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Building the City: Sunk Capital, Sequencing and Institutional Frictions

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Abstract
This paper models a growing city, and focuses on investment decisions and consequent patterns of land use and urban density. We distinguish between formal and informal sector construction. The former can be built tall (at a cost), but structures once built are durable and cannot be modified. Investments are based on expectations about future growth of the city. In contrast, informal structures are malleable and do not involve sunk costs. As the city grows areas will initially be developed informally, and then formally; formal areas are redeveloped periodically. This process can be hindered by land right issues which raise the costs of converting informal to formal sector development. The size and shape of the city are sensitive to the expected returns to durable investments and to the costs of converting informal to formal sector usage. We take the model to data on the built environment for Nairobi, to study urban growth and change between 2004 and 2015 in a context where population is growing at about 4% a year. We study the evolution of building footprints and heights, development at the fringe, infilling, and redevelopment of the formal sector.

Keywords: city, urban, urban growth, slum development, urban structure, urban form, housing investment, capital durability
JEL Classifications: O14; O18; R1; R3

1. Introduction

This paper examines housing development and redevelopment in a growing city. Our focus is a city in a developing country, containing both formal and informal housing sectors. We look at formal sector development and redevelopment, at the allocation (and misallocation) of land between sectors, at the transition between the two, and at the role of property rights and expectations in altering paths of urban development. We develop a model of a growing city in which buildings are durable and investment decisions are taken on the basis of expectations about the future growth of the city. The work builds on the standard monocentric urban model and its dynamic extensions (e.g. Braid 2001). However, most of the urban literature is essentially static – and designed to analyse slowly changing developed country urban areas. The objective of this paper is to capture key features of developing country cities. The paper takes the model to the data for Nairobi, constructing an unusual data set on the built environment in 2004 and 2015, and using it to track the physical transformation of a city which shares common features with other cities in developing countries.

The features captured in the model are as follows. First, the city is growing in both population and area. This means that land rents and patterns of land use are changing through time. Second, the city contains 'formal' or modern structures. Formal buildings involve sunk capital costs, can be built tall, and are hard to modify once constructed. Since they are durable, investment decisions are based on expectations about future land rents, as driven by future incomes and populations. As the city grows there will be periodic demolition and redevelopment of formal areas. Third, the city may also contain informal, or slum structures. Given the technology and materials used in construction, these building are not likely to be built tall, but they can be rebuilt and adjusted after their initial construction; we suppose that capital used in such structures is not sunk, but remains perfectly malleable. Finally, and critical to some of our results, there is a cost of conversion of informal to formal land use that varies over time and even across properties in the city.

We show that as the city grows land will initially be developed with informal structures which are then replaced by formal structures, which will themselves be subject to intermittent redevelopment. The share of urban population in informal structures will generally decline through time. This decline need not be driven by growth of household income: it is instead a consequence of rising land values (and hence a greater return to achieving density by building upwards) as the city expands. Heterogeneity of conversion costs means that informal structures may be very persistent, existing alongside formal structures, and having long-lasting implications for the fabric of the city.

In taking the model to the data for Nairobi, we know the counts and footprint of buildings throughout the urban area for 2004 and 2015 based on tracings of all building polygons from aerial photos. For 2015 we also know heights of these buildings based on LiDAR data. We have high resolution satellite data for 2004 and 2013, from which roads and buildings and neighbourhoods can be extracted by both human and machine counts. We use primarily the polygon and LiDAR data set to analyse how Nairobi transforms from 2004 to 2015 at all locations, tracing demolition, redevelopment, and in-fill. We also have two sources of data on housing and land rents, by location and for a single point in time, and have classifications of formal and 'slum' areas.

Nairobi conforms to the model predictions that (1) in the formal sector, house rents and land prices decline with distance to the centre; (2) building heights decrease with distance to the centre; (3) heights in slum areas are much lower than in the formal sector near the centre and lower throughout; (4) between 1-6 kms from the centre there was major redevelopment between 2004 and 2015 of formal sector buildings into higher new buildings; (5) expansion of the informal sector is towards the city fringe, with intensive demolition very near the centre; and (6) there is widespread intensification of land use, especially on the city fringes with infill of new buildings . We find that development of the informal into formal sector housing mid-city over the 11 years is slow. We explore the cost of converting informal to formal use in the institutional context of Nairobi, to suggest there are high costs of conversion in traditional slums near the centre in Nairobi, meaning that some land is not in highest and best use, with a commensurate welfare loss.

There are four novel aspects to the paper. First is the modelling. While Baird (2001) has a dynamic monocentric model with durable capital, no dynamic model deals with informality, conversion costs, and expectations. Second are the data. While, there is work on the USA using demographic census data to try to analyse redevelopment over of periods of time (Rosenthal and Brueckner, 2009), the work does not utilize data on buildings, with demolition, redevelopment, and intensification (infilling). As far as we know we are the first to utilize changes in direct building information to detail the changes in the urban landscape. Third, we focus on a major developing country city, where population growth is much more rapid than in developed countries and where land market institutions are weak. Fourth are the policy aspects where, in a general equilibrium context, we can think about the role of expectations and conversion costs, and for the latter make inferences from the data about the impact on city development.

The analytical framework makes clear some of the risks faced by a growing city, and the role of policy in addressing these risks. Expectations are fundamentally important in investment decisions, and low expectations of the future development of the city have a major impact in distorting investment levels below their efficient levels. And there are other market failures that deter investment, such as inappropriate regulation and land titling or capital market imperfections. The consequences of such imperfections are long-lasting, given the durability of structures.

The paper is organised as follows. The basic model and core theoretical results are set out in section 2. Section 3 turns to data and analysis of Nairobi. Section 4 looks specifically at Nairobi slums, conversions costs, and misallocation. Section 5 concludes.

2. Theory

In this section we set out the model of a growing city, focusing on investment decisions and consequent patterns of land use and urban density. The analysis is initially developed assuming that prices of housing are exogenous, although changing through time. We then show how prices can be endogenised in a full urban equilibrium.

2.1 Land development:

We consider the problem of developing a particular unit of land, denoted x (which can be thought of as a measure of location and will later be interpreted as distance to the CBD). Developers can use two alternative building technologies, informal or formal, denoted by subscripts $i = I, F$, to build housing with floor-space h_i per unit land. More floor-space can be achieved either by building high, or by increasing building footprint per unit area. In the theory section we will refer to h_i as height. In the empirical section we look at each separately and derive a measure of building volume per unit area (which, given ceiling height, is proportional to floor space).

At some date (say time 0) the present value of earnings from a unit of land at x that has not yet been developed is

$$\begin{aligned} R(x) &= \int_0^{\tau_0} r_0 e^{-\rho t} dt + \int_{\tau_0}^{\tau_1} \{p_I(x,t)h_I(x,t) - C_I(h_I(x,t))\} e^{-\rho t} dt \\ &+ \left\{ \int_{\tau_1}^{\tau_2} p_F(x,t)h_F(x,\tau_1)e^{-\rho t} dt - \left[C_F(h_F(x,\tau_1)) + D(x,\tau_1)\right] e^{-\rho\tau_1} \right\} \\ &+ \sum_{i=2} \left\{ \int_{\tau_i}^{\tau_{i+1}} p_F(x,t)h_F(x,\tau_i)e^{-\rho t} dt - C_F(h_F(x,\tau_i))e^{-\rho\tau_i} \right\}. \end{aligned} \qquad (1)$$

The first term is the present value of rent from undeveloped land (flow rent r_0 which we take to be constant), discounted at rate ρ and calculated up to the date of first development, which is denoted τ_0. The second term gives the present value of earnings from informally developed land. The price of informal housing is $p_I(x,t)$ per unit space and land rent is income from selling space, $p_I(x,t)h_I(x,t)$, minus construction costs.[1] Structures on informal development can be reconfigured at each instant, hence $h_I(x,t)$ may vary through time.

[1] We reserve the word ‘rent’ for income accruing to land, and use the word ‘price’ (per unit space) for housing services, although this is a per period flow, not a capital value.

Construction costs in the informal sector are a flow, occurring continuously through the life of the house; this can be thought of as the rental on the 'lego blocks' or iron sheets used in construction. We denote these costs $C_I\left(h_I(x,t)\right)$, and assume they are increasing and convex in height. The site is occupied by informal settlement for time interval τ_0, τ_1.

Remaining terms in equation (1) give the discounted value of rents earned over the life of formal sector buildings. The first formal sector development of the particular unit of land takes place at date τ_1, and subsequent redevelopments are at dates $\tau_2, \tau_3...$ Formal sector development and redevelopment is 'putty-clay': at the time of development, τ_i, a one-off construction cost $C_F\left(h(x,\tau_i)\right)$ is incurred, with the building height $h(x,\tau_i)$ then fixed. At the date of the next redevelopment there are neither costs to demolishing the structure nor benefits from recycling it back to putty.[2] The first formal sector development, occurring at date τ_1 also incurs a further one-time fixed cost, $D(x,\tau_1)$, of converting to formality. This represents the costs of formal road layout, sewerage, power supply etc. Formal sector development requires reasonably well defined property rights, such as land titling or formal leaseholds on land granted by the government. Obstacles to obtaining these rights may be substantial, particularly in African countries where much land is held traditionally under communal rights. $D(x,\tau_1)$, refers to the combined costs and obstacles, which may vary systematically with x or τ_1, or may vary randomly through the city.

Revenue earned on a unit of land formally developed at date τ_i is $p_F(x,t)h_F(x,\tau_i)$, $t \in (\tau_i, \tau_{i+1})$, with price variable over the life of the development and height fixed at date of construction. It is convenient to define the present value of the price of a unit of formal housing space over its life, relative to its price at date of construction,

$$V(x,i) = e^{\rho\tau_i}\int_{\tau_i}^{\tau_{i+1}}\left[p_F(x,t)/\,p_F(x,\tau_i)\right]e^{-\rho t}dt\,, \quad i = 1,2... \,. \qquad (2)$$

This can be interpreted as the value-to-rent ratio on a newly constructed property, since $V(x,i) = \int_{\tau_i}^{\tau_{i+1}} p_F(x,t)h_F(x,\tau_i)e^{-\rho(t-\tau_i)}dt\,/\left[p_F(x,\tau_i)h_F(x,\tau_i)\right]$, where rent is price times quantity of house on the unit of land. The numerator is the present value of revenue earned on the building discounted to date of construction (equal to the market value of the building), and the denominator is revenue at date of construction.

[2] Building materials in the informal sector have an alternative use, and hence a flow cost to their use in a particular structure. Materials, once used in the formal sector, are entirely sunk and have no alternative use value.

The developer's problem is to maximise $R(x)$, by choosing building height and dates of development and redevelopment. We look first at height decisions.

Building height:

First order conditions for choice of height are:

$$\frac{\partial R(x)}{\partial h_I(x,t)} = \left[p_I(x,t) - C_I{}'(h_I(x,t))\right]e^{-\rho t} = 0, \qquad t \in (\tau_0, \tau_1), \tag{3a}$$

$$\frac{\partial R(x)}{\partial h_F(x,\tau_i)} = \int_{\tau_i}^{\tau_{i+1}} p_F(x,t)e^{-\rho t}dt - C_F{}'(h_F(x,\tau_i))e^{-\rho\tau_i} = 0, \quad i = 1,2.... \tag{4a}$$

Notice that informal sector height is chosen at each instant, $t \in (\tau_0, \tau_1)$, so does not require forward looking behaviour. In the formal sector the choice is at date of construction, and is based on expected future prices. This is central to what follows.

It will sometimes be convenient to derive expressions using specific functional forms. We use iso-elastic construction cost functions, $C_I(h_I) = c_I h_I^{\gamma_I}$, $C_F(h_F) = c_F h_F^{\gamma_F}$ and assume that the cost of a single storey dwelling informal building is less than a single storey formal building, $c_F > c_I$.[3] However, the elasticity of costs with respect to height is greater for informal than formal, so $\gamma_I > \gamma_F$. With these iso-elastic forms the first order conditions for choice of height yield explicit expressions for *h*. From (3a), for informal development

$$h_I(x,t) = \left[\frac{p_I(x,t)}{c_I\gamma_I}\right]^{\frac{1}{\gamma_I - 1}}, \qquad t \in (\tau_0, \tau_1). \tag{3b}$$

For the formal sector, with sunk construction costs, the first order condition (4a) can be written $p_F(x,\tau_i)V(x,i) - C_F{}'(h_F(x,\tau_i)) = 0$ and hence, in the isoelastic case,

$$h_F(x,\tau_i) = \left[\frac{p_F(x,\tau_i)V(x,i)}{c_F\gamma_F}\right]^{\frac{1}{\gamma_F - 1}}, \qquad i = 1,2... . \tag{4b}$$

Dates of development:

Dates of development and redevelopment are also chosen to maximise $R(x)$. For the first development (which we assume for the moment to be informal), the optimal τ_0 is implicitly defined by

[3] Because of different building technologies and also because c_I recurs each unit time, while c_F is one-off.

$$\frac{\partial R(x)}{\partial \tau_0} = r_0 e^{-\rho\tau_0} - \left[p_I(x,\tau_0)h_I(x,\tau_0) - C_I(h_I(x,\tau_0))\right]e^{-\rho\tau_0} = 0\,. \qquad (5a)$$

This first order condition simply equates flow land-rents on undeveloped and informal land. Defining the second of these (i.e. informal sector land-rent at place *x* at date *t*) as $r_I(x,t)$, (5a) becomes:

First development, τ_0: $\quad r_0 = r_I(x,\tau_0)$, (5b)

where $\quad r_I(x,t) \equiv p_I(x,t)h_I(x,t) - C_I(h_I(x,t))$.

The first formal development takes place at date τ_1 satisfying

$$\frac{\partial R(x)}{\partial \tau_1} = \left[p_I(x,\tau_1)h_I(x,\tau_1) - C_I(h_I(x,\tau_1))\right]e^{-\rho\tau_1} - \left[p_F(x,\tau_1)h_F(x,\tau_1) - \rho C_F(h_F(x,\tau_1)) - \{\rho D(x,\tau_1) + D_\tau(x,\tau_1)\}\right]e^{-\rho\tau_1} = 0\,. \qquad (6a)$$

We define $r_F(x,t,\tau_i)$ as revenue net of perpetual capital costs at date *t* on land at *x* that is formally developed at date τ_i so:

Formalisation, τ_1: $\quad r_I(x,\tau_1) = r_F(x,\tau_1;\tau_1) - \{\rho D(x,\tau_1) + D_\tau(x,\tau_1)\}$, (6b)

where $\quad r_F(x,t,\tau_i) \equiv p_F(x,t)h_F(x,\tau_i) - \rho C_F(h_F(x,\tau_i))$.

The first redevelopment of formal land is at date τ_2 satisfying

$$\frac{\partial R(x)}{\partial \tau_2} = p_F(x,\tau_2)h_F(x,\tau_1)e^{-\rho\tau_2} - \left[p_F(x,\tau_2)h_F(x,\tau_2) - \rho C_F(h_F(x,\tau_2))\right]e^{-\rho\tau_2} = 0\,. \qquad (7a)$$

Generalising this for all redevelopments gives:

Redevelopment, τ_i, $i > 1$: $\quad p_F(x,\tau_{i+1})h_F(x,\tau_i) = r_F(x,\tau_{i+1},\tau_{i+1})$. (7b)

Equations (7) say that redevelopment should be undertaken at the date at which the cost saving from postponing capital spending, $\rho C_F(h_F(x,\tau_{i+1}))$, equals the revenue gain from rebuilding at new height, $p_F(x,\tau_{i+1})\left[h_F(x,\tau_i) - h_F(x,\tau_{i+1})\right]$.

For iso-elastic construction technologies, $C_I(h_I) = c_I h_I^{\gamma_I}$, $C_F(h_F) = c_F h_F^{\gamma_F}$, we can use the optimised values of *h* in equations (2b) and (3b) in equations (5) – (7) to generate expressions for the dates at which sites are (re-)developed. The date at which site *x* becomes informally developed, τ_0, given by equation (5b), is implicitly defined by

$$r_0 = \left[\frac{p_I(x,\tau_0)}{c_I \gamma_I} \right]^{\frac{\gamma_I}{\gamma_I - 1}} c_I(\gamma_I - 1). \tag{5c}$$

The date at which informal settlement becomes formalised, τ_1, is given by equation (6b) which using (2b) and (3b) becomes

$$\left[\frac{p_I(x,\tau_1)}{c_I \gamma_I} \right]^{\frac{\gamma_I}{\gamma_I - 1}} c_I(\gamma_I - 1) = \left[\frac{p_F(x,\tau_1)V(x,1)}{c_F \gamma_F} \right]^{\frac{\gamma_F}{\gamma_F - 1}} c_F\left(\frac{\gamma_F}{V(x,1)} - \rho \right) - \rho D(x,\tau_1). \tag{6c}$$

The dates at which successive formal redevelopments of x take place, τ_i, $i > 1$, given by equation (7b) can, using the iso-elastic functional forms, be expressed as

$$\left[\frac{p_F(x,\tau_i)V(x,i)}{p_F(x,\tau_{i+1})V(x,i+1)} \right]^{\frac{1}{\gamma_F - 1}} = \left(1 - \frac{\rho V(x,i+1)}{\gamma_F} \right). \tag{7c}$$

(see appendix for derivation).

2.2 Analysis:

What do we learn from the characterisation of development stages given above? A benchmark case in which prices are growing at constant exponential rates $\hat{p}_I$, $\hat{p}_F > 0$, yields analytical results. The full general equilibrium model that supports constant exponential price growth is outlined section 2.3, but for the present we simply assume these exogenous price paths. We look at the time series development of a particular place, x, and then at the urban cross-section.

Development dynamics:

To draw out results we look first at successive redevelopments of formal areas of the city, and then turn to the city edge and informal development.

