EVAPORATION FROM A PLANE BOUNDARY

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Evaporation is a phenomenon which is of interest to a considerable variety of people. The hydrologist wishes to know the amount of evaporation that will occur from a reservoir; the irrigation engineer needs to be able to predict the evaporation to be expected from reservoirs, ponds, canals, and the soil; the oceanographer must make estimates of the amount of evaporation from the vast surface of the ocean; the meteorologist studies the influence of evaporation from land and sea upon the movement of the atmosphere; and the chemical engineer deals with evaporation in many industrial processes. The needs of these key professions overlap from one field to another, but cover in all an extremely broad area of interest.

Although scientists in each of these fields have contributed to the information available on the subject, considerable difficulty is encountered in making accurate estimates of evaporation for design purposes. The purpose of this paper, therefore, is to present a theoretical analysis of evaporation from a plane boundary together with data taken under both laminar and turbulent conditions. Finally, a generalized analysis is presented which applied to both laminar and turbulent flow and involves only those variables which may be measured either in the field or the laboratory.

Fundamentals of Diffusion

From the kinetic theory of gases it is understood that molecular diffusion involves a random movement of molecules traveling at an extremely high speed. Because of the random nature of the motion, any individual molecule will eventually migrate to a new region, and, since countless molecules are involved, a general diffusion of a given group of molecules will necessarily result. The kinetic energy of the molecules is reflected in the temperature of the fluid. Although the mean velocity of the molecules is of a tremendous magnitude, the mean free path is so extremely short that diffusion by molecular activity alone is a microscopic process. The molar diffusion process, however, involves the turbulent movement of finite fluid masses, each composed of millions of molecules. Like a molecule, a single fluid mass will eventually migrate to a new region. Unlike the molecule, on the other hand, the fluid mass does not retain its original identity, but gradually blends with its new surroundings as it imparts its motion to

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other fluid masses. The general pattern, however, is again completely random, and diffusion on a macroscopic scale is the net result.

The basic definition of the diffusion coefficient D is

$$N = -D \frac{\partial n}{\partial y} \tag{1}$$

where N is the rate of diffusion in the y-direction of the quantity which has the concentration n at any point y. In view of the foregoing discussion it is understandable that the diffusion coefficient should indicate the relative effectiveness of the diffusion process, whether it results from molecular activity alone or from a combination of molecular and molar action. For this reason D in general is the total diffusion coefficient and must represent the sum of the molecular and molar diffusion coefficients ν and ϵ. Hence $D = \nu + \epsilon$. These two diffusion processes, molecular and molar, may operate singly or in combination, depending upon the type of fluid activity. If the motion is laminar ν will predominate, with $\epsilon = 0$. As soon as the flow becomes turbulent, however, the molar diffusion coefficient ϵ predominates, while the molecular diffusion coefficient ν rapidly becomes insignificant with increasing turbulence. Being a very general relationship, eq 1 may be applied to the diffusion of such quantities as momentum, heat, and material.

The general equation of diffusion is $\hspace{2cm}$ (2)

$$\frac{\partial n}{\partial t} + u \frac{\partial n}{\partial x} + V \frac{\partial n}{\partial y} = \frac{\partial}{\partial x}\left(D_x \frac{\partial n}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial n}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial n}{\partial z}\right)$$

where t is time, u and v are the velocity in the x and y directions and D_x, D_y, and D are respectively the diffusions in the x, y, and z directions. This equation applies to diffusion by molecular action, or a combination of both. When the action is entirely molecular, the velocities u and v are zero and the diffusion coefficients are all equal to ν. Hence,

$$\frac{\partial n}{\partial t} = \nabla^2 n \tag{3}$$

the solution of which may be found in the heat transfer literature. However, when the diffusion is steady, primarily molar, and restricted to a zone near a boundary $\frac{\partial n}{\partial t}$, $\frac{\partial}{\partial x}\left(D_x \frac{\partial n}{\partial x}\right)$, and $\frac{\partial}{\partial z}\left(D_z \frac{\partial n}{\partial z}\right)$ are all equal to zero and the resulting relationship

$$u \frac{\partial n}{\partial x} + V \frac{\partial n}{\partial y} = \frac{\partial}{\partial y}\left(D_y \frac{\partial n}{\partial y}\right) \tag{4}$$

is the boundary layer equation with which research workers
have devised extensions of Reynolds analogy.

