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Insecure Debt

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Key words: Repo credit, bank runs, asset liquidity risk.
JEL classification: G01, G21, G28.

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Abstract

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1 Introduction

During the recent US financial boom, credit expansion was boosted by strong demand for safe assets (Caballero and Krishnamurthy, 2009). As savers were willing to pay a safety premium, safe debt was cheap and thus a desirable source of funding (Krishnamurthy and Vissing-Jorgensen, 2012). Banks and shadow banks responded by issuing safer liabilities, such as more short term commercial paper, as well as debt secured on financial collateral, known as *repo*. Once credit and liquidity risk became apparent, intermediaries suffered massive outflows of unsecured debt, which forced fire sales of illiquid assets. In contrast, repo credit raised haircuts but was mostly rolled over up to the eve of default (Gorton and Metrick, 2012; Krishnamurthy, Nagel and Orlov 2012). After Lehmann's default, unsecured lenders suffered heavy losses, while repo lenders were able to repossess and sell the pledged collateral. The induced fire sales played a critical role in propagating distress.

The experience of the crisis has led to sharper scrutiny of liquidity risk, such as the funding of illiquid securitized assets with short term debt and secured financial credit.[1] In this paper we ask two questions. Is there a direct effect of asset liquidity risk on runs, next to fundamental risk? With hindsight, MBS prices fell way too low given real credit losses, as mortgage assets proved too illiquid to be backed by unstable funding. Second, how does the pledging of collateral to repo lenders affects other funding sources? Secured funding proved more stable than unsecured funding. While its role in triggering collateral sales in default is now well recognized (Stein, 2012; Infante, 2013), its direct effect on unsecured debt is not well understood.

We study run equilibria within a global game setting (Morris and Shin, 2003; Goldstein and Pauzner, 2005), introducing asset liquidity risk next to fundamental risk. To model asset liquidity, we introduce a precise characterization of the process of bank default. Traditional models assume all assets are sold immediately to satisfy withdrawals (Diamond and Dybvig, 1984). In reality, once a bank runs out of liquid assets it is forced to declare default, triggering a mandatory stay on remaining lenders to allow orderly liquidation of illiquid assets.[2] Recognizing this feature leads to a novel analysis of run incentives. We show that asset liquidity risk causes a run equilibrium to arise, even if fundamental risk is arbitrarily small. In other words, in this setup almost all runs may be inefficient. The effect of more liquid assets on stability is interesting. As it reduces the chance that the bank runs out of liquidity to repay withdrawals, it favors rollover. However, there is also a relative payoff effect, as both the chance of full repayment in a run as well as the expected repayment after rollover rise. When liquid assets

[1] Next to repo, secured financial credit includes margins on derivative positions.

[2] Bankruptcy law was introduced to solve the free rider problem when all creditors grab assets in an uncoordinated fashion, destroying value in the process.

are low, the relative payoff effect dominates, leading to more runs. This reflects the effect of mandatory stay in bankruptcy, since illiquid assets are not paid out preferentially to those who withdraw early. At some point recovery in bankruptcy becomes very high (the equivalent of better fundamentals), so the probability effect becomes predominant and the frequency of runs decreases . This produces a concave, inverted U-shaped relation between asset liquidity and run frequency. While the bank may reduce instability by offering a high rollover premium, in general it will not choose to leave rents to depositors. Next we introduce secured debt as a funding choice. Once enough liquid assets are securitized and pledged, repo is safe even in default, so it is always rolled over. In the absence of strategic complementarity, insuring risk intolerant lenders is efficient as it reduces funding costs. However, now unsecured debt bears more risk, requiring a higher promised yield. Our contribution is to show that concentrating asset liquidity risk on unsecured debt increases the change of costly runs, unless compensated by a much higher yield. A social planner will reduce the frequency of inessential runs by leaving maximum rollover rents to unsecured creditors. In contrast, a private intermediary will tend to minimize funding costs, while choosing the maximum volume of secured debt. As a result, the private choice of repo debt results in more inefficient runs and default risk than the social optimum. Granting greater security to some lenders is temptingly cheap, but makes other lenders more insecure. [3]

This direct risk effect adds to the known externality associated with repo's fire sales of seized collateral upon default .[4] In our setup there is no externality effect on the private choice of secured funding, which is already at its maximum. However, a higher run frequency that reduces collateral liquidity will induce higher haircuts. We show that this further concentrates risk on unsecured debt, again increasing the frequency of runs.

In conclusion, unregulated secured funding adds risk by causing more runs and costly liquidation, a loss to be traded off against its lower cost. The increased instability creates larger deposit insurance losses, because of repossession of safe assets as well as a higher run frequency.

An implication is that regulatory policy should monitor and constrain the scale of secured funding, in order to reduce instability. On the other hand, a complete welfare assessment of secured debt should take into account its role in satisfying a strong demand for safety. We sidestep this issue by assuming that agents have a safe storage option, but the issue becomes salient in a situation of excess demand for safety. Attracting very risk averse agents at a low cost may increase the scale of investment, both by increasing the scale of funding and

[3]Repo debt represents a reduction in liquid collateral available to withdrawers, so its effect on runs is also nonmonotonic.

[4]On the financial and legal incentives to quickly resell seized collateral, see Perotti (2011) and Duffie and Skeel (2012)}.

reducing the marginal required rate of return. The model has clear limitations as it does not seek to endogenize all features of the intermediary. As most contributions in this literature, we take demandable debt as given. The existence of mandatory liquid reserves is assumed, a most realistic assumption for a bank. These features suggest some unmodeled causes, such as liquidity needs. Introducing a contingent liquidity demand would considerably complicate the analysis while not significantly altering our results.

Related literature

The model analyses asset liquidity risk in the context of bank run models based on Goldstein and Pavner (2005). In their unique equilibrium, run incentives reflect some chance of fundamental risk. Yet many runs are inefficient, as a result of strategic complementarity (Diamond and Dybvig, 1983). We extend this result to the case of risk of large losses in case of early liquidation of bank assets. While some amount of fundamental risk is essential for our result, as it vanishes there remains a positive frequency of inefficient runs. Intuitively, adding interim asset liquidity risk increases the chance that depositors coordinate on a run.

The notion of asset liquidity risk here implies a distinct risk factor, weakly correlated with fundamental risk. The natural interpretation are shocks to adverse selection or to the liquidity available to market participants that are weakly related to the credit risk of bank assets.

The literature explains repo funding in terms of a strong demand for absolute safety, an insight at the heart of recent work on instability and safety demand (Gennaioli et al., 2013; Gorton and Ordoñez, 2014; Caballero and Fahri, 2013; Ahnert and Perotti, 2014.)

Existing work on repo credit (Martin et al 2012, Oehmke, 2014) study the dynamics of repo runs, but does not compare it with debt of the same maturity, so there is no direct interaction effect. These models study the effect of collateral liquidity, while we focus on the rapid sale value of illiquid bank assets. He and Xiong (2011) provide a dynamic model of runs when debt is staggered, where creditors' roll-over decision depends on beliefs about other creditors' subsequent roll-over choice. Kuong (2013) considers the case when unsecured debt responds to higher repo margins by demanding higher required return, and shows that the resulting higher leverage directly affects risk taking by borrowers. Auh and Sundaresan (2014) looks at the effect of repo illiquidity risk on long term debt. In our set up, repo emerges as the preferred choice by investors seeking absolute safety. Our results do not depend on secured debt being demandable, as it is designed to be absolutely safe even in a run. Martin, Skeie and von Thadden (2013) propose that secured credit arise when asset values are non verifiable. Auh and Sundaresan (2014) argue that repo funding demands collateral to avoids violations of absolute priority.[5] They show that a bank may issue repo loans to save on the cost of long

[5]It may be ex post efficient to violate absolute priority, e.g. to ensure proper continuation incentives.

term debt, but will not issue too much when collateral liquidity is low.

Gorton and Ordoñez (2014) elaborate on the insight that information-insensitive claims arise to overcome adverse selection (Pennacchi and Gorton, 1999). Runs triggered by collateral illiquidity may be triggered when it become information sensitive.

A key driver is that investors seeking absolute safety are willing to pay a safety premium. Such a strong investor preferences for safety has now been documented extensively (Gorton Lewellen Metrick (2012), Krishnamurthy Vissing-Jorgensen (2012)), and is leading to a new view of risk attitudes. Recent models also assume that a subset of agents act as (locally) infinitely risk averse (Caballero Fahri (2013), Gennaioli et al (2013)). [6]

Our direct effect of secured credit on stability complements the risk externality resulting from the special bankruptcy treatment for collateralized financial credit. The "safe harbor" status creates a proprietary right directly enforceable on assets, and avoids risks such as excessive issuance or imperfect enforcement that may dilute a claim value. The ability of secured financial creditors to gain immediate access to the collateral is a unique privilege, as it exempts them from mandatory stay.

Legal scholars question whether it is justified to grant superior bankruptcy privileges to repo and derivatives (e.g. Morrison, Roe and Sontchi 2015). Bolton and Oehmke (2011) shows it leads to risk shifting incentives with derivatives. Duffie and Skeel (2012) argue that only cash-like collateral should be excluded from automatic stay, in order to reduce the risk of fire sales. [7] Limiting asset eligibility for safe harbor would limit the scale of encumbered assets and thus their direct effect on instability.

Hanson, Stein, Shleifer and Vishny (2014) show how traditional banks are best at funding less risky but less liquid projects, while shadow banks promise liquidity by pledging liquid assets. In practice the distinction is not sharp, as repo funding issued by commercial banks can be quite significant, also because of central bank refinancing. The degree of balance sheet encumbrance is thus a key stability question for banking supervisors.

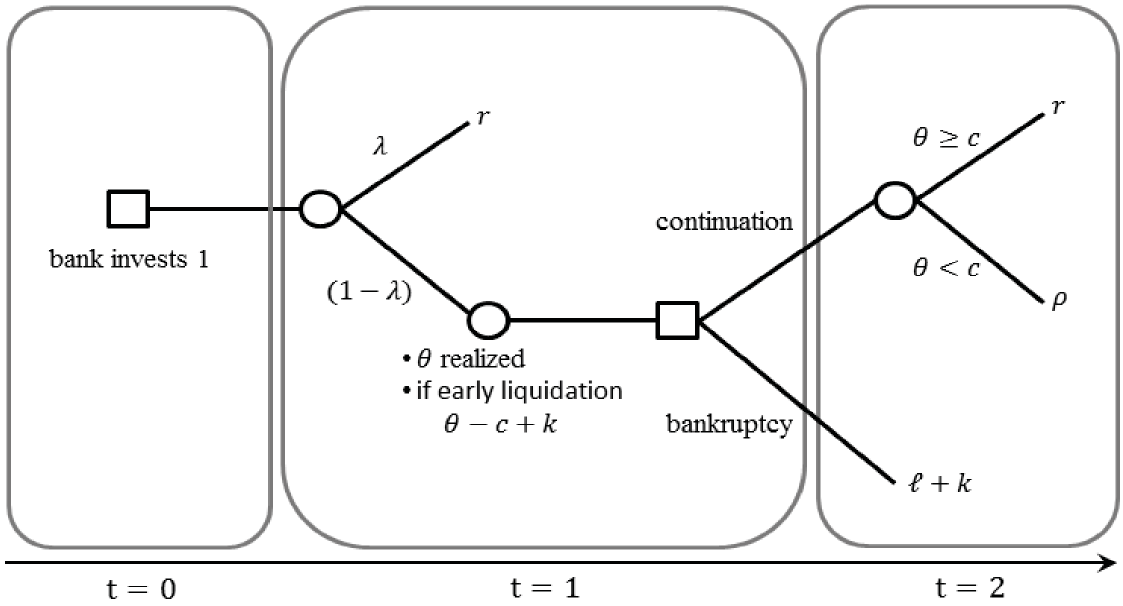
2 The Basic Model

The economy lasts for three periods $t = 0, 1, 2$. It is populated by a bank and a continuum of lenders indexed by i. The intermediary has access to a project that needs one unit of funding in $t = 0$. It raises funds from risk neutral lenders, each of whom is endowed with one unit. Lenders demand a minimum expected return of $\gamma > 1$, reflecting their alternative option. The

[6] Alternatively, all agents may have Stone-Geary preferences demanding a subsistence level of wealth (Ahnert and Perotti (2014)), thus being risk intolerant in some circumstances.

