THE DRAG ON A SMOOTH FLAT PLATE WITH A FENCE

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Abstract

A method is presented which permits the determination of the drag on a smooth flat plate when the boundary layer along it is disturbed by a two-dimensional, sharp edged fence.

This method depends on the knowledge of the drag coefficient of a fence immersed in a boundary layer, and on the friction along the smooth plate in the disturbed boundary layer. The drag coefficient for the fence is calculated using arguments of free streamline theory. The friction along the smooth plate is determined approximately from experimental data.

The results are applied to experimental findings of Wieghardt (1) and satisfactory agreement was found.

List of Symbols

C	Drag coefficient of fence	Dim. [-]
C_1	Reference drag for fence in boundary layer	Dim. [-]
D	Drag per unit width	Dim. [lb_f/ft]
Q	Base pressure coefficient	Dim. [-]
a	Factor of proportionality	Dim. [-]
c	Friction factor of the plate	Dim. [-]
d	Maximum distance of separation streamline from plate	Dim. [in.]
h	Plate height	Dim. [in.]
n	Exponent in velocity distribution law	Dim. [-]
p	Pressure	Dim. [lb_f/ft^2]

List of Symbols (cont'd)

p_{Av}	Average pressure of front of fence	Dim. $[lb_f/ft^2]$
u	Velocity	Dim. [ft/sec]
u_{Av}	Average velocity over $y = h$ in undisturbed boundary layer	Dim. [ft/sec]
x_i	Distance from wind tunnel entrance to point i	Dim. [in.]
y	Vertical distance from plate	Dim. [in.]
$\theta(x_i)$	Momentum thickness at point i	Dim. [in.]
δ	Boundary layer thickness	Dim. [in.]
ρ	Air density	Dim. $[slugs/ft^3]$

subscript	a	refers to wind tunnel free stream
	b	refers to separation streamline
	f	refers to fence
	n	refers to difference between disturbed and undisturbed boundary layer
	o	refers to ideal flow case
	p	refers to plate with disturbed boundary layer
	po	refers to plate with undisturbed boundary layer
	st	refers to stagnation point

1. Introduction

If the boundary layer flow along a smooth flat plate is disturbed by a roughness element, then the total drag on the plate is increased. Most real surfaces are not entirely smooth, therefore it would be useful if the contribution of a roughness element to the drag of the plate could be calculated. This is not a simple task because the drag on a plate with a single roughness element depends on the characteristics of the boundary layer flow as well as on the geometry of the roughness element.

In this paper an analysis is presented which permits the calculation of the drag on a smooth flat plate with a sharp edged fence as roughness element. The analysis depends on experimental data which are taken to determine empirical coefficients.

The choice of a sharp edged fence offers advantages for both analytical and experimental treatment. The flow is essentially two-dimensional. Also, the fence causes the flow to separate at all but the very lowest velocities, and the separation point is always located at the top of the fence. More complex roughness shapes might have separation points whose location depends on the flow velocity. The study of drag due to such roughness elements also benefits from the knowledge gained from the investigation of the sharp edged fence, provided modifications of the arguments, as required by the shapes, are considered.

The problem of drag induced by a single obstacle has been treated by Wieghardt (1) and Tillman (3). They measured the difference D_n between the drag forces on the plate with and without an obstacle by means of a drag balance. Their results were presented as curves of a drag coefficient (per unit width) C_n , defined by

$$C_n = \frac{D_n}{1/2\, \rho\, u_{Av}^2\, h} \qquad (1)$$

where ρ is the density of the fluid and u_{Av} is the average velocity over the height h of the element, plotted against a dimensionless parameter describing the geometry of the roughness element. An example is shown in Fig. 1. Figure 1 as well as all other figures presented by Wieghardt (1), reveal that the body geometry alone is not sufficient to specify the value of C_n , because the drag coefficient changes with the boundary layer thickness δ . Therefore, the results of Wieghardt and Tillman cannot be applied to flow conditions other than the ones used for their experiments. The effects of two dimensional roughness elements on boundary layer flows have been studied, (e.g. (4)), but no further attempts had been made to improve Wieghardt's analysis.

The method presented in this paper is designed to overcome the shortcomings of Wieghardt's empirical analysis by allowing for a variable boundary layer thickness, and a variable mean velocity. The experimental data of Wieghardt are used to check the validity of the proposed method.

