MODEL OF REAL-TIME AUTOMATION AND CONTROL SYSTEMS FOR COMBINED SEWERS

by

Warren Bell, C. B. Winn and G. L. Smith

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FOREWORD

Colorado State University was awarded a grant by the Office of Water Resources Research to study "Metropolitan Water Intelligence Systems." The purpose of the study is to develop criteria and rationale for the establishment of centralized metropolitan water intelligence systems in urbanized and urbanizing areas.

The project consists of three Planned Phases each lasting approximately one year; this report was prepared in Phase I. During Phase I primary attention was focused on real-time automation and control facilities for combined sewers. Basic objectives of Phase I were to:

1. Investigate and describe modern automation and control systems for the operation of urban water facilities with emphasis on combined sewer systems.
2. Develop criteria for managers, planners, and designers to use in the consideration and development of centralized automation and control systems for the operation of combined sewer systems.
3. Study the feasibility, both technical and social, of automation and control systems for urban water facilities with emphasis on combined sewer systems.

Phase I of the research effort consisted of ten tasks. Task 4 has as its objective the development of a Real-Time Automation and Control System (RTACS) model, which will be used as a tool to optimize a selected real world system.

In effect the objective of Task 4 is to formulate and develop the components of an RTACS model, which includes the physical system, the control algorithm including the control logic and any necessary prediction models, and the interfaces between them. To demonstrate the usefulness of an RTACS model, the control logic is to be developed in detail for a reasonable control objective and assumed physical system. The effects of errors, including sensing errors, model errors, and control errors, is to be examined to demonstrate how one would optimize the overall system. In summary, the objective is to demonstrate the feasibility of an automated control system as well as develop the principles for determining the "best" system arrangement.

Existing physical system models are used. The one developed for FWQA by a Triumvirate headed by Metcalf and Eddy is used to provide input to the control algorithm and as the system to be controlled.

* * * * *

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* * * * *

Maurice L. Albertson and George L. Smith are principal investigators and L. Scott Tucker is project manager.

* * * * *

The following technical reports were prepared during Phase I of the CSU-OWRR project, Metropolitan Water Intelligence Systems. Copies may be obtained for $3.00 from the National Technical Information Service, U. S. Department of Commerce, Springfield, VA 22151. (When ordering, use the report title and the identifying number noted for each report.)

Technical Report No. 1 - "Existing Automation, Control and Intelligence Systems of Metropolitan Water Facilities" by H. G. Poertner. (Identifying number to be obtained.)

Technical Report No. 2 - "Computer and Control Equipment" by Ken Medearis. (Identifying number to be obtained.)

Technical Report No. 3 - "Control of Combined Sewer Overflows in Minneapolis - St. Paul" by L. S. Tucker. (Identifying number to be obtained.)

Technical Report No. 4 - "Task 3 - Investigation of the Evaluation of Automation and Control Schemes for Combined Sewer Systems" by J. J. Anderson, R. L. Callery, and D. J. Anderson. (Identifying number to be obtained.)

Technical Report No. 5 - "Social Feasibility of Automated Urban Sewer Systems" by D. W. Hill and L. S. Tucker. (Identifying number to be obtained.)

Technical Report No. 6 - "Urban Size and Its Relation to Need for Automation and Control" by Bruce Bradford and D. C. Taylor. (Identifying number to be obtained.)

Technical Report No. 7 - "Model of Real-Time Automation and Control Systems for Combined Sewers" by Warren Bell, C. B. Winn, and G. L. Smith. (Identifying number to be obtained.)

Technical Report No. 8 - "Guidelines for the Consideration of Automation and Control Systems" by L. S. Tucker and D. W. Hill. (Identifying number to be obtained.)

Technical Report No. 9 - "Research and Development Needs in Automation and Control of Urban Water Systems" by H. G. Poertner. (Identifying number to be obtained.)

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I. PURPOSE, SCOPE AND METHODOLOGY

PURPOSE

The primary purpose of Task 4 is to develop and formulate a model of a real time automation and control system (RTACS) in order to test the importance and relationship of various components to one another. In the prototype an RTACS consists of the following basic components: physical system, sensing elements, control data processor, runoff model physical system, control program, data bank and control elements. The relationship of the components is shown in Figure 1.1.

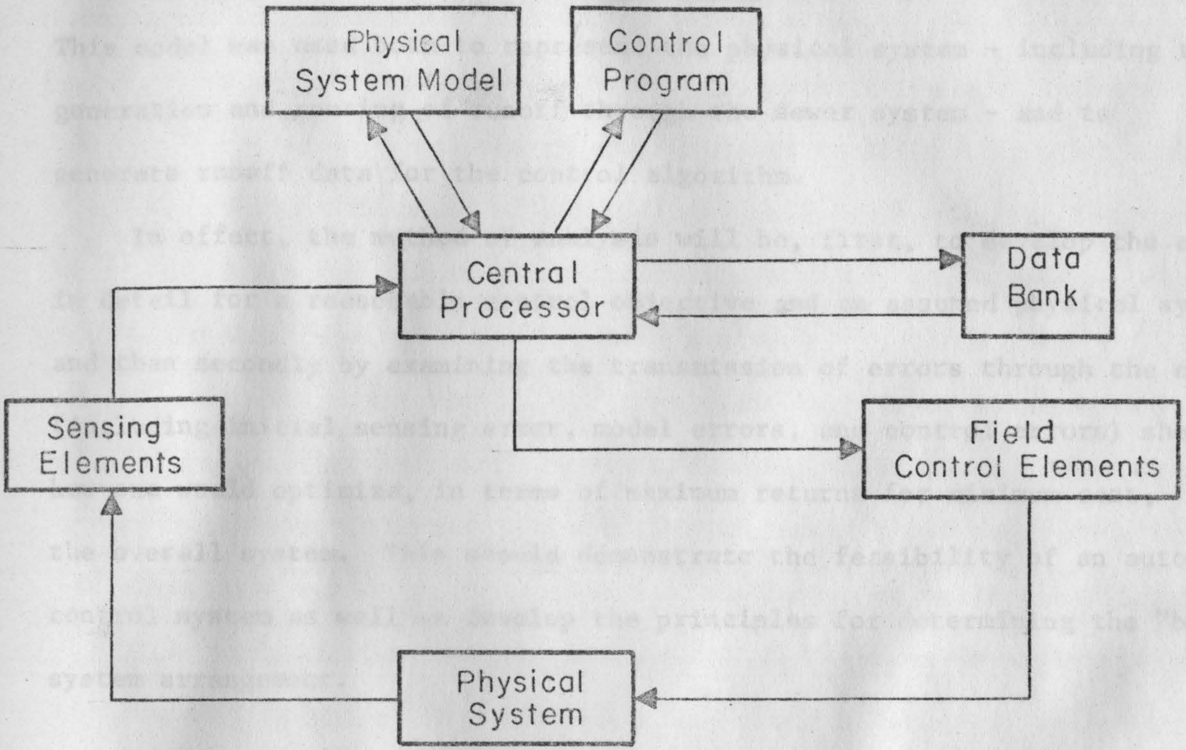
SCOPE

In developing a model of an RTACS, the physical system is represented by the FWQA model. Rainfall data can be used in conjunction with a rainfall-runoff model to predict flows at certain points in a system, and a control program can be developed to direct the operation of the flow control devices. The operation of the physical system model and control program requires a central processor, which in this case will be the CSU computer -- a CDC 6400 unit. The feedback between the various components initially will be accomplished manually, i.e., data from one section will be fed into another and so on.

The RTACS model is to be used to examine such items as spatial and temporal requirements of sensing elements, effect of data errors on effective operation of the system, adequacy of the overall system, and degree of sophistication necessary for the physical system model with respect to the effective operation of the system.

Figure 1.1

COMPONENTS OF REAL TIME AUTOMATED CONTROL SYSTEM



METHODOLOGY

The RTACS model will be formulated using available existing models of combined sewer systems. The principal one used in this study was the one recently developed for FWQA by a Triumvirate headed by Metcalf and Eddy. This model was used both to represent the physical system - including the generation and routing of runoff through the sewer system - and to generate runoff data for the control algorithm.

In effect, the method of analysis will be, first, to develop the control logic in detail for a reasonable control objective and an assumed physical system, and then secondly by examining the transmission of errors through the entire model (including initial sensing error, model errors, and control errors) show how one would optimize, in terms of maximum returns for minimum cost, the overall system. This should demonstrate the feasibility of an automated control system as well as develop the principles for determining the "best" system arrangement.

II. COMPONENTS OF THE RTACS MODEL

A. INTRODUCTION

The objective of control of a combined sewer system is to reduce the amount of pollution of natural receiving waters. This could mean:

1. reduce the amount of flow that bypasses the sewage treatment plant
2. reduce the total amount of pollutants entering the water course
3. reduce the amount of most harmful pollutants entering the water course
4. reduce the amount of pollutants in a manner such that the reduction in the economic cost of pollution is a maximum.

Item 1 does not recognize the variations in the potential damage of various pollutants, the variation in the water quality from different areas, or the fact that too great a flow to the treatment plant may reduce its efficiency and result in an increase in the amount of pollutants discharged.

Item 2 recognizes spatial and temporal differences in water quality, but does not recognize the variations in potential damage of the various pollutants.

Item 3 recognizes the variations in potential damage of the various pollutants and to some extent recognizes their spatial and temporal variation.

Item 4 is essentially the same as Item 3, except that the potential damage is specified in terms of dollars instead of a value judgment. This allows an economic balance to be struck in terms of incremental benefits and the incremental costs of the system required to produce the incremental benefits. With Items 1, 2 and 3, the required degree of pollution reduction and therefore the degree of system control is based on value judgments. (It should be noted that in determining economic values a lot of value judgment may be involved.)

For the purposes of Task 4, use will be made of optimal control theory which allows specified weights to be given to overflows from different sources. As the objective of the study, Item 1 will be used initially because of its relative simplicity; however, in fact it allows some weight to be placed on Item 3, particularly if there is prior data on the variation of the pollutants in the system. With this objective function, the RTACS system will consist of the following components:

1. A model of the physical system that will give a reasonable simulation of the rainfall over an area; that will convert this rainfall into storm runoff to the interceptors; that will route the runoff through the interceptor to the sewage treatment plant or receiving waters; and that contains the capability to control the flow in the interceptor system or to divert the flow to storage in the system or to the receiving waters.

2. A control algorithm which acts on information obtained from the physical system model and then determines the control procedure for the flow in the interceptors of the physical system model.

3. An interface between the physical system model and the control algorithm that simulates the errors in the observation and transmission of data.

B. <u>ROLE OF THE FWQA MODEL</u>

In the RTACS model developed for Task 4 the physical system model is basically the FWQA stormwater management model. This model consists of the following four sections (aside from the executive routines):

1. A rainfall - runoff model,

2. A routing model for flow in sewers,

3. A model for treatment and offline storage, and

4. A receiving water model.

In addition to generating and routing runoff the model also contains the capability to generate and route some pollutants.

The RTACS model uses only the first two parts of the FWQA model. Ideally, both parts 3 and 4 should ultimately be included; part 4 since it is the receiving waters that are ultimately to be protected and part 3 since the control of the sewer flow affects the operation of the various treatment processes. However, it was felt that in Phase I the most usable results could be obtained without parts 3 and 4 of the model, i.e., by considering only runoff, since at the present time real time monitoring of pollutants is not too reliable.

To meet the requirement of variable control, the routing model for flow in sewers, which in its original form has no variable control devices, was modified to permit variable flow control at twenty points. These modifications are discussed in more detail in a later section.

It should be noted at this point that it is not necessary for a designer to use the physical model developed for the RTACS model. Any model capable of reasonably simulating runoff and flow in the sewer system could be used provided that it contains provision for variation of flow control devices. Thus, a city with an already verified model need not develop data for and prove a new model.

To determine the effect of spatial distribution of rain gauges an additional component was added to the physical system model of the RTACS model. The component makes it possible to locate the storm over an area greater than the combined sewer drainage area in the form of a point rainfall grid. The average rainfall over each subarea of the rainfall runoff model is determined by integration (assuming linearity between points) for input to the runoff portion of the physical system. Point rainfall at each rain gauge is determined and "true" and adjusted values (to simulate errors) for each gauge are then printed out. (This later part is merely the interface routine discussed later in this report.)

C. CONTROL ALGORITHM FOR RTACS MODEL

The control logic for the RTACS model is based on optimal control theory. (Optimal control in itself does not allow the designer to ascertain the effects of errors; its use does.) Use of this method of control allows the designer to ascertain the effects of errors in data or control signals or the effect of a less than optimal control algorithm. Furthermore, the control logic is designed to demonstrate the processes necessary to simulate a real world system such as the Minneapolis-St. Paul combined sewer system. In applying the RTACS model elsewhere it would be necessary to develop an optimal control algorithm for the particular location. An objective of this project is to derive the necessary equations, including an example of a numerical solution, in order to simplify the development of such an algorithm elsewhere.

D. PREDICTION MODEL

In addition to the control algorithm itself there is usually some form of prediction model which models part or all of the physical system model. This makes it possible for the control algorithm to act in a "feed-forward" mode (act on assumed future happenings) instead of on the usual feedback mode, i.e., decision based on the present state of the system. Use of a feed forward mode allows maximum benefit to be gained from the physical system storage.

As an addition to the prediction model one needs a model to determine the rainfall over the subareas of the runoff model given the point rainfall in and about the area of concern. This component may use anything from a simple Theissen polygon method of estimation to methods that estimate the changing storm coordinates and growth and decay with time.

E. INTERFACE MODEL

The model of the interfaces between the control algorithm and the physical system of the RTACS model consists of a series of error generating routines to simulate four types of errors:

1. Variation by a constant,
2. Variation by a constant fraction,
3. Uniformly distributed error, and
4. Normally (Gaussian) distributed error.

The program allows these functions to be sequenced in any order to simulate the various parts of the sensing or control system. The process can be repeated to simulate an averaging procedure (repeated inquiry.) Any data generated by the physical system and used by the control algorithm, or vice versa, would be transformed by this model before use by the control algorithm.

III. FUNCTIONS OF THE RTACS MODEL

A. INTRODUCTION

The RTACS model is intended to be an aid in the design of a real time automated control system for combined sewers. The particular design areas in which the model should provide adaptive real world information include:

1. The adequacy of the proposed system to achieve the desired results and the optimal location of control devices.

2. The required accuracy of sensors and control devices.

3. The optimum location and number of sensors.

4. Determining the optimum time period between control adjustments, and/or sensor inquiry.

5. The required accuracy of the prediction component of the control algorithm.

6. The control logic for the system.

To provide each of these items two criteria are necessary:

1. The physical system model must reasonably simulate actual storm events. The better the model the greater the weight that can be placed on the final results.

2. An optimal control logic must be developed for the RTACS model to serve as an absolute standard for comparison of the effects of sensor and controller errors and prediction model errors. (Optimal control logic implies that for a given objective, e.g., minimizing combined sewer overflow diverted to a river, no other control logic will produce a better result.)

Given that these are the functions of an RTACS model, it now must be demonstrated how they can be accomplished.

B. OPTIMIZING THE LOCATION OF FLOW CONTROL DEVICES

Using the optimal control logic for the proposed system, and using the runoff portion of the physical system in a dual role as the prediction model portion of the control algorithm, one can determine the maximum capabilities of the system for any given storm and the specified objective (See Fig. 3.1). It is assumed, of course, that "perfect information" is being used to operate the model. Thus no feedback information is required. Furthermore, any introduction of error into either the prediction model or the control devices will reduce the capabilities of the system.

If the maximum capabilities of the system are not sufficient then some additions will be necessary. This leads to the problem of optimizing the location of the flow control devices. (One that should be considered even if the system is adequate.) This would be essentially a trial and error procedure which is much more feasible on a computer than it is in the field. The control algorithm would have to be altered for each trial. By comparing the physical system output results for each trial one should be able to converge towards an optimal system.

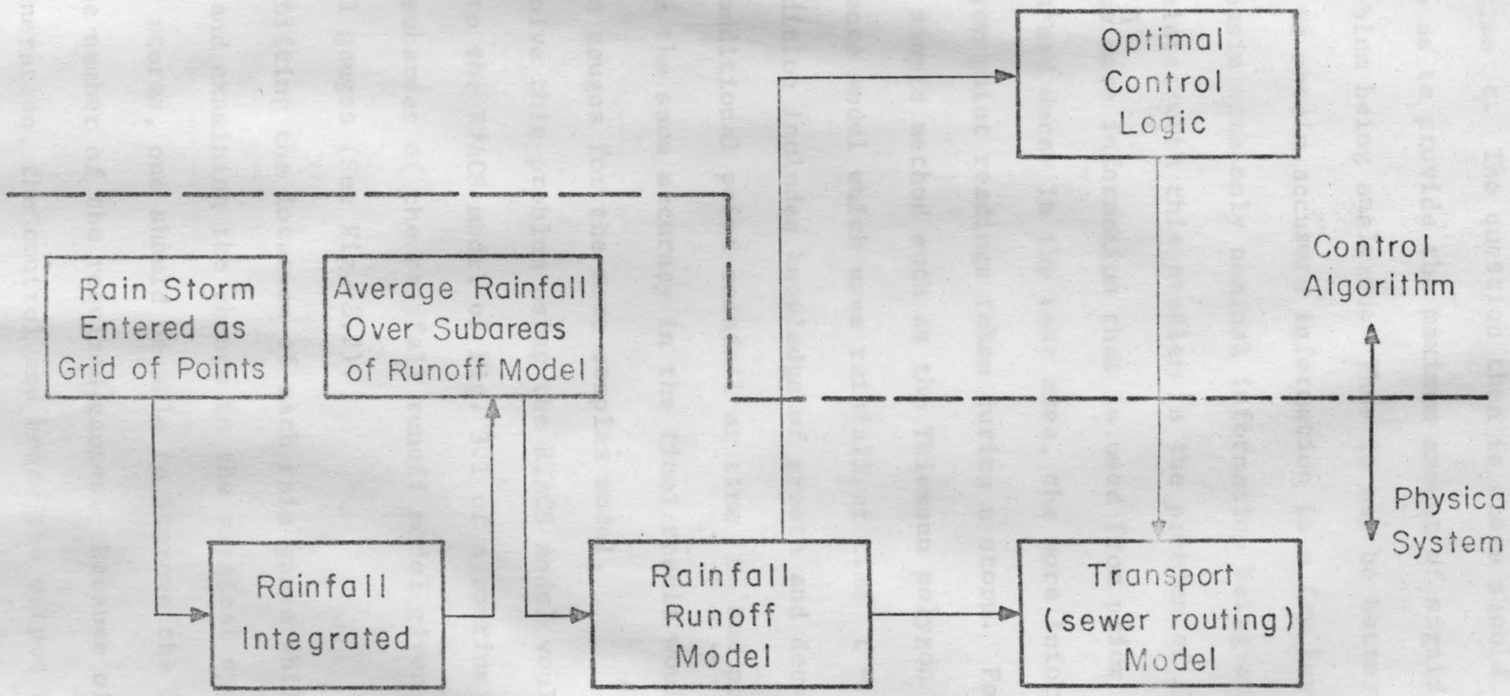
C. OPTIMIZING THE LOCATION AND NUMBER OF SENSORS

The problem associated with optimization of the location and number of sensors is to maximize the amount of information obtained. The sensors one would normally consider would be rain gauges, water levels, flow depths and water quality parameters. Control device positions are predetermined and are thus only an accuracy problem.

The basic problem is that readings for a few points must be regenerated to provide data for the entire field of interest. Rainfall is possibly the best example. Here data from a few points in space must be regenerated

Figure 3.1

RTACS MODEL USED TO DETERMINE MAXIMUM SYSTEM CAPABILITY



to give a reasonable picture of the rainfall over the entire drainage basin at time t. The question then is where should the rain gauges be located so as to provide the maximum amount of significant information to the problem being analyzed. Thus it may be better in achieving the objective to obtain accurate information in a few key areas of the drainage basin with only nominal information being available elsewhere.

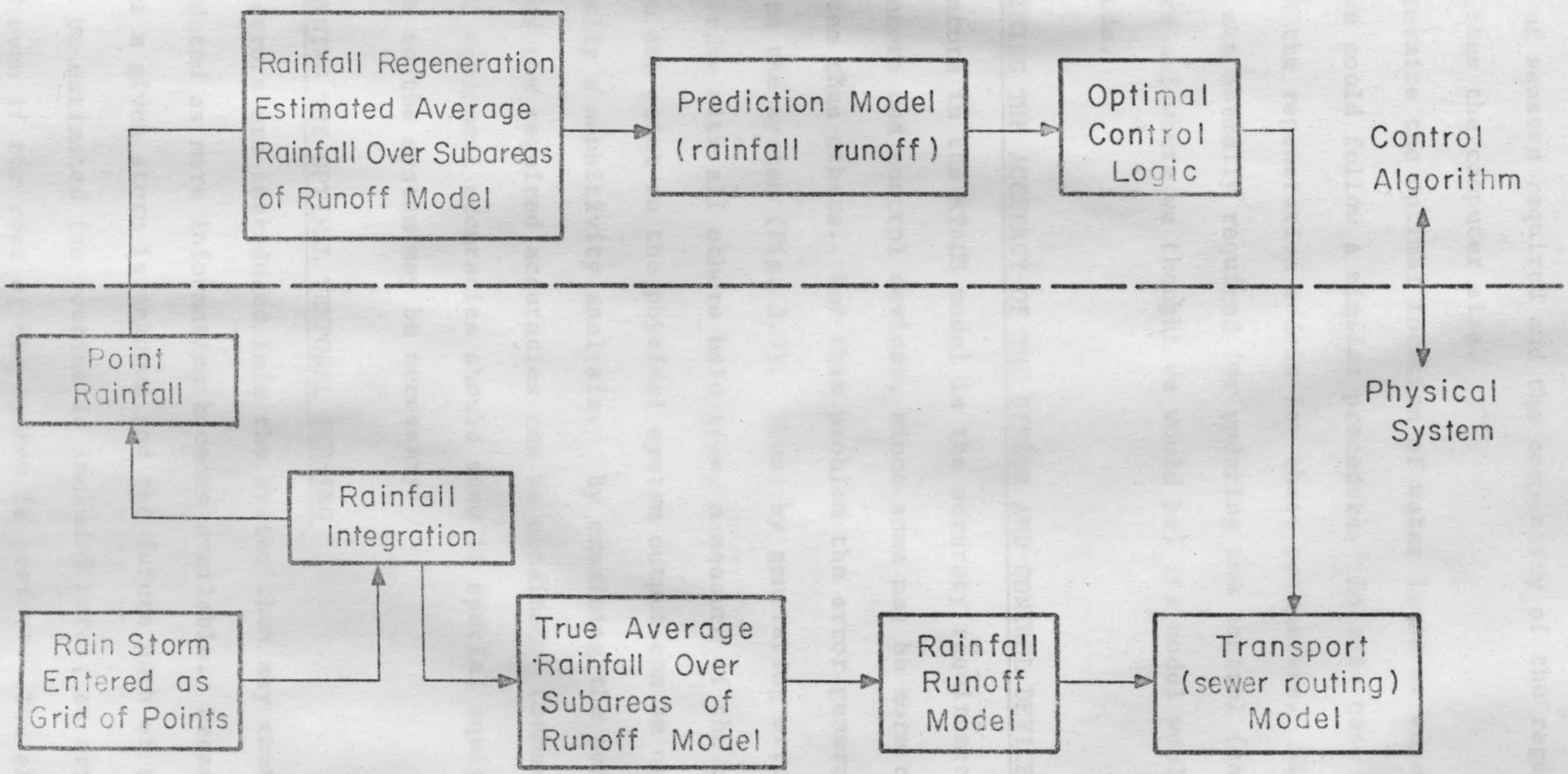
Correlated with this problem is the problem of the regeneration model. The more information that is used from prior storms, e.g. their generation and decay in the test area, the more information that may be derived from point readings taken during a storm. For example, one may use a simple method such as the Thiessen polygon method or a complex time sequence model which uses rainfall at time $t - \delta t$ at point x, y and in addition includes knowledge of growth and decay of storms to estimate additional point rainfall at time t and point $x + \delta x, y + \delta y$. To achieve the same accuracy in the final result would probably require fewer rain gauges for the more complex model.

