SYSTEMATIC TREATMENT OF INFILTRATION WITH APPLICATIONS

by

H. J. Morel-Seytoux

June 1973

ENVIRONMENTAL RESOURCES CENTER

Colorado State University
Fort Collins, Colorado

Completion Report Series
No. 50

SYSTEMATIC TREATMENT OF INFILTRATION WITH APPLICATION

Completion Report

OWRR Project No. B-070-Colorado
Period July 1, 1971 - June 30, 1973

by

Dr. H.J. Morel-Seytoux

Department of Civil Engineering

Colorado State University

submitted to

Office of Water Resources Research
U.S. Department of Interior
Washington D.C. 20240

June 1973

The work upon which this report is based was supported by funds provided by the U.S. Department of Interior, Office of Water Resources Research, as authorized under the Water Resources Research Act of 1964; and pursuant to Grant Agreement No. 14-31-0001-3566.

ENVIRONMENTAL RESOURCES CENTER
Colorado State University
Fort Collins, Colorado

Norman A. Evans, Director

CER73-74HJM51

ABSTRACT

The research described briefly in this completion report has shown that the effects of air movement and air compressibility in soil columns are important. For soils underlain by a relatively impervious layer or by a shallow water table it is found that methods based on Richards' equation would overpredict infiltration rates by factors of two, three or more. Even when air compressibility effects are insignificant as in the case e.g. of an open semi-infinite column, air viscous effects are important. In fact it is shown that the formula of Green and Ampt underestimates the viscous resistance to flow behind the wetting front from 20 to 70 percent depending on soil type.

The use of a theory that properly considers the movement of water and air in the unsaturated zone has the advantage of accounting for observed experimental results that cannot be modeled by the one-phase flow theory. In addition the mathematical problem is actually simplified, not complicated, by the more complete approach. The fact that the problem of Green and Ampt could be solved simply in a few lines whereas it had eluded solution since 1911 is conclusive evidence.

Comparison with experimental results show clearly that the approximations in the methods of solution yield highly accurate and practical estimates of the infiltration quantities of interest.

KEYWORDS: *Infiltration, *Two-Phase Flow, *Air viscous effect, *Air compressibility, Numerical Methods, Experimental results, Research

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF FIGURES

LIST OF FIGURES (contined)

LIST OF TABLES

RESEARCH OBJECTIVES

In this second phase of research on the "Systematic Treatment of the Problem of Infiltration" the objectives were:

a) to convince soil scientists and hydrologists of the necessity and interest of abandoning the traditional unsaturated approach,

b) to simplify the two-phase formulation, particularly the methods of solution, to provide simple equations for everyday hydrologic practice,

c) to pursue the rigorous approach in order to explain all observed phenomena occuring during the simultaneous flow of water and air under natural boundary conditions or under specially imposed ones in the laboratory, and

d) to verify the theory by comparison with experimental data.

Significant progress was achieved regarding items b), c) and d). To a large degree objectives b) and d) have been met. Objective c) can of course never be met fully. Nevertheless the results are extremely encouraging. Regarding item a) only time will tell. However much of this report emphasizes that the two-phase approach simplifies the mathematical problem rather than complicates it. Thus even if it were not necessary (though evidence is clearly to the contrary) it would still be worthwhile to adopt it because of its simplicity.

ACHIEVEMENTS OF CONTRACT

It is not desirable to repeat in this completion report all the results obtained over the past two years and the detailed procedures by which they were obtained. These results and procedures can (or will) be found in one dissertation [41], one thesis [18], one chapter in a book [8], several published papers [25,27,30,35,38] and several submitted papers [3,28,29,31,32,33]. Rather a brief review of the methods of attack and a sample of results will be given.

As a sideline let us mention that the material task of preparing this report was not made easier by the author's absence from Colorado State University while on sabbatical leave at the Institut de Mécanique de Grenoble, Université Scientifique et Médicale. On the other hand these fresh contacts with a dynamic group of new colleagues (whose names will naturally appear in the report) informally or within the framework of a weekly seminar contributed much to the author's learning and teaching experience on the subject.

A. INTRODUCTION

1. Darcy's Law

When I first joined Chevron Oil Field Research Corporation (then California Research Corporation) as a Research Engineer in 1962, following a 27 month military service in the French army, some of which in Algeria, I could remember vaguely from my student past as a Groundwater Fluid Mechanist [19] the existence of a relation grandly called law and attributed to a compatriot named Darcy. That law had the form (more or less):

$$\vec{v} = - K \text{ grad } \Phi = -K\nabla\Phi \tag{1}$$

where K was a coefficient and Φ a potential. I could also remember that by dimensional analysis it had been shown that K was related to density (specific mass) and viscosity in an expression of the form:

$$K = \frac{k\rho_w g}{\mu_w} \tag{2}$$

where k had the dimension of an area and g is the acceleration of gravity. All efforts to relate k to some characteristic length (e.g. grain diameter, pore diameter) having failed, K appeared to the theoretician as a miserable coefficient to be determined experimentally. To quickly forget its lowly origin it seemed best to eliminate it by defining a new potential $\Phi^* = -K\Phi$, whose only, but noble, claim to the name potential is mathematical, being a solution of Laplace's equation.

What a jolt it was when first confronted with the problem of simultaneous flow of water and oil, to discover the elaborate form of Darcy's law for two immiscible fluids (say water and air):

$$\vec{v}_w = - \frac{k_w}{\mu_w} \nabla p_w + \frac{kk_{rw}}{\mu_w} \rho_w \vec{g} \tag{3}$$

$$\vec{v}_a = - \frac{k_a}{\mu_a} \nabla p_a + \frac{kk_{ra}}{\mu_a} \rho_a \vec{g} \tag{4}$$

where $\vec{v}_w$ is the water velocity (in the Darcy sense, that is a volumetric flux vector per unit bulk area), k is the intrinsic permeability (dimension of an area), μ_w, p_w, ρ_w, μ_a, p_a, ρ_a are respectively water or air viscosity, pressure and specific mass, and k_w and k_a are respectively the effective permeability to water and to air. Understandably if two fluids flow simultaneously then two velocities must be defined. Again by velocity we mean the velocity in the Darcy sense. In fact throughout this report we shall only talk about that velocity. Thus no confusion is possible.

If the two phases are continuous then it is intuitively acceptable that one could write for each phase a Darcy's law. Thus, after all, Equations (3) and (4) are not very different from Equation (1) except for two things: (1) instead of a vague potential as in Equation (1) all the terms appearing in these equations have a precise physical meaning (pressure, density, viscosity, etc.) and (2) there appear two new quantities: the effective permeabilities.

2. Relative Permeabilites

Clearly if water alone flows in the porous medium and occupies the entire pore space then the effective permeability to water equals k. When the water content is so low that water no longer moves, then by definition $k_w = 0$. At intermediate water contents k_w will take intermediate values between 0 and k. Since $k_w = k$ at $\theta = \phi$

where ϕ is the porosity it seems natural to define the relative permeability:

$$k_{rw} = \frac{k_w}{k} \tag{5}$$

as a function of the volumetric fraction of the void space occupied by water, namely the saturation:

$$S_w = \frac{\theta}{\phi} \tag{6}$$

so that $k_{rw} = 1$ when $S_w = 1$. Of course one can express k_{rw} as a function of θ just as well. One inconvenience in so doing arises when one wishes to compare shapes of relative permeabilities for different soils having therefore different porosities.

Figure 1 shows curves of relative permeabilities for a water-air system. They were determined experimentally by Vauclin [1971] and Gaudet [1972] using a method described by Smiles et al. [1971]. There are several things worth noting on this Figure. Clearly the relative permeabilities are not linear functions of water saturation (or water content). Second, under natural conditions full water saturation, that is S_w (or S for short) = 1 or $\theta = \phi$, is never reached. In fact it is difficult to achieve in the laboratory. We denote by residual air saturation, S_{ar}, or air content, θ_{ar}, the maximum air saturation or content, for which k_{ra} is still zero. For brevity we shall denote by $\tilde{S}_w$ or $\tilde{S}$ the water saturation at residual air saturation. By definition

$$\tilde{S} = 1 - S_{ar} \tag{7}$$

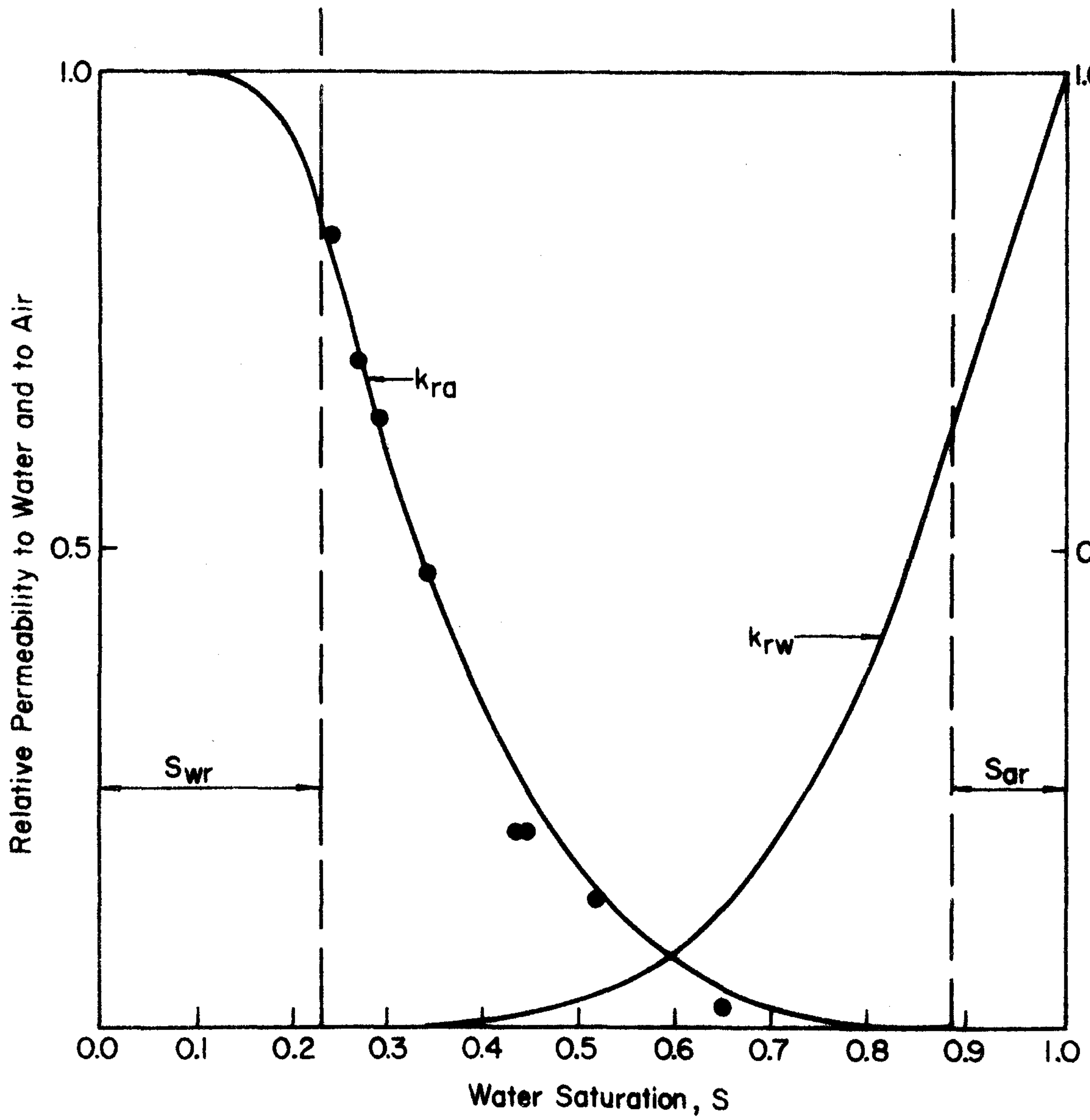

Figure 1. Relative permeabilities to water and air versus water saturation. The • are the experimental points by Gaudet [1972]. The continuous lines correspond to data by Vauclin [1971].

Similarly the sign $\tilde{}$ over any quantity $(\cdot)$ means that the quantity is evaluated at $S = \tilde{S}$. We shall refer to $\tilde{S}$ as the "natural" saturation.

Third it is worth noting that k_{rw} is essentially linear in the high range of water saturation. In the author's experience this seems to be typical of all curves of relative permeability to the wetting phase. On the other hand the curve k_{ra} is sharply curved in the high range of air saturation (low range of water saturation since $S_w + S_a = 1$ by definition). Again this is typical.

It is customary in the petroleum industry to define the mobility (or relative mobility) of each phase as the ratio of effective (or relative) permeability over the viscosity. In particular the total relative mobility is defined as:

$$\Lambda_r = \lambda_{rw} + \lambda_{ra} = \frac{k_{rw}}{\mu_w} + \frac{k_{ra}}{\mu_a} \tag{8}$$

Figure 2 shows a curve of $1/\Lambda_r$ (the total viscosity) versus S_w. Indeed $1/\Lambda_r$ is well named for it has the dimensions of viscosity. It is important to note that the total viscosity has a maximum at a saturation below the natural saturation and this is typical. It means that to sustain a steady total flow rate of water and air at that saturation in a column of soil would require a pressure drop 1.34 times greater than if water alone was flowing. Is it legitimate then to assume that the air viscous resistance to flow is negligible?

3. The Traditional Approach

When in 1966 I joined the water industry, my scant knowledge of hydrology was an obscure recollection of a course taken ten years earlier. Professor Holland (now with the California Water Quality Board) was then

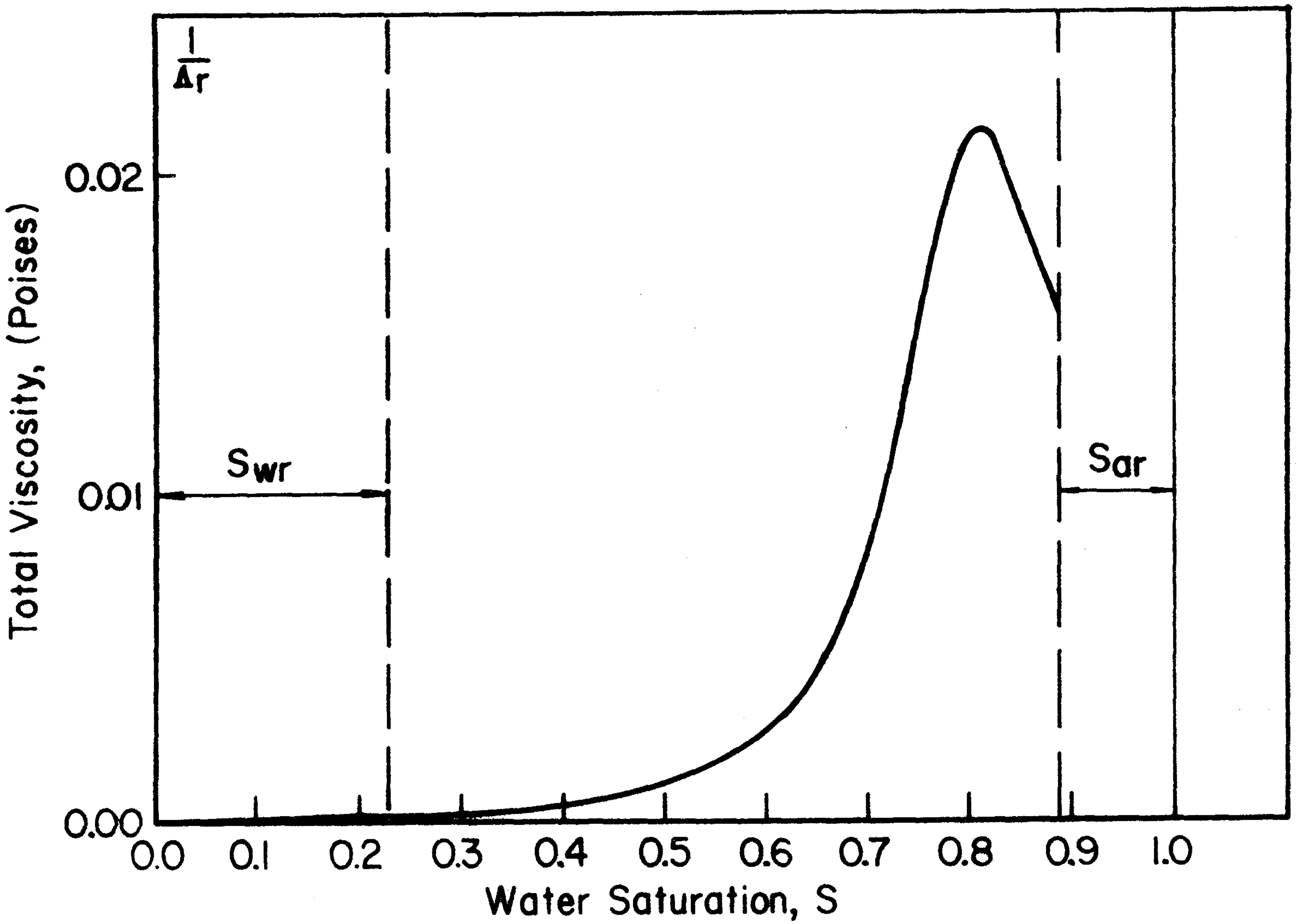

Figure 2. Curve of total viscosity versus water saturation

teaching the course "Advanced Hydrology." As a student enrolled for credit I dutifully carried the required or suggested reading. On the subject of infiltration a recommended reading was Philip's 1957 paper, "The theory of infiltration: 1. The infiltration equation and its solution." After five years of experience in the oil industry [21,22, 23] it was quite another shock in return to go through this paper and desperately look for the equations that would describe air movement. Finally I had to accept the evidence: they were not there. A more careful reading uncovered quickly the artifice which made it unnecessary to consider these equations. However I was not convinced. In addition the introduction of the artificial concept of the "diffusivity" seemed to have hidden the physical problem behind a heavy mathematical smoke screen, much in the same vein as the potential Φ^* mentioned earlier.