The diffusion coefficient has been determined from
measurements of velocity profile. In this paper no attempt
is made to solve Eq 2 but rather a dimensional analysis is
made of the problem together with an experimental determin-
ation of evaporation equations for both laminar and turbulent
flow.

Evaporation Equations

If a gas stream is moving over a boundary (from which
there is evaporation), see Fig. 1, there is developed a bound-
ary layer of momentum diffusion having the thickness δ
which is defined as that region appreciably influenced by the
boundary resistance. The velocity distribution within the
boundary layer, a result of momentum interchange, has been
given considerable study by Prandtl, Karman, and others, re-
sulting in a rather complete theoretical and experimental
treatment. At the boundary the velocity of the gas is zero
and as the distance from the boundary y increases, the vel-
ocity u will increase -- gradually approaching as a limit
the magnitude of the ambient velocity U.

In addition to a boundary layer of momentum diffusion,
there is developed an evaporation layer of vapor diffusion
having the thickness λ. Furthermore, at the boundary the
vapor concentration C_s corresponds to the vapor pressure of
the water at the surface and decreases with increasing dis-
tance from the boundary -- gradually approaching as a limit
the ambient vapor concentration. In view of this considera-
tion, eq 1 may be evaluated for evaporation at the boundary as

$$\mathcal{E}_o = - \nu_e \left(\frac{\partial c}{\partial y} \right)_{y=0} \tag{5}$$

where $\mathcal{E}_o$ is the rate of evaporation perunit area at a given
point, ν_e is the molecular coefficient of diffusion of water
vapor into air, and C is the vapor concentration at the
point y away from the boundary.

An analysis of the variables involved in vapor diffusion
reveals that the total rate of evaporation E from a boundary
over a distance x' depends upon the vapor concentration
ΔC_s at the water surface relative to the ambient concentra-
tion, the ambient velocity U, the kinematic viscosity
of the gas, the diffusivity ν_e of the liquid vapor in the
ambient gas, the distance x from the upstream edge of the

boundary layer, the distance x' from the upstream edge of the evaporation layer, and the surface roughness k. Expressed as a functional relationship this becomes

$$E = \phi_1 \left(\Delta C_s, U, x, x', \nu_f, \nu_e, k \right) \tag{6}$$

which may be expressed in dimensionless form as

$$C_e = \frac{E}{\Delta C_s U x'} = \phi_2 \left(\frac{x'}{x}, S, \sigma, \frac{k}{x'} \right) \tag{7}$$

where C_e is the evaporation coefficient, $S = \dfrac{U x'}{\nu_e}$ is the Peclet number and $\sigma = \dfrac{\nu_f}{\nu_e}$ is the Prandtl number. Although Eq 7 gives considerable insight into the factors involved in evaporation, further information is needed to establish a more tangible relationship.

Because of the similarity between the mechanisms of vapor transfer and momentum transfer, the assumption may be made that C_e is analogous to C_f the drag coefficient (3). Among other things, this analogy is based on the fact that the total amount of evaporation from a surface is equal to the change in vapor flux which takes place in the evaporation layer adjacent to the surface. From the analogy the assumption can be made that the vapor concentration at any point has a power of logarithmic variation with the normal distance from the surface. Based upon this assumption, C_e will have the following forms for laminar flow, turbulent flow (power law), and turbulent flow (log law) respectively.

