MASS DIFFUSION FROM A POINT SOURCE IN A TURBULENT BOUNDARY LAYER OVER A ROUGH SURFACE

by

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ABSTRACT

Turbulent diffusion of dynamically passive and chemically non-reactive matter from a simulated point source within a two-dimensional boundary-layer flow over a rough surface was studied. A rough surface consisting of two-dimensional roughness elements (wooden strips, 0.25 in. high, 0.25 in. wide, and 6.0 ft long, placed 3.0 in. apart normal to the flow direction) was used with the source at three different heights. The free-stream velocity of air for most of the runs was 12.50 ft/sec. Concentrations of the diffusing plume were measured at several downstream distances from the point source located at heights of 0.0625 in., 0.5 in., and 1.0 in. above the rough surface. Anhydrous ammonia was used as the tracer gas.

Both vertical and lateral concentration profiles were found to be self-similar in the fully developed regions of the concentration field. The concentration distribution for such regions is described by two dimensionless universal functions, one for the case of source height h = 0.0625 in. and the other for the cases of source heights h = 0.5 in. and h = 1.0 in. These are obtained by expressing the relative concentration $\frac{C}{C_{max}}$ in terms of position length parameters η and σ of the diffusing plume as length scales.

The vertical and lateral length scales (η and σ respectively) of the plume have been related to the downstream distance from the source by empirical equations valid within the range of the experimental variables.

The Lagrangian similarity hypothesis was tested by means of the experimental data and found to agree reasonably well with the data for the attenuation of maximum ground-level concentration in the longitudinal direction. Experimental results for the growth of the plume height and width with downstream distance from the source are in fair agreement with predictions of the hypothesis. However, in view of the experimental data, further investigations are necessary for a conclusive test of the hypothesis which seems to afford a rational basis for describing the gross characteristics of the diffusing plume within a turbulent boundary layer.

NOMENCLATURE

b,	Batchelor constant
C,	Concentration level of the diffusing gas, M/L^3[*]
C_o,	Concentration at $a = 0$, $y = o$, M/L^3
C_{max},	Maximum value of concentration, M/L^3
H,	h/z_o
h,	Source height, L
k,	Karman constant (k is taken as equal to 0.41)
m_{cp},	$d(\ln C_o) / d(\ln \xi)$
n_{cp},	$d(\ln \sigma) / d(\ln \xi)$ or $d(\ln \eta) / d(\ln \xi)$
$\bar{u}$,	Local mean velocity, L/T
U_a,	Free stream velocity, L/T
x, y, z,	A right-handed coordinate system with origin at the virtual origin of the boundary layer, L
X or x,	Distance along surface downstream from the virtual origin, L
x_o,	Longitudinal distance at which source is located from virtual origin of boundary layer, L
z_o,	Roughness length, L
$\bar{z}$,	Height to center of gravity of probability density distribution for single particle releases, L
θ,	Momentum thickness, L

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[*] The symbols designating dimensions have the following meanings: M - mass, L - length, T - time.

NOMENCLATURE - Continued:

δ,	Boundary-layer thickness, L
η,	Vertical characteristic height of the diffusing plume where $C(x - x_o, 0, \eta) / C_{max} = 0.5$, L
ζ,	$\bar{z}/z_o$
σ,	Horizontal characteristic width of the diffusing plume where $C(x - x_o, \sigma, 0) / C_{max} = 0.5$, L
ξ,	$(x - x_o) / z_o$
γ, α, β,	Exponents, constants

INTRODUCTION

The study of diffusion of scalar contaminants in the lower atmosphere is of great interest of fluid dynamicists and micro-meteorologists. During the last decade, interest in atmospheric diffusion has been heightened by increasing attention given to air pollution in general and to the new hazards introduced by the products of nuclear processes. Considerable efforts have been directed toward the theoretical and experimental investigations on the transport processes of dynamically passive and chemically non-reactive scalar quantities, like heat and mass, released from continuous point and line sources in turbulent shear flows. A great majority of these investigations have been restricted to the case of aerodynamically smooth surfaces. But no systematic attempt has been made to determine the turbulent diffusion characteristics of matter in shear flow fields over rough surfaces under carefully controlled laboratory conditions. The present paper summarizes the results of the experimental invesitigation of Bhaduri [4] to determine mass diffusion characteristics in a turbulent boundary-layer flow over a rough surface.

The theoretical analysis of dispersion of matter released into the boundary-layer type of flow has followed two main approaches, which may be termed the "transfer theory" and the "statistical theory" respectively. In the former, a physical model is implied on the assumption that, in the presence of a gradient of concentration of a substance, the local rate of transport of material by turbulence is proportional to the local gradient, the proportionality factor being referred to as exchange coefficient or eddy diffusivity. Effective application of the idea entails two very difficult problems; namely, the assignment of suitable explicit mathematical forms to the eddy diffusivity in terms of the measurable properties of the flow field and the solution of the governing differential equations with appropriate boundary conditions.

In much of its development, the statistical approach [11] has not rested on any particular physical model of diffusion of matter but is essentially an analytical technique for describing the time history of marked fluid particles or elements in terms of the given statistical properties of the turbulent motion. Statistical theories are restricted to essentially homogeneous turbulence fields and their application in the turbulent boundary-layer type of flows encounters formidable mathematical difficulties. However, the Lagrangian similarity hypothesis, suggested by Batchelor [3] in 1959, and applied to diffusion in turbulent shear flow by Ellison [8] and Cermak [5], seems to be a powerful technique in predicting the gross characteristics of the concentration field through similarity arguments. A few predictions of this hypothesis are experimentally tested in the present investigation.

Valuable information is available, from various field studies [1, 2, 6], of turbulent diffusion phenomena in the lower atmosphere. Some of these data were used by Cermak [5] for comparisons with predictions from the Lagrangian similarity hypothesis. The diffusion phenomena in a turbulent boundary layer depend significantly on the inherent turbulent characteristics of the flow field, which in the atmosphere vary greatly in time and space. The phenomena can be studied in the wind tunnel under controlled conditions. According to the results obtained in reference 5, the wind-tunnel data are similar to field data obtained in the lower atmospheric layer and can be helpful in understanding the complex mass transport process which is intractable by purely mathematical means.

EQUIPMENT AND PROCEDURE

The diffusion study was conducted in a recirculating low-speed wind tunnel with a 6 ft square 30 ft long test section. The rough surface was formed by putting wooden strips 0.25 in. square x 6 ft long spaced 3 in. apart on plywood boards 0.75 in. thick. The plywood boards, with the roughness elements, were placed on the floor of the wind tunnel. A schematic diagram of the test section and test boundary is shown in Fig. 1.

Anhydrous ammonia gas, having a specific gravity of 0.60 relative to the air, was used as the diffusing gas. The gas was fed into the boundary layer at a constant rate through a stainless steel feed probe. The air-ammonia mixture was drawn through a sampling system by inducing negative pressure with a vacuum pump. The metered sample of air and ammonia was passed through an absorption tube containing diluted hydrochloric acid (HC1) which completely absorbed the ammonia from the mixture. Then the absolute quantity of ammonia was determined by means of a photoelectric colorimeter which had previously been calibrated.

Once the flow conditions were established, the vertical cross sections of the diffusing plume were mapped at several downstream distances from the source. The procedure for mapping any plume cross section is shown in Fig. 2.

EXPERIMENTAL RESULTS

Velocity Field

Vertical velocity profiles, $\overline{u}(z)$, were taken at various downstream locations with a mean velocity hot-wire anemometer. In order to compare the velocity field with that obtained by Moore [10] under similar conditions, dimensionless velocity profiles, $\overline{u}/U_a$ vs z/θ, were plotted. The present data compared favorably with those of Moore and are represented by the equation

$$\frac{\overline{u}}{U_a} = 0.51 \left(\frac{z}{\theta}\right)^{0.37} .$$

Details of the data analysis are given by Bhaduri [4].

Mean Concentration Field

The plume geometry and concentration distributions were experimentally determined for several cross sections downwind of the source (located 5 ft 5 in. downstream from the turbulence stimulator) for each fo the three source elevations, $h = 0.0625$ in., 0.5 in. and 1.0 in. The concentration field at each of the cross sections was mapped according to the procedure outlined in Fig. 2. A typical non-dimensional plume cross section is shown in Fig. 3, and vertical concentration profiles are shown in Figs. 4 and 5.

