A SYSTEMATIC TREATMENT OF THE PROBLEM OF INFILTRATION

by

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ENVIRONMENTAL RESOURCES
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ENVIRONMENTAL RESOURCES CENTER
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ABSTRACT

The research described briefly in this completion report has shown that the effects of air movement and air compressibility on infiltration in soil columns are important. For soils underlain by a relatively impervious layer or by a shallow water table errors of the order of 30% can result.

It was found that approximate solutions to the two-phase formulation of the problem of infiltration (i.e. accounting for both water and air movement) give more accurate predictions than exact solution to the unsaturated flow formulation (i.e. accounting only for water movement). In fact the two-phase theory has the significant advantage to account for observed experimental results which cannot be modeled by the one-phase flow theory.

Finally the approximate solutions are not only more accurate but also more economic. The costs of computer runs for prediction of infiltration rates by the analytic approximate method were of the order of one tenth of the costs involved in the numerical finite-difference solution of the two-phase flow equations.

KEYWORDS: *Infiltration, *Two-Phase Flow, *Air Compressibility, Air Counterflow, Numerical Methods

RESEARCH OBJECTIVES

In the original proposal, February 1968, the objectives were listed as follows:

"The overall objective of the research is to develop a mathematical model of infiltration capable of responding to any spatial and temporal pattern of rainfall or its lack. In this form the model would be readily capable of integration into a general model simulating the hydrologic response of a watershed.

In a first phase the objectives will be more limited. In essence the two principal objectives are:

(1) The development of a one-dimensional model of water infiltration into a soil column under realistic conditions of varying water supply at the surface, non-uniform initial mositure conditions, heterogeneous soil characteristics and the effect of hysteresis, and

(2) The development of a less general two-dimensional model of water infiltration to model primarily the influence of spatial variation of the available water supply on infiltration."

ACHIEVEMENTS OF CONTRACT

It is not desirable to repeat in this completion report all the results obtained over the past three years and the detailed procedures by which they were obtained. These results and procedures can be found in 2 theses [1,3], 1 dissertation [2], 1 published paper [4], and several submitted papers [5,6,7,8,9]. Rather a brief review of the methods of attack and a sample of results will be given.

1. <u>Mathematical formulation of the problem</u> - For simplicity the one-dimensional vertical case is discussed. In short, mathematically, the problem of infiltration is one of solving simultaneously the equations of mass conservation for both water and air phases, that is:

$$\phi \frac{\partial(\rho_w S_w)}{\partial t} + \frac{\partial}{\partial z}(\rho_w V\ ) = 0 \tag{1}$$

$$\phi \frac{\partial(\rho_a S_a)}{\partial t} + \frac{\partial}{\partial z}(\rho_a V_a) = 0 \tag{2}$$

subject to the constraints of definition,

$$S_w + S_a = 1 \tag{3}$$

the requirements of Darcy's law generalized to several phases [10], namely:

$$V_w = -k \frac{k_{rw}}{\mu_w} [\frac{\partial \rho_w}{\partial z} - \rho_w g] \tag{4}$$

$$V_a = -k \frac{k_{ra}}{\mu_a} [\frac{\partial \rho_a}{\partial z} - \rho_a g] \tag{5}$$

the capillary relation:

$$p_c(S_w) = p_a - p_w \tag{6}$$

and the proper hydrologic initial and boundary conditions, where the following symbols have been used:

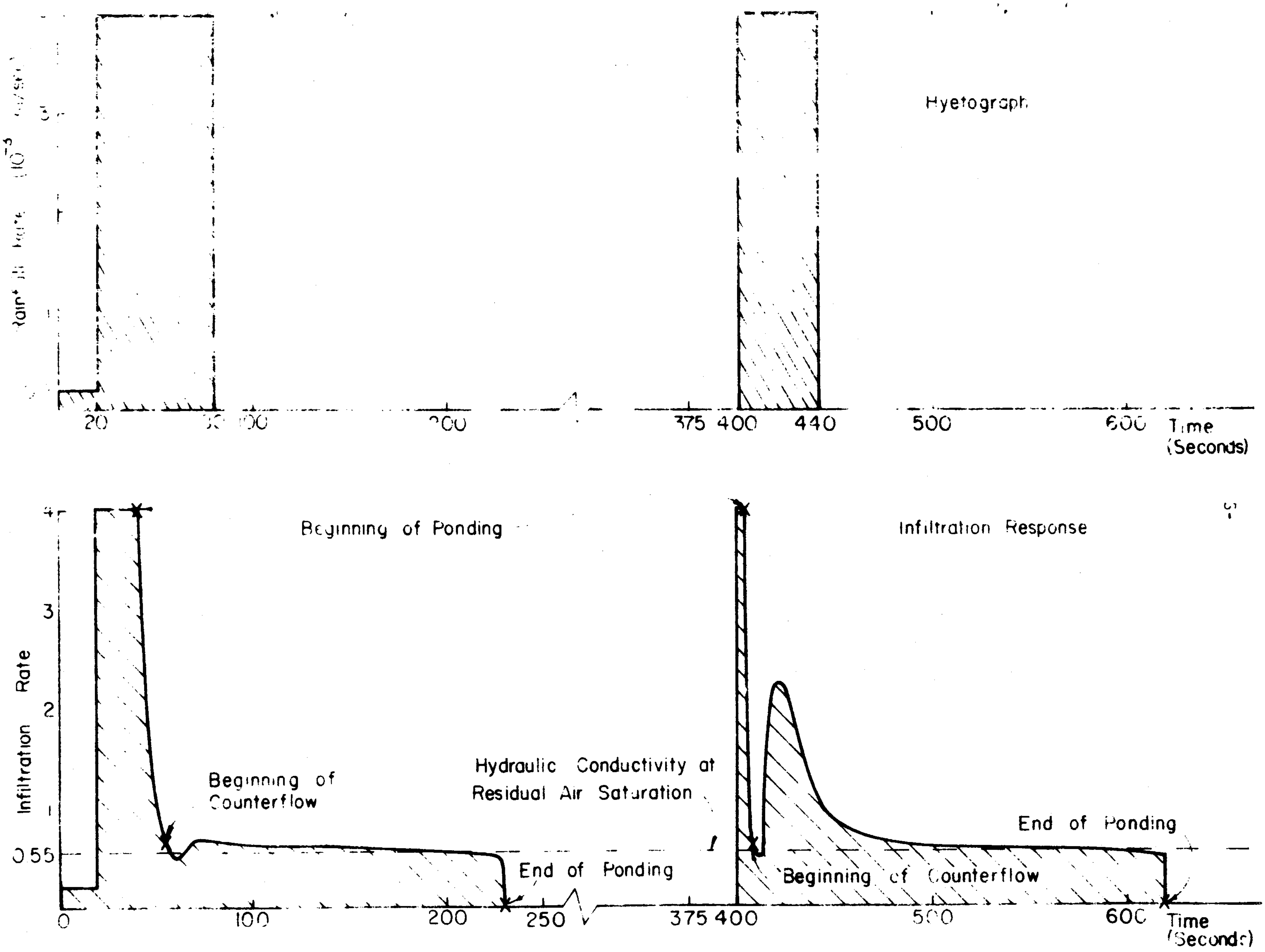
ϕ = porosity
t = time
ρ_w, ρ_a = water, air density
S_w, S_a = water, air saturation
z = vertical coordinate, oriented positive downward
V_w, V_a = water, air velocity in Darcy sense
k = intrinsic permeability
k_{rw}, k_{ra} = relative permeabilities to water, to air
μ_w, μ_a = water, air viscosity
p_w, p_a = water, air pressure
g = acceleration of gravity
p_c = capillary pressure