To solve this problem using the RTACS model would require the addition to the RTACS model of Fig. 3.1 of a routine for generating rainfall over the subareas of the rainfall runoff model given the point rainfall at several gauges (See Fig. 3.2).

By shifting the location of each rain gauge while holding the others constant and examining the change in the physical system output for a series of storms, one should be able to improve the location of, or to reduce the number of the required gauges. Because of the errors in rainfall regeneration, the control and hence the output of the physical system will now be non optimal. The process could be repeated with different regeneration models so that an economical balance could be obtained between

Figure 3.2

RTACS MODEL USED TO DETERMINE OPTIMUM RAIN GAUGE LOCATIONS



the number of sensors required and the complexity of the regeneration model, and thus the computer size.

To determine the optimal location of water level or water quality sensors, one could follow a similar procedure. In this case the effects depend upon the regeneration models for these parameters. Since these parameters are normally required for updating the control (check between where we are and where we thought we would be) the model would be in the feedback mode.

D. DETERMINING THE ACCURACY OF THE SENSOR AND CONTROL DEVICES

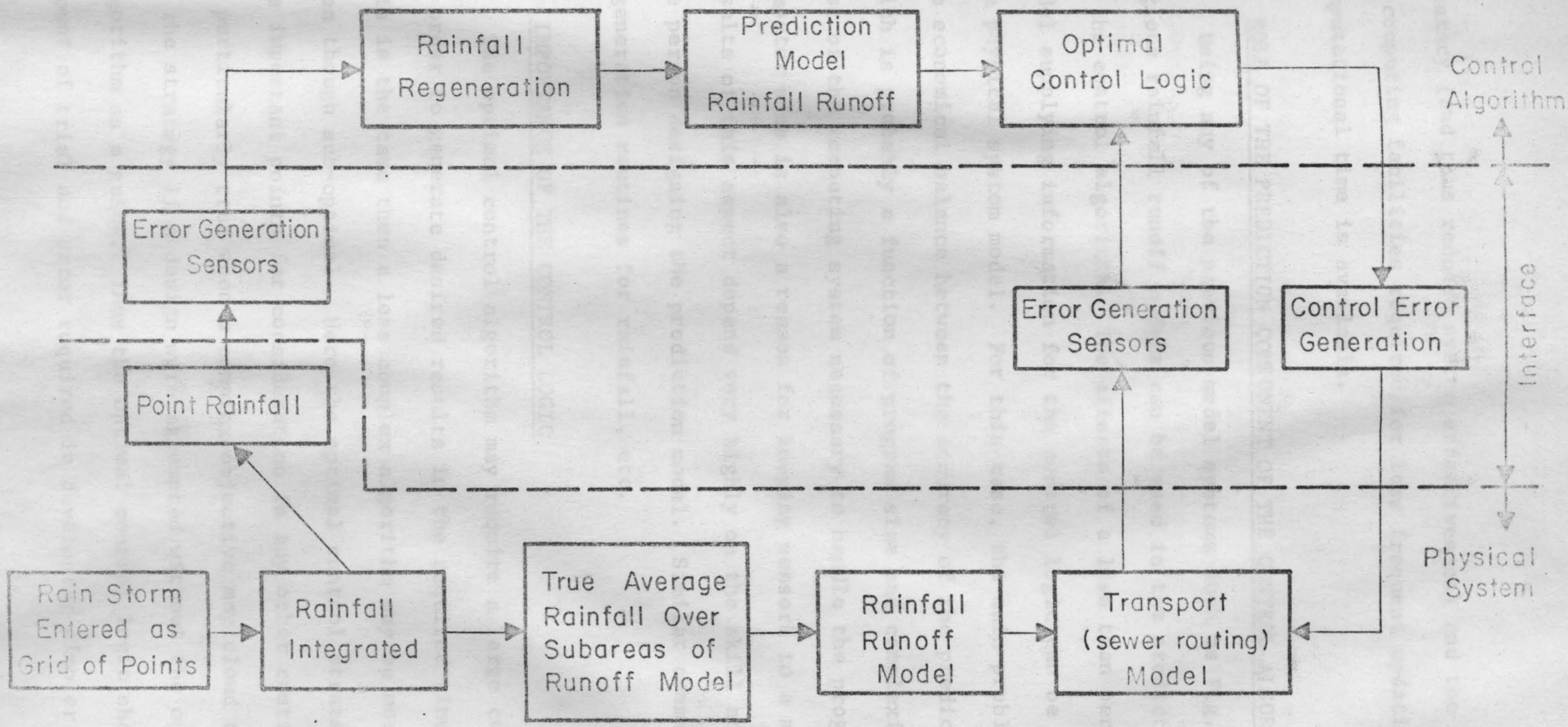
Of concern in the RTACS model is the accuracy requirement of the various sensors and control devices, since some may be more critical to the system than others. For this problem the error generating routines are added to the system (Fig. 3.3). Thus, by generating errors at one point at a time with all others held true, a measure of the effect of errors from one point on the physical system output can be obtained. This is essentially a sensitivity analysis. By examining the sensitivities a measure of the required accuracies can be obtained. Comparing these with normal equipment accuracies should show if special equipment or alterations to the system may be necessary.

E. DETERMINING THE OPTIMAL TEMPORAL SENSING

Once errors are introduced into the system then any control logic must be updated as more information becomes available. Because the duration of a given storm is unknown and the future path of the storm must be estimated (no routine is included here), an error is introduced even if the rest of the system is perfect. The effect of varying the time between system inquiries and updates can be determined by the model. Furthermore, a comparison can be made between the loss of

Figure 3.3

GENERAL RTACS MODEL



accuracy (and thus reduced system effectiveness) and the reduced cost of computing facilities required for less frequent updating, since larger computational time is available.

F. ROLE OF THE PREDICTION COMPONENT OF THE CONTROL ALGORITHM

Using any of the previous model systems such as Fig. 3.1 or Fig. 3.3, various rainfall runoff models can be used in the prediction component of the control algorithm. The effects of a less than perfect prediction model supplying information for the control logic can be determined upon the physical system model. For this case, the main problem is to find the economical balance between the accuracy of the prediction model, which is probably a function of program size and complexity, and the cost of the computing system necessary to handle the program. (Required computer size is also a reason for keeping sensors to a minimum.) The results of this aspect depend very highly on the skill and knowledge of the the person designing the prediction model. Similar comments apply to the regeneration routines for rainfall, etc.

G. IMPORTANCE OF THE CONTROL LOGIC

The optimal control algorithm may require a large computing facility in order to generate desired results in the required time period. If this is the case then a less complex algorithm may be more economical even though sub-optimal. Here the optimal control strategy would supply the important points for consideration in any other control logic. This is particularly true when a complex objective may cloud the main points of the strategy if a design were attempted without the optimal control algorithm as a guide. Thus the optimal control logic should reduce the amount of trial and error required in developing simpler control logic.

H. SUMMARY

The foregoing functions of the RTACS model have been summarized as though they are an independent process. In fact, the optimization of the system (which is the overall objective of using an RTACS model) requires several iterations of the above processes if the most economical system that will satisfy the specified objectives is to be determined.

IV. RTACS COMPONENTS: MODEL ANALYSIS AND THEORETICAL CONCEPTS

A. INTRODUCTION

This section will present the logic used to develop or modify each of the three main components of the RTACS model: the physical system model, the interfaces -- error generating routines--, and the control algorithm.

B. THE PHYSICAL SYSTEM MODEL

This model is basically the Runoff and Transport Sections (plus executive routines) of the FWQA model. As a detailed description of this model is given in the FWQA Research and Development Grant Report titled, "Water Pollution Control Research Series, Storm Water Management Model, Final Report," only the changes made to the Transport Section to provide for increased backwater storage and variation of control devices will be discussed here. For the revised model, only minor modification to the data preparation was necessary. It is assumed in this discussion that the reader has some familiarity with the FWQA model. Lastly, the limitations of the model are discussed in detail in the previously referenced report.

The unaltered FWQA model provides for backwater storage at only two locations and contains no provision for variable control. The reason for the low number of backwater points was a computer storage limitation. Thus in order to increase the number of backwater storage locations it was necessary to decrease the computer storage requirements. This was accomplished in three ways. The first was a reduction in storage requirements by a minor modification to one of the transport section print routines. The second was a reduction in the maximum designation number allowable for conduits from 1000 to 200. (The allowable number of conduits remains unchanged.) The third was a modification to the plug flow option for routing pollutants through backwater storage.

This latter modification consisted of limiting the maximum number of plugs of flow at any backwater storage location to fifty. Thus, if for fifty-one consecutive time steps of the program one backwater storage location has no outflow then the program will abort. As the normal time step length is on the order of two to five minutes this is equivalent to a real time of 100 to 250 minutes, which is more than adequate for most real world cases.

The limitation results from storing the plug flow data in a loop of fifty storage locations. An attempt to use fifty one locations would result in erasing still valid data. The computer program can still run for the maximum number of 150 time steps.

Variable control was added to the program by specifying a time interval DTCHAN (read in as a variable). Thus one computation time step, after each time interval of DTCHAN, new control device position data is read into the program by subroutine TSTRDT. This subroutine computes outflow versus storage curves for standard flood routing procedure (outflow plus storage vs. depth) for the given orifice openings and overflow weir settings. The delay of one calculation time interval was used to better simulate a control change beginning at $t = N \times DTCHAN$ and finishing at $t = N \times DTCHAN + \delta t$.

Because the program adjusts the given depth versus storage relationship so that one of the points of the outflow versus storage curve falls at the weir elevation (thus saving many computational problems) and, because the interpolations are all linear, some slight inconsistency is built into the program, e.g., for one time step storage at a given elevation

X may be equal to Y and at the next time step storage at the same elevation may be Y + δY. Tests to date indicate that this inconsistency is small and relatively unimportant.

In addition to the changes noted above several minor corrections were made to the Transport Section of the program. These changes are listed in Appendix A3 including those modifications necessary to make it feasible for the program to operate on a CDC 6400, e.g., reduction of variable dimensions to a maximum of three.

Appendix A1 lists the modifications required along with flow charts indicating the changes. Appendix A2 lists the revised data requirements and any additional nomenclature used in the revised version.

In order to reduce storage requirements some of the plotting and output routines were deleted from the Executive routines. These deletions are noted in Appendix A3.

All debugging and verification was done on the CDC 6400 computer facility at Colorado State University. The maximum storage requirements for the revised programs are as follows:

1. Executive plus Runoff blocks 66,400 words (octal)
2. Executive plus Transport blocks 107,700 words (octal)

Although it may be possible to reduce computer running time for these models when used with the RTACS model by avoiding recomputation of some initial conditions no real effort was exerted in this direction as it was felt that the same effort would be more productive elsewhere.

An addition necessary to the Physical System model as given by the FWQA model was a separate routine to generate the rainfall over each of the subareas of the rainfall runoff portion of the FWQA model. For an assumed storm configuration moving over the area of interest (larger than the drainage area of the physical system) this model, called program RAIN, computes the average rainfall over each subarea of the runoff model and gives the point rainfall at those points where rain gages are assumed to exist. The latter information is then adjusted by the model for use by the control algorithm according to the method outlined in the discussion of the interface model given in a later section of this report.

The essential information for this model are the coordinates of each of the subareas of the rainfall - runoff model and the configuration of the rainstorm in the form of a grid for a given time instant T along with the required interface model information as discussed in a later section. The model, assuming linearity between points on the grid, integrates to determine the average rainfall for each subarea of the rainfall runoff model. A listing of this model along with the neumonics and data requirements is included in Appendix A4. The major purpose of this model is to make it possible to determine the optimal location of the rain gauges.

C. THE INTERFACE ROUTINES

The interface between the physical system model is simulated by a model which approximates the errors resulting from measurement by sensors or by transmission or reception of sensor or control signals. It was felt that the purposes of the RTACS model could be met by assuming four possible types of errors:

1. Variation by a constant (such as would occur by shifting a scale).

2. Variation by a fixed fraction (such as would occur if a transformer ratio was in error).

3. Variation according to a random variable uniformly distributed between limits a and b.

4. Variation according to a random variable normally (Gaussian) distributed with mean M and standard deviation S. (The last two variations would be representative of measuring and transmission noise).

These errors can be sequenced in the model in any order up to a total of ten operations. Reapplication of the sequence in the model to the original value for a specified number of cycles simulates an averaging procedure such as would occur in case of repeated interrogations of a sensor by a computer facility.

In operation of the RTACS model any information generated by the physical model that is to be used by the control algorithm would first be transformed by the interface model, which would be set up according to the expected characteristics of the sensor and related information transmission system. Similar operations apply to the model for control signals from the control algorithm to the physical system.

The uniformly distributed random errors are generated by subroutine RANF which generates a number between 0 and 1. This number is then scaled and shifted to the range a,b. The normally distributed random errors are generated by subroutine RANSS which generates from a standard normal distribution, i.e. normally distributed with a standard deviation of 1, a number between $-\infty$ and $+\infty$. This number is then scaled and shifted according to the given mean and standard deviation.

A listing of the program along with the neumonics and data requirements is given in Appendix A5. This model is also a part of Program RAIN.

D. THE CONTROL ALGORITHM

This part of the RTACS model can be divided into three subsections:

1. A rainfall runoff model - For this study the rainfall runoff portion of the FWQA model has been used. With the same basic data used in the Physical System part of the RTACS model it is possible to evaluate the effects of sensor and control errors (since it is in effect a perfect model of the physical system); whereas, with basic data changes some measure of the effect of modeling errors can be obtained on the overall control.

2. A rainfall regeneration model - This model takes the point rainfall developed by the program RAIN and re-constitutes the rainfall over each of the subareas of the control algorithm rainfall runoff model. As with other components of the RTACS model there are many ways in which this can be accomplished.

3. The control logic - The approach used in the development of the control logic was to make use of optimal control theory, which is derived from the calculus of variations. Some of the more pertinent points are discussed in the following paragraphs.

Given some function of the state variable that is to be minimized or maximized (e.g., minimize diversions to a river from a combined sewer system; maximize system throughput) and provided that the system constraints can be expressed as a series of differential equations of the form

$$\dot{X} = f(X, U, t)$$

where X is the vector of the state variables, e.g., depths of storage, volumes of overflow, ($\dot{X}$ is the rate of change of X with time), U is the vector of the control variables, e.g., orifice openings, depths of flow over weirs, and

t is time;

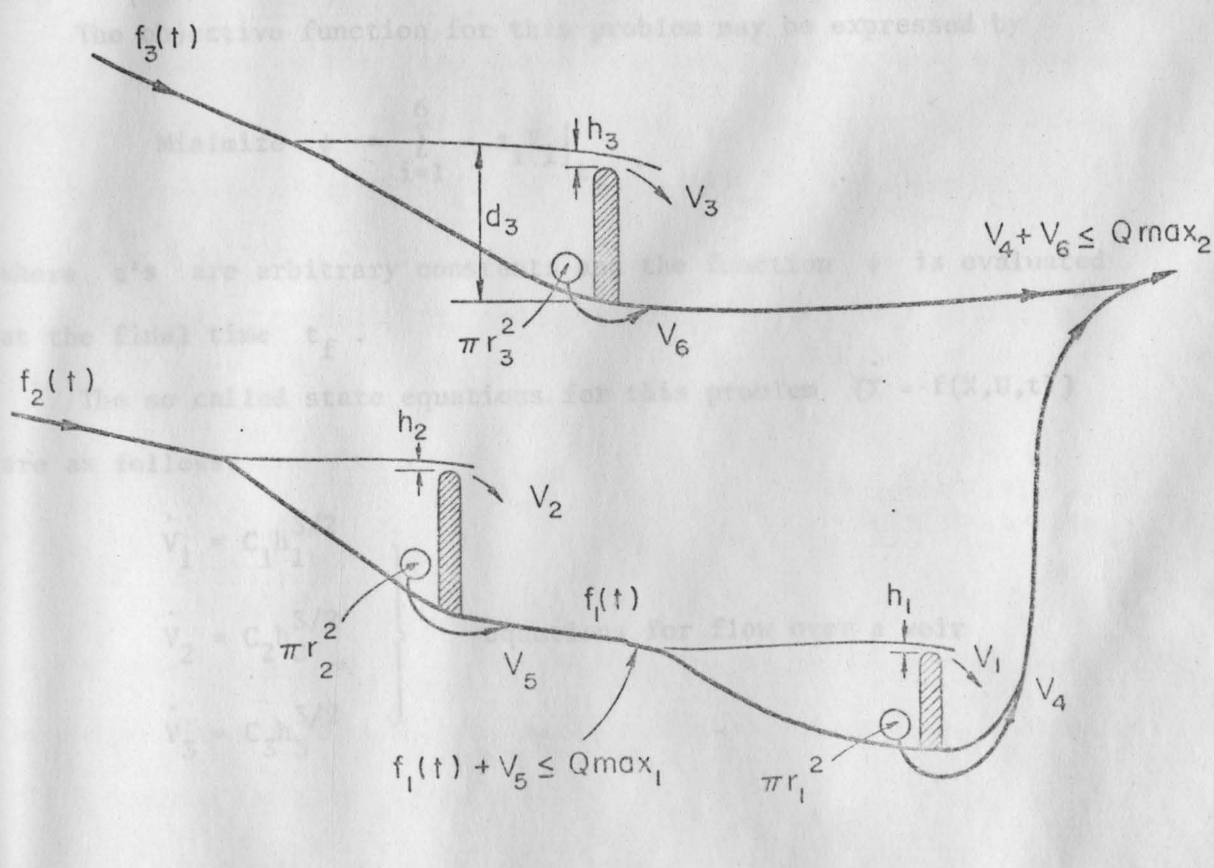
then optimal control theory will lead to the control U(t) that will extremize the objective function.

Before developing the equations further it should be noted that constraints of various forms other than equality can be added to the problem. It is also noteworthy at this point that solution of the problem nearly always requires the use of numerical techniques.

The easist way to develop the theory is to consider a simple problem of three reservoirs shown in Figure 4.1.

Figure 4.1

THE THREE RESERVOIR PROBLEM



In the figure, the state variables V_1 , V_2 and V_3 are the overflow volumes from reservoirs 1, 2 and 3. (These volumes represent diversions from a combined sewer system into a river). State variables V_4 , V_5 and V_6 represent flow through the orifices. (This flow remains within the combined sewer system). The remaining state variables are d_1 , d_2 and d_3 , which are the depths of storage in the reservoirs.

The control variables are the depths of flow over the weirs h_1 , h_2 and h_3 and the radii of the orifices r_1, r_2 and r_3 . The selection of state and control variables is arbitrary. For example, h_1 , h_2 and h_3 could have been the state variables and the control variable could have been $u_i = \dot{h}_i$, the rate of change of depth over the weir.

The known hydrographs representing storm events over the catchment basin are given by $f_1(t)$, $f_2(t)$ and $f_3(t)$ in Figure A.1.

The objective function for this problem may be expressed by

$$\text{Minimize} \quad \phi = \sum_{i=1}^{6} - z_i V_i \Big|_{t_f}$$

where z's are arbitrary constants and the function ϕ is evaluated at the final time t_f .

The so called state equations for this problem $(\dot{X} = f(X,U,t))$ are as follows:

$$\left.\begin{aligned} \dot{V}_1 &= C_1 h_1^{3/2} \\ \dot{V}_2 &= C_2 h_2^{3/2} \\ \dot{V}_3 &= C_3 h_3^{3/2} \end{aligned}\right\} \quad \text{equations for flow over a weir}$$

$$\left.\begin{aligned}\dot{V}_4 &= K_1 r_1^2 d_1^{1/2} \\ \dot{V}_5 &= K_2 r_2^2 d_2^{1/2} \\ \dot{V}_6 &= K_3 r_3^2 d_3^{1/2}\end{aligned}\right\}\quad \text{equations for flow through an orifice}$$

$$\left.\begin{aligned}\dot{d}_1 &= \frac{f_1(t) + K_2 r_2^2 d_2^{1/2} - K_1 r_1^2 d_1^{1/2} - C_1 h_1^{3/2}}{A_1(d_1)} \\ \dot{d}_2 &= \frac{f_2(t) - K_2 r_2^2 d_2^{1/2} - C_2 h_2^{3/2}}{A_2(d_2)} \\ \dot{d}_3 &= \frac{f_3(t) - K_3 r_3^2 d_3^{1/2} - C_3 h_3^{3/2}}{A_3(d_3)}\end{aligned}\right\}\quad \text{equations of continuity for the reservoirs}$$

The functions in the denominators of the last three equations are the area-depth relationships of the reservoirs, evaulated at depths d_i .