$$C_e = S^{\frac{m_1}{2}} \phi_3 \left(\frac{x'}{x}, S, \sigma, \frac{k}{x'} \right) \qquad \text{Laminar} \tag{8}$$

$$C_e = S^{\frac{m_3}{5}} \phi_4 \qquad \begin{array}{l}\text{Turbulent} \\ \text{(Power-law)}\end{array} \tag{9}$$

$$\frac{1}{\sqrt{C_e}} = A_1 + \left(\log S C_e \right) \phi_5 \qquad \begin{array}{l}\text{Turbulent} \\ \text{(log-law)}\end{array} \tag{10}$$

where A_1, the ϕ-values, and the m-values for smooth boundaries will depend predominently upon x'/x and the Prandtl number σ -- the effect of the latter being negligible when σ is close to unity. Furthermore, when x'/x becomes unity, the m-values should also become unity if the assumption of analogy is valid.

If the analogy of the mechanisms of vapor transfer and momentum transfer is carried further, additional deductions can be made by utilizing theoretical or experimental results

of previous investigations. Assuming $\sigma = 0.6$, eqs 8, 9, and 10 become

$$C_e = 0.722 \ S^{\frac{m_1}{2}} \ \phi_6 \qquad\qquad \text{Laminar} \quad (11)$$

$$C_e = 0.057 \ S^{\frac{m_2}{5}} \ \phi_7 \qquad\qquad \begin{array}{l}\text{Turbulent} \quad (12) \\ \text{(Power-law)}\end{array}$$

$$\frac{1}{\sqrt{C_e}} = 3.9 \ A_2 + 5 \left(\log S C_e \right) \phi_8 \qquad\qquad \begin{array}{l}\text{Turbulent} \quad (13) \\ \text{(Log-law)}\end{array}$$

The coefficient 0.722 in Eq 11 is obtained from the theoretical solution of Pohlhausen (2) the coefficients in Eqs 12 and 13 are based on the conversion $C_e = \frac{C_f}{2} \ \sigma^{-\frac{2}{3}}$ obtained experimentally by Kraussold (1). Before Eqs 11, 12, and 13 can be utilized for design purposes, however, A_2 , the m-values, and the functions (as well as the validity of the numerical coefficients) must be established experimentally.

Although Eqs 11, 12, and 13 illustrate that the formation of the evaporation layer is very similar to the formation of the boundary layer, their application is restricted to particular problems in which C_e and S can be either computed or estimated with reasonable accuracy. Such situations are no doubt prevalent in many industrial processes, but when dealing with problems of evaporation into the atmosphere it is seldom that an ambient wind velocity can be determined, because the boundary layer may be hundreds of feet thick and the velocity distribution at some high level may be much more indicative of conditions in the upper air than at the earth's surface. In other words, Eqs 11, 12, and 13 can be used for predicting evaporation from a plane boundary of finite length x surrounded by an ambient velocity U , but it is not possible to determine the evaporation from a surface such as a lake where the length of approach is infinite and the velocity varies indefinitely with distance above the boundary. However, because the variables x and U (together with ν_t) are important primarily because they describe the boundary layer approaching the lake, it is possible to substitute the shear velocity $\sqrt{\tau_o / \rho}$ and the thickness of the laminar sublayer δ' for U and x in Eq 7.

Furthermore, the laminar sublayer thickness depends upon the shear velocity and the kinematic viscosity ν_f so that the following equation may be written

$$E = \phi_9 \left(\sqrt{\tau_o/\rho}, \Delta C_s, \nu_f, \nu_e, x', k \right) \tag{14}$$

and dimensional analysis and a slight rearrangement gives

$$\frac{E}{\nu_e \Delta C_s} = \phi_{10} \left(\frac{x' \sqrt{\tau_o/\rho}}{\nu_e}, \frac{\nu_f}{\nu_e}, \frac{k}{\delta} \right) \tag{15}$$

or

$$Nu' = \phi_{11} \left(B, \sigma, \frac{k}{\delta'} \right) \tag{16}$$

where Nu' is the Nusselts number for evaporation of water into air.

