DROUGHT IMPACT ON REGIONAL ECONOMY

by
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October 1972

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

55

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Hydrology Papers
Colorado State University
Fort Collins, Colorado 80521

October 1972 No. 55

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ACKNOWLEDGMENTS

This paper is the Ph.D. dissertation submitted and defended by the writer in the Civil Engineering Department of Colorado State University during the Spring 1972. The National Science Foundation, through its Grant GK-11564 (Large Continental Droughts) with V. Yevjevich as the principal investigator, provided the financial support leading to research results presented in this study. The basic economic data used in the research was made available by Mr. G. B. Collins from the Office of Environmental Protection Agency, Denver, Colorado.

Jaime Millan

ABSTRACT

DROUGHT IMPACT ON REGIONAL ECONOMY

The characteristics of drought as a natural event and drought as a hazard to the regional economy are studied. Runs as statistical properties of hydrologic sequences are used in an objective definition of droughts. The probability distributions of the longest negative run-length to be found in a sample of size N are reviewed and analytically defined for some simple cases of time dependence. An approximation is introduced for the case of the truncation level being of a linear trend type. The Monte Carlo method in generating large numbers of hydrologic samples is used in conjunction with a model of the regional economy to determine the economic impact of droughts. A programming formulation of a dynamic type interindustry model is used to simulate the regional economy over a selected time horizon in order to allocate the drought shortages and compute its losses following a consistent procedure.

The methodology developed in this study is applied to a case study of the Upper Main Stem of the Colorado River Basin. The results show advantages and flexibility of the model developed for analyzing the alternative policies for regional management of water resources during drought periods.

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PREFACE

It is customary to distinguish damages caused by natural disasters, such as floods and droughts, as direct or indirect. While the direct damages can be obtained by surveys of activities in an already disaster-stricken area, the indirect damages to regional or national economy are less evident and less easy to assess.

Flood events are sudden disaster phenomena which produce risks to flood prone valleys and plains. Insurance policy has been found feasible for floods. Floods are partly subject to prediction, particularly for warning purposes, provided a proper prediction and warning organization exists. It is possible to partly alleviate flood consequences with emergency efforts, and they are often used.

Droughts may be considered natural disasters with some opposite characteristics to floods. Droughts are of the creeping type of disaster. Their effects are built up slowly over a period of time. Since hydrologic deterministic predictions are of a relatively short length, a severe, long drought covering a large area is an unpredictable hydrologic phenomenon in the sense of classical hydrologic predictions. The probability of occurrence of droughts of a given duration, areal coverage and severity must be used instead of classical predictions. Although there is high degree of adaptability of an area to droughts, droughts create long-range negative economic and social consequences. Because a drought affects many sectors of economy in a region, and because these sectors are mutually dependent, then the indirect drought damages may be the same or greater than direct damages.

The research project, Large Continental Droughts, sponsored by the National Science Foundation, Grant No. GK-11564, of which this study is a part, has been approached from several directions. One important aspect is the development of methods for assessing the damages of large continental droughts to a regional economy. This study, as the Ph.D. dissertation by Mr. Jaime Millan, attacks this problem by integrating both the direct and indirect economic damages of droughts with probabilities of droughts in a region. This unique method enables the computation of drought damages without the need for experiencing a sufficient number of severe droughts; whereas, for flood damages, the apostiori surveys of flooded areas lead to a damage function. Because of a lack of practical methods for assessing the economic consequences of large droughts, coupled with probabilities of these droughts, the method outlined in this study is expected to induce practical attempts to sharpen techniques of assessment of potential drought damages in any region. This would induce the search for solutions to problems related to drought disasters.

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CHAPTER I

INTRODUCTION

The initial problem that any investigator of drought faces is definition of the term "drought". Different fields of study hold widely diverse views, most of them subjective, of what constitutes "drought". It is generally understood, however, that the word refers to a deficit or shortage of available water, brought about by the random characteristics of the natural resources that control the distribution of water on the surface of the earth and with time. Such a water shortage is important to man because it may become a hazard-causing event, presenting a threat to the survival of the established biological system or to the present or projected level of various human activities.

Any particular shortage can be, nevertheless, a different threat to different human activities depending on the extent to which those human activities have been developed, and on the adjustment and adaptability that they can exhibit during the shortage. This very point is the source of the diversity of definitions concerning droughts because everyone tends to define drought to the extent that he is affected by the shortage. It seems reasonable, therefore, to define and study first the hazard-causing event and later to investigate the corresponding hazard at the desired level of development and drought adjustments by various human activities.

1.1 Drought: A Natural Event

The occurrence of drought is related to space as well as to time. Severe droughts will affect a whole region, and a delimitation of the study region is necessary, not only in the study of the natural event, but also in determining the hazard or losses that this event can cause to the established human activities.

In a recent study of natural hazard, Kates (1971) describes twelve critical indexes, seven of which are primarily characteristics of the natural event system: spatial distribution, magnitude, frequency, duration, areal extent, forecast capability, and warning time. From these indexes one can apply to drought¾ magnitude expressed as the total volume of the water deficit, duration expressed as temporal periods of continued deficit, frequency expressed as a probability of recurrence in a unit of time or in an average return or recurrent period of time for events of magnitude or duration, and the extent of area studied. For all practical purposes, the forecast capability and the warning time are approximately zero for the present state of knowledge.

To facilitate analysis, it is necessary to adopt an objective definition of drought that permits clear identification of the critical indexes once the series of water supply and water demand are given. The definition adopted in this study has been suggested by Yevjevich (1967) and is based on runs, as statistical properties of sequences, in both time and area. Figure 1.1 represents a discrete series of a random variable X representing the water availability. By selecting an arbitrary value c_0, the discrete series is truncated and two new series, of positive and negative deviations, are formed. The sequence of consecutive negative deviations is called a negative run, and it may be associated with a drought. The sequence of positive deviations is called a positive run and it may be associated with a surplus. It is important to point out that the value c_0 may be either a fixed number or a deterministic or stochastic time series, according to circumstances. When studying the series of natural phenomena, the value c_0 can be the mean of observed values, any quantile, or a multiple of the standard deviation below or above the mean. In the study of water deficits, the quantity c_0 is represented by the

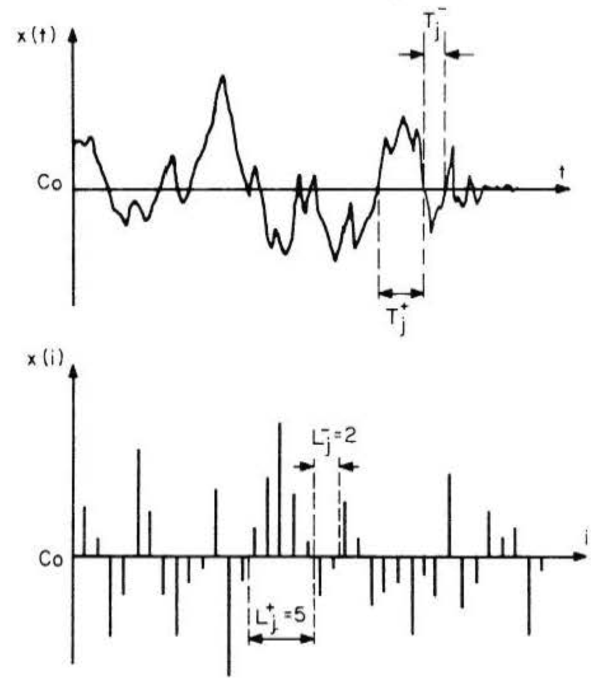

Fig. 1.1 Definitions of runs of continuous (upper graph) and discrete (lower graph) series.

demand series. Among the several droughts that can be observed in a given period, the most critical, i.e. the longest, largest or both, is of special interest because it will create the most severe conditions under which the system would operate.

1.2 Drought: A Hazard

Of particular importance in practice is the investigation of the impact that droughts may produce in the projected or planned economic development of a region. Economic planning is based on many assumptions, among which the water availability may usually be taken for granted. This assumption is often reasonable for a country taken as a whole. The smaller a region, the greater is the probability for random events such as regional floods or droughts which would produce sharp changes, along the points in time, in the economic indicators selected for the planning.

Ordinarily projections of the future economic development of a region involve an increasing trend in water use. Since the total annual water availability as a random variable is a stationary series, a point in time will be reached at which water sets an upper limit to the development of some intensive water-using economic sectors. This point can be determined approximately from the intersection of the projected water demand with the level of supply that can be expected with a given level of confidence. Meanwhile, however, unexpected dry periods can create hazard conditions; and this is the problem that is treated in this study.

When studying the effects of droughts on the regional economy, it is necessary to model both the natural phenomena and the performance of the economy over a relevant planning horizon. A planning horizon is the period of time into the future to be considered in the planning. The selection of this horizon is important because the level of economic activity enjoyed in a particular year is not as relevant as the average level sustainable, with some given adjustments. As Russell (1970) points out, "what may very well be discussed as loss (or a gain) is the difference between some anticipated stream of production over a relevant planning horizon and the actual results achieved".

The economy is an exceedingly complex interdependent system, and any planning model is usually a highly simplified quantitative description of it. Usually, only the relevant features of the economy are modeled, and it always becomes necessary to select what is important for the problem at hand. When analyzing drought damages, a critical factor is the evaluation of (a) the direct losses to sectors of heavy water uses, and (b) of indirect losses that propagate through the economy by virtue of the close interrelations existing between the economic sectors. This can be referred to as or compared to the space dependence. In a similar way, drought effects can propagate into the future by creating a time dependence of results.

1.3 Summary of Objectives of this Investigation

Summarizing, the modeling of drought impact on a regional economy requires the formulation of two separate models and a link between them that permits their joint operation. The first model is concerned with the natural phenomena that are the hazard-causing events, and the second model is concerned with a consistent representation of the regional economy.

When modeling the natural phenomena it becomes possible to generate many new sequences that are statistically indistinguishable from each other but that contain droughts with different characteristics. Chapter II will present a brief summary of the types of processes that can represent the time series of the natural phenomenon. Also in this chapter, a theoretical development is presented which permits estimation of the probability distributions of the length of the longest drought (negative run) in a given period of time for some well known processes. The model of the regional economy must be constructed in such a way as to show clearly the time dependence of each sector and space dependence among the various production sectors, with each sector containing some adjustment measures when faced with water shortage. Chapter III presents a brief introduction to some economic models commonly used to show these effects.

Chapter IV modifies some of the economic models presented in Chapter III, so that they are able to measure how the economy responds to given water availabilities over a time horizon. A detailed procedure for measuring losses is also presented.

Finally, a case study is performed to show the potential of the methodology developed in this study. Chapter V is an organization of basic data, formulating the model for the case study, while Chapter VI illustrates the operation of the model and gives a detailed analysis of the economic performance of the chosen region under the conditions of a stochastic water supply. Conclusions of the study and recommendations for further research are presented in Chapter VII.

CHAPTER II

DROUGHT AS A NATURAL PHENOMENON ITS OCCURRENCE AND PROBABILITIES

The first purpose of this chapter is to identify the random phenomena that control the distribution of water in time and space, and which serve as the basis for the definition of droughts. Mathematical models representing the time series structure of such phenomena are presented. The second purpose is to present an analytical solution for probability distributions of the duration of the longest drought. This is defined as a run in a sequence of N observations.

2.1 Characteristics of Phenomena Used in Defining Droughts

As pointed out in Chapter I, a drought is basically a deficit of water in time, space, or both. For a given region and a period of time, the deficit can be observed in various phenomena. The more relevant of these phenomena are: precipitation at ground level, effective precipitation in the form of precipitation minus evaporation, soil moisture content, ground water levels, runoff, and water stored in various natural or artificial storage spaces. Once the phenomenon, areal extension, time interval, and other variables describing the phenomenon have being selected, a time series of the observed past values of the phenomenon can be constructed and its structure analyzed in order to investigate its stochastic or deterministic-stochastic properties.

In general, a time series can be represented by the sum of a deterministic part, periodic or not, and a stochastic part that can be either dependent or independent. Natural phenomena, such as those mentioned above, ordinarily have periodicities in several of their parameters for time intervals smaller than the year. When the basic period is some multiple of a year, no periodicity has been observed in these phenomena. Therefore, in using annual values, the hypothesis of stationarity can be sustained by the evidence. Annual series of precipitation have ordinarily small or no time dependence. Annual series of runoff, however, have time dependence that can be roughly approximated for most rivers by the first-order linear autoregressive (Markov) model, Yevjevich (1964). In addition to being periodic, monthly and daily observations of natural phenomena have a much larger time dependence than do annual values, and may require more complex models to describe their dependence structure. The time interval selected for this study is one year. This interval corresponds to the characteristics of the proposed economic model. Therefore, only the first-order stationary linear autoregressive models are used in this chapter.

The first-order linear autoregressive model can be expressed for stationary series in the form

$$X_i = \alpha X_{i-1} + \epsilon_i \tag{2.1}$$

in which X_i is the value of the observation at the time i, α is the autoregression coefficient ordinarily estimated by the first serial autocorrelation coefficient, and ϵ_i is an independent stochastic component, and is also independent of X_{i-1}. For a value of α, equal to zero, $X_i = \epsilon_i$, the series become independent. The main characteristic of the first-order linear autoregressive scheme is that the value of the process at any point in time can be expressed only on the outcome in the previous observation, and therefore possesses the properties of the Markovian models.

Accepting the concept of runs as the definition of droughts, the remaining sections of this chapter proceed by review of the state of knowledge of the probabilistic theory of discrete runs, and by presenting an analytical solution for the probability distribution of the duration of the longest run in a given period of time (sample size).

2.2 Brief Review of the Theory of Discrete Runs

In the past, distribution theory of discrete runs for the independent case and for some simple dependence cases has been limited mainly to considering the number of runs, the run length and the first one or two moments of the run-sum for fixed truncation levels. A summary of the present state of knowledge is given by Saldarriaga and Yevjevich (1970), who also obtained an approximation to the distribution of the run-length for the first-order autoregressive (Markov) linear model of dependence by integrating, in an approximate way, the probability distribution function presented in a power-series expansion form.

The distribution of the run-sum is more difficult to obtain and only exact moments for the independent normal variable are given by Downer, Siddiqui and Yevjevich (1967), or in an approximate way for the dependent case by Heiny (1970). The results mentioned so far refer to distributions of runs, independent of the period of observation. The distribution of the length and size of the largest run to be observed in a sample size N is not available except for some simple cases. Millan and Yevjevich (1971) obtained experimentally the distribution for that case by using the Monte Carlo method to generate a large number of samples of given length. Approximate results given by Feller (1957) for the longest run of independent observations were used in checking the accuracy of this statistical experimental approach.

To follow Feller's nomenclature more easily, a success in a Bernoulli trial is identified with any value of the basic process which is below the truncation level, and a failure is identified with any value which is above it. The probability p of a success becomes then the probability of observations being smaller than the truncation level. Feller uses the theory of recurrent events to arrive to an approximation of the probability q_n of no success run of length r in N trials as

$$q_n \approx \frac{1-px}{(r+1+rx)} \cdot \frac{1}{x^{N+1}} \quad (2.2)$$

in which $q = 1 - p$ is the probability of a failure, and

$$x = 1 + qp^r + (r+1)(qp^r)^2 + \ldots$$

Because of the way in which Feller defines a success run, the probability of Eq. 2.2 is equivalent to probability of the longest run in N trials being less than or equal to $r - 1$.

David and Barton (1962) presented a solution for the probability of the longest run of successes in N independent Bernoulli trials conditioned on the occurrence of r_1 successes. Though their solution is original, the problem was previously solved by Whitworth (1896). The solution is based on combinatorial analysis. If M = length of longest run of successes in N trials, r_2 the number of failures, and r_1 the number of successes, the cumulative probability distribution of the random variable M is given by

$$P_N[M \leqslant m | R_1 = r_1]$$

$$= \frac{1}{\binom{N}{r_2}} \sum_{i=0}^{a} (-1)^i \binom{r_2+1}{i} \binom{N-i(m+1)}{r_2} \quad (2.3)$$

in which

$$a = \min \left\{ r_2+1, \left[\frac{N-r_2}{m+1}\right] \right\}$$

and

$$N+1 - r_2 \geqslant m+1 \geqslant \left[\frac{N+r_2+1}{r_2+1}\right]$$

Several existing approximations to Eq. 2.1 are based on property that, when the expanded series is terminated, the absolute error of truncation is less than the first term omitted. Accordingly, Eq. 23. can be rewritten as

$$P[M \leqslant m \mid R_1 = r_1] = 1 - (r_2+1)\frac{r_1^{m+1}}{N^{m+1}} \quad (2.4)$$

$$+ (r_2+1)^2 \frac{r_1^{2m+2}}{N^{2m+2}} - (r_2+1)^3 \frac{r_1^{3m+3}}{N^{3m+3}} + \ldots$$

in which $r_1 + r_2 = N$. According to David (1962), the first three terms of this series give a reasonable accuracy, and this form is suitable for calculation purposes except perhaps for m small and r_1 and r_2 large.

The 100 ϵ percent point of the distribution of Eq. 2.3, at the upper or right-hand tail, with ϵ a small number, is given approximately by

$$m = \frac{\log[-\log(1-\epsilon)] + \log(r_2+1)}{\log N - \log r_1} \quad (2.5)$$

A further approximation is

$$P_N[M \leqslant m \mid R_1 = r_1] \approx \mathrm{Exp}\left[-(r_2+1)\frac{r_1^{m+1}}{N^{m+1}}\right] \quad (2.6)$$

David recommends Eq. 2.6 for large m and for $N \geqq 20$.

2.3 An Analytical Solution for Probabilities of Droughts

This section presents the search for an analytical solution for the probability distribution of the longest negative-run in a discrete sample of size N. If a success occurs when an observation is smaller than the truncation level, and a failure when the observation is greater than the truncation level, the problem is reduced to computing the distributions for dependent or independent Bernoulli trials.

The exact solutions for the case of independent Bernoulli trials and for the case of dependent Bernoulli trials, with their dependence expressed by an irreducible Markov chain with two ergodic states, are first discussed. The solution obtained for this dependent case is also presented as a good approximation for the case in which the basic process follows the first-order linear autoregressive model of dependence.

Independent Bernoulli Trials. The distribution of the conditioned longest run-length to occur in a sample of size N is given by David (1962) as summarized in Section 2.2 of this chapter. To obtain the marginal distribution of m it is necessary only to notice that

$$P_N[M \leq m] = \sum_{r_1=0}^{N} P_N[M \leq m \mid R_1 = r_1] \cdot P_N[R_1 = r_1] \tag{2.7}$$

in which $P_N(R_1 = r_1)$ is the probability of having r_1 successes in N independent Bernoulli trials. This last probability distribution is the well known binomial distribution given as

$$P_N[R_1 = r_1] = \binom{N}{r_1} p^{r_1}(1-p)^{N-r_1} \quad , \qquad \text{for } r_1 = 0, 1, \ldots, N \quad , \tag{2.8}$$

in which p is the probability of a success.

Equation 2.7 then becomes

$$P_N[M \leq m] = \sum_{r_2=0}^{N} \frac{1}{\binom{N}{r_2}} \left\{ \sum_{t=0}^{a} (-1)^t \binom{r_2+1}{t} \binom{N-t(m+1)}{r_2} \right\} \binom{N}{r_1} p^{r_1} (1-p)^{N-r_1} \quad ,$$

which simplifies to

$$P_N[M \leq m] = \sum_{r_1=0}^{N} p^{r_1} (1-p)^{N-r_1} \left\{ \sum_{t=0}^{a} (-1)^t \binom{r_2+1}{t} \binom{N-t(m+1)}{r_2} \right\} \tag{2.9}$$

in which a has the same meaning as in Eq. 2.3, while m is subject to the same limits.

Dependent Bernoulli Trials. The exact distribution of the longest run of successes or failures in N trials for the case of dependent Bernoulli trials is not available for any case of dependence. This section is concerned with obtaining this distribution exactly for the case in which the result of each trial is dependent only on the outcome of the previous trial, i.e. they form an irreducible Markov chain with two ergodic states whose transition probabilities are given by

$$T = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{bmatrix} p_{11} & 1-p_{11} \\ p_{10} & 1-p_{10} \end{bmatrix} \end{array} \tag{2.10}$$

Here, state 1 means a success and state 0 means a failure. Clearly, $p_{11} = p_{10} = p$ for the independent case.

Following Gabriel (1959) it is possible to obtain the joint probability of having S success and C changes in a sequence of N trials. Since this development is basic for the understanding of the approach used in computing the probability distribution of the longest run, it is explained here in detail.

If S success as occur in N trials there will be a number of changes from success on one trial (including the initial trial) to failure on the next trial, and vice versa. It can be easily shown that if a denotes the number of changes from failure to success, b the number of changes from success to failure, and C the total number of these changes, then for C even, a = b = C/2 and for C odd, then b = a + 1, if X_o is a success, and b = a - 1, if X_o is a failure, where X_o is the outcome of the trial previous to the first one.

Lets consider the case of a success at the initial trial, then S successes with C changes will involve b changes from success to failure a changes from failure to success, S - a successes following successes and N-S-b failures following failures. The probability of any one arrangement of S successes and N-S failures with C changes is given by

$$P_{cs} = (1-p_{11})^b \, p_{10}^a \, p_{11}^{S-a} \, (1-p_{10})^{N-S-b} \tag{2.11}$$

The joint probability of S successes and C changes is, therefore, given by the number of such arrangements times the value of P_{cs}. According to Gabriel (1959) any arrangement of N trials with S successes and C changes involves a number a of changes to success, which may occur before any a of the S successes, i.e. in any of $\binom{S}{a}$ different positions. Also b changes occur before failures, of which the first must occur before the first failure, and the rest may be arranged in any of $\binom{N-S-1}{b-1}$ different ways. The total number of changes is, therefore, given by $\binom{S}{a}\binom{N-S-1}{b-1}$, and the probability of S successes with C changes following a successful initial trial is

$$\Pr\left\{S,C|N,\ X_o = 1\right\} = \binom{S}{a}\binom{N-S-1}{b-1} p_{11}^{s}\ (1-p_{10})^{N-S} \left[\frac{1-p_{11}}{1-p_{10}}\right]^{b} \left[\frac{p_{10}}{p_{11}}\right]^{a} \quad (2.12)$$

The number of changes C_1 may be any number between 1 and $N + 1/2 - |2S - N + 1/2|$, except if $S = N$ in which case $C_1 = 0$. In a similar manner for the case of failure in the initial trial

$$\Pr\left\{S,C|N,\ X_o = 0\right\} = \binom{S-1}{b-1}\binom{N-S}{a}\ p_{11}^{S}\ (1-p_{10})^{N-S}\left[\frac{1-p_{11}}{1-p_{10}}\right]^{a}\left[\frac{p_{10}}{p_{11}}\right]^{b} \quad (2.13)$$

The number of changes may be any number between 1 and $N + 1/2 - |2S - 1/2 - N|$, except for $S = 0$ in which case $C_2 = 0$.

From the joint distribution of S and C the cumulative probability distribution of the longest run-length may be derived.

Let us write this joint distribution by

$$P_N[M \leqslant m|X_o = 1] = \sum_{S=0}^{N}\sum_{C=1}^{C_1} P_N[M \leqslant m \mid S,C,X_o = 1] \cdot P_N\ [S,C \mid X_o = 1] , \quad (2.14)$$

and

$$P_N[M \leqslant m \mid X_o = 0] = \sum_{S=0}^{N}\sum_{C=1}^{C_2} P_N[M \leqslant m \mid S,C,X_o = 0] \cdot P_N[S,C \mid X_o = 0] \quad (2.15)$$

in which the subscript N in P_N means P in N trials and C_1 and C_2 are the maximum numbers of changes as given before.

The unconditioned probability, i.e. independent of the initial state, can be obtained as

$$P_N[M \leqslant m] = P_N[M \leqslant m \mid X_o = 1] \cdot P[X_o = 1] + P_N[M \leqslant m \mid X_o = 0] \quad P[X_o = 0] \quad (2.16)$$

in which $P[X_o=1]$ and $P[X_o=0]$ are the probabilities of success and failure at any trial, respectively.

It remains to find the two probabilities, $P_N[M \leqslant m \mid S,C,X_o=1]$ and $P_N[M \leqslant m \mid S,C,X_o=0]$ in order to complete the analytical derivations.

The direct evaluation of these distributions is cumbersome but there is a way to evaluate them which is based on previous developments by David (1962) and Whitworth (1896), who obtained the probability distribution of the largest interval of a line of s elements divided into e intervals. In order to obtain this probability, the authors first obtained the number of ways in which s elements can be arranged into "e" intervals, each of which contains at least one element and the largest of which contains m or less elements. This number is denoted by

$$L\ (s,m,e) = \sum_{i=0}^{a} (-1)^{i} \binom{e}{i} \binom{s-mi-1}{e-1} \quad (2.17)$$

in which

$$a = \min\left\{e,\ [\frac{s-e}{m}]\right\}$$

and

$$s - e + 1 \geqslant m \geqslant [\frac{s+c-1}{e}]$$

The next step relates the number of segments e to the number of success runs, and the number of success runs to the number of changes in N trials having S successes. For the case in which $X_o = 1$ it can be easily verified that for given C changes there is either a or a+1 runs, so the total number of arrangements that can be obtained in such a way that the longest run of successes is less or equal to m in N trials, given, S, C, $X_o = 1$, is

$$N_{sc1} = L\ (S,m,a) + L(S,m,a+1) \quad (2.18)$$

It can be also verified given that $X_o = 1$, the total number of failure runs is fixed and the number of ways in which they can be arranged is $\binom{S-1}{a-1}$, so that

$$P_N[M \leqslant m \mid S,C,X_o = 1]$$

$$= \frac{L(S,m,a)\binom{S-1}{a-1} + L(S,m,a+1)\binom{S-1}{a-1}}{\binom{S-1}{a-1}\binom{S-1}{a-1} + \binom{S-1}{a}\binom{S-1}{a-1}}$$

$$= \frac{L(S,m,e) + L(S,m,e+1)}{\binom{S-1}{a-1} + \binom{S-1}{a}} \qquad (2.19)$$

In the same way, given that $X_o = 0$,

$$N_{sco} = L(S,m,a) \qquad (2.20)$$

$$P[M \leqslant m \mid S,C,X_o = 0] = \frac{L(S,m,e)}{\binom{S-1}{a-1}} \quad , (2.21)$$

Substituting the values of Eqs. 2.19 and 2.29 into Eqs. 2.14 and 2.15, respectively, the cumulative probability distribution function of the longest run of successes may be found. It is shown in Eq. 2.22.

$$P[M \leqslant m] = \left\{ \sum_{S=1}^{N} \sum_{C=1}^{C_1} \frac{L(S,m,a) + L(S,m,a+1)}{\binom{S-1}{a-1} + \binom{S-1}{a}} \binom{S}{a}\binom{N-S-1}{b-1} \left[\frac{1-p_{11}}{1-p_{10}}\right]^b \left[\frac{p_{10}}{p_{11}}\right]^a p_{11}^S (1-p_{10})^{N-S} \right\}$$

$$P[X_o=1] + \left\{ \sum_{S=0}^{N} \sum_{C=1}^{C_2} \frac{L(S,m,a)}{\binom{S-1}{a-1}} \binom{S-1}{b-1}\binom{N-S}{a} \left[\frac{1-p_{11}}{1-p_{10}}\right]^a \left[\frac{p_{10}}{p_{11}}\right]^b p_{11}^s (1-p_{10})^{N-S} \right\} P[X_o=0]$$

$$(2.22)$$

The results obtained are exact for the case in which the Markov chain property applies, namely that the probability of a success or a failure may be expressed only on the outcome of the previous event. It is important, however, to point out that the results of the Markov chain approximate fairly well some other time dependences patterns, in particular when the observations of the basic process follow the properties of Markovian processes. In this case, the time series of observations can be modeled by using the first-order linear autoregressive scheme of Eq. 2.1.