In addition to the state variable equations there are the inequality constraint equations. These may be of two types: inequality constraints on the state and control variables $C(X,U,t) \leq 0$, and inequality constraints on the state variables $S(x,t) \leq 0$.

The control variable inequality constraints for this problem are given by

$$\left.\begin{aligned}f_1(t) + K_2 r_2^2 d_2^{1/2} - Q_{1\,max} &\leq 0 \\ K_1 r_1^2 d_1^{1/2} + K_3 r_3^2 d_3^{1/2} - Q_{2\,max} &\leq 0\end{aligned}\right\}\quad \text{flow limitations within the system}$$

In the above equations Q_{max} is the maximum allowable flow at specified points of the system, i.e., the maximum allowable flow is a function of the hydraulic capacity of the system.

Additional control variable inequality constraints are necessary to ensure that physically impossible controls are not realized. These are as follows:

$$\left.\begin{array}{l} h_1(h_1-d_1) \le 0 \\ h_2(h_2-d_2) \le 0 \\ h_3(h_3-d_3) \le 0 \end{array}\right\}$$ depth of flow over the weirs cannot be negative nor can it be greater than the reservoir depth

$$\left.\begin{array}{l} r_1(r_1-R_1) \le 0 \\ r_2(r_2-R_2) \le 0 \\ r_3(r_3-R_3) \le 0 \end{array}\right\}$$ orifice openings cannot be less than zero, nor can they be greater than some fixed value R_i

The final inequality constraints for this problem are state variable inequality constraints on the depth

$$\left.\begin{array}{l} d_1(d_1-D_1) \le 0 \\ d_2(d_2-D_2) \le 0 \\ d_3(d_3-D_3) \le 0 \end{array}\right\}$$ depth of storage in the reservoir cannot be less than zero or greater than some fixed value D_i

In addition to the above equations the initial depths d_i and overflow volumes V_i are known at $t = 0$ (V_i would normally be zero, since their initial values do not affect the optimization. Also the depths would be obtained by sensors in the field).

The optimal control may be determined by first forming the augmented function given by

$$F = \phi\Big|_{t_i}^{t_f} + \bar{\gamma}^T S^0(x,t)\Big|_{t_1} \int_{t_i}^{t_f} [\lambda^T(\dot{X} - f(X,U,t)) - \pi^T C(X,U,t)$$

$$- \gamma^T S^1(x,t)]dt$$

or

$$F = \phi\Big|_{t_i}^{t_f} + \bar{\gamma}^T S^0(x,t)\Big|_{t_1} + \int_{t_i}^{t_f} (\lambda^T \dot{X} - H)dt$$

where $H = \lambda^T f(X,U,t) + \pi^T C(X,U,t) + \gamma^T S^1(x,t)$, is formed by adjoining to the original objective function, the state equations by means of the Lagrange multipliers $\lambda_i(t)$; the control variable inequality constraints by means of the Lagrange multipliers $\gamma_i(t)$ and the derivatives of the state variable inequality constraints by means of the Lagrange multipliers $\pi_i(t)$. The multipliers $\bar{\gamma}$ apply to a point constraint at the time the state variable constraints first become binding $(S^0(x,t) = S(x,t)$. (Note if $C_K(X,U,t) < 0$ then $\pi_K = 0$. If $S_j(x,t) < 0$ then $\gamma_j = 0$. Therefore, the integral is zero for the initial form). (Note the superscript T means transpose i.e., λ^T is a row vector). The necessary conditions for an optimal control strategy are then:

1. The control equations

$$\frac{\partial H}{\partial u_i} = 0 \qquad i = 1\ldots \text{ number of control variables}$$

2. The adjoint equations

$$\dot{\lambda}_j = -\frac{\partial H}{\partial x_j} \qquad j = 1\ldots \text{ number of state equations}$$

3. The state equations

$$\dot{X}_k = f(X,U,t) \qquad K = 1\ldots \text{ number of state equations}$$

4. The transversality condition

$$d\phi + \sum_{j=1}^{n} \lambda_j dX_j \bigg|_{t_f} = 0$$

(These supply initial conditions at t_f with which to evaluate the adjoint variables)

5. The initial conditions on the state variables

$$X(t_o) = \text{given}$$

6. The continutity equations for a constraint boundary

$$\sum_{i=1}^{n} \lambda_i f_i \bigg|_{t_1^+} = \sum_{i=1}^{n} \lambda_i f_i \bigg|_{t_1^-} + \overline{\gamma}_j \frac{\partial S_j^0}{\partial t_1} \bigg|_{t_1}$$

and

$$\lambda_i \bigg|_{t_1^+} = \lambda_i \bigg|_{t_1^-} - \overline{\gamma}_j \frac{\partial S_j^0}{\partial x_i} \qquad i = 1,\ldots,n$$

where t_1 is the time that the constraint becomes binding. When the constraint ceases to be binding the continuity equations are

$$\sum_{i=1}^{n} \lambda_i f_i \bigg|_{t_2^+} = \sum_{i=1}^{n} \lambda_i f_i \bigg|_{t_2^-}$$

and

$$\lambda_i \bigg|_{t_2^+} = \lambda_i \bigg|_{t_2^-} \qquad i = 1,n$$

where t_2 is the time at which a constraint ceases to be binding. (Note there are cases where the continuity equations on the λ's need to be modified. These are discussed in the detailed example).

7. The state and control inequality constraint equations when they are binding

$$C(X,U,t) = 0$$

$$S(X,t) = 0$$

This type of problem is called a two point boundary value problem. The initial conditions for the state equations are known at time $t = t_o$ whereas, the initial conditions for the adjoint equations are given by the transversality condition at $t = t_f$. As the adjoint equations are a function of the state variables, the state variables are a function of the control, and the control is a function of state variables and Lagrange multipliers solutions of optimal control problems are not simple.

It should be noted that the use of the derivative $S^1(X,t)$ in the H (called the Hamiltonian) is necessary to bring the control into the equations.

$$S^1 = \frac{d}{dt} S(X,t) = \sum_{i=1}^{n} \frac{\partial S}{\partial X_i} \dot{X}_i + \frac{\partial S}{\partial t}$$

and

$$\dot{X}_i = f_i(X,U,t) \; .$$

To complete the illustration for the example given we have as the augmented function

$$F = \sum_{i=1}^{6} - Z_i V_i \Big|_{t_f} - \int_{o}^{t_f} \{ \sum_{i=1}^{6} \lambda_i \dot{V}_i + \sum_{i=7}^{9} \lambda_i \dot{d}_i - H \} dt$$

where the Hamiltonian H is given by,

$$H = \lambda_j f_j(x,u,t) + \pi_i C_i(x,u,t) + \gamma_k S_k^1(x,u,t)$$

or

$$H = \lambda_1 C_1 h_1^{3/2} + \lambda_2 C_2 h_2^{3/2} + \lambda_3 C_3 h_3^{3/2} + \lambda_4 K_1 r_1^2 d_1^{1/2} + \lambda_5 K_2 r_2^2 d_2^{1/2}$$

$$+ \lambda_6 K_3 r_3^2 d_3^{1/2} + \lambda_7 [\frac{f_1(t) + K_2 r_2^2 d_2^{1/2} - K_1 r_1^2 d_1^{1/2} - C_1 h_1^{3/2}}{A_1(d_1)}]$$

$$+ \lambda_8 [\frac{f_2(t) - K_2 r_2^2 d_2^{1/2} - C_2 h_2^{3/2}}{A_2(d_2)}] + \lambda_9 [\frac{f_3(t) - K_3 r_3^2 d_3^{1/2} - C_3 h_3^{3/2}}{A_3(d_3)}]$$

$$+ \pi_1 [f_1(t) + K_2 r_2^2 d_2^{1/2} - Q_{1\ max}] + \pi_2 [K_1 r_1^2 d_1^{1/2} + K_3 r_3^2 d_3^{1/2} - Q_{2\ max}]$$

$$+ \pi_3 (h_1)(h_1 - d_1) + \pi_4 (h_2)(h_2 - d_2) + \pi_5 (h_3)(h_3 - d_3) + \pi_6 r_1 (r_1 - R_1)$$

$$+ \pi_7 r_2 (r_2 - R_2) + \pi_8 r_3 (r_3 - R_3) + \gamma_1 (2d_1 - D_1)(f_1(t) + K_2 r_2^2 d_2^{1/2} - K_1 r_1^2 d_1^{1/2}$$

$$- C_1 h_1^{3/2})/A_1(d_1) + \gamma_2 (2d_2 - D_2)(f_2(t) - K_2 r_2^2 d_2^{1/2} - C_2 h_2^{3/2})/A_2(d_2)$$

$$+ \gamma_3 (2d_3 - D_3)(f_3(t) - K_3 r_3^2 d_3^{1/2} - C_3 h_3^{3/2})/A_3(d_3)$$

From the above the control equations are $\frac{\partial H}{\partial u_i} = 0$. Thus, we obtain the following expressions:

$$0 = \frac{\partial H}{\partial r_1} = [\lambda_4 - \frac{\lambda_7}{A_1(d_1)} - \gamma_1 \frac{(2d_1 - D_1)}{A_1(d_1)} + \pi_2](K_1 2 r_1 d_1^{1/2}) + \pi_6 (2r_1 - R_1)$$

$$0 = \frac{\partial H}{\partial r_2} = [\lambda_5 + \frac{\lambda_7}{A_1(d_1)} - \frac{\lambda_8}{A_2(d_2)} + \pi_1 + \gamma_1 \frac{(2d_1 - D_1)}{A_1(d_1)}](K_2 2 r_2 d_2^{1/2}) + \pi_7 (2r_2 - R_2)$$

$$0 = \frac{\partial H}{\partial r_3} = [\pi_6 - \frac{\lambda_9}{A_3(d_3)} + \pi_2 - \gamma_3 \frac{(2d_3 - D_3)}{A_3(d_3)}](K_3 2 r_3 d_3^{1/2}) + \pi_8 (2r_3 - R_3) \ .$$

$$0 = \frac{\partial H}{\partial h_1} = [\lambda_1 - \frac{\lambda_7}{A_1(d_1)} - \gamma_1 \frac{(2d_1-D_1)}{A_1(d_1)}](\frac{3}{2} C_1 h_1^{1/2}) + \pi_3(2h_1-d_1)$$

$$0 = \frac{\partial H}{\partial h_2} = [\lambda_2 - \frac{\lambda_8}{A_2(d_2)} - \gamma_2 \frac{(2d_2-D_2)}{A_2(d_2)}](\frac{3}{2} C_2 h_2^{1/2}) + \pi_4(2h_2-d_2)$$

$$0 = \frac{\partial H}{\partial h_3} = [\lambda_3 - \frac{\lambda_9}{A_3(d_3)} - \gamma_3 \frac{(2d_3-D_3)}{A_3(d_3)}](\frac{3}{2} C_3 h_3^{1/2}) + \pi_5(2h_3-d_3) .$$

The adjoint equations are $\dot{\lambda} = - \frac{\partial H}{\partial X_m}$

$$\dot{\lambda}_1 = - \frac{\partial H}{\partial V_1} = 0 , \rightarrow \lambda_1 = C_1$$

$$\dot{\lambda}_2 = - \frac{\partial H}{\partial V_2} = 0 , \rightarrow \lambda_2 = C_2$$

$$\dot{\lambda}_3 = - \frac{\partial H}{\partial V_3} = 0 , \rightarrow \lambda_3 = C_3$$

$$\dot{\lambda}_4 = - \frac{\partial H}{\partial V_4} = 0 , \rightarrow \lambda_4 = C_4$$

$$\dot{\lambda}_5 = - \frac{\partial H}{\partial V_5} = 0 , \rightarrow \lambda_5 = C_5$$

$$\dot{\lambda}_6 = - \frac{\partial H}{\partial V_6} = 0 , \rightarrow \lambda_6 = C_6$$

$$\dot{\lambda}_7 = \frac{\lambda_7}{[A_1(d_1)]^2} \{A_1(d_1) \left(\frac{K_1 r_1^2}{2d_1^{1/2}}\right) + (f_1(t) + K_2 r_2^2 d_2^{1/2} - K_1 r_1^2 d_1^{1/2}$$

$$- C_1 h_1^{3/2})\frac{dA_1(d_1)}{dd_1} \} - \{\pi_2 + \lambda_4 - \frac{\gamma_1}{A_1(d_1)} (2d_1-D_1)\} \left(\frac{K_1 r_1^2}{2d_1^{1/2}}\right) + \pi_3 h_1$$

$$\dot{\lambda}_8 = \frac{\lambda_8}{[A_2(d_2)]^2}\{A_2(d_2)\left(\frac{K_2r_2^2}{2d_2^{1/2}}\right) + (f_2(t) - K_2r_2d_2^{1/2} - C_2h_2^{3/2})\frac{dA_2(d_2)}{dd_2}\}$$

$$- \{\lambda_5 + \frac{\lambda_7}{A_1(d_1)} + \pi_1 + \gamma_1\frac{(2d_1-D_1)}{A_1(d_1)} - \gamma_2\frac{(2d_2-D_2)}{A_2(d_2)}\}\left(\frac{K_2r_2^2}{2d_2^{1/2}}\right) + \pi_4h_2$$

$$\dot{\lambda}_9 = \frac{\lambda_9}{[A_3(d_3)]^2}\{A_3(d_3)\left(\frac{K_3r_3^2}{2d_3^{1/2}}\right) + (f_3(t) - K_3r_3^2d_3^{1/2} - C_3h_3^{3/2})\frac{dA_3(d_3)}{dd_3}\}$$

$$- \{\lambda_6 + \pi_2 - \gamma_3\frac{(2d_3-D_3)}{A_3(d_3)}\}\left(\frac{K_3r_3^2}{2d_3^{1/2}}\right) + \pi_5h_3$$

The transversality conditions are given by

$$d\phi + \sum \lambda_i dX_i \Big|_{t_f} = 0$$

thus

$$- z_1dV_1 - z_2dV_2 - z_3dV_3 - z_4dV_4 - z_5dV_5 - z_6dV_6 + \lambda_{1_f}dV_1$$

$$+ \lambda_{2_f}dV_2 + \lambda_{3_f}dV_3 + \lambda_{4_f}dV_4 + \lambda_{5_f}dV_5 + \lambda_{6_f}dV_6 + \lambda_{7_f}dd_1$$

$$+ \lambda_{8_f}dd_2 + \lambda_{9_f}dd_3 = 0$$

or

$$(\lambda_{1_f}-z_1)dV_1 + (\lambda_{2_f}-z_2)dV_2 + (\lambda_{3_f}-z_3)dV_3 + (\lambda_{4_f}-z_4)dV_4$$

$$+ (\lambda_{5_f}-z_5)dV_5 + (\lambda_{6_f}-z_6)dV_6 + \lambda_{7_f}dd_1 + \lambda_{8_f}dd_2 + \lambda_{9_f}dd_3 = 0 \ .$$

From these we obtain

$$\lambda_{1_f} = z_1 \qquad \lambda_{6_f} = z_6$$

$$\lambda_{2_f} = z_2 \qquad \lambda_{7_f} = 0$$

$$\lambda_{3_f} = z_3 \qquad \lambda_{8_f} = 0$$

$$\lambda_{4_f} = z_4 \qquad \lambda_{9_f} = 0$$

$$\lambda_{5_f} = z_5$$

Finally for the constraints we have the continuity conditions when any of the constraints become or cease to be binding:

1. Control variable inequality constraints

$$\sum_{i=1}^{9} \lambda_i f_i \Big|_{t_1^+} = \sum_{i=1}^{9} \lambda_i f_i \Big|_{t_1^-} \; ; \; \sum_{i=1}^{9} \lambda_i f_i \Big|_{t_2^+} = \sum_{i=1}^{9} \lambda_i f_i \Big|_{t_2^-}$$

$$\lambda_i \Big|_{t_1^+} = \lambda_i \Big|_{t_1^-} \; ; \; \lambda_i \Big|_{t_2^+} = \lambda_i \Big|_{t_2^-} : \; i = 1,\ldots,9$$

2. State variable inequality constraints

For the state variable inequality constraints it should be noted that the conditions on the Lagrange Multipliers were earlier listed in their standard form; however, a closer examination of the problem shows that at the entrance to a reservoir depth constraint we do not need to solve for $\bar{\gamma}_j$. Thus λ_j, which is associated with the state variable being constrained is determined from the continuity of the Hamiltonian and the remaining λ's. At the exit from the constraint all the λ's are continuous. The continuity conditions in this case are:

at the entrance

$$\sum_{i=1}^{9} \lambda_i f_i \Big|_{t_1^-} = \sum_{i=1}^{9} \lambda_i f_i \Big|_{t_1^+} ; \qquad \lambda_i \Big|_{t_2^+} = \lambda_i \Big|_{t_2^-} \; : \; i = 1,\ldots j-1, j+1,\ldots 9$$

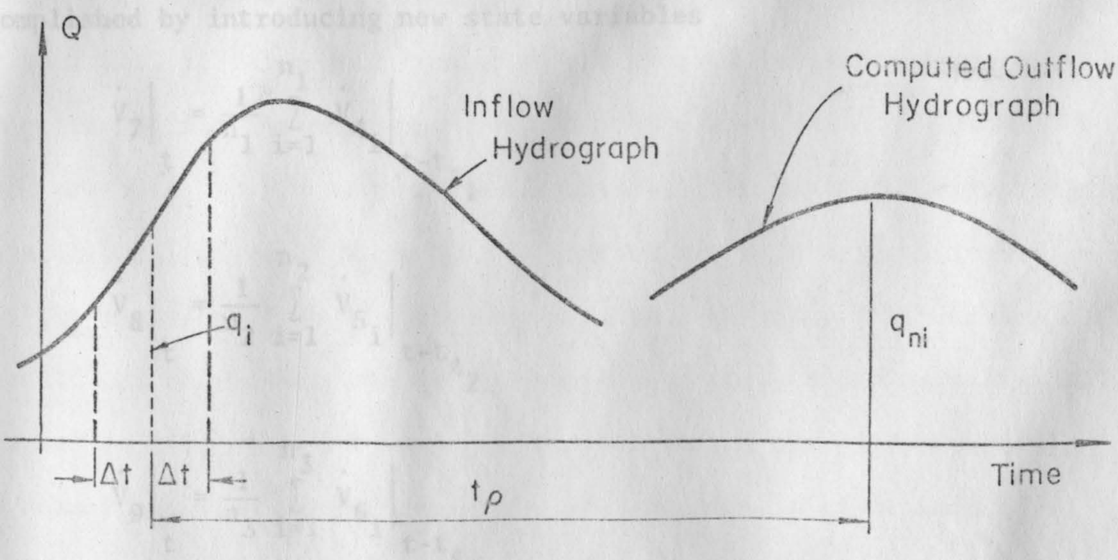
and at the exit

$$\sum_{i=1}^{9} \lambda_i f_i \Big|_{t_2^-} = \sum_{i=1}^{9} \lambda_i f_i \Big|_{t_2^+} ; \; \lambda_i \Big|_{t_2^-} = \lambda_i \Big|_{t_2^+} \; : \; i = 1\ldots 9$$

The foregoing example is not a particularly good representation of a combined sewer system for it makes no allowance for the time delay in flow from one reservoir to the reservoir immediately downstream in the same reach. This difficulty is overcome by use of the Progressive Average Lag method of routing in conduits. This method, developed by Dooge (2), consists of progressively averaging flows in the inflow hydrograph and lagging them by a given time t_ρ to produce points on the outflow hydrograph (see Fig. 4.2).

Figure 4.2

PROGRESSIVE AVERAGE LAG



In Figure 4.2 we have

$$q_{n_i} = \frac{1}{m} \sum_{i=1}^{m} q_i$$

where m is the number of points on the inflow hydrograph to be averaged,

q_i are points on the inflow hydrograph separated by Δt, and

t_ℓ is the lag time.

The values given to n and t_ℓ are quite arbitrary for a given channel and must be evaluated in this model by comparison with the Physical System model (FWQA model-transportation section). Tests have shown that results using the Progressive Average Lag Method agree closely with those obtained by the Method of Characteristics (Ref. 3). Since reasonably good agreement has been obtained between results using the routing method used in the FWQA model and those obtained using the Method of Characteristics, it is safe to assume that routing using the Progressive Average Lag Method should produce an acceptable representation of the FWQA model.

Addition of the routed hydrographs to the simple model can be accomplished by introducing new state variables

$$\dot{V}_7\Big|_t = \frac{1}{n_1} \sum_{i=1}^{n_1} \dot{V}_{4_i}\Big|_{t-t_{\ell_1}}$$

$$\dot{V}_8\Big|_t = \frac{1}{n_2} \sum_{i=1}^{n_2} \dot{V}_{5_i}\Big|_{t-t_{\ell_2}}$$

$$\dot{V}_9\Big|_t = \frac{1}{n_3} \sum_{i=1}^{n_3} \dot{V}_{6_i}\Big|_{t-t_{\ell_3}}$$

into the state equations and revising all the remaining equations accordingly. However, as $\dot{V}_4$, $\dot{V}_5$ and $\dot{V}_6$ can be expressed in terms of existing and control variables and thus likewise $\dot{V}_7$, $\dot{V}_8$ and $\dot{V}_9$, there is no real need to introduce new state variables.

As an example, consider the state equation for $\dot{d}_1$. The modified form is given by

$$\dot{d}_1 = \frac{f_1(t) + \dot{V}_8 - V_1 - V_4}{A_1(d_1)}$$

substituting for $\dot{V}_8$ gives:

$$\dot{d}_i = \frac{f_1(t) + \frac{1}{n_2}\sum_{i=1}^{n_2} \dot{V}_{5_i}\Big|_{t-t_{\ell_2}} - V_1 - V_4}{A_1(d_1)}$$

and finally substituting for $\dot{V}_5$ leads to:

$$\dot{d}_i = \frac{f_1(t) + \frac{1}{n_2}\sum_{i=1}^{n_2} K_2 r_2^2 d_2^{1/2}\Big|_{t=t_\ell} - V_1 - V_4}{A_1(d_1)}$$

In this example $f_1(t)$ was not lagged. However, depending upon its location, it too may be routed by the Progressive Average Lag Method. Using this method the only state equations to be modified are those for state variables $\dot{d}_i$. As the state equations form part of the Hamiltonian, the modifications would occur to the control and adjoint equations.

