DISSERTATION

RESISTANCE TO FLOW IN SAND CHANNELS

Submitted by

Everett V. Richardson

In partial fulfillment of the requirements

for the Degree of Doctor of Philosophy

in Civil Engineering

Colorado State University

Fort Collins, Colorado

June, 1965

CER65EVR27

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ABSTRACT OF DISSERTATION

RESISTANCE TO FLOW IN SAND CHANNELS

A theoretical and laboratory investigation was made of resistance to flow in sand-bed channels. The objectives were to determine the type of flow and energy dissipation in sand-bed channels and develop equations and relations for predicting resistance to flow and mean velocity.

The types of flow, energy dissipation and, thus resistance to flow in sand-bed channels is extremely variable because (1) the configuration of the boundary, (2) the properties of the fluid, and (3) the characteristics of the turbulence are functions of the flow, fluid, and sand characteristics and of the geometry of the channel. The boundary configurations that form in a sand bed are ripples, ripples on dunes, dunes, plane bed, antidunes or chutes-and-pools.

The type of flow in a sand channel with constant discharge and average energy gradient may be steady or unsteady and uniform or nonuniform, depending on the boundary configuration. With the array of boundary configurations found in sand channels, the dissipation of energy may result from grain roughness, form roughness, acceleration of the flow, breaking waves or any combinations of them.

With variable boundary configuration, type of flow and energy dissipation, it is impossible to determine a general equation to predict resistance to flow and mean velocity for all flow conditions. However, if the boundary configuration is known, specific relations and equations are developed for predicting resistance to flow.

For steady uniform flow, the equations are based on integrating the Reynolds equation for turbulent flow. The coefficients in the integrated equation were determined from a study of the velocity distribution and verified using the mean flow variables. For non-uniform and (or) unsteady flow, resistance to flow is determined by applying a correction term to the equation developed for flow over a plane bed. The correction term compensates for the increase in energy dissipation resulting from form roughness, flow acceleration and breaking waves.

The study of the velocity profiles for plane bed flow when there is considerable bed-material movement, determined that there is an inner and outer flow zone. In the inner zone, the slope A and intercept B in the relation $u = A \ln y + B$ are variable. The variation of the slope and intercept are functions of the size and concentration of suspended sediment in the inner zone. In the the outer zone, the slope and intercept are constant.

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June 1965

ACKNOWLEDGMENTS

The writer wishes to express his appreciation to Dr. Daryl B. Simons for his assistance, encouragement, and indulgence during their years of working together in the course of his graduate program and research. Dr. Simons was not only major professor but a generous colleague for a number of years in the study of flow in alluvial channels. The writer is also in debt to Mr. R. W. Carter of the U. S. Geological Survey for the opportunity to study flow in sand channels and to undertake advanced training. Without Mr. Carter's and Dr. Simons' aid and encouragement, this dissertation would not have been possible.

The writer wishes to thank all members of his research committee, Professors J. E. Cermak, D. V. Harris, and S. S. Karaki, and Mr. H. P. Guy and Dr. F. M. Chang of the U. S. Geological Survey for their helpful comments and review of the dissertation.

Acknowledgments and thanks are due to C. F. Nordin, W. W. Sayre, M. R. Williams, and P. C. Benedict of the Geological Survey, and Professors M. L. Albertson, A. R. Chamberlain, J. W. N. Fead, and H. J. Koloseus for their help and encouragement during the course of the research in sand-bed streams.

Finally, but not lastly, the writer wishes to thank Billie, Gail, Tom, and Jerry for their sympathetic indulgence of the time spent in graduate study and on this dissertation.

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PARTIAL LIST OF SYMBOLS

		Units
A	Slope of the $u/\sqrt{gDS}$ versus $\ln y/\xi$ relation ($A = 1/k$)	0
A_*	Slope of the u versus $\ln y$ relation	fps/ln unit
B	Intercept of the $u/\sqrt{gDS}$ versus $\ln y/\xi$ relation	0
B_*	Intercept of the u versus $\ln y$ relation	fps
C	Wave celerity equal to $\sqrt{gL/2\pi \tanh \frac{2\pi D}{L}}$ $C = \sqrt{gD}$ when L is large relative to D and $C = \sqrt{\frac{gL}{2\pi}}$ when L is small relative to D	fps
C_*	Correction term applied to the grain roughness equation for a plane bed to obtain the resistance coefficient for other bed forms $C_* = \sqrt{\frac{(RS)'}{RS}}$	0
C_T	Concentration of bed-material discharge	ppm
C_s	Suspended-sediment concentration	ppm
$C/\sqrt{g}$	The dimensionless Chezy discharge coefficient equivalent to $\sqrt{\frac{8}{f}} = U/\sqrt{gRS}$ where $\sqrt{gRS} = \sqrt{gDS}$ for two-dimensional flow	0
D	Average depth of flow	ft
d	Fall diameter of the bed material	mm or ft

Symbols (continued)

		Units
d_x	Fall diameter of the bed material of which x percent by weight of the sizes are finer, where x is some specified percent	mm or ft
e	Internal energy of the flow	ft^2/sec^2
$\mathbb{F}$	Froude number of the flow, the ratio of average velocity U to wave celerity C. $\mathbb{F} = U/\sqrt{gD}$ when L is large relative to D.	0
f_s	Seepage force caused by the water flowing through the stream bed.	lb/ft^2
f	Darcy-Weisbach friction factor; $f = 8/(C/\sqrt{g})^2 = \frac{8gRS}{U^2}$	0
g	Acceleration of gravity	ft/sec^2
h	Heat in first law of thermodynamics	ft-lb
H	Wave height from trough to crest	ft
L	Wave length from crest to crest, or trough to trough, either water-surface waves or sand waves	ft
P	The wetted perimeter	ft
p	Pressure	lb/ft^2
Q	Discharge of water-sediment mixture	ft^3/sec
q	Stands for either u or U in dimensionless velocity equation	fps

Symbols (continued)

		Units
R	Hydraulic radius (area/wetted perimeter). For wide channels $R \approx D$	ft
$\mathbb{R}$	Reynolds number, UD/ν or UR/ν	0
$\mathbb{R}_*$	Reynolds number based on U_*, i.e., U_*D/ν or U_*R/ν	0
S	Slope of energy grade line, equal to water-surface slope with equilibrium flow	0
S_c	Shape factor of the channel cross section	0
S_p	Shape factor of the sediment particle	0
S_r	Shape factor for the reach of the stream. Sinuosity and change in area with distance	0
T	Temperature	^oC or ^oF
t	Time	Sec. or min.
U_*	Average velocity based on continuity principle	fps
u	Velocity at the point y where y is the distance from the bed	fps
U_*	The shear velocity $U_* = \sqrt{\tau_o/\rho} = \sqrt{gDS}$ or $\sqrt{gRS}$	fps
V	Volume	ft^3
W	Width	ft
y	Distance from the bed surface	ft
γ_s	Specific weight of the sediment particles	lb/ft^3
$\Delta\gamma$	Difference between the specific weight of sediment and water	lb/ft^3

Symbols (continued)

		Units
η	Shape parameter of the form roughness	0
k	Von Karman's kappa	0
μ	Apparent dynamic viscosity of the water-sediment mixture	lb-sec/ft
ν	Apparent kinematic viscosity of the water-sediment mixture	ft /sec
ξ	Height of the grain roughness element	ft
ρ	Mass density of the water-sediment mixture	slug/ft^3
ρ_s	Mass density of the sediment	slug/ft^3
σ	A measure of the gradation of the sediment distribution $\frac{1}{2}\left[\frac{d_{50}}{d_{16}} + \frac{d_{84}}{d_{50}}\right]$	0
τ_o	The shear stress developed on the bed γDS or the average shear stress γRS	lb/ft^2
ψ	Spacing parameter for form roughness	0
χ	Height of form roughness	ft
ω	Fall velocity of sediment particles	fps

CHAPTER I

INTRODUCTION

The need for knowledge of resistance to flow in open channels cannot be overemphasized. Such knowledge is necessary for the design of canals or channels to convey water from the river to irrigate lands; to supply drinking water to cities; to convey the wastes from the city back to the river; or, to convey floods through a city. Also knowledge of flow resistance is required to determine the magnitude of a flood; the amount of water available for beneficial use; or, to determine the elevation a flood of a given magnitude will attain in order to design flood protection works. Many other examples of the need for knowledge of resistance to flow in open channels can be cited.

The nature and magnitude of resistance to flow in open channels under various flow, fluid, and channel conditions are still imperfectly known, though studied for nearly two centuries. The principal reasons for this lack of progress, in comparison to closed conduit flow, are the free surface and unsymmetrical cross section that allow additional degrees of freedom. Although resistance to flow in open channel is imperfectly understood, the engineer, through theory and experiment has built up a fund of information that permits, within limits, the empirical determination of resistance to flow. Unfortunately, the prediction of resistance to flow depends on the experience of the engineer.

Many channels not only have the problem of a free surface and nonsymmetrical cross section but in addition have a deformable and movable alluvial boundary. An alluvial boundary is formed in cohesive or noncohesive materials that have been and can be transported by the flow. The noncohesive alluvial boundary usually consists of sand (0.062 to 2.0 mm), gravel (2.0 to 64 mm), or cobbles (64 to 256 mm) or some combination of these sizes. With an alluvial boundary, the configuration of the bed, the fluid properties, the turbulence of the flow, and the resistance to flow are functions of the boundary material, fluid and flow characteristics, and the channel geometry (Gilbert, 1914, Vanoni, 1946, Elata and Ippen, 1961, Simons and Richardson, 1963).

The objectives of this report are to determine the nature and magnitude of, and reasons for, resistance to flow in sand channels; and to determine equations and relations for predicting resistance to flow and mean velocity. Flow conditions investigated range from beginning of bed material movement to antidunes. The report is restricted to equilibrium flow* in sand channels and uses the data collected by Simons and Richardson (1961) in the 8 ft flume and field data from U. S. Geological Survey reports. It is assumed that the field data was collected for equilibrium flow conditions. Equilibrium flow (the simplest type obtainable when studying flow conditions ranging from beginning of motion to antidunes) may be steady or unsteady, uniform or nonuniform, depending on the bed configuration. The fact that equilibrium flow may be unsteady and nonuniform increases the complexity of the problem.

The report gives background on the nature of flow in sand

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* Equilibrium flow is defined as flow with a constant discharge of the water-sediment mixture and with an average energy gradient that is invariant with time and parallel to the gradient of the bed.

channels. This forms the basis of the analytical considerations and analysis of the data. To provide this background: (a) the different types of resistance to flow that occur in open channels when the flow may be steady or unsteady and uniform or nonuniform are examined; (b) the different bed configurations and flow regimens with their associated flow phenomenon and types of energy dissipation are delineated and discussed; (c) the variables involved for sand channel flow are listed and their role discussed; and (d) finally, a method for predicting what the bed configuration will be for different flow, fluid and bed-material characteristics is presented. A method of predicting the bed configuration is necessary because with the many different types of flow, energy dissipation, and bed configuration that occur in a sand channel, a unique relation between resistance to flow and the variables does not exist. However, if the type of bed configuration is known, a unique relation can be determined.

To obtain equations for the velocity distribution and resistance to flow, the Reynolds equation for turbulent flow is integrated. The integration is accomplished by assuming (a) steady uniform flow, (b) that the viscous stress is very much smaller than the Reynolds stress, and (c) a relation between the Reynolds stress and the velocity distribution. There are insufficient boundary conditions to determine the coefficients in the equation. Therefore, dimensional analysis and study of the data are necessary to determine the coefficients. Dimensional analysis is used to decrease the number of terms involved and group them logically for study.

In the analysis of data, the velocity distribution and resistance to flow are studied for the different bed configurations. The coefficients in the integrated Reynolds equations for the velocity distribution and resistance to flow are determined for those bed configurations that have steady uniform flow. For the bed configurations where the flow is non-uniform, unsteady, or both, the velocity distribution is undefined. To obtain the resistance to flow and mean velocity for the bed configurations with unsteady and (or) non-uniform flow, a correction term was applied to the steady uniform flow equation to account for the energy loss from the acceleration and breaking wave effects. This correction term was also applied to the uniform steady flow case to correct for form roughness effects.

CHAPTER II

DESCRIPTION OF EXPERIMENTS AND DATA

The flume experiments were conducted in a tilting recirculating flume, 150 ft long, 8 ft wide, and 2 ft deep. Slope in this flume could be varied from 0 to 0.013 ft per ft and water discharge from 2 cfs to 22 cfs. Six bed materials were used in the study, their size distributions are given in figure 1. These size distributions are in terms of fall diameter (Colby and Christenson, 1956).

The data from the 8 ft flume study were collected by D. B. Simons and the writer over a four-year period and consists of from 20 to 50 equilibrium runs for each bed material. The general procedure for each run was to recirculate a given discharge of the water-sediment mixture until equilibrium flow conditions were established. Equilibrium flow is defined as flow that has established a bed configuration and slope consistent with the fluid, flow and bed material characteristics over the entire length of the flume neglecting entrance and exit affected reaches. For equilibrium flow, the discharge of the water-sediment mixture is constant, the time average water surface slope of the flow is essentially constant and parallel to the time average bed surface; and the time average concentration of the bed material discharge is constant. Equilibrium flow should not be confused with steady uniform flow because with equilibrium flow the velocity may vary at a point, and from point to point. Steady uniform flow as clasically defined, that is velocity nonvariant with respect to time and space ($\partial u_i / \partial t = 0$, $\partial u_i / \partial x_i = 0$), does not exist in an alluvial channel unless the bed is plane.

After equilibrium flow was established, water surface slope, S; discharge of the water-sediment mixture Q; water temperature, T; depth, D; velocity distribution in the vertical, u; total sediment concentration, C_T; suspended sediment concentration, C_s; and the geometry of bed configuration (length L, height H, spacing ψ, and shape η were determined).

Water-surface slopes were measured with a Lory point gage and a precision level by determining water-surface elevations at definite intervals along the flume. Discharges were measured with calibrated orifice meters located in the return flow pipes. Water temperature was measured to the nearest 0.1 of a degree centigrade with a mercury thermometer. Depth was determined by subtracting mean bed elevation from mean water-surface elevation. Velocity distribution in a vertical was measured with a calibrated Prandtl Pitot tube; however, the mean velocity of the cross section was computed from the discharge and cross-section area data, $U = Q/WD$. Total sediment concentration was measured by traversing the overflow nappe at the downstream end of the flume with a width-depth integrating sampler. Bed configuration was measured by using a point gage and a special sonic depth sounder (Richardson, Simons, and Posakony, 1961).

A complete documentation and description of all of the basic flume data collected from 1956-1962 by the U. S. Geological Survey at Colorado State University will be included in a data report which is in press (Guy and others, 1965).

In addition to the data from the flume experiments, data collected from the following natural streams were used:

Elkhorn River near Waterloo, Nebraska (Beckman and Furness, 1962).

Rio Grande near Bernalillo, New Mexico (Culbertson and Dawdy, 1964, and Nordin, 1964)

Rio Grande near Bernardo, New Mexico (personal communication)

Mississippi River at St. Louis, Missouri (Jordan, 1965)

The size distribution of the bed material for the Elkhorn River near Waterloo, Nebraska is the same as for the 0.28 mm sand in figure 1. The Elkhorn River was the source of the 0.28 mm sand. The median diameter of the bed material d_{50} for the Rio Grande near Bernalillo, New Mexico was 0.29 mm and the measure of gradation σ was 1.60; for the Rio Grande near Bernardo, New Mexico d_{50} was 0.24 mm and σ was 1.62; for the Mississippi River at St. Louis, Missouri d_{50} was 0.38 mm and the average gradation as measured by $\sqrt{d_{75}/d_{25}}$ was 1.5.

CHAPTER III

BACKGROUND OF THE PROBLEM

Resistance to Flow in Open Channels

Resistance to flow in an open channel or in a pipe is a catch-all term that includes all the mechanisms whereby the mechanical energy of the flow is converted to heat and lost to the surrounding medium. From the first law of thermodynamics

$$\left[\frac{P_1}{\rho} + \frac{U_1^2}{2} + gy_1\right] - \left[\frac{P_2}{\rho} + \frac{U_2^2}{2} + gy_2\right] = (e_2 - e_1) - \frac{dh}{pdV} \qquad (1)$$

where the left side of equation 1 is the difference in mechanical energy between two points in steady uniform flow and the right side is the loss in mechanical energy per slug of flowing mass. The first term on the right side of equation 1 is the increase in internal energy of the fluid and the second term is the loss of heat to the boundary between the two sections. The increase in internal energy is small and furthermore not recoverable and neither is the loss of heat recoverable as useful work. Therefore, the two terms are grouped together and may be designated head loss, energy loss, friction loss, or resistance to flow loss.

The loss of mechanical energy (kinetic, pressure, and elevation) in steady uniform turbulent flow results from the viscous shear between the fluid and the boundary, surface resistance; the low pressure downstream of points of separation of the fluid from the boundary, form resistance; and the turbulent velocity fluctuation and eddies which are generated by both surface and form resistance. In addition, when the flow is not uniform and steady, there are additional losses of mechanical energy resulting from acceleration and deceleration of the flow and breaking waves.

Surface Resistance

With surface resistance the boundary may be hydraulically smooth or hydraulically rough depending on whether the flow close to the boundary is laminar or not. The flow is hydraulically smooth if the roughness elements are submerged by laminar flow. The flow is hydraulically rough if the roughness elements are large enough that the flow separates from the micro-elements and laminar flow next to the boundary does not exist.

With a hydraulically smooth boundary, resistance to flow is a function of the viscosity and thus the Reynolds number $\mathbb{R}$. With grain roughness, resistance to flow is independent of viscosity and thus the Reynolds number. The difference between hydraulically rough surface resistance, hereafter called grain roughness, and form roughness is one of scale. With grain roughness the separation zone downstream from the grains, the reduction in pressure in the separation zone, and the size of the eddies in the flow is of a micro-scale, largely confined to a small region in the neighborhood of the grains. However, there effect is felt throughout the flow field. With form roughness the separation zone downstream from the roughness element, the reduction in pressure in the separation zone, and the size of the eddies in the flow is of a macro-scale.

Form Resistance

With form resistance, the flow separates from the macro-boundary. This results in a relatively large separation zone with lower pressure (form drag), a reduction in effective area for the flow and the generation of large scale eddies. Both the eddies and pressure reduction dissipate energy. With form resistance, as with grain roughness, resistance to flow is independent of viscosity.

Acceleration and Deceleration of the Flow

If the macro-roughness is large in relation to the scale of the flow system (width and depth of flow) so that there is considerable acceleration and deceleration of the flow, then the flow is no longer uniform. In this case the so-called roughness elements are in reality changes in cross section. The forces required to produce the acceleration and deceleration of the flow are a drain on the energy of the system. This loss of energy is reflected in an increase in resistance of the flow. The acceleration and deceleration of the flow are gravitational effects and thus resistance to flow will depend on the Froude number in addition to the Reynolds number and the size of the roughness elements. The gravitational effects depend on the relative magnitude of the acceleration and decleration energy losses in relation to the other losses.

Even if the macro-roughness is not large in comparison to the scale of the flow system, there may be considerable acceleration and deceleration of the flow when the Froude number is equal to or greater than one. With this type of flow the bed surface and water surface are in phase and the separation of the flow from the boundary is small unless the Froude number is large. The magnitude of the Froude number for flow separation depends on whether the boundary is rigid or alluvial. For rigid boundaries the water surface is unstable, the flow separates from the boundary and roll waves form, when Froude numbers are larger than 1.6 (Koloseus, 1958). For alluvial boundaries the magnitude of the Froude number, where the flow separates from the boundary and breaking waves occur, depends on the size of the bed material. The coarser the bed material the larger the Froude number must be for flow separation and breaking waves.

If the flow separates from the boundary in accelerating flow, there is considerable dissipation of energy. In addition to the energy loss from acceleration and deceleration of the flow, there is dissipation of energy from form roughness when the Froude number is less than one, and breaking waves and form roughness when the Froude number is greater than one. The resistance to flow from the form roughness and/or breaking wave effects will be considerably larger than that from the acceleration and deceleration of the flow. If the flow does not separate from the boundary in accelerating flow, the dissipation of energy will depend on the grain roughness and the acceleration and deceleration of the flow. The grain roughness effects being the larger of the two.

Breaking Waves

Breaking waves occur when the Froude number of the flow is large enough so that the in-phase water and bed-surface waves reach an instability point and either roll waves (Koloseus, 1958) or breaking waves (Simons and Richardson, 1962a and Kennedy, 1961) form. The eddies and turbulence generated by the breaking waves dissipate much energy. The dissipation of energy with breaking waves depends on the size of the waves, the amount of time and space occupied by the breaking waves and the vigor of their breaking.

For convenience, an over-all resistance factor can be used for acceleration and deceleration and the breaking wave type of flow; but this resistance factor will be a function of the Froude number. Also, there may be discontinuities in any functional relationship involving the gross resistance factor. The discontinuities result from a decrease in the nonuniformity of the flow and the breaking of the waves as depth increases.

Resistance to Flow in Sand Channels

Forms of Bed Roughness

Resistance to flow in a sand channel is complicated by the fact that the bed configuration, and the flow phenomena are determined by the interaction between the flow, fluid and movable bed material. The forms of bed roughness observed in a sand channel, listed in order of their occurrence with increasing stream power $U \gamma DS$, are plane bed without sediment movement, ripples (when $d_{50} \leq 0.6$ mm), ripples on dunes, dunes, plane bed with sediment movement, antidunes and chutes-and-pools. When the d_{50} of the bed material is larger than 0.6 mm, ripples will not form after beginning of motion (Simons and Richardson, 1962a, 1964, Knoroz, 1959), instead, small dunes form after beginning of motion.

There is a transition range in stream power where the bed configuration may range from fully developed dunes to plane bed. Generally, in this transition range the bed consists of long, small amplitude dunes, with length increasing and amplitude decreasing as stream power increases. However, when the bed form consists of dunes, slope or depth and hence stream power can be increased to relatively large values before the bed form changes to a plane bed or antidunes.

Conversely, if the bed form is plane or antidunes, the stream power can be decreased significantly before dunes develope. In this transition range of stream power whether the bed was plane or dunes depended on antecedent conditions.

At certain values of stream power in the transition range the bed form oscillates between dunes and plane bed. This phenomena results from changes in depth or slope or both which occur when the resistance to flow changes with a change in bed form. With a plane bed, resistance to flow is small, whereas with a dune bed resistance to flow is large. With the oscillating bed configuration, when the bed becomes plane, the decrease in resistance to flow decreases depth or slope or both and thus the shear stress γDS on the bed. The reduced value of shear stress is incompatible for a plane bed and dunes form. The increase in resistance to flow with dunes increases depth and/or slope and thus the shear stress. This increased value of shear stress eliminates the dunes and a plane bed results. With these changes in shear stress with a change in bed form, the bed form alternates between dunes and plane bed.

The bed configuration that occur in a sand channel are illustrated in figures 2 and 3. A brief description of each form follows:

<u>Plane bed without sediment movement</u>.-- This bed form, obtained in the flume by screeding the bed is not ordinarily encountered in natural streams. Resistance to flow is the result of grain roughness as for rigid boundary hydraulics.

<u>Ripples</u>.-- Ripples are small scale form roughness elements with spacing about 0.5 to 2.0 ft and heights (crest to trough) of 0.02 to 0.2 ft. Ripple shape is independent of sand size. Although, the

shear stress and stream power required for ripple formation increases with an increase in sand size. Within the accuracy of flume experiments resistance to flow is also independent of sand size. This results from the fact that ripple shapes are independent of sand size, and grain roughness on the backs of ripples is small in comparison to form roughness.

Dunes.-- These are large scale form roughness elements. In the flume dune length ranges from 2.0 ft to 10 ft and in height from 0.2 to 1.0 ft. In the field (Carey and Keller, 1957) dunes measure several hundred feet long and up to 40 ft high in the Mississippi River. The height, shape, and length of dunes, in contrast to ripples, depends on the grain size. It is apparent that dune size increases with an increase in flow depth; in fact, the maximum height of a dune is approximately the average depth.

As mentioned previously, resistance to flow is a combination of grain roughness and form roughness. Dunes are a unique type of form roughness. In fact, large dunes in some parts of a channel may cause appreciable acceleration and deceleration of the flow and thus, may not be form roughness in uniform flow but a change in cross section in nonuniform flow. However, recognizing the possibility of nonuniform effects, a gross resistance factor can be used for the dune bed configuration.

The magnitude of resistance to flow is an interrelated complex function of the grain size, slope and depth. As illustrated, in figure 4 (Richardson and others, 1962) resistance to flow for dunes formed of sands finer than 0.30 mm has little variation with changes in either depth or slope. With sands coarser than 0.30 mm resistance to flow varies with either a change in depth or a change in slope or both. Figure 4 also illustrates that with an increase in sand size there is an increase in resistance to flow. This increase results from an increase in form roughness in addition to the increase in grain roughness. With an increase in sand size there is a decrease in length of the dunes and increase in their angularity (Richardson and others, 1962).

Difference between ripples and dunes.-- There is some controversy in the literature concerning ripples and dunes (Vanoni and others, 1961). The differentiation between the two was made by Kornoz (1959) and Simons and Richardson (1962a, 1963). Others have used the two terms indiscriminately. Ripples are different from dunes in that (1) with ripples there is a decrease in resistance to flow with an increase in depth, whereas with dunes there is not; (2) ripples do not form in material coarser than 0.6 mm, whereas dunes may form in all sizes of alluvial material; (3) with ripples, resistance to flow is independent of the grain size, whereas with dunes it is dependent.

Whether ripples are different from dunes is a moot point except that dunes are the dominant bed form in streams and canals and ripples are the dominant bed form in most laboratory flumes. A difference between the two bed forms means, as Taylor and Brooks (1962) pointed out, that resistance to flow and modeling of alluvial channels cannot be resolved by small-scale laboratory studies.

Plane bed with sediment movement.-- A plane bed with sediment movement has grain roughness type of resistance to flow. However, with the grains moving along the bed the resistance to flow is slightly less than for the static bed case of a plane bed without movement (Vanoni and Nomicos, 1960, Elata and Ippen, 1961, and Simons and Richardson, 1963). Vanoni and Nomicos attribute the decrease in resistance to flow to a damping of turbulence by the suspended sediment; whereas Elata and Ippen indicate that the turbulence was not damped but that its structure was changed.

It was thought that the formation of a plane bed by the flow depended on the Froude number (Simons and Richardson, 1962a), however, it was later discovered that a plane bed was not a function of the Froude number (Simons and Richardson, 1963). Instead, the formation of a plane bed depends on the magnitude of the shear stress in relation to the fall velocity of the bed material. When the fall velocity of the bed material is low (either as a result of fine bed material, or large viscosity of the fluid) the plane bed occurred at relatively low shear stress.

<u>Antidunes</u>.-- Antidunes consist of a series or train of inphase (coupled) symmetrical sand and water waves. The waves do not exist as a continuous train that never changes, but rather as a series of waves that gradually build up with time from a plane bed and water surface. These waves may grow in height until they become unstable and break as the sea surf, or they may grow in height and then gradually subside. As antidunes form they may move upstream, downstream or remain stationary. Their upstream movement is the reason Gilbert (1914) named them antidunes.

In the flume the lengths of the sand and water waves range from 5 to 10 ft. The height of the sand waves ranges from 0.03 to 0.5 ft and the height of the water waves from 1.5 to 2 times the height of the sand waves. In natural rivers surface waves from 2 to 3 ft in height and 10 to 20 ft in length have been observed.

Resistance to flow with antidune results from grain roughness, acceleration and deceleration of the flow, and breaking waves. When breaking waves occur the dissipation of energy, and the gross resistance to flow depends on (1) how often the antidunes form and break, (2) the area of the reach they occupy and (3) the violence of their breaking. When breaking waves do not occur the acceleration and deceleration of the flow increases resistance to flow slightly above that for a plane bed.

Kennedy (1961, 1963) made an extensive study of antidune flow. He found that the minimum wave length is given by $L = 2\pi U^2/g$; that there is a range of depth and slopes where antidunes will form only if an initial surface wave is introduced; and that the Froude number for the occurrence of antidunes decreases as the depth of flow increases or the size of the bed material decreases. This last conclusion was also made by Leopold and Maddock (1953, p. 41), Langbein (1942) and Simons and others, (1963). Simons and others (1963) demonstrated that the antidune activity was related to the fall velocity of the bed material.

Chute-and-pool flow.-- This is an extreme example of nonuniform unsteady flow in an alluvial channel. The flow may consist of a chute (10 to 30 ft long) where the flow is rapid and accelerating. The chute terminates in a hydraulic jump followed by a pool (10 to

30 ft long) where the flow is tranquil and accelerating. These chutes and pools remain stationary with respect to each other, but with reference to the flume, usually move slowly upstream. The gross resistance to flow for this type of flow is large.

Regimes of Flow

On the basis of similarity in the shape of the bed configurations, mode of sediment transport, process of energy dissipation, and phase relation between the bed and water surface Simons and Richardson (1963) divided the flow in sand channels into two flow regimes with a transition between. These two flow regimes with the intermediate transition logically group the flow in sand channels into its three major phases. The flow phenomena, transport and resistance to flow that occur in one regime may be extremely different from that in another. A lack of comprehensive experimental data covering both regimes of flow has contributed to the lack of understanding of sand channel flow and has led to many conflicting statements in the literature. One experimenter or group of experimenters studying only the lower flow regime will arrive at a conclusion that is in conflict with another group studying only the upper flow regime. Because it is not clear that they are studying different phases of the same problem, conflicting concepts naturally develope.

These flow regimes with their associated bed configurations, listed in their order of occurrence with increasing stream power are (Simons and Richardson, 1963, p. 323):

Lower flow regime:

1. Ripples

2. Dunes with ripple superposed

3. Dunes

Transition (bed roughness ranges from dunes to plane bed or antidunes)

Upper flow regime

1. Plane bed

2. Antidunes

    a. Standing waves

    b. Antidune

    c. Peak antidune

3. Chutes and pools

4. Slug flow

Variables

Resistance to flow in an alluvial channel is complicated by the large number of variables, the interdependency of the variables, and the fact that some of the variables may be altered and even determined by the flow. The variables are listed in equation 2 and a brief discussion of the more significant aspects of the major variables follows:

$$U = \phi \left[D,\ S,\ d,\ \sigma,\ \rho,\ g,\ \omega,\ S_r,\ S_c,\ f_s \right] \tag{2}$$

$$\omega = \phi \left[d, \rho_s, \rho,\ g,\ S_p, \mu \right] \tag{3}$$

where

U = average velocity

D = average depth of flow

S = energy gradient

d = fall diameter of the bed material

σ = measure of the size distribution

ρ = mass density of the water-sediment mixture

ρ_s = mass density of the sediment

g = acceleration gravity

ω = fall velocity of the bed material

S_c = shape factor for the cross section

S_r = shape factor for the reach

S_p = shape factor for the sediment particle

f_s = seepage force

μ = viscosity of the water-sediment mixture.

The fall velocity of the bed material has been substituted for viscosity in equation 2 because, after beginning of motion, flow over a sand bed is hydraulically rough. For hydraulically rough flow, viscosity and the Reynolds number $\mathbb{R}$ should not affect resistance. The changes in resistance to flow over sand beds which have been observed with changes in viscosity (Straub, 1954; Hubbell and others, 1956; Vanoni and Brooks, 1957; Hubbell and Al-Shaikh Ali, 1962) are the results of changes in the fall velocity of the bed material, with a corresponding change in the bed configuration, (Hubbell and Al-Shaikh Ali, 1962; Simons and others, 1963; Simons and Richardson, 1963). These studies demonstrated that fall velocity is a very significant variable for the interaction between the fluid and the bed material and can be used to replace the viscosity of the fluid and the shape factor and density of the particle.

In equation 2, the bed material transport, or the bed form can be substituted for the velocity as the dependent variable but cannot be included in the list of variables because they are dependent variables and more often than not are an unknown quatity. A quantitative description of bed form could be substituted for one of the significant variables on the right-side to simplify the problem. Later in the analysis section the type of bed configuration will be used as a variable and one of the variables on the right eliminated. It is interesting to note that Colby (1964) in a study of bed material discharge used the velocity of the flow as the primary independent variable to determine bed material discharge. In this case velocity replaced and integrated the effect of several other independent variables.

Before discussing the effect of the variables on the resistance to flow it is necessary to alter the velocity to obtain either the Darcy-Weisbach friction factor or the Chezy discharge coefficient. Using D, S and g as repeating variables, equation 2 becomes

$$\frac{U^2}{8g\ RS} = \frac{1}{f} = \frac{1}{8}\left(\frac{C}{\sqrt{g}}\right)^2 = \phi\left[S,\ D,\ d,\ \omega,\ \sigma,\ f_s,\ S_r,\ S_c,\ \rho,\ g\right] \quad (4)$$

The other terms are not made dimensionless in order to better discuss the role of each individual item. It has been observed that changing the ratio D/d has a different effect when D is varied than when d is varied even though the ratio has the same numerical values. In the discussion that follows figure 4 is used to illustrate the effects of the variables on resistance to flow.

Slope S.-- The effect of slope is illustrated in figure 4. With depth and bed material constant an increase in slope can change the bed form and resistance to flow. Also, as illustrated in figure 4, resistance to flow will change with a change in slope even when the bed configuration is the same type.

Depth D.-- The effect of depth on resistance to flow is illustrated in figure 4. With bed material, slope and bed form constant, an increase in depth decreases the resistance to flow for a ripple bed and increases resistance to flow for the dune bed for d_{50} coarser than 0.3 mm. Although it is not obvious in figure 4 studies by Colby (1960), Dawdy (1961), Culbertson and Dawdy (1964), and Simons and Richardson (1962b) have conclusively shown that with slope and bed material constant a change in depth changes the bed form and resistance to flow.

Size of bed material d.-- The physical size of the bed material affects (1) the grain roughness and (2) the fall velocity. As illustrated in figure 4 there is an increase in resistance to flow with an increase in grain size. The importance to resistance to flow of the first effect is in direct proportion to the importance of grain roughness with respect to the other types of energy dissipation. The second effect is important in determining what the bed configuration will be for different flow, fluid and channel characteristics. However, the second effect is related to the other variables in equation 3. This effect is discussed in the next section.