Proposition 1: If construction technologies are iso-elastic, prices are growing at constant exponential rates $\hat{p}_I$, $\hat{p}_F > 0$, and agents have perfect foresight then:

(i) The value-to-rent ratio takes constant value V, and the time interval between successive formal redevelopments is constant $\Delta\tau$,

$$V \equiv \int_0^{\Delta\tau} e^{(\hat{p}-\rho)t} dt = \frac{1 - e^{(\hat{p}_F - \rho)\Delta\tau}}{\rho - \hat{p}_F}, \qquad \Delta\tau = \frac{(\gamma_F - 1)}{\hat{p}_F} \ln\left[\frac{\gamma_F}{\gamma_F - \rho V} \right]. \tag{8}$$

(ii) Successive rounds of building are taller by a constant proportional factor.

$$\frac{h_F(x,\tau_{i+1})}{h_F(x,\tau_i)} = e^{\frac{\hat{p}_F \Delta\tau}{(\gamma_F - 1)}} = \frac{\gamma_F}{\gamma_F - \rho V}. \quad (9)$$

(iii) If the rate of growth of prices is the same in all locations, *x*, then so too are *V*, $\Delta\tau$, and height growth.

The first part of this proposition comes from solving equations (2) and (7c), and the second by using this in the first order condition for height, (4b). The third comes from noting that (8) and (9) do not depend on *x*. While value ratios and time intervals do not vary with *x*, the actual dates of redevelopment do, as discussed in the next sub-section.

What about the earlier stages of informal development? The first transition is (we have assumed) from agriculture to informal settlement. This occurs for land at *x* when the price of informal sector housing space reaches the point at which the right hand side of (5c) equals r_0.

The second transition is from informal to formal settlement, and given by date τ_1 that solves (6c). There is a unique transition date if the return to formal development is rising faster than the return to informal settlement (i.e. the right hand side of (6c) increasing faster than the left). If *D* = 0, a necessary and sufficient condition for this is that $\hat{p}_F \gamma_F / (\gamma_F - 1) > \hat{p}_I \gamma_I / (\gamma_I - 1)$. Since $\gamma_F < \gamma_I$, the condition is met unless the price of informal space is increasing much faster than that of formal space. If *D* > 0 and non-increasing with time, then this condition is sufficient, but not necessary. A period of informal settlement exists only if the return to informal settlement at τ_0 is greater than the return to commencing formal settlement, $r_I(x,\tau_0) > r_F(x,\tau_0;\tau_0) - \{\rho D(x,\tau_0) + D_\tau(x,\tau_0)\}$. If not, then initial development will be formal, with date τ_1 implicitly defined by $r_0 = r_F(x,\tau_1;\tau_1) - \{\rho D(x,\tau_1) + D_\tau(x,\tau_1)\}$.

Figure 1 pulls these stages together and illustrates the development path.[4] Building height is given on the vertical axis (in units that are continuous, but not scaled meaningfully), and on the horizontal plane axes are time *t* and location *x* (distance from the CBD). The figure is constructed with house prices increasing exponentially with time and falling exponentially with *x*. We discuss the cross-section – variation across *x* at a given *t* – in the next sub-section, and now look just at the development of a particular location through time, i.e. fix *x* and look along a line sloping up and to the right. Initially (at low *t*) this land is rural. At date τ_0 (specific to location *x*), informal development takes place. The initial height (space per unit area) of informal development increases steadily as increasing p_I causes lego blocks to be rearranged. Formal development takes place at τ_1 and is associated with an increase in

[4] Parameters used in the simulation are given in the appendix.

height, as indicated by the second step. Each subsequent redevelopment occurs at fixed time interval $\Delta\tau$ bringing the same proportionate increase in height. The timing and height of each of these formal investments is based on perfect foresight about the growth of prices and the date of subsequent redevelopments. Notice that this figure tracks urban height (housing floor space) but this, in general, is not the same as population density. Housing space is more expensive closer to the centre, so if demand for space has any elasticity, households chose to consume less. This is captured in section 2.3, which sets out the demand side of the model.

The urban cross-section:

We have so far concentrated on a single location, point *x*, and now place these in the context of a city where *x* measures distance from the CBD, and house prices decrease with distance. For analytical results we suppose that this price gradient is exponential with distance, at rates θ_I, θ_F, i.e. $p_I(x,t) = \bar{p}_I e^{\hat{p}_I t} e^{-\theta_I x}$, $p_F(x,t) = \bar{p}_F e^{\hat{p}_F t} e^{-\theta_F x}$. Once again, micro-foundations are provided in section 2.3.

With $p_I(x,t)$ decreasing in *x*, then (5c), (6c) and (7c) can respectively be interpreted as defining, for each date *t*, the city edge, $x_0(t)$, the location of formalisation, $x_1(t)$, and locations of successive redevelopments, $x_i(t)$, $i > 1$.[5] Using these relationships, Proposition 2 states results on the spacing of different levels of development in the urban cross-section

Proposition 2: If construction technologies are iso-elastic, prices are growing at constant exponential rates, $\hat{p}_I, \hat{p}_F > 0$ and decline with distance at constant rates $\theta_I, \theta_F > 0$, and if agents have perfect foresight, then:

(i) Places at which (re-) development occurs at each date change according to,

a. $dx_0 / dt = \hat{p}_I / \theta_I$,

b. If $D = 0$, $dx_1 / dt = \dfrac{\hat{p}_F \gamma_F (\gamma_I - 1) - \hat{p}_I \gamma_I (\gamma_F - 1)}{\theta_F \gamma_F (\gamma_I - 1) - \theta_I \gamma_I (\gamma_F - 1)}$,

c. $dx_i / dt = \hat{p}_F / \theta_F$, $i > 1$.

(ii) The distance between successive formal sector redevelopments is constant,

$$\Delta x = \Delta\tau \hat{p}_F / \theta_F = \frac{(\gamma_F - 1)}{\theta_F} \ln\left[\frac{\gamma_F}{\gamma_F - \rho V}\right]. \quad (10)$$

Part (i) follows from differentiating (5c)-(7c) using $p_I(x,t) = \bar{p}_I e^{\hat{p}_I t} e^{-\theta_I x}$, $p_F(x,t) = \bar{p}_F e^{\hat{p}_F t} e^{-\theta_F x}$. Part (ii) follows from (i)c with equation (8) for $\Delta\tau$.

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[5] That is, instead of solving (5)-(7) for the date at which a particular location is developed, the equations give the location that undergoes development at a particular date.

Interpretation comes from Figure 1. The urban cross-section at a point in time is indicated by fixing a date and moving along a line sloping upwards to the left towards the CBD, with steps in height occurring at distances $x_i(t)$. Moving towards the centre, locations that have been urban for longer have been through more stages of development and are taller, offering more housing space per unit land. Relatively young/ small cities appear flat, with e.g. just informal and first round formal development, while older/ larger cities will contain central areas that have been redeveloped multiple times.

The figure is constructed with $\hat{p}_I = \hat{p}_F$, and $\theta_I = \theta_F$, this giving the parallel lines for development and redevelopment (see proposition 2.i).[6] This means that that width of the informal area, $x_1 - x_0$, is constant through time (as illustrated) and hence the share of urban land area that is informal falls with time and as the city gets larger. Taking a lower value of $\hat{p}_I$ (so $\hat{p}_F > \hat{p}_I$ and $\theta_I = \theta_F$) then first formal settlement starts sooner (and lower) at each place, and informal settlement diminishes to zero with time. Reducing the value of θ_I (so $\hat{p}_F = \hat{p}_I$ and $\theta_I < \theta_F$) enlarges the informal area, and means that it increases through time, as it both brings forward the date of informal settlement at each place, and delays formal settlement.

While our analytical results are based on constant exponential price paths, we note that it is also possible to numerically compute the perfect foresight equilibrium for more general price paths. These are not reported here, but are work in progress.

Expectations:

Analysis to this point has been based on optimisation with perfect foresight. This provides a benchmark, but a developing city contains many imperfections, and the first we look at is to remove the assumption of perfect foresight. Recall that $V(x,i)$ is the value-to-rent ratio on a newly constructed property, and equations (8) give the perfect foresight values of this and of the expected length of life of the property, $\Delta\tau$. How do results change if construction decisions are based on a value-to-rent ratio that differs from the perfect foresight ratio?

The solid line on Figure 2 is a slice through Figure 1 at t = 200, maintaining $\hat{p}_F = \hat{p}_I$ and $\theta_I = \theta_F$. Given the parameters used, the perfect foresight value-to-rent ratio is V = 37, and the interval between redevelopments is $\Delta\tau$ = 67. The dashed line is constructed on the basis that developers expect a value-to-rent ratio of 28 (imposed at 75% of the perfect foresight value). The transition from rural to informal settlement is unaffected by this, but formal development is based on these less optimistic expectations. As a consequence developers build less tall

[6] Since $\gamma_I > \gamma_F$ both the numerator and denominator of (i*b*) are positive in this case.

and hence buildings become obsolete more rapidly, so the interval between redevelopments drops to $\Delta\tau = 43$.

The welfare cost of this imperfection is measured by its impact on land rents. Equation (1) gives the present value of these rents at a particular point. Integrating (out to $t = 250$) over all urban areas, lower expectations reduce the present value of land rents by 2.3%.[7] It is also useful to have a flow measure by place and time. For informal development this flow measure is simply $r_I(x,t) = p_I(x,t)h_I(x,t) - C_I(h_I(x,t))$. For the formal sector, land rents are the revenue on the vintage that is currently standing minus the construction cost amortised at constant rate over the life of the building, $p_F(x,t)h_F(x,\tau_i) - C_F(h_F(x,\tau_i)\rho / \left[1 - e^{-\rho(\tau_{i+1}-\tau_i)}\right]$, $t \in [\tau_i, \tau_{i+1}]$. Integrating these over the city gives a time path of aggregate rents. Lower expectations reduce rents once formalisation has occurred, although effects vary with time as dates of development are changed, and in some years rents are somewhat higher. On average (out to $t = 250$), lower expectations reduce total land rents by an average of 2.4%, given our parameters.

Spatial variation in formalisation costs:

Locations vary in their distance to the CBD and, potentially, in many other respects. One possibility is that that the cost of formalisation, $D(x,t)$, varies according to place. Figure 3a illustrates the implications of there being an interval of x within which $D(x,t)$ is particularly high. As expected, this extends the period during which the area is occupied by informal settlement. Several other observations are noteworthy. First, a history of informality has a persistent legacy on the area. Formal development starts later, and so therefore does subsequent redevelopment; this impacts on building height which depends on the price (and hence date) of redevelopment. Importantly this means that the rigid steps of height seen in figure 1 will not necessarily apply. Second, a persistent informal area will see housing space per unit area increase through time (perhaps through building taller), so it is possible that it may come to have taller informal buildings than surrounding formal areas (although this is not the case on figure 3a).

The present value loss of land rent and welfare, discounted to date zero, along one of the rows with high conversion costs (row 15) is just 1.2%, relatively small because losses are far in the future. The flow costs are of much larger magnitude, with rents depressed where development has been postponed (for example at point *a*, with rent 38% lower than in the base case) and higher where rents are increased by building taller (a 26% gain at point *b*). There is an analagous pattern in the cross section. Thus point *a* is at year $t = 161$; in the urban

[7] As a percentage of the excess of urban land rent over the rent earned by land in non-urban use, r_0.

cross-section for that year, this area, locked in informal development, yields 38% less land rent than it would have had it been formally developed. This loss of land rent corresponds exactly to loss of urban real income.

An evolving cityscape in which $D(x, t)$ is random is illustrated in figure 3b. All locations see height increase with time, but initial and subsequent formal development takes place at different dates and hence at different times. This means that gradients of height, density and land rents are not monotonically decreasing from the centre in such a city. The probability of a randomly selected formal location being redeveloped in a particular unit of time is $1/\Delta\tau$, which is greater the lower are expectations (as buildings are built too low) and faster the expected rate of price increase.

2.3: Closing the model:

To this point our analysis has focused on construction of the city, given time paths for the price of housing floor-space at each location. We now complete the model by specifying household behaviour and hence the demand for space. This is constructed in a way consistent with the preceding analysis, merely offering a model of price growth in terms of growth in city incomes and productivity. This sub-section may be omitted by readers who want to move directly to empirics.

Households:

At date t a representative urban household living at distance x from the CBD receives income net of commuting costs $w(t)T(x)$, where $w(t)$ is the wage at date t (the same for all households), and $T(x)$ is the fraction remaining after commuting costs. Each household makes a discrete choice between formal and informal sector housing and, for the chosen sector, chooses $s_i(x,t)$ units of housing (i.e. floor-space) at price $p_i(x,t)$ per unit, $i = F, I$. Utility is derived from the quantity of space consumed, its formal/informal status, and consumption of a numeraire good (equal to wage income net of commuting and housing costs).

$$u(x,t) = \max_{I,F} \begin{array}{c} u\left(s_I(x,t),\ \ w(t)T_I(x) - p_I(x,t)s_I(x,t) : I\right) \\ u\left(s_F(x,t),\ \ w(t)T_F(x) - p_F(x,t)s_F(x,t) : F\right) \end{array} \tag{11}$$

The quantity of housing space is chosen to maximise utility. We assume homothetic preferences, so maximised utility is

$$U(x,t) = \max_{F,I}\left[v\left(p_I(x,t) : I\right)w(t)T_I(x),\ \ v\left(p_F(x,t) : F\right)w(t)T_F(x)\right], \tag{12}$$

where v is maximised sub-utility from housing. Free choice of location means that, at any occupied location and housing type, utility equals a common city wide utility level, $\overline{U}(t)$. The price of a unit of housing must be such that this holds, so prices satisfy

$$v\left(p_I(x,t):I\right)=\overline{U}(t)/T_I(x)w(t), \qquad v\left(p_F(x,t):F\right)=\overline{U}(t)/T_F(x)w(t) \ . \tag{13}$$

These prices implicitly given in (13) define the price schedules faced by developers. If preferences are Cobb-Douglas then $v\left(p:I\right)=p^{-\alpha_I}$, $v\left(p:F\right)=\left(p/\phi\right)^{-\alpha_F}$. Housing expenditure is share α_I, α_F, of net income, and parameter $\phi \geq 1$ captures the fact that formal housing may offer higher utility than informal. With this Cobb-Douglas assumption, prices of formal and informal housing satisfy

$$p_I(x,t)=\left(\frac{w(t)T_I(x)}{\overline{U}(t)}\right)^{1/\alpha_I}, \qquad p_F(x,t)=\phi\left(\frac{w(t)T_F(x)}{\overline{U}(t)}\right)^{1/\alpha_F}, \tag{13a}$$

Constant exponential growth of the price of space is achieved by assuming that urban wages relative to outside utility grow at constant rate g. Similarly, constant exponential decline with respect to distance is achieved by the share of income net of commuting declining with distance at rates $\hat{T}_I, \hat{T}_F$, so $p_i(x,t)=\left(\overline{w}e^{gt-\hat{T}_i x}/\overline{U}(t)\right)^{1/\alpha_I}$, $i = I, F$. This gives prices rising at constant rates $\hat{p}_I = g/\alpha_I$, $\hat{p}_F = g/\alpha_F$, and declining with distance, $\theta_I = -\hat{T}_I/\alpha_I$, $\theta_F = -\hat{T}_F/\alpha_F$.

As well as giving choice of housing type and location, these preferences also generate quantities of housing demand per household. They are, by Roy's identity,

$$s_I = -\frac{v'\left(p_I(x,t):I\right)}{v\left(p_I(x,t):I\right)}w(t)T_I(x), \qquad s_F = -\frac{v'\left(p_F(x,t):F\right)}{v\left(p_F(x,t):F\right)}w(t)T_F(x). \tag{14}$$

In the Cobb-Douglas case,

$$s_I = \alpha_I w(t)T_I(x)/p_I(x,t) \ , \qquad s_F = \alpha_F w(t)T_F(x)/p_F(x,t). \tag{14a}$$

Labour and population:

To complete the model, we note that population at a point is *h/s*, total floor area supplied divided by consumption of floor space per household. Total city population at date t is therefore

$$L(t)=\sum_{i=1}^{i\max(t)}\int_{x_{i+1}(t)}^{x_i(t)} h_F(x,\tau_i)/s_F(x,t)dx+\int_{x_1(t)}^{x_0(t)} h_I(x,t)/s_I(x,t)dx. \tag{15}$$

The oldest formal development has been redeveloped the most times (which, at date t, we denote $i\max(t)$). Notice that this expression assumes that the city is linear (or a set of rays), not a disc; adjustment to (15) to capture the latter is straightforward.

The final element is to close the model, either by setting $\overline{U}(t)$ exogenously with $L(t)$ endogenous (open city), or with $L(t)$ exogenous and determining the equilibrium city wide level of utility (closed city). The analysis of the preceding sub-sections follow the open city route, with exogenous growth of urban wages relative to outside utility driving housing price growth.

3. Empirical work on Nairobi

The model gives a number of predictions about both the cross section and dynamic aspects of the building of cities. As noted in the introduction, we use a variety of data sources including high resolution satellite data from two time periods, traced layouts of building footprint polygons, and, for later years, building heights. For the first time that we know of, the evolution of the built environment of a city can be tracked, as well as the predictions of a dynamic model examined empirically. Second the model introduced the notion of conversion costs that could be high and could be heterogeneous across the city. For a city such as Nairobi, based on "accidents" of history, we have an empirical counterpart: informal settlements where conversion costs are high, as well as those where they are likely to be much lower. We explore the role of these conversions costs on the building of Nairobi. In this section the informal sector is defined as officially designated "slums" and we switch to this term. While we proxy the informal sector by slums, it is unclear to us whether slums are an over or underrepresentation of the informal sector based on technology and conversion costs.