The mean concentration distributions at each cross section of the diffusing plume were influenced by several factors:

(a) The ratio of the source elevation to the depth of the boundary layer at the feed point.

(b) The distribution of mean (time averaged) velocity of the flow field.

(c) The characteristic turbulence of the boundary layer.

(d) The size, shape and orientation of the roughness elements (kept constant in this study).

The concentration distributions are represented by a dimensionless exponential form as follows:

$$\frac{C}{C_{max}} = e^{-\gamma\left[\left(\frac{|y|}{\sigma}\right)^{\alpha} + \left(\frac{z}{\eta}\right)^{\beta}\right]}. \tag{1}$$

The two length parameters σ and η which characterize the diffusing plume geometry are defined by: $\frac{C\,[(x - x_o),\ \sigma,\ 0]}{C_{max}} = 0.5$ and $\frac{C\,[(x - x_o),\ 0,\ \eta]}{C_{max}} = 0.5$. The quantities γ, α and β are constants. Both α and β are determined from experimental results and the value of γ is determined by using the definitions of σ and η . It was observed from the experimental data at various downstream locations from the source that the diffusing plume was not fully developed until it reached a certain downstream position. Equation 1 is expected to be valid only in the zones where the concentration field is fully developed and where both lateral and vertical distributions maintain their similarity of form at successive downstream locations.

The asymmetrical vertical plume geometry in the undeveloped regime is a consequence of non-isotropic turbulent diffusion and convective deformation due to the mean velocity gradient. Distribution of material in the horizontal plane from a continuous point source is on the average a close approximation to Gaussian form. This is mainly due to homogeneity of statistical properties of the air flow in the horizontal.

In order to derive an empirical equation of the type of Equation 1, dimensionless plots, C/C_{max} vs y/σ for $z/\eta = 0$, and C/C_{max} vs z/η for $y/\sigma = 0$ and C/C_{max} vs y/σ, for several values of z/η at various downstream distances $(x - x_o)$, were made for the three different source heights employed. From the non-dimensional mean lateral and vertical concentration profiles dimensionless functional expressions for the plume cross sections were developed for the three source heights. Figure 6 shows a typical comparison of the functional representation and the data. The agreement between the isoconcentration contours is within the estimated error in concentration determination. The empirical equations which approximately describe the concentration field are:

$$\frac{C}{C_o} = e^{-0.692\left[\left(\frac{|y|}{\sigma}\right)^{1.92} + \left(\frac{z}{\eta}\right)^{1.70}\right]} \tag{2}$$

for $h = 0.0625$ in. and

$$\frac{C}{C_o} = e^{-0.692\left[\left(\frac{|y|}{\sigma}\right)^{1.80} + \left(\frac{z}{\eta}\right)^{1.87}\right]} \tag{3}$$

for $h = 0.5$ in. and $h = 1.0$ in. Figures 7, 8 and 9 show how the height η and lateral width σ of the diffusing plumes grow with distance downstream from the source. An interesting feature of the curves shown in Figs. 7, 8 and 9 is that they indicate similar growth rates for η and σ.

The attenuation of downstream ground-level concentration C_o for the three cases is shown in Fig. 10 and is found to follow a pattern similar to that for the smooth case [7, 9] except that the magnitude of the concentration at the boundary (top of the roughness elements) is relatively small. Variation of ground-level concentration is further discussed in the light of the Lagrangian similarity hypothesis in the next section.

Experimental Verification of the Langrangian Similarity Hypothesis

One of the main objectives of the present investigation was to test the Lagrangian similarity hypothesis as extended by Cermak [5]. In order to get a thick boundary layer for the flow field, the source was moved downstream to a new location (188 in. from the turbulence stimulator). The boundary-layer thickness at this position was 7.36 in. It was anticipated that this would satisfy the requirement of $\frac{\eta}{\delta} \leq 0.4$. The gas feed rate and flow velocity were maintained at 5.55 mg/sec and 12.5 ft/sec respectively. Data for ground-level concentration in the longitudinal direction were collected for each of the three source heights. Also, vertical (at $y = 0$) and lateral (at $Z = 0$) concentration data were collected at various downstream distances from the source.

Cermak [5] showed that the Lagrangian similarity hypothesis yields the following relations:

$$bk\,\xi = \zeta \ell n\, \zeta - (\zeta - H) + (b - 1)\, H \ell n\, H \tag{4}$$

and

$$m_{cp} = -kb\,\xi \left(\frac{1 + 2 \ell n\, \zeta}{\zeta \quad n^2\, \zeta} \right) \tag{5}$$

where m_{cp} is the slope of the tangent to the points on the curve of $\log C_{max}$ vs $\log \xi$. In order to compare the experimental results with theoretical predictions, values of $-m_{cp}$ were computed by using Eqs. 4 and 5. The longitudinal distribution of C_o vs $(x - x_o)$ was plotted on log-log paper and the slopes $-m_{cp}$ were graphically determined. Figure 11 shows the theoretical and experimental values of $-m_{cp}$ for various source heights. The agreement between the experimental values and theoretical values of $-m_{cp}$ as predicted by Cermak [5] is remarkably close, especially in the fully developed regions of the concentration field.

Another important result of the hypothesis, on the assumption that the plume width and height are related to downstream distance $(x - x_o)$ from the source by σ or η $(x - x_o)^{\ell n_{cp}}$, is that there exists a relation

$$n_{cp} = \frac{bk\ \xi}{\zeta\,\ell n\ \zeta} \tag{6}$$

where n_{cp} is the slope of the tangent to the points on the curve where the logarithm of plume width or height is a function of log ξ. The slopes of the curves, determined by the data, given in Figs. 7, 8 and 9 were measured graphically for various downstream locations to obtain n_{cp} for the experimental data. These slopes are compared with those predicted by Eq. 6 in Fig. 12. Thus, the agreement between the experimental results and predictions of the Lagrangian similarity hypothesis is fair.

In order to make any generalized conclusion regarding validity of the Lagrangian similarity hypothesis, more experimental data for different source heights and flow conditions are necessary. The hypothesis is strictly valid in the constant shear layer zones of the flow field (inner part of the turbulent boundary layer). Indeed, the dimension of the characteristic plume height for the studies described here was always larger than the estimated constant shear-layer thickness; however, η / δ was always less than 0.4.

The values of the roughness parameter z_o were determined graphically from a semi-logarithmic plot of the mean velocity with distance from the boundary and are subject to erros of 10 per cent.

The slopes $-m_{cp}$ and n_{cp} were determined graphically and are, of course, subject to drafting errors. The value of the Batchelor constant has been taken as 0.1. Cermak [5] used this value for correlating experimental results and found that $b = 0.1$ consistently gave good correlations. In the present investigation, the value of $b = 0.1$ did, in fact, give much better correlation than did the value of 0.4 suggested by Ellison [8].

CONCLUSIONS

The principal results derived from this study of turbulent diffusion of mass from a point source in a turbulent boundary layer over a rough surface are the following:

1. When the concentration field is fully developed and the vertical and lateral concentration profiles are self-similar at every successive downstream distance from the source, the concentration distribution at any plume cross section can be determined by a non-dimensional empirical equation of the form

$$\frac{C}{C_{max}} = e^{-0.692\left[\left(\frac{|y|}{\sigma}\right)^{\alpha} + \left(\frac{z}{\eta}\right)^{\beta}\right]} \qquad (7)$$

The values of α and β for the case of source height $h = 0.0625$ in. are 1.92 and 1.70 respectively. For the cases of source heights $h = 0.5$ in. and $h = 1.0$ in., the values of α and β are different and are 1.80 and 1.87 respectively.

2. The experimental results for the attenuation of the axial ground-level concentration agree well with the theoretical predictions of the Lagrangian similarity hypothesis, as extended by Cermak [5] for the three source heights used in the present investigation.

3. The growth of the characteristic plume widths σ and η agrees satisfactorily with values predicted by the Lagrangian similarity hypothesis.

4. Further diffusion measurements to establish the full range of validity of the Lagrangian similarity hypothesis should be made because the hypothesis is simple and powerful and has been able to stand under all tests made so far.