Fall velocity ω.-- The fall velocity of the bed materials is of major importance in determining the bed form. The fall velocity determines the magnitude of the shear stress when the bed material will begin to move (Liu, 1957). The magnitude of the shear stress where ripples change to dunes, dunes change to plane bed, or a plane bed changes to antidunes. The smaller the fall velocity of the bed material the less angular are the dunes and the longer their length. Also, the smaller the fall velocity the smaller the range in shear stress or stream power within which a dune bed occurs. A decrease in fall velocity can change a rippled bed to a dune bed, a dune bed to a plane bed or to antidunes, and can increase the antidune activity. All these changes significantly effect resistance to flow.

The effect of the various factors in equation 3 on fall velocity are well known. However, it must be noted that the viscosity μ is of the water-sediment mixture. Simons and others (1963) have shown that type and concentration of fine sediment significantly effects fluid viscosity and the fall velocity. A 10 percent concentration of the fine sediment (bentonite) increased fluid viscosity 900 percent and decreased the fall velocity of the 0.45 mm sand to the equivalent fall velocity of the 0.28 mm sand. In addition, relations for determining fall velocity are based upon data collected in a quiescent fluid which neglects the unknown effects of turbulence.

Gradation of the bed materials σ.-- Daranandana (1962) compared a uniform sand with a graded sand, both of which had the same d_{50} and found that the gradation of the bed material is a significant factor for resistance to flow. With a ripple, dune or antidune bed configuration resistance to flow is considerably larger for uniform sands.

Seepage force f_s.-- In a natural sand channel there is either inflow or outflow of water through the bank and bed material

This inflow or outflow causes a seepage force on the bed and bank material. This seepage force acts to reduce the buoyant weight of the bed and bank material if there is inflow into the channel and increases the buoyant weight if there is outflow. The inflow forces are large enough in the extreme cases to set up a quick-sand condition where the weight of the material is in equilibrium with the seepage forces. These large seepage forces can have a very definite effect on the bed configuration and resistance to flow in a natural channel. Unfortunately, very little is known quantitatively about the effect of the seepage force either in a flume or natural stream. However, it should be a constant for all flume systems with about the same depth of flow and bed material. Other than to recognize that seepage forces exist and may influence bed forms in a natural stream, these forces will not be considered in this study.

Shape factor for the reach and cross section S_r, S_c.-- The shape factor for the reach is an unknown factor which is included in equation 2 to focus attention on the energy losses resulting from the nonuniformity of the flow in a natural stream caused by the bends and banks. With straight flumes the shape factor for the reach is constant and can be discarded.

The shape factor for the cross section is included in equation 2 to emphasize the fact that wide shallow channels can have multiple roughness with dunes on part of the bed and plane bed or antidunes on another part in the same cross section. Also, it has been observed (Vanoni and Brooks, 1957, Simons and Richardson, 1963) that resistance to flow may be quite different in flumes of different size, other conditions being the same.

Prediction of Form of Bed Configuration

Resistance to flow depends on the form of the roughness elements on the bed. With these roughness elements dependent on the bed material, fluid, flow and channel characteristics it should be possible to develop a relationship between the fluid and flow characteristics and the bed material characteristics from which the bed configuration can be predicted. Knowledge of the bed configuration would simplify the determination of the resistance to flow.

Various relationships between the variables have been proposed for predicting the bed form (Albertson and others, 1958, Simons and Richardson, 1964, Gardi, 1959) but none have been completely satisfactory. A relationship between stream power $U\gamma DS$ the median fall diameter d_{50} of the bed material and bed configuration developed by Simons and Richardson (1964) provides a suitable method of predicting the bed forms. The relationship is given in figure 5. The lines separating the different bed forms are based on a study of the following data:

1. Flume data from the 8 ft flume
2. Elkhorn River (Beckman and Furness, 1962)
3. Rio Grande near El Paso (Fahnestock, personal communication)
4. Rio Grande at Cochiti, near Bernalillo, at Angostura heading (Culbertson and Dawdy, 1964)
5. Punjab canal data (Simons, 1957)
6. West Pakistan (1963) canal data.

CHAPTER IV

ANALYTICAL CONSIDERATIONS

In this section the equations which describe the velocity distribution and those for predicting the resistance to flow are developed on the basis of the turbulent flow Reynolds equation simplified for the steady uniform flow case. Even with the simplification of steady uniform flow there are more unknowns than equations to define them. Therefore, recourse is made to dimensional analysis and experimentation in order to obtain useful equations. Dimensional analysis is a valuable tool in that it will reduce the number of terms, make the results applicable to any system of units employed and will systematize the analysis of data.

Reynolds Equation

The Reynolds equation for turbulent flows of an incompressible fluid

$$\rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (\mu \frac{\partial u_i}{\partial x_j} - \rho \overline{u'_i u'_j}) \tag{5}$$

where

u_i = mean velocity in the i^{th} direction

F_i = body force in the i^{th} direction

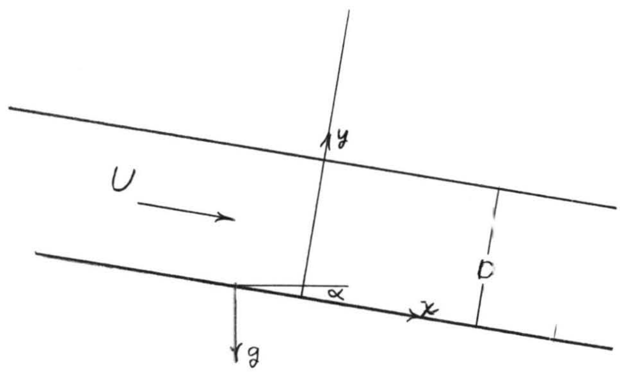
u'_i = the turbulent velocity fluctuations in the i^{th} direction

u, v, w = the velocity components in the x, y, z directions

$\overline{u'_i u'_j}$ = Reynolds stress

In the x direction with constant fluid properties this equation is

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho F_x - \frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] - \rho\left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z}\right] \tag{5a}$$



For the steady, uniform two-dimensional open-channel flow illustrated

$$u = u_{(y)},\quad v = w = 0,\quad \rho\,\overline{u_i'u_j'} = \rho\,\overline{u_i'u_j'}_{(y)},\quad \frac{\partial}{\partial t} = \frac{\partial}{\partial z} = \frac{\partial}{\partial x} = 0$$

then the Reynold's equation becomes

$$0 = \rho\, g \sin\alpha + \mu\frac{\partial^2 u}{\partial y^2} - \rho\frac{\partial \overline{u'v'}}{\partial y} \qquad x$$

$$0 = -\rho\, g \cos\alpha - \frac{\partial p}{\partial y} - \rho\frac{\partial \overline{v'^2}}{\partial y} \qquad y \tag{6}$$

$$0 = 0 - \rho\frac{\partial \overline{v'w'}}{\partial y} \qquad z$$

The boundary conditions for equation 6 are

$$y = D;\quad \overline{u'v'} = \overline{v'^2} = \overline{v'w'} = p = 0$$

$$y = 0;\ u = \overline{u'v'} = \overline{v'^2} = \overline{v'w'} = 0,\ \tau_o = \mu \frac{du}{dy}$$

Also $\tau_o = \gamma D \sin \alpha$ in steady uniform flow.

From the z component of the Reynold's equation and the boundary conditions $\overline{v'w'} = 0$ for all y. That is,

$$\frac{\partial \overline{v'w'}}{\partial y} = 0, \text{ Therefore } \overline{v'w'} = \text{constant.}$$

But at $y = 0$, $\overline{v'w'} = 0$, therefore $\overline{v'w'}$ is 0 for all y.

Integrating the y component

$$p = \gamma \cos \alpha (D - y) - \rho \overline{v'^2}_{(y)} \tag{7}$$

Integrating the x component

$$\mu \frac{\partial u}{\partial y} = -\gamma \sin \alpha\, y + \rho \overline{u'v'} + \gamma D \sin \alpha \tag{8}$$

For small angles of α, $\sin \alpha$ is approximately equal to $\tan \alpha$ the slope of the bed S. With this change equation 8 becomes

$$\gamma DS (1 - y/D) = \mu \frac{du}{dy} - \rho \overline{u'v'} \tag{8a}$$

Equation 8 is as far as the integration of the Reynolds equation can be carried without further knowledge or hypothesis with regard to the Reynolds stress. Various phenomenalogical hypotheses have been advanced concerning the Reynolds stress which have allowed the integration of equation 8.

Some of these hypotheses are as follows

Boussinesq's (1877) eddy viscosity theory

$$\overline{u'v'} = - \xi \frac{du}{dy} \tag{9}$$

Prandtl's (1925) mixing length theory

$$\overline{u'v'} = -l^2 \left|\frac{du}{dy}\right| \frac{du}{dy} \tag{10}$$

Taylor's (1932) vorticity transport theory

$$\frac{\partial \overline{u'v'}}{\partial y} = -l^2 \frac{du}{dy} \frac{d^2u}{dy^2} \tag{11}$$

If l is assumed independent of y then

$$\overline{u'v'} = -\frac{1}{2} l^2 \left|\frac{du}{dy}\right| \frac{du}{dy}$$

Von Karmán's (1930) similarity hypothesis

$$\overline{u'v'} = -k^2 \left(\frac{du}{dy}\right)^4 / \left(\frac{d^2u}{dy^2}\right)^2 \tag{12}$$

where

ξ = a turbulent exchange coefficient

l = is the mixing length for the transferred quatity

k = Empirical dimensionless constant. It is a universal steady uniform flow over a rigid boundary.

The assumptions made in equations 9, 10, 11 and 12 all have serious limitations and inconsistencies. One of the most important limitations is a lack of generality. Although Von Karman's hypothesis is more general than the others, and Prandtl's hypothesis is applicable for steady uniform flow (Pai, 1957, Schlichting, 1960).

With any of the preceding hypotheses equation 8 cannot be integrated to obtain the velocity distribution without further simplification. Hence, the assumption is made that the laminar friction is negligible compared with the Reynolds stress (Schlichting, 1960, p. 477, 514). This assumption has been shown to be valid except in the region close to the boundary by the experiments of Reichardt and Laufer (Schlichting, 1960, p. 466).

With this assumption, equation 8 becomes

$$\gamma DS\ (1 - y/D) = -\rho\ \overline{u'v'} \tag{13}$$

Prandtl, in order to integrate equation 13, assumed further that $l = ky$ and that the shearing stress was constant throughout the flow at its value at the wall. With these simplifications, he obtained for the velocity distribution

$$u = \frac{U_*}{k} \ln y + C \tag{14}$$

Assuming a linear relation, Von Karman was able to integrate equation 13 to obtain a velocity defect equation which was also logarithmic.

The universal constant k in Prandtl's equation and in Von Karman's equation are identical (Schlichting, 1960, p. 490). In rigid boundaries k is a "universal constant" with a value of about 0.40. In alluvial channels k is not constant but is a function of the flow, sediment transport, and the bed configuration. The variation of k for alluvial channels will be discussed later.

Velocity Distribution Equation

Until the relation of $\overline{u'w'}$ with the flow, fluid and channel characteristics is available from experimental data, any assumption or theory concerning the relation will have its limitations. With the present state of knowledge, the validity of any assumption depends on it reproducing the observed logarithmic velocity distribution.

Assume that

$$\overline{u'v'} = \left(\frac{1}{A} y \right)^2 (1 - y/D) \left(\frac{du}{dy}\right)^2$$

In terms of mixing length theory this assumes that

$$l^2 = \left(\frac{y}{A}\right)^2 (1 - y/D)$$

That is, the mixing length has a non-symmetrical distribution, is 0 at the boundary and at the water surface, and has its maximum value in the upper half of the flow. This assumption regarding $\overline{u'v'}$ is as valid as any other as long as the functional relation for $\rho \overline{u'v'}$ is unknown and the resulting equation describes the velocity distribution.

With this assumption equation 13 becomes

$$\frac{\tau_o}{\rho} (1 - y/D) = \left(\frac{y}{A} \right)^2 (1 - y/D) \left(\frac{du}{dy}\right)^2$$

or

$$\frac{du}{dy} = \sqrt{\frac{\tau_o}{\rho}} \; \frac{A}{y} \tag{15}$$

and

$$u = AU_* \ln y + C \tag{16}$$

If $y = \xi/B$ when $u = 0$ then

$$\frac{u}{U_*} = A \ln y/\xi + B \tag{17}$$

where A and B are constants to be evaluated from experimental data. For rigid boundary hydraulics A is 1/k.

Resistance to Flow Equation

An equation for resistance to flow in terms of the dimensionless Chezy coefficient of discharge can be derived by integrating equation 17 over the flow depth. That is

$$U = \frac{1}{D}\int u \quad dy \tag{18}$$

Substituting the value of u from equation 17 into equation 18 and integrating

$$\frac{U}{U_*} = \frac{C}{\sqrt{g}} = A \ln \quad D/\xi \quad + \quad (B - A) \tag{19}$$

Experimental evidence has shown that $(B - A)$ in equation 19 is a function of the geometry of the cross section (Sayre and Albertson, 1963) and that for three-dimensional flows $U_* = \sqrt{g\ RS}$ where R = the hydraulic radius. Some investigators also substitute R for the depth in the relative roughness term D/ξ. Another variation of equation 19 is to combine the $(B - A)$ term (Sayre and Albertson, 1963) to obtain

$$\frac{C}{\sqrt{g}} = A \ln D/\chi \tag{20}$$

Dimensional Analysis

Dimensional analysis of the variables which affect the bed form, velocity distribution and resistance to flow may contribute to an understanding of the problem and systematize the analysis. The variables involved with resistance to flow in an alluvial channel are listed in equation 2. This list can be further simplified by eliminating the shape factors for the reach and cross section and the seepage force as they are constant for the 8 ft flume data. Also the gradation of the bed material is eliminated because the data are not suitable to delineate its effect. With these simplifications equation 2 becomes

$$q = \phi \left[y,\ D,\ S,\ d,\ \omega,\ g,\ \rho \right] \tag{21}$$

where q may be either the velocity at a point or the mean velocity and d is some significant size of the bed material not necessarily the d_{50}. The variable y must be included for the velocity distribution but is eliminated for the mean velocity.

From 21 using the π theorem

$$\frac{q}{U_*} = \phi \left[S,\ \frac{d}{D},\ \frac{\omega}{U_*},\ \frac{y}{D} \right] \tag{22}$$

There are many dimensionless combinations of variables which could be obtained by rearranging equation 22. For example

$$\frac{q}{U_*} = \phi \left[\mathbb{F},\ \frac{D}{d},\ \frac{\omega}{U},\ \frac{y}{d} \right] \tag{23}$$

However, rearranging equation 22 without insight and study of the data would be a sterile exercise.

If bed form is substituted into equation 21 for one of the variables, say the fall velocity, then equation 22 becomes

$$\frac{q}{U_*} = \phi \left[S,\ \frac{D}{d},\ \text{bed form},\ \frac{y}{D} \right] \tag{24}$$

Again there are many possible variations of equation 24. It is proposed in the next sections to analyze the velocity profiles and resistance to flow by grouping the data for the different bed forms. This will be done even though the bed forms are a function of the fluid, flow, and bed material characteristics. Studying only the data for a given bed form will simplify the analysis considerably.

Finally, it may be necessary to include the viscosity in the list of variables in addition to the fall velocity. This may be true if the flow is not fully developed rough turbulent flow, or the fall velocity in quiescent fluid may not account for all the viscous effects of the interaction between a turbulent fluid and the sediment. In this case the Reynolds number in one of its various forms VD/ν , or U_*D/ν should be included in equations 22, 23, or 24 or variation thereof. The general dimensionless equation, variations of which will be studied in the analysis of data, becomes

$$\frac{q}{U_*} = \phi \left[\text{bed form, S}, \quad \frac{D}{d}, \quad \frac{U_* D}{\nu}, \quad \frac{y}{d} \right] \tag{25}$$

Summary

The velocity distribution for steady, uniform, two-dimensional open-channel flow was derived by integrating Reynolds equation for turbulent flow. The assumptions, in addition to those of the preceeding sentence, were: (1) constant fluid properties, (2) viscous stresses are very small in comparison to the Reynolds stress, (3) the angle the flow makes with the horizontal α is small so that sin α = tan α, and (4) the Reynolds stress varies as

$$\rho \, \overline{u'v'} = \rho \left(\frac{y}{A}\right)^2 (1 - y/D) \left(\frac{du}{dy}\right)^2$$

The derived velocity distribution equation is

$$\frac{u}{U_*} = A \ \ln y/\xi \ + B \tag{17}$$

The Chezy discharge coefficients for steady uniform flow derived by integrating equation 17 is

$$\frac{C}{\sqrt{g}} = \frac{U}{U_*} = A \ln D/\xi \quad + \quad (B - A) \tag{19}$$

or

$$\frac{C}{\sqrt{g}} = A \ln D/\chi \quad + \quad (B - A)$$

The term ξ will be used to denote grain roughness only and χ denotes the combination of grain and form roughness.

The evaluation of the constants in equation 17 and 19 will be accomplished by studying the data for each of the bed forms (plane bed, ripples, dunes, antidunes). The dimensionless equation 25 will be used as a guide in the analysis.

CHAPTER V

ANALYSIS OF DATA

In this section the velocity distribution data will be analyzed in order to determine the constants in equation 17 developed in the preceding section. The equations for the velocity distribution and the measured variables that describe the flow, fluid and bed material characteristics will then be used to determine resistance to flow equations. The equations will be developed by studying the flow conditions and measured data for the different bed configurations.

Velocity Distribution

The distribution of horizontal components of velocity in the vertical for flow over a sand channel is as variable as the bed form. With dunes and antidunes, the velocity distribution is constantly changing in time and space. This variation is so large that a statistical study of the averages and variance of the velocity at a point would be needed to define the distribution. With plane bed or a ripple bed the distribution is constant with time and space over most of the depth; and, as with flow over a rigid boundary, the distribution is a logarithmic function of depth. However, for a plane bed, the velocity distribution is different when the bed is moving than when it is static. Also, for both ripples and plane bed Von Karman's constant k is not a universal constant as it appears to be for the rigid boundary case.

Typical velocity distributions for the ripple, dune and plane bed with sediment movement are given in figure 6. The velocity distributions are discussed in more detail in the following sections.

Plane Bed

The point velocities were plotted vs log y to evaluate the constants in equation 17 for the plane bed runs with sediment movement

$$\frac{u}{U_*} = A \ln y/\xi + B \tag{17}$$

From these plots the slope A_* and intercept B_* were determined for the equation

$$u = A_* \ln y + B_* \tag{26}$$

Equating coefficients for the two equations describing the u vs $\ln y$ relation

$$A = A_* / U_* \tag{27}$$

$$B = B_* / U_* + A \ln \xi \tag{28}$$

The ratios A_*/U_* and B_*/U_* can be determined by plotting A_* vs U_* and B_* vs U_* and determining the equation of the line, figures 7 and 8.

The value of A from figure 7 is 3.2 and is equivalent to a k value of 0.31. This value of A is considerably larger than the value of A of 2.5 (k equal to 0.4) for flow over a rigid boundary. There is considerable scatter of the points around the line $A = 3.2$ in figure 7. However, a careful study of the data for each of the plane bed runs showed that the scatter was random. The value of A did not have a systematic variation with depth, energy gradient, size of bed material or bed material concentration.

As noted earlier Vanoni (1946), Einstein and Chien (1954), Ismail (1952), Vanoni and Brooks (1957), and Elata and Ippen (1961) also found that A was larger for flow over a moving sand boundary. They attributed the increase in A to the effect of the moving layer on the turbulence of the flow, and proved that size, concentration, and density of the bed material transport affect A. The magnitude of the increase in A for this data is similar to that noted by the above investigators. However, contrary to the conclusions of Vanoni, Ismail, Einstein and Chein and others A for these data was a constant.

That the value of A for these data was a constant, in contrast to the conclusions of other investigators, may result from:

1. Some of Vanoni's and Ismail's velocity observations, as Laursen and Pin-Nam Lin (1952) pointed out, were for bed configurations other than plane bed. Nordin (1963) showed that variations in the bed configuration can change A considerably.

2. The shallow depths investigated by the other experimenters may have resulted in the entire velocity profile being measured in a layer of extremely large sediment concentration. As Einstein and Chien (1957) have indicated and as will be shown later (fig. 10) the slope of the velocity profile in this layer is extremely variable. Also, the slope in this layer changes with sand size, concentration and shear stress.

3. With experiments conducted under equilibrium plane-bed flow conditions, the concentration of bed-material transport depends on the magnitude of the shear velocity. Also, the occurrence of a plane bed depends on the magnitude of the shear velocity. Similarly, A depends on the shear velocity and the slope of the u vs ln y

relation. If the increase in shear velocity, that causes an increase in sediment transport, causes a corresponding increase in the slope of u vs $\ln y$ relation, or if the change in U_* for a plane bed when the size of the bed material changes results in a corresponding change in the slope of the u vs $\ln y$ relation, then A would be constant.

To determine B and ξ in equation 19, some assumptions regarding either one or both of them must be made in addition to determining the value of B_*/U_*. Note that B_*/U_* is a function of the bed material size. When B was assumed to equal A, it was found that ξ equaled the d_{85} value of the bed material, Table 10.

The assumption that B equals A, and the indications that $A = 3.2$ and $\xi = d_{85}$ was checked by plotting u/U_* vs $\log y/\xi$, figure 9. Despite the wide band of the data the slope of the relation A is equal to 3.2 and the intercept B equals 3.2. Therefore, the assumption that B equals A is valid and the value of A as found from figure 7 is verified. Other assumptions regarding B or ξ may be equally valid but any change in the assumed value for B will alter the value of ξ. Other assumptions regarding B and ξ were tested but the assumption that $B = A$ was the only one that gave a constant ξ. For instance, it was assumed that ξ was the d_{50} bed material size and with this assumption B was a variable.

In figure 9 and 10a there is a very definite change in the slope A of the velocity distribution curves in the vicinity of the bed. The value of A is as large as 11.7 which represents a k value of 0.085 in the region close to the boundary. The magnitude of the increase in A in figures 9 and 10a appears to be a function of the sand size. For the 0.19 mm sand there is not a discernable change in A; with the 0.27 and 0.28 mm sand there is a slight change; and with the 0.45, 0.47 and 0.93 mm sand there is a large change. In addition to the change in A there is a large variation of A in this zone close to the boundary. This zone close to the boundary where there is a large variation in the slope of the velocity profile is called the inner zone. The zone away from the boundary when A is constant is called the outer zone.

The value of A varied from 2.5 to 11.7 or k from 0.4 to 0.085 for the two coarser sands in the inner zone. In the outer zone A was a constant equal to 3.2.

The variation of A with sand size appears contrary to the findings of Vanoni and Brooks (1957, p. 92), where they had larger values of A with the finer bed material. However, the change and variation of A in this inner zone is a function of concentration of the bed material in addition to the size. With the finer sands in the 8 ft flume the concentration of bed material is much lower than the concentration of the coarse bed material. The bed-material concentration in the inner zone for the 0.19 mm sand ranged from 900 to 1,600 ppm; whereas, for the 0.93 mm sand it ranged from 4,500 to 8,000 ppm. The reasons for the lower concentrations in this inner zone with fine sands as the bed material are the lower shear stresses required for the occurrence of a plane bed and the formation of antidunes for fine sands. The value of the shear stress for the plane bed in the 0.19 mm sand ranged from 0.055 to to 0.07 lbs per ft vs 0.27 to 0.31 lbs per ft for the 0.93 mm sand. Breaking antidunes formed in the 0.19 mm sand at shear stresses of 0.08 lbs per ft but did not form in the 0.93 mm sand at the maximum shear stress attainable in the 8 ft flume (0.31 lbs per ft).

The reasons for the change and variation of A in the inner zone are twofold.

1. There is an effect of the size, concentration and density of the sediment particles on the turbulence of the flow. This effect has been well documented by Vanoni (1946), Ismail (1952), Vanoni and Brooks (1957) and Elata and Ippen (1961). The lack of measurements of the turbulent characteristics of the flow has prevented the exact determination of the mechanism of the effect.

2. There is an inner granular shearing stress resulting from the interaction of the sediment grains (Bagnold, 1954). This inner granular stress reduces the fluid shearing stress in the inner zone.

The size of the inner zone is also a function of the bed material size. A study of the velocity profiles indicated that the inner zone was about 0.1 ft in depth for the 0.93 mm sand, 0.07 for the 0.45 and 0.47 mm sand, and 0.04 for the 0.27 and 0.28 mm sand.

The inner zone represents only 10 to 20 percent of the total depth of flow in the 8 ft flume. In the outer zone, which represents 80 to 90 percent of the flow depth, the slope A was a constant, although, a different constant than for the plane bed with little movement. This large outer zone for the studies with large depth in contrast to the small outer zone in studies with shallow depths accounts for the fact that A is a constant in figure 7.

The difference in the velocity distribution for the plane bed with little or no sediment transport and the plane bed with large sediment transport is illustrated in figure 10. For the plane bed with small sediment transport, $A = 2.55$, $B = 8.0$, $k = 0.39$, whereas, for the plane bed with large sediment transport, $A = 3.2$, $B = 3.2$, $k = 0.31$. The larger scatter of the velocity profile data for the large sediment transport case in comparison to the small scatter for the little or no transport case is typical and results from the difficulty in measuring the velocity when there are large concentrations of bed material suspended in the flow. The concentration of bed material discharge for the small sediment transport ranged from 0 ppm to 26 ppm in comparison to 900 ppm to 10,000 ppm for the large. The small transport rates have no effect on A and B and the values are comparable to those for rigid boundaries.

Ripples

As for the plane bed the velocity distribution for the ripple bed configuration followed the logarithmic law. To determine A, B and χ in equation 17 all point velocities were plotted versus the log y. From these plots the slope A_* and intercept B_* in equation 26 were determined. As for the plane bed study, the A_* and B_* values were plotted against the appropriate U_* value, figures 11 and 12. The values of A and B from figures 11 and 12 are

$$A = 3.33 - \frac{0.13}{U_*} \tag{29}$$

$$B = 14.3 + (3.33 - \frac{0.13}{U_*}) \ln \chi \tag{30}$$

Figures 11 and 12 and equations 29 and 30 indicate that k and the roughness vary with the shear velocity. The reason for this dependency results from the fact that the height, length and spacing of the ripples depends on the interaction of the fluid and the bed material. The turbulence of the flow that determines A is a function of the form of the boundary. But the form of the roughness and χ are dependent on the fluid turbulence. This interaction between the turbulence and the roughness depends on the shear stress. Thus, A and the roughness are dependent on the shear velocity a flow parameter.

The scatter in figures 11 and 12 is not exceptionally large for sediment data. One cause of the scatter for the ripple velocity data is that the velocity profiles were not measured from a common base. That is, in any given profile the distance y above the bed may have been from the ripple crest and in another profile from the ripple trough. The slope and intercept of the velocity profile are significantly changed when the measurement base is changed.

A significant aspect of figure 11 (also of figure 7) is the fact that the slope of a line from the origin to a data point represents A and consequently 1/k for that run. The variation in A for the ripple runs was from 0.96 to 3.16 or k from 0.29 to 1.04. This variation in k is the result of the bed roughness and not the effect of the suspended sediment. There is little or no suspended sediment with the ripple runs. Note the clear water for ripples in figure 3a.

The variation in A_* and B_* were studied extensively to determine if a dependency on depth, slope or bed material concentration and size existed. There was no apparent systematic variation with any of these variables.

Equation 30 contains two unknowns so some method is needed to determine χ independently of the velocity profiles in order to establish B. One method of establishing χ is figure 13 where $C/\sqrt{g}$ is plotted against log D. The best fit line through the data is

$$\frac{C}{\sqrt{g}} = 10.7 \log D/\chi \qquad (31)$$

In this equation χ equals 0.06 and A is a constant equal to 4.64. This result is in conflict with the values determined from a study of the velocity profiles. Other methods could be used to determine values of B or χ with the same result of A being a constant. B could be assumed equal to zero from which

$$\ln \chi = - \frac{14.3}{(3.33 - 0.13/U_*)}$$

and

$$\frac{u}{U_*} = (3.33 - \frac{0.13}{U_*}) \ln y + 14.3 \qquad (32)$$

However, for the integrated velocity distribution equation for the determination of the resistance to flow, it was not necessary to assume a value for B or χ as will be shown later.

Dunes

The velocity distribution for dunes is constantly changing in time and space as the bed configuration changes in time and space. The variation of the velocity profile with space is illustrated in figure 14. The data for this figure was obtained by measuring the velocity profiles in the downstream direction over the crest and troughs of two dunes. Note that when the flow is accelerating over the back of a dune the u vs log y curve is almost vertical. Also note, the erratic nature of the curve over the dune troughs.

In addition to the space variation of the velocity, there is a time variation at each vertical. With this variation in time and space, a large number of velocity profiles would have to be analyzed to obtain an average profile. These data are not available. However Sayre and Albertson (1963) showed, with the analysis of baffle roughness, that the average u vs log y curve was logarithmic. Presumably the same would be true for the dune bed.

Antidunes

The velocity profiles for antidunes also vary with time and space. At one time and particular position in space the profile would be similar to the plane bed with sediment movement. At another time or position space, when the antidune is breaking, the velocity profile would represent the slow downstream movement of an extremely turbulent fluid. Antidunes represent a completely unsteady, non-uniform flow condition from which it is not expected that even an average velocity distribution would have meaning.

Summary

The velocity distribution for plane bed with and without appreciable sediment transport and for the ripple bed can, with a suitable choice of coefficients be represented by equation 17

$$\frac{u}{U_*} = A \ln y/\xi + B \qquad (17)$$

For plane bed without appreciable sediment transport

$$A = 2.55$$

$$\xi = d_{85}$$

$$B = 8.0$$

For plane bed with appreciable sediment transport

$$A = 3.2$$

$$\xi = d_{85}$$

$$B = 3.2$$

For ripples

$$A = 3.33 - 0.13/U_*$$

$$B = 14.3 + (3.33 - 0.13/U_*) \ln \chi$$

$$\xi = \chi$$

Values could be assumed for B and then determine χ or for χ and then determine B. However, as will be shown in the next section both B and χ are eliminated from the integrated velocity equation.

The velocity profiles for dunes and antidunes vary with time and space. It is assumed that an average velocity distribution can be defined for flow over dunes and that it would be logarithmic. For antidunes, because they represent the extreme unsteady nonuniform case, it is doubtful that even a statistical velocity distribution would be of value.

The difference between the velocity distribution for ripples and dunes is another proof that there is a fundamental difference between the two bed configurations.

Resistance Coefficient

Resistance to flow in sand channels is the result of <u>grain roughness</u>, <u>form roughness</u>, <u>acceleration and deceleration of the flow</u>, and breaking waves. Only the first two types may occur in steady uniform flow. Accelerating and decelerating flow is always nonuniform and often unsteady, and breaking waves are always nonuniform and unsteady.

Flow over a plane bed has a grain roughness type of resistance to flow. Also, antidunes have grain roughness but in addition they have the unsteady nonuniform effects of acceleration and deceleration of the flow, and breaking waves. Flow over ripples has predominantly form roughness type of resistance to flow. There may be some grain roughness effects but these experiments and those of Knoroz (1959) indicate that grain roughness is small. Flow over dunes has both form roughness and grain roughness types of resistance to flow. In addition flow over roughness elements of the magnitude of dunes, may not be steady and uniform. That is, the size of the dunes with respect to the size of the system (depth and width) may be so large that there is appreciable acceleration and deceleration of the flow. In this case the dunes represent changes in cross section rather than surface irregularities. Flow over dunes is unsteady, even when the discharge is steady, because the moving boundary is causing a change in the velocity at a point with time. This unsteadiness is small and of no consequence for the mean resistance but is a factor in studying the velocity distribution. The problem of nonuniformity and unsteadiness of the flow with dunes and antidunes is further augmented by the fact that with an increase in depth the flow becomes more steady and uniform. Thus, the uniformity and steadiness of the flow is a function of the depth.

In addition to the problems of nonuniform and unsteady flow, the spacing, amplitude, and shape of the roughness elements of the cross section of the channel varies with the fluid, flow and bed material characteristics. Also, with three-dimensional flow, there may be multiple roughness such as dunes on part of the cross section or length of the reach and ripples, plane bed or antidunes on another part of the bed or any combination of these bed configurations.

In addition to multiple roughness, for flow in the transition between the upper and lower flow regimes, the bed flow may oscillate between dunes and plane bed or antidunes. Or if this oscillation doesn't occur, whether the bed configuration is dunes or plane or antidunes will depend on the preceding flow conditions and the rate of change of the flow with time.

With all the problems associated with flow in sand channels (the preceding two paragraphs do not exhaust the possibilities, for instance the sediment being transported by the flow affects the fluid and turbulence characteristics) it is difficult and possibly impossible to write a general functional relationship to describe resistance to flow in sand channels. Also, it probably will be impossible to determine a resistance to flow equation that will determine the velocity within 10 percent under controlled laboratory conditions or 20 to 30 percent for natural channels. Nevertheless, the engineer is faced with the problem of determining the slope and cross section of a conveyance channel to carry a known quantity of water in a sandy material. Or, for natural streams the engineer must determine the quantity of water when only the slope, bed material and cross section are known. The measurement of the slope cross section, and bed material are all subject to error in the natural stream, and, for the conveyance channel, they are subject to change with time as the flow alters the designed channel.

In both the design of conveyance channels and the determination of the flow in a natural stream there will be some knowledge of what the bed configuration should be or was for a given rate of flow. For a conveyance channel, other considerations such as amount of sediment transport, bank stability, etc., will dictate whether ripples, dunes, plane bed or antidunes should be the bed configuration. If the resistance to flow for the desired bed configurations is known, then the channel can be designed, the stream power $\tau_o U$ computed and figure 5 can be used to verify that the desired bed configuration will occur. For the natural channel it is often possible from a study of the gage-height trace, stage-discharge relation, or visual observation of the flow to determine the bed configuration. If the bed configuration cannot be determined it can be assumed and if the resistance to flow is known for that bed configuration the velocity can be computed and figure 5 used to check the validity of the assumed bed configuration.

