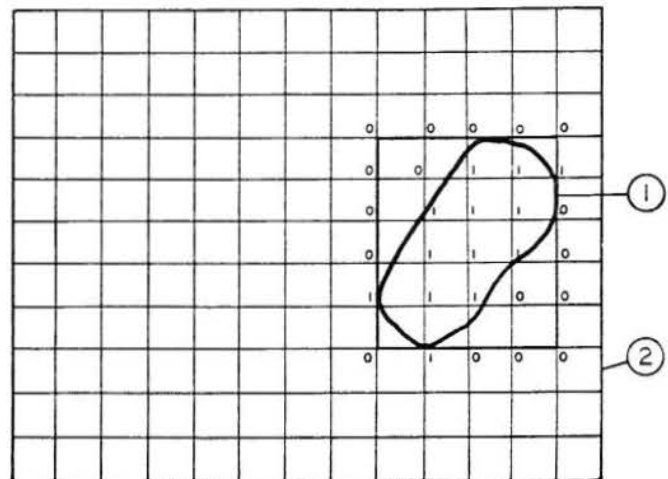


Fig. 4.3 Basin configuration identified for the computer. (1) River basin; (2) Region.

direction. Since in this study, the rotation and transposition of a storm is carried out by rotating and transposing the basin, always keeping the storm fixed, the following text contains the procedure for rotating the basin to make it correspond to an equivalent rotation of a storm.

Suppose D_i is the orientation of a storm, computed in the manner outlined in Section 3.2. Let D_B be an arbitrarily selected fixed line that denotes an axis of the river basin. Also, suppose the orientation D_i to be in the i -th orientation interval J_i of Eq. 3.24. The weights assigned to the basin grid that correspond to the storm orientation D_i are shown in Fig. 4.4. Now, the storm can be rotated in a clockwise direction by $\Delta\theta$ from its original orientation D_i , to a new orientation denoted by D'_i , such that this new orientation lies in the J_{i+1} interval. In order to rotate the basin instead of the storm such that the new orientation of the basin may correspond to rotating the storm to the new orientation D'_i , the basin axis D_B is rotated in the counterclockwise direction by $\Delta\theta$ to a new orientation, say D'_B . Therefore, the new orientation of the basin, D'_B , with the storm fixed, is equivalent to rotating the storm to the orientation D'_i , with the basin fixed. After rotating the basin to D'_B , a new set of numerical weights (0 or 1) can be assigned to the basin grids. This principle of rotating a basin is shown in Fig. 4.4. With this new set of weights assigned to the basin grid, it can again be transposed over the storm isohyets. The same procedure is used for each direction selected from intervals J_1, \dots, J_5 .

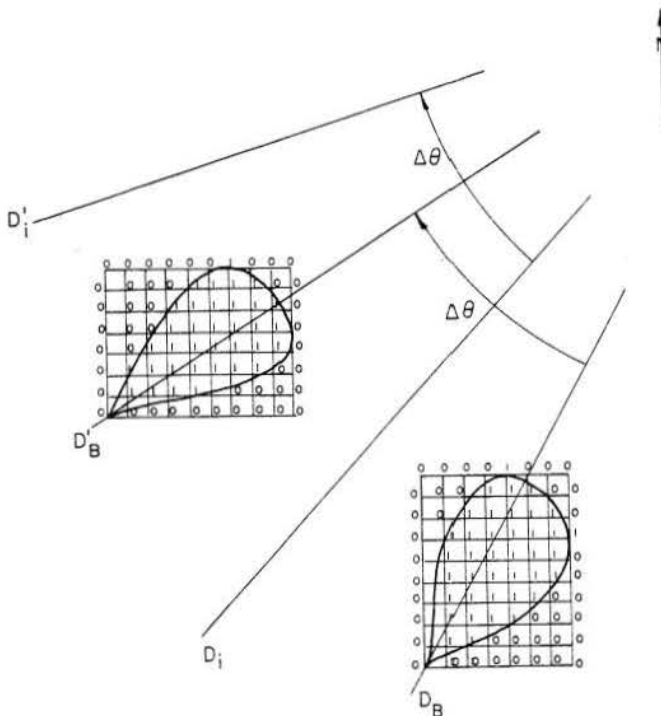


Fig. 4.4 Difference in the numerical identification of a basin configuration, corresponding to different orientations of the basin. D_i - Original storm orientation; D'_i - Storm orientation after it is rotated by $\Delta\theta$; D_B - Original orientation of the basin; D'_B - Orientation of the basin after it is rotated by $\Delta\theta$.

In order to program the transposition of a basin over the observed storm, a reference point in the isohyetal map of the storm is selected; this point is the storm center. The basin is transposed with respect to the storm center and for each position of the basin over the region, the rainfall values at the region grids that coincide with the basin grids are multiplied by the weights either 0 or 1 of the overlapping basin grids. These weighted rainfall values are zero if the grid point is outside the basin. The arithmetic mean of the rainfall within the basin is then computed, which is the value of RF in Eq. 4.2. Other parameters and variables required for the computation of runoff from rainfall are specified at the beginning of the computer program.

The rainfall-to-runoff model is used at this point and the flood descriptor is computed. The same is repeated for all transposed positions of the basin at different grid points of the region. The values of the flood descriptor at each point of the grid are now used in a subroutine CALCNT, which is a standard subroutine that computes isolines for a given matrix input, to interpolate the isolines of the flood descriptor. This is shown schematically in Fig. 4.5. The same procedure is repeated for each selected orientation of the storm. The output results are then used by Eq. 3.32 to compute $P_0(V > v_0)$, i.e., the probability distribution of the flood descriptor.

The computer program as developed for the Goose Creek basin in the East Central Illinois region is given as Appendix I.

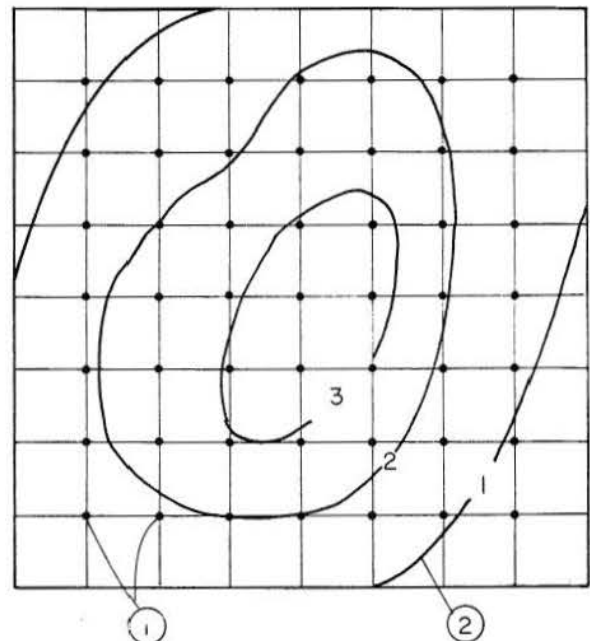


Fig. 4.5 Representation of isolines generated for the computed values of flood descriptor from transposing a storm in an orientation. (1) Transposed positions of storm center over and around the river basin; (2) Isolines of flood descriptor in inches.

APPLICATION OF THE DEVELOPED METHOD

5.1 Brief Description of the Region and the Historic Storms Selected

The East Central Illinois raingage network, which encloses the Goose Creek basin is located in a rural area with a relatively flat terrain. The network covers about 400 square miles, and the raingages are arranged in a near-uniform grid pattern averaging about eight square miles per raingage. Figure 5.1 shows the location of the river basin relative to the Illinois network and the arrangement of the raingages.

To demonstrate the presented methodology, the region surrounding the basin is made to coincide with the East Central Illinois raingage network. For this region it is assumed that the uniform probability distribution hypothesized for the storm center location denoted by (X,Y) holds good. A Beta-probability distribution is fitted empirically for the random variable θ which denotes the storm orientation, based on the data of historic storms on this region.

Five major storms that occurred over the region during the water period October 1, 1956 to September 30, 1958 have been selected to demonstrate the technique. The total rainfall for each storm was greater than 1.35 in. at more than one point on the region. The isohyetal maps of these storms are given in Fig. 5.2, a through e. The dates these storms occurred, the mean total rainfall, and the antecedent precipitation index denoted by API and computed by Eq. 4.3 are given in Table 5.1.

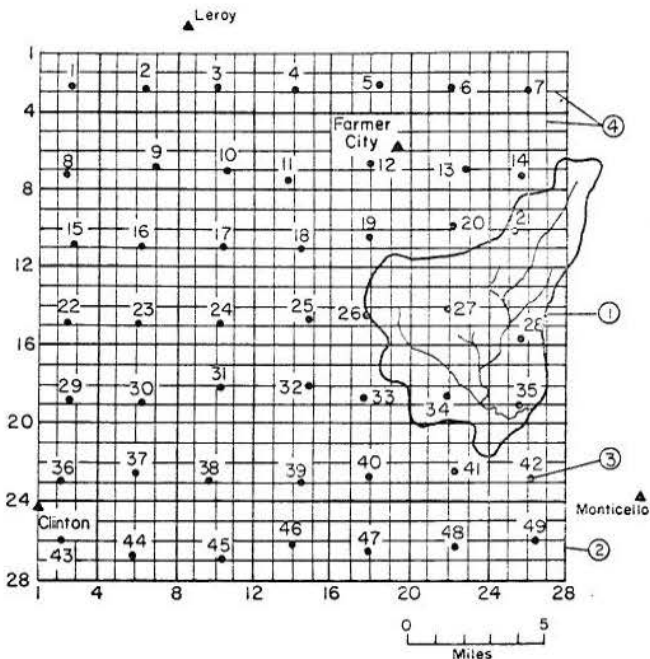


Fig. 5.1 East Central Illinois raingage network. (1) River basin; (2) East Central Illinois Network; (3) Raingage; (4) Grids.