ACKNOWLEDGMENTS

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REFERENCES

1. M. L. Barad, "Project Prairie Grass, A Field Program in Diffusion," R. S. No. 18, Geophysical Research Papers, No. 59, Vols. 1 and 2, Geophysics Research Directorate, Air Force Cambridge Research Center, Bedford, Massachusetts, 1958.

2. M. L. Barad, "Analysis of Diffusion Studies at O'Neill, Atmospheric Diffusion and Air Pollution," Advances in Geophysics, 6:389-398, Academic Press, 1959.

3. G. K. Batchelor, "Note on Diffusion From Sources in a Turbulent Boundary Layer," Cambridge University, Unpublished, 1959.

4. S. Bhaduri, "Mass Diffusion From a Point Source in a Turbulent Boundary Layer Over a Rough Surface," Ph.D. Dissertation, Colorado State University, Fort Collins, Colorado, July 1963, 167 p.

5. J. E. Cermak, "Lagrangian Similarity Hypothesis Applied to Diffusion in Turbulent Shear Flow," Journal of Fluid Mechanics, 15 (1): 49-64, 1963.

6. H. E. Cramer, F. A. Record, and H. C. Vaughn, "The Study of the Diffusion of Gases or Aerosols in the Lower Atmosphere," Massachusetts Institute of Technology, Department of Meteorology, Final Report under Contract No. AF 19(604)-1058, 1958, 70 p.

7. K. S. Davar, "Diffusion From a Point Source Within a Turbulent Boundary Layer," Ph.D. Dissertation, Colorado State University, Fort Collins, Colorado, July 1961, 161 p.

8. T. H. Ellison, "Turbulent Diffusion," Science Progress, 47:495-506, 1959.

9. R. C. Malhotra, "Diffusion From a Point Source in a Turbulent Boundary Layer With Unstable Density Stratification," Ph.D. Dissertation, Colorado State University, Fort Collins, Colorado, 1962, 171 p.

REFERENCES - Continued:

10. W. L. Moore, "An Experimental Investigation of the Boundary Layer Development Along a Rough Surface," Ph.D. Dissertation, Department of Mechanics and Hydraulics, State University of Iowa, Iowa City, Iowa, August 1951, 58 p.

11. G. I. Taylor, "Diffusion by Continuous Movements," Proceedings, London Mathematical Society, 20:196-212, 1920.

LIST OF FIGURES

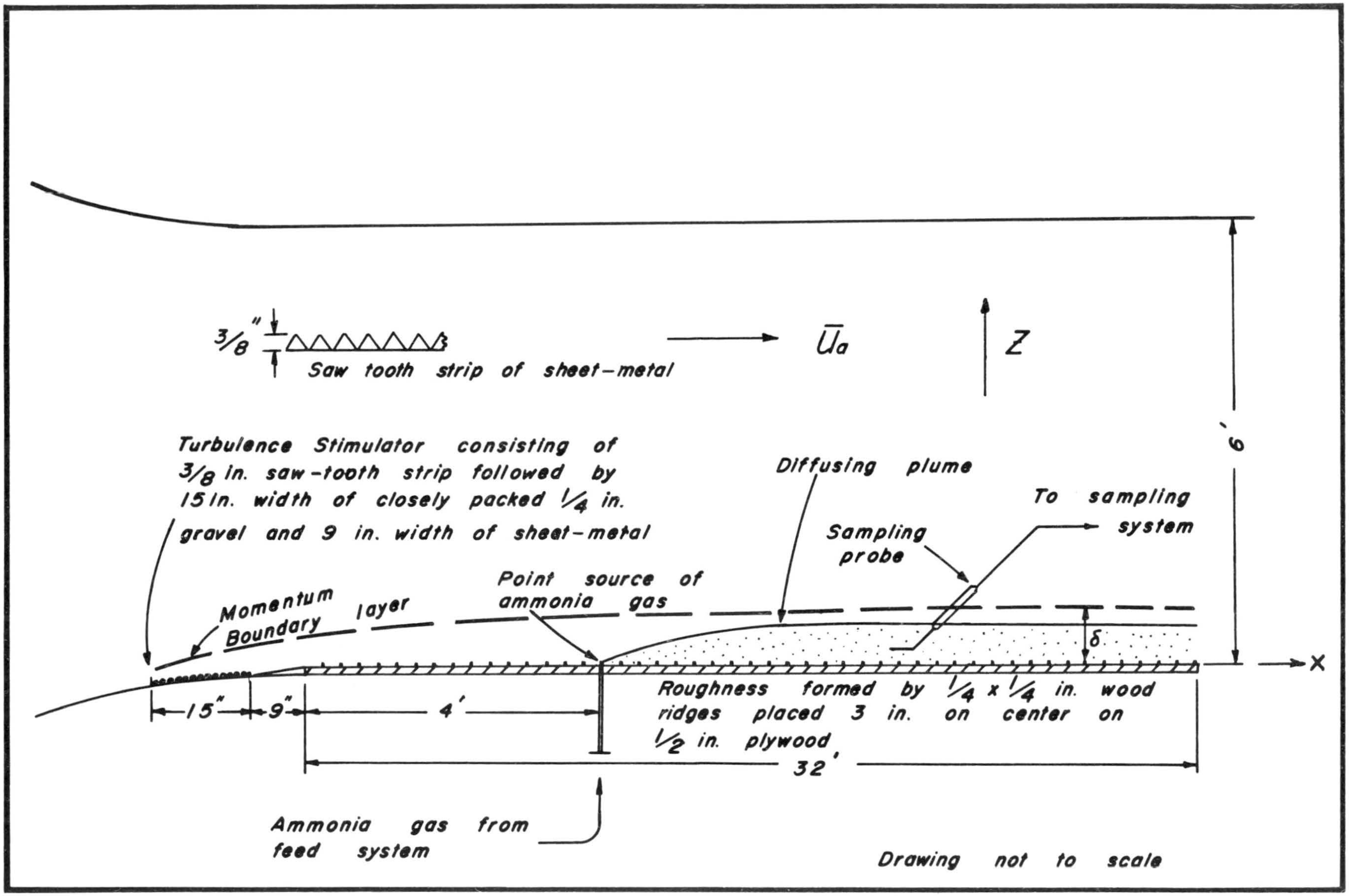
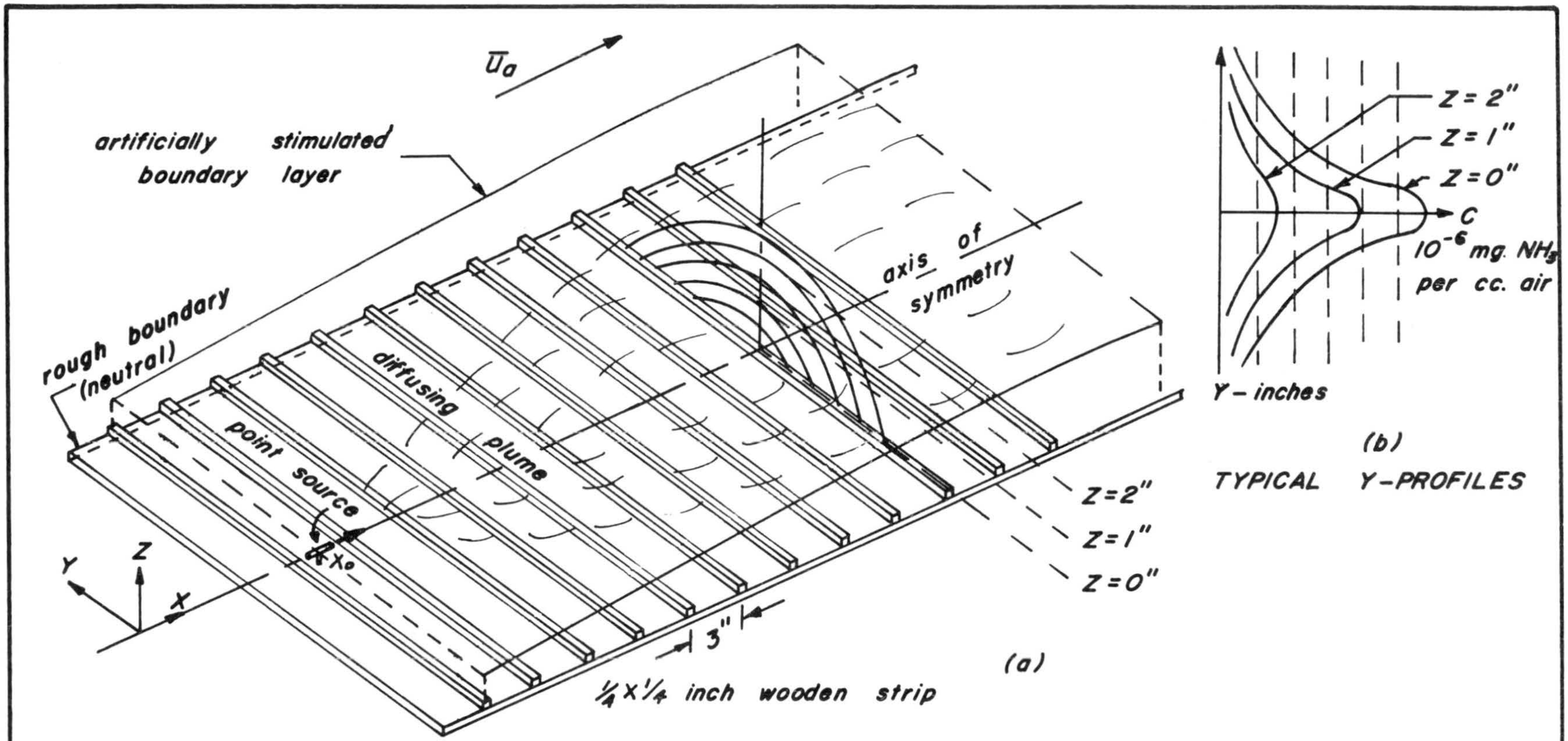