In this section we first describe the data in more detail. Then we present cross-section results which suggest that Nairobi conforms to the basic patterns we see in a monocentric model and that slums present a major issue with very different patterns of rents and usage than the formal sector. Then we analyse the dynamics of how Nairobi's built environment changed from 2004 to 2015, both to explore predictions of the model and to highlight slum and formal sector differentials. Finally, we focus directly on the issue of slums and potential inefficiencies driven by artificially high conversion costs.

3.1 Data

We build a data set for Nairobi which defines characteristics of the built environment at a very fine spatial resolution for 2-3 points in time from several different sources. Most characteristics are mapped into a grid of 150 x 150m square. There are thousands of these in the region; and for the sample we focus on, the 2004 built area of the city, there are 6222 squares.

Our main data, which we refer to as footprint data, is building footprint or roof coverage based on tracing of buildings from aerial photo images (at an even higher spatial resolution) for 2004 and 2015. The 2004 footprint data was received by the Nairobi City Council in 2006 with digitized polygons for every building in the administrative boundary of Nairobi. The data are from the Center for Sustainable Urban Development (CSUD) at Columbia, as far as we can tell mostly based on aerial images taken in February 2003 at a scale of 1:15,000, with later field identification. In January 2015, imagery at (10-20cm resolution) was recorded. The key methodological imagery work has been to overlay the 2004 and 2015 images to determine which building footprints are unchanged since 2004, which buildings were demolished with no replacement on the prior site, which buildings were redeveloped and finally where and to what extent infill occurred with new building on sites where no 2004 buildings existed. Overlay is complicated by variations in the way buildings were traced and specifically located in 2004 versus 2015, as well as by tracing error. The Data Methodology Appendix describes the issue and the algorithm used to overlay and identify types of changes.

This data is supplemented by building height data for 2015 from LiDAR (0.3-1m resolution) which was used to create a Digital Elevation Model and give heights of objects. We also use high resolution SPOT satellite data for the years (circa) 2004 and 2013 to measure road coverage.

For Nairobi we use two classifications of slums. For 2004, a copyrighted land use map prepared jointly with the CSUD at Columbia, Japan International Cooperation Agency (JICA) and the Government of the Republic of Kenya under the Japanese Government Technical Cooperation Program was published and printed by the survey of KENYA 1000 in March 2005. Columbia categorized polygons as slums if they contained small mostly temporary buildings that are randomly distributed in high density clusters, with a statement: "It should also be noted that in some cases the JICA maps labelled these areas as slums on the map and that is the reason we included it here. It was hard to categorize slums so this label was only used when it was clear that this was the type of land use" See Williams, et al. (2014) for their full methodology. Second, in 2011, slums were mapped by IPE Global under the Kenya Informal Settlements program, and we digitized these maps. IPE mapping of settlements was done using satellite imagery and topographic maps with the imprecisely defined criteria. The general idea is that slums are "unplanned settlements" which have some aspects of low house quality, poor infrastructure, or insecure tenure. The 2011 designation has many more slums than in 2004. Most 2011 areas had housing in 2004 not then defined as slums; in most cases these areas subsequently experienced enormous infill of small buildings. It is clear however that the effective definitions differ across years and cannot be used to distinguish new slums or even slums which no longer exist. We generally work with the union of the two and infer slum removal and expansion from changes in the built environment in these areas.

Finally we have cross-section house rent and land value data. We have a cross section of georeferenced household level data from the 2012 ‘Kenya: State of the Cities’ survey by the National Opinion Research Center (NORC). This is the first data set to record *household* rent (with detailed house and some neighbourhood characteristics) in Nairobi for a sample that is stratified between informal and formal areas (based on the 2009 Census, yet another definition of slums). This allows for a cross section of households that is representative of the formal neighbourhoods, and another that is representative of slum areas. Although there have been previous studies of household rents in Nairobi’s slums (Guylani and Talukdar, 2008), they rely on data restricted to slum areas, and so offer no analysis of the relationship between the informal and formal housing markets. In addition to rent data we have, for 2015 property values that have been scraped from property24.co.ke. We focus on the vacant land listings with information on asking price and plot area, for which we have information for 80% of the listings. These data upon mapping only are found in the formal sector.

3.2 Cross-sections: The built features of Nairobi

Standard urban models of the spatial lay-out of cities predict that the intensity with which land is used will decrease with distance from the centre, where land values and house rents are highest (see Duranton and Puga, 2015 for a review of the literature). We develop descriptive regressions and graphs showing the price gradients which drive intensity and then analyse measures of intensity such as building cover and volume and building heights, and how they differ between the formal and informal sector. But there are a number of methodological issues to deal with in the use and mapping of our data

Mapping Nairobi

The first step is to define the spatial extent of Nairobi at two critical dates: 2004 and 2015. Nairobi’s spatial area is a little difficult to characterize because on the fringes there is some degree of sprawl or non-contiguous development and there are also outlying villages near to the city. We adopt a fairly conservative definition of the boundary: that for a (150mx150m) grid cell to be in the city on the outer edge a smoothed (by 900 meter squares) roof cover must be 10% or more of the area. Figures 4a and 4b show the city respectively in dark outline in 2004 and in 2015. For each year we mark the slums as recorded at that time: 2004 in 4a and 2011in Figure 4b. We also mark the radius in red near the CBD in which there are no slums as defined in each time period and the city centre with a yellow star. The city centre is the brightest lit pixel in night lights data in the early 1990’s.

In either Figure 4a or 4b, we see the intensive margin which is the 2004 city. We focus on this margin for examining key aspects of dynamics; our 2004 data do not extend in general to the 2015 border. City shape is not a nice regular circle. It is bounded to the south by an airport and then a large national park and to the immediate north of the centre by a preserved state forest. Related, the major highways tend to run east-west. We can see also that there is a big extensive margin to the city, where city population is growing at almost 4% a year. Apart from spread what we take from the figures concerns slums. As the model predicts, slums are not prevalent near the centre; and the area with no slums near the centre as defined contemporaneously expands considerably between the two years, from a 0.775 to a 2.0 km radius around the centre are gone by 2015. The maps suggest considerable slum expansion at the 2004 fringe of the city, as predicted in the model. Finally we note the large slum of Kibera directly south-west of the centre (ranging from 3-5 kms of the centre). In Section 4, we will focus in part on Kibera.

Land value and house price gradients

In the model, in section 2, house prices and land values decline with distance from the centre because the benefits of lower commuting costs to the centre for those near the centre are capitalized into higher willingness to pay for housing locations and thus also for land near the centre. These higher values near the centre should induce more intensive use of land—higher buildings and more floor space per unit of land built coverage. From NORC we have data on house rents, floor space, and house and neighbourhood characteristics; we know house location and whether that location is classified as being in a slum by the NORC project. We run a hedonic regression of the log of rent per square meter of floor space (as a proxy for house price) on distance from the centre allowing for a slum differential. Results are reported in Table 1 columns 1 and 2. Column 1 has no controls and column 1 has the controls listed in the footnote to the table. Figure 5a graphs how the predicted price per square meter of floor space varies with distance from the centre for a standardized housing unit for formal and slum areas. Formal predicted prices per square meter of floor space decline with distance from the centre, where prices at the centre are 62% higher than at 10 kms out. We stop graphs at 10 kms, since observations start to drop off after that and there is a selection bias on the fringe in terms of omitting less developed areas where the city could have spread in principle, absent barriers of dedicated green space. Slum unit prices vary insignificantly over space. That is a puzzle which we return to in section 4 where we will use the formal sector-slum price differentials to infer inefficiencies created by land market imperfections in Nairobi.

In Table 1 column 3, we report a regression for asking prices of land lots listed in Nairobi in the fall of 2015. All but 3 are in the formal sector as we have defined it. Controls include lot size and “coordinates estimated” meaning we didn’t have an exact street location just a local neighbourhood, which generally indicates an inferior plot. For a standardized piece of land, predicted asking sales prices per square meter are graphed in Figure 5b. Because land its with their limited share in production have to absorb all house price variations, given the price of capital doesn't vary across space, we expect a much larger change in land values with distance from the centre. Indeed inferred unit land values are at the centre are 4.35 times those at 10kms out.

Total cover and volume by sector

The price and land value differentials in Figure 5 drive land use intensity as we move away from the city centre. We now present a set of graphs on the raw data to depict how land is used in the city and the intensity of that use. The Data Methodology Appendix gives details of how we measure coverage and volume for our basic unit of analysis, the 150m x 150m grid square, for slums and the formal sector.

Figure 6a shows the average cover per 150x150 meter grid square at each distance of buildings in formal sector usage and then slum usage for 2004 and 2015, all smoothed over distance1.[8] This average roof cover, or building footprint size is the product of two factors—intensity of cover in each formal sector grid square (to follow in Figure 7) and the fraction of grid squares at each distance that are formal (vs slum or empty). The table at the top of the figure gives the fraction usage by type overall at each distance for 2015, by dividing the numbers in the figure by 22,500. Figure 6a with its table tell us several things. First, overall cover is dominated by formal sector usage. Second, total cover in both the formal and slum sectors increases over time especially further from the centre where later we will see there is enormous infill. Third, slum cover is close to 0 within 2 kms of the centre, as the model would predict. There are next to no slums left there. Finally total cover in both periods—slum plus formal sector doesn't vary much with distance from the centre, or in 2015 even increases in parts. The missing piece is road cover. Roads traced from SPOT satellite images suggest road cover increases substantially as we near the city centre, as predicted by usage models from the 1970's (Vickery and Solow, 1971 and Riley 1974). Near the centre in 2004 over 22% of total area was devoted to roads, which declined to 5% at 10kms out.[9] After 2004, from about 4kms out, there is a lot of road infill, so that by 2013 at 10 kms the 5% road cover in 2004 has increased to 15%

Figure 6b focuses on our key measure: volume, or cubic meters of built space. In 2015 we can directly measure volume, multiplying cover in square meters by LiDAR height in meters to give cubic metres of built space. For 2004 we infer height based on the average 2015

[8] This is STATA local mean smoothing with an Epanechnikov kernel, with default settings.

[9] This is based on tracing of roads from SPOT images in 2004 and 2013. Paved roads of 1-2 lanes are assigned a 10m width and 4 lanes a 20m width. This is conservative. And near the centre excludes sidewalks, which are less existent further out. Dirt roads are excluded.

height of buildings unchanged between 2004 and 2015, in the own and queen 150x 150 m. square neighbourhood. Volume tells us about the overall intensity of land use. Total volume overall falls dramatically as we move away from the centre in the formal sector; and there are big increases over time beyond 1km from the centre. Slum share in volume compared to the formal sector and compared to its share in total cover is quite modest. As we will see shortly that reflects the low heights of slum buildings. Finally note that while total formal sector volume declines with distance from the centre, total slum volume generally rises as does it share, again reflecting the greater presence of slums moving out from the centre. Some tail-off of slum shares after 7 km from the centre we think involves a delay in defining faster growing fringe areas as slums.

Figure 7 examines intensity *within* sectors. Here we have the same measures of coverage and volume by usage at each distance, but rather than dividing by total 150x150m cells in each distance ring, for the formal sector, we divide just by all cells with formal sector usage. Similarly for slums we divide just by the count of slum cells. There is a modest downward bias (about 10%) in both numbers because the 12% of mixed cells appear in counts for both denominators. In principle we could plausibly divide mixed cells into proportions slum vs formal sector, but when we turn to dynamics with changing slum and formal sector coverage within mixed cells, there is no clear way to do this, and bias involved is small.

Figure 7a reveals what we know on the ground. Within slums, especially traditional slums between from 3-6kms from the centre (like Kibera), there is extremely high coverage and little green space —slums have extremely high average coverage about 36% in 2015 at 5kms. Very near the centre, the much lower numbers reflect slum clearance, discussed in Section 4.

Figure 7b repeats the intensity in terms of volume. It reveals a fundamental finding. Height in the formal sector trumps coverage to generate more cubic meters of space. Throughout the city average volume intensity in the formal sector exceeds that in slums in 2015. Volume intensity is very high nearer the centre in the formal sector, declining with distance to about 6kms before flattening out somewhat above slums. Slum volumes are comparatively low and flat reflecting low building heights throughout.

Figure 8 shows directly we inferred by comparing Figures 6 and 7: a measure which effectively gives a cover weighted average of individual building heights (see Appendix). Heights decline from an average of 23 meters near the centre to about 7 meters beyond 6kms from the centre, which is a little under the height (with roof) of a 2 story building. Heights in slums are flat at about 5 meters, approximately a 1+ story building with roof. Figure 8b takes the NORC data for 2012 and estimates for the sample a Poisson model of the number of stories (see Table 1 column 4 for the estimates) in just the residential sector. At the centre buildings average just under 5 stories where 5 is the maximum building size which can operate without reliance on elevators (under uncertain power and repair). Heights in the formal sector decline to about 2 stories at 10km—lower developments are further out. Heights in slums are fairly flat, starting at 1 + stories near the centre and then converging to formal sector heights at 10km. As we modelled in Section 2, slums have a different technology where height is costly and thus building is low.

Heterogeneity over space

We have demarcated everything based on ring averages for distance from the centre. Within distance rings there is considerable heterogeneity in the face of strong patterns in ring averages. To show the heterogeneity, we map 2015 average height and total volume per 150x150 grid square for slum and formal sector areas throughout the 2004 city area from Figure 4a. Figure 9a shows the height map. The formal sector is in blue, with intensity linked to darker shades. The city has a monocentric look with intense heights at the centre, dropping quickly going away from the centre. Throughout the rest of the city there are scatterings and modest clusters of higher heights, but nothing comparable to the centre. Slums generally have low height throughout; see especially Kibera southwest of the centre. We note some other details about the map. Green spaces in the city include a large (to the north east) military airport, the Presidential palace which is not well mapped (and unmapped in 2004) and a large golf course. As noted above to the south there is the main airport just outside the boundary and then the national park; and to the north there is the dark green protected forest. Finally we note to the south in the far-east is a cluster of "slum buildings" with noticeable height. Visually in satellite images, these look like formal sector apartment buildings, an issue of misclassification. Figure 9b shows volume, which displays more heterogeneity and reflects land use details missing from the model. While the centre remains distinct, we note there are large volume areas on the city outskirts especially to the east. These are generally grid squares covered with warehouses and other large flat commercial or industrial buildings, an added complexity in thinking about land use in cities.

3.3 Specifics of dynamic development

Measurement and definitions

In the prior section we looked at the 2004 and 2015 cross-sections which themselves imply by comparison some dynamics of development. Here we overlay our images and break down the changes in total cover and volume within sectors between 2004 and 2015 into change due to demolition (with no replacement), infill, and net redevelopment. Demolition is a building in 2004 where there is no building in 2015, so demolished coverage is lost 2004 cover. We think of demolition as a cousin to redevelopment: spaces cleared but yet to be built upon (or space turned into roads). Infill is new buildings which do now overlap with any 2004 buildings. Net redevelopment involves a 2004 building which has been replaced by a 2015 building(s) overlapping the 2004 footprint. Net redevelopment takes coverage in the new 2015 buildings and subtracts the coverage of old 2004 buildings for each 150 x150 meter square. Net volume redevelopment again assigns heights in 2004 based on neighbourhood averages for unchanged buildings and uses 2015 height information on the new buildings.

So far in displaying patterns by distance from the centre we have used averages for the population raw data at each distance. We do the same here in figures, except for figures we convert to fractions, relative to initial cover or volume. In a distance ring, for cells in formal sector use, by what fraction did overall formal cover or volume decrease or increase due to, say, demolition or infill? We also run regressions. Here we can't do fractions because of a problem of initial zero cover (so no defined fraction) or tiny coverage (yielding astronomical fractions) in many cells. We run total change in any cell on distance rings from the centre with a control for initial coverage or not and extent of coverage, as well as other controls.[10] Tables 2-5 report the predicted infill, redevelopment and demolition (with standard errors) at each distance for a city-wide standardized formal sector or slum sector grid cell as relevant for cover and volume in the formal and slum sectors.

Results

We start with formal sector coverage in Figure 10a and Table 2, which have four outcomes: net redevelopment, demolition, infill and total change. Our focus is on the distance relationship. We plot the distance relationships as fractions of initial cover, while the table has distance predicted totals. Throughout, demolition, as redevelopment waiting to happen, looks flat and modest, albeit from the table there being more near the centre. Net redevelopment of coverage as a fraction is also flat in the Figure 10a, with less right at the centre. In the table with total change there is not much difference initially beyond the first ring but there is a rise as we get more to the city fringe. Note again coverage change is net: a 2015 footprint replacing in general a smaller 2004 one. Infill as a fraction in the figure and as a total increases sharply with distance (consistent with Figure 6a). In summary, there are two takeaways on coverage changes in the formal sector. First, infill dominates the action for coverage change. Second there isn't much change near the centre: the 1km ring around the city centre is locked-in during this 11 year interval, with already tall buildings, historical land marks and set roads.

[10] Controls are listed in the tables and include (a) a control on whether there was any building cover in 2004 in the cell; (b) if so, the extent of grid cover; (c) if so, average 2004 building footprint size; (d) the measures in b and c on average for queen neighbours; (e) own grid square road coverage in 2004; and (f) the fraction of 2004 coverage in own and neighbour cells which are slums.

As before, the most compelling measure involves volume—cubic meters of space which we take to be closely correlated with floor space. Figure 10b and Table 3 show inferred volume changes in the formal sector by source. Here is where we see the redevelopment process analysed in the model. What is immediately apparent is that, in contrast to coverage, infill no longer dominates as the source of change up to 4-5kms from the centre. It is redevelopment of existing buildings into new office towers and apartment buildings with high volumes for the time interval we see in Nairobi. There are higher volumes as fractions or totals from redevelopment up to about 4kms from the centre. After 4-5 kms, volume changes from redevelopment vary little with distance. Again up to 4-5 kms, we have height trumping coverage, now within the formal sector.