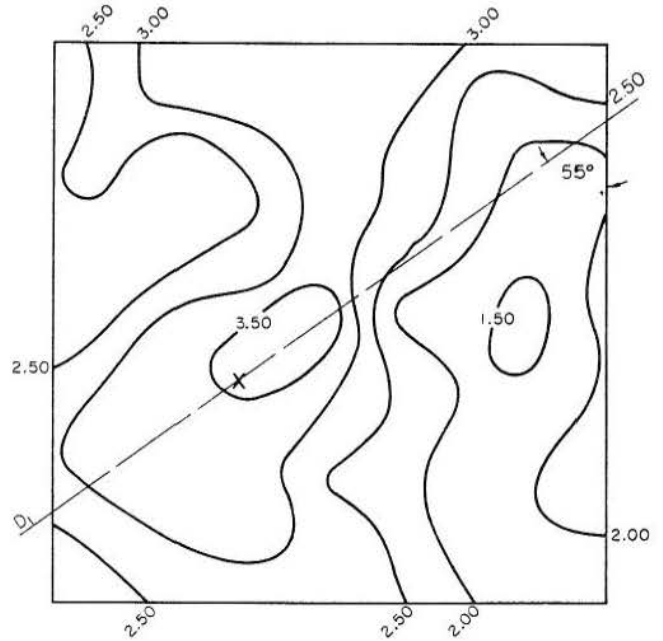


Fig. 5.2a Isohyetals for the storm of April 25-27, 1957. X Storm Center; D_1 Storm Orientation.

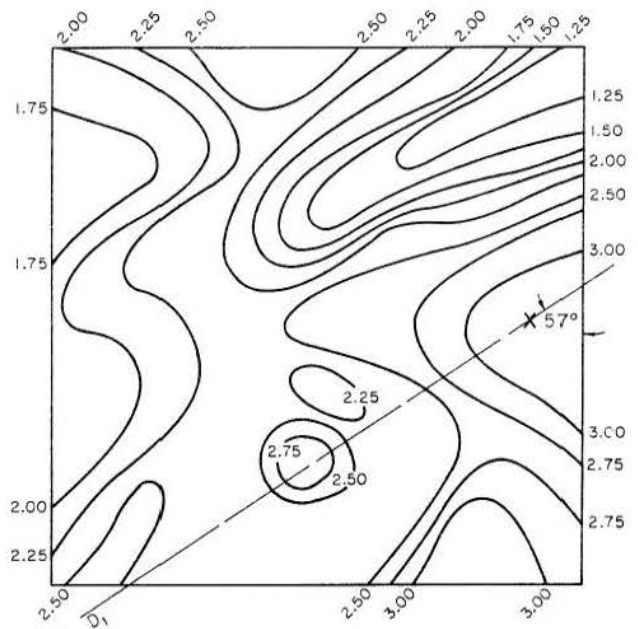


Fig. 5.2b Isohyetals for the storm of June 10, 1958. X Storm center; D_1 Storm orientation.

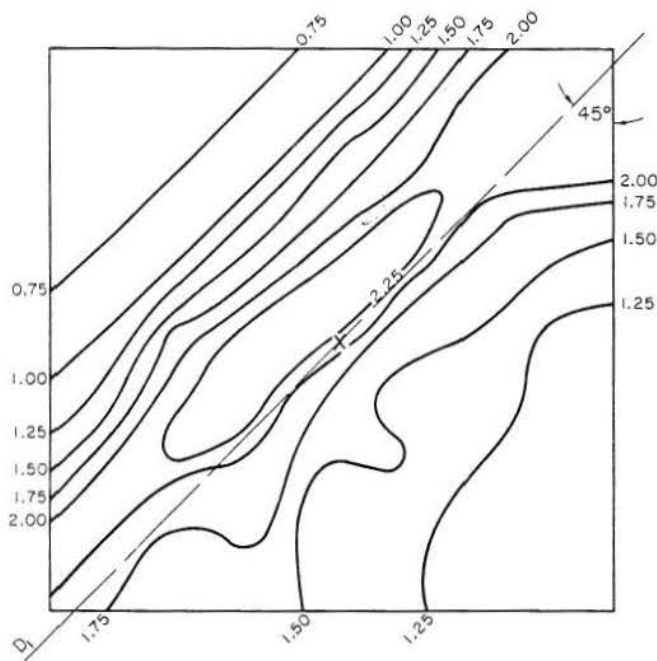


Fig. 5.2c Isohyetals for storm of June 24-25, 1958.
X Storm center; D_1 Storm orientation.

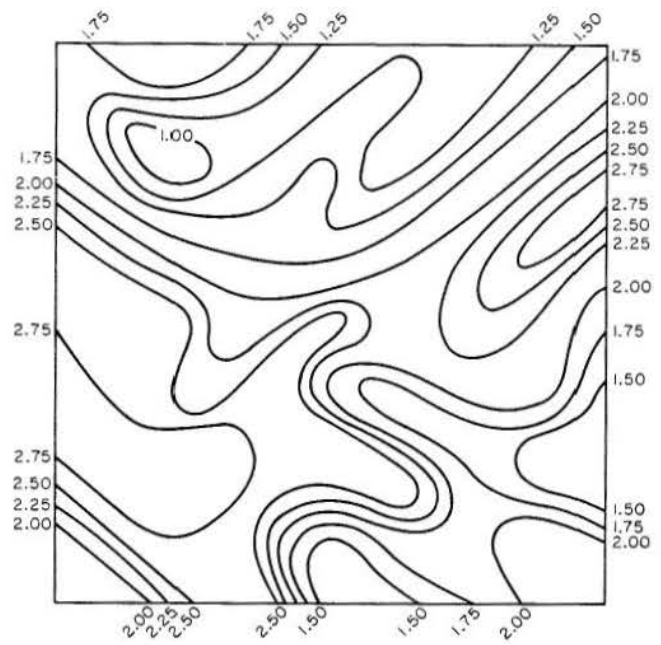


Fig. 5.2d Isohyetals for storm of July 10-11, 1958 -
Cont'd.

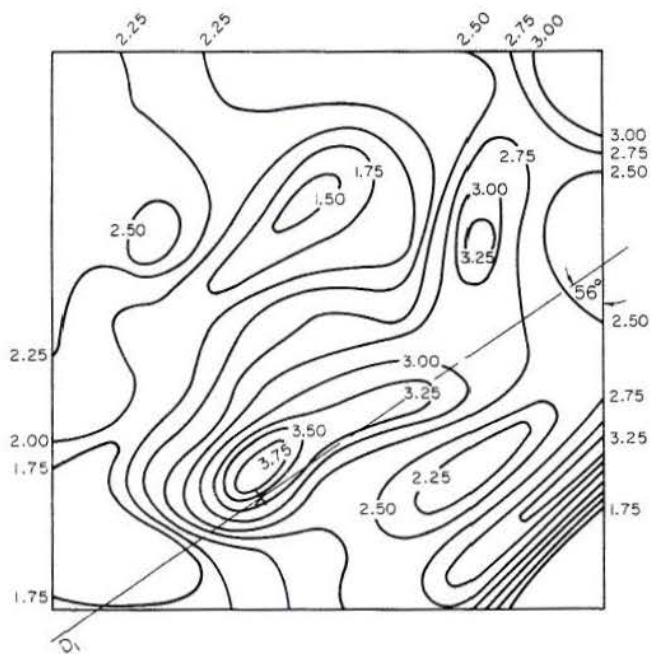


Fig. 5.2d Isohyetals for storm of July 10-11, 1958.
X Storm center; D_1 Storm orientation.

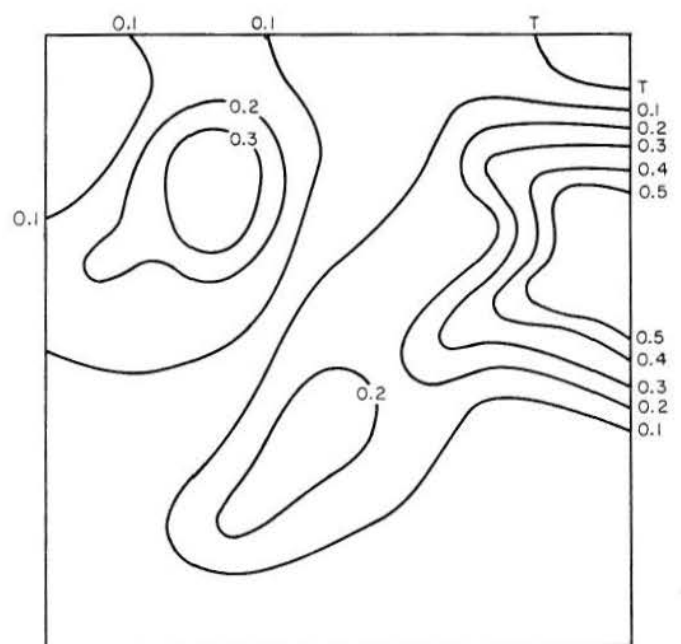


Fig. 5.2e Isohyetals for storm of August 1, 2, 1958.

5.2 Estimation of Rainfall-to-Runoff Model Parameters

The runoff volumes in inches are computed from the recordergraph reprints, produced for the Goose Creek basin. (*) A summary of the input variables required for the rainfall-to-runoff model described in Section 4.1 is given in Table 5.2. The computed runoff volumes in inches for the five storms are also summarized in the same table.

According to Betson, Tucker, and Haller [2], the rainfall to runoff model parameters denoted by a' , b' , c' , f' , n' , should be optimized by using the historic storm rainfall and the corresponding observed runoff values. Since the purpose of using a rainfall to runoff model in this study is only to demonstrate the concepts underlying the methodology developed here, it is not considered necessary to optimize the parameters of the model. However, the parameters are derived by trial and error such that any one computed value does not deviate from the observed value by more than 20% of the observed value in either the positive or negative direction.