Since a success is defined as any time that $X_i < c_o$, and a failure as any time hat, $X_i > c_o$, then the Markov chain property applies whenever the following two equations are satisfied:

$$P[X_{i+1} < c_o \mid X_i > c_o, \ldots, X_{i-n} > c_o] = P[X_{i+1} < c_o \mid X_i > c_o] \qquad (2.23)$$

and

$$P[X_{i+1} > c_o \mid X_i > c_o, \ldots, X_{i-n} > c_o] = P[X_{i+1} > c_o \mid X_i > c_o] \quad . (2.24)$$

Actually, as shown by Heiny (1968), these equations are very well approximated when the basic process X_i can be represented by the first-order linear autoregressive scheme, so in this case

$$P[X_{i+1} < c_o \mid X_i > c_o \ldots, X_{i-n} > c_o] = P[X_{i+1} < c_o \mid X_i > c_o]\,[1 + 0(\rho^2)] \quad (2.25)$$

and

$$P[X_{i+1} > c_o \mid X_i > c_o, \ldots X_{i-n} > c_o] = P[X_{i+1} > c_o \mid X_i > c_o]\,[1 + 0(\rho^2)] \qquad (2.26)$$

with ρ the value of the first serial autocorrelation coefficient of X_i. A function of ρ, say $q(\rho)$, has $0(p^2)$ if $g(\rho)/\rho^2$ remains bounded as ρ tends toward zero. The practical significance of this limit is that the error involved in the approximation is of the order of ρ^2. For small values of ρ, say less than 0.4, the approximation is good as can be verified by numerical values given in the next section.

When the formulas developed for the Markov chain are applied to an autoregressive scheme, the probability p_{11} and p_{10} must be obtained for the autoregressive scheme. These probabilities are

$$p_{11} = P[X_{i+1} < c_o \mid X_i < c_o] = \frac{P[X_{i+1} < c_o,\ X_i < c_o]}{P[X_i < c_o]} \quad , (2.27)$$

and

$$p_{10} = P[X_{i+1} > c_o \mid X_i < c_o] = \frac{P[X_{i+1} > c_o,\ X_i < c_o]}{P[X_i < c_o]} \quad , (2.28)$$

in which $P[X_i < c_o]$ depends only on c and on the type of probability distribution of the process X_i.

TABLE 2.1

COMPARISON OF PROBABILITY DISTRIBUTION FUNCTIONS AS OBTAINED BY EQ. 2.16 AND THE MONTE CARLO METHOD

	m	From formula for P[M = m]	From 3000 samples generated from a Markov Chain	From 5000 samples generated from a Markov Chain	From 3000 samples generated from an Auto-regressive model	From formula for P[M ≤ m]	From 3000 samples generated from a Markov Chain	From 5000 samples generated from a Markov Chain	From 3000 samples generated from an Auto-regressive model
	1	.005025	.0078	.0070	.0055	.05025	.0078	.0070	.0055
	2	.110213	.1050	.1068	.1153	.115238	.1128	.1138	.1208
	3	.263430	.2748	.2674	.2584	.378668	.3876	.3812	.3742
	4	.253103	.2380	.2452	.2586	.631771	.6256	.6264	.6379
	5	.168046	.1736	.1714	.1763	.794817	.7992	.8000	.8142
	6	.095704	.0982	.0972	.0884	.895521	.8974	.8977	.9026
$\rho = 0.1$	7	.051006	.0476	.0486	.0487	.946527	.9450	.9464	.9513
	8	.026382	.0254	.0262	.0250	.972909	.9704	.9728	.9763
	9	.013451	.0128	.0128	.0108	.986360	.9832	.9854	.9871
	10	.006804	.0068	.0070	.0763	.943168	.9920	.9424	.9947
	11								
	12								
	1	.003958	.0049		.0055	.003958	.0049		.0055
	2	.082464	.0809		.0945	.086427	.0858		.1000
	3	.220445	.2289		.2286	.307372	.3147		.3286
	4	.243211	.2428		.2486	.550583	.5575		.5773
	5	.182354	.1766		.1805	.732437	.7341		.7579
	6	.115363	.1164		.1102	.848300	.8511		.8681
$\rho = 0.2$	7	.06749	.0643		.0574	.915795	.4154		.9255
	8	.038040	.0357		.0363	.953837	.9511		.9619
	9	.021040	.0206		.0168	.974880	.9717		.9788
	10	.011523	.0135		.0121	.986403	.9852		.9809
	11								
	12								
	1	.004265	.0061		.005	.004265	.0061		.005
	2	.075077	.0723		.0805	.079342	.0784		.0855
	3	.201957	.2061		.1926	.281294	.2846		.2782
	4	.233217	.2326		.2245	.514516	.5172		.5076
	5	.184518	.1849		.1855	.699034	.7021		.6932
	6	.122770	.1265		.1255	.821804	.8286		.8187
$\rho = 0.3$	7	.075228	.0642		.0684	.897032	.8974		.8876
	8	.044262	.0403		.0487	.941294	.9381		.9363
	9	.025506	.0258		.0234	.966800	.9140		.9547
	10	.014531	.0157		.0218	.981331	.9796		.9816
	11								
	12								

The joint probabilities $P[X_{i+1} < c_o, X_i < c_o]$ and $P[X_{i+1} > c_o, X_i < c_o]$ can be obtained from the bivariate probability distribution of X_i and X_{i+1}. They depend only on the correlation coefficient ρ between the variables, on the value c_o and on the type of distribution. Bivariate distribution functions cannot be evaluated explicitly, but they are widely tabulated. In particular, the bivariate normal distribution is published by the National Bureau of Standards (1959). The accuracy of Eq. 2.16 is tested in the next section.

2.4 Accuracy of the Solution Obtained

The validity of Eq. 2.16 and the accuracy of the approximation obtained, when applied to a process following the first-order linear autoregressive scheme, are tested by using the experimental or Monte Carlo method in generating a large number of samples of given length. Also, results obtained by Eq. 2.16 are compared with results obtained previously by Millan and Yevjevich (1971) in also using the Monte Carlo method. Table 2.1 shows the results obtained for the probability distribution of the longest negative run-length for the values of $N = 25$, $c_o = 0$ and $\rho(0.1, 0.2, 0.3)$, with column 1 showing the probability density function as obtained from by Eq. 2.16, column 2 and 3 showing the same function as obtained by the Monte Carlo method for 3000 and 5000 generated samples, respectively. The agreement between the two approximations of distribution is remarkably good. All deviations fall within the range that can be expected in these cases. A fast and simple check for each frequency obtained by generated samples can be implemented as follows. If n samples are generated, the expected number of samples in interval j is np_j, and the standard deviation is $\sqrt{np_j(1-p_j)}$, because the process can be represented by Bernoulli trials having the probability p_j of a success. The difference between the generated and the computed frequency should be, therefore, smaller than five standard deviations. This means $|p_j - p_j| \leqslant 5\sqrt{p_j(1-p_j)}/\sqrt{n}$. For all distributions presented in Table 2.1, the frequencies obtained by the Monte Carlo method are well within the prescribed limits, as witnessed by a selected group in Table 2.2.

Table 2.2 includes the biggest differences. It is noticed that the largest of them is about three standard deviations and occurred for the autoregressive model with $\rho = 0.3$. In general, the frequency graphs obtained by the Monte Carlo method for the Markov chain are indistinguishable from those obtained for the first-order linear autoregressive scheme. However, there is slight tendency for the Markov chain to approximate better the results of Eq. 2.16 for $\rho = 0.2$ and $\rho = 0.3$. Validity of Eq. 2.16 is verified by the results shown in Tables 2.1 and 2.2. Also the accuracy of approximation to the autoregressive scheme can be considered excellent for all practical purposes in case $\rho \leqslant 0.4$. Considering that the values of ρ for series of annual streamflow and precipitation are generally smaller than 0.5, the solution presented in this chapter can be applied to most relevant cases with sufficient confidence.

2.5 Extensions to Truncation Levels in Form of Trends

Probability distributions presented so far in this

TABLE 2.2

FREQUENCIES OBTAINED FROM GENERATED SAMPLES, WITH $n = 3000$

	p_j	σ	Δmax for the Markov chain	Δmax for the Markov model
$\rho = 0.1$	.253	0.008	.015	.005
	.168	0.007	.005	.008
	.0134	0.002	.0006	.0026
$\rho = 0.2$	.0674	0.004	.003	.010
$\rho = 0.3$	.0145	0.002	.0012	.0073

chapter have been developed considering a constant truncation level. In practice, however, it is important to consider the case of a trend in the truncation level because demands ordinarily increase with time. Let us consider a linear trend in the truncation level $c_i = \alpha + \beta i$, $i = 1,...,N$. It is evident that this scheme produces longer runs than in the case $c_i = \alpha$, and smaller runs than in the case $c_i = \alpha + \beta N$. In a similar way, if $c_i = 1/N \sum_{k=1}^{N} (\alpha + \beta k)$, or the average truncation over the period, there will be a tendency to produce shorter runs in the first half of the period and longer runs in the second half. For the case of a constant truncation it was said before that the location of the longest run is equally likely in any position over the period of N observations. Thus, if the trend in the truncation is replaced by a constant level equal to its average, the decrease of the average length of negative runs in the first half of the period should be compensated by an increase in the average length of long negative runs during the second half of the period. For mild trend slopes the properties of runs are close to properties of runs for the average truncation level of the trend, as demonstrated by using the Monte Carlo method.

Table 2.3 shows the results obtained by generating 1000 samples of size $N_1 = 20$ with X_i following an autoregressive first-order linear model with the parameter $\rho = 0.21$, for a value of βN equal to one half of the standard deviation of the basic process. The selection of α was made in such a way that the average constant truncation would represent a point in the basic process such that $P[X_i \leqslant c_o] = 0.5$. Another test was made for α such that $P[X_i \geqslant c_o] = 0.2$ and the value of βN one standard deviation. The differences between the experimentally obtained frequencies shown in Table 2.3 are all within the expected sampling variation.

Results from the Monte Carlo generation of samples are such that it can be safely concluded that, for mild slope truncation trends, with differences between the end points of the order of one standard deviation of the basic process, the frequency distribution of the longest negative run in a sample of size N can be obtained as an approximation by replacing its trend truncation by a constant truncation level equal to the trend's average value.

TABLE 2.3

COMPARISON OF FREQUENCY DISTRIBUTION OF THE LONGEST NEGATIVE RUN-LENGTH OBTAINED FOR A TIME VARIANT LINEAR TREND TRUNCATION AND A CONSTANT LEVEL EQUAL TO THE TREND AVERAGE

	$P[X_i \leq c_a] = 0.5$		$P[X_i \leq c_a] = 0.2$		
j	$c_i = \alpha + \beta i$	$c_i = c_a$	$c_i = \alpha + \beta i$ $\beta N = 0.5\ \sigma_x$	$c_i = \alpha + \beta i$ $\beta N = \sigma_x$	$c_i = c_a$
0	.013	.014	.036	.026	.038
1	.139	.141	.502	.493	.511
2	.255	.261	.330	.338	.331
3	.223	.237	.096	.105	.086
4	.150	.130	.032	.034	.031
5	.101	.092	.003	.002	.0
6	.063	-058	.0	.001	.0
7	.030	.024	.0	.0	
8	.012	.018	.001	.001	

CHAPTER III

BACKGROUND INFORMATION ON ECONOMIC MODELS

This chapter presents a concise description of the most important techniques employed in an economic analysis to model the complex interrelations between sectors of a regional economy.

In general, such models can be classified in the following categories: structural analysis of the existing economy, prediction of sectoral economic progress through projection of final demand requirements, development of a regional impact studies that permit to assess the impact of a determined policy, and economic planning to achieve a predetermined social objective with given resource constraints by means of optimization techniques. Any such model is concerned with fixing values to certain variables as a function of other variables. Among these variables some are concerned with structural relationships which link together the separate points of the regional economy, and these are called the endogenous variables, while other predetermined variables, called the exogenous variables, affect the endogenous variables but are not affected by them. In addition, there may be some endogenous lagged variables which have been predetermined in earlier periods but given at the moment the model is being applied.

When either simulating or planning the economy with these kinds of models, the first operation is to insert into the model the target or goal variables which ordinarily are determined by the political process. In the process of the realization of the target variables (either by maximization, minimization, or simply by obtaining the satisfactory levels) the model sets values to other variables. Richardson (1969) summarizes a model as

$$q = E[t,x,r],$$

with,
- q = the set of outcomes,
- E = endogenous variables, i.e. the structural relationships which determine correspondence between the independent variables and the set of outcomes,
- t = instrumental variables,
- x = independent uncontrollable variables, and,
- r = stochastic effects.

Here the exogenous variables are divided into controllable or instrumental variables and independent uncontrollable variables. The task of a planner is to use the instrumental variables to achieve the goal proposed.

3.1 Input-Output Fundamentals

Input-output or interindustry analysis has been only recently used for regional water resources planning, Lofting (1963) and (1968), Bargur (1964), Miernyk (1969), Gray (1970), Turner (1970), Schaake (1971) and some others.

The model, however, is not new; it was first proposed by W. Leontief in 1933. The input-output model as proposed by Leontief is based on the premise that it is possible to divide all productive activities in an economy into sectors whose interrelations can be meaningfully expressed in a set of simple input functions.

The bases for the model are given by the transaction table, an example of which is presented in Table 3.1 for an economy consisting of three sectors.

In Table 3.1 (i) corresponds to sector i, $i = 1,2,3,$
- x_{ij} is the amount of production of sector i bought by sector j in dollars,
- X_i, $i = 1,2,3,$ is the total output of sector i in dollars, and
- Y_i, $i = 1,2,3,$ is the final demand from dollars,
- I_i, $i = 1,2,3,$ is the total value of imports made by sector i,
- V_i, $i = 1,2,3,$ is the value added by sector i and it includes wages, profits, taxes and depreciation. The value added plus export minus imports is equivalent to the Gross National Product.

The final demand for sector i is composed of all deliveries by sector i not accounted for in the processing sectors. It is often subdivided into private consumption, consumption by local, state and federal government, inventory accumulation, gross private capital formation, and exports. The projections of final demands are usually made independently of the sector projections because, unlike those, final demands for sector i are practically independent of the final demands and outputs of other sectors. A system which considers final demand as an exogenous sector is considered open to final demands.

TABLE 3.1

INTERINDUSTRY TRANSACTIONS

Inputs	Outputs	Processing Sectors (1)	(2)	(3)	Final Demand	Cross Output
Producing sectors	(1)	x_{11}	x_{12}	x_{13}	Y_1	X_1
	(2)	x_{21}	x_{22}	x_{23}	Y_2	X_2
	(3)	x_{31}	x_{32}	x_{33}	Y_3	X_3
Payment sectors	Imports	I_1	I_2	I_3	-	-
	value added	V_1	V_2	V_3	-	-
Total		X_1	X_2	X_3	-	-

Since the value of production of each sector is equal to the value it pays for its input plus the value added in production, it should be noticed that the total of column j must be equal to the total of row i for i = j for the processing-producing sector in the table. This fact is important in the construction of the table. In brief, the transactions table is a display of all the origins, destinations, and corresponding amounts of all transactions made by the regional economy during a time period, ordinarily taken as one year.

The economic sectors are aggregations of economic activities in such a way that they present certain homogeneity in production techniques and required inputs. Criteria for aggregations are fully discussed in the literature. From the upper left-hand corner of the transaction table, or the section including only the producing and processing sectors, basic equations can be written expressing the fact that the total amount of production from a sector should be equal to the amount that sector sells to the other sectors in the economy plus the amount delivered to final demand. These equations are :

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} + Y_1 &= X_1 \\ x_{21} + x_{22} + x_{23} + Y_2 &= X_2 \\ x_{31} + x_{32} + x_{33} + Y_3 &= X_3 \end{aligned}\right\} \qquad (3.1)$$

From the set of Eq. 3.1 it is possible to build a table of direct coefficients, assuming output a linear function of input, as

$$a_{ij} = \frac{x_{ij}}{X_j}, \; i,j, = 1, 2, 3, \text{ and for } x_{ij} = a_{ij}X_j ,$$

and Eq. 3.1 becomes

$$\left.\begin{aligned} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 &= X_1 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + Y_2 &= X_2 \\ a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + Y_3 &= X_3 \end{aligned}\right\} \qquad (3.2)$$

These sets of linear equations in Eqs. 3.1 and 3.2 can be put into the matrix notation as

$$AX + Y = X, \text{ or } (I-A)X = Y \qquad (3.3)$$

The following assumptions are implicit in the system:

(1) Each commodity, or group of commodities, is supplied by a single industry or sector of production. This further implies (a) only one method is used for producing each group of commodities, and (b) each sector has only a single primary output.

(2) Inputs purchased by each sector are a function only of the level of outputs of that sector.

(3) Additive assumption which implies that the total effect of carrying on several types of production is the sum of the separate effects.

The so-called Leontief matrix (I-A) plays an important role in the input-output theory because the coefficients of its inverse are a measure of the total direct and indirect requirements from the sector j for each dollar of production delivery to final demand by the sector i. Solving for X in Eq. 3.3 then

$$X = (1\text{-}A)^{-1}\ Y \tag{3.4}$$

Equation 3.4 gives the required amount of total production from each sector in order to meet an exogenous set of final demands, Y.

The real contribution of interindustry economic models to solving problems of this study is that they permit one to obtain the total requirements, direct and indirect, from each sector in order to meet a given final demand. This is accomplished by exhibiting the dependence among the sectors built into a model. As an example, consider the chemical industry. In the event of a drought the requirements for water used directly in the process can be met without difficulty by reusing water or by other emergency actions. However, the degree to which this industry depends upon the organic products of traditional agriculture will either decrease production, or increase its cost by having to import substitutes.

3.2 Linear Programming Formulation and Dynamic Models

Although the static Leontief model represents a fairly good example of general equilibrium analysis, it has to operate within the restrictive assumptions mentioned in the previous section and its use is limited to single period projections. A general approach to the problem is that of programming.

The set of equalities given by Eq. 3.2 can be turned into inequalities representing the fact that they actually are constraints, upper or lower, in the process of production. Additionally, availability of primary factors such as land, water, capital, etc., can also impose a limit on the amount of production. A linear programming formulation of the model would be the selection of the production levels for the sectors X_j, i = 1, . . . ,n, in such a way as to optimize a linear objective function of the sector's-output and satisfy the interindustry and primary resource availability constraints. The objective function is ordinarily taken as maximization of total production, total income for the region, value added, or cost minimization.

The following set of equations, Eqs. 3.5 through 3.7 describe analytically the model presented above.

$$\text{Max} \sum_{j=1}^{n} c_j \bar{X}_j \tag{3.5}$$

subject to constraints

$$X_i - \sum_{j=1}^{n} a_{ij} X_j \geqslant Y_i\,, \quad i = 1,...n, \tag{3.6}$$

$$\sum_j f_{hj} X_j \leqslant \bar{F}_h\,, \quad h = n{+}1,...n{+}\ell\ , \tag{3.7}$$

and

$$X_i \geqslant 0, \qquad i = 1,...n\ ,$$

in which c_j values are linear coefficients of the variable, X_j, the objective function, f_{hj} values are linear coefficients representing the amount of resource h demanded in the production of one unit of sector j, and F_h is the total availability of resource h. The linear programming formulation relaxes the first assumption of the static model by introducing alternative ways of producing a given commodity (Gray 1970). The output of sector 1, for instance, can be produced by sectors 1 and 1′, each of which has its own combination of inputs and its own weight in the objective function.

The program would select the optimal productions from each sector. To introduce this new possibility it is necessary only to modify Eq. 3.6 as

$$X_1 + X_1' - \sum_{j=1}^{n} a_{ij} X_j - a_{i1}' X_1' \geqslant Y_1\ ,$$

and (3.8)

$$X_i - \sum_{j=1}^{n} a_{ij} X_j - a_{i1}' X_1' \geqslant Y_i,\ i = 2,...,n$$

Dynamic models have been discussed in the literature by Bargur (1969), Dorfman (1958), Wagner (1951), Miernyk (1970). These models permit projections over a time horizon T instead of just for a single year. This is done by including a set of capital constraints that control the growth of production, and by permitting inventory accumulation and production for consumption in later periods.

Although dynamic models were originally intended for the national economy, they have been successfully applied to regional problems by Bargur (1969) and Miernyk (1970). The dynamic model can be formulated by using the traditional non-substitution approach of Leontief or by using a linear programming framework. Because the second approach is more general, the explanation of two such models as presented by Dorfman (1958) and Wagner (1951) illustrate the use of dynamic models.

The main characteristic of dynamic models is the fact that capital investment requirements are no longer included in the final demand sector but are determined endogenously by the model through the capital coefficient matrices.

In the Dorfman model a new matrix B of elements b_{ij} is introduced accounting for the requirements of capital goods from sector i per unit of production in sector j. A new variable is added per sector. This is $S_i(t)$, or the amount of capital goods from sector i used or available for use by all the other sectors in the economy. The interindustry constraints can be now written as

$$X_i(t) \geqslant Y_i(t) + \sum_{j=1}^{n} a_{ij} X_j(t) + S_i(t) - S_i(t-1) \,, \quad (3.9)$$

with i = 1,...,n, and t = 1,...,T, which reflect the fact that a sector in a period t can produce for final demand, for sectors demand or for increasing the capital stocks.

The additional constraint

$$\sum_{j=1}^{n} b_{ij} X_j(t) \leqslant S_i(t) \,, \quad i = 1,\ldots,n \quad (3.10)$$

means that $S_i(t)$, or the actual stock of capital goods from sector i, must be equal or greater than its requirements from the other sectors.

For Wagner the elements of the matrix B are also capital requirements for industry i, not per unit of production as in Dorfman's model, but per additional unit of capacity in industry, j. New variables are included and represented by the following n-dimensional vectors:

$\underline{C}(i)$ = Initial capacity,
$\underline{\ell}(t)$ = level of capacity building,
$\underline{S}(t)$ = level of stockpiles at the end of period t,
$\underline{C}(t)$ = capacity vector.

In which the notation below the letter indicates vector, The production constraints are expressed as

$$\underline{X}(t) = A\,\underline{X}(t) + Y(t) + \underline{B}\,\ell(t) + \underline{S}(t) - \underline{S}(t-1) = 0 \quad (3.11)$$

or

$$(I-A)\,\underline{X}(t) - Y(t) - B\underline{\ell}(t) - S(t) + S(t-1) = 0 \quad (3.12)$$

with

$$\underline{C}(t) = \underline{C}(1) + \sum_{r=1}^{t-1} \underline{\ell}(r) \,, \quad (3.13)$$

$$\underline{X}(t) - \underline{C}(1) - \sum_{r=1}^{t-1} \underline{\ell}(r) \leqslant 0 \,, \quad (3.14)$$

and

$$Y(t),\ X(t),\ (t),\ S(t),\ C(t) \geqslant 0 \,. \quad (3.15)$$

Equations 3.13 and 3.14 imply that the production may not exceed the available capacity.

Equations 3.11, 3.13 and 3.14 determine the set of constraints in the Wagner model. In general, the model consists of, nT sets of constraints and 3nT decision variables, with nT more decision variables than in the Dorfman model because of the inclusion of stockpiles.

3.3 Recursive Programming

The dynamic model gives an optimal solution. Since the program is solved simultaneously for the total time horizon, it is equivalent to perfect knowledge of the future conditions of the economy, i.e., final demands, prices, etc. When resource constraints are introduced, this will imply that future availability of resources is known. One possible way to model the economy without requiring the knowledge of future availability of resources is to break the time horizon of the problem into a set of sub-problems. For each period, the outputs of the first will become the inputs to the second. This also permits the possibility of technological change and the accommodation of production process for different conditions in resources supply and external trade.

The method of recursive programming can best be summarized as a sequence of mathematical programs which are recursively related. Some of the variables of a given program in the sequence are partly a function of the values of variables in the preceding program. In addition to this internal feedback, the model can utilize information on the effect of external action (external feedback), Day (1963).

The dynamic input-output model has a series of characteristics which make it good for a recursive programming treatment. These characteristics are:

(1) The set of feasible alternatives currently available to decision makers for the level of production or investment, depends on the sequence of decisions already executed.

(2) The choice among these possible acts is considered within a time horizon that is short relative to the economic processes as a whole.

(3) The decision problem is, in general, reformulated and solved at the beginning of each decision period.

(4) The information incorporated into the decision making procedure or "decision operator" does not, in general, incorporate an exact or complete knowledge of the structure connecting current decisions with other relevant variables in the system that affect decisions in succeeding time periods.

The analytical structure can be formulated as

$$\text{Max } [F(X(t)] \ , \tag{3.16}$$

subjected to

$$A(t)\,X(t) \leqslant B(t)$$
$$X(t) \geqslant 0 \ , \tag{3.17}$$

where F( ) is the objective function,
X(t) is the vector of activities,
A(t) is the matrix of technological coefficients, and
B(t) is the right-hand side vector or constraints.

In recursive programming any of the three, F( ), A(t) or B(t), can be a function of the values X(t-1), X(t-2),..., endogenously determined, or of any other exogenous factor.

3.4 Multiregional Models

The models discussed so far pertain only to a single national or regional economy. Exports and imports are all lumped in one sector regardless of their destinations and origins, and production sectors are not discriminated according to their geographical location. This approach can work adequately when the region in consideration is relatively closed, i.e. is self-sufficient and does not depend too much on imports and exports. For analysis of relatively open regions a table of this sort is not sufficient. Considerations must be given to the destination of exports and to the economy of the regions which are doing the importing; therefore, a multiregional model may be required.

The preparation of a multiregional model is, however, extremely complicated because data on origin and destination by industries are not always available, and trade patterns between regions are not rigid. Several simplifications have been made to cope with these problems, some of which are presented here. One solution is to identify inputs and outputs by region instead of by sector, and to concentrate one the interregional trade patterns as Henderson (1961).

Another approach is that of Chenery and Clark (1953) who assumed that each industry would use the same proportion of domestic and imported goods in production. Leontief (1953) developed a Balanced Regional Growth Model in which industries in an economy are classified into three groups: national, regional and local. Distribution coefficients indicate the proportion of each commodity produced by each region. This model can show both the outputs of regional goods and the outputs of national goods, but does not show the origins or destinations of imports and exports.

Leontief and Strout (1961) presented still another model in which some structural equations are presented to supplement the input-output method in the estimation of interregional flow of goods and services. None of these methods, however, can do anything about the rigidity in trade patterns. To accomplish this, a linear programming formulation must be made which permits the selection of optimal interregional trade patterns in such a way that they respond to established demands and resources constraints, and they maximize a pre-established goal. Formulations of this type have been made by Bargur (1969), Isard (1951), Stevens (1958), and Gray (1970). A detailed analysis of these models is not done here since the methodology presented in this study is developed only for a single region. It is important to point out, however, that given adequate data and computer capacity, there is no obstacle of a conceptual nature that does not permit the application of the model presented here to a multiregional economy.

CHAPTER IV

FORMULATION OF A MODEL FOR EVALUATING DROUGHT IMPACT

Having as basis for the analysis the statements made in Chapter I, it is possible now to formulate a model that can capture most of the important features of drought impact and at the same time can reduce the shortcomings and rigidities to a minimum.

The first part of the model is concerned with the consistent projections of total gross outputs and investment levels for a region over a period of time considered to be the planning horizon. The planning horizon is the maximum period over which the projections can have some meaning and at the same time have the opportunity to experience sufficiently severe droughts. These projections are subject to constraints in water availability, as shown later in the Chapter. The second part of the model is concerned with the estimation of losses from drought.