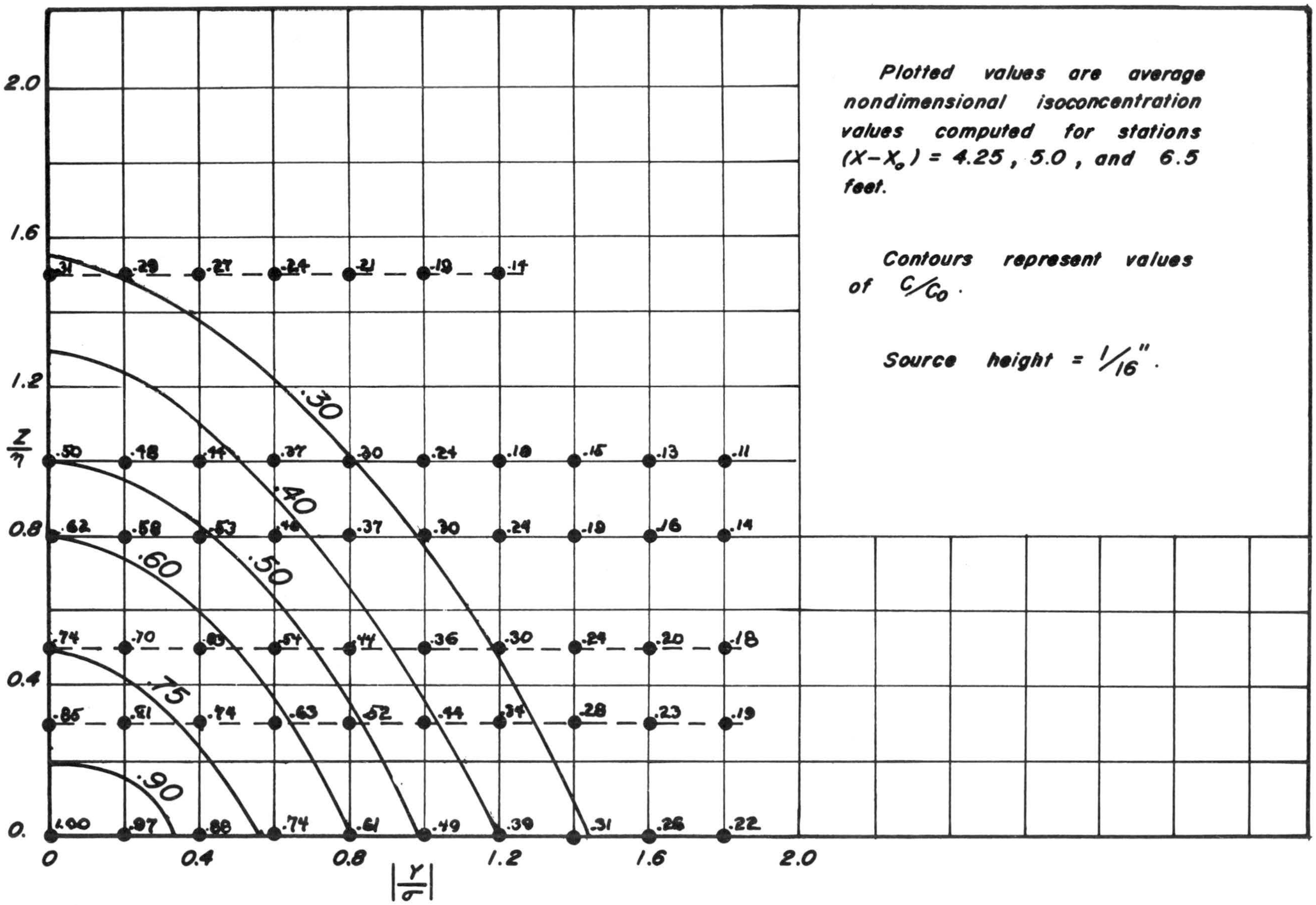
FIG. 1 TEST SECTION GEOMETRY



Procedure for mapping any plume cross section

1. *Y-profiles of concentration measurement were made by traversing the sampling probe parallel to the Y-axis for a fixed height Z. Sampling points were spaced between ¼" to 1" apart; about 20 point measurements were made per Y-profile.*
2. *The Y-profiles were repeated for several heights : Z = 0, 1, 2... ins.*
3. *The measured Y-profiles were superposed as in (b).*
4. *Isoconcentration contours were mapped by reading Y coordinates of points of equal concentration in (b), and transposing to the plume cross section.*

FIG. 2 DEFINITION SKETCH OF PLUME GEOMETRY



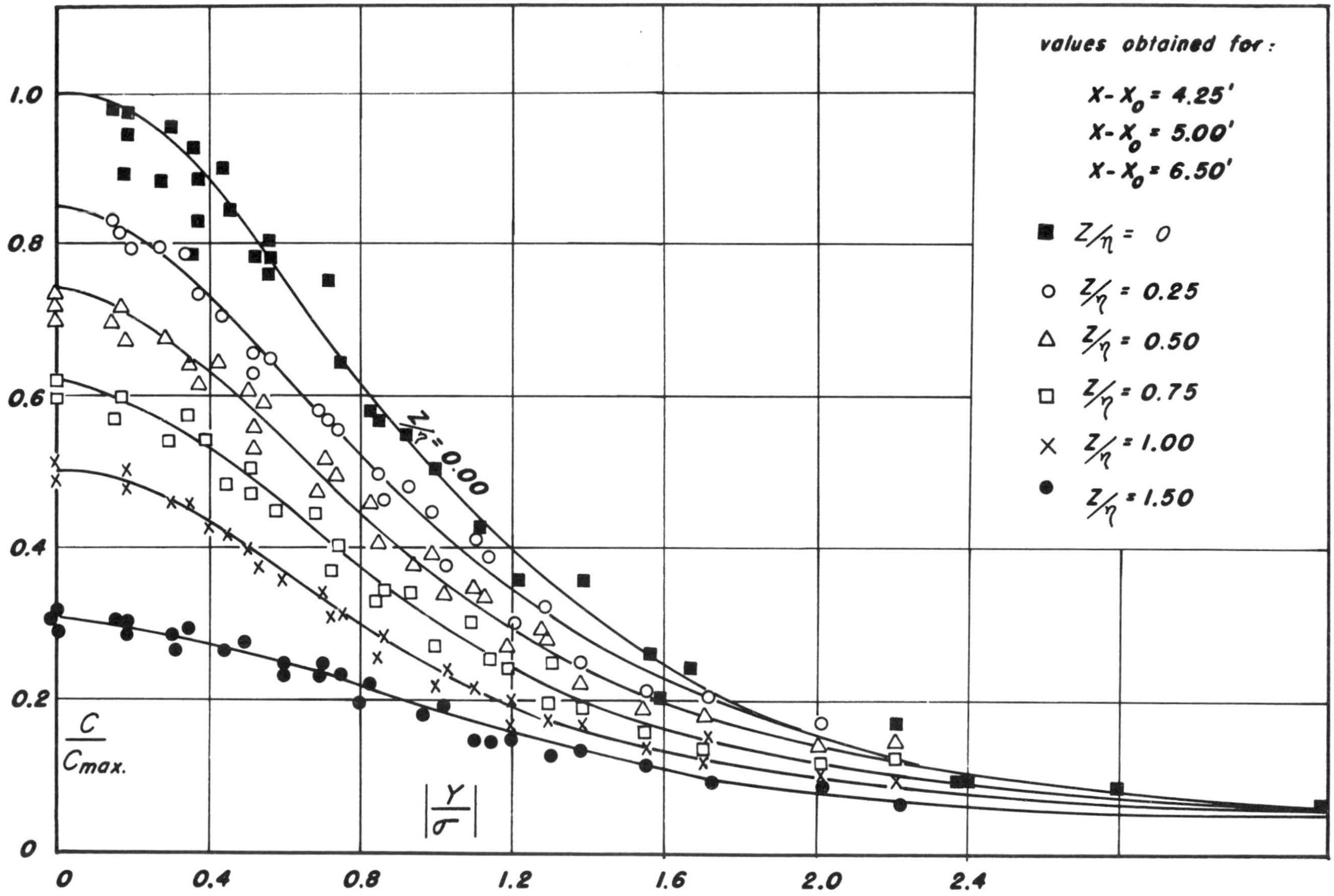
Plotted values are average nondimensional isoconcentration values computed for stations $(X-X_o) = 4.25$, 5.0, and 6.5 feet.