Reading the chapter on Infiltration in V.T. Chow's "Handbook of Applied Hydrology" [16] convinced me further that research on infiltration would be a worthy endeavor. The manner in which infiltration was modeled in the early version of the Standford Watershed Model reinforced this conviction.

4. The Initial Research Thrust

A review (not very complete I must admit now) of the literature had convinced me that the assumption regarding air on which all theoretical studies of infiltration were based over several decades had never been tested. Whereas it is true that the repetition of statements of the type: "it is known that air effects are negligible" do not add to a proof, they do have an intoxicating effect.

The fundamental objective of the first proposal was to determine the magnitude of the air effects by modeling them and to compare the results with those obtained by the traditional approach which neglects them. For various reasons discussed in the Completion Report to OWRR of June 30, 1971 [24], three distinct approaches were followed. In this report it was concluded: "The research has shown that:

a) effects of air movement and compressibility on infiltration are important,

b) approximate solutions to the right equations give more accurate results than "exact" solutions to the wrong ones (visualize the diffusivity equation and Philip's solution), and

c) the approximate solutions (i.e., generated by the Brustkern procedure) are not only more accurate but also more economic."

The details of the three approaches and a more complete discussion of the results can be found in the Chapter "Two-phase flows in Porous Media" of Volume 9, 1973 of "Advances in Hydroscience" [26].

5. The Current Research Direction

Having shown conclusively that, contrary to the assertions of the theoreticians, air effects are often important, the objectives of the next stage of research were:

a) to convince soil scientists and hydrologists of the necessity of abandoning the traditional unsaturated approach,

b) to simplify the two-phase formulation particularly the methods of solution to provide simple equations for everyday hydrologic practice,

c) to pursue the rigorous approach in order to explain all observed phenomena occurring during the simultaneous flow of water and air under natural boundary conditions or under special imposed boundary conditions in the laboratory, and

d) to perform experiments to verify the theory.

B. FOR A REFORMED THEORY OF INFILTRATION. WHY?

In this report on the theory of infiltration and in a general way on unsaturated flow in porous media, several questions will be raised. The title of this section indicates a need for change. It seemed logical to answer first the question: why? Is it desirable to treat the problem of infiltration in a more complete and more rigorous manner by introducing the air effect? First it must be remembered that the current theory does not include these effects. This forgotten fact is discussed in some detail in this chapter. Second it is easy to criticize. Nevertheless even a new theory will not be complete. It appeared important therefore to show that the inclusion of the air effect was probably the most urgent refinement of the theory. This is why as much experimental evidence as possible was reviewed.

1. Inadequacy of the Traditional Theory

For many years, and still today, theoretical studies on infiltration begin like a fairy tale. Once upon a time there was Richards' equation, or else, once upon a time there was the Diffusivity equation! Indeed from the start the story departs from reality because the equations assume that air pressure in the porous medium is everywhere and at all times equal to ambiant atmospheric pressure, which cannot be. Unfortunately the key words of the fairy tale "Once upon a time", which warn the adult and the child not to worry about the realism of the story, no longer appear in the articles.

These statements are of course exaggerated. Nevertheless the reader may find it instructive to consult some recent books, for instance the book of Hillel [1971] and the chapters of Swartzendruber [DeWiest, 1969] and of Philip [Chow, 1969]. He will then find that suction is defined (in absolute value) as water pressure relative to the atmosphere not relative to soil air pressure [Hillel, 1971, pp. 57 and 76; Swartzendruber, 1969, p. 218; Philip, 1969, p. 220] and that in the derivation of Richards' equation the assumption of a constant air pressure is not even mentioned [Hillel, 1971, pp. 109-113; Swartzendruber, 1969, p. 222; Philip, 1969, pp. 220-222].

The fact that the assumption is not mentioned does not mean that the authors are not aware of it. For instance Philip [1969, pp. 226-227] says: "The analysis developed in Section II. C. neglects any effect of pressure differences in the soil air." Philip thinks that in most instances this effect is of no importance "as the pressure differences within the soil atmosphere are trivially small." However, he adds:

> "These considerations do not appear to be generally of much importance in the field, although, as this author is well aware from his personal experiences in the Riverina of Australia, limits to air escape may well affect infiltration into large inundated areas. In fact soil-air pressures have developed which are great enough to lift the pavements of highways passing through the flooded region."

Hillel [1971, p. 243] acknowledges that in Richards' equation "several influences and mechanisms were not considered. To say that the above equation models soil-water flow, one must accept the following additional assumptions," among which, that: "air freely and instantaneously escapes from the system as water accumulates in it." Hillel adds: "Assumptions 5-8 are connected with mechanisms that we haven't yet learned to model successfully, so they must be accepted if we are to study soil-water movement at our present state of knowledge."

In summary the effect of airflow on infiltration has not been studied up to now for two reasons: it is not thought to be important in general and one does not know how to model it. It will be the purpose of the next sections to show that these reasons are not valid.

2. Importance of Air Presence on Infiltration

The influence of the air phase on the soil capacity to absorb water available on the surface has been demonstrated by recent mathematical modeling [Brustkern and Morel-Seytoux, 1970; Phuc and Morel-Seytoux, 1972; Noblanc and Morel-Seytoux, 1972] by laboratory experiments [Free and Palmer, 1940; Horton, 1940; Wilson and Luthin, 1963; Peck, 1965; Kuraz and Kutilek, 1970; Vachaud et al., 1973] and by observations and measures in the field [Feodoroff, 1966; Dixon and Linden, 1972].

Results from mathematical models are suspect for they depend upon the assumptions of the model. Their validity can always be contested.

Field measurements are difficult. They are more subject to errors than laboratory ones. They are more difficult to interpret because many variables cannot be controlled effectively. On the contrary, laboratory measurements are accurate, conditions of the experiment are in general well defined and interpretation of the results is relatively easy. For these reasons, laboratory results are reviewed first.

In their introduction, Free and Palmer [1940] state the reason for their interest in the relation between infiltration and air flow:

> "There also seems to be a rather widespread belief that the movement of soil air is important. However, there are few experimental data that evaluate the effect of entrapment and escape of air upon water movement, and there is an even greater lack of data dealing with the practical importance of the problem." [Free and Palmer, 1940, p. 390].

At the end of the article the authors conclude:

> "It was pointed out that the data secured indicate that many of the field conditions commonly considered responsible for excessive run-off are associated with those conditions in this study which tended to make the release and escape of air difficult." [Free and Palmer, 1940, p. 398].

From a quantitative point of view the study of Free and Palmer on 57cm high sand columns has shown that: "It took from 10 to 100 times longer to wet columns closed at the base than it took to wet columns open at the base." [Free and Palmer, 1940, p. 395].

Horton [1940] is skeptical, to begin, that air effect can be important:

> "It has been alleged, particularly by Russian scientists, that infiltration may be checked or inhibited over large areas, particularly flat steppe terrain, by compression of air within the soil. The author has yet to find a well authenticated example of this phenomenon in the United States." [Horton, 1940, p. 412].

Yet paradoxically among all the effects reviewed by Horton that could influence what he calls the "infiltration capacity", the only one for which he proceeded from mere speculation to actual measurement is precisely the air effect! The results of the experiments performed on soil (25% fine loam, 75% fine sand in weight) jars (8 in. high, 5 in. diameter) showed that:

> "In the second experiment, with capillary tubes within the soil, there was no escape of air through the water surface or around the perimeter, the air pressure within the soil mass remained at zero throughout the experiment and the infiltration-capacity was materially increased, being, at the end of the experiment, about twice as great as in the experiment without provision for escape of air." [Horton, 1940, p. 414].

This experimental result on the air effect is especially intriguing because if Horton's Figure 6 (p. 414) and Figure 2 (p. 405) are compared they look very similar. In fact it seems quite plausible that during the 24 hours between the initial and the wet run soil may have drained to a depth roughly equivalent to the size of the jars used in the experiments. The difference between the initial and the wet run might be simply due to the entrapment of air in this drained upper zone rather than caused by the other mechanisms invoked by Horton. It turns out (further coincidence) that the final infiltration capacity in the initial run is also about twice that of the wet run.

Quite a few years later, Wilson and Luthin [1963] are concerned again with the fact that

> "Theoretical analyses of infiltration [Philip, 1958. Physics of water movement in porous-solids. High Research Boards. Spec. Rept. 40, pp. 147-163] neglect the influence of air on the advancing water front. It is usually assumed that the air escapes readily and atmospheric conditions prevail. It is also assumed that the viscosity and density of air, in comparison to water, are negligible." [Wilson and Luthin, 1963, p. 136].

They conducted experiments in horizontal soil columns to demonstrate the effect of air on infiltration. To achieve this objective they performed four types of imbibition experiments: (1) experiments on homogeneous columns, (2) on heterogeneous columns (downstream soil being 40 or 88 times less pervious than the upstream soil), (3) on homogeneous columns closed at the end, and (4) experiments during which the air pressure just downstream from the wetting front is maintained atmospheric. In 31cm long columns, after 15 minutes, the infiltration rate was for homogeneous columns, heterogeneous columns (permeability ratios of 40 and 88) and closed columns, respectively: 2, 1.5, 1.3 and 0.5 cm^3/min. The air effect has thus reduced infiltration by factors of the order of 3/4, 2/3 and 1/4 respectively. These reductions are quite large. Whereas the case of a closed column is extreme, the cases of heterogeneous open columns are very realistic. As stated by Wilson and Luthin:

> "The use of the diffusion theory becomes even less valid when applied to a layered soil or one of decreasing air permeability with depth. And yet it is just such condition that one normally encounters in the field. Most soils have air permeability that decreases with depth. It may be due to the natural development of the soil, soil layering, or stratification, or it may be due to increases in the soil moisture content, or even to the presence of a water table." [Wilson and Luthin, 1963, p. 142].

Shortly after Wilson and Luthin, Peck [1965] conducted experiments of infiltration on vertical columns closed at the bottom. As Free, Palmer and Horton before him, Peck is concerned with only one aspect of the effect of air on infiltration: the impedance to infiltration due to the compression of the entrapped air. This is why he simulates columns heights of 133, 332, and 490 cm by reservoirs of air located at the end of a short column (about 30 cm). In contrast Wilson and Luthin studied also and mainly the other aspect of the air effect: the impedance to infiltration due to the viscous resistance to air flow. This aspect is not properly accounted for in Peck's experiments. However, for this reason Peck's results should yield values higher (thus optimistic) than those that would be obtained on real columns of same height. Yet Peck states in his Summary:

> "With the slate dust, rates of infiltration were reduced by a factor of up to nine in the bounded columns, which is of the order of values reported in the literature. In the sand, however, a factor of 500 was observed, which is much larger than any value previously recorded and indicates the apparent 'freezing' of water in his material." [Peck, 1965, p. 50].

The optimistic (!) results of Peck show clearly how much air presence can affect infiltration.

A year later, and in the course of very similar experiments, Adrian and Franzini [1966] have found that: "Under certain conditions, namely if the medium is made up of fine enough particles, this pressure build-up can effectively stop the infiltration." [Adrian and Franzini, 1966, pp. 5861-5862]. It is not solely a matter of slowing down the infiltration but sometime of stopping it completely.

As Wilson and Luthin, Kuraz and Kutilek [1970] are not happy with the current status of the theory of infiltration:

> "However, all the equations developed, and all the calculations done consider the infiltration into a profile of an initially constant moisture. The most frequent practical condition is that one where the initial moisture profile cannot be taken even approximately as constant, especially in the mild climatic zone." [Kuraz and Kutilek, 1970, pp. 183-184].

To better understand infiltration conditions in the field, Kuraz and Kutilek undertook experiments of infiltration into a vertical column the water content of which increases with depth. It is worthwhile to note that in the experiments of all the previously quoted authors the initial water contents were uniform and most often the columns were initially dry. In their experiments on closed columns with strongly increasing initial water content with depth they found that:

> "The final value of quasi-steady state is by two orders lower than the hydraulic conductivity of the used sand (0.01 cm/sec), and it can be taken as practically equal zero." [Kuraz and Kutilek, 1970, p. 187].

Recent experiments [Vachaud et al., 1973] on a vertical stratified column of sand have shown without ambiguity the importance of the air effect in this case. In one experiment air can escape laterally through holes. In another experiment air can only escape through the top soil surface. When air cannot escape laterally it compresses in the second (coarser) layer and prevents the transmission of water from the upper to the lower layer. The upper layer saturates at the bottom, fills with water and finally behaves as a layer impervious to the rain. When water can escape laterally, the entire rain infiltrates.

What conclusions can be drawn from this ensemble of laboratory results? At least in the laboratory it is clear that the air presence reduces the infiltration considerably. The order of magnitude of this reduction is such that the current obstinate attitude of the theoreticians of unsaturated flow to neglect this effect is a challenge to plain common sense. Maybe, but what is the situation in the field?

The following field observation is rather eloquent:

> "It has been observed in the field that subsurface drains sometimes start to flow soon after a rain has started and before the infiltrating rain water has percolated to the water table. The flow is probably induced by the confined air that is being drawn out of the soil ahead of the infiltrating water. It is possible that the presence of subsurface drains increase the infiltration rate into soils by permitting free flow of air out. The flow of air out of drains has been frequently observed in the field." [Wilson and Luthin, 1963, p. 143].

The effect reported above is not purely national. As Horton mentioned earlier, it has been noticed in Russia. It has also been observed in France:

> "Finally we had the opportunity to observe in the provinces of Gâtinais and of Puisaye a behaviour which has no connection with other cases described earlier: the top layer, which does not rest upon a characteristically impervious zone, is full of pockets of water of the size of a fist. The soil surface is strongly hit and compacted by the rains. The substratum, porous and pervious, is much better drained.
>
> Against our expectation, drainage by pipes is effective in these soils. One wonders if free water is not held in the top soil due to soil air compression, whose excape upward is forbidden. The drainage pipes apparently play the role of decompression valves. If this hypothesis is verified experimentally, one can imagine that a much less expensive design than the classical one could have the same effect."* [Feodoroff and Guyon, 1966, p. 758].

*Free translation by author of report.

Dixon and Linden have performed measurements in the field to size up the importance of air presence on infiltration in the irrigated area near Fallon Nevada:

> "Soil air pressure and water infiltration were measured during actual and simulated border irrigation of a uniform loam soil having a water table about 2 m beneath the surface.... Infiltration measurements, made under actual and simulated border irrigation, indicated that displaced air pressure, building to a maximum of about 19 cm, reduced total infiltration by about 1/3." [Dixon and Linden, 1972, p. 948].

They add:

> "This research implies that soil air pressure and its infiltration effects are not negligible as is commonly assumed by Darcy-based flow theory and that soil air can be a useful tool for controlling infiltration in some important situations."

It is seen that field observations and measurements agree well with the laboratory results. The mathematical simulations mentioned earlier [2, 35, 38] also gave results in the same direction and with the same order of magnitude. Before discussing the results from these models and the methods used to obtain them it is desirable to review briefly the traditional basis of unsaturated flow theory.

3. Traditional Derivation of Richards' Equation

It is traditional among soil physicists and hydrologists to define soil water pressure, p_w , relative to atmospheric pressure, p_A , and not only to measure it as a water height but also to express it as a water height. This quantity, ψ which has been given a variety of names (suction, tension, potential, etc.) is defined by the relation:

$$\psi = \frac{p_w - p_A}{\rho_w g} \tag{9}$$

For small tensions this quantity ψ can be measured by an apparatus called "tensiometer." Schematic diagrams of this apparatus can be found in all texts on soil science [Swartzendruber, 1969, p. 218] and even in most books on hydrology [Kirkham, 1964, p. 5.13]. It is important to note that on all these diagrams the manometric tube is clearly shown open to the atmosphere. The apparatus thus does measure precisely the quantity defined by Equation (9).

It is also traditional to define Darcy's law generalized to non-saturated flow in the form (for vertical one-dimensional flow):

$$v_w = -K_w \frac{\partial}{\partial z} (\psi - z) = -K_w \frac{\partial \psi}{\partial z} + K_w \qquad (10)$$

where K_w is the unsaturated hydraulic conductivity (a function of water content, θ) and z is the vertical coordinate oriented positive downward. Notations vary slightly depending on authors (e.g. the positive direction for the z coordinate is often chosen upward) but basically all Darcy's laws generalized to unsaturated flow [Philip, 1969, p. 220; Swartzendruber, 1969, p. 219; Hillel, 1972, p. 110; Nielsen et al., editors, 1972, p. 29] are identical to Equation (10).

Expressing mathematically the principle of mass conservation for water or equivalently of volume conservation since water is practically incompressible, one obtains:

$$\frac{\partial \theta}{\partial t} + \frac{\partial}{\partial z} \left[-K_w \frac{\partial (\psi - z)}{\partial z}\right] = 0 \qquad (11)$$

or more explicitly:

$$\frac{\partial \theta}{\partial t} - \frac{\partial}{\partial z} \left(K_w \frac{\partial \psi}{\partial z}\right) + \frac{\partial K_w}{\partial z} = 0 \qquad (12)$$

which is Richards' equation [Philip, 1969, p. 221; Swartzendruber, 1969, p. 222; Hillel, 1972, p. 110; Nielsen et al., 1972, p. 30]. If it can be assumed that the relation $\psi(\theta)$ is uniquely defined, one can define a new function of θ, called "diffusivity" namely:

$$D(\theta) = K_w(\theta) \frac{\partial \psi}{\partial \theta} \tag{13}$$

and Richards' equation transforms into:

$$\frac{\partial \theta}{\partial t} - \frac{\partial}{\partial z}(D \frac{\partial \theta}{\partial z}) + K'_w \frac{\partial \theta}{\partial z} = 0 \tag{14}$$

where K'_w is the derivative of K_w with respect to θ. Traditionally Equation (14) is called the Diffusivity equation.

