Dynamic Hedging using the Realized Minimum-Variance Hedge Ratio Approach – Examination of the CSI 300 Index Futures

Abstract: This paper investigates the dynamic hedging performance of the high frequency data based realized minimum-variance hedge ratio (RMVHR) approach. We comprehensively examine a number of popular time-series models to forecast the RMVHR for the CSI 300 index futures, and evaluate the out-of-sample dynamic hedging performance in comparison to the conventional hedging models using daily prices, as well as the vector heterogeneous autoregressive model using intraday prices. Our results show that the dynamic hedging performance of the RMVHR-based methods significantly dominates that of the conventional methods in terms of both hedging effectiveness and tracking error volatility in the out-of-sample forecast period. Furthermore, the superiority of the RMVHR-based methods is robust in different market structures and different volatility regimes, including China's abnormal market fluctuations in 2015 and the US financial crisis in 2008.

Keywords: Realized Minimum-Variance Hedge Ratio; High-Frequency Data; Out-of-Sample Forecasting; Hedging Effectiveness; Tracking Error; Volatility Regime

1 Introduction

Futures contracts are one of the most popular instruments for hedging risk exposures. Naturally, the optimal hedging strategy is principally of interest for both investors and researchers, and the core issue in improving the effectiveness of a hedging strategy is to accurately estimating the optimal hedge ratio – the optimal proportion of the futures contract held to offset the risks from spot position.

Ideally, when the spot and future prices are perfectly correlated, investors can take a naïve one-to-one hedging strategy (hedge ratio = 1) that holds the opposite positions with equal magnitude in spot and futures and eliminate all price risks as a perfect hedge. In reality, however, perfect hedge may not exist due to basis risks and cross hedging. Therefore, many optimal hedging strategies have been proposed in the literature. The conventional strategies of constructing a constant minimum variance hedge ratio originates from Johnson (1960) and Stein (1961), who choose an optimal futures position to minimize the variance of the spot-futures portfolio. Following them, Ederington (1979) proposes to estimate the constant hedge ratio using an ordinary least squares (OLS) regression of spot returns on futures returns. However, the OLS procedure has been criticized for not taking into account of cointegration and therefore resulting in downward bias in hedge ratios, i.e., under-hedging (c.f. Hill and Schneeweis 1981; Cecchetti et al. 1988; Lien 1996). Later, Ghosh (1993) proposes the error correction model (hereafter, ECM) to estimate the constant hedge ratio based on the cointegration theory. The ECM procedure considers both the long-term equilibrium and the short-term dynamics between spot and futures, and yields better performance over those derived from the OLS procedure (Ghosh, 1995; Ghosh and Clayton 1996). Although still used in some practice for simplicity, an obvious disadvantage of these static hedging models is that they assume the relationship between spot and futures are timeless and therefore ignore the time-varying characteristic of the (co)variance between the spot and futures returns, contradicting the well-known dynamic nature of asset returns.

As evident from many empirical studies (c.f. Koutmos and Tucker 1996; Meneu and Torro 2003), the distribution of spot and futures returns is time-varying, therefore dynamic hedge ratios may be more appropriate for greater risk reduction than the traditional constant hedge ratios (Baillie and Myers 1991; Park and Switzer 1995).

With the development of the generalized autoregressive conditional heteroscedasticity (GARCH) models and its various extensions (Engle 1982; Bollerslev 1986), an extensive framework of bivariate GARCH-type dynamic hedging models have been designed to capture the time-varying (co)variance structure. For instance, the ECM-GARCH model (Kroner and Sultan 1993; Yang and Awokuse 2003) considers the cointegration relationship and characterizes the time-varying covariance of spot and futures; the BEKK-GARCH model (Engle and Kroner 1995) provides a simple extension of the popular univariate GARCH model in Bollerslev (1987); the constant conditional correlation (CCC)-GARCH model (Bollerslev 1990) restricts the correlation structure between spot and futures for computational advantages; the dynamic conditional correlation (DCC)-GARCH model (Engle 2002) provides more flexible correlation structure and simplifies the estimation procedure; and the copula-GARCH model (Hsu et al. 2008; Lai et al. 2009) captures the asymmetric dependency between spot and futures. Overall, the general consensus is that these GARCH-type dynamic hedge ratios outperform the constant hedge ratios both in-sample and out-of-sample, and thus has gained wide applications in practice and rising attention in the literature. However, these GARCH-type models are likely to overestimate the persistence in volatility since relevant sudden changes and regime switches in variance are often ignored (Wei et al. 2011). In addition, the early studies mainly use relatively low frequency data (daily in most cases) to latently characterize the time-varying covariance of spot and futures. Therefore, they cannot capture the intraday variation of prices and are relatively slow in catching up the covariance changes.

The harnessing of high-frequency information and the new development in financial econometrics have enabled significant progress in direct measuring and modeling of covariance, which can be applied to further benefit the dynamic hedge ratios estimations. Koopman et al. (2005) provided evidence of superior informational content of the realized measures using intraday high-frequency data when compared to estimators derived from daily returns. For instance, the realized volatility (RV) calculated as the sum of squared intraday returns provides an unbiased estimator of the quadratic variation (Andersen and Bollerslev, 1998). As a natural extension of the RV into the multivariate case, the realized covariance (RCov) matrix calculated as the sum of the cross products of high-frequency intraday return vectors provides an unbiased estimator of the quadratic covariation (Barndorff-Nielsen and Shephard, 2004). Because the RCov matrix calculation may suffer from market microstructure noise and nonsynchronous trading, some more complicated estimators have been proposed, such as the multivariate realized kernel (Barndorff-Nielsen et al., 2011) and the two-time scale covariance (Zhang, 2011). Unfortunately, the computational complexities of these models impede wide applications in practice. As a more practical alternative, the easily implementable sparse sampling method using high frequencies of data has been employed in empirical applications. Lai and Sheu (2010) proposed the DCC-GARCH-RV model using 15-minutes frequency of data, which encompasses the realized volatility (covariance) in the conditional variance (covariance) functions for spot and futures and shows substantial improvement in hedging performance for the S&P 500 index futures.

Most recently, Markopoulou et al. (2016) proposed the realized minimum-variance hedge ratio (RMVHR) as the ratio of the realized covariance between spot and futures returns divided by the realized variance of futures. Although Markopoulou et al. (2016) show some promising results that RMVHR could improve hedging performance by using high frequency data and finer volatility (covariance) proxies when compared with the conventional low-frequency models, the strength of its potential implications is significantly mitigated, however, by at least three factors. First, they mainly examine the developed market structures such as the United States and the United Kingdom. Given the obvious difference in market structures between the developed and developing markets (c.f. Miao et al. 2017), it is unclear whether this type of approach can also provide improved hedging performance in developing market structure such as China. Second, they only examine a relatively short sample period from 2009 to 2012 without major market crashes or regime switches. Since hedging strategies would be the most important to weather market turbulence, a more thorough examination of the RMVHR-based models under different market conditions is warranted. Third, it would be interesting to comprehensively explore if a combined extension of the GARCH-type models and the RMVHR-based models can provide superior performance than each type of models alone.

In this research, we believe the special characteristics of market structure in China, combined with the market crash and turbulent nature of the Chinese index futures in 2015, provide a unique test bed for investigating the dynamic hedging performance of the RMVHR-based models. In contrast to the dominance of institutional investors in most developed markets such as the United States, retail investors represent a large portion of the investment holdings in China's markets (c.f. Ng and Wu, 2007; Miao et al. 2017). Moreover, China’s market is tightly controlled with numerous trading restrictions such as price-limit rules, margin trading, short selling restrictions and T+1 trading constraints. Growing very rapidly, the Shanghai and Shenzhen Stock exchanges combined has become the second largest stock market in the world by early 2015. Right after China’s market claimed its second place in the world, during a dramatic market crash from June 2015 into early 2016, around $2 trillion of market capitalization was erased, nearly one-third of its value. Following then, intense scrutiny from government and regulators has fiercely questioned the hedging roles of the equity index futures in China's financial market.

The launch of the CSI 300 equity index futures on April 16, 2010 marked a milestone development in the evolution of China's financial market. For the first time, China's financial market provides investors with an essential tool to hedge the systemic risk of holding the market, proxied by the underlying CSI 300 equity index, a free-float weighted index comprises 300 of the largest and most actively traded A-share stocks on the Shanghai and Shenzhen Stock exchanges. While its inception was widely hailed as an effective hedging tool and even a stabilizing force in China's financial markets among both investors and regulators[1], the China Securities Regulatory Commission (CSRC) openly blamed the 2015 stock market collapse on "malicious short-selling" of index futures as "weapons of mass destruction" by speculators and questioning its conventional role as a hedging instrument.