In this section equations and methods are developed for estimating the resistance to flow. The methods are based on using bed configuration as one of the independent variables. As explained in the preceding paragraph, this information is attainable. The method for estimating resistance to flow for the plane bed is to determine the coefficients in equation 19 which was developed from the Reynolds equation. Two methods of estimating resistance to flow for the ripple and dune-bed configurations are developed. One method depends on defining the coefficients in equation 19. The other method, which is also used for antidunes, is to determine a correction term to be applied to the plane bed equation to compensate for the increase in resistance to flow caused by form roughness, flow acceleration and deceleration, and wave breaking. (Hereafter, all of these effects will be referred to as form roughness effects). The correction term and relations for determining it would be different for the three types of bed configuration.

There is some concern about using equations developed assuming steady uniform flow, for unsteady nonuniform flow. Also, there are complications caused by multiple roughness and the transition. However, if space and time averages are used it is possible to determine a gross resistance equation for unsteady nonuniform flow. There will be an error in the results and the equations must be used with care. Also, additional unsteadiness or nonuniformity of the flow which does not arise from the interaction of the fluid and bed material, may have adverse affects on the flow. Additional unsteadiness or nonuniformity may result from changes in discharge with time or from a bend in the channel.

Correction for Form Roughness Effects

The working hypothesis for the development of a correction term to correct the plane-bed equation for the increase in energy dissipation resulting from form roughness is: The increase in resistance to flow as a result of form roughness causes an increase in the product of the hydraulic radius and slope without changing the velocity. That is

$$RS = (RS)' + \Delta RS \tag{33}$$

where

RS = the product of the measured slope and hydraulic radius of the flow.

(RS)' = the product of the hydraulic radius and slope for a plane bed with grain roughness with the same velocity as the flow with form roughness.

ΔRS = the increase in RS resulting from the form roughness.

The equation for determining (RS)' is

$$\frac{C}{\sqrt{g}} = \frac{U}{\sqrt{g\ (RS)'}} = 7.4 \log D/d_{85} \tag{34}$$

or

$$(RS)' = \frac{1}{g} \left(\frac{U}{7.4 \log D/d_{85}}\right)^2 \tag{35}$$

The resistance equation is

$$\frac{C}{\sqrt{g}} = 7.4 \log D/d_{85} \quad C_* \tag{36}$$

where

$$C_* = \sqrt{\frac{g(RS - \Delta RS)}{gRS}} = \sqrt{1 - \frac{\Delta RS}{RS}}$$

C_* is the correction term for the form roughness effects. The functional equation for the study of ΔRS and C_* is

$$\Delta RS \text{ or } C_* = \phi \left[R,\ S,\ \text{Bed form},\ d,\ \rho,\ g \right] \tag{37}$$

The variation of C_* with the shear stress, bed configuration and depth is illustraged in figures 15, 16, 17 and 18. These figures show the difficulty, if not the impossibility, of writing a general equation for the resistance to flow in sand channels. In addition to the large differences in resistance to flow between the bed forms; there is the variation in bed configurations in the transition from the lower to the upper flow regime. In the transition for the same value of shear, the bed form may be dunes, washed-out-dunes, plane or antidune. Consequently resistance to flow and thus C_* has a large variation. The range in shear stress for the transition is larger in finer bed material. With the 0.93 mm sand, the range in shear stress for the transition is small. This indicates that coarse bed material will not have as much variability in roughness as the fine bed material.

It was the transition between the upper and lower flow regime that Brooks (1958) was describing when he concluded that there is no functional relation between shear stress and resistance to flow. However, figures 15 through 18 show that if the bed form, size of bed material and the shear stress on the bed are known, resistance to flow can be determined. That is, if the bed form is known then it should be possible to write a general equation for the resistance to flow applicable to that bed form and sand size. In the following sections the resistance to flow is analyzed for the different bed forms.

Plane Bed

The general equation for steady uniform flow developed from the Reynolds equation was

$$\frac{C}{\sqrt{g}} = A \ln D/\xi + (B - A) \tag{19}$$

From the velocity distribution study of the plane bed with sediment movement it was determined that

$A = 3.2$

$B = 3.2$

$\xi = d_{85}$

With these values equation 19 becomes

$$\frac{C}{\sqrt{g}} = 3.2 \ln D/d_{85} = 7.4 \log D/d_{85} \qquad (38)$$

The validity of equation 38 is illustrated in figure 19 where $C/\sqrt{g}$ is plotted as a function of $\log D/d_{85}$ for the plane bed. The value of A and ξ as determined from the velocity profile data are equally valid for the resistance to flow equation. The linear relation $C/\sqrt{g}$ and D/d_{85} in figure 19 shows that flow over a plane bed with sediment transport is fully developed rough turbulent flow.

Equation 38, which was developed from a study of the velocity distribution and confirmed by the average flow conditions, defines the resistance coefficient for the plane bed with sediment transport in sand channels. However, the value of B for the 8 ft flume data may not hold for other flumes, streams, or channels because B is a function of the cross section. For other cross sections the value of B would have to be determined.

The values of A and B and ξ for plane bed with little or no sediment movement were

$A = 2.56$

$B = 8.0$

$$\xi = d_{85}$$

With these values equation 19 becomes

$$\frac{C}{\sqrt{g}} = 2.56 \ln D/d_{85} + 5.44 \tag{39}$$

or

$$\frac{C}{\sqrt{g}} = 5.9 \log D/d_{85} + 5.44 \tag{40}$$

Ripples

Flow over a ripple bed may be considered steady and uniform. The roughness elements do not cause an appreciable acceleration and deceleration of the flow and may be considered as surface irregularities and not as changes in cross section. Although the irregularities on the bed change with time this change is so slow that it does not represent unsteady flow.

The values of A and B in equation 19 were defined from a study of the velocity profiles as

$$A = (3.33 - 0.13/U_*) \tag{29}$$

and

$$B = 14.3 + A \ln \chi \tag{30}$$

Substituting these values in equation 19 the resistance coefficient for flow over ripples is

$$\frac{C}{\sqrt{g}} = (3.33 - 0.13/U_*) \left[\ln D - 1\right] + 14.3 \tag{41}$$

The elimination of χ from equation 41 was not unexpected. The characteristics of ripples are functions of the flow but independent of the size of the bed material. Because χ is a function of the ripple characteristics, it was possible to eliminate it from equation 19 if a flow variable (in this case U_*) was substituted for it. The accuracy of equation 41 for computing the average velocity is indicated in figure 20.

Another method of determining the coefficients in equation 19 is to plot $C/\sqrt{g}$ vs the log D, figure 13. From this figure the value of χ is 0.06 ft and A = 4.64. That is equation 19 becomes

$$\frac{C}{\sqrt{g}} = 10.7 \log D/0.06 \tag{42}$$

In this case a constant value of k and χ results instead of a flow variable U_* as in equation 41. However, if a relation for k and χ in terms of a flow variable was determined then they could be eliminated from the equation. The accuracy of equation 42 for computing the average velocity is indicated in figure 21.

A third method of determining the resistance coefficient is to correct for the effect of form roughness using ΔRS. To determine a relation for predicting ΔRS, the value of ΔRS for ripple runs was computed using equation 33 and 35. The values of ΔRS so determined were studied using equation 37. This study showed that ΔRS was independent of the bed-material size and was related to RS, figure 22. Therefore using figure 22 and equation 36, the value of $C/\sqrt{g}$ can be determined.

$$\frac{C}{\sqrt{g}} = 7.4 \log D/d_{85} \sqrt{1 - \Delta RS/RS} \tag{36}$$

The accuracy of this method for computing the average velocity is indicated in figure 23.

Although three methods of determining the resistance coefficient are presented, the first method, based on the velocity distribution is considered best. The first method is better than the second in that it is based on a variable k and χ both of which are known to vary with the flow. It is better than the third in that the third method is based on a rather arbitrary assumption that the form roughness effects can be considered only to affect the produce of RS and not the velocity.

Dunes

If the flow over a dune bed configuration is considered steady and uniform, that is, if dunes are considered roughness elements not changes in cross sections, then equation 19 can be used to determine resistance to flow, Unfortunately because of unsufficient data, the coefficients A, B and χ in equation 19 cannot be determined from a study of the velocity distribution. Neither did a study of the $C/\sqrt{g}$ as a function of log D indicate what the values of the coefficients in equation 19 should be. The reason is that the coefficients are such a complex function of the flow and bed material that it is impossible to sort the values of the three variables in such a simple plot.

With no direct measurement of the values of A, B and χ from the measured data, values of A and B were arbitrarily selected and χ computed. Then χ was studied as a function of the flow and bed material. The values for A and B were taken from Keulegan's (1938) paper. The equation is

$$\frac{C}{\sqrt{g}} = 5.75 \log R/\chi + 6.25 \qquad (43)$$

The relation between $C/\sqrt{g}$ and R/χ is given in figure 24; where $C/\sqrt{g}$ is plotted vs the Reynolds number with R/χ as a third variable. This format was used to illustrate that the flow is hydraulically rough. As expected there is a very good relation between $C/\sqrt{g}$ and R/χ for all 4 sands and the field data. However, the relation in figure 24 will not serve to determine resistance to flow unless χ is known.

The value of χ depends on the geometry of the roughness elements. That is, χ may be determined from measurements of the height, length, shape, spacing and pattern of the roughness elements. In functional form the equation is

$$\chi = \phi\ (H, L, d, \psi, \eta) \tag{44}$$

where

H = height of the dunes

L = length of the dune

d = some representative size of the bed material as a grain roughness

ψ = spacing parameter

η = shape parameter of the roughness elements

From rigid boundary studies (Keulegan 1930, Powel, 1946, Sayre and Albertson, 1963, and many others) it has been demonstrated that by measuring the characteristics of the roughness elements χ can be determined. Knowing χ the resistance to flow can be determined from equation 43. This is also true for sand channels when the characteristics of the dunes can be measured. Although, the lack of precision in determining the geometry of the dunes induces considerable error. Also, because dune geometry changes with a change in flow it is difficult to measure dune height, length, spacing and shape. However, because dune geometry is a function of the flow, fluid and bed material characteristics, it should not be necessary to measure the characteristics of the dunes. The value of χ should be related to the characteristics of the bed material, flow and fluid as well as the dune characteristics.

That is

$$\chi = \phi \left[S, R, d, \omega, g, \rho \right] \tag{45}$$

from which

$$\frac{\chi}{R} = \phi \left[\frac{U_*}{\omega}, \frac{R}{d}, S \right] \tag{46}$$

The relation of R/χ to the parameter in equation 46 was studied extensively. It was found that R/χ was a function of U_*/ω and R/d or of S and R/d but that neither considering S in the first relation or U_*/ω in the second improved either relation. Also there was a different relation for each sand size. The reason for this is that the length, height and spacing of the dunes has a different relation with shear stress for each sand size. With τ_o increasing the L/H ratio decreased for the 0.93 mm sand; decreased and then increased for the 0.45 and 0.47 mm sands; and had no systematic variation for the 0.19, 0.27 and 0.28 mm sands. There was considerable scatter of the L/H values for the 0.19 sand. Also for the 0.19 mm sand there is a very narrow range of shear stress within which dunes form, see figure 15.

The relation between R/χ , U_*/ω and R is given in figure 25. Because there is a separate relation for each sand size, values of R are equivalent to R/d. Therefore, R is used for convenience as the third variable in figure 25. There is a good relation between the three parameters for the 0.93, .45, and .47 mm sand. Neither depth nor slope will explain the scatter for the 0.27 and .28 mm sand, and there is a fair relation with depth for the 0.19 mm sand. The velocity computed using equation 46 and figure 25 is compared with the measured velocity in figure 26. The velocities for flow over dunes for the Elkhorn River and Rio Grande are predicted fairly well by the relation but not the velocities for the Mississippi River. This indicates that there is a depth effect that needs study under controlled conditions.

Another method of determining the resistance to flow for dunes is to correct the plane bed equation for the effects of the form roughness, using equation 36

$$\frac{C}{\sqrt{g}} = 7.4 \log D/d_{85} \sqrt{1 - \Delta RS/RS} \qquad (36)$$

It was anticipated that ΔRS would be a function of RS and the size of the bed material for a specified bed configuration. Figure 27 indicates that this is true. However, for the bed material with d_{50} finer than 0.5 mm the relation between ΔRS and RS was not a function of the bed material. The relation for the 0.93 mm sand is to the right of the relation for sands finer than 0.5 mm. Thus, for the same RS value the resistance to flow is less for the coarser sand.

The relation between ΔRS and RS developed from the flume data is valid for rivers when the size of the bed materials are the same. On figure 27 ΔRS vs RS for flow over dunes is plotted for the Elkhorn River near Waterloo, Nebraska and the Rio

Grande near Bernalillo, New Mexico.

The ΔRS vs RS relation developed from the flume data is valid within limits for the large depths of the Mississippi River, figure 28. The mean depth of the Mississippi at St. Louis ranges from 15 to 50 ft (Jordan, 1965). As observed in figure 28 the relation between ΔRS and RS is not linear above $RS = 0.20$. The change in the relation results from the fact that for the larger RS values the dunes are decreasing in height as the depth increases (Jordan, 1965). This decrease in height decreases resistance to flow with a corresponding decrease in ΔRS. The velocity computed using ΔRS and equation 36 is compared with the measured velocity in figure 29.

The good relation between ΔRS and RS and the broad range of conditions for which it is applicable makes the second method of determining resistance to flow the better of the two. This is true in spite of the arbitrariness in the hypothesis that all the form effects are incorporated in changes in R and S and not in U. The first method also involves an arbitrary selection of A and B. A future study where sufficient velocity profiles are taken to evaluate the coefficients in equation 19, may lead to a better resistance equation. Until this is done the ΔRS method or some variation of it (Simons and Richardson, 1965, or Einstein, 1950) where the form effects are evaluated by correcting the depth or slope gives the best results.

Antidunes

Flow with antidunes is unsteady and nonuniform. When the waves are not forming or are not breaking the resistance to flow is then the same as for a plane bed. There is an increase in dissipation of energy with the formation of the waves and when the waves are breaking. This increase in energy dissiptation is reflected in an increase in resistance to flow. This increase in resistance is directly related to the extent of wave formation and wave breaking in time and space. Therefore, it is logical to determine the resistance to flow for antidune flow by correcting the plane bed resistance equation for the form roughness effects. The equation is

$$\frac{C}{\sqrt{g}} = 7.4 \log D/d_{85} \quad C_* \tag{36}$$

The value of C_* is a function of sand size, shear stress, and depth. The relation is given in figure 15, 16, 17 and 18. There is no variation in the value of C_* for the 0.93 sand, see figure 18. The reason for this is that the discharge capacity and slope capabilities of the 8 ft flume were not adequate to obtain breaking waves. Only the standing wave type of antidunes were obtained with the 0.93 mm sand. Resistance to flow with standing waves is only slightly greater than for the plane bed.

The smallest value of C_* was determined by the capacity and slope of the flume. The lowest value of C_* was 0.47 and occurred with the 0.19 mm sand. The lowest value for the 0.27 and .28 mm sand was .55 and for the 0.45 and .47 mm sand was .8. That the lowest value of C_* was a function of the sand size confirms the observation that for the same slope and discharge the number of breaking waves, the violence of their breaking and the frequency of their occurrence increased as the sand size decreased. In fact the chute-and-pool type of flow, which is the most violent type of unsteady flow observed in sand channels did not occur for the 0.45, 0.47 and 0.93 mm sands for the maximum slope and discharge obtainable in the 8 ft flume.

The value of C_* for a constant shear value was a function of depth. C_* increased (decrease in resistance to flow) with an increase in depth. This conclusion is based on the flume studies which have a limited range in depth. However, field observations, where slope is constant, have shown that as depth increased antidune activity decreased. This decrease in antidune action decreases resistance to flow.

The relation between C_*, depth and shear stress in figures 15 through 18 is well defined and the method of correcting the plane bed equation for the nonuniform and unsteady flow effects has considerable merit. Unfortunately, the available laboratory and field data are inadequate to verify and extend the relation for larger depths.

CHAPTER VI

SUMMARY

Resistance to flow in sand channels is extremely variable because (1) the form of the boundary (roughness elements) is a function of the flow, fluid, bed-material characteristics and the geometry of the channel, and (2) the two-phase flow of sand and water changes the fluid properties and changes the structure of the turbulence and thus the velocity distribution and dissipation of energy of the system.

The bed configurations that are formed in an alluvial channel as the result of the interaction between the fluid, flow and sand are ripples, dunes, plane bed, antidunes and chutes-and-pools. These bed configurations and their associated flow phenomena can be divided into a lower flow regime and upper flow regime. This division is based on the similarities of resistance flow, of mode of sediment transport and of mode of energy dissipation. Between the two regimes there is a transition where the bed configuration ranges from those characteristics of the lower flow regime to those of the upper flow regime. In the transition there is no definable relation between the flow, fluid and sand variables and the bed configuration and resistance to flow. However, antecedent conditions may determine the form of the bed.

The regimes of flow, associated bed configurations and range in resistance to flow as they occur in the flume with increasing shear stress or stream power are:

Lower flow regime

Ripples ($7.8 \leq C/\sqrt{g} \leq 12.4$)

Dunes $(7.0 \leq C/\sqrt{g} \leq 13.2)$

Transition $(7.0 \leq C/\sqrt{g} \leq 20)$

Upper flow regime

Plane bed $(16.3 \leq C/\sqrt{g} \leq 20)$

Antidune

Standing waves $(15.1 \leq C/\sqrt{g} \leq 20)$

Breaking waves $(10.8 \leq C/\sqrt{g} \leq 16.3)$

Chutes-and-pools $(9.4 \leq C/\sqrt{g} \leq 10.7)$

Flows with some bed configurations are neither steady nor uniform under the simplest flow conditions. Equilibrium flow is the simplest type of flow that can be studied in a sand channel. With equilibrium flow the water discharge, the average energy gradient and the average sediment transport are not varying with time or space. Even with this restriction the dissipation of energy may be from grain roughness, form roughness, acceleration and deceleration of the flow or breaking waves; or a combination of their processes. The latter two are the result of nonuniform or unsteady flow, or both. In some instances the form roughness elements may be so large in relation to the scale of the channel that they are not roughness elements, but are changes in cross section. Equilibrium flow for both the ripple and plane bed is steady and uniform; for antidunes the flow is always unsteady and nonuniform; and for dunes it may be uniform or nonuniform depending on the scale of the dunes. Equilibrium flow with dunes and ripples is unsteady because the boundary is changing with time, but this change is relatively slow and the unsteadiness usually can be neglected. This variation in type of energy dissipation (or put another way, the fact that even the simplest flow may be uniform and/or unsteady) further complicates the problem of resistance to flow in alluvial channels.

The variables that determine the bed configuration and resistance to flow are the depth of flow; energy gradient; bed-material size, gradation, and fall velocity; and shape of the reach and cross section. The effect of the viscosity of the fluid is limited to its effect on the fall velocity, fall velocity being a primary variable which measures the interaction of the fluid and the bed material.

With the variation in the form of the roughness elements, the large number of variables, the different types of energy dissipation under the simplest flow conditions, and the effect of sand grains moving on the bed and in the flow on the fluid properties and on the turbulence, it is impossible to write a general function to determine resistance to flow. However, if the bed form is known, equations and relations are derived for determining the resistance to flow.

The bed form can be determined in a sand channel from a relation between $\tau_o U$, d_{50} and the bed form. Except in the neighborhood of the change from one bed form to another, this relation differentiates between the various bed forms.

The equations and relations derived for the different bed configurations must be consistent with the type of flow and resistance to flow that exists for each of them. That is, the equations and relations must be derived with cognizance of whether the flow is steady or unsteady; uniform or nonuniform, and whether the dissipation of energy is the result of grain roughness, form roughness, acceleration or deceleration of the flow, or breaking wave; or some combination of them. For these reasons two methods are needed to determine the resistance to flow. For steady uniform flow, the equations of motion were integrated to determine the velocity distribution and the resistance coefficient. For unsteady and/or non-uniform flow, a correction term was developed to be applied to the steady uniform flow equation for a plane bed. This correction term takes into account the additional losses resulting from the acceleration of flow and the breaking waves. In addition, the correction term can be applied for steady uniform flow with form roughness. The term then compensates for the large additional losses resulting from the flow separating from the large roughness elements.

Velocity Distribution

To determine the velocity distribution in a sand channel, the Reynolds equation for turbulent flow was integrated. The integration was accomplished by assuming that the flow was steady and uniform, that the viscous shear was very much smaller than the Reynolds stress and that the Reynolds stress varied as

$$\rho \, \overline{u'v'} = \rho \left(\frac{y}{A} \right)^2 \left(1 - \frac{y}{D}\right) \left(\frac{du}{dy} \right)^2$$

In terms of mixing length theory, the last assumption is that $l^2 = (y/A)^2 (1 - y/D)$. The resulting equation for the velocity distribution is

$$\frac{u}{U_*} = A \ln y/\xi + B$$

where A, B and ξ are coefficients which depend on the bed configuration.

Plane Bed

The coefficients for the plane bed without appreciable sediment movement are:

$A = 2.55$ ($k = 1/A = 0.39$)

$B = 8.0$

$\xi = d_{85}$

These values of A and B compare very well with the values of A and B for the rigid boundary case. The rigid boundary values are 2.5 and 8.5 for A and B.

The coefficients for the plane bed with appreciable sediment movement are:

$A = 3.2$ ($k = 1/A = 0.31$)

$B = 3.2$

$\xi = d_{85}$

The value of A greater than 2.5 (its value for flow over a rigid boundary) is the result of the effect of the concentration of bed material in the flow and density and size of the moving particles on the turbulence of the flow.

A study of the velocity profile for this bed form showed that there is an inner and outer zone. In the inner zone the value of A was variable, whereas in the outer zone A was constant. The value of A equal to 3.2 given above is for the outer zone which comprises 80 to 100 percent of the flow depth. The size of the inner zone ranged from approximately 0 ft to about 0.1 ft, and varied with the sand size. It was 0.1 for the 0.93 mm sand, 0.7 for the 0.45 mm sand, 0.4 for the 0.27 and 28 mm sand and 0 for the 0.19 mm sand.

This variation in size is attributed to the concentration of sand in the inner zone. The higher the concentration, the larger the inner zone and the more variable is A. The variation of A in the inner zone was from 2.5 to 11.7 (k from 0.4 to 0.085).

Ripples

For ripples the velocity distribution equation is

$$\frac{u}{U_*} = (3.33 - \frac{0.13}{U_*}) \ln y + 14.3$$

This equation is the average relation for the velocity distribution for flow over a ripple bed.

The value of k varies with U_* and ranged from 0.4 to 0.7. The k values for the individual runs ranged from 0.29 to 1.04.

The roughness factor χ is eliminated from the velocity equation because the height, spacing and length of the ripples are a function of the flow and independent of the bed material size.

Dunes and Antidunes

The variation of the velocity distribution for these two cases is so large that the distributions are undefined.

Resistance to Flow

Integrated Velocity Distribution Equations

For the plane bed, ripples and dunes, the velocity distribution equation was integrated over the depth to obtain the resistance to flow equation of the Chezy type. The resulting equation is

$$C/\sqrt{g} = A \log D/\xi + (B - A)$$

The constants A, B and ξ (χ for form roughness) were evaluated from the velocity distribution equation and from the mean flow data.

Plane bed.-- The resistance to flow equation for the plane bed with little or no bed material movement is

$$C/\sqrt{g} = 5.9 \log D/d_{85} + 5.44$$

The 5.9 and 5.44 values are in good agreement with the rigid boundary values of 5.75 and 6.25.

The resistance to flow equation for plane bed with appreciable bed material movement is

$$C/\sqrt{g} = 7.4 \log D/d_{85}$$

This equation which was derived from a study of the velocity profiles is in very good agreement with the mean flow data.

Ripples.-- The resistance to flow equation for ripples is

$$C/\sqrt{g} = (7.66 - \frac{0.30}{U_*}) \log D + \frac{0.13}{U_*} + 11$$

The velocity calculated from this equation is in good agreement with the measured velocity.

A study of the mean flow data resulted in the following equation for flow over a ripple bed

$$C/\sqrt{g} = 10.7 \log D/0.06$$

The velocity calculated by this equation is also in good agreement with the measured velocity.

Dunes.-- The coefficients A, B and χ could not be derived from either the velocity distribution or the mean flow data. However, assuming $A = 5.75$ and $B = 6.25$, the resistance to flow equation is

$$C/\sqrt{g} = 5.75 \log R/\chi + 6.25$$

A relation between $C/\sqrt{g}$ and R/χ was developed. Also, it was determined from dimensional analysis that R/χ was a function of U_*/ω, R/d_{50}, and S. A study of the data determined a relation between R/χ, $\frac{U_*}{\omega}$ which was independent of S but dependent on R/d_{50}. From the relation between flow and sediment parameters, R/χ could be determined and then $C/\sqrt{g}$. Velocities computed with this method were in fair agreement with the measured velocities.

Correction Term Equations

The correction term method is based on correcting the plane bed equation for the dissipation of energy resulting from form roughness, acceleration and deceleration of the flow and breaking waves. The equation is

$$C/\sqrt{g} = 7.4 \log D/d_{85}\sqrt{1 - \frac{\Delta RS}{RS}}$$

<u>Ripples</u>.-- For ripples there was a linear relation between ΔRS and RS which was independent of the sand size. The method predicted velocity with good agreement with the measured velocity.

<u>Dunes</u>.-- With dunes there was a linear relation between ΔRS and RS for the data from the flume, Elkhorn River near Waterloo, Nebraska, and Rio Grande near Bernalillo, New Mexico. The relation was independent of sand size when $d_{50} < 0.5$ mm. There was a different but still linear relation for the 0.93 mm sand. The relation for $d_{50} < 0.5$ mm developed from the flume data was also

valid for the Mississippi River at St. Louis, Missouri for RS values less than $.2 \times 10^{-3}$. For larger values of RS, the dunes decrease in height and the relation between ΔRS and RS, although good, was no longer linear.

Antidunes.-- A good relationship existed between the correction term $C_* = \sqrt{1 - \frac{\Delta RS}{RS}}$, the shear stress, depth, and sand size for the flume data. An increase in depth or in sand sizes resulted in an increase in C_*, and consequently a decrease in resistance to flow.

CHAPTER VII

CONCLUSIONS

1. The bed forms that occur in a sand channel are ripples, dunes, plane bed, antidunes and chutes-and-pools.

2. Equilibrium flow with a plane bed or ripples is steady and uniform; with dunes, equilibrium flow may be considered steady, but it may be uniform or nonuniform depending on the size of the dunes in relation to the scale of the flume or stream; with antidunes and chutes-and-pools, equilibrium flow is always nonuniform and unsteady.

3. The conclusion of Simons and Richardson (1963) that there is a fundamental difference between ripples and dunes was substantiated by a study of the velocity distribution and resistance to flow for the two bed configurations.

4. There are four types of energy dissipation for equilibrium flow in a sand channel. With plane bed, energy dissipation results from grain roughness. With ripples, it results from form roughness. With dunes, it results from form roughness and grain roughness if their size is small in comparison to the scale of the system. If dunes are so large that they cannot be considered roughness element, but changes in cross section, then dissipation also results from the acceleration and deceleration of the flow in addition to the grain and form roughness. With antidunes and chutes-and-pools, resistance to flow is grain roughness, acceleration and deceleration of the flow and breaking waves.

5. The velocity-distribution and resistance-to-flow equations for steady uniform flow were derived by integrating the Reynolds equation of motion by assuming that the viscous shear was very much

smaller than the Reynolds stress and that the Reynolds stress varied as

$$\rho \overline{u'v'} = \rho \left(\frac{y}{A} \right)^2 (1 - y/D) \left(\frac{du}{dy} \right)^2$$

6. The velocity distribution for the different bed forms for equilibrium flow that is also steady and uniform is defined by:

a. plane bed with little or no sediment movement

$$\frac{u}{U_*} = 2.55 \ln y/d_{85} + 8.0$$

b. plane bed with appreciable sediment movement

$$\frac{u}{U_*} = 3.2 \ln y/d_{85} + 3.2$$

c. ripples

$$\frac{u}{U_*} = (3.33 - \frac{0.13}{U_*}) \ln y + 14.3$$

7. The velocity distribution for equilibrium flow over a plane bed with appreciable sediment movement has an inner and outer zone. The distribution is logarithmic in both zones. But the slope of the profile in the inner zone varies with the size, density and concentration of bed material. The variation of the slope in the inner zone was from 2.5 to 11.7. In the outer zone the slope was constant value of 3.2. The inner zone occupies from 0 to 20 percent of the flow depth. The inner zone is larger with the coarser sand and larger concentration of bed material in transport.

8. With equilibrium as well as nonequilibrium flow in a sand channel there is a transition for the change from a dune bed to a plane bed or antidunes. In the transition there is a range of

shear stress where the bed may be dunes, plane, have antidunes or washed-out dunes. Also in this range the bed form may oscillate from one bed form to another. In the transition there is not a unique relation between the shear stress, velocity or sediment transport.

9. There is not a general resistance-to-flow equation for equilibrium flow in a sand channel because (1) whether the flow is uniform or nonuniform and steady or unsteady depends on the bed configuration; (2) there are four types of energy dissipation; (3) there is a range in shear stress (transition) where the bed form may range from dunes to plane bed or antidunes.

10. If the bed form is known, the resistance to flow can be determined. The equations are

a. plane bed with little or no sediment movement

$$\frac{C}{\sqrt{g}} = 5.9 \log D/d_{85} + 5.44$$

b. plane bed with appreciable sediment movement

$$\frac{C}{\sqrt{g}} = 7.4 \log D/d_{85}$$

c. ripples

$$\frac{C}{\sqrt{g}} = (7.66 - \frac{0.30}{U_*}) \log D + \frac{0.13}{U_*} + 11$$

d. dunes and antidunes

$$\frac{C}{\sqrt{g}} = 7.4 \log D/d_{85}\sqrt{1 - \frac{\Delta RS}{RS}}$$

For dunes, ΔRS was a function of RS. There was one relation for $d_{50} < 0.5$ mm and another for the 0.93 mm sand. The ΔRS vs RS relation was valid for streams as large as the Mississippi River. For antidunes the correction term $\sqrt{1 - \frac{\Delta RS}{RS}}$ was a function of sand size, shear stress and depth.

CHAPTER VIII

RECOMMENDATIONS FOR ADDITIONAL STUDY

In order to more fully understand the nature of flow in sand-bed channels and to make the derived equations applicable to a broad range of field conditions, the following studies are suggested:

1. The variation of the velocity distribution in the inner and outer zone for plane bed flow should be investigated in greater detail. It is recommended that the distribution of the velocity, sediment concentration, and characteristics of the turbulence be measured for plane-bed flow for different sand sizes. The plane bed should be a function of the bed material and flow characteristics and should not be obtained artificially by cementing the bed. Bagnold's inner granular stress should be considered in this study.

2. The distribution of velocity, sediment concentration, and characteristics of the turbulence in the flow over the dune-bed configuration should be investigated. This study should determine the coefficients in equation 19 and determine the reason why dunes form.

3. The variation of resistance to flow for different shapes of cross sections should be investigated. That is, the equations developed in this study should be studied using a broad range of field and laboratory conditions to determine the value of B in the equations.

4. The variation of the bed forms and resistance to flow for a large range of slopes under controlled laboratory conditions has been well established. The same cannot be ~~said~~ for depth. Therefore, a large range of depths should be investigated under controlled laboratory conditions. The objectives of this study

would be to determine the variation in bed configuration and resistance to flow with depth.

5. It has been demonstrated that fall velocity of the bed material is a very important parameter in determining bed configuration. However, the fall velocity is based on the terminal velocity of an isolated particle in a quiescent fluid. The fall velocity of the bed material in a turbulent field is an unknown quantity. Therefore, extensive studies of the effect and importance of turbulence on fall velocity of a particle are needed. This study may indicate why certain bed configurations form.

6. The reason for the formation of the different bed configurations should be determined and better methods of predicting them developed. For this purpose, the internal mechanics of the flow must be studied.

CHAPTER IX

REFERENCES

Albertson, M. L., Simons, D. B., and Richardson, E. V., 1958, Discussion of mechanics of sediment-ripples formation: Am. Soc. Civil Engineers Jour., v. 84, no. HY1, p. 1558-22-31.

Bagnold, R. A., 1954, Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear: Royal Soc. (London) Philos. Trans., ser. A., v. 225, p. 49-63.

Beckman, E. W., and Furness, L. W., 1962, Flow characteristics of Elkhorn River near Waterloo, Nebraska: U. S. Geol. Survey Water-Supply Paper 1498-B, 34 p.

Boussinesq, T. V., 1877, Theroie de l'ecoulement tourbillant: Mem. Per. par. div. Sav., XXIII, Paris.

Brooks, N. H., 1958, Mechanics of streams with movable beds of fine sands: Am. Soc. Civil Engineers Trans., v. 123, p. 526-594.

Carey, W. C., and Keller, M. D., 1957, Systematic changes in the beds of alluvial rivers: Am. Soc. Civil Engineers Jour., v. 83, no. HY4.

Colby, B. C., and Christensen, R. P., 1956, Visual accumulation tube for size analysis of sand: Am. Soc. Civil Engineers Jour., v. 82, no. HY3, p. 1004-1-17.

Colby, B. R., 1960, Discontinuous rating curves for Pigeon Roost and Cuffawa Creeks in Northern Mississippi: U. S. Dept. of Agriculture, ARS 41-36.

Colby, B. R., 1964, Discharge of sands and mean-velocity relationships in sand-bed streams: U. S. Geol. Survey Prof. Paper 462-A.

Culbertson, J. K., and Dawdy, D. R., 1964, A study of fluvial characteristics and hydraulic variables, Middle Rio Grande, New Mexico: U. S. Geol. Survey Water-Supply Paper 1498-F.

Daranandana, Niwat, 1962, A preliminary study of the effect of gradation of bed material on flow phenomena in alluvial channels: Ph. D. Dissertation, Civil Engineering Dept., Colo. State Univ., Fort Collins, Colorado.