[7] This is equivalent to a "narrow shadow banking model", also invoked in Gorton and Metrick (2012).

Figure 1: Project Timeline



total mass of lenders is large such that perfect competition prevails.

- *Project*

For each unit invested, the project generates a return of $y_t(\omega)$ in $t = 1, 2$, where $\omega \in \{H, L\}$ is the aggregate state. With probability λ the state is revealed at $t = 1$ to be high ($\omega = H$), and the project matures in $t = 1$: $y_1(H) = r > \gamma$. With probability $1 - \lambda$, the state is revealed to be low ($\omega = L$), and the project matures only in $t = 2$. In this case, early liquidation at $t = 1$ could be costly as the project has not fully developed its potential. The early liquidation value has a safe component $k > 0$ plus an uncertain value θ, drawn from a uniform distribution on $\left[0, \overline{\theta}\right]$. Liquidating risky assets at $t = 1$ involves a fixed cost c.

All agents receive private signals on asset liquidity θ at the begin of time 1. In the low state there is some fundamental risk at time 2. Asset returns are the same as in the high state ($y_2(L) = r$) as long as $\theta \geq c > 0$. When asset liquidity is very low, $\theta < c$, the final return is only $y_2(L) = \rho < 1$. As long as c is small, the project is almost riskless if allowed to mature. Note that fundamental risk vanishes as c goes to zero.

The safe portion of the return may be securitized as liquid collateral and held as reserve.[8] If sold at time 1, financial collateral returns an ex ante known price $p \in (0, 1]$, where the discount reflects limited interim liquidity. In the basic model we set $p = 1$, interpreting safe collateral as cash reserves. The more general case is treated in the repo extension, where the price of

[8] The next section considers pledging collateral to lenders seeking higher safety.

collateral at $t = 1$ is critical. To ensure that depositors can be fully repaid in the low state for a sufficiently high liquidation value, we assume that $\overline{\theta} + k > 1 + c$. This assumption and the dependence of $y_2(L)$ on θ create an upper and a lower dominance region in our global game setup, needed to ensure equilibrium uniqueness.

The bank raises funds by issuing unsecured debt with face value d to a subset of mass one of lenders.

- *Lenders' Information Structure*

All agents observe the state ω at the begin of $t = 1$. In the high state the project has matured, so all claims are safe. In the event of a low state $\omega = L$, agents receive individual noisy signals on the early liquidation value of assets in excess of the safe component θ.

This signal is given by

$$x_i = \theta + \sigma\eta_i, \tag{1}$$

where $\sigma > 0$ is an arbitrarily small scale parameter and η_i are i.i.d. across players and uniformly distributed over $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

- *Bank Default and Orderly Liquidation*

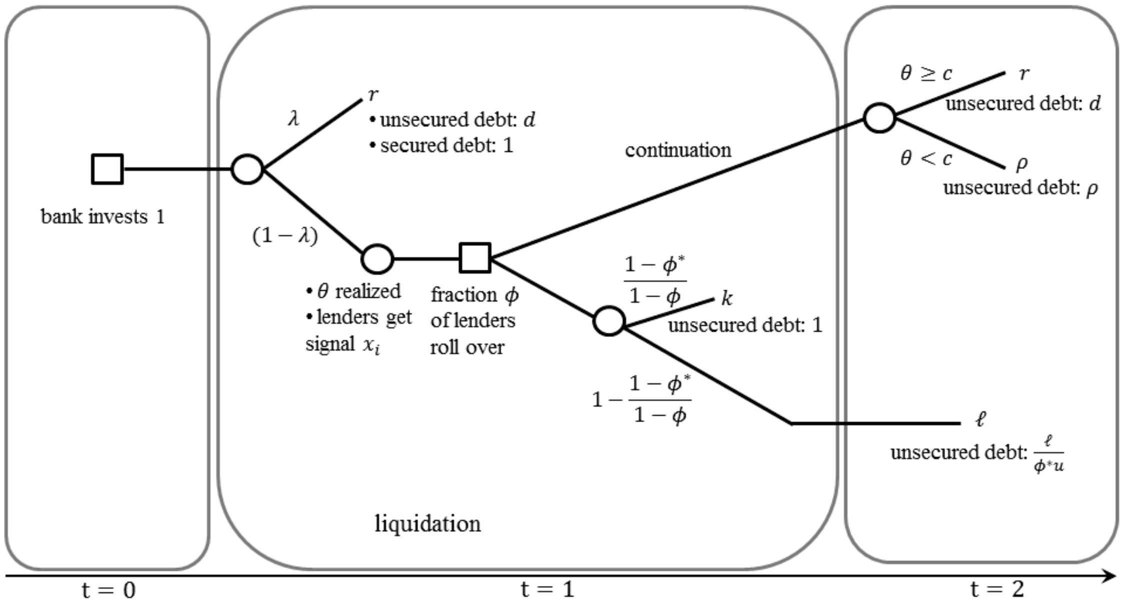
Since all claims are safe in the high state, we focus on the low state $\omega = L$. Upon receiving their signal, lenders may choose to withdraw the principal amount 1. The bank uses its reserves sequentially to meet withdrawals. While the first in the queue are ensured full repayment of principal, once liquid reserves are exhausted the bank is forced to fire sales of illiquid assets. If remaining withdrawals are larger than the net liquidation proceeds, the bank is declared in default. At that point bankruptcy law forces a stay for all unsecured creditors, avoiding the cost c as well as fire sales, and enabling orderly resolution at $t = 2$. This differ from the standard assumption that withdrawals are met by selling all assets immediately, which is clearly less efficient in the case of highly illiquid assets.

Orderly liquidation produces a final value equal to $\ell \geq 0$. At that point, any unpaid depositors are treated equally. This implies a payoff to lenders who rolled over even in case of bank default, as it is the case in reality.

We assume that the project has positive NPV even if always liquidated unker orderly resolution in the low state: $\lambda r + (1-\lambda)(k+\ell) - \gamma > 0$. To ensure equilibrium uniqueness, we also assume that $r - 1 \leq 1 - \ell - k$. This has two natural interpretations. First, the value produced under orderly liquidation is insufficient to fully repay all lenders: $\ell + k < 1$. Second, it implies that a higher asset return (higher r) is associated with higher risk (lower $\ell + k$).

The bankruptcy event can be formally characterized as follows. Let ϕ be the fraction of lenders that roll over in $t = 1$. The first depositors in the running queue are paid out of liquid

Figure 2: Economy Timeline



collateral. If there are depositors left in the queue, the bank is declared bankrupt if and only if

$$1-\phi > \theta - c + k.$$

Here the left hand term indicates the face value demanded by running depositors, and the right hand side the amount available at $t = 1$, namely the net liquidation value of illiquid assets plus the value of the retained collateral. In other words, the bank is bankrupt if after paying out all reserves the net value of selling its non reserve assets exceeds the claims of unpaid withdrawing depositors.

- *Lenders' Payoffs*

When the state is high, lenders are always repaid d. In the low state, their payoffs depend on whether they roll over or withdraw, and whether the bank survives a run.

In a run, the random order of arrival implies that running depositors are repaid out of the liquid reserves with probability $\frac{1-\phi^*}{1-\phi}$, where ϕ^* is such that $1-\phi^* = k$. That is, $1-\phi^*$ is the fraction of lenders running such that all withdrawers receive full repayment out of reserves. With probability $1-\frac{1-\phi^*}{1-\phi}$, the remaining withdrawers receive $\frac{\ell}{\phi^*}$, their share of illiquid asset value under orderly liquidation. Note that they share this value with the creditors who did not run.

So the expected payoff of lenders who do not roll over in $t = 1$ is

$$\pi_U^N(\phi,\theta) = \begin{cases} 1, & \text{if } 1-\phi \le q(\theta-c)+k \\ \frac{1-\phi^*}{1-\phi} + \left(1-\frac{1-\phi^*}{1-\phi}\right)\frac{\ell}{\phi^*}, & \text{if } 1-\phi > q(\theta-c)+k \end{cases},$$

where q is an indicator function that equals 1 if $\theta \ge c$ and 0 if $\theta < c$.

Those who roll over receive d if the bank does not default. In bankruptcy, they are entitled to receive $\frac{\ell}{\phi^*}$ out of the orderly liquidation value. That is,

$$\pi_U^R(\phi,\theta) = \begin{cases} qd + (1-q)\rho, & \text{if } 1-\phi \le q(\theta-c)+k \\ \frac{\ell}{\phi^*}, & \text{if } 1-\phi > q(\theta-c)+k \end{cases}.$$

3 Runs under Asset Liquidity Risk

When lenders receive their signal, they face a complex coordination problem. Their decision to roll over depends on their beliefs about both θ (fundamental uncertainty) and the fraction ϕ of lenders that rolls over (strategic uncertainty).

3.1 Equilibrium Runs

The bank is assessed to be bankrupt if and only if withdrawals $(1-\phi)$ are sufficiently large:

$$1-\phi > q(\theta-c)+k = q(\theta-c)+k, \tag{2}$$

Let $\Pi_U^R(\phi,\theta)$ be the net payoff of lenders who roll over relative to that of running. We have

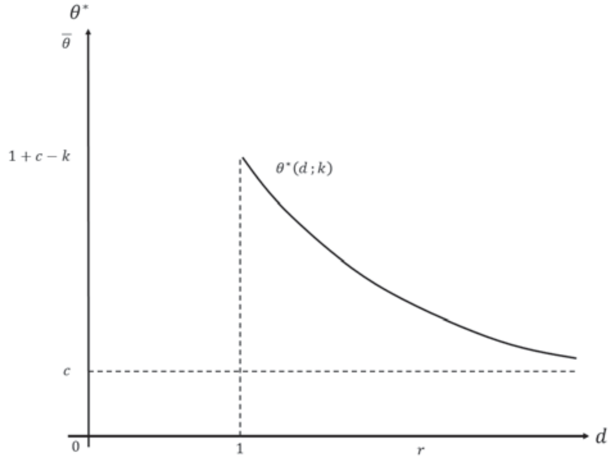
$$\Pi_U^R(\phi,\theta) = \begin{cases} qd + (1-q)\rho - 1, & \text{if } (1-\phi) \le q(\theta-c)+k \\ -\frac{k}{(1-\phi)}\left(1-\frac{\ell}{1-k}\right), & \text{if } (1-\phi) > q(\theta-c)+k \end{cases}. \tag{3}$$

Suppose lenders follow a monotone strategy with a cutoff κ, rolling over if their signal is above κ and withdraw otherwise. Lender i's expectation about the fraction of lenders that roll over conditional on θ is simply the probability that any lender observes a signal above κ, that is, $1-\frac{\kappa-\theta}{\sigma}$. This proportion is less than z if $\theta \le \kappa - \sigma(1-z)$. Each lender i calculates this probability using the estimated distribution of θ conditional on his signal x_i.