Despite its obvious importance and the rising importance of China's market, the effectiveness of hedging strategy using the CSI 300 index futures contracts has been a subject of very limited research. Yang et al. (2012) pioneered in a closely related research field by examining the then newly established CSI 300 index futures surrounding its inception period in 2010. They use a bivariate ECM-GARCH model to study the intraday volatility transmission between the spot and futures markets and show the existence of cointegration, which carries important implications for hedging strategies. Only a limited number of studies have examined the hedging performance of CSI 300 index futures. For instance, Hou and Li (2013) suggest the GARCH-type

[1] On December 5, 2014, Xiao Gang, chairman of the China Securities Regulatory Commission (CSRC) remarked, stock index futures are "sophisticated risk management tools for improving the stock market operation mechanism, providing hedging instruments, improving the investment product market system and promoting stable development of great significance."

models and constant hedge ratio outperform each other in short and long horizons, respectively. More recently, Yan and Li (2018) use the BEKK-GARCH model and show regime switching exists in China's market. Unfortunately, these researches mainly focus on daily data, short examination windows, and provide limited discussion on CSI 300 index futures' hedging performance under different market conditions.[2]

The unique market structure in China and the information-rich environment in 2015 motivate us to examine the information content of intraday data in a dynamic hedging context. Our results show that the RMVHR-based methods significantly dominate that of the conventional methods in terms of hedging effectiveness and the tracking error volatility both in and out-of-sample. The superiority of the RMVHR-based methods is robust during different volatility regimes of China's financial markets, including China's abnormal market fluctuations in 2015. Furthermore, our robustness tests with the S&P 500 index futures confirm that these findings are consistent across different market structures.

This research contributes to the existing literature in at least three important ways.

First, China represents a very unique market structure for testing dynamic hedging performance. In addition to hedging tools, investors in China often view index futures as a vehicle to circumvent onerous trading restrictions in China's stock market such as same-day trading and short-sale ban. To our best knowledge, this is the first study to examine the dynamic hedging performance of CSI 300 index futures by applying intraday high frequency data and the newly proposed realized minimum-variance hedge ratio. We use the intraday five-minute data of CSI 300

[2] Yan and Li (2018) cover a sample period up to June 30, 2015 and only the very beginning of the 2015 futures market turbulence in China.

index and index futures to construct the RMVHR, and employ a variety of time-series models to directly forecast the ratio. The model confidence set test (hereafter MCS test, Hansen et al., 2011) shows that hedging with the directly forecasted hedge ratios is significantly more efficient than with hedge ratios calculated from forecasts of conventional low-frequency models in terms of both the hedging effectiveness and the volatility of tracking errors criteria.

Second, our research provides new insights on the marginal benefits of dynamic hedging performance by incorporating high-frequency information in the realized measures. More specifically, we propose a new method to directly measure the marginal benefits of using the RMVHR and show that directly forecasting it is a more efficient way to utilize the high-frequency intraday information content. In addition to the conventional low-frequency models in the comparison group, we also assess the hedging performance of the DCC-RV-ECM model (Lai and Sheu 2010) and the vector heterogeneous autoregressive (VHAR) model (Busch et al., 2011) that utilize high-frequency data. Because the VHAR model of the realized covariance (RCov) matrix and the heterogeneous autoregressive (HAR) model (Corsi, 2009) of RMVHR utilize exactly the same information set (intraday five-minute returns of spot and futures) and have similar structures, the comparison provides direct measure of the marginal benefit of the RMVHR and illustrates its superiority in utilizing the high-frequency intraday information.

Third, we examine the robustness of our results to different market conditions in the out-of-sample forecast period. Using the nonparametric change point model (Ross et al. 2011), we detect different volatility regimes of the underlying index and show that the superiority of the RMVHR-based methods is robust across different volatility regimes. In addition, we also perform the hedging performance comparisons using the

S&P 500 index futures for robustness tests. Our results confirm that the superiority of the RMVHR-based methods is not restricted to specific market structures.

The remainder of the paper is organized as follows. Section 2 presents the methodology of the RMVHR-based models and its comparison models. Section 3 explains the data and the results are discussed in Section 4. This is followed by a discussion of robustness tests in Section 5. Section 6 concludes the paper.

2. Methodology

2.1 Realized Measures

Let the discretely sampled Δ-period log return be denoted by $r_{t+j\cdot\Delta,\Delta} = \ln p_{t+j\cdot\Delta} - \ln p_{t+(j-1)\cdot\Delta}$, $j = 1, 2, \ldots, M$, $t = 0, 1, 2, \ldots$, where $p_{t+j\cdot\Delta}$ is the high-frequency price observed at time $j\cdot\Delta$ within day t+1 and $M = 1/\Delta$ is the number of sampling intervals per day. The daily realized volatility is defined by the summation of the squared intraday returns as $RV_t(\Delta) \equiv \sum_{j=1}^{1/\Delta} r^2_{t-1+j\cdot\Delta,\Delta}$ (Andersen and Bollerslev, 1998), which converges uniformly in probability to the quadratic variation as $\Delta \rightarrow 0$.

Let $\mathrm{r}_{t+j\cdot\Delta,\Delta} = [r^S_{t+j\cdot\Delta,\Delta}, r^F_{t+j\cdot\Delta,\Delta}]'$ be the column vector of returns, where $r^S_{t+j\cdot\Delta,\Delta}$ is the day (t+1) Δ-period log return of the CSI 300 index and $r^F_{t+j\cdot\Delta,\Delta}$ is the day (t+1) Δ-period log return of the CSI 300 index futures. The daily realized covariance matrix is defined by the summation of the cross products of intraday return vectors as $\mathbf{RCov}^{S,F}_t(\Delta) \equiv \sum_{j=1}^{1/\Delta} \mathbf{r}_{t-1+j\cdot\Delta,\Delta} \cdot \mathbf{r}'_{t-1+j\cdot\Delta,\Delta}$ (Barndorff-Nielsen and Shephard, 2004), which converges uniformly in probability to the quadratic covariation as $\Delta \rightarrow 0$.

The minimum-variance hedge ratio of day t can be calculated as

$HR_t = \frac{Cov(R_t^S, R_t^F)}{Var(R_t^F)} = \rho_t^{S,F} \times \frac{\sqrt{H_t^S}}{\sqrt{H_t^F}}$, where R_t^S and R_t^F are the day t log returns of the spot and the futures, respectively. $\rho_t^{S,F}$ is the day t correlation between the spot and the futures returns, and H^S_t and H^F_t are the day t variances of the spot and the futures, respectively. According to this, the day t realized minimum-variance hedge ratio (RMVHR) is defined as $RMVHR_t(\Delta) \equiv \frac{RCov_t^{S,F}(\Delta)}{RV_t^F(\Delta)}$ (Markopoulou et al., 2016), where $RCov_t^{S,F}(\Delta)$ is the sub-diagonal element of $\mathbf{RCov}_t^{S,F}(\Delta)$, and $RV_t^F(\Delta)$ is the day t realized variance of the futures. For notational simplicity, we omit the notation (Δ) in the realized measures when presenting the forecasting models.

2.2 Forecasting Models

We consider the following time-series models for RMVHR forecasting:

1) The ARMA model: $RMVHR_t = c + \sum_{i=1}^{p} \varphi_i RMVHR_{t-i} + \sum_{j=1}^{q} \vartheta_j \varepsilon_{t-j} + \varepsilon_t$.

2) The ARMA-GARCH model:

$$RMVHR_t = c + \sum_{i=1}^{p} \varphi_i RMVHR_{t-i} + \sum_{j=1}^{q} \vartheta_j \varepsilon_{t-j} + \varepsilon_t,$$
$$\varepsilon_t = \sigma_t e_t,$$
$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \alpha_k \varepsilon_{t-k}^2 + \sum_{l=1}^{n} \beta_l \sigma_{t-l}^2.$$

3) The Regime-switching (RS) model: $RMVHR_t = c_{s_t} + \varphi_{s_t} RMVHR_{t-1} + \varepsilon_t$,

where s_t is the state variable that takes the values 1 and 2. The state transitions are given by a Markov chain with transition probabilities $p_{i,j} = P(s_t = j \mid s_{t-1} = i)$, i, j = 1,2.

4) The ARFIMA model: $\left(1-\sum_{i=1}^{p}\varphi_i L^i\right)(1-L)^d\left(RMVHR_t-\mu\right)=\left(1+\sum_{j=1}^{q}\vartheta_j L^j\right)\varepsilon_t$,

where d is the differencing order and L is the lag operator.

5) The HAR model: $RMVHR_t=\alpha_0+\alpha_d RMVHR_{t-1}+\alpha_w RMVHR_{t-1}^{(w)}+\alpha_m RMVHR_{t-1}^{(m)}+\varepsilon_t$,

where $RMVHR_{t-1}^{(w)}=\frac{1}{5}\sum_{i=1}^{5}RMVHR_{t-i}$, $RMVHR_{t-1}^{(m)}=\frac{1}{22}\sum_{i=1}^{22}RMVHR_{t-i}$ are the past weekly and monthly RMVHRs.

6) The HAR-GARCH model:

$$RMVHR_t=\alpha_0+\alpha_d RMVHR_{t-1}+\alpha_w RMVHR_{t-1}^{(w)}+\alpha_m RMVHR_{t-1}^{(m)}+\varepsilon_t,$$

$$\varepsilon_t=\sigma_t e_t,$$
$$\sigma_t^{\,2}=\omega+\sum_{k=1}^{m}\alpha_k\varepsilon_{t-k}^{\;2}+\sum_{l=1}^{n}\beta_l\sigma_{t-l}^{\;2},$$

where e_t follows skewed-t distribution.

As for the conventional hedging approaches, we include the static OLS and ECM models, the dynamic DCC-GARCH-ECM model, DCC-RV-ECM model and the VHAR model. The former three models completely rely on the daily log returns of the spot (R_t^S) and the futures (R_t^F). The DCC-RV-ECM model incorporates high-frequency based realized covariance matrix (volatilities and correlation) in the DCC framework; while the VHAR model directly models the high-frequency based realized covariance matrix.