4. Derivation of Equations of Flow

The traditional derivation of Richards' equation, that is as presented even in recent texts on the subject, suffers from one major drawback. It omits completely to mention a dubious assumption. Moreover it utilizes a form of Darcy's law which is not general.

In addition (or instead) of the definition of ψ, it is useful to introduce the concept of capillary pressure [DeWiest, 1969, pp. 471-482] for a water (wetting fluid) - air (non wetting) system, namely:

$$p_c = p_a - p_w \tag{15}$$

where p_a is air pressure in the soil. It appears that the introduction of the quantity ψ broadened the ditch between soil physicists and hydrologists on one side and fluid mechanists and petroleum engineers on the other side. Yet ψ and p_c are simply related by the formula of equivalence:

$$\psi = - \frac{p_c}{\rho_w g} + \frac{p_a - p_A}{\rho_w g} = -h_c + \frac{p_a - p_A}{\rho_w g} \tag{16}$$

This formula shows that ψ does not correspond numerically (and in absolute value) to the capillary pressure (expressed as a water height) if soil air pressure differs from atmospheric pressure.

It can be shown [e.g. Chow, 1973, p. 127] that Darcy's generalized law for a compressible fluid, say fluid i, has the form:

$$v_i = -K_i \frac{\partial}{\partial z} \left[\int_{p_o}^{p_i} \frac{dp}{\rho_i g} - z \right] \tag{17}$$

where K_i like K_w has the dimension of a velocity. Equation (17) is more general than Equation (10) but its form is more cumbersome. However, using the formula of equivalence:

$$K_i = \frac{k_i \rho_i g}{\mu_i} \tag{18}$$

it can easily be shown with the rules of calculus [Chow, 1973, p. 127] that Equation (17) takes the simple form:

$$v_i = -\frac{k_i}{\mu_i} \frac{\partial p_i}{\partial z} + K_i \tag{19}$$

which is as concise as Equation (10) and has the advantage of being valid for a compressible fluid.

Application of the law of mass conservation for fluid i, yields:

$$\frac{\partial(\rho_i \theta_i)}{\partial t} + \frac{\partial(\rho_i v_i)}{\partial z} = 0 \tag{20}$$

and use of Equation (19) yields:

$$\frac{\partial(\rho_i \theta_i)}{\partial t} + \frac{\partial}{\partial z}\left[\left(-\frac{k_i}{\mu_i} \frac{\partial p_i}{\partial z} + K_i \right) \rho_i \right] = 0 \tag{21}$$

The equations for water and air are obtained by substituting the subscripts w and a for i. Assuming water to be incompressible, the two equations are:

$$\frac{\partial \theta}{\partial t} - \frac{1}{\mu_w} \frac{\partial}{\partial z} (k_w \frac{\partial p_w}{\partial z}) + \frac{\partial K_w}{\partial z} = 0 \qquad (22)$$

$$\frac{\partial(\rho_a \theta_a)}{\partial t} - \frac{\partial}{\partial z} (\frac{k_a \rho_a}{\mu_a} \frac{\partial p_a}{\partial z}) + \frac{\partial (K_a \rho_a)}{\partial z} = 0 \qquad (23)$$

Using Equations (15), (16) and (18) the equation for water takes the form:

$$\frac{\partial \theta}{\partial t} - \frac{\partial}{\partial z} (D \frac{\partial \theta}{\partial z}) + K'_w \frac{\partial \theta}{\partial z} - \frac{\partial}{\partial z} (\frac{k_w}{\mu_w} \frac{\partial p_a}{\partial z}) = 0 \qquad (24)$$

which does not reduce to the traditional diffusivity equation unless both Equation (23) in toto and the last term of Equation (24) are negligible, that is if air pressure remains the same everywhere at all times. The experiments of Wilson and Luthin [1963] and of Vachaud et al., [1973] show clearly that even for open columns and for rains that do not saturate the soil surface this hypothesis is not legitimate.

5. Errors Due to Uncertainties in the Determination of the Diffusivity Function

This additional problem of the traditional approach using the Diffusivity equation will be briefly mentioned here. A paper [Morel-Seytoux et al., 1973] is devoted to this subject. The conclusion of this paper states:

> "The fundamental flaw of the traditional diffusivity formulation of unsaturated flow in porous media is that it depends on an intrinsic characteristic soil function $D(\theta)$ which is singular at natural saturation. This is not physically realistic and worse precludes both the possibility of an accurate experimental determination and of a precise objective extrapolation scheme for θ values close to and at natural saturation."

6. Conclusions

Laboratory and field results show clearly that a realistic theory of infiltration (or drainage) must include the air effects. The traditional theory currently uses an incomplete equation and completely neglects the equation of mass conservation for air, in spite of the well established character of this principle in fluid mechanics. A more complete theory requires the simultaneous solution of two equations. Apparently the solution of this system is more complex. In the next section, it will be shown that it is not necessarily the case.

C. FOR A REFORMED THEORY OF INFILTRATION. HOW?

In this section basic equations will be derived. Next it will be shown how easily the problem of Green and Ampt unresolved since 1911 can be solved with this new formulation. In addition it will be shown that the assumption of a piston displacement in the Green and Ampt formula leads to errors in the range of 20 to 70 percent, even when the correct capillary term is used in the formula.

1. Basic equations for one-dimensional vertical flow

Darcy's laws for the water and air phases are:

$$v_w = -\lambda_w \frac{\partial p_w}{\partial z} + \lambda_w \rho_w g \tag{25}$$

$$v_a = -\lambda_a \frac{\partial p_w}{\partial z} + \lambda_a \rho_a g \tag{26}$$

where λ_w is the water mobility i.e. $k \frac{k_{rw}}{\mu_w}$. The relative water mobility λ_{rw} which for convenience will be used also is $\frac{k_{rw}}{\mu_w}$.

It will be convenient to define the total velocity v as the algebraic sum of v_w and v_a , namely:

$$v = v_w + v_a \tag{27}$$

At first one may accept this concept as a pure mathematical convenience. It will turn out to be a very fruitful concept leading to remarkably simple results. As experience is gained a physical insight into v will develop.

Similarly one defines the total mobility, Λ as:

$$\Lambda = \lambda_w + \lambda_a \tag{28}$$

the total relative mobility

$$\Lambda_r = \lambda_{rw} + \lambda_{ra} \tag{29}$$

and their inverses, particularly the total viscosity:

$$\frac{1}{\Lambda_r} = \frac{1}{\frac{k_{rw}}{\mu_w} + \frac{k_{ra}}{\mu_a}} \tag{30}$$

One can see that indeed $\frac{1}{\Lambda_r}$ has the dimension of a viscosity since the relative permeabilities are dimensionless. Another useful function is the function denoted f_w defined as:

$$f_w = \frac{\lambda_w}{\lambda_w + \lambda_a} = \frac{\lambda_w}{\Lambda} = \frac{\lambda_{rw}}{\Lambda_r} \tag{31}$$

Again at this stage it may be best to regard the definition of f_w as a mathematical convenience. Later its physical significance will become clear.

By adding Equations (25) and (26) one obtains a first expression for v:

$$v = -\lambda_w \frac{\partial p_w}{\partial z} - \lambda_a \frac{\partial p_a}{\partial z} + \lambda_w \rho_w g + \lambda_a \rho_a g \tag{32}$$

Using the definition of the capillary pressure, Equation (15), water pressure can be eliminated from Equation (32) with the result:

$$v = -(\lambda_w + \lambda_a) \frac{\partial p_a}{\partial z} + \lambda_w \frac{\partial p_c}{\partial z} + \lambda_w \rho_w g + \lambda_a \rho_a g \tag{33}$$

Dividing by Λ (and using Equation (31)) one obtains a first important relation:

$$\frac{v}{\Lambda} = -\frac{\partial p_a}{\partial z} + f_w \frac{\partial p_c}{\partial z} + f_w \rho_w g + f_a \rho_a g \tag{34}$$

Equation (34) is exact. It evolved from the other equations by elementary algebraic operations. The integrated form of Equation (34) with respect to z between two arbitrary levels z_1 and z_2 will be of special interest, namely:

$$\int_{z_1}^{z_2} \frac{v}{\Lambda}\, dz = p_{a1} - p_{a2} + \int_{p_{c1}}^{p_{c2}} f_w \, dp_c + \rho_w g \int_{z_1}^{z_2} f_w \, dz + g \int_{z_1}^{z_2} \rho_a f_a dz \quad (35)$$

By now the exploitation of the concepts of total velocity, total viscosity etc. in relation to Darcy's law is complete. To proceed further fruitfully one must consider the equations of mass conservation, which are for water and for air:

$$\frac{\partial \theta}{\partial t} + \frac{\partial v_w}{\partial z} = 0 \quad (36)$$

$$\frac{\partial \rho_a \theta_a}{\partial t} + \frac{\partial (v_a \rho_a)}{\partial z} = 0 \quad (37)$$

or equivalently:

$$\frac{\partial (\phi S_w)}{\partial t} + \frac{\partial v_w}{\partial z} = 0 \quad (38)$$

$$\frac{\partial (\rho_a \phi S_a)}{\partial t} + \frac{\partial (\rho_a v_a)}{\partial z} = 0 \quad (39)$$

When compressibility effects are negligible the variable ρ_a in Equation (37) or Equation (39) factors out. Then adding Equations (38) and (39) one obtains:

$$\frac{\partial}{\partial t}[\phi(S_w + S_a)] + \frac{\partial v}{\partial z} = 0 \quad (40)$$

If the properties of the medium do not change with time, then one obtains a second important result:

$$\frac{\partial v}{\partial z} = 0 \quad (41)$$

that is the total velocity is invariant in space though not necessarily in time. This result is extremely important because it means that a great deal of information can be obtained regarding the simultaneous flow of water and air by finding the time evolution of v . However, this result was obtained at the price of two assumptions: (1) air behaves as an incompressible fluid and (2) the porosity of the medium does not change with time. Utilizing the space invariance of v , Equation (35) can be rewritten as

$$v = \frac{(p_{w1}+p_{c1}) - p_{a2} + \int_{p_{c1}}^{p_{c2}} f_w dp_c + \rho_w g \int_{z_1}^{z_2} f_w dz + \rho_a g \int_{z_1}^{z_2} f_a dz}{\int_{z_1}^{z_2} \frac{dz}{\Lambda}} \tag{42}$$

It is interesting to note that at a given time the value of v is independent of the choice of the limits of integration z_1 and z_2 . The choice of z_1 and z_2 is thus simply a matter of convenience. In the case of infiltration with ponding at the surface it is convenient to select for level 1 the soil surface $(z = 0)$ where water pressure is known: $p_{w1} = \rho_w gH + p_A$ where H is the ponding depth (possibly variable with time). If a condition of ponding always prevails at the surface $(H > 0)$ and if the capillary pressure just below the surface is within the capillary fringe $(p_{c1} < p_{ce})$ which means that soil air pressure below the surface does not exceed the value: $\rho_w gH + p_A + p_{ce}$, then $p_{c1} + \int_{p_{c1}}^{p_{c2}} f_w dp_c = \int_0^{p_{c2}} f_w dp_c$ since within the capillary fringe $f_w = 1$, and Equation (42) takes the form:

$$v = \frac{\rho_w g H + p_A - p_{a2} + \int_0^{p_{c2}} f_w dp_c + \rho_w g \int_0^{z_2} f_w dz}{\int_0^{z_2} \frac{dz}{\Lambda}} \tag{43}$$

In Equation (43) the air density term is neglected being much smaller than the water gravity term.

In summary the important equations are Equations (36) or (38) and (42). With this background, the problem of Green and Ampt can be discussed next.

2. The significance of the parameters of the law of infiltration of Green and Ampt (1911).

By "law of infiltration" is meant the relation between time (or the cumulative infiltrated volume) and the water flux entering a column of homogeneous soil, of unlimited depth, with a uniform initial moisture content, and under a permanent condition of ponding at the surface. Assuming the existence of a front separating a fully saturated zone from the zone still at the initial water content and using Darcy's law in the saturated zone, Green and Ampt obtained the relation:

$$I = \tilde{K}\,\frac{(H + Z_f + H_f)}{Z_f} = \tilde{K}\,\frac{(H + H_f)}{Z_f} + \tilde{K} = A + \frac{B}{Z_f} \tag{44}$$

where H is the depth of ponding over the surface Z_f is the position of the front and H_f is a kind of average capillary head. Since the publication of the article by Green and Ampt (1911), the two constants that appear in their law have been considered as empirical constants to be determined by experiments. Childs states without ambiguity:

> "Since H_f is not a precisely definable constant for real soils, the formula does not give I in absolute terms. Insofar as the constants A and B of Equation (44) may be determined, in practice, by observations of the

rate of infiltration at two known depths of penetration of the profile, to that extent the formula is to be regarded as empirical."* [Childs, 1969, page 276].

Another author of a more recent text states in a similar vein:

"The Green and Ampt relationships are essentially empirical, since the value of the effective wetting-front suction must be found by experiment. For infiltration into initially dry soil it may be of the order of -50 to -100 cm of water [Green and Ampt, 1911; Hillel and Gardner, 1970]. However, in actual field conditions, particularly where the initial moisture is not uniform, H_f may be undefinable. In many real situations, the wetting front is too diffuse to indicate its exact location at any particular time." [Hillel, 1970 page 143]

To the contrary it will be shown now that these parameters have a very precise physical meaning and can be deduced simply from the soil characteristic curves. Since the column is of indefinite extent the air pressure just ahead of the wetting front can be reasonably estimated at atmospheric pressure, compressibility effects being minor. Equation (43) is applicable and yields:

$$I = \frac{\rho_w g H + \int_0^{\rho_{c2}} f_w dp_c + \rho_w g \int_0^{z_2} f_w dz}{\int_0^{z_2} \frac{dz}{\Lambda}} \tag{45}$$

where the subscript 2 refers to a position in the zone of initial uniform water content just downstream from the front. Since, in addition over most of the wet zone f_w is essentially 1, Equation (45) becomes with good accuracy:

$$I = \frac{\rho_w g H + \int_0^{p_{ci}} f_w dp_c + \rho_w g z_2}{\int_0^{z_2} \frac{dz}{\Lambda}} \tag{46}$$

---

*In the quotations throughout this report the original symbols and equation numbers may have been changed to conform to the notations of this report.

It is important to note that Equation (46) has the elementary form of Darcy's law that is:

$$I = \frac{\Delta\Phi}{R} \tag{47}$$

where $\Delta\Phi$ is the potential drop and R is the viscous resistance. Equation (46) shows that the potential drop can be evaluated practically with accuracy without determining the water content profile. The determination of the water content profile is necessary only to evaluate the viscous resistance. A coarse approximation to this profile is probably sufficient to calculate R. With the Green and Ampt profile, one obtains:

$$I = \frac{\tilde{K}(H + Z_f + \int_0^{h_{ci}} f_w \, dh_c)}{Z_f} \tag{48}$$

therefore the result searched for by identification with Equation (44):

$$H_f = \int_0^{h_{ci}} f_w \, dh_c \tag{49}$$

where h_c is the capillary pressure head (i.e. the capillary pressure expressed as a water height). Figure 3 shows a typical curve of f_w versus h_c, based on experimental data [Vauclin, 1971; Gaudet, 1972].

In summary the precise physical meaning of the undetermined empirical parameter H_f (or B) of Green and Ampt has been found. H_f is related to the soil characteristic curves (capillary pressure, relative permeabilities) and, which is very important, the functional dependence of H_f on the initial water content is known.

In the next sections it will be shown, however, that the Green and Ampt assumed profile is too coarse to evaluate the viscous resistance term. Fortunately a simple and accurate approximation can be

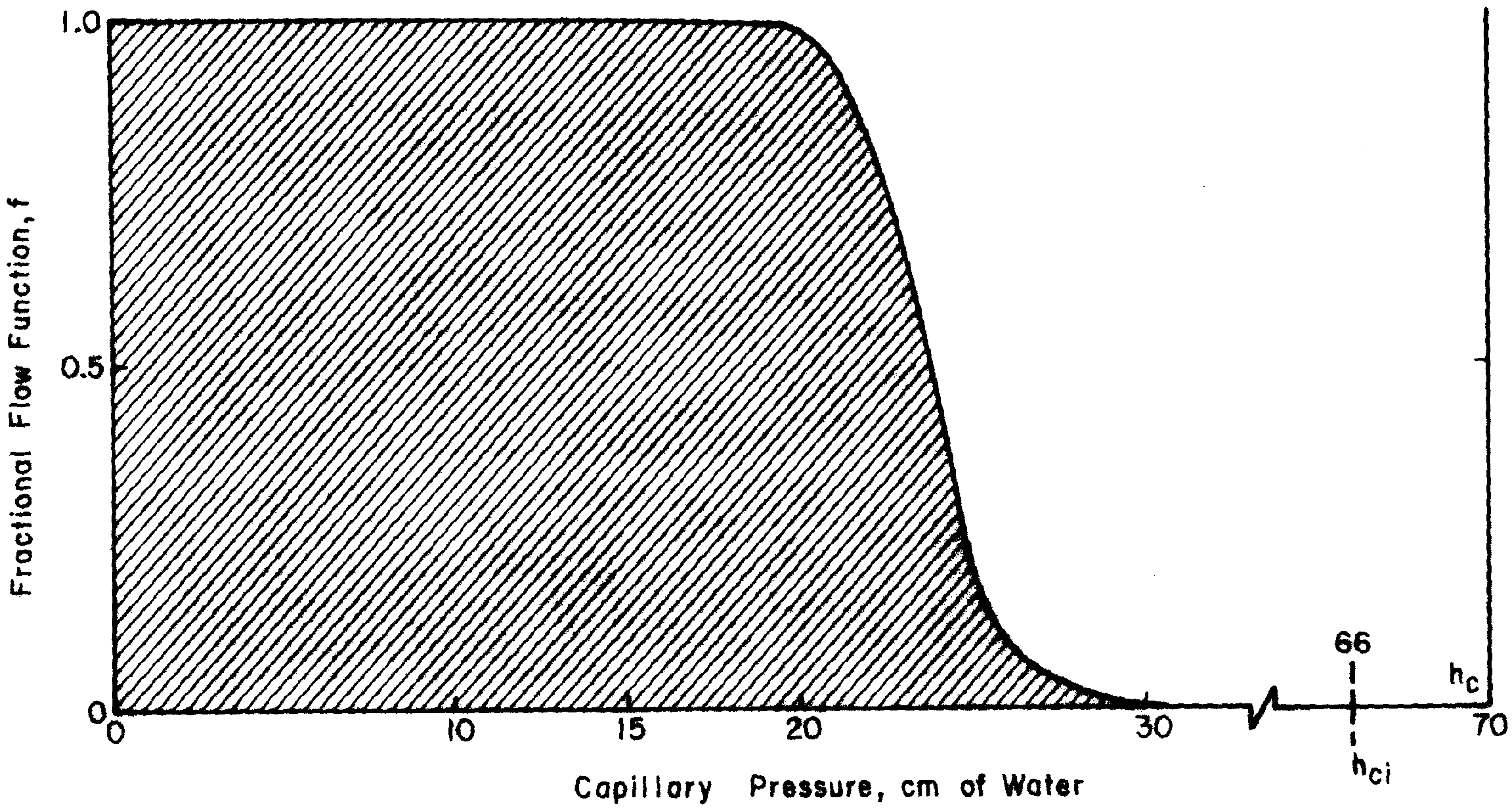

Figure 3. Fractional flow function versus capillary pressure (expressed as a water height). Cross-hatched area expresses the effective capillary drive, in cm of water.

found. First the concept of the fractional flow function is introduced.