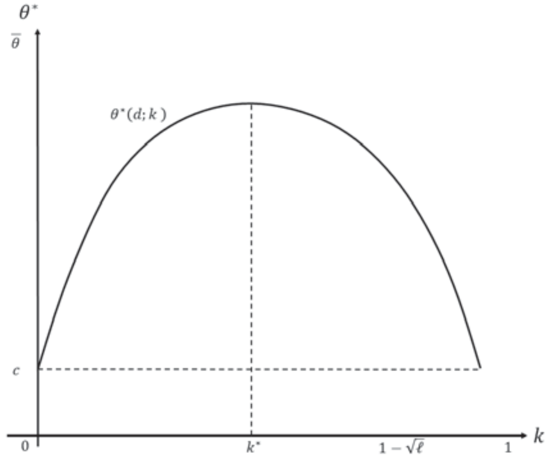
We rely now on the well known result in the literature of global games that as $\sigma \to 0$, this probability equals z for $x_i = \kappa$.[9] That is, the threshold type believes that the proportion of lenders that roll over follows the uniform distribution on the unit interval. As signals

[9] See Morris and Shin (2003) for a comprehensive discussion of the global games literature.

Figure 3: Run Cutoff



(a) Rollover Reward



(b) Collateral

become nearly precise, strategic uncertainty dominates over uncertainty about θ. The equilibrium cutoff can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ. Formally, it is the unique θ^* such that $\int_0^1 \Pi_U^R(\phi, \theta^*)\, d\phi = 0$.

This leads us to our first main result.

Proposition 1 (Run Cutoff) *In the limit $\sigma \to 0$, the unique equilibrium in $t = 1$ has lenders following monotone strategies with threshold θ^* given by*

$$\theta^* = e^{-W\left(\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}\right)} + c - k, \tag{4}$$

where all lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[10]

Proposition 1 allows us to derive how the probability of bankruptcy relates to the bank's financing policy and its collateral. Recall that a lower θ^* is desirable, as it implies less frequent runs.

Corollary 1 (Yield and Collateral Effects on Stability) *θ^* has the following properties:*

(i) It is strictly decreasing and strictly convex in the roll over premium d, and strictly concave in collateral value k.

[10] $W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

(ii) There exists a cutoff $k^* \in \left(0, 1-\sqrt{\ell}\right)$ *such that it is strictly decreasing in* k *for* $k \geq k^*$ *and is strictly increasing in* k *for* $k < k^*$.

Corollary 1 can be more easily interpreted after rewriting (12):

$$\underbrace{\theta^* - c + k}_{\text{debt recovery ratio}} = e^{-W\overbrace{\left(\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}\right)}^{\text{rolling over net-benefit-to-cost ratio}}}. \tag{5}$$

From (13), the signal θ^* makes the threshold lender type just indifferent, balancing the recovery ratio in a run against the rollover premium $d-1$. More k has a stabilizing "probability" effect, the likelihood that the bank has enough liquid assets to repay withdrawals. The second term reflects a "relative payoff" effect, the net benefits of rolling over when there is no bankruptcy relative to the losses incurred in a run.

Corollary 1 (illustrated in Figure 2) offer some insight on the comparative statics. The effect of the rollover premium d is intuitive. Increasing it improves the payoff of rolling over for a given chance of default, and unambiguously reduces the probability of runs. However, promising a large rollover reward reduces the return to the bank in all solvent states.

An increase in financial collateral k has a more complex effect. It has an unambiguous linear "probability" effect, reducing the chance that the bank runs out of reserves in a run and thus leading to a lower θ^*. There is also a "relative payoff" effect, as more safe collateral increases the expected payoff of both rollover and run strategies. The probability effect is dominant when safe collateral is abundant. As it declines, runs are increasingly frequent. However, as runners only receive safe collateral, at some point the relative payoff of withdrawal relative to rollover drops rapidly, producing the hump shaped relationship.[11]. This effect of asset liquidity is quite distinct from fundamental value, which has a monotonic effect on run frequency.

By construction, runs in our model are inefficient if and only if: (i) the bank is not insolvent, $\theta \geq c$, or (ii) the final return, even if lower than r, still exceeds the value of assets under bankruptcy, $\rho > k + \ell$. Proposition 1 tells us that runs occur with positilve probability even if fundamental risk becomes arbitrarily small, in which case almost all runs are inefficient. This is formalized in Corollary 2:

Corollary 2 *Almost all runs are inefficient for* $c \to 0$, *in which case the probability of runs is bounded away from zero:* $\theta^* \geq e^{-W\left(\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}\right)} - k > 0$.

[11]Note that this effect would be stronger if a higher proportion of safe return k implied lower proceeds l in the orderly liquidation process

3.2 The Pricing of Unsecured Debt

This section examines the bank's initial funding choice d. Because the project has positive NPV for any funding choice, we can focus on the stability tradeoff, excluding other effects of its financing structure.

The ex ante expected payoff of lenders as a function of its face value d is

$$V_U(d) = \lambda d + (1-\lambda)\left[\frac{\overline{\theta} - \theta^*(d)}{\overline{\theta}} d + \frac{\theta^*(d)}{\overline{\theta}}(k+\ell)\right]$$

The bank's expected payoff can be written as the return of the project of a solvent bank r net of financing costs and the expected deadweight loss $DW(d)$:

$$\begin{aligned} V_B(d) &= \lambda(r-d) + (1-\lambda)\left(\frac{\overline{\theta} - \theta^*(d)}{\overline{\theta}}\right)(r-d) \\ &= r - V_U(d) - DW(d), \end{aligned} \tag{6}$$

where $DW(d)$ is the total payoff lost in the event of bankruptcy, that is

$$DW(d) = (1-\lambda)\frac{\theta^*(d)}{\overline{\theta}}(r-k-\ell). \tag{7}$$

3.2.1 Socially Optimal Pricing

As a benchmark, we characterize the optimal financing contract chosen by a social planner. The social planner chooses the face value d that maximizes the aggregate payoff subject to the participation constraint of the bank and its lenders:

$$\begin{aligned} &\max_d \; r - DW(d) \\ &\text{subject to} \\ &V_B(d) \geq 0,\; V_U(d) \geq \gamma. \end{aligned} \tag{8}$$

In other words, the optimal financing policy minimizes the chance of runs (a deadweight loss) subject to agents' participation constraints. Since $-DW(d)$ is increasing in d, the social planner would increase d as much as possible.

Increasing d above lenders' breakeven level offers lenders some rent to encourage rollover, henceforth defined as "rollover rent". The maximum rollover rent is reached when bank's participation constraint is binding at $d = r$, as all asset value is promised to depositors rolling over. Since the bank's participation constraint binds, it follows that the lenders' participation constraint does not bind. Proposition 2 characterizes the socially optimal financing policy.

Proposition 2 (Optimal Funding) *The socially optimal financing contract requires the bank to offer the maximum possible rollover rent ($d^o = r$).*

Intuitively, the social planner care about minimizing losses due to early withdrawals, and thus boosts the incentive to roll over by offering the maximum possible roll over yield.

3.2.2 Private Pricing

The bank's problem is to choose the rollover reward d that maximize its payoff subject to the participation constraint:

$$\begin{aligned} &\max_{d} V_B(d) \\ &\text{subject to} \\ &V_U(d) \geq \gamma. \end{aligned} \tag{9}$$

In making this choice, the bank trades off the cost of financing d against the expected deadweight loss from runs.

Proposition 3 (Private Inefficiency) *The probability of bankruptcy under the socially optimal funding structure is always lower than under the private funding choice: $\theta^*(d^o) < \theta^*(d^*)$.*

While the social planner minimizes the probability of runs by choosing the maximum feasible rollover value $d^o = r$, the private choice of d^* is lower than the social optimum value, leading to a higher threshold $\theta^*(d^*) > \theta^*(d^o)$ and thus more frequent runs.

Proposition 4 characterizes the optimal private funding choice.

Proposition 4 (Private Pricing) *The bank's financing policy is characterized as follows:*

(i) The privately optimal choice of d^* either holds lenders to their participation constraint, or leads to a positive rollover rent characterized by $-\frac{\partial DW(d^*)}{\partial d} = \frac{\partial V_U(d^*)}{\partial d}[1-\mu^*]$, where μ^* is the Lagrange multiplier associated with lenders' participation constraint.

(ii) There exists a cutoff $\lambda_1 \in [0,1)$ such that, if $\lambda > \lambda_1$, the bank offers no rollover rents to its lenders.

The face value d balances lower funding costs against a higher deadweight loss. When λ is sufficiently high, runs are rare so the private choice of funding is a corner solution.

4 Runs with Repo

We now introduce repo funding, secured with financial collateral. Assume that next to risk-neutral lenders there exists a set of infinitely risk averse lenders, willing to lend if and only if assured to be paid in full in all states. In exchange for absolute safety, these "repo lenders" accept a lower return equal to 1, the rate that they could earn on safe storage. The measure of each set of lenders is sufficiently large, ensuring perfect competition and allowing the bank to finance the project with only one type of lender.

We now interpret k as the safe portion of asset returns at $t = 2$, which can be collateralized at $t = 0$. This produces safe collateral that may be pledged to repo lenders. Now the value of collateral p at the interim date $t = 1$ becomes relevant. We assume its price in the low state is p, so that the value of safe collateral k at $t = 1$ is pk. Realistically, we assume that a fraction of collateral $k - \underline{k} > 0$ must be retained by the bank (e.g. to meet routine withdrawals), so the maximum amount that can be pledged is $\underline{k}$.[12]

We denote by u and s the fraction of unsecured and secured debt issued by the bank, such that $u + s = 1$. As more$s$ reduces the amount of safe return available to other lenders, unsecured creditors require a higher rollover value to compensate for larger losses in default.

As we will see, although secured debt can be made so safe that it never runs, it concentrates risk on unsecured lenders, potentially triggering more runs.

4.1 Equilibrium Runs

As repo lenders seek absolute safety, they will demand a haircut h since at $t = 1$ financial collateral may have value p, known at $t = 0$.