7) The OLS model: $R_t^S=\alpha+\beta R_t^F+\varepsilon_t$.

8) The ECM model: $R_t^S=\alpha+\beta R_t^F+\gamma\left(R_{t\text{-}1}^S-\theta R_{t-1}^F\right)+\varepsilon_t$,

where $\left(R_{t\text{-}1}^{S}-\theta R_{t-1}^{F}\right)$ is the error correction term that characterizes the long-term equilibrium between spot and futures.

The OLS model and the ECM model are static models, and the estimated parameter β is the (constant) hedge ratio.

9) The DCC-GARCH-ECM model:

$$
\begin{aligned}
&R_t^S=\mu^S+\gamma^S\left(R_{t\text{-}1}^S-\theta R_{t-1}^F\right)+\varepsilon_t^S,\\
&R_t^F=\mu^F+\gamma^F\left(R_{t\text{-}1}^S-\theta R_{t-1}^F\right)+\varepsilon_t^F,\\
&\begin{pmatrix}\varepsilon_t^S\\ \varepsilon_t^F\end{pmatrix}\mid\psi_{t-1}\sim N(0,\mathbf{H}_t),
\end{aligned}
$$

where $\psi_{t\text{-}1}$ is the information set up to day (t-1) and $\mathbf{H}_t$ is the conditional covariance matrix modeled as:

$$
\begin{aligned}
&\mathbf{H}_t=\begin{pmatrix}H_t^S & H_t^{S,F}\\ H_t^{S,F} & H_t^F\end{pmatrix}=\begin{pmatrix}\sqrt{H_t^S} & 0\\ 0 & \sqrt{H_t^F}\end{pmatrix}\times\begin{pmatrix}1 & \rho_t^{S,F}\\ \rho_t^{S,F} & 1\end{pmatrix}\times\begin{pmatrix}\sqrt{H_t^S} & 0\\ 0 & \sqrt{H_t^F}\end{pmatrix}=\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t,\\
&H_t^S=\beta_0^S+\beta_1^S\varepsilon_{t-1}^{S\;2}+\beta_2^S H_{t-1}^S, \qquad (*)\\
&H_t^F=\beta_0^F+\beta_1^F\varepsilon_{t-1}^{F\;2}+\beta_2^F H_{t-1}^F, \qquad (*)\\
&\mathbf{R}_t=diag\left(\mathbf{Q}_t\right)^{-\frac{1}{2}}\mathbf{Q}_t diag\left(\mathbf{Q}_t\right)^{-\frac{1}{2}},\\
&\mathbf{Q}_t=(1-\alpha-\beta)\bar{\mathbf{Q}}+\alpha\mathbf{z}_{t-1}\mathbf{z}_{t-1}{}'+\beta\mathbf{Q}_{t-1}, \qquad (*)
\end{aligned}
$$

where $\mathbf{z}_t=\begin{pmatrix}\varepsilon_t^S/\sqrt{H_t^S}\\ \varepsilon_t^F/\sqrt{H_t^F}\end{pmatrix}$ is the standardized residual vector, and $\bar{\mathbf{Q}}$ is the unconditional correlation matrix of the spot and the futures returns. α and β are nonnegative scalars with $\alpha+\beta\leq 1$.

10) The DCC-RV-ECM model has similar formulation compared to the DCC-GARCH-ECM model, with modifications in the three equations of 9) that are

marked with (*):

$$
\begin{aligned}
H_t^S &= \beta_0^S + \beta_1^S RV_{t-1}^S + \beta_2^S H_{t-1}^S, && (*)\\
H_t^F &= \beta_0^F + \beta_1^F RV_{t-1}^F + \beta_2^F H_{t-1}^F, && (*)\\
\mathbf{Q}_t &= (1-\alpha-\beta)\bar{\mathbf{Q}} + \alpha \mathbf{RCorr}_{t-1}^{S,F} + \beta \mathbf{Q}_{t-1}, && (*)
\end{aligned}
$$

where $\mathbf{RCorr}_t^{S,F}$ is the realized correlation matrix whose sub-diagonal element is calculated as $RCorr_t^{S,F} = \dfrac{RCov_t^{S,F}}{\sqrt{RV_t^S}\cdot\sqrt{RV_t^F}}$.

11) The VHAR model:

The matrix logarithm transformation method is adopted to guarantee the positive definiteness of the forecasted covariance matrix. Specifically, define $\mathbf{A}_t = \text{logm}(\mathbf{RCOV}_t^{S,F})$ and define $\boldsymbol{X}_t = \text{vech}(\mathbf{A}_t) = \left(X_t^S, X_t^{S,F}, X_t^F\right)'$. The VHAR model is constructed as:

$$
\begin{aligned}
\begin{pmatrix} X_t^S \\ X_t^{S,F} \\ X_t^F \end{pmatrix} &= \begin{pmatrix} \alpha^S \\ \alpha^{S,F} \\ \alpha^F \end{pmatrix} + \begin{pmatrix} \beta_{11}^{(d)} & \beta_{12}^{(d)} & \beta_{13}^{(d)} \\ \beta_{21}^{(d)} & \beta_{22}^{(d)} & \beta_{23}^{(d)} \\ \beta_{31}^{(d)} & \beta_{32}^{(d)} & \beta_{33}^{(d)} \end{pmatrix} \begin{pmatrix} X_{t-1}^S \\ X_{t-1}^{S,F} \\ X_{t-1}^F \end{pmatrix} \\
&+ \begin{pmatrix} \beta_{11}^{(w)} & \beta_{12}^{(w)} & \beta_{13}^{(w)} \\ \beta_{21}^{(w)} & \beta_{22}^{(w)} & \beta_{23}^{(w)} \\ \beta_{31}^{(w)} & \beta_{32}^{(w)} & \beta_{33}^{(w)} \end{pmatrix} \begin{pmatrix} X_{t-1}^{S\ (w)} \\ X_{t-1}^{S,F(w)} \\ X_{t-1}^{F\ (w)} \end{pmatrix} + \begin{pmatrix} \beta_{11}^{(m)} & \beta_{12}^{(m)} & \beta_{13}^{(m)} \\ \beta_{21}^{(m)} & \beta_{22}^{(m)} & \beta_{23}^{(m)} \\ \beta_{31}^{(m)} & \beta_{32}^{(m)} & \beta_{33}^{(m)} \end{pmatrix} \begin{pmatrix} X_{t-1}^{S\ (m)} \\ X_{t-1}^{S,F(m)} \\ X_{t-1}^{F\ (m)} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^S \\ \varepsilon_t^{S,F} \\ \varepsilon_t^F \end{pmatrix},
\end{aligned}
$$

where $X_{t-1}^{S\ (w)} = \frac{1}{5}\sum_{i=1}^{5} X_{t-i}^S$, $X_{t-1}^{F\ (w)} = \frac{1}{5}\sum_{i=1}^{5} X_{t-i}^F$, $X_{t-1}^{S\ (m)} = \frac{1}{22}\sum_{i=1}^{22} X_{t-i}^S$, $X_{t-1}^{F\ (m)} = \frac{1}{22}\sum_{i=1}^{22} X_{t-i}^F$ $X_{t-1}^{S,F(w)} = \frac{1}{5}\sum_{i=1}^{5} X_{t-i}^{S,F}$, $X_{t-1}^{S,F(m)} = \frac{1}{22}\sum_{i=1}^{22} X_{t-i}^{S,F}$.

The inverse of the vech() function and the matrix exponential transformation is then applied to get the prediction of the covariance matrix.

3. Data Description

Our empirical data are five-minute ($1/\Delta = 48$) prices of the CSI 300 index and index futures from January 4, 2012 to December 29, 2017, covering a total of 1456 trading days in China's market.[3] We chose the five-minute sparse sampling approach following the majority of previous studies (c.f. Lai and Sheu 2010) as it provides a good trade-off between accuracy and market microstructure noise (nonsynchronous trading). The trading time of the CSI 300 index futures was 9:15am – 11:30am, 13:00pm – 15:15pm before 2016. Since January 1, 2016, China Financial Futures Exchange has adjusted the opening and closing times for the CSI 300 index futures to 9:30am and 15:00pm, respectively, to match those of the CSI 300 index. Thus in this empirical research, we use the five-minutes prices between 9:30am – 11:30am and 13:00pm – 15:00pm for both the CSI 300 index and the CSI 300 index futures, deleting all price records in the non-overlapping periods.

[Insert Figure 1 Here]

Figure 1 displays the time series plots of the log daily prices for the CSI 300 index and the CSI 300 index futures in the whole sample period. It shows that the log daily prices of the CSI 300 index futures are very close to those of the CSI 300 index in most of the trading days, and that the Chinese stock market has observed both relative tranquil and extremely volatile periods during our sample period. This observation inspires us to test the robustness of our results to different market conditions, which will be explained later.

[3] There are 1458 trading days from January 4, 2012 to December 29, 2017. However, trading on January 4, 2016 and January 7, 2016 closed much earlier, due to the circuit breaker mechanism being triggered. Thus these two days are deleted from our sample.

[Insert Figure 2 Here]

Figure 2 displays the time series plots of the realized volatilities for the CSI 300 index (RV_t^S) and the CSI 300 index futures (RV_t^F) as well as the realized covariance between the spot and the futures ($RCov_t^{S,F}$) in the whole sample period. It shows that the realized volatility of the CSI 300 index futures has a similar pattern as that of the CSI 300 index, although it is more volatile. Both the realized volatility series and the realized covariance series are relatively tranquil during the period from January 4, 2012 to the end of 2014, but are very turbulent around the year of 2015. Such pattern necessitates our robustness check in different volatility regimes.