In formulating the first part of the model, a two-step approach is followed. The first step makes a general formulation within the framework of a multiperiod input-output linear programming model. After the limitations of this approach are discussed, the second model is formulated, with simplifications made when necessary and having as a main characteristic a recursive solution for each period in time and the introduction of alternative activities with the purpose of relaxing some of the restrictive assumptions of input-output models.

4.1 Formulation of a Multiperiod Dynamic Model

The formulation of a general model which includes both supply and demand and which can serve the purposes of this study is based on dynamic input-output models of regional growth. The model selects an optimal path of investments and activity outputs, during a time period T, for selected sectors in the economy for a region. The solution is optimal in the sense that it maximizes an objective function. This function for the case of drought impact may be the regional income which is a function of the total gross output of each sector.

At this point it is convenient to introduce the nomenclature:

$X_j(t)$, j = i...,n, is the total gross output of the sector j in year t, $t = 1,\ldots,T$,

$Y_i(t)$, i = 1,...,n, is the final demand for year t, $t = 1,\ldots,T$,

$S_i(t)$, i = 1,...,n, is the total capital stock of goods, i, available for the economy at the beginning of period t, $t = 1,\ldots,T$,

$g_{ij}[X(t)]$ is a function indicating the amount of production from sector i in time t required to produce $X_j(t)$ of sector j, and

$g'_{ij}[X(t)]$ is a function indicating the capital stock of goods from sector i in time t required to produce $X_j(t)$ units of sector j.

From now on, the subindex t is assumed to vary as $t = 1,\ldots,T$ and the subindex i as $i = 1,\ldots,n$, unless it is otherwise specified. The general problem is to maximize

$$\sum_{t=1}^{T} f_t\ [\underline{X}(t), \underline{S}(t)],$$

Subject to the production capital and water constraints.

Production Constraints. This set of constraints reflects the fact that the total output of industry i in year t can be used for consumption in the current year t, $Y_i(t)$, for current production in the period t,

$$\sum_{j=1}^{n} g_{ij}[(X_j(t))]$$

making the assumption that the output of industry i needed in the production of industry j, is a function only of the output of industry j, and for the net addition to the stock of capital good $S_i(t+1)$ - $S_i(t)$. The production constraint can be written now as

$$\sum_{j=1}^{n} g_{ij}[X_j(t)] + S_i(t+1) - S_i(t) + Y_i(t) \leqslant X_i(t)\ , \tag{4.1}$$

This can be rewritten as

$$X_i(t) - \sum_{j=1}^{n} g_{ij}[X_j(t)] - S_i(t+1) + S_i(t) \geqslant Y_i(t) \tag{4.2}$$

Capital Constraints. These constraints are

$$\sum_{j=1}^{n} g'_{ij}\ [X_j(t)] \leqslant S_i(t+1), \tag{4.3}$$

which imply that excess capacity can be built, and

$$S(t+1) \leqslant (1+\alpha) \sum_{t=1}^{n} g'_{ij} [X_j(t-1)] \qquad (4.4)$$

which implies that investment in a sector cannot jump from a year to the next.

Water Constraints. These constraints point out that the continuity in water flow must be met and that the possibility of water storage in the region must be accepted. If water use and water supply can be assumed to be concentrated at one point, then a nodal type representation can illustrate the mass balance. Figure 4.1 shows this for year t. The terminology in Fig. 4.1 is as follows:

- $I(t)$ the amount of water entering the region from the upper basins,
- $F(t)$ the total fresh water originated within the region,
- $S_t(t)$ the storage in the region at the beginning of time t,
- $\overline{G}(t)$ the groundwater maximum withdrawals,
- $G_a(t)$ the assigned groundwater with a particular use, for instance for rural domestic water supply,
- $G_w(t)$ the unassigned groundwater,
- $M(t)$ the imports of water,
- $W(t)$ the maximum surface fresh water withdrawals,
- $E(t)$ the exports of water,
- $D_f(t)$ the demand for fresh water,
- $C_f(t)$ the consumption of fresh water,
- $C_g(t)$ the consumption of ground water,
- $D_w(t)$ the demand for waste water,
- $C_w(t)$ the consumption of waste water,
- $O(t)$ the outflow to other basins,
- $Q(t)$ the minimum water flow requirements within the region,
- $A(t)$ the reservoir releases in period t, and
- $L_o(t)$ the minimum deliveries to downstream users.

The scheme of Fig. 4.1 presents a general picture of all water availabilities in the region. If the region consists of more than one major basin and use cannot be assumed concentrated in one point, then it is necessary to know the geographical distribution of the economic activity inside the basins and for each one to construct a scheme like that shown in Fig. 4.1. The particular regional conditions dictate the composition of the water mass balance. Meanwhile let us

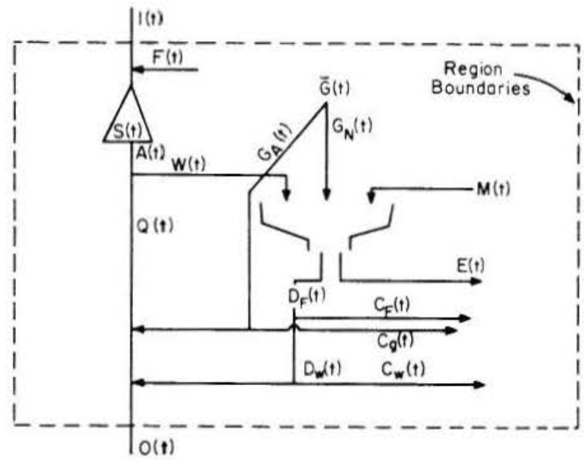

Fig. 4.1 General scheme representing water supplies and uses in a region.

consider the node shown in Fig. 4.1 or a combination of similar nodes in the general case.

The demand for fresh water should be expressed as a function of the total gross outputs of all water using sectors in the region. A function of the type $Wu_j = f_j(X_j)$, for $j = 1,...,n$, should be defined in which Wu_j is the fresh water used by industry j at a level of production X_j.

The water consumption of each industry should be determined in order to identify the waste water available for reuse. If the knowledge of the industrial location allows it, waste water can be allocated for other users; in other words, some sectors can meet their demands first by using the waste water and thereafter by using the fresh water. In this case waste water demand functions can also be established as $V_j = fw_j(X_j)$.

In this way constraints can be set up such as the ground water constraints

$$G_a(t) + G_w(t) \leqslant \overline{G}(t) \ , \qquad (4.5)$$

fresh water demand constraints

$$G_w(t) + W(t) + M(t) \geqslant \sum_{j=1}^{n} f_j[X_j(t)] + E(t) \ , \quad (4.6)$$

waste water demand constraints

$$G_w(t) + W(t) + M(t) - C_f(t) \geqslant \sum_{j=1}^{n} f_{wj} [X_j(t)] \qquad (4.7)$$

low flow constraints

$$A(t) - W(t) \geqslant Q(t) \ , \qquad (4.8)$$

also downstream water rights

$$0(t) \geq L_o(t) \ , \qquad (4.9)$$

Combining Eqs. 4.5 and 4.6 produces

$$\sum_{j=1}^{n} f_j[X_j(t)] \leq W(t) + \overline{G}(t) - G_w(t) + M(t) - E(t). \qquad (4.10)$$

From Eq. 4.8

$$A(t) - W(t) \geq Q(t)$$

$$W(t) = A(t) - Q(t)$$

W(t) becomes the maximum surface fresh water availability. Introducing it into Eq. 4.10, then

$$\sum_{j=1}^{n} f_j[X_j(t)] \leq A(t) - K_c(t) \qquad (4.11)$$

with the notation

$$K_c(t) = - Q(t) + \overline{G}(t) - G_a(t) + M(t) - E(t).$$

Equation 4.11 does not discriminate between surface water and ground water supplies. When this discrimination is wanted, $G_n(t)$ should be made a decision variable and Eq. 4.5 should be used independently of Eq. 4.6.

The operation of the reservoir requires the following constraints: (a) the volume of water released during any period cannot exceed the volume of the reservoir at the beginning of the period plus the inflows into the reservoir during the period, given by

$$A(t) \leq S_t(t) + F(t) + I(t) \ . \qquad (4.12)$$

(b) the content of the reservoir at the beginning of any period cannot exceed the amount left over from the previous period, given by

$$S_t(t) \leq S_t(t\text{-}1) + F(t\text{-}1) + I(t\text{-}1) - A(t\text{-}1) \ . \qquad (4.13)$$

(c) the content of the reservoir at the end of any period cannot exceed the capacity of the reservoir, given by

$$S_t(t) + F(t) + I(t) - A(t) \leq \overline{S_t} \qquad (4.14)$$

or

$$S_t(t+1) \leq \overline{S_t}$$

In summary, it is necessary with the introduction of 5T constraints to take into consideration the water constraints including the storage and the ground water. The variables F and $\overline{S_t}$ are exogenous to the program, but St(t), Gn(t), and A(t) are decision variables.

For the waste water section of the model, Eq. 4.7 can be rewritten as

$$\sum_{j=1}^{n} f_{wj}[X_j(t)] \leq A(t) + K_c(t) - \sum_{j=1}^{n} K_j f_j[X_j(t)] \quad ,$$

or

$$\sum_{j=1}^{n} f_{wj}[X_j(t)] + \sum_{j=1}^{n} K_j f_j [X_j(t)] \leq A(t) + K_c(t) \ . \qquad (4.15)$$

The downstream commitments can be expressed by

$$A(t) - \sum_{j=1}^{n} K_j f_j [X_j(t)] - \sum_{j=1}^{n} K_j' f_{wj} [X_j(t)] \geq L_o(t), \qquad (4.16)$$

in which K_j is the consumptive use coefficient for fresh water for sector j, and K'_j is the consumptive use coefficient for waste water for sector j.

To summarize, the water constraints are reduced to 7T constraints:

- T constraints expressing ground water constraints of Eq. 4.5,
- T constraints expressing the fresh water demands of Eq. 4.11,
- 3T constraints modeling the storage operation of Eqs. 4.12, 4.13, and 4.14.
- T constraints representing the waste water demands, and
- T constraints representing the downstream water rights.

The foregoing discussion of water constraints permits the modeling of the total water balance in the basin, but involves the postulation of several assumptions and the computation of several production functions and use coefficients. At this point it is important to make a summary of these assumptions and relationships:

(1) The water consumption is assumed to be centralized at a single point; in other words, all sectors are assumed to withdraw from a common pool with the exception of the assigned water and the waste water demands. This assumption is necessary in

any modeling of this nature, because it is very complicated to obtain exact distributions of demands per sector and in space. The practical significance of this assumption is that water rights are not considered, and water is allocated to the most efficient users. In a planning regional macromodel like this one, this assumption is reasonable.

(2) The total storage in the basin is assumed to be concentrated at one point. When all major reservoirs are located in series along the main river stem this assumption does not present any problem operationally, Hall and Dracup (1971). In case the reservoirs are located in a different way, the assumption is still adequate, because the reservoir design is not a decision variable in this problem, and the assumption (1) assumes already concentrated demands at one point. It is important, however, to obtain an estimate of the total annual regulation capacity of all reservoirs in the region as the volume of an equivalent reservoir. The storage capacities of reservoirs for the within-the-year regulations are not included.

(3) Two production functions are postulated, one for fresh water, and the other for waste water. An additional number of assumptions must be made concerning the nature of these production functions which depend on the data available.

(4) Groundwater availabilities are considered to be the safe yield of aquifers in the region. Certain water demands, like rural domestic and some municipal domestic, can be assigned to ground water.

(5) The only variable left to be defined is F, the total fresh water availability. This so-far elusive variable can be very well represented with the virgin or natural flow leaving the region. The U. S. Water Resources Council (1968) has computed the time series for this variable for major basins and subbasins in U.S.A. These series can be the basis for the computation of the drought events and their corresponding probabilities, as shown later.

The main interest of this study is the use of the model for investigating drought impacts on regional economy. While assuming the objective function as given above to be valid for an economic growth model, such as the model presented, the attention is given to water constraints in the model, its effects on the optimal solution, and the meaning of the results for public policy-making referring to water resources.

Programming Results and Limitations. The programming problem consists of selecting output levels, the production delivery to meet capital requirements within the region, water storage levels and reservoir operation for each of the T years considered. The values of these decision variables must meet the constraints and maximize a preassigned objective function. A total of 2NT + 2T decision variables are required if only one activity is permitted per sector. If substitution is introduced and two different technologies are possible for each sector, one having the traditional water technology, and the other having a much more efficient water technology-the introduction of which will save water but will represent a larger capital investment, the model will have the choice of introducing the new technology or confronting a water deficit. The consideration of alternatives increases the number of decision variables by 2NT but will keep the same number of constraints, Chapter III.

The number of constraints is now T(2N + 7) in which NT are production constraints, NT are capital constraints, and 7T are water constraints.

The solution of the dynamic model for the planning period T can be found once a perfect knowledge of all exogenous variables is obtained or assumed for the whole period T. A formulation of this type, besides being computationally unfeasible because of the dimensions of the model, assumes that the regional economy can perform as if it knew when the drought was going to strike. This solution presents the optimal allocation of production and investment in accordance with the selected objective function, but it must be regarded as an ideal allocation rather than the one the regional economy is actually prepared to make. Because of inertia in investment, behavioral constraints and imperfect knowledge of resources and prices, the economy will instead follow a suboptimal path. Therefore, economic allocations must be made with a limited knowledge of the future, or at most to the next couple of years.

4.2 A Recursive Model

To overcome difficulties of the multiperiod model, the programming model can be solved for each point in time in a recursive fashion as suggested by Day (1963). By setting up the constraints for a new period based on the solution of the present period, a greater efficiency can be expected by reducing the dimensionality of the program. Besides, a more realistic path of outputs and investments is obtained. The formulation of the regional model as a recursive programming problem implies a series of assumptions and modifications to the multiperiod model. The formulation is first made for an arbitrary period t. It will then be followed by a flow chart showing the intertemporal relationships.

The problem is then to maximize the total income for the region in a period. This total income is expressed as a function of the total gross outputs from all sectors in the same period. Concerning the adequacy of the selected objective function, the following discussion is appropriate.

The objective function as presented in the model performs several important tasks. It is used to allocate total gross outputs of all sectors in such a way that a regional measure of performance is maximized. Indirectly and together with water constraints this function will also be a critical factor in water allocation to different sectors when the total water demands exceed the total water supplies. Furthermore, the differences between the constrained and unconstrained values of the objective function are used as one measure of the impact of drought in the regional economy, as explained in section 4.3 of this chapter. Of several objective functions proposed in similar models in the past, the following can be the best suited for the objectives of this study. The total value added is expressed as the sum of all payment rows except the imports row in the interindustry transaction table, and the regional income is expressed as the sum of the wages, profit and other income rows. The regional income is considered a better indicator of regional performance. Unlike the value added it does not include taxes paid and depreciation that can inflate the value of losses. There are some objections in the literature, Young and Gray (1971), to the use of value added or income per incremental unit of water supply as a measure of the value of water. The authors claim that the value (opportunity cost) of productive inputs other than water are ignored, leading to overestimation of water value. These objections should be valid to a certain extent in the case studied, if the model is used to justify expansions in water-using sectors or in water supplies that required long-term adjustments in the regional economy, based on the income per incremental unit of water supply at a given point in time. Fortunately, this is not the case in this analysis of drought impact. Losses are measured as differences from otherwise planned income over the time horizon, and adjustments are made only in the short term without involving a permanent rearrangement of other productive inputs. Furthermore, alternative activities can assure that rigidities of the model do not lead to a big overestimation of losses, or because of the same rigidities, losses underestimated for water using sectors when the model requires a smaller amount of fixed inputs than otherwise would be realistic. The objective function then become the maximization of

$$\sum_{j=1}^{n} F_j [X_j(t)] \ , \qquad (4.17)$$

subject to constraints.

The usual lower bound in final demands can be written as

$$X_i(t) - \sum_{j=1}^{n} g_{ij} [X_j(t)] - \Delta S_i(t) \geqslant Y_i(t) \quad , \qquad (4.18)$$

If alternative production activities are allowed, being represented by the vector $\overline{X}_{ai}(t)$ containing the alternative activities for sector i, and having m as the total number of activities, then Eq. 5.18 can be written as

$$X_i(t) + \overline{X}_{Ai}(t) - \sum_{j=1}^{m} g_{ij} [\overline{X}_j(t)] - \Delta S_i(t) \geqslant Y_i(t) \ . \qquad (4.19)$$

The alternative activities are important because they are one way of guaranteeing that the rigidities involved in the original input-output assumptions (no input process substitution, no technological change, and linearity in production functions) do not result in an overestimation of losses in the case of a drought. In principle these alternative activities can be classified into three groups for the purpose of this study. Group 1 is composed of all alternatives intended to obtain a better water application and water savings by reduction of conveyance losses. This group of alternatives reduces the losses at the cost of capital expenditures in the implementation of a new conveyance technology. Group 2 is composed of all alternatives that modify the original water production function to avoid unrealistic lower levels of production when faced with scarce water resources. Group 3 is concerned with all those activities that use the same water technology, but whose production function is modified so as to apply less water-using inputs. It is through the alternative activities that the adjustment mentioned in Chapter I is accomplished.

For a better explanation of the possible adjustments in sectors production let us consider the isoquant curve of Fig. 4.2 showing all possible combinations of resources, R_1 and R_2 giving the same level of output for a given sector. The curve D shows that it is possible, at least in theory, to maintain the same output level by substituting resources inputs, the relative prices of resources, however, may cause the occurrence of different profit and income for different combinations of inputs. In case that resource R_2 becomes scarce, it would still be possible to maintain the same level of production by using more of resource R_1, though it might be more expensive. Lets consider steam power production for example. When water becomes scarce, shortrun substitution can be made so to consume more coal and less water. In a similar way, R_2 may represent all water or water using inputs while R_1 may represent other inputs; the alternatives selected for the case study in Chapter V present a good illustration of these adjustments.

Fig. 4.2 Isoquant of production.

The behavioral constraints reflecting the inertia forces in the economy can be represented with upper and lower flexibility constraints in production changes as

$$X_i(t) \leq (1 + \overline{\beta}_i)\, X_i(t-1)\ , \qquad (4.20)$$

and

$$X_i(t) \geq (1 - \underline{\beta}_i)\, X_i(t-1)\ , \qquad (4.21)$$

in which $\overline{\beta}_i$ and $\underline{\beta}_i$ are coefficients which fix the maximum increases (decreases) in production; they can be related to elasticity in demand, Day (1963). The β coefficients control rapid changes of production that can affect the structure of prices and demand in the economy. Such coefficients are difficult to estimate. Upper bounds in production can also be established by fixing maximum growth rates in final demands fore each sector. In this case there are two sets of final demands or production constraints.

The following Eq. 4.22 represents the lower bounds, or the minimum deliveries to final demand in which $Y_i^L(t) = (L_i + 1)\ Y_i(t-1)$. L_i the minimum growth rate, and $Y_i(t-1)$ represents the actual final demands in the period (t-1). Equation 4.23 represents the maximum deliveries to final demand in which $Y_i^U = (U_i + 1)\ Y_i(t-1)$, and U_i is the upper bound in the growth rate for sector i as determined by the planner.

$$X_i(t) + \overline{X}_{Ai}(t) - \sum_{j=1}^{m} g_{ij}\ [X_j(t)] - \Delta S_i(t) \geq Y_i^L(t)\ , \qquad (4.22)$$

and

$$X_i(t) + \overline{X}_{Ai}(t) - \sum_{j=1}^{m} g_{ij}\ [X_j(t)] - \Delta S_i(t) \leq Y_i^U(t)\ . \qquad (4.23)$$

The capital constraints formulated for the multiperiod model can be made more explicit, now that alternative activities have been considered. Equation 4.3 may be written as

$$\sum_{j=1}^{m} g'_{ij}\ [\overline{X}_j(t)] - \Delta S_i(t) \leq S_i(t-1)\ . \qquad (4.24)$$

The introduction of the alternative activities is controlled by another capital constraint. This constraint guarantees that there will not be a sudden change in technology, but that the new technology will be introduced gradually and as a result of drought conditions which make it profitable for the various sectors to do so. The constraint will imply that the total amount of capital goods from sector i in year t in alternative activities should be less than a given fraction k of the total capital goods from sector i invested the previous year in the whole region. It is represented by

$$\sum_{j=n+1}^{m} g'_{ij}\ [X_j(t)] - \sum_{j=n+1}^{m} g'_{ij}\ [X_j(t-1)] \leq K \sum_{j=1}^{m} g'_{ij}\ [X_j(t-1)]\ . \qquad (4.25)$$

A new constraint is necessary to reflect the fact that, once a new technology has been introduced, to a certain extent its levels can not be decreased. In other words, there is no possibility of disinvestment. This equation is

$$\sum_{j=n+1}^{m} g'_{ij}\ [X_j(t)] \geq \sum_{j=n+1}^{m} g'_{ij}\ [X_j(t-1)]\ . \qquad (4.26)$$

This formulation can work when the new capital goods can be used by many sectors, as is the case of new irrigation technologies for the agricultural sectors. Whenever capital goods can not be shared among sectors, it is necessary to allow for one constraint per using sector.

Finally, another constraint is necessary in order to avoid the accumulation of excess capacity in the region. If the maximum excess capacity or inventory of capital goods from sector i is represented by a fraction (1 - Ex) of the total goods available, then it is necessary that the following constraints be met:

$$\sum_{j=1}^{m} g'_{ij}[X_j(t)] \geq [S_i(t-1) + \Delta S_i(t)] \quad Ex \,. \qquad (4.27)$$

The Water Constraints for the recursive model are fewer in number and simpler in form than for the multiperiod model, because reservoir contents are known at the end of each period. Thus,

$$G_a(t) + G_w(t) \leq \overline{G}(t) \,, \qquad (4.28)$$

and

$$\sum_{j=1}^{m} f_j[X_j(t)] \leq G_w(t) + M(t) + I(t) + F(t) + S_t(t) - S_t(t+1) \,, \qquad (4.29)$$

in which $S_t(t+1)$ is the reservoir storage at the end of period, t and constitutes the only new decision variable for the water section of the model. The constraint

$$S_t(t+1) \leq \text{Max. } S_t(t) \quad , \qquad (4.30)$$

with max $\overline{S}_t(t)$ the maximum storage capacity available in year t, insures that the variable $S_t(t+1)$ remains bounded.

The constraints regarding the waste water and minimum flows can be written in the same way:

$$\sum_{j=1}^{m} f_{wj}\,[X(t)] + \sum_{j=1}^{m} K_j\, f_j\, [X_j(t)] \leq G_w(t) + M(t) + I(t) + F(t) + S_t(t) - S_t(t+1) \,, \qquad (4.31)$$

and

$$F(t) - S_t(t+1) + S_t(t) - Q(t) - E(t) + I(t) + M(t) + G_w(t) - \sum_{j=1}^{m} K_j\, f_j\, [X_j(t)] - \sum_{j=1}^{m} K'_j\, f_{wj}\, [X(t)] \geq L_o(t) \,. \qquad (4.32)$$

4.3 Estimation of Losses

As indicated in Chapter I, the estimation of total losses from droughts to the regional economy is a difficult task, and there is no guarantee that any model can faithfully reproduce all the adjustments that the economy is in a position to make in a case of drought. Even considering the many restrictive assumptions that have to be imposed, no model can claim a perfect reproduction of reality with the exception of reality itself. With this initial warning in mind, some of the more important features of the regional economy are presented in this study within the restrictive assumptions presented in this chapter. An estimation of losses is also presented, based on the discussion in Chapter I and on the possibilities of the model.

As it was pointed out before, losses are measured as the time streams of differences with the otherwise possible values of the economic indicators used as reference projections. Economic indicators for the model are the values of total regional income and total gross outputs for each of the economic sectors being considered. Losses can be identified in several ways in the model such as:

(i) Reduction in total gross outputs by water using sectors because of the water shortage and according to the postulated water production functions.

(ii) Changes in total gross outputs in other sectors due to the adjustments in water using sectors. These changes can take place in two ways: reduction of total gross outputs because of reduced intersector demand accompanying this reduction in water using sectors, and because of reduction in final demands due to reduced income in water using sectors.

(iii) Reduction in final demand, reflecting inabilities to meet the lower bound constraint with the reduced level of output.

(iv) Overall change in value added or income for the region.

In order to compute these indicators, a consistent procedure is followed in the model. When water is not a constraint to the economy, the model will proceed to select the optimal levels of outputs, investment, deliveries to final demand and regional income (in a particular time period) and the reference projections for the whole time horizon in the case of unconstrained projections. When available water is not sufficient to meet the water constraints, it is necessary to adjust the total gross outputs so the constraint can be satisfied. This task is performed automatically by the optimization algorithm, which will reduce output levels in such a way that water will go in preference to the most profitable uses according to the objective function and water production functions. Meanwhile, the algorithm will also select alternative activities whenever it is profitable to include them in the solution.

The case may occur in which it is no longer possible to reduce the sector's total gross outputs without failing to meet the lower bound constraint in deliveries to final demand. In this case it is not possible to meet simultaneously the water constraint and the lower bound in deliveries to final demand for some sectors. The program cannot obtain a feasible solution unless the last constraint is adjusted downward for some sectors.

The device used to make these adjustments if the introduction of a new set of variables, $Z_i(t)$, to the left-hand side of the lower bound constraints. Equation 4.32 then becomes

$$X_i(t) + \overline{X}_{Ai}(t) - \sum_{j=1}^{m} g_{ij}\ [X_j(t)]$$
$$- \Delta S_i(t) + Z_i(t) \geqslant Y_i^L(t)\ . \qquad (4.33)$$

When the variable $Z_i(t)$ assumes a positive value, it is equivalent to having the value of $Y_i^L(t)$ reduced by the same amount. The introduction of these variables, (instead of just having the final demands as a decision variable), is convenient because they will give automatically the deviation from minimum deliveries to final demand. Also by using adequate coefficients for the Z's in the objective function (large negative values), it will be possible to guarantee that values of the Z's will enter into the solution only when all other possibilities have been exhausted. The order and pattern in which the Z's should be increased can be controlled by the type of functional form the Z's take in the objective function.

Figure 4.3, shows a pattern which makes it increasingly difficult to increase Z_i once it has been already attained a certain amount. This pattern also sets up a maximum value for Z_i which is the value at the minimum final demands. The broken line shows how this function can be piecewise linearized by dividing the range of the variable Z_i into the variables $Z_{1,i}$ and $Z_{2,i}$. This is done in such a way that $Z_{1,i} \leqslant A$, $Z_{2,i} \leqslant Y_i^L - A$, and $Z_i = Z_{1,i} + Z_{2,i}$. The convex scheme selected guarantees that the slope S_2 will be greater than the slope S_1. Detailed discussion for the conditions for a global optimum are given in the next section of this Chapter.

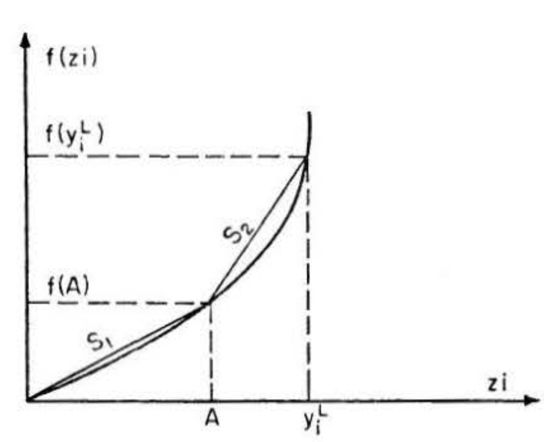

Fig. 4.3 Linearization of loss function.