As the payoff of secured lenders at $t = 1$ in case of a run is $\pi_S = ph$, to ensure full repayment the minimum haircut h^* demanded at $t = 0$ solves $ph^* = 1$.

Under a pledge of sh^* units of financial collateral, secured lenders are completely safe as long as $sh^* = sp^{-1} \leq \underline{k}$, which implies a higher bound on total secured debt $s \leq p\underline{k}$. Under this condition, repo lenders never wish to run at $t = 1$.

Consider now the unsecured lenders' rollover decision. Adjusting payoffs from the previous section, the bank is bankrupt if and only if withdrawals $1 - \phi$ are sufficiently large:

$$(1 - \phi)\, u > q\,(\theta - c) + p\,(k - sh^*) = q\,(\theta - c) + pk - s, \tag{10}$$

Let $\Pi_U^R(\phi, \theta)$ be the net payoff of unsecured lenders who roll over relative to that of running.

[12]This minimum reserve requirement recalls the Liquidity Coverage Ratio norm under Basel III.

We have

$$\Pi_U^R(\phi,\theta) = \begin{cases} qd + (1-q)\rho - 1, & \text{if } u(1-\phi) \leq q(\theta - c) + p(k - sh) \\ -\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \frac{\ell}{1-pk}\right), & \text{if } u(1-\phi) > q(\theta - c) + p(k - sh) \end{cases}. \tag{11}$$

As in the previous section, we focus on strategic uncertainty as signals become nearly precise. At the equilibrium cutoff, the threshold type must be indifferent between rolling over and withdrawing given uniform beliefs about ϕ. That is, it is the unique θ^* that solves $\int_0^1 \Pi_U^R(\phi,\theta^*)\, d\phi = 0$.

This leads us to Proposition 5.

Proposition 5 (Run Cutoff with Repo) *In the limit $\sigma \to 0$, the unique equilibrium in $t = 1$ has unsecured lenders following monotone strategies with threshold θ^* given by*

$$\theta^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - (pk - s)\,, \tag{12}$$

where all unsecured lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[13]

Proposition 5 allows us to derive the relation between the probability of bankruptcy and secured credit:

Corollary 3 (Effects of Repo on Stability) *θ^* has the following properties:*

(i) It is strictly concave in s.

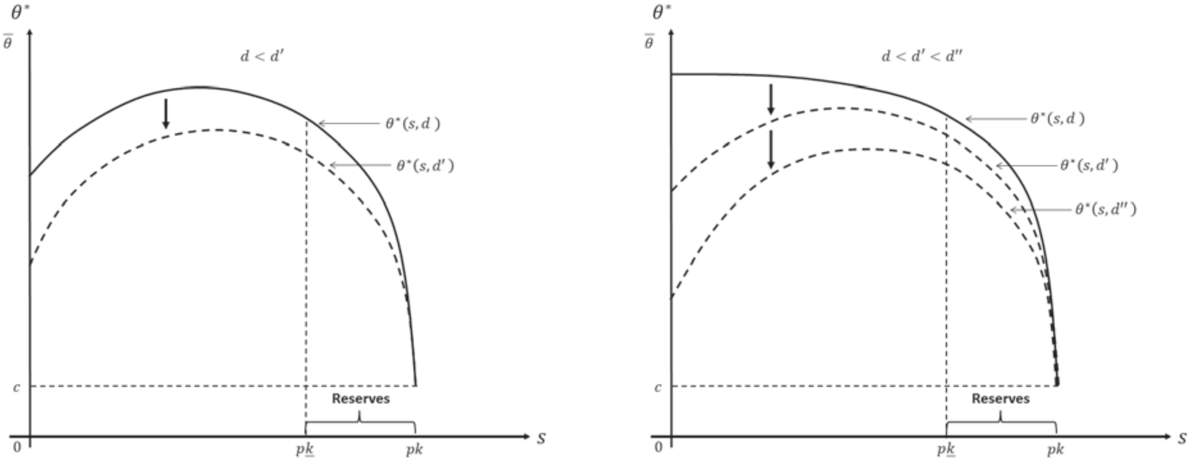
(ii) It is first strictly increasing then strictly decreasing in s if the value of safe collateral is high ($pk \geq 1/2$). If the value of safe collateral is low ($pk < 1/2$), it is first strictly increasing then strictly decreasing in s if the rollover yield is high ($d > \underline{d} > 1$), and strictly decreasing if the rollover yield is low ($d \leq \underline{d}$).

These results can be more easily interpreted after rewriting (12):

$$\underbrace{\frac{\theta^* - c + (pk - s)}{1-s}}_{\text{unsecured debt recovery ratio}} = e^{-W\overbrace{\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)}^{\text{rolling over net-benefit-to-cost ratio}}}. \tag{13}$$

[13] $W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

Figure 4: Yield and Repo Effects on Stability



(a) High Collateral $\left(pk \geq \frac{1}{2}\right)$ (b) Low Collateral $\left(pk < \frac{1}{2}\right)$

The effect of secured debt s on θ^* is concave, while the sign of its derivative is ambiguous. More s has a direct effect to reduce the recovery ratio of unsecured debt, making the bank more likely to go bankrupt for any given fraction of funds withdrawn. But more secured debt reduces the collateral available to withdrawers, decreasing the opportunity cost of rolling over. For most parameter values this produces a hump shaped relationship, while it is declining when both the rollover reward d and collateral value are low. This result is intuitive as more secured debt represents a drop in available collateral, so it mirrors the nonmonotonic effect of k on run incentives in the previous section.

Ultimately, the effect of s depends on the choice of the rollover premium d. Intuitively, a higher rollover premium lowers and flattens the run threshold curve in s, the same effect it has on the run curve in k (see previous section). The slope of the threshold curve in s at $s = 0$ depends on both k and d. When assets have low liquidity risk (k is large, so that $pk \geq \frac{1}{2}$), a small increase in secured debt above $s = 0$ leads to a higher risk of runs (as the probability effect dominates), so the threshold θ^* is hump shaped. [14]. Figure 3(a) shows such a case. The probability of runs at first rises in s, reflecting the dominance of the risk concentration effect when available collateral is declining. It is then decreasing when the payoff effect becomes more prominent, as very little collateral is available to runners. As d is set higher, $\theta^*(s, d)$ shifts lower.

In contrast, when asset liquidation risk is high (that is, k is sufficiently low) and the rollover reward offered to unsecured debt d is very low, the run threshold has a very high intercept and is declining in repo debt. The reason is that repo debt subtracts liquidity, and discourages

[14]This is the specular effect of increasing the amount of safe collateral k

withdrawals as most of the value is in illiquid assets that are shared with non withdrawing lenders. While maximizing s may be the preferred private choice, increasing d would be more effective. This case is illustrated in Figure 3(b).

4.2 The Funding Choice

We can now examine the bank's funding choice (s, d). Because the project has positive NPV for any funding choice, we can focus on the cost/stability tradeoff.

The expected payoff of unsecured lenders as a function of its face value d is

$$V_U(s,d) = \lambda d + (1-\lambda)\left[\left(\frac{\overline{\theta} - \theta^*(s,d)}{\overline{\theta}}\right) d + \frac{\theta^*(s,d)}{\overline{\theta}} \frac{pk - s + \ell}{1-s}\right] \tag{14}$$

The bank's payoff can be written as the return of the project of a solvent bank r net of financing costs, minus the expected deadweight loss $DW(s,d)$:

$$\begin{aligned} V_B(s,d) &= \lambda\left[r - d(1-s) - s\right] + (1-\lambda)\left(\frac{\overline{\theta} - \theta^*(s,d)}{\overline{\theta}}\right)\left[r - d(1-s) - s\right] \\ &= r - s - (1-s)V_U(s,d) - DW(s,d), \end{aligned} \tag{15}$$

where $DW(s,d)$ is the total payoff lost in the event of bankruptcy, that is

$$DW(s,d) = (1-\lambda)\frac{\theta^*(s,d)}{\overline{\theta}}(r - pk - \ell). \tag{16}$$

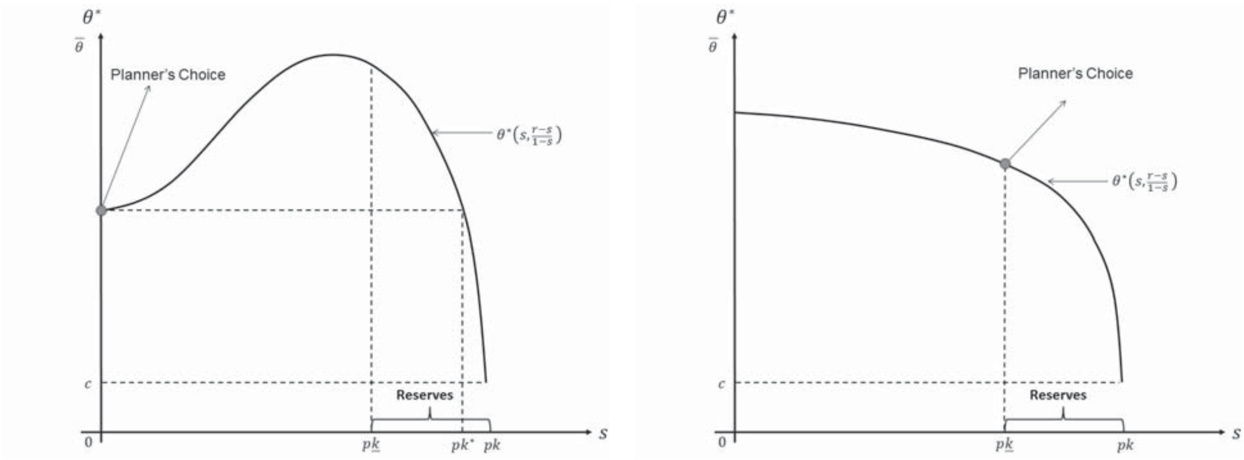
4.2.1 Socially Optimal Funding

The socially optimal financing contract chooses a pair (s, d) that maximizes the aggregate payoff subject to the participation constraint of the bank and unsecured lenders:

$$\begin{aligned} &\max_{s,\,d}\; r - DW(s,d) \\ &\text{subject to} \\ &\;V_B(s,d) \geq 0,\; V_U(s,d) \geq \gamma,\; s \in [0, p\underline{k}]. \end{aligned} \tag{17}$$

This is equivalent to minimize the probability of bankruptcy $\theta^*(s,d)$ — and thus the deadweight loss of runs — subject to agents' participation constraints. Since $-\theta^*(s,d)$ is increasing in d, the social planner would increase the rollover premium d as much as possible for any s. As the bank's participation constraint is binding at $d = \frac{r-s}{1-s}$, this is the maximum face value that may be chosen.

Figure 5: Socially Optimal Funding



(a) High Project Return ($r > \underline{r}$) (b) Low Project Return ($r \leq \underline{r}$)

Since the bank breaks even at the social optimum and the positive has positive NPV, lenders receive positive rollover rents (i.e., their participation constraints do not bind). Thus, the social optimum is achieved by minimizing $\theta\left(s, \frac{r-s}{1-s}\right)$ subject to $s \in [0, p\underline{k}]$. The social planner can indirectly increase the rollover reward $d = \frac{r-s}{1-s}$ by increasing the amount of secured debt s, as this relaxes the bank's participation constraint. However, this may come at the cost of directly increasing the probability of bankruptcy. Proposition 6 below shows that the social planner either issues no secured debt or the maximum amount possible, which results from the strict quasi-concavity of $\theta\left(s, \frac{r-s}{1-s}\right)$.