The height differentials for infill, redevelopment and unchanged buildings in the formal sector in 2015 are shown in Figure 12. Redevelopment heights between 1 and 6 kms out are higher than either for unchanged or infill buildings, as the model predicts. Right near the centre, redevelopment is at a lower height than existing buildings. We think this occurs because it is difficult to find enough contiguous land and buildings ready for demolition in order to have enough of a footprint to put up a tall building. Near the centre with limited and scattered vacant space in 2004, infill is done with low building height, rather than new office towers. It is comes disproportionately near the centre from parking lot infill. [11].

Coverage change in slums compared to the formal sector has some distinct differences in Figure 11a and Table 4. There is relatively a lot of demolition near the centre as old slums are torn down, we assume in anticipation of redevelopment. Second, net redevelopment in slums is pretty flat throughout the city until the fringes. Finally, for the total change as a *fraction* of initial coverage, there is a lot more action in the formal sector than defined slums, reflecting a lower opportunity for in-fill in slums with already intense coverage.

For slums coverage and volume patterns are similar throughout (given little spatial variation in heights of slum buildings), given pretty invariant heights. Volumes are given in Table 5 and Figure 11b. We do note patterns are somewhat distorted as we will see next in Section 4 because of cleared slums near the centre.

4. Slum redevelopment and lack thereof

We have no good way to define what a slum is in 2004 versus in 2015. The official definitions in the two time periods do not match up and both contain clear classification errors under any reasonable criterion. So it is hard to be definitive about slum change in terms

[11] Based on random samples of 50 infill spaces at different distances, 32% at 0-1.5kms are parking lots while further out it is 10-12%.

of adding versus subtracting officially designated slums per se. Instead we simply look at what is going on – in terms of demolition, redevelopment and infill – in the union of slums in 2004 and 2011. So far we saw that slum volumes in typical grid squares tended to rise with distance from the centre, while they declined sharply in the formal sector. We also observed that remaining slums near the centre are very modest in extent. In this section, we focus on what we perceive to be high costs to conversion near the centre which have kept slums at an intermediate distance from redeveloping and the opportunity costs of this inhibition. There are several pieces to this inference.

First there is intense pressure to redevelop. Figure 13 and the corresponding Table 6 look at the *fraction* of slum coverage per slum grid square demolished and redeveloped very near the centre based on regression coefficients where we can deal with small numbers which get smoothed in the figures (and here we avoid the infill fractions). Close in demolition is 100% and still high from 1-2 kms. The redevelopment fraction is also somewhat higher at 1-2 and 2-3 kms, compared to further out.

Figure 14 repeats Figure 4a to show the substantial house price differentials for a standardized unit of housing, where the gap is significantly different from zero until 7kms out. This suggests a misallocation represented in the model, based on quality differentials. In Table 1, we saw that the slum price gradient for an average quality house was flat, based on a slum indicator variable and that interacted with distance. In predicting rents for a 'same quality' unit in the formal and slum sectors, we controlled for many house characteristics and some neighbourhood ones. However we think there are unobservable characteristics which are generally worse in slums near the centre and better further out and which represent the quality differential between slum and formal housing we allowed in the model. The indicator variable and its distance interaction capture this. A particular example is very poor infrastructure and lay-out within slums nearer the centre, inherited by their development decades ago and affected by corruption (see below), while slums further out were developed later under different conditions.

In particular, based on the IPE study, we know the coverage of slum land under government versus private ownership. This is given in Figure 15, where near the centre slums are mostly government owned while further out they are privately owned. This could mean all slums near the centre have been government owned, or that all former private ones have been redeveloped. Most of the literature is has focused on these remaining traditional slums near the centre under government land ownership. We think government ownership impedes infrastructure investment in the land by slum lords, while in private slums returns on improving land now and looking to the future can be captured by the land owners. In column 5 of Table 1, we add an indicator for slum land owned by the government. That has a significant negative coefficient and wipes out the significance of the slum-formal sector distance differential. [12]

Slum lands near the centre whose redevelopment is inhibited by conversion costs reflect two inefficiencies. First, land is trapped in low quality usage, as reflected in the rent differentials by distance. Redeveloping slum lands near the centre would eliminate the low quality conditions and generate a welfare gain related to the price differentials.[13] Second is the technology difference and inability to build high in slums which in the model is represented in land value differentials, which we do not observe.

We have linked government ownership to high conversion costs, as well as low quality infrastructure. Why? To illustrate we consider the literature on Nairobi slums and the specific example of Kibera. The 1000 acres in Kibera was awarded to Nubians soldiers in 1912, albeit without formal title. They immediately occupied a portion of the land but at independence their claims (but not tenancy) were revoked, and land reverted in theory to the government. The large portion of Kibera not occupied by Nubians was settled on by others and had titles illegally allocated by local chiefs and bureaucrats. Studies on Nairobi's slum housing market suggest that slum lords who operate housing earn high rates of return despite paying bribes to maintain their possessary rights, mostly live outside the slums, and are mostly politically connected public officials (Gulyani and Talukdar 2008, Syagga et al 2002, and Joireman and Vanderpoel 2011). These political figures, absent a major buy-out of their possessary rights, have no interest in redevelopment: the land is not theirs to redevelop; they don't pay for it; and they can't profit from sale of improved land. However they can run a profitable slum business with land in its current use. Cleaning up this mess in traditional slums is the conversion cost. When opportunity costs of land get high enough, then these cost may be overcome, as represented by the demotion and redevelopment of slums very near the centre in Figure 13. But large areas like Kibera look particularly intractable.

Suppose we take the 1000 acres in Kibera and set its average distance from the centre at 4kms. Then by comparing unit house prices in Kibera with the formal sector, we can tentatively infer potential land values in Kibera to compare with the values under formal sector usage, to capture the quality differential problem.[14] To translate slum prices into land

[12] The ownership base case in the regression includes Nairobi county land, road or riparian reserve, some slums which are part government and part private mostly all far from the centre, and uncategorized. Nairobi county land is reputed to be much better managed by the local government.

[13] There is a complication: heterogeneity in the population. Slums with lower quality housing near the centre contain low income people (with relatively higher willingness-to-pay for low quality housing). Such residents may also have differential commuting costs, with lower willingness-to-pay for access to office jobs in the city centre. They may disproportionately work nearby to their residence in the informal sector or commute to factory and warehouse jobs on the city fringes. However this also represents misallocation.

[14] The unit cost function is $p = A[(1-\beta)^{\sigma} r^{1-\sigma} + \beta^{\sigma} p_k^{1-\sigma}]^{\frac{1}{1-\sigma}}$ where from Shepard's Lemma the share of land in

prices, we have to use the same CES unit cost function in each sector with standard parameters and calibration based on formal sector house prices and land values in our data. A calculation suggests that converting Kibera to formal sector usage would result in a $1b (USA) increase in land values. That calculation does not capture technological differences between slum and formal sector housing including the lost value from not being able to build high (although altering CES parameters to differ for the slum sector so it is more land intensive with less substitutability has little impact on land value magnitudes). The magnitude suggest that there is a substantial welfare loss from lack of redevelopment and that the surplus in values could be used to buy-out vested interests of slum lords hindering formalization of Kibera lands, as well as helping with relocation. One solution might be to give longer term residents ownership of their units and land, allowing redevelopers to buy them out in a timely (and voluntary) fashion; but that solution would require settling with slum lords.

5. Conclusions

The model and data both suggest that in the formal sector house rents and land prices decline with distance to the centre; consequently building heights decrease with distance to the centre. Heights in slum areas are much lower than in the formal sector near the centre and lower throughout the city. However intensity of land cover within slums is very high. Slums account for a small fraction of total housing space overall at any distance from the centre, but a fraction that mostly rises modestly with distance from the centre. Between 1-6 kms from the centre from 2004 to 2015 there is major redevelopment of 2004 formal sector buildings into higher height new buildings. Expansion of the informal sector is towards the city fringe, with intensive demolition very near the centre. We find that there is high intensification of land use with infill of new buildings through much of the city especially on the fringes. We find that development of the informal into formal sector housing mid-city over the 11 years is slow.

In the model we explore the role of expectations in altering (re)development paths. Under-estimating future demand growth leads to stunted city heights and spatial size. In the model we explore the cost of converting slum to formal use; and in the data for the common

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revenue is $s_l = (Ar/p)^{1-\sigma}(1-\beta)^{\sigma}$ for p_k the price of capital. Invoking the literature, suppose we set $\beta = 0.7$, $\sigma = 0.5$, and $s_l = 0.4$ at 4kms and convert rental prices to house stock values using a depreciation rate of 2%, an interest rate of 8.5% and a house value rate of appreciation of 8.5% (the last two based on conditions in Nairobi). We use standardized land value and house rent prices (converted to a stock price) data at 4kms from the gradients in Figure 4. We solve the unit cost function and share equation to get the unknowns, A and p_k . To solve for slum land values we insert the slum unit house value based on the gradient in Figure 4a at 4kms into the unit cost function.

institutional context of Nairobi, we explore misallocation of land between slums and formal sector usage, based of conversion contexts arising from poor institutions. We argue that slum ‘ownership’ by government means unresolved land right issues and corruption with vested slum interests of political figures, with a significant welfare loss.

Appendix for section 2

Derivation of (5c), (6c, (7c) is facilitated by noting that:

$$r_I(x,t) \equiv p_I(x,t)h_I(x,t) - \rho C_I(h_I(x,t)) = \left[\frac{p_I(x,t)}{\rho c_I \gamma_I}\right]^{\frac{\gamma_I}{\gamma_I - 1}} \rho c_I(\gamma_I - 1)$$

$$r_F(x,t,\tau_i) \equiv p_F(x,t)h_F(x,\tau_i) - \rho C_F(h_F(x,\tau_i)) = \left[\frac{p_F(x,\tau_i)\beta(x,i)}{c_F \gamma_F}\right]^{\frac{\gamma_F}{\gamma_F - 1}} \rho c_F\left(\frac{p_F(x,t)\gamma_F}{\rho p_F(x,\tau_i)\beta(x,i)} - 1\right)$$

Parameter values in figure 2 are: $c_F = 2$, $c_I = 0.004$, $\gamma_F = 2$, $\gamma_I = 4$, $\rho = 0.04$, $\hat{p}_I = \hat{p}_I = 0.02$, $\theta_F = \theta_I = 0.1$, $r_0 = 10$, , $\bar{p}_F = 5, \bar{p}_I = 4$. Simulation is done with time running to *t* = 800, and reported up to *t* = 250. In figure 3, *D*=100 or *D*=5000 (10> *x* <20).

Data Methodology Appendix

This Appendix has two components. The first deals with measures on cover/footprint and volume we use to analysis. The second gives the algorithm used to extract unchanged buildings, redeveloped buildings and infill from the overlay of 2004 and 2015 depiction of building polygons.

Measures of cover and volume

Our unit of analysis is 150x150m grid squares. For calculating cover within the grid square in a usage, each of these is broken into 50 3m by 3m cells and use type classified by what is at the centroid of the 3m square. There are three uses: vacant land, slum area and formal. Each 3x3 square is given the type of cover there in whichever time period. For each 150x150 square we sum across the 50 cells to get for example total building cover in each type. If for example a 150m by 150m gird has only formal sector buildings the square meter coverage can take values of 9, 18, 27, etc. up to 450. And the same for areas that are always slums. Most 150x150 squares are either all slum or all formal sector. However there are about 12% which are mixed grid squares, for which we record the cover or volume of slum and formal separately.

For average coverage in a grid square in the formal sector, before smoothing in a year in a given distance ring, the total area of all cover in 3x3 squares is summed up for all 150x150 meter squares whose centroid falls in a narrow distance ring. That sum is then divided by the **total** number of 150x150 grid squares in that distance band. The same procedure follows for slums. For Volume for 2015, for each 3x3m square which is formal sector, we have the height of the building whose cover is over the centroid of that square. So volume for that 3x3 square is 9 times the height in meters of the building from LiDAR data. We then sum across the grid squares occupied with formal usage for 150x150m grid squares in each distance ring and then average by the total number of 150x150 meter grid squares in the ring. For 2004 we have no height data. To infer 2004 heights, we use what we think is an upper bound on height: the height of unchanged buildings, where we presume demolished buildings between 2004 and 2015 are likely to be of lower height than those which survive. To assign a height to a 3mx3m square in 2004 in formal sector usage, we take the average height in 2015 of all buildings that were there in 2004 for all 3x3m formal sector unchanged buildings in the own 150x150m grids square and its 8 queen neighbours. Height is the height assigned to each 3x3m square in usage in a distance ring from the centre averaged over all such cells, to effectively get a coverage weighted average of individual building heights.

How do we measure change between 2004 and 2015? For demolition, at the 3x3m level the square is defined as demolition if its centroid is covered by a 2004 building which has been replaced by open space. Demolished coverage is lost 2004 cover; demolished volume is assessed as before using the average height of unchanged buildings in the neighbourhood. Infill is new buildings which do now overlap with any 2004 buildings; a 3x3m square is infill if its centroid is covered by such a building on 2015 where there was no building in 2004. Infill cover and volume are assessed from 2015 data. Net redevelopment in coverage takes coverage in the new 2015 buildings and subtracts the coverage of old 2004 buildings. So for each 150m150m meter square we have for redeveloped buildings, we have total coverage in 2004 measured at the 3x3m level (centroid covered by the old 2004 building(s)) and we have total coverage in 2015 measured at the 3x3m squares (centroid covered by the new replacement 2015 building(s)). Net redevelopment at the 150x150sqaure is the difference. In general, the same buildings are drawn in 2015 to have modestly more coverage than in 2004 so coverage change is likely to be an upper bound. Net volume change again assigns heights in 2004 to the 3x3m coverage based on neighbourhood averages for unchanged buildings and uses 2015 height information on the new buildings.

Overlaying Buildings

We match buildings across time by overlaying 2015 and 2004 building polygon data in order to track the persistency, demolition, construction and reconstruction of buildings over time. Since buildings are not identified across time our links rely on a shape matching algorithm. For each building, the algorithm determines whether it was there in the other period, or not, by comparing it with the buildings that overlap in the other time period.

This task is not straightforward, since the same building can be recorded in different ways depending on the aerial imagery used, whether building height was available, and the idiosyncrasies of the human digitizer.

Data and definitions

For 2004 we use a building dataset received from the Nairobi City Council with digitized polygons for every building, roughly 340,000 in the administrative boundary of Nairobi. For 2015 we use a similar dataset that was created by Ramani Geosystems using imagery (10-20cm resolution) and LiDAR (0.3-1m resolution). We have 2015 data for a wider extent, and consequently many more buildings, about 1.14 million. The LiDAR data in 2015 were used to measure heights of objects. With use of the aerial imagery and heights in 2015, a 3D model was created by hand, and rooftops extracted from this model.

Here we define the nomenclature that we use. First, a *trace* is the collection of polygon vertices that make up its outline. A *shape* is the area enclosed by the trace, and it can be thought of as a representation of the rooftop of a building. A *cavity* is an empty hole completely enclosed in a shape. A *candidate pair* is the set of any two shapes in different time periods which spatially intersect. A *link* is the relationship between a set of candidates in one period to a set of candidates in the opposite time period.

Pre-processing

Before running our shape matching algorithm we clean up the data sets. First we take care of no data areas. There are some areas that were not delineated in 2004, including the Moi Air Base, and the Nairobi State House. We drop all buildings in these areas for both 2004 and 2015. We drop roughly 1,500 buildings from the 2015 data, and 100 buildings from the 2004 data. Next we deal with overlapping shapes. While the 2004 data has no overlapping shapes, in the 2015 data there are some shapes that overlap. This is most often the same building traced multiple times. We identify all such overlapping polygons and discard the smaller version, until no overlaps remain. We drop about 1,400 buildings from the 2015 data this way. We also decide to drop small shapes, in part because the 2015 data has many very small shapes, while the 2004 data does not. In order to avoid complications of censoring in the 2004 data, we simply drop all shapes that have an area of less than $1m^2$. We drop 2 small buildings in 2004, and 462 small buildings in 2015.

Another issue is that buildings are often defined as contiguous shapes in 2004, but broken up in 2015. For the majority of buildings we cannot aggregate the broken up pieces in 2015 since it is hard to identify such cases in general. To match these cases across time we rely on our one to many, and many to many matching algorithms defined below. However, in the specific case where a building is completely enclosed in another the task is much easier. First, we find all cavities present in each period, then we take all building shapes that overlap with the cavities in the same time period. After identifying all shapes that intersect a cavity, we redefine both shapes, the original shape containing the cavity and the shape intersecting it, as a single new shape.

Shape Matching Algorithm

After the pre-processing of each cross-section is complete, we run our shape matching algorithm to establish links between buildings across time periods. For any given building we consider 5 possible scenarios; that it has a link to no building, that it has a link to one building (one to one match), that it has a link to multiple buildings (one to many), that it is part of a group of buildings that match to one building (many to one), or that it is a part of a group of buildings that matches to a group of buildings (many to many). We follow and approach similar to Yeom et al (2015) however, due to the inherent difficulty of inconsistent tracings we contribute to their method by introducing the one to many and many to many approaches. We assign each link a measure of fit that we call the overlay ratio. We then choose optimal links based on the overlay ratio. Finally, we categorize links as matched or not using a strict cut-off on the overlay ratio of 0.5. Other cu-offs such as 0.4, 0.6 and 0.7 produced more errors in categorization.