The estimated rainfall to runoff model parameters are $a' = 12.70$, $b' = 0.45$, $c' = 4.00$, $f' = 6.15$, and $n' = 1.225$. The computed runoff values based on these parameters and the observed runoff values are summarized in Table 5.3.

Based on the estimated values of the five parameters, Eqs. 4.1 and 4.2 can be written as

$$RI = 4.0 + (12.7 + 6.15 \cdot SI) e^{-0.45 \cdot API} \quad (5.1)$$

$$SRO = (RF^{1.225} + RI^{1.225})^{1/1.225} - RI \quad (5.2)$$

Equations 5.1 and 5.2 are now used in computing the runoff volumes for different transposed positions of the storms.

5.3 Estimation of the Probability of Occurrence of Floods in a Year

The East Central Illinois raingage network is divided into 28 grids each along the x-axis and the y-axis, as shown in Fig. 5.1. The Thiessen polygon for

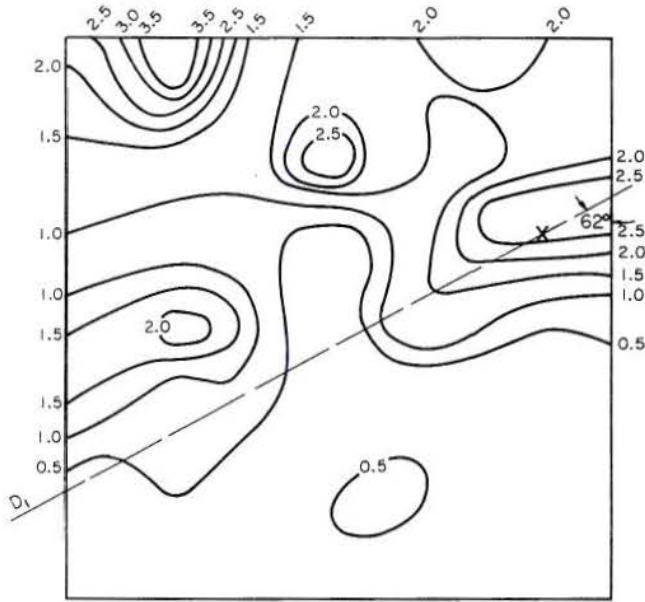


Fig. 5.2e Isohyetals for storm of August 1, 2, 1958 - Cont'd. X Storm center; D_1 Storm orientation.

TABLE 5.1
Selected Historic Storms

Storm Number	Storm Dates	Mean Rainfall in Inches	API Index
1	April 25-27, 1957	1.941	1.195
2	June 10, 1958	2.742	0.736
3	June 24-25, 1958	1.470	1.253
4	July 10-11, 1958	5.025	.733
5	August 1-2, 1958	1.386	1.265

TABLE 5.2

Information Needed in Estimating the Rainfall-to-Runoff Model Parameters

Storm Dates	Mean Rainfall in Inches	API	Week Number	S.I.	Observed Runoff Volume in Inches
April 25-27, 1957	1.941	1.195	17	-.24	1.356
June 10, 1958	2.742	.736	24	.60	1.236
June 24-25, 1958	1.470	1.253	25	.70	0.7
July 10-11, 1958	5.025	.733	28	1.00	3.458
August 1-2, 1958	1.386	1.265	31	1.00	.74

(*) Recordergraph reprints were obtained through personal correspondence with the U.S. Geological Survey, Champaign, Illinois.

TABLE 5.3

Observed and Computed Runoff Values for the Five Historic Storms

Storm Dates	Observed Runoff Volume in Inches	Computed Runoff Volume in Inches
April, 25-27, 1957	1.356	1.071
June 10, 1958	1.236	1.494
June 24-25, 1958	0.700	0.722
July 10-11, 1958	3.459	3.039
August 1-2, 1958	0.740	0.662

the raingage network is represented in Fig. 5.3. Since the raingages are distributed in a fairly uniform grid pattern within the raingage network, the Thiessen polygons for the raingages are also fairly uniform in size. Each Thiessen polygon contains 16 grid points: Four along the x-axis and four along the y-axis. Due to the symmetry of the grid points within each Thiessen polygon, the rainfall input to each grid point is transferred within the computer program by assigning the rainfall values to each raingage.

The range of a storm orientation is divided into three intervals, which are (0 - 1/2), (1/2 - 3/4) and (3/4 - 1). Only three intervals are selected to reduce the bulk of calculations and computer output, since the presented methodology is only demonstrative. The directions that the isohyets tended to elongate toward were used to determine the orientation of each of the five storms. Equation 3.13 is not used here in determining the storm orientation. The storm center for each storm is arbitrarily selected as the point of maximum rainfall. The storm orientation and the location of the center for each storm are depicted in the isohyetal map of the respective storm, given in Fig. 5.2, a through e.

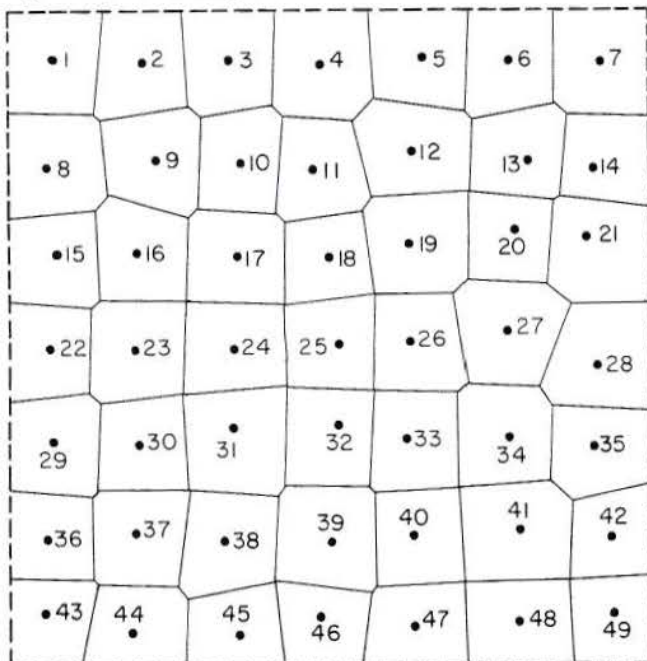


Fig. 5.3 Thiessen polygons for East Central Illinois Network.

The transposition of storms is carried out by transposing the Goose Creek basin over the isohyets of storms in the raingage network. An arbitrary axis selected across the basin is used as a reference line for orienting the basin to different storm orientations. This arbitrary line is inclined at 36° in the clockwise direction from the grid pattern of the East Central Illinois raingage network, as shown in Fig. 5.4. The East Central Illinois network is inclined at 19° in the clockwise direction to the established N-S line, as shown in Fig. 5.4. Therefore, the basin is at 55° inclination from the N-S line.

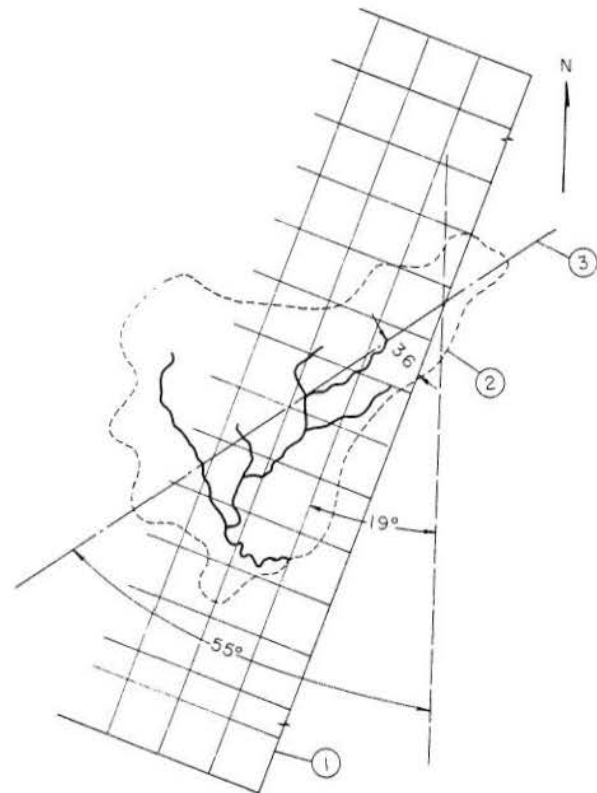


Fig. 5.4 Orientation of the arbitrary axis for the Goose Creek Basin from the N-S line. (1) Network boundary; (2) Goose Creek Basin; (3) Arbitrary axis of the basin.

The three orientations of the basin are so selected that corresponding to each orientation of the basin, all orientations of the five storms fall in one of the three orientation intervals. The three orientations used for the basin are 55° , 109° and 154° . This selection provided facility in using the computer program, i.e., for each orientation interval; all the five storms having orientations in that interval were transposed in one run of the program. Therefore the total runs required for the computer program are only three, one for each orientation interval. The 55° is the original orientation of the basin, the 109° orientation is obtained by rotating the basin by 54° in a clockwise direction, and the orientation of 154° is obtained by rotating the basin by 99° in a clockwise direction from the original orientation.

Based on these orientations of the basin, the equivalent orientation of the historic storms and the respective orientation intervals in which the orientations fall are summarized in Table 5.4. All orientations are given with reference to the N-S line.