Proposition 6 (Optimal Funding with Repo) *The socially optimal contract (s^o, d^o) sets the rollover yield d^o such that the bank breaks even: $d^o = \frac{r-s^o}{1-s^o}$. The bank issues either only unsecured debt $(s^o, d^o) = (0, r)$, or the maximum possible amount of secured debt $(s^o, d^o) = \left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$. If the project return is high ($r > \underline{r} > 1$), issuing secured debt is socially optimal if and only if the reserve requirement is sufficiently low ($\underline{k} > \underline{k}^* \in (0, k)$). If the project return is low ($r \leq \underline{r}$), issuing only unsecured debt is never socially optimal.*

Note that the threshold curve $\theta\left(s, \frac{r-s}{1-s}\right)$ under the socially optimal choice of maximizing the rollover premium is no longer concave, since now d is increasing in s. As before, its slope at $s = 0$ depends on d and thus on the project return. Proposition 6 indicates that whenever the project return is high ($r > \underline{r} > 1$), a small increase in secured debt above $s = 0$ always leads to a higher risk of runs. In this case (illustrated in Figure 5(a)), the threshold curve is first strictly increasing then decreasing. As a result, the social planner chooses to issue no secured debt unless the reserve requirement is very low.

The threshold $\theta\left(s, \frac{r-s}{1-s}\right)$ may also be downward sloping from $s = 0$ when the project return is low (that is, $r \leq \underline{r}$), which limits the size of the rollover premium. In this case it is best to maximize the use of repo, as subtracting liquid collateral from runners discourages withdrawals. Figure 5(b) show such a case.

It is worth noting that the result of Proposition 6 does not imply that secured debt could not add value if $\underline{k} \leq \underline{k}^*$. If the project had positive NPV if and only if some secured debt is used ($r - \gamma < 0$), then it could be financed only if some secured debt is used. Specifically, if

$$\lambda r + (1-\lambda)(pk + \ell) - (1 - p\underline{k})\gamma - p\underline{k} > 0,$$

then the project can be financed provided that the bank issues enough secured debt.

4.2.2 Private Funding Choice

The bank's problem is to choose a funding structure (s, d) that maximize its payoff subject to the participation constraint:

$$\begin{aligned} &\max_{s,\, d} V_B(s,d) \\ &\text{subject to} \\ &V_U(s,d) \geq \gamma,\ s \in [0, p\underline{k}]\,. \end{aligned} \tag{18}$$

In choosing its optimal funding structure, the bank faces a tradeoff between the cost of financing and the expected deadweight loss. The cost of financing is decreasing in the face value of unsecured debt d. As the unsecured lenders' required payoff is greater than for secured lenders, increasing the proportion of secured debt reduces the average cost of financing. However, lower d makes runs more likely, which increases the expected deadweight loss.

Proposition 7 (Private Inefficiency with Repo) *The probability of bankruptcy under the socially optimal funding structure is always lower than under the bank's financing policy:* $\theta^*(s^o, d^o) < \theta^*(s^*, d^*)$.

In graphic terms, because the private choice of d is lower than for the social planner, it produces an upward shift of the $\theta^*(s^*, d)$ curve with a higher intercept at $s = 0$. The curve also exhibit increasing concavity. In conclusions, the private choice of s^* is either equal or higher than the social optimum value. Even when it is equal, it is combined with a lower value for d, as shareholders prefer to earn more in solvent states than reducing further the chance of runs. This leads to a higher threshold $\theta^*(s^*, d)$, and thus more frequent runs than the social optimum.

Proposition 8 characterizes the optimal private funding choice.

Proposition 8 (Private Funding with Repo) *The bank's financing policy is characterized as follows:*

(i) There exists a cutoff $\lambda_1 \in [0,1)$ *such that, if* $\lambda \geq \lambda_1$*, the bank's financing policy* (s^*, d^*) *has the bank borrowing either by issuing only unsecured debt (*$s^* = 0$*) or by issuing the maximum possible amount of secured debt (*$s^* = p\underline{k}$*). When* $s = 0$*, the bank offers no rollover premium, and the unsecured lenders' participation constraint is binding.*

(ii) There exists a cutoff $\lambda_2 \in (0,1)$ *such that, if* $\lambda > \lambda_2$*, unsecured lenders' participation constraint binds, i.e.,* $V_U(s^*, d^*) - \gamma = 0$*.*

(iii) There exists a cutoff $\lambda_3 \in [0,1)$ *such that, if* $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$*, the bank borrows by issuing the maximum possible amount of secured debt (*$s^* = p\underline{k}$*), and offers no rollover premium to unsecured lenders.*

Intuitively, a lower chance of illiquidity induces the bank to favor lower funding costs over the risk of runs. Similarly, it induces a lower rollover premium.

5 Deposit Insurance

In this section, we extend our model to include the possibility that a third party, such as a regulator, provides deposit insurance (DI) to unsecuder lenders. Consistent with real practice, we model DI as a minimum payment of $\pi \in [0,1]$ for unsecured lenders in all states.

In the presence of DI, the payoff of unsecured lenders who do not roll over in $t = 1$ is

$$\pi_U^N(\phi, \theta) = \begin{cases} 1, & \text{if } u(1-\phi) \leq q(\theta - c) + p(k - sh) \\ \frac{1-\phi^*}{1-\phi} + \left(1 - \frac{1-\phi^*}{1-\phi}\right) \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & \text{if } u(1-\phi) > q(\theta - c) + p(k - sh) \end{cases},$$

while that of those who roll over is

$$\pi_U^R(\phi, \theta) = \begin{cases} qd + (1-q)\max\{\rho, \pi\}, & \text{if } u(1-\phi) \leq q(\theta - c) + p(k - sh) \\ \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & \text{if } u(1-\phi) > q(\theta - c) + p(k - sh) \end{cases}. \tag{19}$$

Therefore, unsecured lender's net payoff of rolling over relative to that of running is

$$\Pi_U^R(\phi, \theta) = \begin{cases} qd + (1-q)\max\{\rho, \pi\} - 1, & \text{if } u(1-\phi) \leq q(\theta - c) + p(k - sh) \\ -\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}\right), & \text{if } u(1-\phi) > q(\theta - c) + p(k - sh) \end{cases}. \tag{20}$$

Similar to Diamond and Dybvig (1984), if the regulator provides full insurance, $\pi = 1$, then it is a dominant strategy to roll over regardless of the uncertain liquidation value of the assets θ and the fraction of unsecured lenders that roll over ϕ. That is, full insurance fully deters runs and achieves efficiency. If the amount of DI is such that $\pi \leq \min\left\{\frac{\ell}{1-pk}, \rho\right\}$, the payoffs are the same as those without the presence of DI and all the previous results go through.

We are thus left with the following two cases: $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi \leq \max\left\{\frac{\ell}{1-pk}, \rho\right\}$ and $\max\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$. As before, the equilibrium cutoff θ^*_{DI} can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ: $\int_0^1 \Pi_U^R\left(\phi, \theta^*_{DI}\right) = 0$.

This leads us to Proposition 9:

Proposition 9 (Run Cutoff with DI) *Suppose* $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$. *In the limit* $\sigma \to 0$, *the unique equilibrium in* $t = 1$ *has unsecured lenders following monotone strategies with threshold* θ^* *given by*

$$\theta^*_{DI} = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\max\left\{\frac{\ell}{1-pk}, \pi\right\}\right)}\right)} + c - (pk - s)\,, \tag{21}$$

where all unsecured lenders roll over if $\theta > \theta^*$ *and do not roll over if* $\theta < \theta^*$.

The results in Corollary 3 below follow from Proposition 9.

Corollary 4 (DI Effect on Stability) *If* $\pi = 1$, *then there is no run in the presence of DI. If* $\pi \leq \min\left\{\frac{\ell}{1-pk}, \rho\right\}$, *the probability of bankruptcy with DI and without DI are the same:* $\theta^*_{DI} = \theta^*$. *If* $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$, *the probability of bankruptcy with DI is at least as low as that without DI:* $\theta^*_{DI} = \theta^*$ *for* $\pi \leq \frac{\ell}{1-pk}$ *and* $\theta^*_{DI} < \theta^*$ *for* $\pi > \frac{\ell}{1-pk}$, *in which case* θ^*_{DI} *is strictly decreasing in* π.

The results above show that for any given private funding choice, an increase in the level of DI from π to $\pi' > \pi$ reduces the probability of bankruptcy (provided that π is sufficiently large). The natural question that arises is whether the same result holds taking into the dependence of the bank's funding choice on the level of DI.

If the high state is sufficiently likely (λ large enough), then Proposition 8 tells us that the bank issues the maximum possible amount of secured debt, $s^* = p\underline{k}$, and the face value of unsecured debt is determined by unsecured lenders' participation constraint $V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$. An increase in π, *directly reduces* the probability of bankruptcy as $\theta^*\left(p\underline{k}, d^*; \pi'\right) < \theta^*\left(p\underline{k}, d^*; \pi\right)$, which increases unsecured lenders' expected payoff $V_U\left(p\underline{k}, d^*; \pi'\right) > V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$. Thus, the bank's is able to reduce the face value of debt to $d^{*\prime} < d^*$ such that $V_U\left(p\underline{k}, d^{*\prime}; \pi'\right) =$

$V_U(p\underline{k}, d^*; \pi) = \gamma$, which *indirectly increases* the probability of bankruptcy: $\theta^*(p\underline{k}, d^{*\prime}; \pi) > \theta^*(p\underline{k}, d^*; \pi)$. Corollary 4 below shows that the direct effect dominates when λ is sufficiently large.

Corollary 5 (DI Effect on Private Inefficiency) *Suppose $\frac{\ell}{1-pk} < \pi < 1$ and λ is sufficiently large. Then under the private funding choice with DI, both the face value of unsecured debt d^* and the probability of bankruptcy $\theta^*(p\underline{k}, d^*)$ are strictly decreasing in π.*

The intuition behind the result in Corollary 4 is simple. If illiquidity is sufficiently rare, unsecured debt's payoff is highly sensitive to its face value. Therefore, a small decrease in d offsets the gains brought about by decreases in probability of bankruptcy. As a result, the bank is unable to significantly reduce the face value of unsecured debt following an increase in the level of DI.

6 Conclusion

This paper examines run incentives under both asset liquidity and fundamental risk. We obtain an unique run equilibrium thanks to a precise characterization of the bank default process. While existing models assume that withdrawals are satisfied by asset sales, in reality less liquid assets cannot be sold immediately without huge losses. To avoid a hasty termination of real projects, bankruptcy law forces an automatic stay once the borrower runs out of liquid assets, so that illiquid assets are sold under orderly resolution. In this setup we are able to show that asset liquidity risk may cause runs even as fundamental risk vanishes.