3. The fractional flow function

The water flow rate v_w is only a fraction of the total flow rate v . Formally the fractional flow function is naturally defined as:

$$F_w = \frac{v_w}{v} \tag{50}$$

Nothing is gained by this mere definition. However, a more explicit expression for F_w can be obtained by elementary algebraic operations [Chow, 1973, page 141] with the result:

$$F_w = f_w\{1 + \frac{\lambda_a p_c'}{v} \frac{\partial S}{\partial z} + \frac{\lambda_a}{v} \Delta\rho g\} \tag{51}$$

where $\Delta\rho = \rho_w - \rho_a$. For convenience in writing F_w can be condensed as:

$$F_w = f_w + \frac{G_w}{v} - \frac{E_w}{v}\frac{\partial S}{\partial z} \tag{52}$$

where G_w and E_w are two functions of saturation, defined as:

$$G_w = \lambda_a f_w \Delta\rho g = \frac{kk_{ra}}{\mu_a} f_w \Delta\rho g \tag{53}$$

$$E_w = \lambda_a f_w p_c' = \frac{kk_{ra}}{\mu_a} f_w p_c' \tag{54}$$

which are nonnegative, both taking the value zero at $S = \tilde{S}$ and at $S = S_{wr}$.

Using Equation (50) the equation of water conservation Equation (38) takes the form:

$$\frac{\partial(\phi S)}{\partial t} + \frac{\partial}{\partial z}(vF_w) = 0 \tag{55}$$

If again the assumptions of incompressibility for air and of indeformability for the porous medium are made, Equation (55) takes the form:

$$\phi \frac{\partial S}{\partial t} + v \frac{\partial F_w}{\partial z} = 0 \tag{56}$$

or

$$\phi \frac{\partial S}{\partial t} + vF'_w \frac{\partial S}{\partial z} = 0 \tag{57}$$

where F'_w is shorthand notation for $\left.\frac{\partial F_w(S,t)}{\partial S}\right|_t$. From Equation (57) one deduces readily that the velocity at which a given saturation propagates through the porous medium is:

$$\left(\frac{dz}{dt}\right)_S = \frac{v}{\phi} F'_w \tag{58}$$

The explicit formula for F'_w is:

$$F'_w = f'_w + \frac{G'_w}{v} - \frac{E'}{v} \frac{\partial S}{\partial z} - \frac{E \frac{\partial^2 S}{\partial z^2}}{v \frac{\partial S}{\partial z}} \tag{59}$$

Equations (58) and (59) tell that the instantaneous velocity of propagation of a given saturation is not only a function of the value of that saturation but also of the slope of the saturation profile and of the curvature of the profile at that saturation. However, if the capillary terms are neglected in Equation (59), that is in Equation (57), then the instantaneous velocity of propagation is only a function of the value of the saturation. If the capillary terms are neglected then Equation (57) takes the form:

$$\phi \frac{\partial S}{\partial t} + (vf'_w + G'_w) \frac{\partial S}{\partial z} = 0 \tag{60}$$

and for horizontal flow:

$$\phi \frac{\partial S}{\partial t} + vf'_w \frac{\partial S}{\partial x} = 0 \tag{61}$$

which is the Buckley-Leverett equation [Buckley and Leverett, 1942; DeWiest, 1969, page 490].

4. Prediction of imbibition in a horizontal column

The problem is the same as in section 2 except that the column is horizontal and that a condition of saturation at the entrance is maintained with the use of a porous plate. Proceeding as before from Equation (42) [Chow, 1973, page 144] one obtains for v the expression:

$$v = \frac{\rho_w g \int_H^{h_{ci}} f_w dh_c}{\int_0^L \frac{dx}{\Lambda}} \tag{62}$$

where H now represents the capillary pressure on the upstream side of the porous plate and L is the column length. At this stage the problem is again with the evaluation of the viscous resistance. The effective capillary drive is known exactly.

Again to evaluate the viscous resistance only an approximate solution of the saturation equation is needed. Instead of Equation (57), Equation (61) can be used. Though capillary effects on the shape of the profile have been neglected the equation still describes properly the viscous interactions between the two flowing phases. On the other hand the piston flow model of Green and Ampt fails to model this interactive effect.

The solution of the Buckley-Leverett equation is obtained readily [DeWiest, 1969, page 491]. From this solution (and temporarily neglecting the effect of the porous plate on the resistance to flow) one obtains by substitution in Equation (62):

$$v = \frac{k\rho_w g \int_H^{h_{ci}} f_w dh_c}{\frac{W}{\phi} \int_{S_{BL}}^{\tilde{S}} \frac{f_w'' ds}{\Lambda_r}} \tag{63}$$

where W is the cumulative volume of imbibition (expressed as a depth of water) and S_{BL} is the value of saturation just upstream of the Buckley-Leverett front. Equation (63) can be rewritten in the form:

$$v = \frac{\tilde{K} H_f}{\beta\left(\frac{W}{\tilde{\theta}-\theta_i}\right)} \tag{64}$$

where θ_i is the initial water content, H_f is the integral in the numerator of Equation (63) and where β can be viewed as a correction factor to the formula of Green and Ampt (since $W = (\tilde{\theta}-\theta_i)Z_f$) . For the sand used in the experiments the calculated value of β was 1.36. This means that the use of the Green and Ampt formula in the horizontal case would have overpredicted the imbibition rate with a relative error of 36 percent, even though the effective capillary drive (H_f) is known exactly. Of course, the word error is used here purposefully because Figure 4 shows a comparison of the experimental infiltration rates and of the ones calculated by an equation of the type of Equation (63) but corrected for the presence of the porous plate impedance [Morel-Seytoux et al., 1973]. The relative error on Figure 4 is less than 5 percent, for a $\beta = 1.36$. Clearly for $\beta = 1$, i.e. with no correction, the overprediction would have exceeded 36 percent. Since the error in the Green and Ampt formula comes entirely from the assumption of piston

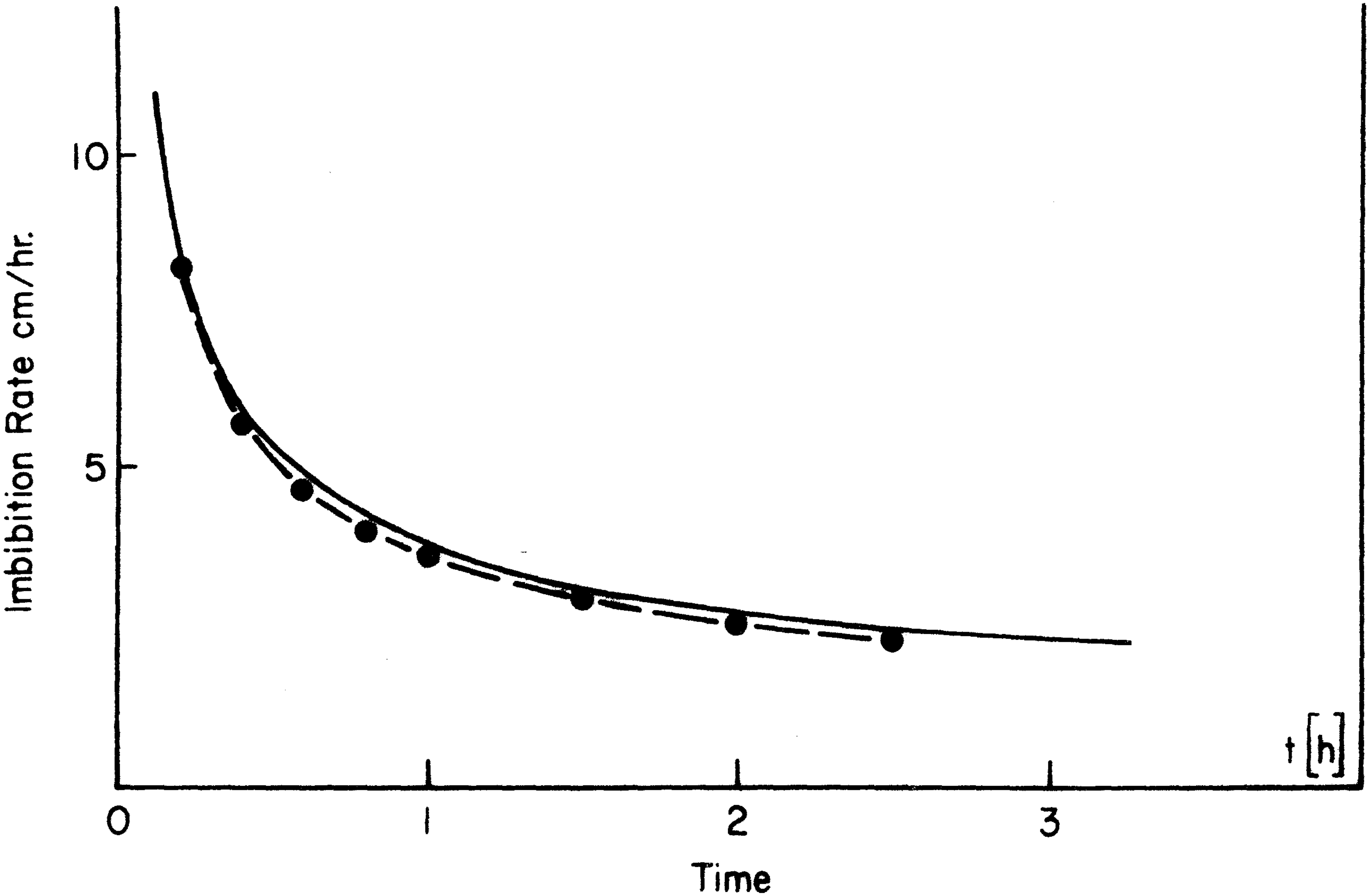

Figure 4. Comparison of observed (·) and predicted (solid line) imbibition rates

displacement behind the front (thus $\beta \equiv 1$) and not from errors in the estimate of the effective capillary drive which is now known exactly, the approximate but highly accurate evaluation of the viscous resistance term using a Buckley-Leverett type of water content profile provides a measure of the error in the Green and Ampt formula. More constructively this approximation yields an accurate value of the correction factor.

5. The viscous resistance correction factor

The correct form of the Green and Ampt formula is:

$$I = \frac{1}{\beta}\left\{\frac{\tilde{K}\left(H+H_f + \dfrac{W}{\tilde{\theta}-\theta_i}\right)}{\dfrac{W}{\tilde{\theta}-\theta_i}}\right\} \tag{65}$$

where all the terms have been defined before. H_f and β can be calculated from the soil characteristic curves, according to Equation (49) for H_f and by the following formula for β [Morel-Seytoux and Khanji, 1973]:

$$\beta = \frac{(\tilde{\theta}-\theta_i)}{\mu_w} \int_{\theta_{BL}}^{\tilde{\theta}} \frac{-f''d\theta}{\dfrac{k_{rw}}{\mu_w} + \dfrac{k_{ra}}{\mu_a}} \tag{66}$$

In this formula f'' means $d^2 f_w/d\theta^2$ and the relative permeabilities are defined relative to $\tilde{k}_w$ rather than k. In other words at $\theta = \tilde{\theta}$, $k_{rw} = \tilde{k}_{rw} = 1$ by definition.

Values of β have been calculated for six soils. The results are shown in Table 1 [Morel-Seytoux, 1973]. Clearly the order of magnitude of the correction is significant.

Table 1

Values of the viscous correction factor in the formula of Green and Ampt for various soils

Soil type	Isère sand	Plainfield sand	Columbia sandy loam	Guelph loam	Ida silt loam	Yolo light clay
β	1.36	1.40	1.45	1.30	1.10	1.70
$\frac{1}{\beta}$	0.73	0.7	0.7	0.8	0.9	0.6

6. Conclusions

In this chapter it has been shown that even when air can escape freely from the porous medium ahead of a wetting front the air effects are already important. The effect in this case is caused by the non-negligible increase of the total viscosity over the water viscosity for saturations below and close to the natural saturation. It has also been shown that by a more physical approach, i.e. an approach that truly considers the simultaneous movement of water and air, the mathematical problem is actually simpler. The fact that the problem of Green and Ampt could be solved simply in a matter of a few lines [27] whereas it had defied solution since 1911 should be conclusive evidence.

In addition air exit ahead of the wetting front may be impeded due to the presence of less pervious or completely impervious layers. Additional reductions of infiltration will occur due to the compressibility of air, and due to the interaction of compressibility and capillary barrier effects. In the next chapter the effect of air compressibility will be investigated in the absence of capillary barrier effects.

D. AIR COMPRESSIBILITY EFFECTS

The major theoretical thrust of the project in this regard has been toward the extension of the Brustkern procedure [2,24] because the work of Noblanc [24,30,35] and Phuc [24,38] indicated that the approximations involved were of little consequence in the prediction of infiltration. This is a happy result because the Brustkern approach is simpler, and its computer costs are lower and it is more readily generalizable than the Noblanc procedure.

First it was necessary to eliminate the limitations of Brustkern's work with respect to boundary conditions at the soil surface (only ponding in the work of Brustkern), to initial conditions (uniform and immobile profile in the work of Brustkern), to hysteresis (not considered in the work of Brustkern) and to lower boundary conditions (impervious boundary for finite depth column in the work of Brustkern). This means that many constraints had to be relaxed and it has been done successfully as will be illustrated later. However, the most interesting and difficult extension was to consider the effect of heterogeneities in soil characteristics. This has been done with a maximum provision of six layers in the computer program and it has been run for two layers. The results are reasonable but they have not been tested yet against analytic or experimental results. Much of this work is documented in the Ph.D. dissertation of Mr. J. Sonu. Sample results from Mr. Sonu's work are now presented.

1. Comparison with experimental data

First it may be worthwhile to show that the new computer program can perform all calculations feasible with the (old) Brustkern program.

Figure 37, Reference 8, page 175 shows a comparison of experimental results by McWhorter [8,41] using oil as the wetting fluid, and of calculations by Sonu. The porous medium in the experiments was a sand for which the capillary pressure, drying and wetting, and the relative oil permeability curves as well as the sand intrinsic permeability and porosity were determined experimentally. The only missing piece of information for the data to be complete was the relative air permeability curve. The relative air permeability used in the computer simulation was derived from the capillary pressure and relative water permeability curves following the method suggested by Brooks and Corey [41].

In the case of an open end column the experimental and calculated curves have a typical monotonically decreasing behavior. The agreement between the two curves is fair. Because the air permeability curve was not actually measured the difference between the two curves may be due to this factor. More probably, since the curves for the 185 cm closed column match very well during the first 10 minutes, the difference is due to too large a time step between the time 3 and 6 minutes. Note that large time steps in the Brustkern procedure do not create instability, but they affect accuracy.