TABLE 5.4

Storm Orientations Used in Transposition

Year	Storm Dates	INTERVAL	INTERVAL	INTERVAL
		$45^\circ < \phi \leq 90^\circ$	$0 < \phi \leq 45^\circ$	$90^\circ < \phi \leq 180^\circ$
	Storm Orientation when basin is at 55°	Storm Orientation when basin is at 1°	Storm Orientation when basin is at 136°	
1957	April, 25-27	$55^\circ + 19^\circ = 74^\circ$	20°	155°
1958	June, 10	$57^\circ + 19^\circ = 76^\circ$	22°	157°
1958	June, 24-25	$45^\circ + 19^\circ = 64^\circ$	10°	145°
1958	July, 10-11	$56^\circ + 19^\circ = 75^\circ$	21°	156°
1958	August, 1-2	$62^\circ + 19^\circ = 81^\circ$	27°	162°

Remarks: The orientations in Fig. 5.2, a through e, are given with respect to the network boundary. Therefore 19° are added to all values in column 3, to make the storm orientation correspond to the N-S line.

The isolines of the flood descriptor (runoff volume) have been interpolated by using a standard subroutine CALCNT. These isolines are represented in Fig. 5.6, a through c, for the storm of April 25-27, 1957; Fig. 5.7, a through c, for the storm of June 10, 1958; Fig. 5.8, a through c, for the storm of June 24-25, 1958; Fig. 5.9, a through c, for the storm of July 10-11, 1958, and Fig. 5.10, a through c, for the storm of August 1-2, 1958.

The selection of a region (East Central Illinois raingage network) for the Goose Creek basin was governed more by the availability of a dense raingage network surrounding the basin than by any other considerations. However, the region does have a fairly

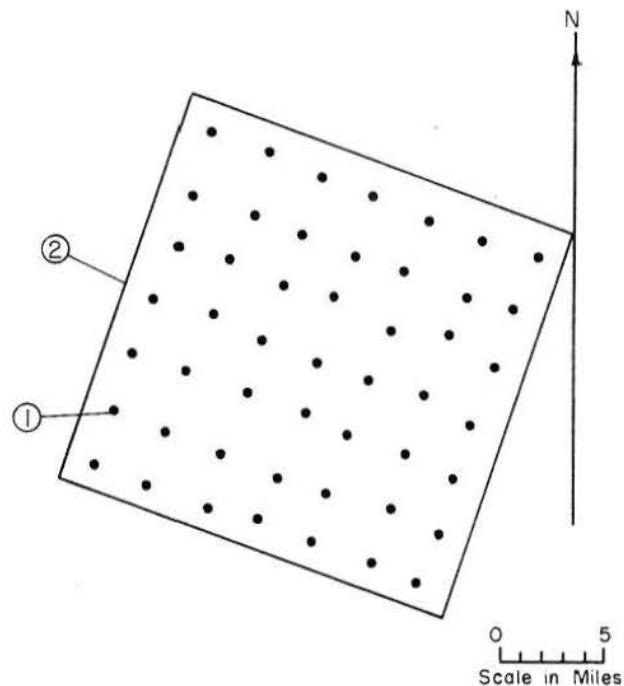


Fig. 5.5 East Central Illinois raingage network, 400 sq. mi., 49 recording gages, 1955-1968 (Illinois State Water Survey). (1) Recording raingage; (2) Network boundary.

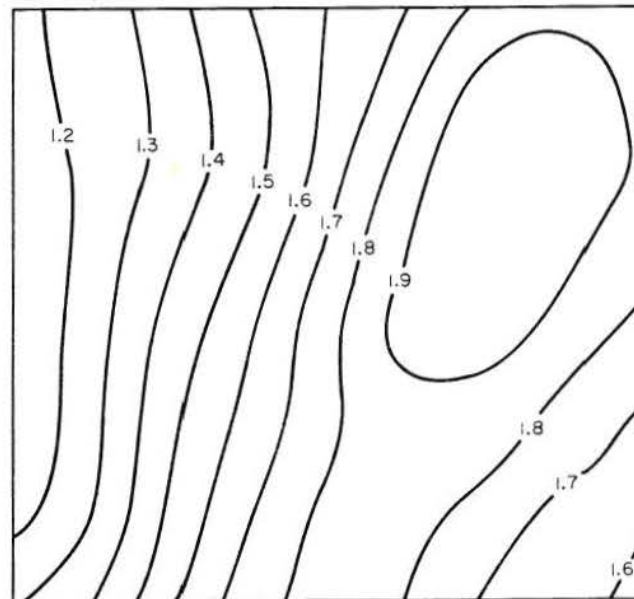


Fig. 5.6a Runoff volume contours for storm of April 25-27, 1957. Basin at 55° orientation.

uniform topography and because it is only 400 sq. mi. in area, it is meteorologically homogeneous. Since no raingage outside the region exists, the rainfall records from historic storms are confined to the region. As a result, the transposition of the Goose Creek basin could not be carried out over the entire region. To visualize this restriction, imposed by the non-availability of data outside the region, see Fig. 5.11.

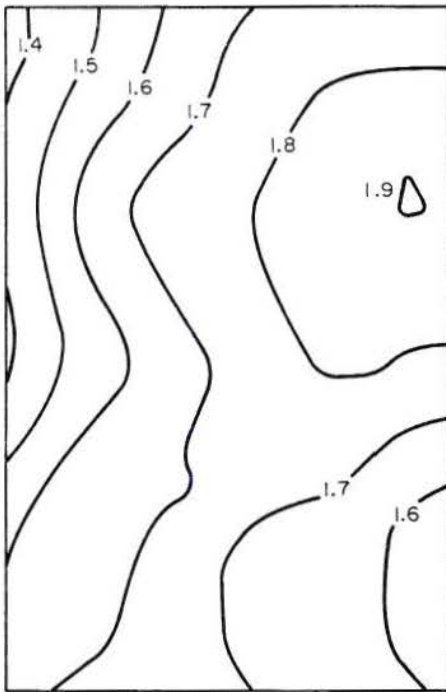


Fig. 5.6b Runoff volume contours for storm of April 25-27, 1957. Basin at 1° orientation.

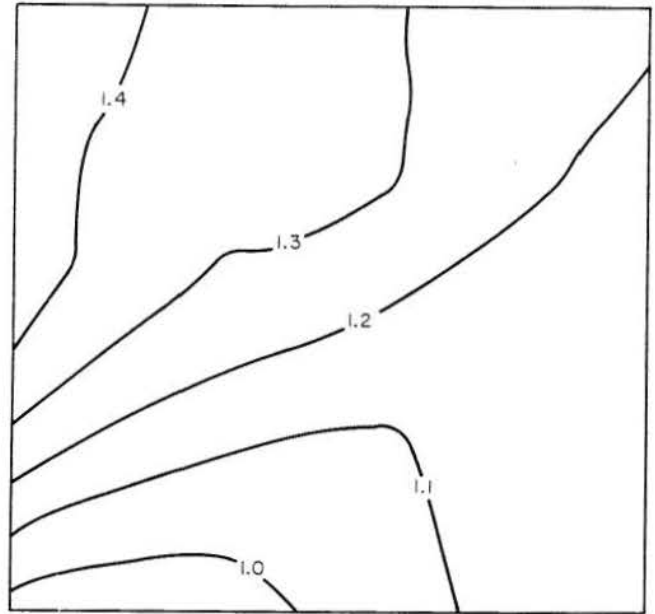


Fig. 5.7a Runoff volume contours for storm of June 10, 1958. Basin at 55° orientation.

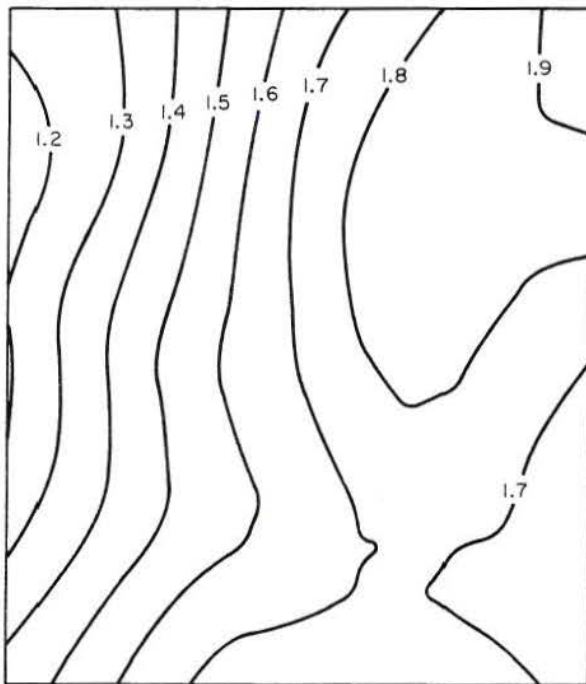


Fig. 5.6c Runoff volume contours for storm of April 25-27, 1957. Basin at 136° orientation.

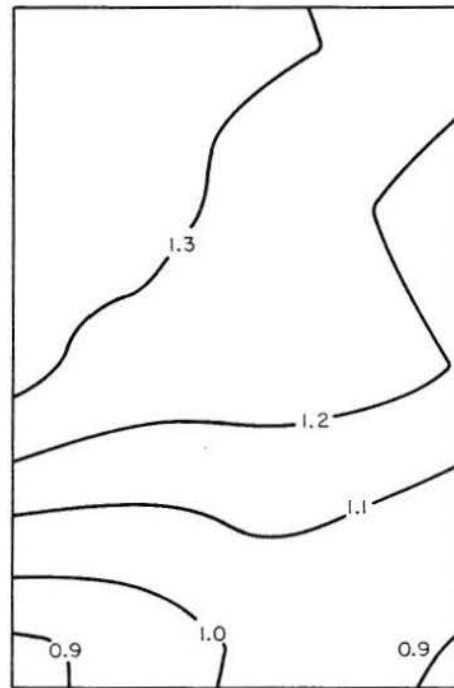


Fig. 5.7b Runoff volume contours for storm of June 10, 1958. Basin at 1° orientation.