The rationale behind this scheme is that after the sector's final demand has been reduced to a certain point further reduction will become increasingly difficult, because of behavioral and other restraints, and other sectors will be affected instead. The problem has been compensated for to a certain extent by the optimization itself. That will reduce the sector's final demand in a way that maximizes the income. The scheme which is intended to be simulated by the nonlinear penalty has the possibility of having different preferences for different sections or portions of the final demands. For instance, it is possible that the region is willing to give up exports for certain sectors in a given year. However, it won't so easily give up the local consumption, because producers may feel confident about keeping their export markets, and imports for substitution in local markets are not available at reasonable prices. The converse could be also true. It is also possible that the local community has different elasticities of demand for certain products, and in the process makes a change in the relative prices. An analysis of the relative price elasticities of demand can determine the extent of these changes, but it is difficult to manage prices in the model proposed here. Only indications of the approximate value of the parameters (S_1, S_2 and A) can be given based on a detailed analysis of the composition of final demands, their expected growth, and regional preferences.

The interaction in time is best explained by using the flow chart of Fig. 4.4.

For a period t two exogenous events have the responsibility of driving the economy, these are the final demand and the water availability. The production constraints for year t are endogenously affected by the inventory levels and production levels at

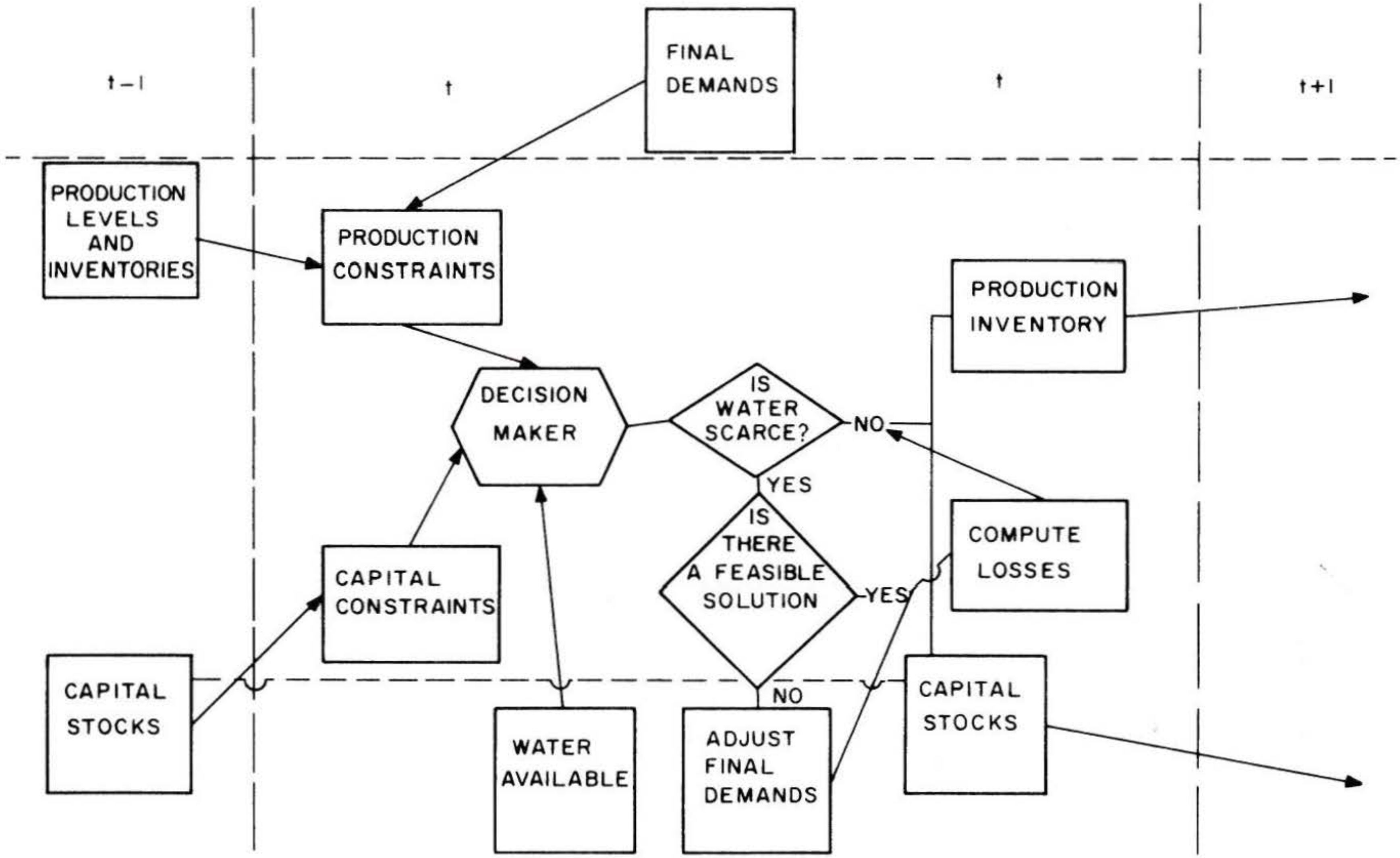

Fig. 4.4 Flow chart describing the time interation of the recursive model.

year (t - 1). Also, the capital constraints are determined by the existence of capital stocks, which in itself is a function of previous capital stocks, investment levels in period (t - 1), and a depreciation rate that can be exogenously fixed. In the previous presentation, inventories were not shown because they were considered together with capital, but the distinction can be made if necessary. Once the different variables of the programming problem have been updated, either by exogenous or endogenous means, the task of the program is to select optimal levels of production and investment.

Two alternative results are possible concerning the water constraints: they are either binding or not binding. If they are not binding, then the program proceeds to update levels of capital stock and inventories and to determine the constraints for the next period. When the water constraints are active and binding, there are two possible outcomes again: there is a feasible solution, or there is none. In the first case the difference between the objective function values for the constrained and the unconstrained cases gives a measure of income foregone because of scarce water resources. When there is no feasible solution, constraints must be relaxed in order to get one. This will imply reduction in deliveries to final demand and can be considered as a failure to accomplish the growth rates advocated in the plan. Since there is one alternative activity for each sector with a more advanced water technology, the program always has the possibility of turning to the new technology. However, this sometimes implies an investment and, therefore, the rate of introduction of the new technology is controlled by some capital constraints.

4.4 Special Features in Linear Programming

This section is concerned with two features of linear programming that are important in the formulation of the model if the functions of section 4.3 are linear. The first topic deals with the conditions for optimality required in the piecewise linearization of penalties for unmet final demands, and the second topic refers to an efficient use of the so-called "parametric programming" in such a way that total computation time is reduced, making feasible the realization of a large number of runs with the model.

In section 4.3, it was suggested that the penalty function should have the form shown in Fig. 4.3. If the problem is formulated in such a way as to minimize $-\sum C_jX_j + f(z)$ instead of to maximize $\sum^{n} C_jX_j - f(z)$, the penalty function becomes convex and causes the slopes in the piecewise linearization to be monotonically increasing, i.e., $S_2 > S_1$. Hence, if the objective function is to be minimized, the variable $Z_{1,i}$ is always utilized to its fullest extent before $Z_{2,i}$ enters the solution. Moreover, this convexity in the objective function will guarantee that a global minimum can be normally obtained using the simplex algorithm.

If the penalty function is not a convex function, the scheme presented here is not applicable, and a more general treatment, such as the ones presented in Beale (1968) and Maass and others (1962), is required. This method consists of expressing the value Z_i as a weighted sum of the values 0, A, and $Y_i(t)$, thus expressing it as

$$Z_i = 0\beta_o + A\,\beta_1 + Y_i^L(t)\,\beta_2 \qquad (4.34)$$

where the weights, β_o, β_1, and β_2 are required to be non-negative and to have as their sum unity. From an infinite number of combinations of β's that satisfy these restrictions, only those are acceptable for which no more than two of the weights are positive, and for which the two positive weights are adjacent. They correspond to the chords in Fig. 4.3 with the value of a chord at any point being a weighted average of values of its end points. Then

$$f(Z) = 0\beta_o + A\,S_1\,\beta_1 + [A\,S_1 + (Y_i^L - A)\,S_2]\,\beta_2 \qquad (4.35)$$

The conditions expressed above are met automatically when the functions are convex, otherwise a special procedure must be followed. For the case discussed here this method requires one more variable but less constraints than the original method presented in section 4.3.

The solution of the model over the time horizon T requires solving T linear programs that have a common objective function and a common matrix of coefficients, A, differing only on the right-hand side vector b. To solve the whole problem each time would be to waste information already available in the previous period. A more efficient method can be obtained by using recursively the primal simplex algorithm and the dual simplex algorithm as suggested by Takeuchi (1972).

The following notation and developments are taken from Hillier and Lieberman (1967). In matrix form, the linear programming problem is to maximize

$$X_o = C\underline{X} \qquad (4.36)$$

subject to $A\underline{X} \leqslant \underline{b}$, and $\underline{X} \geqslant 0$

After introducing a column of slack variables, $\underline{X}_s$, the constraints become

$$[A,I] \begin{bmatrix} \underline{X} \\ \underline{X}_s \end{bmatrix} = \underline{b}, \text{ and } \begin{bmatrix} \underline{X} \\ \underline{X}_s \end{bmatrix} \geqslant 0 \qquad (4.37)$$

n which I is the indentity matrix. A basic solution is a solution of m equations

$$[A,I] \begin{bmatrix} \underline{X} \\ \underline{X}_s \end{bmatrix} = \underline{b}\ , \qquad (4.38)$$

in which n of the elements of the vector $\begin{bmatrix} \underline{X} \\ X_s \end{bmatrix}$ the nonbasic variables, are set equal to zero. Eliminating these variables by equating them to zero leaves a set of m equations with m unknowns (the basic variables). This set of equations is denoted by

$$B\ \underline{X}_B = \underline{b}\ , \qquad (4.39)$$

in which equation the basic matrix B is an mxm matrix obtained by eliminating the columns corresponding to coefficients of non-basic variables from [A,I], the basic solution $\underline{X}_B$ can be found

$$B^{-1}\ B\ \underline{X}_B = B^{-1}\ \underline{b}$$

or

$$\underline{X}_B = B^{-1}\ \underline{b}\ . \qquad (4.40)$$

Usually the objective function is included in the tableau as row (0) in the following way. Maximize X_o subject to

$$X_o - C\underline{X} = 0$$

and

$$[A,I] \begin{bmatrix} \underline{X} \\ \underline{X}_s \end{bmatrix} = b \qquad (4.41)$$

This can be arranged so as to maximize X_o, subject to

$$\begin{bmatrix} 1 & -C & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} X_o \\ \underline{X} \\ \underline{X}_s \end{bmatrix} = \begin{bmatrix} 0 \\ \underline{b} \end{bmatrix}, \qquad (4.42)$$

By the same procedure, let B_0 be the expanded basic matrix

$$B_0 = \begin{bmatrix} 1 & -C_B \\ 0 & B \end{bmatrix}$$

$$\begin{bmatrix} X_o \\ \underline{X}_B \end{bmatrix} = B_o^{-1} \begin{bmatrix} 0 \\ b \end{bmatrix} \qquad (4.43)$$

$$B_0^{-1} \begin{bmatrix} 1 & -C & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} X_o \\ \underline{X} \\ X_s \end{bmatrix} = B_o^{-1} \begin{bmatrix} 0 \\ \underline{b} \end{bmatrix}$$

making the complete set of equations that would have been obtained by the simplex method in order to identify $\begin{bmatrix} X_o \\ \underline{X}_B \end{bmatrix}$,

$$B_0^{-1} = \begin{bmatrix} 1 & C_B\ B^{-1} \\ 0 & B^{-1} \end{bmatrix},\quad B_0^{-1} \begin{bmatrix} 1 & -C & 0 \\ 0 & A & I \end{bmatrix}$$

$$= \begin{bmatrix} 1 & C_B\ B^{-1}A - C & C_B\ B^{-1} \\ 0 & B^{-1}\ A & B^{-1} \end{bmatrix}$$

and the complete set of equations is

$$\begin{bmatrix} 1 & C_B\ B^{-1}\ A - C & C_B B^{-1} \\ 0 & B^{-1} A & B^{-1} \end{bmatrix} \begin{bmatrix} X_o \\ \underline{X} \\ \underline{X}_s \end{bmatrix}$$

$$= \begin{bmatrix} C_B B^{-1} B \\ B^{-1} b \end{bmatrix} \qquad (4.44)$$

From this it can be easily seen that if the only changing factor is the vector b, the left-hand side does not have to be computed each time, but only the right hand side. Changing the vector b can make the solution unfeasible, but it still is optimal. When the change of b leads to negative values in $B_0 b$, the solution is unfeasible. But since $C_B\ B^{-1}A - C$ and $C_B\ B^{-1}$ are non-negative because of the definition of optimality, the dual problem is feasible and one can proceed to solve it.

4.5 Limitations

The proposed methodology to evaluate the impact of drought on a regional economy is subject to a number of limitations imposed by assumptions implicit in modeling the economy. In the first place, the decision to make the final demands an exogenous variable will make the model vulnerable to unforeseen changes in the export markets and in general will force fixed patterns of trade for the region. This limitation could have been avoided to a certain extent with multiregional models, but the burden of data collection for their implementation made them unfeasible in this study.

Data availability also places restrictions on the type of functions to be used in representing different production functions and capital functions. In general, only linear relationships can be obtained with the consequent constant returns to scale implication. The fixed-proportion production functions of the Leontief type are avoided to a certain degree by introducing alternative ways of production.

When projections are made, the present production functions are assumed to hold in the future. Actually, technology changes with time, and there are some techniques available, Miernyk (1970), that permit future projections of production functions based on the assumption that average technology some time in the future will be equivalent to the technology used by some of the most efficient sectors at present.

Finally, limitations are imposed by assumptions concerning the patterns of water use distribution in the region, as it is presented in the relevant parts of section 4.1.

CHAPTER V

APPLICATION OF THE MODEL TO A CASE STUDY

This chapter relates to necessary data collection, assumptions made that permit the application of the model to a region, and the procedures proposed. The selection of region depended on data availability and on the present or envisioned threats that droughts can cause to that region's economy. Several assumptions which determine the type of relationships and the extension of the model are made explicit as soon as they are required in the development presented in this chapter.

5.1 Selection of the Region

Because of difficulties outlined in Chapter IV in obtaining all the economic data and information required for this type of models, and because the time and resources available for this study do not make the realization of a survey practical, it is necessary to rely on previous hydrologic and economic studies, and on published data, updating and adapting them to the particular region and conditions. Table 5.1 provides a start in selecting the region by presenting data available from a selected number of previous studies.

The particular objective of this study and the model characteristics permit a further selection screening of regions presented in Table 5.1. First, it is important that the time series of the total water available for the region can be obtained. When the region coincides with a major river basin or sub-basin and a time series of total flows is available at the outlet of the basin, this series then serving the purpose of total surface water availability will facilitate the analysis. Also, a river basin with most of its water supply from surface water can avoid unnecessary complications. The number of feasible regions is then reduced to those presented in Table 5.2.

A final condition basic to the study is the occurrence of droughts constituting a present or forecast threat to the economic development of the region. At this point, it must be recognized that there is probably no region that meets all requirements specified above at the present time. A compromise is necessary in selecting a basin that can best explain and illustrate the potentials of the method.

After a careful process of elimination, the Upper Main Stem Sub-basin of the Colorado River was chosen for investigation. The Upper Main Stem (U.M.S), with its map shown in Fig. 5.1, gives a best combination of the following factors:

(1) A detailed interindustry transaction table for the year 1960 and a good estimation of water use by sectors are available for the region, Udis (1968).

(2) Even though statistics on capital use by sectors are scarce, this is a problem faced by most of the other regions. Besides, the capital goods originated in the basin are served almost in their entirety by one sector (construction) for which approximate data can be adapted from the coefficients computed for the West by Bargur (1969).

(3) The question, "Is the capital sector in the U.M.S. big enough to justify the application of dynamic models?", can be answered satisfactorily after comparing the available statistics on gross private domestic investment from the U.M.S. with the nation as a whole and with West Virginia for which a detailed dynamic model was used successfully by Miernyk (1970). Gross private domestic investment at the national level is 10 percent of the total final demands, with a very small percentage of that originating outside the country. Gross investment in West Virginia is less than 6 percent of total final demands, and more than half of capital inputs originated outside the state. The application of the dynamic model to the West Virginia economy, however, introduced significant improvements over the static model. As Miernyk points out: "the relatively small magnitudes should not obscure the importance of capital transactions, however. Investment is a strategic variable in the economic development of a region as such deserves capital attention. Also in a few sectors capital sales account for a substantial proportion of total transactions". As a comparison, the gross private domestic investment for the U.M.S. is about 10 percent of total final demands, and only about 40 percent originates outside the region. The construction sector final demand is almost completely devoted to investment.

(4) Very little ground water used in the region, most of it by the rural domestic sector.

(5) The time series of virgin flows is available. The actual and projected exports to other basins can be also obtained from the Colorado State Engineer's Office, and there seems to be little apparent problem in the reuse of water.

(6) Concerning the possible impact of a drought on the water-oriented economy of the region, it must be emphasized that although the Colorado River is one of the most highly regulated rivers in the world

TABLE 5.1

AVAILABILITY OF REGIONAL DATA FOR SELECTED INPUT-OUTPUT STUDIES

Region	Base Year	No. of Sectors	Data On Capital Coef-fients	Water Coef-fients
California, Lofting III (1963)	1947	31	No	Yes
California, Lofting IV (1968)	1958	24	No	Yes
California, Davis (1968)	1958	15	No	Yes
West, 8 states, Davis (1968)	1963	15	No	Yes
California, Bargur (1969)	1963	19	Yes	Yes
West, Bargur (1969)	1963	19	Yes	Yes
California, Ireri (1970)	1958	26	No	Yes
Arizona, Ireri (1970)	1958	26	No	Yes
Texas, Canion (1968)	1958	30	No	Yes
Colorado River Basin - Gila, Udis (1967)	1960	36	*	Yes
Lower Main Stem of River Colorado	1960	30	*	Yes
Little Colorado	1960	25	*	Yes
Upper Main Stem of River Colorado	1960	31	*	Yes
San Juan Basin	1960	28	*	Yes
Green River	1960	22	*	Yes
West Virginia, Miernyk (1970)	1965	48	Yes	No
U.S., Polenske (50 states)	1963	78	*	No
Colorado, Smith (1971)	1965	-	No	No

*Means that data is available on gross private capital formation.

TABLE 5.2

DATA AVAILABLE ON SELECTED REGIONS

Name of basin	TGO	Inter-industry transac-tion	Capital sells	Water coefficients	Water availability	Percent of surface water use	Storage capacity
Gila	Y	Y	Y	Y	B	30%	H
Lower Main Stem	Y	Y	Y	Y	G	30%	H
Little Colorado	Y	Y	Y	Y	G	30%	-
Upper Main Stem	Y	Y	Y	Y	G	99%	L
San Juan Basin	Y	Y	Y	Y	G	99%	H
Green River	Y	Y	Y	Y	G	99%	H
Arkansas Valley	Y	1	N	N	B	40%	H
San Luis Valley	Y	1	N	N	G		-
Northeastern Colorado	Y	1	N	N	G	80%	H
Denver Metropolitan Area	Y	1	N	N	G	80%	H

TGO = total gross outputs, Y = available, N = not available, H = high, M = medium, L = low, G = good estimates can be made, B = bad, 1 = Colorado coefficients.

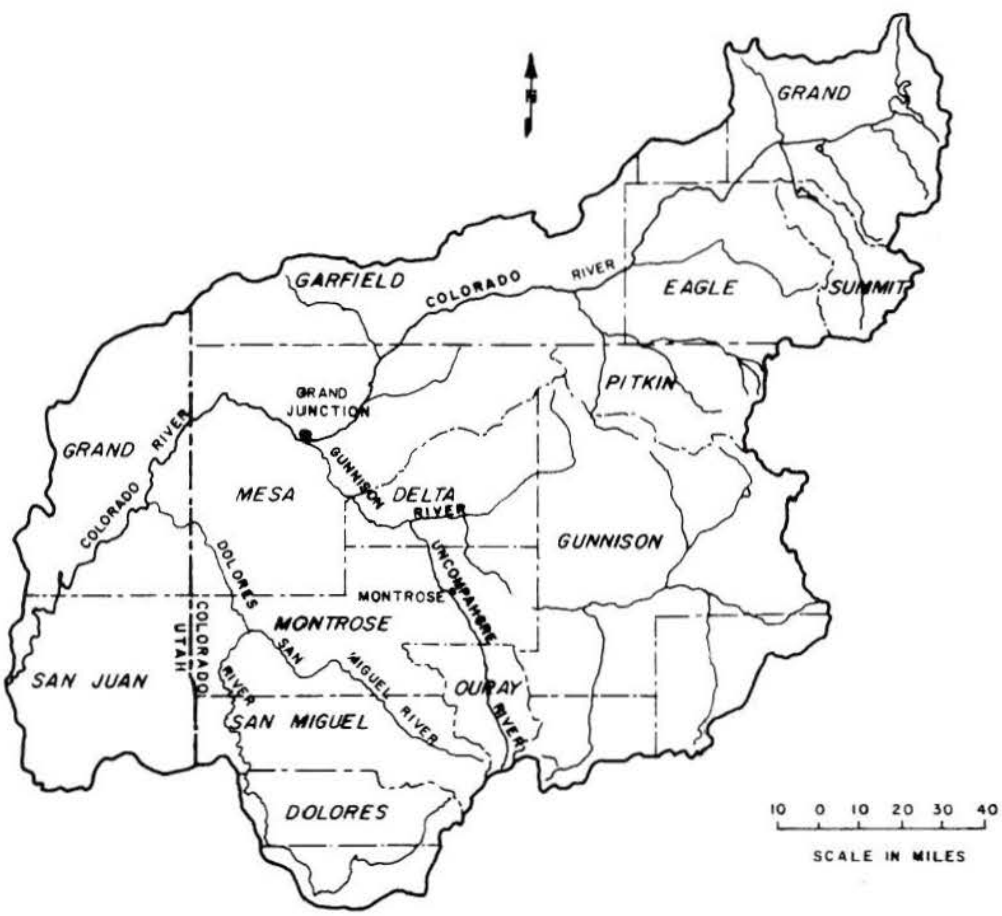

Fig. 5.1 Map of the upper main stem sub-basin of the Colorado River.

almost no regulation of significant size exists in the selected sub-basin. The high variability in annual flows represents a real threat to the sub-basin's economy, as can be inferred from the following quote from the "Nation's Water Resources" referring to the Upper Colorado Basin: "Foremost among the present water problems are the sustained periods of drought with consequent low safe water yields. No longer can the region anticipate full use of 6.7 billions of gallons per day (7.5 millions of acre feet per year) apportioned to it under the Colorado River Compact. Consumptive uses of Colorado River water in Colorado and New Mexico will approach their regional allotments under the Upper Colorado River Compact when authorized projects are constructed" Water Resources Council (1968).

5.2 The Upper Main Stem Basin

The Upper Main Stem Sub-basin of the Colorado River extends westward from the Continental Divide in Central Colorado. It covers about 26,000 square miles in Colorado and in Grand and San Juan counties in eastern Utah (See Fig. 5.1). The largest city in the sub-basin is Grand Junction, Colorado; other cities are Montrose, Glenwood Springs, Aspen, Fraser, and Dillon. The sub-basin has long been a center of mining activity, and in recent years uranium has been of particular importance. There are some projects for the future exploitation of the oil shale in the region, especially in Garfield County. Because of the excellent ski facilities, outdoor recreation during the winter season has become of economic importance. Irrigated agriculture is intensive in the sub-basin, and range livestock is by far the most important agricultural industry. According to Udis (1967), most of the agricultural establishments are small. In 1960 the population of the sub-basin was 128,079 and has since steadily increased. The percentage of urban population in the same year was around 34 percent, and the employment was distributed as: 29.83 percent in agriculture, 7.43 percent in mining, and 17.94 percent in services. A detailed description of the economic base and estimated future trends are given in Udis (1967), from which most of information presented here was taken.

5.3 Data for the Economic Projections

The basic economic data used for this example is taken from a study by Udis and others (1967) undertaken for the Federal Water Pollution Central Administration. The FWPA study used data from several published sources and carried out a survey of the basin economy for the year of 1960. In order to make the data from FWPA compatible with the model presented in this study, it was necessary to disaggregate and aggregate sectors in a way different than they were originally. Substantially new estimates were obtained based on published data and expert opinion as explained in Appendix A. Also, data on capital coefficients was adapted from a University of California study, Bargur (1969).

The available data did not permit establishing non-linear relationships for the interindustry model. The water production functions and, therefore, all equations developed in the previous chapter become linear. Also, projections are made from the year 1960 into the future, and all monetary measures of production are expressed in 1960 constant dollars.

(a) Interindustry Transactions and Sectors Aggregation - The interindustry transactions table available for the U.M.S. from Udis (1967) is not in a form that can be easily adapted for use in the proposed model. For one thing, the transactions table has a great deal of detail in the manufacture, trade, and service sectors, making it difficult to use the programming formulation because it excessively increases the dimensions of the problem. Also, it increases the burden of complementary data on capital formation and water use. On the other hand, the table includes forage in the livestock and dairy sectors, complicating the application of the scheme developed to allocate irrigation water shortages and the formulation of alternative activities for the livestock sector. Table 5.3 shows the new classification adopted and the related Udis (1960) and Bargur (1963) classifications. A major consideration in making the new classification was the fact that the larger water users, direct and indirect, should be kept in separate sectors. A new sector was introduced, the forage, based on detailed census data (1959), revision of working papers from Udis report (1967), and on Bureau of Reclamation Project reports (1965). Table 5.4 shows the final form of the transactions table ready to use for the model.

(b) Data on Capital Sectors - As was the case for interindustry transactions, the capital coefficients can be constructed either by using information available from published studies and statistics or by using direct survey. The capital coefficients can have different meaning, however, according to the definition of what constitutes capital goods and according to uses of the particular model. For our model, coefficients (b_{ij}) constitute the amount of total capital goods, including inventory produced by industry "i" which is required by industry "j" to

TABLE 5.3

SECTOR CLASSIFICATION FOR THE MODEL

Sector	Related 1960 (Udis) Classification	Related 1963 (Bargur) Classification
1. Range and feeder livestock	1,2*	1
2. Dairy	3	3
3. Food and field crops	4	4
4. Forrage and pastures	1,2	8
5. Fruit	6	7
6. Forestry and all other aggie	7,8,5	8,6
7. Mining	9,10,11,12,13	9
8. Food and kindred products	14	11
9. Manufacturing	15,16,17,18,19	16
10. Trade and transportation	20,21,22,23,27	18
11. Utilities	28,29	17
12. Services	24,25,26,31	19
13. Construction	30	10

*adjusted

produce one unit of output $b_{ij} = s_{ij}/X_j$ where s_{ij} is the amount of capital goods bought by industry j from industry i. These capital coefficients should not be confused with the well known capital-output ratio which can be represented by

$$b_j = \sum_{i-1}^{n} b_{ij} = S_j/X_j.$$

The capital-output ratio from industry "j" is the ratio of the total cost of capital goods to the value of output at capacity. Data on capital coefficients for the nation as a whole, Fauccet (1966), and for a few states for given years are available in the literature. For a specific region and base period it is necessary, however, to update and adapt known data. The procedure for doing this is diverse and varied, according to the amount and quality of additional data on which indexes and rates of change are based. Bargur (1969) presented a detailed review of these techniques together with an extensive literature on the sources of basic data. In general, the procedures can be made as complex as desired, the criterion being the accuracy sought in the coefficients.