The setup enables to evaluate the effect of secured financial credit, which reduces liquid assets available for withdrawals. Targeting such debt to risk intolerant investors reduces the cost of funding. However, as secured credit claims the safest assets, it makes each unit of unsecured debt more exposed to risk, requiring a higher rollover yield. This shifts the signal threshold for asset liquidity, leading to more self protecting runs (Goldstein and Pauzner, 2005). In equilibrium, more secured debt results in more frequent unsecured runs. We show that when illiquidity is sufficiently rare, intermediaries maximize the use of repo funding even if they recognize the increased risk of unsecured debt runs. The socially optimal choice would either avoid issuing secured debt, or use its lower cost to create rolloever rents for other creditors.

The direct risk creation effect of repo debt described here complements the indirect effect from collateral fire sales, which reduces collateral liquidity. We show that this forces higher repo haircuts, which further shifts the run threhsold, reinforcing the direct risk allocation effect. While the private choice of secured funding is excessive, its lower cost may induce more credit for marginal projects. This may have a procyclical effect on credit volume as well as on risk incentives.

A key question for future research concerns the effect of encumbered assets on stability when disclosure is limited. This reinforces market segmentation between traditional bank funding and its secured transactions, such as derivatives (Acharya and Bisin, 2013). Imprecise disclosure may create Knightian uncertainty and self fulfilling panics (Caballero Khrisnamurthy, 2008), even before private information on fundamental values becomes information sensitive (Gorton and Ordoñez, 2014).

Finally, the introduction of a more accurate bank default process implied that run incentives depend in part on the liquidity of bank assets. In future work we plan more attention to the role of mandatory cash reserves.

Appendix

Proof of Proposition 1. Goldstein and Pauzner (2000) and Morris and Shin (2003) prove this result for a general class of global games, including those where θ is drawn from a uniform distribution on $\left[\underline{\theta},\overline{\theta}\right]$, the noise terms η_i are i.i.d. accross players and drawn from a uniform distribution on $\left[-\frac{1}{2},\frac{1}{2}\right]$, and that satisfy the following additional conditions: (i) for each θ, there exists $\phi^* \in \mathbb{R}\cup\{-\infty,\infty\}$ such that $\Pi_U^R(\phi,\theta)>0$ if $\phi>\phi^*$ and $\Pi_U^R(\phi,\theta)<0$ if $\phi<\phi^*$; (ii) $\Pi_U^R(\phi,\theta)$ is nondecreasing in θ; (iii) there exists a unique θ^* that satisfies $\int_0^1 \Pi_U^R(\phi,\theta^*)\,d\phi=0$; (iv) there exists $\overline{D}$ and $\underline{D}$ with $\sigma<\min\left\{\overline{\theta}-\overline{D},\underline{D}-\underline{\theta}\right\}$, and $\epsilon>0$ such that $\Pi_U^R(\phi,\theta)\leq-\epsilon$ for all $\phi\in[0,1]$ and $\theta\leq\underline{D}$ and $\Pi_U^R(\phi,\theta)>\epsilon$ for all $\phi\in[0,1]$ and $\theta\geq\overline{D}$; and (v) continuity of $\int_0^1 w(\phi)\,\Pi_U^R(\phi,\theta)\,d\phi$ with respect to signal θ and density w. Except for (iii), $\Pi_U^R(\phi,\theta)$ clearly satisfies (i), (ii), (iv) and (v).

We now show that (iii) is also satisfied. Let $\Delta(\theta;d)\equiv\int_0^1\Pi_U^R(\phi,\theta)\,d\phi$. Since $\Delta(\theta;d)<0$ for all d and $\theta<c$, then if θ^* exists it must be that $\theta^*\geq c$. Moreover, since $\Delta(\theta;d)$ is strictly increasing in θ for $\theta\geq c$, we must show that $\Delta(c;d)\leq 0$ for all d (otherwise for some d we have $\Delta(\theta;d)\geq\Delta(c;d)>0$ for all $\theta\geq c$ and no θ^* would satisfy $\Delta(\theta^*;d)=0$). We also have that (a) $\Delta(c;d)$ is strictly increasing in d, (b) d is bounded by r (in which case the bank's participation constraint binds), and (c) $\Delta(c;r)=k\left(1-\frac{\ell}{1-k}\right)\ln\frac{k}{e^{-\frac{r-1}{1-\frac{\ell}{1-k}}}}\leq(<)\,0$ if $e^{-\frac{r-1}{1-\frac{\ell}{1-k}}}\geq(>)\,k$. Therefore, for $\frac{r-1}{1-\ell-k}\leq 1$ we have

$$e^{-\frac{r-1}{1-\frac{\ell}{1-k}}}>1-\frac{r-1}{1-\frac{\ell}{1-k}}=(1-pk)\left(1-\frac{r-1}{1-\ell-k}\right)+k\geq k,$$

which implies that for all d we have $\Delta(c;d)\leq\Delta(c;r)<0$. In addition, for all d we have $\Delta(\theta;d)>0$ for θ sufficiently large such that there exists $\theta^*>c$ that satisfies $\Delta(\theta^*;d)=0$. Finally, there is a unique such θ^* as $\Delta(\theta;d)$ is strictly increasing in θ for $\theta\geq c$.

For the derivation of the cutoff θ^*, note that condition $\int_0^1\Pi_U^R(\phi,\theta^*)\,d\phi=0$ is equivalent to

$$k\left(1-\frac{\ell}{1-k}\right)\ln(\theta^*-c+k)+(\theta^*-c+k)(d-1)=0. \tag{A.1}$$

After some algebra, (A.9) can be rewritten as

$$\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}=-\ln(\theta^*-c+k)\,e^{-\ln(\theta^*-c+k)}. \tag{A.2}$$

Let $W(\cdot)$ be the inverse function of $y=xe^x$ for $x\geq-1$ (known as the Lambert W function), that is, $x = W(y)$. Combined with (A.10) this implies

$$\theta^* = e^{-W\left(\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}\right)} + c - k,$$

which establishes the result. ■

Proof of Corollary 1. Implicitly differentiating $y = W(y)\, e^{W(y)}$ results in

$$W' = \frac{W}{(W+1)\, y} = \frac{e^{-W}}{1+W} > 0,$$
$$W'' = W'^2 \left(\frac{-2-W}{1+W}\right) < 0.$$

This allows us to compute

$$\frac{\partial \theta^*}{\partial d} = \frac{-e^{-W} W'}{k\left(1-\frac{\ell}{1-k}\right)} < 0,$$
$$\frac{\partial^2 \theta^*}{\partial d^2} = \frac{e^{-W}\left(W'^2 - W''\right)}{\left[k\left(1-\frac{\ell}{1-k}\right)\right]^2} > 0,$$
$$\frac{\partial \theta^*}{\partial k} = -1 + \frac{W^2}{(d-1)(W+1)}\left[1 - \frac{\ell}{(1-k)^2}\right].$$

If $(1-k)^2 \leq \ell$, then $\frac{\partial \theta^*}{\partial k} < 0$ and W is increasing in k, which implies $\frac{\partial^2 \theta^*}{\partial k^2} < 0$. If $(1-k)^2 > \ell$, then $\frac{\partial \theta^*}{\partial k}$ is positive for k close enough to 0 and W is decreasing in k, which implies that $\frac{\partial^2 \theta^*}{\partial k^2} < 0$. Therefore, there exists $k^* \in \left(0, 1-\sqrt{\ell}\right)$ such that $\frac{\partial \theta^*}{\partial k} = 0$, with $\frac{\partial \theta^*}{\partial k} > 0$ for $k < k^*$ and $\frac{\partial \theta^*}{\partial k} < 0$ for $k > k^*$. ■

Proof of Corollary 2. This result is shown in the proof of Proposition 1. ■

Proof of Proposition 2. We show that the bank's participation constraint must bind at a solution d^o, in which case $d^o = r$. Suppose not, that is, $V_B(d^o) > 0$. The aggregate payoff $r - DW(d)$ is clearly increasing in d. The bank's payoff is strictly concave in d as

$$\overline{\theta}\frac{\partial V_B^2(d)}{\partial d^2} = 2(1-\lambda)\frac{\partial \theta^*}{\partial d} - (1-\lambda)(r-d)\frac{\partial^2 \theta^*}{\partial d^2} < 0,$$

which in turn implies $V_B(d)$ is either (1) decreasing or (2) increasing and then decreasing since

$$\overline{\theta}\frac{\partial V_B(d)}{\partial d} = -\left[\overline{\theta} - (1-\lambda)\theta^*\right] - (1-\lambda)(r-d)\frac{\partial \theta^*}{\partial d} - \left(\overline{\theta} - 1\right)$$

is negative for $d = r$. If $\frac{\partial V_B(d)}{\partial d} \leq 0$ for all d, then $V_B(d)$ is monotone decreasing. If $\frac{\partial V_B(d)}{\partial d} > 0$ for some d', then there exists d'' such that $\frac{\partial V_B(d)}{\partial d} = 0$. Since $V_B(d)$ is strictly concave in

d, $\frac{\partial V_B(d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(s,d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = r$. Therefore, the social planner can increase d^o until $V_B(d^o)$ binds: this increases the aggregate payoff while still satifying the constraints, which contradicts d^o being a solution. ■

Proof of Proposition 3. Suppose that $\theta^o(d^o) \geq \theta^*(d^*)$. Since we assume the project has positive NPV=, the bank's payoff under (18) is greater than zero. But then a contrat with d marginally greater than d^* satifies both participation constraints in (17) and results in $\theta^o(d^o) \geq \theta^*(d^*) > \theta^*(d)$. But this contradicts d^o being a solution to (17). ■

Proof of Proposition 4. The first order necessary conditions (FOC) are

$$-\frac{\partial DW(d)}{\partial d} = \frac{\partial V_U(d)}{\partial d}(1-\mu), \tag{A.3}$$

$$\mu\left[V_U(d) - \gamma\right] = 0, \tag{A.4}$$

$$V_U(d) \geq \gamma, \tag{A.5}$$

$$\mu \geq 0. \tag{A.6}$$

Since $V_B(d)$ is strictly concave (see Proof of Proposition 2), any d satisfying the FOC is a global maximizer, which shows (i).