Dawdy, D. R., 1961, Depth-discharge relations of alluvial streams-discontinuous rating curves: U. S. Geol. Survey Water-Supply Paper 1498-C, 16 p.

Einstein, H. A., 1950, The bed load function for sediment transportation in open channel flows: U. S. Dept. of Agriculture Tech. Bull. 1026, 70 p.

Einstein, H. A., and Barbarossa, N. L., 1952, River channel roughness: Am. Soc. Civil Engineers Trans., v. 117, p. 1121-1146.

Einstein, H. A., and Chien, Ning, 1954, Effects of heavy sediment concentration near the bed on velocity and sediment distribution: California Univ. Inst. Eng. Research, Missouri River Div. sediment ser. 8, 45 p.

Elata, C., and Ippen, A. T., 1961, The dynamics of open-channel flow with suspensions of neutrally buoyant particles: Mass Inst. of Tech., Dept. of Civil and Sanitary Engineering Tech. Rept. No. 45.

Garde, R. J., 1959, Total sediment transport in alluvial channels: Ph. D. Dissertation, Civil Engineering Dept., Colo. State Univ., Fort Collins, Colorado.

Gilbert, G. K., 1914, The transport of debris by running water: U. S. Geol. Survey Prof. Paper 86, 263 p.

Guy, H. P., Simons, D. B., and Richardson, E. V., 1965, Summary of alluvial channel data from flume experiments: U. S. Geol. Survey Prof. Paper 462-L (in press).

Hubbell, D. W., and others, 1956, Investigations of some sedimentation characteristics of a sand bed stream: U. S. Geol. Survey open-file report, Lincoln, Nebraska.

Hubbell, D. W., and Al-Shaikh Ali, Khalid S., 1962, Qualitative effects of temperature on flow phenomena in alluvial channels: U. S. Geol. Survey Prof. Paper 450-D, p. 21.

Inglis, Sir Claude, 1948, Historical note on empirical equations developed by engineers in India for flow of water and sand in alluvial channels: Internat. Assoc. for Hydraulic Structures Research, 2nd meeting, Stockholm.

Ismail, H. M., 1952, Turbulent transfer mechanism and suspended sediment in closed channels: Am. Soc. Civil Engineers Trans., v. 117, p. 409-446.

Jordan, P. R., 1965, Fluvial sediment of the Mississippi River at St. Louis, Missouri: U. S. Geol. Survey Water-Supply Paper 1802 (in press).

Kennedy, J. F., 1961, Stationary waves and antidunes in alluvial channels: California Inst. Tech. report no. KH-R-2.

Kennedy, J. F., 1963, The mechanics of dunes and antidunes in erodible-bed channels: Fluid Mechanics Jour., v. 16, part 4, p. 521-544.

Keulegan, G. H., 1938, Laws of turbulent flow in open channels: U. S. Nat. Bur. Standards Jour. Research, v. 21, p. 708-741.

Kharrufa, Najib, S., 1962, Flume studies of steep flow with large graded natural roughness elements: Ph. D. Dissertation, Civil Engineering Dept., Utah State Univ., Logan, Utah.

Knoroz, V. S., 1959, The effect of the channel macroroughness on its hydraulics resistance: Inst. Gidrotekaniki, v. 62, p. 75-96. In Russian, translated by Ivan Mittin, U. S. Geol. Survey, Denver, Colorado.

Koloseus, H. J., 1958, The effect of free-surface instability on channel resistance: Ph. D. Dissertation, Civil Engineering Dept., State Univ. of Iowa, Iowa City, Iowa.

Koloseus, H. J., and Davidian, J., 1961, Flow in an artificially roughened channel: U. S. Geol. Survey Prof. Paper 424-B, p. 25.

Kramer, H., 1935, Sand mixtures and sand movement in fluvial models: Am. Soc. Civil Engineers Trans., v. 100, p. 798-838.

Lane, E. W., and Carlson, E. J., 1953, Some factors affecting the stability of canals constructed in coarse granular materials: Internat. Hydrol. Convention Proc., p. 37-48.

Langbein, W. B., 1942, Hydraulic criteria for sand waves: Am. Geophys. Union Trans., v. 23, p. 615-618.

Laursen, E. M., and Pin-Nam Lin, 1952, Discussion of turbulent transfer mechanism and suspended sediment in closed channels: Am. Soc. Civil Engineers Trans., v. 117.

Leopold, L. B., and Maddock, T. Jr., 1953, The hydraulic geometry of stream channels and some physiographic implications: U. S. Geol. Survey Prof. Paper 252.

Leopold, L. B., and Langbein, W. B., 1962, The concept of entropy in landscape evolution: U. S. Geol. Survey Prof. Paper 500-A.

Liu, H. K., 1957, Mechanics of sediment ripple formation: Am. Soc. Civil Engineers Jour., v. 83, no. HY2, p, 1197-1-23.

Nagabhushanaiah, H. S., 1961, Separation flow downstream of a plate set normal to a plane boundary: Ph. D. Dissertation, Civil Engineering Dept., Colo. State Univ., Fort Collins, Colorado.

Nordin, C. F., 1964, Vertical distribution of velocity and suspended sediment, Middle Rio Grande, New Mexico: U. S. Geol. Survey Prof. Paper 462-B, 20 p.

Pie, S. L., 1957, Viscous flow theory-turbulent flow: New York, Van Nostrand.

Powel, R. W., 1946, Flow in a channel of definite roughness: Am. Soc. Civil Engineers Trans., v. 111, p. 531-566.

Prandtl, L., 1925, Ueber die ausgebildeteturbulenz: ZAMN 5.

Richardson, E. V., Simons, D. B., and Posakony, G. J., 1961, Sonic depth sounder for laboratory and field use: U. S. Geol. Survey Cir. 450.

Richardson, E. V., Simons, D. B., and Haushild, W. L., 1962, Boundary form and resistance to flow in alluvial channels: Internat. Assoc. of Scientific Hydrol. Bull., v. vii, no. 1.

Sayre, W. W., and Albertson, M. L., 1963, Roughness spacing in rigid open channels. Am. Soc. Civil Engineers Trans., v. 128.

Schlichting, H., 1960, Boundary layer theory: New York, Mc-Graw-Hill.

Shields, I. A., 1936, Application of similarity principles and turbulence research to bed-load movement: A translation from the German by W. P. Ott and J. C. van Vehelin, U. S. Soil Conserv. Service Coop. Lab., California Inst. of Technology, p. 21.

Simons, D. B., 1957, Theory and design of stable channels in alluvial material, Ph. D. Dissertation, Civil Engineering Dept., Colo. State Univ., Fort Collins, Colorado.

Simons, D. B., and Richardson, E. V., 1961, Studies of flow in alluvial channels-basic data from flume experiments: U. S. Geol. Survey, Colorado State Univ., CER61EVR31, Fort Collins, Colorado.

Simons, D. B., and Richardson, E. V., 1962a, Resistance to flow in alluvial material: Am. Soc. Civil Engineers Trans., v. 127, p. 927-1006.

Simons, D. B., and Richardson, E. V., 1962b, The effect of bed roughness on depth-discharge relations in alluvial channels: U. S. Geol. Survey Water-Supply Paper 1498-E.

Simons, D. B., and Richardson, E. V., 1963, Form of bed roughness in alluvial channels: Am. Soc. Civil Engineers Trans., v. 128.

Simons, D. B., and Richardson, E. V., 1964, A study of variables affecting flow characteristics and sediment transport in alluvial channels: Trans. Federal Inter-Agency Sedimentation Conf., Jackson, Miss., (in press).

Simons, D. B., Richardson, E. V., and Albertson, M. L., 1961, Studies of flow in alluvial channels, flume studies using medium sand (0.45 mm): U. S. Geol. Survey Water-Supply Paper 1498-A, 76 p.

Simons, D. B., Richardson, E. V., and Haushild, W. H., 1963, Studies of flow in alluvial channels, some effects of fine sediment on flow phenomena: U. S. Geol. Survey Water-Supply Paper 1498-G.

Straub, L. G., 1954, Transportation characteristics of Missouri River sediment: U. S. Corps of Engineers, Missouri River Div. sediment ser., no. 4, Minneapolis, Minnesota.

Taylor, G. I., 1954, Transport of vorticity and heat through fluids in turbulent motion: Royal Soc. (London) Philos. Trans., ser. A., v. 135, p. 685.

Taylor, R. H., and Brooks, N. H., 1962, Discussion of resistance to flow in alluvial channels: Am. Soc. Civil Engineers Trans., v. 127, p. 927-1006, p. 246-256.

Vanoni, V. A., 1946, Transport of suspended sediment by water: Am. Soc. Civil Engineers Trans., v. 111, p. 67-102.

Vanoni, V. A., and Brooks, N. H., 1957, Laboratory studies of the roughness and suspended load of alluvial streams: California Inst. of Technology rept. E-68, 121 p.

Vanoni, V. A., and Nomicos, G. N., 1960, Resistance properties of sediment-laden streams: Am. Soc. Civil Engineers Trans., v. 125, p. 1140-1175.

von Karman, T., 1930, Mechanische aennlichkeit and turbulenz: Nach. Gesell. Wiss. Goettingen, Math. Phys. Klasse, Heft 5. Also NACA TM611, 1931.

West Pakistan, 1963, Canal and head works data observation program 1962 data tabulation: Harza Engineering Co. Internat., Chicago, Illinois.

White, C. M., 1940, Equilibrium of grains on bed of streams: Royal Soc. (London) Philos. Trans., ser. A., v. 174, p. 322-334.

APPENDIX

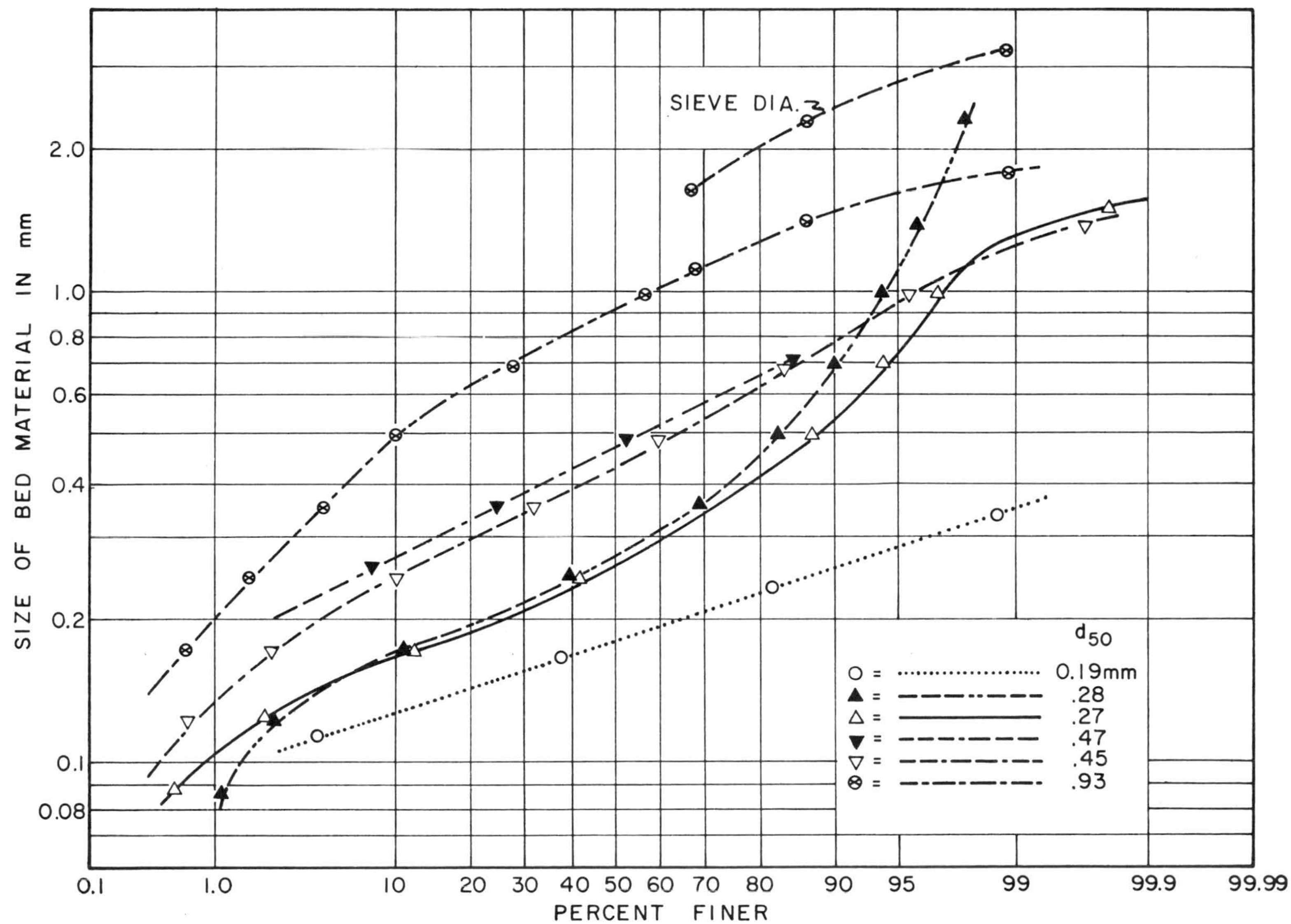


Fig. 1 Size distribution curves for the sands used in the 8ft flume.

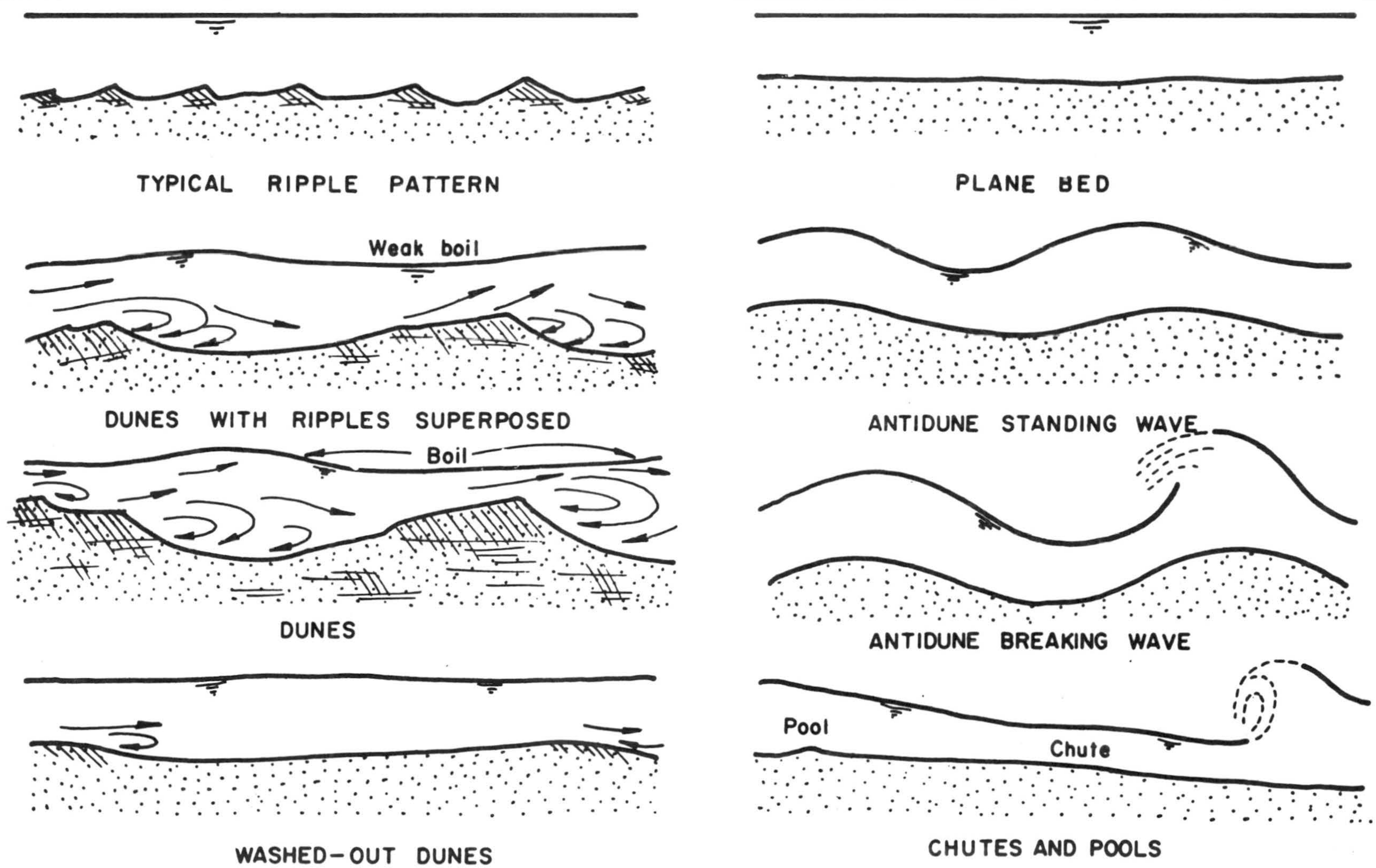
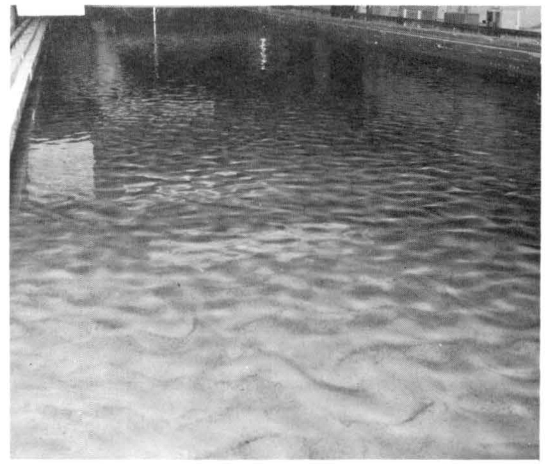
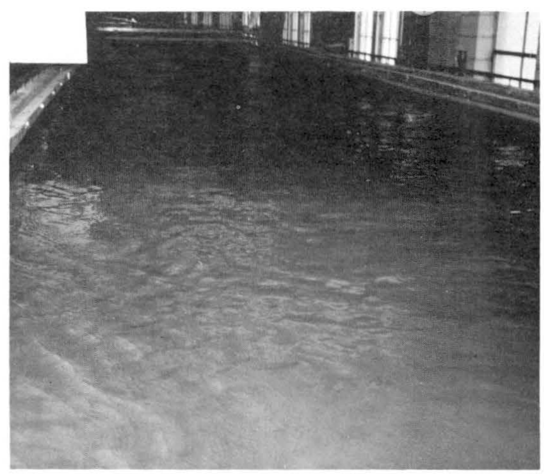


Fig. 2 Forms of bed roughness in sand channels



Bed surface　　　　　　　　　　Water surface

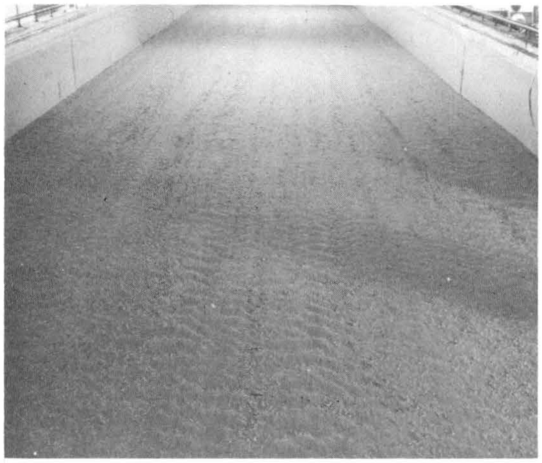
Ripple bed configuration. Run 9, 0.28 mm sand.

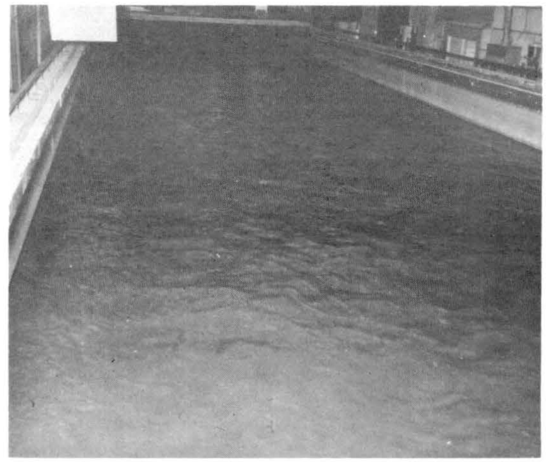


Bed surface　　　　　　　　　　Water surface

Dune bed configuration. Run 55, 0.27 mm sand.

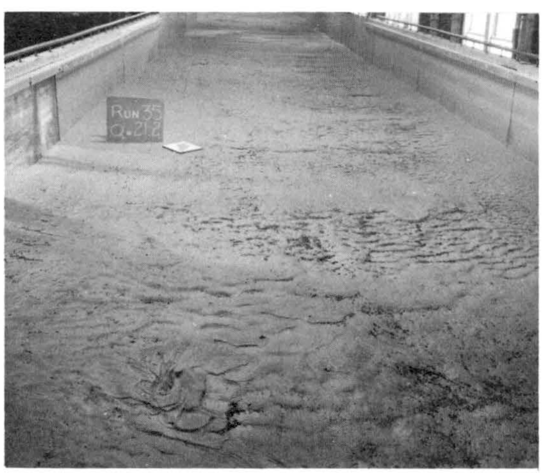
Fig. 3a. Photographs of the bed and water surface for the ripple and dune bed configuration.

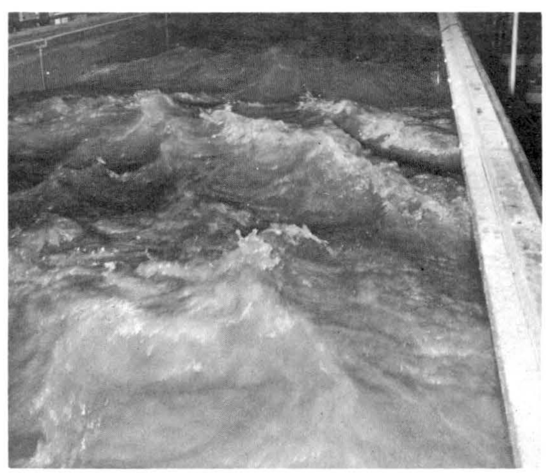




Bed surface Water surface

Plane bed configuration. Run 28, 0.28 mm sand.





Bed surface Water surface

Antidune bed configuration. Run 35, 0.28 mm sand.

Fig. 3b. Photographs of the bed and water surface for the plane and antidune bed configuration.

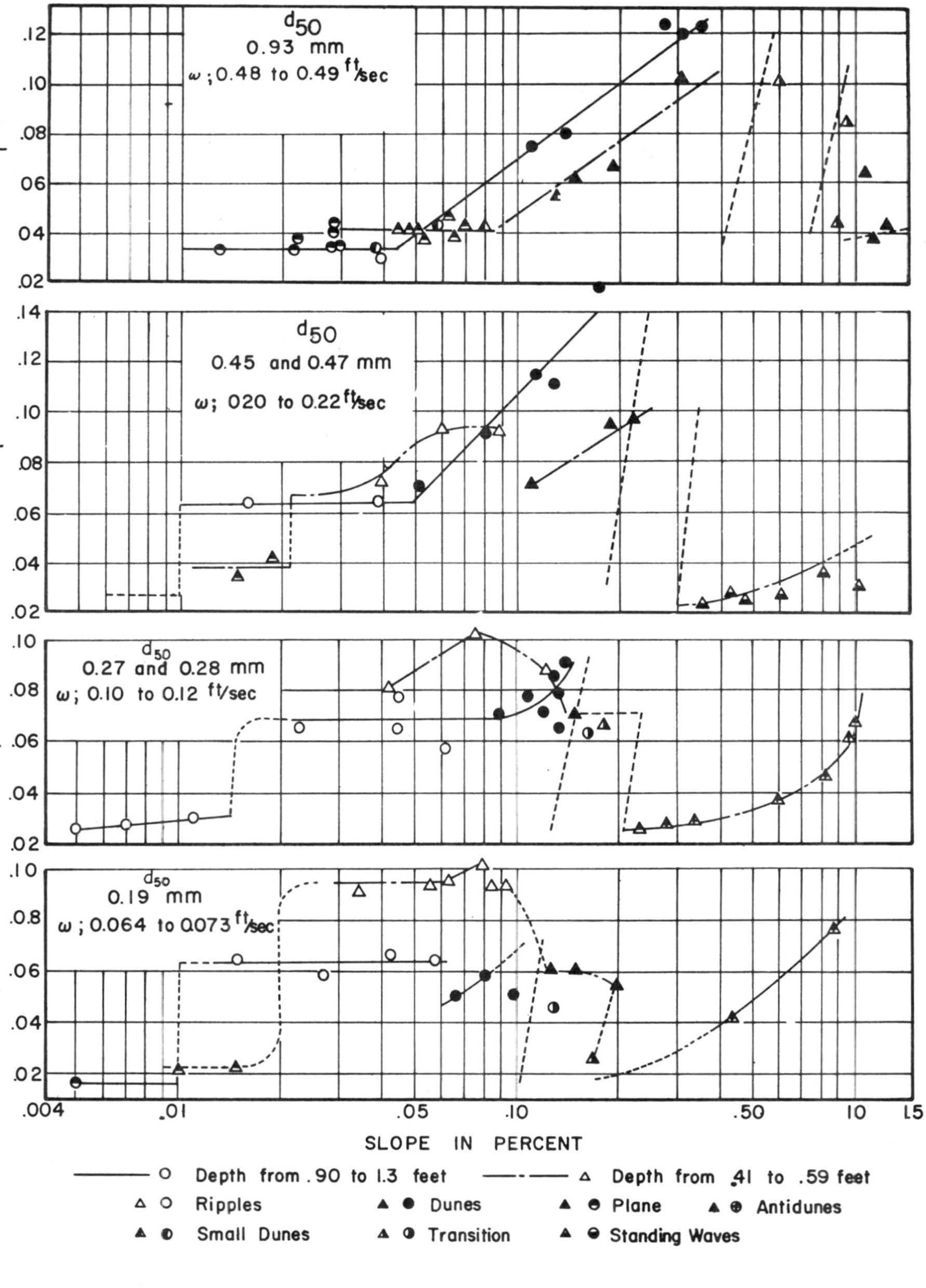


Fig. 4 Change in Darcy-Weisbach f with slope, depth and fall velocity of the bed material, 8 ft. flume data. ($f = 8/(C/\sqrt{g})^2$)

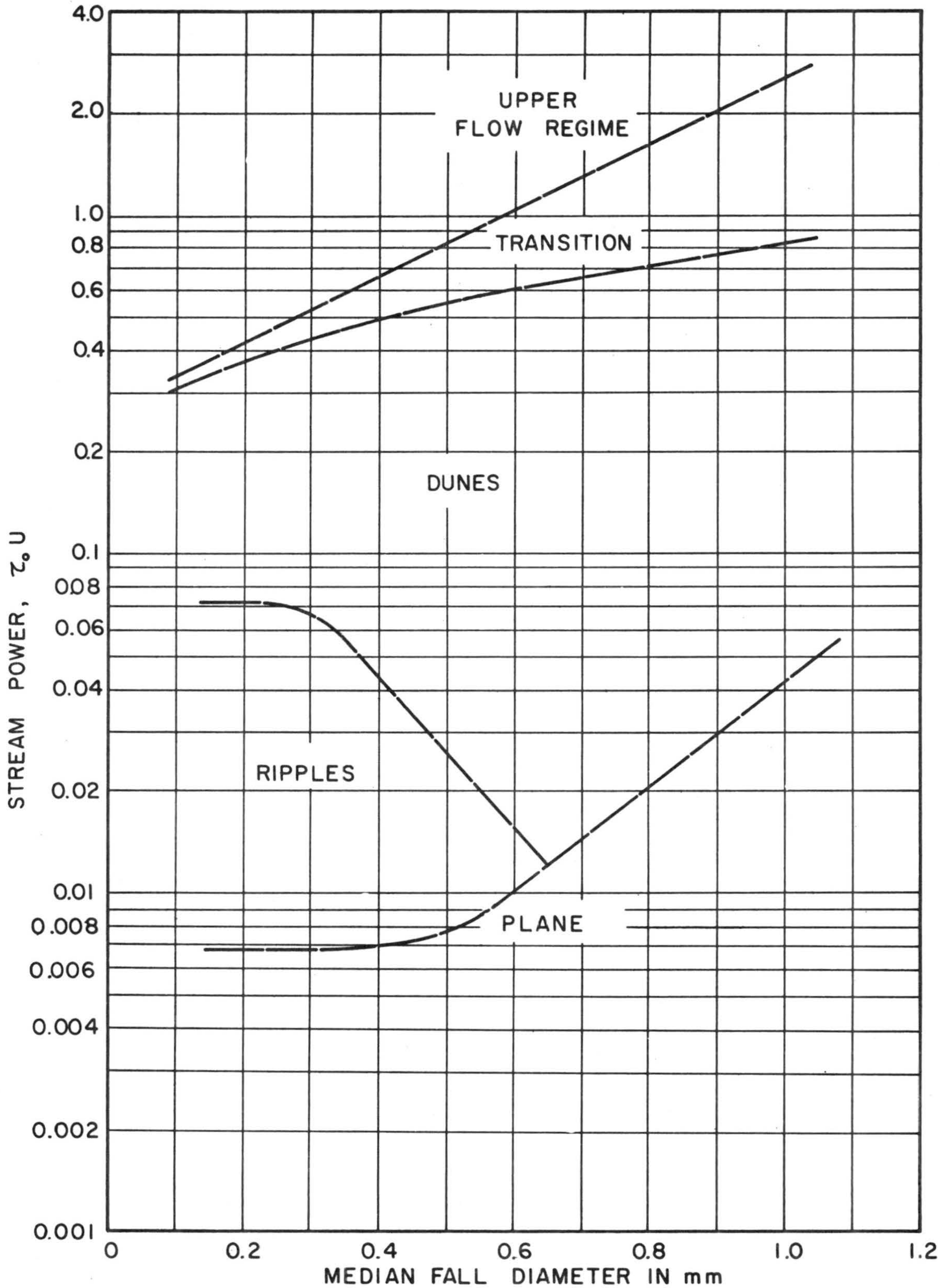


Fig. 5 Relation between stream power, median fall diameter and bed configuration.

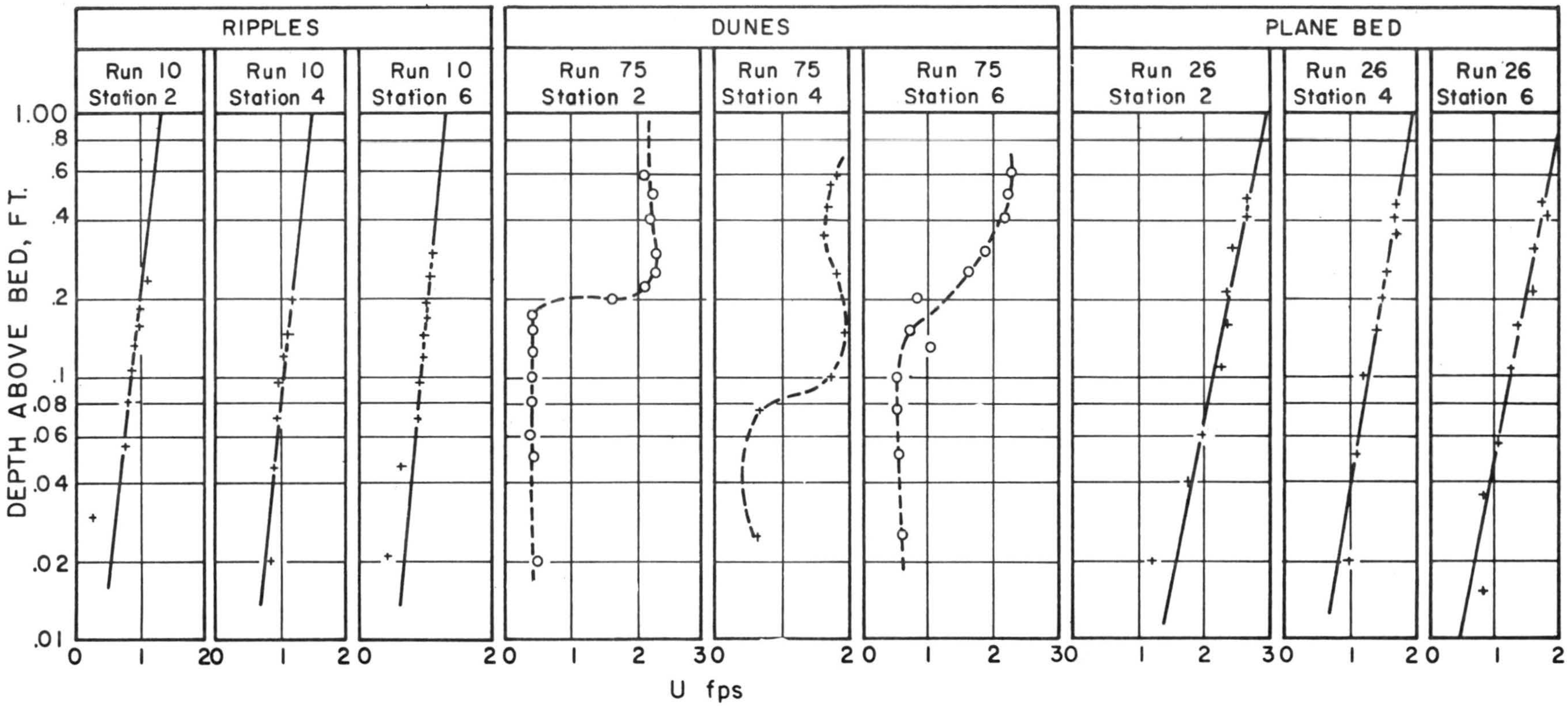


Fig. 6 Typical velocity distribution curves.