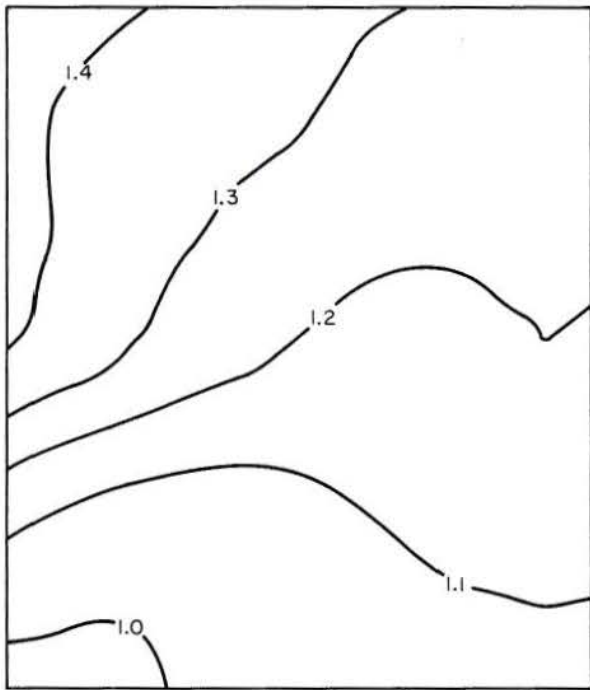


Fig. 5.7c Runoff volume contours for storm of June 10, 1958. Basin at 136° orientation.

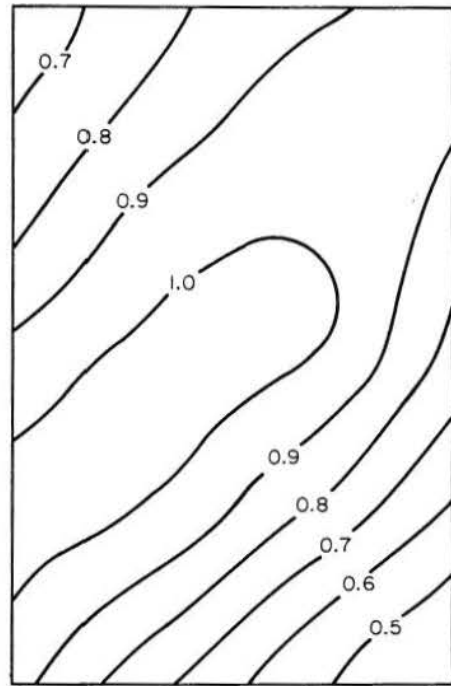


Fig. 5.8b Runoff volume contours for storm of June 24-25, 1958. Basin at 1° orientation.

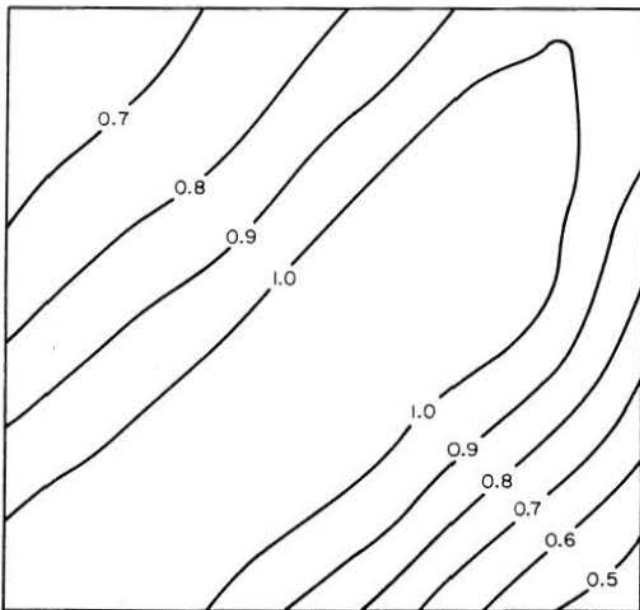


Fig. 5.8a Runoff volume contours for storm of June 24-25, 1958. Basin at 55° orientation.

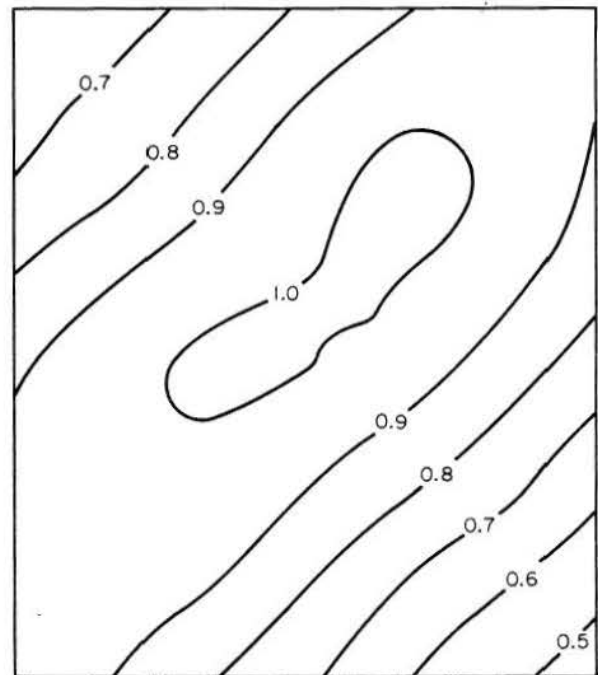


Fig. 5.8c Runoff volume contours for storm of June 24-25, 1958. Basin at 136° orientation.

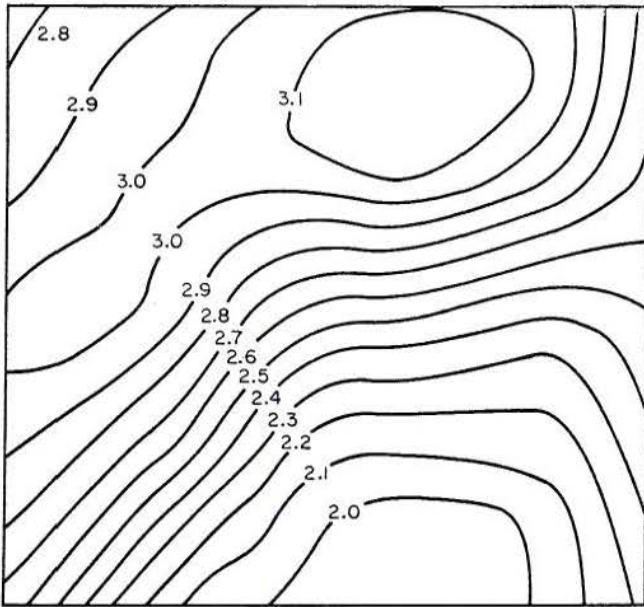


Fig. 5.9a Runoff volume contours for storm of July 10-11, 1958. Basin at 55° orientation.

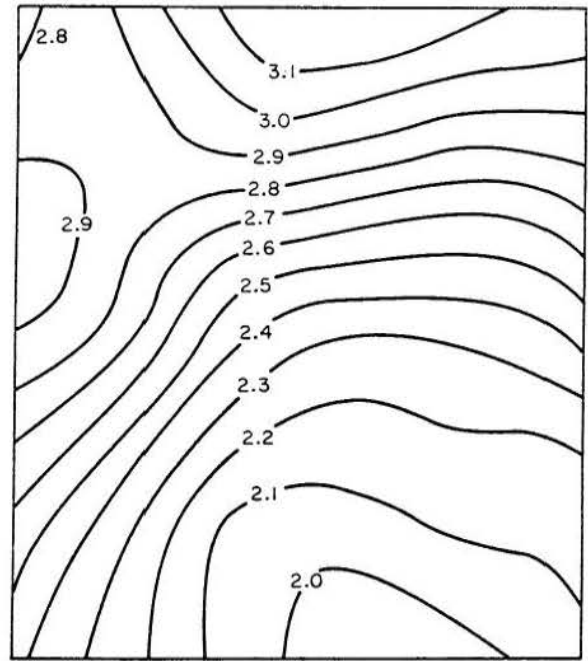


Fig. 5.9c Runoff volume contours for storm of July 10-11, 1958. Basin at 136° orientation.

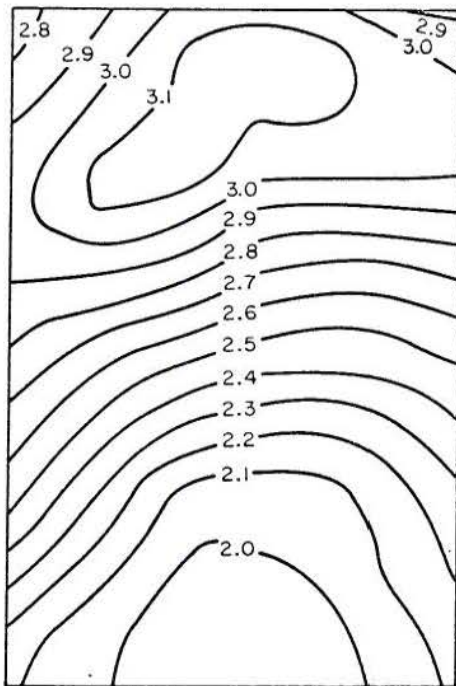


Fig. 5.9b Runoff volume contours for storm of July 10-11, 1958. Basin at 1° orientation.