Candidates

For all buildings A in the first time period, and B in the second time period we identify the set of candidates:

$$CP = \{(A, B);\ Area(A \cap B) \neq 0\}$$

For each candidate pair we find the ratio of the intersection area over the area of each shape, so if shapes A and B intersect, we find $r_{AB} = \frac{Area(A \cup B)}{Area(A)}$ and $r_{BA} = \frac{Area(A \cap B)}{Area(B)}$

We link all shapes which do not belong to a candidate pair to the empty set.

One to One Matching

First we consider candidate pairs to be links on their own. For each pair, we calculate the overlay ratio as the intersection area over union area, so if A and B are candidate pair, we find:

$$R_{AB} = \frac{Area(A \cap B)}{Area(A \cup B)} = \frac{Area(A \cap B)}{Area(A) + Area(B) - Area(A \cap B)}$$

One to Many Matching

For each time period separately, we identify all candidate pair links for which their intersection to area ratio is above threshold θ. For shape A we define a group $= \{B; r_{BA} \geq \theta\}$. Now we calculate the overlay ratio of one to many links as the intersection area over union area ratio:

$$R_{AG} = \frac{Area(A \cap \bigcup_{B \in G} B)}{Area(A \cup \bigcup_{B \in G} B)} = \frac{\sum_{B \in G} Area(A \cap B)}{\sum_{B \in G} Area(A \cup B)}$$

Many to Many Matching

Here we have two cases, one when the shapes are fairly similar, which we capture in previous sections (one to one, or many to one). The other is inconsistent shapes that form the same structure. To capture these we consider both time periods at the once, we clean the candidate pair list, keeping links for which either ratio is above a threshold θ_1:

$$LC = \{(A, B);\ r_{AB} \geq \theta_1\ or\ r_{BA} \geq \theta_1\}$$

Then we condition to only keep shape for which the total ratio intersection is above threshold θ_2, so shape A will be included if $\sum_{B \in \{x|(A,x) \in LC\}} r_{AB} \geq \theta_2$. Now we are left with a new candidate list, which we convert to sets $LC = \{(\{A\}, \{B\})\}$ and start merging them:

$$if\ G_i \cap G_j \neq \emptyset\ or\ H_i \cap H_j \neq \emptyset: LC = \{(G_i \cup G_j, H_i \cup H_j)\} \cup LC/\{(G_i, H_i), (G_j, H_j)\}, \mathrm{i} \neq \mathrm{j}$$

We keep doing this until we can no longer merge any two rows. At this point we calculate the overlay ratio of many to many links as the intersection area over union section ratio:

$$R_{GH} = \frac{Area(\bigcup_{A \in G} A \cap \bigcup_{B \in H} B)}{Area(\bigcup_{A \in G} A \cup \bigcup_{B \in H} B)}$$

ICP Translation

We encounter a problem when the two shapes or groups of shapes are similar but do not overlap well, this usually stems from the angle at which the images were taken, and is especially prevalent with tall buildings. To address this issue, we translate one trace towards the other, and then recalculate the overlay ratio. As in Besl and McKay (1992), we use the iterative closest point (ICP) method to estimate this translation. To perform the ICP we ignore any cavity points as we found they often cause less suitable translation. We found that for similar shapes this will optimize the intersection area.

Optimal Linking

In the end, we rank all links by their overlay ratio. We iteratively keep the link with the highest overlay ratio, or discard it if at least one of the buildings in the link has already been confirmed in a separate link. From the list of optimal links, we define a link to be a match if its overlay ratio, or the overlay ratio after ICP translation is above 0.5. We then define all matched candidates as unchanged, and the remaining candidates as redeveloped. All buildings that were not considered as candidates are defined as infill, if from 2015, and demolished, if from 2004.

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Figure 1: Urban development with perfect foresight

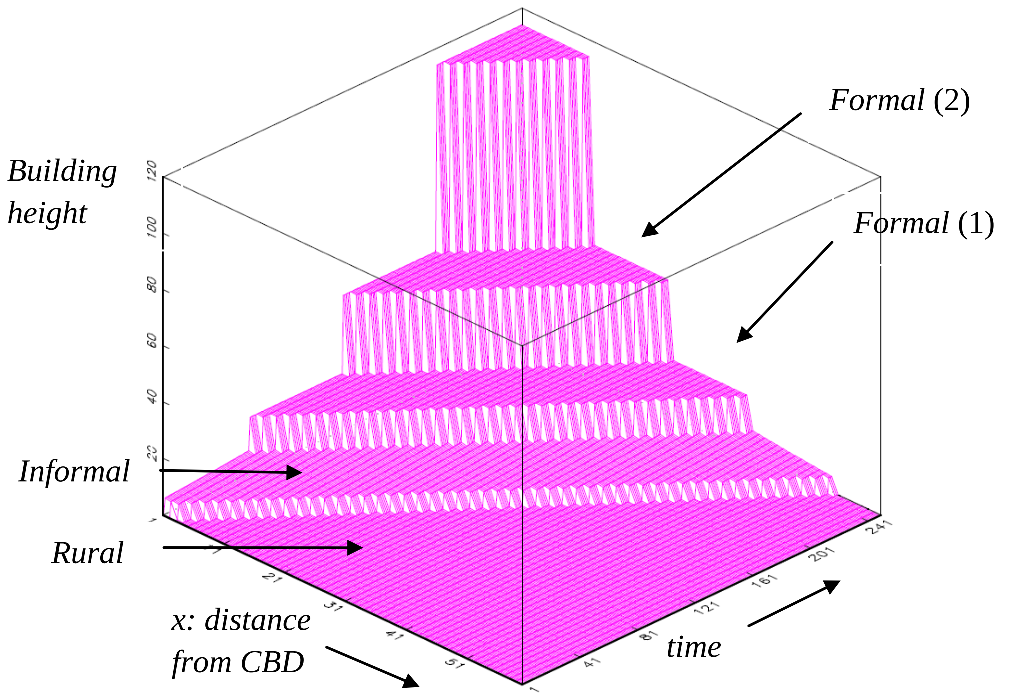


Figure 2: Expectations: height profile of city at *t* = 200.

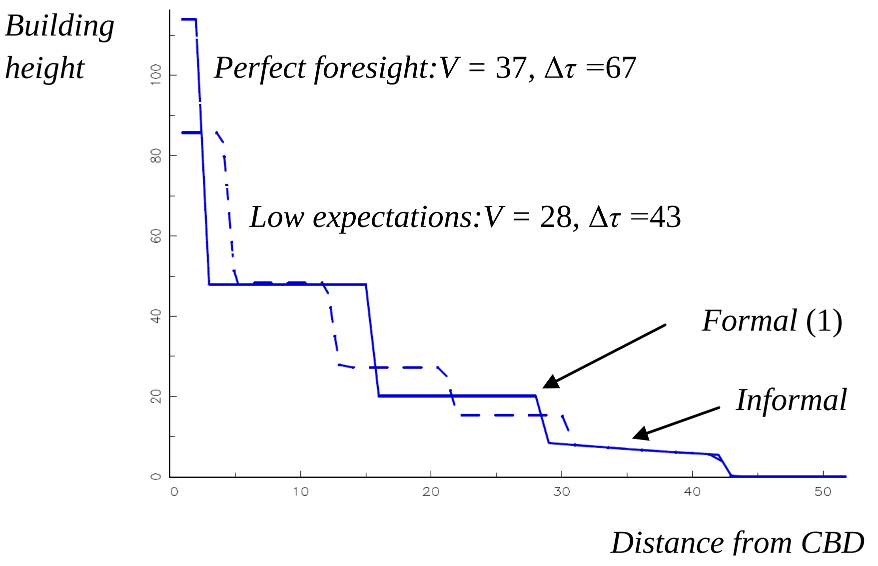


Figure 3a: Costs of formalisation

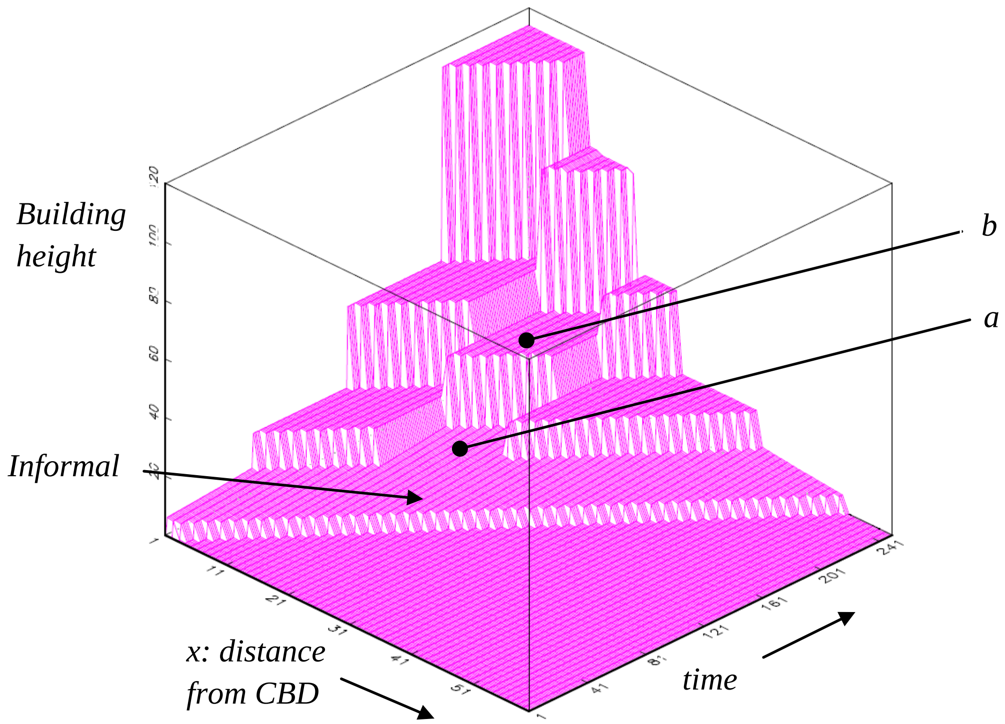


Figure 3b: Random variation in formalisation costs

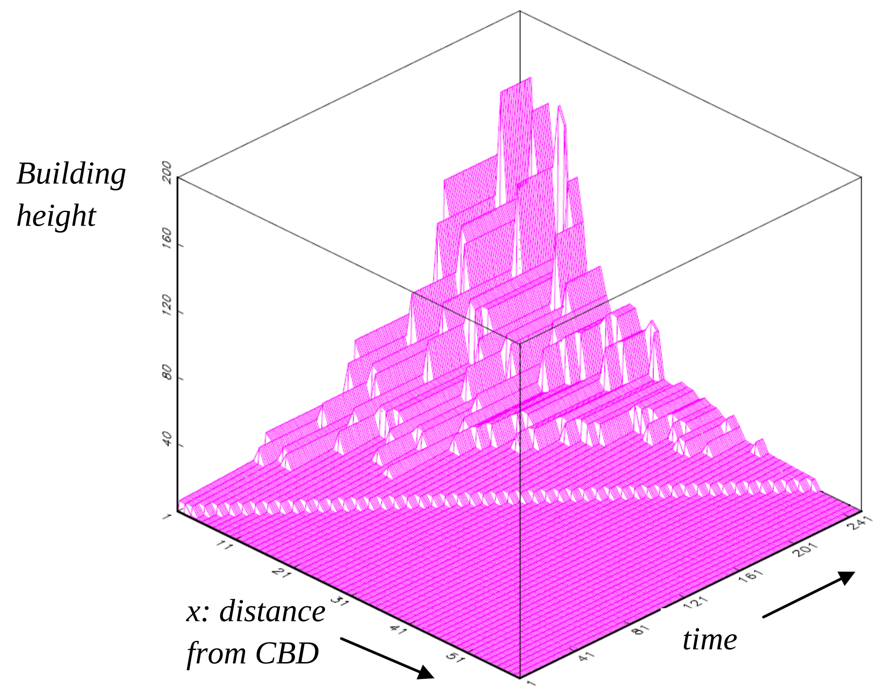
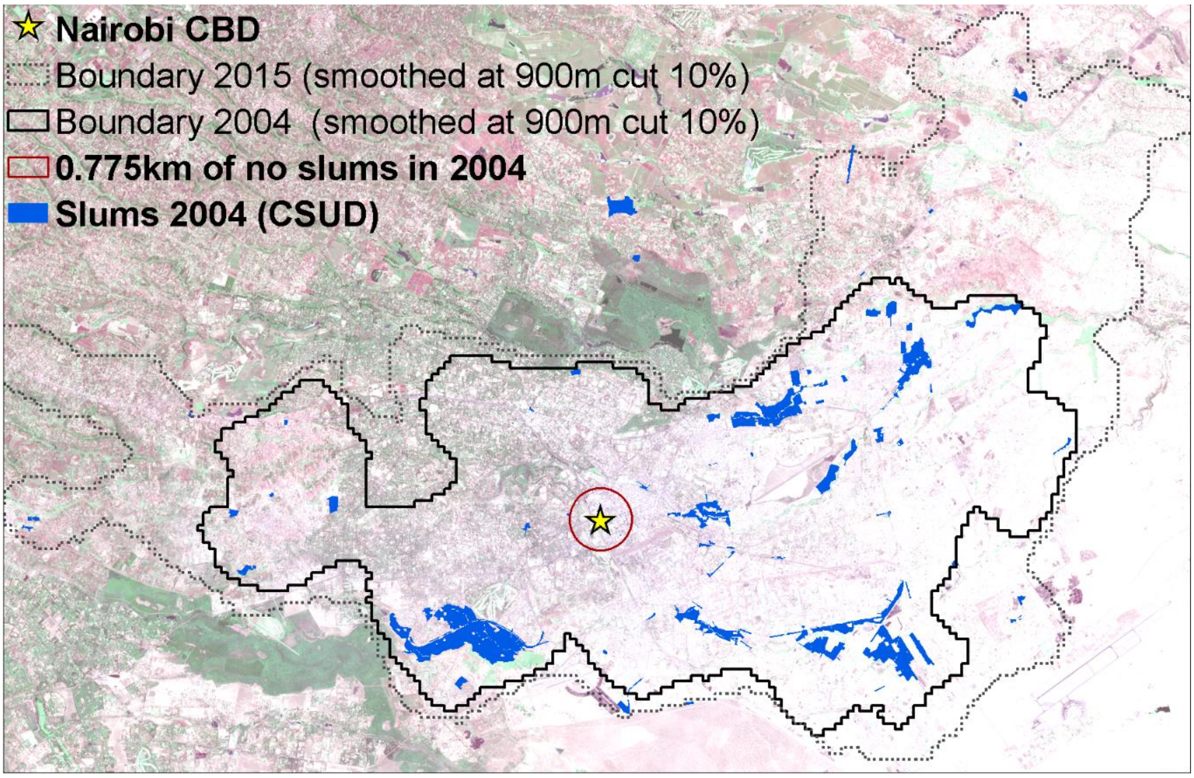


Figure 4 City shape **a. City in 2004**



b. City in 2015

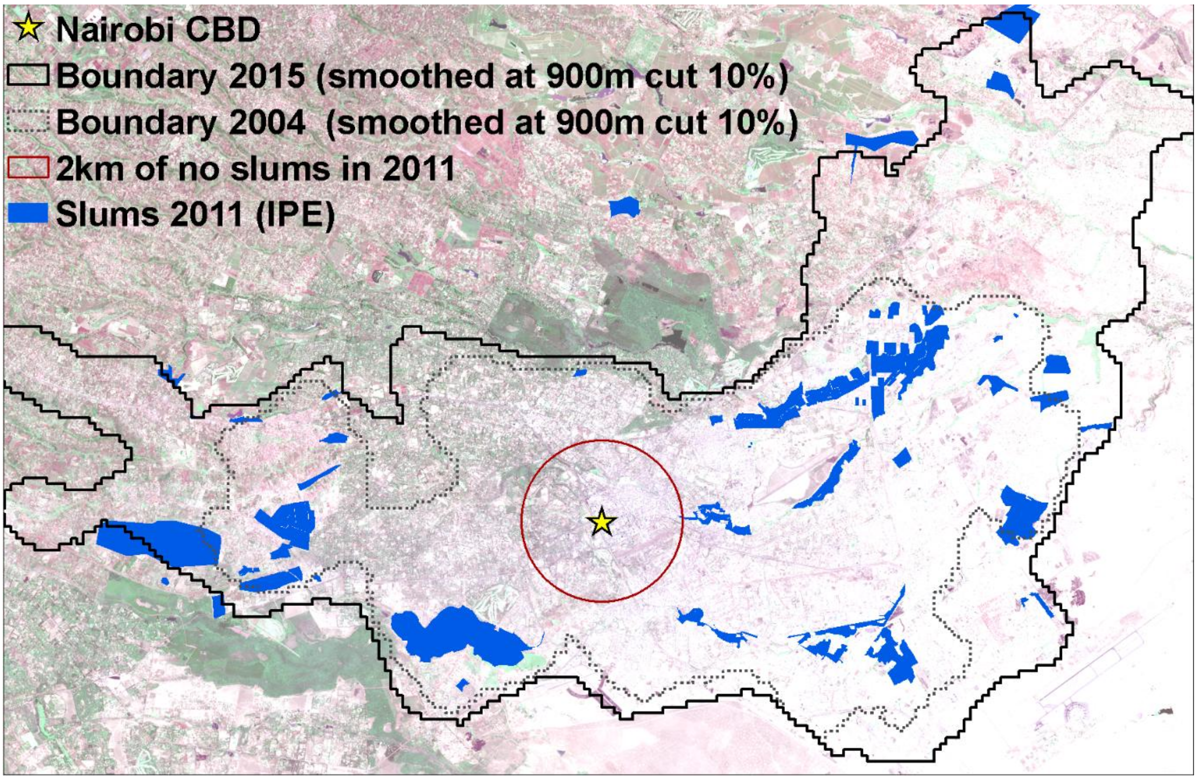
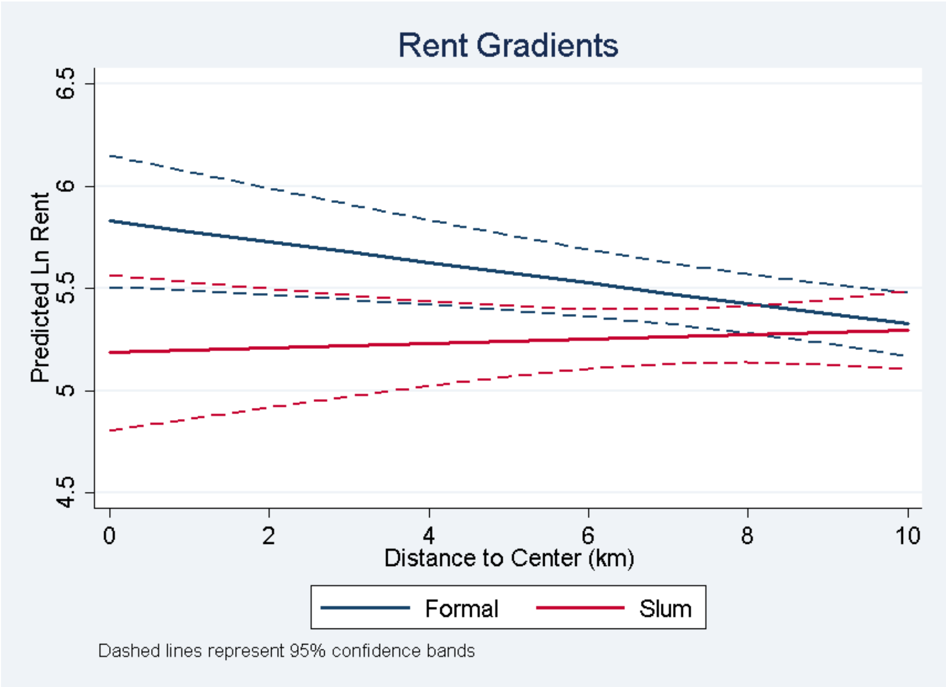