It goes without saying that the problem is not simple. Three approaches were pursued for the following reasons. First, it was realized that it probably (almost certainly) would not be feasible to solve the above complete system of equations exactly and to expect the corresponding model to be used economically in day to day hydrologic practice. On the other hand it was felt that simplified models would not receive acceptance in the profession unless it could be shown that they provided acceptably good answers. Thus one method was followed in which no simplifications were made. The resulting model is not meant to be used operationally but to serve as a standard against which to test other procedures. This procedure will be named after the student who implemented it and referred to as the PHUC procedure for brevity. To the contrary, in another procedure, as many simplifying assumptions as possible were made which seemed to be physically acceptable a priori. This procedure (the BRUSTKERN procedure), if successful, is meant to become an economic technique in everyday hydrologic practice. For fear of having gone too far a sort of middle of the road procedure was followed (the NOBLANC procedure). It is the purpose of the next sections to discuss each of the three procedures and to show results obtained by each method.

2. The PHUC procedure - It is a finite-difference approach to the solution of the problem. It makes no assumptions at all, except those inherent in the finite-difference algebraic approximations of a system of partial differential equations. In its present state the PHUC program can handle all hydrologically meaningful initial and boundary conditions, including effects of water movement, air movement, air compressibility and hysteresis but not of heterogeneities in soil conditions. The versatility of this program is well illustrated in Figure 1 which shows the infiltration response curve to two bursts of rainfall, for a soil of hydraulic conductivity of 0.55 x 10^{-3} cm/sec and a depth 40 cm above an impervious boundary. The initial saturation of the soil was assumed uniform and equal to residual water saturation (0.2). Naturally, the initial conditions for saturation at the start of the second burst is not one of uniform saturation. Saturation profiles at some important times are shown in Figure 2 as well as the cumulative water heights for rainfall, infiltration and ponding.

3. The BRUSTKERN procedure - This procedure reduces the system of two complex partial differential equations to a system of only one (simpler) partial differential equation and one strictly algebraic equation, namely:

$$\phi \frac{\partial S_w}{\partial t} + \bar{V} \frac{\partial F_w}{\partial z} = 0 \qquad (7)$$