[Insert Table 1 Here]

Table 1 reports descriptive statistics for the realized volatilities (RV_t^S and RV_t^F), the realized covariance ($RCov_t^{S,F}$), and the realized minimum-variance hedge ratio ($RMVHR_t$) of the CSI 300 index and index futures over the entire sample period. We can see that the realized volatility of the CSI 300 index futures has higher standard deviation than that of the CSI 300 index, indicating that the CSI 300 index futures is more volatile. The ADF and PP test statistics show that these four realized measures are all stationary, and thus can all be directly modeled. The Ljung-Box test statistics show that these four realized measures all exhibit up to 20th order serial correlation, and thus the long-memory models may be appropriate choices to model the RMVHR and the RCov matrix.

4. Hedging Performance Comparison

We set the period from January 2, 2014 to December 29, 2017 (975 trading days) as the out-of-sample forecast period, and perform one-step-ahead rolling window forecast. That is, we use the period from January 4, 2012 to December 31, 2013 (2 years, 481 trading days) as the first estimation window, to make forecasts for January 2, 2014. The estimation window is then rolled forward, and we use the period from January 5, 2012 to January 2, 2014 as the second estimation window, to make forecasts for January 3, 2014. The estimation window keeps rolling forward, until we have made forecasts for all the 975 out-of-sample trading days.

Based on these forecasts, we perform dynamic hedging of the CSI 300 index futures, and calculate the following two hedging performance indicators:

(1) Hedging Effectiveness (HE) (Ederington, 1979): $HE = E\left(HE_t\right)$, where $E()$ means taking expectation. $HE_t = 1 - \frac{\sigma^2_{HP,t}}{\sigma^2_{UP,t}}$, where $\sigma^2_{UP,t}$ is the day t variance of the unhedged portfolio, and is calculated as the realized variance of the CSI 300 index (RV_t^S); $\sigma^2_{HP,t}$ is the day t variance of the hedged portfolio, and is calculated using the realized variances of the CSI 300 index and index futures (RV_t^F), and the realized covariance of the spot and the futures ($RCov_t^{S,F}$)[4]: $\sigma^2_{HP,t} = RV_t^S - 2\hat{\beta}_t RCov_t^{S,F} + \hat{\beta}_t^2 RV_t^F$, with $\hat{\beta}_t$ being the forecasted minimum-variance hedge ratio for day t.

HE assesses the hedged risk reduction relative to the unhedged portfolio variance. Higher HE is preferred since it means that the portfolio risk has been largely reduced. It is closely related to the tracking error measures (c.f. Kofman and McGlenchy 2005) and is commonly used for hedging performance measure

[4] Following Markopoulou et al. (2016), we utilizes high-frequency data to generates the HE_t series, which enables statistical significance tests such as the multi-model MCS test (Hansen et al., 2011) and the pairwise DM test (Diebold and Mariano, 1995).

in the literature (Lee and Chien 2010, Hou and Li, 2013, Sheu and Lee 2014).

(2) Tracking Error Volatility (TEV) (Roll, 1992): $TEV = std\left(TE_t\right)$, where $std()$ means taking standard deviation, $TE_t = R_t^{HP} - R_t^S$ is the day t tracking error, R_t^{HP} and R_t^S are day t return of the hedged portfolio and day t return of the index, respectively.

TEV assesses how close the hedged portfolio is to a perfect hedge and is widely used in the industry. It measures the volatility of the difference between the performance of spot and the hedged portfolio. A high TEV value indicates a less hedged portfolio. Therefore, a lower TEV is preferred to remain neutral to the risk of the underlying index as the benchmark. In the extreme case of a perfect hedge when the spot and future prices are perfectly correlated, the TEV would be equal to 0.

Table 2 reports the hedging performance of all the models in the out-of-sample forecast period from January 2, 2014 to December 29, 2017. It is divided into two panels. Panel I displays results for those models that directly model the RMVHR. Panel II displays results for those models that model the daily returns (covariance matrix). The performance of the naïve method that uses a hedge ratio equal to 1 is also reported in Panel II. In each panel, the hedging performance indicators are listed in the first column, while the models are specified in the second row. In addition, we perform the model confidence set (MCS test, Hansen et al., 2011) using the HE_t series and the TEV_t series[5] to identify models with significantly superior hedging

[5] We calculate TEV every 22 days in the forecast period so as to construct the TEV_t series for the statistical significance tests (MCS test and DM test).

performance (significantly higher HE and significantly lower TEV). The corresponding MCS test *p*-values are reported in parenthesis, and those greater than 0.1 indicate that the corresponding method survives in the model confidence set $\hat{M}_{90\%}$ and is significantly superior than the other methods.

[Insert Table 2 Here]

Our results show that the HE measure and the TEV measure lead to consistent conclusions. From Table 2 we can see that when HE is considered, the numeric numbers in Panel I are mostly larger than those numbers in Panel II. When TEV is considered, the numeric numbers in Panel I are mostly smaller than those numbers in Panel II. Therefore, the dynamic hedging performance of the CSI 300 index futures using RMVHR dominates that of the conventional methods in the out-of-sample forecast period in general. Specifically, the ARMA model and the ARMA-GARCH model of RMVHR have the largest HE among all the twelve hedging methods. These two models, together with the ARFIMA model of RMVHR, have significantly higher hedging effectiveness than the other methods, evidenced by their MCS test *p*-values. Therefore, when larger variance reduction is preferred, these three ARMA-type models of RMVHR significantly dominate the other models. On the other hand, the RS model of RMVHR has the lowest TEV among all the twelve hedging methods. Furthermore, its corresponding MCS test *p*-value is 1, while all the other methods have *p*-values of 0. Therefore, when the volatility of tracking errors is considered, the RS model of RMVHR significantly dominates the other methods.

Additional insights include: 1) In Panel II, the DCC-RV-ECM model has higher HE than the DCC-GARCH-ECM model. We perform the Diebold-Mariano test (DM test, Diebold and Mariano 1995) to check the statistical significance of the hedging performance difference. The DM-statistic of 19.42 shows that incorporating the information in the realized covariance matrix can significantly improve the variance reduction effectiveness of the DCC-GARCH-ECM model. 2) In Panel II, the VHAR model has higher HE and lower TEV than the DCC-RV-ECM model. While performing the DM test to compare these two models, we calculate the statistics of 19.60 and 5.53 with the HE_t series and the TEV_t series, respectively. Thus, the VHAR model significantly outperforms the DCC-RV-ECM model in terms of the variance reduction effectiveness and the volatility of tracking errors. Since these two models both utilize the realized covariance matrix, we argue that directly modeling the realized covariance matrix can better utilize the intraday information and further improve the hedging performance. 3) The HAR model of RMVHR in Panel I has higher HE than the VHAR model of RCov in Panel II. We perform the DM test and the DM-statistic of 2.35 indicates that the difference is significant at the 5% significance level. Since these two models utilize exactly the same information set (intraday five-minute returns of spot and futures) and have similar structures, we conclude that constructing the RMVHR and directly forecasting it is significantly superior in utilizing intraday information in terms of variance reduction effectiveness.

5. Robustness Checks

5.1 Different Market Conditions

To further test the robustness of the above results to different market conditions, we use the nonparametric change point model (NPCPM) (Ross et al. 2011) to detect the different volatility regimes of the CSI 300 index in the forecast period. The NPCPM detects the shifts in the volatility by sequential application of Mood’s test

(Mood, 1954), which is a nonparametric test for comparing the variances of two samples. Since the Mood's test assumes the independence of observations, we filter the original return series using a GARCH(1,1) model with student-t innovations following Ross (2013), and use the standardized residuals for the sequential Mood's tests.

Assume the two samples for variance comparison are $\left(r_{1,1}, r_{1,2}, \ldots, r_{1,a}\right)$ and $\left(r_{2,1}, r_{2,2}, \ldots, r_{2,b}\right)$, where a+b=T. The Mood's test statistic can be calculated as: $M = \sum_{i=1}^{a}\left[rank\left(r_{1,i}\right) - \frac{T+1}{2}\right]^2$, where $rank\left(r_{1,i}\right)$ is the rank of $r_{1,i}$ in the combined sample of length T. By comparing the standardized Mood's test statistic with the simulated thresholds reported in Ross et al. (2011), we can decide whether the null hypothesis of equal variance is rejected. The NPCPM applies sequential Mood's tests in the following manner to detect the volatility change points:

1) Divide the out-of-sample period into two contiguous samples. The first sample contains the initial 22 (a month) observations, and the second sample contains the remaining 953 (975-22=953) observations.
2) Perform the Mood's test on these two samples.
3) If the null hypothesis of equal variance is not rejected, prolong the first sample by 1 observation, and thus the second sample contains the remaining 952 observations. Perform the Mood's test on these two updated samples.
4) Repeat procedure 3) until the null hypothesis is rejected, which means a volatility change point has been detected. Flag this change point and repeat procedures 1)-3) starting from the first observation after the change point.