Contours represent values of C/C_0.

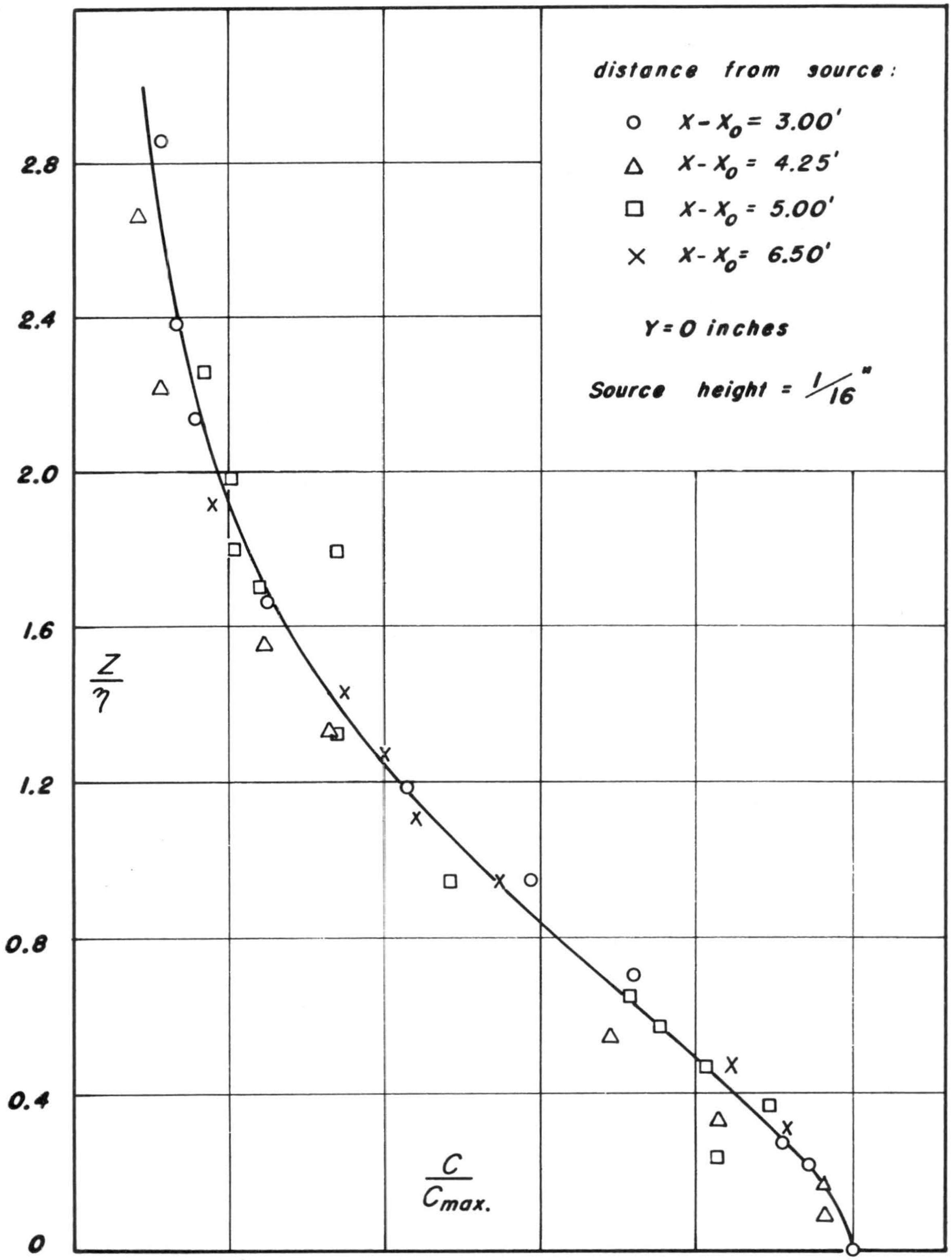
Source height = 1/16".

A TYPICAL

FIG. 3 NON-DIMENSIONAL PLUME CROSS SECTION — h = 1/16"



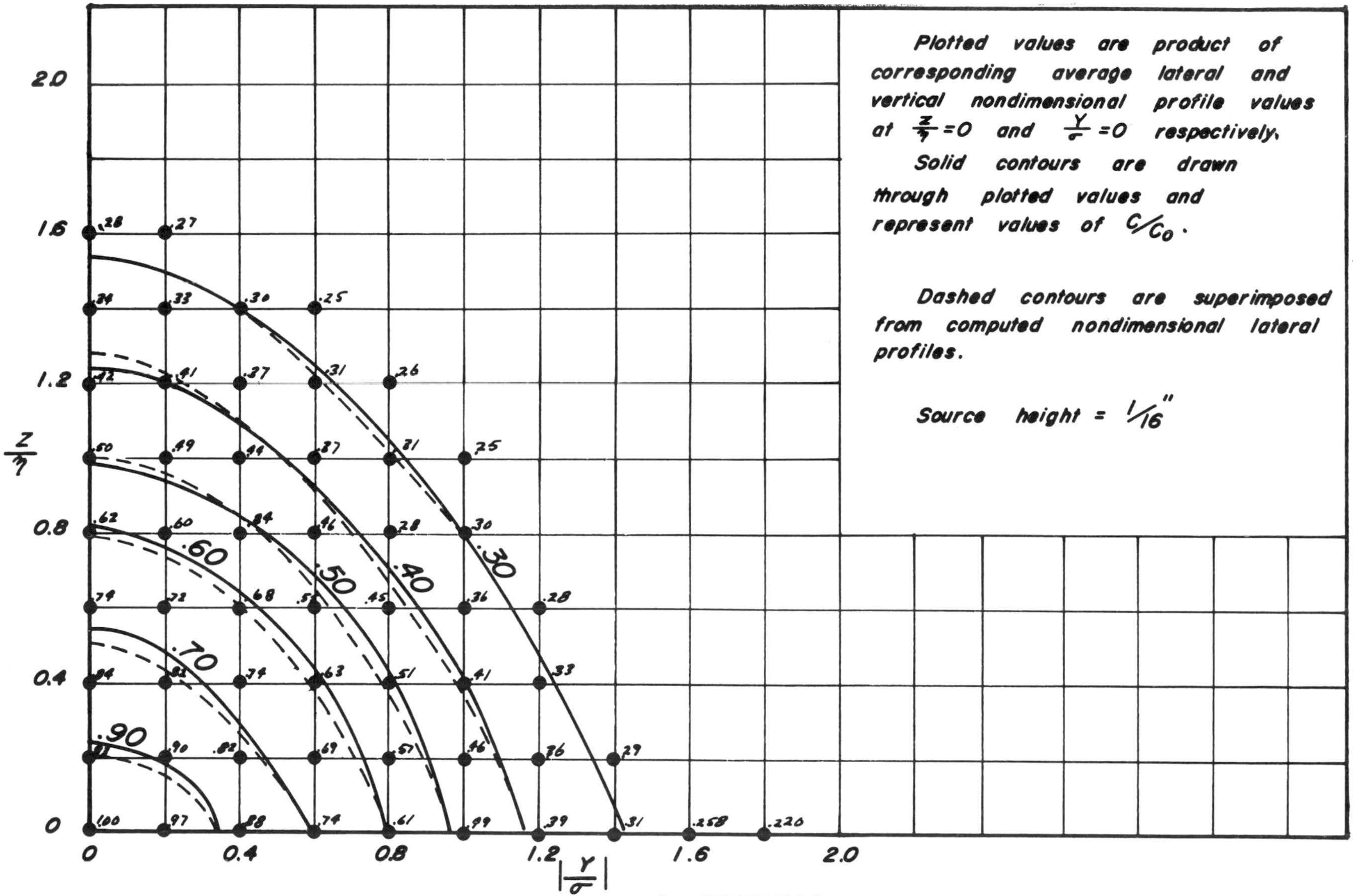
TYPICAL
FIG. 4 NON-DIMENSIONAL LATERAL CONCENTRATION PROFILES FOR VARIOUS VALUES OF Z/η — $h = 1/16''$