In the case of the 185 cm closed end column the agreement is excellent for the first 10 minutes. Figure 5 shows calculated water saturation profiles. At time 784 sec (about 13 minutes) when the agreement between the two infiltration curves is still very good, it can be seen that desaturation is taking place in the column. Note that at this time air counterflow is occurring but no air escapes yet because the air saturation at the surface is still residual. At time

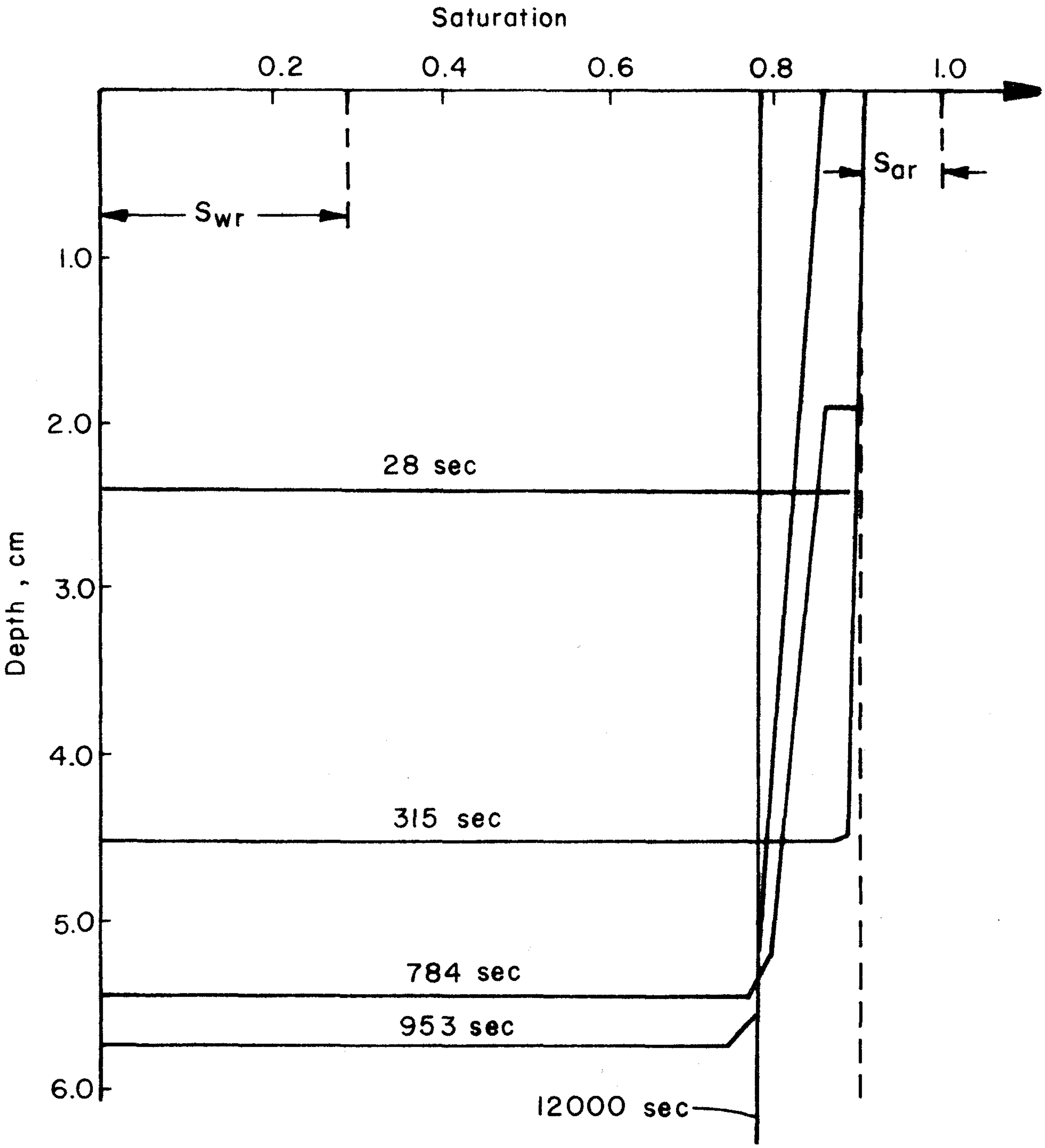

Figure 5. Evolution of saturation profile for a 185 cm, closed end, Poudre sand column under constant ponding of 0.8 cm. Initially the saturation is zero throughout the column

953 sec (about 16 minutes) air has escaped through the top of the column. The exact moment of air breakthrough was not caught in the simulation but it can be inferred that it occurred approximately at time 14 minutes.

The experimental curve after 14 minutes shows a fast recovery. The simulated curve does not. It was observed during the experiments that the upper portion of the soil column was disturbed by the air upward thrust. We speculate that after time 14 minutes the medium is no longer homogeneous but rather consists of two layers; one disturbed layer with greatly improved permeability on top of a deep layer of undisturbed soil. The rapid rise in infiltration corresponds to the rush of water into the more pervious layer. Once this layer saturates, the characteristic behavior of the infiltration curve is again that corresponding to ponding above the original soil column. Indeed the two curves join again for large times. Since the simulation after time 14 minutes was continued under the assumption that the medium was undisturbed, the two curves cannot be expected to match. On the other hand the fact that the two curves do reattach for large times seems to indicate that only a fraction of the soil column is perturbed. From the simulation results of Figure 5 one can infer that the approximate depth of the disturbed layer is of the order of 6 cm. It is felt that this comparison of the experimental data with the simulation results is extremely encouraging.

Figure 6 shows the relative importance of the various driving (or retarding) mechanism of flow, prior to air escape. The "Integral"

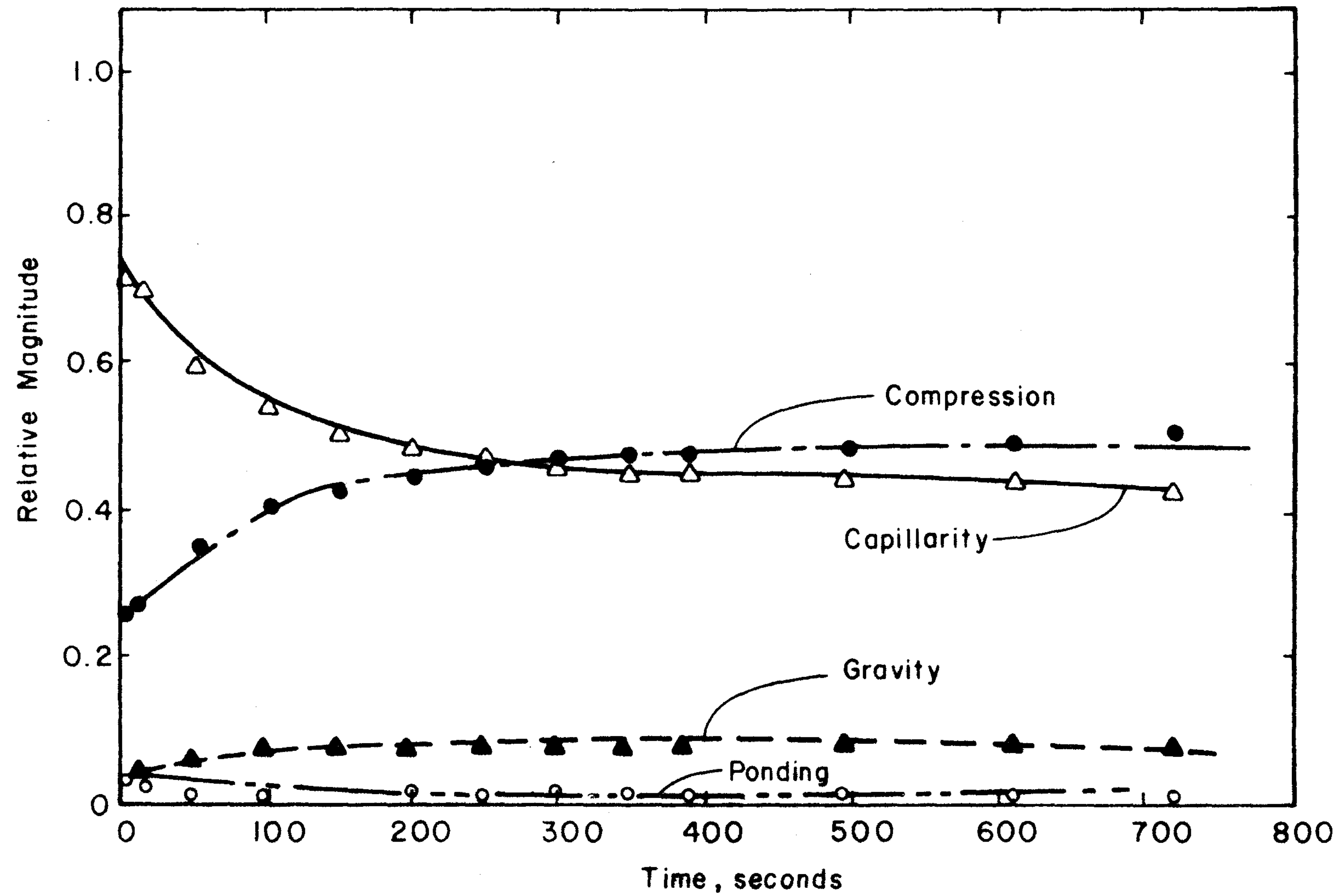

Figure 6. Relative magnitude of the four different kinds of driving mechanisms of flow, in a 185 cm deep column of Poudre sand with closed bottom

equation [Eq. (123), Ref. 8, p. 158] for $\bar{v}$ has the appealing elementary form of Darcy's law, namely:

$$\bar{v} = \Gamma \Delta \Phi \tag{67}$$

where $\bar{v}$ is the (total) flow, Γ is the (instantaneous) conductance and $\Delta\Phi$ is the (instantaneous) potential drop. From Equation (123) one can see that in the case of a ponding condition the potential drop (the numerator of the right-hand side) is made of four terms which are easily recognizable:

1. a ponding term: $\rho_w gH$, where H is the depth of ponding,

2. a compressibility (or elastic) term: $p_c(S_u) + p_A - p_{a2}$, where S_u is the saturation at the soil surface (on the soil side), p_A is atmospheric pressure and p_{a2} is the air pressure ahead of the wetting front. (Note that in this case this term is negative and reduces the overall potential drop. An expression for p_{a2} as a function of time is given as Eq. (137), Ref. 8, p. 160),

3. a capillary term: $\int_1^2 f_w \, dp_c$, and

4. a gravity term: $\Delta\rho g \int_1^2 f_w \, dz$.

On Figure 6 the magnitude (i.e. absolute value) of each term relative to the sum of all terms (in absolute value) is shown as a function of time. At time past 700 sec, the compression term adds up practically to the other terms and cancels them in the $\bar{v}$ equation. Around that time $\bar{v}$ must be close to zero. There must exist a time when $\bar{v}$ first reaches zero and when this happens the infiltration rate also drops to zero, momentarily until air escapes from the surface. Figure 6 is very informative, because it tells (or can tell) which approximate form of the saturation equation is acceptable and when. In the Brustkern precedure it is always basically assumed that the gravity term is the predominant term to shape the saturation profile. When it is not so, as shown on Figure 6 for the first 700 seconds, a much better determination of the saturation profile (but hardly different infiltration rate) would be obtained by solving the saturation equation without gravity. That equation is not Richards' equation for the horizontal case because the compressibility effect is of the same order as the capillary one. Thus Equation (123) is not just important because it is needed to calculate $\bar{v}$ (an absolute necessity in the Brustkern, Noblanc and Sonu procedures) but it provides a continual check on the relative magnitude of the driving mechanisms.

2. Relaxation of boundary and initial conditions

In the work of Brustkern the initial uniform saturation had to be immobile. This constraint has been relaxed. Figure 7 shows a comparison of infiltration rates for the case of a zero initial saturation and a mobile 50 percent initial saturation.

In the work of Brustkern it was found that hysteresis would come to play even during infiltration, but this effect was not incorporated in the calculations. Now it is and Figure 8 shows a comparison. In Figure 37, Reference 8, page 175, the effect of hysteresis was not included. With effect of hysteresis the fit with the experimental data will be improved but only for a few minutes prior to the disturbance of the soil upper zone.

Another important relaxation of the limitations of Brustkern work is in the type of lower boundary conditions that can be handled. In the work of Brustkern the lower boundary was an impervious boundary. Now

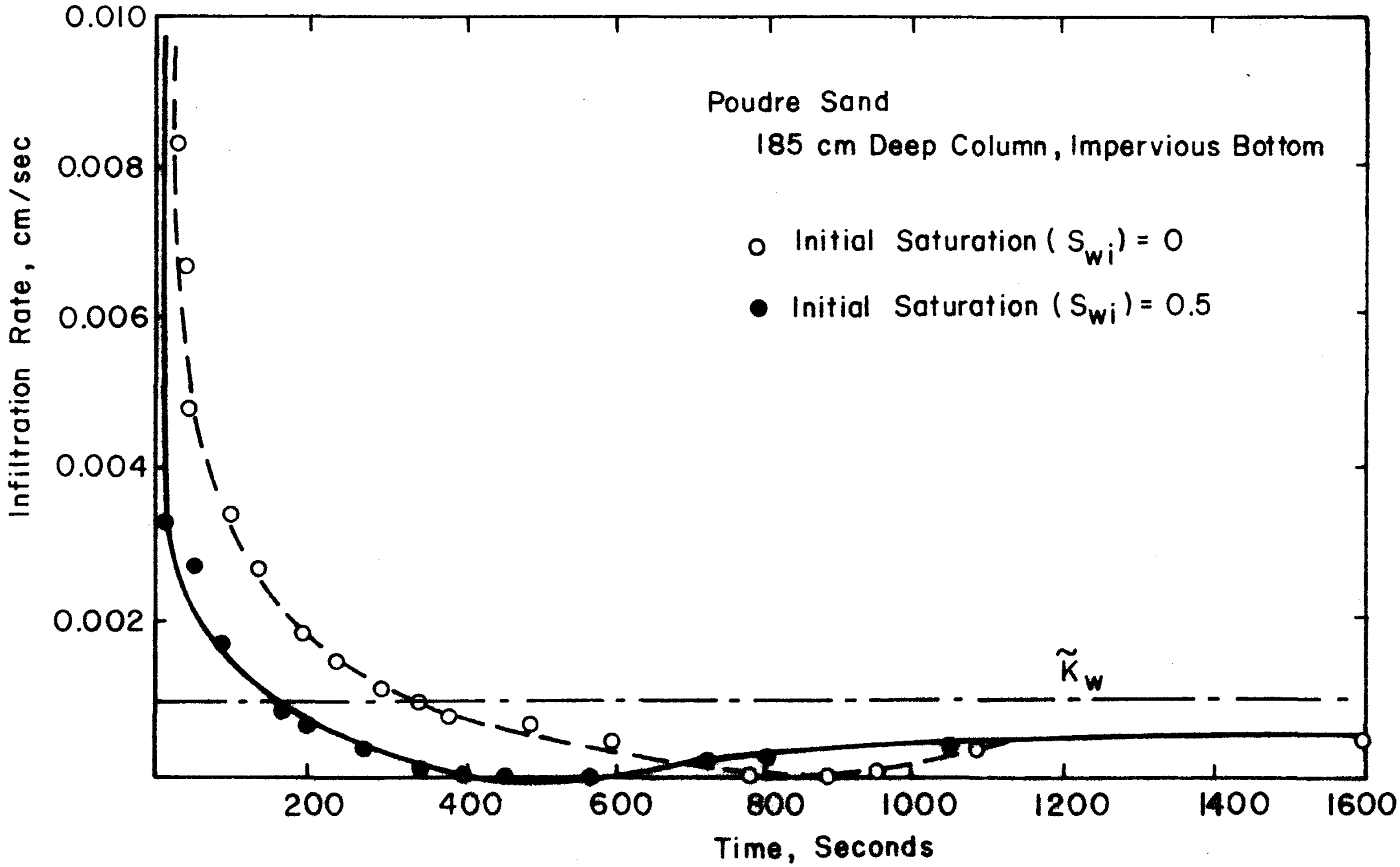

Figure 7. Infiltration rates for different uniform initial saturation profile

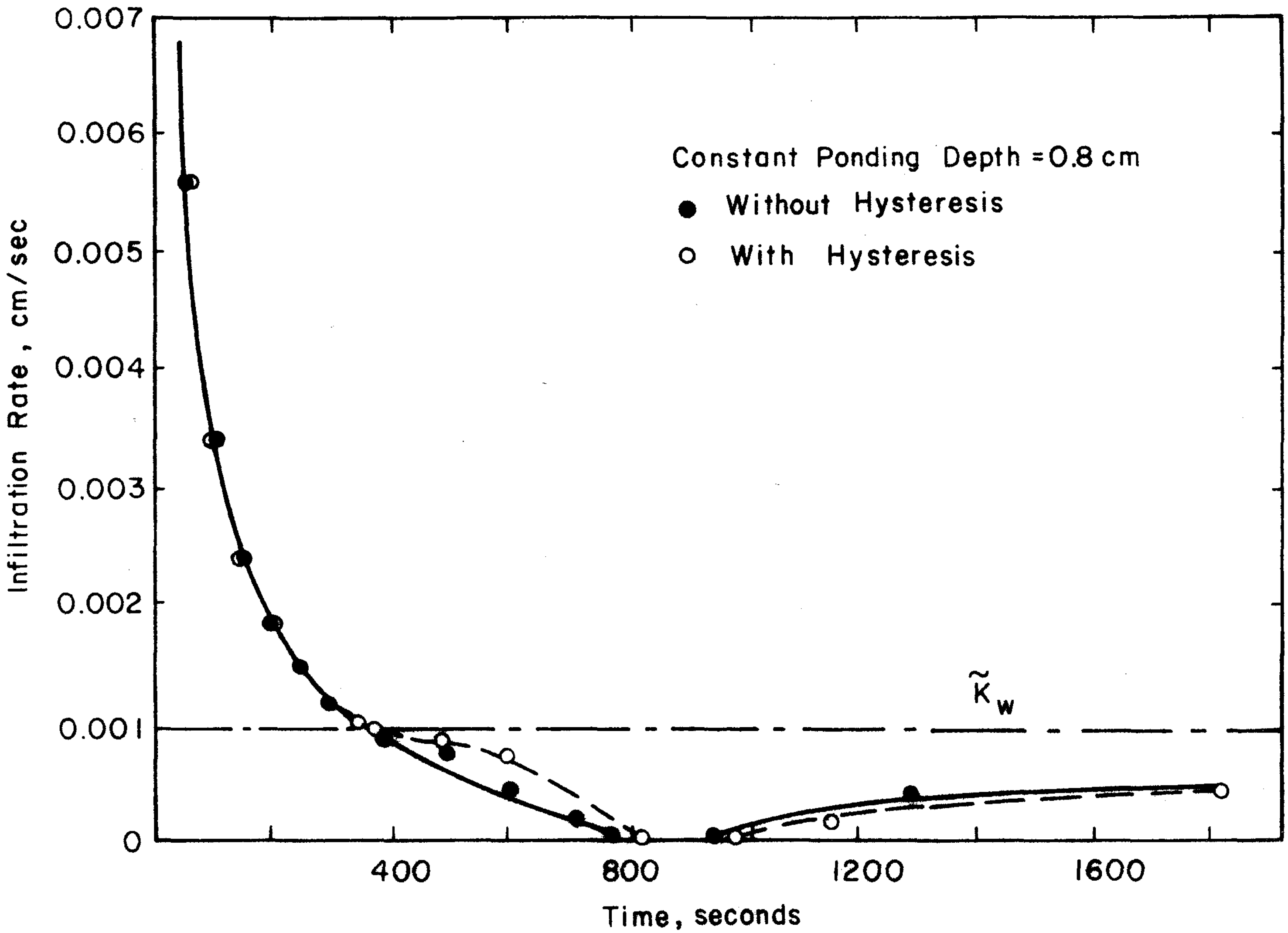

Figure 8. Effect of hysteresis on infiltration rate for a 185 cm deep, closed bottom, column of Poudre sand

a condition of flux across the lower boundary can be imposed and with the relaxation also of the condition that the initial profile must be uniform it is possible to predict infiltration rates under various water table conditions. For illustration two cases will be considered. In both cases a constant ponding condition is maintained at the upper surface. In one case water is drawn from the bottom of the saturated zone at a rate of 0.002 cm^3 per sec per cm^2 of bulk soil cross section or 0.002 cm/sec, in the other it is injected at the bottom at a rate of 0.001 cm/sec. Figure 9 shows the evolution of the saturation profile, for the first case, water penetration at the surface, drainage at the bottom and drawdown of the water table level. Figure 10 shows the rise of soil air pressure with time and the infiltration rate curve for this case. Figure 11 shows the evolution of the saturation profiles when the water table rises and Figure 12 the air pressure and infiltration curves. The comparisons of Figures 9 and 11 and of Figures 10 and 12 are very interesting. Within the framework of the two-phase theory (or more specifically the liquid-gas theory) the interpretation of the results is immediate. Without any doubt here is a case where the introduction of the air phase simplifies rather than complicates the solution of the mathematical problem.