Infiltration response to successive bursts of rainfall

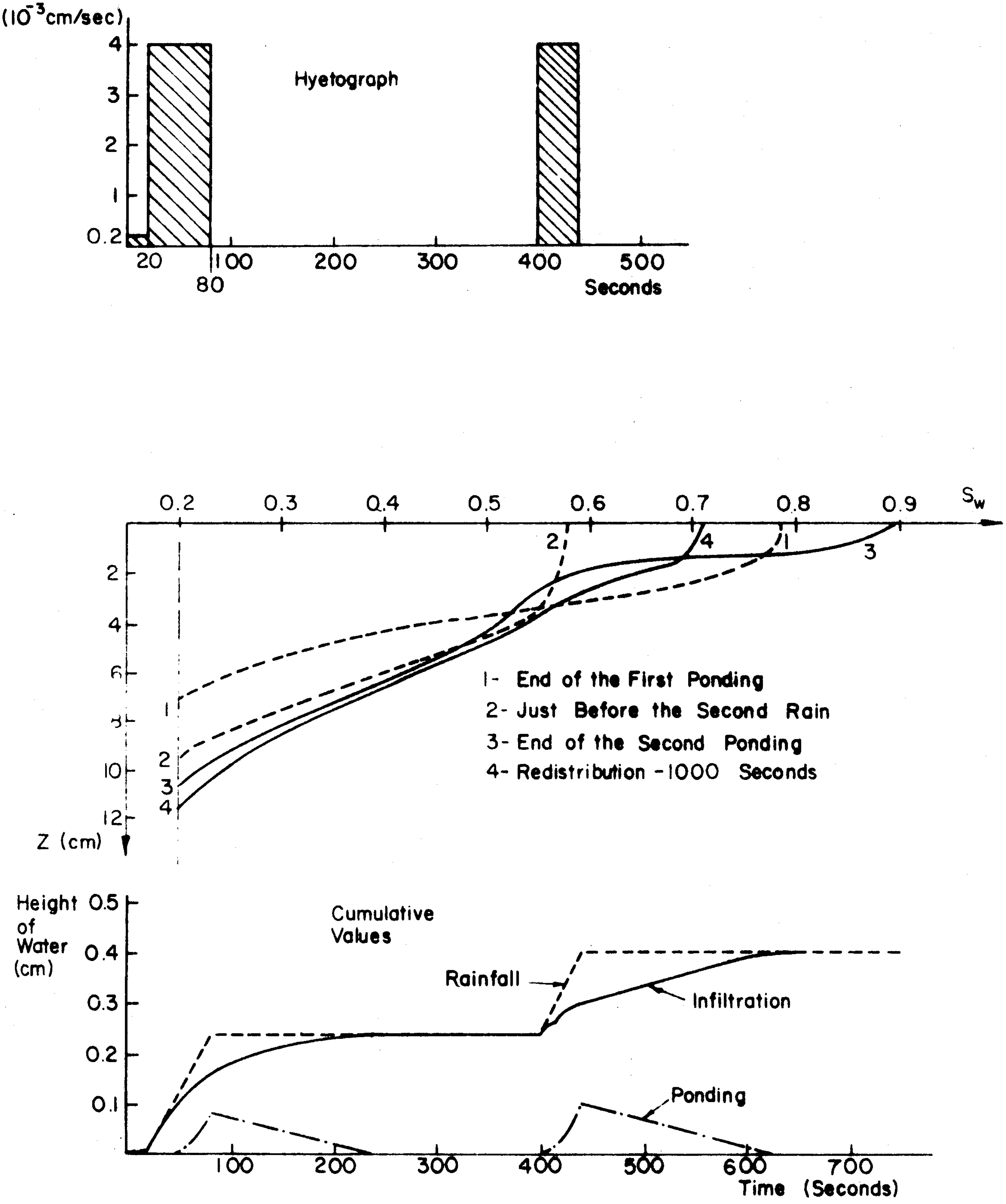


Figure 2 Saturation and Infiltration Responses to Rainfall

and

$$\overline{V} = \frac{(p_{a1}-p_{a2}) + g\rho_w \int_1^2 f_w dz + \int_1^2 f_w\, dp_c}{\int_1^2 \frac{dz}{k\left(\frac{k_{ra}}{\mu_a} + \frac{k_{rw}}{\mu_w}\right)}} \tag{8}$$

where the additional following symbols have been used:

V = total algebraic velocity i.e., $V_w + V_a$

$\overline{V}$ = average total velocity

F_w = fractional flow function neglecting capillarity [6,8]

p_{a1}, p_{a2} = air pressure at location 1 (ground surface), at location 2 (ahead of wetting front)

f_w = fractional flow function when both gravity and capillarity are neglected [4,10].

The new system of equations (7) and (8) is particularly simple because Eq. (7) can be solved explicitly [40] and Eq. (8) involves integration of known functions of saturation, coordinate and time. It is therefore simpler to solve this system than Richard's equation [11] even though the effects of air movement and air compressibility, albeit in a gross way, are included in the system. The superiority of this procedure is well demonstrated by the results of Figure 3. The case investigated is that of a constant ponding of 5 cm of water over soil columns of various depths above an impervious boundary for a uniform initial water saturation equal to residual saturation. It is clear from Figure 3 that the soil depth is a factor in the shape of the infiltration curve. On the other hand, the solution of Richard's equation always corresponds to the solution $D = \infty$ i.e. semi-infinite medium until the wetting front reaches the boundary. After such time the

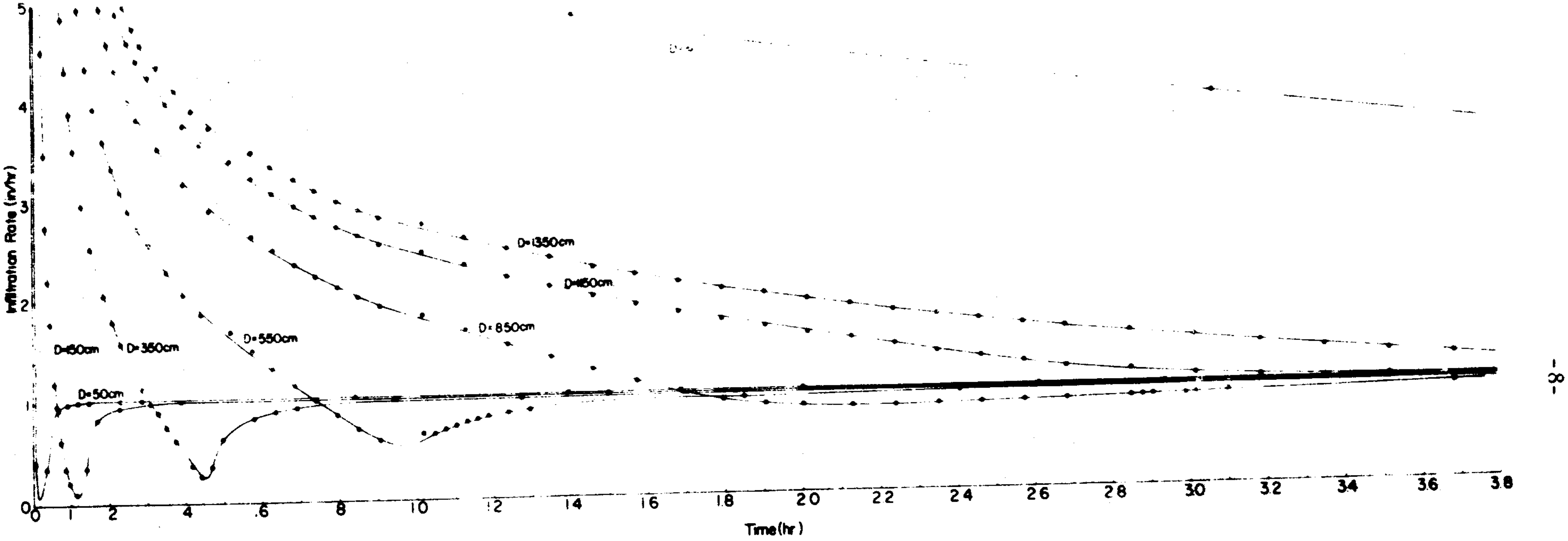


Figure 3. Comparison of Infiltration Rates When Depth of Water Table (D) Varies

infiltration rate rapidly drops to zero. With the Richard's equation the presence of the lower impervious boundary cannot be felt until the wetting front reaches it. Thus until such time occurs the solution cannot be different from that for a semi-infinite medium. If air movement and air compressibility are ignored, as they are in Richard's equation, it is immaterial whether the uniform saturation extends a few inches or a few miles ahead of the wetting front; the same infiltration rate is calculated. In fact, however, this is not the case. As shown in Figure 3 the presence of the lower impervious boundary is felt rapidly and significantly. The air compression wave travels much more rapidly than the wetting front. The air pressure build up slows down the infiltration rate. A point is reached when it is so large that the air breaks through. The air pressure is reduced and as a result the infiltration rate picks up. Fairly rapidly an equilibrium is reached when compressibility is no longer a factor and the infiltration rate equals the volumetric rate of air escape.