[Insert Figure 3 Here]

Figure 3 displays the volatility regimes detected by the NPCPM in the out-of-sample period from January 2, 2014 to December 29, 2017. There are three volatility regimes. The first regime is from January 2, 2014 to November 3, 2014, altogether 203 trading days. We refer to it as the low volatility regime (L) since the CSI 300 index is very tranquil during this period. The second regime is from November 4, 2014 to August 31, 2016 (448 trading days). We refer to it as the high volatility regime (H) since the CSI 300 index is extremely volatile during this period. This regime corresponds to China's abnormal market fluctuations in 2015. The last regime is from September 1, 2016 to December 29, 2017 (324 trading days). We again refer to it as the low volatility regime (L) due to its similarity with the first regime.

[Insert Table 3 - 4 Here]

We perform hedging performance comparison on each of these three volatility regimes and report the results in Tables 3-4. Comparing these two tables, we can see that the hedging effectiveness is always lower during the low volatility regime than during the high volatility regime, with the only exception of the naïve method. This observation confirms the appropriateness of our partition of volatility regimes to some extent. Inspecting each of these two tables, we confirm that our observations in Table 2 are all supported in Table 3 for the low volatility regimes, and are mostly supported in Table 4 for the high volatility regime, which we summarize as follows.

1) The RMVHR based models have higher HE and lower TEV than those of the conventional methods in general in both the low volatility regimes and the high volatility regime.

2) The ARMA-type models of RMVHR and the RS model of RMVHR are significantly superior in terms of the variance reduction effectiveness and the volatility of tracking errors respectively, regardless of the volatility regime considered.

3) Incorporating the information in the realized covariance matrix into the DCC-GARCH-ECM model significantly improves the variance reduction effectiveness, regardless of the volatility regime considered.

4) Directly modeling the realized covariance matrix with the VHAR model can better utilize the intraday information than the DCC-RV-ECM model and further significantly improve the hedging performance, regardless of the volatility regime considered.

5) Constructing the RMVHR and directly forecasting it is significantly more efficient in utilizing the intraday information during the low volatility regimes. However, this conclusion does not hold in the high volatility regime. Nevertheless, by replacing the normal innovations in the HAR model with the GARCH-skewed-t innovations, the HAR-GARCH model in Panel I has lower TEV than the VHAR model. The significance of the improvements in 3) - 5) is justified by the DM test statistics. To conserve space, the results are not tabulated and are available upon request.

5.2 Different Market Structures

To examine whether the above results are extendable to different market structures, we use the S&P 500 index and index futures for robustness test. Five-minute prices from January 2, 2004 to December 31, 2015 are used as sample data, altogether 2915 trading days. The out-of-sample forecast period starts from January 3, 2006, covering 2452 days. Accordingly, the fixed-length rolling window is 463 days, and the first window is from January 2, 2004 to December 30, 2005. The time series plots of the log daily prices and the realized volatilities and covariance are displayed in the Appendix.

[Insert Figure 4 Here]

Furthermore, we applied the nonparametric change point model to detect the different volatility regimes of the S&P 500 index in the forecast period. Figure 4 displays the three detected volatility regimes. The first regime is from January 3, 2006 to April 9, 2007, altogether 295 trading days. We refer to it as the low volatility regime (L) since the S&P500 index is very tranquil during this period. The second regime is from April 10, 2007 to October 30, 2009 (641 trading days). We refer to it as the high volatility regime (H) since the S&P 500 index is extremely volatile during this period. This regime corresponds to the subprime crisis. The last regime is from November 2, 2009 to December 31, 2015 (1516 trading days). We again refer to it as the low volatility regime (L) due to its similarity with the first regime.

[Insert Tables 5-7 Here]

Tables 5-7 report the hedging performance comparisons in the whole out-of-sample forecast period and in different volatility regimes, respectively. We can see that the observations from China's market also hold in the US market. Specifically, 1) The RMVHR-based models have higher HE and lower TEV than those of the conventional methods in general in all the volatility regimes. 2) The HAR model of RMVHR is significantly superior in terms of both the variance reduction effectiveness and the volatility of tracking errors, regardless of the volatility regime considered. 3) The DCC-RV-ECM model significantly outperforms the DCC-GARCH-ECM model in terms of both the variance reduction effectiveness and the volatility of tracking errors, regardless of the volatility regime considered. 4) The VHAR model significantly outperforms the DCC-RV-ECM model in terms of both the variance reduction effectiveness and the volatility of tracking errors, regardless of the volatility regime considered. Therefore, we conclude that the superiority of the RMVHR based methods are robust to different market structures, although the superior model in different markets might differ.

As evidenced by the Ljung-Box Q-statistics in Table 8 and the autocorrelation plots in Figure 5, there exist different levels of long-term serial correlation of the realized minimum-variance hedge ratio ($RMVHR_t$) in US and China's markets. We can clearly see that although the RMVHR in both markets exhibit up to 30^{th} order serial correlation, the level of autocorrelation is much stronger in US than in China's market. A possible explanation is that as a developed market, the US market has much smaller volatility,[6] and requires less adjusting of the hedge ratio.[7] Accordingly, the RMVHR-based models that characterize the long-memory property (ARFIMA, HAR and HAR-GARCH) have better hedging performance than that of the other models in US market, among which the HAR model is superior. On the other hand, the long-memory RMVHR-based models do not have clear superiority in China's market.

6. Concluding Remarks

[6] The mean of RV_t^S and RV_t^F in US market is 0.8653 and 1.2878 in our empirical period, much smaller compared to that of 1.6101 and 2.1515 (see Table 1) in China's market.

[7] The standard deviation of $RMVHR_t$ in US market is 0.1056 in our empirical period, much smaller compared to that of 0.1724 (see Table 1) in China's market.

The optimal hedge ratio is crucial for investors and portfolio managers. This paper evaluates the performance of the dynamic hedging methods that employ information content from high-frequency prices of spot and futures over the conventional hedging models. We examined a number of popular time-series models and used forecasts of the RMVHR to perform dynamic hedging on the CSI 300 index futures and the S&P 500 index futures. We also included the static OLS and ECM models, the VHAR model, the dynamic DCC-GARCH-ECM model based on daily returns, and the DCC-RV-ECM model using five-minute prices for comparison. In addition, we detected different volatility regimes in the forecast period using the nonparametric change point model (Ross et al. 2011). Using the hedging effectiveness and the tracking error volatility as criteria, we conducted hedging performance comparison in the out-of-sample forecast period as well as in each detected volatility regime.

Our results show that the dynamic hedging performance of the RMVHR-based models dominates that of the conventional methods in different market structures and in all the volatility regimes, including China's abnormal market fluctuations in 2015 and the US financial crisis in 2008. Our research also shed new lights on the conventional hedging models. For instance, incorporating information in the realized measures from high-frequency data improves the dynamic hedging performance. In addition, the VHAR model that directly models the realized covariance matrix better utilizes the intraday information and outperforms the DCC-RV-ECM model.

Our research provides insightful information for investors, risk managers, and researchers and shows that dynamic hedge ratios with intraday high frequency information can provide substantial benefits to risk managers and hedgers. Future work would involve exploring forecast combination techniques to further improve the forecasting capability of RMVHR and the dynamic hedging performance.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (grant number 71671084).

References

Andersen, T. G., & Bollerslev, T., 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 34(9), 885-905.

Baillie, R. T., & Myers, R., J., 1991. Bivariate GARCH estimation of the optimal commodity hedge. *Journal of Applied Econometrics*, 6(2), 109-124.

Barndorff-Nielsen, O. E., & Shephard, N., 2004. Econometric analysis of realised covariation: high frequency based covariance, regression and correlation in financial economics. *Econometrica*, 72(3), 885–925.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., & Shephard, N., 2011. Multivariate realised kernels: Consistent positive semi-definite estimators of the covariation of equity prices with noise and non-synchronous trading. *Journal of Econometrics*, 162(2), 149-169.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3), 307-327.

Bollerslev, T., 1987. A conditionally heteroskedastic time series model for speculative prices and rates of return. *The review of economics and statistics*, 542-547.

Bollerslev, T., 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics & Statistics*, 72(72), 498-505.

Busch, T., Christensen, B. J., & Nielsen, M. Ø., 2011. The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock and bond markets. *Journal of Econometrics*, 160(1), 48-57.

Cecchetti, S. G., Cumby, R. E., & Figlewski, S., 1988. Estimation of the optimal futures hedge. *The Review of Economics and Statistics*, 623-630.

Corsi, F., 2009. A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7(2), 174-196.

Diebold, F. X., & Mariano, R. S., 1995. Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 13(3), 253-263.

Ederington, L. H., 1979. The hedging performance of the new futures markets. *Journal of Finance*, 34(1), 157-170.

Engle, R. F., & Kroner, F. K., 1995. Multivariate simultaneous generalized ARCH. *Econometric Theory*, 11(1), 122-150.

Engle, R. F., 2002. Dynamic conditional correlation: a simple class of multivariate GARCH models. *Journal of Business & Economic Statistics*, 20(3), 339-350.

Engle, R. F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987-1007.

Ghosh, A., & Clayton, R., 1996. Hedging with international stock index futures: an intertemporal error correction model. *Journal of Financial Research*, 19(4), 477-491.

Ghosh, A., 1993. Hedging with stock index futures: Estimation and forecasting with error correction model. *The Journal of Futures Markets*, 13(7), 743–752.

Ghosh, A., 1995. The hedging effectiveness of ECU futures contracts: forecasting evidence from an error correction model. *Financial Review*, 30(3), 567-581.

Hansen, P. R., Lunde, A., & Nason, J. M., 2011. The model confidence set. *Econometrica*, *79*(2), 453-497.