The last relaxation is relative to the upper boundary condition. Instead of being limited to a condition of ponding at the surface, any sequence of rainfall events can be simulated. Figure 13 shows a hyetograph and the resulting infiltration rates. Figure 14 shows the evolution of the saturation profiles.

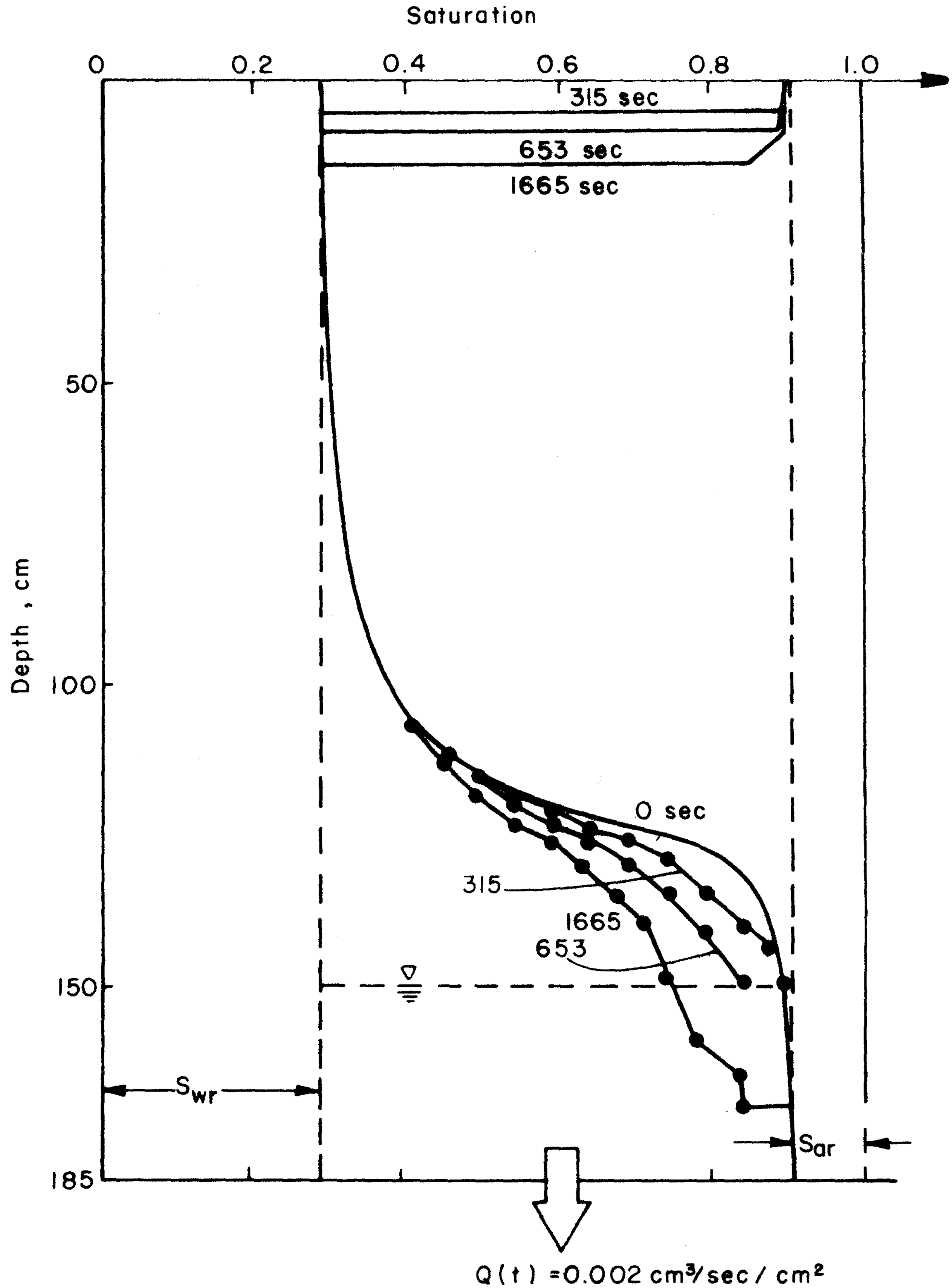

Figure 9. Evolution of saturation profile for a withdrawal rate of 0.002 $cm^3/(sec\text{-}cm^2)$

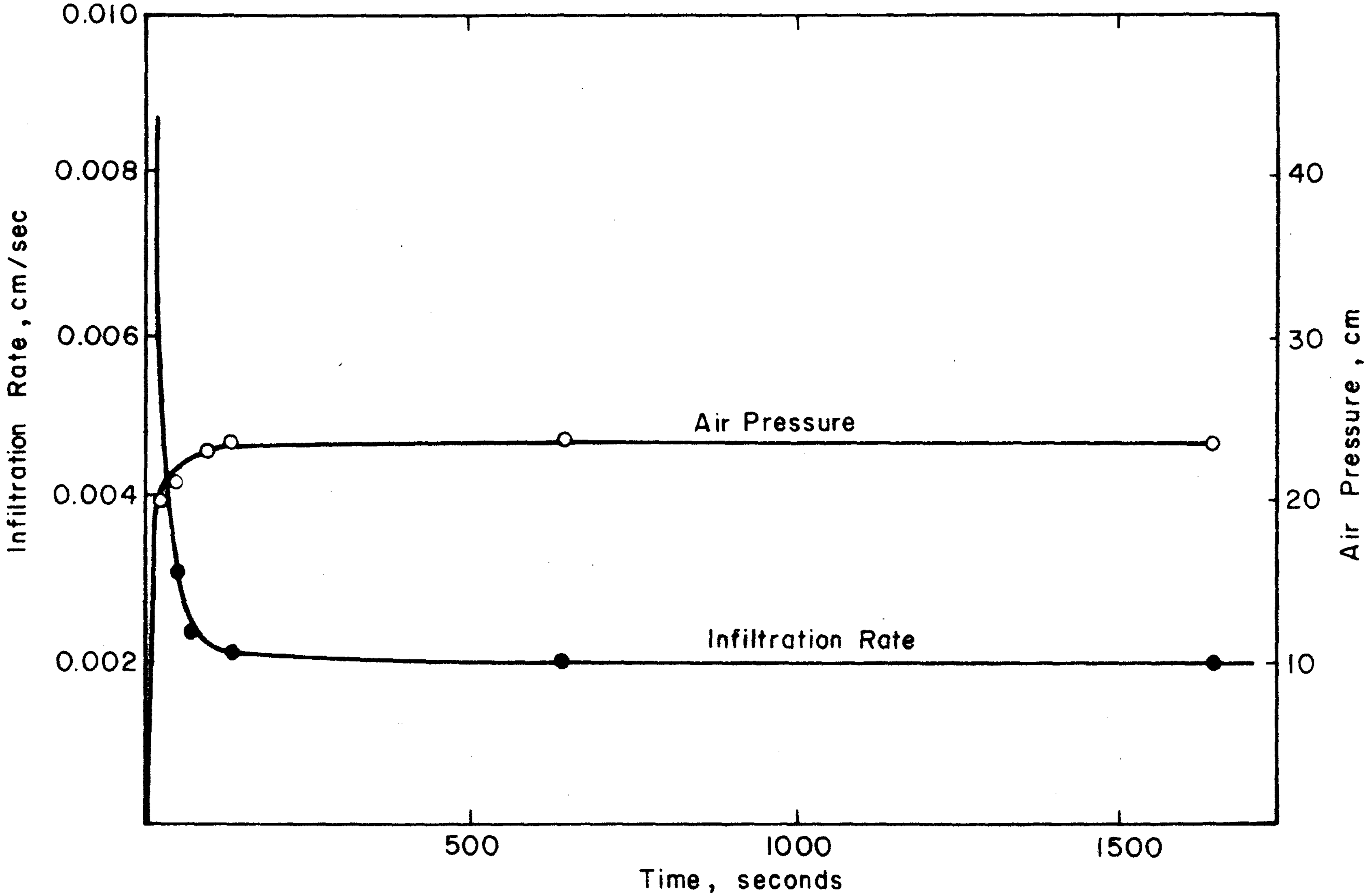

Figure 10. Curves of infiltration rate and air pressure versus time for a withdrawal rate of 0.002 $cm^3/(sec\text{-}cm^2)$

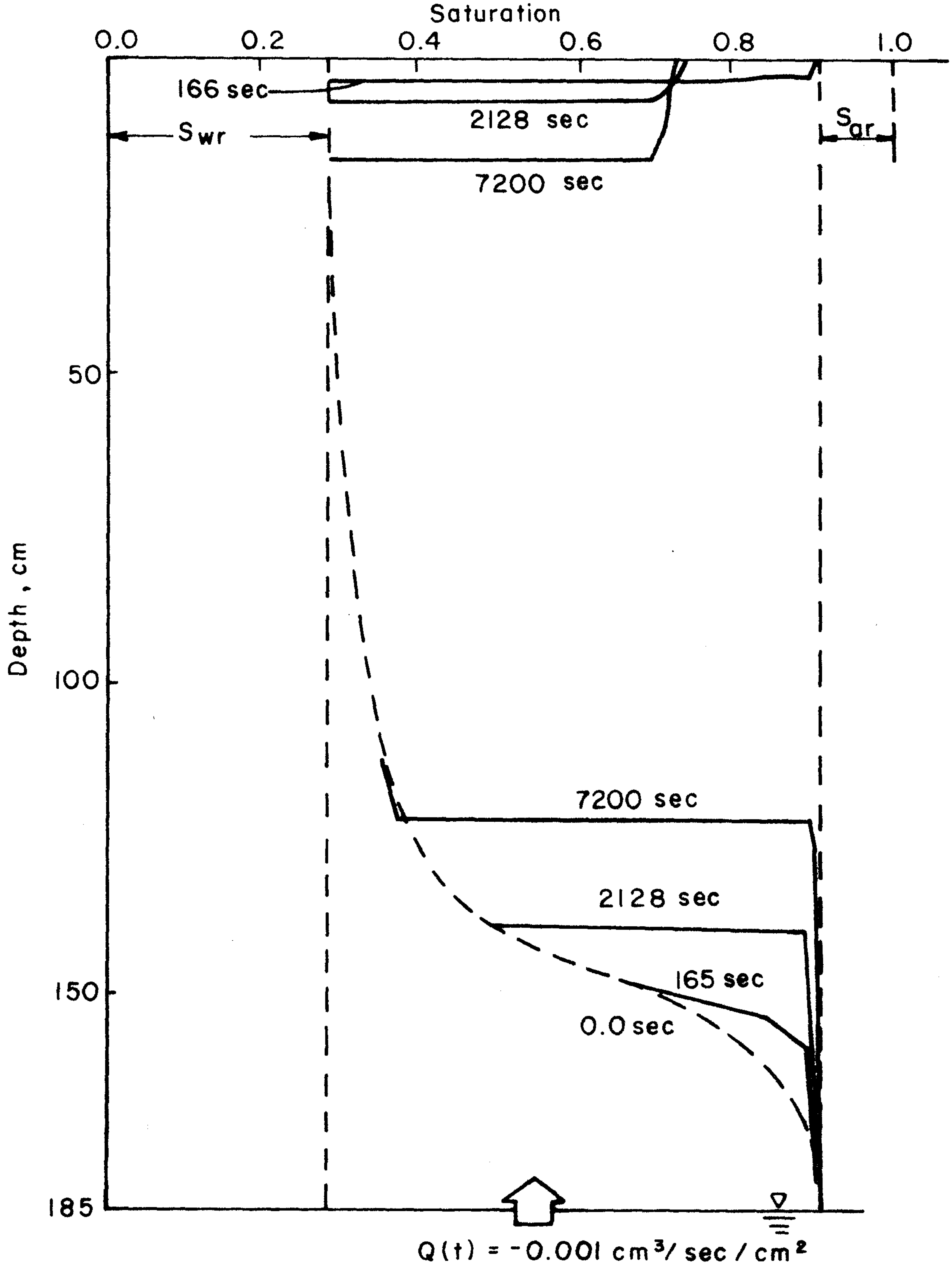

Figure 11. Evolution of saturation profiles for an infection rate of 0.001 $cm^3/(sec\text{-}cm^2)$

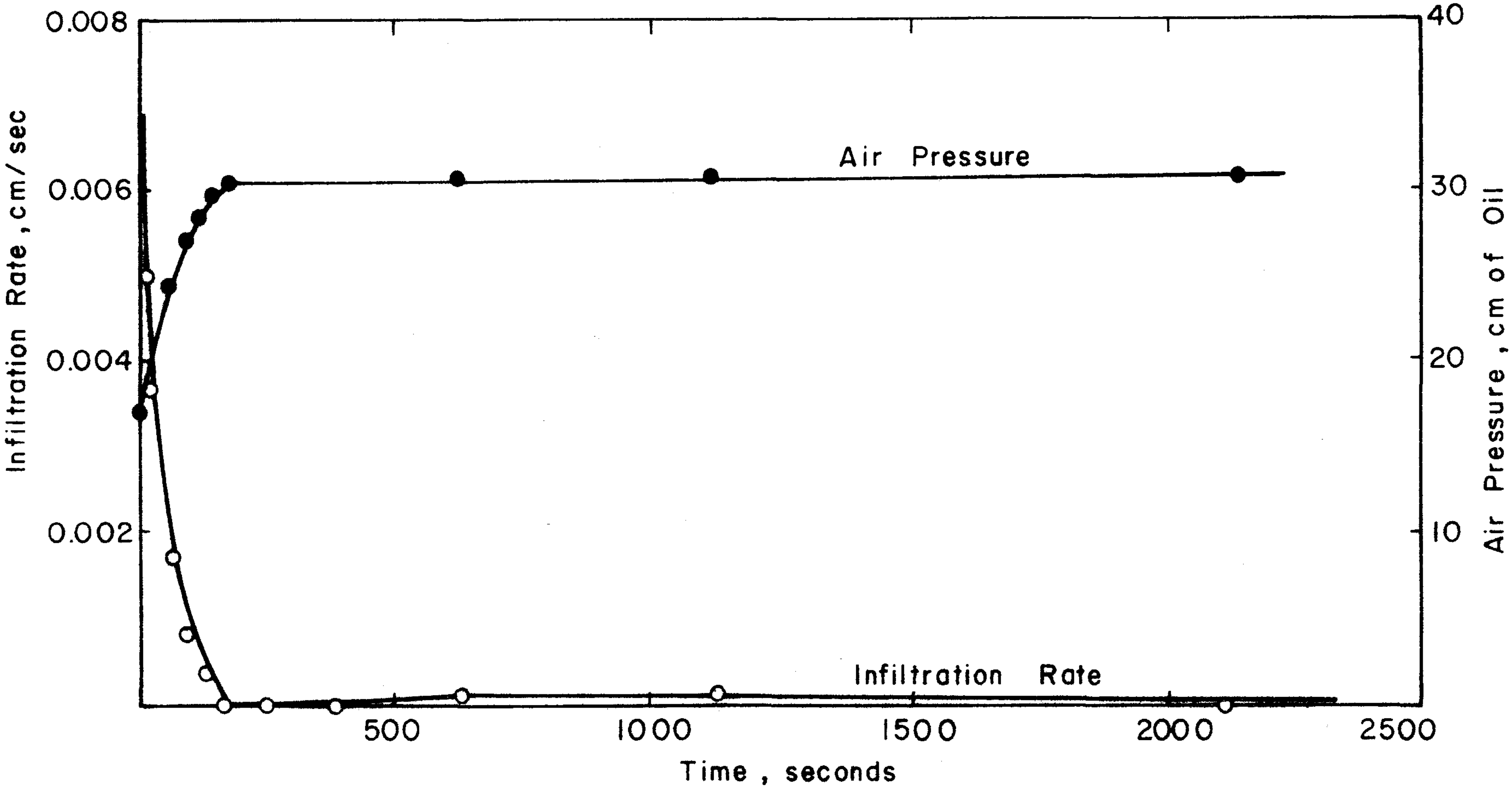

Figure 12. Curves of infiltration rate and air pressure versus time for an infection rate of 0.001 $cm^3/sec\text{-}cm^2$)