Experimental Equipment

Velocity distribution and evaporation data were taken for both laminar and turbulent boundary layers, x was varied from 12 in. to 60 in., x' was varied from $\frac{1}{2}$ in. to 48 in., the wind velocity was varied from 1.0 fps to 25 fps, and the ambient relative humidity was observed. The velocity was measured with a hot-wire anemometer and the evaporation was measured in a battery of serological pipettes which were connected by rubber tubing to the porous porcelain evaporating surfaces which were 2 in. wide. On each side of the central evaporating strip was a buffer strip 2 in. wide to reduce the influence of lateral diffusion. To determine the temperature of the water surface, number 40 copper-constant thermocouples were imbedded in the evaporating surface. The humidity was determined with wet and dry thermocouples. The boundary itself was 23 in. wide by 70 in. long with a lucite nosepiece tapered over its 12-in. width to a sharp edge. To prevent lateral flow of air across the boundary, 6-in. vertical shields were fastened to each side of it. Down the center of the boundary were placed the evaporating strips. In order to create a turbulent boundary layer from the start, a removable $\frac{1}{4}$-in. tape was fastened at right angles to the flow 2 in. upstream from the hydrodynamical leading edge.

Analysis of Results

Measurements of the velocity distribution established the validity of the Blasius relationship for the laminar boundary layer, and the logarithmic law for the turbulent boundary layer. Measurements of evaporation were made for both the

laminar and turbulent boundary layers throughout a hundred-
fold range in the value of S, and more than a ten-fold
range in that of x'/x. The magnitudes of x'/x were pur-
posely chosen in an approximate geometric progression to
facilitate the analysis of the data. By plotting the experi-
mental data as C_e versus S it was possible to determine
empirically the unknown functions in Eqs 8 and 10.

For the laminar boundary layer it was found that $\phi_6 = 1.0$

and $m_1 = -\left(\frac{x'}{x}\right)^{-0.06}$ so that Eq 11 becomes

$$C_e = 0.722 \; S^{-\frac{1}{2}} \left(\frac{x'}{x}\right)^{-0.06}$$

(17)

Of particular significance is the fact that within the limits
of the data $0.722 \; \phi_6$ is independent of x'/x. Apparently,
only the exponent of S in eq 11 is influenced by a vari-
ation in x'/x and this variation is of such form that when
$x'/x = 1$ Eq 11 is

$$C_e = 0.722 \; S^{-\frac{1}{2}}$$

(18)

Despite the experimental scatter and the arbitrary man-
ner in which the final functional form was determined, Fig. 2
shows that the data in general follow rather closely the
derived relationships. It is particularly noteworthy that
Eq 18 is identical to the theoretical results of Pohlhausen,
thereby substantiating much of the theory and many of the
assumptions used in deriving Eq 11. There is no doubt, for
example, that the only diffusion coefficients which are
important in the laminar boundary layer and the laminar
evaporation layer are the molecular coefficients ν_f and ν_e.

For the turbulent boundary layer, the data were plotted
logarithmically and an attempt was made to determine the
functions in much the same manner as for laminar conditions.
This procedure, however, did not show the data to support Eqs
9 and 12 and no satisfactory modification of the equations
could be devised.

When the evaporation data taken in the turbulent bound-
ary layer were plotted semi-logarithmically to evaluate A_2
and ϕ_8 in Eq 13, it was found that the equation could not be
forced to fit the data with any degree of satisfaction. On
the other hand, it was a simple matter to determine the func-
tions A_1 and ϕ_5 in Eq 10 which best fit the data. These

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are $A_1 = -3$ and $\phi_5 = 6.6\left(\frac{x'}{x}\right)^{-0.15}$ so that Eq 10 becomes

$$\frac{1}{\sqrt{C_e}} = 6.6\left(\frac{x'}{x}\right)^{-0.15} \log C_e S - 3 \tag{19}$$

This equation is plotted with the data in Fig. 3 and although there is considerable scatter of the data there is no systematic trend. Therefore, in spite of the fact that the constants in Eq 19 do not agree with the constants in Eq 13, the log relationship may be considered to represent the data reasonably well.