Hill, J., & Schneeweis, T., 1981. A note on the hedging effectiveness of foreign currency futures. *Journal of Futures Markets*, 1(4), 659-664.

Hou, Y., & Li, S., 2013. Hedging performance of Chinese stock index futures: An empirical analysis using wavelet analysis and flexible bivariate GARCH approaches. *Pacific-Basin Finance Journal*, 24, 109-131.

Hsu, C. C., Tseng, C. P., & Wang, Y. H., 2008. Dynamic hedging with futures: A copula‐based GARCH model. *The Journal of Futures Markets*, 28(11), 1095-1116.

Johnson, L. L., 1960. The theory of hedging and speculation in commodity futures. *The Review of Economic Studies*, 27(3), 139-151.

Kofman, P., & McGlenchy, P., 2005. Structurally sound dynamic index futures hedging. *Journal of Futures Markets*, *25*(12), 1173-1202.

Koopman, S. J., Jungbacker, B., & Hol, E. 2005. Forecasting daily variability of the S&P 100 stock index using historical, realised and implied volatility measurements. *Journal of Empirical Finance*, 12(3), 445–475.

Koutmos, G., & Tucker, M., 1996. Temporal relationships and dynamic interactions between spot and futures stock markets. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, 16(1), 55-69.

Kroner, K. F., & Sultan, J., 1993. Time-varying distributions and dynamic hedging with foreign currency futures. *Journal of Financial & Quantitative Analysis*, 28(4), 535-551.

Lai, Y. S., & Sheu, H. J., 2010. The incremental value of a futures hedge using realized volatility. *The Journal of Futures Markets*, 30(9), 874-896.

Lai, Y., Chen, C. W., & Gerlach, R., 2009. Optimal dynamic hedging via copula-threshold-GARCH models. *Mathematics and Computers in Simulation*, 79(8), 2609-2624.

Lee, H., & Chien, C., 2010. Hedging performance and stock market liquidity: evidence from the Taiwan futures market. *Asia-Pacific Journal of Financial Studies*, 39(3), 396-415.

Lien, D. 1996. The effect of the cointegration relationship on futures hedging: A note. *Journal of Futures Markets*, 16(7), 773–780.

Markopoulou, C. E., Skintzi, V. D., & Refenes, A. N., 2016. Realized hedge ratio: Predictability and hedging performance. *International Review of Financial Analysis*, 45, 121-133.

Meneu, V., & Torro, H., 2003. Asymmetric covariance in spot‐futures markets. *Journal of Futures Markets*, 23(11), 1019-1046.

Miao, H., Ramchander, S., Wang, T., & Yang, D., 2017. Role of index futures on China's stock markets: Evidence from price discovery and volatility spillover. *Pacific-Basin Finance Journal*, 44, 13-26.

Mood, A. M., 1954. On the asymptotic efficiency of certain nonparametric two-sample tests. *The Annals of Mathematical Statistics*, 25(3), 514-522.

Ng, L., & Wu, F., 2007. The trading behavior of institutions and individuals in Chinese equity markets. *Journal of Banking & Finance*, 31(9), 2695-2710.

Park, T. H., & Switzer, L. N., 1995. Bivariate GARCH estimation of the optimal hedge ratios for stock index futures: a note. *Journal of Futures Markets*, 15(1), 61-67.

Roll, R., 1992. A mean/variance analysis of tracking error. *The Journal of Portfolio Management*, summer, 13-22.

Ross, G. J., 2013. Modelling financial volatility in the presence of abrupt changes. *Physica A: Statistical Mechanics and its Applications*, 392(2), 350-360.

Ross, G. J., Tasoulis, D. K., & Adams, N. M., 2011. Nonparametric monitoring of data streams for changes in location and scale. *Technometrics*, 53, 379-389.

Sheu, H., & Lee, H., 2014. Optimal futures hedging under multichain Markov regime switching. *The Journal of Futures Markets*, 34(2), 173-202.

Stein, J. L., 1961. The simultaneous determination of spot and futures prices. *American Economic Review*, 51(5), 1012-1025.

Wei, Y., Wang, Y., & Huang, D., 2011. A copula–multifractal volatility hedging model for CSI 300 index futures. *Physica A: Statistical Mechanics and its Applications*, 390(23), 4260-4272.

Yan, Z., & Li, S., 2018. Hedge ratio on Markov regime-switching diagonal Bekk-Garch model. *Finance Research Letters*, 24, 49-55.

Yang, J., & Awokuse, T. O., 2003. Asset storability and hedging effectiveness in commodity futures markets. *Applied Economics Letters*, 10(8), 487-491.

Yang J., Yang Z. H., & Zhou, Y. G., 2012. Intraday price discovery and volatility transmission in stock index and stock index futures markets: evidence from China. *The Journal of Futures Markets*, 32(2), 99-121.

Zhang, L., 2011. Estimating Covariation: Epps Effect, Microstructure Noise. *Journal of Econometrics*, 160(1), 33-47.

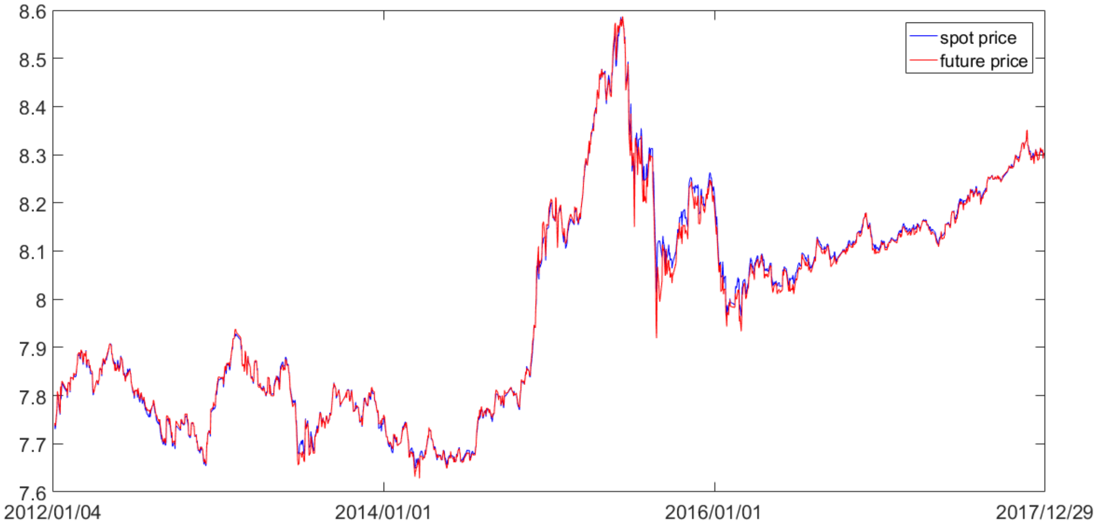

Figure 1. Time series plots of the log daily prices for the CSI 300 index and the CSI 300 index futures from January 4, 2012 to December 29, 2017.

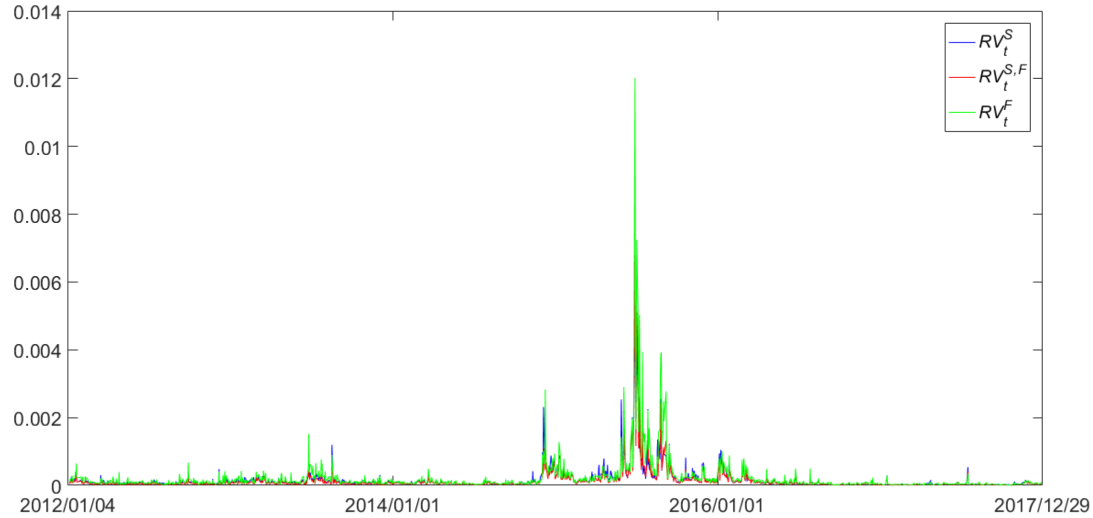

Figure 2. Time series plots of the realized volatilities for the CSI 300 index and the CSI 300 index futures, and the realized covariance between the spot and the futures from January 4, 2012 to December 29, 2017.