(runs are for 0.45 and 0.47 mm sands)

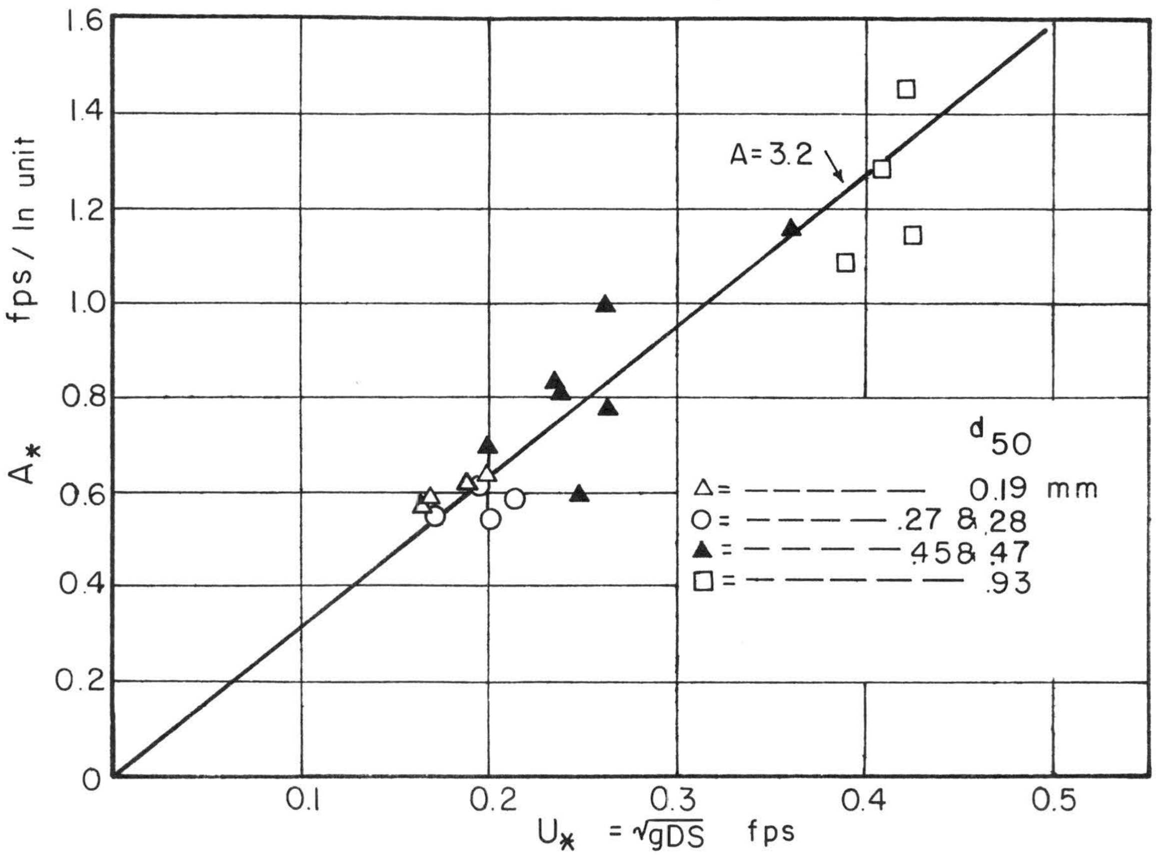


Fig. 7 Relation between the slope A_* of the velocity distribution in the vertical and the shear velocity U_* for plane bed flow with sediment movement.

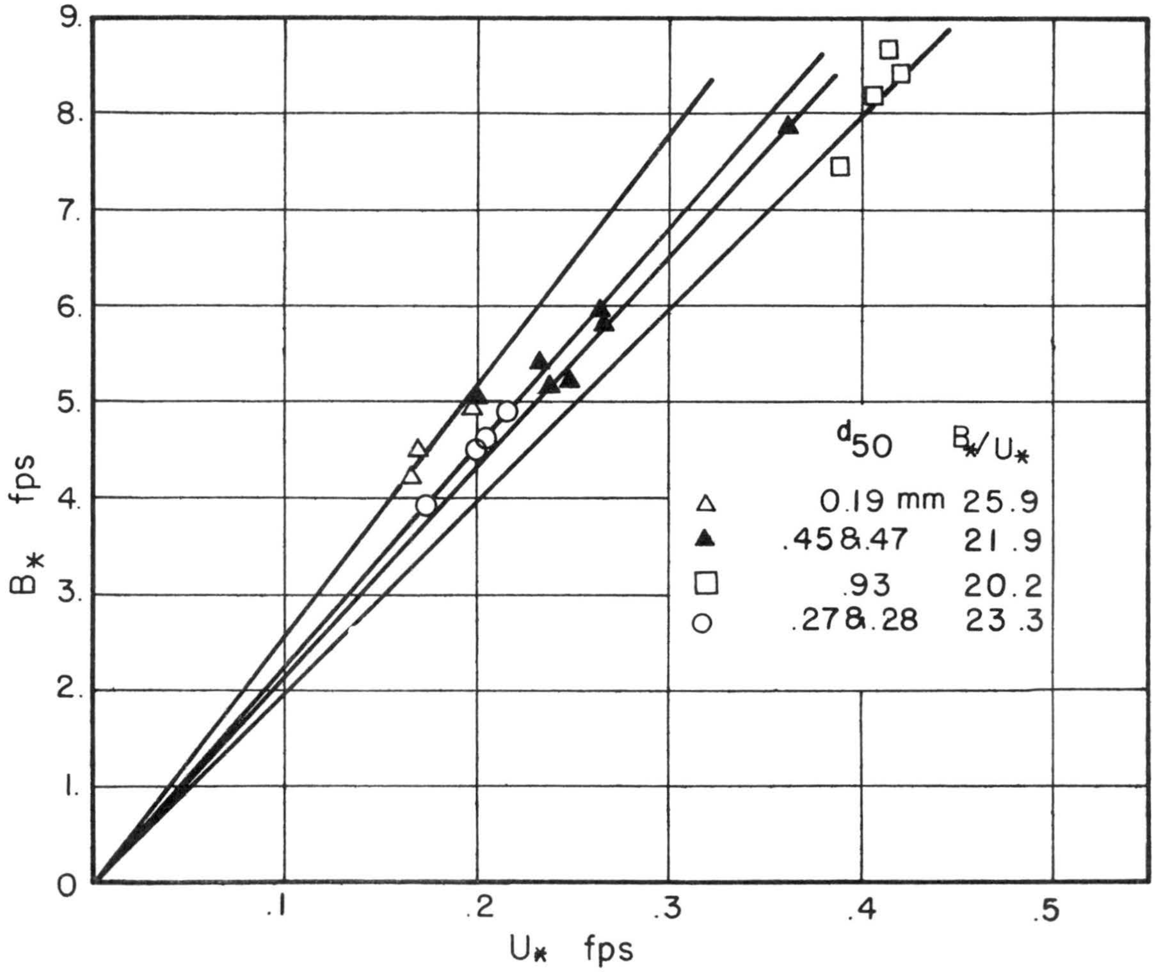


Fig. 8 Relation between the intercept B_* of the velocity distribution in the vertical with U_* for plane bed flow with sediment movement

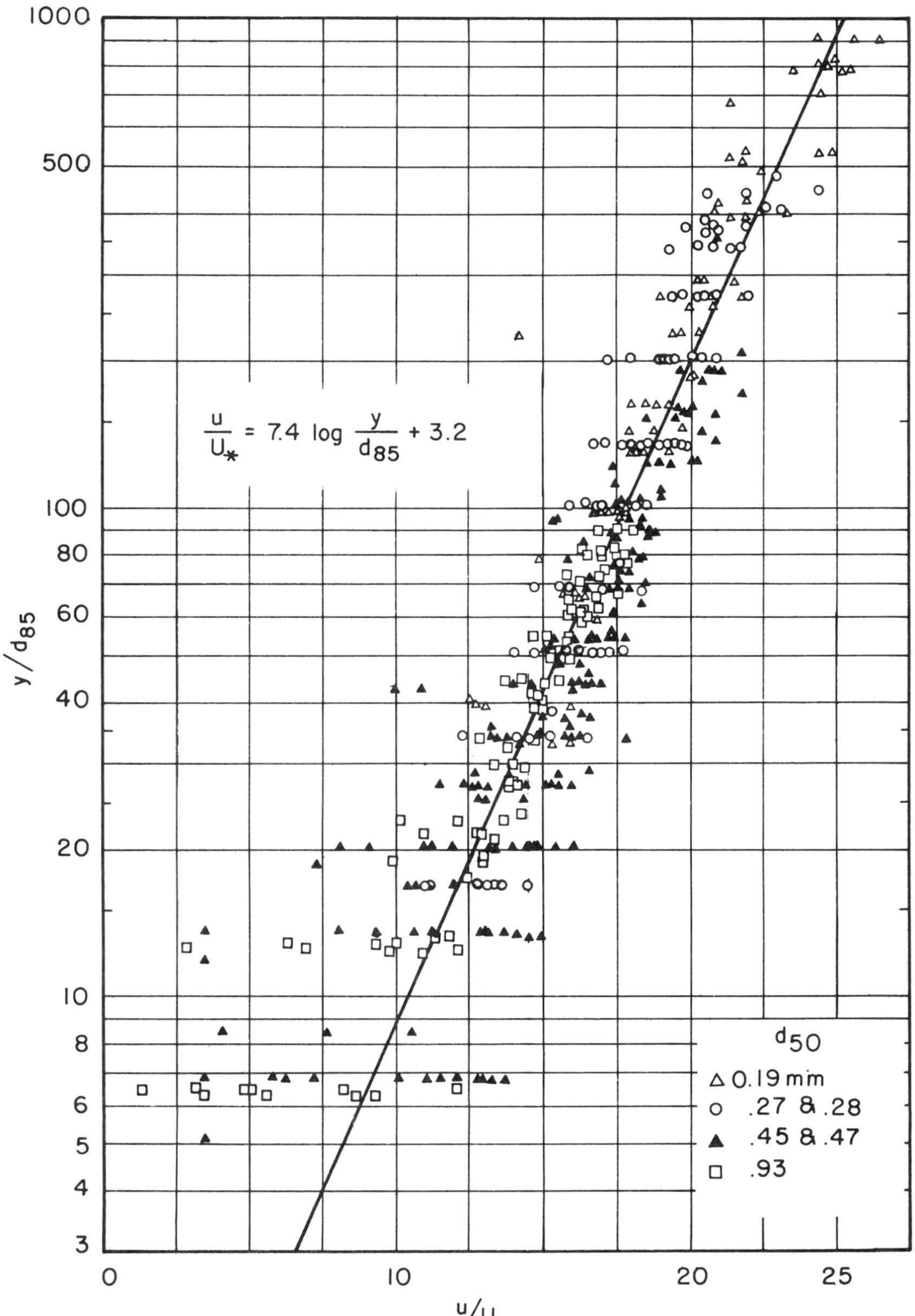


Fig. 9 Relation between u/U_* and y/d_{85} for the plane bed with sediment transport in the 8 ft. flume

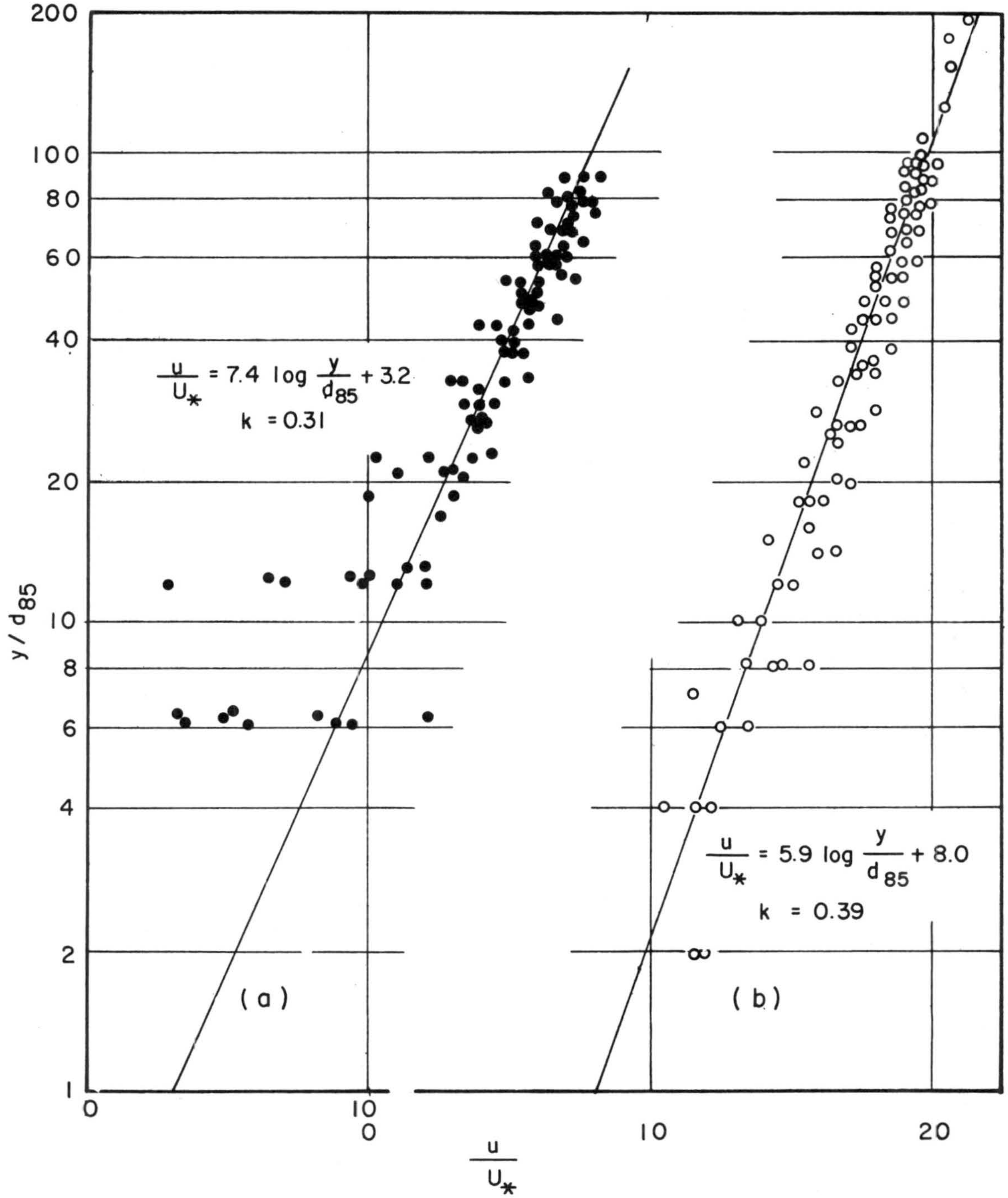


Fig. 10 Comparison of the relation between u/U_* and y/d_{85} for plane bed flow with (a) and without (b) appreciable sediment movement (0.93 mm sand)

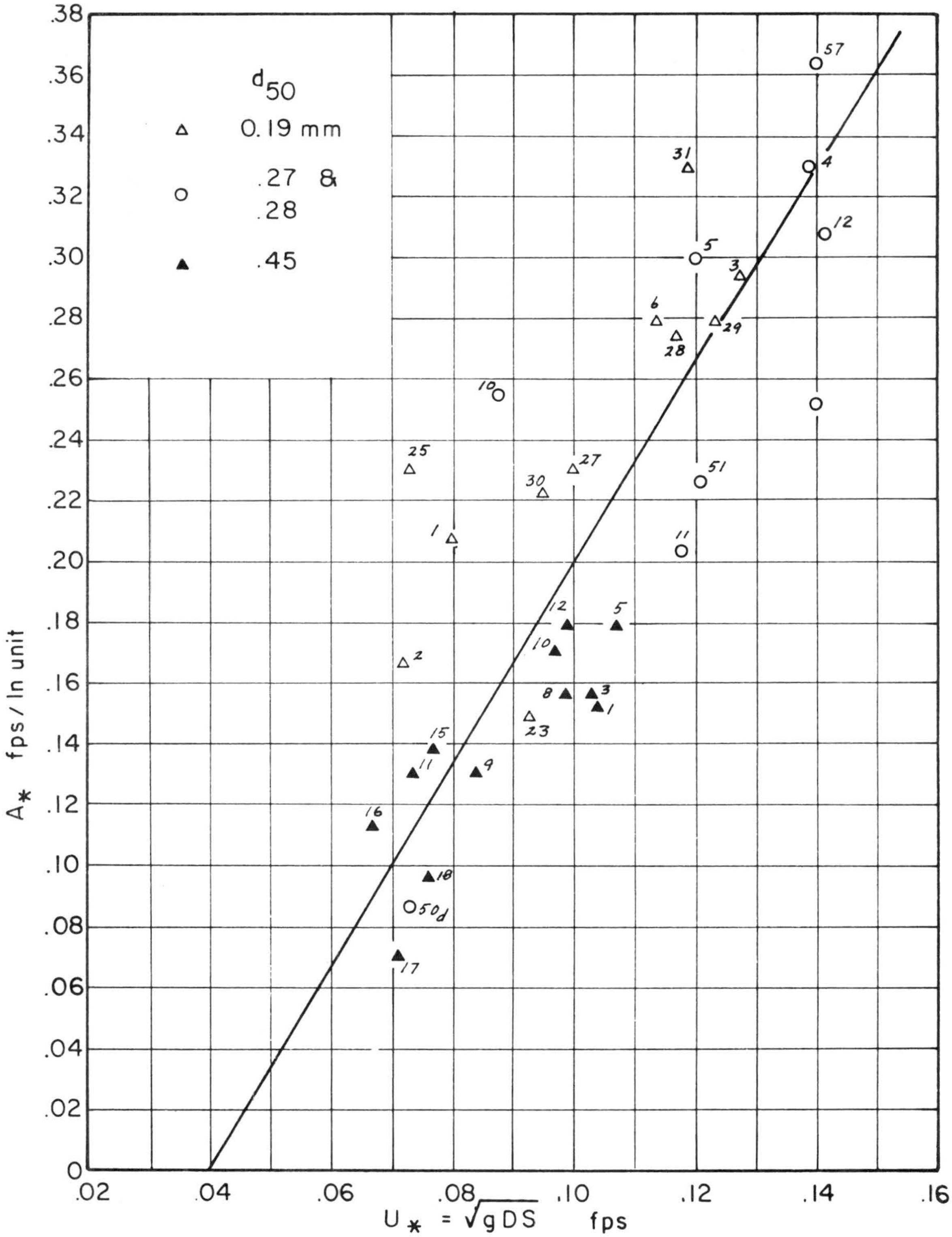


Fig. 11. Relation between the slope A_* of the velocity distribution in the vertical and the shear velocity U_* for ripple bed configuration. Third variable is run number.

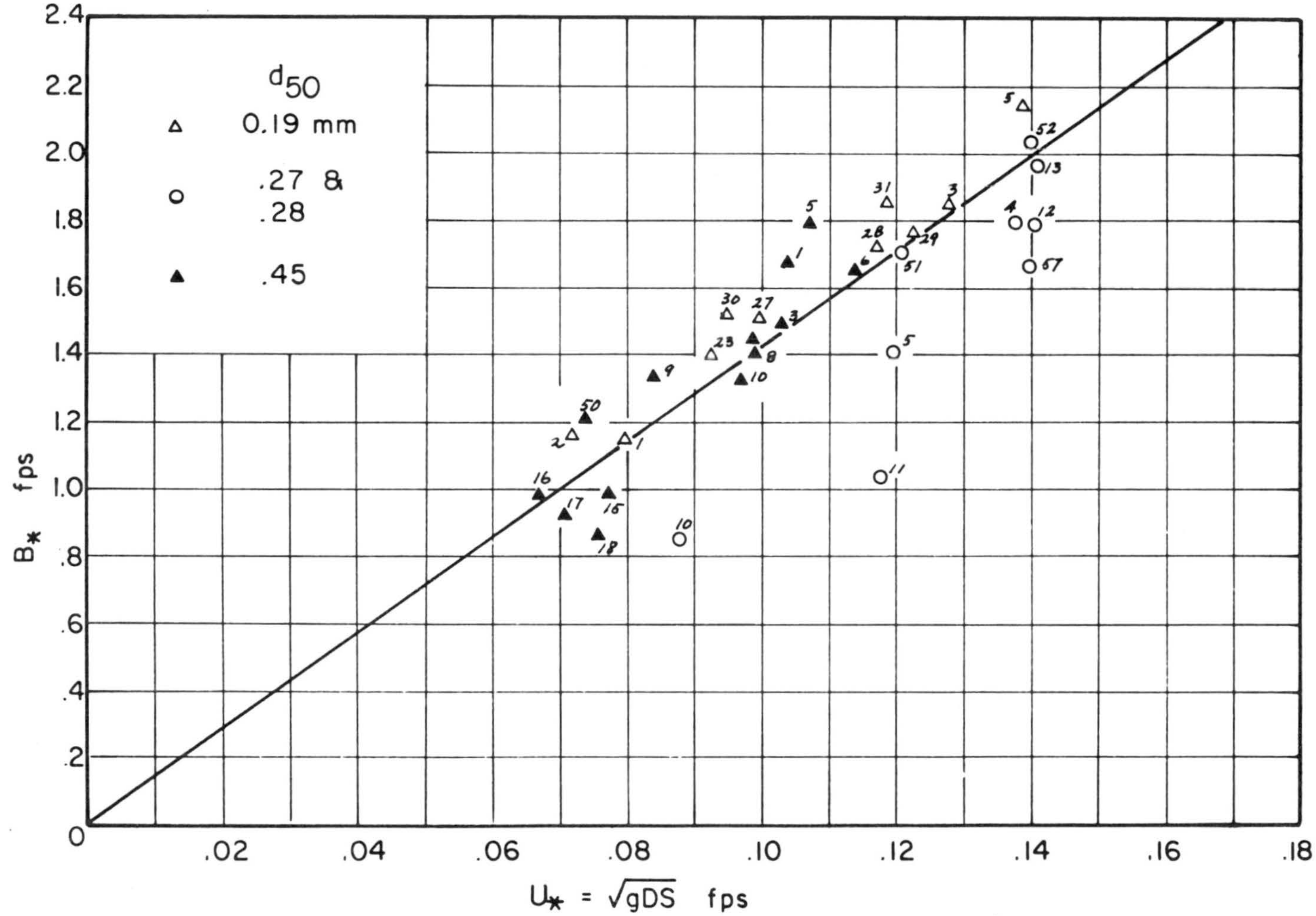


Fig. 12. Relation between the intercept B_* of the velocity distribution in the vertical and the shear velocity U_* for the ripple bed configuration. Third variable is run number.

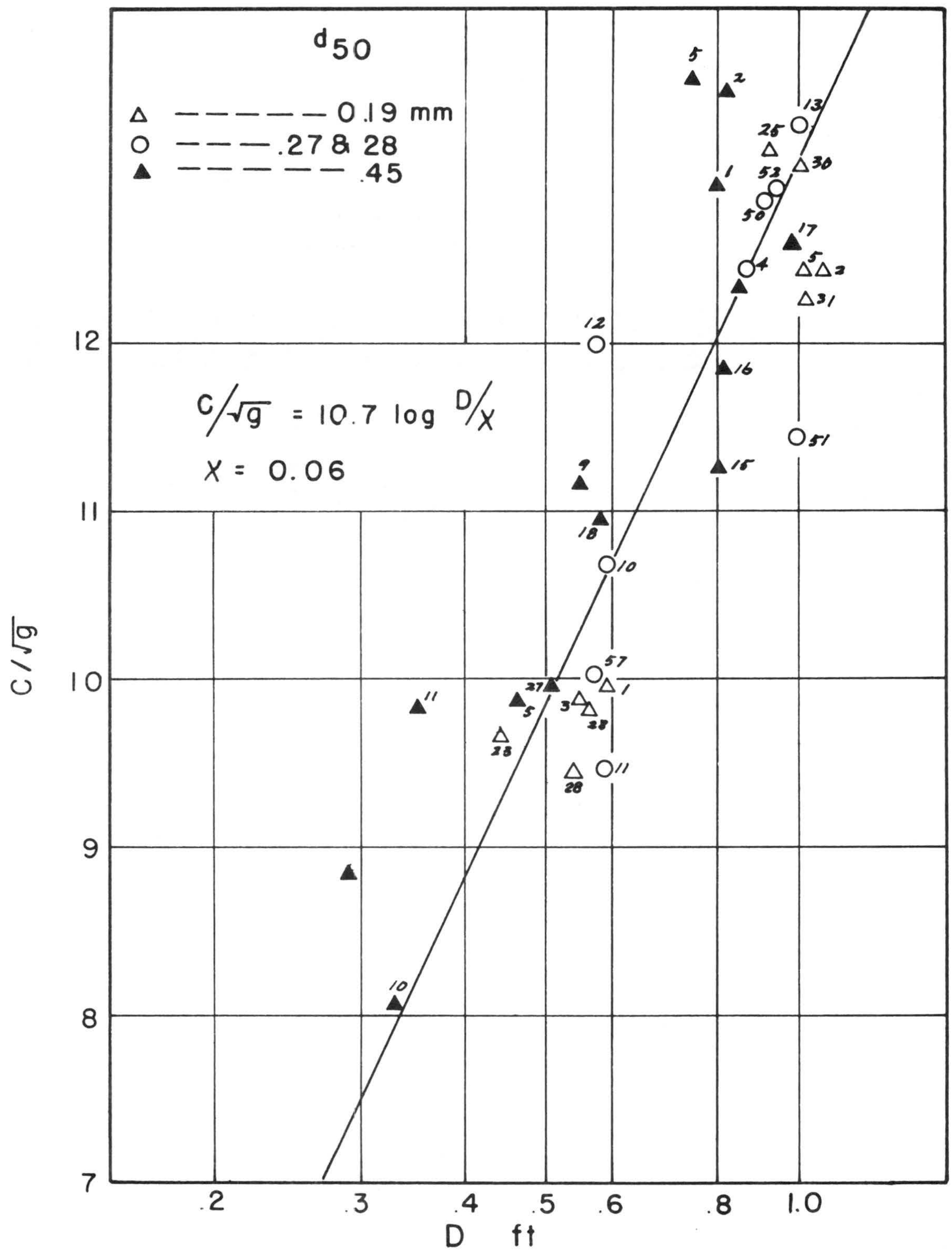


Fig. 13. Variation of $C/\sqrt{g}$ with depth of flow for the ripple bed configuration. Third variable is run number.

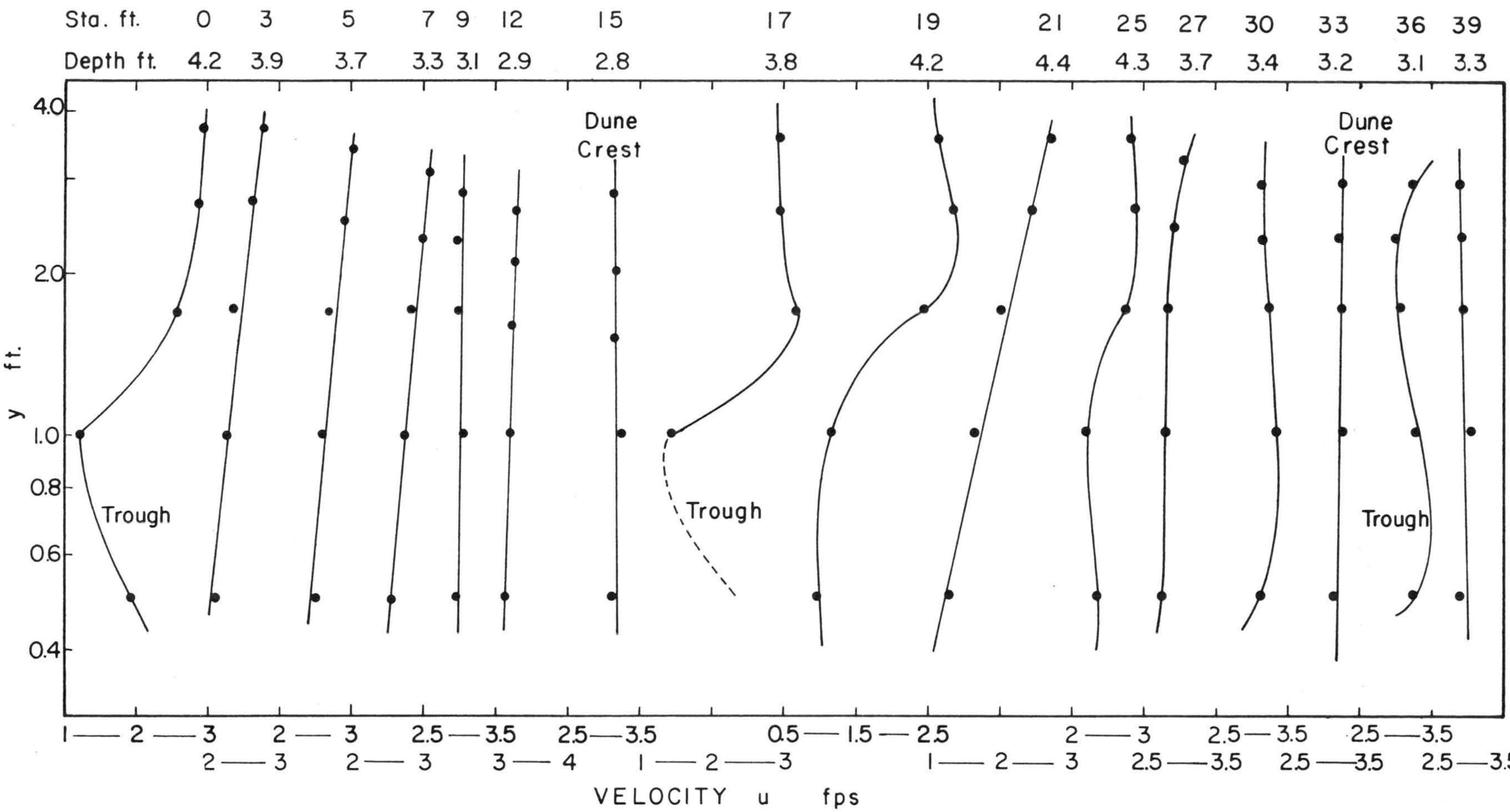


Fig. 14 Variation of the velocity profiles in the downstream direction over a dune bed Rio Grande near Bernardo, New Mexico.

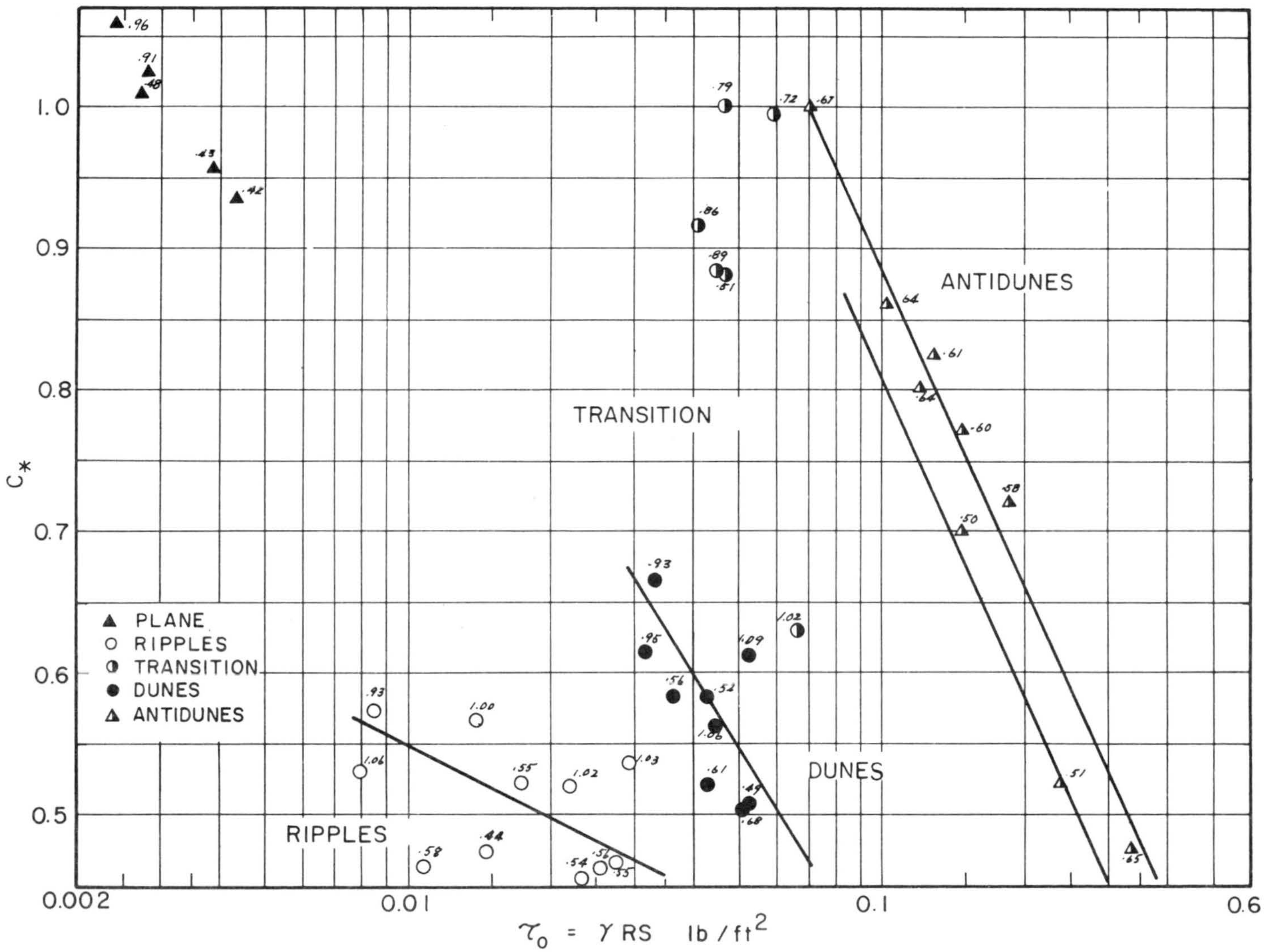


Fig. 15. Relation between C_* , τ_o bed configuration and depth for the 0.19 mm sand. Third variable is depth of flow.