A TYPICAL

FIG. 5 NON-DIMENSIONAL VERTICAL

CONCENTRATION PROFILE—h=1/16", Y=0"



Plotted values are product of corresponding average lateral and vertical nondimensional profile values at $\frac{Z}{\eta} = 0$ and $\frac{Y}{\sigma} = 0$ respectively.

Solid contours are drawn through plotted values and represent values of C/C_0.

Dashed contours are superimposed from computed nondimensional lateral profiles.

Source height = 1/16"

FIG. 6 A TYPICAL NON-DIMENSIONAL PLUME CROSS SECTION — h = 1/16"

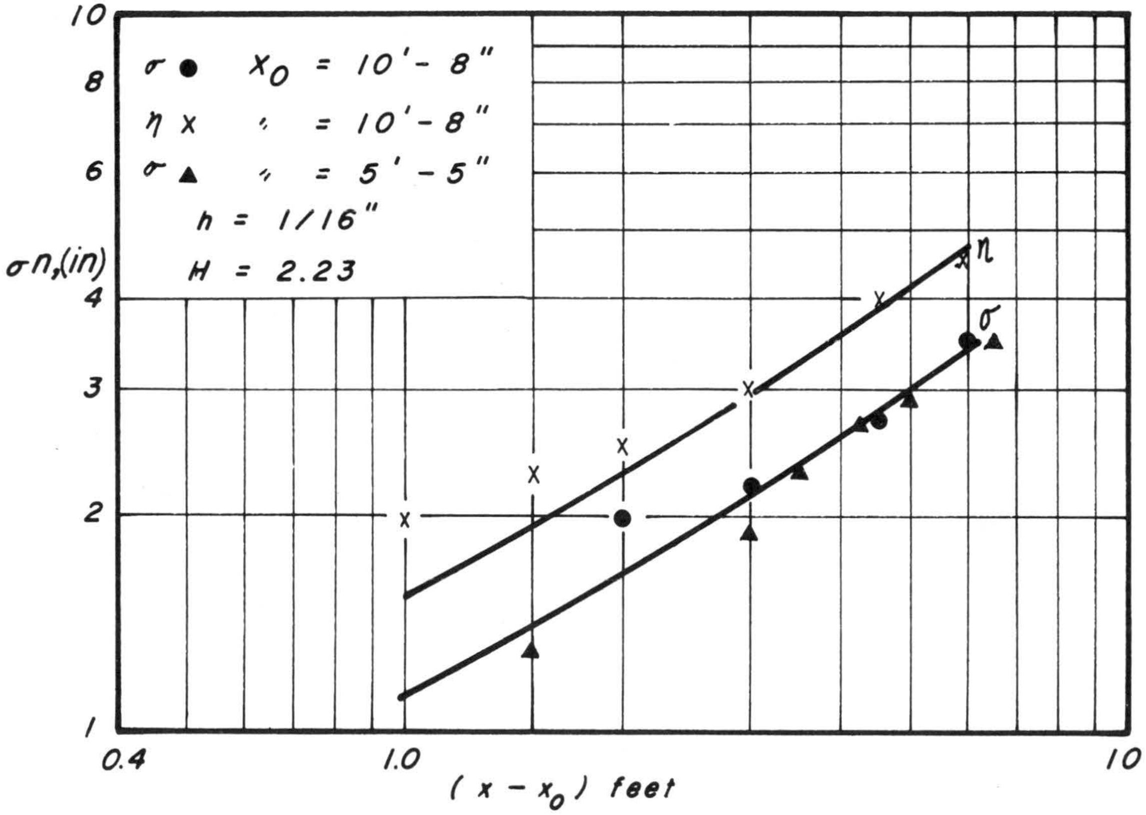


FIG. 7 VARIATION OF σ AND η WITH DISTANCE FROM GAS SOURCE

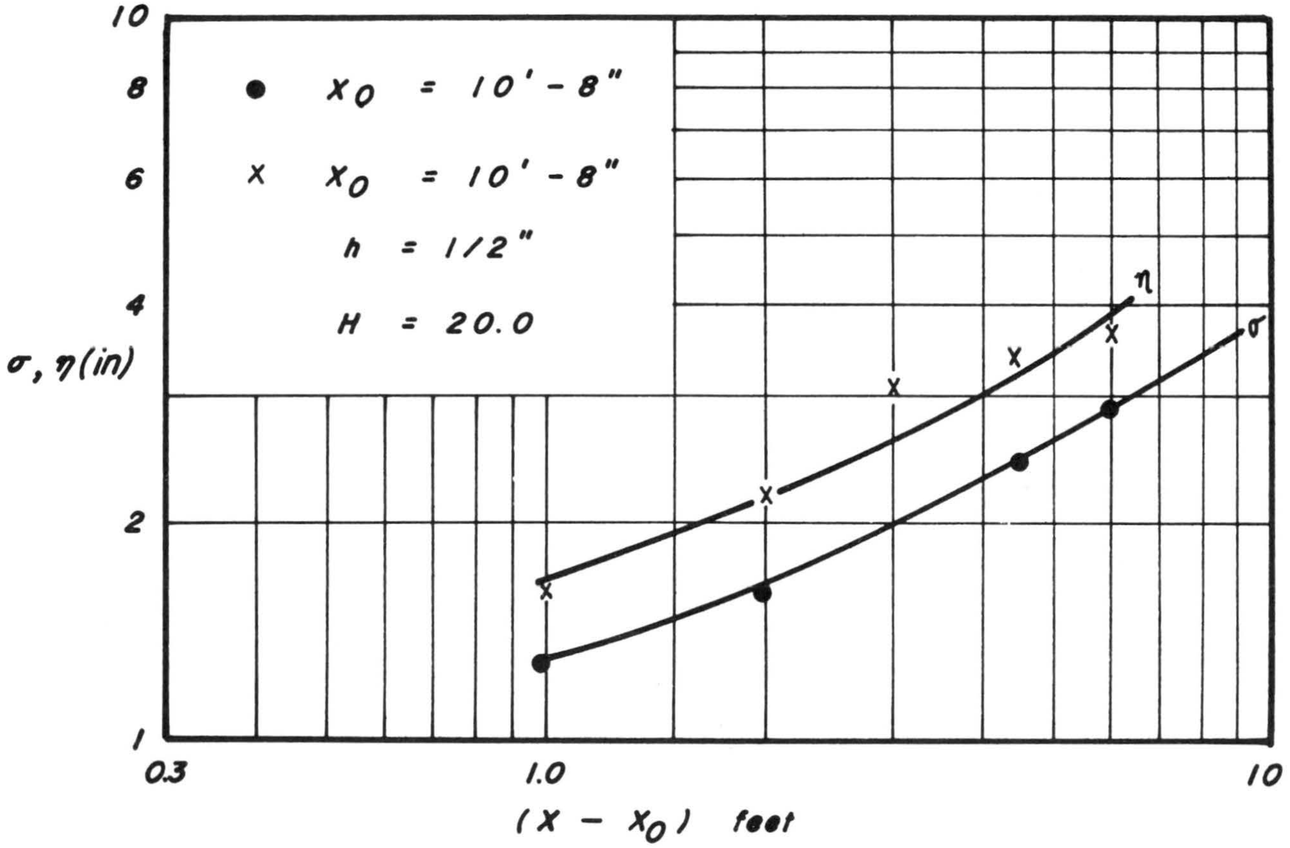


FIG. 8 VARIATION OF σ AND η WITH DISTANCE FROM GAS SOURCE

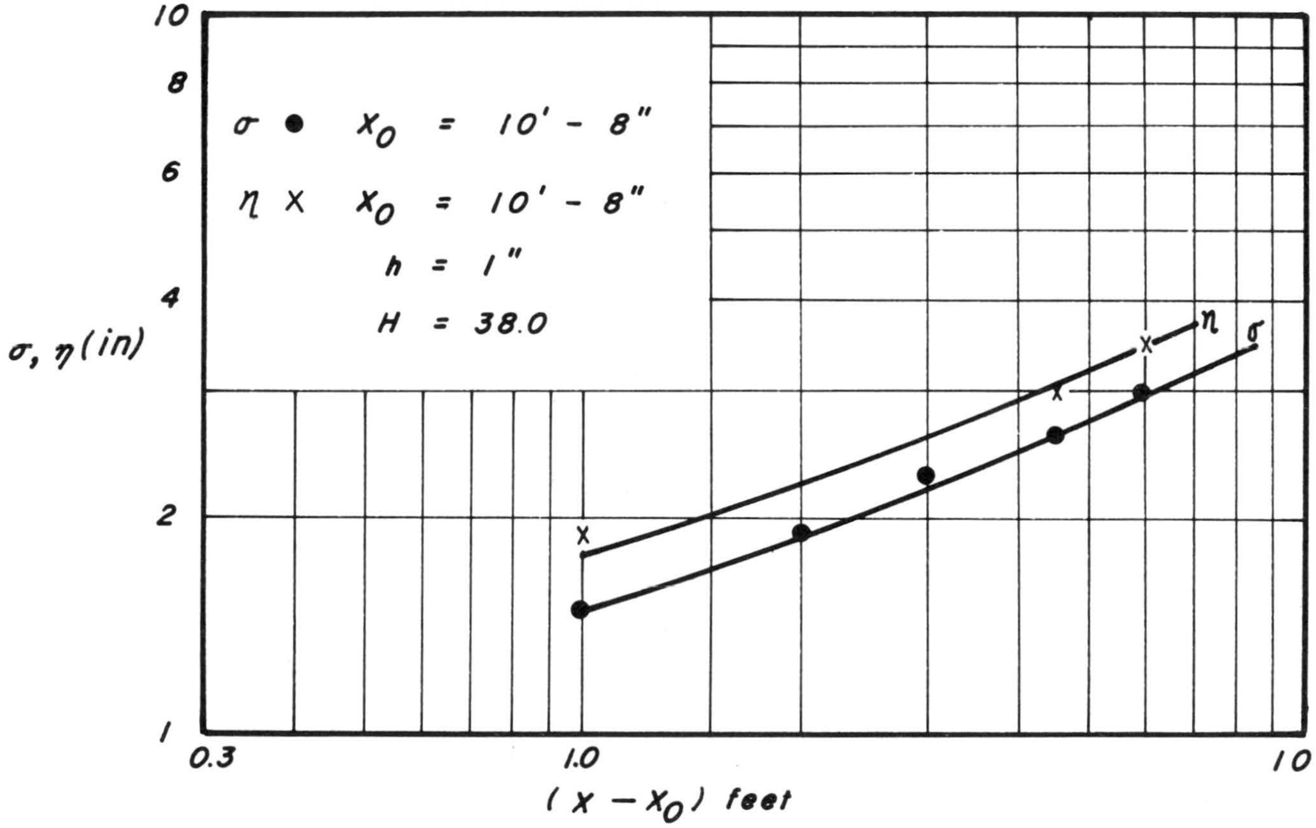


FIG. 9 VARIATION OF σ AND η WITH DISTANCE FROM GAS SOURCE