Miernyk (1970), who obtained capital coefficients for the West Virginia economy by using a survey of industries, discriminates between replacement capital and expansion capital coefficients. An expansion capital coefficient, b^e_{ij} is defined as capital requirement by sector j from sector i per unit of increase of capacity in j. A replacement capital coefficient is defined as the amount of capital per unit of output necessary to replace the wornout capacity. Replacement coefficients were based on the average life of equipment and plants for the U.S.A. It is not known with certainty how sensitive the model can be to increased variations in the capital coefficients. This is not a concern in this study, because for the purpose of the present investigation it is sufficient to state

TABLE 5.4

TRANSACTIONS TABLE FOR THE NEW SECTORS OF THE U.M.S.

Buying Sectors / Selling Sectors	1	2	3	4	5	6	7	8	9	10	11
1. Livestock	3598	0	0	0	0	0	0	3459	0	0	0
2. Dairy	0	0	0	0	0	0	0	2055	0	0	0
3. Food and field crop	43	0	0	0	0	0	0	1619	0	0	0
4. Forage crops	8260	1000	0	0	0	635	0	0	0	0	0
5. Fruit	79	53	0	0	0	0	0	859	0	0	0
6. Forestry and other ag.	9	1	2	0	0	1	0	786	1884	0	0
7. Mining	0	0	0	0	0	0	18010	2	389	34	906
8. Food and kindred products	648	136	0	0	0	265	0	98	0	1294	0
9. Manufacturing	22	1	424	532	94	130	1459	255	409	7966	202
10. Trade and transportation	990	188	342	723	204	205	9531	332	1090	8821	211
11. Utilities	88	23	30	116	28	36	1508	256	495	2318	1238
12. Services	1102	216	522	1039	2909	545	692	190	349	7087	624
13. Construction	0	0	0	0	0	0	76	17	133	898	362
Payment Sectors											
14. State and Federal	832	105	29	1794	15	545	1129	1147	1086	3979	819
15. Local Government			272		271	120	987	254	619	2419	1804
16. Wages	2411	84	225	944	42	913	28502	8118	5618	37318	6523
17. Profits and other income	8784	637	2780	1873	1964	1170	6356	1425	1121	16272	2456
18. Inventory change	413	0	0	0	0	0	1591	772	808	9646	6362
19. Depreciation	1330	242	336	1348	214	186	9868	480	1279	7002	2792
20. Imports C.R.B.	139	35	95	2753	38	143	8970	0	8440	3087	951
21. Imports others			686		404	580	29607	1457	5414	21828	3180
22. T.G.O.	28748	2725	5743	11122	6234	5476	118287	19143	29134	131188	28476

TABLE 5.4 (cont.)

TRANSACTIONS TABLE FOR THE NEW SECTORS OF THE U.M.S.

Selling Sectors / Buying Sectors	12	13	Final Demands: State & Federal	Local	Domestic	Inventory Change	Gross Capital	Exports CRB	Exports Other	TGO
1. Livestock	0	0	0	0	712	0	93	0	20886	28748
2. Dairy	0	0	0	0	54	0	0	0	616	2725
3. Food and field crop	0	0	728	0	48	0	0	186	8159	5793
4. Forrage crops	40	0	1187	0	0	0	0	0	0	11122
5. Fruit	0	0	17	0	508	0	0	897	3830	6243
6. Forestry and other ag.	0	0	2	0	766	0	0	183	1839	5474
7. Mining	28	1316	67343	165	709	1140	2178	1537	24630	118287
8. Food and kindred	285	0	48	103	10185	1045	0	168	4868	19143
9. Manufacturing	500	2234	671	736	2132	1229	176	723	9249	29134
10. Trade and transportation	906	2015	630	2406	48273	11121	1972	7867	32400	131188
11. Utilities	2188	338	613	673	10047	6304	30	1258	850	28496
12. Services	2101	1554	4721	3887	21243	2674	679	1038	13093	67549
13. Construction	332	23757	3255	2003	7726	12408	35539	7064	0	93630
Payments										
14. State and Federal	2360	238	9491	323	42435	0	0	194	3067	68476
15. Local	868	130	11707	874	10360	0	0	207	959	38493
16. Wages	16951	15088	35093	15894	636	0	1326	6087	1092	179280
17. Profits and other income	21301	6176	627	1218	7942	0	0	756	49	82487
18. Inventory change	4863	12411	0	0	0	0	0	0	0	36866
19. Depreciation	2989	1231	0	0	0	0	0	0	0	29897
20. Imports C.R.B.	643	2277	449	627	1625	126	1683	93	767	30057
21. Imports others	10181	24265	8765	8528	72316	28958	31790	23926	16024	288928
22. T.G.O.	67549	93630	149957	35502	237657	65480	75466	52184	138688	1295320

clearly that the capital formation as an endogenous sector is important in modeling the economy and that considerable research is being done in this area by competent investigators.

From a rapid inspection of figures on the gross private capital formation column of the transactions table, it can be seen that the only sector with significant capital sales is the construction sector. Inventories are small for all sectors and practically non-existent for agricultural sectors. Based on this consideration, it was decided to treat all inventories and capital goods as part of the final demand except for the construction sector for which capital coefficients will be adapted from Bargur (1969). Table 5.5 shows these coefficients.

(c) Alternative Activities for the Upper Main Stem - A number of possible production alternatives could be adopted in order to make more efficient use of available water for the region in case of droughts. In principle, the possible alternatives could be classified into three groups. The first group includes all those alternatives which result in an improved water application efficiency and water savings by reduction of conveyance losses. The second group is composed of possible alternatives which change the water production function in such a way that a reduction in water applied is not reflected in a linear decrease in production. The third group contains all those alternatives that change the basic disposition of the sector components in such a way that a more efficient water use is obtained.

Alternative activities concerning water savings. Considerable losses occur in the application of irrigation water in the U.M.S. basin. The current use pattern does not seem to encourage savings by either a better application or a lining of canals and laterals unless a shortage of water forces irrigators to do so. Data on actual losses are difficult to obtain for the basin. Bureau of Reclamation project data show that from total deliveries about 30 percent is for consumptive use, around 40 percent consists of seepage and evaporation losses in canals and laterals, and about 30 percent is lost due to inefficient application. Data given by Stewart (1969) seem to agree with the Bureau data. D. L. Miles (1971), Extension agricultural engineer at Colorado State University, estimates that, of the total losses, only about one fourth are due to seepage and the rest to inefficient water application and evaporation. In a study of the Grand Valley area, Skogerboe (1971) finds similar results and estimates that irrigation efficiency could be improved from 30 to 60 percent considering the efficiency as the ratio of water delivery from the laterals to the water used consumptively by crops. Skogerboe also found that lining the canals and laterals would lead to further savings of about 10 percent of the water at intakes, and gives some figures about the cost of lining.

From the previous discussion, it seems reasonable to assume that irrigation efficiency could be increased to the extend that total diversions represent only twice the consumptive use, and that a 10 percent reduction in losses could be obtained by lining canals and laterals. In this case, the values of C_j, the ratio of water intake to consumptive use for sector j, can be established as 2.0 for basic activities and 1.67 for alternative or new activities.

Costs of lining canals and laterals were taken from estimates given by D. L. Miles. They are as follows: About 80 feet of ditch per acre are needed with an average price of $1.75 per foot (1971 prices) and about 30 feet of canal with an average of $7.00 per foot. These prices were deflated to 1960 price using the Irrigation and Hydraulic cost indexes for the West from the U.S. Bureau of Reclamation:

	1960	1971
Canal structures	.89	1.21
Lateral structures	.92	1.18

and the deflated cost is then

$$\text{Canals} = \frac{7.00 \times 0.89}{1.21} =$$
$$7.00 \times 0.735 = \$5.15 \text{ per foot}$$
$$\text{Laterals} = \frac{1.75 \times 0.92}{1.18} =$$
$$1.75 \times 0.78 = 1.35 \text{ per foot.}$$

From this data, 80. x 1.35 + 30. x 5.15 can be assigned as the cost of lining, or 108 + 154 = $262 per acre. Leaving the canals to the irrigation districts, we will then consider only lining of laterals, or $108 per acre. This, given the differences in topography, compares well with costs in Arizona prices given by Young (1968), which are about $62 not including canals.

In order to allocate these costs to different sectors, it is necessary to estimate the average value of production per acre. Bureau of Reclamation projects for the area for the year 1960 give the results in Table 5.6

TABLE 5.5

BASIC DATA FOR THE U.M.S. SUB-BASIN

		Capital coefficient	Growth rates in final demand 1960-80	Growth rates in final demand 1980-2010	Water depletion coefficient AF/$1,000	C_j	Water intake coefficients	Percentages of economic activity located in region A as shown in Fig. 5.2
1.	Livestock	.090839	.47	1.55	.23	2.0	.43	.63
2.	Dairy	.0526700	-.20	-.1	.083	2.0	.166	.42
3.	Food and field crops	.031298	.31	.26	6.97	2.0	13.94	.36
4.	Forrage	.02768	0.0	.0	70.5	2.0	141.0	.60
5.	Fruits	.003400	1.42	1.74	4.0	2.0	8.0	.05
6.	Forestry and other ag.	.06230	1.60	.91	1.37	2.0	2.74	.58
7.	Mining	.3780	.11	0.	0.0208	6.0	.125	.5
8.	Food and kindred	.055400	2.25	.62	0.0095	6.0	.057	.5
9.	Manufacturing	.1489	-.55	3.18	.0206	6.0	.125	.5
10.	Trade and transportation	.439842	3.19	2.35	.0043	6.0	.026	.5
11.	Utilities	3.422316	1.56	1.66	.0147	6.0	.118	.5
12.	Services	1.273300	4.68	3.38	.0022	6.0	.0132	.5
13.	Construction	0.015600	1.76	1.09	.0178	6.0	.1068	.5
14.	Livestock - alternate	0.090834			.230	2.0	.46	
15.	Food and forage (crop reduction)	0.031248			6.55	2.0	13.10	
16.	Food and forage (lining)				6.97	1.67	11.60	
17.	Forrage (crop reduction)	.02768			59.6	2.0	118.40	
18.	Forrage (lining)				70.5	1.67	118.00	
19.	Fruits (lining)				4.0	1.67	6.70	
20.	Other Ag. (lining)				1.37	1.67	2.29	

TABLE 5.6

ALLOCATION OF LINING COSTS TO SECTORS

Sector	Value of Crop per Acre	Total Cost per 1000 Output Units, in Dollars Lateral Alone
3	138	782
4	77	1400
5	413	261
6	320	337

In order to allocate these costs, it is first necessary to determine the useful life of a structure, which is taken to be 20 years. Then it is assumed that the government subsidy will be 50 percent of any lining cost made in annual payments, with the average balance being approximately 1/2 of the initial cost, Udis (1967).

The coefficients of capital goods demanded from the construction sector are given in column 3 of Table 5.6, since the total work can be assumed to be performed by the local construction sector.

The cost of new facilities to the water using sectors will imply adjustments in four coefficients: the services sector coefficient should be increased to take care of payments from interest on loans, the payments to local irrigation districts should also be slightly modified to account for construction costs of off-the-farm installations, the depreciation coefficients should be increased, and the final check out has to be made with the income coefficient.

Depreciation can be calculated according to any of the accepted accounting procedures. For this example the straight line depreciation is used, and each year is charged 1/20 of the total construction cost as shown in Table 5.7.

At this point it is important to notice that most changes are made within the payments sector, and that they do not affect the processing sector coefficients. The only coefficient of the processing sector to be changed is the services coefficient. Following the assumption of local district payments, the increase in this coefficient due to the new technology can be approximated by taking 8 percent annual interest on 1/4 of the present value of the total cost. Table 5.8 shows these increases.

Again, the sum of all coefficients for a given column should add up to unity, and the balance is made with the income coefficient.

TABLE 5.7

CHANGES IN DEPRECIATION COEFFICIENTS

Sector	Increase in Depreciation Coefficients
3	0.039
4	0.070
5	0.013
6	0.017

TABLE 5.8

CHANGES IN SERVICES COEFFICIENTS

Sector i	Increase in Services Coefficients
3	.0156
4	.0280
5	.0052
6	.0067

Alternatives concerning water application. Alternatives of the second type were selected to represent the fact that, in case of shortage, different irrigation schedules can be implemented in such a way that they lead to substantial water savings without having a corresponding linear decrease in the total

TABLE 5.9

NEW WATER COEFFICIENTS

Sector	Water Application percent	Yield percent	New Water Coefficients percent
3	61	65	94
4	71	85	84

output for the sector. The timing of irrigation is an important determinant in crop yield, and missing irrigations at different times in the growing season will lead to different crop yields as pointed out by Anderson (1971), Young et al (1972), Hall (1970), and Bidwell (1971). In theory, a multitude of alternatives could be set up to take into account applying the shortage at different periods and letting the algorithm select the optimal one. In the case studied, this problem can be approached by selecting only the alternative that would represent the actual situation in the U.M.S., U.S.B.R. (1946), that when a drought strikes, the last one or two irrigations are deleted rather than the previous ones.

Data from Young et al (1971) were taken to illustrate this point and are presented in Table 5.9.

The new water coefficients are obtained as

$$W_i^N = W_i \frac{\text{water application in percent}}{\text{yield in percent}} ,$$

with W_i the old water coefficient and W_i^N the new water coefficient.

An application of this type permits a smaller reduction in the yield for a given reduction in water application, while the inter-industry transactions and the investments are the same. In other words, more inputs are used for less outputs, with the only exception being the income sector and local district sector for water charges.

The new coefficients can be computed as follows. Let Y^p be the potential yield, then requirements from sector i are equal to

$$x_{ij} = a_{ij} Y_j^p ,$$

and the new coefficient will be

$$a_{ij}^N = \frac{a_{ij}^o}{\text{yield in percent}} \frac{Y_j^p}{Y_j^p}$$

Again the difference has to be allocated to the income sector.

Alternatives considering marketing variations. For sector 1 (livestock) an alternative of the third group is also selected. This alternative is based on the concept that it is possible to temporarily modify the marketing structure of the livestock sector in order to save forage, and therefore water in case of drought. It must be realized that many of these changes can be performed, and that considerations of most of them can be made by increasing the number of alternatives. For purposes of this study, only one possible case is considered as an illustration.

The livestock industry in the U.M.S. is composed of three major sub-sections, or programs: (1) program consisting of sheep and livestock other than cattle, (2) program selling the calves before they are yearlings, and (3) program proceeding with yearlings. The interest may be in investigating the impact of deciding not to keep the yearlings when the drought hits in order to save the forage.

Taking data from the census of agriculture of 1959, it was found that, for the counties forming the U.M.S. basin, calves constitute approximately 72 percent of all livestock. From these figures, the percentage value of yearlings of the value of total livestock is approximately 39 percent. Gee and Robinson (1969) give prices and cost for different livestock programs in western Colorado. Let Y_p in dollars be the selling price of yearlings and C_p in dollars the selling price of calves. Assuming that all the calves are sold to other regions with the result of zero production in yearlings in U.M.S. then the total value of production is affected by the factor $K_1 = 0.39\ (Y_p - C_p)/Y_p$.

The prices given by Gee and Robinson (1969) are Y_p = \$186 and C_p = \$142. This gives

$(Y_p - C_p)/Y_p = 0.24$, and K_1 becomes approximately 0.09. To compute the amount of forage saved by carrying through this measure, from Gee and Robinson (1969), this amount to about 35 percent, which gives a total reduction in forage equal to 0.35 x 39/100 = 0.15. It yields, in turn, a new forage coefficient $a^N = 0.85a^o/0.91$. Since the new output is only 0.91 of the old output, the remaining coefficients have to be adjusted upward by dividing them by 0.91, and the balance is made up with the income coefficients. The results of these modifications are given in Table 1 of the Appendix, which shows the old and the new coefficients. Since yearlings are not retained, the sales of sector 1 to sector 1 should go down, and, given the reduction in production, the coefficient could be assumed to remain the same.

(d) Projections of Final Demands - Depending on available resources and on the objectives of a particular study, a projection may range from a rough extrapolation of the final demand vector in aggregate form to a careful projection of every entry in each of its columns. In general, final demands projections are made by extrapolating trends found in the time series analysis of data available on household and public expenditures, and by assuming that trade with the outside world will follow a preestablished pattern. Some investigations have used the analysis of related series like personal income figures or have adapted national trends given by the Bureau of Labor Statistics. The projections of final demands is an important phase of the analysis, because the results are going to be highly dependent on them as the exogenous part of the model. In general, several projections are postulated ranging from optimistic to pessimistic, or according to a desired goal of growth.

When the knowledge of economics of the region allows, trends are adjusted, based on the judgment of forecasters, with upper limits imposed on forecasts. For the Upper Main Stem a set of projections was made by Udis (1967) by using mainly judgment, expert opinion, and comparison of per capita final demand in the region. Two sets of projections were made, one for the year 1980 and another for the year 2010. Based on these projections, annual average rates of growth were computed and are shown in Table 5.5. In this model for the U.M.S. the rates of growth for the period 1960 to 1980 were taken as the minimum or lower rates, except when rates of growth for the period 1980 to 2010 were smaller than the rate for 1960 to 1980. In this case, rates for 1980 to 2010 were taken as the lower rates in recognizing a possible anticipation of the slowdown in the sector demands. A similar approach was taken in setting the upper limits for growth in final demands. More realistic projections would imply considerable economic research beyond the scope of the present study. Therefore, the given rates should be looked at mainly as an illustration.

5.4 Water Use Patterns

The formulation of the model requires the definition of relations expressing fresh water intake and consumptive use as functions of the total gross output per sector. For an individual industrial or agricultural establishment, these relations are known, to a certain degree, when the details of the water technology are also known. For the whole sector, however, the aggregated production functions cannot be easily obtained. It is necessary to use a regression procedure between gross outputs, in constant dollars, with water use data for the particular years in which they are available. Data on water use can be obtained from several sources, among which the U.S. Geological Survey (1965) and the Bureau of Census (1967) provide statistics for most industrial sectors. These statistics are given at intervals of a least five years, thus providing few points to fitting anything more than a straight line for the relationship of the total gross outputs (TGO) to the water use. For irrigated agricultural sectors estimates of consumptive water use are ordinarily made with the evapotranspiration requirements per crop per acre. For certain projects, however, a time series of water application per crop is available from the Bureau of Reclamation (1965).

Water used by sectors in the U.M.S. For the case of the Upper Main Stem Sub-basin of the Colorado River, Udis (1967) obtained estimates of consumptive use of water by a combination of the previous methods and a review of related publications on the region. From these estimates, water coefficients are computed, assuming a linear relationship between water use and the total gross output. The consideration of drought impact makes it necessary to consider the estimation of water intake and of water reuse also. To consider only the consumptive use of water is equivalent to assuming that nonconsumptive water can be fully utilized, and that no shortage can arise from intake conditions only. It is clear that this is an ideal situation and that it underestimates water needs. To use the intake coefficient alone, on the other hand, neglects the possibilities of water reuse. It is equivalent to a gross overestimation

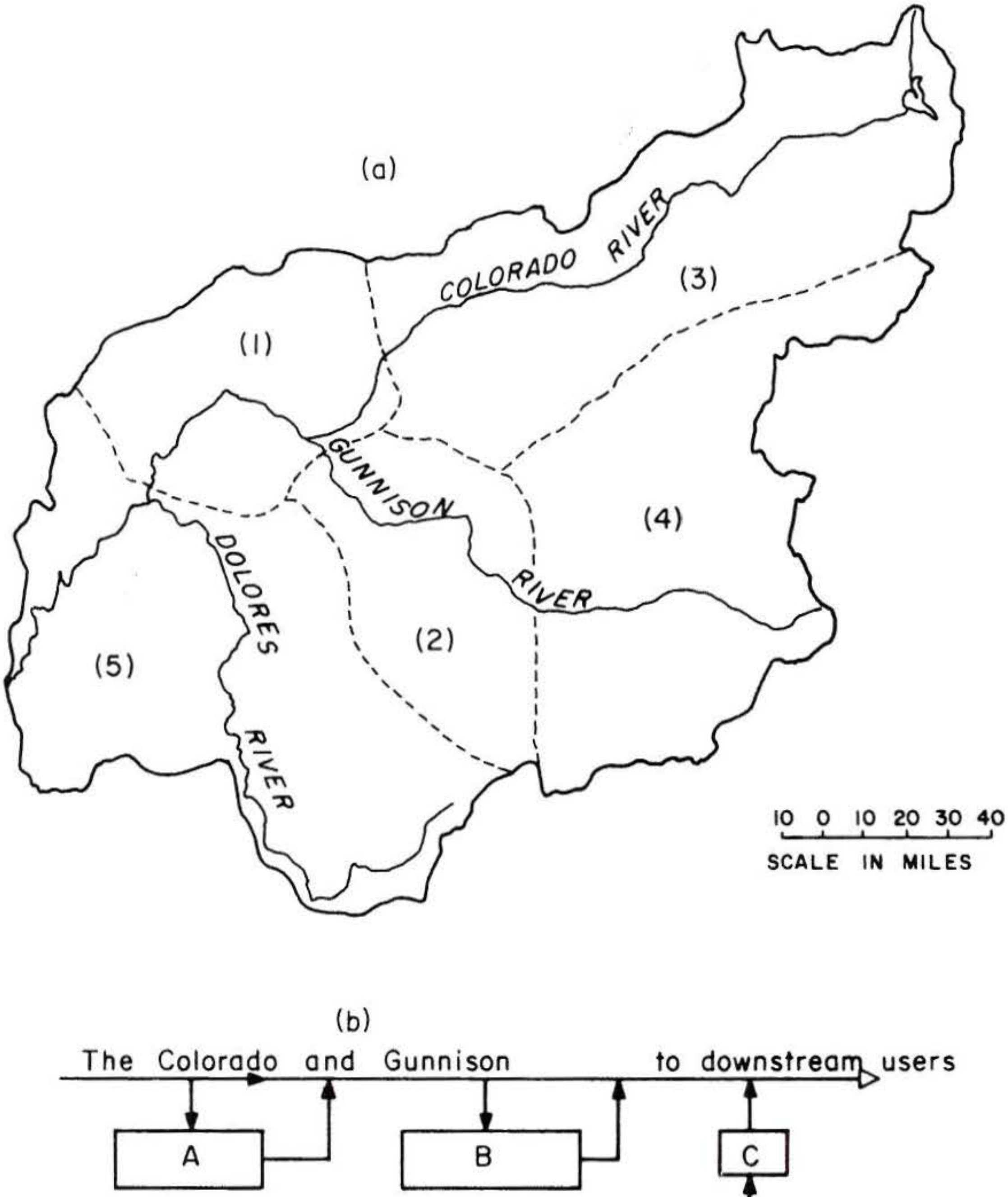

Fig. 5.2 Patterns of water reuse selected for the upper main stem of the Colorado River.

of water needs. A more realistic approach can be taken if detailed geographical distribution of water using sectors can be obtained in such a way that return flows can be established and water reuse patterns determined.

Water intake coefficients can be estimated by multiplying the consumptive use coefficients by a factor that takes into account conveyance losses and application inefficiencies. The Nations Water Resources (1968) gives this factor a value of C = 2.0 for crop sectors and C = 6.0 for other sectors. Stewart (1969), Bureau of Reclamation projects (1965), and expert opinions agree that water is inefficiently used in the basin, and that the actual factor for crops is around 3 rather than 2. However, experts agree also, that a factor of 2 can be obtained simply by removing application inefficiencies, Skogerboe (1971), and that this is the factor that should be considered in any projections. To do otherwise would imply a perpetuation of local inefficiencies and overestimation of real shortages. Table 5.5 shows the actual consumptive coefficients, the intake, consumptive use factor, and the computed intake coefficients for the sectors in the Upper Main Stem Basin. Intake coefficients for alternative activities have been corrected for the proposed water savings in Section 5.3.

For the purpose of this study, a simplified water reuse scheme is proposed. More complicated schemes can be formulated only with better data. The scheme is equivalent to having an intake coefficient modified to take into account the reuse, and can be summarized as follows. The agricultural water use of the Upper Main Stem Sub-basin (Fig. 5.1) can be divided into five sub-systems as shown in Fig. 5.2, having the respective percentages of water use as obtained from Udis (1967). It can be assumed that return flows from regions 2, 3, and 4 can be reused in region 1, and are also available together with return flows from regions 1 and 5 for meeting deliveries to the Lower Basin. Actually, there is an amount of reuse in sub-systems 2 and 3, but this can be adjusted for, as shown later. The next step is to divide the sub-basin into three major sub-systems, A, B and C, as shown in Fig. 5.2. Sub-system A is composed of the equivalent amount of economic activity that does not reuse water. Sub-system B is composed of the equivalent amount of economic activity that uses return flows from the sub-system A. Subsystem C is composed of the equivalent amount of economic activity that does not use return flows, with its return flows not being reused either. If the proportion of economic activity from sector j in region **A** is denoted by P_{jA}, then the total equivalent intake water, corrected for the water reuse in year t, can be represented by

$$\sum_{j=1}^{m} C_j W \; X_j(t) - \sum_{j=1}^{m} P_{jA} \, W_j \, (C_j - 1) \, X_j(t) \quad , \tag{5.1}$$

in which C_j is the intake flow to consumption ratio, W_j is the consumptive use coefficient, and $X_j(t)$ is the total gross output for jector j in year t.

The new equivalent coefficient W' is then

$$W_j' = C_j W_j - P_{jA} \, W_j \, (C_j - 1) = W_j (C_j - P_{jA} \, (C_j - 1)). \tag{5.2}$$

Estimates of P_{ja} were made by using data from the census of agriculture (1960) for the agricultural sectors. For all other sectors, it was estimated that approximately 50 percent of the activity was located in region A. This estimate is not important, since more than 95 percent of the total water use in the sub-basin is in agriculture. Table 5.5 shows the P_{ja} values in the analysis.

Water exports, compact requirements and other demands. There are several projects, most of them located in the eastern slope of the Rocky Mountains, that divert water from the Upper Main Stem Sub-basin of the Colorado River. Among the existing diversions, the Colorado-Big Thompson project, the Denver Water Board diversions and the Arkansas Valley take most of the 470,000 acre feet per year estimated in 1965 by the Water Resources Council (1968). The amount of total diversions is expected to grow in a considerable way in the near future. Figure 5.3 shows the water exports up to the year 2020 projected by the Water Resources Council. These projections imply the export of a sizable amount of the total water available in the sub-basin which could constitute a critical factor for its economy in case of drought occurrences.

The other water demands to be projected in an exogenous way include municipal and rural water supplies. The Water Resources Council (1968) estimates the rural domestic uses for the Upper Main Stem Sub-basin as 1.9 million gallons per day in 1965, 75 percent of which was obtained from ground water. Municipal water supply projections are shown in Fig. 5.4 as estimated by the Council.

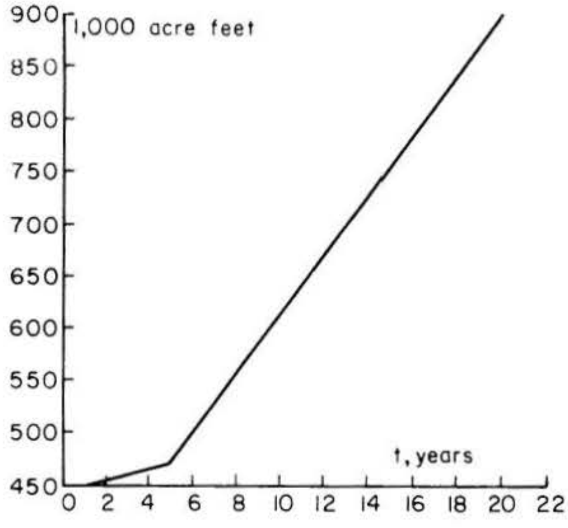

Fig. 5.3 Projected water exports from the upper main stem sub-basin of the Colorado River over the 20 years, 1960-1980.