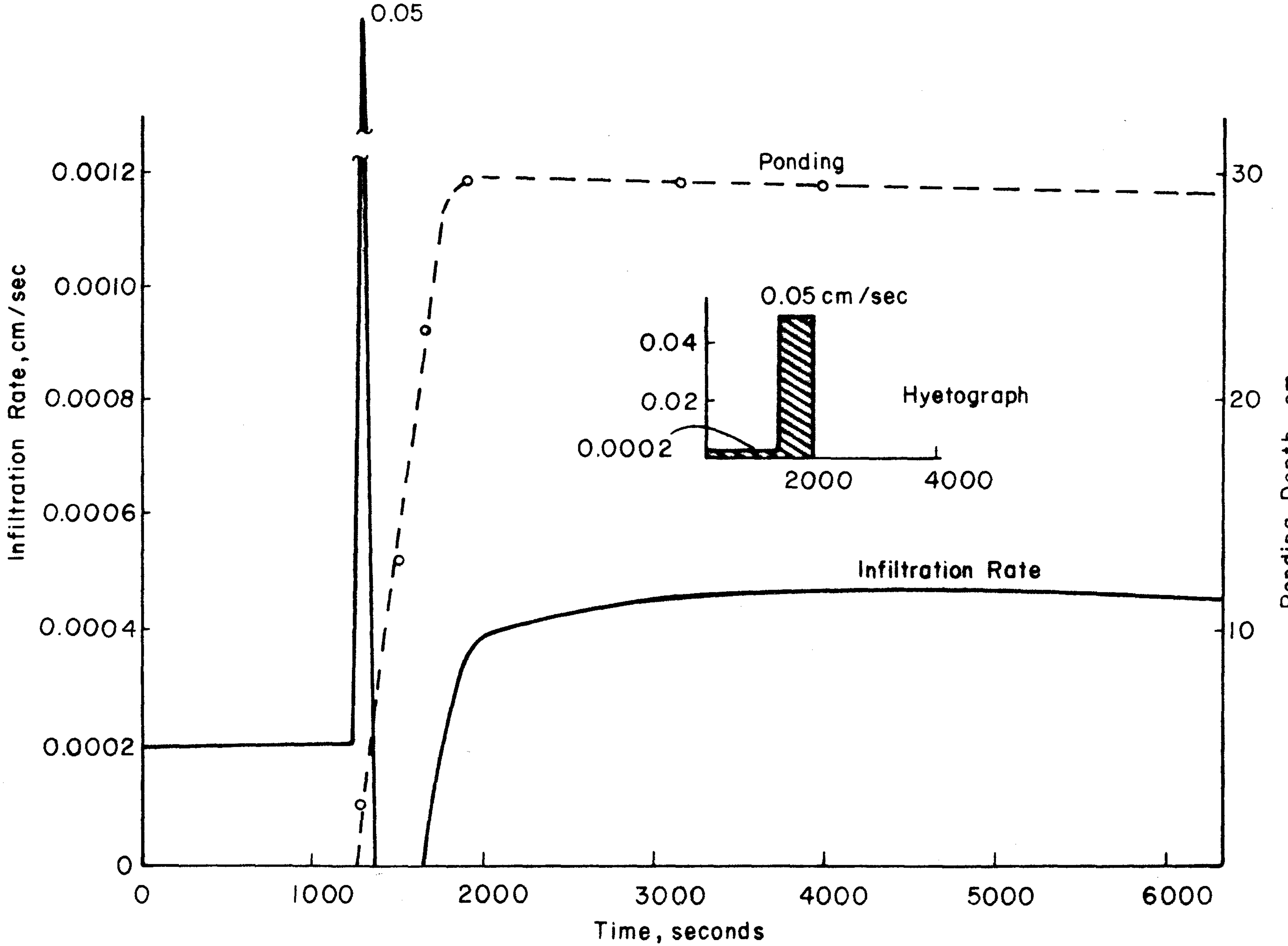

Figure 13. Infiltration rate and ponding depth response to a given hyetograph in a 185 cm deep, closed bottom, column of Poudre sand

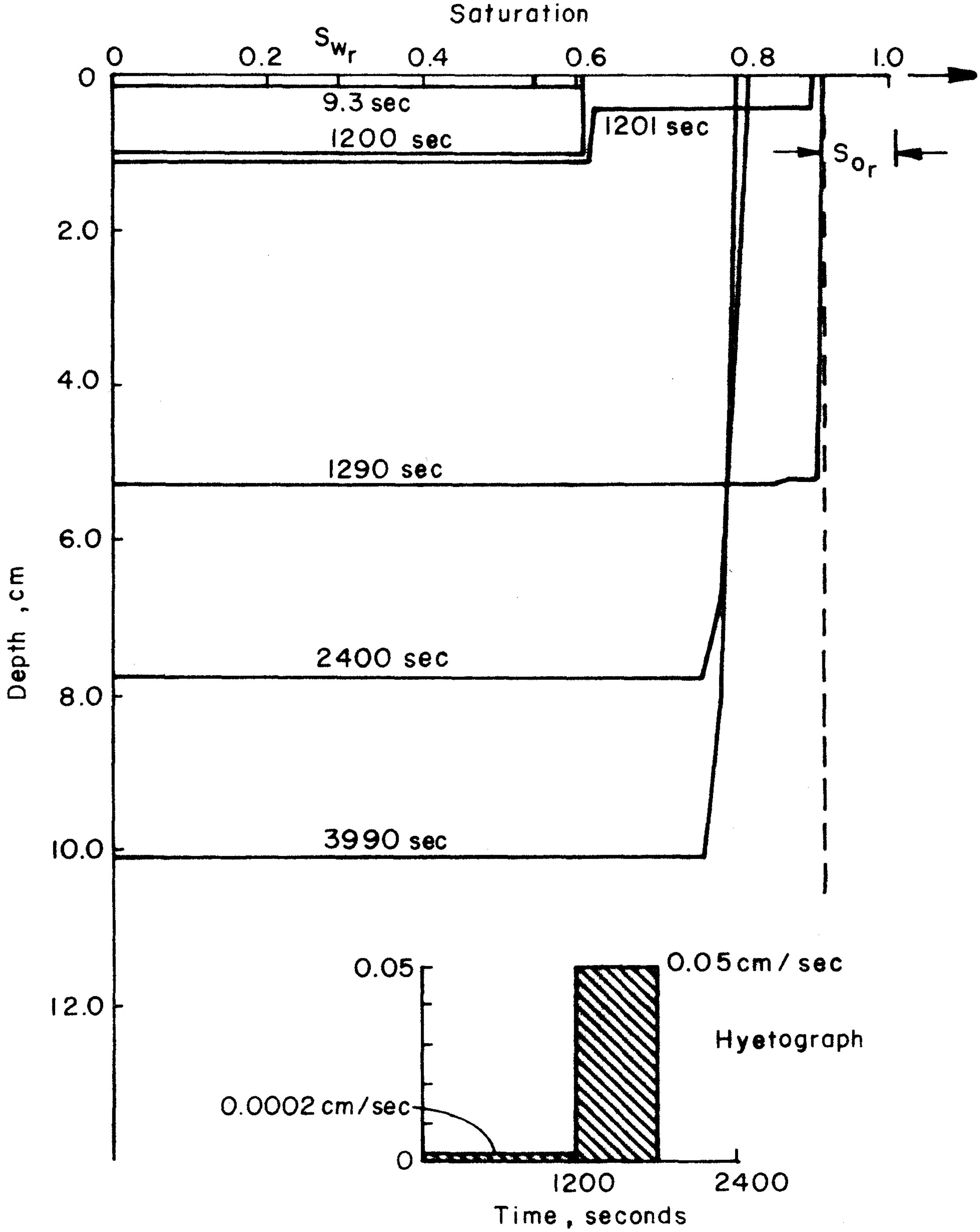

Figure 14. Evolution of saturation profiles corresponding to a given hyetograph in a 185 cm deep, closed bottom, column of Poudre sand

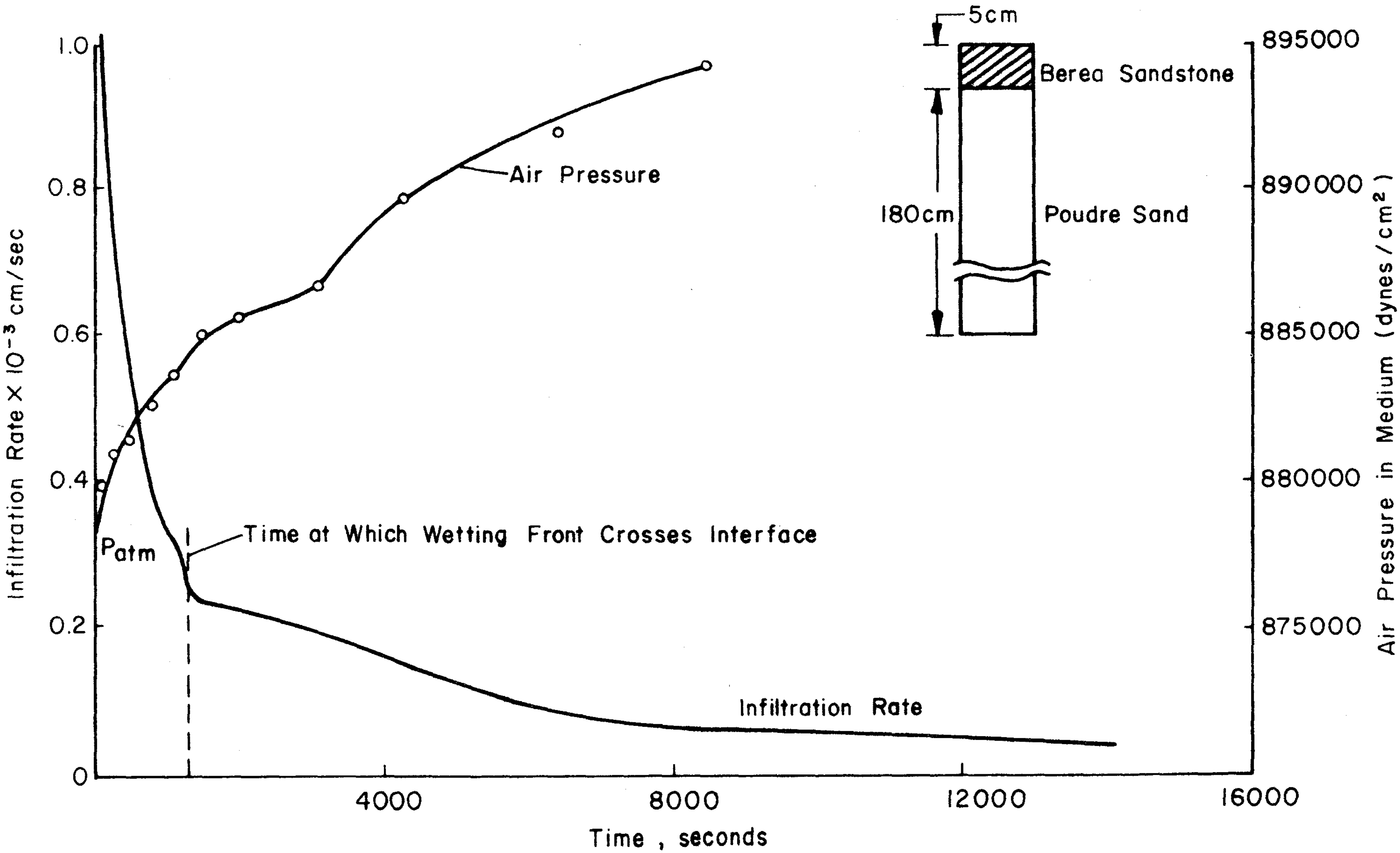

Figure 15. Time variation of infiltration rate and air pressure for a fine-coarse system

E. EFFECT OF HETEROGENEITIES

The rudiments of the approach taken to describe water and air movement in heterogeneous soils are presented in Reference 8, pages 179-192. In fact a complete discussion would be much more lengthy and the implementation of the theory into a general efficient computer program is a difficult task. It has been done and only sample results are reported here. Two situations are considered. In the first case a fine soil (Berea sandstone characteristics) stands over a coarse soil (Poudre sand characteristics). In the second case the position of the two soils is reversed. In both cases, the boundary conditions are: at the top surface a condition of constant ponding (0.8 cm) and at the bottom one of no-flow, and initially the entire column is air-dry.

In the case of a fine layer (5 cm, Berea sandstone) on top of a coarse layer (180 cm, Poudre sand) or fort short in the "fine-coarse" case, it is interesting to note (Figure 15) that as the wetting front passes the interface the infiltration rate drops rapidly, but that later, as the wetting front proceeds in the coarse medium, the decline in infiltration rate is slower than it was in the fine soil. The reason for the short but sharp decline in infiltration rate as the front passes the interface is the practically instantaneous drop in capillary drive. The capillary term $\int_1^2 f_w \, dp_c$ is a constant depending solely on the characteristics of the fine soil, as long as the wetting front remains in the fine soil. Once the wetting front has passed the interface and the fine soil has practically saturated, the capillary term is again practically a constant, now depending solely on the characteristics of the coarse soil. The transition from the fine soil value to the coarse soil value is very rapid. The very rapid reduction in capillary drive produces an abrupt drop in infiltration rate. Once the capillary drive term has stabilized to its new lower value, the infiltration rate declines more slowly than it did in the fine sand because the instantaneous resistance (demoninator of right-hand side of [Eq. (123), Ref. 8, p. 158]), increases more slowly in the coarse soil than in the fine soil because its intrinsic permeability is higher.

Figure 16 shows the evolution of the saturation profiles in the fine-coarse system. Note the desaturation in the coarse medium while the fine layer still saturates. Finally the fine layer starts to desaturate around 14,000 sec. However the reasons for desaturation in the two layers are different. In the coarse layer the desaturation is due to the relative ease of transmission of water in this medium compared to the fine soil. In the fine layer the desaturation is due to the air compression.

For the coarse (4 cm, Poudre sand) fine (396 cm, Berea sandstone) system, Figures 17 and 18 display the results. As the wetting front passes the interface the capillary drive gets a kick. The resistance to flow increases rapidly however due to the lower permeability of the sandstone. The net result is a short-lived stabilization of the infiltration rate followed by a rapid decline in infiltration rate (see Figure 17). It is interesting to note that the inability of the fine layer to transmit the flux that enters the coarse one after around 400 sec causes the coarse soil to fill from the bottom. Note the position of a (small) upward moving wetting front in the coarse medium at time 464 sec.