Figure 5. House rents and land prices for standardized units

a. House rents per sq meter square



b. Land prices per square meter land

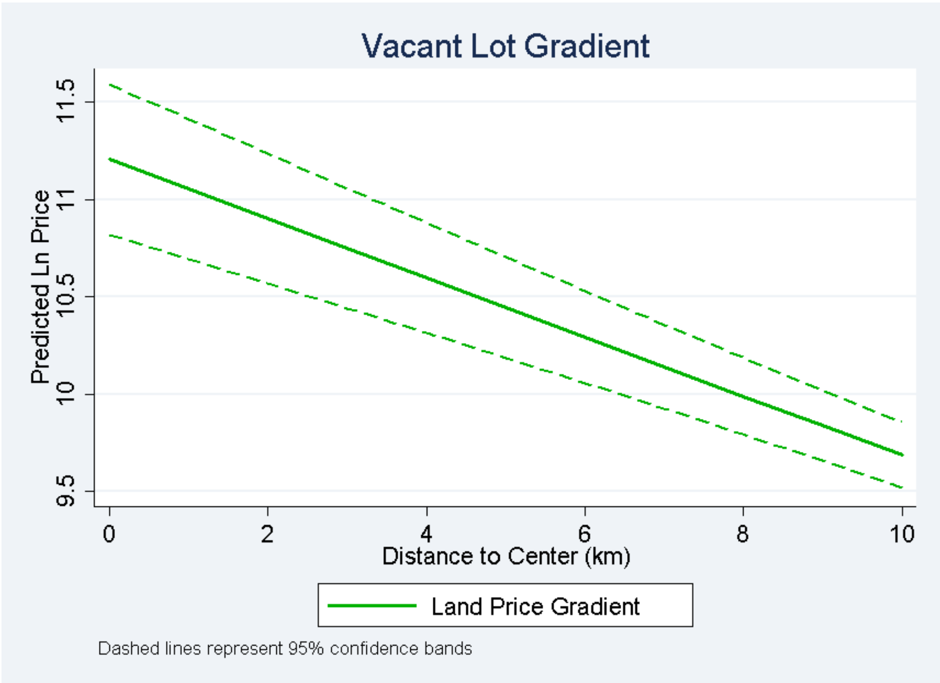
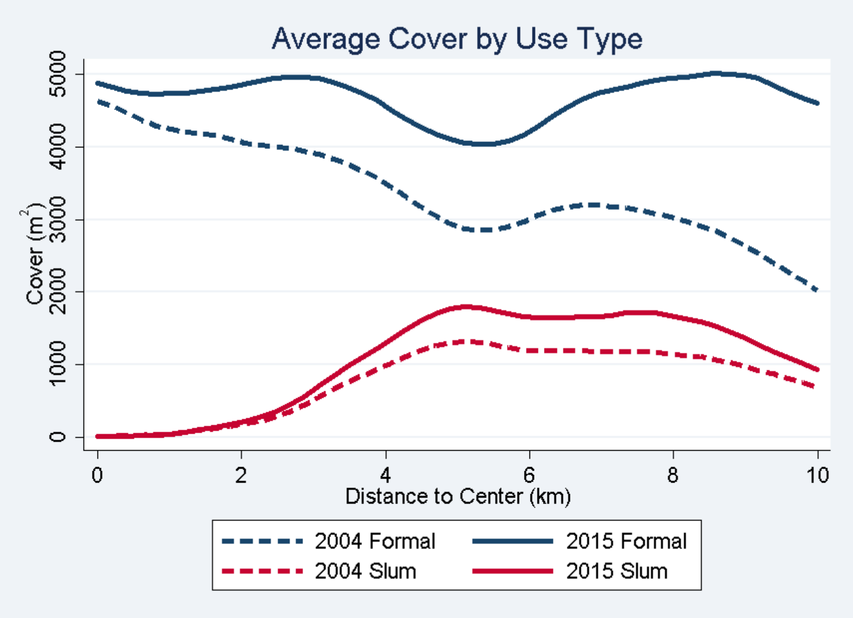


Figure 6 Average (over all grid squares): cover and volume per grid square

a. Cover

	Distance	1	2	3	4	5	6	7	8	9	10
2015, avg. fraction grid sq. cover in:	Formal	0.21	0.22	0.21	0.20	0.19	0.19	0.21	0.22	0.22	0.24
	slum	0.00	0.01	0.03	0.06	0.08	0.07	0.08	0.07	0.06	0.06



b. Volume

Distance	1	2	3	4	5	6	7	8	9	10
2015, ratio slum to formal : vol.	0.01	0.02	0.07	0.12	0.21	0.25	0.25	0.24	0.20	0.18

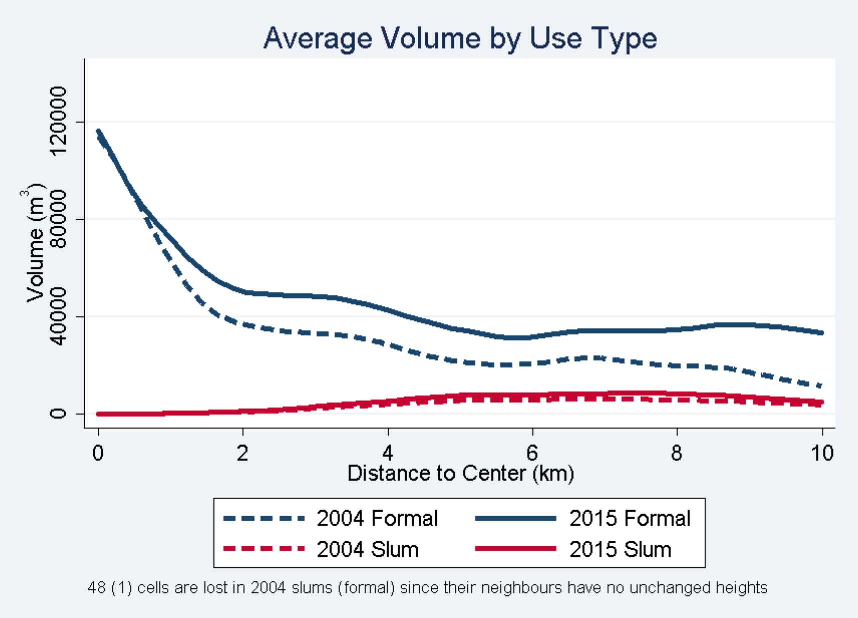
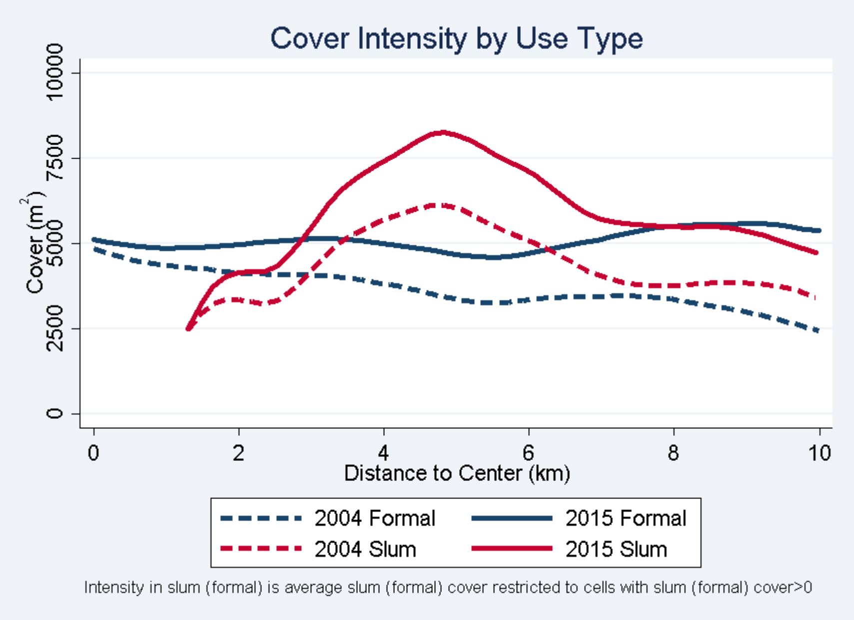


Figure 7 Intensity within usage: Average grid square within each usage

a. Cover

	Distance	1	2	3	4	5	6	7	8	9	10
Fraction in sector grid sq covered	Formal	0.22	0.22	0.23	0.22	0.21	0.21	0.23	0.24	0.25	0.27
	slum	0.07	0.19	0.23	0.34	0.36	0.32	0.25	0.24	0.24	0.25



b. Volume

Distance	1	2	3	4	5	6	7	8	9	10
Slum to formal vol. intensity	0.16	0.40	0.51	0.65	0.88	0.94	0.79	0.72	0.67	0.64

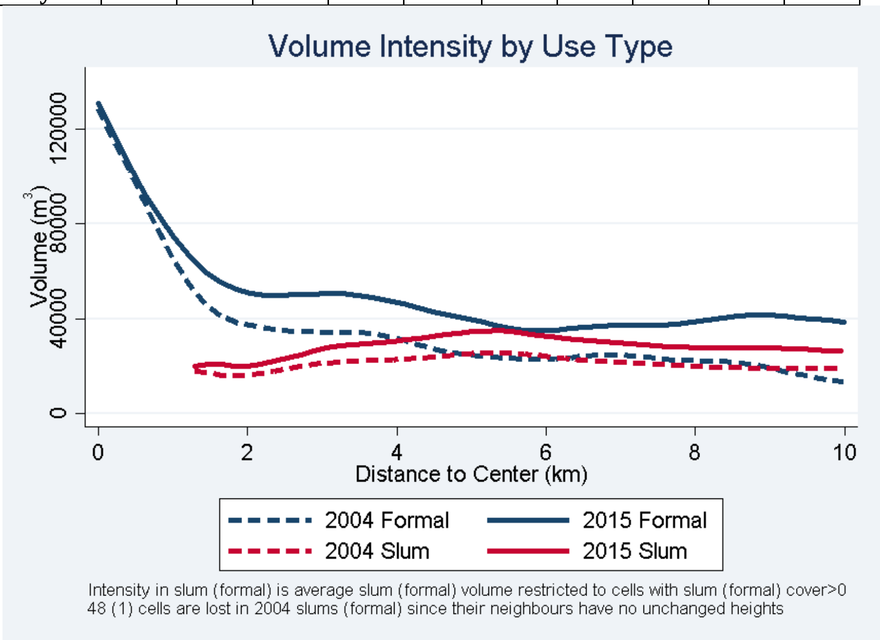
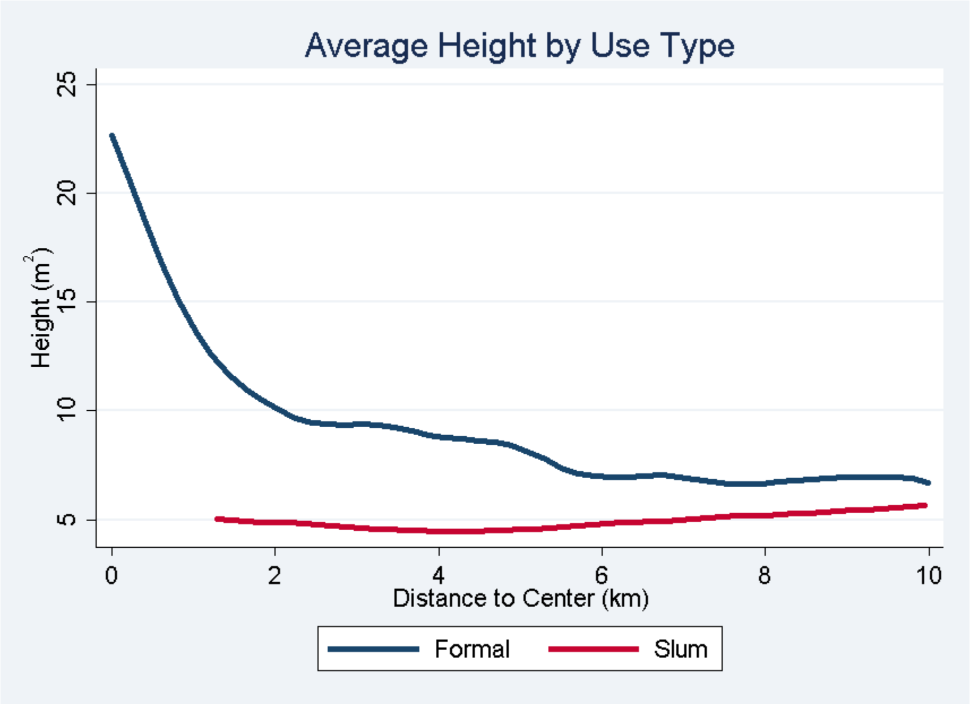


Figure 8 Average heights

a. Overall for city: Average in meters (weighted by cover) [city building data]



b. Predicted height: Residential, count of floors [NORC data]

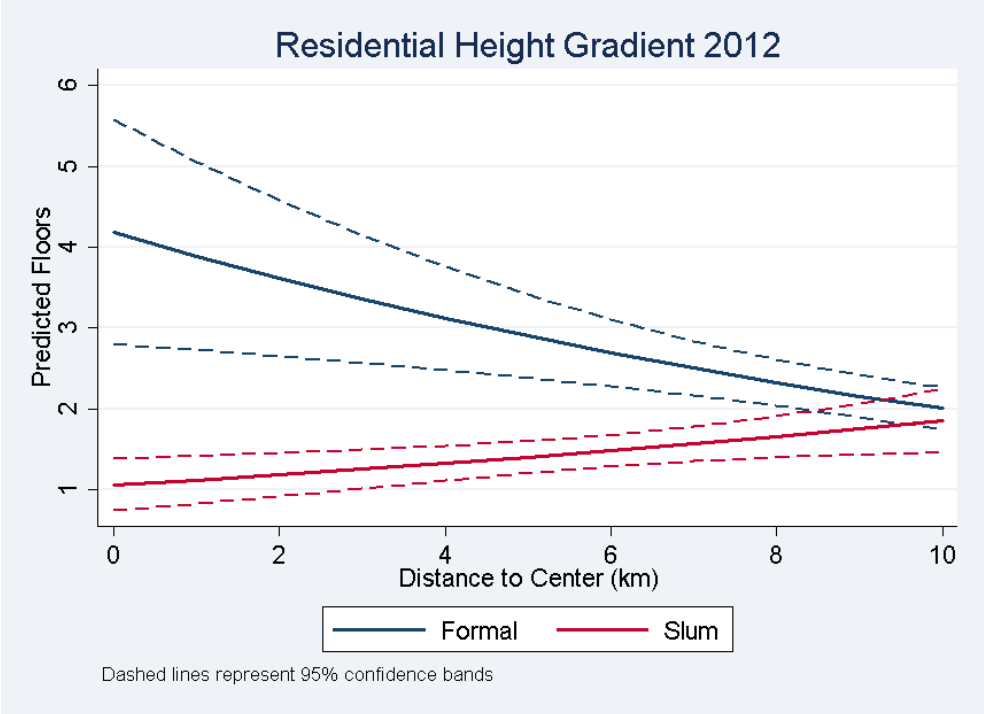
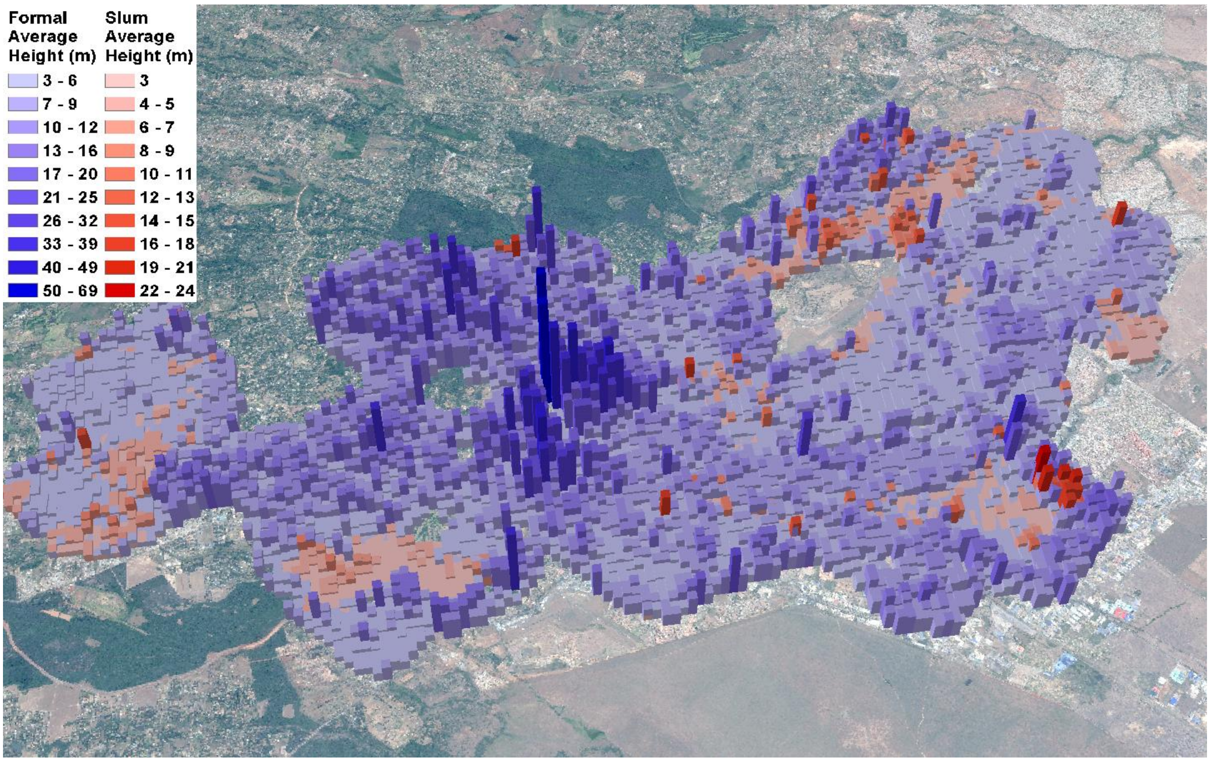