2. Evaluation of Drag

The contribution of the fence to the total drag of the plate consists of two parts. The first part is the drag which is caused by the fence directly. This is the fence drag, D_f , which is, for the fence considered in this paper, almost identical with the resultant of the pressures on the fence. The second part results from the viscous shear force on the plate itself. This part is called the plate drag, D_p , and is quite distinct from the drag on the plate existing in the undisturbed boundary layer, (since it reflects the changes of the flow field caused by the fence). The increase in drag D_n which is caused by the fence can be written formally as:

$$D_n = D_f + D_p - D_{po} \tag{2}$$

where D_{po} denotes the drag on the plate with undisturbed boundary layer. The quantity D_{po} can be calculated, for a boundary layer without pressure gradient from well established techniques (Schlichting (3)). No equivalent information exists for the drag on a fence D_f or for the drag D_p on the plate with fence. These two quantities must be determined by experiment.

To determine D_n , Eq. (2), it is advantageous to sub-divide the distorted boundary layer into five regions as shown in Fig. 2. The flow in each zone can be described qualitatively by using results obtained by Rouse et al (5, 6), by the observations of Nagabhushanaiah (7) and by the results of the present study. In region 1, the boundary layer flow is not influenced by the presence of the fence, and considerations pertinent to boundary layers along smooth flat plates apply. The fence starts to change the flow in zone 2. The motion near the plate is retarded, causing an increase in pressure. A maximum pressure is attained at the upstream face of the fence. At the edge of the fence the flow is strongly accelerated and separates from the edge. At the point of separation there is a decrease in pressure so that in region 3 the pressure in the boundary layer is below ambient. The low pressure along the separating streamline (or rather in the narrow zone of intense turbulence that contains portions of the fluid which come from the fence edge) is transmitted downward to the plate, since the slow eddying motion below the separating streamline can only sustain very small vertical pressure gradients. Along the separating streamline the pressure gradually increases and the velocity decreases. The resulting pressure gradient across the separating streamline causes a deflection of the streamline towards the plate, and at some distance downstream from the fence the separation streamline re-attaches to the plate. The re-attachment point marks the beginning of zone 4, the zone of redevelopment of the boundary layer. Ultimately, in zone 5, the boundary layer again exhibits the features of the undisturbed boundary layer of zone 1, but with a different boundary layer thickness, reflecting the effect of the fence.

The experimental data of Wieghardt (1) indicate that the difference in drag between the disturbed and the undisturbed boundary layer reaches a constant value, independent of the length of the drag plate, after the length exceeds a certain value. This implies that the friction coefficient of the redeveloped boundary layer at any given point downstream from the fence has the same value as that of the undisturbed boundary layer at the same point.

Therefore, the solution of Eq. (2) requires only the calculations of the drag contributions in the zones 2, 3 and 4. Zones 1 and 5 do not contribute to the increase in drag due to the fence. Contributions D_p and D_{po} are henceforth understood as pertaining only to zones 2 to 4.

In the subsequent sections, details of the solution for Eq. (2) are described. After a short description of the experimental equipment, the problem of the drag D_f on a fence submerged in a boundary layer is treated. In section 4, the drag D_p on the plate is evaluated from experimental data. The results are combined in section 5 to give a solution of Eq. (2) for the sharp edged fence, and the final result is compared with the experimental data of Wieghardt (1) shown in Fig. 1.

3. Experimental Equipment and Procedure

All experimental data were obtained in the large wind tunnel, located at Colorado State University. This tunnel has a cross sectional area of 6 x 6 ft^2 and a test section length of 88 ft. The ceiling of the tunnel was adjusted so that the pressure gradient in the tunnel test section was zero without fences. With fences, a pressure drop of less than 2 percent of the stagnation pressure developed at the ceiling. The fences consisted of steel plates with a machined sharp edge. Fence heights of 0.5, 1, 1.5 and 2 inches were used, with velocities from 11 to 72 fps. Pressure taps with a hole of 1/16 in. diameter were provided at 1/4 in. intervals over the height of the plates. The ceiling had piezometer openings every 8 ft along the center line. The velocities were determined with a pitot-static tube. The pressures were measured with an electronic manometer (Transonics Equibar Type 120).