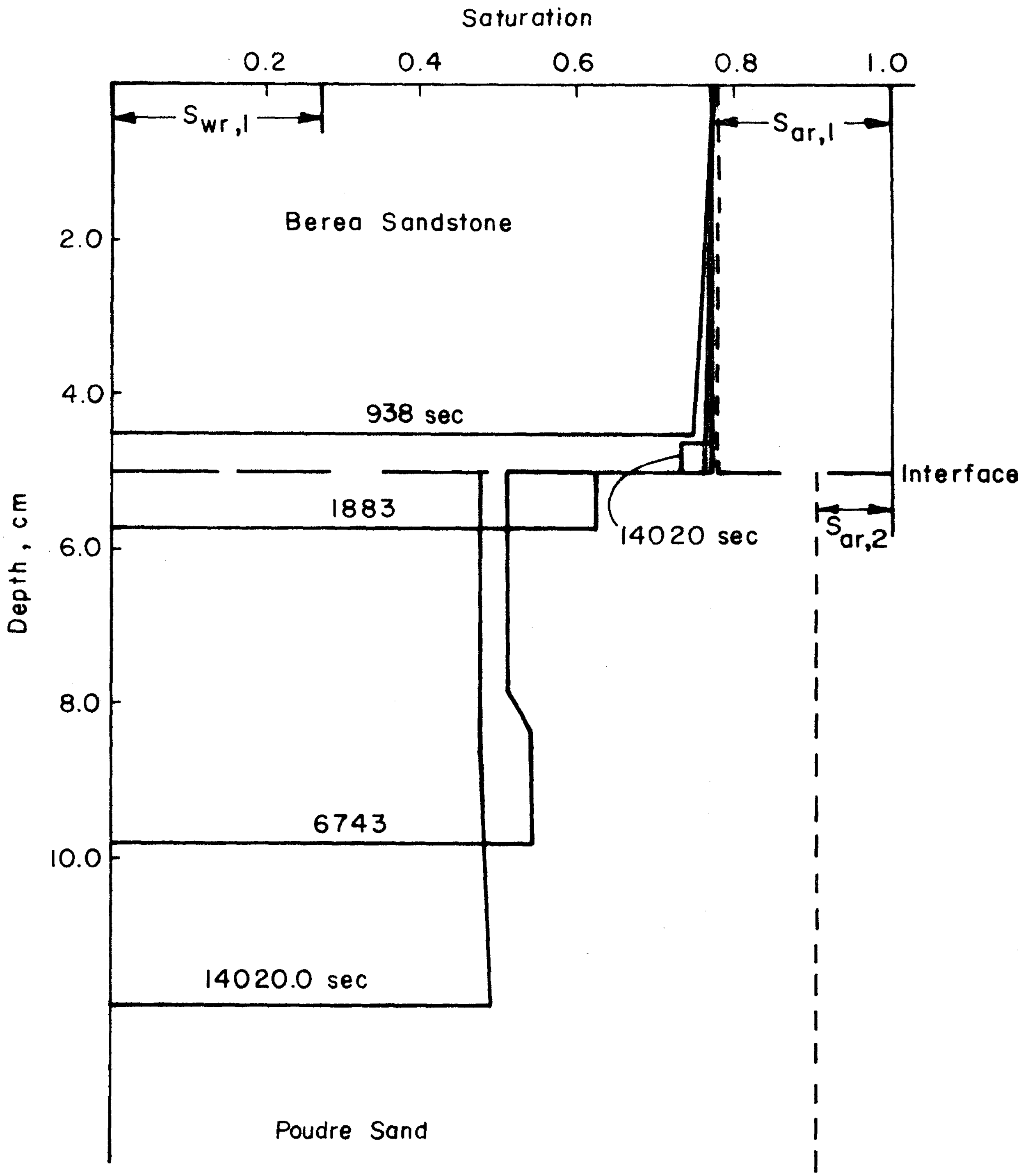

Figure 16. Evolution of saturation profile for fine-coarse system

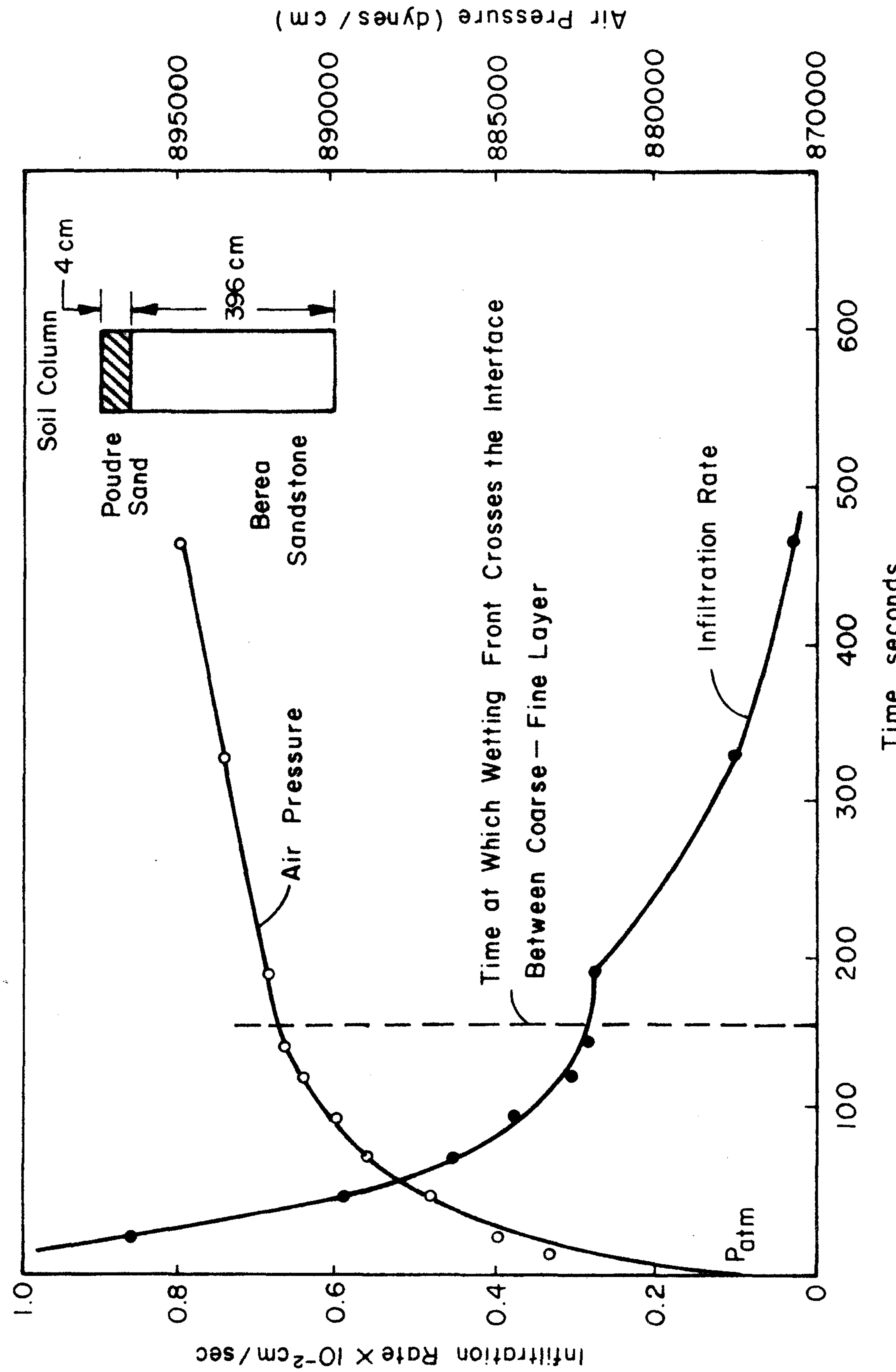

Figure 17. Time variation of infiltration rate and air pressure for coarse-fine system

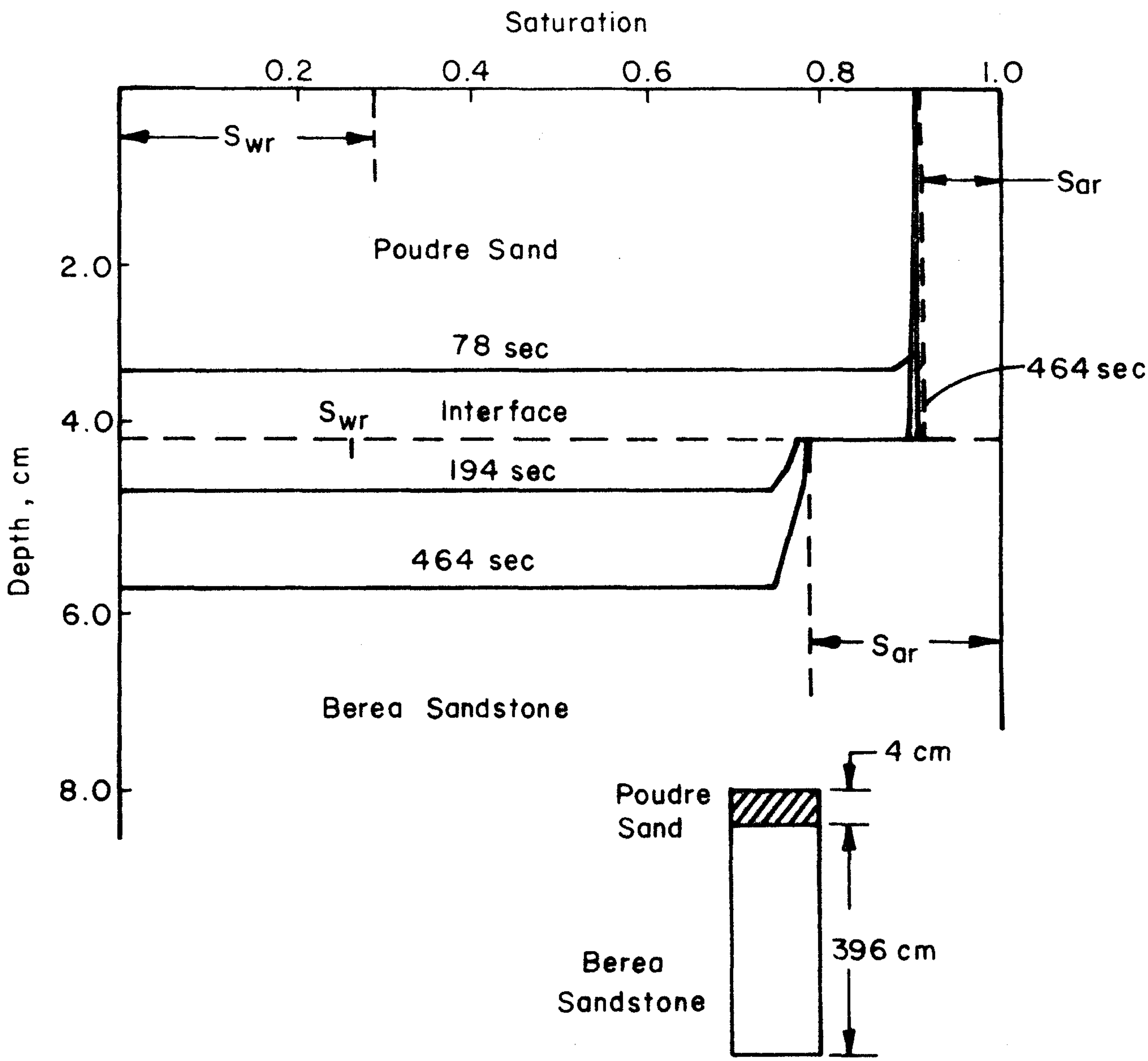

Figure 18. Evolution of saturation profiles for coarse-fine system

F. SUMMARY OF RESULTS

The research described briefly in this completion report has shown that the effects of air movement and air compressibility in soil columns are important. For soils underlain by a relatively impervious layer or by a shallow water table it is found that methods based on Richards' equation would overpredict infiltration rates by factors of two, three or more. Even when air compressibility effects are insignificant as in the case of an open semi-infinite column, air viscous effects are important. In fact, it is shown that the formula of Green and Ampt underestimates the viscous resistance to flow behind the wetting front from 20 to 70 percent, depending on soil type.

The use of a theory that properly considers the movement of water and air in the unsaturated zone has the advantage of accounting for observed experimental results that cannot be modeled by the one-phase flow theory. In addition the mathematical problem is actually simplified, not complicated, by the more complete approach. The fact that the problem of Green and Ampt could be solved simply in a few lines, whereas it had eluded solution since 1911, is conclusive evidence.

Comparison with experimental results show clearly that the approximations in the methods of solution yield highly accurate and practical estimates of the infiltration quantities of interest.

G. REFERENCES

1. Adrian, D. D. et J. B. Franzini. 1966. Impedance to Infiltration by Pressure Build-up Ahead of the Wetting Front. Journal of Geophysical Research, Vol. 71, No. 24, December 1966, pp 5857-5862.

2. Brustkern, R. L. and H. J. Morel-Seytoux. 1970. Analytical Treatment of Two-Phase Infiltration. Journal of the Hydraulics Division, American Society of Civil Engineers Proceedings, Vol. 96, No. HY12, December 1970, pp. 2535-2548.

3. Brustkern, R. L. and H. J. Morel-Seytoux. 1973. Description of Water and Air Movement During Infiltration. Submitted for publication to Journal of Hydrology.

4. Buckley, S. E. and M. C. Leverett. 1942. Mechanism of Fluid Displacements in Sands. Trans. A.I.M.E., Vol. 146, pp 107-116.

5. Childs, E. C. 1969. An Introduction to the Physical Basis of Soil Water Phenomena. John Wiley and Sons, London, 493 p.

6. Chow, V. T., editor. 1964. Handbook of Applied Hydrology. McGraw Hill, New York.

7. Chow, V. T., editor. 1969. Advances in Hydroscience. Vol. 5, Academic Press, 305 p.

8. Chow, V. T., editor. 1973. "Advances in Hydroscience," Vol. 9, Academic Press, New York.

9. DeWiest, R. J. M. editor. 1969. "Flow Through Porous Media," Academic Press, New York, 530 p.

10. Dixon, R. M. et D. R. Linden. 1972. Soil Air Pressure and Water Infiltration Under Border Irrigation. Soil Science Society of America Proceedings, Vol. 36, No. 5, September-October 1972, pp. 948-953.

11. Feodoroff, A. et M. Gugon. 1966. Remarques sur le choix d'un systeme d'assainissement des sols adapté aux conditions du Milieu. Comptes-Rendus de l'Académie d'Agriculture de France du 8 juin 1966, pp. 754-760.

12. Free, G. R. et V. J. Palmer. 1940. Interrelationship of Infiltration, Air Movement, and Pore Size in Graded Silica Sand. Soil Science Society of America Proceedings, Vol 5, pp. 390-398.

13. Gaudet, J. P. 1972. Effet de l'écoulement de l'air sur une infiltration d'eau dans une colonne de sol verticale, Rapport de D.E.A., Université Scientifique et Médicale de Grenoble.

14. Hillel, D. 1971. Soil and Water. Physical Principles and Processes. Academic Press, New York, 288 p.

15. Horton, R. E. 1940. An Approach Toward a Physical Interpretation of Infiltration Capacity. Soil Science Society of America Proceedings, Vol. 5, pp. 399-417.

16. Kirkham D. 1964. Soil Physics. Chapter 5 in Handbook of Applied Hydrology, V. T. Chow, editor. McGraw Hill, New York, pp. 5.1-5.26.

17. Kuraz, V. et M. Kutilek. 1970. Vertical Infiltration of Water Soil of a Non-uniform Moisture. Czechoslovak National Committee of the International Commission on Irrigation and Drainage. International Water Erosion Symposium. Prague, pp. 183-197.

18. Lefevre, B. Pseudo Two-dimensional Experiments of Infiltration with Air Effects. Tentative title. M.S. Thesis in preparation.

19. Morel-Seytoux, H. J. 1961. Effects of Boundary Shape on Channel Seepage. Technical Report No. 7, November 1961, Department of Civil Engineering, Stanford University, 100 p.

20. Morel-Seytoux, H. J. 1964. Domain Variations in Channel Seepage Flow. Journal of Hydraulics Division, ASCE, March 1964. pp. 55-79.

21. Morel-Seytoux, H. J. 1965. Analytical-Numerical Methods in Waterfloodings Predictions. Society of Petroleum Engineers Journal, Vol. 5, No. 3, September 1965, pp. 247-258.

22. Morel-Seytoux, H. J. 1966. Unit Mobility Ratio Displacement Calculations for Pattern Floods in Homogeneous Medium. Society of Petroleum Engineers Journal, Vol. 6, No. 3, September 1966, pp. 217-227.

23. Morel-Seytoux, H. J. 1967. A Study of Quasi-linear Noncapillary Two-phase Flow in Porous Media. Development in Mechanics, Vol. 4, Proceedings of the 10th Midwestern Mechanics Conference, Johnson Publishing Company, pp. 1321-1335.

24. Morel-Seytoux, H. J. 1971. A Systematic Treatment of the Problem of Infiltration. Completion Report Series No. 23, Environmental Resources Center, Colorado State University, June 30, 1971, 22 p.

25. Morel-Seytoux, H. J. 1973. Modéle d'Infiltration Polyphasique. French-English Symposium on Unsaturated Flow in Porous Media, January 24-25, 1973, Reading, England.

26. Morel-Seytoux, H. J. 1973. Two-phase Flows in Porous Media. Chapter in Vol. 9 d'Advances in Hydroscience, V. T. Chow, editor, Academic Press, pp. 119-202.

27. Morel-Seytoux, H. J. 1973. Sur la signification des paramètres de la loi d'infiltration de Green et Ampt. Comptes Rendus de l'Académie des Sciences. Serie A. Hydrologie Séance du 21 Mai 1973.

28. Morel-Seytoux, H. J. 1973. Pour une théorie réformée de l'infiltration. 1ere partie: Pourquoi? Cahiers Hydrologiques de l'ORSTOM, Vol. X, No. 2, 1973.

29. Morel-Seytoux, H. J. 1973. Sur l'intérêt d'une correction a la loi d'infiltration de Green et Ampt. Comptes Rendus de l'Académie des Sciences. Serie A. Hydrologie.

30. Morel-Seytoux, H. J. and A. Noblanc. 1971. Infiltration Predictions by a Moving Strained Coordinates Method. Paper presented at the Symposium on Soil Water Physics and Technology, Rehovot, Israel, August 29-September 4, 1972. To appear in Vol. 4, Ecological Studies, Springer, 1973.

31. Morel-Seytoux, H. J., J. Khanji and G. Vachaud. 1973. Prediction of Imbibition in a Horizontal Column. Submitted for publication to Soil Science Society of America, 34 p. Engineering Research Center Unpublished Rept., CEP72-73HJM47.

32. Morel-Seytoux, H. J., J. Khanji and G. Vachaud. 1973. Prediction Errors Due to Uncertainties in the Determination and Extrapolation of the Diffusivity Function. May 1973, submitted for publication to Soil Science Society of America, Engineering Research Center Unpublished Report, CEP72-73HJM48.

33. Morel-Seytoux, H. J., and J. Khanji. 1973. Derivation of a Law of Infiltration. June 1973, to be submitted for publication to Water Resources Research Journal, Engineering Research Center, Unpublished Report, CEP72-73HJM49.

34. Nielsen, D. R. et al., éditeurs. 1972. Soil Water. American Society of Agronomy, Madison, Wisconsin, 175 p.

35. Noblanc, A. et H. J. Morel-Seytoux. 1972. Perturbation Analysis of Two-Phase Infiltration. Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY9, September 1972, pp. 1527-1541.

36. Peck, A. J. 1965. Moisture Profile Development and Air Compression During Water Uptake by Bounded Porous Bodies: 3. vertical columns. Soil Science Journal, Vol. 100, No. 1, pp. 44-51.

37. Philip, J. R. 1969. Theory of Infiltration. In "Advances in Hydroscience," Vol. 5, V. T. Chow, éditeur, Academic Press, pp. 215-305.

38. Phuc, L. V. and H. J. Morel-Seytoux. 1972. Effect of Soil Air Movement and Compressibility on Infiltration Rates. Soil Science Society of America Proceedings, Vol. 36, No. 2, March-April 1972, pp. 237-241.

39. Richards, L. A. 1931. Capillary Conduction of Liquids in Porous Media. Physics, New York, Vol. 1, pp. 318-333.

40. Smiles, D. F., G. Vachaud and M. Vauclin. 1971. A Test of the Uniqueness of the Soil Moisture Characteristic During Transient, Nonhysteretic Flow of Water in a Rigid Soil. Soil Science Society of America Proceedings, Vol. 35, pp. 534-539.

41. Sonu, J. 1973. Water and Air Movement in Bounded Layered Soil. Ph.D. dissertation, Department of Civil Engineering, Colorado State University, Fort Collins, Colorado.

42. Swartzendruber, D. 1969. The Flow of Water in Unsaturated Soils. In "Flow Through Porous Media." DeWiest, editor, Academic Press, New York, pp. 215-292.

43. Vachaud, G., M. Vauclin, J. Khanji et M. Wakil. 1973. Effect of Air Pressure on Water Flow in an Unsaturated Stratified Vertical Column of Sand. Water Resources Research Journal, Vol. 9, No. 1, February 1973, pp. 160-173.

44. Vauclin, M. 1971. Effets dynamiques sur la relation succion-teneur en eau lors d'écoulements en milieu non saturé. Thèse de Docteur-Ingénieur, Université Scientifique et Médicale de Grenoble, 111 pages.

45. Wilson, L. G. and J. N. Luthin. 1963. Effect of Airflow Ahead of the Wetting Front on Infiltration. Soil Science Journal, Vol. 96, No. 2, August 1963, pp. 136-143.