Figure 9 Maps of city height and volume

a. Height



b. Volume

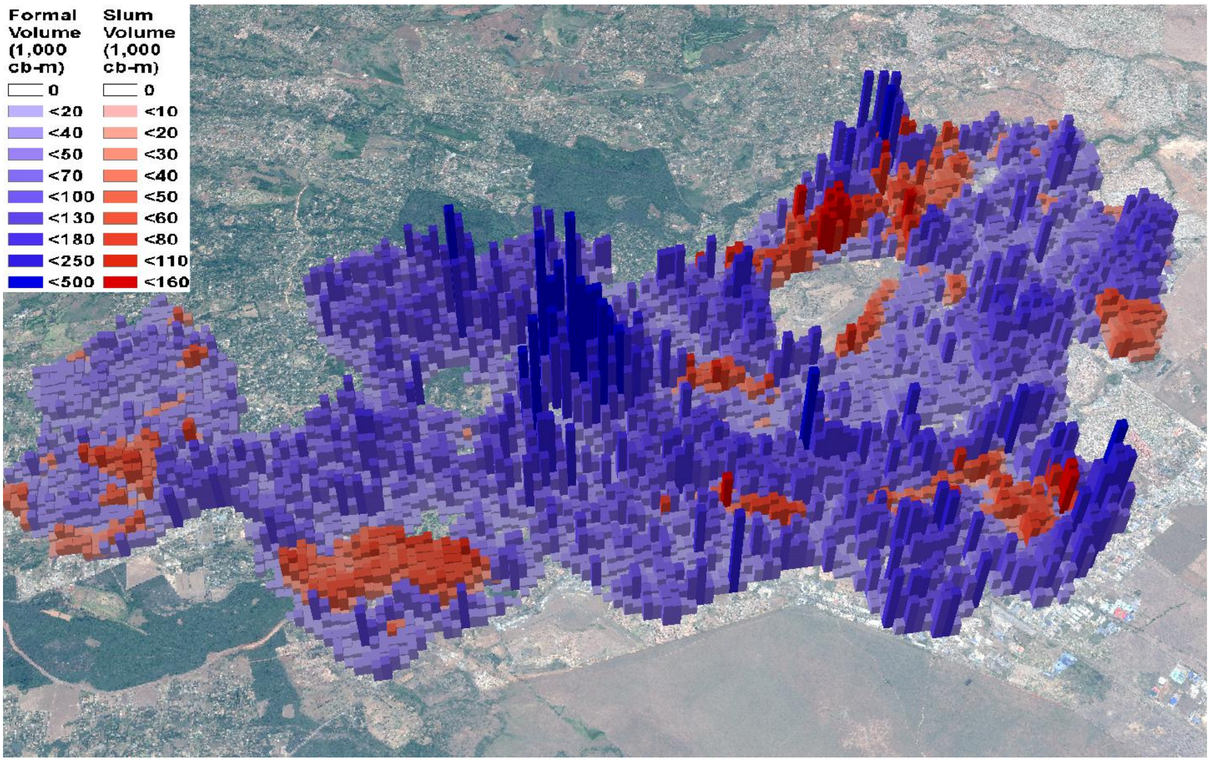
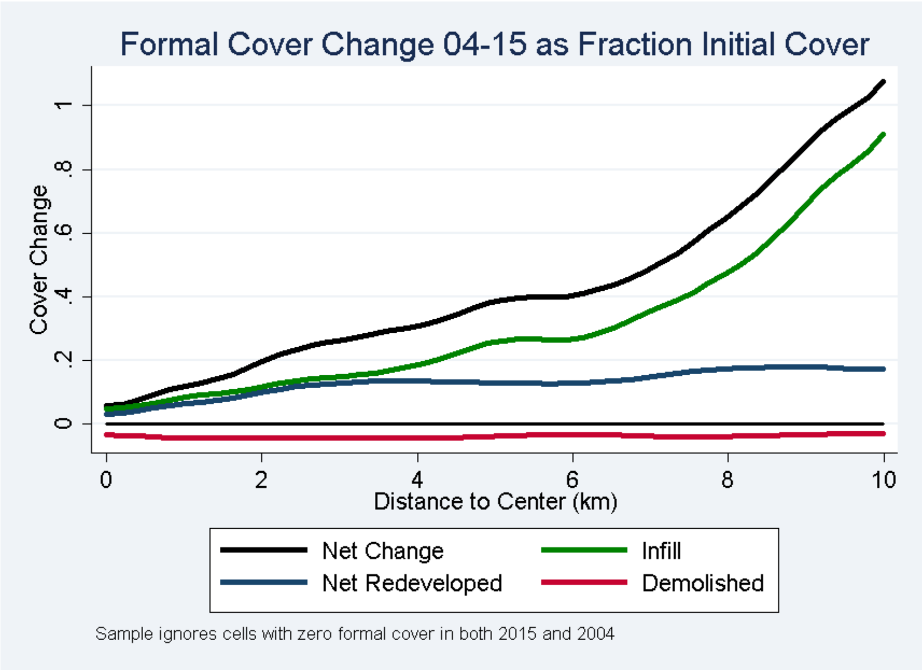


Figure 10: Cover and volume change decomposition: formal sector

a. Cover



b. Volume

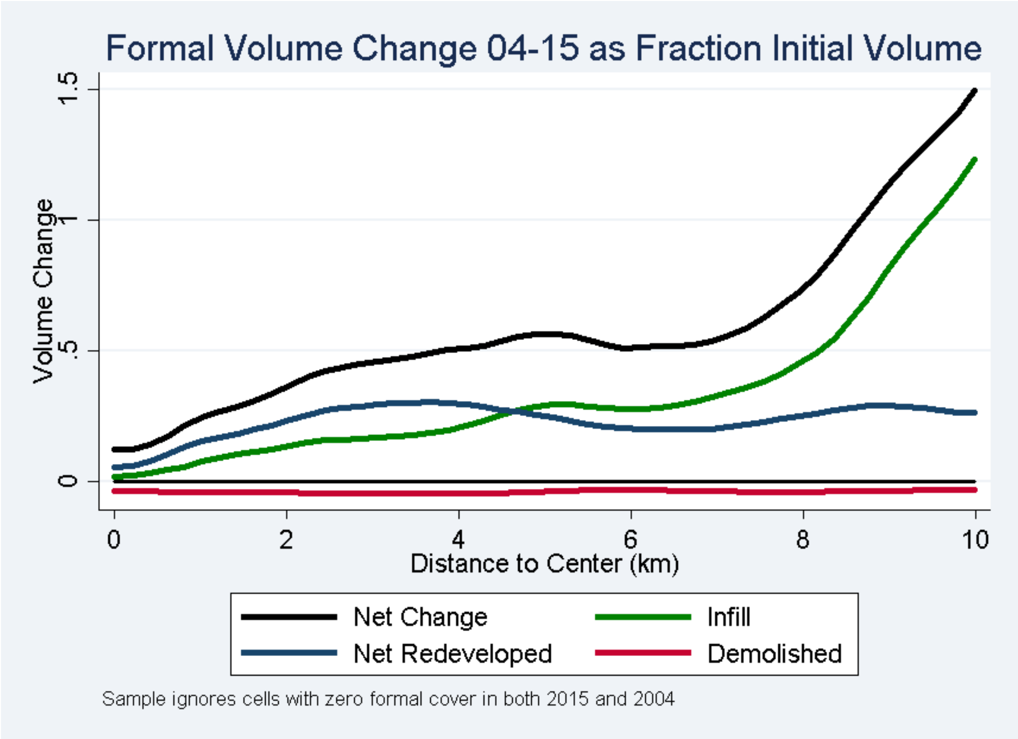
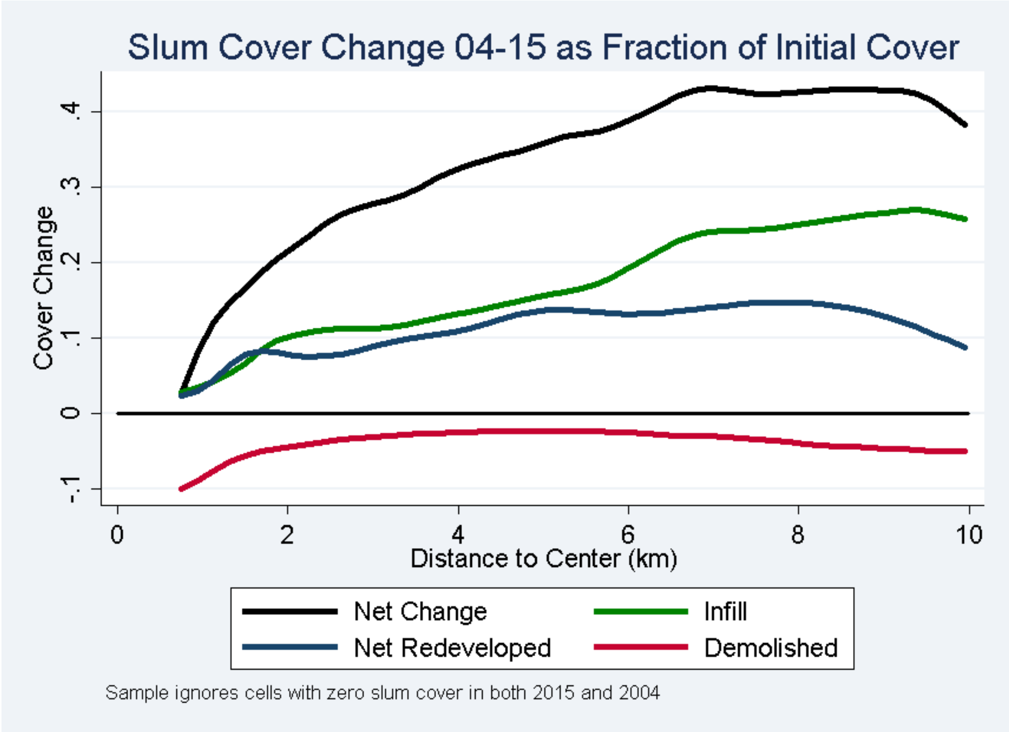