The drag coefficients of the fences were measured with plate heights of 1, 1.5 and 2 in. The fences were fastened to the wind tunnel floor at different distances from the tunnel entrance. For measuring the velocity profiles in the zone of redevelopment, four fence heights were used. The fences were installed at a distance of 41 ft downstream from the test section entrance. For this part of the study only velocities of 14 and 20 fps were used.

4. The Drag Coefficient of a Fence Submerged in a Boundary Layer

The separation of the flow at the edge of the fence is the main factor that determines the drag on the fence. In this respect the flow about a fence submerged in a boundary layer is similar to the motion around a plate which is placed perpendicular to a uniform flow. It is reasonable to expect that the considerations which permit the evaluation of the drag on the plate in the case of the free stream can also be applied to the drag on a fence in the boundary layer. The determination of the drag of the fence in a boundary layer shall therefore be based on results found for the plate in the free stream.

The approach to a determination of the drag on a plate in a free stream is based on ideal fluid models. Ideal fluid models for the flow around a plate are shown in Fig. 3. It is well known that the model of Fig. 3a yields a drag of zero. That is, the drag coefficient C_f is 0, where C_f is defined by the equation:

$$C_f = \frac{D_f}{1/2\, \rho\, u_o^2\, h} \tag{3}$$

In this expression D_f is the drag per unit width on the fence and h is the fence height. The velocity u_o is the velocity at infinity. The free streamline model of Kirchhoff and Helmholtz (e.g. Birkhoff (8)) in Fig. 3b gives a drag coefficient $C_o = 0.88$. This value is considerably smaller than the drag coefficient for viscous flow around the fence in the free stream of the wind tunnel. For this case, Fage and Johansen (9) found a value of about 2.0. Instead of u_o they used the velocity u_a of the undisturbed stream in the core of the wind tunnel as reference velocity.

The difference between Kirchhoff's model and experiments with real fluids is mainly the result of the drop in pressure behind the wall. Kirchhoff considered the pressure constant along the plate (the axis of symmetry of the ideal flow model) and equal to the pressure at infinity. This assumption is not verified by experiments with real fluids. Therefore, in later analyses, a variable pressure p_b behind the wall was introduced which is expressed by the pressure coefficient Q :

$$Q = \frac{p_o - p_b}{1/2\, \rho\, u_o^2} \tag{4}$$

The pressure p_b has to be determined from other than free streamline considerations. Models which extend Kirchhoff's results to flow with variable downstream pressures have been proposed by Riabouchinski (cited in (8)), Gilbarg (cited in (8)) and Roshko (10). Roshko's model appears to be the most realistic one for the air flow considered here. It is therefore included in Fig. 3c. This model assumes a wake pressure which is constant over a short distance behind the wall, and then gradually returns to the free stream pressure. The details of the drag calculations for this flow are discussed in Roshko's paper (12). For the case of the fence in a boundary layer, it is sufficient to know that the dependency of the drag coefficient on the pressure directly behind the plate in the free stream (or "base pressure"), can be expressed by the relation (given by Birkhoff (8)).

$$C_f = C_o (1 + Q) \tag{5}$$

where C_o is the Kirchhoff drag coefficient, which equals 0.88.

The meaning of Eq. (5) is apparent from consideration of Bernoulli's Equation along the separation streamline. For the separation streamline,

$$p_o + \frac{1}{2} \rho u_o^2 = p_b + \frac{1}{2} \rho u_b^2 \ . \tag{6}$$

It follows that

$$Q + 1 = \frac{u_b^2}{u_o^2}$$

and

$$C_o = \frac{D_f}{1/2 \rho u_b^2 h} \ . \tag{7}$$

Equation (6) and (7) imply that the drag coefficient in the ideal flow case remains constant if the velocity u_b along the streamline, rather than the velocity at infinity, is chosen as the reference velocity for the calculation of the drag coefficient.

The usefulness of this conclusion extends even further. Valcovici (cited in Birkhoff (8)) shows that if the velocity u_b is applied as a reference for the case of an ideal flow bounded by parallel walls (shown in Fig. 3d), then the drag coefficient deviates insignificantly from the value C_o . Hence, the use of u_b and C_o as computed from Eq. (6) serves the dual function of correcting for wall effects and for base pressure variations provided, of course, that viscous effects around the fence in the wind tunnel only influence the base pressure p_b . The wind tunnel data of Fage and Johansen (9) and Arie and Rouse (6) shown in Fig. 4 indicate that this is the case. The agreement of their experimental data with Eq. (5) (as well as with Roshko's calculated relation) is good. Equation (5) therefore applies to wakes where Karman vortices can form, and for wakes where the generation of periodic eddies is prevented artificially by a solid plane along the axis of symmetry of the wake. Thus, the drag coefficient for real flows in the free airstream of a wind tunnel can be determined from Eq. (5) if the base pressure coefficient Q is known. Unfortunately no methods other than experiments are available to calculate Q .