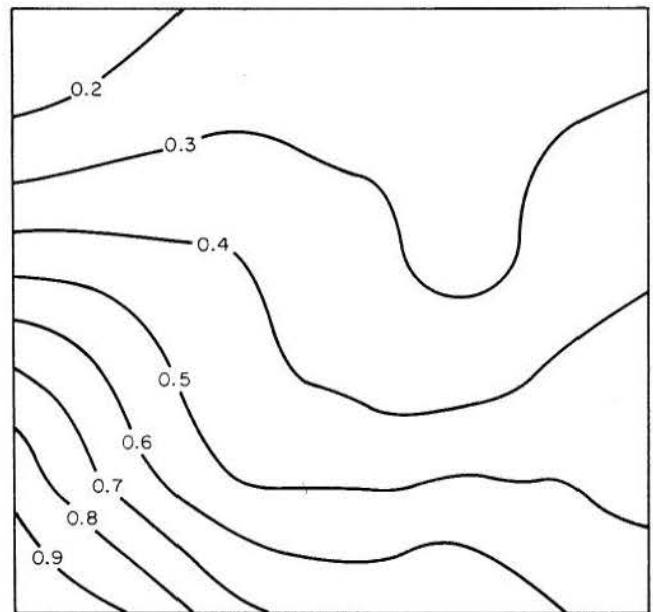


Fig. 5.10a Runoff volume contours for storm of August 1-2, 1958. Basin at 55° orientation.

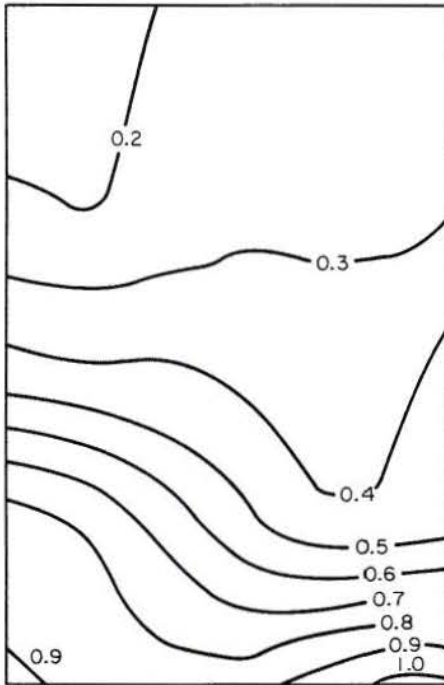


Fig. 5.10b Runoff volume contours for storm of August 1-2, 1958. Basin at 1° orientation.

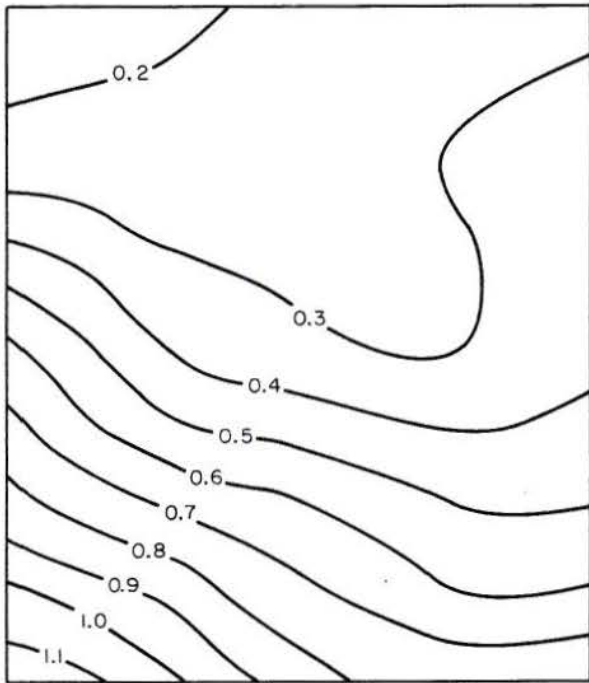


Fig. 5.10c Runoff volume contours for storm of August 1-2, 1958. Basin at 136° orientation.

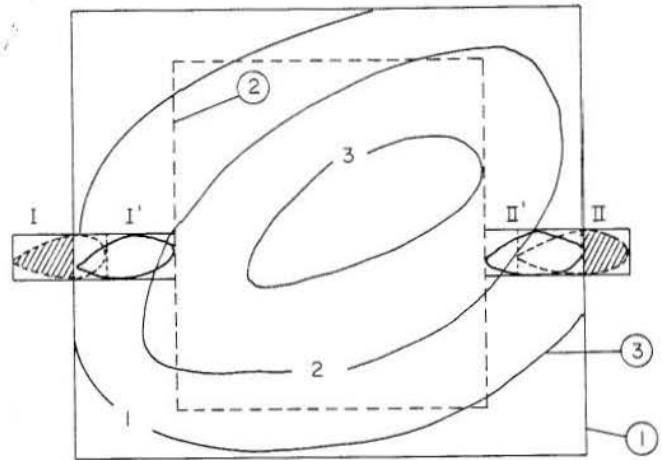


Fig. 5.11 Transposition limits of the basin within the region. (1) Network boundary; (2) Inner boundary denoting limits of transposing the basin within the region; (3) Rainfall isohyets in inches; I, II - Transposed positions of the basin, where part of the basin extends beyond the region; I', II' - Initial and final positions of the basin over the region, beyond which the basin cannot be transposed.

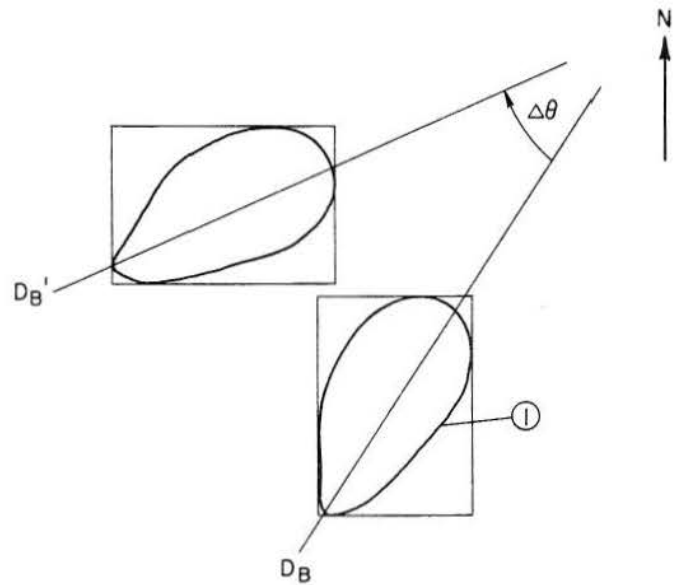


Fig. 5.12 Change in the rectangle circumscribing the basin corresponding to a change in the basin orientation. (1) River basin; D_B - Original orientation of the basin; D'_B - Orientation of the basin after it is rotated by $\Delta\theta$.

In Fig. 5.11, the basin is first placed at a position denoted by I. In this position, part of the basin extends beyond the region into an area where there is no rainfall data available. Therefore the starting position of transposition as practically feasible is denoted by I' in the figure. Similarly, when the basin is in the position denoted by II, part of it extends beyond the network region; therefore the last position in which the basin can be transposed is denoted by II'. Based on these restrictions, a new boundary must be drawn within the region that shows the limits up to which the Goose Creek basin can be transposed. This inner boundary is shown in Fig. 5.11.

When the basin is rotated in some other direction, the size of the rectangle circumscribing the basin compared to the rectangle circumscribing the basin when the basin was in the previous orientation changes. This change is schematically shown in Fig. 5.12. The basin oriented in a new direction is again transposed over the region. As a result, the size of the inner boundary depicting the limits to which the basin can be transposed, changes. The difference in the inner boundaries corresponding to the different orientation of the basin is evident from the change in the sizes of the areas within which the isolines of flood volumes have been generated as given in Figs. 5.6 through 5.10 for the five storms. For any one storm, for instance, the storm of April 25-27, 1957, Figs. 5.6 a through c have different areas over which the flood descriptor isolines are interpolated. The reason for these different areas is that each area corresponds to a different orientation of the basin.

In view of the above limitations, Eq. 3.32, used to estimate the probability distribution of the selected flood descriptor, is modified for purposes of computation as follows.

The area $A_0^j(\theta_i)$ in Eq. 3.32 denotes the area of the isoline of flood descriptor v_0 , when the j-th storm is oriented in the θ_i direction. Also the area of the entire region denoted by A_R is always a constant. Since the actual transposition cannot be carried out over the entire region, then the area enclosed by the boundary inside the region (representing

the transposition limits of the basin) is smaller than the area A_R . This area of the inner boundary is denoted by A_R^* . Also the area enclosed by the isoline of the flood descriptor v_0 generated for j-th storm oriented in θ_i direction selected from an orientation interval J_i , is less than or equal to the actual area $A_0^j(\theta_i)$, so it is denoted by $A_0^j(\theta_i)^*$.

Consider the ratio r_{ji} , given as

$$r_{ji} = \frac{A_0^j(\theta_i)}{A_R} \quad (5.3)$$

in which the subscript j denotes the storm and the subscript i denotes the orientation interval.

Let the estimate of the ratio r_{ji} be denoted as \hat{r}_{ji} . Then \hat{r}_{ji} can be computed as

$$\hat{r}_{ji} = \frac{A_0^j(\theta_i)^*}{A_R^*} \quad (5.4)$$

Substituting Eq. 5.4 into Eq. 3.32, Eq. 3.32 can be expressed as

$$\hat{P}[V \geq v_0] = \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^s \hat{r}_{ji} F[J_i] \quad (5.5)$$

for purposes of computation.

The computations in estimating $\hat{P}[V > v_0]$ for different values of v_0 are given in Table 5.5, a through e. The different values selected for v_0 are 3 in., 2.5 in., 2 in., 1.5 in., and 1 in. The probability of occurrence of floods, where the occurrence of floods is defined for the corresponding value of v_0 , is given in Table 5.5, a through e. The value of the length of the time interval from which the five storms are selected is two years, therefore $T = 2$ is used in estimating the parameter λ of the Poisson distribution.