The presence of the dip in the infiltration rate was at first intriguing. Though it was expected that the infiltration rate would be affected by the presence of the impervious lower boundary the dip came as a surprise. However, after reflection, the presence of the dip seemed perfectly understandable physically. Then in at least one paper [12] reporting experiments of infiltration in finite soil columns the data displayed the dip. A new glance at old results of double ring infiltrometer tests conducted by CSU staff to study the recharge potential of river beds showed unmistakably (?) the existence of such a dip (Figure 4). The word "unmistakably" is used in a "tongue in cheek" context. One may question the accuracy of the measurements. Most field

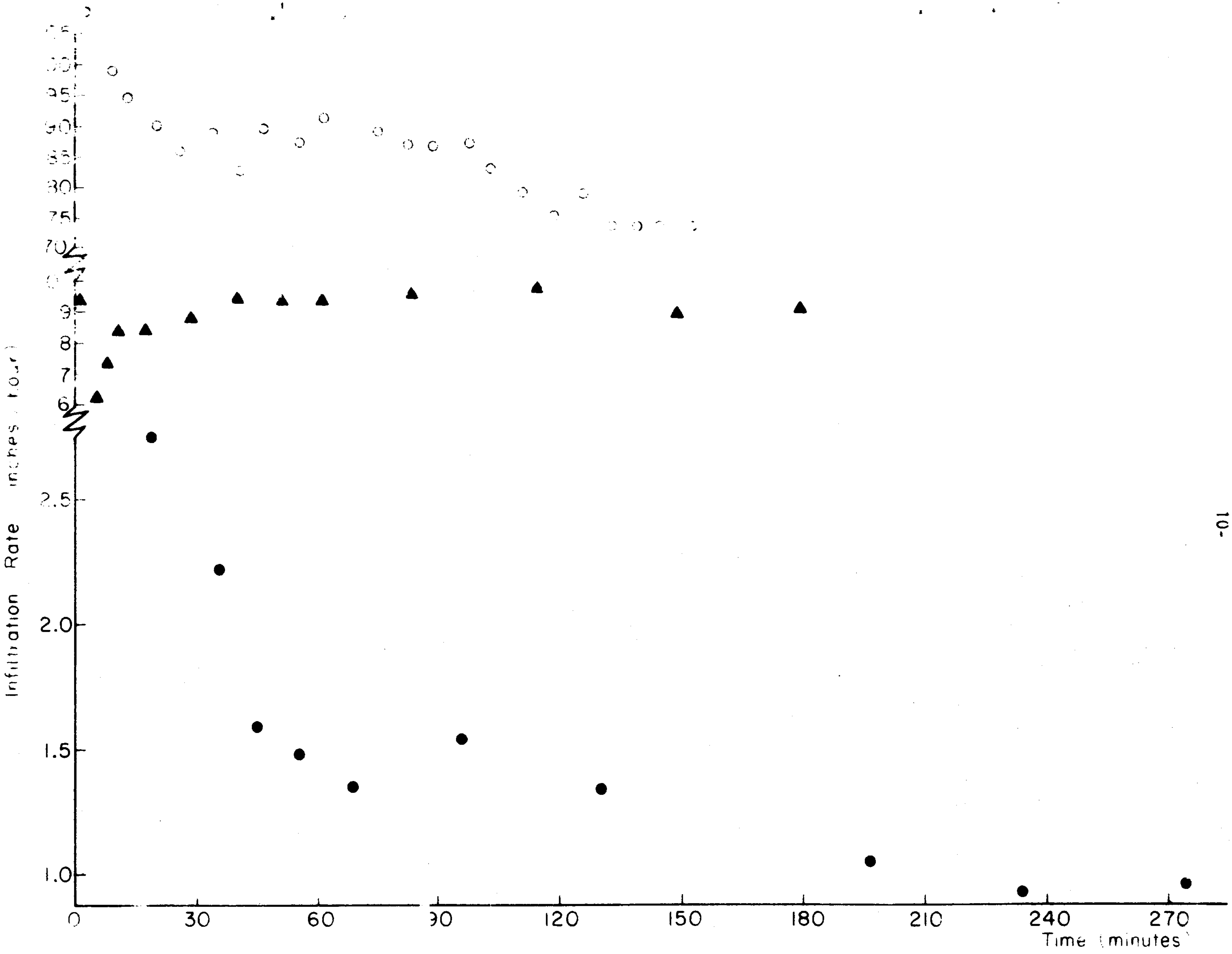


Figure 4. Infiltrometer data.

experimenters have tended to disregard the fluctuations in the data and fit monotonically decreasing curves to the data. Based on the results of the current study it is felt that the "dip" in the infiltration rates is real. By the same token if the "dip" is real then so must be the "hump" that follows it. This hump is not predicted by the BRUSTKERN procedure. The hump could be present even for a medium that is perfectly homogeneous in the horizontal plane. If that is the case the BRUSTKERN procedure in its present form is treating the compressibility effect too crudely to predict it. It is possible however that the hump is due to the superposition of say two response curves corresponding to the same depth but to two different hydraulic conductivities. Whereas the superposition of two monotonically decreasing functions is of same nature the superposition of curves of the type shown on Figure 3 can have a hump. Finally recently Dr. D. McWhorter,* [13], provided the results of experiments of infiltration under constant head of ponding for various soil columns, shown in Figure 5. Even though the infiltration results of Figures 3 and 5 are for different soils (both sands, however) and for different wetting fluids (Dr. McWhorter used oil in his experiments) the time of the dips for the depth 150 cm (Figure 3) and 185 cm (Figure 5) are of the same order of magnitude, respectively 8 and 16 minutes. The corresponding numbers for the depth 350 cm (Figure 3) and 393 cm (Figure 5) are 28 and 65. A quick correction (assuming linearity) for depth gives the following ratios:

$$\frac{16}{7} \times \frac{150}{185} = 1.9 \qquad \frac{65}{28} \times \frac{350}{393} = 2.1$$

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* Assistant Professor of Agricultural Engineering, Colorado State University.