Equation (5) does not apply directly to experimental data if the fence is submerged in a boundary layer. A graph showing the dependence of our experimental values of C_f on Q is shown in Fig. 4. The experimental points do not fall on the straight line given by Eq. (5). The deviation is explained by considering Eq. (6). Bernoulli's Equation in the form of Eq. (6) does not apply in a boundary layer. The separation streamline is identical to the stagnation streamline. For the fence submerged in the boundary layer, the stagnation pressure on the fence is not equal to the stagnation pressure in the free stream because, in the boundary layer, the Bernoulli sum varies from streamline to streamline.

The present experiments and those of Nagabhushanaiah (7) show that the pressure distribution on the upstream face of the fence has a maximum value, which corresponds to the stagnation pressure. The location of the stagnation point depends on the degree of submergence of the fence in the boundary layer. No experimental data exist which permit an evaluation of the location of the stagnation point. Experimental data of Nagabhushanaiah (7) indicate that for a given free stream velocity the distance of the stagnation point from the plate is proportional to the fence height provided that $h/\delta > 0.1$. The factor of proportionality for u_a = 9 to 12 fps lies between 0.6 and 0.7. The present experiments confirm these results. However, it was found that the factor of proportionality depended qualitatively on the velocity of the ambient airstream. Experiments showed that the factor of proportionality decreased with decreasing velocity.

A knowledge of the location of the stagnation point is of value only if it is possible to infer from it the magnitude of the stagnation pressure. Since this is not feasible, no attempts were made to determine the location exactly. However, since the distance from the horizontal plate to the point of maximum pressure is proportional to the fence height, the distance from the plate to the corresponding streamline at the end of the zone 1 is expressed also as the proportion, a, of the fence height h.

In the undisturbed boundary layer, the velocity distribution can be expressed by a power law of the form:

$$\frac{u}{u_a} = \left(\frac{y}{\delta}\right)^{1/n} . \tag{8}$$

This power law relationship was used also by Wieghardt (1) with $n = 7$. Furthermore, the pressure in the boundary layer is approximately equal to the ambient pressure p_a. Consequently, the velocity along the streamline which becomes the stagnation streamline near the fence, is given by Eq. (8) with $y = ah$. Hence, for the fence submerged in the boundary layer the relation which is equivalent to Eq. (6) is:

$$p_a + \frac{1}{2} \rho u_a^2 \left(\frac{ah}{\delta}\right)^{2/n} = p_b + \frac{1}{2} \rho u_b^2 ,$$

and

$$\left(\frac{u_b}{u_a}\right)^2 = Q + \left(\frac{ah}{\delta}\right)^{2/n} . \tag{9}$$

In analogy to Eq. (5), the expression for C_f becomes:

$$C_f = C_1 \left| \left(\frac{ah}{\delta}\right)^{2/n} + Q \right| \tag{10}$$

In this equation, a is a factor which is smaller than one, and C_1 is a reference drag coefficient, which should be independent of velocity and of Q.

The arguments leading to Eq. (10) depend on the assumption that a streamline is defined along which the contribution of the turbulence to the Bernoulli Equation can be neglected. Since the turbulence in the flow upstream of the fence has a much lower intensity than that near the separation point, along the separation streamline, this hypothesis might be questioned. However, along the separation streamline, the total turbulent energy,(i.e., energy influx by convection and generation and energy efflux by convection and dissipation into heat) is approximately zero. This was shown by Fage and Johansen (9) who found that Eq. (6) was valid for their data taken for a plate in the free stream of the wind tunnel. This was also confirmed by Rouse (5) for the case of a plate with splitter plate in the free stream. The reason for the agreement lies in the fact that above the separation streamline, energy is lost in the region of high turbulent shear created by the separation, while underneath the separation streamline the standing eddy requires continuous addition of energy to keep it moving. The energy can only be supplied across the separation streamline. And since there is no discontinuity in vertical energy distribution, a line must exist between the standing eddy and the high shear zone along which energy is neither gained nor lost. This line must be very close to, or identical to the separation streamline. Therefore, Eq. (10) can be accepted with confidence, provided that the numerical values of C_1 , a , and of Q are known.