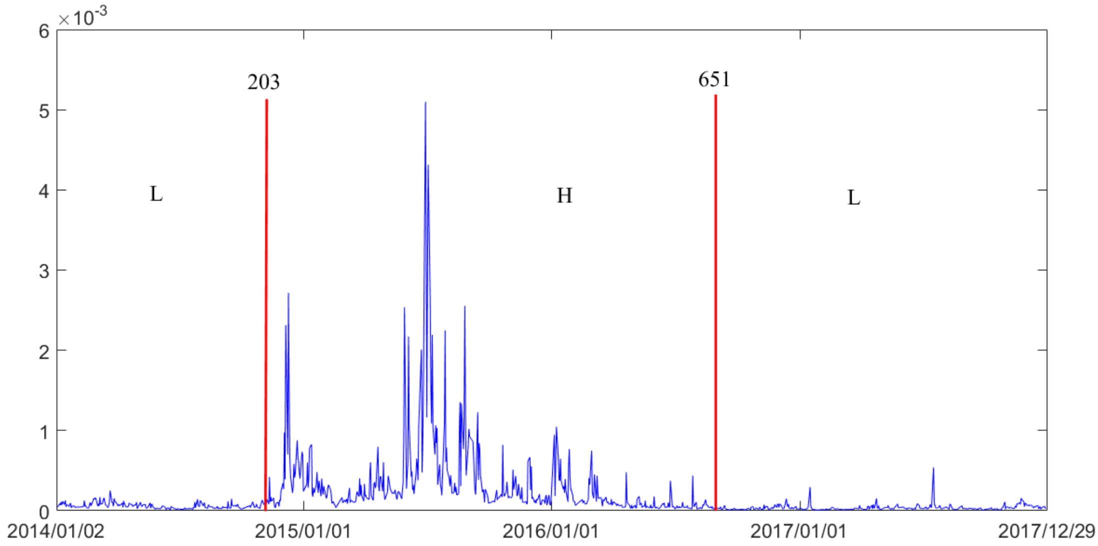

Figure 3. Volatility regimes detected by the NPCPM in the out-of-sample period from January 2, 2014 to December 29, 2017. (CSI 300)

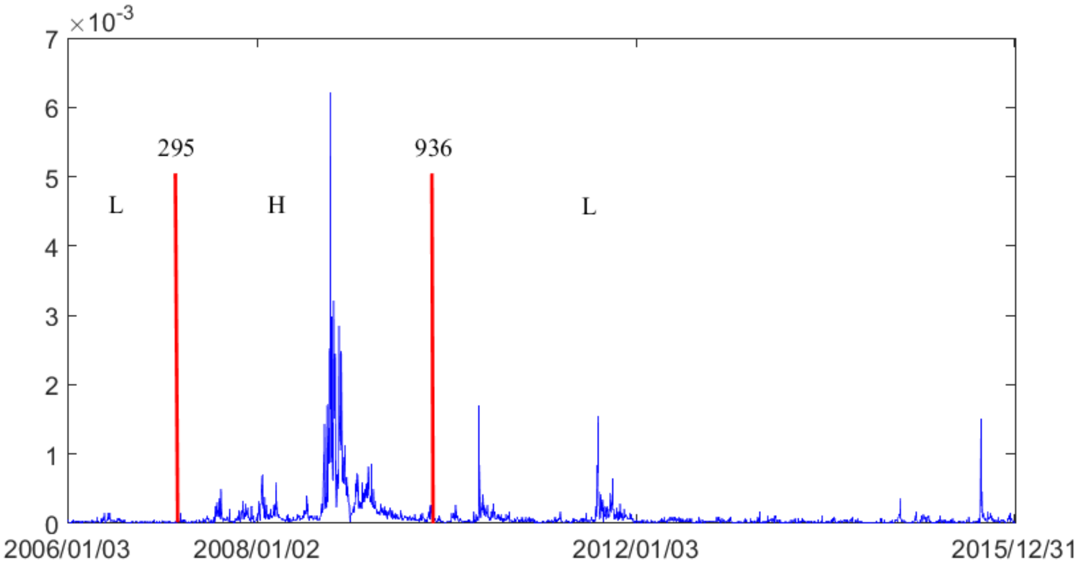

Figure 4. Volatility regimes detected by the NPCPM in the out-of-sample period from January 3, 2006 to December 31, 2015. (S&P 500)

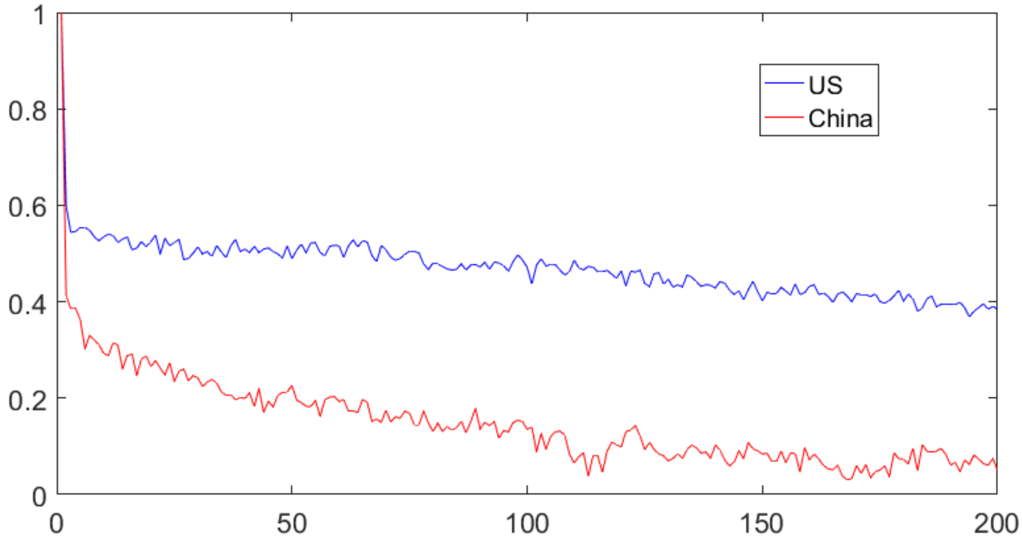

Figure 5. Autocorrelation of the RMVHR in US and China’s markets for lags 1 to 200.

Table 1. Descriptive statistics for the realized volatilities (RV_t^S and RV_t^F), the realized covariance ($RCov_t^{S,F}$), and the realized minimum-variance hedge ratio ($RMVHR_t$) of the CSI 300 index and index futures from January 4, 2012 to December 29, 2017.

	$RMVHR_t$	RV_t^S	RV_t^F	$RCov_t^{S,F}$
Mean	0.6600	1.6101	2.1515	1.3713
Standard Deviation	0.1724	3.3081	5.6002	3.4555
Skewness	0.1303	7.2779	10.7185	10.0274
Kurtosis	3.4574	76.4803	172.1786	146.6007
ADF	-3.1386***	-10.7050***	-12.7407***	-12.4644***
PP	-30.4818***	-16.8493***	-17.3488***	-16.5660***
LB(5)	1007.8***	2719.2***	2627.6***	2695.7***
LB(10)	1698.6***	3847.8***	3708.5***	3668.4***
LB(20)	2891.2***	5575.6***	5015.1***	5007.5***

Note: JB represents the Jarque-Bera normality test statistics, ADF represents the Augmented-Dickey-Fuller test statistics, PP represents the Phillips-Perron test statistics, LB(k) represents the Ljung-Box Q-statistics for k^{th} order serial correlation, *** represents the significance level of 1%. The orders of magnitude for the mean and the standard deviation of RV_t^S, RV_t^F and $RCov_t^{S,F}$ are 10^{-4}.

Table 2. Hedging performance comparison in the out-of-sample forecast period from January 2, 2014 to December 29, 2017 for CSI 300.

Panel I: modeling the RMVHR						
	RS	ARMA	ARMA-GARCH	ARFIMA	HAR	HAR-GARCH
HE	58.3634% (0.014)	58.7288% (0.656)	58.7288% (1.000)	58.6469% (0.554)	58.5888% (0.057)	58.5556% (0.057)
TEV	1.1758% (1.000)	1.2302% (0.000)	1.2303% (0.000)	1.3151% (0.000)	1.3266% (0.000)	1.3063% (0.000)
Panel II: modeling the daily returns (covariance matrix)						
	OLS	ECM	DCC-GARCH-ECM	DCC-RV-ECM	VHAR	NAIVE
HE	55.2131% (0.006)	55.2960% (0.014)	45.0022% (0.000)	49.5666% (0.000)	58.5016% (0.014)	43.4252% (0.000)
TEV	1.5308% (0.000)	1.5258% (0.000)	1.2859% (0.000)	1.4721% (0.000)	1.3213% (0.000)	1.8932% (0.000)

Note: TEV represents the tracking error volatility, HE represents the hedging effectiveness. The numbers in parenthesis are MCS test *p*-values.

Table 3. Hedging performance comparison in the low volatility regime from January 2, 2014 to November 3, 2014, and from September 1, 2016 to December 29, 2017 for CSI 300.

Panel I: modeling the RMVHR						
	RS	ARMA	ARMA-GARCH	ARFIMA	HAR	HAR-GARCH
HE	58.0378% (0.068)	58.3540% (0.602)	58.3542% (0.602)	58.3797% (1.000)	58.3610% (0.602)	58.3191% (0.602)
TEV	0.5681% (1.000)	0.6000% (0.000)	0.6001% (0.000)	0.6347% (0.000)	0.6362% (0.000)	0.6234% (0.000)
Panel II: modeling the daily returns (covariance matrix)						
	OLS	ECM	DCC-GARCH-ECM	DCC-RV-ECM	VHAR	NAÏVE
HE	54.1327% (0.068)	54.1596% (0.068)	37.4661% (0.000)	44.9733% (0.000)	58.1988% (0.068)	44.7205% (0.000)
TEV	0.7864% (0.000)	0.7851% (0.000)	0.9570% (0.000)	0.8919% (0.000)	0.6547% (0.000)	0.9052% (0.000)

Note: TEV represents the tracking error volatility, HE represents the hedging effectiveness. The numbers in parenthesis are MCS test *p*-values.