Figure 11 Cover and volume changes, slums

a. Cover



a. Volume

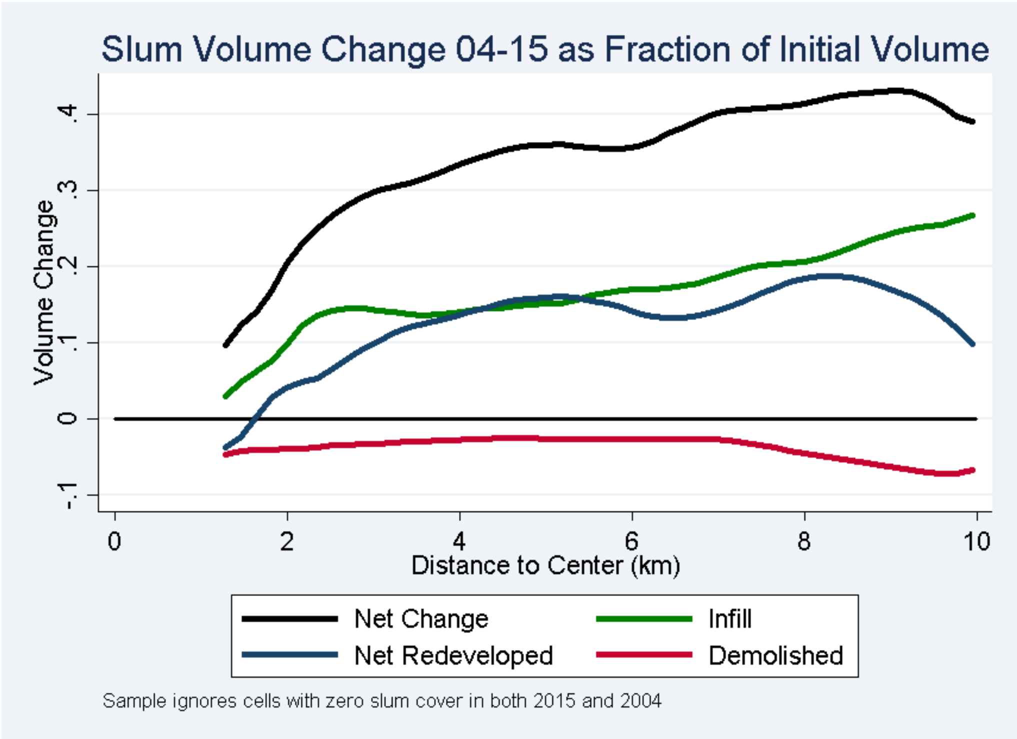


Figure 12. Height comparisons: Infill, redevelopment, versus unchanged

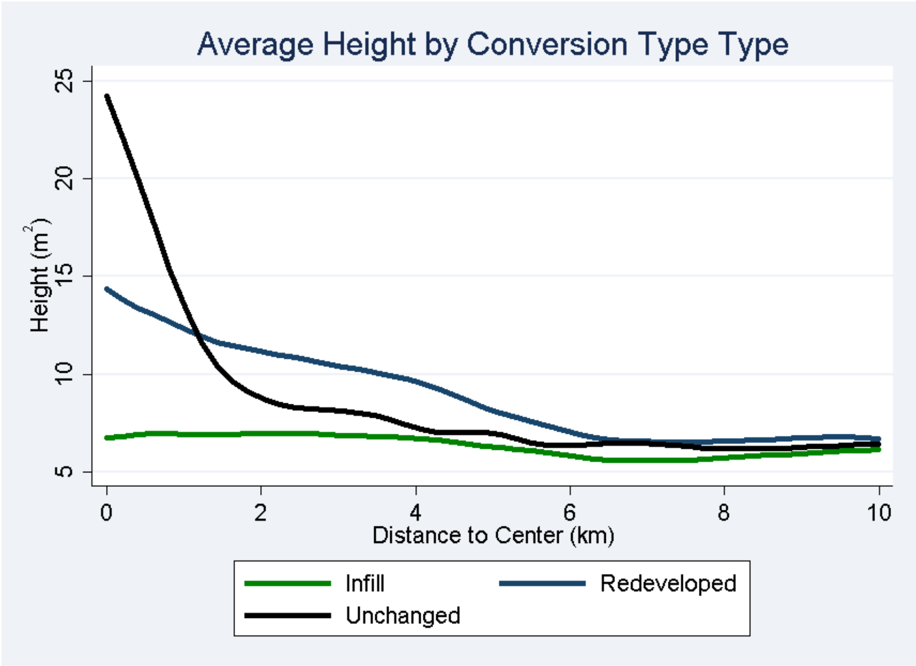


Figure 13. Slum clearance and redevelopment

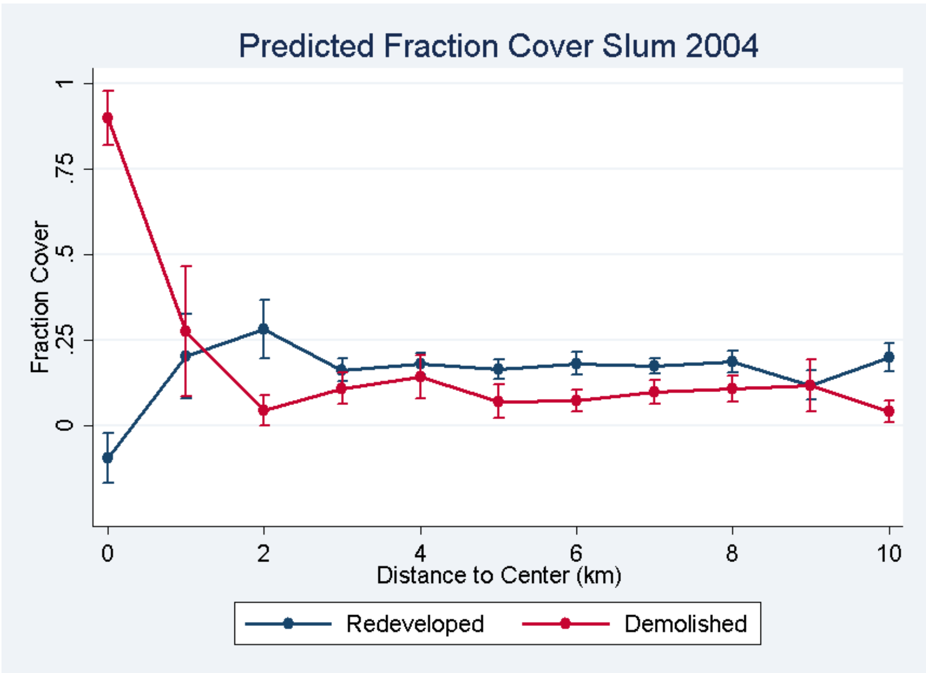


Figure 14. The formal sector-slum rent gap per sq m of standardized housing

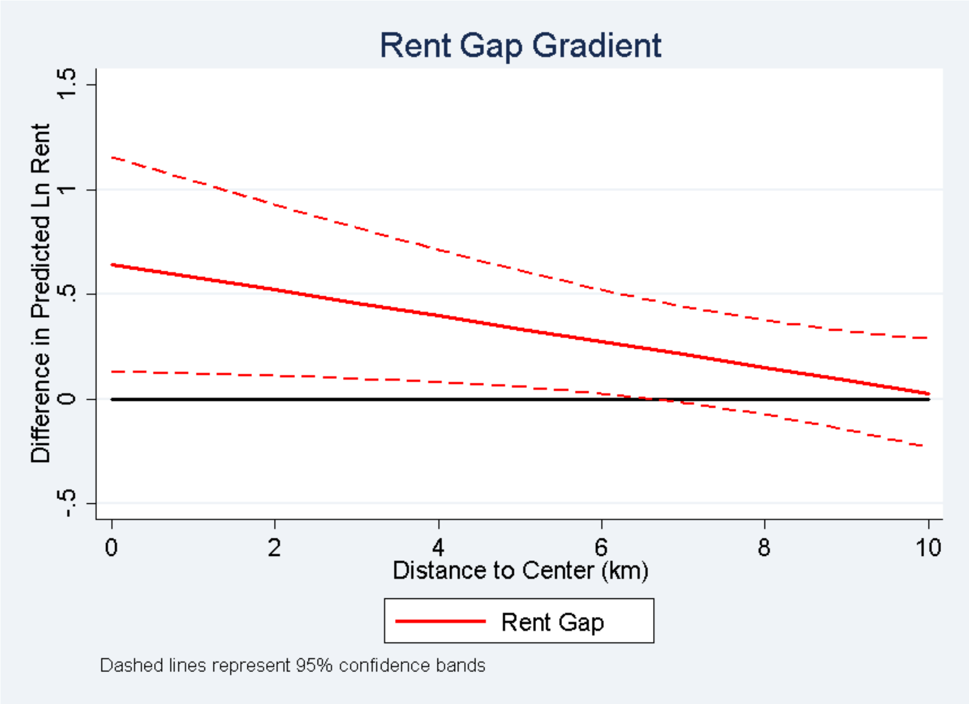


Figure 15. Slum ownership

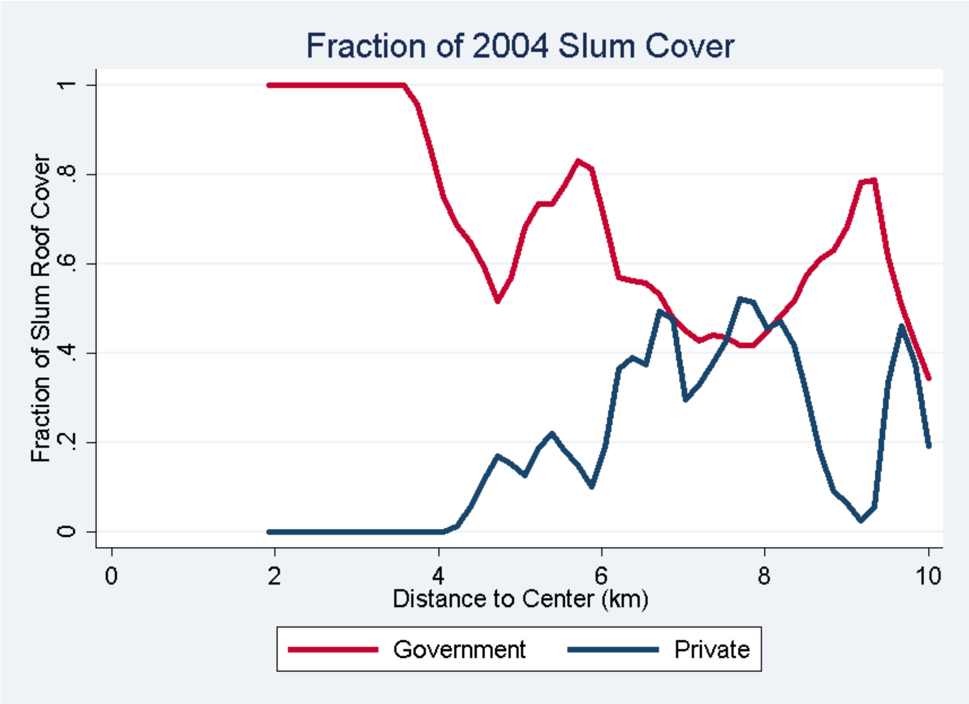


Table 1. Prices, values and heights

	(1)	(2)	(3)	(4)	(5)
	Ln house rent per m-sq	Ln house rent per m-sq	Ln house rent per m-sq	Ln land value (KSh/m-sq)	Poisson on #Floors
main					
Distance to Centre	-0.0748***	-0.0503***	-0.0501***	-0.152***	-0.0735***
	(0.0171)	(0.0177)	(0.0176)	(0.0151)	(0.0182)
Slum=1	-1.422***	-0.643**	-0.273		-1.369***
	(0.246)	(0.261)	(0.279)		(0.228)
Slum=1 X Distance to Centre	0.122***	0.0615**	0.0317		0.129***
	(0.0315)	(0.0302)	(0.0322)		(0.0288)
On Government Land=1 X Slum=1			-0.319**		
			(0.153)		
On Private Land=1 X Slum=1			-0.0851		
			(0.145)		
Lot size				-0.0276***	
				(0.00829)	
Coordinates estimated=1				-0.320**	
				(0.124)	
Constant	6.223***	5.952***	5.922***	10.74***	1.432***
	(0.166)	(0.570)	(0.566)	(0.103)	(0.169)
Controls	No	Yes	Yes	No	No
Listing Month	No	No	No	Yes	No
Observations	1008	928	928	561	1111
R-squared	0.121	0.290	0.302	0.570	

* p<0.10, ** p<0.05, *** p<0.01

Controls Include: Ln EA Population Density ln(people/km-sq); Tenancy agreement is written formal; Number of Bath Rooms; Piped water to dwelling; Piped water within 50m; Toilet facility in house; Connected to Electricity; Wall is made from iron sheet or tin; Wall is made from brick stone or block; Roof is made from iron sheet; Floor is made from earth or clay; Access road is paved or tarmacked; Access road is gravel; Dwelling or room in multi-story ; Piped water to dwelling; One flood in recent rainy season; 2-3 floods in recent rainy season; 3+ floods in recent rainy season; # Floors;

Table 2 Change (04-15) in Formal Cover

	(1)	(2)	(3)	(4)
	Net Change	Infill	Net Redeveloped	Demolished
km to Center Bin=0	542	466	165	-130
	(88)	(63)	(43)	(25)
km to Center Bin=1	683	499	318	-177
	(58)	(39)	(38)	(20)
km to Center Bin=2	943	602	386	-142
	(50)	(35)	(26)	(10)
km to Center Bin=3	1152	689	520	-161
	(46)	(32)	(27)	(10)
km to Center Bin=4	1278	835	464	-156
	(59)	(44)	(26)	(15)
km to Center Bin=5	1385	898	470	-105
	(69)	(53)	(27)	(9)
km to Center Bin=6	1420	937	496	-115
	(74)	(54)	(30)	(9)
km to Center Bin=7	2006	1467	606	-162
	(100)	(83)	(31)	(18)
km to Center Bin=8	2273	1635	616	-115
	(99)	(84)	(41)	(11)
km to Center Bin=9	2717	2112	569	-111
	(137)	(123)	(34)	(13)
km to Center Bin=10	2327	1700	772	-48
	(151)	(133)	(45)	(7)
km to Center Bin=11	3305	1911	1296	-65
	(177)	(118)	(77)	(11)
km to Center Bin=12	4417	3197	1118	-75
	(696)	(581)	(200)	(37)
Observations	5697	5697	5697	5697
R-Squared	0.18	0.20	0.13	0.04

Controls Include: Dummy=1 if Cover 2004 > 0 X Fraction of Cell that is Roof Cover in 2004; Dummy=1 if Cover 2004 > 0 X Average Size of 2004 Buildings in Cell; Dummy=1 if Cover 2004 > 0; Fraction of Cell that is Road Cover 2004; Fraction of 2004 Cover in Cell and Queen Neighbours that is Slum; Fraction of Cell and Queen Neighbours that is Roof Cover; Average Size of 2004 Buildings in Cell and Queen Neighbours;

Note - sample is for gridcells that have some formal cover in 2004 or 2015

Table 3. Change (04-15) in Formal Volume

	(1)	(2)	(3)	(4)
	Net Change	Infill	Net Redeveloped	Demolished
km to Center Bin=0	8952	2465	4307	-2327
	(4059)	(691)	(1765)	(455)
km to Center Bin=1	12682	4804	8174	-1829
	(1713)	(682)	(1256)	(240)
km to Center Bin=2	13199	5697	7401	-1214
	(993)	(499)	(632)	(89)
km to Center Bin=3	17109	6557	10547	-1452
	(930)	(431)	(695)	(105)
km to Center Bin=4	16277	7571	7957	-1216
	(942)	(470)	(634)	(104)
km to Center Bin=5	12410	6827	4923	-739
	(824)	(499)	(420)	(61)
km to Center Bin=6	12493	6758	5507	-778
	(836)	(492)	(465)	(64)
km to Center Bin=7	14179	9058	5358	-1140
	(900)	(612)	(406)	(142)
km to Center Bin=8	18862	11446	6807	-788
	(1116)	(706)	(684)	(74)
km to Center Bin=9	22056	16089	5249	-803
	(1753)	(1385)	(582)	(111)
km to Center Bin=10	14274	9339	4966	-300
	(1060)	(799)	(409)	(55)
km to Center Bin=11	19090	9194	7433	-403
	(1470)	(704)	(677)	(77)
km to Center Bin=12	19280	11824	5191	-451
	(4486)	(2966)	(1353)	(160)
Observations	5696	5677	5696	5696
R-Squared	0.04	0.12	0.04	0.05

Controls Include: Dummy=1 if Cover 2004 > 0 X Fraction of Cell that is Roof Cover in 2004; Dummy=1 if Cover 2004 > 0 X Average Size of 2004 Buildings in Cell; Dummy=1 if Cover 2004 > 0; Fraction of Cell that is Road Cover 2004; Fraction of 2004 Cover in Cell and Queen Neighbours that is Slum; Fraction of Cell and Queen Neighbours that is Roof Cover; Average Size of 2004 Buildings in Cell and Queen Neighbours; Note - sample is for gridcells that have some formal cover in 2004 or 2015

Table 4. Change (04-15) in Slum Cover

	(1)	(2)	(3)	(4)
	Net Change	Infill	Net Redeveloped	Demolished
km to Center Bin=0	668	-230	549	-342
	(270)	(196)	(64)	(57)
km to Center Bin=1	936	454	488	-236
	(215)	(165)	(81)	(59)
km to Center Bin=2	1817	697	614	-27
	(164)	(101)	(108)	(27)
km to Center Bin=3	1803	1067	465	-144
	(162)	(127)	(44)	(28)
km to Center Bin=4	1837	784	661	-96
	(135)	(68)	(57)	(19)
km to Center Bin=5	2329	1114	730	-125
	(183)	(123)	(61)	(44)
km to Center Bin=6	1622	1020	436	-99
	(163)	(110)	(46)	(23)
km to Center Bin=7	1648	882	635	-140
	(115)	(81)	(39)	(16)
km to Center Bin=8	1528	850	519	-202
	(144)	(106)	(48)	(29)
km to Center Bin=9	786	524	236	-189
	(201)	(149)	(45)	(48)
km to Center Bin=10	2509	1559	1140	-69
	(249)	(198)	(96)	(10)
km to Center Bin=11	1600	836	656	-146
	(241)	(150)	(96)	(29)
km to Center Bin=12	1744	136	1212	-80
	(120)	(81)	(39)	(23)
Observations	1272	1272	1272	1272
R-Squared	0.20	0.20	0.29	0.05

Controls Include: Dummy=1 if Cover 2004 > 0 X Fraction of Cell that is Roof Cover in 2004; Dummy=1 if Cover 2004 > 0 X Average Size of 2004 Buildings in Cell; Dummy=1 if Cover 2004 > 0; Fraction of Cell that is Road Cover 2004; Fraction of 2004 Cover in Cell and Queen Neighbours that is Slum; Fraction of Cell and Queen Neighbours that is Roof Cover; Average Size of 2004 Buildings in Cell and Queen Neighbours;

Note - sample is for gridcells that have some slum cover in 2004 or 2015

Table 5. Change (04-15) in Slum Volume

	(1)	(2)	(3)	(4)
	Net Change	Infill	Net Redeveloped	Demolished
km to Center Bin=0	1633	1109	1151	-943
	(1067)	(896)	(561)	(278)
km to Center Bin=1	8100	3923	2280	63
	(2630)	(1682)	(733)	(165)
km to Center Bin=2	8421	4988	2719	-513
	(935)	(758)	(320)	(130)
km to Center Bin=3	7180	3237	2838	-424
	(798)	(318)	(334)	(145)
km to Center Bin=4	10352	5003	3624	-568
	(1077)	(696)	(383)	(230)
km to Center Bin=5	7287	4202	2143	-564
	(1145)	(511)	(297)	(148)
km to Center Bin=6	8288	3516	3807	-560
	(716)	(351)	(315)	(56)
km to Center Bin=7	8582	4151	3959	-1475
	(1015)	(634)	(478)	(338)
km to Center Bin=8	4618	3365	1599	-1317
	(1472)	(1047)	(340)	(544)
km to Center Bin=9	10277	6003	5079	-381
	(1224)	(863)	(508)	(65)
km to Center Bin=10	7018	3312	3095	-639
	(1052)	(694)	(388)	(128)
km to Center Bin=11	7293	424	5708	-564
	(822)	(475)	(275)	(148)
km to Center Bin=12	.	.	.	.
	(.)	(.)	(.)	(.)
Observations	1257	1245	1257	1257
R-Squared	0.08	0.12	0.16	0.05

Controls Include: Dummy=1 if Cover 2004 > 0 X Fraction of Cell that is Roof Cover in 2004; Dummy=1 if Cover 2004 > 0 X Average Size of 2004 Buildings in Cell; Dummy=1 if Cover 2004 > 0; Fraction of Cell that is Road Cover 2004; Fraction of 2004 Cover in Cell and Queen Neighbours that is Slum; Fraction of Cell and Queen Neighbours that is Roof Cover; Average Size of 2004 Buildings in Cell and Queen Neighbours;

Note - sample is for gridcells that have some slum cover in 2004 or 2015

Table 6. Slum Fraction of Gridcell in 2004 Changed

	(1)	(2))
	Redeveloped	Demolished
km to Center Bin=0	-0.095	0.906
	(0.0370)	(0.0363)
km to Center Bin=1	0.203	0.264
	(0.0629)	(0.0970)
km to Center Bin=2	0.281	0.015
	(0.0435)	(0.0223)
km to Center Bin=3	0.162	0.081
	(0.0165)	(0.0173)
km to Center Bin=4	0.182	0.101
	(0.0161)	(0.0173)
km to Center Bin=5	0.164	0.055
	(0.0143)	(0.0132)
km to Center Bin=6	0.181	0.044
	(0.0174)	(0.0104)
km to Center Bin=7	0.175	0.067
	(0.0117)	(0.0123)
km to Center Bin=8	0.186	0.077
	(0.0157)	(0.0118)
km to Center Bin=9	0.119	0.082
	(0.0216)	(0.0230)
km to Center Bin=10	0.200	0.039
	(0.0208)	(0.0097)
km to Center Bin=11	0.296	0.019
	(0.0415)	(0.0216)
km to Center Bin=12	0.461	-0.088
	(0.0156)	(0.0153)
Observations	1199	1199
R-Squared	0.12	0.18

Controls Include: Dummy=1 if Cover 2004 > 0 X Fraction of Cell that is Roof Cover in 2004; Dummy=1 if Cover 2004 > 0 X Average Size of 2004 Buildings in Cell; Dummy=1 if Cover 2004 > 0; Fraction of Cell that is Road Cover 2004; Fraction of 2004 Cover in Cell and Queen Neighbours that is Slum; Fraction of Cell and Queen Neighbours that is Roof Cover; Average Size of 2004 Buildings in Cell and Queen Neighbours;

Note - sample is for gridcells that have some slum cover in 2004

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