The value of C_1 reflects the non-uniformity of the pressure distribution over the fence. This becomes clear if Eq. (3) is interpreted as the ratio of the average pressure p_{Av} to the maximum resultant pressure, i.e., the stagnation pressure p_{st} . Similarly, C_o in Eq. (5) can be defined by

$$C_o = \frac{p_{Av} - p_b}{p_{st} - p_b} \tag{11}$$

The same equation, with C_o replaced by C_1 , expresses the reference drag of Eq. (10). The experimental distribution of pressures on a fence in a free stream show a shape similar to the theoretical distribution of Kirchhoff, and a value of $C_o = 0.88$ is reasonable. This is expressed in the curve of Fig. 4 which shows the agreement of Eq. (5) with measurements. For the fence submerged in the boundary layer, the pressure distribution has a maximum near the top of the fence rather than on the bottom and the equality, $C_1 = C_o$ would be coincidental. Instead, it has been found for all experimental data that $C_1 = 0.95$, with a variation of no more than $\pm$ 0.02. Here, C_1 was calculated using Eq. (11).

The value of a could be determined either from the measured values of $p_{st} - p_a$ or from Eq. (10) by using experimental values of Q. However, a difficulty arises in defining Q because of the difficulties in determining the reference pressure p_a. If the wind tunnel is adjusted so that for a given velocity the pressure gradient along the test section without a fence is zero, then a pressure drop is caused by the fence, and p_a is different upstream and downstream from the fence. Arie and Rouse (6) avoided the difficulty of correcting for the pressure drop by introducing a false ceiling which produced a pressure gradient of zero with the obstruction installed. Neither Nagabhushanaiah (7) nor the present data used such corrective measures. Instead, p_a was taken as the static pressure in the upstream part of the wind tunnel test section where the pressure gradient was still zero. The value of Q which was obtained with this p_a has been used for Fig. 4.

From Fig. 4, an average relationship between C_D and Q is obtained by drawing a straight line through the data points. The equation of the straight line is:

$$C_f = 1.60\,Q \quad . \tag{12}$$

Equation (12) has a counterpart in the geometry of the wake. Birkhoff (8) quotes an analysis of Reichardt, who assumed that energy extracted from the flow by the fence drag is used to expand the wake to a maximum width, d. This led to an approximate relation

$$\frac{d}{h} = \frac{C_f}{Q} \tag{13}$$

which was found to agree with experiments for low values of Q . Nagabhushanaiah (7) has demonstrated with his data the dependency of d on h . His results are reproduced in Fig. 5. It is seen that for h/δ values corresponding to $Q < 0.5$, a relationship

$$d = 1.60\,h \tag{14}$$

is a good approximation to the data. For values of Q larger than 0.5, Eqs. (12), (13) and (14) cease to apply.

A solution of C_f in terms of known parameters, and at the same time a numerical value for a can be found by inserting Eq. (12) into Eq. (10). If $C_1 = 0.95$ is used, rearrangement yields:

$$C_f = 0.235 \left(\frac{ah}{\delta}\right)^{2/n} . \tag{15}$$

The coefficient a is found by plotting $\left(\frac{ah}{\delta}\right)^{2/n}$ versus the measured C_f . The boundary layer thickness δ was determined analytically from the law of the smooth flat plate for velocity distributions which obey the power law Eq. (8) with $n = 7$ (Schlichting (2) p. 537). The result is shown in Fig. 6. The experimental data are well represented by the relation:

$$C_f = 1.05 \left(\frac{h}{\delta}\right)^{2/7} . \tag{16}$$

Comparison of Eqs. (15) and (16) shows that $a = 0.06$. Also, it is seen that Eq. (16) is valid for all experimental data, i.e., it is valid over a wider range than Eq. (12), and applies also to values of $h/\delta < 0.1$ below which a constant value of a could not have been expected. The velocities range from 11 to 72 fps, and no systematic deviation from Eq. (16) is observed with change in velocity.