Table 4. Hedging performance comparison in the high volatility regime from November 4, 2014 to August 31, 2016 for CSI 300.

Panel I: modeling the RMVHR						
	RS	ARMA	ARMA-GARCH	ARFIMA	HAR	HAR-GARCH
HE	58.7464% （0.016）	59.1696% （1.000）	59.1696% （0.834）	58.9614% （0.194）	58.8569% （0.016）	58.8339% （0.006）
TEV	1.6221% （1.000）	1.6948% （0.000）	1.6950% （0.000）	1.8148% （0.000）	1.8323% （0.000）	1.8055% （0.000）
Panel II: modeling the daily returns (covariance matrix)						
	OLS	ECM	DCC-GARCH-ECM	DCC-RV-ECM	VHAR	NAÏVE
HE	56.4839% （0.005）	56.6327% （0.006）	53.8673% （0.005）	54.9698% （0.006）	58.8577% （0.006）	41.9014% （0.000）
TEV	2.0920% （0.000）	2.0847% （0.000）	1.6890% （0.000）	1.9449% （0.000）	1.8161% （0.000）	2.6158% （0.000）

Note: TEV represents the tracking error volatility, HE represents the hedging effectiveness. The numbers in parenthesis are MCS test *p*-values.

Table 5. Hedging performance comparison in the out-of-sample forecast period from January 3, 2006 to December 31, 2015 for S&P 500.

Panel I: modeling the RMVHR						
	RS	ARMA	ARMA-GARCH	ARFIMA	HAR	HAR-GARCH
HE	76.1409% (0.005)	77.2818% (0.005)	77.2768% (0.005)	78.2452% (0.020)	78.2574% (1.000)	78.1892% (0.005)
TEV	1.0749% (0.000)	1.0770% (0.000)	1.0772% (0.000)	1.0384% (0.000)	1.0343% (1.000)	1.0427% (0.000)
Panel II: modeling the daily returns (covariance matrix)						
	OLS	ECM	DCC-GARCH-ECM	DCC-RV-ECM	VHAR	NAIVE
HE	70.7956% (0.000)	70.5539% (0.000)	69.3481% (0.000)	72.1657% (0.000)	78.2285% (0.020)	68.0626% (0.000)
TEV	1.2826% (0.000)	1.2866% (0.000)	1.2889% (0.000)	1.1670% (0.000)	1.0355% (0.000)	1.3347% (0.000)

Note: TEV represents the tracking error volatility, HE represents the hedging effectiveness. The numbers in parenthesis are MCS test *p*-values.

Table 6. Hedging performance comparison in the low volatility regime from January 3, 2006 to April 9,2007, and from November 2, 2009 to December 31, 2015 for S&P 500.

Panel I: modeling the RMVHR						
	RS	ARMA	ARMA-GARCH	ARFIMA	HAR	HAR-GARCH
HE	70.1863% (0.008)	71.6688% (0.008)	71.6624% (0.008)	72.9252% (0.014)	72.9394% (1.000)	72.8630% (0.008)
TEV	0.7883% (0.007)	0.7838% (0.000)	0.7840% (0.000)	0.7401% (0.007)	0.7388% (1.000)	0.7392% (0.407)
Panel II: modeling the daily returns (covariance matrix)						
	OLS	ECM	DCC-GARCH-ECM	DCC-RV-ECM	VHAR	NAIVE
HE	64.3656% (0.000)	64.0526% (0.000)	63.5997% (0.000)	65.4364% (0.000)	72.9009% (0.014)	61.3731% (0.000)
TEV	0.9436% (0.000)	0.9461% (0.000)	0.9317% (0.000)	0.9278% (0.000)	0.7452% (0.000)	0.9761% (0.000)

Note: TEV represents the tracking error volatility, HE represents the hedging effectiveness. The numbers in parenthesis are MCS test *p*-values.

Table 7. Hedging performance comparison in the high volatility regime from April 10, 2007 to October 30, 2009 for S&P 500.

Panel I: modeling the RMVHR						
	RS	ARMA	ARMA-GARCH	ARFIMA	HAR	HAR-GARCH
HE	92.9642% （0.003）	93.1400% （0.218）	93.1391% （0.176）	93.2756% （0.460）	93.2822% （1.000）	93.2370% （0.224）
TEV	1.6333% （0.000）	1.6445% （0.000）	1.6448% （0.000）	1.6064% （0.000）	1.5977% （1.000）	1.6182% （0.000）
Panel II: modeling the daily returns (covariance matrix)						
	OLS	ECM	DCC-GARCH-ECM	DCC-RV-ECM	VHAR	NAIVE
HE	88.9621% （0.000）	88.9218% （0.000）	85.5887% （0.001）	91.1779% （0.003）	93.2802% （0.679）	86.9622% （0.000）
TEV	1.9448% （0.000）	1.9514% （0.000）	1.9767% （0.000）	1.6676% （0.000）	1.5925% （0.000）	2.0317% （0.000）

Note: TEV represents the tracking error volatility, HE represents the hedging effectiveness. The numbers in parenthesis are MCS test *p*-values.

Table 8. Ljung-Box Q-statistics for k^{th} order serial correlation of the realized minimum-variance hedge ratio ($RMVHR_t$) in US and China's markets.

Lags	1	5	10	15	20	25	30
US	1036.2***	4576.6***	8860.3***	12930.3***	16987.5***	21027.6***	24969.1***
China	244.3***	1007.8***	1698.6***	2337.6***	2891.2***	3375.2***	3820.8***

Note: *** represents the significance level of 1%.

Appendix

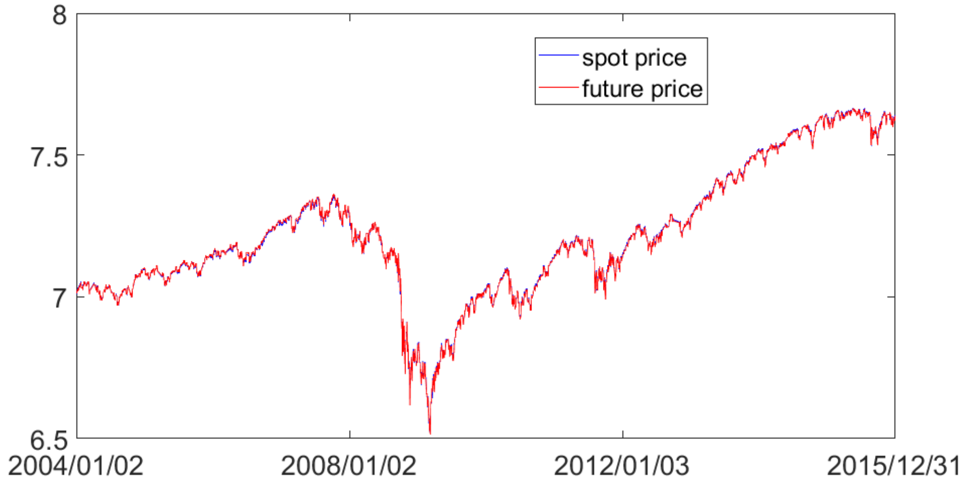

Figure A.1. Time series plots of the log daily prices for the S&P 500 index and the S&P 500 index futures from January 2, 2004 to December 31, 2015

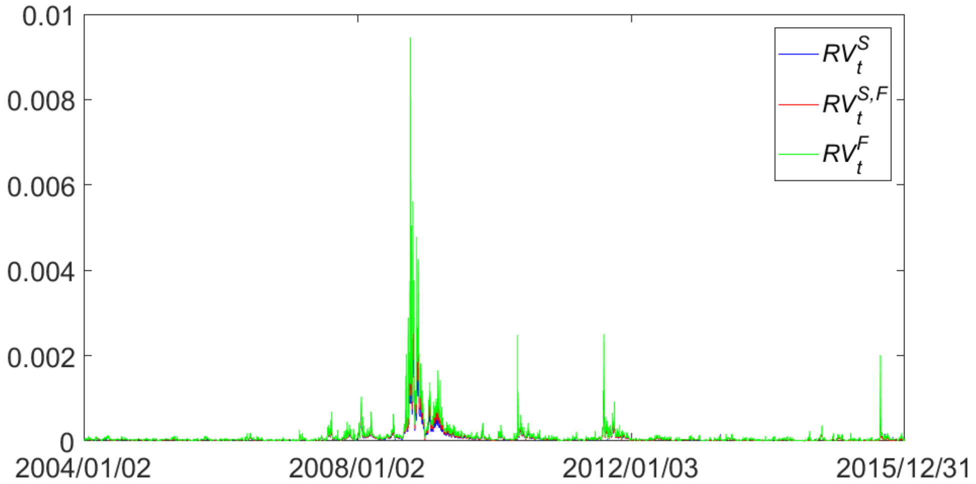

Figure A.2. Time series plots of the realized volatilities for the S&P 500 index and the S&P 500 index futures, and the realized covariance between the spot and the futures from January 2, 2004 to December 31, 2015