The addition of the Progressive Average Lag Method to the example is still not sufficient for optimal control of the physical system model, because, at this stage of development, it does not simulate accurately the nonuniform flow conditions of the FWQA model. In other words it does not represent backwater effects due to flow control in a conduit. Figure 4.3 illustrates how the FWQA model simulates a backwater condition.

In Figure 4.3, r is the ratio of the storage at time t to maximum possible storage in the reservoir. The reservoir shown outside the conduit has its dimensions defined by the conduit. To simulate backwater, entering flow QI is divided into two portions as shown. QO1 goes directly to the reservoir while QO2 is routed through the conduit in the normal procedure for pipe flow routing of the FWQA model. The logic behind this method can be demonstrated by considering Figure 4.4. At t_1 the flow QI must travel the full length of the conduit before it becomes backwater storage. At time t_2 the flow QI becomes backwater storage almost immediately. At time $t_1 < t_3 < t_2$, the time for the flow to enter backwater storage is less than that for the flow to travel the full conduit length, but is still greater than zero. In this case the division shown in Figure 7 gives a reasonable representation. The FWQA report (4) indicates that results obtained by the FWQA method of backwater simulation compares quite favorably with those obtained by the method of characteristics.

Figure 4.3

SIMULATION OF BACKWATER STORAGE IN THE FWQA MODEL

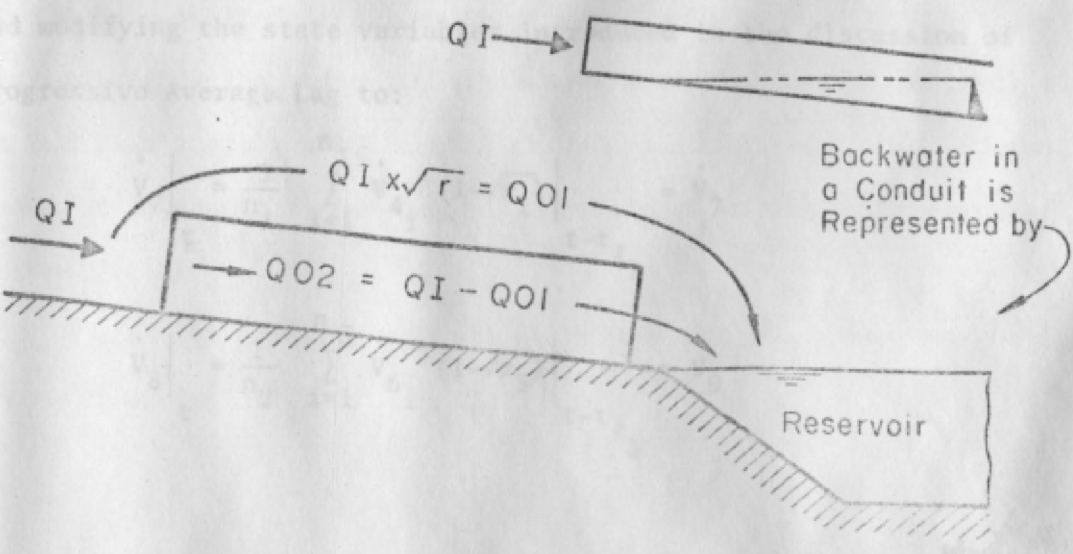
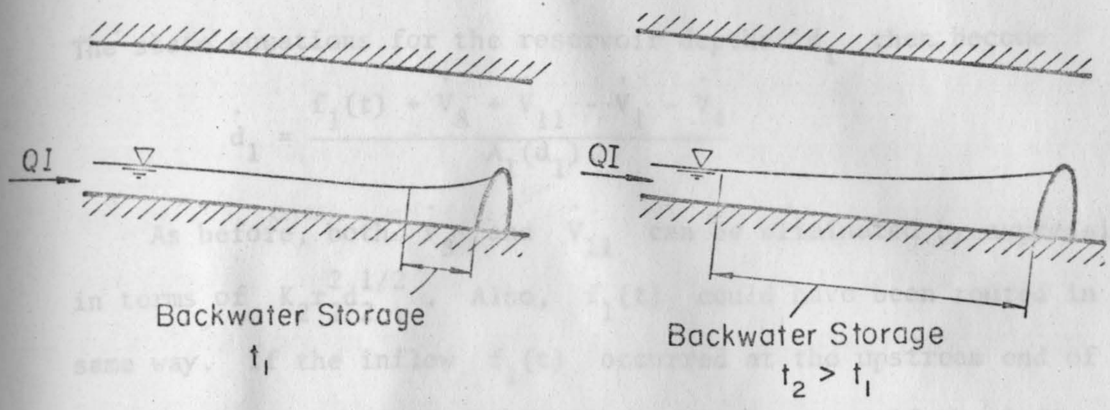


Figure 4.4

COMPARISON OF BACKWATER STORAGE AT DIFFERENT TIMES



This modification to the example, which has only one pipe length between reservoirs, may be accounted for by introducing additional state variables as follows:

$$\dot{V}_{10} = V_4 \sqrt{r_1}$$

$$\dot{V}_{11} = \dot{V}_4 \sqrt{r_2}$$

$$\dot{V}_{12} = \dot{V}_4 \sqrt{r_3}$$

and modifying the state variables introduced in the discussion of Progressive Average Lag to:

$$\dot{V}_7\Big|_t = \frac{1}{n_1} \sum_{i=1}^{n_1} \dot{V}_{4_i} \left(1-\sqrt{r_1}\right)\Big|_{t-t_{\ell_1}} = \dot{V}_7$$

$$\dot{V}_8\Big|_t = \frac{1}{n_2} \sum_{i=1}^{n_3} \dot{V}_{6_i} \left(1-\sqrt{r_3}\right)\Big|_{t-t_{\ell_3}} = \dot{V}_9$$

$$\dot{V}_9\Big|_{t} = \frac{1}{n_3}\sum_{i=1}^{n_3}\dot{V}_{6_i}\,(1-\sqrt{r_3})\Big|_{t-t_{\ell_3}} = \dot{V}_9$$

The state equations for the reservoir depths d_i then become

$$\dot{d}_1 = \frac{f_1(t) + \dot{V}_8 + \dot{V}_{11} - \dot{V}_1 - \dot{V}_4}{A_1(d_1)}$$

As before, both $\dot{V}_8$ and $\dot{V}_{11}$ can be eliminated by expressing them in terms of $K_2 r_2^2 d_2^{1/2}$. Also, $f_1(t)$ could have been routed in the same way. If the inflow $f_1(t)$ occurred at the upstream end of the conduit, such routing would be necessary; however, if the inflow occurred downstream near the control device no such routing would be required.

In the simulation of the FWQA backwater profile the hydraulic characteristics of the optimal control model should be nearly identical to those of the physical system model (FWQA) therefore making it possible to optimize the control of the physical model. Although the system described here consists of three control points only, it does contain all of the essential ingredients of a much larger system.

The equations given above for including time delay in the flow routing are written in a form that would apply to numerical solution. More correctly the summation signs should be replaced by integral signs. In addition there may be further necessary conditions for an optimal solution in cases of time delay. This aspect is receiving further investigation.

A more detailed description of optimal control theory may be found in reference 1.

E. NUMERICAL SOLUTION OF THE CONTROL ALGORITHM

The major problem in the solution of the system of equations for the simplified three-reservoir model is the fact that if the orifice radii are not at their maximum or minimum limits then the associated Lagrange multipliers π_6 , π_7 or π_8 are zero and the control drops out of the control equation (i.e., r_i is no longer explicit in the equations). One way to solve this problem is to perturb the value of r at each step in time, solve for all the Lagrange multipliers, compute values of the Hamiltonian for both the unperturbed and perturbed cases (i.e., compute $\frac{\partial H}{\partial r_i}$ numerically) and then compare these values with integrated value of the Hamiltonian. r_i is then adjusted by linear extrapolation so that the Hamiltonian computed at time t will agree with the integrated value of the Hamiltonian.

A further complication arises from the fact that this is a two-point boundary value problem. The initial values of the state variables are known at time $t = 0$ whereas the final values of the Lagrange multipliers are known at time $t = t_f$. To circumvent this problem the state equations must fitst be integrated to $t = t_f$ along an assumed control history, using the unperturbed and perturbed values of r_i (note - in computing the effects of the perturbed values of r_i on the unperturbed values of V_k and d_j the perturbation is made from the unperturbed trajectory at each time step, i.e., if at time $t = t_\ell$ r is perturbed then values of the perturbed V_k and d_j at time $t_{\ell+1}$ are calculated on the basis of the unperturbed V_k and d_j at time t_ℓ and the perturbed value of r at time t_ℓ). Once the state equations have been integrated to time t_f then the Lagrange multipliers λ_i are integrated backward in time along the state variable trajectory (perturbed and unperturbed values are computed). The values of the Hamiltonian for the perturbed and unperturbed trajectories are computed and compared with the integrated value of the Hamiltonian and the control adjusted accordingly. (Note the values of the weir setting are adjusted when integrating the state equations forward. At the end of each full ineraction they are then set at their maximum values, i.e., $h_i = 0$). This procedure is repeated until the computed and integrated values of H agree within a preset tolerence.

At this point it is worth while to show that adjusting r_i so that computed Hamiltonian at time t is equal to the Hamiltonian integrated from time t_f to t is equivalent to trying to driver $\frac{\partial H}{\partial r}$ to zero.

Consider $\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial u}\frac{dr}{dt} + \frac{\partial H}{\partial x}\frac{dx}{dt}$

if the solution is optimal then necessarily $\frac{\partial H}{\partial r} = 0$. Furthermore,

$$\frac{\partial H}{\partial t} = \frac{d\lambda}{dt} f + \lambda\frac{\partial f}{\partial t} \; ;$$

$$\dot{\lambda} = -\lambda \frac{\partial f}{\partial x} \; ; \; \frac{dx}{dt} = f \; ; \quad \text{and}$$

$$\frac{\partial H}{\partial x} = \lambda\frac{\partial f}{\partial x}$$

Substituting these last four equations into the first yields

$$\frac{dH}{dt} = -\lambda(\frac{\partial f}{\partial x})f + \lambda\frac{\partial f}{\partial t} + 0 + \lambda(\frac{\partial f}{\partial x})f$$

or

$$\frac{dH}{dt} = \lambda\frac{\partial f}{\partial t}$$

Thus in order for the integrated value of the Hamiltonian and the value of the Hamiltonian computed directly at time t to be equal, $\frac{\partial H}{\partial u} = 0$ at all points between t_f and t .

A simplified flow chart of the control program is shown in Fig. 4.5.

Figure 4.5 SIMPLIFIED FLOW CHART OF THE NUMERICAL SOLUTION OF THE NECESSARY CONDITIONS FOR OPTIMAL CONTROL

Integrate State Equations Forward

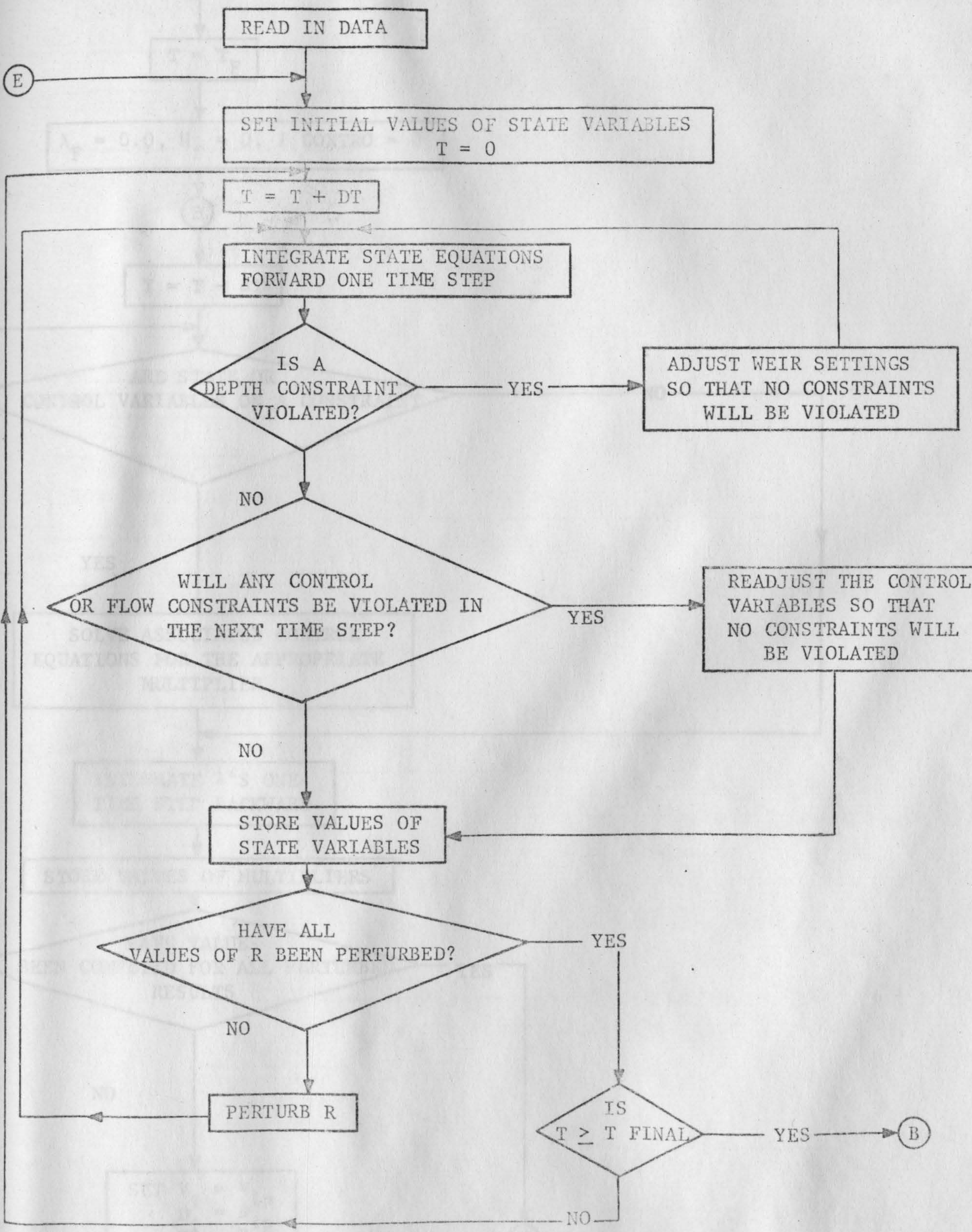


Figure 4.5 (Continued)

Integrate H and λ's backward and solve for π's and γ's

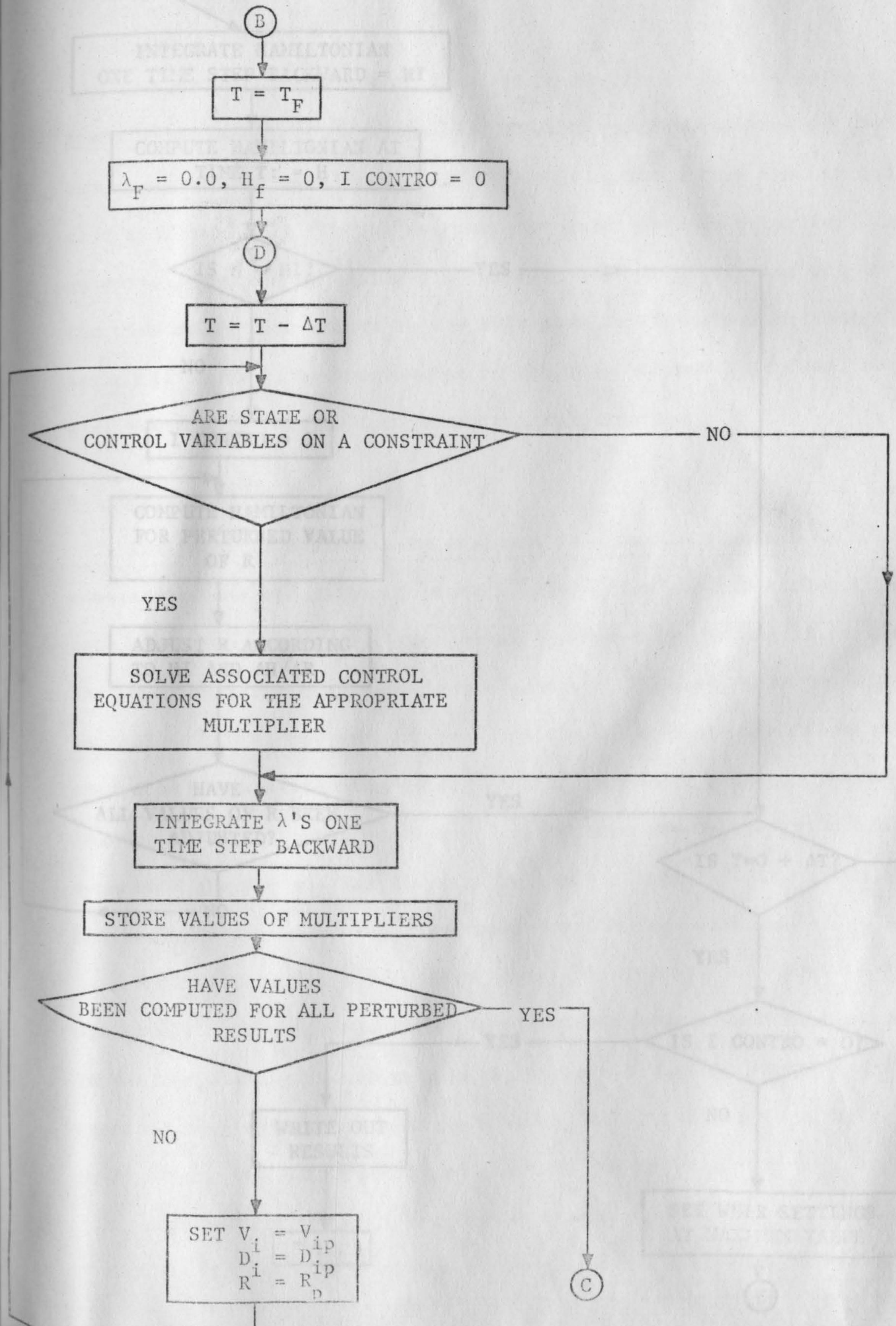
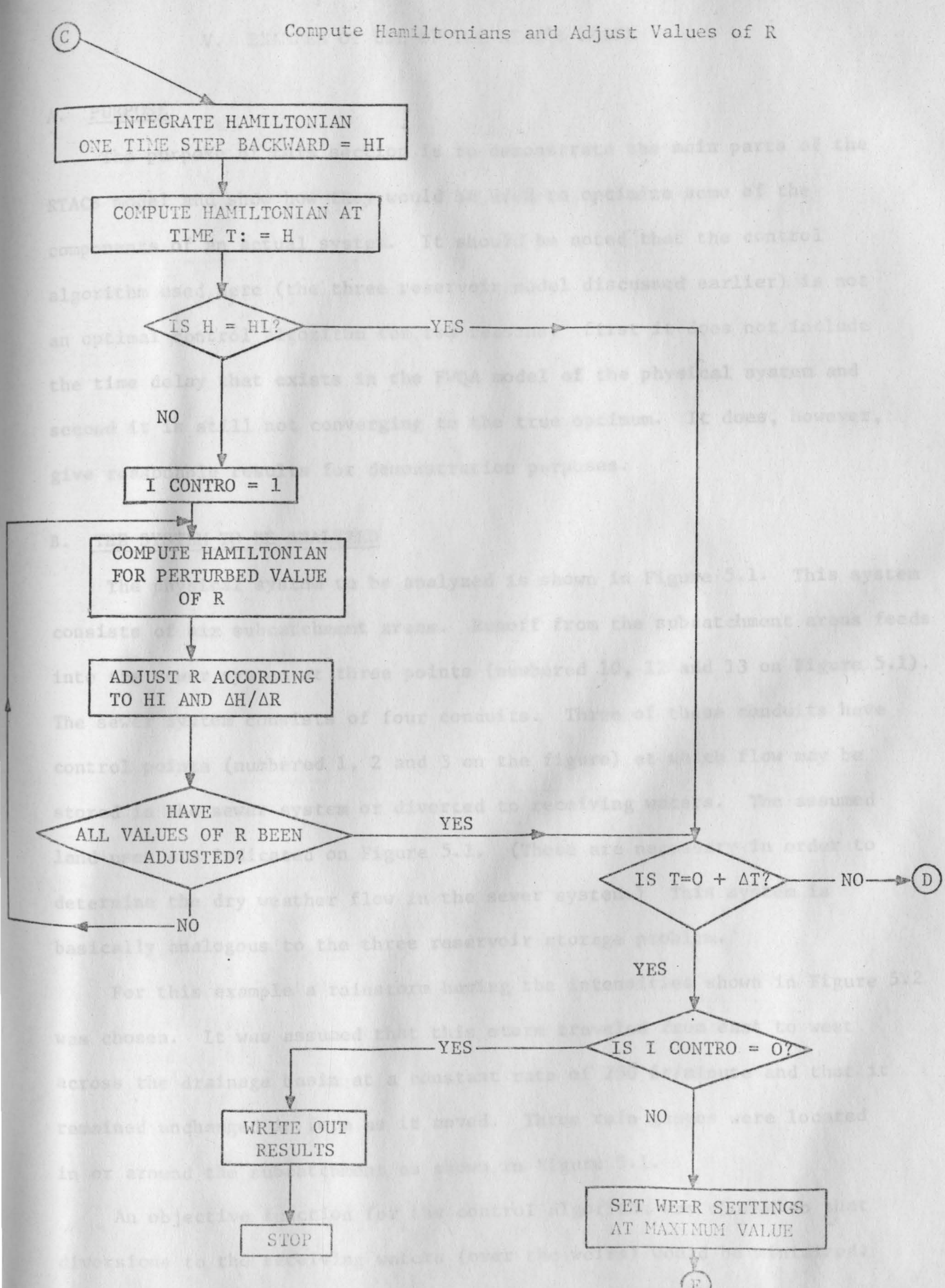


Figure 4.5 (Continued)

Compute Hamiltonians and Adjust Values of R



V. EXAMPLE OF USE OF THE RTAC'S MODEL

A. PURPOSE

The purpose of this section is to demonstrate the main parts of the RTACS model and show how they would be used to optimize some of the components of an actual system. It should be noted that the control algorithm used here (the three reservoir model discussed earlier) is not an optimal control algorithm for two reasons: first it does not include the time delay that exists in the FWQA model of the physical system and second it is still not converging to the true optimum. It does, however, give reasonable results for demonstration purposes.