Equation (16) is a simple result obtained from arguments based on results of free streamline theory and experimental observations. It is valid only for the sharp edged fence. However, the basic reasoning should apply to other shapes provided that modifications are introduced which account for possible shifts of the separation point.

5. The Wall Friction of the Disturbed Boundary Layer

Mean velocity data were used to evaluate the distribution of the wall friction coefficient in a boundary layer distorted by a fence. The mean velocity profiles were taken in the course of an investigation on diffusion in the reattached boundary layer (12). The low test velocity of 14 fps gave a flow which was not entirely stable in the reattached region. On different days, slightly different profiles were measured at the same distance downstream from the fence. The data, therefore, exhibit considerable scatter. However, they will suffice for the estimate of the difference in wall friction between the disturbed and undisturbed boundary layer.

No experimental values of the friction coefficient in the standing eddy zone downstream of the fence are available. In this zone the velocity near the plate is directed upstream. Hence, the contribution to the plate drag is negative. On the other hand, downstream from the point where the separation streamline reattaches to the floor, the velocity is directed downstream, and a positive contribution to the drag results. Thus, there exists at some distance downstream from the fence a point x_2 at which the total drag between it and the end point x_1 of the undisturbed boundary layer of zone 1 is exactly equal to the drag of the fence D_f. Consequently, only the plate drag downstream from x_2 contributes to the total drag of the plate with a fence submerged in the boundary layer.

In view of the fact that only zones 2, 3 and 4 of the disturbed boundary layer contribute to the increase in drag, a point x_3 can be defined which marks the end of zone 4, or more precisely, the end of the part of zone 4 where the plate friction coefficient c_p is different from the friction coefficient for the undisturbed boundary layer, c_{po}.

It will now be shown that the distance $x_1 - x_2$ is proportional to the fence height h and that points x_2 and x_3 coincide approximately. For this purpose, the drag of the plate is evaluated from measured vertical velocity distributions by means of the momentum equation (Schlichting (2) p. 160). It is assumed that no net pressure force exists. Then one obtains:

$$D_f + D_p(x) = \rho u_a^2 \quad \theta(x) - \theta(x_1) \tag{17}$$

where

$$D_p(x) = \int_{x_1}^{x} C_p \, dx \, \frac{1}{2} \rho u_a^2 \tag{18}$$

is the friction drag on the plate between points x_1 and x, and $\theta(x)$ and $\theta(x_1)$ are the momentum thicknesses at x and x_1 respectively, where the momentum thickness is defined by

$$\theta(x) = \int_{o}^{\delta} \frac{u}{u_a} \left(1 - \frac{u}{u_a}\right) dy \quad .$$

Here, u is the local velocity parallel to the plate. Combining Eqs. (3), (with $u_o = u_a$), (17), and (18), it is found that:

$$\theta(x) - \theta(x_1) = \frac{1}{2} C_f \cdot h + \frac{1}{2} \int_{x_1}^{x} c_p \, dx \ . \tag{19}$$

Equation (19) can be used for determining x_2 and x_3. By definition, at x_2

$$\theta(x_2) = \frac{1}{2} C_f h = \theta(x_1) \quad . \tag{20}$$

Experimental data were used for finding x_2. For all data, the difference $\theta(x) - \frac{1}{2} C_f h$ was determined and plotted vs. x/h in Fig. 6. For the computations, the drag coefficient was obtained from Eq. (16). The plot shows considerable scatter. But it can be inferred that the distance x_2 is at about 30 to 35 h downstream from the fence where Eq. (20) is satisfied.

In Eq. (19) the effect of the pressure gradient was neglected. There is no doubt that this effect causes a large error near the reattachment point. Therefore, near the reattachment point (which is, according to experimental data of Nagabhushanaiah (7), located at a distance of 12.5 h downstream from the fence regardless of velocity and fence height), the calculated momentum thickness is much lower than expected from neglecting only the wall friction in the standing eddy zone. However, experimental results indicate that at 35 h , the pressure gradient is essentially zero again. For the calculation of the difference in drag, we assume that $x_2 = 35\ h$, regardless of the velocity, and any effects of a pressure drop are negligible.