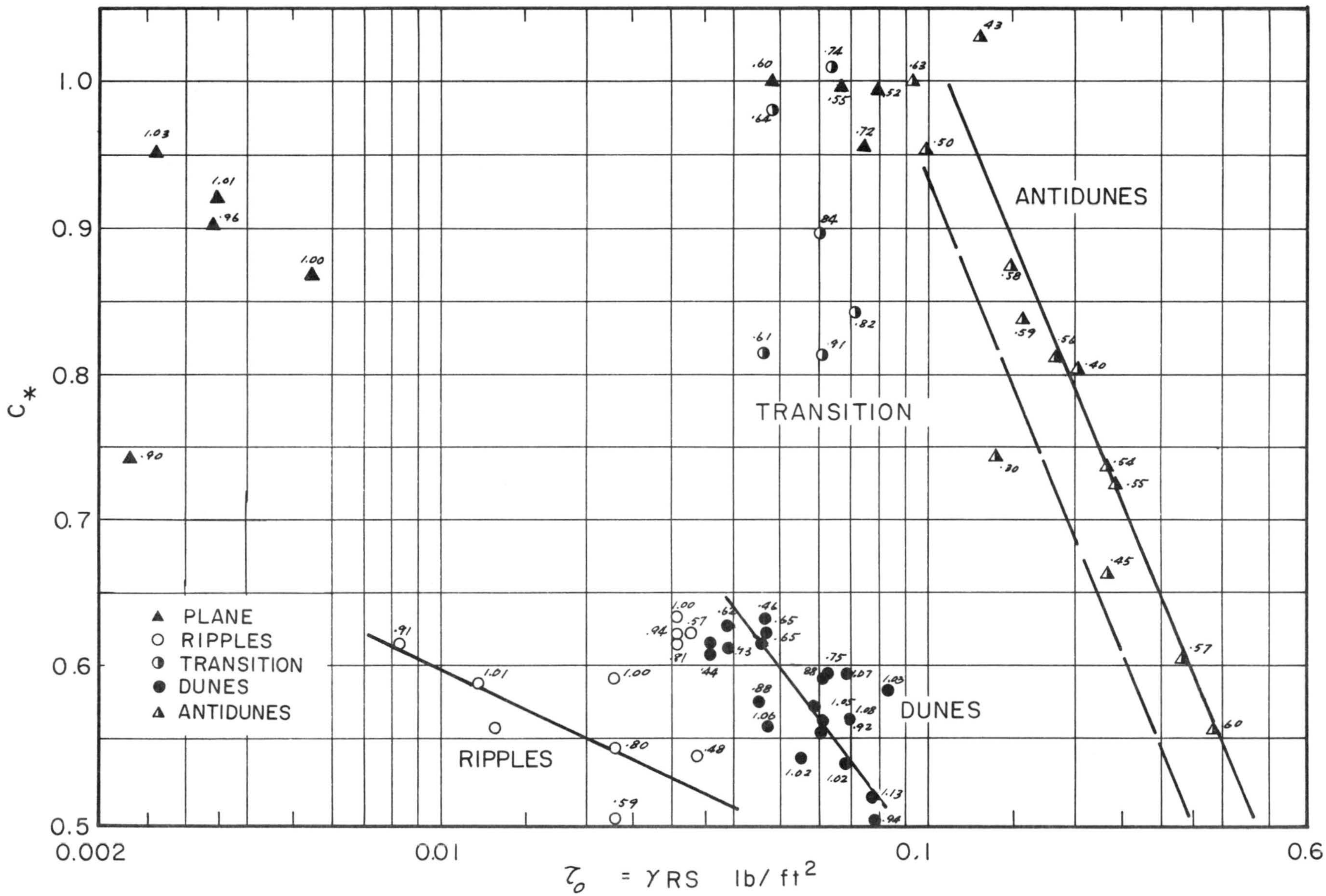


Fig. 16. Relation between C_* , τ_o and depth for 0.27 and 0.28 mm sand. Third variable is depth of flow.

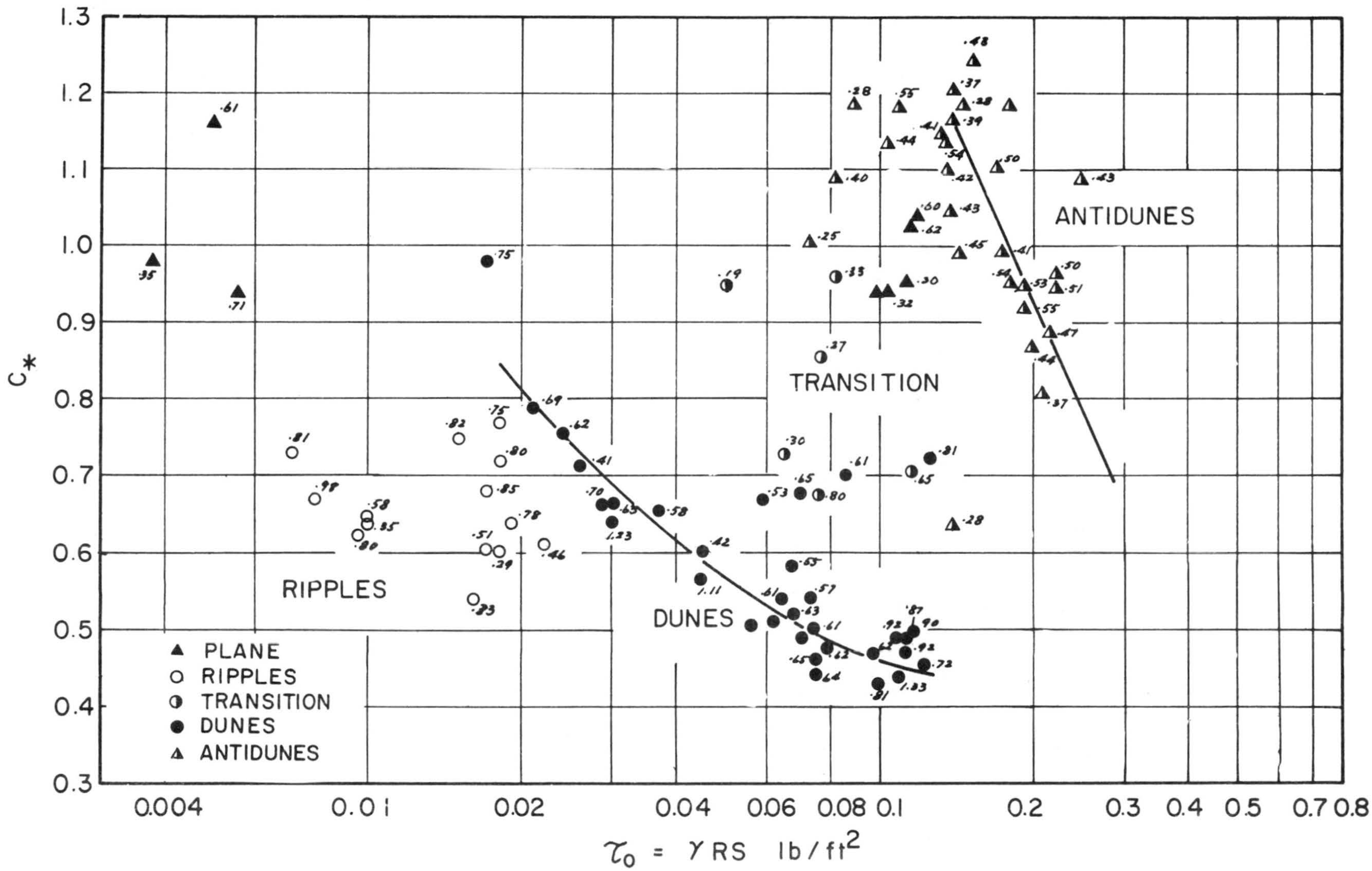


Fig. 17. Relation between C_*, τ_o and depth for 0.45 and 0.47 mm sand. Third variable is depth of flow.

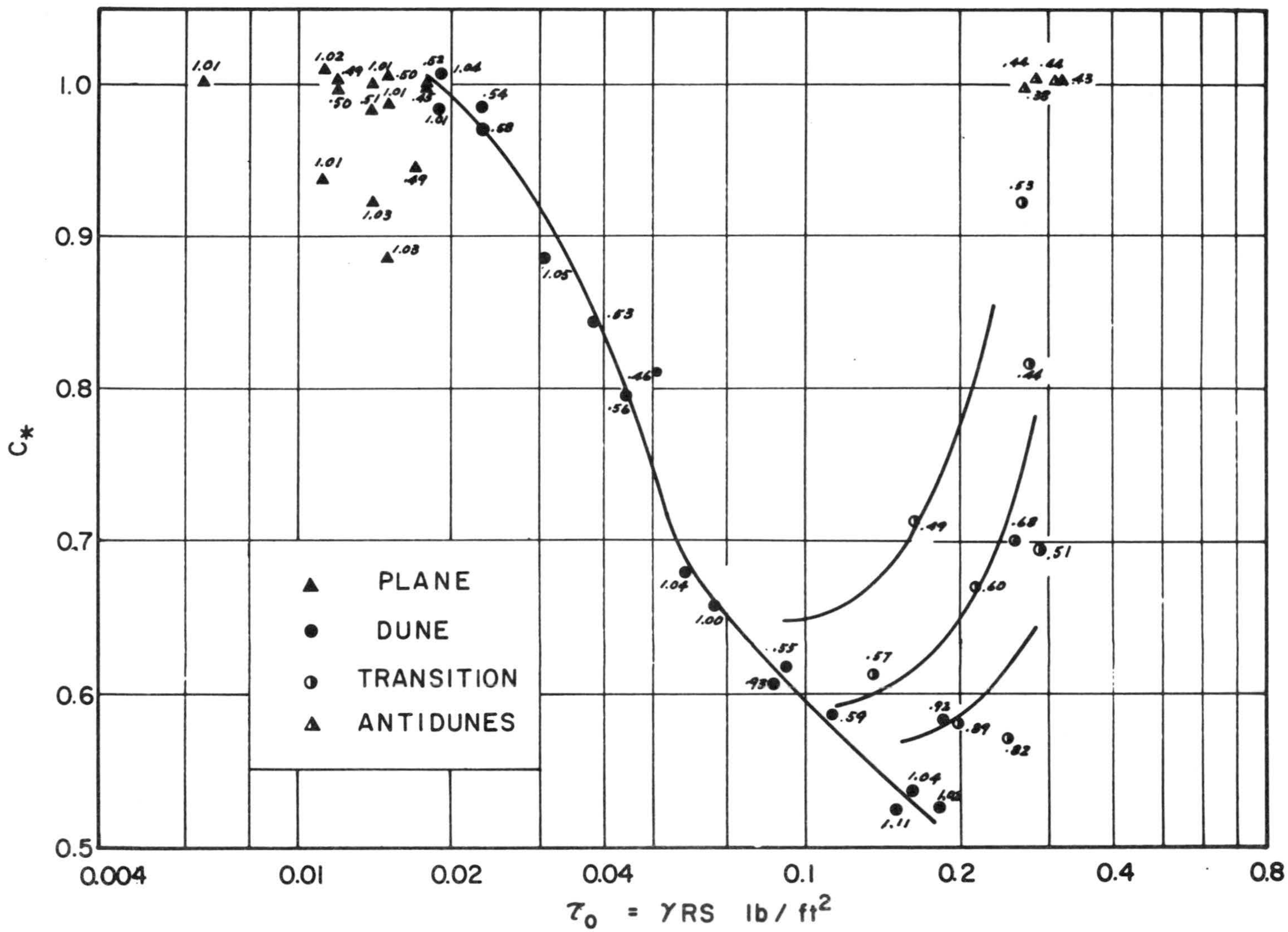


Fig. 18. Relation between C_* and τ_o depth for the 0.93 mm sand. Third variable is depth of flow.

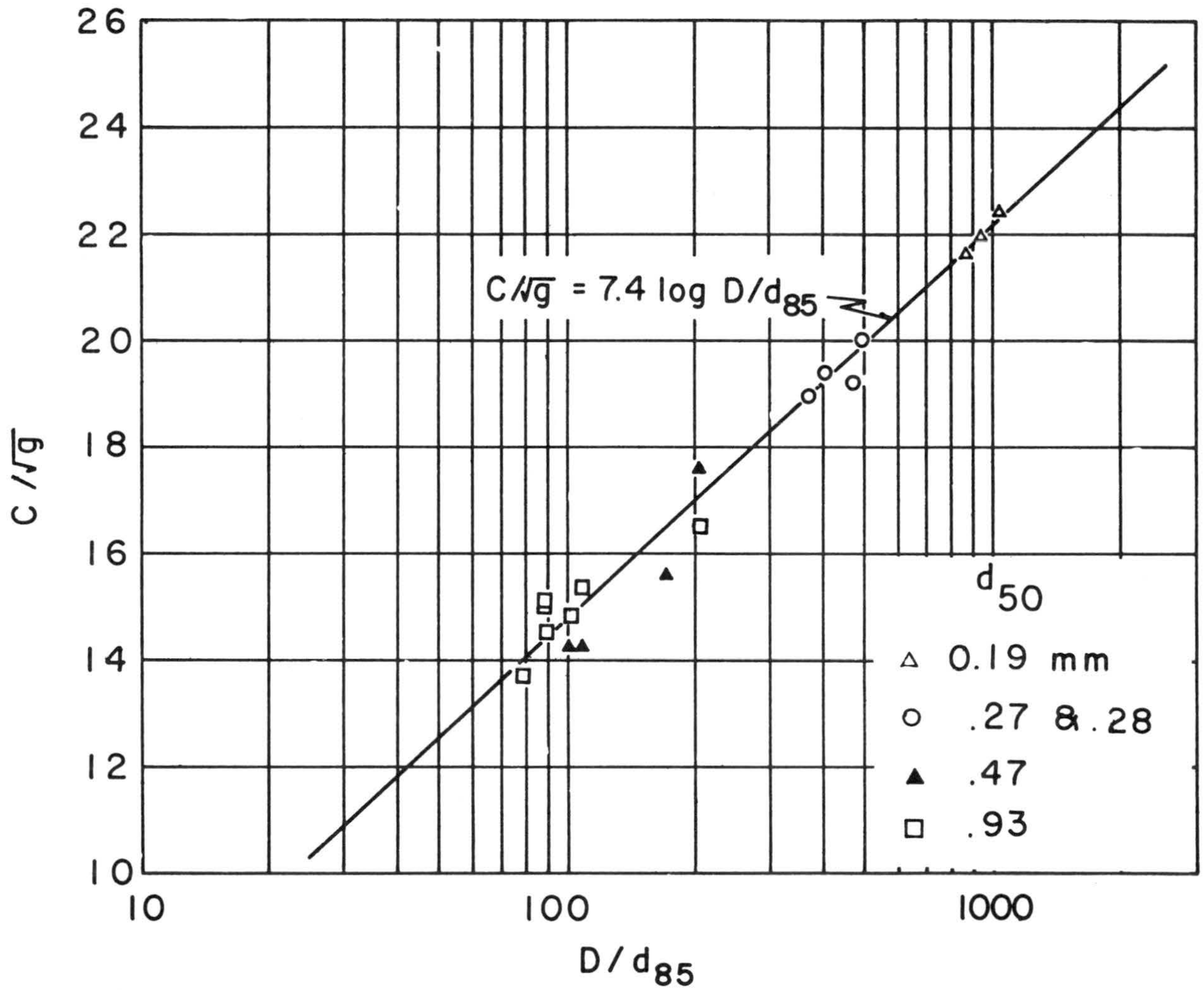


Fig. 19 Variation of $C/\sqrt{g}$ with D/d_{85} for plane bed with sediment movement.

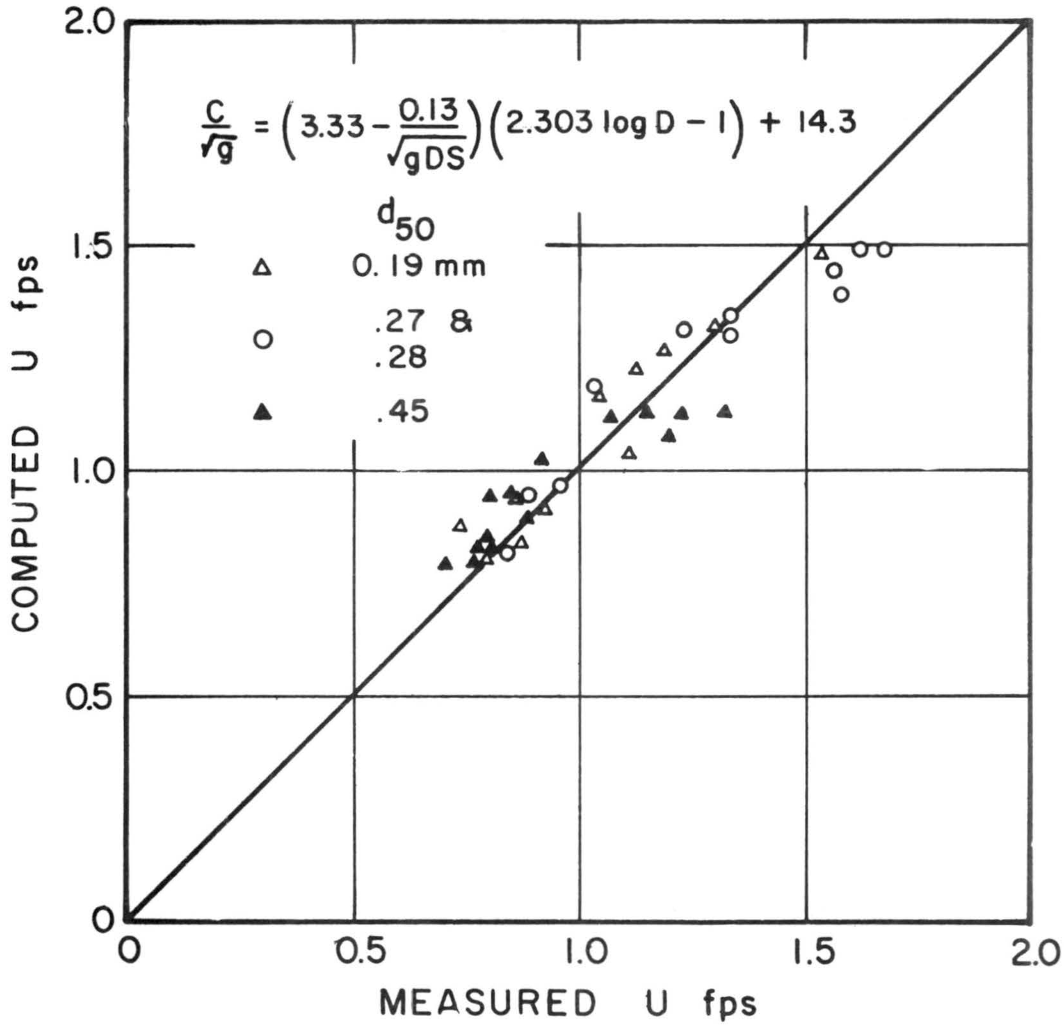


Fig.20 Comparison of the computed velocity with the measured velocity for ripples. (Eq. 41)

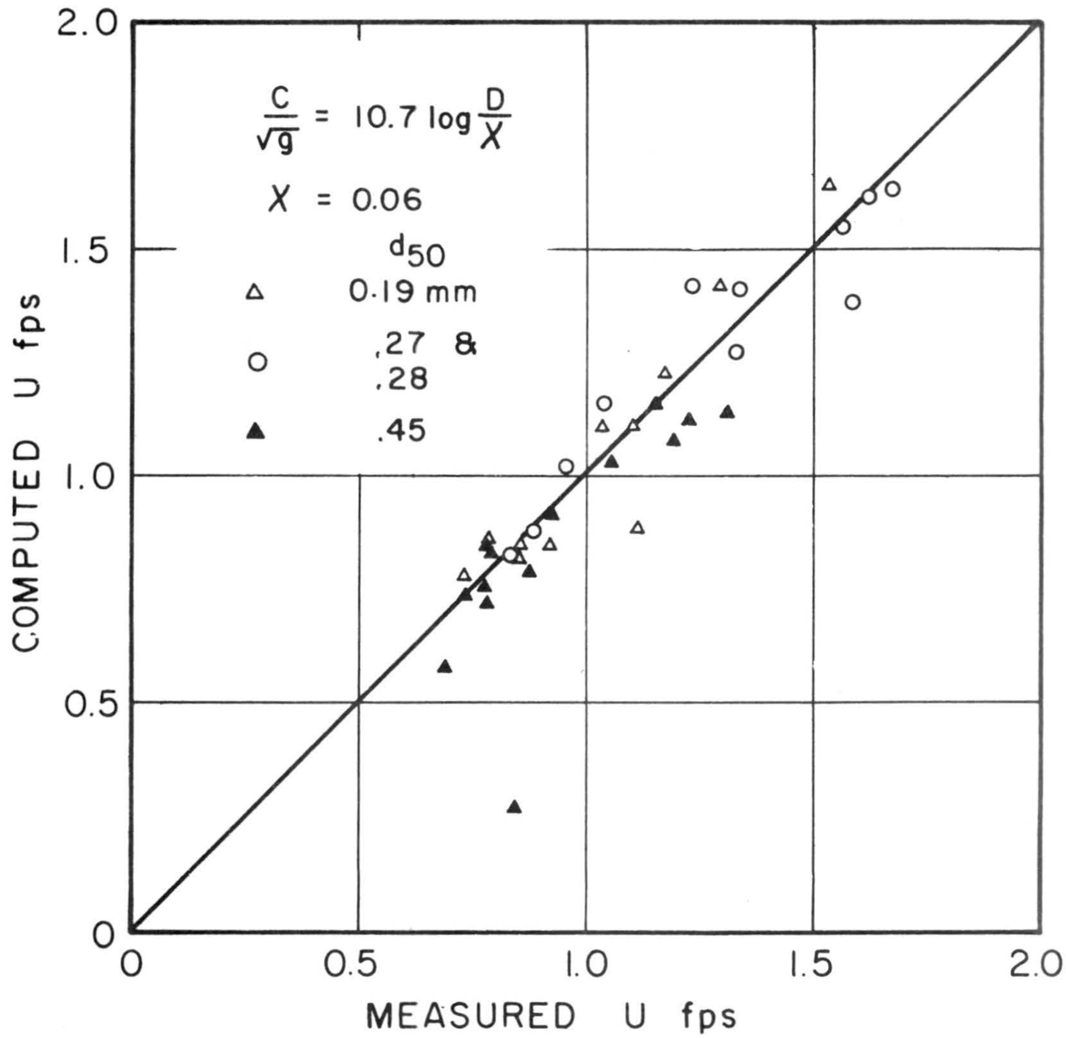


Fig. 21 Comparison of the computed velocity with the measured velocity for ripples. (Eq. 42)

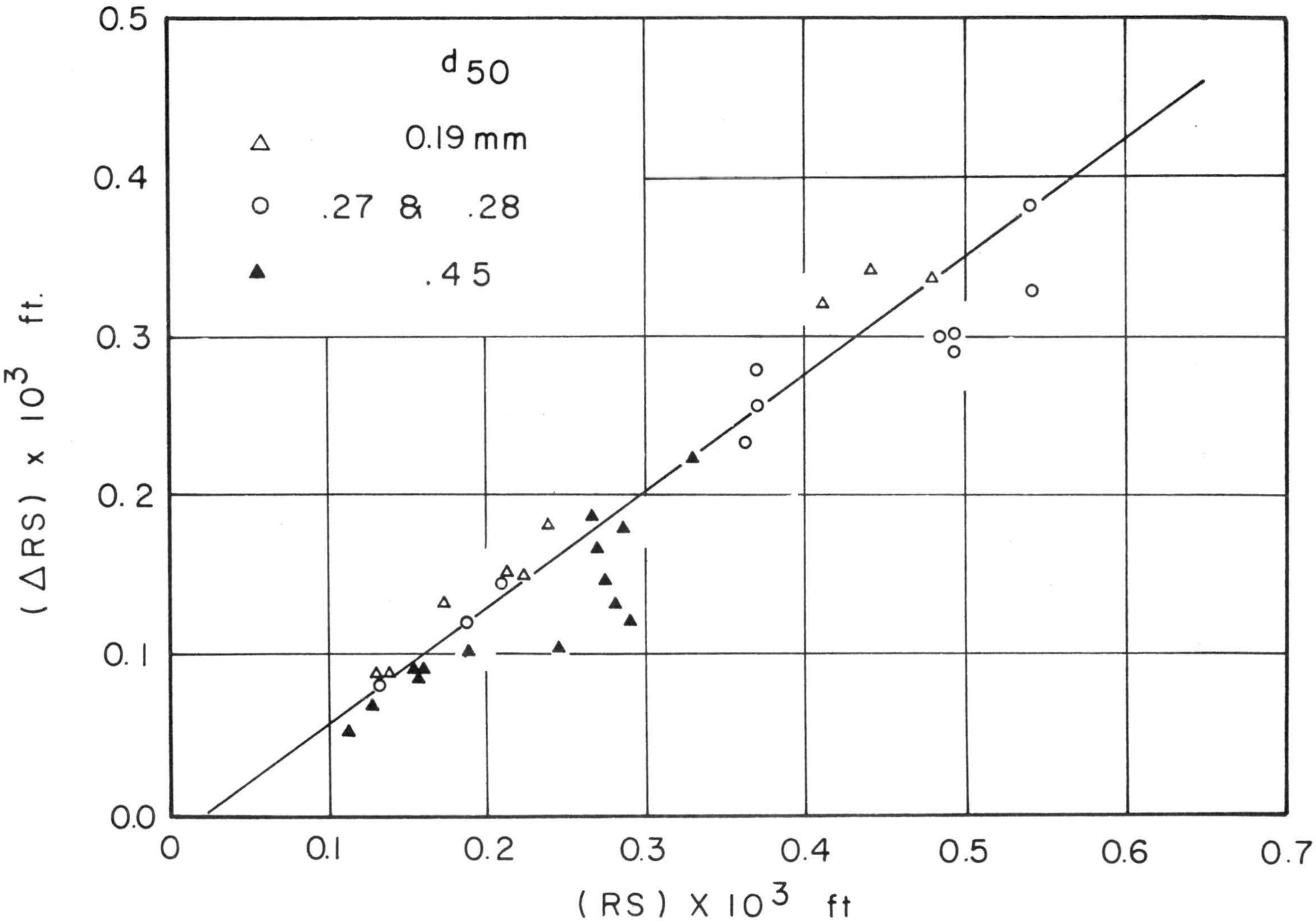


Fig. 22 Relation between ΔRS and RS for ripples

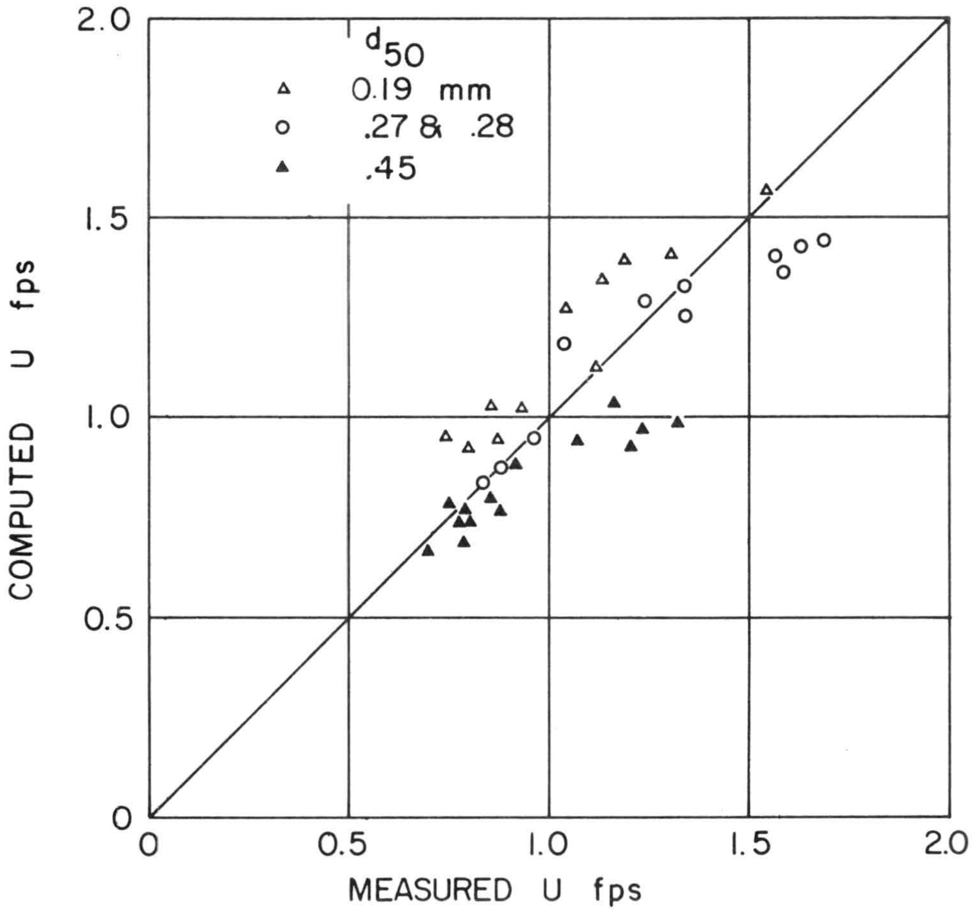


Fig. 23 Comparison of the computed velocity with the measured velocity (eq. 36)

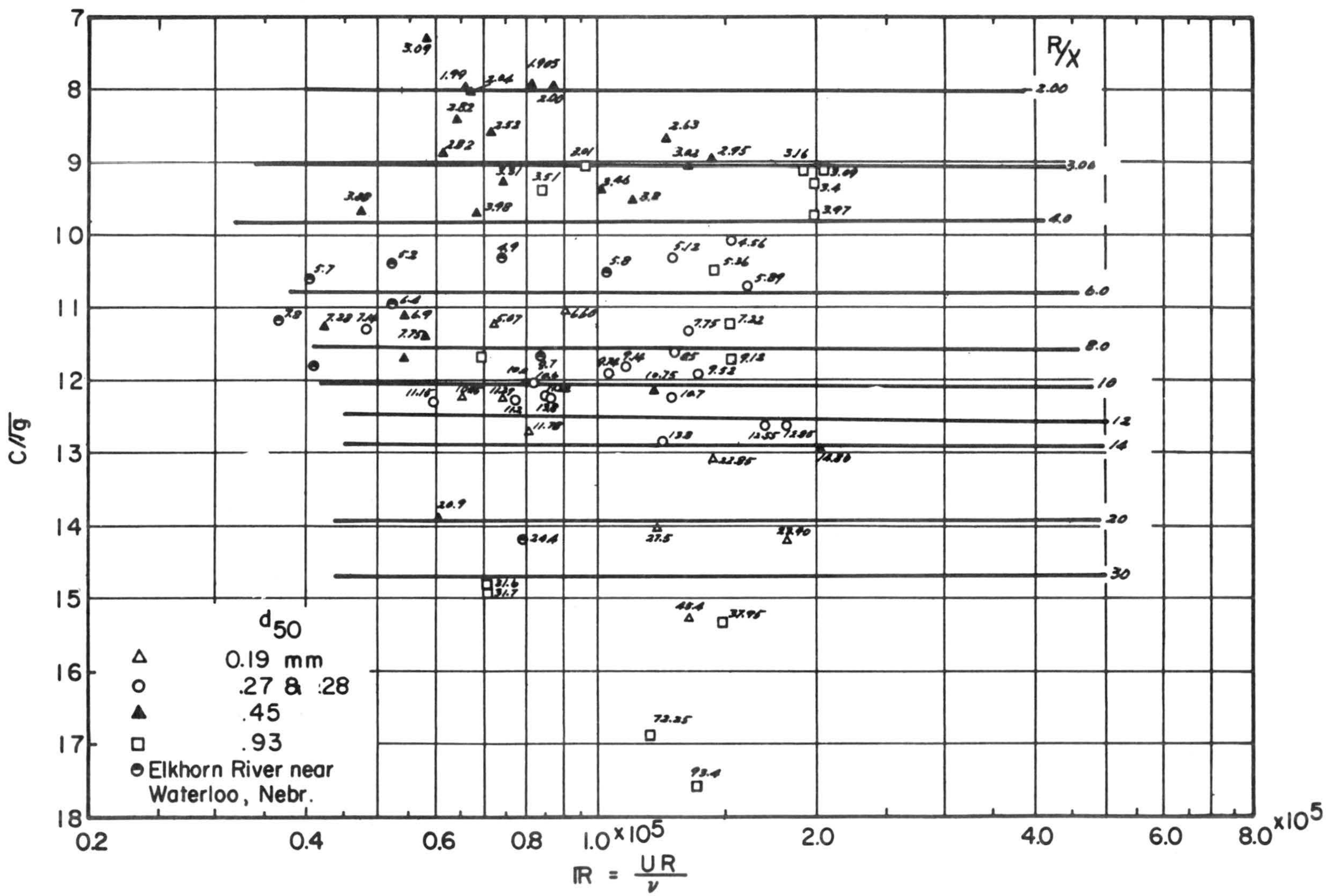


Fig. 24 Relation between $C/\sqrt{g}$, IR and R/X when $C/\sqrt{g} = 5.75 \log R/x + 6.25$.