B. THE SYSTEM TO BE ANALYZED

The physical system to be analyzed is shown in Figure 5.1. This system consists of six subcatchment areas. Runoff from the subcatchment areas feeds into the sewer system at three points (numbered 10, 12 and 13 on Figure 5.1). The sewer system consists of four conduits. Three of these conduits have control points (numbered 1, 2 and 3 on the figure) at which flow may be stored in the sewer system or diverted to receiving waters. The assumed land uses are indicated on Figure 5.1. (These are necessary in order to determine the dry weather flow in the sewer system.) This system is basically analogous to the three reservoir storage problem.

For this example a rainstorm having the intensities shown in Figure 5.2 was chosen. It was assumed that this storm traveled from east to west across the drainage basin at a constant rate of 250 ft/minute and that it remained unchanged in form as it moved. Three rain gauges were located in or around the subcatchment as shown in Figure 5.1.

An objective function for the control algorithm was chosen so that diversions to the receiving waters (over the weirs) would be minimized.

Figure 5.1

THE PHYSICAL SYSTEM

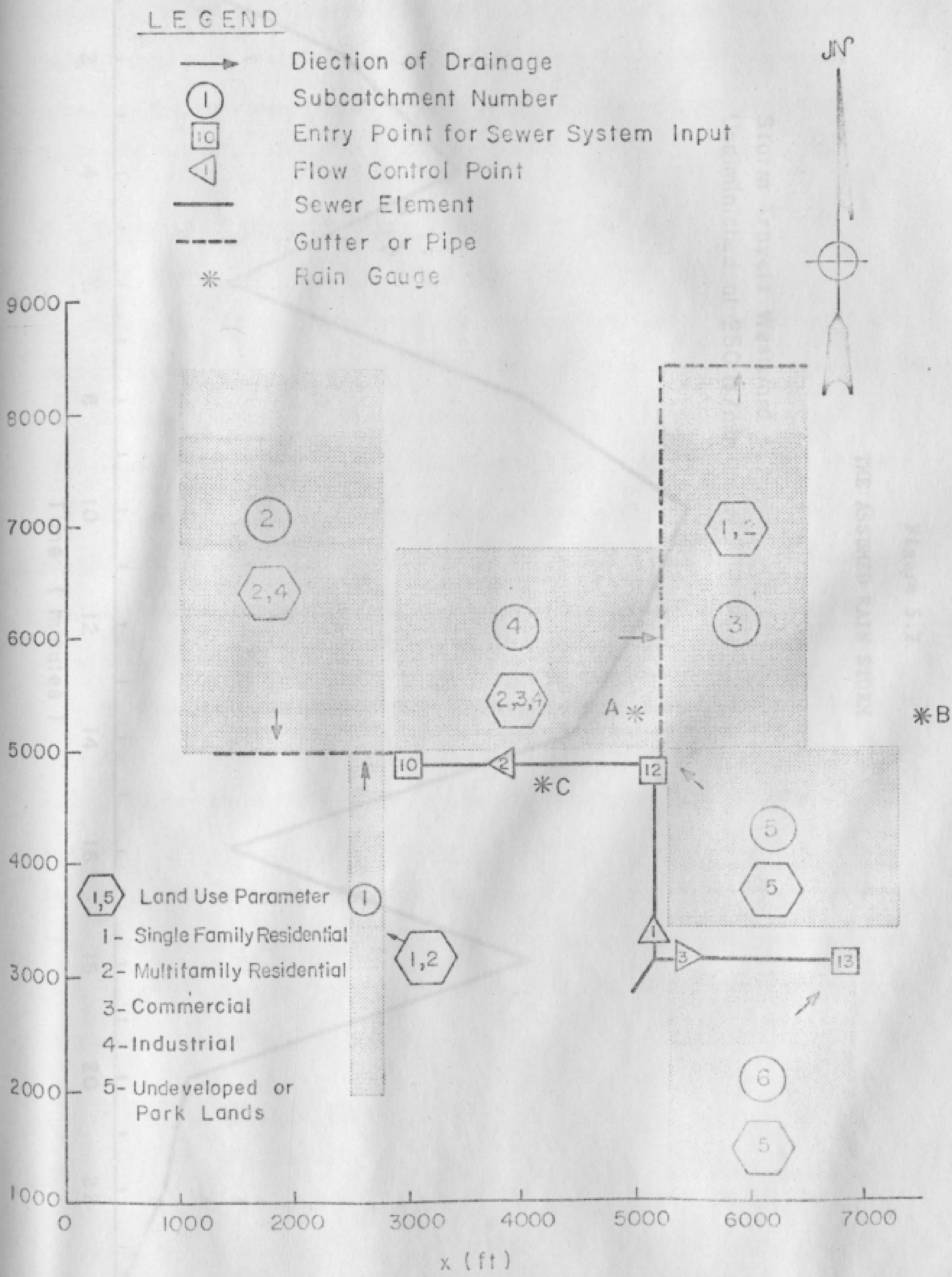
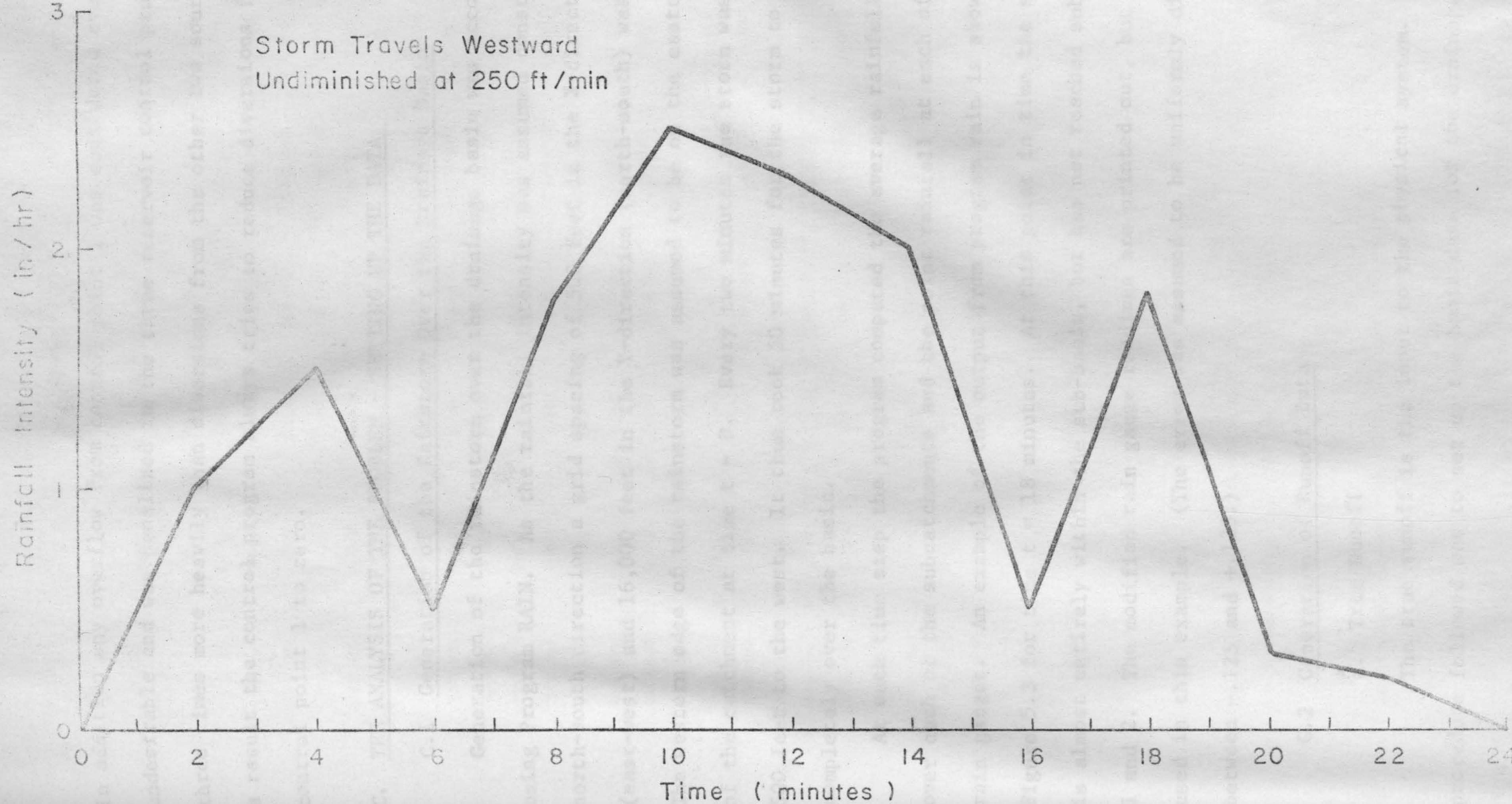


Figure 5.2

THE ASSUMED RAIN STORM



In addition any overflow from control point 1 was considered even more undesirable and was penalized in the three reservoir control program three times more heavily than diversions from the other two sources. As a result the control program always tries to reduce diversions from control point 1 to zero.

C. THE ANALYSIS OF THE PROBLEM - SETTING UP THE DATA

C.1 Generation of the Rainstorm Over the Drainage Basin

Generation of the rainstorm over the drainage basin was accomplished by using Program RAIN. As the rainfall intensity was assumed constant in the north-south direction a grid spacing of 500 feet in the X-direction (east-west) and 16,000 feet in the Y-direction (north-south) was chosen. The western edge of the rainstorm was assumed to be on the eastern edge of the catchment at time t = 0. Every two minutes the storm was moved 500 feet to the west. It thus took 50 minutes for the storm to pass completely over the basin.

At each time step the program computed the average rainfall intensity over each of the subcatchments and the point rainfall at each of the three rain gauges. An example of the output from program rain is shown in Figure 5.3 for time t = 18 minutes. At this point in time the storm is almost entirely within the sub-basin, but has not reached subcatchments 1 and 2. The modified rain gauge readings are printed out, but were not used in this example. (The error was assumed to be uniformly distributed between -.125 and +.125.)

C.2 Generation of Runoff Data

a. True Runoff

The true runoff is the input to the physical system. The procedure followed was to set up the basic data for the drainage basin for

Figure 5.3

Typical Output from Program Rain

GIVEN RAINFALL PATTERN AT TIME = 18.00 MINUTES

X	Y	RAINFALL (IN)	X	Y	RAINFALL (IN)
0.000	0.000	0.000	0.000	16000.000	0.000
500.000	0.000	0.000	500.000	16000.000	0.000
1000.000	0.000	0.000	1000.000	16000.000	0.000
1500.000	0.000	0.000	1500.000	16000.000	0.000
2000.000	0.000	0.000	2000.000	16000.000	0.000
2500.000	0.000	0.000	2500.000	16000.000	0.000
3000.000	0.000	0.000	3000.000	16000.000	0.000
3500.000	0.000	1.000	3500.000	16000.000	1.000
4000.000	0.000	1.500	4000.000	16000.000	1.500
4500.000	0.000	.500	4500.000	16000.000	.500
5000.000	0.000	1.800	5000.000	16000.000	1.800
5500.000	0.000	2.500	5500.000	16000.000	2.500
6000.000	0.000	2.300	6000.000	16000.000	2.300
6500.000	0.000	2.000	6500.000	16000.000	2.000
7000.000	0.000	.500	7000.000	16000.000	.500
7500.000	0.000	1.800	7500.000	16000.000	1.800
8000.000	0.000	.300	8000.000	16000.000	.300
8500.000	0.000	.200	8500.000	16000.000	.200
9000.000	0.000	0.000	9000.000	16000.000	0.000
9500.000	0.000	0.000	9500.000	16000.000	0.000
10000.000	0.000	0.000	10000.000	16000.000	0.000
10500.000	0.000	0.000	10500.000	16000.000	0.000
11000.000	0.000	0.000	11000.000	16000.000	0.000
11500.000	0.000	0.000	11500.000	16000.000	0.000

THE AVERAGE RAINFALL OVER EACH OF THE PHYSICAL SYSTEM CATCHMENTS IS LISTED BELOW

CATCHMENT NO.	RAINFALL (IN)
1	0.000
2	0.000
3	2.257
4	.937
5	1.935
6	1.875

THE UNADJUSTED RAIN GAUGE READINGS ARE -

RAIN GAUGE NO.	RAINFALL (IN)
1	1.800
2	.300
3	1.100

THE RAIN GAUGE READINGS MODIFIED BY ERRORS ARE LISTED BELOW

RAIN GAUGE NO.	RAINFALL (IN)
1	1.787
2	.276
3	1.118

the Runoff portion of the FWQA model. The basic data for the subcatchments is shown in Figure 5.4. The gage number refers to the hyetographs for each subcatchment. These hyetographs are shown in Figure 5.5 (a-f) and were determined by program RAIN. The true runoff to the physical system, calculated at two minute intervals, is shown in Figure 5.6 (a-c). This output data was stored on a magnetic tape for input to the transport section of the FWQA model.

b. Runoff Data for the Control Algorithm

Five different runoff inputs were used for the three reservoir control program. For each case the same subcatchment data was used as was used in the physical system thus making the runoff model in the control algorithm a "perfect" model. The only difference in the five cases was the rainfall intensity input.

Two rainfall regeneration models were used to convert the data from an individual rain gauge to rainfall over the entire basin. The first model assumed that the rainfall intensity recorded at a rain gauge was constant over the entire basin, i.e. each subcatchment has the same hyetograph, (note for each case only one rain gauge was considered to exist in the system.) This model was used for each rain gauge. The hyetographs and resulting runoff computed by the runoff model is shown in Figures 5.7 - 5.9. The runoff data was punched on cards for input to the control program.

The second rainfall regeneration model assumed that the rainfall intensity recorded at rain gauge B would occur over each of the subcatchments at time $t + T_i$ where T_i was defined for each subcatchment and is the time for the storm to move from over the rain gauge to the center of subcatchment i. The time delays assumed are listed in Table 5.1.

Figure 5.4a

Physical System Subcatchment Data

SUBAREA NUMBER	GUTTER OR MANHOLE	WIDTH (FT)	AREA (AC)	PERCENT IMPERV.	SLOPE (FT/FT)	RESISTANCE FACTOR IMPERV.	RESISTANCE FACTOR PERV.	SURFACE STORAGE (IN) IMPERV.	SURFACE STORAGE (IN) PERV.	INFILTRATION RATE (IN/HR) MAXIMUM	INFILTRATION RATE (IN/HR) MINIMUM	INFILTRATION RATE (IN/HR) DECAY RATE	NO
1	31	300.	21.	40.0	.070	.013	.250	.062	.184	3.00	.52	.00115	1
2	31	1800.	140.	40.0	.070	.013	.250	.062	.184	3.00	.52	.00115	2
3	41	1200.	94.	80.0	.070	.013	.250	.062	.184	3.00	.52	.00115	3
4	42	1800.	98.	80.0	.020	.013	.250	.062	.184	3.00	.52	.00115	4
5	12	1600.	73.	85.0	.020	.013	.250	.062	.184	3.00	.52	.00115	5
6	13	1600.	73.	85.0	.020	.013	.250	.062	.184	3.00	.52	.00115	6

TOTAL NUMBER OF SUBCATCHMENTS, 6

TOTAL TRIBUTARY AREA (ACRES), 500.00

Figure 5.4b

GUTTER NUMBER	GUTTER CONNECTION	WIDTH (FT)	LENGTH (FT)	SLOPE (FT/FT)	SIDE SLOPES L	SIDE SLOPES R	MANNING N	OVERFLOW (IN)
31	10	4.8	260.	.020	1.0	1.0	.025	6.00
41	42	4.0	320.	.070	1.0	1.0	.025	6.00
42	12	4.5	600.	.041	0.0	0.0	.012	-0.00

TOTAL NUMBER OF GUTTERS/PIPES, 3

Figure 5.5
TRUE RAINFALL INTENSITY OVER SUBCATCHMENTS

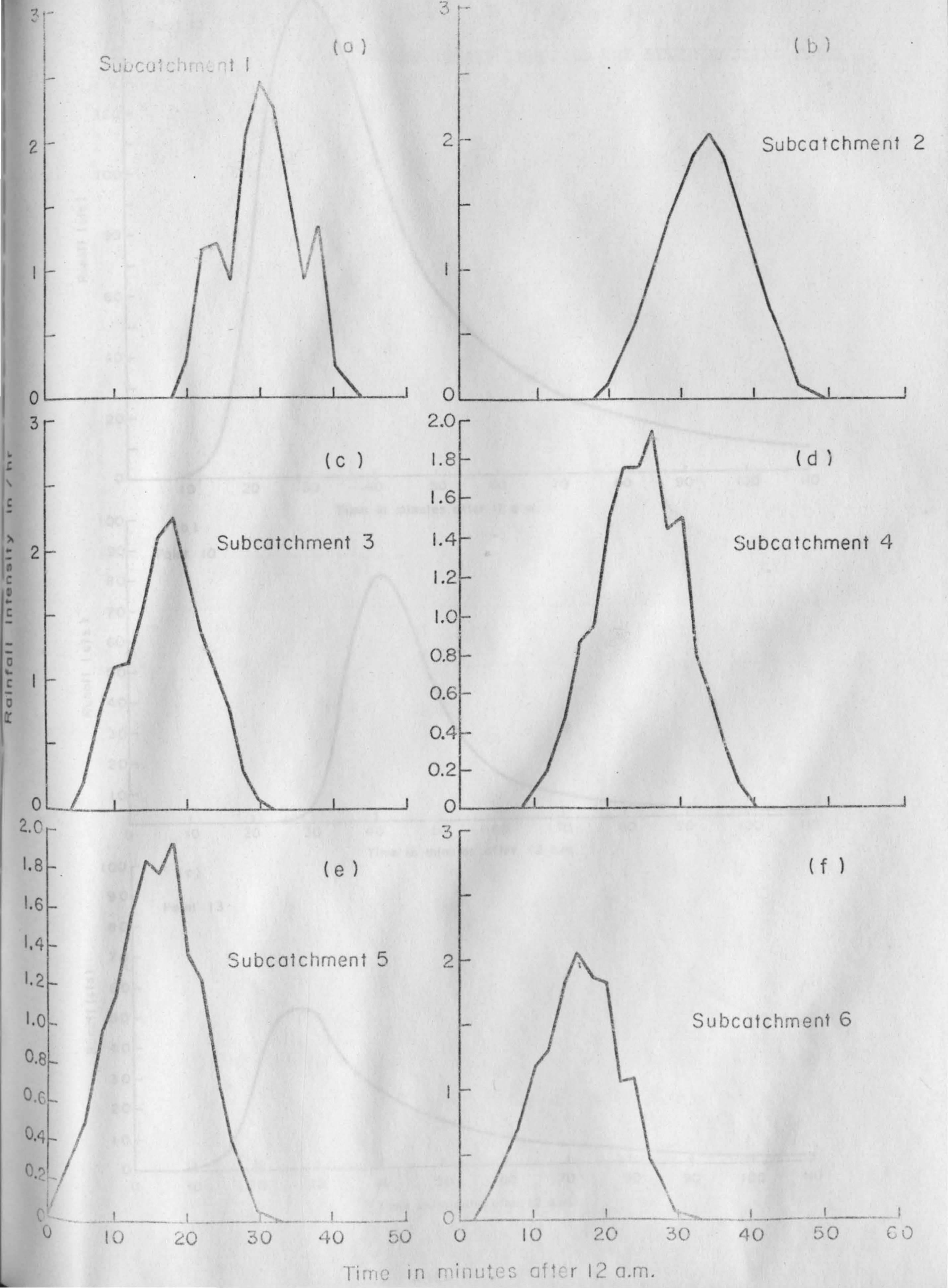


Figure 5.6

TRUE RUNOFF INPUT TO THE SEWER ROUTING MODEL

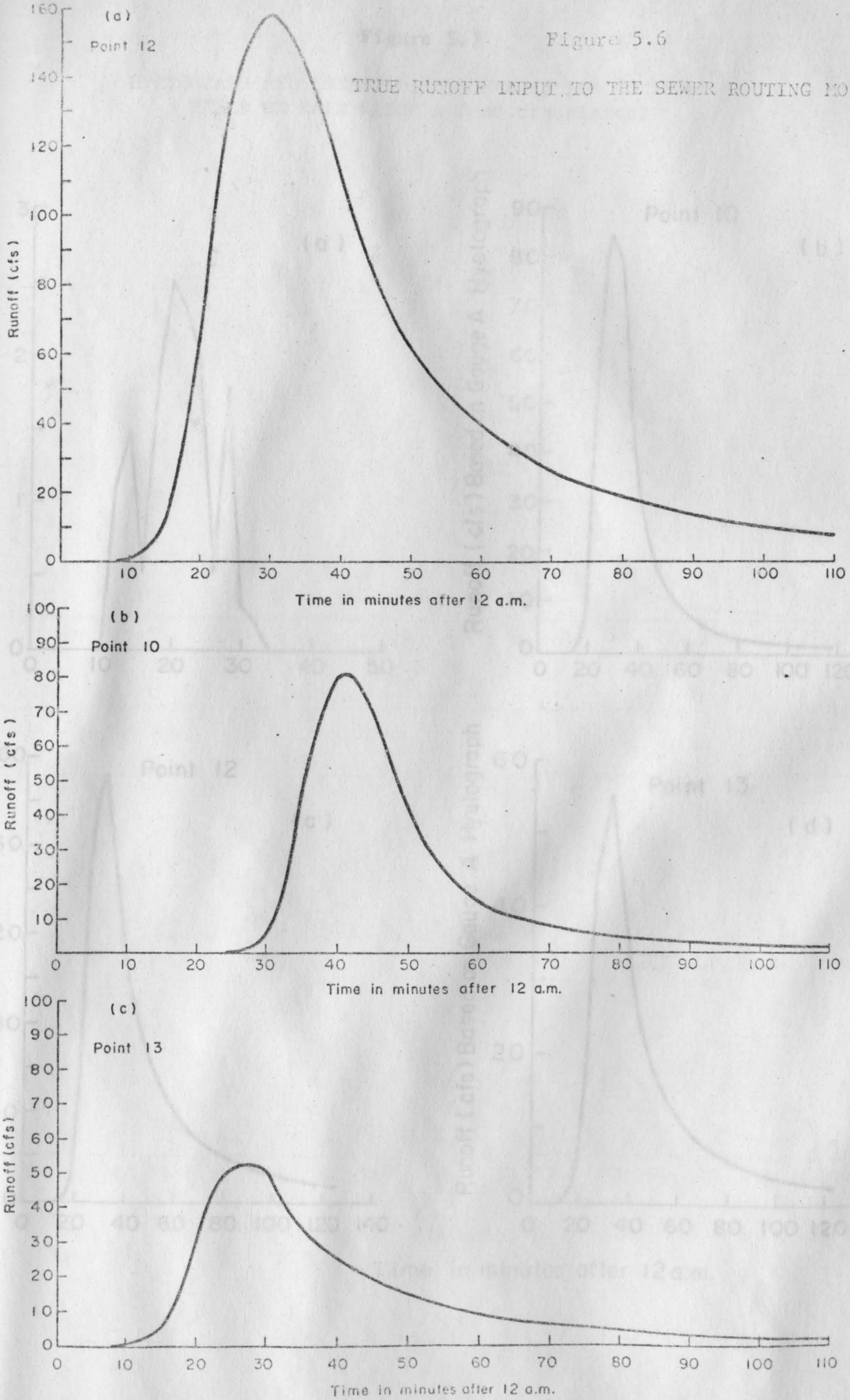


Figure 5.7

HYETOGRAPH AND COMPUTED RUNOFF USED FOR CONTROL
BASED ON RAIN GAUGE A (not translated)