It remains to determine the friction coefficients for a distance larger than 35 h downstream from the fence. Several methods are available for determining the friction coefficient. Karman's momentum equation, for zero pressure gradient flow along a flat plate, holds only where the boundary layer approximations are valid, that is, in the region far downstream from the point of reattachment. In the zone of redevelopment of the boundary layer, an average value of $c_p = 0.0025$ was found from the slope of the plot of momentum thickness vs. distance.

A second method starts with the assumption that the "inner law" holds for the velocity distribution near the wall. This method is difficult to apply near the reattachment point, but further downstream it gave an approximately constant friction coefficient of 0.0025.

The third method consists of the use of the Ludwieg-Tillman Equation (14), which has been used successfully to determine friction coefficients near separation and in pressure gradient flows (see for example Sandborn and Kline (15)). The results of the calculations are shown in Fig. 7. This figure shows that the friction coefficient increases rapidly downstream from the point of reattachment of the separation streamline. By $\frac{x}{h} = 35$ a value of c_p approximately equal to the final value of $c_p = c_{po} = 0.0025$ is obtained. This implies that $x_2 = x_3$. Hence, with Eqs. (2) and (3) the final result is:

$$\frac{D_n + D_{po}}{\frac{1}{2}\rho\, u_a^2\, h} = C_f \tag{21}$$

In equation (21) $D_{po} = \int_{x_1}^{x_1 + 35\ h} c_{po}(x)\, dx \cdot \frac{1}{2}\rho\, u_a^2$, where $c_{po}(x)$ = friction coefficient for the undisturbed boundary layer. One obtains with Eq. (16)

$$\frac{D_n}{\frac{1}{2}\rho u_a^2 h} = 1.05 \left(\frac{h}{\delta}\right)^{2/7} - \frac{1}{h}\int_{x_1}^{x_1+35h} c_{po}\, dx \qquad (22)$$

In Eq. (22), c_{po} and δ pertain to the undisturbed boundary layer. The ambient velocity u_a as well as the fence height h are known quantities. Therefore, the additional drag due to a fence in a boundary layer can, in the first approximation, be calculated from Eq. (22).

6. Calculation of the Drag on a Fence in a Boundary Layer

A check of Eq. (22) against directly measured drag data can be obtained from the data of Wieghardt (1) which are reproduced in Fig. 1. In Fig. 1, a sharp edged fence corresponds to $t/h = 0$. The average velocity u_{Av} of Wieghardt was based on a 1/7 power law for the velocity distribution. It is given by

$$u_{Av}^2 = u_a^2 \, \frac{7}{9} \left(\frac{h}{\delta}\right)^{2/7}$$

Hence, one obtains from Eq. (22) and Eq. (1):

$$C_n = 1.35 - \frac{1.29}{h} \left(\frac{\delta}{h}\right)^{2/7} \int_{x_1}^{x_1+35h} c_{po}\, dx$$

For Wieghardt's data, c_{po} is found to be approximately constant and equal to 0.003. Thus,

$$C_n = 1.35 - 0.135 \left(\frac{\delta}{h}\right)^{2/7} .$$

Only the fence heights of Wieghardt of 22 mm and of 53 mm can be used, since the validity of Eq. (22) for $h/\delta > 1$ is not established. Then, for $h = 53$ mm C_n is calculated to 1.20 and for $h = 22$ mm C_n becomes 1.16. These values are in agreement with Wieghardt's results, as indicated in Fig. 1.

7. Conclusions

1. The determination of the drag coefficient of a fence sumberged in a

boundary layer can be reduced to the problem of determining the base pressure behind the fence. The base pressure is found by referring the drag coefficient to a theoretical velocity along the separation streamline downstream from the fence. This velocity can be computed from Bernoulli's Equation by assuming that the energy along the separation streamline is equal to the energy of a streamline in the undisturbed boundary layer located at a distance ah from the wall. The coefficient a was found to be approximately equal to 0.06.

2. The wall friction on the smooth flat plate downstream from the fence consists of a negative part in the standing eddy zone and a positive part downstream from the reattachment point of the separation streamline. As shown by the experimental data, these two contributions cancel approximately over a distance of 35 h . Further downstream, the friction coefficient is approximately equal to that found at the same point for the undisturbed boundary layer.

3. The results for wall friction and fence drag can be used to predict the added drag which a plate suffers when a two dimensional fence obstructs the boundary layer. It is believed that the method can be extended to other shapes of obstructions if appropriate empirical relations are found for the drag coefficient and the equilibrium length for skin friction.

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