For (ii), note that

$$\frac{\partial DW(d)}{\partial d} = \frac{(1-\lambda)}{\overline{\theta}}\frac{\partial\theta^*(d)}{\partial d}(r - pk - \ell), \tag{A.7}$$

$$\frac{\partial V_U(d)}{\partial d} = 1 - \frac{(1-\lambda)}{\overline{\theta}}\left[\theta^*(d) + \frac{\partial\theta^*(d)}{\partial d}(d - pk - \ell)\right]. \tag{A.8}$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.14) approach 0 ((A.18) approximates 0) and 1 ((A.19) converges to 1), respectively. Therefore, there are only two possibilities: either the left-hand side of (A.14) (strictly decreasing in λ) is smaller than the right-hand side (strictly increasing in λ) for all $\lambda \geq \lambda_1 = 0$, or there exists $\lambda(d) \in (0,1)$ such that the left-hand side of (A.14) is smaller than the right-hand side if $\lambda > \lambda(d)$ and at least as great if otherwise. If the former is true for all d, then (A.14) can only be satified if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_1 = \sup\{\lambda(d) : d \in Y\}$, then (A.14) can only be satified if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_1 \in [0,1)$ such $\mu > 0$ if $\lambda > \lambda_1$, which in turn implies that $V_U(d) - \gamma = 0$ (from (A.15)). ■

Proof of Proposition 5. As in the proof of Proposition 1, it suffices to show that (iii) is also satisfied. Let $\Delta(\theta; s, d) \equiv \int_0^1 \Pi_U^R(\phi, \theta)\, d\phi$. Since $\Delta(\theta; s, d) < 0$ for all (s, d) and $\theta < c$, then if θ^* exists it must be that $\theta^* \geq c$. Moreover, since $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$, we must show that $\Delta(c; s, d) \leq 0$ for all (s, d) (otherwise for some (s, d) we have

$\Delta(\theta; s, d) \geq \Delta(c; s, d) > 0$ for all $\theta \geq c$ and no θ^* would satisfy $\Delta(\theta^*; s, d) = 0$). It is straightforward to show that (a) $\Delta(c; s, d)$ is strictly increasing in d, (b) d is bounded by $\frac{r-s}{1-s}$ (in which case the bank's participation constraint binds), (c) $\Delta\left(c; s, \frac{r-s}{1-s}\right)$ is decreasing in s if $\frac{r-1}{1-\ell-pk} \leq \frac{1}{pk}$, and (d) that $\Delta(c; 0, r) = pk\left(1 - \frac{\ell}{1-pk}\right) \ln \frac{pk}{e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}}} \leq (<)\, 0$ if $e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} \geq (>)\, pk$. Therefore, for $\frac{r-1}{1-\ell-pk} \leq 1$ we have

$$e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} > 1 - \frac{r-1}{1-\frac{\ell}{1-pk}} = (1-pk)\left(1 - \frac{r-1}{1-\ell-pk}\right) + pk \geq pk,$$

which implies that for all (s, d) we have $\Delta(c; s, d) \leq \Delta\left(c; s, \frac{r-s}{1-s}\right) \leq \Delta(c; 0, r) < 0$. In addition, for all (s, d) we have $\Delta(\theta; s, d) > 0$ for θ sufficiently large such that there exists $\theta^* \geq c$ that satisfies $\Delta(\theta^*; s, d) = 0$. Finally, there is a unique such θ^* as $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$.

As in the proof of Proposition 1, the cutoff θ^* can be derived by solving $\int_0^1 \Pi_U^R(\phi, \theta^*)\, d\phi = 0$, which is equivalent to:

$$\frac{pk-s}{1-s}\left(1 - \frac{\ell}{1-pk}\right) \ln \frac{\theta^* - c + pk - s}{1-s} + \frac{\theta^* - c + pk - s}{1-s}(d-1) = 0. \tag{A.9}$$

After some algebra, (A.9) can be rewritten as

$$\frac{d-1}{\frac{pk-s}{1-s}\left(1 - \frac{\ell}{1-pk}\right)} = -\ln \frac{\theta^* - c + pk - s}{1-s} e^{-\ln \frac{\theta^* - c + pk - s}{1-s}}. \tag{A.10}$$

Let $W(\cdot)$ be the inverse function of $y = xe^x$ for $x \geq -1$ (known as the Lambert W function), that is, $x = W(y)$. Combined with (A.10) this implies

$$\theta^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - (pk - s).$$

■

Proof of Corollary 3. Differentiating θ^* with respect to s shows (i):

$$\frac{\partial \theta^*}{\partial s} = e^{-W}\left[e^W - 1 - \frac{1-pk}{pk-s} \frac{W}{W+1}\right], \tag{A.11}$$

$$\frac{\partial^2 \theta^*}{\partial s^2} = -\frac{e^{-W} W(-2-W)}{(W+1)^3} \frac{(1-pk)^2}{(pk-s)^2(1-s)} < 0. \tag{A.12}$$

We now show (ii). Since $\lim_{s \to pk} \frac{\partial \theta^*(s,d)}{\partial s} = -\infty$ and $\theta^*(s, d)$ is strictly concave in s, it follows that $\theta^*(s,d)$ is strictly decreasing in s if $\frac{\partial\theta^*(0,d)}{\partial s} \leq 0$, and first strictly increasing then decreasing in s if $\frac{\partial\theta^*(0,d)}{\partial s} > 0$. Let us write $\frac{\partial\theta^*(0,d)}{\partial s} = e^{-W(d)}\beta(d)$, where $\beta(d)$ is the term inside the brackets in (A.11).

We have that $\beta'(d) = W'(d)\left[e^{W(d)} - \frac{1-pk}{pk}\frac{1}{(W(d)+1)^2}\right] > 0$ whenever $pk \geq \frac{1}{2}$. Since $\beta(1) = 0$, it follows that $\beta(d) > 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} > 0$ for all $d > \underline{d} = 1$ and $pk \geq \frac{1}{2}$, which implies $\theta^*(s,d)$ is first strictly increasing then decreasing.

For $pk < \frac{1}{2}$, we have that $\beta'(d) < 0$ for d close enough to 1 and $\beta'(d) > 0$ for d high enough, which implies there exists $d' > 1$ such that $\beta'(d') = 0$. Our next step is to show that $\beta(d)$ is strictly quasi-convex. We do so by showing that the strict single crossing functions $W'(d)e^{W(d)}$ and $-W'(d)\frac{1-pk}{pk}\frac{1}{(W(d)+1)^2}$ satisfy strict signed-ratio monotonicity, which implies $\beta'(d)$ is a strict single crossing function (Qua and Strulovici, 2012). Two functions $f(d)$ and $g(d)$ satisfy strict signed-ratio monotonicity if whenever $f(d) > 0$ and $g(d) < 0$, $-\frac{g(d)}{f(d)}$ is strictly decreasing and whenever $f(d) < 0$ and $g(s) > 0$, $-\frac{f(d)}{g(d)}$ is strictly decreasing. We take $g(d) = -W'(d)\frac{1-pk}{pk}\frac{1}{(W(d)+1)^2}$ and $f(d) = W'(d)e^{W(d)}$. Since $f(d)$ is always positive, we only need to consider the case in which $g(d) < 0$. In this case, $-\frac{g(d)}{f(d)} = \frac{\frac{1-pk}{pk}\frac{1}{(W(d)+1)^2}}{e^{W(d)}}$ is clearly strictly decreasing since the numerator is strictly decreasing while the denominator is strictly increasing.

Therefore, $\beta'(d) > 0$ for $d > d'$ and $\beta'(d) < 0$ for $d < d'$. Since $\beta(1) = 0$, it follows that $\beta(d) < 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} < 0$ for $1 < d \leq d'$. Because $\beta(d) > 0$ for d sufficiently high, there exists $\underline{d} > d'$ such that $\beta(\underline{d}) = 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} = 0$. As a result, we have $\beta(d) \leq 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} \leq 0$ for $d \leq \underline{d}$ ($\theta^*(s,d)$ is strictly decreasing) and $\beta(d) > 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} > 0$ for $d > \underline{d}$ ($\theta^*(s,d)$ is first strictly increasing then decreasing). ■

Proof of Proposition 6. We first show that the bank's participation constraint must bind at a solution (s^o, d^o). Suppose not, that is, $V_B(s^o, d^o) > 0$. The aggregate payoff $r - DW(s,d)$ is increasing in d, while the bank's payoff is either one of the following: (1) decreasing, or (2) increasing and then decreasing. To see this, note that

$$\overline{\theta}\frac{\partial V_B(s,d)}{\partial d} = -(1-s)\left[\overline{\theta} - (1-\lambda)\theta^*\right] - (1-\lambda)(r - d(1-s) - s)\frac{\partial\theta^*}{\partial d} - \left(\overline{\theta} - 1\right)(1-s)$$

is negative for $d = \frac{r-s}{1-s}$ and $\overline{\theta}\frac{\partial^2 V_B(s,d)}{\partial d^2} < 0$. If $\frac{\partial V_B(s,d)}{\partial d} \leq 0$ for all d, then $V_B(s,d)$ is monotone decreasing. If $\frac{\partial V_B(s,d)}{\partial d} > 0$ for some d', then there exists d'' such that $\frac{\partial V_B(s,d)}{\partial d} = 0$. Since $V_B(s,d)$ is strictly concave, $\frac{\partial V_B(s,d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(s,d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = \frac{r-s}{1-s}$. Therefore, the social planner can increase d^o until $V_B(s^o, d^o)$ binds: this increases the aggregate payoff while still satifying the constraints, which contradicts (s^o, d^o) being a solution.

The result that the bank's participation constraint binds along with our assumption that the project has positive NPV implies that the unsecured lenders' participation constraint does not bind. Thus, the social social planner's problem can be equivalently rewritten as $\min\limits_{s\in[0,p\underline{k}]} \theta^*\left(s, \frac{r-s}{1-s}\right)$.

We now show that $\theta^*\left(s, \frac{r-s}{1-s}\right)$ is strictly quasi-concave in s, implying a corner solution $s^o \in \{0, p\underline{k}\}$. We do so by showing that the strict single crossing functions $e^{-W}\frac{1-pk}{pk-s}\frac{W}{W+1}$ and $-e^{-W}\left(e^W - 1 - \frac{W}{W+1}\right)$ satisfy strict signed-ratio monotonicity, which implies $-\frac{\partial\theta^*\left(s,\frac{r-s}{1-s}\right)}{\partial s} = e^{-W}\left[\frac{1-pk}{pk-s}\frac{W}{W+1} - \left(e^W - 1 - \frac{W}{W+1}\right)\right]$ is a strict single crossing function (Qua and Strulovici, 2012). Two functions $f(s)$ and $g(s)$ satisfy strict signed-ratio monotonicity if whenever $f(s) > 0$ and $g(s) < 0$, $-\frac{g(s)}{f(s)}$ is strictly decreasing and whenever $f(s) < 0$ and $g(s) > 0$, $-\frac{f(s)}{g(s)}$ is strictly decreasing. We take $g(s) = -e^{-W}\left(e^W - 1 - \frac{W}{W+1}\right)$ and $f(s) = e^{-W}\frac{1-pk}{pk-s}\frac{W}{W+1}$. Since $f(s)$ is always positive, we only need to consider the case in which $g(s) < 0$. In this case, it must be that $-\frac{g(s)'f(s)-g(s)f(s)'}{f(s)^2} < 0$. Indeed, we have

$$
\begin{aligned}
-g(s)' f(s) + g(s) f(s)' &= \frac{(1-pk)}{(pk-s)^2}\left[1 - \frac{\left(e^W - 1\right)(W+2)}{W(W+1)}\right] \\
&< \frac{(1-pk)}{(pk-s)^2}\left[1 - \frac{W+2}{W+1}\right] < 0,
\end{aligned}
$$

where the first inequality results from the following version of Bernoulli's inequality: $e^x > 1 + x$ for $x > 0$.