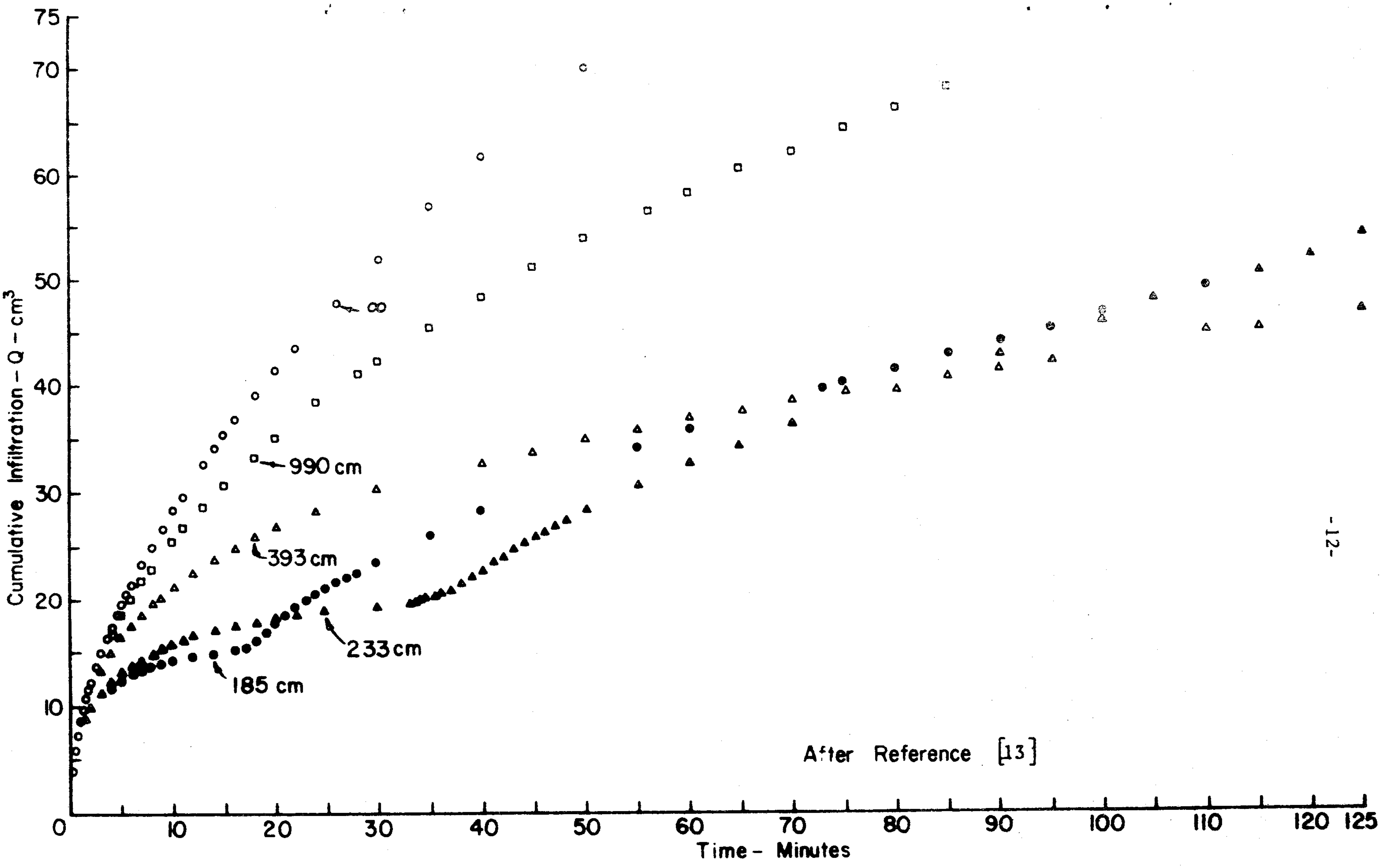

Figure 5 – Comparison of cumulative infiltration volume when depth of water table varies.

it is clear that close agreement exists qualitatively and in order of magnitude.

Note again that the data show clearly the presence of a dip in infiltration rates and indicate also the existence of a hump. These data obtained under carefully controlled laboratory conditions confirm the value of the BRUSTKERN procedure and show that the dips and humps of the field tests must be taken seriously.

4. The NOBLANC procedure - As with the BRUSTKERN procedure the system of two complex partial differential equations is reduced to a system of only one partial differential equation and one strictly algebraic equation. The complete partial differential equation for saturation is of the form:

$$\phi \frac{\partial S_w}{\partial t} + \bar{V} F_w' \frac{\partial S_w}{\partial z} + \frac{k}{\mu_a} \frac{\partial}{\partial z} \left[E \frac{\partial S_w}{\partial z} \right] = 0 \qquad (9)$$

where the ' denotes differentiation with respect to saturation and a new symbol was introduced for brevity: $E(s) = k_{ra} f_w p_c'$. A new moving coordinate system is introduced with origin, $\zeta(t)$, relative to the original and fined z coordinate, namely:

$$Z = z - \zeta(t) . \qquad (10)$$

In this new coordinate system eq. (9) becomes:

$$\phi \frac{\partial S_w}{\partial t} + \left(\bar{V} F_w' - \phi \frac{d\zeta}{dt} \right) + \frac{k}{\mu_a} \frac{\partial}{\partial Z} \left(E \frac{\partial S}{\partial Z} \right) = 0 \qquad (11)$$

To the extent that (in the front region) the rate of deformation of the profile is small compared to the rate of translation of the profile, then it is legitimate to neglect the first term in Eq. (11). Finally the solution for the profile in the front region is:

$$z = \int_{S_{wi}}^{S_w} \frac{k\ E(S)\ dS}{\mu_a\{\phi \frac{d\zeta}{dt} (S-S_{wi}) - \overline{V}[F_w(S) - F_w(S_{wi})]\}} \qquad (12)$$

The quantity $d\zeta/dt$ is known from a matching condition with a solution of Eq. (9) valid away from the front region. As in the BRUSTKERN procedure an explicit solution for the partial differential equation is available. Thus only numerical quadratures are needed. The integral equation is still given by Eq. (8).

In the BRUSTKERN procedure there exists a sharp front in the solution for the saturation profile, see figure 6. Figure 7, obtained by the NOBLANC procedure indicates that for large times (i.e., after a few hours) the profile in the wetting zone is indeed an extremely sharp front. For large times the BRUSTKERN procedure must be valid. In the early stages and particularly for shallow depths to an impervious boundary the profiles do not show steep wetting fronts (Figure 8). As a result the infiltration curves are noticeably different. However, compared to the high value of hydraulic conductivity at residual air saturation which by the currently accepted theory is supposed to be the asymptotic limit of the infiltration rate, the two curves are not very different (Figure 9). A comparison of results for infiltration predictions by the BRUSTKERN and Phuc methods is shown on Figure 10 for a semi-infinite medium. The agreement is quite good.

5. Summary of Results - The research has shown that:

a) effects of air movement and air compressibility on infiltration are important,

b) approximate solutions to the right equations give more accurate results than exact solutions to the wrong ones (visualize Richard's equation and Phillip's solution) [14], and

c) the approximate solutions (i.e., generated by the BRUSTKERN procedure) are not only more accurate but also more economic.