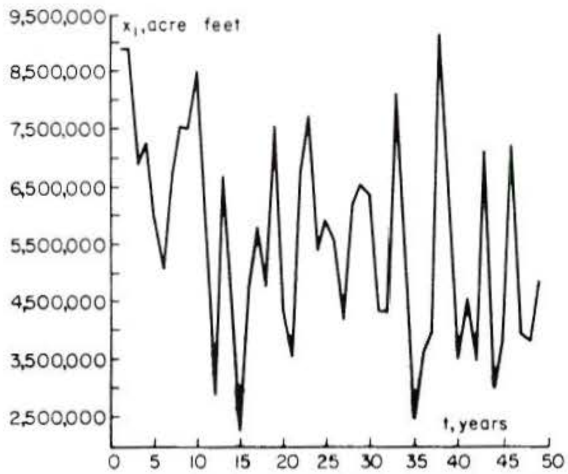

Fig. 5.4 Historical annual reconstituted or virgin streamflows of the Colorado River near Cisco, Utah.

A more critical demand from the basin is the water that the basin is required to deliver to the lower basin of the Colorado River by virtue of the compact regulating the allocation of water among the upper and lower basin states. The upper basin is required to deliver at Lee's Ferry, Arizona, 75 million acre-feet of water in any successive 10 year period. It has been argued by the Upper Colorado Commission (1968) that there is not sufficient water available in the river to meet these requirements and at the same time permit the full utilization of the water apportioned to the upper basin. If this claim is true, it would make the deliveries by the compact a crucial point in the analysis to be presented here. Water for the upper basin was allocated by states according to the Upper Colorado River Compact of 1948. Since the three major sub-basins of the Upper Colorado River are located in four different states, it was not possible to determine directly what portion of total deliveries should be supplied by each sub-basin. For purposes of this study, 75 million acre-feet were allocated in a proportional way to the mean flow of each sub-basin, and a total of 33.5 million acre-feet were assigned to the upper main stem.

In order to compute the 10-year total deliveries, it is necessary to know the actual deliveries from the sub-basin in the previous period of ten years before the initiation of the projections with the model. These deliveries were taken from the U.S. Geological Survey publications are given in Table 5.10. The 10-year total is 46,918,000 with the average 4.69 million acre-feet per year.

TABLE 5.10

WATER DELIVERIES TO COMPACT FOR 1951-1960 PERIOD

Year	
50-51	3,921,000
51-52	7,707,000
52-53	4,037,000
53-54	2,329,000
54-55	3,241,000
55-56	3,604,000
56-57	8,486,000
57-58	6,350,000
58-59	3,111,000
59-60	4,132,000
TOTAL	46,918,000

5.5 Analysis of Water Availability

The annual flows of the Colorado River, corrected for diversion, evaporation, etc., at a gauging station near the outlet of the basin, were selected as the time series of total water availability. Other variables for measuring water supply, such as precipitation, do exist. However, such measurements are scarce at the places where most of the water use is concentrated with less than 9 inches a year, so that

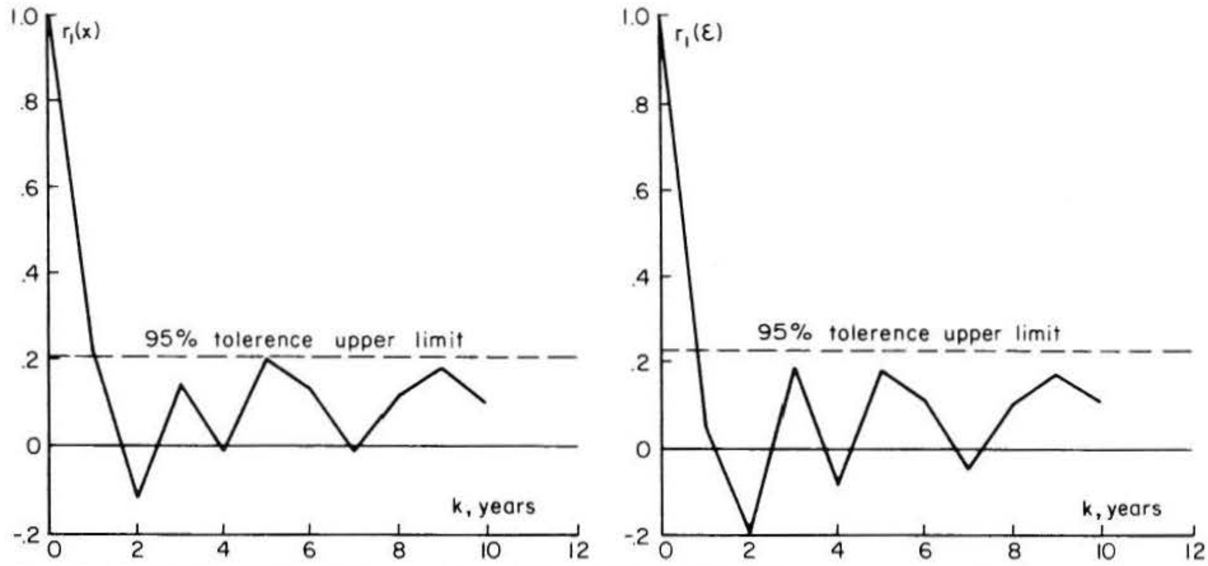

Fig. 5.5 Autocorrelation functions for the original (left) and the whitened series (right) of annual series of streamflows of the Colorado River near Cisco, Utah.

TABLE 5.11

TESTS OF FITTED DISTRIBUTION FUNCTIONS

Function	Computed chi-square	chi-square critical
Normal	13.53	28.90
Lognormal with 3 parameters	132.8	27.60
Pearson type III function	30.37	27.60

precipitation variation can hardly modify the results obtained for a model in which the water requirements are measured in terms of water diverted from streams.

The stream gauging station selected for the analysis is the Colorado River near Cisco, Utah, U.S.G.S. number 918050, with 49 years of continuous records. Virgin flows were reconstituted by the staff of the Hydrology and Water Resources Program of the Civil Engineering Department at Colorado State University.

Basic statistics. Annual flow series is shown in Fig. 5.4, and the basic statistics as estimates of population parameters are:

Mean, $\overline{X}$ = 5,568,098 acre-feet,
Standard deviation, S_x = 1,823,552 acre-feet,
Coefficient of variation, c_v = 0.3275,
Skewness, C_s = 0.15,
Excess, E = -0.9232, and
First serial correlation coefficient, r_1 0.21.

The autocorrelation function of the annual series is shown in Fig. 5.5, both for the historical series (X) and for series (ϵ), i.e., the autocorrelation function of the new series ϵ_i formed after the dependence of the first-order autoregressive model has been removed according to

$$\epsilon_i = X_i - r_1 X_{i-1} \quad , \qquad (5.3)$$

together with the upper tolerance limit at 95 percent level, for the autocorrelation coefficients of an independent process. It is evident that the hypothesis of independence for the ϵ_i-series should be readily accepted. The first-order autoregressive linear model of independence very well approximates the dependence of the series.

Three probability distribution functions were fitted to data with the chi-square results shown in Table 5.11. The chi-square critical has the one-tail rejection region of 5 percent.

From this analysis, it can be concluded that the best fit is by the normal distribution function, with the following dependence model:

$$X_i = \bar{X} + r_1(X_{i-1} - \bar{X}) + S_x \sqrt{1 - r_1^2} \quad \xi_i \ , \qquad (5.4)$$

in which X is the mean of observations,

S_x is the standard deviation of the original series,

r_1 is the first serial correlation coefficient, and

ξ is a deviate from a normal distribution with the mean zero and the standard deviation unity.

The proposed model permits preservation of the first three moments and the first serial correlation coefficient in the generation of large samples.

5.6 Formulation of Programming

The formulation of a model for a year t as a programming problem requires the definition of 32 variables and 46 constraints. For the sake of clarity, the nomenclature used and a summary of the constraints are given before a detailed analysis of this formulation is made.

The variables are denoted by the symbol X (t), in which the subscript j stands for the variable number and t for the time period considered. The following notations are given to the various variables:

$X_j(t)$, j = 1,...,13, refers to the total gross output of the principal or original activities;

$X_j(t)$, j = 14,...,20, refers to the total gross output of the alternative activities;

$X_j(t)$, j = 21, is the variable representing the portion of the construction sector output devoted to capital goods;

$X_j(t)$, j = 22,...,31, are variables designed to adjust the final demands when necessary to maintain a feasible solution;

$X_j(t)$, j = 32, is the variable denoting the content of the reservoir at the end of a period t;

$C_j(t)$ are the coefficients of objective function.

There are thirty-one constraints that are smaller than or equal to the prescribed conditions:

Constraints 1 through 13 represent the upper limits in deliveries to final demand;

Constraints 14 and 15 refer to capital goods;

Constraints 16 to 18 relate to water availabilities;

Constraints 19 through 28 relate to maximum adjustments to final demands; and

Constraints 29 through 31 control the size of the alternative activities of the second and third groups.

There are fifteen constraints that are greater than or equal to the prescribed conditions;

Constraints 32 through 44 represent lower limits in deliveries to final demands and,

Constraints 45 and 46 are capital goods constraints.

Objective function. The sector's outputs are determined by maximizing the total income for the region,

$$\sum_{j=1}^{32} C_j \, X_j(t) \ , \qquad (5.5)$$

in which

C_j, j = 1,...,20, are the income coefficients for the sectors,

$C_{21} = C_{32} = 0$, and

C_j, j = 22, 31, are the penalties chosen for adjusting the lower bounds of final demands.

The penalties are selected as the slopes of the piecewise approximations to the nonlinear function as described in Chapter V.

A decision has to be made regarding which sectors are chosen to permit adjusting their final demands. It is not realistic, nor necessary, that lesser water using sectors become affected directly and indirectly in their production by droughts to the point of requiring an adjustment. The adjustment is made, therefore, only on intensive water users either directly or indirectly with the exception of sector five (fruits). The importance of this sector in the future development of the region and its highly efficient use of water will likely limit such adjustment to a minimum. Two segments are chosen as maximum, and the total number depends on the composition of final demands and on the subjective preference that can be given to different sectors by the planner, as explained in Chapter V. For this example, the two slopes selected are -10 and -20. The absolute values of slopes is selected in such a way that they are large in comparison with the income coefficients, and that the second slope makes it increasingly difficult to reduce the final demands. Table 5.12 gives a summary of slopes selected, the proportion of final demands that they affect, and the variables selected to make the adjustments. The sectors are selected by including the crop sectors as direct users, while livestock, dairy

TABLE 5.12

SELECTED SECTORS FOR FINDING DEMANDS REDUCTIONS

Sector	Variables	A in percent of Y_i^L
1	X_{22}, X_{23}	67
2	X_{24}	100
3	X_{25}, X_{26}	60
4	X_{27}	100
6	X_{28}, X_{29}	50
8	X_{30}, X_{31}	40

and food and kindred products are selected as the large indirect users of water. Sixty-seven percent of the final demand from sector 1 is included in the milder slope. This portion includes most of the exports, and it is assumed that local consumption and a smaller portion of exports are reduced only as a last resort. A similar criterion is applied to sectors 3 and 6, while only one slope is required for sectors 2 and 4 because of its limited final demand. Food and kindred products are allowed a reduction up to 40 percent in the milder slope, because of their heavy dependence on sectors 1 and 2.

The coefficients C_j for j = 22, through 31 are then C_j = -10 for j = 22, 24, 25, 27, 28, and 30, while C_j = -20 for j = 23, 26, 29, and 31.

Constraints. Following the developments in Chapter V, the constraints can be classified into the following six groups.

(1) The first set of constraints refers to the interindustry relations by considering the upper limits in deliveries to final demands. Having the alternative activities defined in Section 3 of this chapter, $\overline{X}_{Ai}(t)$ is a vector containing alternative activities corresponding to sector i.

The constraints are

$$X_i(t) + \underline{X}_{Ai}(t) - \sum_{j=1}^{20} a_{ij} X_j(t) \leq Y_i^U(t) \qquad \text{for } i = 1,...,6$$

$$X_i(t) - \sum_{j=1}^{20} a_{ij} X_j(t) \leq Y_i^U(t) \qquad \text{for } i = 7,...,12$$

$$\text{and } X_{13}(t) - \sum_{j=1}^{20} a_{ij} X_j(t) - X_{21}(t) \leq Y_{13}^U(t) \qquad (5.6)$$

(2) A second set of constraints consist of Eqs. 4.32 through 4.44, representing interindustry relations by considering the lower limits to final demands. By introducing a vector $\underline{X}_{ADi}(t)$ which contains the adjustments in final demands, then

$$X_i(t) + \underline{X}_{Ai}(t) - \sum_{j=1}^{20} a_{ij} X_j(t) + \overline{X}_{ADi}(t) \geq Y_i^L(t) \qquad \text{for } i = 1,..., 6 \text{ and } 8$$

$$X_5 + \overline{X}_{A5} - \sum_{j=1}^{20} a_{ij} X_j(t) \geq Y_5^L(t)$$

$$X_i(t) - \sum_{j=1}^{20} a_{ij} X_j(t) \geq Y_i^L(t) \qquad (5.7)$$

$$\text{for } i = 7, 9, 10, 11, 12$$

$$\text{and } X_{13}(t) - \sum_{j=1}^{20} a_{ij} X_j(t) - X_{21}(t) \geq Y_{13}^L(t)$$

(3) For capital constraints, it is important to consider the replacement of capital goods: otherwise, there will be an underestimation of the total capital goods required by the regional economy. The capital coefficients account only for capital goods required by the expansion of industries. In a study by Carter (1970), in which the introduction of new technology

is modeled for the United States during the period 1947-1960, there are some figures on minimum and maximum rates of scrappage for the economy. These figures are from 6 to 9 percent per annum for maximum rate and from 2 to 3 percent per annum for minimal rates. Considering an average rate of 5 percent as an appropriate value for computing the amount of replacement capital that is necessary, Eq. 4.24 should be modified to

$$\sum_{j=1}^{20} a_{14,j} X_j(t) - X_{21}(t) \leqslant 0.95\ S(t-1). \quad (5.8)$$

A maximum rate of introduction of new technologies can also be estimated from these figures as 6 percent if the difference between the maximum and the minimum rates of scrappage is considered as the rate in which the old technology is being replaced by the new technology.

Among the proposed alternatives only those activities which concern the introduction of new irrigation technology are relevant for Eq. 4.25, since they are the only activities requiring fundamentally different capital investments. Equation 4.25 then becomes

$$\begin{aligned} & a_{15,16} X_{16}(t) + a_{15,18} X_{18}(t) + ... \\ & + a_{15,20} X_{20}(t) \leqslant 1.6\ [a_{15,16} X_{16}(t-1) + ...] \\ & + 0.6\ [a_{15,3} X_3(t-1) + ...] \end{aligned} \quad (5.9)$$

In the same way, Eq. 4.26 can be rewritten as

$$\begin{aligned} & a_{15,16}X_{16}(t) + a_{15,18} X_{18}(t) + ... \\ & + a_{15,20} X_{20}(t) \geqslant a_{15,16} X_{16}(t-1) \\ & + a_{15,18} X_{18}(t-1) + ... + a_{15,20} X_{20}(t-1), \end{aligned} \quad (5.10)$$

and Eq. 4.27 as

$$\sum_{j=1}^{20} a_{15,j} X_j(t) - Ex\ X_{21}(t) \geqslant S(t-1)\ Ex\ , \quad (5.11)$$

with Ex being the minimum percentage of the total capacity that can be used. Again, as in the determination of the rate of introduction for new technologies, the value of this parameter should be determined in order to prevent the accumulation of unused capacity. A preliminary estimate of ninety-five percent is considered here.

(4) The water constraints in this example are reduced to three. The first one requires that the effective intake be less than the water availabilities and can be written as

$$\begin{aligned} & \sum_{j=1}^{20} W'_j X_j(t) + E(t) + D_d(t) \\ & \leqslant F(t) + X_{32}(t-1) - X_{32}(t)\ ; \end{aligned} \quad (5.12)$$

by arranging the terms it becomes,

$$\begin{aligned} & \sum_{j=1}^{20} W'_j X_j(t) + X_{32}(t) \\ & \leqslant F(t) - E(t) - D_d(t) + X_{32}(t-1) \end{aligned} \quad (5.13)$$

The second constraint requires

$$X_{32}(t) \leqslant \bar{S}_t(t)\ , \quad (5.14)$$

in which $\bar{S}_t(t)$ is the maximum storage available at the time t. The third constraint results from the compact, and can be written as

$$\begin{aligned} & F(t) - E(t) - D_d(t) - \sum_{j=1}^{20} W_jX_j(t) - X_{32}(t) + X_{32}(t-1) \\ & - F(t-10) + D_c(t) \geqslant 33{,}500{,}000\ , \end{aligned} \quad (5.15)$$

in which $D_c(t-1)$ is the total amount delivered to the compact in the previous ten years ending at the year t-1.

Rearranging terms, Eq. 5.15 becomes

$$\begin{aligned} & \sum_{j=1}^{20} W_jX_j(t) + X_{32}(t) \leqslant F(t) - F(t-10) - E(t) - D_d(t) \\ & + D_c(t-1) - 33{,}500{,}000 + X_{32}(t-1)\ . \end{aligned} \quad (5.16)$$

To study the flexibility of this constraint, a case will be investigated by considering deliveries during the period of 20 years instead of deliveries during the period of 10 years.

(5) The last set of constraints requires the variables used to adjust to the final demands in order that they become smaller than or equal to the upper limits imposed on them.

(6) For the alternative activities to the second and third group, it is necessary to assure that the reduction in yields of production implicit in them is realized. Sector 1 for instance, uses an alternative X_{14} of the third group, which involves a 9 percent reduction in yield. The value X_{14} should therefore be controlled by

$$X_1 + \frac{1}{0.91} X_{14} \leqslant X_1^*\ . \quad (5.17)$$

in which X_1^* is the total gross output of sector 1 when it is produced by the original sector alone. This value is obtained from a run of the program without drought conditions.

CHAPTER VI

ANALYSIS OF RESULTS OF THE MODEL

The results of the application of the methodology proposed in this study to the Upper Main Stream Sub-basin of the Colorado River are presented in this Chapter. The analysis includes a detailed description of the way in which the economic model works in case of droughts to give the estimated value of losses. Also, a probabilistic analysis of the performance of the sub-basin economy under different conditions of water deliveries to downstream users is presented.

6.1 Selection of Time Horizon

The length of the time horizon depends on the period of time into the future for which the assumptions made in the formulation of the model can be considered to remain valid. Also, the time horizon should be long enough to have the opportunity to experience severe droughts. Economic projections, especially when they are linear, should not be made for too long a period without a measure of caution. Technology changes and interregional trade patterns are subject to unforeseen modifications. Projections longer than twenty years with the model presented here could have a range of error so high that it could invalidate the results of the analysis. With this word of caution in mind, and after making several computations of drought probabilities for different time horizons, it was concluded that a time horizon of 20 years can well meet both conditions and therefore was adopted.

6.2 Unconstrained Projections with the Model

For the selected time horizon and the data and parameters described in the previous chapters, a set of unconstrained projections was made with the model. The unconstrained projections consist of the series of values the economic indicators would obtain if there were enough water in the region over the time horizon considered. Figure 6.1 shows the projections of total income, net water intake and water depletions for the 20-year selected time horizon which starts with the base year of 1960. The shape of the curve showing the growth in water depletions and water intakes is controlled mainly by the projections of water exports (linear), which grow much faster than the water for local needs over the span of time considered in this study.

In addition to the regional income, total gross outputs for every sector and levels of investment in capital goods are also computed. All unconstrained projections are kept in storage since they are the basis on which to measure the performance of the model once water constraints are introduced.

In the absence of resource constraints, all final demands will be satisfied at their upper levels. In fact, only the upper bound in final demand constraints and the excess capacity constraint prevent the model from assigning unrealistically large values to the sectors outputs. In programming terminology this implies that all less than or equal constraints will be tight and all greater than or equal constraints will be loose.

6.3 Projections Subject to Water Constraints

When water constraints are taken into consideration, they may become the controlling factors in the determination of projections. Total gross outputs from sectors may be reduced in order to adjust themselves to the water availabilities. Reductions in income and in deliveries to final demand may also occur.

To illustrate in detail how the model presented in Chapter V works for the Upper Main Stem Sub-basin, one streamflow sequence 20 years long was generated using the statistical model of Eq. 5.2, and it is shown in Fig. 6.2. The selected sample contains two years in which the flow was below the unconstrained intake requirements shown in Fig. 6.1, and one year in which the constraint on downstream requirements (compact) was binding. The probability of having two years or more in a row below the requirements can be readily computed by using the formula developed in Chapter II for the corresponding parameters. This probability happens to be 0.029, which is very low. It gives a measure of the frequency of events that long, but it tells very little about the magnitude of the shortage.

Outcomes from the simulation of the economy for the selected sample are summarized in Tables 6.1, 6.2 and 6.3. Table 6.1 shows the sectors' outputs and their corresponding decreases as compared with unconstrained projections for the years in which a shortage occurs. By inspecting Table 6.1 it can be noticed that during the year No. 9, major reductions in the total gross outputs occurred to livestock, forage, food and kindred products, and to

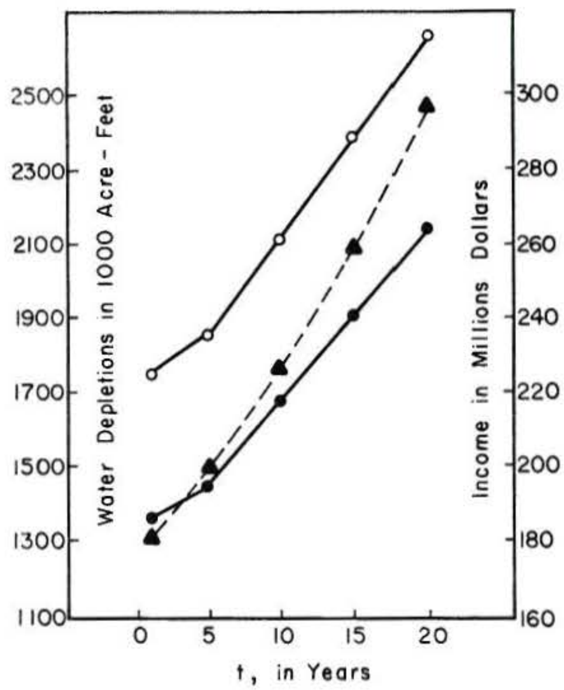

Fig. 6.1 Unconstrained Projections: (1) water depletion, (2) net water intake, and (3) total income generated.

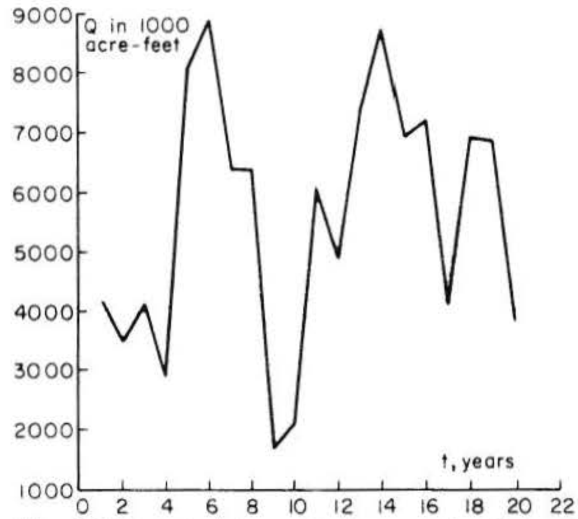

Fig. 6.2 Generated demonstration sample of river flows of 20 year length, which was used to illustrate the performance of the economic model under drought conditions.

construction. This pattern is logical; most of the water used in the sub-basin is devoted to irrigating forage, with relatively low return per unit of water used, and this sector should be the first to adjust to the shortage. Reductions in other crop sectors follow according to their water efficiencies.

The indirect effects of the drought are clearly shown by inspecting the total gross outputs from sectors 7 through 13, which are the non-agricultural sectors. The reduced outputs in agricultural sectors lead to reduced outputs in food and kindred products, trade, services and construction, which reflect the space dependence mentioned in Chapter I. The sector most affected is, naturally, food and kindred products because its main source of inputs is constituted by the agricultural output in the region. Construction is affected also to a considerable extent, because the reduced output in all other sectors delays investment in new construction and replacement. This is not a permanent reduction, however, and as soon as the economy recovers from the drought it will make the necessary investments to catch up with the projected production levels as can be noticed from Table 6.1 for the year No. 10.

Table 6.1 also shows the way in which alternative activities enter into the solution to alleviate the shortage to a certain extent. In the year No. 9 the whole production of sector 1 is carried on by the alternative activity which made possible a larger income with the reduced amount of forage available; also, the entire production of the forage sector was conducted by the alternative activity which considers lesser water application, while all of the new irrigation technology was allocated to sector 5, fruits.

The shortage for the year No. 10 of the sample was not as critical as the year No. 9, and only the forage and livestock sectors were affected. These two sectors had to introduce their respective alternative activities, but in a limited extent. Table 6.2 shows a summary of the changes in income and in water use over the whole period. Water shortages were declared only during years 9 and 10 of the sequence, and the deficit in the compact deliveries that was forecast from preliminary analysis of the supply and demand sequences did not materialize due to the adjustments and to the consumption of less than the expected amount of water, permitting delivery of more water to the compact during years 9 and 10 than was anticipated. The effects of the shortage, however, extended until year 13 because of the adjustments that the production of the construction sector had to experiment in order to account for the additional demand involved on the introduction of new canal lining technology.

The first row in Table 6.2 shows the net reduction in income for the region, and the second

TABLE 6.1

TOTAL GROSS OUTPUTS FOR PRODUCTION SECTORS DURING DROUGHT

Number Year 9	Original Sector	Type 3 Alternative	Type 2 Alternative	Type 1 Alternative	Total For Sector	Net Reduction
Sector 1	0.	30200.	0.	0.	30200.	3016.
Sector 2	2878.	0.	0.	0.	2878.	313.
Sector 3	5984.	0.	0.	0.	5994.	282.
Sector 4	0.	0.	10375.	0.	10375.	2416.
Sector 5	6794.	0.	0.	404.	7188.	243.
Sector 6	6386.	0.	0.	0.	6386.	334.
Sector 7	102988.	0.	0.	0.	102988.	37.
Sector 8	20689.	0.	0.	0.	20889.	2863.
Sector 9	35921.	0.	0.	0.	35921.	171.
Sector 10	166984.	0.	0.	0.	166934.	230.
Sector 11	33190.	0.	0.	0.	33150.	84.
Sector 12	94642.	0.	0.	0.	94602.	354.
Sector 13	70768.	0.	0.	0.	70708.	2303.
Number Year 10						
Sector 1	27472.	5718.	0.	0.	33190.	565.
Sector 2	3280.	0.	0.	0.	3250.	-0.
Sector 3	6336.	0.	0.	0.	6336.	0.
Sector 4	11086.	0.	1552.	78.	12715.	274.
Sector 5	7577.	0.	0.	0.	7577.	-1.
Sector 6	6870.	0.	0.	0.	6870.	-4.
Sector 7	103315.	0.	0.	0.	103315.	-35.
Sector 8	24109.	0.	0.	0.	24109.	-3.
Sector 9	37216.	0.	0.	0.	37216.	-62.
Sector 10	172301.	0.	0.	0.	172301.	-61.
Sector 11	33900.	0.	0.	0.	33900.	-13.
Sector 12	98979.	0.	0.	0.	98979.	-45.
Sector 13	77194.	0.	0.	0.	77194.	2355.