Finally, we determine the conditions under which each corner solution is optimal. Since $\theta^*\left(s, \frac{r-s}{1-s}\right)$ is strictly quasi-concave and $\lim\limits_{s\to pk}\frac{\partial\theta^*\left(s,\frac{r-s}{1-s}\right)}{\partial s} = -\infty$ it follows that θ^* is strictly decreasing in s if $\frac{\partial\theta^*(0,r)}{\partial s} \leq 0$, and first strictly increasing then decreasing in s if $\frac{\partial\theta^*(0,r)}{\partial s} > 0$, where

$$
\frac{\partial\theta^*(0,r)}{\partial s} = e^{-W}\left[e^W - 1 - \frac{1}{pk}\frac{W}{W+1}\right]. \tag{A.13}
$$

Let us write $\frac{\partial\theta^*(0,r)}{\partial s} = e^{-W(r)}\beta(r)$, where $\beta(r)$ is the term inside the brackets in (A.13). Following the same steps in the proof of Corollart 3 (for the case when $pk < \frac{1}{2}$), we can show that $\beta(r)$ is strictly quasi-convex and there exists $\underline{r} > 1$ is such that $\beta(\underline{r}) = 0$, with $\beta(r) \leq 0 \Rightarrow \frac{\partial\theta^*(0,r)}{\partial s} \leq 0$ for $r \leq \underline{r}$ and $\beta(r) > 0 \Rightarrow \frac{\partial\theta^*(0,r)}{\partial s} > 0$ for $r > \underline{r}$.

Therefore, it follows that $(s^o, d^o) = \left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $r \leq \underline{r}$. For $r > \underline{r}$, there exists s' such that $\theta^*\left(s, \frac{r-s}{1-s}\right)$ is strictly increasing for $s < s'$ and strictly decreasing for $s > s'$. From the proof of Proposition 1, we know that $\theta^*(0,r) > c$. Since $\lim\limits_{\underline{k}\to k}\theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right) = c$, there exists a $\underline{k}^* \in \left(\frac{s'}{p}, k\right)$ such that $\theta^*(0,r) = \theta^*\left(p\underline{k}^*, \frac{r-p\underline{k}^*}{1-p\underline{k}^*}\right)$, with $\theta^*(0,r) \leq \theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $\underline{k} \leq \underline{k}^*$ and $\theta^*(0,r) > \theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $\underline{k} > \underline{k}^*$. ■

Proof of Proposition 7. Suppose that $\theta^o(s^o, d^o) \geq \theta^*(s^*, d^*)$. Since we assume the project has positive NPV, the bank's payoff under (18) is greater than zero (the bank can garantee a positive payoff by choosing $s = 0$). But then a contrat (s^*, d) with d marginally greater than d^* satifies both participation constraints in (17) and results in $\theta^o(s^o, d^o) \geq \theta^*(s^*, d^*) > \theta^*(s^*, d)$. But this contradicts (s^o, d^o) being a solution to (17). ■

Proof of Proposition 8. We first show (i). We use the Principle of Iterated Suprema to break the bank's problem into two stages, that is, we solve $\max_{d\in D}\left[\max_{s\in S} V_B(s,d)\right]$, where $S = [0, p\underline{k}]$ and $D = \{d : V_U(s^*(d), d) \geq \gamma\}$.

The next step is to show that $V_B(s,d)$ is quasiconvex in s if λ sufficiently high, which implies that there is not interior solution to problem $\max_{s\in S} V_B(s,d)$. This is done by showing that $\frac{\partial V_B(s,d)}{\partial s} = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s} - \frac{\partial DW(s,d)}{\partial s}$ is a single crossing function, which is equivalent to $V_U - 1 - (1-s)\frac{\partial V_U}{\partial s}$ and $-\frac{\partial DW}{\partial s}$ satisfying signed-ratio monotonicity (Qua and Strulovici, 2012). Two functions $f(s)$ and $g(s)$ satisfy signed-ratio monotonicity if whenever $f(s) > 0$ and $g(s) < 0$, $-\frac{g(s)}{f(s)}$ is decreasing and whenever $f(s) < 0$ and $g(s) > 0$, $-\frac{f(s)}{g(s)}$ is decreasing. We take $g(s) = -\frac{\partial DW(s,d)}{\partial s}$ and $f(s) = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s}$. Since $f(s)$ is always positive, we only need to consider the case in which $g(s) < 0$. In this case, it must be that $-\frac{g(s)' f(s) - g(s) f(s)'}{f(s)^2} < 0$. We have

$$
\begin{aligned}
&\overline{\theta}\left[-g(s)' f(s) + g(s) f(s)'\right] = \\
&\quad (1-\lambda)\theta^{*\prime\prime}(r - pk - \ell)\left\{(d-1)\left[\overline{\theta} - (1-\lambda)\theta^*\right] + (1-\lambda)\theta^{*\prime}\left[d(1-s) - (pk - s + \ell)\right]\right\} \\
&\quad - (1-\lambda)\theta^{*\prime}(r - pk - \ell)\left\{(1-\lambda)\theta^{*\prime\prime}\left[d(1-s) - (pk - s + \ell)\right] - 2(1-\lambda)\theta^{*\prime}(d-1)\right\} \\
&\qquad = (1-\lambda)(r - pk - \ell)(d-1)\left[\theta^{*\prime\prime}\left(\overline{\theta} - (1-\lambda)\theta^*\right) + 2(1-\lambda)\theta^{*\prime}\theta^{*\prime}\right].
\end{aligned}
$$

The sign of the above expression is determined by the term inside brackets, which is strictly decreasing in λ. For any given s, it is negative if λ is sufficiently close to 1, so that we are left with two possibilities: either it is nonpositive for all λ, or there exists $\lambda(s) \in (0,1)$ such that it is nonpositive if $\lambda \geq \lambda(s)$ and positive if otherwise. If the former is true for all s, then $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1 = 0$. Suppose there exists s such that the latter is true and denote X the set of all such s. Then $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1 = \sup\{\lambda(s) : s \in X\}$. Combining both cases we have that there exists a cutff $\lambda_1 \in [0,1)$ such that $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1$, which in turn implies that we must have a corner solution: $s^* \in \{0, p\underline{k}\}$.

We now turn to the problem $\max_{d \in D} V_B(s^*, d)$. The first order necessary conditions (FOC) are

$$-\frac{\partial DW(s^*, d)}{\partial d} = \frac{\partial V_U(s^*, d)}{\partial d}[1 - s^* - \mu], \tag{A.14}$$

$$\mu[V_U(s^*, d) - \gamma] = 0, \tag{A.15}$$

$$V_U(s^*, d) \geq \gamma, \tag{A.16}$$

$$\mu \geq 0. \tag{A.17}$$

To conclude the proof of (i) we need to show that any d satisfying the FOC is a global maximizer. This follows from

$$\bar{\theta}\frac{\partial V_B^2(s, d)}{\partial d^2} = 2(1 - s)(1 - \lambda)\frac{\partial \theta^*}{\partial d} - (1 - \lambda)(r - d(1 - s) - s)\frac{\partial^2 \theta^*}{\partial d^2} < 0,$$

which implies that $V_B(s^*, d)$ is (strictly) concave in d.

We now show (ii). Note that

$$\frac{\partial DW(s^*, d)}{\partial d} = \frac{(1 - \lambda)}{\bar{\theta}}\frac{\partial \theta^*(s^*, d)}{\partial d}(r - pk - \ell), \tag{A.18}$$

$$\frac{\partial V_U(s^*, d)}{\partial d} = 1 - \frac{(1 - \lambda)}{\bar{\theta}}\left[\theta^*(s^*, d) + \frac{\partial \theta^*(s^*, d)}{\partial d}\left(d - \frac{pk - s^* + \ell}{1 - s^*}\right)\right]. \tag{A.19}$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.14) approach 0 ((A.18) approximates 0) and $1 - s^*$ ((A.19) converges to 1), respectively. Since s is bounded above by $p\underline{k} < 1$, the right-hand side of (A.14) is bounded away from 0. Therefore, there are only two possibilities: either the left-hand side of (A.14) (strictly decreasing in λ) is smaller than the right-hand side (strictly increasing in λ) for all λ, or there exists $\lambda(s^*, d) \in (0, 1)$ such that the left-hand side of (A.14) is smaller than the right-hand side if $\lambda > \lambda(s^*, d)$ and at least as great if otherwise. If the former is true for all d, then (A.14) can only be satified if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_2 = \sup\{\lambda(s^*, d) : d \in Y\}$, then (A.14) can only be satified if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_2 \in (0, 1)$ such $\mu > 0$ if $\lambda > \lambda_2$, which in turn implies that $V_U(s^*, d) - \gamma = 0$ (from (A.15)).

We finally show (iii). Suppose $\lambda > \max\{\lambda_1, \lambda_2\}$. In this case we know from (i) that there are two possible candidates for the bank's choice of secured debt: either $s^* = p\underline{k}$ or $s^* = 0$. We also know that unsecured lenders' participation constraint binds. Therefore, the bank's implied payoffs are given by

$$V_B(p\underline{k}, d^*(p\underline{k})) = r - p\underline{k} - (1 - p\underline{k})\gamma - DW(p\underline{k}, d^*(p\underline{k})), \tag{A.20}$$

$$V_B(0, d^*(0)) = r - \gamma - DW(0, d^*(0)). \tag{A.21}$$

The difference is given by

$$V_B(p\underline{k}, d^*(p\underline{k})) - V_B(0, d^*(0)) = p\underline{k}(\gamma - 1) - [DW(p\underline{k}, d^*(p\underline{k})) - DW(0, d^*(0))], \tag{A.22}$$

which is positive for λ sufficiently close to 1. Thus, there are two cases to consider: either (A.22) (strictly increasing in λ) is nonnegative for all $\lambda \geq \lambda_3 = 0$, or there exists $\lambda_3 \in (0, 1)$ such that (A.22) is nonnegative if $\lambda > \lambda_3$, and negative if $\lambda < \lambda_3$. Therefore, we conclude that if $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$, then the bank's financing policy has the bank borrowing by issuing only secured debt ($s^* = p\underline{k}$) and d^* is such that $V_U(s^*, d) - \gamma = 0$. ■

Proof of Proposition 9. Proof is analogous to those of Propositions 1 and 5. ■

Proof of Corollary 4. See discussion in text. ■

Proof of Corollary 5. For λ large enough, unsecured lenders' participation constraint binds: $V_U(p\underline{k}, d^*; \pi) = \gamma$. Thus, the overall change in θ^* caused by an increase in π can be found by differentiating both sides with respect to π, whihc yields:

$$\frac{\partial\theta^*(d,\pi)}{\partial d}d' + \frac{\partial\theta^*(d,\pi)}{\partial\pi} = \frac{\frac{(1-\lambda)\theta^*(d,\pi)}{\overline{\theta}}\left[d' - \frac{1-pk}{1-p\underline{k}}\right] - d'}{\frac{(1-\lambda)}{\overline{\theta}}\left[-(d-1) - \frac{1-pk}{1-p\underline{k}}(1-\pi)\right]}.$$

Since the denumerator on the right-hand side is negative and $d' < 0$, the overall expression is negative for λ large enough. ■

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