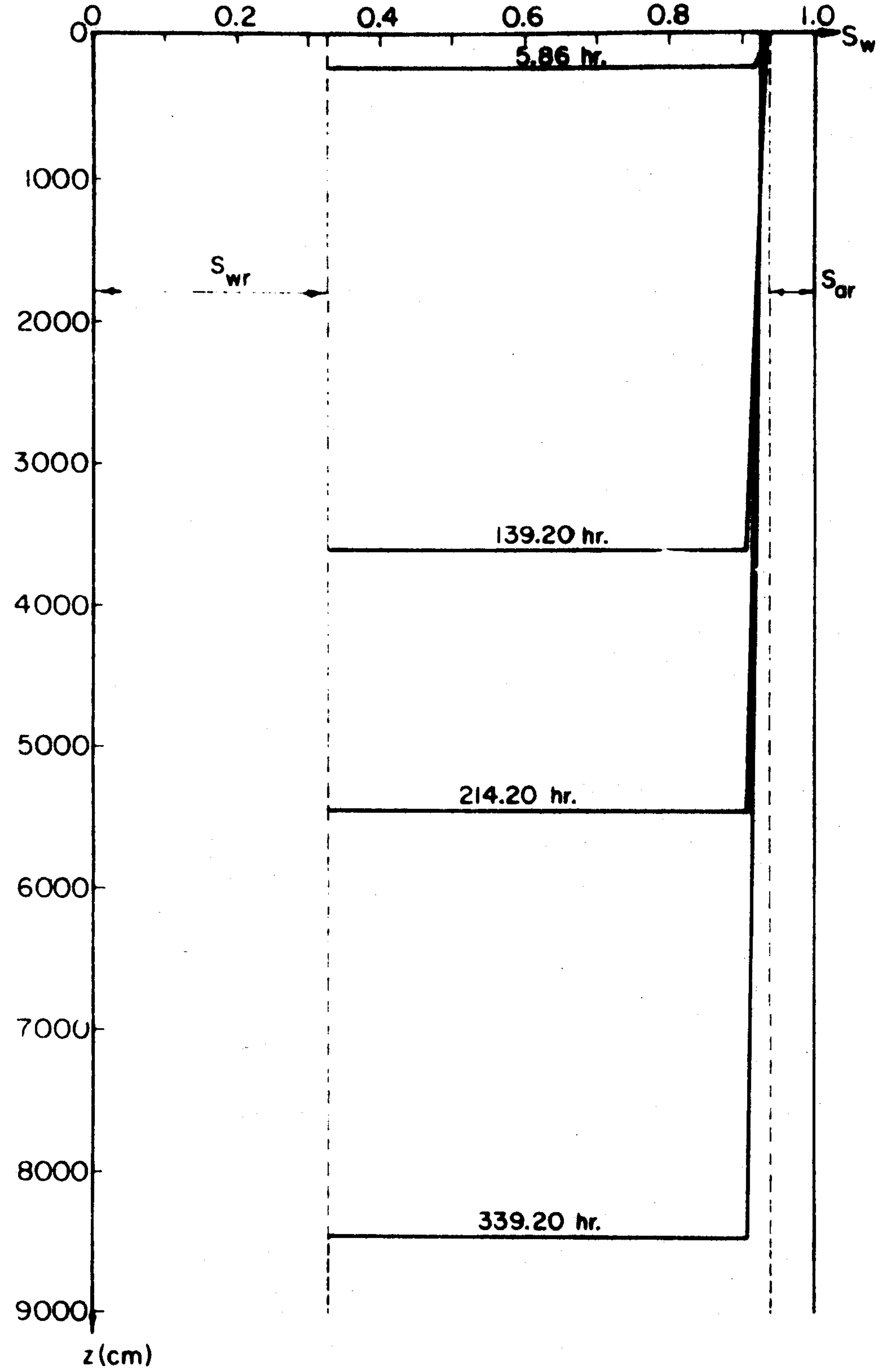

Figure 6. Advance of the Saturation Profile in a Semi-Infinite Medium

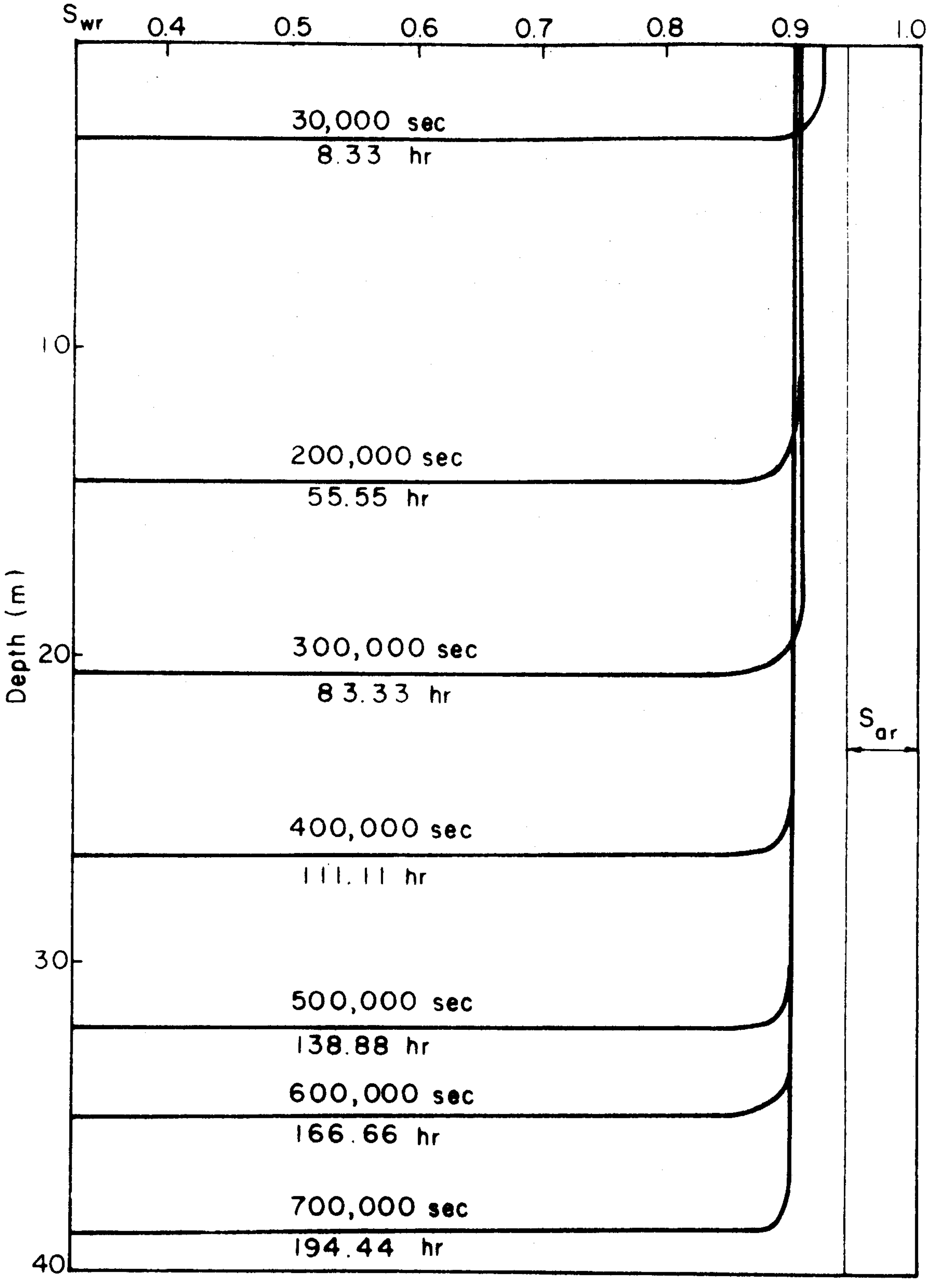

Fig. 7. Saturation profile in a semi-infinite medium.