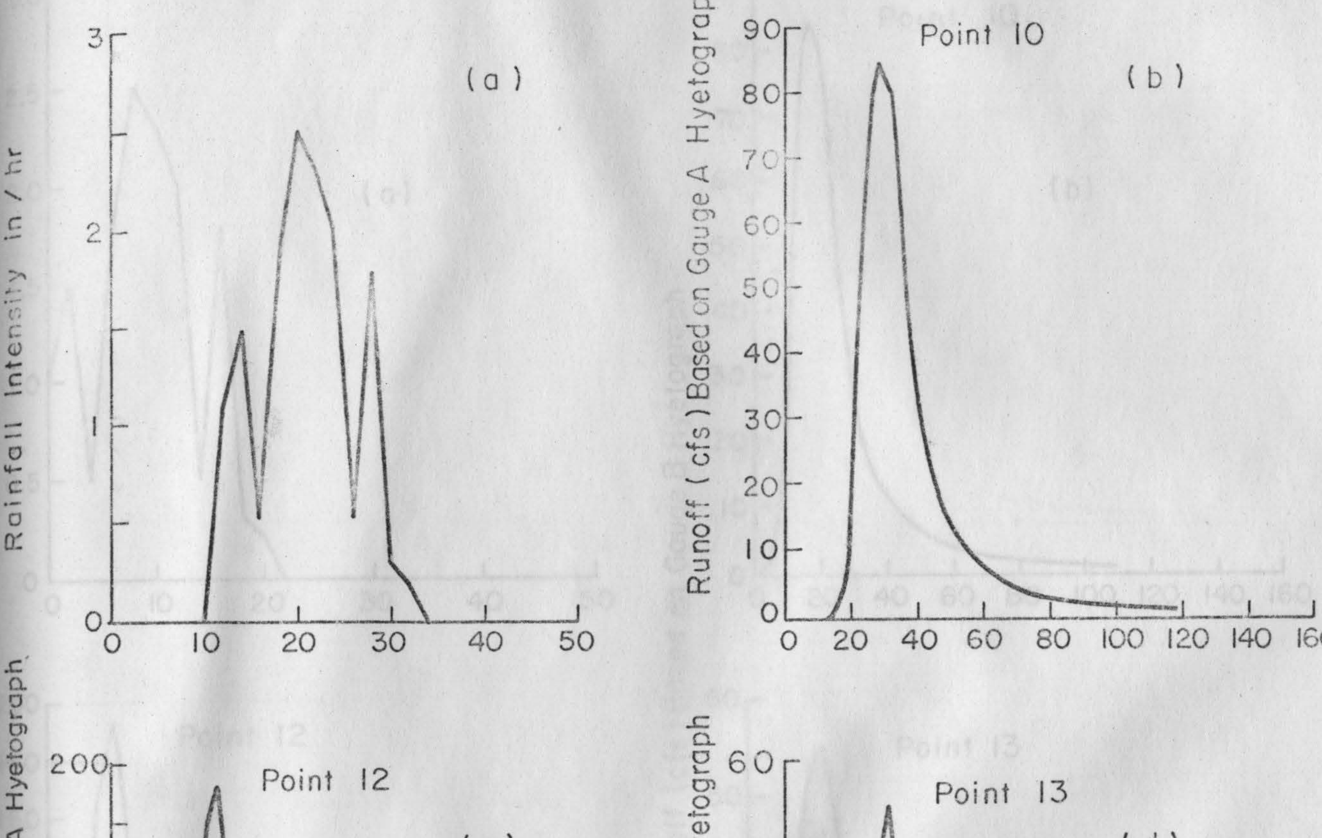
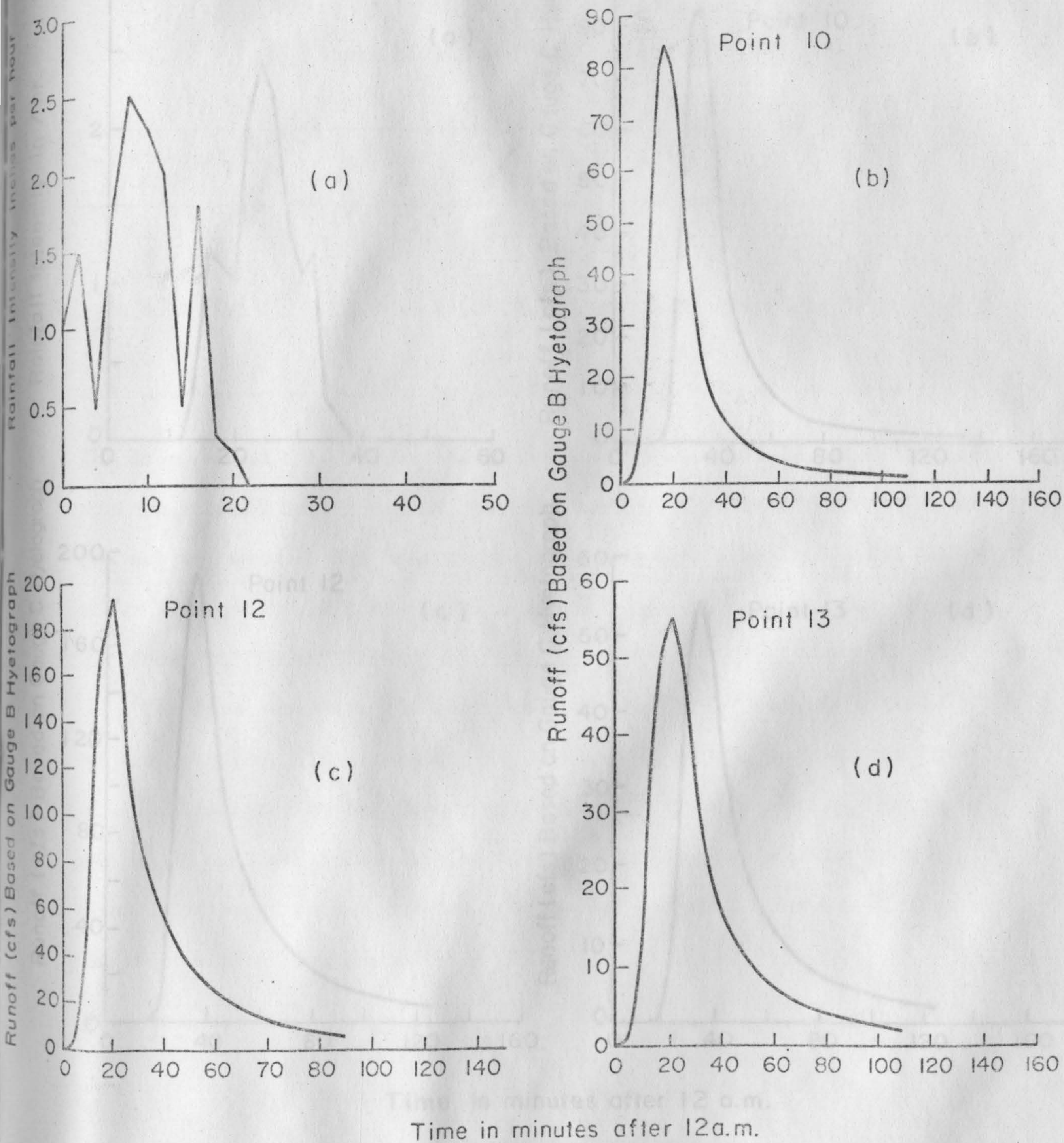
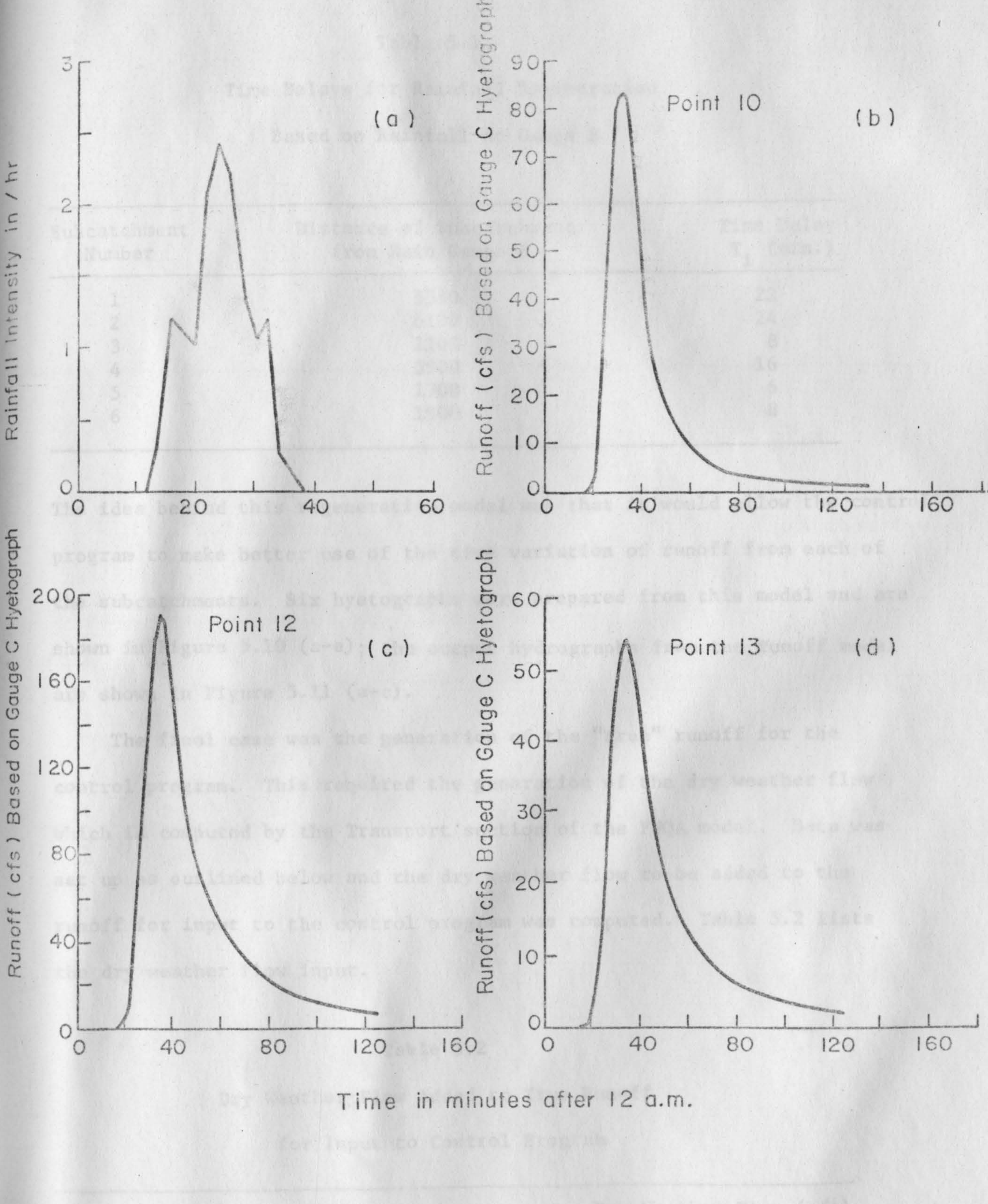


Figure 5.8

HYETOGRAPH AND COMPUTED RUNOFF USED FOR CONTROL
BASED ON RAIN GAUGE B (not translated)





Rainfall Intensity in / hr
(a)
0
1
2
3
0
20
40
60
Runoff (cfs) Based on Gauge C Hyetograph
Point 10
(b)
0
10
20
30
40
50
60
70
80
90
0
40
80
120
160
Runoff (cfs) Based on Gauge C Hyetograph
Point 12
(c)
0
40
80
120
160
200
0
40
80
120
160
Runoff (cfs) Based on Gauge C Hyetograph
Point 13
(d)
0
10
20
30
40
50
60
0
40
80
120
160
Time in minutes after 12 a.m.

Table 5.1

Time Delays for Rainfall Regeneration

Based on Rainfall at Gauge B

Subcatchment Number	Distance of Subcatchment from Rain Gauge B	Time Delay T_i (min.)
1	5350	22
2	6100	24
3	2100	8
4	3900	16
5	1700	6
6	1900	8

The idea behind this regeneration model was that it would allow the control program to make better use of the time variation of runoff from each of the subcatchments. Six hyetographs were prepared from this model and are shown in Figure 5.10 (a-e); the output hydrographs from the runoff model are shown in Figure 5.11 (a-c).

The final case was the generation of the "true" runoff for the control program. This required the generation of the dry weather flow which is computed by the Transport section of the FWQA model. Data was set up as outlined below and the dry weather flow to be added to the runoff for input to the control program was computed. Table 5.2 lists the dry weather flow input.

Table 5.2

Dry Weather Flow Added to True Runoff

for Input to Control Program

Flow Entry Point	Dry Weather Flow (cfs)
10	7.0
12	12.6
13	.31

HYETOGRAPHS AT RAIN GAUGE B TRANSLATED TO SUBCATCHMENTS

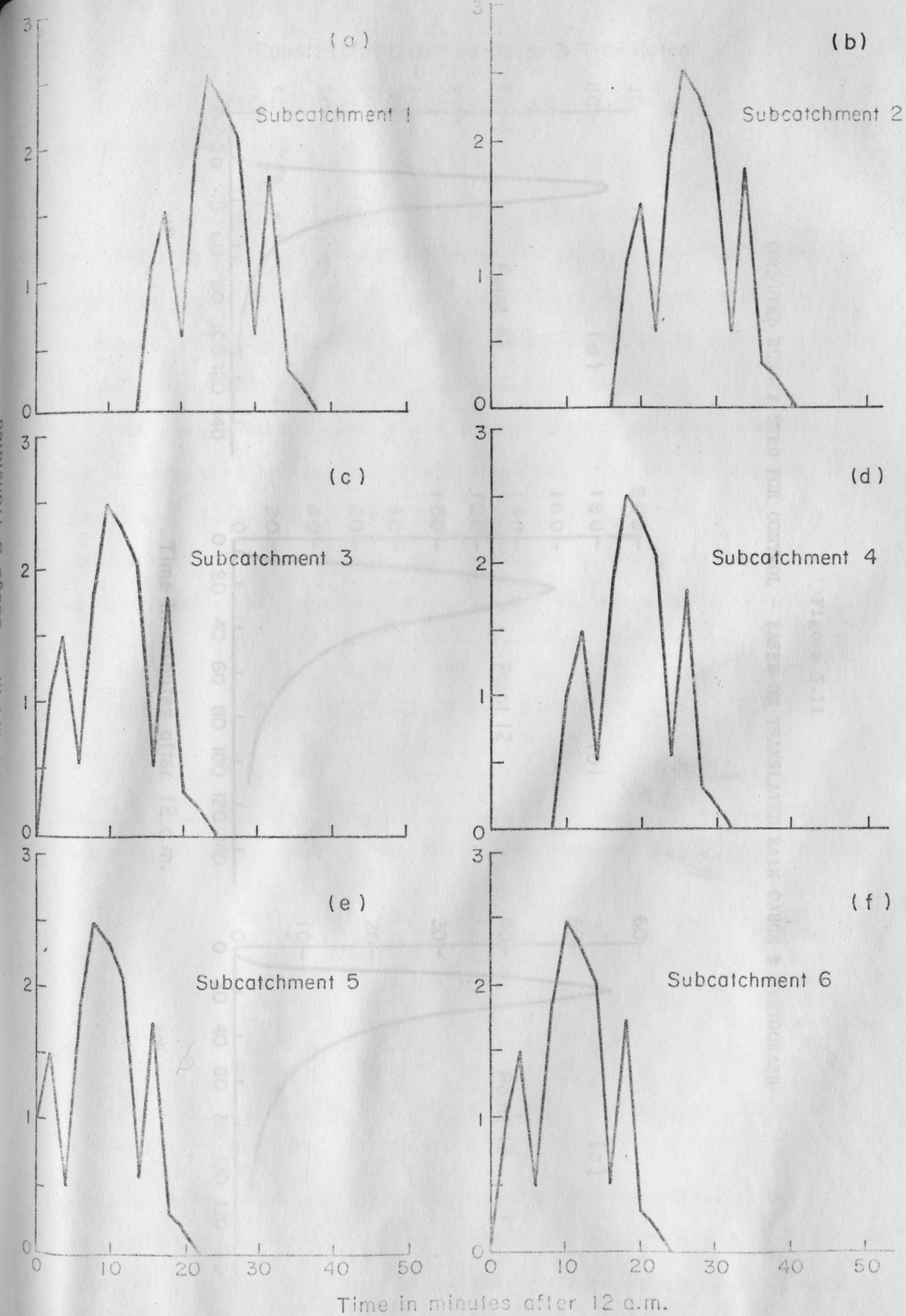


Figure 5.11

COMPUTED RUNOFF USED FOR CONTROL - BASED ON TRANSLATED RAIN GAUGE B HYETOGRAPH

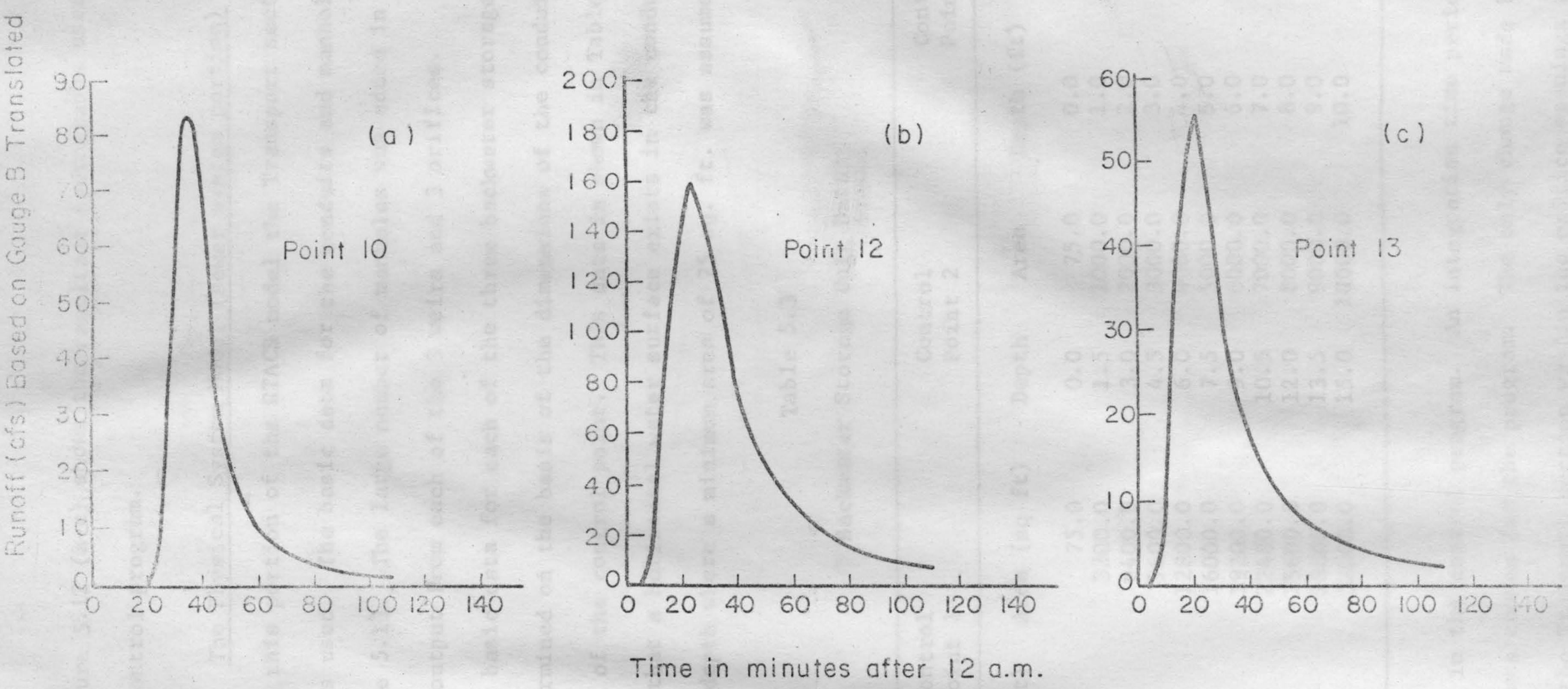


Figure 5.12 (a-c) shows the resultant hydrographs used as input to the control program.

C.3 The Physical System Model (sewer system portion)

For this portion of the RTACS model the Transport section of the FWQA model was used. The basic data for the conduits and manholes is listed in Figure 5.13. The large number of manholes was added in order to obtain printed output from each of the 3 weirs and 3 orifices.