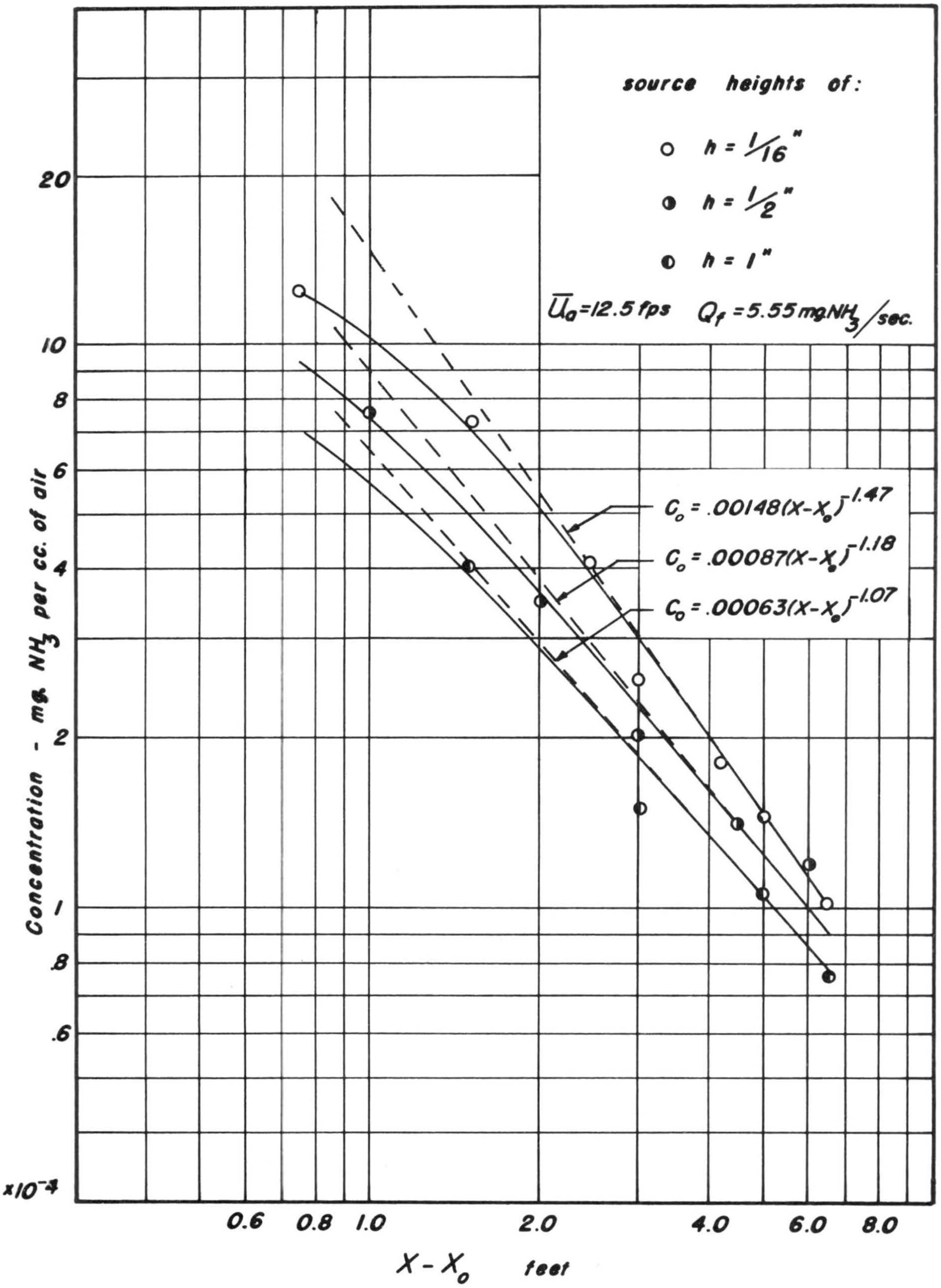


FIG. 10 EFFECT OF SOURCE HEIGHT ON BOUNDARY CONCENTRATION

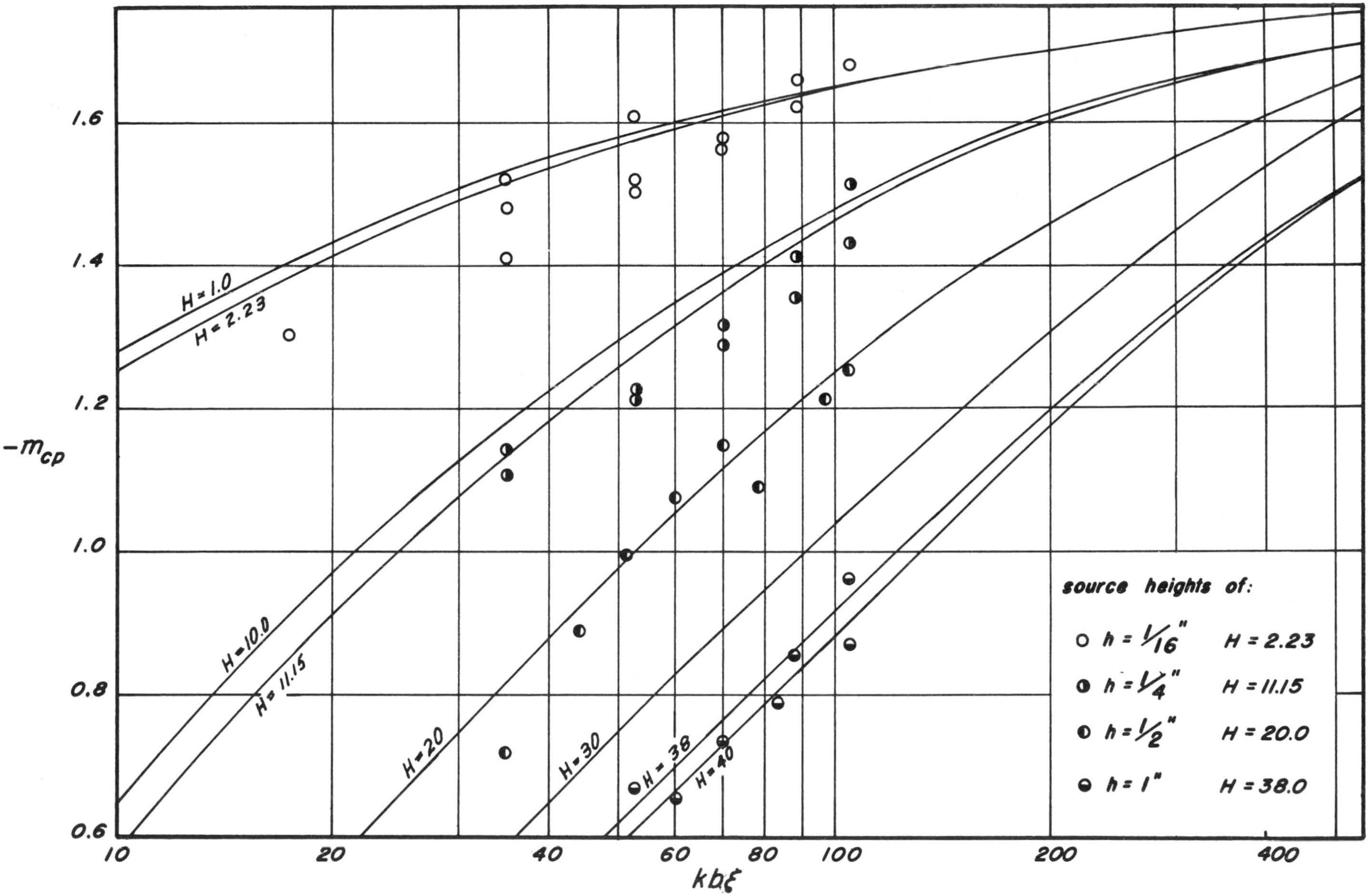


FIG. 11 COMPARISON OF THEORETICAL AND EXPERIMENTAL SLOPES OF C_o vs. $(X-X_o)$ CURVES

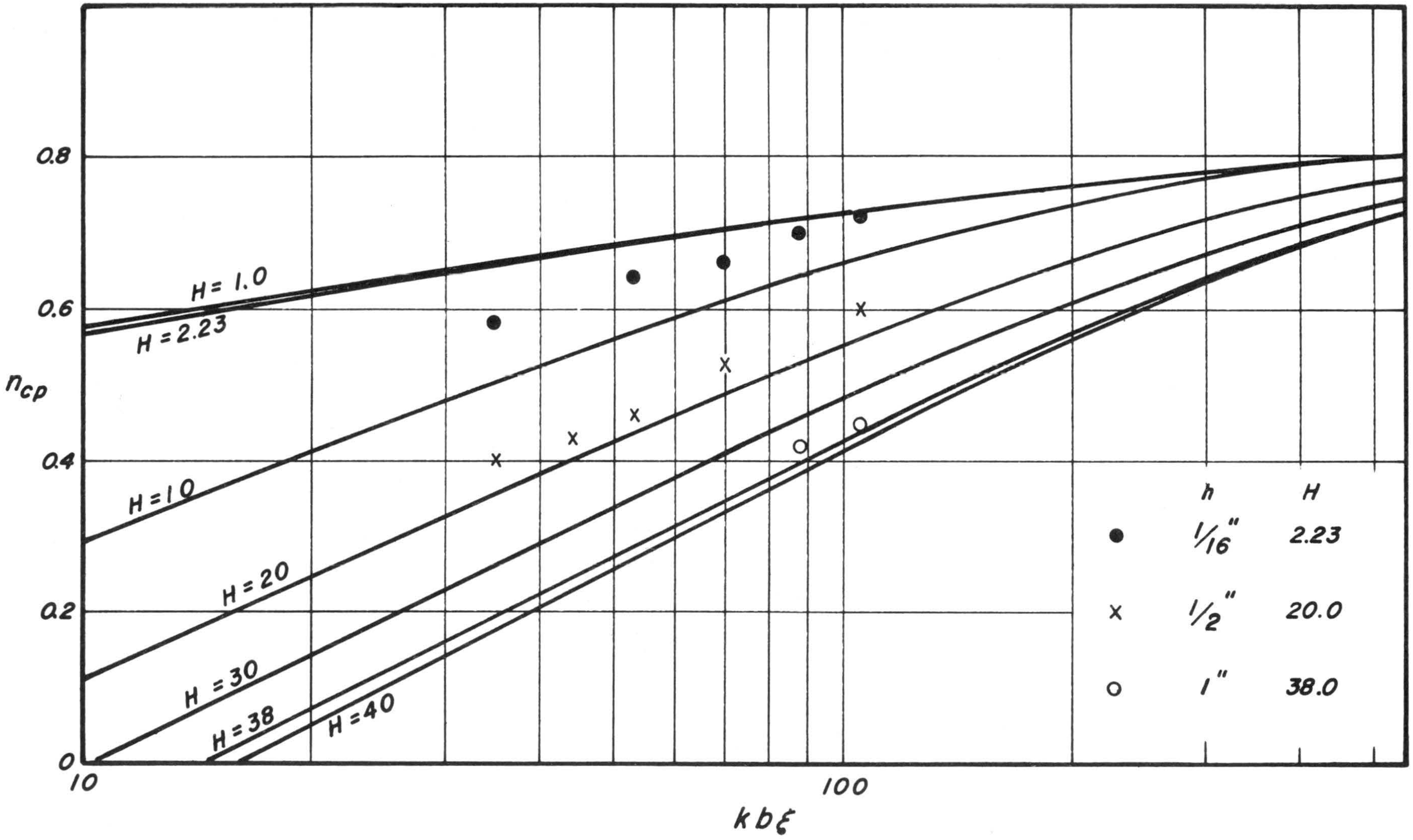


FIG. 12 COMPARISON OF THEORETICAL AND EXPERIMENTAL SLOPES OF σ, η VS $(x - x_0)$ CURVES