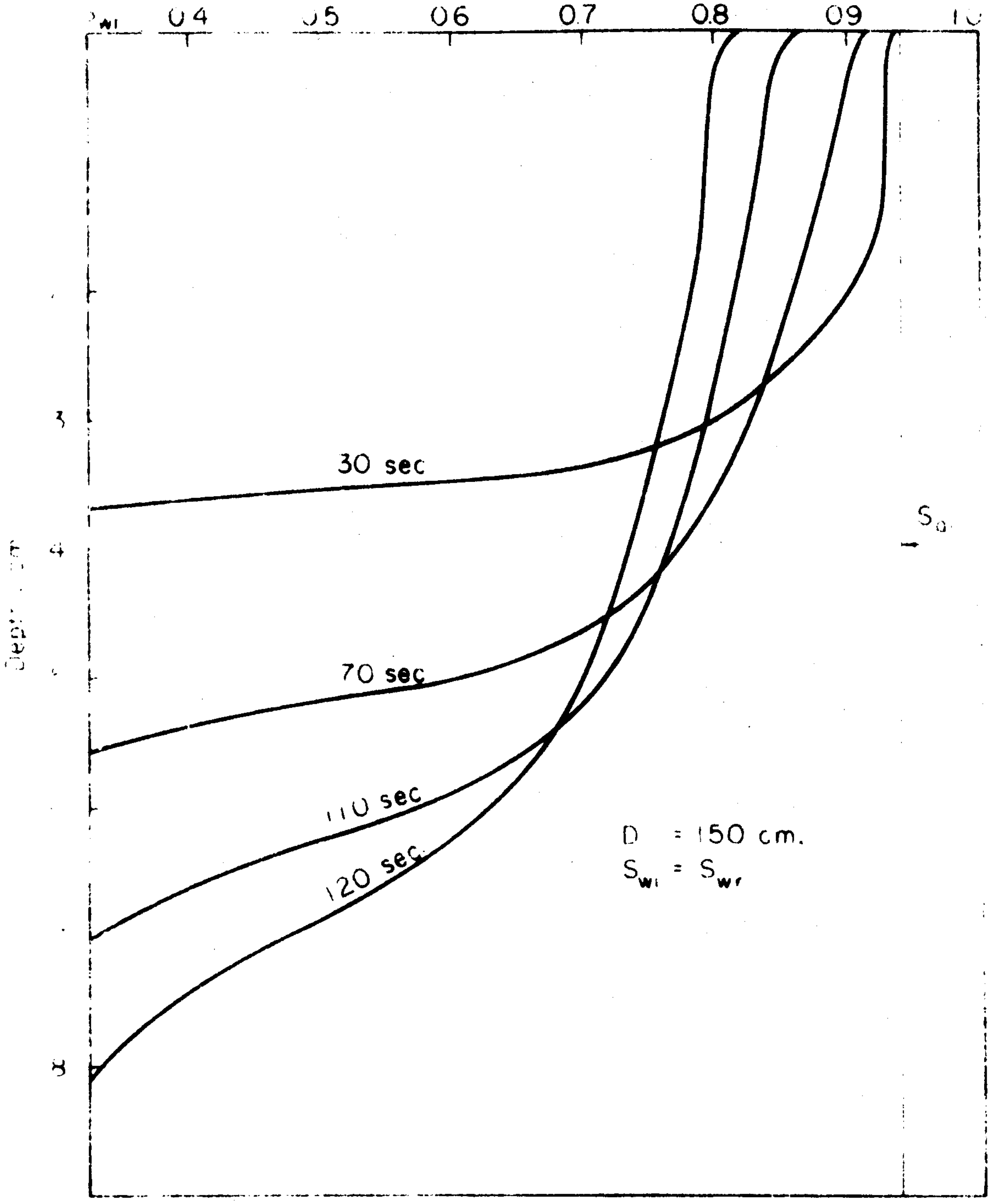

Fig. 8. Saturation profile. Water table at 150 cm

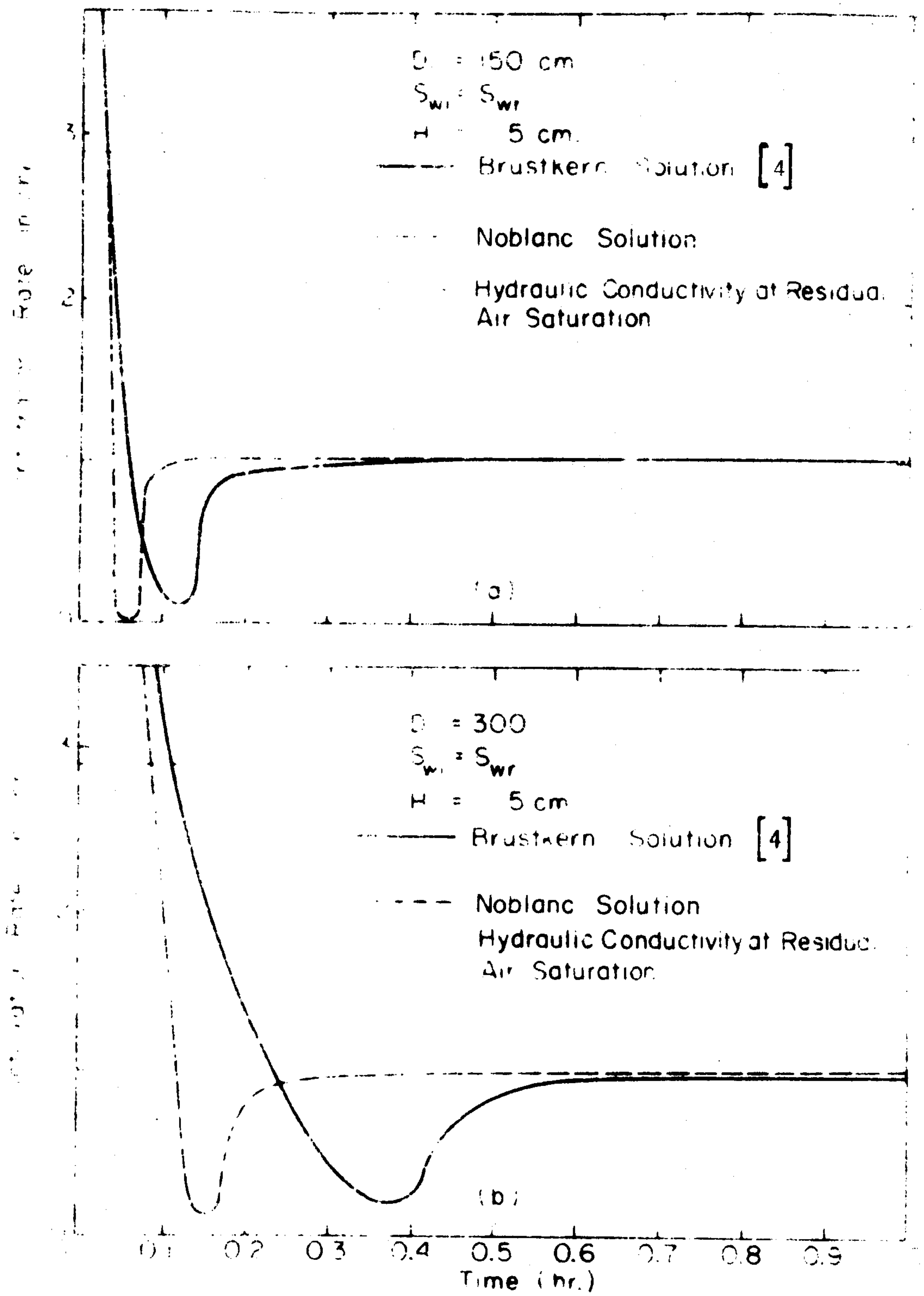

Figure 9 Comparison of Infiltration Rates as Obtained by the Brustkern Procedure [12] and by the Noblanc Procedure (Water Table at Depths of 150 and 300 cm)

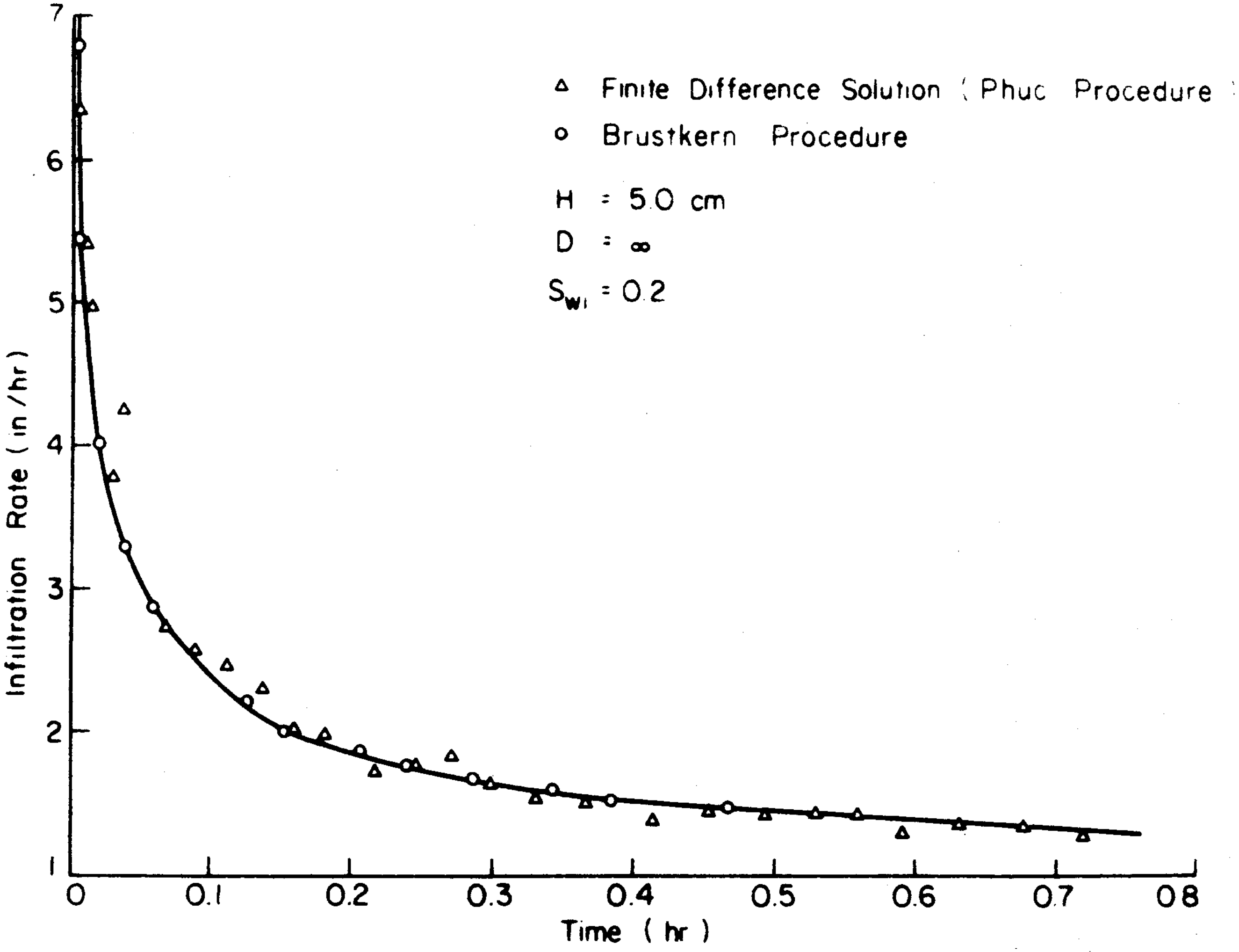

Figure 10. A Comparison of Infiltration Rates as Obtained from a Finite Difference Solution and Brustkern's Solution.

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