The basic data for each of the three backwater storage locations was determined on the basis of the dimensions of the conduit directly upstream of the control point. This data is shown in Table 5.3. It assumes that a horizontal water surface exists in the conduit except at zero depth where a minimum area of 75 sq. ft. was assumed to avoid

Table 5.3

Backwater Storage Unit Data

Control Point 1		Control Point 2		Control Point 3	
Depth (ft)	Area (sq ft)	Depth	Area	Depth (ft)	Area (sq ft)
0.0	75.0	0.0	75.0	0.0	75.0
1.0	3200.0	1.5	1000.0	1.0	1000.0
2.0	6400.0	3.0	2000.0	2.0	2000.0
3.0	9600.0	4.5	3000.0	3.0	3000.0
4.0	12800.0	6.0	4000.0	4.0	4000.0
5.0	16000.0	7.5	5000.0	5.0	5000.0
6.0	19200.0	9.0	6000.0	6.0	6000.0
7.0	22400.0	10.5	7000.0	7.0	7000.0
8.0	25600.0	12.0	8000.0	8.0	8000.0
9.0	28800.0	13.5	9000.0	9.0	9000.0
10.0	32000.0	15.0	10000.0	10.0	10000.0

problems in the control program. An integration time period of two minutes was chosen for the program. The only change made between different runs of the Transport section was in the orifice openings and weir heights.

Figure 5.12

TRUE RUNOFF USED FOR CONTROL - DRY WEATHER FLOW ADDED

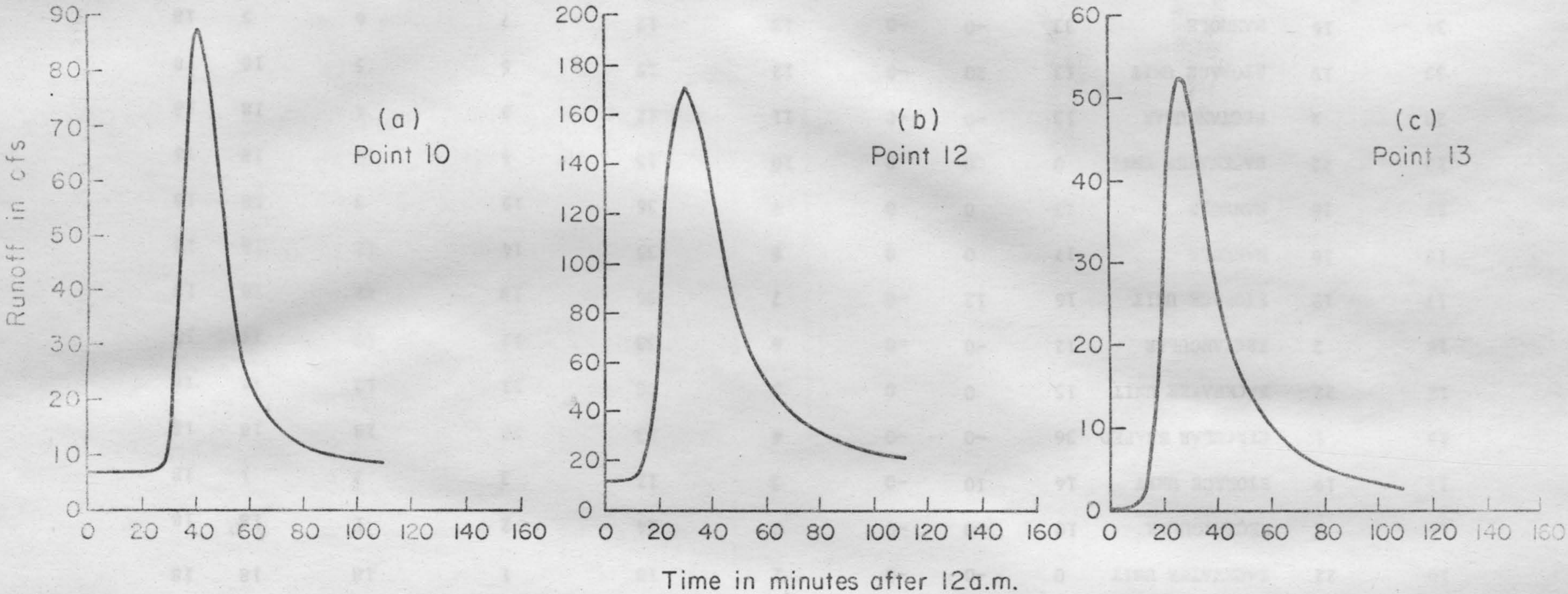


Figure 5.13

Basic Conduit and Manhole Data for Transport Section of FWQA Model

SNAFOUVER USA GENERATE DATA FOR OPTIMAL CONTROL

EXTERNAL ELEMENT NUMBER	TYPE	DESCRIPTION	UPSTREAM ELEMENTS 1	2	3	INTERNAL ELEMENT NUMBER	ELEMENT COMPUTATION SEQUENCE: EXTERNAL NUMBER	INTERNAL NUMBER	INTERNAL ELEMENT	UPSTREAM NUMBERS	
10	22	BACKWATER UNIT	0	-0	-0	1	10	1	18	18	18
14	2	RECTANGULAR	10	-0	-0	2	14	2	1	18	18
11	19	STORAGE UNIT	14	10	-0	3	11	3	2	1	18
15	1	CIRCULAR SHAPED	36	-0	-0	4	13	10	18	18	18
12	22	BACKWATER UNIT	15	0	0	5	20	11	10	18	18
16	2	RECTANGULAR	12	-0	-0	6	33	12	10	11	18
17	19	STORAGE UNIT	16	12	-0	7	34	13	12	18	18
18	16	MANHOLE	17	0	0	8	35	14	12	18	18
19	16	MANHOLE	17	0	0	9	36	15	3	18	18
13	22	BACKWATER UNIT	0	0	0	10	15	4	15	18	18
20	2	RECTANGULAR	13	-0	-0	11	12	5	4	18	18
33	19	STORAGE UNIT	13	20	-0	12	16	6	5	18	18
34	16	MANHOLE	33	-0	-0	13	17	7	6	5	18
35	16	MANHOLE	33	-0	-0	14	18	8	7	18	18
36	16	MANHOLE	11	-0	-0	15	19	9	7	18	18
37	16	MANHOLE	11	-0	-0	16	37	16	3	18	18
38	16	MANHOLE	18	34	-0	17	38	17	8	13	18

Basic Conduit and Manhole Data for Transport Section of FWQA Model

SNAFOUVER USA GENERATE DATA FOR OPTIMAL CONTROL
NUMBER OF ELEMENTS = 17
NUMBER OF TIME INT = 55
TIME INTERVAL = 120.0 SECONDS.

ELEMENT PARAMETERS

EXT. ELE. NUM.	TYPE	DESCRIPTION	SLOPE (FT/FT)	DISTANCE (FT)	MANNING ROUGHNESS	GEOM1 (FT)	GEOM2 (FT)	GEOM3 (FT)	NUMBER OF BARRELS	AFULL (SQ.FT)	QFULL (CFS)	QMAX (CFS)	SUPER-CRITICAL FLOW WHEN LESS THAN 95(FULL
10	22	BACKWATER UNIT	-0.00000	-0.00	-0.0000	0.000	-0.000	11.000	1.0	0.000	0.000	0.000	
14	2	RECTANGULAR	.01500	1000.00	.0130	15.000	10.000	-0.000	1.0	150.000	4379.864	4904.662	YES
11	19	STORAGE UNIT	-0.00000	-0.00	-0.0000	0.000	-0.000	36.000	1.0	0.000	0.000	0.000	
15	1	CIRCULAR SHAPED	.00100	1500.00	.0130	16.000	-0.000	-0.000	1.0	201.062	1836.311	1983.215	NO
12	22	BACKWATER UNIT	-0.00000	-0.00	-0.0000	0.000	-0.000	17.000	1.0	0.000	0.000	0.000	
16	2	RECTANGULAR	.00500	2000.00	.0130	10.000	8.000	-0.000	2.0	80.000	1104.103	1258.917	NO
17	19	STORAGE UNIT	-0.00000	-0.00	-0.0000	0.000	-0.000	19.000	1.0	0.000	0.000	0.000	
18	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	
19	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	
13	22	BACKWATER UNIT	-0.00000	-0.00	-0.0000	0.000	-0.000	33.000	1.0	0.000	0.000	0.000	
20	2	RECTANGULAR	.00500	2000.00	.0130	10.000	5.000	-0.000	1.0	50.000	569.636	621.400	NO
33	19	STORAGE UNIT	-0.00000	-0.00	-0.0000	0.000	-0.000	34.000	1.0	0.000	0.000	0.000	
34	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	
35	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	
36	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	
37	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	
38	16	MANHOLE	-0.00000	-0.00	-0.0000	0.000	-0.000	-0.000	1.0	0.000	0.000	0.000	

The true runoff as calculated in 5.C.2(a) was always used as the input to the Transport section. As a basis of future comparison one run was made using the true runoff and setting all the orifices at their maximum opening and all the weirs at the maximum level used in the control program (13.5 ft., 8.75 ft. and 8.5 ft. for control points 1, 2 and 3 respectively.) The diverted overflows are shown in Table 5.4. Note that in this case there is overflow at all three control points.

Table 5.4

Physical System Overflows

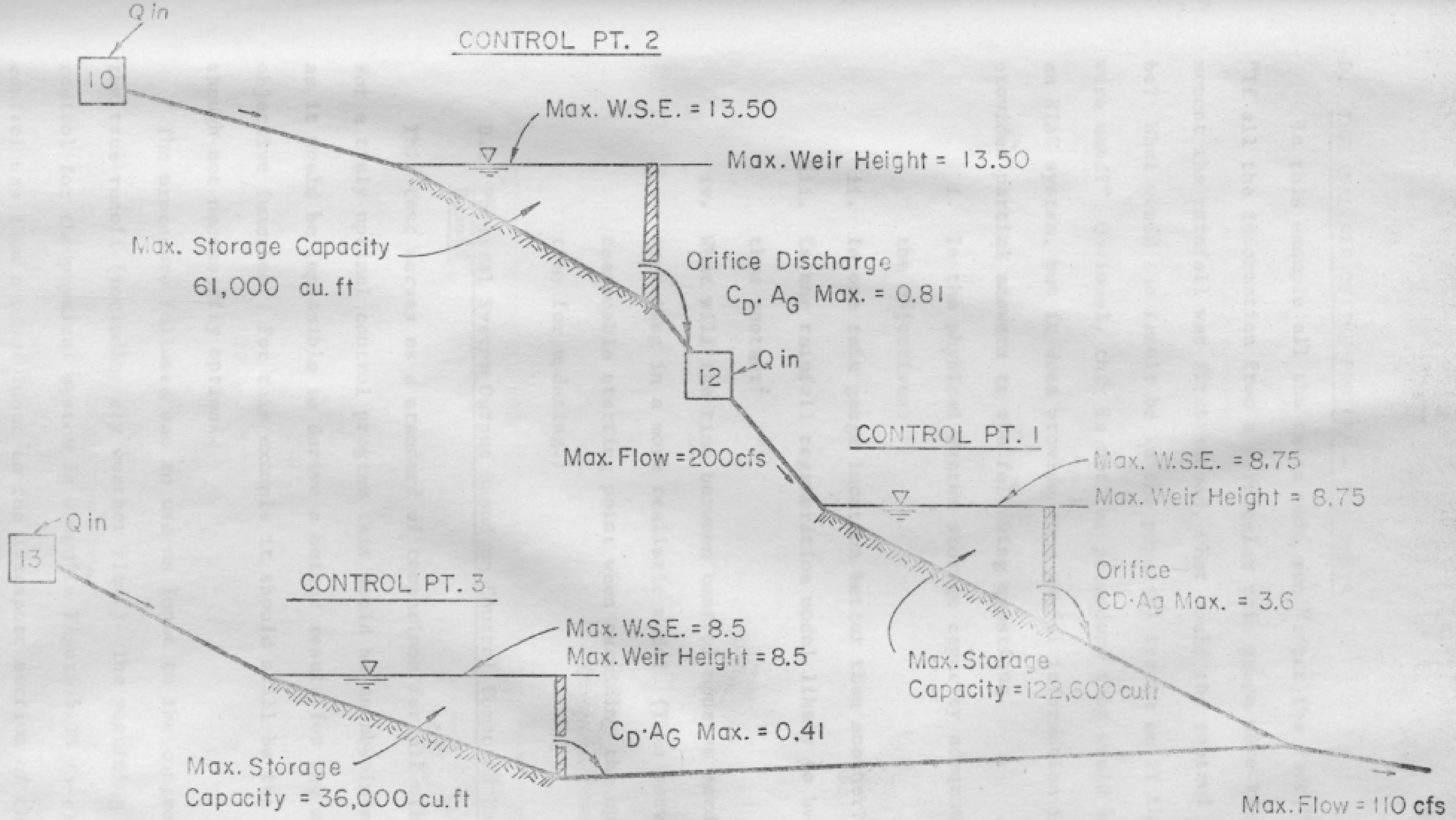
All Controls Set at Maximum Limits

Control Point	Overflow (cu. ft.)	Maximum Depth (ft.)
1	2510	8.94
2	2430	13.78
3	6100	8.78

C.4 <u>The Control Program</u>

The control program used in this example is designed to minimize the overflows from the three reservoir system shown in Figure 5.14. The capacity of the reservoirs is the same as that at each of the back-water storage location in the Transport section of the physical system model. The flow and depth constraints are shown in Figure 5.14. The maximum allowable depths were reduced from the maximum conduit height to allow a factor of safety on the control. The input data to this program consisted of initial depths, reservoir depth-area curves, constraint limits and the input hydrographs determined in 5.C.2(b).

As noted earlier the control program is still not developed to the stage where it converges properly to the optimal result for the three reservoir case, but it does provide a reasonable control.


Q in
10
CONTROL PT. 2
Max. W.S.E. = 13.50
Max. Weir Height = 13.50
Max. Storage Capacity
61,000 cu. ft
Orifice Discharge
$C_D \cdot A_G$ Max. = 0.81
12
Q in
Max. Flow = 200cfs
CONTROL PT. 1
Max. W.S.E. = 8.75
Max. Weir Height = 8.75
Orifice
CD·Ag Max. = 3.6
Max. Storage
Capacity = 122,600 cu.ft
Q in
13
CONTROL PT. 3
Max. W.S.E. = 8.5
Max. Weir Height = 8.5
$C_D \cdot A_G$ Max. = 0.41
Max. Storage
Capacity = 36,000 cu. ft
Max. Flow = 110 cfs

D. THE ANALYSIS OF THE PROBLEM - THE TESTS

In this example all the tests were run "after the fact," that is: "If all the information from a particular rain gauge were known at the moment the rainfall was first sensed, what would the control procedure be? What would the result be in the physical system be if this control were used?" Obviously this is not the procedure that would be used in an RTAC system, but it does provide some useful information in that it provides partial answers to the following questions.

i. Is the physical system storage capacity adequate to meet the objectives?

ii. Is one rain gauge location better than another?

iii. Is one rainfall regeneration model likely to be better than another?

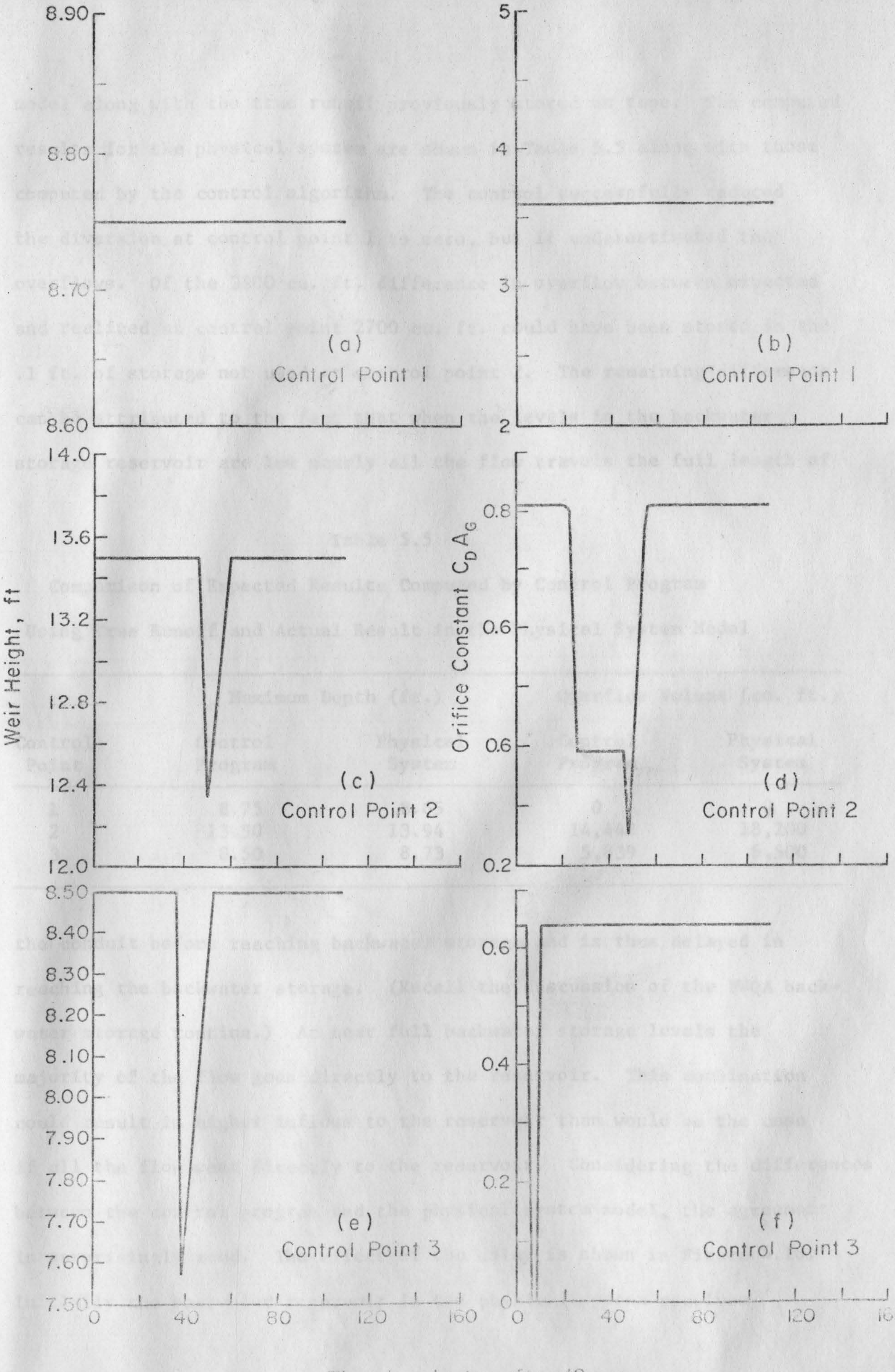
iv. What will the time between control updates need to be when operating in a more realistic mode? (This serves as a reasonable starting point when examining the optimal time for updating.)

D.1 Physical System Output Based on Control Computed Using the True Runoff

This test serves as a standard of comparison for all other tests. For a truly optimal control program this would be an absolute standard as it would be impossible to derive a better result for the specified objective function. For this example it should still be the best result though not necessarily optimal.

The procedure followed was to use as input to the control program the true runoff (including dry weather flow.) The resulting specified control for the physical system is shown in Figure 5.15 (a-c). This control was then used as input to the Transport section of the FWQA

THE SPECIFIED CONTROL BASED ON TRUE RUNOFF



model along with the true runoff previously stored on tape. The computed results for the physical system are shown in Table 5.5 along with those computed by the control algorithm. The control successfully reduced the diversion at control point 1 to zero, but it underestimated the overflows. Of the 3800 cu. ft. difference in overflow between expected and realized at control point 2700 cu. ft. could have been stored in the .1 ft. of storage not used at control point 2. The remaining difference can be attributed to the fact that when the levels in the backwater storage reservoir are low nearly all the flow travels the full length of

Table 5.5

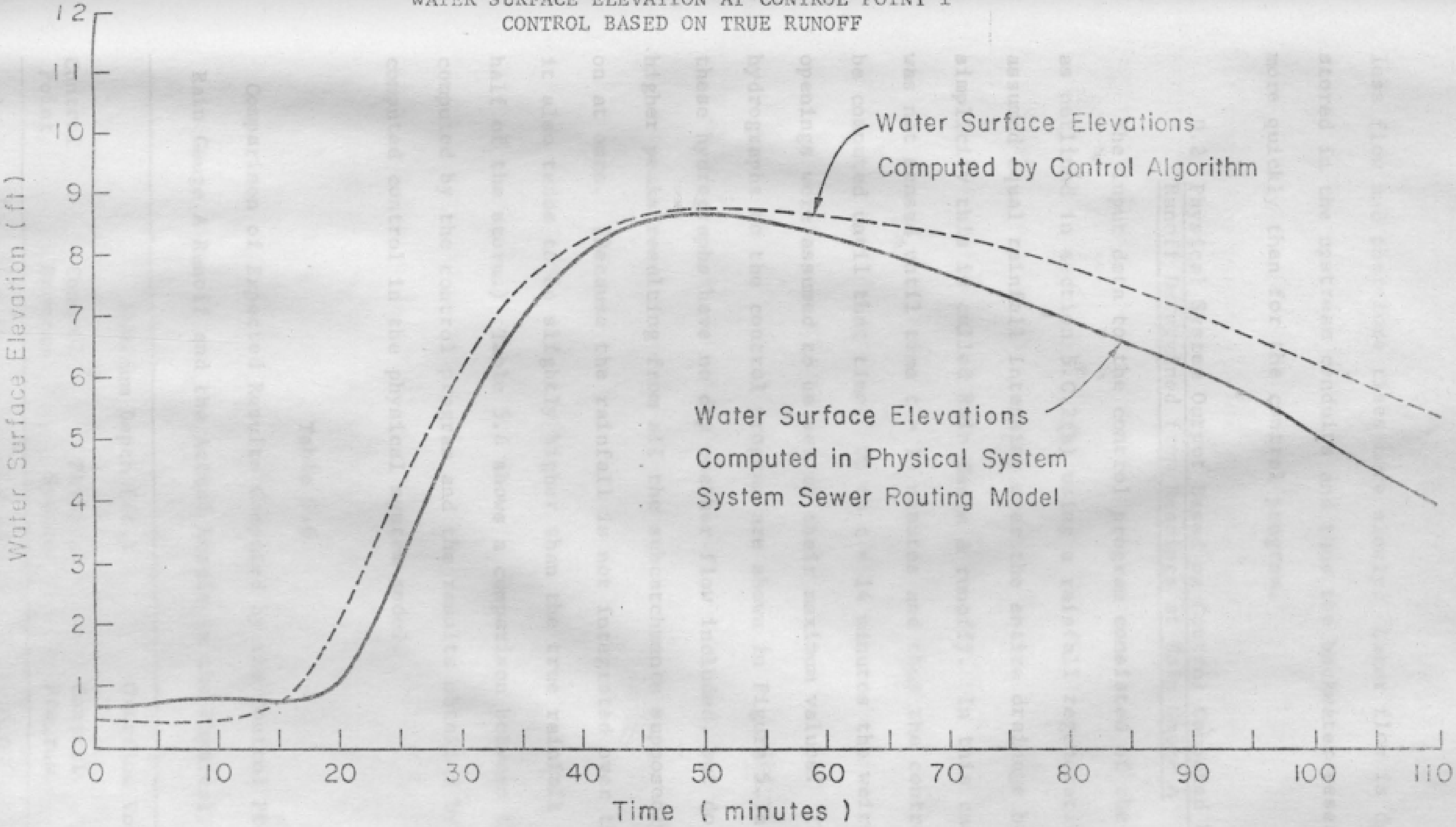
Comparison of Expected Results Computed by Control Program Using True Runoff and Actual Result in the Physical System Model

	Maximum Depth (ft.)		Overflow Volume (cu. ft.)	
Control Point	Control Program	Physical System	Control Program	Physical System
1	8.75	8.65	0	0
2	13.50	13.94	14,442	18,200
3	8.50	8.73	5,839	6,500

the conduit before reaching backwater storage and is thus delayed in reaching the backwater storage. (Recall the discussion of the FWQA backwater storage routine.) At near full backwater storage levels the majority of the flow goes directly to the reservoir. This combination could result in higher inflows to the reservoir than would be the case if all the flow went directly to the reservoir. Considering the differences between the control program and the physical system model, the agreement is surprisingly good. The effect of the delay is shown in Figure 5.16. Initially the backwater reservoir in the physical system receives

Figure 5.16

WATER SURFACE ELEVATION AT CONTROL POINT 1
CONTROL BASED ON TRUE RUNOFF



less flow and therefore rises more slowly. Later flow is delayed and stored in the upstream conduits and thus the backwater reservoir drops more quickly than for the control program.