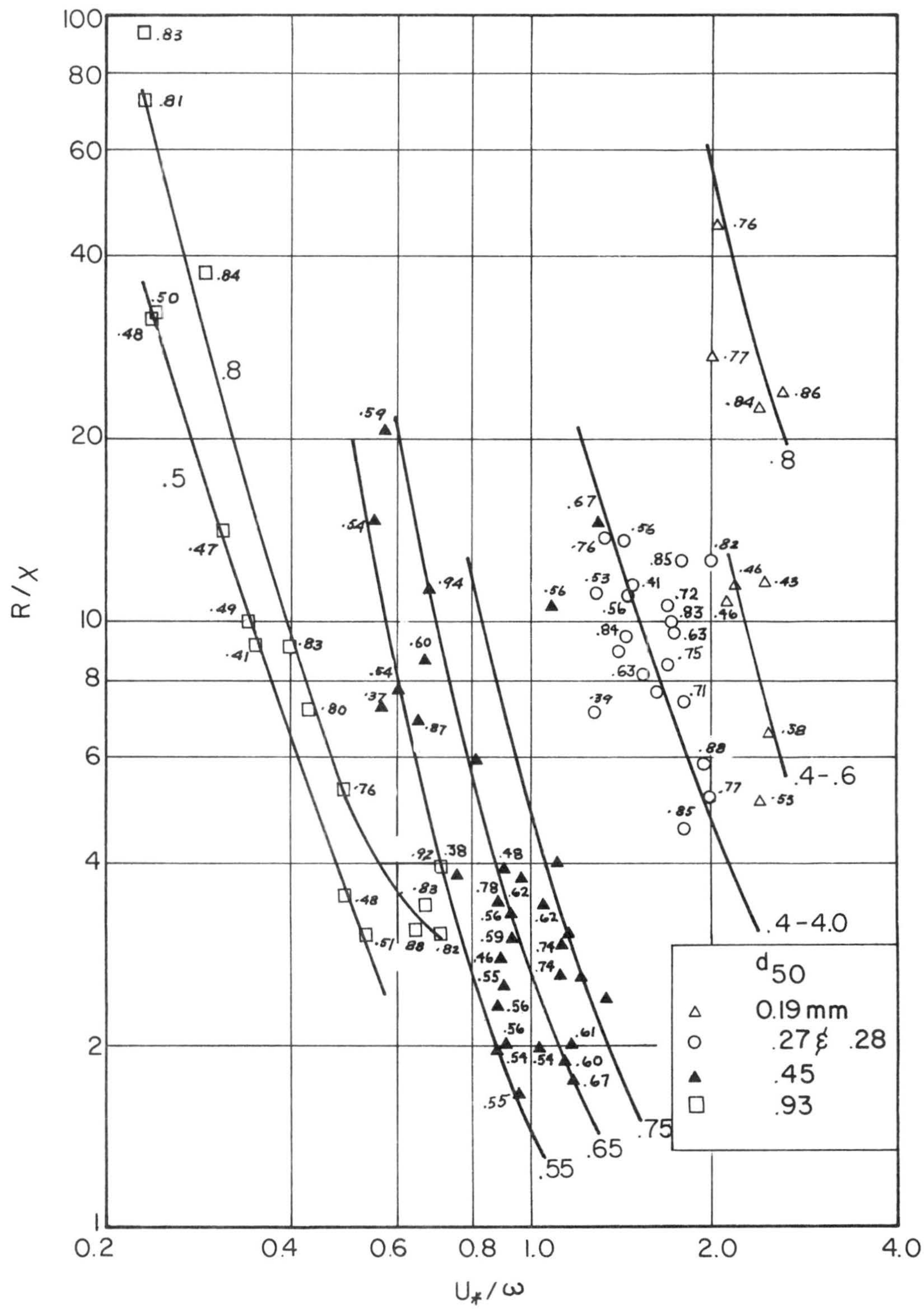


Fig. 25 Variation of R/χ with U_*/ω for the dune bed. Hydraulic radius is the third variable.

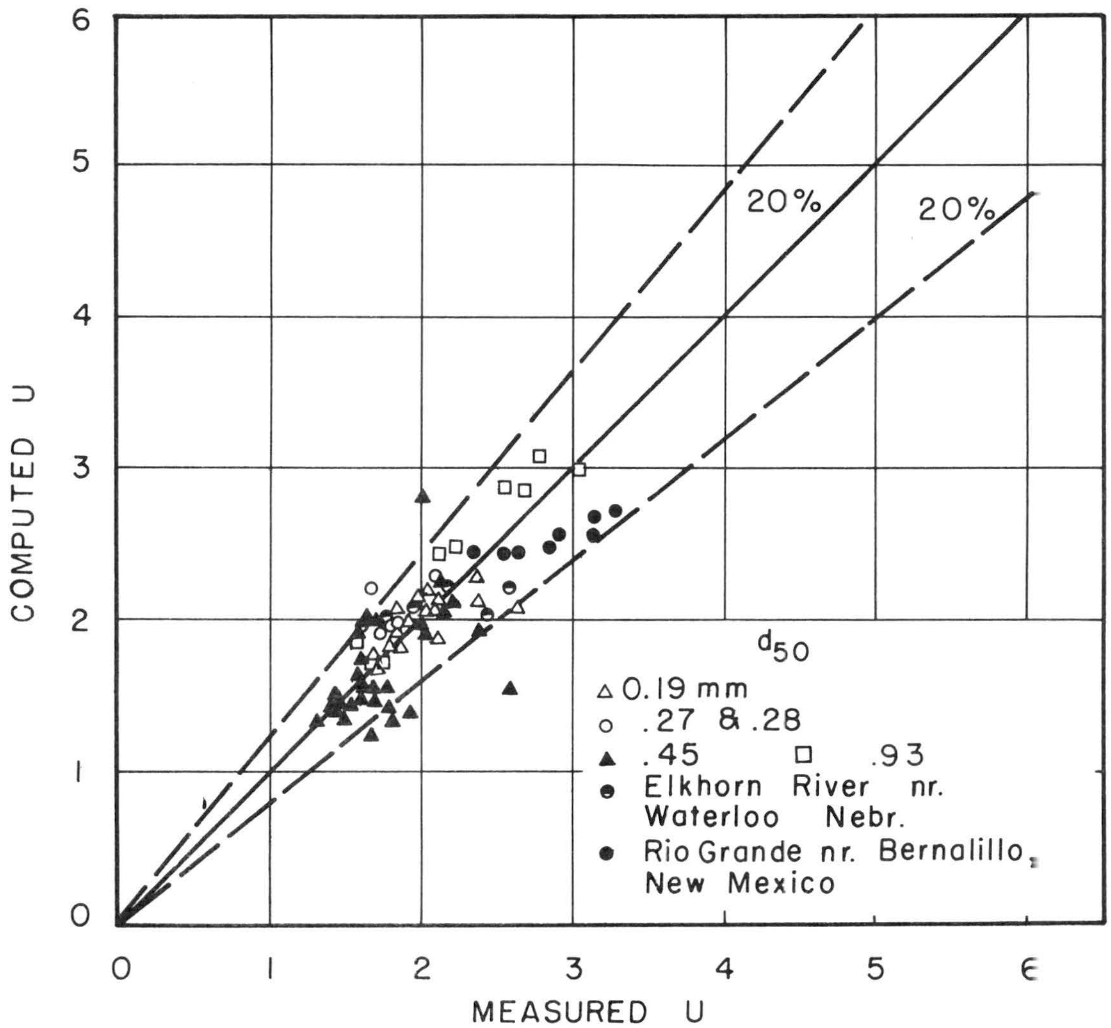


Fig 26 Comparison of the computed velocity w th the measured velocity. (eq. 43)

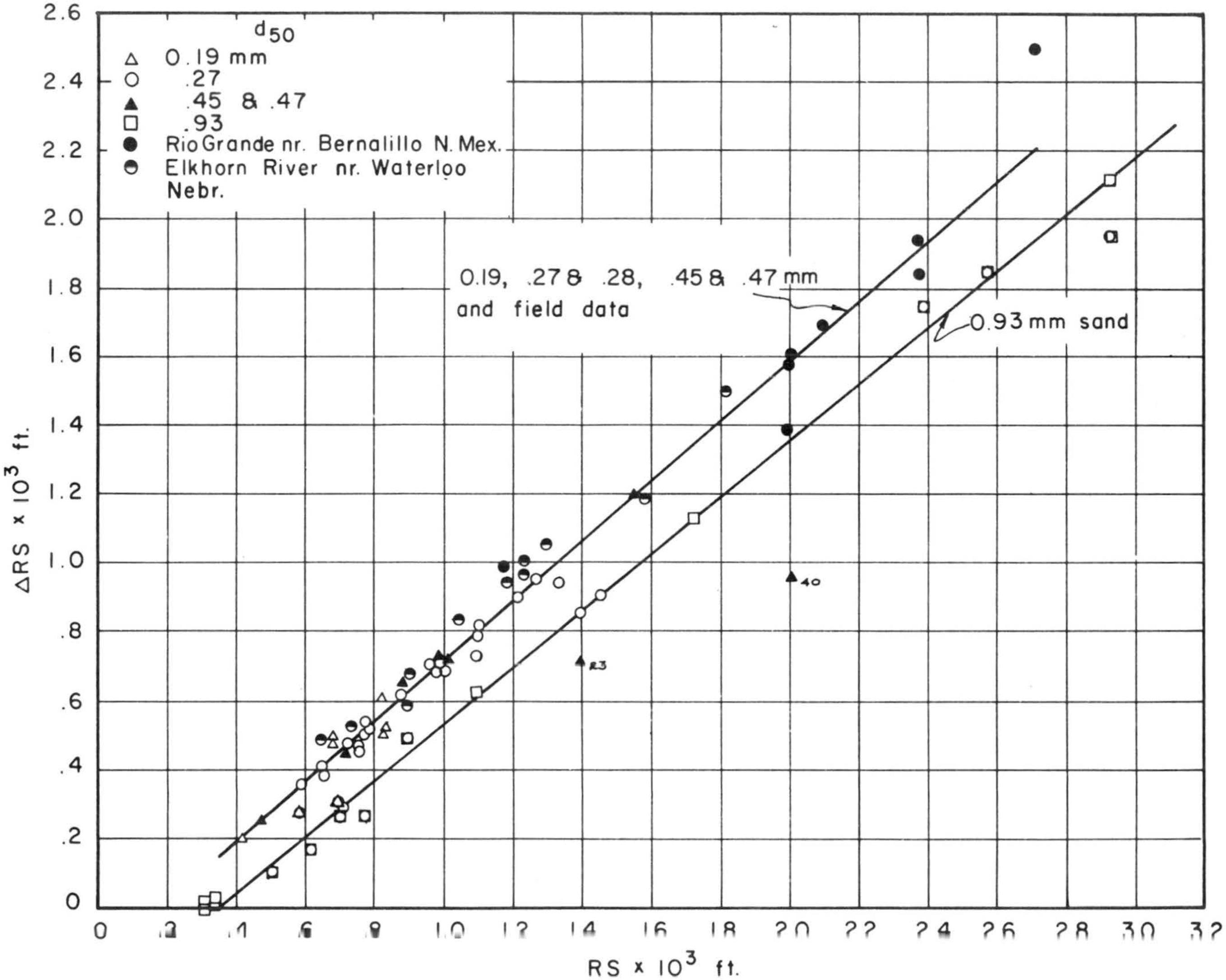


Fig. 27 Relation between ΔRS with RS for the 8 ft. flume and field data.

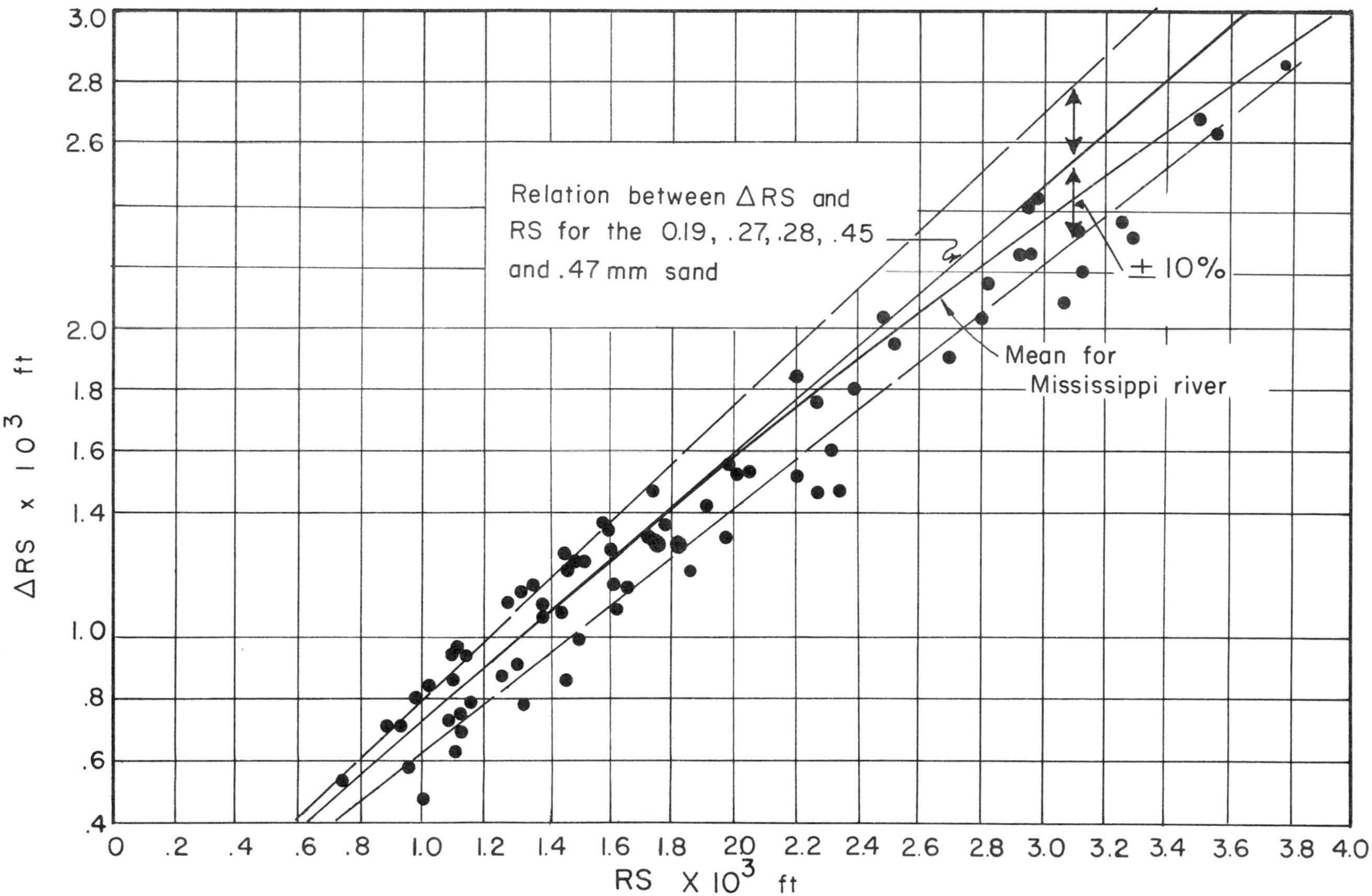


Fig. 28 Relation between ΔRS and RS for the Mississippi River at St. Louis, Missouri

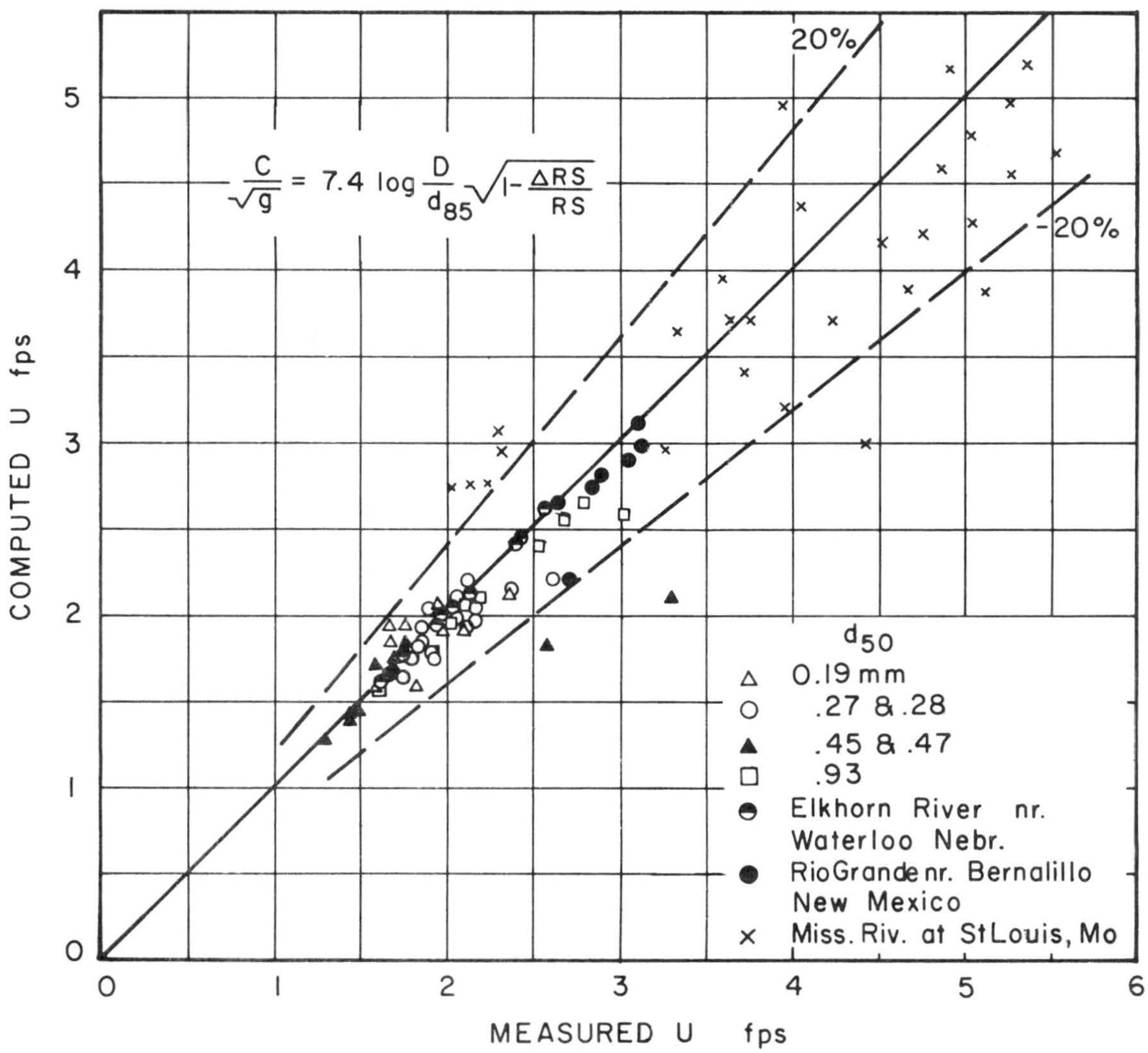


Fig. 29 Comparison of computed velocity with the measured velocity for dunes using the ΔRS method.

TABLE 1 - BASIC DATA AND COMPUTED PARAMETERS FOR 0.19 MM SAND

Bed[1] Form	Run	D ft	R ft	U fps	S X10^2	τ_0 lb/ft^2	$\frac{C}{\sqrt{g}}$	U_* fps	IR X10-5	IR$_*$ X10^{-2}	RS ft	ΔRS ft	C_*	χ ft
P	24B	0.96	0.78	0.86	0.005	0.0024	24.28	0.0354	0.604	2.480	----	-----	1.[illegible]2	-----
P	24A	.91	.74	.88	.006	.0028	23.25	.0378	.578	2.480	----	-----	1.[illegible]6	-----
P	22A	.48	.43	.78	.010	.0027	21.01	.0373	.279	1.42	----	-----	1.[illegible]1	-----
R	2	1.06	.85	.79	.015	.0079	12.42	.0642	.505	4.100	0.128	0.0918	.[illegible]1	0.0725
P	22B	.43	.39	.87	.016	.0039	19.45	.0447	.295	1.52	----	-----	.[illegible]8	-----
P	26	.30	.28	---	.017	.0030	-----	.0392	----	9.97	----	-----	-----	-----
R	25	.93	.76	.87	.018	.0085	13.17	.0666	.602	4.60	.138	.0923	.[illegible]4	.0577
P	22C	.42	.38	.89	.018	.0043	18.96	.0470	.297	1.57	----	-----	.[illegible]6	-----
R	30	1.00	.80	1.11	.028	.0140	13.08	.0850	.758	6.82	.224	.1520	.[illegible]7	.0528
R	1	.58	.51	.74	.034	.0108	9.95	.0746	.295	2.97	.173	.1350	.[illegible]6	.1170
R	31	1.02	.82	1.30	.043	.0220	12.26	.1083	.914	7.80	.364	.2654	.[illegible]0	.0748
R	27	.55	.48	.93	.057	.0171	9.88	.0826	.393	3.48	.212	.1513	.[illegible]4	.1123
R	5	1.33	.82	1.54	.058	.0297	12.44	.1240	1.06	8.54	.478	.3390	.[illegible]8	.0683
R	23	.44	.39	.85	.061	.0148	9.63	.0876	.291	3.00	.238	.1840	.[illegible]76	.1026
D	32	.95	.77	1.79	.066	.0317	14.03	.1270	1.21	8.60	.501	.3101	.[illegible]17	.0345
D	8	.93	.76	1.99	.070	.0332	15.27	.1310	1.34	8.80	.533	.2958	.[illegible]66	.0205
R	28	.54	.47	1.04	.079	.0232	9.46	.1091	.429	4.49	.370	.2930	.[illegible]54	.1294
D	33	1.06	.85	1.96	.083	.0440	13.09	.1505	1.45	11.35	.703	.4809	.[illegible]61	.0372
R	29	.56	.49	1.13	.084	.0257	9.81	.1152	.503	5.08	.412	.3234	.[illegible]63	.1175
R	3	.55	.48	1.18	.092	.0276	9.86	.1192	.426	4.25	.441	.3437	.[illegible]69	.1150
D	11	1.09	.86	2.35	.099	.0531	14.22	.165	1.82	12.80	.846	.8127	.[illegible]12	.0359
T	13	.89	.73	3.09	.100	.0456	20.19	.154	2.05	10.20	.730	.156	.[illegible]86	-----
T	14	.86	.71	3.22	.106	.0414	20.72	.1558	2.08	10.10	.753	.122	.[illegible]17	-----
T	15	.79	.66	3.46	.112	.0461	22.40	.1546	2.07	9.28	.740	----	1.[illegible]05	-----
D	34	.52	.46	1.68	.127	.0364	12.26	.1372	.655	5.34	.584	.3842	.[illegible]85	.0420
T	12	1.02	.82	2.69	.130	.0665	14.75	.185	2.01	13.90	1.070	.644	[illegible]30	-----
D	6	.61	.53	1.67	.130	.0430	11.22	.149	.726	6.47	.689	.5003	[illegible]23	.0731
D	7	.68	.56	1.78	.140	.0506	11.01	.162	.905	8.25	.815	.6067	[illegible]04	.0879
D	35	.52	.46	1.81	.147	.0422	12.28	.148	.743	6.08	.680	.4471	[illegible]84	.0408
T	16	.72	.61	3.84	.156	.0594	21.96	.175	2.09	9.54	.937	.005	[illegible]97	-----
T	10	.51	.45	2.89	.170	.0477	18.37	.157	1.17	6.36	.766	.171	[illegible]81	-----
D	9	.49	.43	2.10	.194	.052	12.73	.164	.805	6.29	.835	----	[illegible]16	0.365
A	17	.67	.57	4.14	.196	.070	21.78	.190	2.12	9.75	1.12	.011	1[illegible]05	-----
A	18	.64	.55	4.33	.300	.103	18.79	.231	2.14	11.50	1.65	.413	[illegible]366	-----
A	19	.64	.55	4.33	.350	.120	17.40	.249	2.12	12.20	1.92	.684	[illegible]303	-----
A	39	.61	.53	4.58	.390	.129	17.80	.258	2.17	12.20	2.33	.733	[illegible]329	-----
A	20	.60	.52	4.62	.460	.149	16.63	.278	1.22	12.90	2.39	.955	[illegible]77	-----
A	21	.50	.44	4.03	.542	.148	14.49	.277	1.58	10.90	2.38	1.21	[illegible]00	-----
A	38	.58	.51	4.74	.582	.186	15.46	.309	2.12	13.80	2.96	1.42	[illegible]22	-----
A	36	.51	.45	3.81	.845	.237	10.88	.349	1.45	13.30	3.81	2.77	[illegible]521	-----
A	37	.65	.56	4.20	.950	.332	9.92	.414	2.03	20.00	5.32	4.15	[illegible]69	-----

[1]P = plane, R = ripples, D = dunes, T = transition, A = antidunes

TABLE 2 - BASIC DATA AND COMPUTED PARAMETERS FOR 0.27 MM SAND

P	49	1.03	0.82	0.73	0.005	0.0026	20.05	0.0364	0.434	2.16	----	----	0 [illegible]51	-----
P	50A	.96	.78	.79	.007	.0034	18.93	.0420	.493	2.62	----	----	[illegible]300	-----
R	50D	.91	.74	.84	.018	.0083	12.83	.0655	.514	4.01	0.133	0.0822	[illegible]518	0.0536
R	51	.99	.80	1.24	.046	.0230	11.44	.109	.833	7.27	.369	.2601	[illegible]542	.1005
R	52	.94	.75	1.63	.065	.0307	12.93	.126	1.03	7.97	.493	.3018	[illegible]522	.0526
D	54	.93	.75	1.83	.084	.0398	12.82	.144	1.23	9.68	.644	.4030	[illegible]611	.0551
D	53	1.02	.82	1.91	.108	.0553	11.36	.169	1.34	11.84	.887	.6306	[illegible]537	.1058
R	57	.48	.43	1.33	.126	.0338	10.08	.132	.454	4.47	.541	.3834	[illegible]539	.1052
D	56	.75	.63	1.85	.126	.0589	11.57	.160	.955	8.36	.795	.5308	[illegible]576	.0759
D	55	1.08	.85	2.06	.130	.0689	10.09	.189	1.54	14.10	1.11	.8147	[illegible]514	.1863
T	45	.84	.70	3.25	.138	.0602	18.52	.176	1.98	10.70	.964	.188	[illegible]898	-----
D	43	1.13	.88	2.13	.140	.0767	10.69	.198	1.61	15.00	1.22	.900	510	.1493
D	44	1.03	.82	2.62	.163	.0835	12.63	.208	1.82	14.45	1.34	.941	582	.0637
D	42	.94	.76	2.09	.167	.0792	10.34	.202	1.28	12.40	1.27	.956	495	.1482
T	46	.74	.62	3.68	.167	.0646	20.06	.182	2.04	10.07	1.04	----	1[illegible]011	-----
D	58	.46	.41	1.83	.185	.0473	12.31	.156	.598	5.07	.756	.4539	631	.0368
A	47	.63	.54	4.32	.280	.0945	19.51	.221	2.00	9.34	1.51	0	1[illegible]000	-----
A	48	.59	.51	4.60	.493	.157	16.15	.284	1.96	12.07	1.49	.448	838	-----
A	39	.55	.48	4.93	.813	.243	13.86	.356	1.69	12.20	3.91	1.86	725	-----
A	41	.45	.40	4.28	.952	.237	12.18	.351	1.24	10.20	3.81	2.18	661	-----
A	40	.60	.52	4.45	1.022	.382	10.65	.414	1.67	15.60	5.28	3.67	553	-----

[1]P = plane, R = Ripples, D = dunes, T = transition, A = antidunes

TABLE 3 - BASIC DATA AND COMPUTED PARAMETERS FOR 0.28 MM SAND

P	6	0.90	0.74	0.53	0.005	0.0023	15.44	0.0345	0.268	1.75	----	----	[illegible]742	-----
P	7	1.01	.81	.82	.007	.0035	19.20	.0424	.523	2.71	----	----	.920	-----
P	8A	1.00	.80	.97	.011	.0055	18.21	.0532	.579	3.18	----	----	.868	-----
R	8B	1.01	.81	.96	.023	.012	12.43	.0775	.563	4.55	0.187	0.1218	.588	0.0634
R	10	.59	.51	.88	.041	.013	10.69	.0821	.365	3.41	.209	.1445	.555	.0909
R	5	1.00	.80	1.34	.045	.023	12.45	.1078	.901	7.24	.361	.2341	.592	.0818
R	13B	1.00	.80	1.68	.062	.031	13.30	.126	1.35	8.48	.493	.2940	.635	.0593
R	4	.86	.71	1.56	.069	.031	12.45	.125	.886	7.09	.485	.3018	.614	.0592
R	11	.59	.51	1.04	.073	.023	9.46	.109	.432	4.14	.369	.2783	.494	.1407
D	33	1.06	.85	1.86	.090	.047	11.93	.157	1.38	11.60	.766	.5259	.559	.0892
D	1	.88	.72	1.80	.100	.045	11.81	.152	1.10	9.29	.717	.4794	.575	.0788
R	12	.57	.50	1.58	.108	.033	12.00	.132	.658	5.49	.541	.3307	.623	.0500
D	14	.62	.54	1.74	.116	.039	12.29	.142	.776	6.33	.626	.3784	.628	.0482
D	20	1.05	.84	2.16	.120	.062	12.04	.180	1.82	12.5	1.01	.6828	.566	.0839
D	2	.92	.75	2.06	.131	.061	11.61	.178	1.28	11.0	.984	.6778	.557	.0882

Continued

TABLE 3 - BASIC DATA AND COMPUTED PARAMETERS FOR 0.28 MM SAND--Continued

Bed[1] Form	Run	D ft	F ft	U fps	S $X10^2$	τ_o lb/ft^2	$\frac{C}{\sqrt{g}}$	U_* fps	IR $X10^{-5}$	IR_* $X10^{-2}$	RS ft	ΔRS ft	C_*	χ ft
D	21	1.07	0.85	2.38	0.131	0.069	12.61	0.189	1.70	13.5	1.11	0.7188	0.593	0.0677
D	19	.65	.56	1.90	.134	.046	12.24	.156	.865	7.11	.756	.4666	.618	.0406
D	16	1.02	.82	2.11	.134	.068	11.28	.188	1.43	12.75	1.10	.7858	.532	.1108
T	23	.91	.74	3.02	.134	.062	16.89	.178	1.85	10.9	----	-----	.815	-----
D	17	.65	.56	1.92	.136	.047	12.28	.157	.868	7.08	.766	.4696	.621	.0499
D	3	.88	.72	2.17	.136	.061	12.21	.178	1.27	10.5	.984	.6391	.591	.0672
T	18	.61	.53	2.45	.141	.047	15.81	.155	1.05	6.63	----	-----	.814	-----
T	30	.64	.55	3.06	.142	.049	19.28	.159	1.35	7.00	----	-----	.981	-----
D	34	.44	.39	1.56	.150	.036	11.28	.137	.483	4.24	.583	.3596	.618	.0546
P	22	.60	.52	3.11	.153	.049	19.39	.160	1.23	6.36	----	.643	1.001	-----
D	15	.75	.63	2.14	.158	.062	11.95	.179	1.04	8.68	.995	.9585	.594	.0646
T	24	.82	.68	3.35	.172	.073	17.26	.194	1.88	10.9	----	-----	.846	-----
P	25	.72	.61	3.79	.199	.076	19.19	.198	1.86	9.74	----	-----	.957	-----
P	28	.55	.48	3.57	.229	.068	18.95	.188	1.38	7.28	----	-----	.994	-----
P	29	.52	.46	3.77	.278	.080	18.60	.203	1.42	7.66	----	-----	.982	-----
A	26	.50	.44	3.88	.328	.100	17.91	.216	1.39	7.73	1.66	.146	.955	-----
A	32	.58	.51	4.69	.470	.150	16.97	.278	1.74	10.3	2.39	.566	.874	-----
A	27	.43	.39	4.50	.533	.130	17.44	.238	1.43	7.55	2.07	.14	1.033	-----
A	31	.56	.49	4.76	.593	.182	15.69	.306	1.67	10.7	2.90	.995	.811	-----
A	35	.54	.47	4.93	.815	.239	13.94	.351	1.68	11.9	3.83	1.76	.735	-----
A	37	.30	.28	3.48	.820	.143	12.80	.272	.722	5.64	2.29	1.03	.743	-----
A	38	.40	.36	4.77	.930	.209	14.42	.328	1.25	8.63	3.35	1.16	.805	-----
A	36	.57	.40	4.69	1.007	.337	10.75	.402	1.72	14.8	5.03	3.18	.607	-----