TABLE 6.2

SUMMARY OF OUTCOMES FOR THE SHORTAGE YEARS, IN THOUSANDS OF DOLLARS AND IN THOUSANDS OF ACRE-FEET

Year	9	10	11	12	13	Total
Net income loss	5446	19.35	13.49	-43.	13.	6006.
Indirect income loss	1588	-629.	13.49	-43.	13.	898.
Water intake shortage	409 (28%)	52 (0.5%)	-	-	-	461.
Water depletion shortage	291	35	-	-	-	316.

TABLE 6.3

FINAL DEMANDS FOR THE FIRST YEAR OF THE SHORTAGE IN THOUSANDS OF DOLLARS

Sector	Lower Limit	Actual Value	Upper Limit
1	22687	22687	24801
2	658	658	664
3	4196	4196	4234
4	1187	384	1187
5	6112	6112	6221
6	3219	3219	3426
7	97602	99373	99373
8	17325	17325	20145
9	14258	19805	19805
10	129053	139638	139638
11	22679	22881	22881
12	63954	71565	71565
13	22200	23622	23622

row shows the reduction in income caused indirectly to sectors other than agriculture. The indirect losses amounted to over 25 percent of the total losses during the first year of the drought but are reduced to only 15 percent when the whole period is considered. This is due, as mentioned before, to the recovery of the construction sector during the post-drought period. Actually, the shortage was relatively severe for the first drought year, since it amounted to almost 28 percent of the normal intakes for the basin, but the reduction in income was mild, less than 3 percent of the total income and about 15 percent of the agricultural income.

Another important feature of the model is the satisfaction of final demands during the drought. Table 6.3 shows the upper, lower and actual final demands for the year no. 9. It can be noticed that final demands for all agricultural sectors and food and kindred products were at their lower limit, with the exception of final demands for the forage sector which was reduced below the limit in order to assure feasibility in the linear programming solution. Final demands for all other sectors were at their upper bounds since they are very low consumers of water (directly and indirectly), and water has a large marginal value. Their outputs are not reduced because of the water shortage but because of the reduction in outputs of other sectors.

For the shortage in the year No. 10, all sectors are at their upper bounds in final demands with the exception of livestock, due to its reduced production and also to its reduced amount to final demands.

As a final comment it should be pointed out that some water storage is available for the region during the time of the projections, but the shortage occurred just before the storage units were scheduled to enter into operation, and the region could not benefit from it. The water storage availability will nevertheless ameliorate the expected damage, as can be seen from the results given in the next section. The importance of the alternative activities in ameliorating the impact of drought was investigated by running the hydrologic sample shown in Fig. 6.2 through the economic model without including the alternative activities. The results of this run are shown in Tables 6.4 and 6.5. Comparison of Tables 6.1 and 6.4 permits us to see that the application of the alternative activity of the second type to sector 4 allows avoiding a loss of $1,652,000.00 dollars of production in sector 4. The additional forage available, together with the alternative activity in sector 1, avoids a reduction in total gross output for the livestock sector of nearly five and a half million dollars. Comparison of Tables 6.2 and 6.5, shows that the net effect of the alternative activities was to reduce the income losses by about 36 percent for the year of the shortage and by about 25 percent for the whole period. Indirect income losses were reduced by about 50 percent. Finally, the introduction of alternative activities avoided adjustments in final demands for the livestock, dairy and the forage sectors.

6.4 Severity of Droughts Generated

The description made in the previous section included only one realization of the process representing the water availability and, even though an estimate of the probability of the duration of the shortage can be made, it is not possible to obtain the probability statements about the size of the shortage or the amount of damages without having recourse to the experimental method in generating many samples of size equal to the time horizon. The first question that arises when using the experimental method is the adequacy of the proposed model to reproduce the "critical observed droughts," i.e., the more severe droughts observed during the historical record. Several investigators claim, many times without sufficient arguments, Maldenbrot and Wallis (1969) and others, that short memory models such as the ones presented in this study to not reproduce the critical periods observed, mainly because they do not preserve the value of a highly controversial statistic called Hurst's "h", named after Hurst (1956). This statistic has a high sampling variability and there is no conclusive evidence, in spite of claims to the contrary, that hydrologic phenomena, in particular annual observations, actually exhibit the so-called Hurst phenomenon.

The historical drought is said to be reproduced when, after many generations of hydrologic samples with size equal the length of the historical record, the size and length of the largest drought is on the average equal to the size and length of the largest drought observed in the historical record. As it was demonstrated by Millan and Yevjevich (1971) for many rivers and precipitation stations around the world, the first-order linear autoregressive scheme is sufficient to reproduce the critical drought, both in length and in size, for annual observations. Moreover, the historical sample is only one realization of the process. To reject a generating scheme only on the grounds of its capability of reproducing the historical drought, is to neglect the possibility that the

TABLE 6.4

TOTAL GROSS OUTPUTS FOR PRODUCTION SECTORS DURING DROUGHT EXCLUDING ALTERNATIVE ACTIVITIES

Number Year 9	Original Sector	Type 3 Alternative	Type 2 Alternative	Type 1 Alternative	Total For Sector	Net Reduction
Sector 1	24702.	0.	0.	0.	24702.	8514.
Sector 2	2193.	0.	0.	0.	2193.	998.
Sector 3	5962.	0.	0.	0.	5962.	314.
Sector 4	8680.	0.	0.	63.	8748.	8748.
Sector 5	7139.	0.	0.	0.	7139.	292.
Sector 6	6353.	0.	0.	0.	6353.	367.
Sector 7	102920.	0.	0.	0.	102920.	105.
Sector 8	20441.	0.	0.	0.	20441.	3111.
Sector 9	35581.	0.	0.	0.	35581.	511.
Sector 10	166237.	0.	0.	0.	166237.	977.
Sector 11	33009.	0.	0.	0.	33009.	225.
Sector 12	93723.	0.	0.	0.	93723.	1233.
Sector 13	66462.	0.	0.	0.	66462.	6549.
Number Year 10						
Sector 1	31946.	0.	0.	0.	31946.	1809.
Sector 2	3246.	0.	0.	0.	3246.	4.
Sector 3	6330.	0.	0.	0.	6330.	6.
Sector 4	12468.	0.	0.	63.	12468.	521.
Sector 5	7569.	0.	0.	0.	7569.	7.
Sector 6	6872.	0.	0.	0.	6872.	-6.
Sector 7	103370.	0.	0.	0.	103370.	-90.
Sector 8	24066.	0.	0.	0.	24066.	40.
Sector 9	37281.	0.	0.	0.	37281.	-127.
Sector 10	172291.	0.	0.	0.	172291.	-51.
Sector 11	33902.	0.	0.	0.	33902.	-15.
Sector 12	98919.	0.	0.	0.	98919.	15.
Sector 13	81029.	0.	0.	0.	81029.	-6190.

TABLE 6.5

SUMMARY OF OUTCOMES FOR DROUGHT YEARS WITHOUT CONSIDERING ALTERNATIVE ACTIVITIES

Year	9	10	11	12	13	Total
Net income losses	8592.95	-668	13.37	-	-	7952.60
Water intake shortage	409 (28%)	52	(.5%)	-	-	461.
Water depletion shortage	291	35	-	-	-	316.

TABLE 6.6

LONGEST NEGATIVE RUNS FOUND IN THE HISTORICAL RECORD USED IN THIS STUDY

Truncation C_o in Acre-feet	$P(X \leq C_o)$	Longest Negative Run-Length Sample	Expected	Largest Negative Run-Sum in the Sample	Expected
5568098	0.4988	4	5.75	$4.136800\sigma_x$	$5.50\sigma_x$
5580964	0.5	4	5.75	$4.44190\sigma_x$	$5.50\sigma_x$
4909596	0.4	4	4.47	$2.54169\sigma_x$	$4.0\sigma_x$
4429305	0.3	4	3.47	$1.63012\sigma_x$	$2.86\sigma_x$
3929107	0.2	2	2.56	$0.87337\sigma_x$	$1.00\sigma_x$

historical sample contains droughts more severe than the average drought expected from such processes in the given period of time. If this actually happens, then preserving the historical drought would lead to a gross overestimation of the drought conditions. The problem is not simple and by no means has been settled, because many variables are involved and no general procedure to follow is available. For the time being, it is necessary to study each case in particular, and the following discussion is pertinent only to the example presented in this study.

The 49 years of virgin river flows available were searched for the critical droughts at several truncation levels, with the results presented in Table 6.6

By using Eq. 2.16 it is possible to compute the expected longest negative run-length for the statistics of the historical record as presented in Chapter V. Also, the size of the expected largest negative run-sum can be read from Millan and Yevjevich (1971), who obtained it by using the Monte Carlo method. Comparing results of Table 6.6, it can be readily verified that the sizes of the critical droughts will be a somewhat greater on the average than the historical one when using the autoregressive model. This shows that there is no danger of underestimating the critical drought for the example of this study.

6.5 Probability Analysis of Drought Impact

Preliminary runs, obtained by using historical annual flows, show that deliveries by the compact are a critical factor, and that for a particular year the total water in the river is not sufficient to meet those requirements. The main causes of this situation can be found in the particular set of flows delivered by

TABLE 6.7

PERFORMANCE OF FOUR POLICIES TO SATISFY THE COLORADO COMPACT

Compact Policy	Percentage of Infeasible or Total Failure Realizations
1	42.5
2	23.3
3	21.4
4	8.3

the compact during the ten years previous to the initiation of predictions with the model, and in the assumption that exports to the eastern slope of the Rocky Mountains were given the first priority.

Since the compact requirements were assumed to be allocated among the upper sub-basins based on average flows for each sub-basin with their total deliveries of 75 million acre-feet in a period of ten years, several other policies were simulated for comparison. Allocation of compact requirements among the upper sub-basins is not clearly described in the Colorado River Compact of 1948. It must be understood that the 75 million acre-feet must be delivered by the total of the three upper sub-basins, and that a complete treatment would require a multi-regional model, the simultaneous generation of river flows in the three sub-basins by preserving their cross-correlations, and the operation of the big storage units of Flaming Gorge and Navajo Reservoir in the Green and San Juan Rivers. This is an interesting study in itself, which is beyond the scope of this section included only to demonstrate the capabilities and operation of the model developed.

Keeping this limitation in mind, four different ways to meet the compact requirements were tested: (1) 75 million acre-feet delivered in each ten-year period are allocated among the upper sub-basins according to their mean flows: (2) the same requirements as under (1) but allocated among the upper sub-basins according to the annual flows that are exceeded 90 percent of the time, (3) 112.5 million acre-feet, delivered in 15-year periods, are allocated according to the mean flow; and (4) the same requirements under (3) but allocated according to the flows that are exceeded 90 percent of the time. More favorable policies for the U.M.S. are justified because it has more variability in flows and the smaller storage capacity of all three sub-basins.

A total of 1000 hydrologic samples, each 20 years long, were generated to test the above policies. Every year, the flows generated were corrected for the potential water depletions as shown in Fig. 6.1, and deliveries by the compact were computed. Every time that the total water available in the river was smaller than the water needed to satisfy the compact conditions in a sample was called an unfeasible realization and a counter was set up. Results are shown in Table 6.7.

An analysis of data in Table 6.5 clearly shows that the enforcement policy No. 1 results in total failure a large proportion of the time. Actually, the total number of failures will be somewhat smaller when adjustments and reductions in water depletion below the optimal are permitted in the model. However, they are still a problem because when distribution of losses is wanted a total failure is difficult to quantify. Based on these results, additional simulations were realized for policies 1 and 4. Policy 1 was chosen because it was more in accordance with the written compact and policy 4 was selected because it was felt that the large storage capacity of Lake Mead and Glen Canyon could permit periods longer than 10 years to provide the requirements of the compact. Additional numbers of policies could be tested once the assumptions behind each policy are stated clearly.

Distributions of the longest negative run in 20 years having as the truncation level the demand series were computed according to Eq. 2.22 and are shown in Table 6.8

As pointed out in Chapter II, when the demand series has a small trend, the probability distribution of the longest run-length can be very well approximated by the distribution of the longest run-length having as the truncation level the average

TABLE 6.8

PROBABILITY DISTRIBUTION OF THE LONGEST RUN LENGTH FOR THE TRUNCATION LEVEL EQUAL TO THE AVERAGE DEMAND

j	0	1	2	3	4
$P[\ell \leq j]$	0.607	0.960	0.447	0.909	1.000

demand over the time horizon. Simulation confirms this approximation for the case discussed here. By inspecting Table 6.8 it can be noticed that no shortage at all will occur with a probability of around 0.6. It remains to study the performance of the two chosen policies with respect to the compact.

Assuming again that all water exports to the Eastern Slope from the Upper Colorado River will be met and having unconstrained water demands for the region, a series of simulations was carried on to test the distribution of the longest run length of total failures to satisfy the compact, the distribution of the longest run length of years in which the compact constraint would be binding and the distribution of the number of such runs in a period of twenty years.

One thousand samples were generated and results are shown in Tables 6.11 and 6.12.

The following comments are pertinent with respect to the computed distributions. From the 1000 samples generated for policy 1, there are 537 that did experience compact constraint and 459 that did include a total failure. The remaining 78 samples experienced some sort of compact constraint that did not include a total failure and whose losses could be quantified. The number of samples which would have to be run through the model would be somehow different because of the adjustments to water storage and because of deficit conditions that did not result in compact deficits. What this pattern tells is that the distribution of total losses for policy 1 has two heavy tails, no loss and total loss, and a very weak part in between. Also the distribution of the number of runs shows that around 86 percent of the time there would be only one run or less of deficits to the compact in the 20 years studied.

To investigate the economic effects of the drought, 100 samples were generated and the corresponding results are presented in Table 6.9. Averages or variances are not computed because of difficulties in quantifying a total loss.

The losses are arranged in ascending order in Table 6.9; there were 38 samples for which no losses do occur, and 50 samples reporting a complete failure. With only 100 samples generated and no feasible way to quantify the losses due to total failure, it is difficult to put these results into a probability distribution of losses; nevertheless, some remarks can be made. As can be seen in Fig. 6.3, a good relationship exists between water deficit and total economic losses, as was expected. For small shortages, the indirect losses are negative but small, which can be easily explained by the additional income generated by the introduction of the new technology. There are 8 samples for which a shortage was declared in the preliminary computations that registered zero losses when the model was actually run. This was due to the effect of adjustments and to the consideration of storage, as the Curecanti unit was assumed to start operations in year No. 9 of the sequence.

Performing a similar type of analysis for the results obtained with compact policy 4, it was found that from the 1000 samples generated in the simulation only 126 indicated a total failure, while 153 showed the compact constraint binding. As expected, such a policy would be more feasible, because the present compact policy has a high risk, almost 50 percent, of failing to meet the compact requirements. Policy 4 would avoid a large reduction in the economic activity. The results of the economic analysis are summarized in Table 6.10. Out of 100 samples generated, 74 show no loss. From these 74 samples, 18 have shown the compact constraint binding or registered an intake deficit in the preliminary analysis. Under policy 4 only 11 samples register a total failure. It can be noticed by comparing Tables 6.9 and 6.10 that the small deficits are the same for both policies, implying that they are caused by a deficit in the water intake and not by the compact constraints. Also, much bigger values are obtained for some of the computed losses. These

TABLE 6.9

RESULTS FROM THE SIMULATION OF THE UPPER MAIN STEM SUB-BASIN ECONOMY FOR 100 HYDROLOGIC SAMPLES EACH OF 20 YEARS SIZE, AND COMPACT POLICY 1

Total Income Loss in Thousand of Dollars	Indirect Income Loss in Thousand of Dollars	Water Deficit in Thousand of Acre-Feet
22.95	-.23	2.45
745.09	-42.35	64.03
1229.92	141.98	114.32
1711.82	207.75	119.76
2177.34	-5.08	175.44
2650.21	349.32	182.01
6791.77	691.27	505.86
9517.48	1698.90	634.02
11464.19	1578.74	789.26
13866.25	2806.80	819.06
18179.13	3867.37	997.13
29356.90	6187.86	1691.57

TABLE 6.10

RESULTS FROM THE SIMULATION OF THE U.M.S. SUB-BASIN FOR COMPACT POLICY 4

Total Income Loss	Indirect Income Loss	Water Deficit
745.09	-42.35	64.03
1229.92	141.98	114.32
1711.82	207.75	114.76
2177.39	-5.08	175.44
2297.80	-40.70	188.08
2930.16	-48.23	238.31
4503.46	1.20	359.68
5343.12	724.77	453.50
9517.48	1698.90	634.00
15055.51	3349.90	849.78
22547.64	5400.44	1067.81
23372.92	7383.10	1087.02
36464.82	11103.05	1799.77
47094.08	13484.38	2179.12

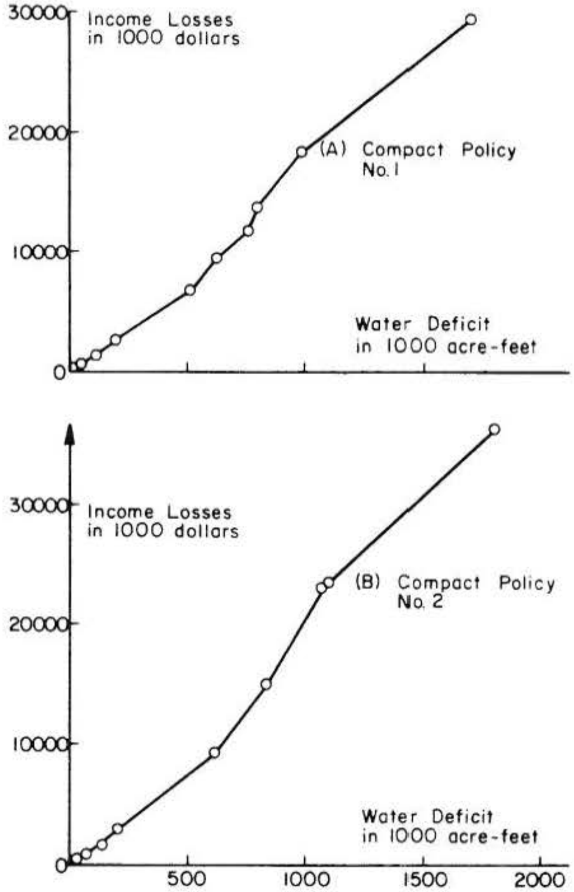

Fig. 6.3 Water deficit versus income losses.

samples would have been declared a failure when considering policy 1.

Computations are also made for the case in which the compact constraint is removed. Statistics were collected, however, on the amount of water that the sub-basin fails to deliver to the compact, on the number of runs of failure years, on the longest of such runs and on the starting year of the longest run. The frequency histograms of statistics are given in Figs. 6.4, 6.5, 6.6 and 6.7 with the following comments.

(1) In Figs. 6.4 and 6.5 there are 85 samples with zero losses and 43 samples with no compact deficits. From 15 samples that showed losses, only nine include also a compact deficit, as compared with the results of the previous constrained case. Also, from 57 samples showing compact deficits, only 49 may be classified as total failures when the compact constraint is enforced, and most samples show no water intake shortage.

(2) Table 6.8 shows a probability of about 0.6 for not having an intake deficit. This probability is increased to about 0.85 when the storage capacity of the Curecanti unit, of around 800,000 acre-feet is introduced in year No. 9 of the operation.

(3) Distributions of intake and compact deficits, as shown in histograms of Fig. 6.4, are approximately exponential. This should be expected, since the probabilities of small deficits are greater than of large deficits. The mean water intake deficit is small and its coefficient of variation is large, mainly due to the large probability of zero deficit. The average compact deficit is considerably greater with a smaller coefficient of variation. There is, however, a probability of about 0.97 that the total compact deficit during the 20 years period is smaller than the average flow available from the sub-basin in one year.

(4) Figure 6.6 shows the frequency histogram of the first year of the compact deficit. No deficit is observed during the first six years. This can be explained by the fact that the water delivered by the compact in the previous 10 years was well above the requirements. Otherwise, the first years of deficit seem to be uniformly distributed over the remaining period from year 7 to year 20.

(5) The distribution of the number of runs shows a probability of about 0.82 that there is only one run, and a probability of about 0.99 that there are at most 2 runs. Also, with 50 percent probability, the longest run will be smaller than 2. There is, however, a tendency to have long runs whenever a deficit occurs.

It is obvious from the previous analysis that there is a high risk for the U.M.S., of the order of 50 percent, of not being able to meet all projected local demands, export and compact requirements at least once in every 20 years studied. This risk highly depends, however, on the projected water export and on the decision on how to implement the compact agreement. A relaxation of the compact conditions leads to a remarkable reduction of the risk level without seriously affecting the downstream water rights, mainly because of the large storage capacities on the Colorado River. Figure 6.7 shows the histograms of the length of the longest run of water intake deficits and of the number of such runs in the 20 years considered. Results for the length of the longest run agree well with the theoretical distributions shown in Table 6.8. Furthermore, only 7 samples exhibited 2 runs, which was the largest number of runs observed in any sample. In this case, and in similar cases with low truncation levels, the longest or largest drought to be observed during the planning period may be the natural event responsible for most of the total water

TABLE 6.11

FREQUENCY DISTRIBUTIONS OF LONGEST RUN OF TOTAL FAILURES AND OF LONGEST RUN OF CONSTRAINED YEARS FOR COMPACT DELIVERIES UNDER POLICIES 1 AND 4

	Policy 1		Policy 2	
j	Total Failure	Compact Constraints	Total Failure	Compact Constraints
0	.541	.463	.876	.847
1	.088	.109	.035	.044
2	.088	.082	.023	.022
3	.058	.070	.016	.024
4	.053	.057	.017	.018
5	.036	.043	.012	.013
6	.034	.031	.005	.011
7	.029	.035	.006	.007
8	.018	.032	.007	.005
9	.024	.026	.003	.009
10	.010	.020	-	-
11	.008	.010	-	-
12	.003	.007	-	-
13	.008	.007	-	-
14	.002	.008	-	-

TABLE 6.12

FREQUENCY DISTRIBUTION OF THE NUMBER OF RUNS OF TOTAL FAILURES AND OF THE NUMBER OF RUNS OF CONSTRAINED YEARS FOR COMPACT DELIVERIES UNDER POLICIES 1 AND 4

	Policy 1		Policy 4	
j	Total Failure	Compact Constraint	Total Failure	Compact Constraint
0	.541	.463	.876	.847
1	.348	.400	.109	.130
2	.092	.119	.014	.021
3	.018	.015	.001	.002
4	.001	.003	-	-

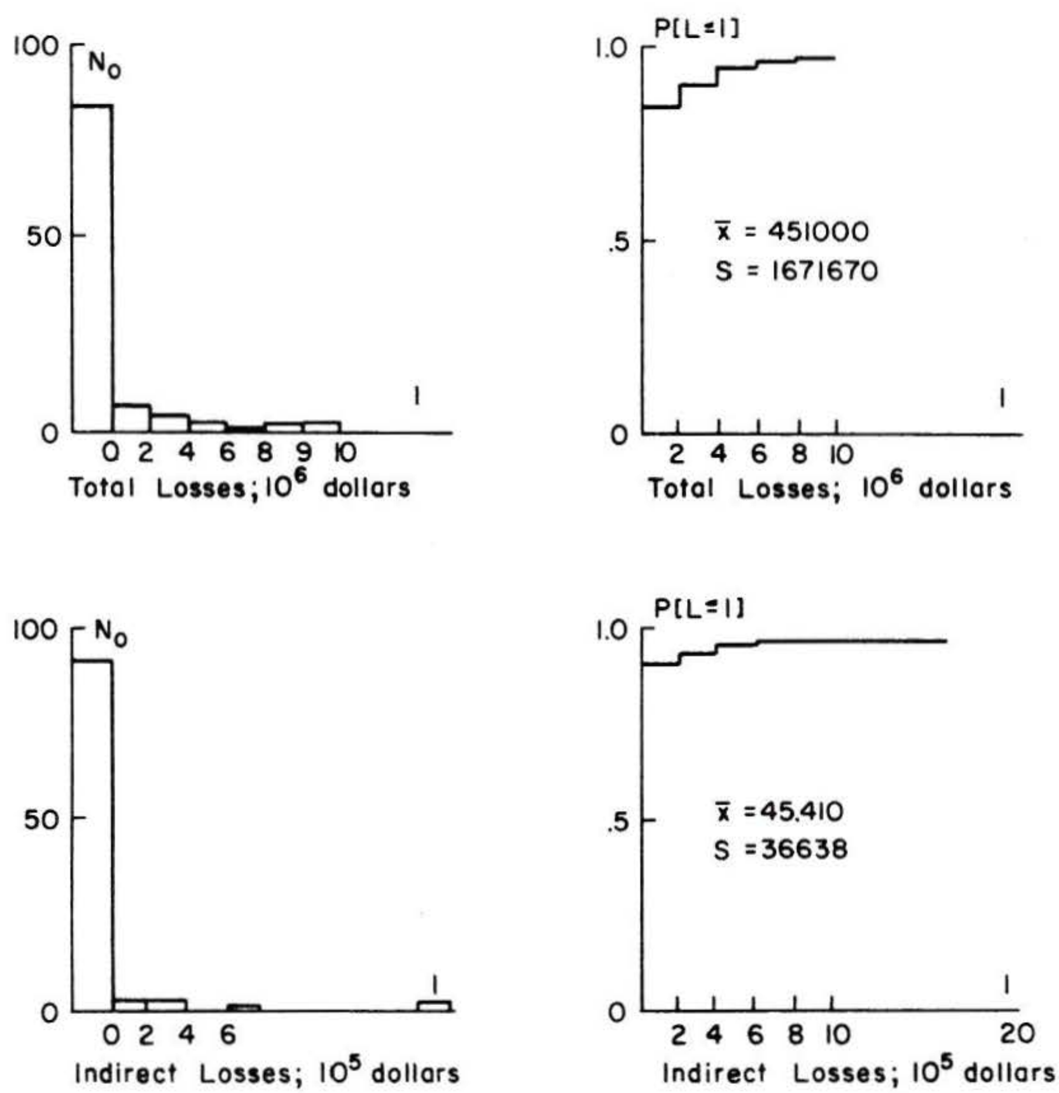

Fig. 6.4 Histograms of total and indirect losses.

deficit. A check was made on the percentage of the total income losses generated during the critical drought, and it accounted to nearly 99 percent of it. The impact of the smallest drought, whenever it occurred, was minor.

For higher levels of development and different storage capacities the relative impact of the largest drought may be smaller, but it will still be of interest to identify the characteristics of the most severe condition the system could experience.

An appraisal of the results obtained is warranted with suggestions for some further research. It is necessary to generate more hydrologic samples in order to determine more accurately the distribution of losses for any policy to be implemented. The number of samples that can be generated is limited, however, by the limitations of computer time available. Also of importance is the quantification of the total loss. A minimum amount of water will be always consumed by the local economy, regardless of the conditions of the compact constraint, while at the same time keeping the track of deficits in water delivered to downstream users. Another alternative would be to allocate part of the deficit to the export to the eastern slope, to keep a minimum consumption in sub-basins and to allocate the rest of the deficit to the downstream users.