D.2 Physical System Output Based on Control Computed Using the Runoff Determined from Readings at Rain Gauge A

The input data to the control program consisted of the runoff computed as outlined in section 5.C.2(b) using a rainfall regeneration model that assumed equal rainfall intensity over the entire drainage basin (for simplicity this is called Rain Gauge A runoff). In this case the rainfall was not sensed until time t = 14 minutes and thus the control could not be computed until that time. Up to t = 14 minutes the weirs and orifice openings were assumed to be set at their maximum values. The input hydrographs to the control program are shown in Figure 5.7a. Note that these hydrographs have no dry weather flow included, but do have slightly higher peaks resulting from all the subcatchments supposedly being rained on at once. (Because the rainfall is not integrated over the subcatchment it also tends to be slightly higher than the true rainfall for the first half of the storm.) Table 5.6 shows a comparison between the results computed by the control program and the results obtained by using the computed control in the physical system model.

Table 5.6

Comparison of Expected Results Computed by the Control Program Using Rain Gauge A Runoff and the Actual Result in the Physical System Model

	Maximum Depth (ft.)		Overflow Volume (cu. ft.)	
Control Point	Control Program	Physical System	Control Program	Physical System
1	8.68	8.74	0.0	0.0
2	13.50	14.07	933	9330
3	8.50	9.05	6334	9260

The results for control point 2 are interesting as in general they are better than the results from computed control based on true runoff. This brings home the problem of having a non-optimal control program for comparison purposes, i.e. bad data input to a non-optimal control program may lead to better results than "perfect data" input to a non-optimal control program. Conversely, in designing a real-life system, knowledge of the effects of information errors may allow simplifications to the overall control algorithm, i.e. considering this example (or the next one to be discussed) it would appear that no time delay would need to be included in the control program if the rainfall data and resulting computed runoff would compensate. In all of these examples there may be another problem evident; the FWQA model computes its outflows on the basis of interpolating between points on a curve. Changing the control would cause changes in backwater storage depth and thus vary the accuracy of interpolated results. In addition each time the weir control is changed there is a change in the depth-storage relationship and the routing equations since the FWQA model always has three points below the top of the weir and seven above it and one at the weir height. Earlier tests indicated that the effect would be small; however, these tests were done with much larger flows compared to the present example. This factor will need more investigation.

D.3 Physical System Output Based on Control Using the Runoff Determined from Readings at Rain Gauge B

This example used the same rainfall regeneration model used in D.2 above. In this case the rain gauge first senses the storm at t = 0. Because Rain Gauge A and Rain Gauge B both sense identical rainfall hyetographs, but at different times, the computed runoff input to the control program is identical to that for Rain Gauge A. Also because the backwater storage levels in the physical system are almost constant during the first twenty minutes (thus initial conditions are almost identical) the computed control is identical to that for the previous example; however, it begins at time t = 0. The computed control input to the physical system model is shown in Figure 5.17(a-c). The comparison between expected and realized results is shown in Table 5.7.

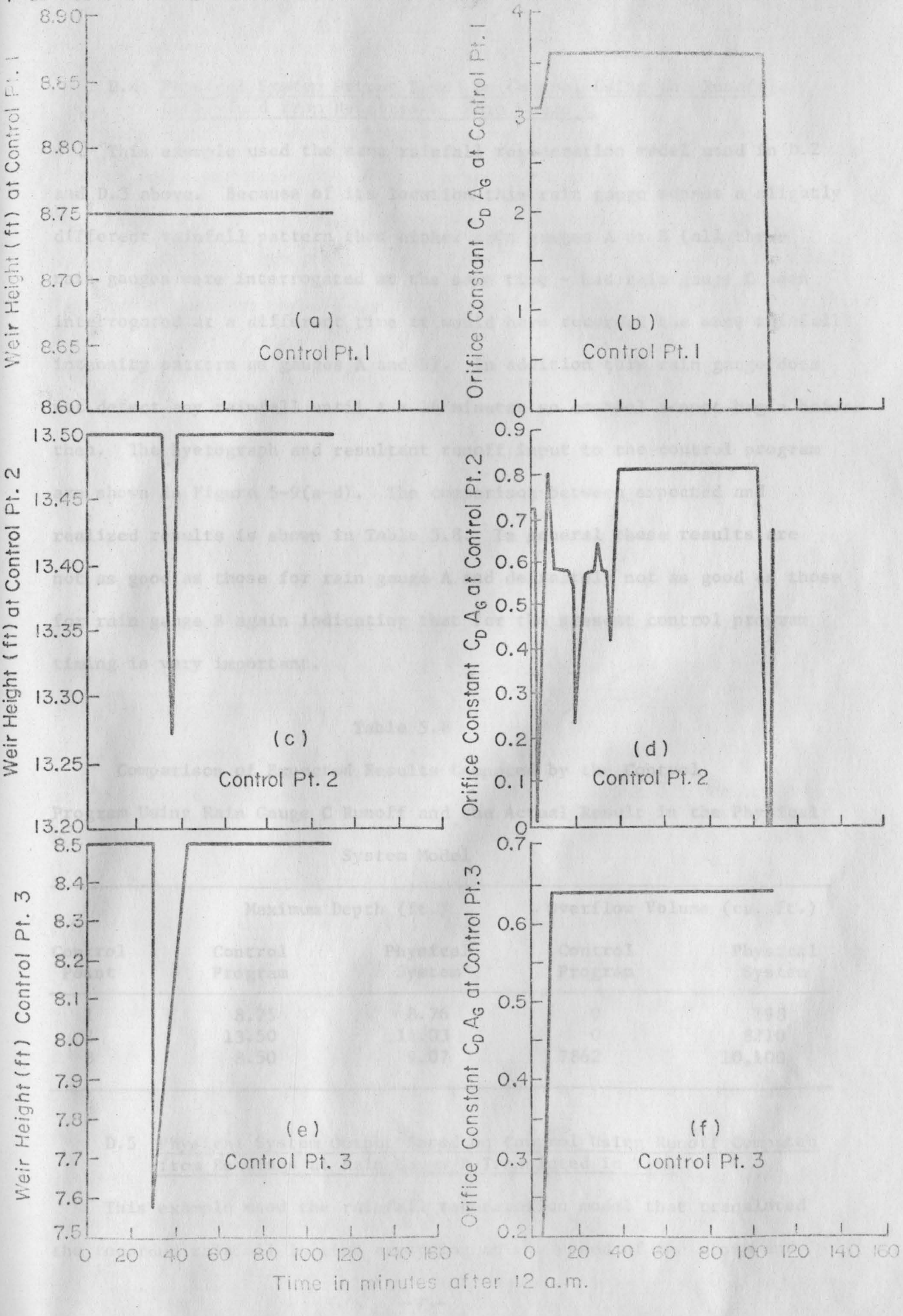
Table 5.7

Comparison of Expected Results Computed by the Control Program Using Rain Gauge B Runoff and the Actual Result in the Physical System Model

Control Point	Maximum Depth (ft.) Control Program	Maximum Depth (ft.) Physical System	Overflow Volumes (cu. ft.) Control Program	Overflow Volumes (cu. ft.) Physical System
1	8.68	8.81	0	527
2	13.50	13.85	933	3670
3	8.50	8.87	6334	5830

Although there is overflow at control point 1, this is still the best overall result and points out futther the need for an optimal control algorithm. Even for this simple system a designer would have a very difficult time determining the salient points for a control program. Both this example and the previous one had higher maximum flows than the true runoff as input to the control program. In the previous case the control was not initiated until 14 minutes after the storm had been in the drainage basin indicating that timing is very important. Would the same results apply for another storm?

COMPUTED CONTROL - BASED ON RUNOFF COMPUTED FROM RAIN GAUGE B HYETOGRAPH



D.4 Physical System Output Based on Control Using the Runoff Determined from Readings at Rain Gauge C

This example used the same rainfall regeneration model used in D.2 and D.3 above. Because of its location this rain gauge senses a slightly different rainfall pattern than either rain gauges A or B (all three rain gauges were interrogated at the same time - had rain gauge C been interrogated at a different time it would have recorded the same rainfall intensity pattern as gauges A and B). In addition this rain gauge does not detect any rainfall until t = 16 minutes so control cannot begin before then. The hyetograph and resultant runoff input to the control program are shown in Figure 5-9(a-d). The comparison between expected and realized results is shown in Table 5.8. In general these results are not as good as those for rain gauge A and definitely not as good as those for rain gauge B again indicating that for the present control program timing is very important.

Table 5.8

Comparison of Expected Results Computed by the Control Program Using Rain Gauge C Runoff and the Actual Result in the Physical System Model

	Maximum Depth (ft.)		Overflow Volume (cu. ft.)	
Control Point	Control Program	Physical System	Control Program	Physical System
1	8.75	8.76	0	198
2	13.50	14.03	0	8210
3	8.50	9.07	7862	10,100

D.5 Physical System Output Based on Control Using Runoff Computed from Readings at Rain Gauge B Translated in Time

This example used the rainfall regeneration model that translated the recorded rainfall in time according to the speed of the storm and the distance of the subcatchment. This should approximate more closely the "true" runoff; however, because it does not include the dry weather flow it under-estimates the runoff (compare Figure 5.6 and 5.11). The net result is that the control program expects no overflow at control points 1 and 2 and therefore leaves all controls at their maximum limits. The physical system results for this control are noted earlier in Table 5.4.

E. WHAT CONCLUSIONS COULD A DESIGNER DRAW FROM THIS EXAMPLE

Assuming that the designer were to use the same control program then it would appear that he would get reasonable results using data from rain gauge B and the simplest rainfall regeneration model for storms similar to the one tested. However, in an RTAC system he is not operating after the fact and therefore must consider the effect of the delay in the storm moving over the rain gauge before he can determine his control strategy. For the storm used in this example there would have been a delay of nearly 20 minutes before enough information would be available. Thus, if he were taking data at rain gauge B his resultant control computed at time t = 20 would most likely be worse than examples D.2 and D.3 above which essentially said they had all the rainfall information at time t = 14 and t = 16 respectively. Computing new control strategies at ten minute intervals after t = 0 would probably not improve the result as at the first time interval not enough rainfall information would be available to indicate that overflow is going to take place. He has two alternatives available to improve his final result: 1) keep rain gauge B in its present location and improve his rainfall regeneration model to estimate future rainfall, i.e. based on information up to t = T what will be the future rainfall intensity at t > T; 2) rain gauge B can be relocated eastward so that rainfall intensity information is available before the storm reaches the drainage basin.

Whichever alternative were selected it would appear that an interval of 15-20 minutes would be reasonable between control updates. Therefore, in the second stage of development, which would be operation of the RTACS model in its feed-back, feed-forward mode the designer might relocate rain gauge B eastward, use a rainfall regeneration model that says that rainfall intensity recorded at the rain gauge is felt over the entire drainage basin T minutes later, and update his control every 15 minutes.

VI. CONCLUSIONS AND RECOMMENDATIONS

From the foregoing it would appear that there is good reason to continue development of optimal control strategy for an RTACS model to serve as a basis of comparison for the effects of non optimality and to serve as an aid in developing simplified control strategies for more complex systems.

Even without optimal control it appears that useful information can be derived from an RTACS model for use in optimizing an RTAC system. Considering the potential cost of such a system and the cost of computer-time in simulation, it would appear that simulation could be very easily justified.

Future work should concentrate on the development of optimal control strategy, particularly the development of the theory necessary to include time delay in the flow routing and in the development of numerical techniques to solve the resulting equations.

In addition, the effect of the possible errors introduced by the changing depth storage relations and the linear interpolations in the FWQA Transport Model should be investigated.

Although considerable work has been done on rainstorm regeneration some research should be carried out to determine those methods most feasible for use in an RTAC system.

REFERENCES

1. Citron, S. J., Elements of Optimal Control, Holt Rinehart and Winston Inc., New York, 1969.

2. Dooge, J. C. and Harley, B. M., "Linear Routing in Uniform Open Channels," Proc. International Hydrology Symposium Sept. 6-8, 1967, Fort Collins, Colorado, pp. 8.1-8.7.

3. Harris, Garth S., Status Report on Development of a Computer Program to Route Runoff in the Minneapolis - St. Paul Interceptor Sewers, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, December 1968.

4. Storm Water Management Model, Water Pollution Control Research Series, Environmental Protection Agency, Water Quality Office, Washington, D. C. July 1971, Three Volumes.

APPENDIX A.1

Changes required to introduce variable control into the FWQA Storm Water Management Program.

1. These changes consist of three basic parts.

   a. those changes necessary to introduce variable control for the two backwater storage locations originally existing with the FWQA storm water management program.

   b. those changes necessary to reduce the computer storage requirements for the backwater storage routines of the FWQA Storm Water Management Program.

   c. those changes that would reduce computer storage requirements that were not related to the changes backwater storage routines.

The actual changes made are listed in Figure A1.1. All the changes of parts a and b above were made to the Transport group of subroutines. Those of part c consisted of: a. reduction in the length of the input-output buffers of the Executive program (this reduction is a function of the individual computer system); b. elimination of the curve plotting subroutines of the executive program; (These two changes although minor result in a reduction in computer storage of about 8000_8 words); c. substitution of an unlabled common block for a labled common block in subroutine PRINT of the Transport section of the program. (This resulted in a computer storage reduction of $20{,}000_8$ words.

Table A.1.1 outlines the changes made to subroutines TRANS and subroutine TSTRDT to introduce variable control for two or more backwater storage locations.

In order to reduce the required storage for the FWQA model backwater routines the following alteration was made to the variables BODIN, SSIN, COLIN, VOLIN, and VOLOUT.

Consider the variable BODIN. In the unaltered program the dimensions were BODIN (2,150). This variable is part of the routine for plug flow through backwater storage and as such is used as shown in Figure A.1.2(a). At each time step (KTSTEP) a new variable is entered into the array. Once a plug has passed completely through backwater storage the values stored in locations "i" less than LPREV(KSTOR) are no longer required. Thus the only values that need be stored in the computer memory are those between LPREV(KSTOR) and KTSTEP. If it is assumed that the number of plugs at any one backwater storage location does not exceed 50 the required values can be stored in the computer in the following manner.

At each KTSTEP time step the required new value is stored at a location given by

$$\text{INC} = \text{KTSTEP} - \left(\frac{\text{KTSTEP} - 1}{50}\right) \times 50$$

For integer numbers the computer truncates the term in brackets so that if

	$\text{KTSTEP} \leq 50$	$(\ ) = 0$
	$51 \leq \text{KTSTEP} \leq 100$	$(\ ) = 1$
	$101 \leq \text{KTSTEP} \leq 150$	$(\ ) = 2$

Thus the computer storage locations are made to form a continuous loop see Fig. A.1.2(b) and for a given backwater storage location the computer storage requirements necessary to simulate plug-flow of pollutants is reduced by nearly two-thirds.

Figure A.1.1 lists all the changes of this form that were made to the Transport section of the program.

Table A1.1

Changes to Subroutines TRANS and TSTRDT to Allow Variable Control at Two Backwater Storage Locations

a. <u>In subroutine TRANS</u>

Modification or Addition	Statement No.	Comment
i. READ IN EXECUTION DATA	Statement 106	- unaltered
READ (5,900) DT, EPSIL, DWDAYS, DTCHAN	Statement 127	- altered
C DTCHAN = TIME BETWEEN CHANGES IN CØNTRØL	Statement 127+1	added
ii. READ DATA FØR STØRAGE ELEMENTS	Statement 182	- unaltered
DØ 700 I = 1, NSTØR	Statement 183+1	added
700 KCHAN(I) = 0	" 183+2	added } see definition Appendix 2
KSTAR = 0	" 183+3	added
TCHANGE = DTCHAN	" 183+4	added
IF(NSTØR.GT.20) GØ TØ 9000	184	- altered
IF(NSTØR.GT.0) CALL TSTRDT	185	- unaltered
iii. BEGIN MAIN LOOPS OF PROGRAM	" 320	- unaltered
OUTER LOOP ON TIME, INNER LOOP ON ELEMENTNO)	" 325	- unaltered
IF(TIME - .01.LT.TCHANGE) GØ TØ 3000	329+1	added
TCHANGE = TCHANGE + DTCHAN	329+2	added
CALL TSTRDT	329+3	added
3000 KMINS = KMINS + INT(DT)/60	330	altered

b. <u>In subroutine TSTRDT</u>

Modification or Addition	Statement No.	Comment
DIMENSIØN ADEPTH(20,11) AASURF(20,11) CDEPTH (11), CASURF (11)	30	altered - area depth relationshi stored for all reservoirs
DØ 8888 KSTØR = 1, NSTØR	35	unaltered 8888 = end of loo on storage elemen
IF(KSTAR.EQ.0) GØ TØ 7001	35+1	added-KSTAR ≠ 0 implies time greater than zero
IF(KCHAN(KSTØR).EQ.1) GØ TØ 2901	35+2	added-KCHAN=1 implies reservoir with variable control. Time >0
GØ TØ 8888	35+3	added-if T>0 and reservoir does not have weir and orifice contro no changes are to be mad
7001 CØNTINUE	35+4	added

Statements 35+1 and 35+2 lead to two different procedures. The first refers to the initial set up of data for all reservoirs of any type. The second refers to reservoirs having variable control and occurs at the time control is changed: for the most part this procedure is a subset of the first procedure.

Consider the first procedure, i.e. jump to statement 7001.

	Modification or Addition	Statement No.	Comment
7001	CØNTINUE . . .	35+4	added
	READ RESERVOIR PARAMETERS	104	unaltered
	a. FOR IRREGULAR (NATURAL) RESERVOIR	114	unaltered
	READ(5,12) (ADEPTH(KSTOR,II),AASURF(KSTOR,II) ,II=1,11)	123	area depth relations now permanently stored for all irregular reservoirs
	WRITE(5,12) (ADEPTH(KSTOR,II),AASURF(KSTOR,II) ,II=1,11)	127	write statement changed to agree with statement 123
	APLAN(KSTØR) = AASURF(KSTØR,11)	130	dimensions of AASURF changed
	IF(ADEPTH(KSTØR,11).LT.DEPMAX(KSTØR) GØ TØ 903	131	dimension of ADEPTH changed
	DO 7002 I = 1,11	239+1	This addition is required for call to subroutine TINTRP which requires singly dimensioned arrays.
	CDEPTH(I) = ADEPTH(KSTOR,I)	239+2	
7002	CASURF(I) = AASURF(KSTOR,I)	239+3	
	CALL TINTRP (CDEPTH,CASURF,11,DEPTH,AREA, KFLAG)	245	CDEPTH and CASURF replace ADEPTH and AASURF which are multiply dimensioned.
	CALL TINTRP(CDEPTH,CASURF,11,DEPTH,AREA, KFLAG)	257	same as statement 245
	KCHAN(KSTØR) = 1	390+1	KCHAN is used to prevent re-reading data at future time steps.

Modification or Addition	Statement No.	Comment
IF(KSTAR.EQ.1) GO TO 8888	390+2	KSTAR is used to prevent re-reading data at future time steps
KSTAR = 1	408+1	added prevents re-reading of initia data

Consider the second procedure, i.e. jump to 2901 note that if KCHAN ≠ 1 (implying reservoir does not have weir and orifice control) then control transfers to the end of the do-loop. The jump to statement 2901 skips the reading and writing of reservoir parameters.

C OUTLET BY GRAVITY WITH VARIABLE WEIR AND ORIFICE	204	added comment statement
2901 READ(5,529) WEIRHT, WEIRL, CDAOUT, ORIFHT	204+1	This statement reads in changes in control at time N x DTCHAN + dt
GØ TØ 3000	204+2	Shifts control around read and write statements

Statement 3000 is the beginning of computations for flood routing parameters. For the remainder of the subroutine the changes noted above apply. (i.e., statement numbers greater than 234.