[1]P = plane, R = ripples, D = dunes, T = transition, A = antidunes

TABLE 4 - BASIC DATA AND COMPUTED PARAMETERS FOR 0.45 MM SAND

P	14	0.61	0.53	0.81	0.015	0.0050	16.06	0.041	0.307	1.55	----	-----	1.159	-----
R	17	.98	.79	.80	.016	.0079	12.60	.064	.472	3.77	0.127	0.0697	.672	0.0621
R	16	.81	.67	.79	.017	.0071	11.86	.060	.395	3.00	.112	.0519	.732	.0718
P	13	.35	.32	.65	.019	.0038	14.65	.043	.146	9.44	----	-----	.988	-----
P	15A	.71	.60	.89	.015	.0056	16.70	.0539	.361	1.34	----	-----	.939	-----
R	15B	.80	.67	.79	.023	.0096	11.24	.0705	.384	3.42	.154	.0944	.623	.0925
R	18	.58	.51	.78	.031	.0099	10.97	.0714	.292	2.68	.158	.0922	.646	.0773
R	2	.82	.68	1.20	.036	.015	13.51	.089	.591	4.38	.246	.1077	.749	.0374
R	3	.85	.70	1.16	.039	.017	12.37	.094	.597	4.84	.274	.1474	.680	.0610
R	9	.55	.48	.88	.040	.012	11.16	.078	.315	2.79	.189	.1032	.673	.0677
R	1	.80	.67	1.23	.042	.018	12.96	.095	.564	4.36	.280	.1348	.720	.0464
R	5	.75	.63	1.32	.047	.018	13.6	.097	.602	6.11	.292	.1201	.767	.0177
R	11	.35	.32	.70	.049	.0098	9.82	.071	.165	1.67	.157	.0916	.644	.0767
D	4	.69	.59	1.44	.057	.021	13.87	.104	.602	4.35	.336	.1250	.792	.0283
R	8	.51	.45	.93	.060	.017	9.97	.093	.312	3.12	.269	.1696	.606	.1030
D	7	.70	.60	1.43	.078	.029	11.69	.123	.631	5.43	.470	.2622	.664	.0690
R	10	.33	.30	.75	.088	.016	8.08	.0924	.162	1.99	.265	.1882	.537	.1436
R	6	.46	.41	1.07	.088	.022	9.89	.108	.307	3.09	.331	.2261	.612	.0962
R	12	.29	.27	.85	.106	.018	8.85	.0960	.170	1.92	.286	.1822	.602	.1064
D	19	.41	.37	1.30	.112	.026	11.23	.115	.422	3.73	.411	.2007	.715	.0509
D	21	.96	.78	1.58	.114	.056	9.38	.169	1.03	11.00	.887	.6611	.504	.2250
D	22	1.00	.80	1.70	.124	.062	9.51	.179	1.12	11.80	.995	.7362	.509	.2150
D	25	.42	.38	1.47	.189	.045	9.66	.152	.477	4.94	.717	.4544	.605	.0978
D	20	.61	.53	1.68	.193	.064	9.27	.182	.748	8.10	1.03	.7262	.541	.1600
D	23	.65	.56	2.57	.247	.086	12.19	.212	1.20	9.89	1.40	.7089	.701	.0522
D	24	.62	.54	1.76	.289	.097	7.88	.224	.812	10.3	1.56	1.208	.473	.2839
D	40	.81	.67	3.32	.301	.126	12.99	.255	2.01	15.4	2.02	.9663	.722	.0453
A	39	.55	.48	4.71	.364	.109	19.70	.238	2.04	10.3	----	-----	1.181	-----
P	26	.34	.31	5.38	.366	.071	27.82	.192	1.43	5.09	----	-----	1.844	-----
A	28	.40	.36	3.52	.366	.082	16.99	.206	1.06	6.68	----	-----	1.087	-----
T	29	.30	.28	1.89	.369	.065	10.37	.182	4.56	4.39	----	-----	.731	-----
A	31	.44	.39	4.24	.432	.105	18.05	.223	1.43	7.82	----	-----	1.137	-----
T	27	.33	.30	2.99	.436	.082	14.44	.206	.786	5.42	----	-----	.961	-----
T	36	.19	.18	2.04	.446	.050	12.63	.161	.331	2.61	----	-----	.950	-----
A	41	.54	.47	5.05	.466	.137	18.89	.265	2.12	11.10	----	-----	1.140	-----
T	30	.27	.25	2.47	.492	.077	12.76	.199	.532	4.28	----	-----	.859	-----
A	35	.25	.24	2.80	.494	.073	14.80	.195	.574	4.00	----	-----	1.010	-----
A	34	.28	.26	3.73	.546	.089	17.40	.214	.836	4.79	----	-----	1.192	-----
A	33	.27	.25	4.60	.607	.095	20.69	.221	.958	4.61	----	-----	1.443	-----
A	38	.50	.44	5.38	.619	.170	18.09	.296	2.12	11.7	----	-----	1.105	-----
A	37	.43	.39	5.54	.620	.151	19.91	.279	1.93	9.72	----	-----	1.244	-----
A	32	.37	.34	5.03	.656	.139	18.85	.269	1.50	8.02	----	-----	1.208	-----
A	45	.28	.26	2.50	.862	.138	9.31	.269	.586	6.30	----	-----	.636	-----
A	44	.28	.26	4.78	.898	.146	17.42	.275	1.13	6.50	----	-----	1.181	-----
A	42	.31	.29	5.36	.986	.179	18.64	.304	1.44	8.16	----	-----	1.184	-----
A	43	.43	.39	6.18	1.01	.246	17.45	.356	2.15	12.4	----	-----	1.087	-----

[1]F = plane, R = ripples, D = dunes, T = transition, A = antidunes

TABLE 5 - BASIC DATA AND COMPUTED PARAMETERS FOR THE 0.47 MM SAND

D	46	1.11	0.87	1.64	0.084	0.045	10.69	0.153	1.10	10.2	0.727	0.4950	0.564	0.1479
D	47	.75	.63	1.60	.042	.017	17.33	.092	.741	6.77	.263	.0102	.980	-----
D	48	1.23	.93	1.55	.052	.030	12.34	.125	1.06	8.55	.485	.2823	.642	.0812
D	49	1.33	.98	2.00	.173	.108	8.47	.234	1.42	16.6	1.70	1.37	.435	.4080
R	85	.78	.65	1.13	.047	.019	11.38	.019	.561	4.92	.304	.1801	.638	.0838
D	93	.62	.54	1.45	.072	.024	13.00	.112	.632	4.88	.389	.1653	.758	.0365
D	92	.63	.54	1.43	.090	.030	11.39	.125	.581	5.07	.485	.2705	.665	.0696
D	91	.58	.51	1.53	.117	.037	11.08	.138	.549	4.96	.591	.3359	.657	.0738
D	82	.64	.55	1.60	.248	.075	7.54	.210	.840	11.6	1.37	1.099	.443	.3315
D	51	.62	.54	1.62	.236	.079	8.02	.203	.683	8.56	1.28	.9992	.467	.2705

Continued

TABLE 5 - BASIC DATA AND COMPUTED PARAMETERS FOR THE 0.47 MM SAND--Continued

Bed[1] Form	Run	D ft	R ft	U fps	S $\times 10^2$	τ_o lb/ft^2	$\frac{C}{\sqrt{g}}$	U_* fps	$\mathbb{R}$ $\times 10^{-5}$	$\mathbb{R}_*$ $\times 10^{-2}$	RS ft	ΔRS ft	Θ_*	χ ft
D	52	.55	0.48	1.81	0.222	0.067	9.74	0.185	0.689	7.06	1.06	0.6999	0.584	0.1205
D	73	.61	.53	1.67	.222	.074	8.59	.195	.713	8.33	1.18	.8827	.502	.2108
D	74	.65	.56	1.58	.215	.075	8.04	.197	.675	8.43	1.21	.9450	.463	.2744
D	76	.63	.54	1.69	.203	.068	7.997	.187	.662	7.40	1.11	.8080	.520	.2705
D	75	.64	.55	1.60	.204	.070	8.41	.190	.647	7.69	1.12	.8513	.489	.2345
D	53	.57	.50	1.77	.235	.073	9.11	.194	.583	6.38	1.17	.8252	541	.1618
D	77	.65	.56	1.68	.199	.070	8.88	.143	.619	5.27	.635	.3424	678	.1986
D	96	.53	.47	1.94	.201	.059	11.15	.174	.472	4.23	.940	.5145	672	-----
D	94	.81	.67	1.74	.237	.099	7.68	.225	.910	11.8	1.57	1.280	430	.3769
D	83	.91	.74	2.14	.200	.092	9.80	.218	1.33	13.6	1.48	1.056	532	.1816
D	54	.92	.75	2.08	.240	.112	8.66	.241	1.25	14.5	1.80	1.405	.469	.2850
D	56	.90	.74	2.14	.242	.112	8.95	.240	1.43	16.0	1.79	1.362	.488	.2505
D	55	.74	.62	2.04	.237	.092	9.35	.217	1.01	10.7	1.46	1.050	.530	.1790
D	57	.87	.72	2.20	.259	.116	9.02	.245	1.33	14.8	1.86	1.390	.494	.2384
D	58	.90	.74	2.11	.233	.108	8.68	.236	1.25	14.0	1.73	1.317	.487	.2814
T	95	.80	.67	2.39	.180	.075	12.16	.197	.788	6.50	1.21	.6549	.675	-----
D	78	.72	.61	2.00	.320	.122	7.98	.250	.876	10.8	1.94	1.535	.456	.3055
T	59	.65	.56	2.96	.326	.114	12.11	.242	1.38	11.3	1.82	.9092	.707	-----
P	60	.62	.54	4.28	.342	.115	17.61	.244	1.96	11.2	----	----	.024	-----
P	61	.61	.53	4.36	.355	.117	17.71	.245	1.92	10.8	----	----	.041	-----
P	71	.32	.30	3.21	.531	.099	14.26	.227	.816	5.77	----	----	.943	-----
P	72	.32	.30	3.26	.550	.103	14.23	.231	.746	5.28	----	----	.940	-----
P	70	.30	.28	3.41	.640	.112	14.21	.240	.795	5.60	----	----	.958	-----
A	63	.43	.39	4.48	.570	.139	16.79	.268	1.51	9.01	----	----	.048	-----
A	64	.41	.37	4.76	.578	.133	18.10	.262	1.40	7.69	----	----	.151	-----
A	65	.42	.38	4.63	.571	.135	17.52	.265	1.31	7.52	----	----	.098	-----
A	66	.45	.40	4.34	.575	.144	15.86	.272	1.26	7.88	----	----	.992	-----
A	80	.39	.35	4.91	.643	.140	18.09	.269	1.20	6.54	----	----	.171	-----
A	81	.55	.48	4.85	.634	.190	15.44	.313	1.69	10.9	----	----	.927	-----
A	62	.54	.47	4.89	.622	.182	15.85	.307	2.05	12.9	----	----	.955	-----
A	67	.53	.47	4.91	.646	.189	15.73	.313	1.70	10.8	----	----	.946	-----
A	79	.55	.48	4.82	.651	.195	15.15	.317	1.58	10.4	----	----	.909	-----
A	84	.41	.37	4.69	.740	.171	15.69	.298	1.41	8.95	----	----	.993	-----
A	69	.43	.39	4.48	.734	.179	14.79	.304	1.41	9.56	----	----	.925	-----
A	68	.53	.47	4.95	.740	.217	14.82	.335	1.89	12.7	----	----	.889	-----
A	98	.44	.39	4.51	.821	.200	13.93	.321	.714	5.08	----	----	.879	-----
P	100	.51	.45	5.28	.790	.222	15.56	.338	1.80	11.8	----	----	.947	-----
A	99	.50	.44	5.32	.806	.221	15.66	.338	1.19	7.58	----	----	.962	-----
A	97	.37	.34	4.07	.960	.204	12.57	.324	1.07	8.54	----	----	.813	-----

[1]P = plane, R = ripples, D = dunes, T = transition, A = antidunes

TABLE 6 - BASIC DATA AND COMPUTER PARAMETERS FOR 0.93 MM SAND

P	19	1.01	0.81	1.00	0.0129	0.0065	17.25	0.058	0.743	4.31	----	----	1.005	-----
P	26A	1.02	.82	1.32	.0219	.0112	17.39	.076	.974	5.61	----	----	1.010	-----
P	25	1.01	.81	1.22	.0220	.0111	16.08	.0759	.898	5.58	----	----	.938	-----
P	27	1.01	.81	1.47	.0280	.014	16.52	.0855	1.17	6.79	----	----	1.003	-----
P	26	1.03	.82	1.37	.0283	.014	15.75	.0864	1.00	6.33	----	----	.923	-----
P	20	1.03	.82	1.32	.0284	.015	15.19	.0865	.993	6.51	----	----	.887	-----
P	21	1.01	.81	1.49	.0295	.015	16.94	.0879	1.13	6.66	----	----	.988	-----
D	18	1.01	.81	1.66	.0370	.019	16.95	.0984	1.18	6.98	0.301	0.0972	.984	0.0112
D	28	1.04	.83	1.75	.0373	.019	17.59	.099	1.37	7.76	.304	.3413	1.055	.0089
P	29	.50	.44	1.16	.0426	.012	15.09	.078	.459	3.09	----	----	.999	-----
P	22	.49	.43	1.15	.0430	.012	14.94	.077	.446	2.98	----	----	1.005	-----
P	30	.51	.45	1.25	.0497	.014	14.61	.085	.507	3.45	----	----	.9835	-----
P	31	.50	.44	1.36	.0537	.015	15.47	.087	.507	3.25	----	----	1.049	-----
D	15	1.05	.84	1.93	.0590	.031	15.36	.126	1.49	9.70	.493	1.065	.8857	.0221
P	23	.49	.43	1.30	.0615	.017	13.99	.0924	.508	3.69	----	----	.9470	-----
P	32	.52	.46	1.50	.0640	.018	15.49	.0975	.585	3.89	----	----	1.025	-----
P	24	.49	.43	1.46	.0682	.018	14.91	.0974	.571	3.81	----	----	1.009	-----
D	14	.58	.51	1.60	.0710	.023	14.88	.107	.703	4.71	.356	.1957	.972	.0161
D	34	.54	.47	1.64	.0800	.023	14.82	.11	.701	4.71	.376	.1150	.985	.0149
D	16	1.04	.83	2.03	.112	.058	11.77	.173	1.53	13.1	.930	5.000	.6798	.0910
D	35	.53	.47	1.80	.130	.038	12.88	.1405	7.22	5.64	.613	1.745	.8454	.0333
D	17	1.00	.80	2.10	.136	.068	11.23	.187	1.53	13.6	1.09	6.177	.6567	.1104
D	33	.56	.49	1.83	.145	.044	12.06	.151	.807	6.67	.708	.2609	.7947	.0490
D	5	.93	.76	2.21	.183	.087	10.49	.212	1.45	13.9	1.40	8.643	.6170	.1416
D	10	.46	.41	1.88	.192	.049	11.77	.159	.695	5.88	.785	2.699	.8101	.0450
D	37	1.11	.87	2.54	.275	.149	9.16	.278	1.94	21.2	2.40	17.42	.5237	.2755
D	36	.55	.48	2.04	.304	.091	9.39	.217	.844	8.97	1.46	2.031	.6134	.1370
D	6	1.04	.83	2.68	.313	.162	9.29	.289	2.00	21.6	2.59	18.44	.5377	.2440
D	7	.59	.51	2.14	.339	.113	9.05	.236	.966	10.6	1.73	11.34	.5869	.1692
D	38	1.02	.82	2.78	.356	.182	9.10	.307	2.06	22.7	2.93	21.14	.5270	.2655
D	11	.92	.75	3.02	.393	.184	9.74	.308	2.00	20.5	2.95	19.49	.5618	.1884
T	8	.57	.50	2.46	.430	.134	9.37	.263	1.06	11.3	----	----	.6114	-----
T	12	.89	.73	3.12	.437	.199	9.76	.321	2.03	20.9	----	----	.5804	-----
T	13	.82	.68	3.37	.587	.249	9.39	.358	2.03	21.6	----	----	.5704	-----
T	9	.49	.43	2.89	.600	.161	9.94	.288	1.11	11.1	----	----	.7128	-----
T	3	.60	.52	3.43	.65	.211	10.39	.331	1.54	14.8	----	----	.6695	-----
T	1	.68	.58	4.10	.71	.257	11.45	.367	2.16	19.4	----	----	.7030	-----
T	2	.53	.47	5.20	.92	.269	13.98	.373	2.16	15.5	----	----	.9233	-----
T	4	.51	.45	3.83	.94	.287	10.38	.369	1.51	14.6	----	----	.6943	-----
T	41	.44	.39	4.45	1.12	.273	11.78	.376	1.67	14.1	----	----	.8178	-----
A	42	.44	.39	5.81	1.16	.282	15.11	.382	2.12	13.9	----	----	1.051	-----
A	40	.38	.35	5.11	1.23	.269	13.79	.393	1.64	12.0	----	----	.9786	-----
A	43	.44	.39	5.86	1.26	.307	14.62	.398	2.18	14.8	----	----	1.0176	-----
A	39	.43	.39	6.07	1.28	.312	15.21	.402	2.22	14.6	----	----	1.049	-----

[1]P = plane, R = ripples, D = dunes, T = transition, A = antidunes

TABLE 7 - BASIC DATA AND COMPUTED PARAMETERS FOR THE ELKHORN RIVER NEAR WATERLOO, NEBRASKA (Upper Cableway Section)

Bed Form[1]	Measure No.	D ft	R ft	U fps	S $\times 10^2$	τ_o lb/ft^2	$\frac{C}{\sqrt{g}}$	U_* fps	$\mathbb{R}$ $\times 10^{-5}$	$\mathbb{R}^*$ $\times 10^{-2}$	RS ft	ΔRS ft	C_*	χ ft
T	567	3.76	3.74	2.87	4.19	0.977	12.81	0.224	0.993	7.76	1.56	1.154	0.508	
D	568	4.97	4.95	5.11	4.26	1.31	19.62	.260	2.26	11.49	2.10	.9170	.750	
P	569	4.92	4.80	5.22	4.43	1.32	19.94	.262	2.24	11.23	2.13	.9067	.758	
T	570	6.65	6.57	4.28	3.79	1.55	15.19	.282	2.63	17.32	2.47	2.227	.590	
T	571	5.75	5.71	4.86	4.50	1.50	16.87	.288	2.59	15.37	2.58	1.497	.647	
T	572	5.92	5.87	4.78	4.31	1.58	16.71	.286	2.62	15.69	2.54	1.489	.643	
T	573	6.58	6.51	4.32	4.47	1.81	14.13	.306	2.51	17.79	2.91	2.110	.523	
T	574	6.55	6.46	3.86	4.73	1.95	12.30	.314	2.31	18.78	3.06	2.423	.455	
P	578	7.16	7.10	6.22	3.56	1.58	21.77	.286	4.21	19.34	2.54	.9132	.800	
T	580	6.20	6.15	4.14	4.39	1.69	14.03	.295	2.42	17.28	2.70	1.959	.524	
T	581	5.97	5.92	3.89	4.52	1.67	13.22	.294	2.19	16.58	2.68	2.022	.496	
P	582	5.13	5.09	4.79	3.22	1.02	20.82	.230	2.10	10.09	1.643	.6041	.795	
T	583	4.52	4.48	3.75	3.48	.973	16.74	.224	1.45	8.65	1.56	.9015	.649	
D	587	4.30	4.27	2.58	4.23	1.13	10.66	.242	1.09	10.23	1.82	1.503	.415	
T	589	6.44	6.37	4.96	3.10	1.23	19.67	.252	----	-----	1.97	.9225	.729	
T	591	5.07	5.02	3.30	3.87	1.21	13.20	.250	1.69	12.83	1.94	1.446	.504	
T	599	4.01	4.00	3.26	3.67	.916	14.95	.218	1.29	8.63	1.48	.9477	.589	
P	628	3.07	3.06	3.79	4.03	.770	19.05	.199	.630	3.31	1.23	.4915	.774	
P	630	6.19	6.11	5.44	3.51	1.34	20.68	.263	2.12	10.24	1.15	.8773	.769	
P	632	6.08	6.02	5.51	3.70	1.39	20.56	.268	2.20	10.68	2.23	.9163	.767	
P	633	5.65	5.58	5.32	4.32	1.50	19.14	.278	1.89	9.88	2.40	1.156	.719	
P	634	5.73	5.66	5.23	4.33	1.53	18.61	.281	1.96	10.53	2.45	1.250	.700	
P	635	5.70	5.63	5.01	4.51	1.58	17.49	.286	1.89	10.81	2.54	1.437	.658	
T	636	5.46	5.36	4.55	4.75	1.59	15.90	.286	1.62	10.15	2.54	1.616	.602	
P	637	4.65	4.60	4.57	4.37	1.26	18.01	.254	1.28	7.44	2.00	1.042	.692	
T	638	3.99	3.96	3.82	3.97	.981	16.97	.225	.964	5.68	1.57	.8686	.668	
P	640	2.90	2.89	3.68	3.72	.671	19.79	.186	.776	3.92	1.07	.3678	.811	
T	642	3.92	3.90	3.99	3.84	.936	17.97	.222	1.15	6.41	1.53	.7638	.701	
T	644	3.02	3.01	2.78	4.07	.765	14.04	.198	.775	5.52	1.22	.8196	.571	
T	646	2.71	2.70	2.48	4.33	.730	12.78	.194	.618	4.85	1.17	.8447	.526	
D	648	2.84	2.82	2.08	4.38	.771	10.40	.200	.524	5.04	1.24	1.013	.426	0.540
D	650	2.84	2.82	2.14	4.24	.746	10.92	.196	.525	4.81	1.19	.9529	.447	.438
P	651	4.44	4.42	4.95	3.68	1.02	21.62	.229	2.03	9.37	1.63	.4831	.838	
D	652	3.42	3.38	2.45	4.04	.852	11.67	.210	.847	7.26	1.37	1.070	.467	.388
D	656	2.21	2.20	1.60	2.88	.396	11.19	.143	.369	3.30	.635	.4916	.474	.307
D	657	3.44	3.43	2.05	3.64	.780	10.25	.200	.746	7.28	1.24	1.032	.410	.695
D	658	3.10	3.09	2.42	2.91	.560	14.24	.170	.794	5.58	.898	.5959	.579	.127
D	662	2.05	2.05	2.00	4.32	.553	11.83	.169	.410	3.46	.887	.6573	.508	.222
T	664	3.75	3.73	2.85	4.25	.985	12.61	.226	1.06	8.43	1.59	1.188	.500	
D	666	2.28	2.27	1.95	4.61	.653	10.60	.184	.402	3.80	1.05	.8384	.449	.400
D	673	1.41	1.40	1.76	5.22	.456	11.43	.154	.185	1.62	.737	.5406	.515	.177
P	710	7.28	7.20	6.61	3.32	1.49	23.78	.278	4.87	20.47	2.40	.5954	.867	
P	876	4.38	4.34	6.00	4.67	1.26	23.44	.256	2.50	10.68	2.04	.3489	.910	

[1] P = plane, R = ripples, D = dunes, T = transition, A = antidune

TABLE 8 - BASIC DATA AND COMPUTED PARAMETERS FOR THE RIO GRANDE NEAR BERNALILLO, NEW MEXICO (Section A)

Bed Form[1]	Date	D ft	R ft	U fps	S $\times 10^2$	τ_o lb/ft^2	$\frac{C}{\sqrt{g}}$	U_*	RS ft	ΔRS ft	χ ft
T	4-25-52	2.47	2.43	4.06	0.089	0.144	14.9	0.266	2.16	1.27	
U	5-12-52	3.63	3.54	6.57	.084	.210	20.0	.314	2.97	0.87	
U	6-17-52	3.79	3.69	5.96	.083	.219	17.7	.318	3.06	1.34	
T	6-20-52	3.49	3.40	5.09	.079	.202	15.8	.298	2.69	1.43	
T	6-26-52	2.76	2.70	3.71	.076	.160	12.9	.260	2.05	1.35	
L	7-24-52	2.69	2.64	2.84	.080	.157	10.8	.263	2.11	1.70	0.592
L	~~6-2-53~~	2.56	2.51	3.11	.095	.149	11.2	.277	2.38	1.84	.347
U	5-3-58	3.68	3.58	6.91	.080	.213	20.9	.308	2.86	1.05	
U	5-13-58	4.46	.432	6.88	.080	.256	18.9	.339	3.46	1.30	
U	5-21-58	4.11	3.98	7.82	.079	.236	22.4	.323	3.14	0.22	
U	5-27-58	4.80	4.64	7.71	.080	.276	20.6	.352	3.71	1.03	
U	6-4-58	4.34	4.21	6.92	.083	.250	19.3	.341	3.49	1.28	
U	6-10-58	3.43	3.35	6.27	.074	.199	19.6	.290	2.48	0.24	
U	6-13-58	2.67	2.62	6.10	.076	.156	21.6	.256	1.99	----	
U	6-18-58	2.96	2.90	5.06	.076	.172	17.0	.269	2.20	0.87	
U	6-25-58	3.40	3.32	6.50	.080	.197	20.4	.296	2.66	0.54	
L	4-6-60	2.56	2.51	3.04	.083	----	11.6	.262	2.08	1.61	.282
L	5-24-60	3.44	3.28	2.71	.083	----	8.94	.303	2.72	2.47	.142
L	6-22-60	2.93	2.90	2.89	.082	----	10.4	.278	2.38	1.95	.541
T	4-27-61	2.64	2.60	3.16	.083	----	11.9	.266	2.16	1.65	
T	5-3-61	3.12	3.05	3.99	.083	----	13.8	.289	2.53	1.75	
T	5-19-61	2.33	2.29	3.62	.085	----	14.3	.253	1.95	1.24	
L	4-29-53	2.15	2.12	2.65	.095	.128	10.4	.254	2.01	1.59	.404
L	5-5-53	1.25	1.24	1.66	.095	.074	8.5	.195	1.18	0.99	.505
L	6-1-53	2.66	2.61	3.58	.095	.158	12.5	.286	2.48	1.83	
L	6-4-53	2.48	2.44	3.12	.095	.147	11.3	.276	2.32	1.82	.324
L	6-17-53	2.14	2.11	2.34	.095	.127	9.2	.256	2.00	1.38	.644

L--Lower flow regime (dunes), T--Transition, U--Upper flow regime (Plane bed or antidunes)

TABLE 9 - BASIC DATA AND COMPUTED PARAMETERS FOR MISSISSIPPI RIVER AT ST. LOUIS, MISSOURI[1]

Date	D ft	R ft	U fps	S X10^4	$\frac{C}{\sqrt{g}}$	U_* fps	RS ft	ΔRS ft	χ ft
5-8-50	37.3	35.7	5.26	0.997	15.6	0.338	3.56	2.64	0.793
5-22-50	37.2	35.6	5.54	.924	17.0	.325	3.29	2.30	.482
6-5-50	37.3	35.7	5.18	.909	16.0	.323	3.25	2.35	.718
8-7-50	28.0	27.1	3.27	.428	16.9	.193	1.16	.785	.382
9-5-50	28.4	27.4	3.27	.399	17.5	.187	1.09	.717	.303
9-25-50	24.7	23.9	3.26	.399	18.6	.175	.954	.574	.165
4-17-51	41.3	39.4	7.37	1.06	20.1	.366	4.18	2.43	.155
5-21-51	40.4	38.6	5.39	.982	15.4	.349	3.79	2.86	.989
6-18-51	39.1	37.4	4.89	.822	15.5	.315	3.07	2.30	.923
7-16-51	54.8	51.6	7.26	1.26	15.9	.458	6.50	4.93	1.105
7-22-51	56.7	53.4	7.53	1.39	15.3	.489	7.42	5.72	1.425
7-30-51	49.9	47.2	5.37	.351	14.9	.360	1.66	3.15	1.474
9-17-51	45.0	42.7	5.03	.632	17.1	.295	2.70	1.91	.552
9-24-51	39.4	37.6	4.33	.530	10.0	.249	1.99	1.32	.836
10-8-51	34.5	33.2	3.66	.341	19.2	.191	1.13	.687	1.795
10-15-51	33.0	31.8	3.36	.355	17.6	.191	1.13	.758	.336
11-13-51	33.5	32.1	3.97	.312	22.2	.179	1.00	.469	.527
11-26-51	34.6	33.2	4.08	.400	19.7	.207	1.33	.780	.148
12-3-51	31.0	29.9	3.75	.370	19.8	.189	1.11	.630	.127
2-18-52	27.9	27.0	4.24	.544	19.4	.218	1.47	.857	.135
4-23-52	46.5	44.2	7.86	1.16	19.4	.406	5.13	3.20	.221
4-30-52	46.9	44.7	7.69	1.10	19.3	.398	4.92	3.07	.410
6-16-52	33.1	31.8	3.63	.545	15.3	.238	1.73	1.32	.855
7-17-52	33.5	32.2	3.66	.502	16.1	.228	1.62	1.17	.619
8-19-52	30.3	29.2	3.37	.428	16.8	.201	1.25	.866	.427
9-29-52	21.4	20.8	2.24	.428	13.3	.169	.890	.705	1.225
10-20-52	21.6	21.0	2.35	.355	13.2	.155	.746	.540	.583
12-8-52	20.4	19.9	2.14	.487	12.1	.177	.969	.801	1.80
12-29-52	20.3	19.7	2.15	.516	11.9	.181	1.02	.848	2.03
1-13-53	19.7	19.2	2.04	.574	10.9	.188	1.10	.941	2.98
2-11-53	27.1	26.2	2.98	.530	14.1	.212	1.39	1.11	1.12
2-16-53	25.4	24.6	2.90	.603	13.2	.219	1.48	1.19	1.41
2-24-53	31.8	30.5	3.90	.545	16.8	.232	1.66	1.16	.445
3-16-53	32.3	31.1	3.96	.487	17.9	.221	1.51	.991	.283
4-6-53	40.2	38.4	4.69	.603	17.2	.273	2.32	1.61	.480
4-27-53	33.0	31.7	4.42	.589	18.0	.245	1.87	1.21	.283
5-25-53	28.1	27.1	3.94	.603	17.2	.229	1.63	1.09	.349
6-8-53	27.4	26.5	3.90	.632	16.8	.232	1.67	1.18	.387
7-13-53	30.1	29.0	4.00	.632	16.5	.243	1.83	1.29	.468
7-20-53	27.9	26.0	3.54	.603	15.5	.229	1.57	1.20	.642
8-10-53	27.6	26.6	3.61	.661	15.2	.238	1.76	1.30	.742
9-14-53	20.2	19.7	2.59	.559	13.8	.188	1.10	.851	.956
9-28-53	18.4	18.0	2.40	.516	13.9	.173	.929	.708	.837
10-19-53	18.2	17.8	2.40	.516	14.0	.172	.918	.699	.798
12-28-53	17.7	17.3	2.30	.661	12.0	.192	1.14	.941	1.73
1-28-54	15.3	15.0	2.01	.749	10.6	.190	1.12	.960	2.63
2-8-54	16.1	15.8	2.12	.807	10.4	.203	1.28	1.11	3.00
3-22-54	19.8	19.3	2.63	.720	12.5	.211	1.39	1.64	1.585
4-19-54	25.5	24.7	3.42	.720	14.3	.239	1.78	1.36	.988
4-27-54	27.5	26.6	3.72	.720	15.0	.248	1.92	1.43	.799
6-7-54	34.2	32.9	4.95	.691	18.3	.271	2.27	1.47	1.12
7-12-54	28.9	27.9	3.74	.720	14.7	.254	2.01	1.52	.968
7-26-54	24.1	23.3	3.07	.603	14.4	.213	1.40	1.07	.945
9-21-54	22.8	22.1	2.50	.647	11.7	.214	1.43	1.20	2.465
10-18-54	31.3	30.1	4.22	.734	15.8	.267	2.21	1.52	.654
12-1-54	19.7	19.2	2.52	.720	11.9	.211	1.38	1.15	.409
2-24-55	34.0	32.7	5.11	.720	18.6	.275	2.35	1.48	.227
3-30-55	26.0	25.9	3.82	.793	14.9	.257	2.05	1.53	.797
5-24-55	22.0	22.1	2.81	.691	12.7	.222	1.53	1.25	1.947
12-27-55	17.0	16.7	2.33	.822	11.1	.210	1.37	1.161	2.369
2-21-56	18.9	16.5	2.20	.822	10.5	.209	1.36	1.17	3.000
3-27-56	19.8	19.3	2.65	.763	12.2	.218	1.47	1.21	1.755
4-24-56	25.6	27.6	3.65	.836	13.4	.272	2.31	1.83	1.55
5-9-56	27.7	26.3	3.34	.836	12.5	.268	2.20	1.84	2.138
7-9-56	27.5	26.6	3.49	.749	13.8	.253	1.99	1.56	1.298
8-6-56	22.5	21.2	2.98	.822	12.4	.240	1.74	1.47	1.823
10-2-56	17.2	16.8	2.30	.938	10.2	.225	1.58	1.37	3.510
11-5-56	15.8	15.5	2.31	.938	10.7	.216	1.45	1.24	2.583
1-21-57	15.2	14.9	2.18	.982	10.0	.217	1.46	1.27	3.340
3-20-57	17.6	17.2	2.61	.932	11.5	.227	1.60	1.34	2.115
3-26-57	24.5	23.8	3.50	.874	13.5	.259	2.08	1.64	1.310
4-9-57	32.2	31.0	4.76	.903	15.9	.300	2.80	2.03	.801
4-22-57	28.9	27.9	4.08	.903	14.3	.285	2.52	1.95	1.094
5-7-57	28.1	27.1	3.76	.845	13.8	.272	2.29	1.81	1.322
5-20-57	36.9	35.3	5.03	.991	15.0	.336	3.50	2.68	1.041
6-3-57	32.2	31.0	4.05	.962	13.1	.310	2.98	2.43	1.938
6-25-57	33.0	31.7	4.06	.932	13.2	.308	2.95	2.40	1.921
10-7-57	18.5	18.1	2.63	.816	12.1	.218	1.41	1.21	1.750
4-14-58	27.9	26.9	3.81	.845	14.1	.270	2.27	1.76	1.162
4-29-58	24.5	24.1	3.41	.816	13.5	.252	1.97	1.55	1.340
5-19-58	22.1	21.5	3.15	.727	13.5	.233	1.56	1.32	1.120
6-17-58	35.0	33.6	5.29	.932	16.7	.317	3.13	2.19	.520
7-18-58	31.1	29.9	4.62	.991	15.0	.309	2.96	2.25	.882
7-25-58	44.4	42.2	6.43	1.05	17.0	.378	4.43	3.16	.558
9-8-58	22.6	22.0	3.00	.728	13.2	.227	1.60	1.28	1.333

[1] Bed form is dunes

Continued

TABLE 9 - BASIC DATA AND COMPUTED PARAMETERS FOR MISSISSIPPI RIVER AT ST. LOUIS, MISSOURI--Continued

Date	D ft	R ft	U fps	S X10^4	$\frac{C}{\sqrt{g}}$	U_* fps	RS ft	ΔRS ft	χ ft
9-29-58	23.4	22.7	3.41	0.583	16.5	0.207	1.32	0.910	0.366
11-18-58	21.1	20.5	3.15	.787	13.8	.228	1.61	1.25	.976
3-23-59	30.6	29.5	4.51	.991	14.7	.307	2.92	2.24	.983
4-6-59	35.8	34.3	4.92	1.11	14.1	.350	3.81	3.02	1.460
5-4-59	29.5	28.5	3.60	.874	12.7	.283	2.49	2.04	2.111
5-13-59	32.4	31.2	4.45	.903	14.8	.301	2.82	2.15	1.020
6-13-59	26.3	25.5	3.40	.699	14.2	.240	1.78	1.39	1.065
7-1-59	24.5	23.7	3.23	.612	15.0	.216	1.45	1.08	.779

Table 10. Values of ξ when B is assumed equal to A.

Sand size			
d_{50} mm	d_{85} ft	B_*/U_*	ξ ft
0.19	0.00078	25.9	0.00083
.27, .28	.0015	23.3	.0018
.45, .47	.0030	21.9	.0029
.93	.0049	20.2	.0050