Chapter VI

SUMMARY AND RECOMMENDATIONS FOR FURTHER RESEARCH

6.1 Summary

Contemporary techniques used to estimate the critical design flood descriptor for a river basin from data on historic storms are subjective. The critical value of a flood descriptor, as computed by such techniques, represents a physical upper limit of the flood descriptor for which there is little or no phenomenological justification. For example, flood descriptors, i.e., instantaneous flood peak or total flood volume, etc., are random variables and, therefore, follow a characteristic probability distribution which may depend on the geographic location of the river basin, physical characteristics of the river basin, or on meteorological influences on the river basin. Therefore, the probability of exceedence of a value of flood descriptor, selected from the upper tail of its probability distribution is always greater than zero.

A methodology, based on a theoretical framework, is presented in this study for estimating the probability of exceedence of a preselected flood magnitude, based on random characteristics of storms. Many random variables can be defined on random characteristics of storms, but as a first approximation only a three-dimensional random vector is defined here. The random vector is comprised of the storm center location denoted by the subset random vector (X,Y) and the storm orientation denoted by the random variable θ . The estimation procedure, as developed from the theoretical formulation of the problem, requires transposition of historic storms to $(X=x, Y=y)$ and at an orientation $\theta=\theta$ to estimate the function $P_0[V > V_0 | X=x, Y=y, \theta=\theta]$ over and around a given river basin enclosed by a region. The historic storms are transposed at different positions over and around the basin and in different orientations. Each orientation is selected from one orientation interval; the sum of all the orientation intervals is the entire range of the values assumed by the random variable θ .

The technique developed in this study to transpose historic storms over and around a river basin is computer oriented. The application of the developed estimation model and the storm transposition technique is demonstrated on the Goose Creek basin located in the East Central Illinois raingage network. To simplify the computations, a simple rainfall-to-runoff model is selected to generate the flood descriptor at each transposed position of a storm. The selected flood descriptor is the total flood volume. However, the estimation model and the transposition technique are general and can be applied to any flood descriptor.

6.2 Limitations

The estimation model and the transposition of historic storms as it is developed requires availability of sufficient data about historic storms. In other words, the region that contains the given river basin must have a dense network of recording raingages to provide hyetographs of rainfall at each raingage. Such a raingage network ensures records of high rainfall

intensities from any historic storm. With continuous records of rainfall from all historic storms available, the flood descriptor selected as the instantaneous maximum peak of the entire hydrograph generated from a storm could be computed. However, if the total flood volume is used as a flood descriptor, then it is sufficient to have only the total rainfall at each raingage. In other words, the isohyetal map of total rainfall from a storm is sufficient.

In absence of a dense network of raingages, the areal distribution of rainfall from a storm cannot be ascertained, and the random variables defined in this study for random characteristics of storms would not adequately represent such characteristics because of lack of data.

There should be a dense raingage network inside the region containing the river basin as well as a dense raingage network beyond the region so that the river basin can overlap the rainfall information on the extended network when the storm is transposed on the basin. If there is not a dense raingage network beyond the region, then the storm in some transposed positions will only partially cover the basin, thereby limiting the transposition to only part of the observed storm.

Lastly, the estimation model as it is presented here is only for rain storms. This restricts the application of this model to those basins that have both rainfall and snowmelt floods. Furthermore, the concepts presented here in the estimation model can be only applied to small basins because in small basins the changes in storm patterns in a region surrounding the basin are not very significant. Therefore, the probability distribution functions fitted or hypothesized for different random variables defined on storms would not change from one location to another in the region.

6.3 Recommendations for Further Research

The methodology as developed is only a first step toward presenting an approach based on random characteristics of storms for estimating the probability of floods occurring in a unit time interval from the historic storms observed in a region that contains the basin. A three dimensional random vector is defined for the random characteristics of storms. The random vector is comprised of the storm center location denoted by (X,Y) and the storm orientation denoted by θ . For future research, other random variables should be incorporated in order to expand application of the model. For example, the total rainfall yield at some point from a storm is a random variable. The probability distribution of this random variable can be studied over the region and it can be incorporated into the probabilistic model.

The soil moisture level prior to the commencement of a storm is a function of the historic rainfall; hence, it is a random variable. The probability distribution of the soil moisture level could be studied and also included in the estimation model.

If the instantaneous flood peak is selected as a flood descriptor, it is essential to consider hourly rainfall values in order to compute the resultant hydrograph. This can be achieved by the presented technique in specifying hourly rainfall amounts at each raingage instead of the total rainfall. For each transposed position of the storm over the basin, the hourly values could be used in any rainfall-to-runoff model and the flood hydrograph could be generated. The maximum instantaneous peak could then be selected

from the hydrograph. If this is repeated for all transposed positions, a grid of instantaneous peaks would be obtained; from these peaks the isolines could be interpolated.

Another recommendation is to explore various other estimators used to estimate the function $P_0[V > v_0 | X=x, Y=y, \Theta=\theta]$, and arrive at the "best" estimator based on their statistical properties, e.g., variance, and unbiasedness, etc.

APPENDIX I

The computer program for storm transposition as developed for the East Central Illinois raingage network is given in this appendix.

A brief explanation to facilitate the use of this program is given below.

Data Input Statements

<u>Card No.</u>	<u>Column No.</u>	<u>Explanation of the Variables</u>
1	1-4	NGR - Total number of storms to be transposed
2	1-5	NJ1 - j-th coordinate of the first raingage
	6-10	NJ2 - j-th coordinate of the last raingage
	11-15	NI1 - i-th coordinate of the first raingage
	16-20	NI2 - i-th coordinate of the last raingage
	21-25	NGPJ - Total number of grid points in each Theissen polygon (constant) in the j-th coordinate direction
	26-30	NGPI - Total number of grid points in each Theissen polygon (constant) in the i-th coordinate direction
3	1-72	DIR(I) - Specify the orientation of the basin with respect to the N-S line (alphabetic characters)
4	1-5	KKI - Total number of basin grid points in the i-th direction

<u>Card No.</u>	<u>Column No.</u>	<u>Explanation of the Variables</u>
	6-10	KKJ - Total number of basin grid points in the j-th direction

The following cards are punched such that one card corresponds to one set of grid point weights in the j-th direction. Therefore the total number of cards would be KKI, where $1 \leq KKI \leq 20$ and $1 \leq KKJ \leq 20$.

<u>Max. no. of cards</u>	<u>3,6...,51</u>	<u>A(I,J), J=1, KKJ</u>	<u>Basin grid weights (0 or 1)</u>
is 20			

The following set of cards is repeated for each storm to be transposed.

1	1-72	The title of the problem and the dates of storm occurrence
2	1-6	API - Antecedent precipitation index
	7-12	SI - Season index

The following cards specify the total rainfall observed at each raingage. The total number of cards would be $(NI2 - NI1 + 1)$, such that $1 \leq (NI2 - NI1 + 1) \leq 7$, and $1 \leq NJ2 \leq 7$.

<u>Max. no. of cards</u>	<u>1-10,11-20 ..., 61-70</u>	<u>P(I,J), J=NJ1, NJ2</u>	<u>Total observed rainfall</u>
is 7			
10	1-5	ISC - i-th coordinate of the storm center	
	6-10	JSC - j-th coordinate of the storm center	
11	1-70	Title to be printed in the microfilm plot.	

```

PROGRAM GUPTA(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,FILMPL)
DIMENSION P(7,7),GP(28,28),TITLE(18),DIR(18),ROF(43,43)
DIMENSION A(20,20),SRV(14,16)
DIMENSION TTL(10)
C   NGR-TOTAL NO. OF STORMS TO BE TRANSPOSED
C   NJ1-NJ2 J-TH COORDINATES OF FIRST AND LAST RAINGAGES RESPECTIVELY
C   NI1-NI2 I-TH COORDINATES OF FIRST AND LAST RAINGAGES RESPECTIVELY
C   THE COORDINATE VARIABLES FOR THE RAINGAGES GIVE A PROVISION TO
C   DELETE THE RAINGAGES WITH TRACES OF RECORDED RAINFALL
C   NGPJ-NO OF GRID POINTS (CONSTANT) IN EACH POLYGON IN J-TH DIRECT.
C   NGPI-NO OF GRID POINTS (CONSTANT) IN EACH POLYGON IN I-TH DIRECT.
C   DIR(I) IS THE TITLE IDENTIFYING THE DIRECTION OF THE BASIN FOR
C   TRANSPOSITION
C   INPUT VALUES FOR RAINFALL-RUNOFF MODEL PARAMETERS
Z=12.70
B=0.45
C=4.000
D=6.15
XN=1.225
C   DATA INPUT STATEMENTS
READ(5,4) NGR
4  FORMAT(I4)
READ(5,5) NJ1,NJ2,NI1,NI2,NGPJ,NGPI
5  FORMAT(7I5)
READ(5,17)(DIR(I),I=1,18)
17 FORMAT(18A4)
C   KKI =NO OF BASIN GRID POINTS IN I-DIRECTION
C   KKJ =NO OF BASIN GRID POINTS IN J-DIRECTION
C   A(I,J)=WTS. AT BASIN GRID POINTS
READ(5,18)KKI,KKJ
18 FORMAT(2I5)
DO 19 I=1,KKI
READ(5,20)(A(I,J),J=1,KKJ)
20 FORMAT(17F3.0)
19 CONTINUE
C   TITLE FOR EACH STORM TRANSPOSED . ALSO IDENTIFY THE STORM DATES
C   P(I,J) IS TOTAL RAINFALL AT IJ--RAINGAGE
DO 101 JKL=1,NGR
READ(5,3)(TITLE(I),I=1,18)
3  FORMAT(18A4)
READ(5,2) API,S1
2  FORMAT(2F6.3)
DO 6 I=NI1,NI2
READ(5,7)(P(I,J),J=NJ1,NJ2)
7  FORMAT(7F10.5)
6  CONTINUE
C   TRANSFER OF PRECIP TO GRID POINTS.
INC=0
DO 8 I=NI1,NI2
ILLM=1+(NI1-1)*NGPI+INC
IULM=ILLM+(NGPI-1)
DO 9 IK=ILLM,IULM
JNC=0
DO 10 J=NJ1,NJ2
JLLM=1+(NJ1-1)*NGPJ+JNC
JULM=JLLM+(NGPJ-1)
DO 11 JK=JLLM,JULM
GP(IK,JK)=P(I,J)
11 CONTINUE
JNC=JNC+NGPJ
10 CONTINUE
9  CONTINUE
INC=INC+NGPI
8  CONTINUE
C   INPUT OF BASIN GRID POINTS FOR NDIR DIRECTIONS.
WRITE(6,32)(TITLE(I),I=1,18)
32 FORMAT(*1*,15X,18A4)
C   TRANSPOSITION OF STORMS
S=0.
DO 22 I=1,KKI
DO 22 J=1,KKJ
S=S+A(I,J)