The implementation, however, of any of the allocations proposed above raises some practical problems which involve decision making and political processes at the highest level. The main purpose of

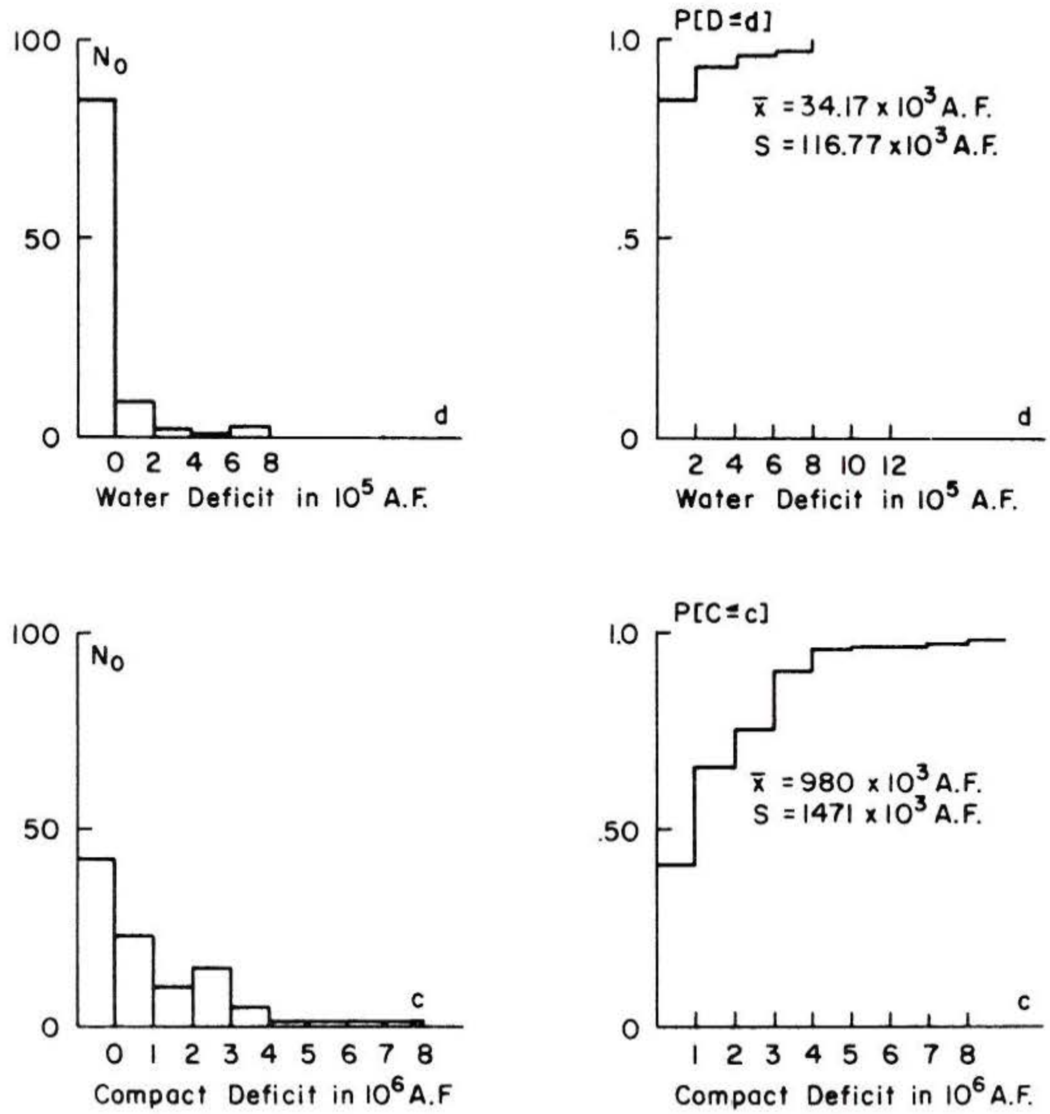

Fig. 6.5 Histograms of intake and compact deficits.

this study has been a presentation of techniques that could serve the analysis of this type of problem and at the same time to show its applicability to a particular region. It is beyond the scope of this study to try to determine certain kinds of value judgments necessary in any analysis of this type, particularly those which are the prerogative of a decision maker. Only after the "rules of the game" have been clearly determined by the decision maker and the formulation of the model has been accepted, or at least understood, the model builder can present to the decision maker a set of results that can answer some of the questions. The results obtained so far in this study show the methodology presented to be feasible, and that the model does in a reasonable way what originally was intended. The methodology is sufficiently flexible to permit the testing of the impact of different policies and the sensitivity of results to basic assumptions and to the effect of model parameters.

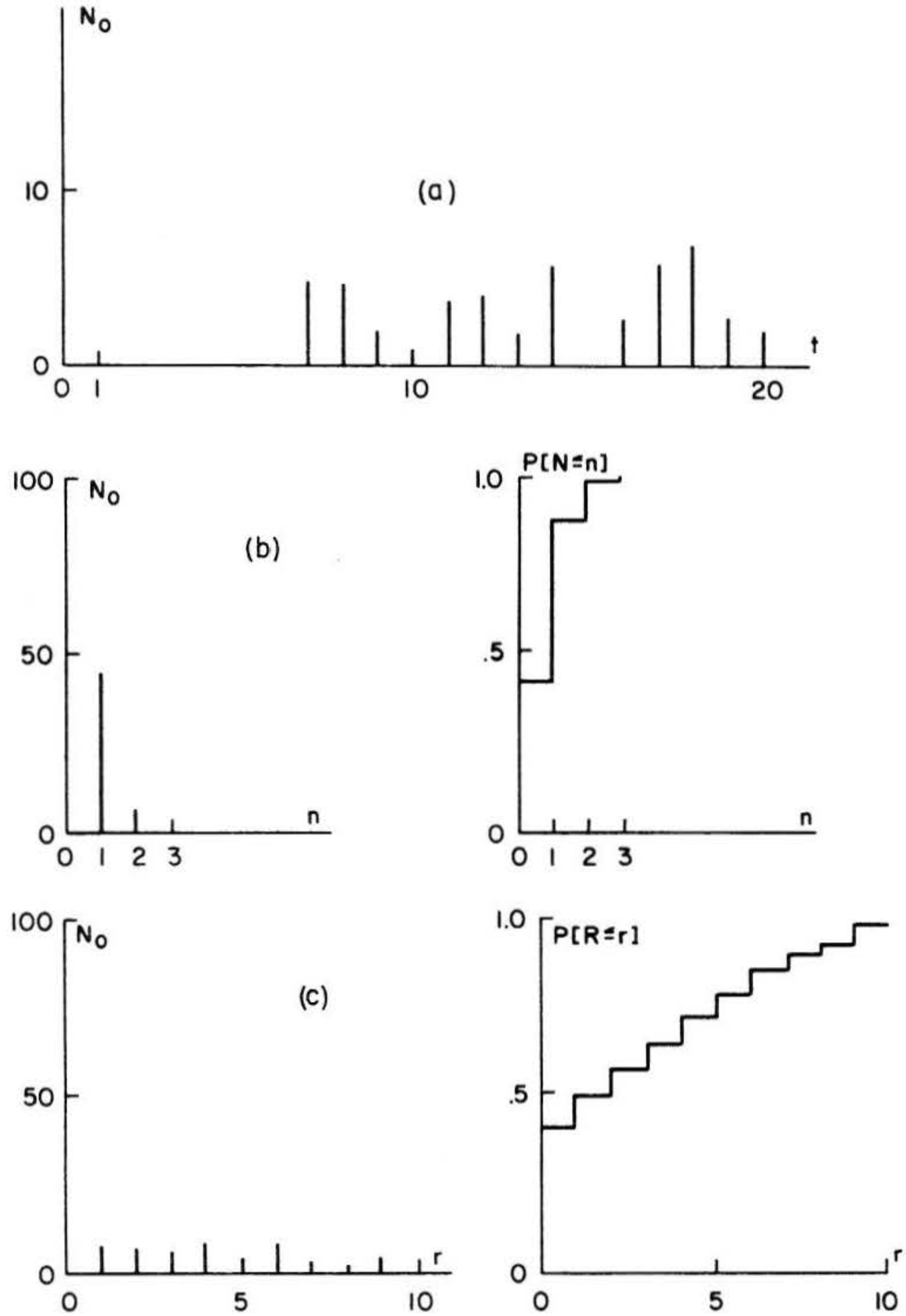

Fig. 6.6 Histograms of runs of compact deficits, (a) initial year of deficit, (b) number of deficit runs, (c) longest deficit run.

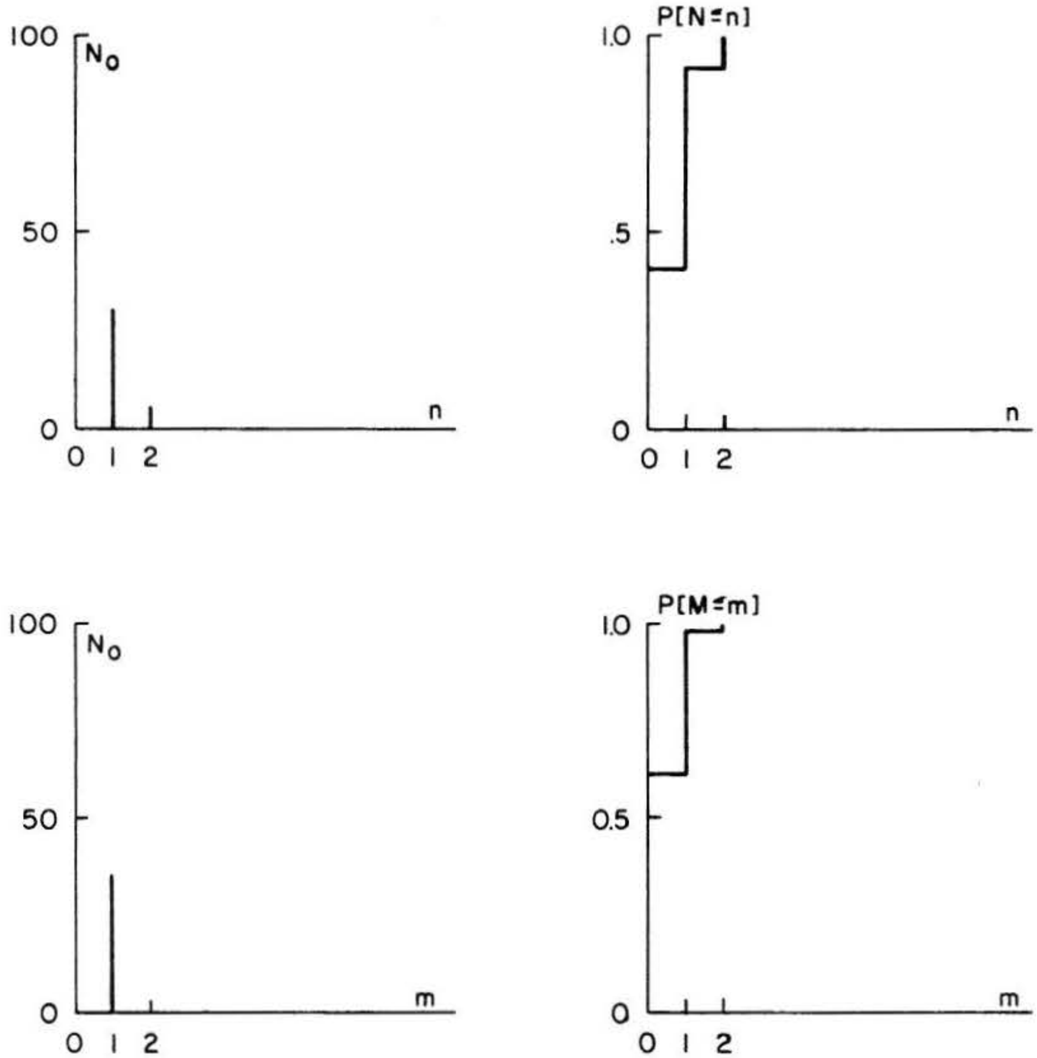

Fig. 6.7 Histograms of the number of intake deficit runs (upper graph) and of the longest intake deficit run observed (lower graph).

CHAPTER VII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

7.1 Summary

The characteristics of drought as a natural event and as a hazard to the regional economy are studied. Runs as statistical properties of sequences are used for the definition of droughts. The probability distribution of the length of the longest run to be found in a sample size N is presented for time series for which the occurrence of a dry or a wet year in the year t may be expressed only as a function of a wet or a dry year in the year t-1. The solution obtained in this study is presented as a good approximation for the case of series which follow the first-order linear autoregressive model of dependence. An approximation is also suggested for the case of truncation level in the form of a linear trend.

It is not easy, even for the more simple cases, to obtain analytical distributions of the water deficit volume and of losses they cause to the economy. It is possible, however, to obtain good estimates of these distributions by using the experimental or Monte Carlo method in generating a large number of hydrologic samples of size N in conjunction with a simulation of the regional economy.

Regarding the severity of droughts generated by the experimental method, three situations may result when they are compared with the historical drought: the historical drought is at the lower tail of distribution of the largest droughts to be found in a sample of size of the historical record; the historical drought is in the middle region of this distribution; and the historical drought is at the upper tail of it. In the first case, the use of experimental method resulted in the generation of droughts which are more severe than the historical drought, while in the third case it resulted in the generation of less severe droughts. No general agreement exists among hydrologists about the properties of droughts that should be reproduced by a generation method; in the absence of a generally accepted methodology, each case should be studied as a particular case. For the time series of river flows analyzed in this study, it was found that a simple model, the first-order linear autoregressive model, reproduces well not only the structure of the historical time series of annual values, but also reproduces the critical drought characteristics. This should be expected for annual series of precipitation and runoff with relatively small time dependences. In case a more sophisticated model is required to preserve the historical drought characteristics, a study of physical background for the model, together with a regional study of historical droughts, should be made before a decision on the model is taken.

The scheme selected to model the complex interdependences among the existing economic sectors shows a satisfactory performance in the case of the selected region. Given the limitations of data availability and the restrictive assumptions imposed on the model, a special procedure is adopted in allocating the water shortage and in measuring the losses in a consistent way. Also, the introduction of alternative activities permits a relaxation of some rigid assumptions of the originally conceived model. Of particular importance is the consideration of losses over the complete time horizon of projections, thus permitting the incorporation of time dimension.

The joint use of the Monte Carlo method for water availability and the simulation of the regional economy permitted running many drought conditions through the model to find their corresponding impacts. The results shown, however, are valid only under the conditions of economic data and assumptions applied in this study. Many parameters in the model were estimated only crudely, while some other parameters are subject to a high uncertainty. Though it is possible to treat several characteristics of the economic model by a stochastic approach, the complexity of the model increases so much that it would be difficult to analyze the additional information. What is usually done in this case is to test the sensitivity of the results to changes in the basic assumptions and in parameters used to define some weak points of the model.

Finally, it should be emphasized that this model, like any other model of its kind, operates under some highly restrictive assumptions and its results must be viewed with caution. However, it does include a series of important factors neglected in previous studies of the same type, and provides the basis on which to develop a more rational assessment of the impacts of water shortages at the regional level.

7.2 Conclusions

The following conclusions may be drawn for the case example studied:

(1) The risk of a water deficit for at least one

year of the time horizon is relatively small, compact requirements aside. When both the compact requirements and the water export projection are introduced, the risk of a deficit increases to about fifty percent. This result, however, is based on what perhaps unrealistic assumptions regarding the allocation of the compact requirements among the upper sub-basins. A more comprehensive analysis would require a joint model of the water supply from all three upper sub-basins.

(2) Given a very large total storage capacity in the Colorado River Basin, (35.2 million acre-feet in the upper region of the basin only), more flexible compact rules are feasible. As an illustration, a compact rule permitting delivery of an average of 7.5 million acre-feet per year over a span of 15 years instead of 10 years reduces the risk of failure of water supply in the Upper Main Stem of the Colorado River to a tolerable level. Moreover, the reduced water export projections, or a simple allocation of water shortage also to export reduces this risk even further.

(3) For a given economic projection in the Colorado River water using regions, several policies regarding water allocation can be tested by using the model presented in this study, or with an extension of it. The methodology is sufficiently flexible to permit an evaluation of the impact of different policies once the rules governing the allocation of drought shortages were defined.

(4) The consideration of new storage capacity (the Curecanti unit) reduces the risk of water shortage from an original 40 percent to only 15 percent. Also, the introduction of new canal lining technology and other alternative activities contributes to alleviate the shortage in case of droughts.

(5) The implementation of the model was based mostly on published data, some of them of questionable reliability. Economic surveys of the region would help obtain some specific data, especially those regarding capital investment and water technologies. The projections of final demands should also be the subject of a more detailed economic study.

(6) Though drought shortages are allocated mainly according to the income maximization principle, the existence of constraints permits modeling particular conditions that can introduce rather substantial changes to the market allocation.

7.3 Recommendations for Further Research

The present study opens several possibilities for further research:

(1) In the field of probability theory, it may be of interest to give a more rigorous treatment to distributions of runs for non-constant truncation level.

(2) In the field of stochastic hydrology, the distributions obtained for the critical drought could be used to derive a criterion that may help in the selection of the hydrologic model or a better estimation of its parameters.

(3) The basic methodology presented in this study should be subject to a detailed sensitivity analysis that can permit the reduction of both the number of constraints and the number of parameters, and by extending it to the multiregional case.

(4) The major conclusion of the case study, admittedly conditioned by the assumptions of the model, do indicate that there is a very high risk of not being able to meet the compact requirements with the projected level of local developments and water exports to the Eastern Slope of the Rocky Mountains. These results imply that either the water export projections or the rules imposed by the compact are unrealistic if all interests for the Colorado River water are taken into account. The model presented here may provide a basis for better planning the allocations of the water of the Upper Main Stem of the Colorado River by testing the drought impact of alternative policies.

(5) As is the case with most regional studies concerned with interindustry analysis, many questions are still unsettled regarding the level of aggregation in the economic sectors and the stability of projections in the face of changing trade patterns. Also, it may be profitable to perform research by introducing the new technologies of production following the lines suggested by Carter (1970), and by using, to a limited extent, the new irrigation technology.

(6) Other resource limitations, such as land and labor constraints, were ignored in the analysis. They can be introduced in the model whenever they are important in modeling the regional economy.

(7) Finally, the only losses discussed in this study were losses directly associated with the quantity of water consumed. Nothing is said about its quality or about non-consumptive uses such as recreation. Economic losses due to degraded water quality during drought may constitute, in some cases, a sizable share of the total losses. Water oriented recreation constitutes an important part of the economy of Western Colorado and the occurrence of drought may have an appreciable impact. If the above mentioned factors are ignored, the estimated of total losses from drought for the Upper Main Stem of the Colorado River may have a negative bias. Inclusion of environmental effects of this kind in drought damage evaluation open a challenging area of future research.

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APPENDIX A

DESEGREGATION OF LIVESTOCK, DAIRY AND FORAGE SECTORS

Lack of adequate economic data in the study of the Upper Main Stem of the Colorado River does not permit a good desegregation of the economic sectors of livestock, dairy and forage. It was necessary, therefore, to rely on indirect methods based on secondary information from census data, from Bureau of Reclamation projects, and on judgments of experts who worked in this region.

Total Gross Output for New Sectors. The first attempt was to determine the total gross output of the new sectors. To accomplish this approach three independent procedures were applied.

Procedure 1 used information from the U. S. Bureau of Reclamation (1963) study on the Grand Valley trade area, composed of Delta, Mesa and Montrose counties, where more than 70 percent of the economic activity of the region is concentrated. The following are the total gross outputs obtained:

Forage	$ 4,540,685
Dairy	2,037,890
Livestock	– 18,037,511
	$ 24,616,086

These figures give a value of about 20 percent of the combined activity of pasture and forage for the whole region. By multiplying this percentage by the total equivalent for the three activities in the whole region, then $35,499,000 x 0.20 = $7.2 millions for forage, of which approximately $6.3 million should be allocated to livestock. With this value, an unrealistically large water depletion coefficient is produced for the range livestock and dairy sectors, when the total water consumption given by Udis (1967) is allocated to these three sectors.

Considering livestock and forage as separate sectors, a depletion water coefficient of about 75 gallons per dollar of output for livestock and around 20,000 gallons per dollar for forage can be assumed, based on data for other sub-basins in the Udis study. Solving for water depletion coefficients using the total amounts of output given before results in a depletion coefficient for forage of around 35,000 gallons per dollar of output, which is a relatively large requirement.

Procedure 2 assumes 23,000 gallons per dollar of output as a reasonable magnitude for the water depletion coefficient in the region, given the relative inefficiencies in the region. The total gross output for forage is obtained by solving the following equation,

$$W_{\ell+f} X_{\ell+f} = W_{\ell}(X_{\ell+f} - X_f) + W_f X_f \quad , \qquad \text{(A-1)}$$

in which $X_{\ell+f}$ is the joint output of the combined sector X_f is the output of the forage sector and W refers to the water coefficients. Equation A-1 is a balance of water used by the joint sector as distributed among the sectors considered separately. In a similar way, equation A-1 can be solved for the joint sector of forage and dairy, by replacing the livestock sector in eq. A-1 with corresponding values of the dairy sector.

The solution of eq. A-1 for the joint range livestock and dairy sector produces $9.80 million and $1.42 million as the total gross output of the forage sector included in the range livestock sector and in the dairy sector, respectively.

Finally, procedure 3 estimates total value of production from forage based on the total acreage for the year 1960, Anderson (1967), and on the average production per acre as given by the Bureau of Reclamation (1965) for selected projects in the region. The computed values are shown in Table A.1.

A discussion of the above estimates for selecting the most correct value leads to the conclusion that the first estimate is too low, because the Grand Valley trade area contains most of the feeder livestock in the region. Feeder livestock uses sources of feed other than forage so that it has a higher estimate. The first estimate will be somewhat higher if the yields given in the Bureau of Reclamation projects are above the average for the total region. Based on these considerations, the second estimate is selected for the study

Allocation of Row Transactions. The revised sales are determined from the old livestock and dairy sectors by computing what of their proportions can be allocated to the forage sector. Table A.2 shows these sales for old and new sectors. The explanation of this table is as follows: (1) sales from livestock to

TABLE A.1

VALUE OF FORAGE PRODUCTION IN THE U.M.S. SUB-BASIN

Crop	Acres in 1960	Value per Acre	Total Production in $\$10^6$
Alfalfa	115,705	70	8.00
Other hay	90,000	30	2.70
Irrigated pasture	142,000	16.50	2.34
Total			13.04

livestock amount to $5,538,000 in the old sector of which $2,780,000 are sales to range livestock for feed and livestock, Anderson (1967, pages 43,..), and $2,758,000 for sales to feeder livestock of which only a "limited amount" is for forage. Assuming arbitrarily that this amount is of the order of 15 percent, and assuming further that the amount of range livestock purchases from range livestock as forage is about 45 percent, using the same weight that is given in the statement by Anderson (1967), a figure of $3,598,000 is obtained as the amount sold by the livestock to the new livestock sector.

(2) Sales from old livestock L^o, to dairy, according to Anderson (1967), consist of forage.

(3) Sales of L^o to food and kindred products are 100 percent livestock.

(4) Sales from L^o to other agriculture are 100 percent from the forage sector.

(5) Sales from L^o to services are forage products.

(6) Deliveries of L^o to final demand are discriminated as $1,152,000 in payments for agricultural conservation practices and subsidies, which can be allocated to forage and the rest to livestock sector.

(7) Sales from dairy (sector 2) to livestock (sector 1) are small and can be allocated mainly to forage, as are sales from dairy to other agriculture (sector 6).

(8) Sales from dairy to food and kindred products (sector 8) are 100 percent dairy.

(9) Deliveries from dairy to final demand are mostly dairy products, with the exception of only $35,000 worth of products to be allocated to the forage sector.

Allocation of Column Transactions. Once the allocation of rows is completed the values of the total gross outputs for the three new sectors can be determined. It remains to allocate the columns in such a way that the total gross outputs from these columns check with the total gross outputs obtained from the rows. Identification of transactions of processing sectors are made in the same way as transactions for the rows with one important addition. In order to obtain the same row totals the balance is made with the income sector or the sector formed by the profits and wages sectors. The allocations are made according to the following comments pertinent to Table A.3.

(1) Allocations for sector 1 and 2 are the same as obtained for the rows.

(2) The amount sold by sector 3 (food and field crops) goes entirely to livestock.

(3) Amounts sold by forage to livestock and dairy can be obtained once the respective amounts of forage sales, including the old livestock and dairy sectors, are subtracted from the forage total gross output obtained in the previous section.

(4) Sector 5 (fruits) does not sell to forage, which is also valid for sector 6 (other agriculture) and sector 8 (food and kindred products).

(5) Sales from manufacturing to the forage sector are assumed to be in the same proportion to total gross output as the sales to other related sectors, such as feed and field crops.

(6) Remaining allocations are based on ranch budgets for Western Colorado obtained from Henry Gronewoller (1971) (extension economist, farm management Colorado State University). The following percentages are obtained for sectors shown in Table A.4.

TABLE A-2

DESEGREGATION OF TRANSACTION TABLE ROWS

Sector	1	2	3	4	5	6	7	8	9	10	11	12	13	Final Demands	TGO
1. Livestock old	5538	64	0	0	0	350	0	3459	0	0	0	40	0	22843	32294
2. Livestock new	3578	0	0	0	0	0	0	3459	0	0	0	0	0	21691	28748
3. Dairy old	160	0	0	0	0	285	0	3055	0	0	0	0	0	705	3155
4. Dairy new	0	0	0	0	0	0	0	2055	0	0	0	0	0	670	2725
5. Forrage	8260	1000	0	0	0	635	0	0	0	0	0	40	0	1187	11122

TABLE A-3

DESEGREGATION OF TRANSACTION TABLE COLUMNS

	1	2	3	4	5	6	7
	Livestock Original	Livestock Desegregation	Old Dairy	Dairy Desegregation	Forage From Livestock	Forage From Dairy	Total Forage
1. Livestock	5538	3598	64	0	0	0	0
2. Dairy	160	0	0	0	0	0	0
3. Food and Feed	43	43	0	0	0	0	0
4. Forage	0	8260	0	1000	0	0	0
5. Fruits	79	79	53	53	0	0	0
6. Other Ag.	9	9	1	1	0	0	0
7. Mining	0	0	0	0	0	0	0
8. Food and kindred	648	648	136	136	0	0	0
9. Manufacturing	485	22	70	1	463	64	532
10. Trade and Transportation	1582	990	319	188	592	131	723
11. Utilities	178	88	49	23	90	26	116
12. Services	1950	1102	407	216	848	191	1039
13. Construction	0	0	0	0	0	0	0
14. State and Federal	684	832	15	105	1588	206	1794
15. Local	1736		296				
16. Wages	3321	2411	123	89	910	34	944
17. Profit and other income	10330	7244	844	637	-	-	1873
18. Inventory change	413	413	0	0	0	0	0
19. Depreciations	2463	1330	457	242	1133	215	1348
20. Imports	2675	139	321	35	2536	217	2753
21. TGO	32294	28748	3155	2725			11122
Income Coefficient*	.425	.390	.305	.266			.254

*Columns 16 and 17 divided by TGO

TABLE A.4

ALLOCATION OF COLUMN TRANSACTIONS AMONG LIVESTOCK AND FORAGE SECTORS

Sector	Percent Livestock	Percent Forage
10 Trade and Transportation	59	41
11 Utilities	48	52
12 Services	53	47
14 and 15 Government	34	66
16 Wages	72.5	27.5
19 Depreciation	53	47
20 and 21 Imports	10	90

KEY WORDS: Drought, economy, water demand, water supply, water availability, objective function, reservoir, rate of growth, cost production, planning.

ABSTRACT: The characteristics of drought as a natural event and drought as a hazard to the regional economy are studied. Runs as statistical properties of hydrologic sequences are used in an objective definition of droughts. The probability distributions of the longest negative run-length to be found in a sample of size N are reviewed and analytically defined for some simple cases of time dependence. An approximation is introduced for the case of the truncation level being of a linear trend type. The Monte Carlo method in generating large numbers of hydrologic samples is used in conjunction with a model of the regional economy to determine the economic impact of droughts. A programming formulation of a dynamic type interindustry model is used to simulate the regional economy over a selected time horizon in order to

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