In comparing the three functional relationships, the laminar, the turbulent power law, and the turbulent logarithmic law, it is evident that the laminar relationship Eq 17 is the most accurately determined and probably applies over a wider range of S than represented by the data. Eq 9, however, cannot be arranged to fit the data. In regard to the logarithmic relationships, there is considerable doubt whether Eq 19 applies outside the range of the data, although it is probable that an equation of similar form applies at high values of the Peclet number.

Finally, all of the data for both the laminar and the turbulent boundary layers are plotted in Fig. 4 which shows that the analysis of Eq 16 is reasonably accurate. The Prandtl number σ is constant at 0.6 and apparently, the relative roughness is at most of only secondary importance. By simple curve fitting Eq 16 becomes

$$Nu' = 0.5 B^{4/5} \tag{20}$$

which applies with reasonable accuracy at least over the range of the data presented in this study.

The fact must be emphasized that the entire foregoing analysis is based upon a boundary layer which has formed over a smooth surface. In case the viscous influence in the laminar sub-layer is either modified or completely destroyed by irregular terrain, the problem not only involves a length parameter characterizing the geometry of the roughness, but it requires an analysis of an altogether different nature. In view of these considerations it is quite understandable that considerable difficulty results in attempting to correlate the measurements from an evaporation pan, which in itself is a boundary irregularity of a very pronounced nature, with the evaporation to be expected from a plane surface.

Acknowledgements

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Nonmenclature

A_1 and A_2	Coefficients which depend upon x'/x.
B	Dimensionless evaporation parameter $\dfrac{x'\sqrt{\tau_o/\rho}}{\nu_e}$ which does not include the ambient velocity.
C	Local concentration of water vapor.
ΔC	Difference between the ambient concentration of water vapor and the total local concentration.
ΔC_s	Difference between the ambient concentration of water vapor and the concentration at the evaporating surface.
C_e	Coefficient of evaporation.
C_f	Coefficient of resistance.
D	General coefficient of diffusion.
$\mathcal{E}_o$	Rate of evaporation per unit area of surface.
E	Rate of evaporation over the distance x' per unit width of surface.
k	Boundary roughness.
m_1 and m_2	Exponents which depend upon x'/x and S.
N_u'	Nusselts number for diffusion of water vapor into air and the coefficient of evaporation which does not include the ambient velocity.
n	Concentration of any quantity.
N	Rate of diffusion of any quantity.
R	Reynolds Number $\dfrac{Ux}{\nu_f}$.
S	Péclet number, a dimensionless evaporation parameter $\dfrac{Ux'}{\nu_e}$.
t	Time.
u	Local velocity in the x-direction.
U	Ambient velocity in the x-direction.

x	Distance from the upstream edge of the boundary.
x'	Distance from the upstream edge of the evaporating surface.
y	Normal distance from the boundary.
δ	Thickness of the boundary layer.
δ'	Thickness of the laminar sub-layer.
ϵ	General molar coefficient of diffusion.
λ	Thickness of the evaporation layer.
σ	Prandtl Number $\dfrac{\nu_f}{\nu_e}$.
ν_e	Molecular coefficient of diffusion of water vapor.
ν_f	Molecular coefficient of diffusion of momentum.
ρ	Mass density of air.
τ_o	Intensity of shear at the boundary.
$\sqrt{\tau_o/\rho}$	Shear velocity.

Figure Captions

Fig. 1 Schematic representation of boundary layer and evaporation layer.

Fig. 2 Composite plot of C_e as a function of S and x'/x for laminar boundary layer.

Fig. 3 Composite plot of C_e as a function of SC_e and x'/x for turbulent boundary layer.

Fig. 4 Combined plot of Nu' as a function of B .