```

```

22 CONTINUE
   READ(5,23) ISC,JSC
23 FORMAT(2I5)
   WRITE(6,40) (DIR(KZ),KZ=1,18)
40 FORMAT(///,10X,18A4)
   WRITE(6,81) ISC,JSC
81 FORMAT(///,10X,*COORDINATES OF CENTER OF THE OBSERVED STORM*,5X,*Y
   I=*,15,3X,*X=*,15)
   WRITE(6,35)
35 FORMAT(///,10X,*THE LOCATION OF STORM CENTER OVER THE BASIN GRID W
   I.R.T. NEW COORD. SYSTEM AND RESULTING RUN OFF VOL. IN INCHES*)
   JLLM=1+(NJ1-1)*NGPJ
   ILLM=1+(NI1-1)*NGPI
   IJLM=NGPI*NI2
   JULM=NGPJ*NJ2
   KI1=ILLM
   KI2=KI1+KKI-1
   IND=1
30 KJ1=JLLM
   KJ2=KJ1+KKJ-1
27 CONTINUE
   SUM=0.
   DO 25 I=KI1,KI2
   DO 26 J=KJ1,KJ2
   SUM=SUM+GP(I,J)*A(1-KI1+1,J-KJ1+1)
25 CONTINUE
26 CONTINUE
   RF=SUM/S
   IC1=KI1-ISC+28
   JC1=JSC-KJ1+15
   RI=C+(Z+5I*0)*EXP(-1.*B*API)
   SR0=((RI**XN)+(RF**XN)**(1./XN))-RI
   ROF(IC1,JC1)=SR0
   IF(IND.EQ.1) GO TO 99
   GO TO 98
99 JMAX=JC1
   IND=2
96 KJ1=KJ1+1
   KJ2=KJ1+KKJ-1
   IF(KJ2.GT.JULM) GO TO 28
   GO TO 27
28 CONTINUE
   JMIN=JC1
   IF(IND.EQ.2) IMIN=IC1
   IND=6
   KI1=KI1+1
   KI2=KI1+KKI-1
   IF(KI2.GT.IJLM) GO TO 51
   GO TO 30
51 IMAX=IC1
   WRITE(6,52) IMAX,IMIN,JMAX,JMIN
52 FORMAT(///,3X,*THE MAX Y COORD=*,13,3X,*THE MIN Y COORD=*,13,3X,*Th
   IE MAX X COORD=*,13,3X,*THE MIN X COORD=*,13)
   WRITE(6,56) (JK,JK=JMIN,JMAX)
56 FORMAT(///,3X,17I7)
   DO 57 IK=IMIN,IMAX
   WRITE(6,58) IK,(ROF(IK,JK),JK=JMIN,JMAX)
58 FORMAT(/,15,17F7.3)
57 CONTINUE
   WRITE(6,67)
67 FORMAT(*1*)
   MX=JMAX-JMIN+1
   NY=IMAX-IMIN+1
   DO 71 JJ=1,MX
   DO 72 II=1,NY
   SRV(JJ,II)=ROF(IMIN+II-1,JMIN+JJ-1)
72 CONTINUE
71 CONTINUE
   II=NY

```

```

208 WRITE(6,78) II,(SRV(JJ,II),JJ=1,MX)
78  FORMAT(/,I5,17F7.3)
    II=II-1
    IF(II.EQ.0) GO TO 209
    GO TO 208
209 CONTINUE
C   FOLLOWING STATEMENTS COMPUTE THE NEEDED VARIABLES FOR SUBROUTINE
C   C A L C N T CALLED INORDER TO OBTAIN MICROFILM PLOT OF ISOLINES
    RFMAX=0.0
    RFMIN=SRV(1,1)
    DO 203 II=1,NY
    DO 203 JJ=1,MX
    IF (SRV(JJ,II).LT.RFMIN) RFMIN=SRV(JJ,II)
    IF (SRV(JJ,II).GT.RFMAX) RFMAX=SRV(JJ,II)
203 CONTINUE
    NFMIN=RFMIN*10.0
    NFMAX=RFMAX*10.0
    FLO=NFMIN
    FLO=FLO/10.
    HI=NFMAX
    HI=(HI/10.)*0.1
    WRITE(6,205) FLO,HI
205  FORMAT(//////////,10X,*MIN. VALUE=*,F6.3,3X,*MAX. VALUE=*,F6.3)
    READ(5,215) (TTL(NB),NB=1,7)
215  FORMAT(7A10)
    FINC=0.1
    NSET=0
    NDOT=0
    NHI=0
    CALL PWRT(120,990,TTL,65,1,0)
    CALL CALCNT(SRV, MX, NY, FLO, HI, FINC, NSET, NHI, NDOT)
    CALL FRAME
101 CONTINUE
    END

```

BIBLIOGRAPHY

1. Benson, M. A., "Evolution of Methods for Evaluating the Occurrence of Floods," U.S.G.S. Water Supply Paper 1580-A, 1962.
2. Betson, R. P., Tucker, R. L., and Haller, F. M., "Using Analytical Methods to Develop a Surface-Runoff Model," Water Resources Research, Vol. 5, No. 1, February 1969, 103-111.
3. Bruce, J. P., "Preliminary Estimates of Probable Maximum Precipitation over Southern Ontario," The Engineering Journal, Vol. 40, No. 7, 1957, 978-984.
4. Bruce, J. P., "Storm Rainfall Transposition and Maximization," National Research Council of Canada, Hydrology Symposium, No. 1, November 1959, 162-171.
5. Brunt, A. T., "Maximum Possible Rainfall -- Little Nerang Creek, Queensland," Australian Meteorological Magazine, No. 14, September 1956.
6. Byers, H. R., "Methods of Estimating the Maximum Flood," paper presented at the Meeting of the American Meteorological Society, Chicago, Illinois, April 1959.
7. Huff, F. A., "Time Distribution of Rainfall in Heavy Storms," Water Resources Research, Vol. 3, No. 4, Fourth Quarter 1967, 1007-1019.
8. Kohler, M. A., and Linsley, R. K., "Predicting the Runoff from Storm Rainfall," U.S. Weather Bureau Research Paper, No. 34, September 1951.
9. Kohler, M. A., and Richards, M. M., "Multicapacity Basin Accounting for Predicting Runoff from Storm Precipitation," Journal of Geophysical Research, Vol. 67, No. 13, December 1962, 5194-5196.
10. Linsley, R. K., Kohler, M. A., and Paulhus, J. L. H., Hydrology for Engineers. McGraw-Hill Book Company, New York, 1958.
11. Myers, V. A., "Criteria and Limitations for the Transposition of Large Storms over Various Size Watersheds," Symposium on Consideration of Some Aspects of Storms and Floods in Water Planning, Texas Water Development Board, Report 33, November 1966, 45-66.
12. Myers, V. A., "Meteorology of Hypothetical Flood Sequences in the Mississippi River Basin," Hydrometeorological Section, Hydrologic Service Division, Washington, D.C., Hydrometeorological Report, No. 35, December 1959.
13. Paulhus, J. L. H., and Gilman, C. S., "Evaluation of Probable Maximum Precipitation," Transactions American Geophysical Union, Vol. 34, No. 5, 1953, 701-708.
14. U.S. Army Corps of Engineers, "Flood Hydrograph Analysis and Computations," EM-1110-2-1405, August 31, 1959.
15. U.S. Army Corps of Engineers, "Generalized Standard Project Rainflood Criteria, Southern California Coastal Streams," Hydrologic Engineering Center, Sacramento, California, March 1967.
16. U.S. Army Corps of Engineers, "Standard Project Flood Determinations," Civil Engineering Bulletin, Vol. 52, No. 8, 1952.
17. U.S. Bureau of Reclamation Manual, "Water Studies," Vol. 4, 1951.
18. Yevjevich, V. M., "Misconceptions in Hydrology and Their Consequences," Water Resources Research, Vol. 4, No. 2, April 1968, 225-232.

KEY WORDS: Storms, storms description, transposition of storms, floods, flood frequencies, flood probability distributions.

ABSTRACT: Contemporary literature in hydrology usually contains the concepts of maximum probable precipitation and maximum probable flood along with methods used to arrive at these limits. These limits signify some physical upper limits for precipitation and flood, however it is difficult to find physical justification for existence of these limits and more so the methods used to compute them. Also, the use of the word 'probable' is incorrect because these 'probable limits' are not assigned any probabilities.

In view of the misconceptions that prevail in such existing concepts, this study attempts to develop a practical methodology with a theoretical framework for estimating the probability of occurrence of floods in a unit time interval,

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based on the random characteristics of storms. In general, many random characteristics can be defined for a storm, but as a first step only a three-dimensional random vector has been defined for the random characteristics of storms. The random vector is comprised of the coordinates of storm center location and storm orientation. The developed estimation methodology uses all information on historic storms observed in a region that contains the river basin.

For carrying out the estimation as required by the estimation methodology, a computer oriented technique has been developed. Application of this technique is demonstrated on the Goose Creek basin in East Central Illinois.

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